



UNIVERSITÉ CATHOLIQUE DE LOUVAIN

PH.D. THESIS

**Radar Signature Simulator
Based on Large Eddy Simulation
of Wake Vortices in Turbulent
and Stratified Atmospheres**

PAR
DMITRY A. KOVALEV

Thèse présentée en vue de l'obtention du grade de docteur en sciences de
l'ingénieur

Composition du Jury

Prof. Danielle Vanhoenacker-Janvier (Supervisor)	UCLouvain, Belgium
Prof. Christophe Craeye (Chair)	UCLouvain, Belgium
Prof. Grégoire Winckelmans	UCLouvain, Belgium
Mr. Frédéric Barbaresco	Thales Land and Air Systems, France
Prof. Alexander Yarovoy	Delft University of Technology, The Netherlands
Dr. Laurent Delobbe	Institut Royal Météorologique, Belgium

Louvain-la-Neuve
2019

Acknowledgments

I would like to express the deepest appreciation to Prof. Danielle Vanhoenacker-Janvier for many years of great advice. Danielle, thank you for your patience, profound knowledge in the vast field of electromagnetic waves propagation and always having the time for discussions and providing me with valuable feedback to my work. Without your guidance and persistent help, this thesis would not have been possible.

I am deeply grateful to the Thermodynamics and Fluid Mechanics group of UCLouvain, Prof. Grégoire Winckelmans, Philippe Billuart and Dr. Matthieu Duponcheel for their work on the Large Eddy Simulations that I used in my thesis. Grégoire, thank you for your insightful explanations about the world of turbulence and wake vortices.

I would particularly like to thank Thales and Mr. Frédéric Barbaresco for their providing me with the radar measurements of the wake vortices which allowed to validate the results of my work. I am deeply grateful to the Université Pierre-et-Marie-Curie and Meteo France who provided valuable data to validate my work on the earliest stages. I owe my deepest gratitude to Dr. Carlos Pereira who was working on the initial version of the radar simulator which was further developed in my thesis.

I would like to sincerely thank the thesis jury, Prof. Christophe Craeye, Prof. Alexander Yarovoy, and Dr. Laurent Delobbe for accepting to be member of this jury. I also thank them for their time, valuable advice, comments and corrections that they proposed during and after my private defense. Their precious remarks and advice were greatly useful to improve the quality of this thesis.

Finally, I gratefully appreciate the financial support of UCLouvain that made it possible to complete my thesis. I would like to express my gratitude to my colleagues, Alberto, Alessandro, Philippe, Laurent, Jean, Babak, Mojtaba, Pasha for their help and for always being there. I thank Sumit Karki who provided me with very valuable hints on how to improve the styling of the final print of this manuscript. Many thanks go to the technical and administrative staff at the ICTEAM institute, especially to Brigitte Dupont, Souley Djadjandi, and François Hubin for helping me, Marie-Christine Vandingenen, Isabelle Dargent and Christel Derzelle for their administrative help and prompt responses.

I should mention generous support from my friends. I am deeply grateful to Zhenya for the message he sent me more than four years ago about the open

position at UCLouvain. I want to thank Zhenya and Sergei for the fruitful discussions in informal settings.

I owe a very important debt to my family for their love, continuous encouragement, and unwavering support. I am particularly grateful to my father and grandparents who from the very early years supported and encouraged me in my explorations.

I would not have finished this thesis without the support, warm encouragement and help of my beloved wife Maria and kids Mark and Agnia. Thank you for always being there, for your care, unconditional love, and support.

Abstract

A pair of counter-rotating vortices is created after roll-up of the wake of flying aircraft. Normally, wake vortices (WVs) cannot be observed visually unless they capture some smoke, clouds, or aircraft contrails. Nevertheless, wake vortex (WV) turbulence is a serious hazard for the following aircraft especially during taking off and landing. The aircraft encountering a wake vortex system that has not dissipated enough, may be subjected to a strong rolling moment or to a strong downwash velocity causing a drop in lift. To prevent such a risk, safety separations between aircraft were imposed by the International Civil Aviation Organization (ICAO) at the beginning of the 70s. With the exponentially growing air traffic, those safety separations become too constraining for large and busy airports, limiting their capacity and flexibility, and causing many delays on departures and arrivals.

In order to support the development of new systems, such as weather dependent separation, measurement tools are required to provide in real time information about the WVs intensity and their location relative to the glide path. The main sensors proposed are lidar and radar. They are complementary to detect WVs in all weather conditions: lidar is used in clear air and radar in rain. It is important to determine accurately the limitations of the radar detection and the parameters influencing them.

This thesis provides a tool that allows the simulation of the WVs detection by radar in clear atmosphere and in the presence of rain. It can be used to optimize the radar parameters, to develop algorithms for the detection of WVs and to estimate their parameters. The novelty of the approach resides in the use of 3D fields from Large Eddy Simulations (LES) of the WVs in the turbulent stratified atmosphere, for the calculation of the radar backscattered signal. The LES output is used to simulate the time series and the Doppler spectra of the signal returning to the radar. The simulated signal is then analyzed to retrieve the position and the strength of the WVs.

The platform developed and the models/methods tested, constitute a useful tool for the optimization of the radar configuration and parameters, and also for the improvement of the radar link budget. It could also be used, alongside with other algorithms, for a benchmark of methods for estimating the WVs strength. The radar simulator developed provides a useful tool for the optimization of the algorithms dedicated to the detection/classification of the WVs, using the backscattered radar signal.

The main contributions of this thesis can be summarized as follows:

1. Electrical characterization of the scattering medium based on the LES.
2. Development of a simulator of the radar signal backscattered on the WVs using LES 3D fields. It allows simulation of the time series of the signal backscattered in clear air as well as the direct synthesis of the backscattered signal's Doppler spectrum in the presence of rain.
3. Implementation of a methodology for estimation of the WVs radar cross-section that enables the analysis of the detectability of the WVs under different weather conditions.
4. Analysis of the radar signatures of the WVs to establish a relation between the different parts of the Doppler signatures and the physical phenomenon causing those contributions.
5. Verification of the methodology to retrieve the atmospheric turbulence strength using the LES.
6. Implementation of two different methods to analyze the WVs strength. It is shown that the WVs' strength can be accurately retrieved from the radar measurements.

Contents

Acknowledgments	iii
Abstract	v
Contents	vii
List of Acronyms	xi
1 Introduction	1
1.1 Problem statement	1
1.2 Context of research	3
1.3 Content of the thesis	4
1.4 State of the art	5
1.5 Detection of the WVs under clear air conditions	5
1.5.1 Measurements	6
1.5.2 Theoretical models of electromagnetic waves scatter from the WVs	11
1.6 Detection of the WVs in presence of rain	15
1.6.1 Measurements	15
1.6.2 Theoretical models of electromagnetic waves scattering by the WVs	17
1.7 Main shortcomings and what this thesis proposes	18
2 Characterization of turbulent and stratified atmospheres, and of WVs evolution, without and with rain	21
2.1 Large eddy simulations of turbulent stratified atmospheres, and of added wake vortices	22
2.1.1 LES of clear-air turbulent and/or stratified atmospheres	22
2.1.2 LES of WVs added to clear-air turbulent and stratified atmospheres	25
2.1.3 LES of atmospheric turbulence with rain	34
2.1.4 WVs added to turbulent atmosphere in the presence of rain	41
2.2 Electrical characterization of the medium and scattering of elec- tromagnetic waves	44
2.2.1 Refractive index of the atmosphere	44

2.2.2	Spectrum of the refractive index in the turbulent medium	45
2.2.3	Refractive index spectrum at the transition between the inertial subrange and the viscous dissipative range . . .	50
2.2.4	Characteristics of rain	56
2.3	Conclusions	57
3	Principles of radar measurements of atmospheric phenomena	59
3.1	Radar basic principles	60
3.1.1	Radar equation and signal-to-noise ratio	60
3.1.2	Pulse-Doppler radar concept	62
3.2	Radar measurements of distributed scatterers	66
3.2.1	Geometrical configuration of radar measurements	66
3.2.2	Distributed scatterers	69
3.2.3	Radar cross-section of the WV and turbulence in clear air	71
3.2.4	Radar cross-section of the wake vortex in the presence of rain	73
3.2.5	Simulation of the signal backscattering in clear air . . .	77
3.2.6	Simulation of the backscattered signal's Doppler spec- trum in the presence of rain	82
3.3	Conclusions	85
4	Doppler spectrum evaluation and processing	87
4.1	Spectral analysis of the backscattered signal	87
4.1.1	Estimation of the Doppler spectrum from generated time series	88
4.1.2	Estimation of the sub-grid turbulence contribution to the Doppler spectrum width.	93
4.1.3	Doppler spectrum moments	94
4.2	Doppler spectrum of clear air turbulence	95
4.3	Doppler spectrum of wake vortices in clear air	96
4.4	Doppler spectrum of wake vortices in rain	99
4.5	Retrieval of the turbulence intensity	99
4.6	Estimation of the wake vortex circulation	103
4.6.1	Direct estimation from the Doppler spectrum width . .	103
4.6.2	Indirect approach based on descend velocity tracking . .	105
4.7	Conclusions	105
5	Results of simulations of backscattered signal and its Doppler spectrum	107
5.1	Retrieval of the turbulence intensity under clear air conditions	107
5.1.1	Estimation of the EDR with the radar simulator	108
5.1.2	EDR retrieved from the X-band radar measurements . .	110
5.2	Retrieval of wake vortex parameters in clear air	113
5.2.1	Doppler spectrum evolution in time	114
5.2.2	Estimation of the wake vortex circulation	116
5.2.3	RCS of the wake vortex in clear air	122
5.2.4	Impact of the noise on the accuracy of the WV strength estimation	126

5.3	Retrieval of wake vortex parameters in the presence of rain . .	128
5.3.1	Qualitative comparison of the WV radar signatures simulated and measured with an X-band radar	129
5.3.2	RCS of the wake vortex in the presence of rain	132
5.3.3	Estimation of the wake vortex circulation	134
5.3.4	Backscattering in the W band	135
5.4	Conclusions	138
6	Conclusions and perspectives	139
6.1	Summary	139
6.2	Main achievements	141
6.3	Perspectives	142
	List of publications	143
	Bibliography	145

List of Acronyms

ABL	atmospheric boundary level
AWGN	additive white Gaussian noise
B-H	Burnham-Hallock
CDG	Charle de Gaulle
DNS	direct numerical simulation
EDR	energy dissipation rate
EM	electromagnetic
FMCW	frequency-modulated continuous-wave
ICAO	International Civil Aviation Organization
JU	Joint Undertaking
LES	Large Eddy Simulations
LNA	low-noise amplifier
PRF	pulse repetition frequency
PSD	power spectral density
RASS	Radio acoustic sounding system
RCS	radar cross-section
RHI	range-height indicator
SGS	sub-grid scale
SNR	signal-to-noise ratio
TASS	Terminal Area Surveillance System
TFL	Thermodynamics and Fluid mechanics

TKE turbulent kinetic energy

WV wake vortex

WVs wake vortices

Chapter 1

Introduction

After introduction of the problem, this chapter describes the context in which the research started and the context of this thesis. It also gives the state of the art of WVs detection and simulation, in clear air and rainy conditions. It concludes by the shortcomings of the existing models and the strategy followed by this thesis to improve the situation.

1.1 Problem statement

After roll-up of the wake of an aircraft, a pair of counter-rotating vortices is created. This phenomenon, called WV turbulence, can constitute a hazard for any other aircraft behind the first, especially during takeoff and landing phases (Krause, 2003). Indeed, the aircraft encountering a wake vortex system that has not dissipated enough may be subjected to a strong rolling moment (when encountering one of the vortices) or to a strong downwash velocity (when flying in between the two vortices), causing a drop in lift (See Fig. 1.1).

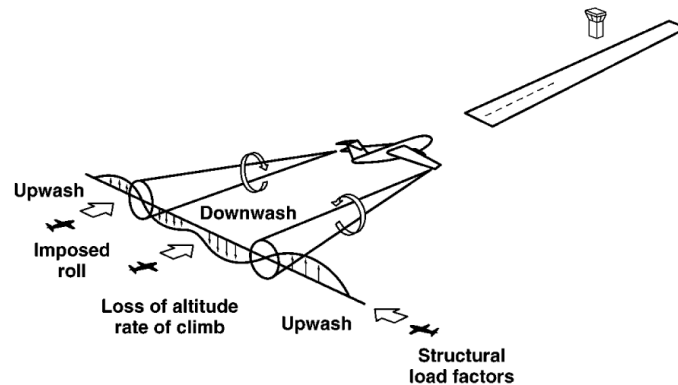


Figure 1.1: Possible encounters with WVs by a following aircraft. The figure is taken from (Rossow, 1999).

Figs. 1.2 and 1.3a illustrate the vortex roll-up in the smoke observed by Huffaker et al. (1971) and at the NASA Flight Research Center, respectively. Fig. 1.3b illustrates the WVs interaction with ground.

To prevent such risks of encountering a wake, safety separations between aircraft were imposed by International Civil Aviation Organization (ICAO) in the early 1970s. With the current exponentially growing air traffic, those safety separations become too constraining for large and busy airports, limiting their capacity and flexibility, and causing many delays in departures and arrivals.

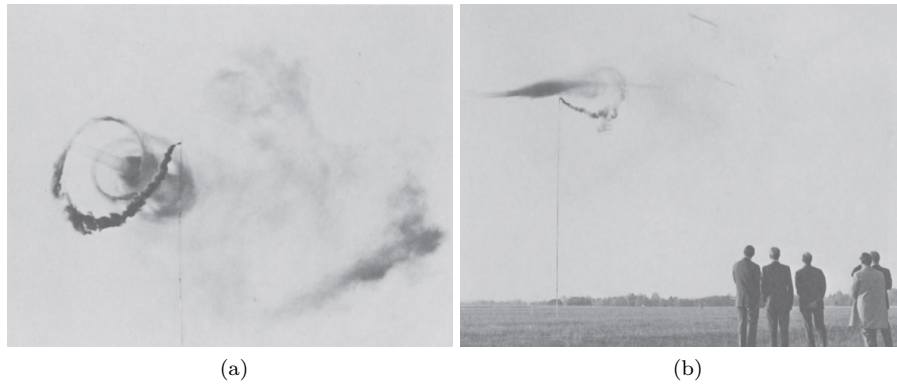


Figure 1.2: Earliest works on WVs detection – wake vortex in smoke. This figure is taken from (Huffaker et al., 1971).

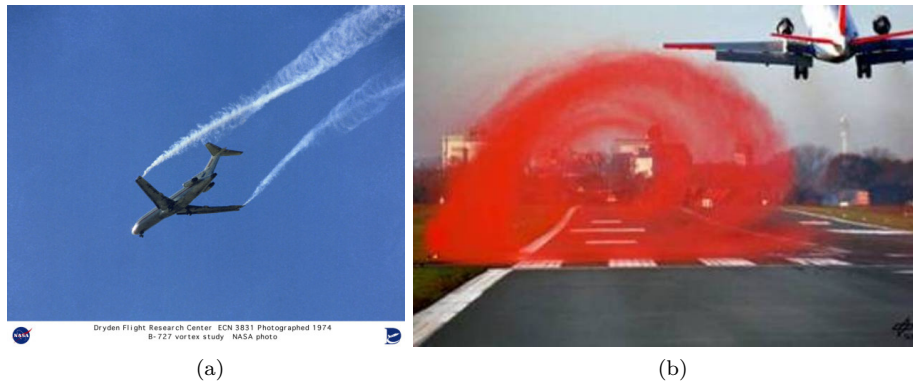


Figure 1.3: Visualization of the WVs: (a) by “smoke generators installed on the tips of the aircraft’s wings” (obtained during a series of flight tests performed in 1973 – 1974 at the NASA Flight Research Center (FRC)); (b) illustration of the WVs interaction with ground. Picture (a) is taken from NASA website (2019) and picture (b) is from DLR website (2019).

To optimize the airport/runway throughput while maintaining an acceptable level of safety, the separation minima were redefined in 2013 by splitting

the ICAO Heavy and Medium categories into ‘Upper’ and ‘Lower’ aircraft parts, resulting in 6 categories instead of the 4 previous ones (EUROCONTROL, 2013). This new categorization (RECAT-EU) has been deployed in Paris Charles de Gaulle airport since the spring of 2016 (DGAC, 2016). A similar system (RECAT) has been implemented by the FAA in the US (Cheng et al., 2016).

In order to support the development of new systems for a safe decrease of aircraft spacing (such as weather-dependent separation systems), measurement tools that can provide information about the wake vortex situation around the airport, in real time, is also required. The main sensors used for this issue are lidar and radar. Lidars operate in clear air (Holzäpfel et al., 2003; Köpp et al., 2005) but they require a sufficient concentration of aerosols in the air and they are blind in rain or fog conditions. Radars are able to see scattered electromagnetic (EM) waves on raindrops or fog particles (Barbaresco et al., 2008; Seliga and Mead, 2009); but the power of the scattered EM waves returning to the radar is quite weak in clear air (Gilson, 1992). To cover all weather conditions, both tools are likely to be used together (Oude Nijhuis et al., 2018). For reliable work, it is necessary to know the limits of detectability for the sensors under different weather conditions.

1.2 Context of research

A description of the wake vortices was provided, for the first time, by F. W. Lanchester in 1907 (Lanchester, 1908). Since then, this problem was gradually attracting more and more attention to ensure the aviation safety and satisfy the demand for increasing the airports capacity. Li (2010) and Liu (2013) discriminate four phases of the research activities on the WVs:

1. During the first phase (1900s – 1950s), the basic theories describing the WVs were formed.
2. The second phase (1960s – 1980s) emerged with the increasing number of types of aircraft. The WV was then investigated from the point of view of aviation safety. A reference point was a Symposium on aircraft wake turbulence, held in Seattle, in September 1970 (Olsen, 1970). One of the main topics discussed during the symposium was the aspects of the interaction between the wake vortices and the aircraft. The first systems for the detection of the WVs appeared during this period (Burnham et al., 1971; Hallock, 1991).
3. The third phase (1990s – 2005s) was initiated by a new FAA (Federal Aviation Administration, USA) program on the WVs. During this period, intensive studies of the WVs were conducted in the USA and in Europe. Many different types of systems were suggested for WV monitoring: Radar (Gilson, 1992; Myers, 1997; Mackenzie, 1997; Marshall and Myers, 1996; Marshall et al., 1997; Hanson and Marcotte, 1997; Iannuzzelli et al., 1998; Shephard et al., 1994; Neece et al., 2005), Lidar (Köpp et al., 2005;

Frehlich and Sharman, 2005; Köpp, 1994), Sodar (Zhang et al., 2003; Rubin, 2005), and Radio acoustic sounding system (RASS) (Rubin, 2000; Morris et al., 2004). During this phase, questions related to the wake vortex decay, wake vortex ground effect and the impact of atmospheric conditions on the wake vortex were addressed.

4. The fourth phase (2005 – now) has been driven by two large projects: SESAR in Europe and NGATS in the USA, aiming at developing new sophisticated air transportation systems.

The work presented in this manuscript was performed in the frame of UFO project (2016) and SESAR 12.02.02 Joint Undertaking (JU). The goal of these projects, related to the radar sensor, is to have one single radar that is used to estimate all the relevant parameters of the atmosphere that influence the air traffic and the parameters of the WV itself. This work is aimed at supporting the development of the radar sensors that will be employed to monitor the WVs.

1.3 Content of the thesis

The objective of this work is to provide a tool that allows the simulation of the wake vortex detection in clear atmosphere and in the presence of rain. This tool can be used for the optimization of the radar parameters and the development of the algorithms and methods for the detection of wake vortices and the estimation of their parameters. The analysis of the radar backscattering is based on the use of 3D fields from LES developed by Thermodynamics and Fluid mechanics (TFL) group of UCLouvain in the frame of UFO project (2016) and SESAR 12.02.02 JU. The LES is used to simulate the time series and the Doppler spectra of the backscattered signal returning to the radar. The simulated signal is then analyzed to retrieve the position and the strength of the WV. The work performed is described in the following chapters.

The rest of this chapter overviews the experimental work and theoretical models dedicated to the detection of the WVs in clear atmosphere and rain. It concludes with the current status of the problem, stresses the main shortcomings of the present theoretical frameworks used for the simulation of the radar signal backscattered by the WV and describes the novelties brought by this work.

Chapter 2 presents the results of the LES developed by TFL group of UCLouvain. The fields of physical parameters are transformed to the parameters providing the electrical characterization of the scattering medium. Those parameters are used to simulate the signal measured by the radar.

Chapter 3 recalls the principles of the Doppler radar measurements of distributed targets. It develops a methodology to simulate the time series of the backscattered radar signal on the WV in clear air and to synthesize the Doppler spectrum of the signal backscattered on the WV in rain. The geometrical configurations of the measurement performed in the frame of UFO project (2016) and SESAR 12.02.02 JU are given in this chapter.

Chapter 4 provides the theoretical basis needed to retrieve the parameters of the atmosphere and the WV from the radar measurements: the energy dissipation rate (EDR) of the atmospheric turbulence, which characterizes its intensity, and the WV circulation, which characterizes the strength of the wind field induced by the wake vortex. The retrieval of these parameters is based on the evaluation of the Doppler spectrum of the backscattered signal.

Chapter 5 presents the results obtained. The EDR of the atmospheric turbulence is estimated from the Doppler spectrum width. The WV detection in clear atmosphere is simulated to evaluate the possibility of estimating the WV strength from the backscattered signal under clear air and rainy conditions.

Chapter 6 summarizes the contributions of this work and provides an overview of future perspectives of the work in this field.

1.4 State of the art

The first radar measurements of WVs were performed at the beginning of the 1970s with a Doppler radar employed for the detection of EM waves scattered on suitable tracers injected in the WV and on the natural snow tracing the wake vortex wind flow (Easterbrook and Joss, 1971). Subsequently, the feasibility of using a radar to detect the WV in clear atmosphere was shown with a frequency-modulated continuous-wave (FMCW) radar (Noonkester et al., 1974; Noonkester and Richter, 1980).

During the last five decades, numerous experimental efforts were made for the detection of the WVs as well as theoretical works aimed to explain the experimental data and provide a better understanding of the wake vortex phenomenon and the mechanisms of EM wave scattering on it. Recently, successful detection of the WV in both clear air and rain was shown, using an X-band radar (Barbaresco et al., 2007, 2008, 2016). Using a W-band radar (Seliga and Mead, 2009), a meter-scale resolution was achieved when measuring the signal backscattered on the WV in the presence of rain. While the mechanism of scattering of the EM waves on the WV in rain is well-known, there is still no consensus on the exact nature of the scattering of the EM waves on the WV in clear air.

The rest of this chapter overviews the experimental work and theoretical models dedicated to the detection of the wake vortex in clear atmosphere and rain. It then concludes with the current status of the problem, stresses the main shortcomings of the present theoretical frameworks used for the simulation of the radar signal backscattered on the WV and describes the novelties brought by this work.

1.5 Detection of the WVs under clear air conditions

The radar detection of WVs under clear atmospheric conditions is reported in the literature, in various frequency ranges: VHF, UHF, L, S, C, X band

and even W band. Table 1.1 provides the correspondence between the radar-frequency bands and the letters classically designating them in accordance with (IEEE, 2003). The respective wavelengths are also recalled.

1.5.1 Measurements

Noonkester and Richter (1980) reported the first detection of wake vortices in clear atmosphere. The observations were made with an S-band FMCW radar operating at 2.8 – 3 GHz, that was placed at the International Airport Lindbergh Field, San Diego, California, during field tests in 1973 (preliminary results (Noonkester et al., 1974) were not published as pointed by Strauch (1979) and Chadwick et al. (1981)). The radar used an antenna with a 35 dB gain and a half-power beamwidth of 4°; the power transmitted was 200 W (Richter, 1969), (Richter et al., 1971). The antenna was pointed vertically, providing measurements in the range from 80 m to either 550 or 700 m with a range resolution of about 2.5 m. The radar was placed 200 m away from the end of the runway at a distance of 30 m from the extended runway center line. The radar worked in the range-only mode without resolving the velocity of the scatterers. The paper reports that 292 wake vortex-related echoes were observed in a one-month period. Fig. 1.4 provides an example of the observed echo. It was assumed that the large shear present between the wake vortex and the ambient wind carrying it, creates sufficient turbulence in the area with high gradient of the radio refractive index created by the WV. The turbulent eddies having sizes around 5 cm (the Bragg wavelength for the radar used) generate radar backscatter. A few possible explanation of the sinusoidal height dependence of the scattered echoes measured over time (see Fig. 1.4) are given. Nevertheless, the exact nature of this dependence was not identified.

Later, Campbell et al. (1980) and Chadwick et al. (1981) reported detection of the backscattering returns associated with the turbulent WV of large jet aircraft at short ranges as a by-product of their development of a clear-air S-band FMCW radar for detection of meteorological hazards at airports. The transmitted peak power was 200 W and the range resolution was 100 m. A few experiments were conducted in 1979 at the Denver’s Stapleton Interna-

Table 1.1: Letter designations for radar-frequency bands (IEEE, 2003) and respective wavelengths.

Band designation	Nominal frequency range	Nominal wavelength range
VHF	30 – 300 MHz	10 – 1 m
UHF	300 – 1000 MHz	1 – 0.3 m
L	1 – 2 GHz	30 – 15 cm
S	2 – 4 GHz	15 – 7.5 cm
C	4 – 8 GHz	7.5 – 3.75 cm
X	8 – 12 GHz	3.75 – 2.5 cm

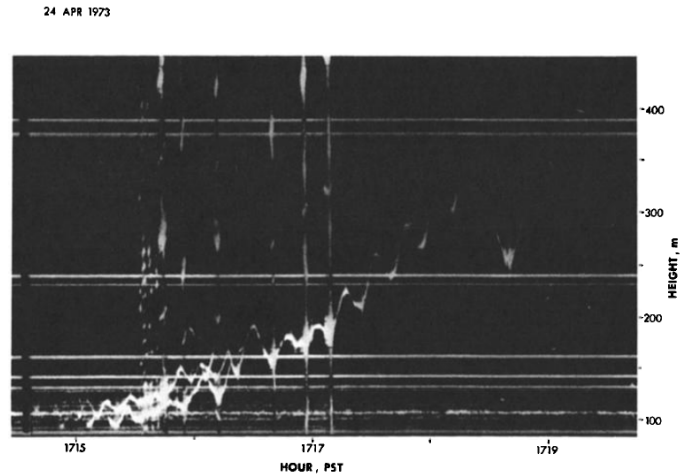


Figure 1.4: Example of the echo which is considered to be from a wing tip wake vortex. The sinusoidal echo occurred “at increasingly higher elevations as it drifted over the radar (increasing time)”. This figure is taken from (Noonkester and Richter, 1980).

tional Airport, showing that wake vortex-related echoes can be detected by the radar “when the atmospheric conditions are right”. An accurate definition of these conditions is however not given. Unlike in the Noonkester’s experiments, the radar was operating at low resolution in range (100 m) and in the range-Doppler mode, providing resolution of the scatterers’ radial velocities. The radar operated with a maximum range of 1000 m and the antenna looking along the runway axis at the elevation angles ranging from 20° to 25° .

The authors reported that, in the presence of the strong cross wind, the wake vortices could not be detected. Contrary, with a light wind along the runway, the WV could be easily observed for most of the large aircraft. However, as authors pointed out, the physical phenomena causing the backscattering could not be identified. Chadwick et al. (1981) proposed an explanation for the sinusoidal variations of the backscattered signal with height: the sinusoidal nature of the returns observed by Noonkester and Richter (1980) is supposed to be due to the inability of the range-only radar to measure Doppler frequency unambiguously with range.

Nespor et al. (1991) conducted experiments at White Sands Missile Range, New Mexico, to study the feasibility of including additional functionality (such as WV detection) to the Terminal Area Surveillance System (TASS) (Proctor, 1987). A C-band phased array pulse-Doppler radar was used to detect the WV under clear air conditions. The radar emitted 1 MW peak power and used an antenna with a beamwidth of 1° ; the pulse width was varying between $0.25 \mu\text{s}$ and $1 \mu\text{s}$ corresponding to a resolution between 37.5 m and 150 m. Two geometrical configurations were employed: (1) the antenna beam oriented transversally with respect to the aircraft flight direction and (2) the antenna beam axially aligned with the WV behind the aircraft. Detection was however

only achieved on axial view. Since only a small A-7 jet was present at the time of the trials, the radar measurements were obtained for weak wake vortices, relative to the WVs of large aircraft. The authors found that the radar cross-section of the WV tends to increase as the WV descends, assuming that it is caused by the expansion of the WV that occupies a larger fraction of the radar resolution volume as it decays and dissipates with time.

Gilson (1992) reports an extensive series of measurements performed at Kwajalein Atoll in the Marshall Islands in May and June of 1990, during which the reflectivity of the WV was measured at VHF, UHF, L, S and C bands. Though, no detection was achieved in the Ka band (27 – 40 GHz frequency band corresponding to 1.1 – 0.75 cm range of wavelengths). The radars' antenna beams were oriented transversally to the aircraft flight paths. Due to a high peak power for all radars used (7 MW, 5 MW, 2 MW, 2 MW, and 3 MW for VHF, UHF, L, S and C bands respectively), the detection of the WV was possible for distances of up to 55 km (for comparison, the power emitted by the Ka-band radar was 25 kW only). The WV were detected behind the C-5A aircraft (which is 75 m in length and has a wingspan of 68 m) from one hundred meters to tens of kilometers. Fig. 1.5 shows an example of the S-band range-Doppler slice through the WVs of the C-5A aircraft.

The authors found that the frequency dependence of the reflectivity is largely flat with a decrease at the C band and that the radar cross-section (RCS) of the WV decreases with height. Both Nespor et al. (1991) and Gilson (1992) report that the reflectivity in the area of the WV increases with respect to the ambient air, due to the turbulence induced by the WV. They also report an increase of WV's reflectivity over time, assuming that the turbulence induced by the WV spreads over larger distances and occupies the larger fractions of the radar resolution volume. Gilson also observed that the strength of the backscattered signal has a strong dependence on climate, with strongest level of backscattering in tropics near sea level. This probably implies that

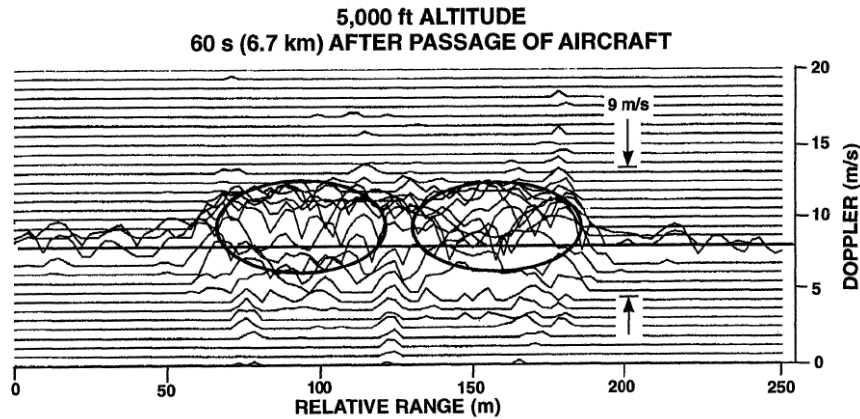


Figure 1.5: Example of the S-band range-Doppler slice through the WVs of the C-5A aircraft. This figure is taken from (Gilson, 1992).

the moisture content significantly influences the reflectivity of the developed turbulent wake vortices.

Shephard et al. (1994) report two trials performed in 1991 and 1993 during which the wake vortex detection was investigated by using pulse-Doppler radars at S and X bands respectively. Measurements in S band were obtained with a range resolution of 50 m and pulse repetition frequency (PRF) of 2 kHz for a HS-748 aircraft, which is a medium-sized aircraft with a wingspan of around 31 m. Trials with X-band radar used targets of opportunity overflying the radar location on their way to London Heathrow airport. The range to the aircraft path was around 4 to 5 km. Two measurements of the WV created by a Boeing 747 were reported at ranges of about 4 km. It is noticed that the presence of clouds created a strong clutter obscuring the weak echoes from the WV. As concluded by authors, the exact mechanism of the backscatter is uncertain but it is assumed that it could be caused by airborne particles that are disturbed by the WV.

Marshall et al. (1997) conducted a series of field experiments in the Wallops Island in 1995. Authors tried to demonstrate the detectability of WVs by an X-band radar having a wavelength of 3 cm. The C-130 employed in the experiments was equipped with wingtip smoke generators to visualize the wake vortices. Despite the accurately planned experiment and lengthy data analysis, no conclusion on the ability of the X-band radars to detect the WV was obtained. Authors indicate two main factors preventing the successful detection: (1) low emitted peak power – it was foreseen that 2 kW would be needed to measure reflectivities of WVs estimated at levels as low as -135 dBm^{-1} (the volumetric reflectivity is measured in m^2/m^3) but only 700 W were achieved during the experiment, due to a component failure; (2) unsuitable parabolic antenna with high side lobes contaminated the measurements by the ground clutter. Even though wake vortices were not detected, this work provides valuable insights that can be used in the future field experiments.

Mackenzie (1997) reports the results of WV detection by a C-Band radar conducted in parallel with Marshall et al. (1997) on Wallops Island. Despite the strong influence of the clutter, which blocked the detection of the WV with the X-band radar (Marshall et al., 1997), a clear evidence of the WV detection was obtained with a C-band radar when the position of the antenna beam coincided with the WV observed, visible due to the smoke tracer used. The measured WV's reflectivity was in agreement with the previously reported observations.

Hanson and Marcotte (1997) and Iannuzzelli et al. (1998) developed a bistatic continuous-wave X-band radar combined with an acoustic pumping system to enhance the detectability of the WV. The system was employed during trials conducted at Baltimore–Washington International Airport in 1996 and was able to measure the Doppler signatures of a C-130 aircraft's WV.

Barbaresco et al. (2007) made radar measurements of WVs with an X-band radar during trials at Paris Orly Airport in 2006. A low-power radar (20 W peak power), with a range resolution of 40 m and angular resolution of 2.7° in azimuth and 4° in elevation, was able to detect wake vortices at distances up to 700 m.

Two geometrical configurations were employed for the measurements: (1)

a staring antenna mode – the radar looks almost perpendicularly to the runway – providing measurements for 5 radar resolution volumes ranging from 520 m to 720 m, and (2) a scanning antenna mode, in which the radar scans an azimuth sector of 45° at the scanning rate of $8^\circ/\text{s}$ to track the wake vortex speed components over time. As observed in the measurements, the Doppler signatures of the wake vortex contain some regularly spaced slopes in the time-Doppler domain. As explained by the authors, those slopes are induced by the WV roll-up structure (the same as modeled by Myers (1997); Shariff and Wray (2002); Li et al. (2010); Vanhoenacker-Janvier et al. (2012) and described in the following section), and characterize the age and the spiral geometry of the WV. The measurements show the presence of Doppler spectral components with radial velocities up to ± 26 m/s, while the highest tangential velocity within the WV is estimated to be less than 10 – 15 m/s (the theoretical model considered). As noticed by F. Barbaresco (private communication, 2018), the airport meteorological services confirmed that they have measured 100 km/h wind bursts (close to 26 m/s) when the WV were in proximity to anemometers. Barbaresco et al. (2008) conducted additional radar trials in the Thales premises in Limours, near Orly Airport. During the measurements, WVs of landing aircraft were observed at altitudes around 1000 to 1500 m. The power emitted by the radar was increased to 75 W compared to 20 W for the previous trial. The radar was placed below the glide. It performed scans in vertical plane, deviating the antenna beam by 45° from the vertical line at a rate of $8^\circ/\text{s}$ (with the plane of scan perpendicular to the runway). Wake vortices Doppler signatures similar to (Barbaresco et al., 2007) were observed.

Fig. 1.6 shows examples of the WV's Doppler signatures obtained by Barbaresco et al. (2008). So, in Fig. 1.6(left), two WVs signatures are seen on the consecutive scans with the interval of around 4 s. In Fig. 1.6(right), the signature of the WVs has changed at the second scan.

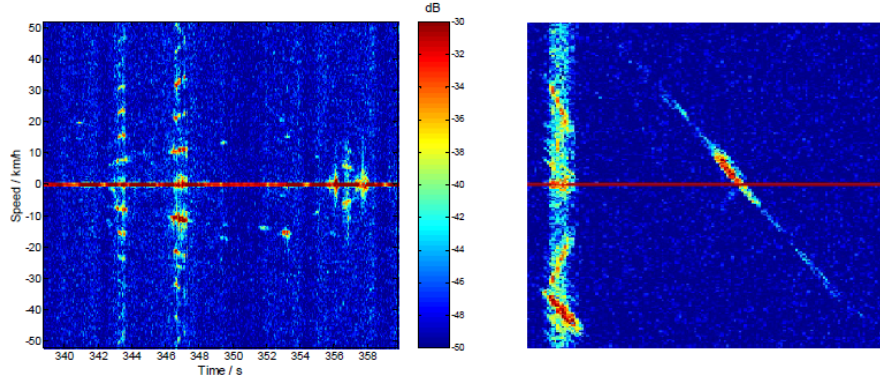


Figure 1.6: Doppler signatures of the WVs in clear atmosphere: (left) low turbulence intensity, (right) high turbulence intensity. These figures are taken from (Barbaresco et al., 2008).

From the analysis of the results provided by Barbaresco et al. (2007, 2008),

a few interesting conclusions might be drawn:

1. The WVs Doppler signatures were observed during a very short time interval: around 6 – 7 seconds for the staring mode and around 10 seconds for the scanning mode. There can be two reasons for that: (a) a strong cross wind that removes the WVs from the radar resolution volume located at a given range or (b) a short life time of the spiral pattern that causes the scattering.
2. As pointed out by authors, the atmospheric conditions during the second trial (Barbaresco et al., 2008) were characterized by a strong turbulence and the presence of a strong cross wind. The measurements under these conditions show that the Doppler signature has a different shape in two adjacent radar scans separated in time by 5 – 6 s (see Fig. 1.6(right)). The distributed Doppler components (slopes in the time-Doppler domain) in the first scan changed to the Doppler spectrum concentrated around mean radial velocity (approximately around zero) and spread in the interval from around +10 m/s to –10 m/s of Doppler velocities. These results, as noticed by authors, obtained under very high turbulence kinetic energy of the background atmosphere, might imply that the life time of the spiral pattern is short and the spiral scattering pattern diffused by the background atmospheric turbulence and self-induced turbulence of roll-up very fast. Nevertheless, absence of the additional information does not allow to confirm these conclusions.

Detection of the WVs in clear air was shown using frequencies from VHF up to X band. The level of the backscattered signal is very weak, which makes it difficult to detect the WVs on long distances. Initial experimental works generally suggest that the wake vortex induced turbulence is created at scale sizes of about 5 – 10 m (a so-called outer scale of the turbulence at which the energy is entered in the turbulence spectrum – see Chapter 2 for more details) during the roll-up process and then break down into the smaller eddies until the smallest sizes at which the WVs dissipates its energy. The Bragg scale irregularities caused by this turbulence enable the EM waves backscattering.

The following section provides an overview of the theoretical research dedicated to the mechanisms of the EM waves backscattering on the WVs in clear atmosphere.

1.5.2 Theoretical models of electromagnetic waves scatter from the WVs

Early experimental works showed that the WVs can be detected by a radar in clear atmosphere; however, the exact mechanisms of the EM energy scattering on the WVs was not identified: although, it was assumed that refractive index irregularities in the WVs result in Bragg scattering, it was not yet specified which parts of the WVs caused the observed radar returns. Some authors also indicated that the engine's combustion products could cause the (Rayleigh) scattering of the EM waves. So, a theoretical analysis was needed to establish

the underlying mechanisms of the electromagnetic waves backscattered on the WVs.

Marshall and Myers (1996) used a 2D model (the NASA TASS (Proctor, 1987)) in order to estimate the radar reflectivity of the wake vortex. Authors assumed that the EM scatter from the WVs is due to the Bragg scattering on the refractive index irregularities produced by the vortex induced turbulence. According to the results obtained, authors estimated that the boundary part (the WVs oval) of the descending WVs can be detected by radars at the X band and below. As follows from simulations for clear air, a characteristic pattern of the reflectivity of the wake vortex is observed due to the absence of the reflectivity in the cores. For the simulations in the presence of fog (Rayleigh scattering on the fog particles), the reflectivity is not equal to zero in the wake vortices cores.

Myers (1997) uses in his dissertation, a wake vortex passive conservative simulation (WV-PCS) in order to simulate the RCS and Doppler spectra of the WVs. A new mechanism is identified. This mechanism does not take into account turbulence within the WVs system. The author argues that in the TASS model used in previous research, the EDR levels are unrealistically large due to the use of Smagorignski sub-grid scale model (Zeman, 1995). It was concluded that the scattering of the electromagnetic waves on the WVs is not due to the incoherent Bragg scattering on turbulence but rather to the coherent Bragg scattering from the spiral structure of the WVs laminar flow (Myers et al., 1999). The results of reflectivity estimation are in quantitative agreement with the previously reported experimental data. As shown, the new mechanism results in a sharply peaked Doppler spectrum for higher frequencies (see S band in Fig. 1.7).

However, the considered “laminar flow mechanism” does not take into account turbulence within the wake vortex system, neither the atmospheric turbulence, which significantly influences the WV’s life time, nor the turbulence self-produced by the WVs, which results in large EDR within the WV oval.

Shariff and Wray (2002) developed a model to analyze the reflectivity of the WVs in clear air observed during the experiment of Gilson (1992). Refractive index of a wake vortex system is obtained as a combination of three mechanisms: (1) the radial density of a 2D wake vortex, (2) 3D fluctuation in the WVs cores and (3) the adiabatic transport of atmospheric fluid in a 2D oval surrounding the pair of vortices while it descends in a stratified atmosphere. The structure of the WV results in a gradient between the ambient air and the WVs oval. It is concluded that for mechanism (3) - scattering on the oval surrounding the wake vortices agree with experiments conducted by Gilson (1992), however theoretical predictions have a cut-off when the EM incident angle is away from normal, which was not observed in the experiment of Gilson. As concluded by authors, the absence of the cut-off in the experiment is likely caused by the turbulent boundary of the WVs oval that is not included in the model. Moreover, it is concluded that for mechanisms (1) - scattering from the radial density of the WVs and (3) - scattering from 2D oval surrounding the WVs, the estimated reflectivity is of the same order. The model proposes almost a flat dependence of the WVs reflectivity on the frequency.

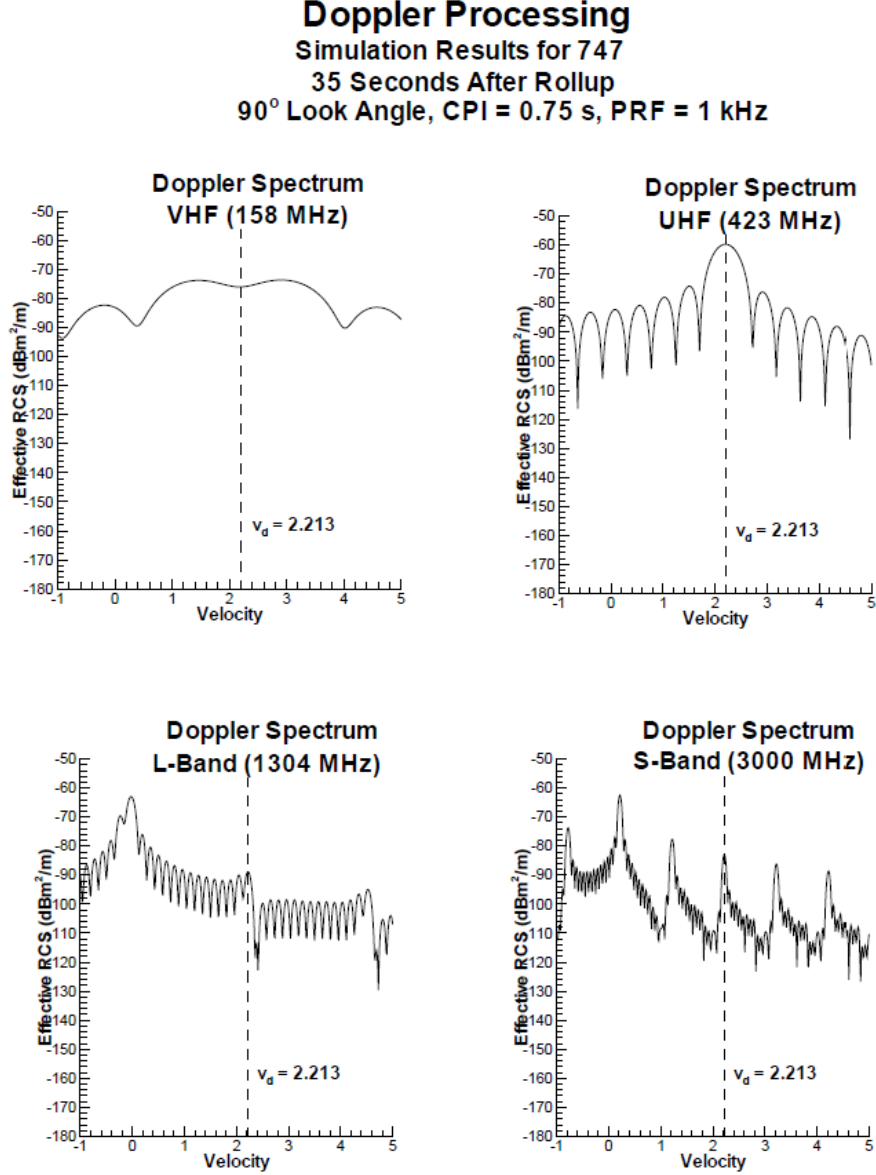


Figure 1.7: Doppler spectra for WVs of Boeing 747. The antenna beam looks perpendicular to the aircraft's flight path. The simulations are shown for 35 s after roll-up. This figure is taken from (Myers, 1997).

Iannuzzelli (1998) used an X-band continuous-wave bistatic radar along with an acoustic pumping system to study detection of wake vortices in clear air. To support the interpretation of their experimental data, they ran numerical simulations to obtain the information on the reflectivity distribution of

velocities induced by wake vortices in a turbulent field. The code used solves steady-state nonlinear equations for the velocity, density, turbulent kinetic energy, and turbulence dissipation rate. Nevertheless, the model considered does not include the water vapor content of the atmosphere. The output of the simulation provided distribution of the velocity field of the WVs system, distribution of the refractive index structure constant and distribution of turbulent kinetic energy across the WVs region. Although the predicted vortex trajectories agreed with the data, the estimated values of the refractive index structure constant seemed to be much larger than the ones predicted by experiments and other theoretical research. The distribution of the turbulent kinetic energy in the considered model does not seem to be affected by the interaction of the WVs with the ambient turbulence. The model's predictions provides insights into which parts of the wake vortex can produce the observed radar echoes: the cores of the wake vortex and the border of the WVs oval have higher velocities, turbulent kinetic energy, and refractive index structure parameters values than the other parts of the WVs. However, as concluded by authors, the coincidence of the regions with high velocities and high refractive index structure constant make it harder to distinguish which effect is primarily responsible for the observed radar signatures.

Li et al. (2010) and Vanhoenacker-Janvier et al. (2012) extended works of Myers (1997) and Shariff and Wray (2002), assuming that the atmospheric gradient of water vapor is wound up into a spiral by each vortex, which causes scattering at high frequencies. Li et al. (2010) investigated the effect of the density and weather variation of the dielectric constant on the reflectivity of the wake vortex, in the frequency range from 1 MHz to 10 GHz. He found that the density variation is the dominant factor to the total reflectivity of the wake vortex at low frequencies (less than 100 MHz), while in the high-frequency band, the dominant factor turns out to be the water vapor variation. The convection effect makes the water vapor variation more and more evident as the evolution time increases, so the reflectivity of the WVs caused by this factor becomes larger and larger. Vanhoenacker-Janvier et al. (2012) used a model combining two mechanisms causing refractive index gradients: radial density gradient in the vortex cores and the transport of the atmospheric fluid in the oval surrounding the vortices. The results are in agreement with those previously reported by Shariff and Wray (2002) and Li et al. (2010).

The models available in the literature can be divided into two groups: (1) Bragg scattering on the turbulence induced by the WVs and (2) Bragg scattering on the spiral density pattern of the WVs. While both models do not exclude each other, the ones relying on the spiral structure of the WVs do not take into consideration the influence of the turbulence self-induced by the wake vortex and, as a consequence, they lack to predict the real life time of the developing wake vortex and its real structure at later times. Those models also imply a high dependence of the WVs RCS on the incident angle of the EM wave arriving on the WVs. This is however in contradiction with what is shown in experimental works (Noonkester and Richter, 1980; Nespor et al., 1991; Gilson, 1992): the RCS of the WVs does not depend on the incident angle.

From the measurements obtained by Barbaresco et al. (2007, 2008, 2016),

the WVs Doppler signatures obtained just after the formation of the WVs, can indeed be caused by its spiral structure (during a very short time interval after the formation of WVs) until it is diffused by the induced turbulence.

Finally, all the models available in this review are 2D models which lack the realistic simulation of the backscattered radar signal. The gap between experimental and theoretical work performed up to now suggests that more practical efforts are needed to combine measurements with theoretical models in order to establish the scattering mechanisms during the whole life time of the wake vortex as well as the estimate of the limits of operability of the radar in clear air.

1.6 Detection of the WVs in presence of rain

Detection of WVs using a radar was first obtained by Easterbrook and Joss (1971), who was observing the scattering of EM waves on natural precipitations (snow). Using radars for detection of WVs in rain, fog or snow did however not receive attention in the radar community during a long time, neither from an experimental nor from a theoretical point of view. A renewed interest appeared in the years 2000. The main advantage of the WVs detection under rainy conditions is that the level of the backscattered signal is much stronger when compared to the clear air case. The following section presents the experimental and theoretical works dedicated to the WVs radar detection in the presence of rain or fog.

1.6.1 Measurements

Barbaresco et al. (2007, 2008) made radar measurements of the WVs with an X-band radar during trials at Paris Orly Airport in 2006 and 2007. A 20 W peak-power radar was able to detect wake vortices in clear air and in rain (with intensity of 1.19 mm/h) at ranges up to 700 m. The configuration of the measurements is given in Section 1.5.1. The measured WV's Doppler signatures contain regularly spaced slopes in the time-Doppler domain (see Fig. 1.8) in both weather conditions: clear air and rain. As interpreted by the authors, these slopes are induced by the roll-up of the aircraft wake.

Seliga and Mead (2009) reported the first measurements of WVs obtained with a W-band radar at ranges of 0.5 to 1 km. Meter-scale detection was achieved by using a low power (100 mW) solid-state W-Band (94 GHz) radar, primarily designed for the monitoring of the tall vessels in the Boston Harbour, neighbor to the airport. The radar employed a 1.2 m diameter antenna with a half-power beamwidth of 0.18° providing 1.5 m angular resolution at 500 m range. Measurements clearly showed enhanced droplet concentration between the wake vortices, and a significant reduction of the concentration within the wake vortex cores. These spatial structures were observed up to 60 s after the passage of the aircraft. A very high range resolution provided a fine evaluation of the spatial distribution of velocities (Fig. 1.9) which potentially enables the estimation of the WVs strength and WV's core radii.

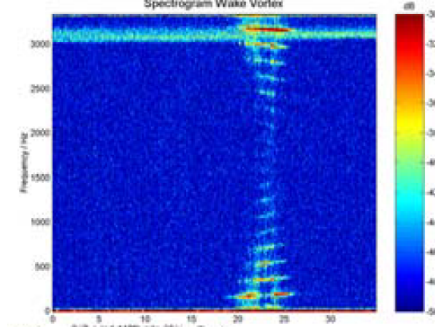


Figure 1.8: WVs signature in the Doppler spectrum changing in time. The horizontal axis represents time (a span of 35 s). The vertical axis represents the Doppler frequency (corresponds to a span of Doppler velocities with 0 m/s at the top and at the bottom of the figure and ± 26 m/s in the middle of the figure). The color represents the Doppler spectrum amplitude. This figure is taken from (Barbaresco et al., 2007).

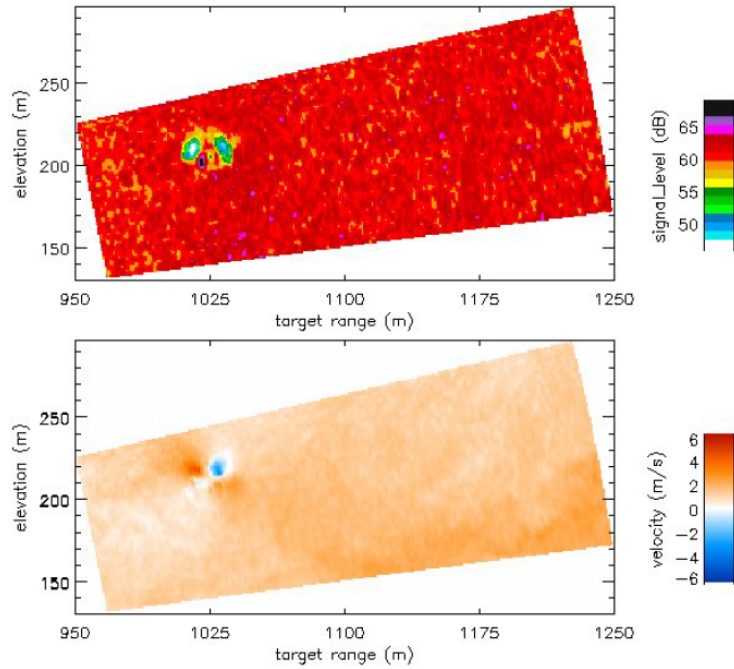


Figure 1.9: WVs detected by the W-Band radar on March 26, 2009. “The upper RHI is the peak spectral power level of the signal within each range bin, and the lower one is an RHI of the peak Doppler velocity”. This figure is taken from (Seliga and Mead, 2009).

During SESAR P12.2.2 XP0 trials at Paris Charle de Gaulle (CDG) Airport, the wake vortex Doppler signatures were observed with a vertically oriented radar's antenna beam in the planes crossing the glide of the landing/taking-off heavy and medium aircraft (total 34 planes) (Barbaresco et al., 2012) (see also (Barbaresco, 2012)). The observation time of the wake vortices varied from a few seconds to 250 s. The time of observation was limited by two main factors: the low backscattered signal for low rain rates and the shift of the WVs out of the sector of observation, due to the wind.

Barbaresco et al. (2016) reported results of a second measurement campaign in CDG (XP2 in 2015 – 2016), made in the framework of SESAR program. The aim was to validate the wake-vortex detection algorithms and circulation retrieval during long trials, supported by cross-wind and EDR measurements. A scanning mode with three beams steered in elevation was employed to observe the mean Doppler velocity inversion in a plane transverse to the WVs trace. The Doppler spectrum widths calculated for the measured Doppler spectra of the WVs were confirmed to be a good estimation of the wake-vortex strength, preserving the order of aircraft categories: 1.9 m/s for A380; 1.7 m/s for A340; 1.25 m/s for A300; 0.85 m/s for A320; for comparison, the width of the Doppler spectrum estimated for the radar resolution volume containing rain only was around 0.2 m/s.

1.6.2 Theoretical models of electromagnetic waves scattering by the WVs

Neece et al. (2005) considered two numerical models for a Ka-band (35 GHz) radar system to investigate the method of detection and the resolution requirements needed to estimate accurately the WVs parameters. Their work is based on the models developed by Marshall and Mudukutore (1996) and Mudukutore et al. (1998). The use of both compressed and uncompressed short pulses is compared to evaluate the wake vortex structure. Authors conclude that, while the use of pulse compression allows to increase the range resolution and keeps the same level of energy of the emitted pulses, it results in the leakage of the signals from the adjacent radar resolution volumes.

Liu (2013) developed a methodology to simulate the radar signatures of the WVs in presence of rain. The atmosphere was considered as neutral (no stratification) and without turbulence. The X- and W-band radar parameters were used to simulate radar returns from the WVs as a sum of the contributions from the raindrops in the WVs area. He concluded that the Doppler spectrum width is a good indicator of the wake vortex strength.

Li et al. (2016) and Li et al. (2017) considered the problem of retrieving WVs strength in fog and rainy conditions respectively. In foggy conditions, the strength of the wake vortex is estimated using the Doppler spectrum of the echo signal. For the rainy conditions, it is proposed to use a side-looking radar, performing a scan in a vertical direction to detect the presence of the WVs. The proposed method is based on the identification of the so-called characteristic-size raindrops, which make major contribution to the backscattered signal, i.e. have the largest RCS. The velocity of these raindrops are estimated from the

Doppler spectra, and further used to retrieve the strength of the wake vortex.

Recently, Shariff (2016) proposed an approach to make the wake vortices visible to the existing weather radars for the aid to the air traffic controllers or research purposes. He proposes to inject water spray into the wake vortex of a heavy aircraft during a landing phase. The radar reflectivity of the spray trace was simulated using parameters of existing weather radars. The results showed that a large enough signal-to-noise ratio (SNR) can be obtained at distances up to 11 km behind the aircraft. This fulfills the requirements of the possible use of this technique in the wake vortex separation.

1.7 Main shortcomings and what this thesis proposes

While the mechanism of the backscattering of the EM waves on the raindrops is well-studied, the exact nature of the EM waves backscattering on the WVs in clear atmosphere is still disputed. Existing models for clear air scattering have following drawbacks:

1. models available in the literature are simplified 2D models;
2. most of the models do not take into account interaction between the wake vortices and surrounding turbulent atmosphere, which significantly influence the WVs structure and its life time;
3. there is little theoretical research on the estimation of the backscattered signal's Doppler spectrum;
4. there is no analysis of the WVs strength retrieval from the radar signal backscattered on the WVs in clear atmosphere;
5. absence of the estimation of the WV's RCS based on realistic 3D models that take into account the turbulent nature of the WVs; weak level of the backscattered signal necessitates an accurate estimation of the WV's RCS.

Recent theoretical works suggest backscattering of the radar signal on the spiral structure of the WVs. However, the surrounding turbulent atmosphere, which significantly influences the WVs structure and WV's life time, as well as the turbulent nature of the WVs itself, are not taken into account. Some evidence from the experimental works (Barbaresco et al., 2007, 2008) supports this theory, but those spiral structures seem to be diffused by the turbulent atmosphere very fast. Probably, the scattering regime is switching in time from the scattering on the spiral structure of the WVs (at the early stages of the WVs evolution) to the Bragg scattering on the generated turbulence.

For the simulation of the WVs in the presence of rain, the main weakness of the models available is that they are 2D simplified models that do not take into account atmospheric turbulence. As seen from the measurements, especially at high frequencies (Seliga and Mead, 2009), there is a strong evidence that

turbulence plays an important role in the structure of the WVs Doppler signatures. Therefore, to simulate Doppler signatures of the WVs, which are of great importance, for instance for the development of the classification algorithms, it is important to have more realistic simulations.

This work proposes to use the results of realistic 3D simulations of WVs in turbulent stratified atmosphere obtained by Billuart, Duponcheel and Winckelmans in the framework of UFO project (2016) and SESAR 12.2.2 JU (Kovalev et al., 2018). For the first time, the output of these 3D simulations is used to investigate both, the WV's RCS and the radar Doppler signatures of the WVs for two states of the atmosphere:

- WVs added in turbulent stratified atmosphere without rain;
- WVs added in turbulent atmosphere with rain.

The objective of this work is threefold:

1. to simulate the radar signatures of the WVs under clear air and rainy conditions taking into account a turbulent nature of the interaction between the WVs and the surrounding turbulent stratified atmosphere;
2. to estimate the possibility of the retrieval of the WVs strength from the radar signatures;
3. to estimate the WVs RCS in clear air and in the presence of rain and to give a tool to the designers to calculate the limit of detectability of the WVs and to improve the radar sensitivity.

The research strategy of this thesis is as follows. The 3D LES fields of physical parameters of the atmosphere with added WVs are used to simulate the complex time series of the backscattered signal. Then, the Doppler spectrum of this signal is computed to estimate the distribution of the radial velocities of the scatterer. This information is used to evaluate the parameters of the WVs: their location and strength. The estimates are compared with the reference values of the WVs available directly from the LES to validate the simulator and evaluate the possibility of the accurate estimation of the WVs parameters from the backscattered radar signal. Finally, simulated WV's signatures (WVs' RCS, the backscattered signal's Doppler spectrum) are compared with those reported in the literature.

Chapter 2

Characterization of turbulent and stratified atmospheres, and of WVs evolution, without and with rain

The simulation of the radar detection of the wake vortices and atmospheric turbulence necessitates realistic models of the medium. Fluid mechanics simulation of the turbulent atmosphere uses numerical integration of Navier-Stokes equations. This approach is known as direct numerical simulation (DNS) (see, for example, McComb (1990)). Nevertheless, DNS is computationally expensive since it requires resolving a very wide range of time and length scales which affect the flow field. Its cost prohibits its use in the simulation of practical engineering systems, such as the ones considered in this thesis. The LES, is a very efficient method to reduce the numerical expense. It was initially proposed by Smagorinsky (1963) and explored further by Lilly (1967) and Deardorff (1970). This chapter presents the results of the Large Eddy Simulation of turbulent stratified atmospheres, as well as the results of LES of WVs added in such turbulence, in view of the calculation of the radar backscattered field. These LES simulations were developed in the framework of UFO project (2016) (LES of turbulent and stratified atmosphere, also with rain) and in the framework of SESAR 12.02.02 Joint Undertaking (SESAR 12.02.02, 2016) (LES of WVs in clear air turbulent and stratified atmospheres, and of WVs in turbulent and rainy atmosphere).

The first part of this chapter (Sections 2.1.1 to 2.1.4) presents the results obtained by the TFL group of UCLouvain for the LES of turbulent atmospheres and of added wake vortices. The second part of the chapter (Sections 2.2.1 – 2.2.4) links the physical characteristics of the simulated LES cases with the parameters that determine the electromagnetic properties of the scattering

medium.

2.1 Large eddy simulations of turbulent stratified atmospheres, and of added wake vortices

The objective of the Large Eddy Simulations in the present work is to provide realistic parameters of the medium in order to simulate the radar measurements of atmospheric turbulence and wake vortices in the absence and in the presence of rain.

First, clear air ambient turbulence was simulated, providing turbulent fields of velocity, temperature, pressure, and humidity for various turbulence intensities (energy dissipation rate, denoted hereafter as ϵ or EDR) and various stratification levels characterized by the Brunt-Väisälä frequency N (Section 2.1.1). Then, the output parameters of the LES simulations are used to introduce rain droplets falling in the turbulent stratified atmosphere (Section 2.1.3).

Second, a WVs was added to the simulations of clear-air turbulence (Section 2.1.2) and, finally, the simulation of the WV in the turbulent atmosphere in presence of rain was performed, to provide to the radar simulator the information about the raindrops concentration and their velocities (Section 2.1.4). Most of the content presented in Sections 2.1.1 – 2.1.4 is extracted from the reports of UFO project (2016) and SESAR 12.02.02 JU (SESAR 12.02.02, 2016).

2.1.1 LES of clear-air turbulent and/or stratified atmospheres

This subsection presents the basic concept and the underlying hypotheses used for the simulation of the stratified turbulent atmosphere. It briefly discusses the results obtained and the database of LES cases that are further used by the radar simulator. Most of the content presented in this section is extracted from the report of the UFO project (M. Duponcheel contribution to (Vanhoenacker-Janvier et al., 2014)).

Large eddy simulations for clear air

The simulation variables, representing the atmospheric properties, are decomposed into a mean vertical profile and the deviation from this profile. This enables the simulation of the variables deviations in the spectral periodic domain. The simulated variables are:

- the potential temperature, $\Theta = T(p_{ref}/p)^{(\gamma-1)/\gamma}$, which is the temperature that a parcel of air would have after being brought isentropically from its pressure p to a reference pressure p_{ref} ; here, T is the static temperature in K; $(\gamma - 1)/\gamma = R/c_p$, where c_p is the specific heat capacity of air at the constant pressure and R is the gas constant.

- the specific humidity, ϕ , that represents the amount of water vapor per kilogram of humid air (kg_v/kg_h).
- the pressure p .

In order to take the transport of humidity into account, the spectral solver developed by De Visscher et al. (2013a) was further used by Duponcheel (UFO project, 2016). The governing equations for the deviations are the incompressible Navier-Stokes equations supplemented by a sub-grid scale (SGS) model, and the Boussinesq approximation for the buoyancy effects. Those equations are solved in the Fourier spectral domain. “The time integration is carried out using the third order low storage Runge-Kutta scheme of Williamson (1980). The RVMS proposed by Jeanmart and Winckelmans (2007) as an improvement over SGS viscosity models (such as the classical Smagorinsky (1963) model)” is used in this work (SESAR 12.02.02, 2016). The RVMS model is designed to dissipate energy only at the smallest resolved scales and it uses an effective viscosity which is also computed using the smallest resolved scales. Those scales are obtained by applying a high-pass filter to the velocity field. The temperature and humidity SGS fluxes are obtained assuming unity turbulent Prandtl and Schmidt numbers. The influence of the ground and the boundary layer is not represented in the simulations.

From the deviation fields of potential temperature and humidity and the imposed mean profiles, one can reconstruct the global time-dependent profiles for the atmospheric fields. Once the global profiles for the potential temperature and pressure are computed, the static temperature can be obtained from the definition of the potential temperature:

$$T = \Theta \left(\frac{p}{p_{ref}} \right)^{\frac{\gamma-1}{\gamma}}. \quad (2.1)$$

The database provided to the radar simulator is delivered when the turbulence has reached a statistical equilibrium state. This state is characterized by the mean Brunt-Väisälä frequency, by the mean energy ϵ_f injected per unit time by the forcing term, and by the size of the domain L . In stratified atmosphere, the turbulent kinetic energy injected at the largest scales (ϵ_f) is dissipated by viscous stress (ϵ) and by the conversion to the potential energy (ϵ_B): $\epsilon_f = \epsilon + \epsilon_B$. The Brunt-Väisälä frequency represents the number of oscillations per unit time that a parcel of air would perform around its equilibrium position if it was moved away from this equilibrium by a small perturbation:

$$N = \sqrt{\frac{g}{\bar{\Theta}_0} \frac{d\bar{\Theta}}{dz}}, \quad (2.2)$$

where g is the acceleration of gravity.

Parameters of the simulation and simulated cases

Simulations performed consider the atmospheric conditions encountered during aircraft approach: a very-weakly to weakly stratified atmosphere combined to

atmospheric turbulence ranging from weak to strong. It should be mentioned that very strong unstable conditions may also be encountered in the lower layers of the atmosphere; yet the periodic setup that was used to produce the LES does not allow to simulate such unstable conditions.

The clear-air turbulence was computed at a resolution of $N_x = N_y = N_z = 256$ grid points in each direction of the LES cube. The size of the domain is used as the outer length scale, since the turbulence is forced at scales proportional to the domain size. The length of the domain was set to $L = 200$ m in order to have a sufficiently large domain with a relatively fine resolution $\Delta x = L/N_x = 0.78$ m. The output parameters of the LES simulations are the fields of atmospheric parameters: pressure, temperature, relative humidity, and velocity. They are provided at each of 256^3 LES nodes every second for a duration of 60 s of physical time. The fields of pressure, temperature, and relative humidity are scalar fields. The velocity field is a vector field represented by a three-component vector (u, v, w) at each LES node, where u , v , and w are the components of the local wind velocity along x -, y -, and z -axes respectively.

Six LES cases were defined as the combinations of 3 levels of energy dissipation rates, ϵ , and 3 stratification levels, N . In addition to these 6 cases, a case with $\epsilon = 3.4 \cdot 10^{-3} \text{ m}^2/\text{s}^3$ was simulated to reproduce the turbulent conditions of the atmosphere that dominated during the second measurement campaign of UFO project (2016). The parameters of these simulated cases are summarized in Table 2.1. The index number of each LES case with clear-air turbulence (without rain) precedes with a letter t in order to distinguish the LES cases in the absence and in the presence of the WV. The levels of energy dissipation rate range from low $\epsilon = 10^{-5} \text{ m}^2/\text{s}^3$ to high $\epsilon = 10^{-2} \text{ m}^2/\text{s}^3$ whereas the stratification levels are restricted to the low values: $N < 1.3 \cdot 10^{-2} \text{ s}^{-1}$. These values of N are typical for low altitude stratification. For convenience, the values of ϵ in Table 2.1 are also provided in a so-called “1/3” form that is often used in meteorology.

In order to reconstruct the physical atmospheric properties from the simulations, the parameters of the mean profiles have to be defined as well as the altitude at which the atmospheric properties are reconstructed, since only the simulated fluctuations are periodic, not the mean profiles. These parameters are only defined in the reconstruction post-processing so that they can be modified to generate different databases using the same simulation data. The only value that cannot be changed is the dimensionless Brunt-Väisälä frequency,

Table 2.1: Matrix of the LES simulations of turbulent and stratified atmospheres.

$\epsilon, \text{ m}^2/\text{s}^3$	$\epsilon^{1/3}, \text{ m}^{2/3}/\text{s}$	Brunt-Väisälä frequency, Hz			
		$2.55 \cdot 10^{-3}$	$8.91 \cdot 10^{-3}$	$1.27 \cdot 10^{-2}$	$1.8 \cdot 10^{-3}$
$1 \cdot 10^{-5}$	0.0215	Case t1	Case t4	Case t6	—
$5 \cdot 10^{-4}$	0.0793	Case t2	Case t5	—	—
$3.4 \cdot 10^{-3}$	0.15	—	—	—	Case t7
$1 \cdot 10^{-2}$	0.215	Case t3	—	—	—

which fully characterizes the turbulence:

$$\tilde{N} = Nt_0 = N \left(\frac{L^2}{\epsilon} \right)^{1/3}, \quad (2.3)$$

where $t_0 = \left(\frac{L^2}{\epsilon} \right)^{1/3}$ is the reference time. The Nt_0 values of the 7 LES cases range from 0.4 to 20.

The humidity is treated as a fully passive scalar (i.e. due its low concentration it has no dynamic effect on the fluid motion itself); therefore, it does not determine the flow regime and does not appear in the definition of the flow parameters. Mean values for Belgium, as reported by della Faille and van Swieten (2009), were used for the humidity profiles. An altitude of $z_0 = 500$ m was used in this work to be sufficiently far from the ground with respect to the domain length, $L = 200$ m (since the effect of the ground is not taken into account). The same potential temperature $\theta_0 = 283$ K at the altitude of the box was used in all cases. The variations of Brunt-Väisälä frequencies are directly related to the variations of the temperature gradient (see Eqs. (2.1) and (2.2)).

Example of reconstructed fields

Mean profiles for the specified altitude and location plus variations simulated by the LES for Case t3 are represented in an arbitrary vertical xz -plane in Fig. 2.1 and along an arbitrary vertical line in Fig. 2.2. The same fields are shown for Case t4 in Figs. 2.3 and 2.4. Case t3 has the largest turbulent kinetic energy (TKE) among all simulated LES cases ($\epsilon = 1 \cdot 10^{-2} \text{ m}^2/\text{s}^3$), which results in a large range of velocity variations. The velocities of Case t3 are about 10 times larger than those of Case t4 (see Figs. 2.1a, 2.1b and Figs. 2.3a, 2.3b). Due to the high TKE, the larger variations in the humidity field are also observed in Case t3 in comparison with Case t4 (note that the same vertical gradient of humidity is used for all cases in the reconstruction process). At the same time, the temperature fluctuations are larger in Case t4 than in Case t3 (see Figs. 2.1c, 2.3c and Figs. 2.2a, 2.4a) since the stratification of Case t4 is about 3.5 times larger than the ones of Case t3 (see Table 2.1). Yet, for all the quantities, the turbulent fluctuations are significantly smaller than the mean values as clearly shown by the vertical profiles shown in Figs. 2.2 and 2.4 where the fluctuations can hardly be seen, except in the relative humidity.

2.1.2 LES of WVs added to clear-air turbulent and stratified atmospheres

The aircraft WV system obtained after roll-up, as simulated in this work, consists of a pair of counter-rotating vortices with circulation Γ_0 and $-\Gamma_0$ and separated by a distance b_0 . A homogeneous isotropic turbulence is first generated and converged to create a realistic background stratified turbulence as explained in Section 2.1.1. Next, the vortex pair generated by a heavy aircraft is superimposed into the turbulence.

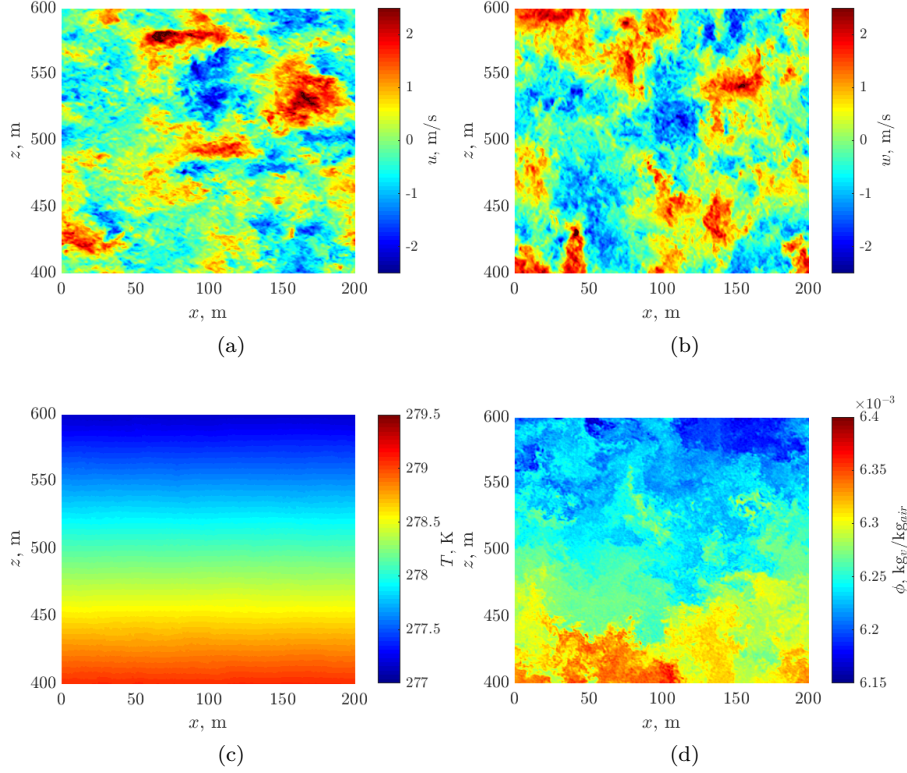


Figure 2.1: Visualizations of reconstructed fields in an arbitrary vertical plane for Case t3: (a) u -velocity (horizontal component along x -axis), (b) w -velocity (vertical component along z -axis), (c) static temperature, and (d) humidity.

The following sections describe the numerical set-up used for simulation of the WV in the turbulent stratified atmosphere and presents some examples. Most of the content presented in this section is extracted from the report of the SESAR 12.02.02 JU (SESAR 12.02.02, 2016) (P. Billuart contribution to (SESAR 12.02.02, 2016) and also to (Kovalev et al., 2018)).

Numerical set-up

In order to simulate the wake vortices in realistic conditions, background turbulent fields for each case have been first generated and converged statistically in the computational domain. The computational box size is set to $4b_0$, which corresponds to a few hundred meters for a heavy aircraft (e.g. the wingspan b_0 of the order of 50 m). Whatever the WVs simulated, the computational domain is too small to consider the Coriolis term (Rossby number $\ll 1$) in the momentum equation; it is thus neglected.

Once the stratified turbulence has been produced, the two counter-rotating vortices (aligned with the x -axis) are superimposed to the background turbulent

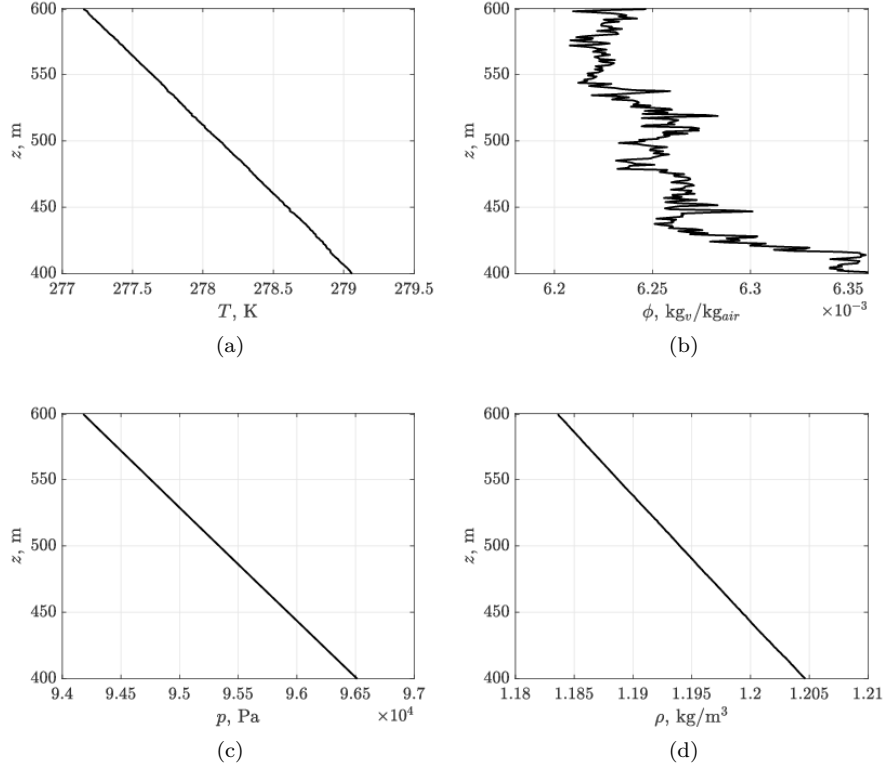


Figure 2.2: Profiles of the simulated fields along an arbitrary vertical line for Case t3: (a) temperature, (b) humidity, (c) pressure, and (d) density.

velocity field. A low order algebraic profile, also called Burnham-Hallock (B-H) in the aircraft community, is used as the initial profile of the vortices velocity. This model is given by a tangential velocity field:

$$v_\theta(r) = \frac{\Gamma_0}{2\pi r} \frac{r^2}{r^2 + r_c^2}, \quad (2.4)$$

where r is the distance from the WV core center, r_c is the effective core size (i.e. the radius of maximum induced velocity). The value of r_c is set to $r_c/b_0 = 0.05$, which is typical of what is used when modeling aircraft wake vortices with that model. The dependence of the tangential velocity calculated using Eq. (2.4) is shown in Fig. 2.5. The velocity reaches its maximum at $r = r_c = 2.5$ m. The Reynolds number based on the vortex circulation is very high: $Re_\Gamma = \Gamma_0/\nu \simeq 10^7$.

The LES box has a size of $4b_0$ with a uniform grid size $\Delta \approx 0.78$ m. The core size ratio $r_c/b_0 = 0.05$ chosen ensures that the most relevant dynamics of the vortices are well captured ($r_c/\Delta > 3$) (De Visscher, 2012). It is important to ensure that the WV is in equilibrium with the background turbulence. This occurs at the time of the maximum energy dissipation rate (see Fig. 2.6): $t^* =$

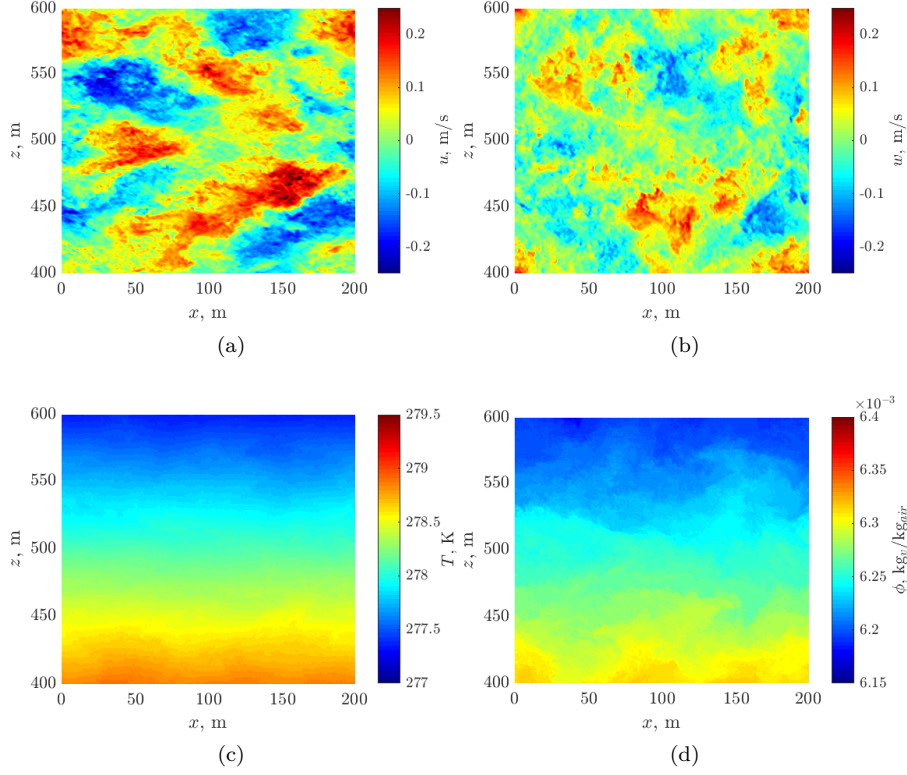


Figure 2.3: Visualizations of reconstructed fields in an arbitrary vertical plane for Case t4: (a) u -velocity, (b) w -velocity, (c) static temperature, and (d) humidity.

$t/t_0 = 2.5$ for Case 1 and 3, and $t^* = 4$ for Case 2. The simulations are then run for two more t_0 before evaluating the vortex parameters, to ensure statistical convergence.

This introduction of adding two vortices in a stratified turbulent box can be characterized by three dimensionless parameters :

$$\epsilon_f^* = \frac{\epsilon_f b_0}{V_0^3}, \quad N^* = N t_0, \quad L^* = \frac{L}{b_0}, \quad (2.5)$$

where $V_0 = \frac{\Gamma_0}{2\pi b_0}$ and $t_0 = \frac{b_0}{V_0}$, the characteristics velocity and time scales of vortices, and $L = 200$ m is the size of the domain. The first parameter characterizes the level of turbulence ϵ_f that corresponds to the amount of energy dissipated per unit time when the stratified turbulence is statistically converged. For atmosphere with a low stratification ($N^* < 1$), it is equal to the EDR which, for such high Reynolds simulations, corresponds entirely to the SGS dissipation; therefore, $\epsilon_f = \epsilon$. The second parameter is an image of the level of stratification, comparing the Brunt-Väisälä frequency to the descent

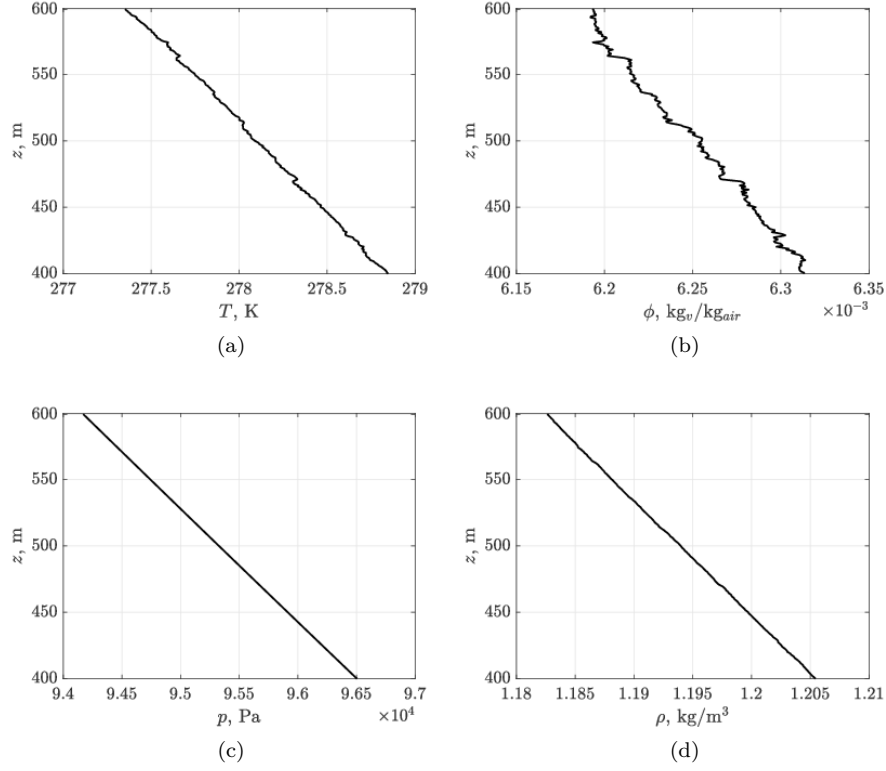


Figure 2.4: Profiles of the simulated fields along an arbitrary vertical line for Case t4: (a) temperature, (b) humidity, (c) pressure, and (d) density.

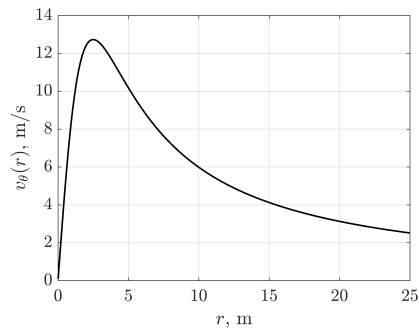


Figure 2.5: Initial profile of the vortices velocity.

time of the vortices. The third one determines the maximum wave-length mode that the code is able to capture. It therefore limits the range of instabilities that the code will be able to simulate. For all the present cases, the box size is $L^* = 4$ so that Crow instabilities (Crow, 1970) cannot develop. Indeed, for

Table 2.2: Dimensionless parameters of the three simulated cases used for the radar simulator. Cases t2, t4, and t5 are used as initial forcing turbulence for Cases 1, 2, and 3 respectively.

Cases	N^* [-]	ϵ^* [-]	L^* [-]
1	0.1	1.21×10^{-2}	4
2	0.35	2.42×10^{-4}	4
3	0.35	1.21×10^{-2}	4

Table 2.3: Dimensional parameters of the three simulated cases used for the radar simulator. Cases t2, t4, and t5 are used as initial forcing turbulence for Cases 1, 2, and 3 respectively.

Cases	$N[s^{-1}]$	$\epsilon[m^2/s^3]$
1	2.54×10^{-3}	5×10^{-4}
2	8.9×10^{-3}	1×10^{-5}
3	8.9×10^{-3}	5×10^{-4}

a $r_c/b_0 = 0.05$, only wavelengths larger than $5.6 b_0$ are unstable. Helical and elliptic instabilities are however captured.

The three cases used in the radar simulator for clear air conditions are summarized in Tables 2.2 and 2.3. Each simulation is fully characterized by the dimensionless parameters in Eq. (2.5). Each reconstruction is characterized by a set of dimensional/physical parameters. These parameters are also provided in Tables 2.2 and 2.3 for each case. As seen in the table, the parameters chosen to reconstruct the physical fields are the same for each simulation; this reconstruction was carried out using typical values for heavy aircraft ($\Gamma_0 = 400 m^2/s$, $b_0 = 50 m$), an initial height z_0 of 500 m and typical atmospheric conditions. With the reconstruction parameters chosen, the characteristic velocity V_0 and time scale t_0 of vortices are equal to 1.27 m/s and 39.3 s, respectively.

Examples of the WVs parameters calculated from the LES simulations

When the vortices are introduced in the simulation, the vortex pair starts to descend at a velocity $V_0 = \Gamma_0/(2\pi b_0)$. The turbulence surrounding the vortices is affected by two main phenomena. First, the background turbulence is stretched by the vortex system, generating intense transverse vortical structures. Second, due to the stratification, baroclinic vortical structures are generated at the border of the Rankine oval. This extra source term of vorticity arises from the difference in density between the warmer fluid carried by the vortices inside the Rankine oval and the cooler fluid outside. Subsequently, small-scales instabilities develop inside those structures, leading to a fully turbulent vortex system. The turbulence and the associated eddy dissipation rate increase until reaching a statistical equilibrium with the vortex system (see Fig. 2.6).

From that moment on, the flow becomes representative of a real vortex system as obtained after roll-up and can be used for radar cross-section simulation. This is highlighted in red on the graphs. After reaching this equilibrium, at

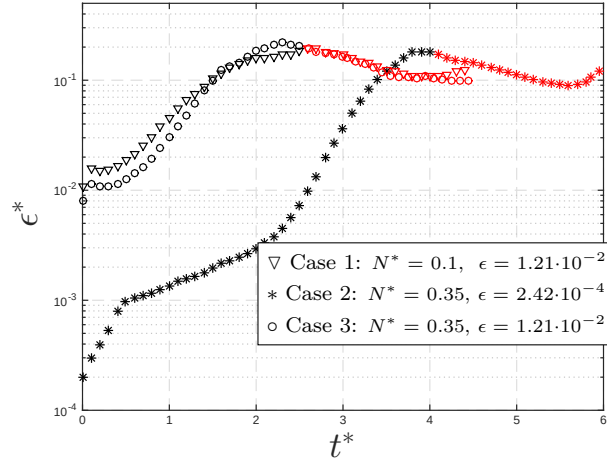


Figure 2.6: Time evolution of the energy dissipation rate for the three investigated cases. Red markers highlight the intervals when the simulations represent a real vortex system as obtained after a roll-up. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016) and also to (Kovalev et al., 2018).

about $2.5t_0$ for Cases 1, 3 and at $4t_0$ for Case 2, the circulation of both vortices decreases smoothly (Fig. 2.9), with the vortex system sustaining its own turbulence. The descent velocity as well as the maximum tangential velocity (see Fig. 2.7) of the vortices hence also decreases.

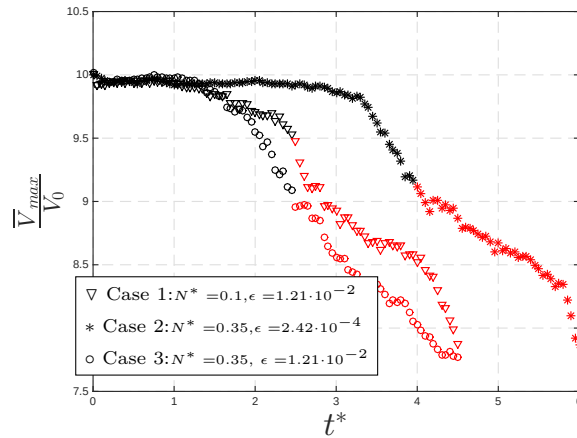


Figure 2.7: Time evolution of the maximum tangential velocity of the WV normalized by the initial descent velocity. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016) and also to (Kovalev et al., 2018).

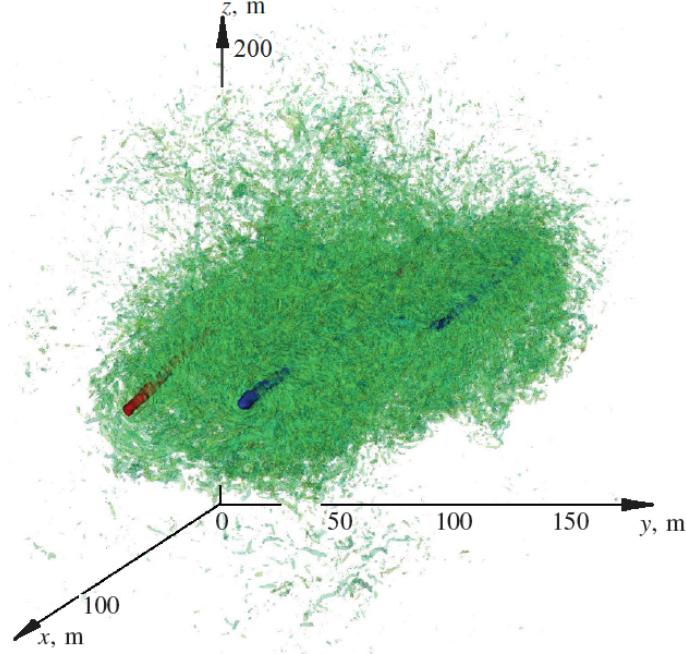


Figure 2.8: Iso-surface of $|\omega|b_0^2/\Gamma_0 = 40$ (opacity 100%) and $|\omega|b_0^2/\Gamma_0 = 4$ (opacity 20%) colored by the axial vorticity ω_x for Case 3 ($N^* = 0.35$, $\epsilon^* = 1.21 \times 10^{-2}$) at time $t^* = 4$. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016) and also to (Kovalev et al., 2018).

In the LES, the total circulation of each vortex, $\bar{\Gamma}_{tot}(t)$, is defined as the average, over all x_i LES slices (here 256 slices), of the maximum of the circulation profile $\Gamma(x_i, r, t)$ measured in that slice (with r measured from the local WV center); this maximum is $\Gamma_{tot}(x_i, t)$, and it corresponds to the total circulation of the vortex as measured in that slice. It is positive for the left wake vortex (a clockwise rotating vortex) and negative for the right wake vortex (a counter-clockwise rotation vortex).

Fig. 2.8 shows a 3D visualization of Case 3 at time $t^* = 4.0$. Two iso-surfaces of the normalized vorticity $|\omega|b_0^2/\Gamma_0$ are used to illustrate the wake vortex core tubes ($|\omega|b_0^2/\Gamma_0 = 40$) and the turbulent flow around the developed wake vortices ($|\omega|b_0^2/\Gamma_0 = 4$). As shown in Fig. 2.9 (which depicts the time evolution of the dimensionless vortex circulation $\bar{\Gamma}_{tot}^* = \bar{\Gamma}_{tot}(t)/\Gamma_0$), at $t^* = 4$, the total circulation has decayed by approximately 30 %.

The intensification of the turbulence surrounding the vortices is stronger compared to Case 2 with a weaker turbulent background. The stratification also creates baroclinic vortical structures. Those structures quickly interact with the turbulent vortices, enhancing the decay of the vortices compared to the less stratified case at same level of turbulence, i.e. Case 1 (see Fig. 2.9). This baroclinic vorticity is also of opposite sign relative to the primary vortices and thus tends to slow down the vortices as seen in Fig. 2.10 that depicts the

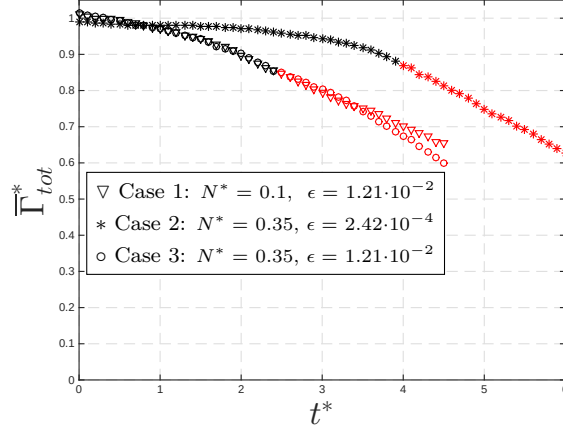


Figure 2.9: Time evolution of $\bar{\Gamma}_{tot}^*$ for the three investigated cases. The reported values are averaged between the left and right wake vortices. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016) and also to (Kovalev et al., 2018).

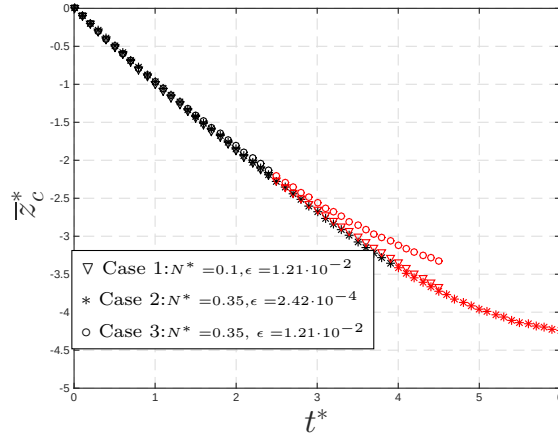


Figure 2.10: Time evolution of $\bar{z}_c^*$ for the three investigated cases. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016) and also to (Kovalev et al., 2018).

dimensionless vertical position of the vortex tubes: $\bar{z}_c^* = z_c(t)/z_0$.

For non-stratified or weakly stratified atmosphere (Case 1), the circulation is proportional to the product of the descent velocity (V_c) and the distance between the WV cores (b_c). Fig. 2.11 illustrates this ratio: for Case 1, the circulation calculated from the WV velocity field is equal to that calculated

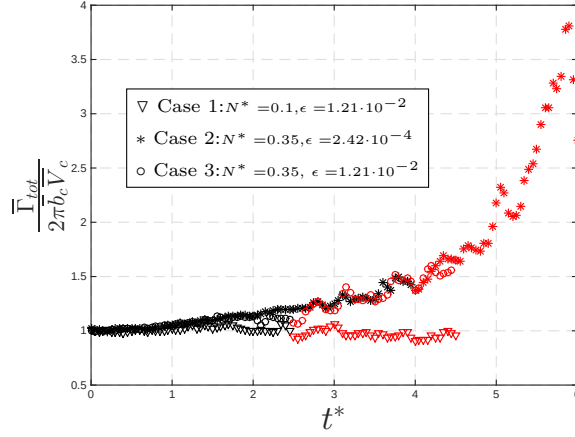


Figure 2.11: Time evolution of the ratio between $\bar{\Gamma}_{tot}/(2\pi\bar{b}_c)$ and the WV descent velocity $\bar{V}_c$. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016) and also to (Kovalev et al., 2018).

from V_c and b_c , while for the more stratified Cases 2 and 3, the ratio deviates significantly from unity.

Fig. 2.12 shows the humidity field of Case 2 at time $t^* = 5$ in a vertical cross-section of the computational box combined with streamlines of a WV system mean flow represented in a reference frame that sinks at the descent velocity V_0 . To obtain the streamlines, w and v components of the velocity field were averaged along the x -axis, and the WVs descent velocity (V_0) was added to the w component. The black crosses represent the center of the vortex tubes (averaged along the x -axis). Those were located by identifying the position of the minimum pressure. The labels on the vertical axis correspond to the altitude of the vortices below their release altitude z_0 , normalized by b_0 . During its descent, the vortex system carries the lower humidity, which is situated at a high altitude, downward.

2.1.3 LES of atmospheric turbulence with rain

The physics of rain is particularly complex since it involves phase changes of the water content of the atmosphere: evaporation and condensation, interactions between the falling droplets, and also interactions between the droplets and the air. The full two-phase flow simulation of rain is beyond the scope of the present work and a simplified model was adopted. The methodology is similar to the one used by Liu et al. (2013) to simulate the radar measurement of wake vortices in the presence of rain. Most of the content presented in this section is extracted from the report of the UFO project (M. Duponcheel contribution to (Vanhoenacker-Janvier et al., 2014)).

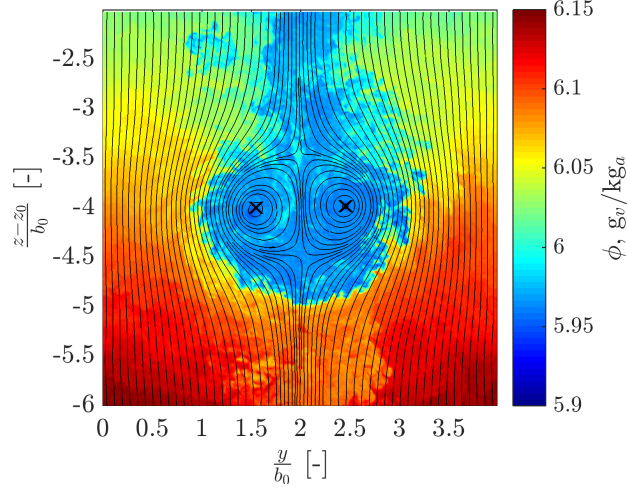


Figure 2.12: Humidity field (specific humidity, g_v/kg_a) combined with streamlines of a WV system mean flow represented in a reference frame that sinks at V_0 for Case 2 ($N^* = 0.35$, $\epsilon^* = 2.42 \times 10^{-4}$) at time $t^* = 5$.

Basic assumptions

In order to simplify the problem, the following assumptions were made (Vanhoenacker-Janvier et al., 2014):

- The possible phase changes are neglected: the atmosphere is supposed to be saturated in water vapor so that the amount of liquid water contained in the droplets does not change.
- The interactions between droplets and the breakdown of the droplets are neglected; the mass of each droplet does not change.
- The shape of the raindrop is being deformed by a drag force (Thurai et al., 2007); the deformation depends on the size of the raindrop (see Fig. 2.13). Nevertheless, in the present work, to simplify the simulation, it is assumed that the raindrops are spherical for all raindrops diameters considered.
- It is supposed that the motion of the droplets does not influence the motion of the air whereas the droplets are slowed down by the drag force due to their relative motion in the air. This assumption is justified later in this section given the low volume fraction occupied by rain droplets.

These hypotheses allow to model the raindrops as point particles with a fixed mass, that are subjected to gravity and drag forces. This also creates a one-way coupling between the simulation of the air turbulence, which is not influenced by the raindrops, and the motion of the particles which undergo a drag force depending on the air velocity.

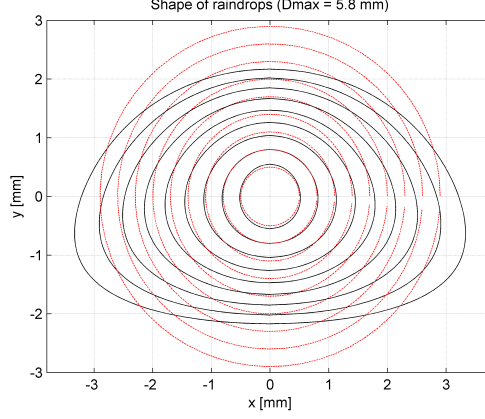


Figure 2.13: Shapes of the falling raindrops.

Equation of raindrop's motion

The motion of droplets as point-like particles is governed by Newton's second law:

$$m_p \frac{d\mathbf{v}_p}{dt} = \mathbf{F}_g + \mathbf{F}_D , \quad (2.6)$$

where m_p and $\mathbf{v}_p$ are the mass and the velocity of the droplet respectively, $\mathbf{F}_g = -m_p g \mathbf{e}_z$ is the gravity force, and $\mathbf{F}_D$ is the drag force. Assuming the spherical shape of the droplets regardless of their diameter, the drag force reads:

$$\mathbf{F}_D = \frac{1}{2} \rho_{air} \frac{\pi D^2}{4} C_D |\Delta \mathbf{v}|^2 \mathbf{e}_{\Delta \mathbf{v}} , \quad (2.7)$$

where ρ_{air} is the density of the air, $\Delta \mathbf{v} = \mathbf{v}_{air} - \mathbf{v}_p$ is the velocity of the droplet with respect to the velocity of the airflow, $\mathbf{v}_{air}$, and $\mathbf{e}_{\Delta \mathbf{v}} = \Delta \mathbf{v} / |\Delta \mathbf{v}|$ is a unit vector pointing in the direction of $\Delta \mathbf{v}$. The drag coefficient C_D is generally expressed in term of the terminal velocity V_T . The latter is the raindrop's falling velocity at which the drag force balances the gravity force in still air ($\mathbf{F}_g = \mathbf{F}_D$):

$$\rho_{water} \frac{\pi D^3}{6} g = \frac{1}{2} \rho_{air} \frac{\pi D^2}{4} C_D V_T^2 . \quad (2.8)$$

Introducing in Eq. (2.7) the expression of C_D extracted from Eq. (2.8), the motion equation:

$$\frac{d\mathbf{v}_p}{dt} = \frac{g}{V_T^2} |\Delta \mathbf{v}| \Delta \mathbf{v} - g \mathbf{e}_z . \quad (2.9)$$

The motion of a particle is hence obtained by integrating in time the system of two ordinary differential equations:

$$\begin{cases} \frac{d\mathbf{x}_p}{dt} = \mathbf{v}_p \\ \frac{d\mathbf{v}_p}{dt} = \frac{g}{V_T^2} |\Delta \mathbf{v}| \Delta \mathbf{v} - g \mathbf{e}_z , \end{cases} \quad (2.10)$$

$$(2.11)$$

where $\mathbf{x}_p$ is the droplet's position.

Terminal velocity of the raindrops

Various models for the raindrop's terminal velocity as a function of the droplet diameter are proposed in the literature (e.g. in (Lovejoy and Schertzer, 2008)). This work uses the expression initially proposed by Atlas and Ulbrich (1974) and further modified by Lovejoy and Schertzer (2008) to take into account the variation of the air density:

$$V_T(D) = [\alpha_1 - \alpha_2 e^{-\alpha_3 D}] \left(\frac{\rho_0}{\rho} \right)^{0.4}, \quad (2.12)$$

with $\alpha_1 = 9.65$ m/s, $\alpha_2 = 10.3$ m/s and $\alpha_3 = 0.6$ [mm⁻¹]. The factor term $(\rho_0/\rho)^{0.4}$ is a correction proposed by Lovejoy and Schertzer (2008) to consider the variation of air density in the drag term of Eq. (2.6). Nevertheless, this relation yields negative terminal velocities for particles with a diameter smaller than 0.108 mm. In order to ensure positive terminal velocities, Duponcheel (Vanhoenacker-Janvier et al., 2014) proposed to modify the curve at small diameters by replacing it by a quadratic extension:

$$V_T(D) = \begin{cases} \beta D^2 \left(\frac{\rho_0}{\rho} \right)^{0.4}, & \text{if } D \leq D_0 \\ [\alpha_1 - \alpha_2 e^{-\alpha_3 D}] \left(\frac{\rho_0}{\rho} \right)^{0.4}, & \text{if } D > D_0 \end{cases} \quad (2.13)$$

where β and D_0 are determined so that the extension is tangent to the initial curve. The corrected curve is depicted in Fig. 2.14 for two altitudes: a reference altitude $\rho = \rho_0$ and for the altitude simulated in this work $\rho = \rho(z=500 \text{ m}) = 1.194$ kg/m³.

The difference between the two curves is relatively small; therefore, the raindrops' terminal velocities, determined by the balance between the gravity force and the drag force, do not change over considered interval of heights.

Simulation of falling raindrops in a turbulent atmosphere

The simulation of the fall of the raindrops is performed in parallel with the simulation of the turbulence which provides the velocity field of the air required to compute the relative velocity of the droplets. Because of the one-way coupling, the simulation of the turbulence is unaffected by the presence of the raindrops. The periodicity used for the simulation of the turbulence is also enforced to the raindrops. This means that the number of raindrops in the simulation remains constant in time and that no new particles need to be injected at every time-step of the LES simulation.

The transport of the point particles was implemented in the spectral solver developed by Duponcheel in the framework of UFO project (2016). The simulation of rain in a turbulent atmosphere uses the simulation cases of turbulent atmosphere presented in Section 2.1.1. The positions of the raindrops are

initialized with a uniform distribution; the number of raindrops and their diameters are taken from the drop size's distribution which is a function of the rain rate.

Raindrop size distribution

The raindrops' concentration for each drop size is defined by the distribution $N_R(D)$ [$\text{m}^{-3}\text{mm}^{-1}$] which represents the number of raindrops of size between D and $D+dD$ per unit volume. Several models exist to describe the drop size's distribution $N_R(D)$ as a function of the rain rate R . As referred by Liu et al. (2013), the most common distribution for the European temperate climate is the Gamma distribution:

$$N_R(D) = N_{R0} D^\mu e^{-\Lambda D} [\text{mm}^{-1}\text{m}^{-3}], \quad (2.14)$$

with the calibration of Jiang et al. (1997) for European climate : $\mu = 2$, $N_{R0} = 64500 R^{-0.5} [\text{mm}^{-3}\text{m}^{-3}]$, $\Lambda = 7.09 R^{-0.27} [\text{mm}^{-1}]$. When the calibrated Gamma distribution is integrated, one obtains the following expression for the total density of raindrops :

$$N_T = \int_0^\infty N_R(D) dD = N_{R0} \int_0^\infty D^2 e^{-\Lambda D} dD = 2 \frac{N_{R0}}{\Lambda^3} [\text{m}^{-3}]. \quad (2.15)$$

Fig. 2.15 presents the selected raindrop size distribution for various rain rates. The corresponding total raindrops density as a function of the rain rate is depicted in Fig. 2.16.

Using the Gamma distribution from Eq. (2.14), the total volume fraction of raindrops is:

$$\Phi_p = 10^{-9} \int_0^\infty \frac{\pi D^3}{6} N_R(D) dD = 20 \cdot 10^{-9} \pi \frac{N_{R0}}{\Lambda^6} [-]. \quad (2.16)$$

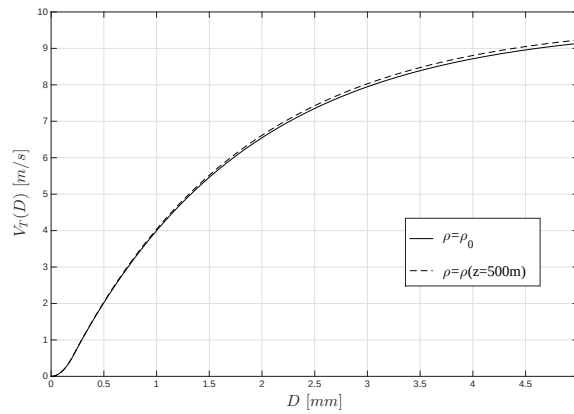


Figure 2.14: Raindrop terminal velocity at two altitudes : $\rho = \rho_0$ and $\rho = \rho(z = 500 \text{ m})$. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016).

The graph of this quantity as a function of the rain rate is depicted in Fig. 2.16. For $R = 5$ mm/h, the volume fraction is $\Phi_p = 2 \cdot 10^{-7}$. According to Elghobashi (1994), at such a low volume fraction, the particles do not influence the flow and this justifies the one-way coupling.

The total number of raindrops that are present in the LES volume is given by $N_T L^3 = 4.8 \cdot 10^9$ for a rain rate of 5 mm/h with $N_T \approx 600$. Tracking of such a large number of raindrops is computationally expensive. A reduced number of particles is thus simulated. A reasonable amount of particles that can be added to the turbulence simulation is equal to the number of grid points of the LES simulation, $N_G = 256^3$; therefore, the number of the simulated raindrops is reduced by a factor of $4.8 \cdot 10^9 / 256^3 \approx 286$. This results in the reduced total raindrop density $N'_T = 600 / 286 \approx 2.1 \text{ m}^{-3}$. In this model, each simulated raindrop is not affected by the number of raindrops presented nor by their distribution since the raindrops do not interact with each other and do not influence the background turbulence. This gives a freedom on the selection of the raindrop's set that is simulated for the use in the radar simulator.

Simulated cases

Two examples of simulations were carried out using different raindrop size's distributions. The first one (Case t1-r1) reproduces the Gamma distribution (Eq. (2.14)) for a rain rate $R = 5$ mm/h but with a reduced total number of raindrops N_G . The second one (Case t1-r2) uses a uniform distribution of sizes between 1 and 2 mm. Such uniform simulations can be used to estimate statistics of the raindrop parameters as a function of the raindrop size. For instance, since the number of the simulated raindrops is reduced, the number of the large raindrops (if simulated with a Gamma distribution) will be too small (see Fig. 2.15); therefore, the set of the raindrops simulated with the Gamma distribution will not be a good representation of the raindrops' velocities dis-

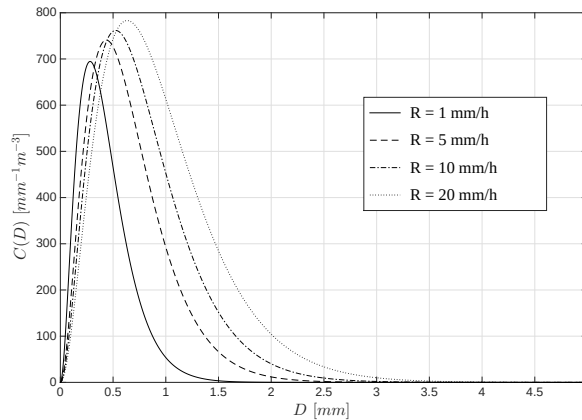


Figure 2.15: Gamma distribution for several rain rates R . This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016).

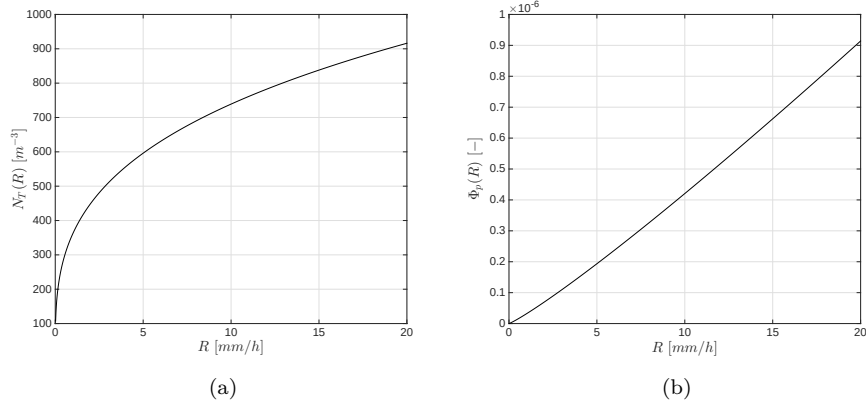


Figure 2.16: Total raindrop density (a) and volume fraction (b) for Gamma distribution and various rain rates. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016).

tribution. The uniform distribution allows simulation of a sufficient number of raindrops of all considered diameters to represent reliably the statistics of the raindrops' parameters. This uniform distribution can be then recombined as a function of the rain rate and the actual raindrop distribution (Vanhoeacker-Janvier et al., 2014). Both simulations use the atmospheric properties of Case t1 (see Section 2.1.1). Fig. 2.17 shows the raindrop distributions for both simulated cases. The values of the distributions are much smaller than that of

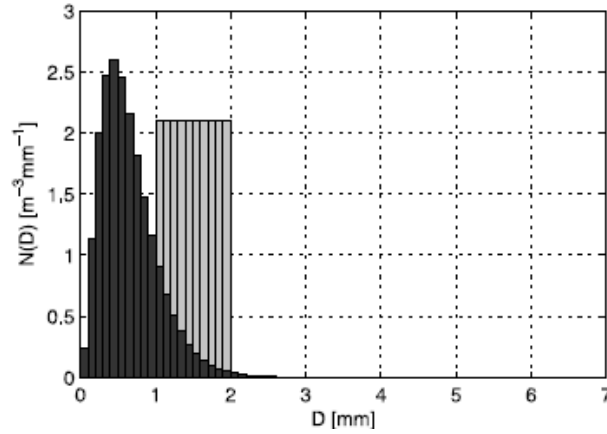


Figure 2.17: Raindrop size distribution used in Rain 1 (black) and Rain 2 (gray) simulations. This figure is from M. Duponcheel et al. contribution to (Vanhoeacker-Janvier et al., 2014).

the actual distributions (see Fig. 2.15) because of the reduced total raindrop density $N'_T = 2.1 \text{ m}^{-3}$ compared to the actual density $N_T \approx 600 \text{ m}^{-3}$.

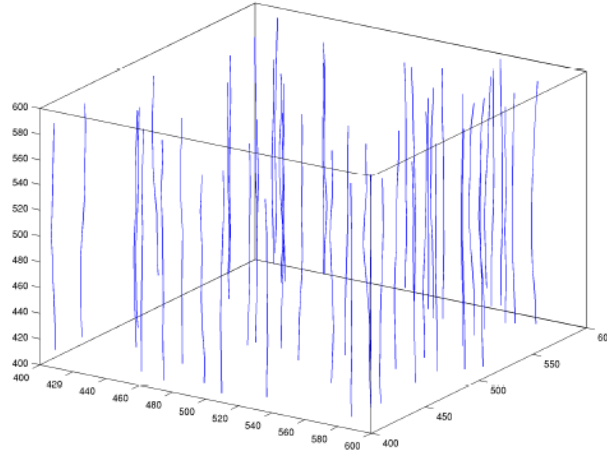


Figure 2.18: Visualizations of a few raindrop trajectories as obtained in Case t1-r1 simulation. This figure is from M. Duponcheel et al. contribution to (Vanhoeacker-Janvier et al., 2014).

Examples of raindrop trajectories are visualized in Fig. 2.18 for Case t1-r1. Since the turbulence is weak (the rms velocity of the turbulent flow is about 0.07 m/s), the trajectories of the droplets only slightly deviate from the vertical. Fig. 2.18 may give the impression that the raindrops enter the top of the domain and leave at its bottom. This is due to the fact that only one crossing of the domain is shown in the figure whereas, because of the periodicity, each particle can cross the domain several times during the whole simulation period.

The results of the LES with rain presented in this section are not further used in the present work because no combined radar and rain rate measurements are available. Nevertheless, these LES simulations are ready to be used in the future; for example, to simulate the extraction of the EDR from the radar signal backscattered on the rain droplets.

2.1.4 WVs added to turbulent atmosphere in the presence of rain

For the simulations of the WVs in the presence of rain, the set of raindrops of a given diameter is not handled as discrete elements as presented in Section 2.1.3 but as raindrops concentration in $\text{mm}^{-1}\text{m}^{-3}$. Unlike in (Liu et al., 2013), the phase-averaged velocity is defined everywhere, even in low concentration areas where the probability to find a droplet is very low.

In rainy weather, the atmosphere is in general neutral, i.e. not stratified. The simulations with rain were thus performed assuming no stratification ($N^* = 0$). The code solving the equations of raindrops is a hybrid solver. The raindrop velocity field is solved in spectral space, as for the air flow velocity field, while the concentration field is solved using a Lagrangian method (Winck-

elmans (2004), Cottet and Koumoutsakos (2000), Koumoutsakos (2005)).

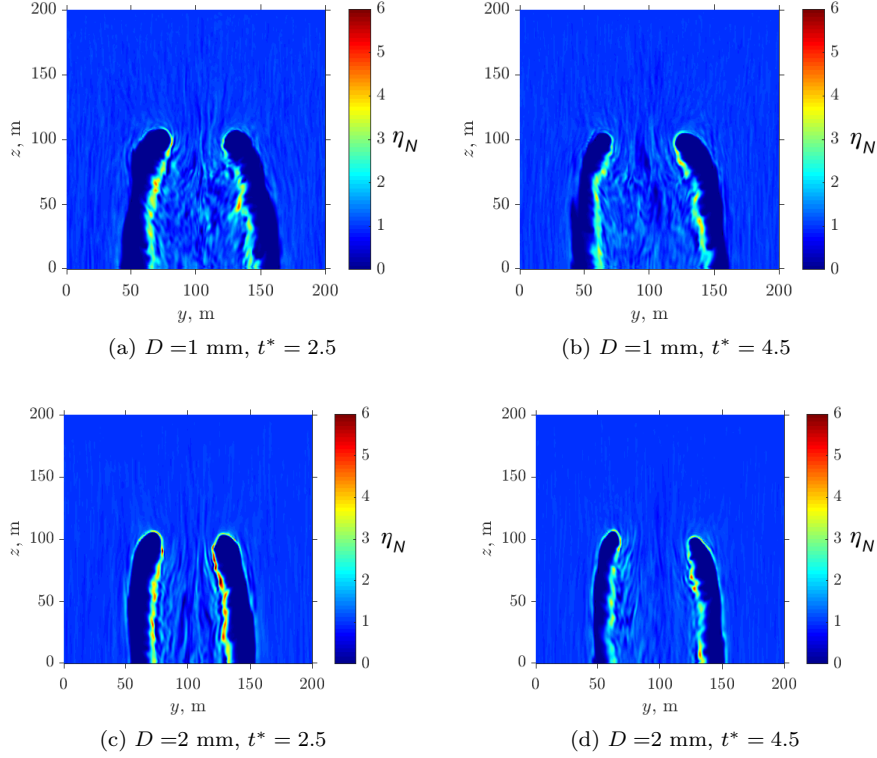


Figure 2.19: Distribution of the relative concentration of the raindrops for cases 1-1r and 1-2r in a vertical cross-section of the LES (η_N) box at the beginning and at the end of the physically meaningful intervals for the case considered. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016).

The only cases for which the terminal velocity of the raindrops is significantly higher than the vortex descent velocity are simulated. A concentration then develop below both vortices, while the concentration above them will remain relatively equal to the concentration in the still air. “Contrary to the UFO project (2016), a periodic domain in the vertical direction can no longer be assumed for particles. Otherwise, the wake concentration would go back to the vortices through the upper surface, which is not physical”(SESAR 12.02.02, 2016).

Due to the budget constraints of the SESAR project, only a few simulations considering a set of four raindrop diameters: 1 mm, 2 mm, 3 mm, and 4 mm were simulated in a vortex system. Table 2.4 provides the characteristics of the LES for which the presence of the raindrops was simulated. The wake vortices descend at the constant speed, $V_0 = \Gamma_0/(2\pi b_0) \approx 1.27$ m/s; therefore, their vertical coordinate z_c linearly decrease with time. Since the atmosphere is not stratified in this case, the ratio $\Gamma_{tot}/(2\pi b_c V_c) \approx 1$ over all intervals of

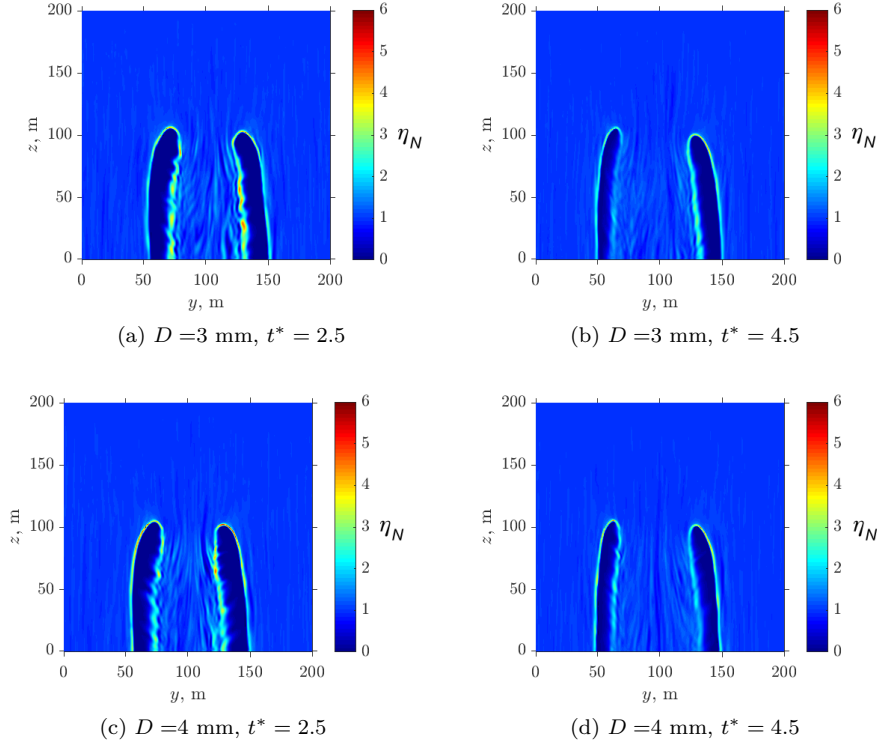


Figure 2.20: Distribution of the relative concentration of the raindrops for cases 1-1r and 1-2r in a vertical cross-section of the LES (η_N) box at the beginning and at the end of the physically meaningful intervals for the case considered. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016).

the simulation.

The relative concentrations of raindrops $\eta_N(D_i, x, z)$ provided by LES at the beginning and at the end of the physically meaningful intervals (see Fig. 2.6) are shown in Figs. 2.19 – 2.20 for diameters (D_i) used in the simulation. The raindrops are forced out of the WV cores, forming areas with almost zero concentration of the raindrops. The concentration of bigger raindrops is less

Table 2.4: Parameters of the LES cases with WV in the presence of rain.

LES case	raindrop diameter, mm	$\epsilon, m^2/s^3$	N, Hz
Case 1-1r	1	$1 \cdot 10^{-5}$	0 (non-stratified)
Case 1-2r	2	$1 \cdot 10^{-5}$	0 (non-stratified)
Case 1-3r	3	$1 \cdot 10^{-5}$	0 (non-stratified)
Case 1-4r	4	$1 \cdot 10^{-5}$	0 (non-stratified)

affected by the turbulence than the concentration of the smaller ones.

2.2 Electrical characterization of the medium and scattering of electromagnetic waves

The radar is a remote sensor that emits EM waves and receives them backscattered on some object. The received signal is then analyzed in order to extract the information about the scattering object. The following sections introduce the mechanisms of EM waves scattering and provide electrical characterization of the medium represented by the fields of physical parameters simulated by LES. Sections 2.2.1 to 2.2.3 analyze the electrical properties of the clear air. Section 2.2.4 considers the characteristics of the rain that influence the backscattering of the EM field.

2.2.1 Refractive index of the atmosphere

The solution of Maxwell's equations shows that in free space, the transverse electromagnetic waves emitted by the radar propagate along straight lines at a constant velocity equal to the speed of light since the dielectric permittivity ε_0 and magnetic permeability μ_0 are constant and related to the speed of propagation as $c = (\varepsilon_0\mu_0)^{-1/2}$ (Adler et al., 1968). The dielectric permittivity of air, ε , changes with altitude due to the gravity and stratification of the atmosphere. For the troposphere, and for the atmosphere in general, $\varepsilon > \varepsilon_0$. This results in the propagation of EM waves at a speed $v < c$ along curved lines. Taking into account that the permeability of air is $\mu = \mu_0$, the refractive index of air is defined as:

$$n = \frac{c}{v} = \sqrt{\frac{\varepsilon}{\varepsilon_0}}. \quad (2.17)$$

In this sense, the refractive index is a measure of the bending of the EM wave from the straight line when it propagates through some medium with non-constant n .

The troposphere can be considered as a stratified mixture of dry air and water vapor which density and polarization define the dielectric permittivity. Molecules of water vapor are polar; therefore they produce their own electric field that extends far beyond the atomic spacing. Molecules of dry air do not possess their own permanent dipole moments; nevertheless, they become polarized when an external electric field (radar signal) is applied. In the absence of the external force, these dipoles are randomly oriented due to the thermal agitation. The external electrical field aligns randomly oriented dipole moments, and the oscillation of the external EM field causes the oscillation of the induced local dipole moments; this acts as a source of EM radiation thus causing scattering of the original EM field.

Calculation of the refractive index

The refractive index is proportional to the density of the molecules and their polarization. It can be shown that refractive index can be expressed via the pres-

sure, temperature, and water vapor content of air (see, for example, Tatarskii (1971)). Since for air, values of n are close to unity, it is more convenient to use a so-called radio refractivity N :

$$N = (n - 1) \cdot 10^6. \quad (2.18)$$

In this work, the formula proposed in (ITU-R, 2017) is used to calculate the radio refractive index:

$$n = 1 + N \cdot 10^{-6} = 1 + \left(\left(\frac{77.6}{T} \right) \cdot \left(P + 4810 \frac{q}{T} \right) \right) \cdot 10^{-6}, \quad (2.19)$$

where P and q are the atmospheric pressure and the water vapor pressure in hPa, and T is the absolute temperature in K. The pressure, temperature and humidity fields are provided by the LES: values of each field are available at each of the 3D LES grid nodes with a grid resolution of 0.78 m and temporal resolution of 1 s.

Figs. 2.21 – 2.22 show examples of the distributions of the refractive index (n) and the absolute values of its gradient ($|\nabla n|$) in a vertical cross-section of the LES. All coordinates are given relative to the LES cube. Fig. 2.21 presents Case t2 (a,c) and Case 1(b,d) for turbulent stratified atmosphere with and without wake vortices respectively. The intensity of the turbulence for Case t2, $\epsilon = 5 \cdot 10^{-4} \text{ m}^2/\text{s}^3$, corresponds to the intensity of the background forcing turbulence of Case 1.

Fig. 2.22 presents Case t4 (a,c) and Case 2 (b,d) for turbulent stratified atmosphere without and with wake vortices respectively. The intensity of the turbulence for Case t4, $\epsilon = 1 \cdot 10^{-5} \text{ m}^2/\text{s}^3$, corresponds to the intensity of the background forcing turbulence of Case 2. Spatial variations of the refractive index for Case t4 caused by turbulence ($\epsilon = 1 \cdot 10^{-5} \text{ m}^2/\text{s}^3$) are significantly smaller than the gradient of n due to stratification (see Fig. 2.22.a), while in Case t2 more intense turbulence ($\epsilon = 5 \cdot 10^{-4} \text{ m}^2/\text{s}^3$) significantly disturbs the gradient of the refractive index (Fig. 2.21a). In both Case 1 (see Fig. 2.21b) and Case 2 (see Fig. 2.22b), the WV disturbs the stratification and creates a strong gradient of n along the WV oval (see Fig. 2.21d and Fig. 2.22d for Case 1 and Case 2 respectively). The values of n in Case 2 (Fig. 2.22b) are larger than in Case 1 (Fig. 2.21b). This is due to the fact that the wake vortices in Case 2, at $t^* = 4$, has descended to a lower altitude than in Case 1 (see Fig. 2.10).

2.2.2 Spectrum of the refractive index in the turbulent medium

At large scales, the spatial distribution of the refractive index values is determined by the stratification of the clear atmosphere or the perturbation of the atmosphere in the presence of the WVs. At smaller scales, the values of the refractive index are influenced by small-scale turbulent motion of air. From the theory of turbulence, the kinetic energy of the turbulent field is distributed between eddies having different scales: the energy transfers through the spectrum, from large-scale eddies to small-scale ones. A characteristic scale L_0 ,

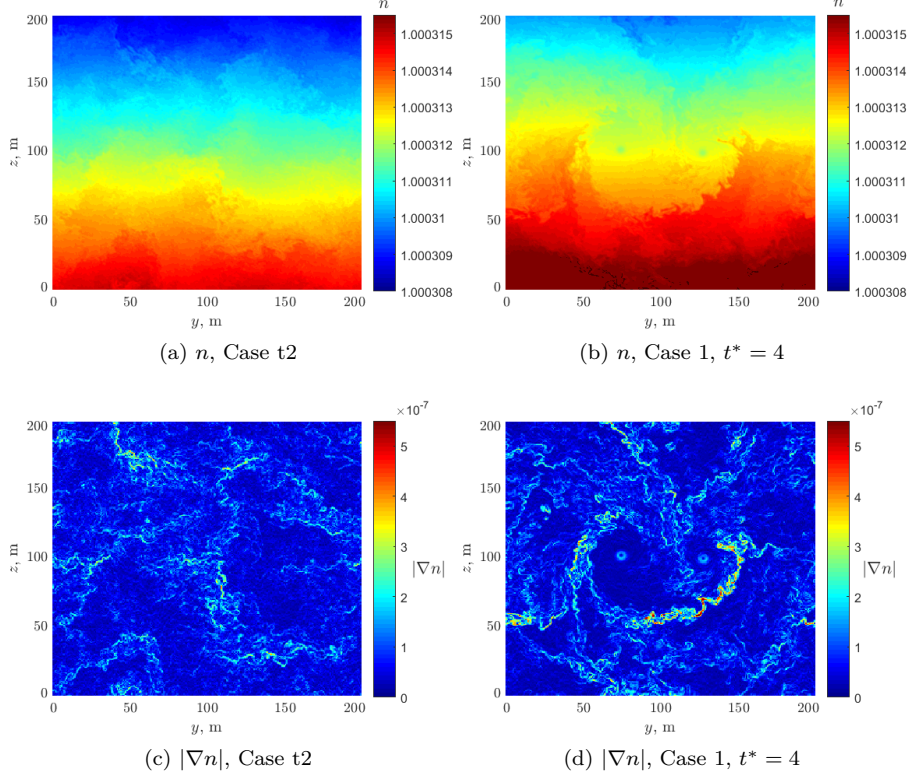


Figure 2.21: Distribution of (a,b) the refractive index and (c,d) the absolute values of its gradient in a cross-section of the LES. (a,c) Case t2 – stratified turbulent atmosphere in the absence of the WV; (b,d) Case 1 – stratified turbulent atmosphere in the presence of the WV at $t^* = 2.5$. Case t2 ($\epsilon = 5 \cdot 10^{-4} \text{ m}^2/\text{s}^3$, $N = 2.54 \cdot 10^{-3} \text{ s}^{-1}$) is used as initial forcing turbulence for Case 1.

which is called the outer scale of turbulence, equals to the largest eddies in the turbulent flow. The scale of the smallest eddies in the turbulent flow is characterized by a Kolmogorov length scale η_0 :

$$\eta_0 = \left(\frac{\nu^3}{\epsilon} \right)^{1/4}, \quad (2.20)$$

where ν is the kinematic viscosity of air (a value of $1.45 \cdot 10^{-5} \text{ m}^2/\text{s}$ is used in this work). Scales between L_0 and η_0 constitute the inertial subrange of the turbulence spectrum. A finite inertial subrange exists if the Reynolds number is higher than some critical value (typically $10^6 - 10^7$) (Tatarskii, 1971). The shape of the turbulence energy spectrum in the inertial subrange is defined by the so-called $k^{-5/3}$ law (Kolmogorov, 1941):

$$E(k) = K_0 \epsilon^{2/3} k^{-5/3}, \quad (2.21)$$

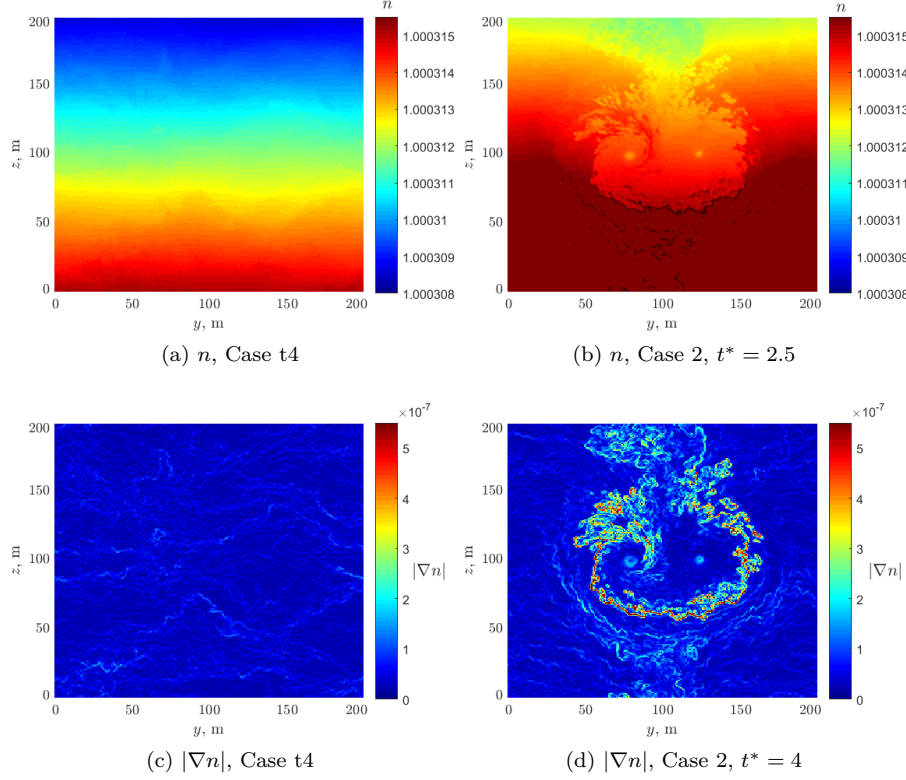


Figure 2.22: Distribution of (a,b) the refractive index and (c,d) the absolute values of its gradient in a cross-section of the LES. (a,c) Case t4 and (b,d) Case 2 represent stratified turbulent atmosphere with and without wake vortices respectively. Case t4 ($\epsilon = 1 \cdot 10^{-5} \text{ m}^2/\text{s}^3$, $N = 8.9 \cdot 10^{-3} \text{ s}^{-1}$) is used as initial forcing turbulence for Case 2.

where K_0 is the Kolmogorov coefficient ranging between 1.4 and 1.8, which slightly depends on Reynolds number. In this range, a fragmentation of the larger eddies occurs: due to the large Reynolds numbers, the eddies at the scale L_0 are unstable and therefore split into smaller eddies that are characterized with smaller Reynolds number. Each fragmentation reduces the Reynolds number of the resulting eddies and the fragmentation process continues until some critical Reynolds number is reached at the Kolmogorov length scale. At this scale, the eddies are stable: they do not split to smaller ones and their energy is transformed into heat due to the viscous force. Energy and viscous ranges are called stable range of the turbulence while the inertial subrange is called unstable reflecting the splitting of the bigger eddies (inhomogeneities) into the smaller ones. Fig. 2.23 shows the energy spectra for the LES cases simulated in the absence of wake vortices: in the inertial subrange that is captured by the LES, the spectra shapes follow the $k^{-5/3}$ law; the scales of turbulence lie

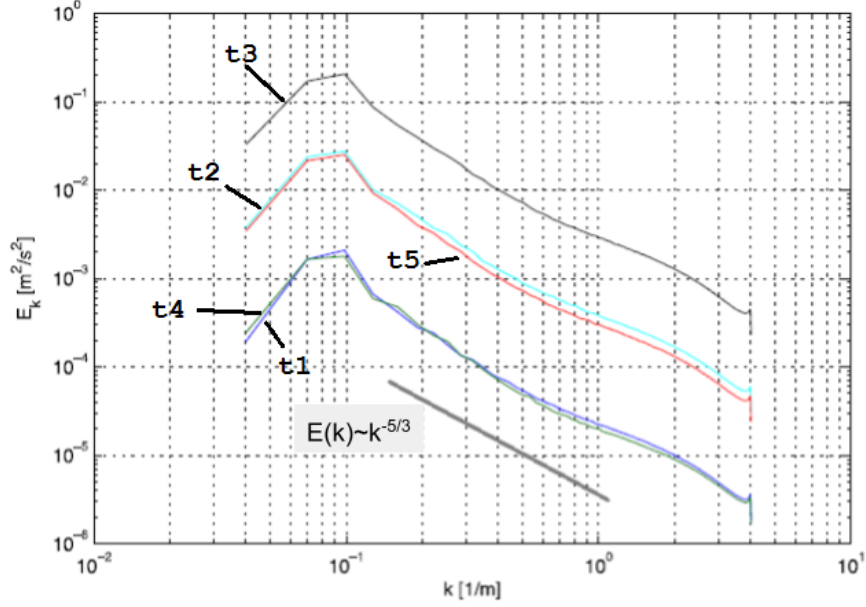


Figure 2.23: Turbulence energy spectra for cases without wake vortices: Case t1 (blue), Case t2 (cyan), Case t3 (black), Case t4 (green) and Case t5 (red). This figure is taken from (Vanhoeacker-Janvier et al., 2014).

within range from approximately 100 m to 1.57 m. The absolute values of the spectra is proportional to the values of energy dissipation rate for each case. In the inertial subrange, the spectral density of the refractive index is defined as follows (Tatarskii, 1971):

$$\Phi_n(k) = 0.033 \cdot C_n^2 k^{-11/3}, \quad (2.22)$$

where $k = 2\pi/l$ is the wave number corresponding to the scale l and C_n^2 is the refractive index structure parameter representing the intensity of the turbulence. C_n^2 directly influences the intensity of the scattered signal as it is explained in Chapter 4. The following section develops the calculation of C_n^2 with LES data.

Refractive index structure parameter C_n^2

Meteorological fields and turbulence are not stationary processes; statistical properties of these fields depend on the altitude above the ground. To analyze the spatial structure of these fields, a method of structure functions introduced by Kolmogorov (1941) is used. Instead of considering the values of the random process itself, the structure function considers a difference between the random process values at two closely located points of observation. The second order structure function for n is defined as follows:

$$D_n(\mathbf{r}_2, \mathbf{r}_1) = \left\langle [n(\mathbf{r}_2) - n(\mathbf{r}_1)]^2 \right\rangle, \quad (2.23)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the positions at which n is analyzed; $\langle \rangle_r$ serves for spatial averaging. Structure functions are used in place of the correlation functions for stationary processes. A generalization of stationarity for random fields is homogeneity. A random process is homogeneous if its mean value is constant and its correlation function does not change with simultaneous displacement of the pair of points of observation in the same direction and on the same value. A homogeneous field is isotropic if its correlation function depends only on the distance between points of observation and not on the direction. Despite the fact that, at large scales, the field of n is neither homogeneous nor isotropic, at small scales (in the inertial subrange), it can be considered as locally homogeneous and isotropic. For locally isotropic and homogeneous turbulence, the structure function depends only on the distance δ between the given points (that lie within a local subspace which is locally homogeneous and isotropic):

$$D_n(\mathbf{r}_2, \mathbf{r}_1) = D_n(|\mathbf{r}_2 - \mathbf{r}_1|) = D_n(\mathbf{r} + \delta, \mathbf{r}) = C_n^2 \delta^{2/3}. \quad (2.24)$$

For all spatial inhomogeneities with scales belonging to the inertial subrange of the turbulence spectrum, the refractive index structure parameter C_n^2 can be calculated using the definition of the structure function for turbulent field (see Eq. (2.23)):

$$C_n^2(\mathbf{r}) = \frac{\langle [n(\mathbf{r} + \delta) - n(\mathbf{r})]^2 \rangle_r}{|\delta|^{2/3}}, \quad (2.25)$$

where $\mathbf{r}$ is the position vector and δ is the vector defining the spatial separation between the points at which the spatial averaging is calculated. The dependence of the C_n^2 on the position ($\mathbf{r}$) reflects the inhomogeneity and anisotropy of n at large scales (see Figs. 2.21 and 2.22), while locally at small scales (distances of the order of a few LES grid cells) the field of n is considered homogeneous and isotropic.

In the present work, the C_n^2 parameter calculated at each LES grid node takes the 6 closest nodes (at a distance Δ equal to the size of the LES grid cell) and averages the square difference between values of n at + and - the LES resolution, taken two-by-two along the 3 axes (see Fig. 2.24: the highlighted pairs of the LES nodes used to estimate C_n^2 at a given node are connected with solid lines). So, in this case $|\delta| = 2\Delta$. Mean values of C_n^2 for LES cases without WV are varying in the range of $10^{-15} - 10^{-14} \text{ m}^{-2/3}$ that corresponds to “intermediate turbulence” (Davis, 1966).

Examples of the distribution of C_n^2 in a vertical cross-section of the LES box are provided in Fig. 2.25 for Cases 1, 2 and 3. The WVs are clearly visible, with high values of C_n^2 at the limit of the core and at the limit of the oval surrounding the vortices. High values of C_n^2 are caused not only by strong turbulence within the WV oval but also by high values of the humidity and temperature gradients (Doviak and Zrnić, 1984). Case 3 has the same stratification level as Case 2 ($N = 8.9 \cdot 10^{-3} \text{ s}^{-1}$) but the EDR of the forcing background turbulence is significantly larger ($\epsilon = 5 \cdot 10^{-4} \text{ m}^2/\text{s}^3$ for Case 3 and $\epsilon = 1 \cdot 10^{-5} \text{ m}^2/\text{s}^3$ for Case 2). The interaction with more intense turbulence in

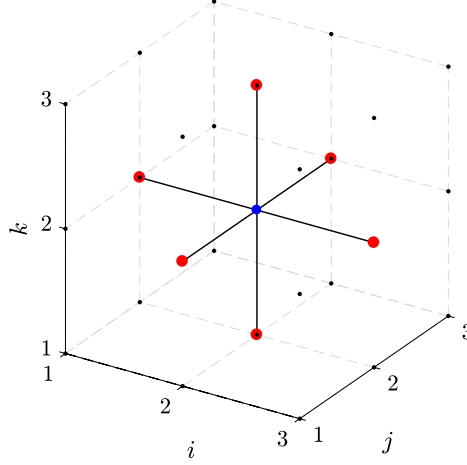


Figure 2.24: Illustration of C_n^2 calculation. Red markers highlights the pairs of the LES nodes used to estimate C_n^2 at the central LES node with coordinates (2,2,2).

Case 3 deforms the WV oval and spread the large values of C_n^2 around the WV system (see Figs. 2.25c, 2.25e and Figs. 2.25d, 2.25f). From the comparison between two cases with the same level of energy dissipation rate (Cases 1 and 3, $\epsilon = 5 \cdot 10^{-4} \text{ m}^2/\text{s}^3$), but different stratification level (stratification of Case 3 is about 3.5 higher than those of Case 1) it is seen that higher stratification prevents the significant deformation of the WV oval and also produces more uniform spread of high C_n^2 values around the WV.

2.2.3 Refractive index spectrum at the transition between the inertial subrange and the viscous dissipative range

The Bragg scattering is caused by variations of the clear air refractive index having a spatial scale of half the radar wavelength, which is about 1.5 cm for the X-band radar. To ensure backscattering on the refractive index inhomogeneities (caused by turbulent eddies), the following condition should be met

$$\frac{\lambda}{2} \geq l_0, \quad (2.26)$$

where λ is the radar wavelength and l_0 is the inner scale, characterizing the transition scale between the inertial subrange and the viscous dissipative range of the turbulence spectrum. For higher radar frequencies (smaller wavelengths), there are no significant eddies with dimension of the order of the Bragg wavelength and the backscattered signal reduces quickly. The inner scale l_0 is typically considered as a threshold for the radar detectability of the turbulence in clear air; therefore, for the detection of the turbulence in clear air, the smallest

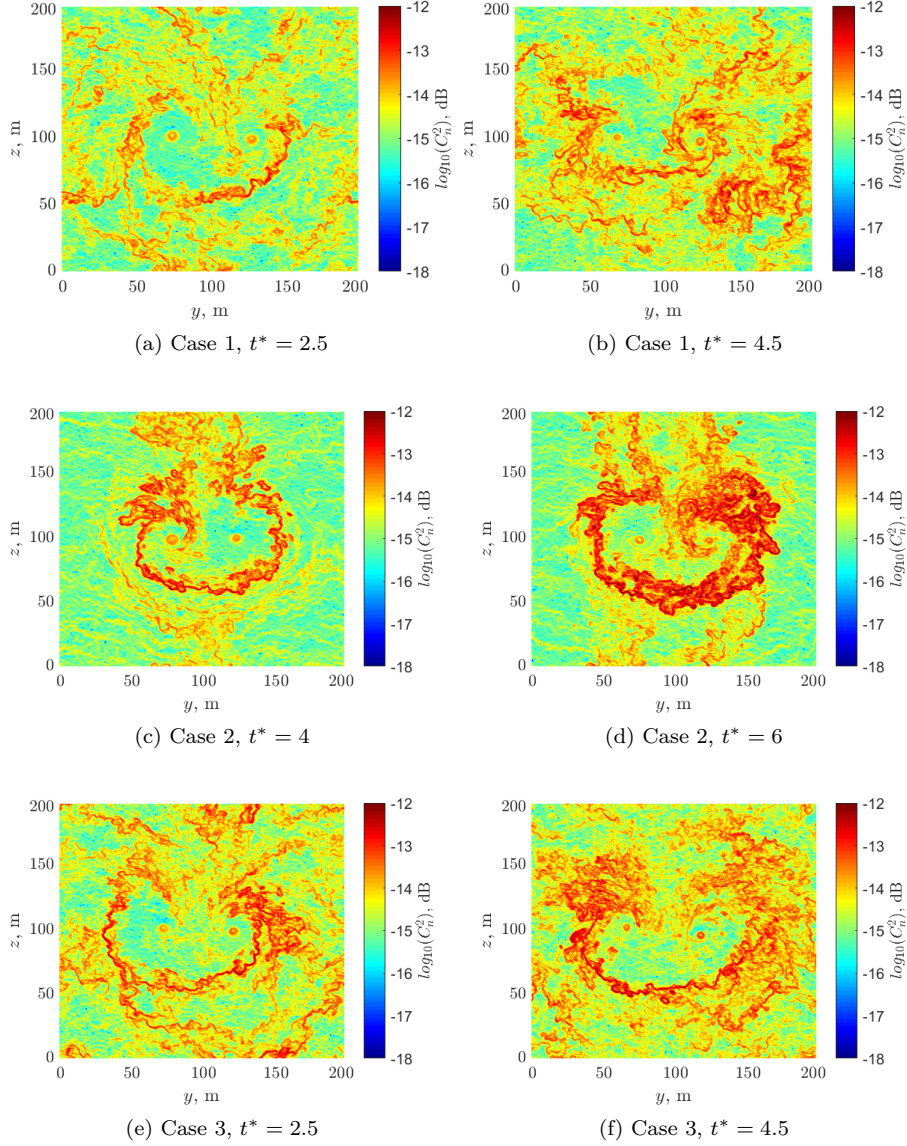


Figure 2.25: $\log_{10}C_n^2$ distribution in a vertical cross-section of the LES box at the beginning and at the end of the physically meaningful intervals for considered cases.

radar wavelength should be greater than $2l_0$. The value of l_0 was estimated by Hill and Clifford (1978) for the temperature spectrum of the turbulence as

$$l_0 \simeq 7.41\eta_0 = 7.41 \left(\frac{\nu^3}{\epsilon} \right)^{1/4}. \quad (2.27)$$

The inner scales of the turbulence simulated in the LES are listed in Tables 2.5 and 2.6 for LES cases with and without WV respectively. As follows

Table 2.5: Estimation of the inner scale, l_0 , for the LES cases without WV. Cases t2, t4, and t5 are used as initial forcing turbulence for Cases 1, 2, and 3 respectively.

LES case	$\epsilon^{1/3}$, $\text{m}^{2/3}\text{s}^{-1}$	l_0 , cm
Cases t1, t4, t6	0.022	3.1
Cases t2, t5	0.08	1.2
Case t7	0.15	0.7
Case t3	0.215	0.6

Table 2.6: Estimation of the inner scale, l_0 , for the LES cases with wake vortices.

LES case	$\epsilon^{1/3}$, $\text{m}^{2/3}\text{s}^{-1}$	l_0 , cm
Cases 1 (=t2), 3 (=t5) (initial turbulence)	0.08	1.2
Case 2 (=t4) (initial turbulence)	0.022	3.1
Cases 1, 3 (developed turb. WV)	0.2–0.165	0.6–0.7
Case 2 (developed turb. WV)	0.195–0.158	0.6–0.7

from the evaluation of l_0 for the X-band radar, the Bragg wavelength belongs to the transition region between inertial and dissipative ranges. Therefore, to estimate the level of the refractive index spectrum at the Bragg scale, it is needed to take into account the attenuation of the spectrum level in the dissipation range. For that purpose, Eq. (2.22) is modified by introducing an attenuation factor in the refractive index spectrum. In this work two forms of the refractive index spectrum at the transition between inertial and dissipative ranges are considered as described hereafter.

Tatarskii's spectral model

To model the refractive index spectrum in proximity to the inner scale, Tatarskii (1971) proposes the following expression:

$$\Phi_n(k_B) = 0.033C_n^2k_B^{-11/3}e^{-k_B^2/k_m^2}, \quad (2.28)$$

where $k_B = 2\pi/l_B$ is the wavenumber that correspond to the Bragg length scale l_B and k_m is the wavenumber that corresponds to the smallest inhomogeneities in the velocity field of the turbulence:

$$k_mk_0 \simeq 5.92. \quad (2.29)$$

Combining Eqs. (2.27) and (2.29), the spectrum of the refractive index at the transition zone between inertial subrange and viscous dissipative range reads:

$$\Phi_n(k_B) = 0.033C_n^2k_B^{-11/3}e^{-1.57\nu^{3/2}\epsilon^{1/2}k_B^2}. \quad (2.30)$$

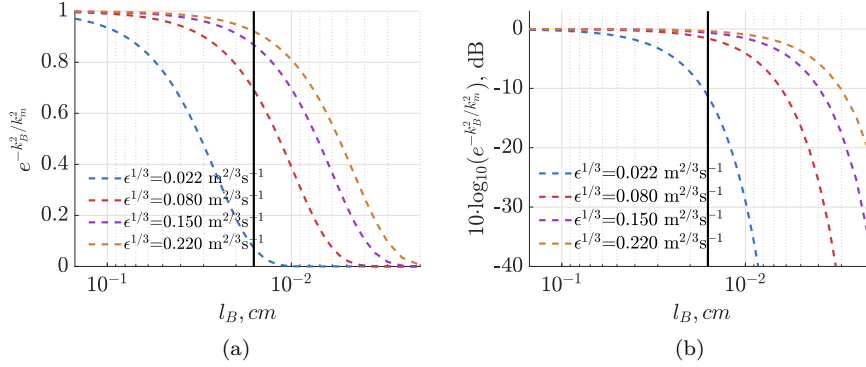


Figure 2.26: Tatarskii's cut-off as an attenuation factor for refractive index spectrum for different values of ϵ : (a) linear scale, (b) logarithmic scale. The vertical black line represents the Bragg wavelength for the radar wavelength of 3.2 cm.

A dependence of the attenuation exponent on the inhomogeneities length scale for a set of energy dissipation rates used in this work is shown in Fig. 2.26. The vertical black line represents the value of the Bragg wavelength (1.6 cm) for the radar wavelength of 3.2 cm. So, for cases with high ϵ (0.15 and $0.22 \text{ m}^2/\text{s}^{-1}$), the attenuation of the refractive index spectrum is negligible at $l = 1.6 \text{ cm}$ (less than -1 dB). For lower values of ϵ ($0.08 \text{ m}^2/\text{s}^{-1}$), the attenuation is about -2 dB. The turbulence with these length scales may be assumed to lie within the inertial subrange. For the LES case with the lowest energy dissipation rate ($0.022 \text{ m}^2/\text{s}^{-1}$), a high attenuation indicates that the length scales of the order of 1.6 cm enter into the dissipation range (attenuation of Φ_n is about -10 dB).

The inner scale however depends on ϵ and Fig. 2.27 shows that the eddy dissipation rate varies in the developed turbulent wake vortex system. The WV system has local ϵ values in the WV oval that are much larger than that of the initial forcing turbulence indicated in Table 2.6. Even in the background part, outside of the WV oval and downstream, the energy dissipation rate is significantly higher than initially, as indicated hereafter. An example of the $\epsilon^{1/3}$ values in a cross-section of the LES box is provided for Cases 1, t4, and t7 in Fig. 2.27. For an initial turbulence forcing with $\epsilon_f^{1/3}$ of $0.08 \text{ m}^2/\text{s}^{-1}$, typical (median) values of 0.2 and as high as $1.7 \text{ m}^2/\text{s}^{-1}$ ($l_0 = 0.58$ and 0.12 cm respectively) occur in the WV oval (nevertheless, the upper limit for the scale of the Fig. 2.27 is set to $0.7 \text{ m}^2/\text{s}^{-1}$ in order to improve the readability of the figure); and typical values of $0.1 - 0.14 \text{ m}^2/\text{s}^{-1}$ (l_0 of $1 - 0.76 \text{ cm}$) occur outside and downstream of the WV oval. As seen from Tables 2.5 and 2.6, the $\epsilon^{1/3}$ caused by WV varies in the range $0.158 - 0.2 \text{ m}^2/\text{s}^{-1}$ which corresponds to the energy dissipation rate of the highly turbulent atmosphere. In Fig. 2.28, an example of the attenuation exponent is plotted for the snapshots of Cases 1, t4, and t7 (Fig. 2.27). This figure clearly shows that, in the WV oval (see Figs. 2.28c and 2.28d), the attenuation exponent is close to unity, even if the

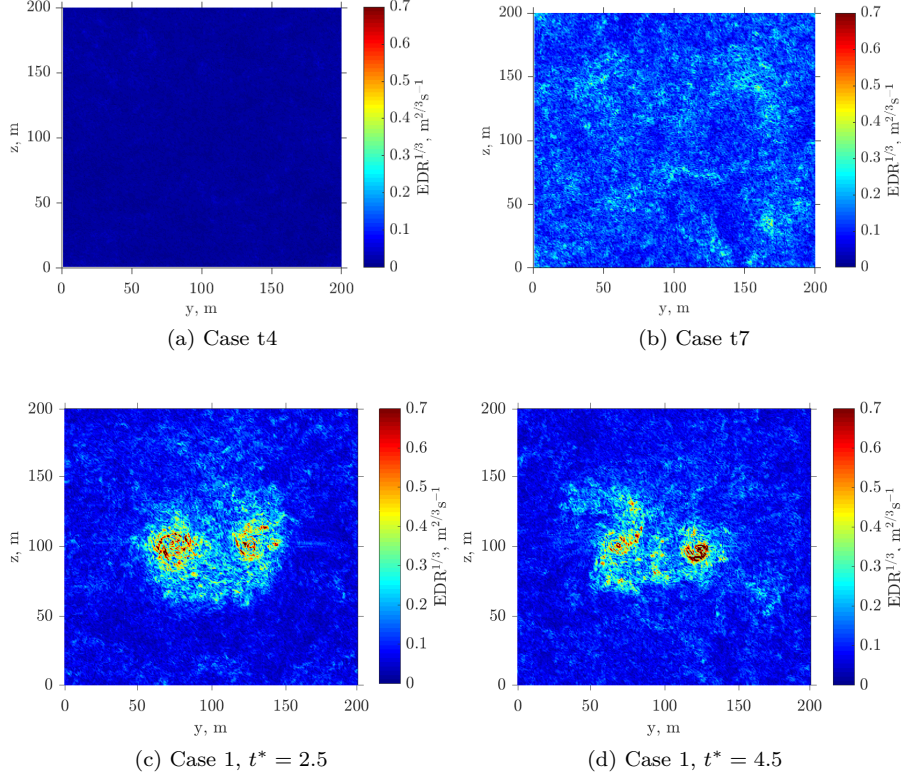


Figure 2.27: Distribution of $\epsilon^{1/3}$ in a cross-section of the LES for (a) Case t4 and (b) Case t7 and for Case 1 at (c) $t^* = 2.5$, (d) $t^* = 4.5$.

exponent decreases in the background. So, it can be concluded that the small-scale turbulence in LES cells around WV oval lie within the inertial subrange.

The WVs creates a strong gradient of the humidity and temperature fields at the border of the WV oval (see Fig. 2.12). Along with the increased intensity of the turbulence (see Table 2.6), it results in a significantly higher level of C_n^2 within the WV system (up to $10^{-13} - 10^{-12} \text{ m}^{-2/3}$) comparing to the clear air turbulence ($10^{-15} - 10^{-14} \text{ m}^{-2/3}$).

Hill's spectral model

Eq. (2.28) was proposed by Tatarskii (1971) in absence of experimental data on the level of the refractive index spectrum in proximity to the inner scale. Later, relying on experimental research, Hill (1978) discovered that in the transition between the inertial subrange and the viscous dissipative range, the spectrum has a so-called bump. The numerical solution of the Hill (1978) model is approximated by Frehlich (1992) as:

$$\Phi_n(k_B) = 0.033 C_n^2 k_B^{-11/3} g(\eta_0 k_B), \quad (2.31)$$

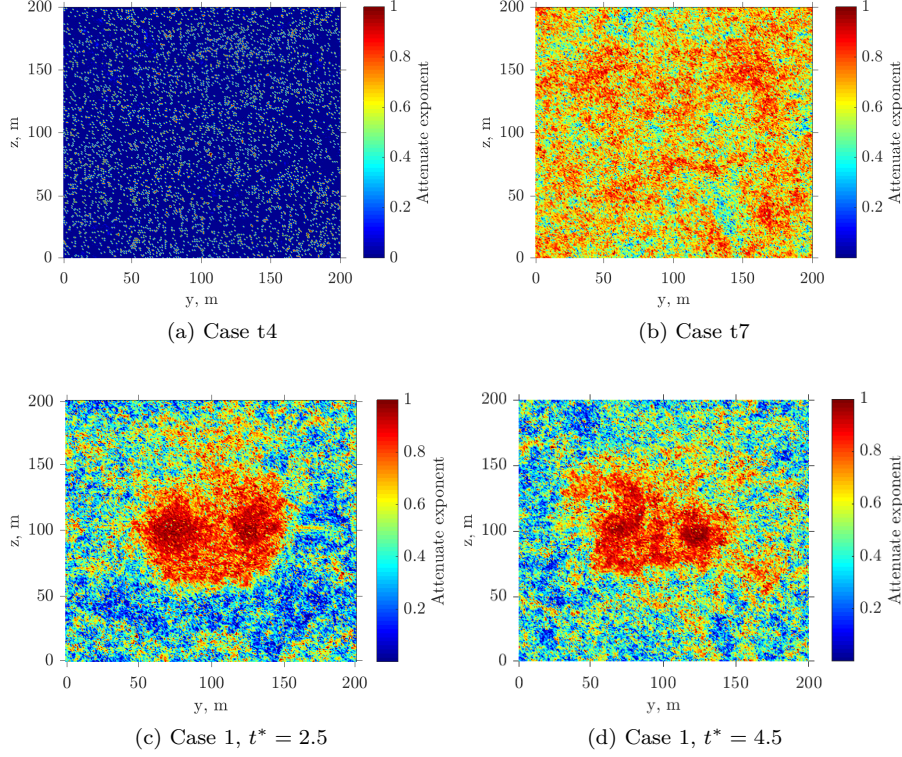


Figure 2.28: Distribution of the attenuation exponent in a cross-section of the LES for (c) Cases t4 and (d) t7 and for Case 1 at (a) $t^* = 2.5$, (b) $t^* = 4.5$.

where the attenuation factor $g(\eta_0 k_B)$ is:

$$g(k) = \left[1 + \sum_{n=1}^K a_n k^n \right] \exp(-\delta k) , \quad (2.32)$$

where, for $K = 4$, the values of the constants providing the best fit for the Hill spectral model are: $\delta = 1.1090$, $a_1 = 0.70937$, $a_2 = 2.8235$, $a_3 = -0.28086$, and $a_4 = 0.08277$. Fig. 2.29 provides the dependence of $h(\cdot)$ on the Bragg wave number, k_B , scaled by the Kolmogorov length scale η_0 .

As seen from Fig. 2.29, for the values of ϵ typical for the WV oval, the Bragg wavelength of the X-band radar is right at the bump value of the attenuation factor. The difference between Tatarskii's cut-off and Hill's bump is relatively small for low levels of ϵ , while for high levels of ϵ the difference is significant. Further in this work, the spectrum of the refractive index, n , is computed with Eqs. (2.30) and (2.31) in order to take into consideration the proximity of the Bragg scale to the transition between the dissipative and the inertial range of the turbulence spectrum.

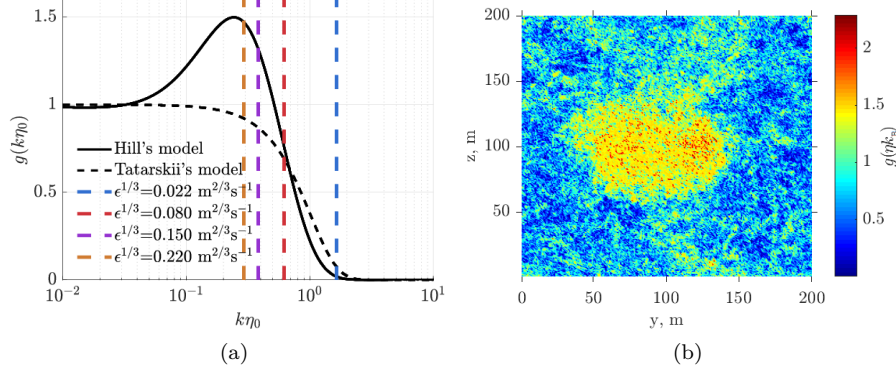


Figure 2.29: Hill's bump and Tatarskii cut-off as an attenuation factors for refractive index spectrum for different values of ϵ : (a) plotted against the wavenumber scaled with the Kolmogorov length scale, (b) distribution of the attenuation exponent in a cross-section of the LES for Case 1 at $t^* = 3.75$. The vertical lines represent the Bragg wave numbers for the radar wavelength of 3.2 cm for different levels of the energy dissipation rate.

2.2.4 Characteristics of rain

For the frequencies higher than 1 GHz, the scattering of EM waves on the rain droplets dominates over the scattering of the EM on the atmospheric turbulence. Therefore, in rainy conditions only raindrops are involved in the simulation of the backscattered signal as it is described in the following chapter. This subsection briefly introduces the mechanism of EM scattering on the rain droplets and recalls the parameters that LES provides as input data for the radar simulator.

Raindrops as dielectric particles

To enable the LES to simulate the WV in a turbulent atmosphere in the presence of rain, it is assumed that the rain droplets have a spherical shape (see Section 2.1.3). This is a fair approximation for relatively small raindrops with radius of up to 2 mm (see Fig. 2.13). Larger raindrops are significantly deformed by a drag force. Nevertheless, polarization effects that occur when the EM wave is scattered on the non-spherical particle are not considered in this work; therefore, it is assumed in the radar simulator, as well as in the LES, that all simulated raindrops have a spherical shape. The mechanism of the scattering of EM waves on the spherical particles depends on the ratio between the raindrop size (its radius, r) and the wavelength of the EM wave (λ). In this work, the radar wavelength for the considered X-band is of the order of a few centimeters (typically 3 cm), while the size of the raindrops is of the order of a few millimeters; therefore:

$$kr \ll 1, \quad (2.33)$$

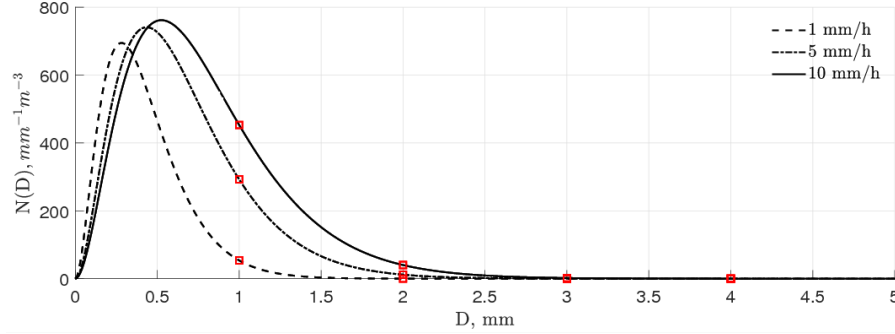


Figure 2.30: Raindrop size distributions for a few selected rain rates. Square markers label the values of raindrop diameters simulated in the LES.

where $k = 2\pi/\lambda$ is the wave number and r is the radius of a spherical raindrop. The product kr is often called the electrical size of the particle since $kr = 2\pi r/\lambda$ is the circumference of the sphere expressed in wavelengths of the EM wave. For $kr \ll 1$, the regime of the scattering is called a Rayleigh scattering. More details on electrical properties of the raindrops will be given in the following chapter.

Output parameters of the LES in the presence of rain

As output data, the Large Eddy Simulations with wake vortices provides the fields of the raindrops concentrations (for diameters of 1, 2, 3, and 4 mm) and the distribution of the local velocities at each node of the LES grid. Fig. 2.30 shows examples of raindrop size's distributions calculated with Eq. (2.14) for various rain rates of 1, 5, and 10 mm/h; the drop sizes provided by LES are highlighted with the red markers. Since the strength of the backscattered EM waves is proportional to the size of the raindrops, it is crucial to have large raindrops represented in the simulation rather than the smaller ones (for further details, see section 3.2.3). Further chapter develops the methods used for the simulation of the backscattered signal using the LES data.

2.3 Conclusions

The LES provides 3D fields of physical parameters which characterize the scattering medium. In the first step, clear air turbulent stratified atmosphere is simulated. Then, the wake vortices are added to the LES to obtain a realistic simulation of the WV interacting with the surrounding turbulent atmosphere. With reasonable assumptions, the presence of the rain is then simulated for the cases with and without the WV. Since simulated variables are decomposed into their mean profiles and deviations from those means, it is possible to reconstruct the atmospheric fields at different altitudes or with different profiles without a need to rerun the LES.

LES cases of the output data set are characterized by two main parameters, namely the energy dissipation rate, which characterizes the intensity of the turbulence, and the Brunt-Väisälä frequency, which is the measure of the atmosphere stratification. To simulate the scattering of EM waves on the WV, it is needed to transfer from the atmospheric parameters: pressure, temperature and specific humidity, to the electrical characteristics of the medium, introduced by the refractive index.

The LES does not resolve the small scales of the turbulent flow. Therefore, it does not provide the physical parameters with the resolution of the Bragg scales at which the scattering of the EM waves occurs. However, the estimation of the refractive index spectrum with the sub-grid energy dissipation rate enables simulation of the radar signal backscattering, as described in the following chapter. Two spectral forms are used in this work to estimate the refractive index spectrum at the Bragg scale of the X-band radar: Tatarskii's cut-off and Hill's bump. It is shown that the presence of the WV extends the inertial subrange of the turbulence spectrum enabling the backscatter of the X-band EM waves from the refractive index irregularities induced by the WV.

In the presence of rain, the scattering of the EM waves on the raindrops dominates over scattering on the irregularities of the radio refractive index. The LES provides data on the concentration of the raindrops of different sizes as well as on their velocities. This data is used to simulate the scattering of EM waves on the raindrops as on the dielectric spherical particles.

Chapter 3

Principles of radar measurements of atmospheric phenomena

Over the last decades, radars became a widely used remote sensors, able to provide accurate information about the weather and atmospheric phenomena: rain rate, turbulence intensity, atmospheric gravity waves, wind velocity, wind shear, storms, tornadoes, wake vortices etc.

Recent works indicate that radar will be one of the tools to monitor the WV (Oude Nijhuis et al., 2018). It is especially effective under rainy conditions (Barbaresco et al., 2016), but may also be used under clear-air weather and foggy conditions as a supplementary tool to lidar.

Radars provide remote sensing of the atmospheric phenomena at very large ranges in C and S bands. However, the long range is not the main requirement for a system aimed at the detection of the WV. Among the main advantages provided by the X band are high resolution relative to the C and S bands and a smaller size of the sensor. Accessibility of a wider bandwidth in the X band provides high resolution measurements. Additionally, due to the lower wavelength it is possible to obtain a narrow beamwidth with a relatively small antenna.

In the present work, the analysis of the detectability of the WV and estimation of the WV parameters is based on the simulation of the backscattered radar signal. This necessitates a development of methods for the simulation of the backscattered EM waves under different weather conditions for which the mechanisms of scattering of electromagnetic waves on the WV are different.

This chapter provides the description of the radar basic principles and details related to the measurements performed with a pulse-Doppler radar (Section 3.1). Then it introduces the parameters of the radar models used and the geometrical configurations of the measurements (Sections 3.2.1), along with the special aspects of the observation of the distributed atmospheric phenomena (Section 3.2). The methodology for calculation of the WV RCS is developed in Section 3.2.3 for clear air conditions and in Section 3.2.4 for the rainy condi-

tions. Finally, Section 3.2.5 develops the simulation of the backscattered radar signal and Section 3.2.6 provides a method for the synthesis of the Doppler spectrum of the radar signal backscattered on the WV in the presence of rain.

3.1 Radar basic principles

This section introduces basic principles of radar measurements for a pulse (Doppler) monostatic radar. In this introductory section, the analysis is given for a single scatterer. The radar measurement of the distributed targets is considered in Section 3.2.

3.1.1 Radar equation and signal-to-noise ratio

Fig. 3.1 illustrates the basic principles of radar measurements for a single point-like scatterer: the radar emits EM pulses in the direction of observation using a directional antenna to focus the emitted EM energy in the direction of interest. If there is an object located in that direction, it scatters back some portion

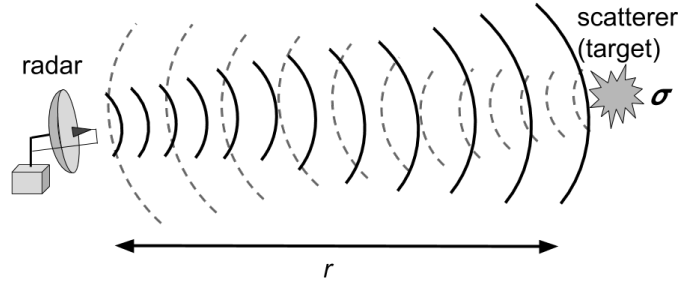


Figure 3.1: Illustration of the basic principles of radar measurements.

of EM power. The object scattering the radar signal is traditionally called a target in a radar field. The backscattered EM field propagates back to the radar where it is collected via the receiving antenna and is transferred further to the receiver for initial processing such as filtering, amplifying etc. In the receiver, the signal is normally converted into a digital form, and is further analyzed in a processing unit to estimate the parameters of the scatterer observed.

The main factors influencing the radar measurements are combined in the so-called radar equation, which provides an estimate of the power, P_r , received by the radar:

$$P_r = \frac{P_t G^2 f^4(\theta, \phi) \lambda^2 \sigma}{(4\pi)^3 r^4 L}, \quad (3.1)$$

where P_t is the peak power of the transmitted pulse, G is the antenna gain (assuming that the same antenna is used for both transmitting and receiving), $f(\theta, \phi)$ is the normalized antenna pattern in the direction of the target (θ, ϕ) ($f(\theta_0, \phi_0) = 1$ in the direction (θ_0, ϕ_0) normal to the antenna aperture), λ is the radar wavelength, r is the distance between the radar and the target, L is the attenuation of the radar signal due to the losses on the propagation path

(attenuation of the signal in rain, snow, clear air gases etc.), and σ is the RCS, which is the measure of the amount of the EM energy that is reflected by the target.

The radar equation provides an estimate of the power of the received signal at the output of the antenna port. Nevertheless, the receiver detects the backscattered signal if its level exceeds some threshold which is defined by the level of the noise in the receiving channel. The sources of noise are classified in two groups: the internal noise which is generated in the electrical circuits of the receiver system and the external noise, such as cosmic noise, solar noise, noise from the closely located structures etc. The internal noise is caused by the thermal motion of the electrons in the receiver's elements (Bringi and Chandrasekar, 2001). The presence of noise in the receiving channel is a fundamental limitation in radar measurements.

The noise is usually modelled as an additive white Gaussian noise (AWGN) having a total power N_0 in the bandwidth of the receiver, N_0 has a zero mean and a variance $\sigma^2 = N_0/2$. For the case of two-sided spectrum, It is estimated as follows:

$$N_0 = k_b T_0 B , \quad (3.2)$$

where $k_b = 1.38 \cdot 10^{-23}$ Joules/K is the Boltzmann's constant, T_0 is the equivalent noise temperature in degrees Kelvin (K), and $B \approx 1/\tau_p$ is the receiver bandwidth. The equivalent noise temperature of the system, T_0 , is commonly specified at the input of the low-noise amplifier (LNA). This is the first active element at the input of the receiver used to increase the level of the weak input signal so as it could be detected. To neglect the contribution of the subsequent elements of the receiver to the total noise power, it is needed to have the gain of the front LNA significantly large ($G_{LNA} \gg 1$) (see, for example, Bringi and Chandrasekar (2001)). Although the total noise temperature T_0 , referred to the input of the LNA, depends on the noise temperatures and gains/losses of all elements of the receiver along with the external noise temperature collected by the antenna (see, for example, (Bringi and Chandrasekar, 2001, Section 5.8)), it is often assumed that T_0 is the ambient temperature typically given as 290 K.

The ratio between the power of the received signal (Eq. (3.1)) and noise power (Eq. (3.2)) is called an SNR taken at the input of the receiver (which normally corresponds to the input of the LNA):

$$SNR_{in} = \frac{P_r}{k_b T_0 B} = \frac{P_t G^2 f^4(\theta, \phi) \lambda^2 \sigma}{(4\pi)^3 r^4 L k_b T_0 B} . \quad (3.3)$$

Nevertheless, the overall estimate of the SNR is usually given at the output of the receiver:

$$SNR_{out} = SNR_{in} / NF , \quad (3.4)$$

where NF is the noise figure characterizing the level of the receiver's internal noise. The NF , which is mainly determined by the noise figure of the LNA, specifies the degradation of the signal-to-noise ratio at the output of the receiver, SNR_{out} , relative to the signal-to-noise ratio taken at the input of the receiver, SNR_{in} . The ratio given in Eq. (3.3) along with Eq. (3.4) enables the

analysis of the power budget of radar measurements and the evaluation of the detectability of the signal scattered by the target in the presence of noise.

3.1.2 Pulse-Doppler radar concept

Section 3.1.1 gives the basic equations of the pulse radar. This type of radar estimates the distance to the target which is equal to $c\tau/2$, where c is the speed of light and τ is the delay between emitted and received pulses. If a directive antenna is used, the estimate of the location is then also available and the position of the scatterer can be refined with the tracking of the scatterer coordinates over time, for instance with a Kalman filter in case the trajectory of the scatterer is linear (Brookner and Cynamon, 2000) or other types of filters for a non-linear movement of the scatterer. In this way, the spatial position of the target in the subsequent observations is used to estimate the target's velocity and acceleration as the first and the second order derivatives of the tracked trajectory respectively. The spatial resolution of the radar is then directly linked to the antenna pattern. Nevertheless, in many cases it is needed to know the instant (radial) velocity of the scatterer. The direct measurements of the radial velocity uses the Doppler effect due to which the frequency of the signal scattered on the moving object is proportional to the velocity of this object. To enable the measurements of the frequency change, more precisely the rate of change of the full phase of the pulse $d\phi(t)/dt = w$, where w is the angular frequency, pulse radars need to track the changes of the pulses phase. Such radars are called pulse-Doppler radars, where Doppler stresses the use of the Doppler effect employed to measure instant radial velocity of the scatterer, besides range.

A pulse-Doppler radar is the classical type of weather or meteorological radar which represents a powerful tool for sensing the atmospheric phenomena. These radars emit a train of consecutive phase-coherent EM pulses and analyze the rate of the change of scattered pulses' phase to obtain the estimate of the scatterer's radial velocity. Keeping the phase of all emitted pulses coherent allows the tracking of the phase change of the consecutive pulses scattered back to the radar. As shown hereafter, the rate of phase change is used to evaluate the radial velocity of the scatterers which can be raindrops, clear-air refractive index irregularities, snow, fog particles etc.

Fig. 3.2 illustrates the structure of the radar signal: during the dwell time, the radar emits M pulses with a pulse repetition time of T . The radar receives the pulses scattered by a point-like target located at a distance of $c\tau/2$, where c is the speed of light and τ is the time necessary for an EM wave to travel from the radar to the scatterer and back. A pulse received at a time $mT + \tau$, where m is the consecutive number of the pulse in the burst, corresponds to the pulse emitted at a time of mT and reflected back by a scatterer. The pulses of Fig. 3.2 are given in the form of a power envelop.

Radar range resolution

The parameters of the target, its range and Doppler velocity, are estimated from the received backscattered signal. These estimates are obtained with a

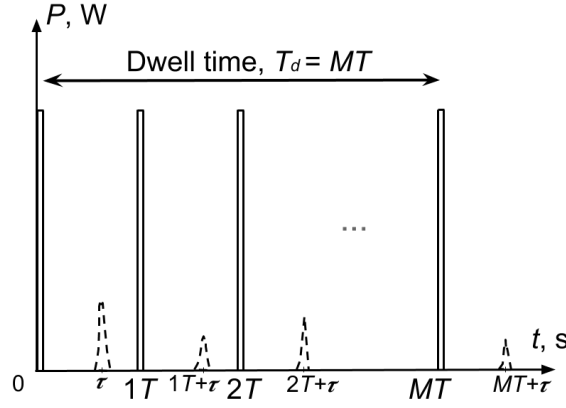


Figure 3.2: Schematic representation of the train of emitted (solid) and received (dashed) pulses.

limited resolution in range, which is specified as a minimum distance between two closely located targets for which the signals backscattered on these targets can be distinguished in the receiver. As the detection of the pulse is based on the correlation (convolution) between the emitted and the received pulses, the conventional way to define the resolution is to fix it at the level of half-width of the pulse's correlation function which is $\tau_p/2$ for a rectangular-shaped pulse, where τ_p is the duration of the EM pulse. In this case, the range resolution is specified as:

$$\Delta r = \frac{c\tau_p}{2} . \quad (3.5)$$

So, for a typical value of $\tau_p = 10 \mu s$, the corresponding resolution is 150 m, which is quite low to observe the small-scale atmospheric phenomena such as small-scale turbulence or the WVs. An increase of the resolution by a factor of 10 to 100 necessitates a use of pulses that are 10 to 100 times shorter inducing a bandwidth increase and an SNR decrease by the same factor. To maintain the same SNR (see Eq. (3.3)), the peak power of the emitted pulses should be increased by 10 to 100, which is unpractical.

To overcome this limitation, each emitted pulse is modulated in order to increase its bandwidth and provide the range resolution needed. Such modulation is called a pulse compression. As the detection of the pulse is based on the matched processing, i.e. the correlation between the emitted and the received pulses, and the width of the correlation function inversely proportional to the signal bandwidth, the resolution in range is also inversely proportional to the pulse bandwidth. So, for the pulses with the same duration τ_p but having a bandwidth B_c the range resolution is determined by

$$\Delta r = \frac{c}{2B_c} , \quad (3.6)$$

where B_c is the bandwidth of the modulated pulse. A processing gain equal to $G_{compr} = \tau_p B_c$ is often used in the literature to demonstrate the gain obtained by using the pulse compression. Nevertheless, it should be noticed that this gain is only related to the increase of the range resolution due to the narrow autocorrelation function. This holds true if the energy of the pulse does not change (P_t and τ_p are not changed). The SNR at the output of the matched receiver does not change when compressed pulses are used and is defined as:

$$SNR = \frac{P_t G^2 \lambda^2 \sigma G_{compr}}{(4\pi)^3 r^4 k T_0 B_c} = \frac{P_t G^2 \lambda^2 \sigma \tau_p B_c}{(4\pi)^3 r^4 k T_0 B_c} = \frac{P_t G^2 \lambda^2 \sigma \tau_p}{(4\pi)^3 r^4 k T_0}, \quad (3.7)$$

where $\tau_p = 1/B$ is the duration of the pulse, which is inverse proportional to the bandwidth of the non-modulated pulse. Eq. (3.7) reflects the fact that when the bandwidth of the pulse is increased, the bandwidth through which the receiver collects the noise is increased as well since radar's receiver should use filters that pass increased bandwidth of the pulse; therefore, the bandwidth of the noise at the output of the receiver (before matched processing) is increased to B_c . So, the pulse compression helps to increase the range resolution but the signal-to-noise ratio remains the same as for the unmodulated pulse. This holds true if τ_p and the peak power P_t are not changed for the modulated pulse.

For a pulse-Doppler radar (or in pulse radars in general), if the distance to the target $r' > cT/2$, then the echo of the m^{th} transmitted pulse is received after transmission of the $(m+1)^{th}$ pulse. Therefore, echo of m^{th} pulse reflected from a target at the distance $r' > cT/2$ and $(m+1)^{th}$ pulse reflected by the target located at $r'' < cT/2$ cannot be distinguished unless some different internal modulation is applied to each of the consecutive pulses in the burst. Therefore, the period of pulses repetition, T , determines the maximum distance at which the echoes from the target can be unambiguously measured:

$$R_{max} = \frac{cT}{2}. \quad (3.8)$$

Doppler effect and Doppler velocity

The Doppler effect, which causes a change of the initial frequency of the signal when it is scattered by a moving object, enables the measurement of the instant radial velocity of a target. The phase of the pulse, scattered by a target at range $r(t)$ from the radar and received back, is given as:

$$\phi(t) = \phi_0 - \frac{4\pi r(t)}{\lambda}, \quad (3.9)$$

where λ is the radar wavelength, ϕ_0 is the initial phase of the emitted signal. If the target illuminated by the radar is moving with a radial velocity v_r with respect to the radar, the rate of change of phase is:

$$\frac{d\phi}{dt} = -\frac{4\pi}{\lambda} \cdot \frac{dr(t)}{dt} = v_r \cdot \frac{4\pi}{\lambda}. \quad (3.10)$$

In Eq. (3.10), it is assumed that the target is moving toward the radar; therefore, the derivative of $dr(t)/dt$ is negative. Since the rate of phase change is

equal to the angular frequency w_D :

$$\frac{d\phi}{dt} = w_D = 2\pi f_D , \quad (3.11)$$

where f_D is the Doppler frequency defined as:

$$f_D = \frac{2v_r}{\lambda} . \quad (3.12)$$

The Doppler frequency, f_D determines the frequency shift affecting the carrier frequency of the pulse ($f_c = c/\lambda$) when it is scattered on the moving target. If the scatterer moves towards the radar, $f_D > 0$, and when the scatterer moves away from the radar, $f_D < 0$. If the scatterer moves perpendicular to the line connecting the radar and the scatterer, then $f_D = 0$ since the radial velocity is zero.

Similarly to the maximum unambiguous range (Eq. (3.8)), there is a maximum radial velocity that can be unambiguously measured by a pulse-Doppler radar. The estimation of the scatterer's radial velocity is based on the measurement of the phase change between successive pulses backscattered by a target. Therefore, if the radial speed of the scatter is so large that the change of phase between two successive pulses is greater than π then these measurements provide an ambiguous estimate of v_r . Therefore, the maximum velocity, V_{max} , unambiguously measured is determined from Eq. (3.10) as:

$$\frac{V_{max}4\pi}{\lambda} \cdot T = \pi , \quad (3.13)$$

and, finally:

$$V_{max} = \frac{\lambda}{4T} = \frac{\lambda f_{PRF}}{4} , \quad (3.14)$$

where f_{PRF} is the pulse repetition frequency.

Eqs. (3.8) and (3.14) show that the choice of the pulse repetition frequency of the emitted train of pulses is a compromise between the maximum radial velocity and the range that can be measured unambiguously: increasing the f_{PRF} allows estimation of higher radial velocities of the scatterers, while it decreases the maximum unambiguous range, and contrary, reduction of f_{PRF} increases R_{max} but decreases V_{max} . This is illustrated by the product $R_{max}V_{max}$ that is independent of f_{PRF} :

$$R_{max}V_{max} = \frac{c\lambda}{8} . \quad (3.15)$$

Eq. (3.15) shows that for a given pulse repetition frequency, the increase of the maximum measurable values of the range and velocity is proportional to the radar wavelength.

The analysis given in this section is performed for a single point-like scatterer; nevertheless, atmospheric phenomena are scatterers distributed in space or so-called volume scatterers, which are analyzed further in Section 3.2.

3.2 Radar measurements of distributed scatterers

The radar signal backscattered on the atmospheric phenomena is a composite of echoes from a very large number of closely located scatterers, which are the refractive index irregularities in clear-air atmosphere (caused by the atmospheric turbulence or by the presence of the WVs) or by the raindrops (in still air or in the presence of the WVs). The distributed nature of the scatterers necessitates a more careful consideration of the radar resolution volume and its influence on the measurements.

This section first considers the geometries of the radar measurements used in this work (Section 3.2.1) and the special aspects of the pulse-Doppler radar measurements of the distributed targets (Section 3.2.2). Then, it develops the calculation of the distributed scatterers' RCS for clear-air atmosphere and for rain, both in the presence and in the absence of the WVs (Sections 3.2.3 and 3.2.4 for clear-air and rainy conditions respectively). Finally, Section 3.2.5 develops the methodology for the simulation of the time series of the signal backscattered in clear air as well as for the direct synthesis of the backscattered signal's Doppler spectrum in the presence of rain.

3.2.1 Geometrical configuration of radar measurements

Over the past decades radars became unavoidable tools for weather observation, due to their ability to provide remote sensing of large regions of atmosphere with high resolution and accuracy. There are two main types of weather radars: wind profiler radars and meteorological radars. Wind profilers use vertically oriented antenna to remotely sense the wind velocity, atmospheric turbulence intensity, and atmospheric gravity waves in the atmospheric boundary level (ABL). Meteorological radars perform a horizontal scanning with the antenna typically aimed at low elevation angles to monitor low layers of the atmosphere for sensing the precipitation (rain, snow, fog etc.), storms etc.

In the framework of UFO project (2016), one of the goals was to demonstrate the ability to estimate the turbulence intensity with the vertically oriented X-band Doppler radar developed by the Laboratoire ATmosphere, Milieux, Observations Spatiales (LATMOS) from Université Pierre et Marie Curie (UPMC) and with the horizontally scanning X-band radar developed by Thales. Additionally, the scanning radar is supposed to be used for retrieval of the wind velocity. The ultimate goal is to use a single sensor (a horizontally scanning radar) for detecting the WV, measuring its strength and estimate the parameters influencing the WV life time, such as wind velocity and energy dissipation rate of the background atmospheric turbulence.

The simulator developed in this work is validated first via retrieval of the energy dissipation rate of the atmospheric turbulence and then is used to simulate the detection of the WV and retrieval of the WV's strength. The following subsection provides a description of the geometrical setups and the parameters of the radar models used in this work.

Table 3.1: Radar parameters

Radar parameters	LATMOS CURIE
Emitted power	60 W
Frequency	9.42 GHz
Repetition pulse time	4.8 μs
Number of pulses coherently integrated	300
Pulses in a burst	1228800
Dwell time, T_d	5.9 s
Range cell size	22.5 m
Aperture angle	1.9°
Antenna gain	40 dBi
Elevation angles, θ	90°

Turbulence intensity measurements

The parameters of the radar model used in the present work are the same as the ones of the radar used in UFO project (2016). The main parameters of the vertical profiler (CURIE radar) used for the simulation of measurements of the turbulence intensities are listed in Table 3.1. The CURIE X-band Doppler radar developed by LATMOS (Al Sakka et al. 2009) operates at 9.42 GHz. It uses a parabolic-offset antenna to minimize the effects of secondary beams and ground clutter. This sensor is a low power pulsed radar; therefore, to increase the SNR, it uses a relatively high pulse repetition frequency to allow coherent integration of the consecutive pulses (300 pulses in the considered mode).

Fig. 3.3 illustrates the geometrical configuration of the simulations. The profiler radar's antenna beam is pointed vertically providing measurements for the altitudes in the range of 70 – 720 m above the radar with a range resolution of 22.5 m. In order to simulate the radar measurements for a given range and altitude, the LES cube is placed as depicted in Fig. 3.3. The wind is modelled by adding its velocity to the LES local wind field.

WV measurements

The parameters of the radar model used for WV detection (see Table 3.2) are chosen to be equal to the parameters of the X-band Thales radar that was used in the trials of the SESAR 12.02.02 joint initiative for detection of the WV at the Charles de Gaulle airport in 2015 (Barbaresco et al., 2016). The radar model has a range resolution of 6 m in short range mode, achieved by pulse compression, that allows for the discrimination of wake vortices. The far range mode has a 60 m resolution. The geometrical configuration of the simulation, nevertheless, differs from (Barbaresco et al., 2016) in order to demonstrate the tracking of the descending WV.

Fig. 3.4 illustrates the geometry used for the simulation: the radar is located at the side of the glide with the distance r to the runway axis equal to 1100

m. Two geometrical setups, named hereafter as setups A and B, are used in the simulator for extraction of the WVs strength from the radar measurements. For setup A, the size of the radar resolution volume in range (Δr_{rc}) is 60 m and the elevation angle of the antenna beam is fixed at 5.3° to monitor the WV at the altitude of the glide ($h_1 = 100$ m). For setup B, the range resolution is 6 m and the radar performs a scan of the LES box in elevation (with a step of 0.27°) to track the WV descent.

An additional geometry setup C is used in order to simulate the Doppler spectrum of the W-band radar signal backscattered on raindrops within the WV system. The results of simulations are compared qualitatively with those obtained by Seliga and Mead (2009). For this setup, the radar performs a scan of the LES box in elevation with a step of 0.09° . The parameters of the radar

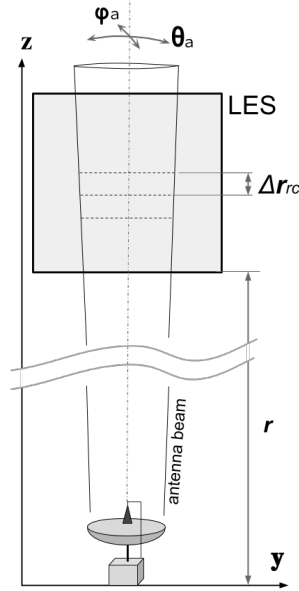


Figure 3.3: Geometrical configurations of the simulation used for retrieval of the energy dissipation rate from the vertical profiler's radar signal.

Table 3.2: Parameters of the X-band radar.

Parameter name	Value
Frequency, f_c	9.5 GHz
Pulse duration, τ_{pls}	$2 \mu s$
Pulse repetition time, T	$50 \mu s$
Number of pulses in a burst	512
Dwell time, T_d	25.6 ms
Radar cell size in range, Δr_{rc}	60 m; 6 m
Antenna beam width at 3dB, $\varphi_a = \theta_a$	3.1°

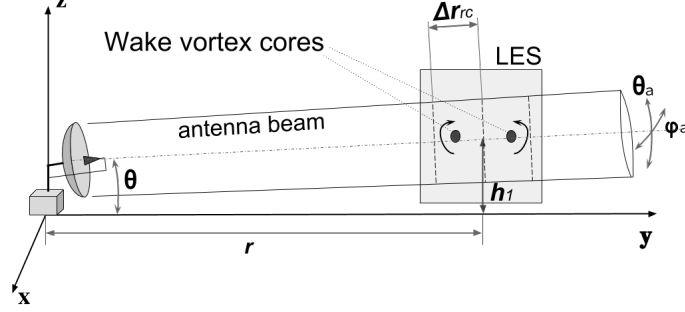


Figure 3.4: Geometrical configuration of the simulation used for detection of the WV.

are the same as in (Seliga and Mead, 2009) (Table 3.3).

It is assumed that the radar uses a digital antenna beam forming (for all geometrical configurations).

3.2.2 Distributed scatterers

In the receiver, the pulse scattered on the distributed scatterers is normally observed as a continuous signal due to its backscattering on the continuum of closely located individual scatterers. The backscattered signal arrives at the receiving antenna with the delay corresponding to the time necessary for the radar pulse to travel to the front of the volume scatterer and back. Normally, the front of the atmospheric volume scatterers begins immediately at the surface of the antenna. It can be a region with rain or simply the clear-air turbulence around the radar.

The illustration of the scattered signal at the output of the filter matched to the emitted pulses is given in Fig. 3.5 (dashed curve). Since individual echoes from closely located scatterers cannot be resolved, the complex amplitude of the scattered pulse for a given resolution volume is collected by sampling the received signal at the discrete time delays τ with a period of T ; the samples locations for a given radar resolution volume are depicted with square markers in Fig. 3.5; the backscattered signal is shown in the form of a power at the

Table 3.3: Parameters of the W-band radar.

Parameter name	Value
Frequency, f_c	94 GHz
Pulse repetition time, T	50 μs
Number of pulses in a burst	512
Dwell time, T_d	25.6 ms
Radar cell size in range, Δr_{rc}	1 m
Antenna beam width at 3dB, $\varphi_a = \theta_a$	0.18°

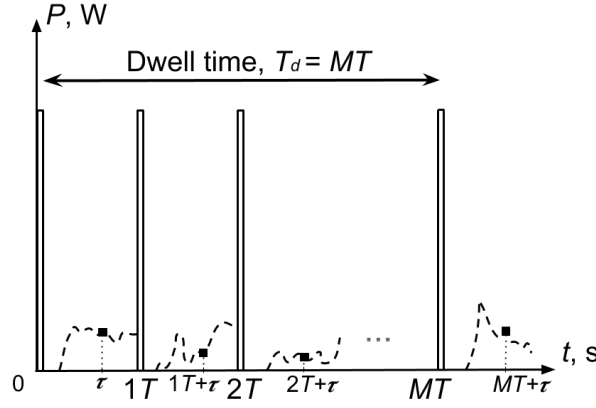


Figure 3.5: Schematic representation of the train of emitted pulses (solid) and received signals (dashed). The received signals are presented at the output of the matched filter. The square markers specify the delays at which the samples corresponding to the range resolution volumes centered at $c\tau/2$ are collected for further processing.

output of the matched filter. The output of the matched filter is proportional to the convolution between the received signal and the filter's impulse response matched to the emitted pulses. Each sample collected corresponds to a resolution volume located at $\tau c/2$ within which the scatterers contribute significantly to the sample. The dimensions of this volume and contribution of each particular scatterer is determined by a range weighting function. This function is defined by the autocorrelation of the matched filter transfer function.

Fig. 3.6 demonstrates a radar resolution volume, the weighting functions and a single scatterer. The range weighting function is specified by the amplitude of the matched filter's transfer function and the shape of the transmitted pulse. Angular weighting is related to the antenna radiation pattern $f^2(\theta, \phi)$. The location of the radar resolution volumes for the corresponding sample is called a gate.

The locations of the adjacent gates are usually chosen so that the range weighting functions centers, which correspond to the adjacent resolution volume centers, intersect each other at the level of -3 dB. This is a conventional criteria of the resolution of the two closely located individual targets. It should be taken into account that the interval over which the sample for a given gate is calculated (by the convolution between the matched filter's impulse response and the input continuous signal), is equal to the pulse duration, τ_p ; nevertheless, due to the pulse compression, only scatterers located within the space interval determined by Eq. (3.6) around the range $c\tau/2$ are significantly influencing the sample of the received signal at the delay τ . This holds true for clear-air scatterers since the order of the RCS does not vary significantly for the closely

located (unresolved) individual scatterers. However, this may not be valid for the raindrops because the order of the RCS for raindrops can vary in a large interval since it depends heavily on the raindrop's size. That can result in the significant influence of the big raindrops on the sample of the adjacent gates.

Despite the fact that the range weighting function significantly influences the contribution of the each scatterer to the reflected signal, it is assumed in this work that weighting function is equal to 1 over the whole resolution volume. As a consequence, the possible contribution of the raindrops to the samples of the neighboring radar resolution volumes are neglected.

The total RCS for a given radar resolution volume is calculated as a sum of RCS of all contributors (refractive index inhomogeneities or rain droplets) within that volume:

$$\sigma = \sum_{p=1}^P \sigma_p , \quad (3.16)$$

where σ_p is the RCS of the individual scatterer within the radar resolution volume (see Fig. 3.6). The following sections develops the calculation of the distributed scatterers' RCS for clear-air atmosphere and for rain, both in the presence and in the absence of the WVs (Sections 3.2.3 and 3.2.4 for clear-air and rainy conditions respectively).

3.2.3 Radar cross-section of the WV and turbulence in clear air

A very weak level of the backscattered power makes it difficult to detect the WV with the radar in clear-air atmosphere (Gilson, 1992), (Barbaresco et al., 2007). The power of the backscattered signal is proportional to the scatterer's RCS (see Eq. (3.1)). The estimation of the RCS enables the evaluation of

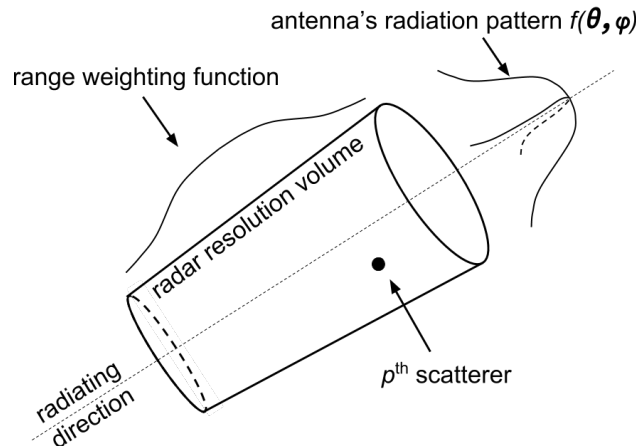


Figure 3.6: Illustration of the weighting functions of the radar resolution volume.

the limits of detectability of the WV in clear air or clear-air turbulence itself (Eq. (3.3)).

All the models reported in the literature classify the mechanisms of scattering of the electromagnetic waves on the WVs in two groups: scattering on the laminar structure of the evolving WVs as performed by Myers et al. (1999), Shariff and Wray (2002), Li et al. (2010), Vanhoenacker-Janvier et al. (2012) and scattering on the turbulence generated by the WV, as proposed by Nespor et al. (1991), Gilson (1992), Marshall and Myers (1996).

The calculation of the WV RCS based on the laminar structure of the WV in clear air (as in (Myers, 1997; Shariff and Wray, 2002; Li et al., 2010; Vanhoenacker-Janvier et al., 2012)) is not used in this work, because, in view of the state-of-art LES simulations of the WV in clear atmosphere, it is supposed that the laminar structure of the WV can persist only during a limited time interval after the formation of the WV (Kovalev et al., 2018) and is not realistic for the developed turbulent WV. So, the Bragg scattering of the radar signal on the turbulence is considered, assuming that the aircraft wake vortex creates a cascade of turbulent energy similar to atmospheric turbulence. High values of C_n^2 enable the detection of the WVs by the radar. The methodology of the estimation of the clear-air turbulence RCS is the same as for the WV in clear atmosphere.

Methodology of estimation of the wake vortex radar cross-section

The refractive index (n) is calculated for the clear air from pressure, temperature and relative humidity fields provided by the LES (see Section 2.2.1). Then, a refractive index structure parameter (C_n^2) is calculated (see Section 2.2.2) and used to estimate the RCS as described hereafter.

The RCS is calculated for a volume of one radar cell as follows (Doviak and Zrnić, 1984):

$$\sigma = \sum_{p=1}^P \eta_p V_p , \quad (3.17)$$

where η_p is the reflectivity [m^{-1}] of the p^{th} LES cell, having a volume V_p (V_p is equal for all LES cells and is approximately 0.5 m^3), and P is the total number of LES cells located within a given radar resolution volume.

It can be shown (Doviak and Zrnić, 1984) that reflectivity of the clear-air turbulence for backscattering case (where the angle between incident and scattered EM waves is π) is defined as:

$$\eta = 8\pi^2 k_B^4 2^{-11/3} \Phi_n(k_B) , \quad (3.18)$$

where $\Phi_n(k_B)$ is the refractive index spectrum defined by Eq. (2.22) for the assumption that the Bragg wavelength lies within the inertial subrange of the turbulence spectrum. Substituting Eq. (2.22) into Eq. (3.18) and taking into account that $k_B = 2\pi/\lambda_B$, where $\lambda_B = \lambda/2$ is the Bragg wavelength, Eq. (3.18) transforms into the well-known form (Ottersten, 1969):

$$\eta = 0.38 \lambda_B^{-1/3} C_n^2 . \quad (3.19)$$

For the X-band radar, the Bragg wavelength becomes comparable to the Kolmogorov length scale and therefore it falls into the transition zone between inertial subrange and viscous dissipative range of the refractive index spectrum. Thus, it is needed to account for the decrease of the refractive index variance spectrum in that region (See Section 2.2.3, Eq. (2.30)). Applying Eq. (2.31) to Eq. (3.18), the reflectivity of the clear-air turbulence or turbulence induced by the WV (in the transition between inertial subrange and dissipative range) is calculated for p^{th} LES cell as:

$$\eta_p = 0.38\lambda_B^{-1/3} [C_n^2]_p g(\eta_0 k_B) . \quad (3.20)$$

To calculate the total RCS of given resolution volume, the contributions from all LES cells in Eq. (3.20) are weighted with the value of the antenna pattern at the angular position of the p^{th} LES cell w_p^2 (it is assumed that antenna patterns w_p are equal in azimuth and elevation). A range weighting function, characterized by the receiver filter impulse response and the envelope of the transmitter pulse, is assumed to be constant over the size of the radar cell in range. A non-constant envelope can easily be added to the simulator if needed. Therefore, the RCS of the radar resolution volume is:

$$\sigma = 0.38\lambda^{-1/3} V_p \sum_{p=1}^P [C_n^2]_p w_p^2 g(\eta_0 k_B) . \quad (3.21)$$

Eqs. (3.21) and (3.3) allow the estimation of the signal-to-noise ratio in order to assess the detectability of the WV under different radar parameters and geometrical configurations of the measurements.

Fig. 3.7 provides an example of distributions of the LES cells' RCS, calculated as $\eta_p \cdot V_p$, with Eq. (3.21) ($V_p \approx 0.5 \text{ m}^3$). This distribution is given in a cross-section of LES Case 1. Two regions A (containing the left WV core) and B (outside the WV) are considered to illustrate the difference between the RCS calculated for the volumes with high and low EDR: the median values of $\epsilon^{1/3}$ is $0.185 \text{ m}^{2/3}\text{s}^{-1}$ for region A, and $0.075 \text{ m}^{2/3}\text{s}^{-1}$ for region B. The regions are extended in the x -axis direction to form cubes having dimensions of $60 \times 60 \times 60 \text{ m}^3$, which roughly correspond to the size of the radar resolution volume used for geometrical setup A. The RCS of these volumes is -81.9 dBsm for A and -85.4 dBsm for B. The difference of 3.5 dB comes from the decrease of the attenuation function $g(\eta_0 k_B)$ by a factor of approximately 2 corresponding to the $\epsilon^{1/3}$ decrease from 0.185 to $0.075 \text{ m}^{2/3}\text{s}^{-1}$ (Fig. 2.29). Further analysis of WV RCS in function of the radar and meteorological parameters is given in Chapter 5.

3.2.4 Radar cross-section of the wake vortex in the presence of rain

The mechanism of the EM scattering on the spherical (non-absorbing) raindrops and consequently the level of raindrop's RCS depend on the electrical size of the raindrop kr_d (Eq. (2.33)), where $k = 2\pi/\lambda$ and r_d is the raindrop radius. Fig. 3.8 illustrates the dependence of the RCS of a spherical particle,

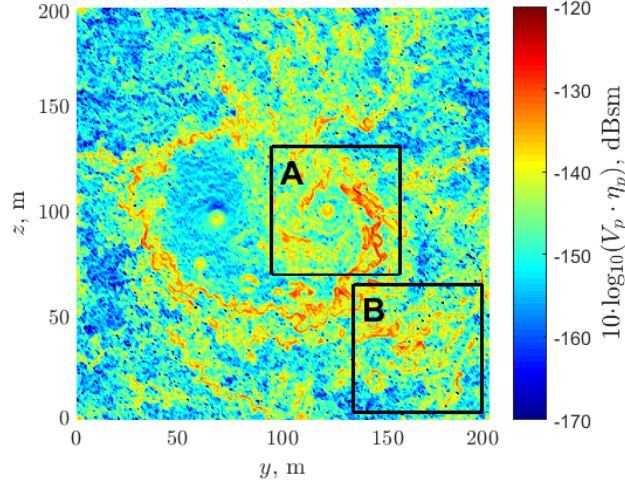


Figure 3.7: Distribution of the RCS of the LES cells having a volume of approximately 0.5 m^3 .

normalized to its cross-section πr_d^2 on the electrical size of the particle normalized by 2π . There are three distinctive regions, depending on the kr_d (or r_d/λ used in Fig. 3.8): the Rayleigh region ($r_d/\lambda < 0.1$), the Mie region ($0.1 < r_d/\lambda < 2$), and the optical region for which the raindrop RCS is equal to its cross-section πr_d^2 . The limits of the regions are determined approximately. While the Mie scattering theory is based on the solution of Maxwell's equations taking into account the incident, scattered and internal EM fields, and can be used to calculate the RCS of the raindrop of any size, the Rayleigh approximation is normally used for the region where $kr_d \ll 1$ (or $r_d/\lambda \ll 0.1$).

For the X-band radar, the ratio $r_d/\lambda < 0.1$ belongs to the Rayleigh region, and the RCS of a raindrop is calculated as follows (Doviak and Žrnić, 1984):

$$\sigma = 7.11\pi r_d^2 (2\pi r_d/\lambda)^4. \quad (3.22)$$

Eq. (3.22) shows that the RCS of the raindrop depends heavily on its diameter in the Rayleigh region; therefore the RCS of the raindrops varies in a very wide range.

For $r_d/\lambda > 0.1$, the RCS of a spherical raindrop can be estimated by calculating the Mie series as (Sauvageot, 1992):

$$\sigma = \frac{\lambda^2}{4\pi} \left| \sum_{n=1}^{\infty} (-1)^n (2n+1) (a_n(\rho) + b_n(\rho)) \right|^2, \quad (3.23)$$

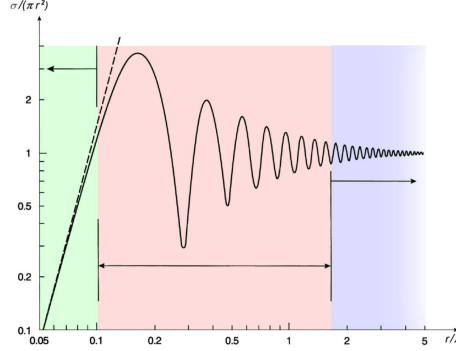


Figure 3.8: Dependence of the normalized RCS of the raindrop on the ratio r_d/λ . The region highlighted in green is the Rayleigh region; red - the Mie region; purple - the optical region. This figure is taken from <http://www.radartutorial.eu/>.

where $\rho = kr_d$, $a_n(\rho)$ and $b_n(\rho)$ are given by:

$$a_n(\rho) = \frac{J_n(\rho)}{H_n^{(2)}(\rho)}, \quad (3.24)$$

$$b_n(\rho) = \frac{-[\rho J_n(\rho)]'}{[\rho H_n^{(2)}(\rho)]'}, \quad (3.25)$$

where J_n and $H_n^{(2)}$ are the n^{th} order spherical Bessel function of the first kind and the n^{th} order spherical Hankel function of the second kind, respectively. The prime denotes the differentiation with respect to ρ .

The shape of a falling raindrop is being deformed by a drag force (see Fig. 2.13). Hence in principle, the RCS depends on the orientation of the raindrops relative to the incident EM wave. However, in the present work, it is assumed that all considered raindrops have a spherical shape and polarimetric effects, which occur when electromagnetic wave is scattered on a non-symmetric raindrop, are neglected.

Since the RCS of the raindrop is proportional to r_d^6 , the bigger raindrops contribute more to the scattered signal than the smaller ones. Therefore, for more accurate synthesis of the backscattered radar signal, it is important to introduce in the simulation the raindrops with large diameters. The LES with the WV in the presence of rain provides fields of the relative raindrops distributions for the diameters of 1, 2, 3, and 4 mm (see Section 2.1.4). The raindrop's size distributions are presented in Fig. 3.9a,b for the rain rates of 1 and 10 mm/h respectively. Fig. 3.9c,d provides the raindrop's size distributions weighted by the RCS of raindrops; the black curves represent the distributions calculated with Eq. (2.14); the red curves show the approximation introduced by the raindrops diameters used in the LES simulations. It is clearly visible that the raindrops sizes used in the LES poorly represent the original distribution of the raindrops. However, the raindrop RCS is proportional to r_d^6 and, as

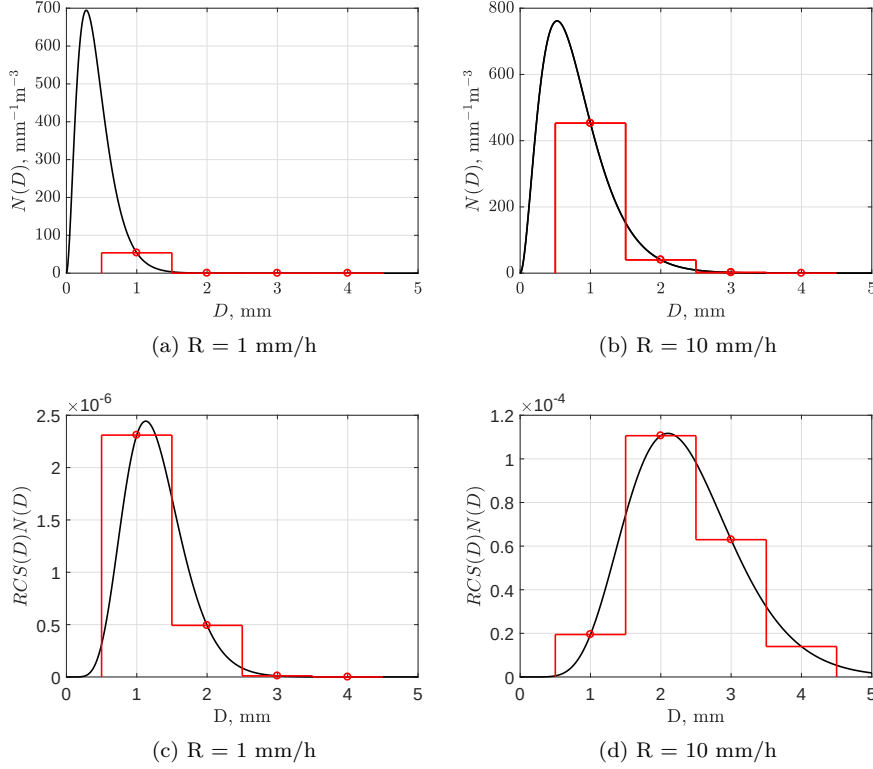


Figure 3.9: (a,b) Raindrop's size distributions for rain rates 1 and 10 mm/h and (c,d) raindrop's size distributions weighted by the RCS of the raindrops. The black curves are original distributions (Eq. (2.14)) without (a,b) and with (c,d) weighting by the raindrops RCS (Eq. (3.22)); red curves represent the approximation of the corresponding dependencies by the raindrops diameters available from the LES simulations.

seen from Fig. 3.9c,d, the set of raindrops diameters used in LES fairly represents the original distribution weighted by the raindrops RCS calculated with Eq. (3.22).

To estimate the error introduced by the approximation of the raindrop's size distribution for different rain rates, the total RCS of all raindrops in the approximated distribution is calculated as an integral of the weighted distribution (the surface under the red curves in Fig. 3.9c,d) and then it is subtracted from the total RCS calculated using the original distribution (surface under the black curves in Fig. 3.9c,d). The result is presented in Fig. 3.10 as a dependence of the error ($10 \cdot \log_{10}(RCS_{error})$) on the rain rate.

For a moderate rain, the error of estimation of RCS is about 0 dB (no error). For low and very high rain rates, the absolute value of the error of the RCS estimation reaches 1 dB. So, the LES with the raindrop's diameters

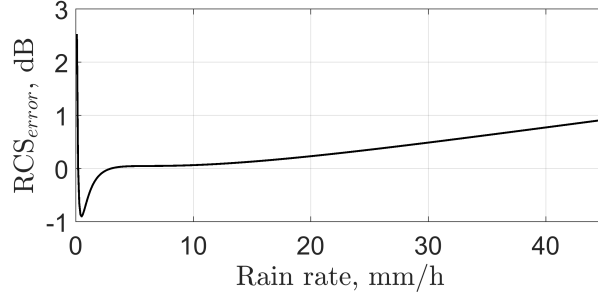


Figure 3.10: Error of the total RCS estimation due to the limited number of raindrop diameters simulated in LES.

chosen provides a flexibility to simulate different intensities of the rain.

3.2.5 Simulation of the signal backscattering in clear air

A fully developed turbulent WV creates a cascade of turbulent energy similar to atmospheric clear-air turbulence. This enables the detection of the WV and measurement of the WV parameters. The RCS of the WV (or clear-air turbulence) calculated with the LES data allows to perform the analysis of the power budget of the radar measurements and estimate the radar detectability of the WV. However, to estimate the velocity field induced by the WVs (which allows retrieval of the WV strength) or turbulent kinetic energy of the turbulence, it is needed to analyze the Doppler frequency shifts of the backscattered signal. As shown hereafter, this is achieved via the simulation of the time series of the backscattered signal or via direct synthesis of the power spectral density (or Doppler spectrum) of the backscattered signal. While simulation of the time series provides more flexibility for the signal processing, the direct simulation of the Doppler spectrum reduces the computational power needed for simulation. This section develops the method to simulate the backscattered complex-valued signal time series with LES simulations of the clear air turbulent stratified atmosphere (with or without WV).

The backscattered signal can be obtained by calculating the so-called scattering integral (Doviak and Zrnić, 1984):

$$I(t) = A \int W(\mathbf{r}) f_{\theta\phi}^2(\mathbf{r}) n'(\mathbf{r}, t) e^{-jk_B r_t} dV, \quad (3.26)$$

where $W(\mathbf{r})$ is the range weighting function, $f_{\theta\phi}^2(\mathbf{r})$ is the antenna pattern (power), $n'(\mathbf{r}, t)$ is the refractive index of the irregularities within the radar cell, $e^{-jk_B r_t}$ is the phase factor defined by the Bragg wavenumber $k_B = 4\pi/\lambda$ and the distance r_t , A is an energetic constant that necessitates the knowledge of various radar parameters:

$$A = \frac{G}{\lambda r_0^2} \sqrt{\frac{P_t}{2R}}, \quad (3.27)$$

where G is the antenna gain, λ is the radar wavelength, r_0 is the distance between the radar and the center of the radar resolution cell, P_t is the emitted power, R is the receiver resistance.

Eq. (3.26) assumes the validity of the Born approximation, which holds true when the EM waves propagate in the medium where the fluctuations of the refractive index, n , are small within a wavelength, λ , (Doviak and Zrnić, 1984), i.e.:

$$\nabla n \ll \frac{2\pi}{\lambda} . \quad (3.28)$$

When Eq. (3.28) is true (which is the case for the turbulence in clear-air atmosphere), it can be assumed that the contribution to the EM field caused by scattering is small relative to the refracted field that would be observed in the absence of the turbulence. Therefore, the Born approximation neglects the scattering of the scattered field (multiple scattering), which simplifies the solution of the Maxwell's equations to obtain Eq. (3.26).

The discrete nature of the output of the LES imposes the transformation of the integral into a sum over the radar cell volume for which the product $W(\mathbf{r})f_{\theta\phi}^2(\mathbf{r})$ is significant. The numerical calculation of the scattering integral necessitates the value of the refractive index irregularities $n'(\mathbf{r}, t)$ with a resolution better than the Bragg wavelength (since $e^{-jk_B r_t}$ rapidly oscillates within the radar resolution volume). For classical simulators using 2D models for the wake vortices, the resolution could possibly be adapted to the wavelength of the radar. The resolution of the present 3D LES, adequate to capture the most relevant dynamics of the medium to large scales of the vortices and the associated turbulence, is however not fine enough to capture the Bragg wavelength scale. The latter is 50 times smaller than the LES resolution, making a full calculation of the scattering integral impossible. It is of course not possible to increase the LES resolution by a factor of 50 which would require 125.000 extra grid points and extend the LES computer time to unrealistic duration. Muschinski et al. (1999) proposed the use of LES-generated fields of atmospheric parameters to construct time series of the complex-valued wind-profiler signals for the radar working at 915 MHz. The generated time series were processed to derive the Doppler velocities of the scatterers and the result was validated using the wind velocities available directly from the LES set.

In the absence of an atmospheric model with sufficient resolution, Muschinski et al. (1999) proposed to use a parametrized model for the calculation of the scattering integral to obtain the following time series of the backscattered signal (3.26):

$$I(t) = A\sqrt{0.033k_B^{-11/3}} \sum_{p=1}^N W_p \sqrt{[C_n^2(t)]_p} V_p e^{-j\varphi_p(t)}, \quad (3.29)$$

where C_n^2 is the refractive index structure parameter, W_p is the value of $W(\mathbf{r})f_{\theta\phi}^2(\mathbf{r})$ in the LES cell of volume V_p , $\varphi_p(t)$ is the phase of the signal backscattered by the p^{th} grid cell, and N is the number of LES grid cells within the volume of a given radar cell. The calculation of C_n^2 is given in Section 2.2.2 and calculation of $\varphi_p(t)$ is described in the following subsection.

To derive Eq. (3.29), Taylor’s hypothesis of the “frozen turbulence” (Taylor, 1938) is used for the relatively short dwell times. It assumes that the scattering pattern within the LES cell remains “frozen” and is advected in time by the local velocity field; C_n^2 is assumed to be constant over the dwell time. If the dwell time is greater than the LES time step, a proper use of Eq. (3.29) necessitates the interpolation of the values of n and the velocity field for the time corresponding to the radar pulses (Muschinski et al., 1999).

The model of Muschinski uses two main hypotheses:

- First, the initial phases φ_{0p} of the LES grid cell contributing to the sum over the radar cell are statistically independent.
- Second, the LES sub-grid turbulence obeys the Kolmogorov scaling, i.e. it belongs to the inertial subrange of the turbulence spectrum, at the Bragg scale.

The first hypothesis is verified because the distance between LES grid points is large with respect to the wavelength. The second hypothesis is valid for the frequency of 915 MHz, since for this frequency the Bragg wavelength is of the order of 16 cm, which belongs to the inertial subrange of turbulence. However, for the X-band radar model used in this work, half the radar wavelength is about 1.6 cm. This is of the order of magnitude of the smallest inhomogeneities of the refractive index in clear air that are generally considered as having dimensions between a few centimeters and a few millimeters. This necessitates a careful estimation of the inner scale of turbulence and the refractive index variance spectrum as it is proposed in Section 2.2.3. The amplitude of the time series in Eq. (3.29) is directly proportional to the square root of the refractive index spectrum (Muschinski et al., 1999). Therefore, taking into account the full form of the refractive index spectrum developed in Section 2.2.3, for Tatarskii’s cut-off (Eq. (2.30)) , the formula to calculate the backscattered signal for the X-band becomes:

$$I(t_0 + kT) = A \sqrt{0.033 k_B^{-11/3}} \sum_{p=1}^N W_p \sqrt{[C_n^2(t_0 + kT)]_p} V_p \cdot e^{-k_B^2/k_m^2} e^{-j\varphi_p(t_0 + kT)}, \quad (3.30)$$

where k_m is the wavenumber that corresponds to the smallest inhomogeneities in the velocity field of the turbulence (see Section 2.2.3). For the Hill’s bump, the formula to calculate the backscattered signal for the X-band is:

$$I(t_0 + kT) = A \sqrt{0.033 k_B^{-11/3}} \sum_{p=1}^N W_p \sqrt{[C_n^2(t_0 + kT)]_p} V_p \cdot g(\eta_0 k_B) e^{-j\varphi_p(t_0 + kT)}, \quad (3.31)$$

where attenuation factor $g(\eta_0 k_B)$ is calculated with Eq. (2.32).

As the LES output data are 3D snapshots of meteorological parameters at discrete time values, time t in Eqs. (3.29) is replaced by a discrete values of kT , where T is the pulse repetition time of the radar; the initial time t_0 corresponds to the time of the given LES snapshot.

Attenuation of the refractive index spectrum in the transition zone between the inertial subrange and viscous dissipative range (see Section 2.2.3) causes a

significant decrease of the power of the backscattered radar signal. An analysis of the impact of the respective values of inner scale and Bragg wavelength on the signal detected by the radar is performed in Chapter 6.

Calculation of the phase of the backscattered EM pulses

The phase contribution of the p^{th} LES grid cell to the total sum in Eqs. (3.29) and (3.30) for the k^{th} pulse in the burst is calculated as follows

$$\varphi_p(t_0 + kT) = \varphi_{0p} + \mathbf{k}_{Bp} \cdot \mathbf{v}_p(t_0)kT, \quad (3.32)$$

where t_0 is the time of the LES snapshot, φ_{0p} is the initial phase, $\mathbf{k}_{Bp}$ is the Bragg wavenumber vector directed from the radar antenna center to the p^{th} LES grid node, $\mathbf{v}_p(t_0)$ is the local velocity at the LES grid node at t_0 , kT is the time shift from the initial time of the LES snapshot, t_0 .

The calculation of the time-series of the scattered signal with Eqs. (3.29) and (3.30) is based on the assumption that all Bragg's scatterers behave like point-scatterers with infinite lifetime. However, the refractive index irregularity pattern consists of a continuous turbulent field that is reorganized after some time period at the Bragg length scale ($\lambda/2$). At large scales, eddies of size l have characteristic velocity $u(l)$ and time scale $\tau(l) = l/u(l)$. The scale $\tau(l)$ characterizes an eddy turnover time after which the eddy loses a significant fraction of its kinetic energy that transfers further to the eddies of smaller sizes. The energy transfer rate is defined as ε (energy dissipation rate) (Taylor, 1935):

$$\varepsilon \propto \frac{u^2(l)}{\tau} = \frac{u^3(l)}{l}. \quad (3.33)$$

From Eq. (3.33), it follows that within the LES cell having a size of Δ , the characteristic velocity has values of order of $(\varepsilon\Delta)^{1/3}$. This velocity is the r.m.s. velocity of the turbulent motion within the cell; it indicates the speed at which the scattering template is reorganized within the LES cell. In view of that, proper use of Eq. (3.32) necessitates a random change of the phases of Bragg scatterers after a timescale of the order of $\lambda/(\varepsilon\Delta)^{1/3}$. Muschinski et al. (1999) estimate the upper boundary of this time during which the Bragg scatterers pattern with a scale Δ remains coherent. The latter is called the “phase correlation timescale”, τ_φ , and is estimated as the time interval after which the r.m.s. phase difference between two scatterers separated by a distance Δ is equal to π :

$$\tau_\varphi(\Delta) = 0.16 \frac{\lambda}{(\varepsilon \Delta)^{1/3}}. \quad (3.34)$$

For the three LES cases (Table 2.2), the spatial resolution of $\Delta \simeq 78$ cm and the wavelength of 3.16 cm, the phase correlation timescale τ_φ corresponding to the forcing background turbulence is shown in Table 3.5 for the LES cases with the WV and in Table 3.4 for LES cases with clear air turbulence only.

As discussed before in Section 2.2.3, the fully developed turbulent WV have much higher values of ε than the one used as the forcing background turbulence. For all cases, the mean value of ε increases to $3.9 \cdot 10^{-3} - 8 \cdot 10^{-3} \text{ m}^2/\text{s}^3$, which corresponds to τ_φ in the range from 35 to 27 ms.

As seen from Tables 3.5 and 3.4, the dwell time used for detection of the WV (see Table 3.2) does not exceed the lowest value of τ_ϕ presented by the simulated LES cases; however for the estimation of the turbulence intensities, long dwell times are employed by the radar models used (see Table 3.1). If the dwell time is greater than τ_ϕ , then it is needed to reset the phases ϕ_{0p} (see Eq. (3.32)) of the first pulse in the burst for which $kT > \tau_\phi$. After that, the phases of the following pulses are calculated using newly generated values of ϕ_{0p} until the next τ_ϕ interval is lapsed.

Table 3.4: Estimation of the phase correlation timescale (τ_ϕ) and the inner scale (l_0) for the LES cases without WV. Case t2, t4, and t5 are used as initial forcing turbulence for Cases 1, 2, and 3, with WV, respectively.

LES case	$\epsilon^{1/3}$, $\text{m}^{2/3}\text{s}^{-1}$	τ_ϕ , ms
Cases t1, t4, t6	0.022	255
Cases t2, t5	0.08	69
Case t3	0.215	26
Case t7	0.15	37

Table 3.5: Estimation of the phase correlation timescale (τ_ϕ) and the inner scale (l_0) for the LES cases with WV.

LES case	$\epsilon^{1/3}$, $\text{m}^{2/3}\text{s}^{-1}$	τ_ϕ , ms
Cases 1, 3 (initial turbulence)	0.08	69
Case 2 (initial turbulence)	0.022	254
Cases 1, 3 (developed turb. WV)	0.2–0.165	27–33
Case 2 (developed turb. WV)	0.195–0.158	28–35

The methodology developed in this section is based on the use of the LES simulations of the turbulent stratified atmosphere with and without the WVs. It allows simulation of the complex-valued time series of the signal backscattered on the refractive index irregularities caused by clear-air turbulence or turbulence induced by the WV. The subsequent analysis of the backscattered signal is provided in Chapter 4.

Comparison of Tatarskii's cut-off and Hill's bump

To compare two forms of the refractive index spectrum, the attenuation factors were calculated for all LES nodes within a given radar resolution volume with Eq. (2.32) and the exponential attenuation from Eq. (2.30) for Hill's bump and Tatarskii's cut-off respectively. The geometry configuration of the vertical profiler is used (see Section 3.2.1). The histograms showing the distribution of the attenuation factors within a given radar cell are shown in Fig. 3.11. As seen from the figures, the difference between Tatarskii's cut-off and Hill's bump is larger for higher levels of ϵ . The so-called bump in Hill's spectral model results in increase of the backscattered signal's amplitude by approximately 30% relative to the Tatarskii's cut-off (for cases t3 and t7). Fig. 3.12 shows an

example of the time series of the backscattered signal calculated with Eq. (3.31) for Case t7.

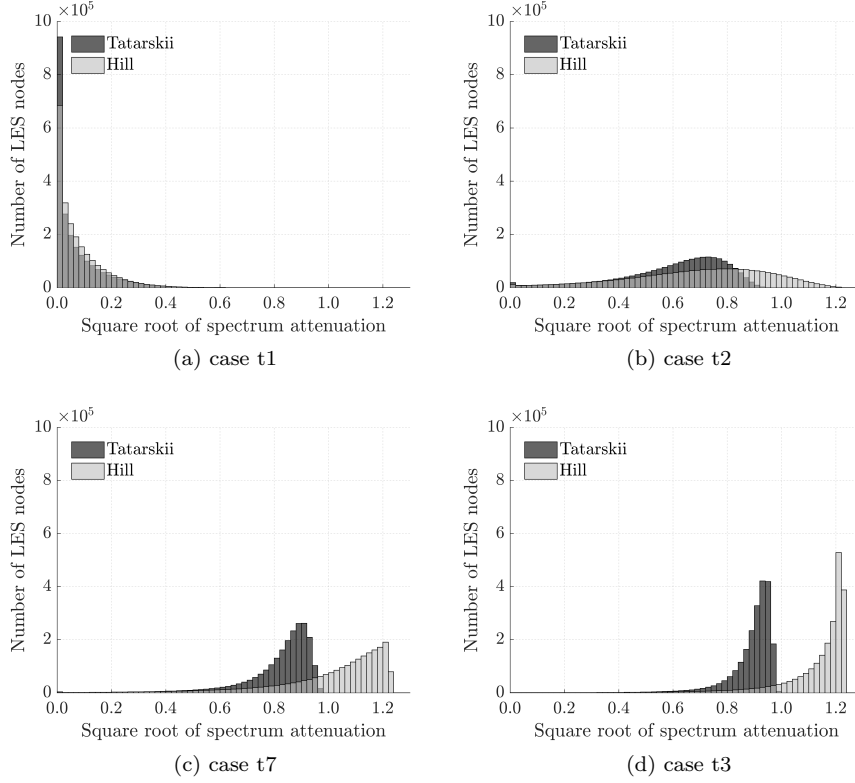


Figure 3.11: Distribution of the attenuation factors for a given radar resolution volume.

3.2.6 Simulation of the backscattered signal's Doppler spectrum in the presence of rain

For frequencies higher than 1 GHz, the backscattering of electromagnetic waves on raindrops dominates over the backscattering on atmospheric turbulence. The signal backscattered on raindrops, which are captured by the WV, can be used for detection of the WV and estimation of its parameters (Barbaresco et al., 2016; Liu et al., 2013).

This section describes how the Doppler spectrum of the radar signal backscattered on the raindrops in the WV is simulated, using LES output data.

Since LES provides raindrops concentration at LES nodes, without the information about each particular raindrop, the direct simulation of the time series of the backscattered signal is impossible. The synthesis of the Doppler spectrum with LES data is explained in the present section.

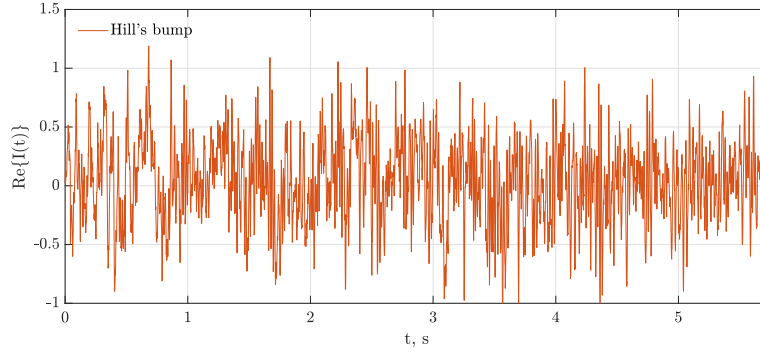


Figure 3.12: Time series of the backscattered signal simulated for the vertical profiler using LES case t7.

The Doppler spectrum of the scattered signal, $S(r_k, v_n)$, for a radar resolution volume located at the distance r_k is estimated for each Doppler velocity v_n using the radar equation (see Eq. 3.1)

$$S(r_k, v_n) = A\sigma_{k,v_n} , \quad (3.35)$$

where σ_{k,v_n} is the RCS of all raindrops moving with a radial velocity v_n within a given radar resolution volume k and A is the amplitude coefficient given by

$$A = \frac{P_t G^2 \lambda^2 N_p}{(4\pi)^3 R_k^4 L} , \quad (3.36)$$

where P_t is the transmitter peak power, G is the antenna gain, N_p is the number of pulses in a burst, R_k is the distance between the radar and the center of the k^{th} radar cell, and L is the attenuation of the signal caused by the internal and external losses.

Attenuation of the radar signal due to the presence of rain on the signal propagation path is calculated as (ITU-R, 2005)

$$L_R = kR_r^\alpha \quad (3.37)$$

where α and k are coefficients that depend on the frequency and polarization of the emitted wave, R_r is the rain rate. L_R is given as the attenuation of the signal in dB on 1 km of the path (specific attenuation). For $f = 10$ GHz, $\alpha = 1.2156$ and $k = 0.01129$. As an example, the attenuation of the signal at 9.5 GHz is equal to 0.08 and 0.19 dB/km for rain rates of 5 and 10 mm/h respectively. This allows to assume that the attenuation of the signal due to rain is negligible, except for very high rain rates.

The RCS of the raindrops moving with a radial speed v_n in the k^{th} radar cell is calculated as (Liu et al., 2013; Doviak and Zrnić, 1984)

$$\sigma_{k,v_n} = \sum_{D=D_{min}}^{D_{max}} \sigma(D) N(D) \Delta D \sum_i \sum_j \eta_N(D, i, j) \quad (3.38)$$

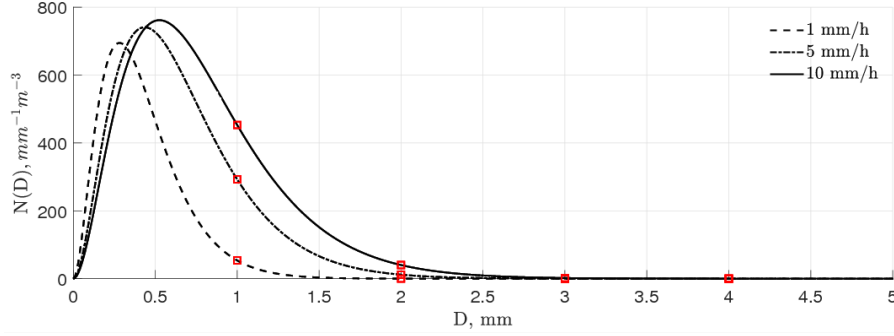


Figure 3.13: Raindrop size distributions for a few selected rain rates. Square markers label the values of raindrop diameters simulated in the LES.

where $\sigma(D)$ is the RCS of a raindrop with diameter D . The product of $N(D)\Delta D$ represents the number of raindrops with a diameter between $D - \Delta D/2$ and $D + \Delta D/2$ per unit volume of still air, i, j are the indices of the LES grid cells within the k^{th} radar cell, $\eta_N(D, i, j)$ is the relative concentration of raindrops having diameter between $D - \Delta D/2$ and $D + \Delta D/2$. Summation in Eq. (3.38) is performed over all diameter classes provided by the LES.

So, the value of the Doppler spectrum of the signal backscattered on the raindrops located within k^{th} radar cell is calculated for each velocity bin v_n as follows

$$S(r_k, v_n) = A \sum_{D=D_{min}}^{D_{max}} \sigma(D) N(D) \Delta D \sum_i \sum_j \eta_N(D, i, j) \quad (3.39)$$

For the LES generated in this work, due to the budget constraints of the SESAR project, only four raindrops diameters 1, 2, 3, and 4 mm are simulated to approximate the raindrop size distribution $N(D)$. This implies that $\Delta D = 1$ mm. As referred by Liu et al. (2013), the most common distribution for the European temperate climate is the Gamma distribution (given by Atlas and Ulbrich (1974)):

$$N(D) = N_0 D^2 e^{-\Lambda D}, \quad (3.40)$$

with the calibration of Jiang et al. (1997) for European climate : $\mu = 2$, $N_{R0} = 64500 R^{-0.5} [\text{mm}^{-3} \text{m}^{-3}]$, $\Lambda = 7.09 R^{-0.27} [\text{mm}^{-1}]$, where R_r is the rain rate (mm/h). Fig. 3.13 provides an example of raindrop size distributions for rain rates of 1, 5 and 10 mm/h. It has to be taken into account that larger raindrops have larger RCS; therefore, their contribution into the final sum in Eq. (3.39) is greater than from the smaller raindrops. That is why, the LES is set to simulate raindrop concentration for the bigger raindrops. The model developed provides a possibility to simulate any specified rain rate. It should be noted that choosing another distribution of the raindrop size distribution only has a second-order effect on the simulated Doppler spectrum.

The relative concentration of raindrops $\eta_N(D, i, j)$ provided by LES is shown in Figs. 2.19 – 2.20 for diameter classes used, for two instants. It is

seen that the concentration of bigger raindrops is less affected by the turbulence than the concentration of the smaller raindrops. The procedure of the Doppler spectrum simulation can be summarized into the following steps:

1. Initialization of the radar parameters and its location; initialization of the LES cube location relative to the radar.
2. Extraction of the LES nodes located within a given radar resolution volume.
3. Computation of the radial velocity and RCS for all raindrop classes at each LES node within a given radar resolution volume.
4. Calculation of the Doppler spectrum of the backscattered radar signal for the k^{th} radar cell with Eq. (3.39) for the selected value of the rain rate.

3.3 Conclusions

In clear air, the aircraft wake vortices create cascades of turbulent energy similar to the clear atmosphere turbulence. Fluctuation of the refractive index in the turbulent medium causes a Bragg scattering of the electromagnetic waves emitted by the radar.

This section presented the methodology to simulate the backscattered signal based on the use of the LES simulations of the turbulent stratified atmosphere with and without the WVs. It allows simulation of the complex-valued time series of the signal backscattered on the refractive index irregularities. A special attention is given to the fact that, for the X-band radar wavelength, the Bragg length scale lies within a transition region between the inertial subrange and dissipative range of the atmospheric turbulence spectrum. In this region, turbulence transforms into heat due to the viscosity forces. This leads to a significant decrease of the backscattered signal power. Nevertheless, the presence of the WV increases the intensity of the turbulence; it results in a significantly higher level of C_n^2 within the WV system comparing to the clear air turbulence and therefore a higher level of the backscattered EM power. For both spectral closure forms: Tatarskii's cut-off and Hill's bump, the level of the refractive index spectrum at the Bragg length scale is of the same order as in the inertial subrange. For Hill's spectrum, the spectrum at the Bragg scale exceeds its level in the inertial subrange due to the presence of the so-called bump. The spectral analysis of the backscattered signal is provided in the following chapter.

In rain, the scattering of EM waves occurs on the rain droplets whose RCS heavily depends on their diameter. LES provides the raindrops concentration in the WV relative to still air, allowing a direct synthesis of the Doppler spectrum of the backscattered radar signal. The LES with several raindrop diameter classes provides flexibility to simulate different intensities of the rain rate.

For all weather conditions, the LES allows estimation of the WV RCS which is used to evaluate the signal-to-noise ratio. This enables the analysis of the detectability of the WV under different weather conditions and optimization of the radar parameters and geometrical configurations of the measurements.

Chapter 4

Doppler spectrum evaluation and processing

This chapter provides the theoretical basis needed to retrieve the parameters of the atmosphere and the WV from the radar measurements: the energy dissipation rate of the atmospheric turbulence, which characterizes its intensity, and the WV circulation, which characterizes the strength of the wind field induced by the WVs. The retrieval of these parameters is based on the evaluation of the Doppler spectrum of the backscattered signal.

Section 4.1 develops the estimation of the Doppler spectrum and considers specific aspects related to the limits of the Doppler spectrum resolution caused by the unresolved sub-grid scales of the LES. Sections 4.2 – 4.4 provide examples of the Doppler spectra simulated for clear air turbulence, of the WVs in clear atmosphere, and the WVs in the presence of rain. Section 4.5 proposes a method to retrieve the energy dissipation rate of the atmospheric turbulence and section 4.6 describes two methods employed for the estimation of the wake vortex circulation.

4.1 Spectral analysis of the backscattered signal

In order to extract the distribution of the velocities within the radar resolution volume, it is needed to calculate the power spectral density of the backscattered signal which is generally called the Doppler spectrum. The power spectral density (PSD), or Doppler spectrum, of a stationary random process can be estimated with nonparametric, parametric or subspace methods. In this work, only nonparametric methods are considered, more specifically the periodogram and the modified periodogram methods. The circulation retrieval is based on the use of the WV Doppler spectrum.

4.1.1 Estimation of the Doppler spectrum from generated time series

The power spectral density, or Doppler spectrum, is estimated by calculation of a periodogram (see, for instance, (Bringi and Chandrasekar, 2001)), with the following formula:

$$\hat{S}(f_k) = \frac{1}{F_s N} \left| \sum_{n=0}^{N-1} s(n) e^{-j2\pi n k/N} \right|^2, \quad (4.1)$$

where $s(n)$ is the time series of the signal, N is the number of samples of the signal, $f_k = F_s k/N$ are the frequencies at which the Doppler spectrum is estimated, F_s is the sampling frequency that is equal to the pulse repetition frequency in a pulse-Doppler radar.

Fig. 4.1 shows an example of a Doppler spectrum calculated with Eq. (4.1). This section uses LES Case 1 (see Table 2.3) to illustrate the estimation of the Doppler spectrum. The abscissa in Fig. 4.1 and in all further representations of the Doppler spectra are shown in the form of a Doppler velocity:

$$V_{Doppler} = \frac{f_k \lambda}{2}, \quad (4.2)$$

where λ is the radar wavelength. Linear and logarithmic scale representations are used to enhance the main peak and the sidelobes, respectively. To concentrate on the analysis of the Doppler spectra produced by the WV, the noise is not added to the time series of the simulated backscattered signal.

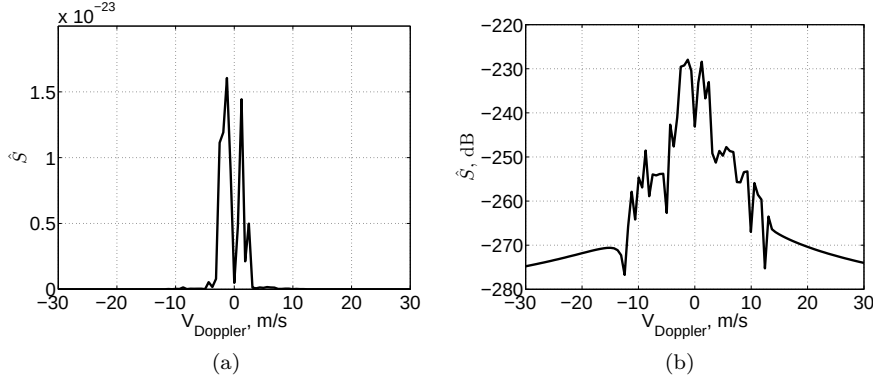


Figure 4.1: Linear (a) and logarithmic (b) representations of the WV Doppler spectrum.

The periodogram provides an estimate of the true PSD based on a limited number of samples of the random process. The quality of this estimate is assessed via its consistency (Hayes, 1996). Applied to the PSD, the estimate is consistent if it is asymptotically unbiased:

$$\lim_{N \rightarrow \infty} \mathbf{E} \left\{ \hat{S}(f_k) \right\} = S(f_k), \quad (4.3)$$

where $\mathbf{E}\{\cdot\}$ denotes a statistical averaging over the ensemble of realizations, and the variance of the estimate at a given frequency is zero:

$$\lim_{N \rightarrow \infty} \text{Var} \left\{ \hat{S}(f_k) \right\} = 0. \quad (4.4)$$

Let's first examine the bias. The expected value of the periodogram is equal to the convolution of the true PSD with the Fourier transform of the Bartlett window (triangular), $W_b(f_k)$ (Hayes, 1996):

$$\mathbf{E} \left\{ \hat{S}(f_k) \right\} = \frac{1}{2\pi} S(f_k) * W_b(f_k), \quad (4.5)$$

where $*$ denotes the convolution operation.

Windowing causes an effect called spectral leakage: estimates of the PSD at a given frequency contain contributions from the other spectral components of the signal due to the finite level of the window's sidelobes (Hayes, 1996). Therefore, the periodogram estimate is biased. Nevertheless, as the number of the samples $N \rightarrow \infty$, the shape of the Bartlett window spectrum converges toward an impulse and the periodogram provides asymptotically unbiased estimation of the true power spectral density of the random process. Fig. 4.2 shows the FFT of $W_b(f_k)$ for two durations of the window: $N = 512$ and $N = 512000$.

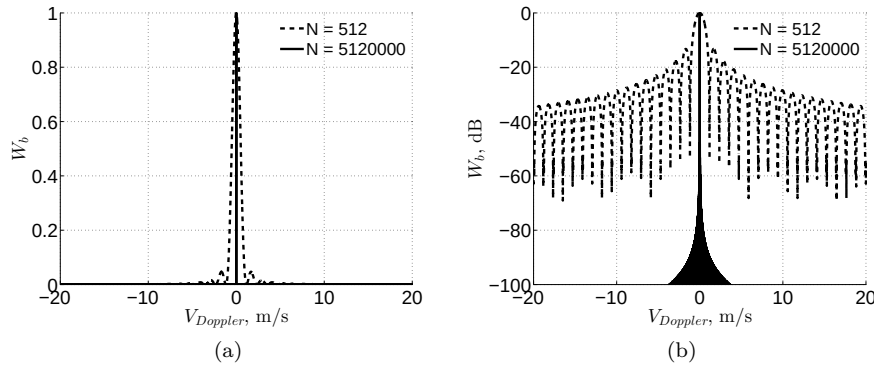


Figure 4.2: Linear (a) and logarithmic (b) representations of the Bartlett window spectrum.

In radar applications, however, N is limited: mechanical scanning radars (mechanical rotation of the antenna) can only observe the scattering of the pulses from a given direction during a limited amount of time. Moreover, the scattering pattern of the refractive index inhomogeneities within the radar resolution volume is stationary over a limited duration, depending on the state of the atmosphere and the age of the WVs. To mitigate the bias of the estimate, a modified periodogram is used (Hayes, 1996): the signal is multiplied by a window function in the time-domain before computing the FFT, in order to

smooth the edges of the signal:

$$\hat{S}(f_k) = \frac{1}{F_s N} \left| \sum_{n=0}^{N-1} w(n) s(n) e^{-j2\pi nk/N} \right|^2, \quad (4.6)$$

where $w(n)$ is the window function. For a rectangular window, Eq. (4.6) reduces to Eq. (4.1). Fig. 4.3 compares the Doppler spectrum calculated with a rectangular window and a Hamming window. The side lobes of the Hamming window are lower than those of the rectangular window. The windowing effect will be further discussed in Chapter 5.

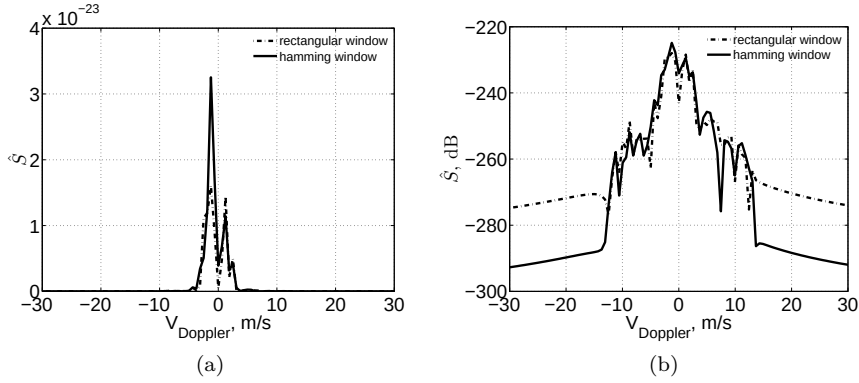


Figure 4.3: Linear (a) and logarithmic (b) representations of the WV Doppler spectrum calculated with the modified periodogram method using a hamming window.

The second consistency criterion concerns the variance of the estimator. For the periodogram, the variance of the estimated power spectrum at a specific frequency does not go to zero as $N \rightarrow \infty$ (Hayes, 1996). The variance of the periodogram is equal to the average power spectral level at each frequency f_k , irrespective of the duration of the dwell time (Hayes, 1996; Muschinski et al., 1999):

$$\lim_{N \rightarrow \infty} \text{Var} \left\{ \hat{S}(f_k) \right\} = S^2(f_k). \quad (4.7)$$

This explains the “spiky” nature of the PSD estimated with the periodogram. Spectral estimates at a given frequency f_k are exponentially distributed around the mean value: they obey a chi-square distribution with 2 degrees of freedom (real and imaginary parts of the signal) (Blackman and Tukey, 1958).

A way to overcome this problem in real radar measurements is to average in time consecutive Doppler spectra obtained for the same radar resolution volume. In the simulation, a consistent estimate of the Doppler spectrum (with a zero variance of the estimates) can be obtained through an ensemble average of spectra since the simulator provides the possibility to generate various realizations of the scattered signal for the same radar resolution volume and for the same period of time. To obtain this ensemble average, the scattered signal

is computed M times (M is the size of the ensemble), using a single set of LES nodes within the radar cell. The initial phases of the contributions of the LES nodes to the backscattered signal are randomly generated for each realization (Muschinski et al., 1999). The Doppler spectrum is calculated for each realization and the obtained Doppler spectra are averaged. The comparison between the results of the ensemble averaging and a single realization is shown in Fig. 4.4. Ensemble averaging provides a smooth estimate of the Doppler

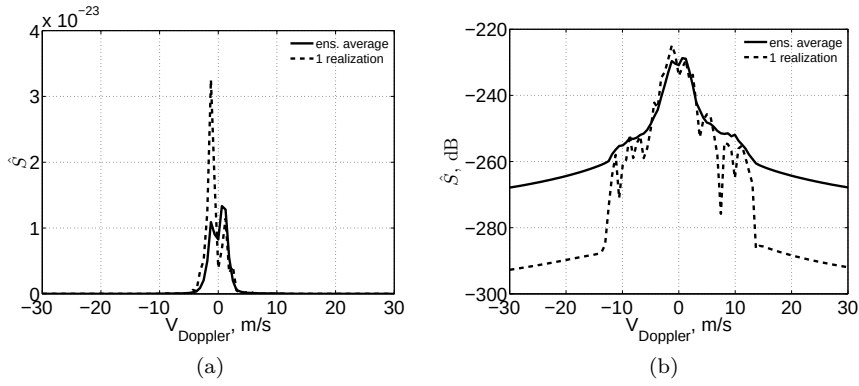


Figure 4.4: Linear (a) and logarithmic (b) representations of the Doppler spectrum calculated with the modified periodogram method (Hamming window) for ensemble averaging (solid line) and a single realization (dashed line).

spectrum at the price of an increase of the window's sidelobes. A consistent estimate of the Doppler spectra is obtained by the generation of various realizations of the scattered signal and then averaging of the Doppler spectra calculated for each realization. Each simulated time series is obtained with randomly generated initial phase φ_{0p} .

Nevertheless, the generation of the ensemble of scattered signal realizations is computationally expensive. Muschinski et al. (1999) proposed a closed form for the consistent estimation of the Doppler spectrum (with a zero variance), based only on the LES parameters:

$$S(\omega_n) = \sum_{p=1}^N a_p^2 Z_{np} Z_{np}^* , \quad (4.8)$$

where $a_p = \sqrt{[C_n^2]_p V_p g(\eta_0 k_B)}$ (C_n^2 is the refractive index structure constant and V_p is the volume of the p^{th} LES grid cell), $\omega_n = 2\pi n/T_d$ are the frequencies of the Fourier series for $n = \dots, -2, -1, 0, 1, 2, \dots$, T_d is the dwell time, Z_{np} are the Fourier amplitudes:

$$Z_{np} Z_{np}^* = \begin{cases} \frac{2}{((\omega_p - \omega_n)T_d)^2} [1 - \cos((\omega_p - \omega_n)T_d)] , & \omega_p \neq \omega_n \\ 1 , & \omega_p = \omega_n \end{cases} \quad (4.9)$$

where $\omega_p = \mathbf{k}_{Bp} \cdot \mathbf{v}_p$ ($\mathbf{k}_{Bp}$ is the Bragg wavenumber vector directed from the radar antenna center to the center of the p^{th} LES grid cell, $\mathbf{v}_p(t_0)$ is the

velocity of the LES grid cell at t_0). After a simple trigonometric transformation, Eq. (4.9) becomes:

$$Z_{np}Z_{np}^* = \begin{cases} \text{sinc}^2\left(\frac{(w_p - w_n)T_d}{2}\right), & \omega_p \neq \omega_n \\ 1, & \omega_p = \omega_n \end{cases} \quad (4.10)$$

For 512 pulses used to compute the Doppler spectra, $n = -255, \dots, -2, -1, 0, 1, 2, \dots, 256$ and $T_d = 512 \cdot T$, where T is the period of pulses repetition in a train of pulses (see Tables 3.1, 3.2, and 3.3). According to Eq. (4.10), the contribution of each LES node to the Doppler spectrum is weighted by a sinc function squared. Fig. 4.5 shows Eq. (4.10) computed for $n = 256$ and $n = 275$. The dot markers in the figure correspond to the contribution from LES nodes. For $n = 256$ (Fig. 4.5a), there are many LES nodes having Doppler velocity in range from -5 to 3 m/s, so the dots fuse in a solid curve.

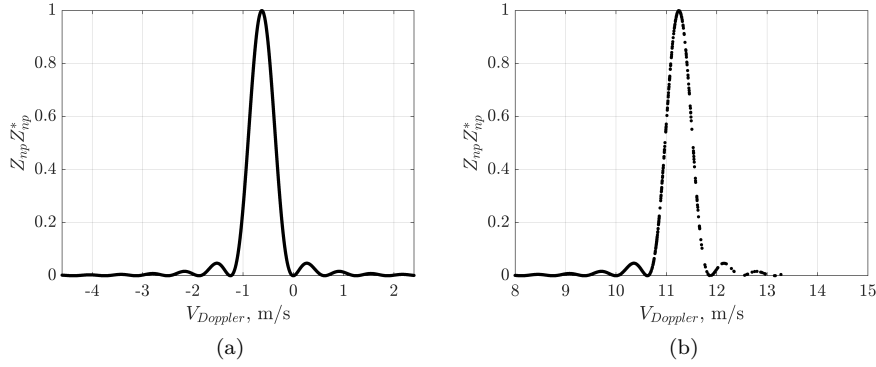


Figure 4.5: $Z_{np}Z_{np}^*$ computed for (a) $n = 256$ and (b) $n = 275$. Dot markers represent contribution from the LES nodes.

Note that in Fig.4.5, the sinc function is centered at w_n for a given n and the abscissa is presented in a form of the Doppler velocity (see Eq. (3.12)):

$$V_{Doppler} = w_p \cdot \frac{1}{2\pi} \cdot \frac{\lambda}{2}. \quad (4.11)$$

The two Doppler spectrum estimators are compared: the Doppler spectrum calculated directly with the closed form Eq. (4.8) is compared with the ensemble average of the Doppler spectra calculated over 100 independent time series of the radar signal. These time series are generated with Eq. (3.31) using randomly generated initial phases φ_{0p} in Eq. (3.32) to simulate 100 realizations of time series. The Doppler spectra are estimated for each realization and averaged. Fig. 4.6 clearly shows that the ensemble averaged Doppler spectrum is not distinguishable from the spectrum calculated with Eq. (4.8) and has a reduced variance with respect to a single realization. In the following sections, the closed form Doppler spectrum is used.

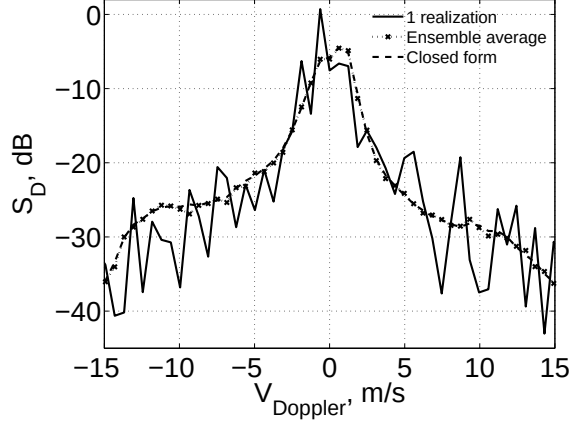


Figure 4.6: Doppler spectrum of the scattered signal (Case 1). The ensemble average is calculated over 100 realizations. The closed form represents the Doppler spectrum estimated with Eq. (4.8). All Doppler spectra are normalized to the area under their graph.

4.1.2 Estimation of the sub-grid turbulence contribution to the Doppler spectrum width.

As LES does not provide the information needed to calculate the contribution of the sub-grid turbulence to the simulated Doppler spectrum, Muschinski et al. (1999) show that this contribution to the Doppler spectrum width can be estimated as follows

$$\Delta\sigma_v = \frac{2\pi}{k_B} = \frac{\lambda}{2\tau_\varphi} = 3.12(\varepsilon \Delta)^{1/3}, \quad (4.12)$$

where k_B is the Bragg wave number, τ_{phi} is the correlation time scale specified in Section 3.2.5, ε is the energy dissipation rate, and Δ is the linear size of the LES cube.

Tables 4.2-4.1 show the broadening of the Doppler spectrum ($\Delta\sigma_v$) caused by the sub-grid turbulence. These estimates are given for the background forcing turbulence as well as for the strong increase of the EDR caused by the developed turbulent WV, which results in an increase of $\Delta\sigma_v$ for all cases.

The resolution of the Doppler spectrum is

$$\Delta v_r = \frac{\lambda}{2T_d}, \quad (4.13)$$

where T_d is the dwell time used for the Doppler spectrum calculation. Comparing Eqs. (4.12) and (4.13), it can be concluded that, in order to keep the contribution of the sub-grid turbulence smaller than the Doppler spectrum resolution, the dwell time should be less than the phase correlation timescale (see Table 3.5). A further increase of the Doppler spectrum resolution is useless

because the sub-grid turbulence is not resolved by the LES. So, for the radar parameters used for simulation of the WV detection (Table 3.2), the width of the velocity bin $\Delta v_r = 0.62$ m/s is greater than broadening of the Doppler spectrum caused by the sub-grid turbulence contribution in any of the LES cases with the WV (see Table 2.3).

For a vertical profiler, using a long dwell time to increase the SNR, the resolution of the simulated Doppler spectra can be far less than the broadening.

Table 4.1: Estimation of the broadening of the Doppler spectrum ($\Delta\sigma_v$) due to the sub-grid turbulence contribution for the LES cases without WV. Case t2, t4, and t5 are used as initial turbulence for Cases 1, 2, and 3 respectively.

LES case	$\epsilon^{1/3}$, $\text{m}^{2/3}\text{s}^{-1}$	$\Delta\sigma_v$, m/s
Cases t1, t4, t6	0.022	0.06
Cases t2, t5	0.08	0.23
Case t3	0.215	0.62
Case t7	0.15	0.43

Table 4.2: Estimation of the broadening of the Doppler spectrum ($\Delta\sigma_v$) due to the sub-grid turbulence contribution for the LES cases with WV.

LES case	$\epsilon^{1/3}$, $\text{m}^{2/3}\text{s}^{-1}$	$\Delta\sigma_v$, m/s
Cases 1, 3 (initial turbulence)	0.08	0.23
Case 2 (initial turbulence)	0.022	0.06
Cases 1, 3 (developed turb. WV)	0.2–0.165	0.57–0.47
Case 2 (developed turb. WV)	0.195–0.158	0.56–0.45

4.1.3 Doppler spectrum moments

The Doppler spectrum provides a power weighted estimate of the radial velocities distribution within a given radar resolution volume. This distribution is characterized by the moments of the estimated Doppler spectrum, namely, mean power, mean Doppler velocity, and variance of the Doppler velocity usually called spectrum width. Before calculation of the mean and variance of Doppler velocity spectrum, the spectrum should be normalized by its energy. So, the expression of the normalized Doppler spectrum at a given radial velocity v_n is:

$$\bar{S}(r_k, v_n) = \frac{S(r_k, v_n)}{\sum_{n=1}^{N_p} S(r_k, v_n) \cdot \Delta v} , \quad (4.14)$$

where range r_k specifies the radar resolution volume for which the Doppler spectrum is calculated, Δv is the velocity resolution of the Doppler spectrum and N_p is the total number of pulses in the burst of the emitted radar signal.

The first moment of the normalized Doppler spectrum gives the mean radial

velocity:

$$\bar{v}_r = \sum_{n=1}^{N_p} v_n \bar{S}(r_k, v_n) \cdot \Delta v, \quad (4.15)$$

and the second central moment gives the width of the Doppler spectrum:

$$\sigma_v^2 = \sum_{n=1}^{N_p} (v_n - \bar{v}_r)^2 \bar{S}(r_k, v_n) \cdot \Delta v. \quad (4.16)$$

These first three moments of the radial velocities distribution allow the analysis of the parameters of the observed atmospheric phenomena, such as the turbulence intensity or the wake vortex strength. It should be noticed that, due to the presence of the noise, the Doppler spectrum moments are calculated using the values of the spectrum above a certain threshold, specified by the noise level of the radar used. The noise will significantly impact the estimated moments. An analysis of the impact of the noise on the estimated parameters is given in Chapter 5.

4.2 Doppler spectrum of clear air turbulence

The power of the backscattered signal significantly depends on the level of the refractive index spectrum at the radar Bragg wavelength (see Section 2.2.3). Fig. 4.7 provides examples of the Doppler spectra simulated with the parameters and geometry of a vertical profiler employed for the clear atmosphere turbulence detection (see Section 3.2.1).

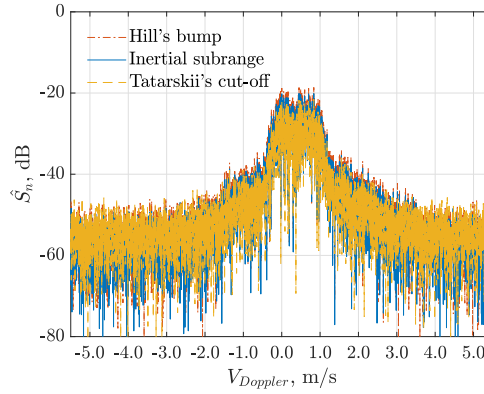


Figure 4.7: Doppler spectra of turbulence for three models of the refractive index spectrum: (blue) inertial subrange, (yellow) Tatarskii's cut-off, and (orange) Hill's bump at the transition between inertial and viscous dissipative ranges (case t2).

The three different spectra shown correspond to the different models of the refractive index spectrum used to simulate the backscattered signal: inertial

subrange (Eq. 2.22), Tatarskii's cut-off (Eq. (2.30)), and Hill's bump Eqs. (2.31) – (2.32). To evaluate the relative change of the DS level, all signals are normalized to the energy of the signal simulated assuming that the Bragg wavelength belongs to the inertial subrange. As seen from Fig. 4.7, Tatarskii's cut-off suggests that for Case t2, the decrease of the DS level is more pronounced relative to the Hill spectral model.

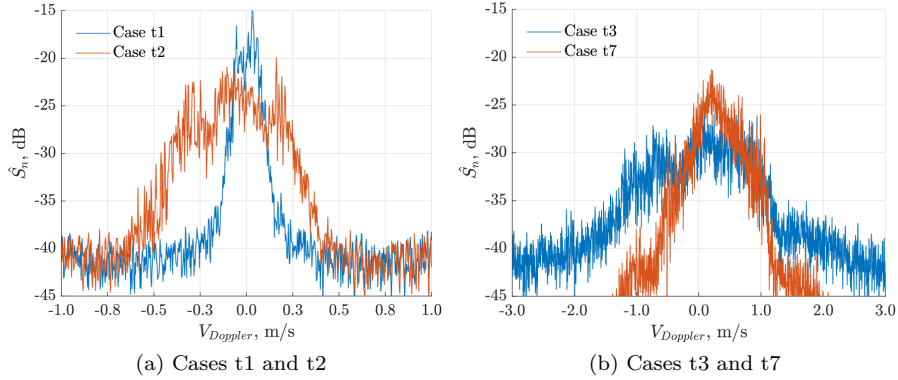


Figure 4.8: Doppler spectra of turbulence in clear air.

Doppler spectra shown in Fig. 4.7 are calculated with Eq. (4.1) and therefore are biased in terms of the variance of the spectral estimates for a given Doppler frequency. To reduce the variance of the estimates and provide a better evaluation of the DS width, a certain number of Doppler spectra consecutive in time are averaged. Fig. 4.8 shows examples of the DS averaged over 10 consecutive trains of scattered pulses. The Hill's model of the refractive index spectrum is used to produce DS for four LES cases. To facilitate the comparison between the DS widths, all simulated signals are normalized by their energies. It is seen that the width of the Doppler spectrum is proportional to the EDR of turbulence for a given LES case (see Table 2.1). The method which uses the DS width to retrieve the energy dissipation rate of the turbulence is given in Section 4.5.

4.3 Doppler spectrum of wake vortices in clear air

This section provides an example of the Doppler spectrum of the WV simulated with geometrical configuration A (see Section 3.2.1), which is illustrated in Fig. 4.9: the radar cell is aligned with the WV core center; the size of the radar cell in range is about 60 m; the half power width of the symmetrical antenna beam is 3.1° . Positioning the WV core at the center of the radar cell is chosen in order to be able to compare the WV circulation estimated from the Doppler spectrum to the exact value as measured in LES (see Section 4.6). Fig. 4.10 represents a distribution of the local velocity and C_n^2 in horizontal

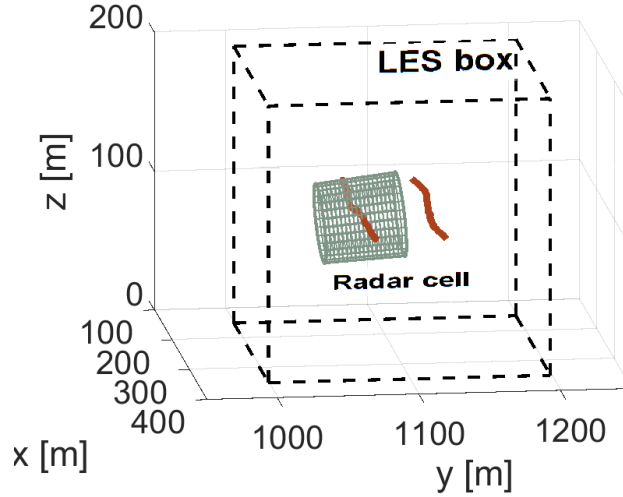


Figure 4.9: Alignment of the radar cell relative to the wake vortex core. Black dashed lines - borders of the LES box; red solid curves - positions of the wake vortex cores; gray - borders of one radar cell.

(Figs. 4.10c, 4.10d) and vertical cross-sections of the LES (Figs. 4.10a, 4.10b). Each distribution is an average over 80 adjacent LES layers taken from the center of the LES, in the Z- and X-directions for the horizontal and vertical cross-sections respectively. The borders of the radar cell centered at the initial position of the left WV (as shown in Fig. 4.9) are depicted with dashed lines.

Contributions to the Doppler spectrum width

For the radar resolution volume depicted in Fig. 4.9, a time series of scattered signal is generated. Each time series' sample corresponds to the complex amplitude of the electromagnetic pulse scattered on the refractive index inhomogeneities. The simulated radar burst contains $N = 512$ pulses, therefore 512 samples of the scattered signal are generated using one LES frame. Taylor's hypothesis of "frozen turbulence" (Taylor, 1938) is used: due to the relatively short dwell time (25.6 ms), it is assumed that the scattering pattern within the LES cell remains constant (C_n^2 is constant over the dwell time) and is advected by the local wind field during the 25,6 ms.

Fig. 4.11 represents the Doppler Spectra (ensemble averaged with applying rectangular window) of the signals backscattered by the left wake vortex in case 1 ($EDR = 5 \cdot 10^{-4} \text{ m}^2 \text{s}^{-3}$): the right part of the spectrum corresponds to the upper part of WV for which local inhomogeneities recede the radar, and to the lower part corresponds to the left part of the Doppler spectrum, for which local inhomogeneities approach the radar. The directions are opposite for the right wake vortex. The Doppler spectra of the signal scattered from the area of LES that contains the WV present distinctive features:

- The central peak of each spectrum is located at zero Doppler velocity

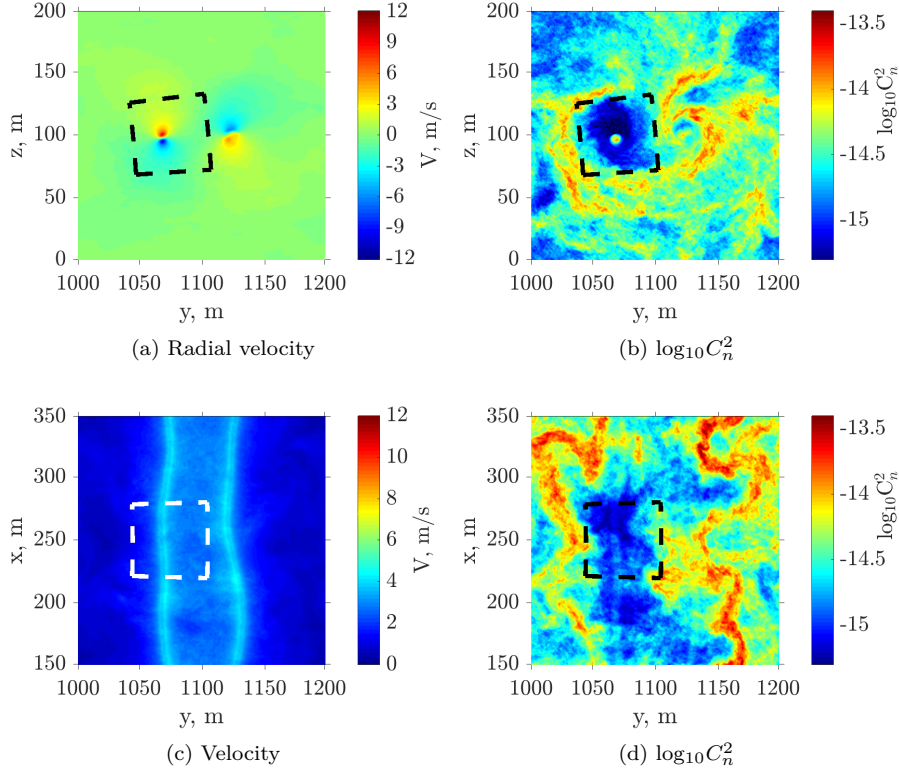


Figure 4.10: Distribution of (a) radial velocity, (b,d) C_n^2 , and the intensity of the velocity (c) for Case 1 (clear air) at $t^* = 3.75$. Borders of the radar cell are shown with dashed lines.

because the mean radial velocity is equal to zero in the LES cell. No crosswind is added to the LES wind velocity field;

- The width of the central peak (part a in Fig. 4.11), corresponding to the lowest velocities, is representing the spread due to the background turbulent field velocities and is symmetrical;
- The side parts of the spectrum, corresponding to higher velocity values, represent the distribution of the radial components of the wind field induced by the WV (parts b in Fig. 4.11). Those parts are roughly symmetric around the peak of the spectrum because each radar cell contains a single vortex.

It should be noticed that the Doppler spectra shown in Fig. 4.11 are normalized to the area under their graphs. As shown in Section 3.2.3, the RCS of the WV increases with time. This results in the increase of the scattered signal power. Consequently, the maximum value of the Doppler spectrum also increases. Normalization is done in order to correctly compare the widths of the

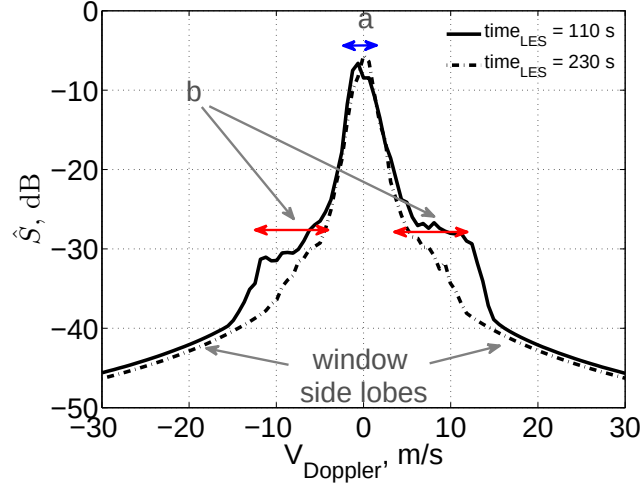


Figure 4.11: Illustration of the contributions to the Doppler spectrum.

Doppler spectra at different moments in time. Consequently, the side parts of the Doppler Spectrum contain the information about the strength of the wind field in the WVs and could therefore be used to estimate the WV circulation. Nevertheless, those spectral components lie 20 to 25 dB below the central peak. This makes their detection difficult above the noise floor.

4.4 Doppler spectrum of wake vortices in rain

The same geometrical setup as employed in the previous section is used to illustrate the DS simulated for the detection of the WV under rainy conditions. The methodology of the DS simulation is given in Section 3.2.6. Fig. 4.12 shows the radial velocity distribution in a cross-section of the LES cube with the borders of the radar cell shown with dashed lines. In order to explain the structure of the simulated DS, Fig. 4.12b uses the relative concentration of the raindrops given in Fig. 4.13 to blank the area of the LES with a near absence of raindrops. The simulated Doppler spectrum is shown in Fig. 4.14. Due to the low relative concentration (nearly absence) of the raindrops in the area with positive Doppler velocities (negative velocity on the graphs), the Doppler spectrum is not symmetric.

4.5 Retrieval of the turbulence intensity

The simulator developed in this work is validated first by a comparison of the EDR values retrieved from the simulated backscattered radar signal against the EDR values directly available from the LES. The simulator is used to check whether the measurements obtained in the X band are unbiased, and thus reliable, for a variety of EDR values and atmospheric stratification levels

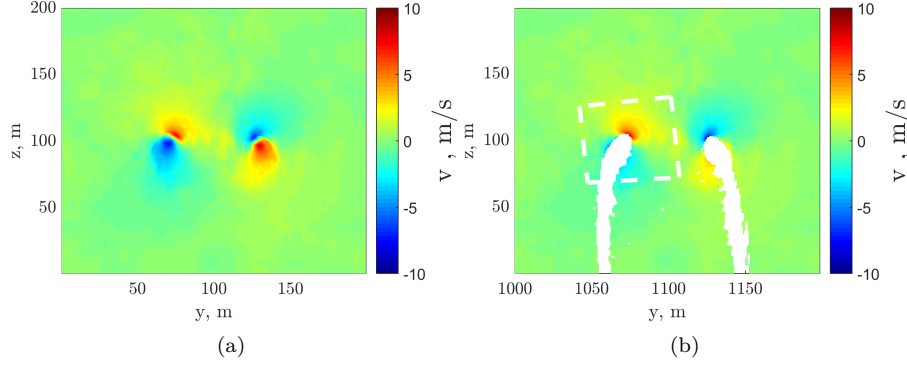


Figure 4.12: Distribution of the radial velocities in a cross-section of the LES cube: (a) without and (b) with mask calculated from the relative concentration of the raindrops.

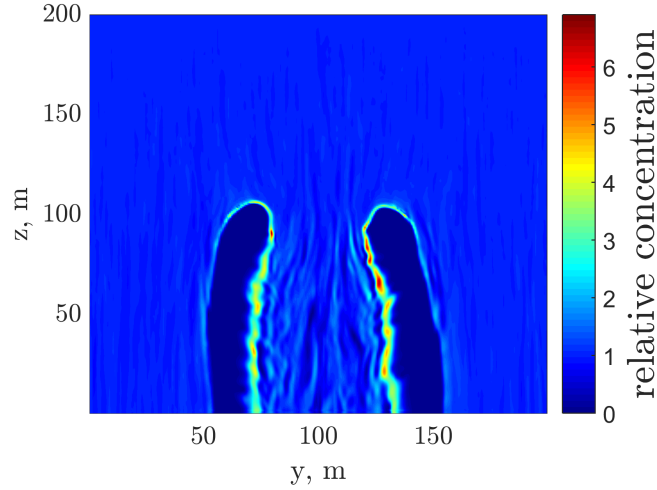


Figure 4.13: Relative concentration of the raindrops with a diameter of 2 mm.

characterized by a Brunt-Väisälä frequency. In addition, the simulations can give some confidence to affirm that the turbulence intensity values are derived from atmospheric turbulent motions and not from radar noise signals.

EDR is a measure for the viscous dissipation of TKE, which transfers from large scales to small scales within the inertial subrange via eddies (see, for instance, Pope, 2011). From Kolmogorov's hypotheses, for locally homogeneous and isotropic turbulence, the turbulent energy spectrum $E(\kappa)$ of three-dimensional wind velocities in the inertial subrange is partitioned among the eddies in a universal form (Kolmogorov, 1941; Sutton, 1953; Pope, 2011):

$$E(\kappa) = K_0 \epsilon^{2/3} \kappa^{-5/3} \quad , \quad (4.17)$$

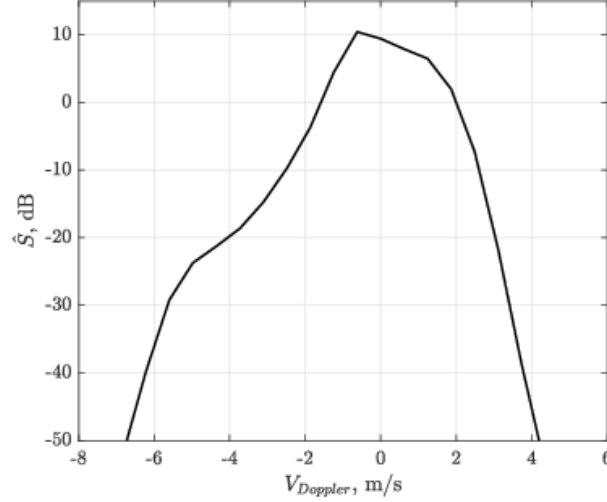


Figure 4.14: Example of the WV Doppler spectrum simulated in the presence of rain for a radar resolution volume containing a single WV.

where $\kappa = 2\pi/\lambda$ is the wavenumber with wavelength λ , ϵ is the energy dissipation rate, and K_0 is a universal Kolmogorov constant that is determined from experiments (Sreenivasan, 1995) (ranging between 1.4 and 1.8). In addition to Eq. (4.17), there are one-dimensional turbulence energy spectra, and also the second order structure function, which are provided in Section 2.2.2 with the extension to the transition zone between the inertial and viscous dissipation ranges. Integration of the turbulent energy spectrum leads to an analytic relation that is used to estimate an EDR value, based on the variance of velocities from Doppler radar measurements (Brewster and Zrnić, 1986).

In this work, the EDR is retrieved from the width of the Doppler spectrum of the backscattered signal. To retrieve the energy dissipation rate, the expression proposed by White et al. (1999) is used:

$$\epsilon_{rad} = \sigma_{turb}^3 (4\pi K_0)^{3/2} J^{-3/2}, \quad (4.18)$$

where σ_{turb} is the spread of the Doppler spectrum caused by turbulence, $K_0 = 1.6$ is the Kolmogorov constant, and J is defined as:

$$J = 12\Gamma(2/3) \int_0^{\pi/2} \int_0^{\pi/2} \sin^3(\theta) (b^2 \cos^2\theta + a^2 \sin^2\theta + \frac{L^2}{12} \sin^2\theta \cos^2\varphi)^{1/3} d\theta d\varphi, \quad (4.19)$$

where Γ is the Gamma function, $L = V_T t_d$. Here, V_T is the wind speed transversal to the radar beam and t_d is the radar dwell time. The simulated radar-based estimation of the EDR uses the calculation of the first and second moments of the scattered signal Doppler spectrum above a given threshold, estimated for the selected spectrum estimator and window (Kovalev et al., 2016).

The total broadening of the Doppler spectrum is expressed as a sum of

different terms (Hocking, 1985; Nastrom and Eaton, 1997):

$$\sigma_{DS}^2 = \sigma_{turb}^2 + \sigma_s^2 + \sigma_b^2 + \sigma_{nt}^2, \quad (4.20)$$

where σ_{turb} is the spread of the spectrum caused by turbulence, σ_s is the broadening due to the wind shear, σ_b is the so-called “beam broadening” (Hocking, 1983), and σ_{nt} is the non-turbulent fluctuations broadening (caused, for instance, by atmospheric gravity waves). Since no wind shear is present in the LES, this term can be neglected. The “beam broadening” is caused by a uniform wind across a finite width of the radar beam (Doviak and Zrnić, 1984). Fig. 4.15 illustrates the effect of “beam broadening”. Scatterers located within the radar resolution volume, having the same velocity but located in different parts of the antenna pattern function, will have different radial velocities (projection of the scatterer velocity on the shortest path between the radar and the scatterer). This causes a spread of the Doppler spectrum.

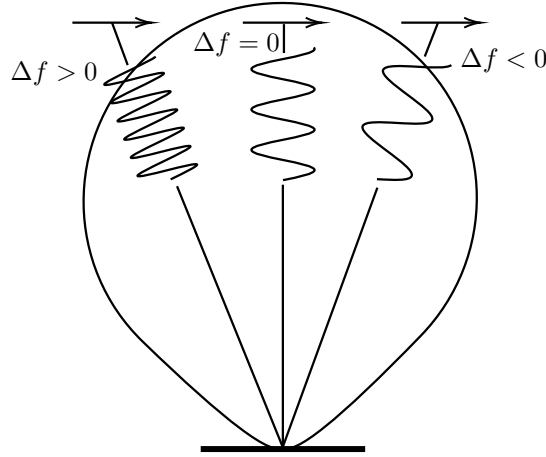


Figure 4.15: Illustration of the “beam broadening”: scatterers having the same velocity but seen at different angles, have different radial velocities and hence contribute with different Doppler frequency shifts Δf to the backscattered signal.

For vertically oriented beams this term is smaller than for the slant beams (Nathanson et al., 1999). Moreover, for narrow beam radars, the “beam broadening” is the largest correction factor (Nastrom and Eaton, 1997). Therefore, the estimation of the Doppler spectrum width caused by the turbulence within the radar resolution volume is:

$$\sigma_{DS}^2 = \sigma_{turb}^2 - \sigma_b^2. \quad (4.21)$$

The beam broadening term for a vertically oriented antenna is calculated as follows (Chu, 2002):

$$\sigma_b^2 = \frac{V_T^2 \theta_a^2}{2 \ln(4)}, \quad (4.22)$$

where θ_a is the half power half width of the antenna pattern function and V_T is the horizontal wind speed.

The influence of other terms introduced in Eq. (4.20), such as wind shear or influence of the range weighting function, on the accuracy of the atmospheric parameters estimation can be analyzed with the simulator.

4.6 Estimation of the wake vortex circulation

Two methods are used to estimate the WV circulation. For simulation A, the circulation of the WV is estimated from the distribution of the radial velocities within the radar resolution volume. Those distributions are represented by the Doppler spectrum of the backscattered signal. For geometrical setup B, the estimation is based on the tracking of the WV descent velocity and the distance between WV cores.

4.6.1 Direct estimation from the Doppler spectrum width

The estimation of the wake vortex circulation is based on the method proposed by Rubin (2000), who used a radar-acoustic system to detect the wake vortices. It is shown that the circulation is proportional to the second moment of the Doppler spectrum raised to the power 2/3:

$$\Gamma_{radar} = k \left\{ \int_{-V_{max}}^{-V_{min}} V^2 [S(V)]^{\frac{2}{3}} dV + \int_{V_{min}}^{V_{max}} V^2 [S(V)]^{\frac{2}{3}} dV \right\}, \quad (4.23)$$

where $S(V)$ is the Doppler spectrum normalized to the area under its graph (the total energy of the train of pulses), $V_{max} = \Gamma(r_c)/(2\pi r_c)$ is the maximum velocity induced by one WV (also close to the maximum found in the LES velocity field), V_{min} is chosen so as to exclude the broadening of the Doppler spectrum caused by the turbulence, $k = 2\pi r_c/V_{min}$.

However, the Rubin model assumes that the wake vortex circulation Γ_{tot} is linked to V_{max} by $\Gamma_{tot} = 2\pi r_c V_{max}$, thus a top-hat wake vortex model (i.e. uniform vorticity within r_c , and zero outside). In the B-H vortex model used as initial condition in this work, $\Gamma(r)$ varies smoothly from zero to Γ_{tot} , and is equal to $\Gamma_{tot}/2$ at r_c (SESAR 12.02.02, 2016). Although the circulation distribution of the developed turbulent WV no longer follows the simple B-H model, it is found that $\Gamma(r_c)$ is also roughly equal to $\Gamma_{tot}/2$, and that this holds fairly well for the representative time intervals, see red part of Fig. 4.16. To take this variation into account, it is proposed to modify Eq. (4.23) by using $\Gamma_{tot}/2 \simeq 2\pi r_c V_{max}$. The coefficient k was evaluated using $r_c = 0.05 \cdot b_0 = 2.5$ m. The actual change of r_c over time with respect to the initial spacing between WV cores (b_0) is shown in Fig. 4.17: r_c deviates from the initial value by +10% to -20%, depending on the LES case.

Compared to Rubin (2000), the interval of integration in Eq. (4.23) is changed to $[-V_{max}, -V_{min}]$ and $[V_{min}, V_{max}]$ because the Doppler spectrum

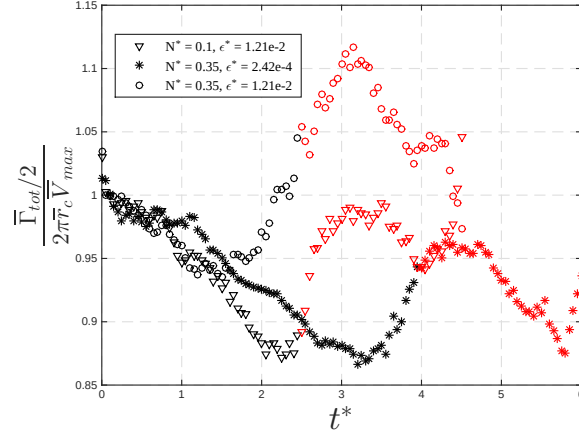


Figure 4.16: Time evolution of the ratio between $\bar{\Gamma}_{tot}/2$ and $\bar{\Gamma}(r_c) = 2\pi\bar{r}_c\bar{V}_{max}$, as measured from the LES data. Red markers highlight the intervals when the simulations represent a real vortex system as obtained after a roll-up. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016) and also to (Kovalev et al., 2018).

is not symmetric: the negative and positive parts of the spectrum represent different parts of the wake vortex system.

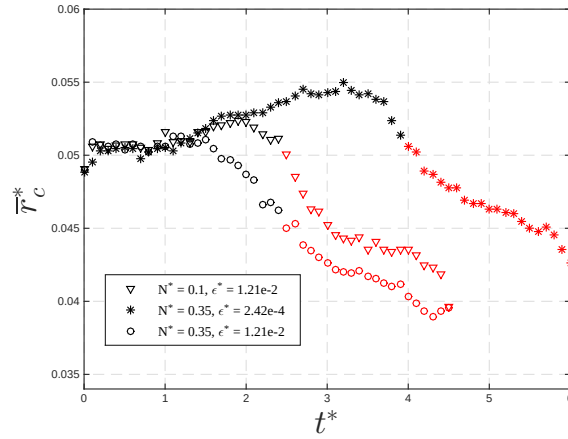


Figure 4.17: Time evolution of the ratio $\bar{r}_c^* = \bar{r}_c/b_0$, as measured from the LES data. Red markers highlight the intervals when the simulations represent a real vortex system as obtained after a roll-up. This figure is from P. Billuart et al. contribution to (SESAR 12.02.02, 2016) and also to (Kovalev et al., 2018).

4.6.2 Indirect approach based on descend velocity tracking

Geometrical setup B is similar to setup A (see Section 3.2.1) but the altitude of the LES box tracks the altitude of the descending WV. The radar performs a scan of the LES box in elevation with a step of 0.27° . It is assumed that the position of the LES box does not change during each scan. The range resolution is 6 m. For each radar resolution volume, the Doppler spectrum is estimated with Eq. (4.8).

In the absence of stratification, which is normally observed in the presence of rain, the relationship between the WV circulation and its descent velocity is defined as follows

$$\bar{\Gamma}_{tot}(t) = 2\pi\bar{b}(t)\bar{V}_c(t), \quad (4.24)$$

where $\bar{V}_c(t)$ is the WV descent velocity and $\bar{b}(t)$ is the distance between the WV cores. This also works well for low stratification, see Fig. 2.11. It however is no longer true for higher stratification as the WV are also subjected to a buoyancy effect. The validity of Eq. (4.24) is thus limited to the low stratification levels.

It should be noticed that De Visscher et al. (2013b) show that the influence of the stratification can be accurately modeled. For cases with significant stratification, the model they propose could potentially be used for estimating the WV circulation from the measurement of the WV descent velocity.

4.7 Conclusions

The analysis of the backscattered signal is based on the estimation of its PSD, which is also called a Doppler spectrum in the case of the Doppler velocities measurements. This spectral distribution is an estimate of the distribution of radial velocities of the scatterers within the radar resolution volume, weighted by the reflectivity of these scatterers. This chapter introduced how the consistent estimate of the PSD can be obtained from the simulated radar signal or from the LES parameters directly. The estimated PSD and its moments are then used to estimate the parameters of the turbulent atmosphere (the energy dissipation rate) and the parameters of the wake vortex: its strength and location. The following chapter presents the results obtained with the developed simulator.

Chapter 5

Results of simulations of backscattered signal and its Doppler spectrum

The previously described methodology for the simulation of the backscattered radar signal and its Doppler spectrum is used to produce the radar signatures of the WV and atmospheric turbulence. Simulated Doppler spectra are then processed as described in Chapter 5 to extract the turbulence strength and the location and strength of the wake vortices.

Section 5.1 employs the geometrical configuration of the radar profiler with vertically oriented antenna to provide results of the EDR retrieval from the backscattered radar signal. The Doppler spectrum widths are calculated for the simulated spectra and then used to estimate the EDR of the atmospheric turbulence simulated with LES. The radar measurements, available for the geometrical configuration employed in this work, are used to assess the results obtained with the radar simulator. Section 5.2 provides results of the WV detection in clear atmosphere. Two methods previously described in Chapter 5 are applied to evaluate the circulation of the WV from the simulated Doppler spectra. The errors of the estimation are analyzed for the different parameters that impact the estimation of the WV strength. Section 5.3 provides results of the WV detection under rainy conditions. The Doppler spectra simulated are qualitatively compared with the radar Doppler signatures obtained during trials made by Thales in the framework of SESAR 12.02.02 JU.

5.1 Retrieval of the turbulence intensity under clear air conditions

To verify the accuracy of the radar simulator, the EDR is estimated from the Doppler spectrum for the various LES cases (see Table 2.1), and compared to the EDR that was used as input for the LES. Results of the measurement

campaign conducted within UFO project (2016) are then analyzed to assess the results obtained with the radar simulator.

5.1.1 Estimation of the EDR with the radar simulator

To simulate the signal backscattered on the atmospheric turbulence, the geometrical configuration specified in Section 3.2.1 is used. The simulation scenario reproduces the conditions of the trials made within UFO project (2016) with the UPMC radar. The relative location of the LES cube and the radar resolution module are set up in the following way.

1. The LES cube is placed at a given altitude and it is ensured that the radar resolution volume for a given range is located within the LES cube;
2. A wind with a magnitude of 2 m/s is added to the LES wind field to reproduce the conditions under which the radar measurements with the UPMC vertical profiler were obtained;
3. To simplify the calculations, the LES cube remains at the same location but the radar coordinates are changing to simulate the advection of the turbulence by the wind relatively to the radar.
4. The trajectory of the radar movement is illustrated in Fig. 5.1 showing a top view of the LES cube. At the initial moment, the radar is placed at the location 1, with coordinates (x_1, y_1) . It is assumed that the direction of the wind is aligned with the x -axis; therefore $x_2 - x_1 = 2$ m. When the radar is moved to position i , the LES parameters are interpolated using $i - 1$, i , and $i + 1$ frames of the LES. During a 1-second period between i and $i + 1$, the complex amplitudes of the backscattered pulses are calculated as described in Section 3.2.5. Each second the radar is moved to the next location and the procedure is repeated.
5. At the moment N , when the radar resolution volume moves out from the LES cube, the location of the radar is reset to position $N + 1$, as shown in Fig. 5.1. At each reset, the phase of the next pulse is reset as described in Section 3.2.5. This movement is repeated iteratively until the location of the radar reaches position $M \cdot N$. The radar is then moved back to its initial position 1 with some random shifts added to the coordinates (x_1, y_1) . The procedure described above is repeated again until the needed duration of the backscattered signal time series is obtained.

Using the procedure described above, 1 hour of scattered signal time series is generated with a radar configuration corresponding to the CURIE radar. A small pulse repetition time ($4.8 \mu s$) is used to allow a coherent integration of 300 consecutive pulses, which provides about 24.8 dB gain of SNR. So, the pulse repetition time of a burst after coherent integration is 1.44 ms and 4096 pulses are used to calculate the Doppler spectrum. 1 hour of the simulated signal results in 610 Doppler spectra; each 10 consecutive Doppler spectra are averaged to reduce the variance of the spectral estimates and the Doppler spectrum widths of the resulting spectra are calculated as described in Section 4.1.3. This

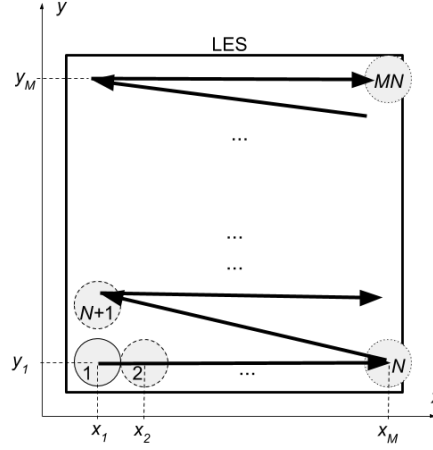


Figure 5.1: Illustration of the radar movement (top view) which imitates the presence of the wind. The circles represent the borders of the radar resolution volumes.

corresponds to 61 samples of EDR that are retrieved using Eqs. (4.18) – (4.20). In Fig. 5.2a, the four different values of $EDR^{1/3}$ used as input for the LES are plotted against the values of $EDR^{1/3}$ retrieved from the radar simulator assuming that the Bragg wavelength lies in the inertial subrange (Eq. (3.29)).

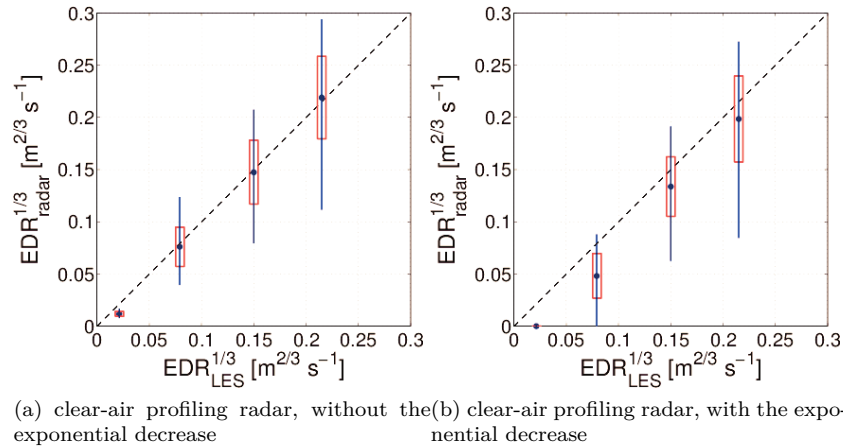


Figure 5.2: EDR estimated from simulated clear-air profiling radar backscattering, versus LES EDR input value, (a) without the exponential decrease in the refractive index spectrum. (b) with the exponential decrease. Blue lines depict the range between the minimum and maximum values of the estimated EDR. Blue dots are the mean values of the estimations for each case. Red boxes display the standard deviation of radar simulator estimates of EDR values.

The simulated data shows that the method considered in Section 4.5 accurately retrieves the EDR values from the Doppler spectrum width. Some underestimation occurs for the LES case with the lowest level of EDR. For that case, the width of the Doppler spectrum becomes comparable to the antenna beam broadening term, which is subtracted from the Doppler spectrum width before the calculation of the EDR.

In a second simulation (Fig. 5.2b), the hypothesis of the Bragg wavelength being in the inertial subrange is released. The full spectrum of the refractive index is used (Eq. 2.30), including the dissipative range. Tatarskii’s cut-off is used to model the attenuation of the refractive index spectrum in the transition between the inertial and dissipative ranges. When the Bragg wavelength is in the dissipative region, the “peak-to-threshold” value decreases, while keeping all other simulation parameters nearly constant. The Doppler spectrum width seen above the threshold is then reduced. This leads to an underestimation of EDR. Fig. 5.2b demonstrates this underestimation of EDR when it is retrieved from the simulated Doppler spectrum. The results for the LES cases with the weakest turbulence, show that there is no appreciable scattering for 1.6 cm Bragg wavelength because it belongs to the dissipative range. Table 5.1 summarizes the estimation accuracy in linear scale. The simulator confirms thus the difficulties for a clear-air X-band radar to retrieve accurate EDR values, in particular for EDR values smaller than $0.15 \text{ m}^{2/3}\text{s}^{-1}$. Further, it is demonstrated that the sensitivity of this EDR retrieval technique for clear-air X-band radar can be estimated by using the simulator developed.

EDR ^{1/3} [$\text{m}^{2/3} \text{ s}^{-1}$]	0.215	0.15	0.0793	0.0215
inertial subrange	1.08	.98	.997	0.22
dissipative range	0.81	0.73	0.25	0

Table 5.1: Accuracy of the EDR estimation defined as $\text{mean}(EDR_{\text{radar}})/EDR_{\text{LES_input}}$. Inertial subrange and dissipative range rows correspond to the use of refractive index spectrum without (Eq. (2.22)) and with (Eq. (2.30)) exponential decrease respectively.

5.1.2 EDR retrieved from the X-band radar measurements

During the measurement campaign in April – May 2014 at the Toulouse airport, various radar and Lidar data were made available, as well as meteorological data from various sources, the most complete source being issued from high resolution Weather Forecast model of Meteo France (MHRPS, HARMONIE) (HARMONIE, 2016). The data from a vertically pointing CURIE X-band radar from LATMOS (Al-Sakka et al., 2009) is validated using Meteo France meteorological data.

Description of the Toulouse airport dataset used for the validation

A 2-months trials sensors campaign has been performed at Toulouse Blagnac Airport in the framework of the UFO project (Barbaresco et al., 2015). Various sensors and meteorological data were available:

- X-band Radar/1.5 micron Lidar sensors
- X-band Radar & Lidar Profilers by LATMOS (France)
- In-Situ Measurements of Technical University Braunschweig (Germany)
- Mode-S EHS Downlink Data post-processed by Dutch Meteorological Institute (KNMI). Those data are issued from Enhanced Surveillance (EHS) terminal area radar gathered by Air Traffic Control, and contain wind and temperature observed by the aircraft during their flight.
- High Resolution Weather Forecast Models (MHRPS, HARMONIE (2016)) made available by Météo France.

A fair evaluation of the radar estimates necessitates the knowledge of the state of the atmosphere and more specifically $\varepsilon^{1/3}$ and N (the Brunt-Väisälä frequency which characterizes the atmosphere stratification level), in order to determine to which LES case it refers. In this work, meteorological data is used to provide a reference level for the EDR estimates retrieved from the radar measurements. Only days where the High Resolution Weather Forecast Models data are available were used because $\varepsilon^{1/3}$ is directly available and N can be calculated; this makes a total of 8 clear days in April and 4 clear days in May during which the LATMOS CURIE radar data are also available. For those days, the time series of potential temperature, humidity, temperature, Turbulent Kinetic Energy (used for estimation of $\varepsilon^{1/3}$), Brunt-Väisälä frequency and EDR from Météo France data have been plotted in order to select the periods of time corresponding to the cases selected for LES simulation (Table 2.1). An example of such plots is presented in Fig. 5.3 for April 10, 2014. As seen from the example provided, during diurnal hours the turbulent kinetic energy increases, reaching its maximum while the stratification of the atmosphere stabilizes at given levels.

Radar measurements and meteorological data

The example shown in Figs 5.4 is representative of the days available during the measurement campaign: the CURIE radar only detects clear air turbulence when $\varepsilon^{1/3}$ is large (LES cases 2, 3 and 5) and N is low. This happens during the daily hours, between 10 am and 6 pm (see Figs. 5.4). The level of reflected power should be large enough to allow an accurate estimation of $\varepsilon^{1/3}$ from the Doppler spectrum. Therefore, only the radar data showing a sufficiently large SNR ratio are considered for the validation of the simulator.

Since the power level of the signal received by the radar varies in a wide range, it is necessary to introduce a SNR threshold to cut off the measurements with a SNR below the threshold. The SNR is directly obtained from the radar

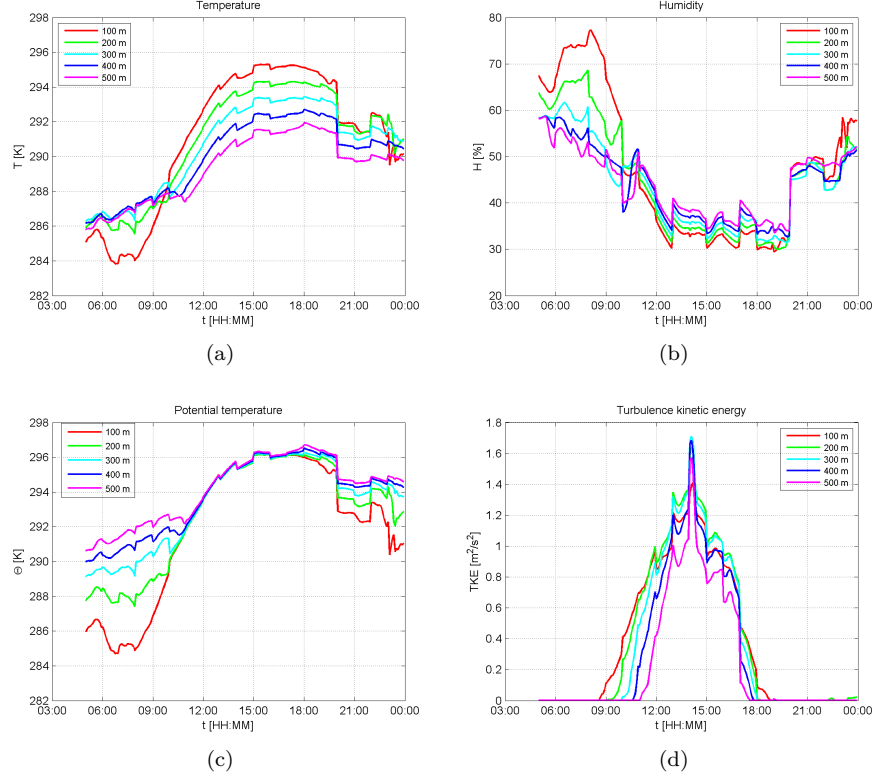


Figure 5.3: Temperature, humidity, potential temperature, and turbulent kinetic energy versus time, at various altitudes, for April 10, 2014.

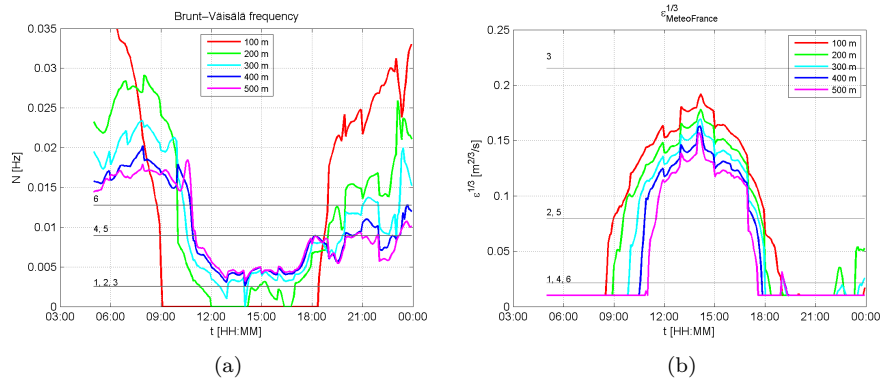


Figure 5.4: Brunt-Väisälä frequency and the energy dissipation rate ($\epsilon^{1/3}$) versus time, at various altitudes, for April 10, 2014. The horizontal lines represent the values of the parameters used for LES simulation of the various cases presented in Table 2.1.

measurements because both signal and noise levels are available. Time series of SNR for 3 adjacent levels of altitude were averaged in order to reduce the variations. Analysis of the mean values of EDR retrieved from the radar data shows that all measurements with high SNR correspond to the $\text{EDR}^{1/3} = 0.15 \text{ m}^{2/3}\text{s}^{-1}$. The dependence of the EDR estimation accuracy on SNR for the X-band radar is shown in Fig. 5.5. Constant levels in the graphs indicate the absence of measurements with high SNR for high altitude levels (400 m and 500 m). Assuming that the meteorological data provide a reference level of

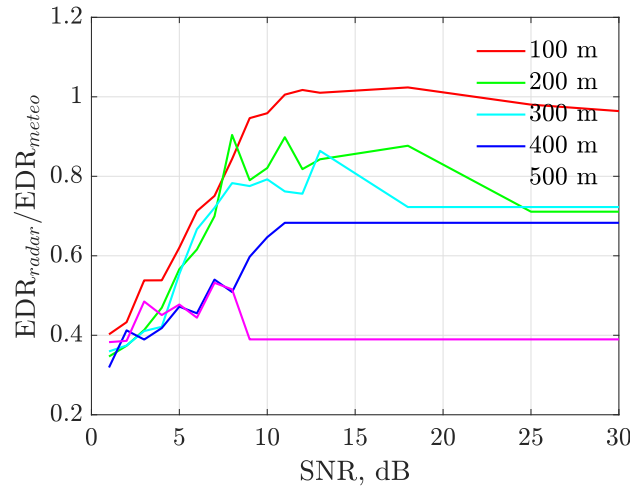


Figure 5.5: The accuracy of the EDR estimation versus SNR. The statistics obtained on all 12 days available.

the EDR, it is concluded that for low altitudes (around 100 m), the EDR can be accurately estimated if the SNR is larger than 10 dB. The small number of measurements having SNR greater than 10 dB at higher altitudes, does not allow to estimate EDR at those altitudes.

5.2 Retrieval of wake vortex parameters in clear air

Following sections present results of the WV circulation estimation based on the methods introduced in Chapter 5. In Sections 5.2.1 – 5.2.2, backscattered signals are analyzed without the presence of noise. This enables to view the potential accuracy of the methods considered in Chapter 5. The very low RCS of the WV in clear atmosphere however necessitates the analysis of the influences of the SNR on the accuracy of the parameters retrieved from the radar measurements. For that purpose, the RCS of the WV in clear air is analyzed in Section 5.2.3 and compared with the values reported in the literature. The impact of the noise on the accuracy of the WV strength estimation is then given in Section 5.2.4.

5.2.1 Doppler spectrum evolution in time

Various instants are used for each LES case to illustrate the evolution of the Doppler spectrum during the decay of wake vortices (see Figs. 5.6, 5.7, and 5.8 for Cases 1, 2, and 3 respectively). The geometry presented in Section 3.2.1 is used for the simulation of the DS; therefore, each Doppler spectrum is calculated for the signal backscattered on the Bragg scatterers within an approximate radar resolution volume of $60 \times 60 \times 60 \text{ m}^3$, whose center is aligned with the location of the (left) WV core at the time it is added to the LES (see Fig. 4.9).

For each case, the spectra are introduced in two forms to (1) facilitate the illustration of the increase of the RCS of the developed turbulent WV and (2) illustrate the decrease of the Doppler spectrum total width over time. Before calculation of the Doppler spectrum, the simulated signals are normalized by the energy of the signals at a given time, t^* , to ease the comparison of the total width of the Doppler spectra of the simulated signal at various instants (Figs. 5.6a – 5.8a). As seen, the total width of the DS decreases over time for each case, while the WV dissipates its energy interacting with surrounding turbulent atmosphere. In Figs. 5.6b – 5.8b, all signals are normalized by the energy of the signal at the initial time considered: $t^* = 2.5$ for Cases 1 and 3 and $t^* = 4$ for Case 2. Then, the Doppler spectrum is calculated. Consequently, this helps to illustrate how the distribution of the EM energy among the scatterers is changing over time for each LES case. As seen from the results provided, the absolute values of the Doppler spectra peaks increase over time relative to the initial moment of t^* . As shown later in Section 5.2.3, this is caused by the fact that the RCS of the WV increases over time, while turbulence induced by the presence of the WV occupies larger parts of the radar resolution volume. Zero time ($t_{LES} = 0$, $t^* = 0$) corresponds to the moment at which the wake vortices are added to the LES. As seen in Fig. 2.9, the circulation reduces

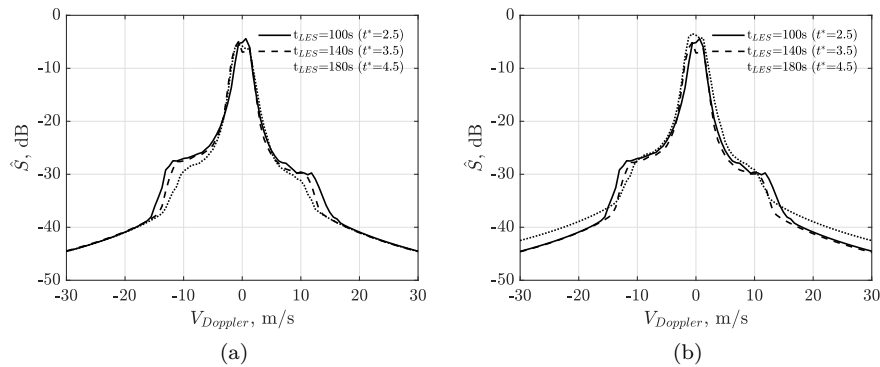


Figure 5.6: Ensemble averaged Doppler spectra for Case 1: (a) all signals are normalized by their energies before calculation of the Doppler spectrum; (b) all simulated signals are normalized by the energy of the signal at $t^* = 2.5$ prior to calculation of their Doppler spectra;

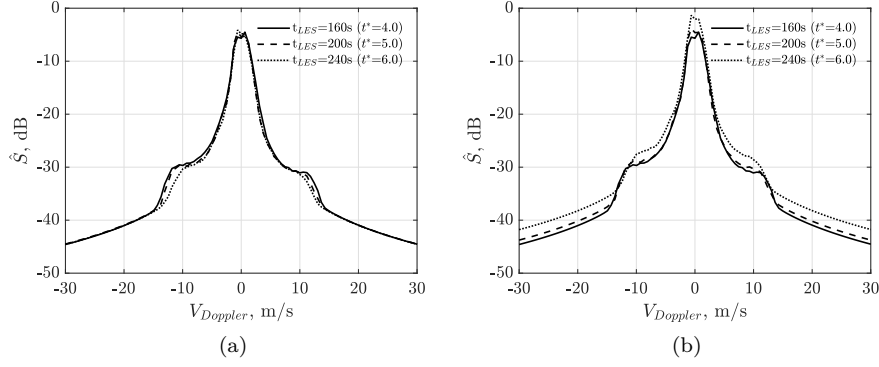


Figure 5.7: Ensemble averaged Doppler spectra for Case 2: (a) all signals are normalized by their energies before calculation of the Doppler spectrum; (b) all simulated signals are normalized by the energy of the signal at $t^* = 4$ prior to calculation of their Doppler spectra.

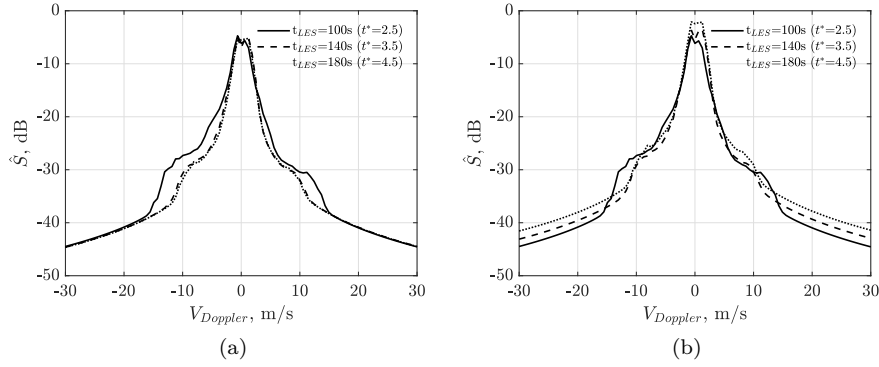


Figure 5.8: Ensemble averaged Doppler spectra for Case 3: (a) all signals are normalized by their energies before calculation of the Doppler spectrum; (b) all simulated signals are normalized by the energy of the signal at $t^* = 2.5$ prior to calculation of their Doppler spectra.

when the age of the wake vortex increases because it exchanges its energy with the surrounding medium. Since the EDR level is relatively low for the LES cases used in this work for the simulation of the presence of the WV, it is hard to analyze the impact of the EDR of the background atmosphere on the time of the WV decay. It is seen from Figs. 2.9 and 2.9, that for the periods considered (red markers in the graphs), the level of the EDR within the WV oval is of the same order for all cases, as well as the circulation of the WV decreases by the same amount over considered intervals. As seen from Figs. 5.6 – 5.8, the total width of the Doppler spectra decreases more or less by the same amount for all cases: from around 12.5 m/s to around 10 m/s (see the width

of the DS at the levels of -28 to -30 dB in Figs. 5.6a – 5.8a). The relative differences in the level of the DS sidelobes between Cases 1,3 and Case 2 (in the range of -30 to -25 dB) is rather caused by the inconsistency of the spatial reflectivity within the WV area as shown hereafter in Section 5.2.3. The WV RCS and its temporal and spatial evolution indeed depend on the parameters of the background atmosphere, its EDR and Brunt-Väisälä frequency.

Fig. 5.9 shows the changes of the Doppler spectra for case 1, with $\text{EDR} = 5 \cdot 10^{-4} \text{ m}^2\text{s}^{-3}$ and $N^* = 0.35$ ($N = 8.9 \cdot 10^{-3} \text{ s}^{-1}$) and case 2, with $\text{EDR} = 1 \cdot 10^{-5} \text{ m}^2\text{s}^{-3}$ and $N^* = 0.35$ ($N = 8.9 \cdot 10^{-3} \text{ s}^{-1}$). When the WV decays in time by dissipating its energy through the interaction with the turbulent background atmosphere, the side parts of the Doppler spectrum “disappear” as indicated by the black dashed lines (representing the level of -28 dB below the maximum of the Doppler spectrum) in Fig. 5.9.

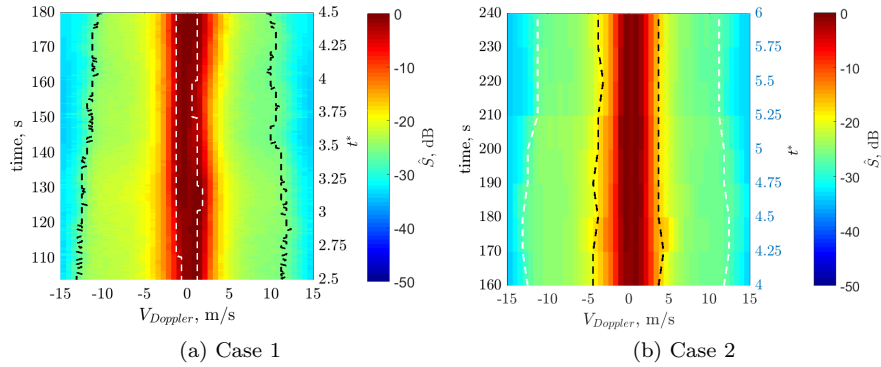


Figure 5.9: Doppler spectrum change over time. All individual Doppler spectra are normalized to their maximum values. Black and white dashed curves track in time, respectively, the -28 dB and -19 dB levels of the Doppler spectrum. For case 1, 342 consecutive LES samples are used to generate Doppler spectra. For case 2, only 10 LES samples are available with the interval of 10 s.

At the same time, the width of the DS main peak narrows slowly as represented by the decrease of the width of the zone restricted by white dashed lines (-19 dB) in the same figure. Local increases of the main peak width can be explained by the temporary increase of the turbulent kinetic energy (and consequently, the EDR) of the turbulence in the volume of the WV while it evolves and interacts with the surrounding turbulent atmosphere. Fig. 2.4b represents Case 2 for which EDR is small ($10^{-5} \text{ m}^2\text{s}^{-3}$). The decrease of the side parts of the spectrum is smaller and the main lobe stays stable, because the exchange of the energy with the surrounding medium is reduced.

5.2.2 Estimation of the wake vortex circulation

In order to compare the circulation retrieved from a given radar cell to the circulation measured from the LES velocity field, the radar cell center is aligned

with the vortex core center. The LES reference $\Gamma(x_i, r_{max}, t)$ is sought at each x_i , where r_{max} is equal to half the radar cell width in range. Then, it is averaged over x_i (within the radar resolution volume) to obtain $\Gamma_{tot}(t)$. For the considered LES cases, the circulation decreases as shown in Fig. 5.10.

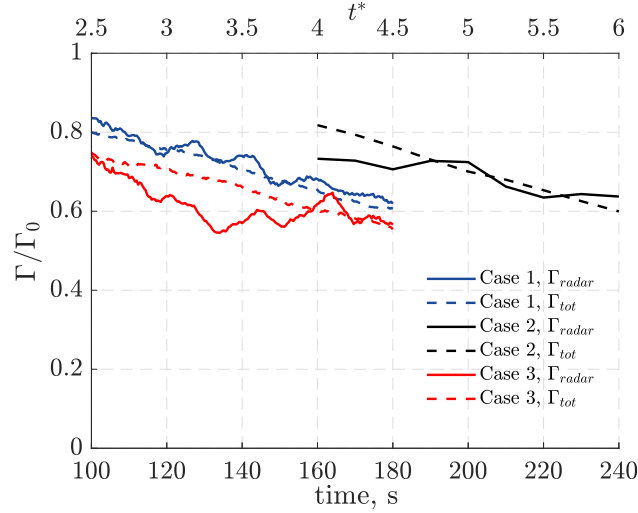


Figure 5.10: Temporal evolution of the wake vortices normalized circulation retrieved from the radar simulator (solid) and obtained directly from the LES (dashed).

The radar estimates of the circulation at each LES time step are obtained from the radar cells centered at the initial positions of the left and right WV cores (at $t^* = 0$). Then, the average between the left and right WV estimates is calculated. The results provided in Fig. 5.10 show a good agreement with the circulation calculated directly from the LES field.

In the LES, $t^* = 0$ corresponds to the injection of the wake vortices into the initial stratified turbulence. The moment at which the LES reaches a statistical equilibrium - the state at which the energy dissipation rate reaches its maximum - indicates the time after which the LES provides a physically meaningful simulation of fully developed turbulent wake vortices in the atmosphere. Those moments differ among the LES cases and, consequently, the estimation of Γ_{tot} starts accordingly (the red parts in Fig. 2.9, lasting for a total time of $2t_0$).

The limits of integration (V_{min}) in Eq. (4.23) significantly influence the accuracy of the estimation of Γ_{tot} . For all cases, a constant upper limit $V_{max} = 12.8$ m/s is used. Nevertheless, the value of V_{max} at each time step can be accurately estimated from a total width of the Doppler spectrum.

The lower limits V_{min} are chosen so as to exclude the influence of the Doppler spectrum broadening caused by the turbulence induced by the WV: 1.1 m/s for Cases 1 and 3, and 0.9 m/s for Case 2. These values of V_{min} are estimated by minimizing the error between the estimate, Γ_{radar} , and the reference circulation from the LES, Γ_{tot} . However, in real systems V_{min} should be estimated from the radar signal backscattered by the turbulence induced by

the WV.

The influence of V_{min} on the accuracy of the circulation estimation (with fixed V_{max}) is shown in Fig. 5.11a in the form of a normalized root-mean-square error:

$$NRMSE = \sqrt{\frac{\sum_{i=1}^N [\Gamma_{radar}(t_i) - \Gamma_{tot}(t_i)]^2}{\sum_{i=1}^N \Gamma_{tot}(t_i)^2}}. \quad (5.1)$$

It first follows a type of hyperbolic law due to the presence of the $1/V_{min}$ factor in the k coefficient of Eq. (4.23). It then departs from a hyperbolic behavior and eventually reaches a minimum error for some optimal V_{min} value. These are the values mentioned above. After reaching the minimum, the curve increases because of the quadratic form of the NRMSE definition.

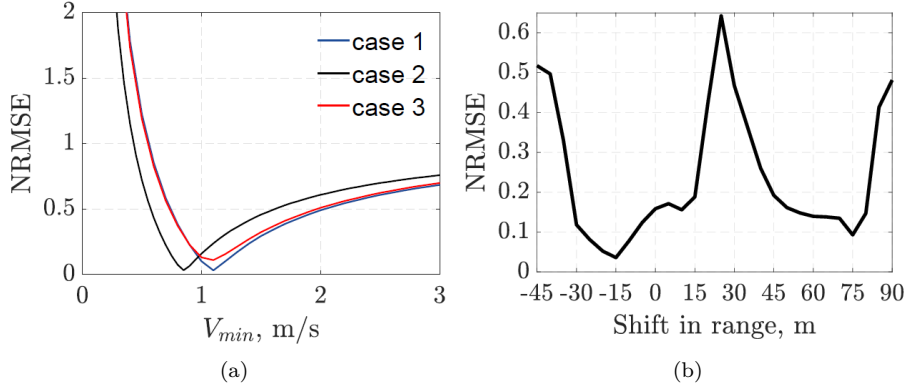
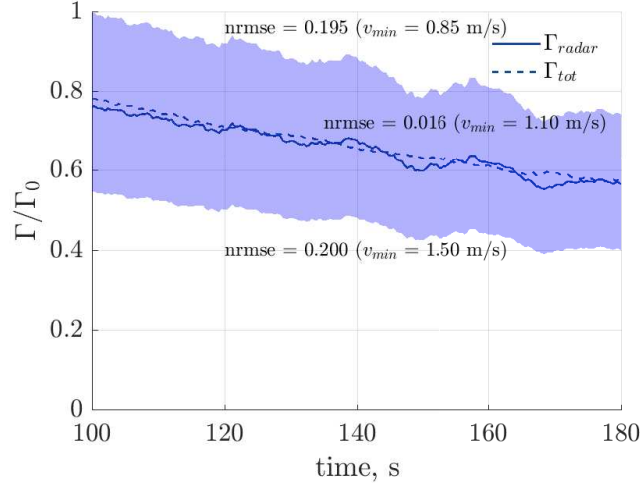


Figure 5.11: Influence of (a) V_{min} (chosen to exclude the broadening of the spectrum due to turbulence) and (b) the misalignment of the radar resolution volume relative to the WV core on the NRMSE (Case 1).

To evaluate the influence of the misalignment of the radar resolution volume center relative to the WV core, the location of the radar cell center is changed in range from -45 m to 90 m. For each shift in range, the circulation within a given radar cell is computed and Eq. (5.12) is used to calculate the error of the estimation (see Fig. 5.11b). It is seen that this error is minimal as long as the shift of the radar cell center is within ± 15 m relative to the center of the WV core. Deviations of the local minima of the NRMSE from the initial positions of the WV cores (range shifts 0 m and 50 m for the left and right WV respectively) are due to the fact that the wake vortices do not remain symmetrical and centered at their initial positions as they evolve in the turbulent stratified atmosphere.

To evaluate the deviation of estimates of the WV circulation from their actual true values which corresponds to a given value of NRMSE, the circulation of the WV is estimated for optimal value of V_{min} and also for two other values of V_{min} : 0.85 m/s and 1.5 m/s resulting in NRMSE of 0.195 and 0.2 respectively. The results are given in Fig. 5.12. It is seen that for NRMSE of about 0.2, the estimates of the circulation deviate by about 20% from their true values.

Figure 5.12: NRMSE for various V_{min} .

It can be concluded that the method tested here provides a fairly accurate estimation of the WV circulation for the cases investigated.

Results of the circulation retrieval shown in Fig. 5.10 are obtained with Eq. (4.8) for the Doppler spectrum estimation. It is therefore assumed that a consistent estimate (in terms of variance) of the Doppler spectrum is available. This helps to evaluate the potential performance of the method applied to retrieve the circulation of the WV. Nevertheless, in real systems, there is no access to the true Doppler spectrum. A smooth estimate of the spectrum can be obtained from a short time series of the scattered signal at the price of a decreased spectral resolution. For that purpose, other nonparametric spectral estimators, such as a Welch estimator, can be applied (Hayes, 1996). In this work, only periodogram and modified periodogram methods providing the highest spectral resolution among nonparametric estimators, are considered. Fig. 5.13 shows the results of the circulation estimation based on Doppler spectrum of one realization of the scattered train of pulses. The deviation of the estimate from the true value of Γ_{LES-rc} has the same order as the estimated value. To decrease the variance of the estimate, one can average in time the Doppler spectra calculated for consecutive bursts of pulses scattered from the same radar cell. Under the assumption of stationarity of the scattering pattern during that short interval of time, this can substitute, to some extent, the ensemble averaging considered above. Averaging of the Doppler spectra in time reduces the variance of the estimate (see Fig. 5.13) but an increased time duration will be necessary, in real radar systems, to get the estimate of each scanned radar cell. An example of the circulation retrieved from the time average of 15 successive Doppler spectra over a duration of about 0.4 s (each burst consist of 512 pulses with a pulse repetition interval of 50 μ s) is shown in Fig. 5.13 with black solid line: it converges to the estimate obtained using Doppler spectra calculated with Eq. (4.8).

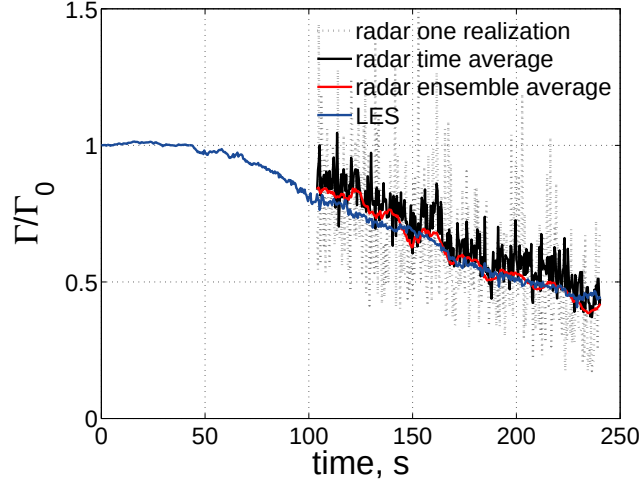


Figure 5.13: Estimation of the circulation from one Doppler spectrum (gray dotted curve) and from time average of the 15 successive Doppler spectra (black curve).

setup B

Another method that can be used for the estimation of the WV circulation is described in Section 4.6.2. It is based on the evaluation of the WV's descend velocity and the distance between the WV cores, which are used to calculate the circulation with Eq. (4.24).

Fig. 5.14 illustrates the distribution of the Doppler spectra width among radar cells in one elevation scan. The position of the WV cores is estimated by the local maximum of the Doppler spectrum width at each scan (see Fig. 5.15). The discontinuities in the estimated coordinates (y, z) are caused by : (1) a relatively coarse radar range resolution (6 m) and (2) the fact that the radar cell with the maximum Doppler spectrum width does not necessarily correspond to the actual location of the WV core. To reduce the variance of the estimated WV coordinates, the width of the Doppler spectrum in a given radar cell is obtained as an average over the 25 closest radar cells in elevation and range (in an area of 5×5 radar resolution volumes centered at a given radar cell).

For geometrical setup B, the coordinates of the WV cores are estimated as explained in Section 4.6.2. Then, the distance between the WV cores is computed and the descent velocity is estimated using three successive measurements for each WV core. Fig. 5.16 shows results of the circulation estimation obtained using Eq. (4.24). Γ_{tot} shown in Fig. 5.16 is calculated from the descending velocity of the WV and the distance between the WV cores which are directly available from the LES.

The parameters $\bar{V}_c$ and $\bar{b}_c$ of the descending WV are estimated with a reasonable accuracy (with some outliers due to the discontinuities in the estimation of the WV coordinates). As seen from Fig. 5.16, while the estimates

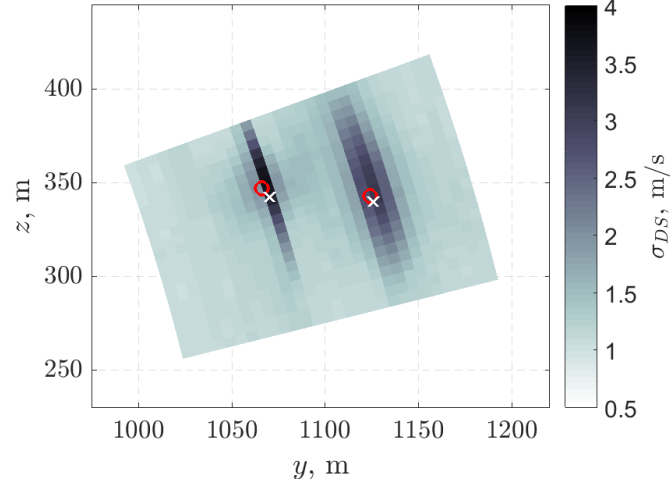


Figure 5.14: Distribution of the Doppler spectra width in one elevation scan for Case 1 at $t^* = 3.75$. The white crosses show the position of the WV cores as measured in the LES; the red circles - those as estimated by the radar.

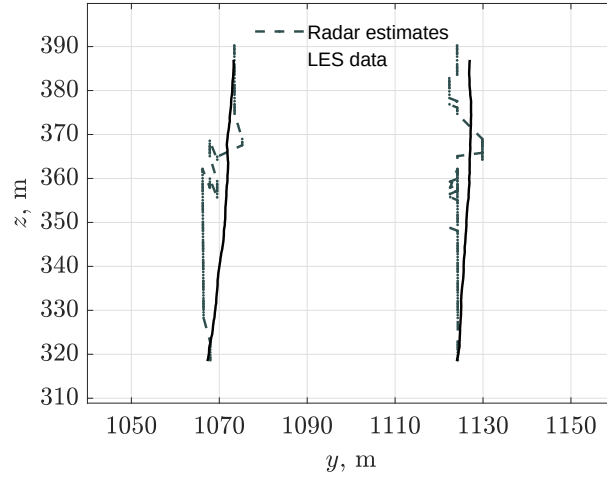


Figure 5.15: Time evolution of the coordinates of the WV cores: dashed line with markers - estimated from Doppler spectrum width distribution; solid line - LES data.

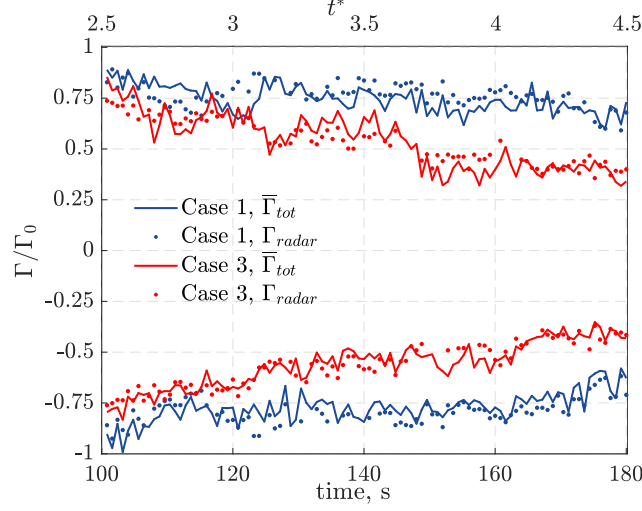


Figure 5.16: Circulation estimated from the descent velocity of the WV.

of the descend velocity and the distance between WV cores are quite accurate, the method provides accurate estimates of $\bar{\Gamma}_{tot}$ for low stratification only (Case 1). For higher stratifications, a significant deviation of the estimated value is observed, as expected. Both methods can provide an accurate estimate of the WV strength. However, they both rely on the Doppler spectrum width estimates that means that these methods are very sensitive to the SNR. Following Section 5.2.3 provides the analysis of the WV's RCS that are then used to assess the impact of the SNR on the accuracy of the WV strength estimation, which is provided in Section 5.2.4.

5.2.3 RCS of the wake vortex in clear air

Fig. 5.18 shows a dependence of the RCS on the shift of the radar cell in range relative to the core center of the left wake vortex. The position of the left wake vortex, at the moment when it is added to the LES, corresponds to $\Delta r=0$ in Fig. 5.18. The size of the radar cell in range is 60 m; the half power width of the antenna beam is 3.1° , providing a radar cell width approximately equal to the range size of the radar resolution volume, both in elevation and azimuth.

The wake vortices are being deformed over time due to the interaction with the surrounding turbulent atmosphere. As a result, the position of the wake vortex cores at a given coordinate along the WV initial axis (axis of the WV at the moment when it is added to the LES) change in time. These shifts are not sufficient for case 2 due to the low turbulence intensity but can reach tens of meters in other cases (see Fig. 5.17f – Case 3).

The RCS calculated for LES without WV are provided for two cases: (1) the reflectivity is calculated with Eq. (3.19); (2) the reflectivity is calculated with Eq. (3.20). This is done in order to demonstrate a decrease of the RCS for the turbulent scales located outside the inertial subrange. The most drastic fall

of the RCS (about 15 dB) is observed for case 2 that has the lowest turbulence intensity of the background atmosphere among all considered cases.

The refractive index parameter C_n^2 , which determines the level of the RCS, depends not only on the intensity of turbulence but also on the gradient of the relative humidity and temperature (Tatarskii, 1971; Doviak and Zrnić, 1984). The presence of the WV produces a strong gradient of these atmospheric parameters at the WV oval and, as a result, a high level of C_n^2 . Fig. 5.17 provides examples of the C_n^2 distribution in a cross-section of the LES. Each slice shown in Fig. 5.17 is taken from the center of the LES box.

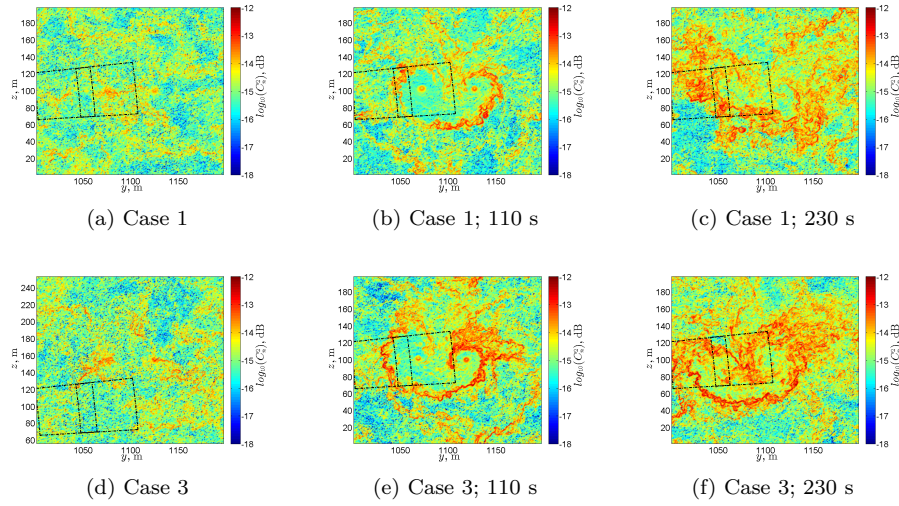


Figure 5.17: C_n^2 distribution for cases 1 (a-c) and 3 (d-f). (a) and (d) correspond to the LES without WV. The borders of the radar cells are shown with black lines; left and right radar cells correspond to $\Delta r = -40$ m and $\Delta r = 0$ m respectively.

The following can be concluded based on the analysis of RCS and C_n^2 for simulated LES cases:

- The RCS at the WV oval has the highest value due to the strong gradient of the relative humidity and temperature.
- Interaction with the background turbulent atmosphere influences the life time of the WV and as a consequence determines the speed of the RCS change in time.
- Higher stratification level leads to higher RCS; the maximum RCS is observed in LES case 2 that has the lowest intensity of the background turbulence and the highest level of stratification among all simulated LES cases.
- The presence of the WV significantly increases the RCS of the turbulent atmosphere. So, for Case 2, the increase of the RCS is around 20 – 25 dB.

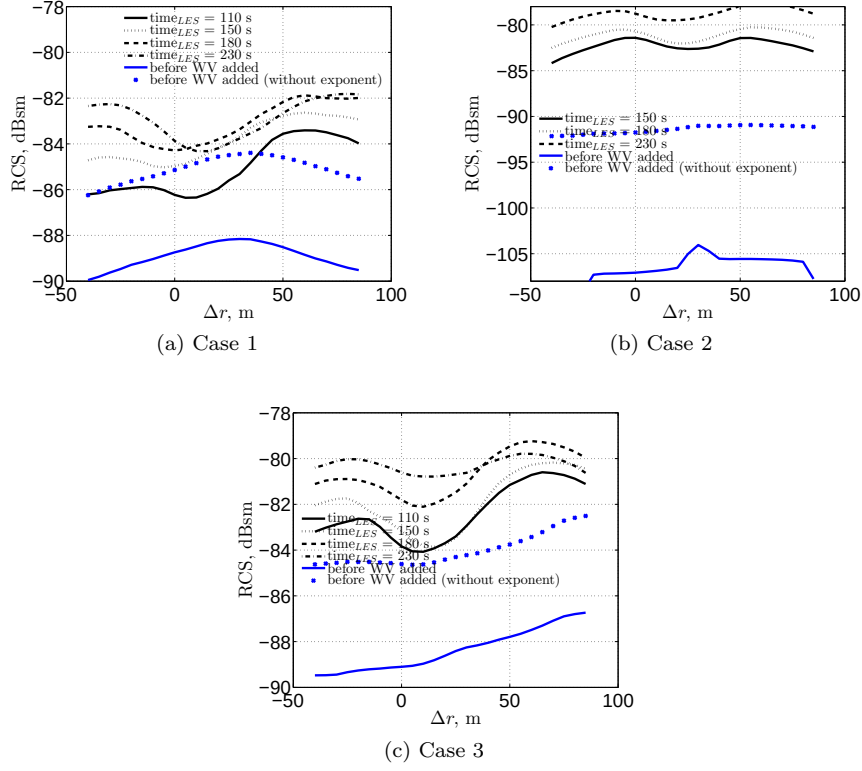


Figure 5.18: RCS of the radar cell volume. Blue solid line depicts the RCS of turbulence without the WV. Blue dotted line shows the RCS of turbulence calculated under assumption that the Bragg wavelength belongs to the inertial subrange (Eq. (3.19)).

Dependence of the WV RCS on the frequency

Fig. 5.19 presents the dependence of the RCS on the frequency. The RCS is calculated for the radar resolution volume which is aligned ($\Delta r = 0$ m) with the left WV core position at the time when the WV are added into the LES. The simulations show that the absolute values of the WV RCS increase as the frequency increases from 100 MHz to 10 GHz. Then, the WV RCS starts decreasing very fast as the Bragg wavelength enters the viscous dissipation range of the turbulence spectrum. The increase of the RCS with the frequency, which is proportional to $\lambda^{-1/3}$, reaches its maximum at a frequency that corresponds to the transition from the inertial subrange to viscous dissipation range. This critical frequency mainly depends on the intensity of the turbulence that is produced by the WV.

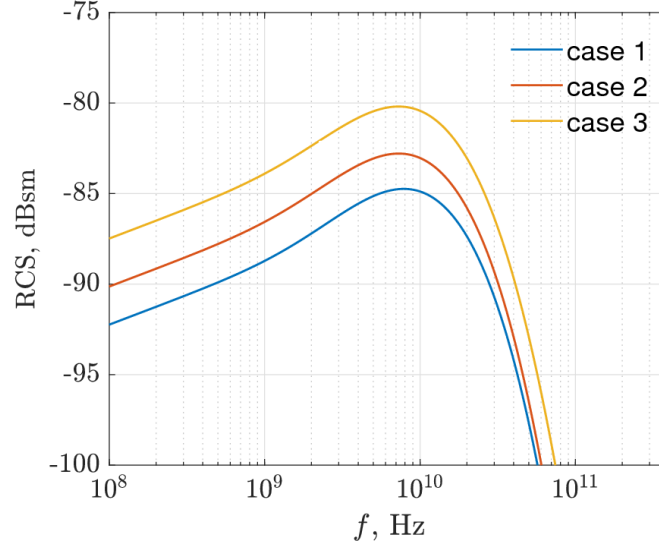


Figure 5.19: WV RCS calculated with Eq. (3.20) versus frequency ($t^* = 3.75$).

Overview of the modeled and measured WV RCS reported in the literature

This section briefly overviews the RCS values of the WV in clear atmosphere that were previously reported in the literature.

In Nespor et al. (1991), a C-band phased array radar was used to detect the WV under clear air conditions. The reported measurements of RCS are in range from -65 dBsm to -80 dBsm, corresponding to $10\log_{10}(C_n^2)$ ranged from -116.7 dB to -135.4 dB. It was found that the RCS tends to increase as the WV descends, assuming that it is caused by the expansion of the WV: it occupies more of the radar resolution volume as it decays and dissipates with time.

The RCS of the WV was measured in Gilson (1992) at VHF, UHF, L, S and C bands, at distances behind the C-5A aircraft from one hundred meters to tens of kilometers. The RCS of the WV in C-band is varying from -107 dBsm/m to -80 dBsm/m for the distances behind the aircraft up to 10 km. It was found that a frequency dependence of the RCS is largely flat with a decrease at the C-band. Both Gilson (1992) and Nespor et al. (1991) report increased reflectivity due to turbulence in WV and an increase of reflectivity with time.

In Marshall and Myers (1996), a two-dimensional model (the NASA TASS Proctor (1987)) is used in order to estimate radar reflectivity of the wake vortex. According to the obtained estimate, it is assumed that the boundary part (oval) of the descending WV can be detected by radars at X-band and below. The refractive index structure constant, C_n^2 , for C-130 aircraft (30 s after the roll up) reaches -116 dB at the oval of the WV, decreasing to -148 dB at the WV core.

In Myers et al. (1999), it is assumed that the scattering of the electro-

magnetic waves on the WV is not due to the incoherent Bragg scattering on turbulence but rather to the coherent Bragg scattering from the spiral structure of the laminar flow in the WV. The considered “laminar flow mechanism” does not assume turbulence within the wake vortex system. Simulation results show increase of the WV RCS with time: from -100 dBsm to -80 dBsm for S-band; from -110 dBsm to -85 dBsm for L-band; from -110 dBsm to -85 dBsm for VHF and UHF bands. The results are provided for the interval from 20 s to 115 s after roll-up of the wake vortex. In Shariff and Wray (2002), the RCS estimation is based on the simulation of the refractive index. The refractive index is obtained for three mechanisms: (A) Radial density gradient in a 2D vortex (B) 3D fluctuations in the vortex cores’ (C) Adiabatic transport of atmospheric fluid in a 2D oval surrounding the pair of vortices. The estimated RCS values for the oval surrounding the vortices (mechanism C) agree with the experiments of Gilson (1992). The model proposes almost a flat dependency of the RCS versus frequency. The RCS of the WV oval is estimated to be in the range of -80 dBsm to -60 dBsm for all frequencies from 100 MHz up to 10 GHz. The effect of the density and weather variation of the dielectric constant on the RCS of the wake vortex is investigated in Li et al. (2010), from 1 MHz to 10 GHz. It was found that the density variation is the dominant factor to the total RCS of the wake vortex at low frequencies; while in the high frequency band, the dominant factor turns to be the water vapor variation. The estimated WV RCS is varying in the range from -150 dBsm to -70 dBsm depending on the frequency and WV age. Vanhoenacker-Janvier et al. (2012) consider two mechanisms causing refractive index gradients: radial density gradient and the transport of the atmospheric fluid in the oval surrounding the vortices. The RCS is estimated for lower frequencies and the values agree with Li et al. (2010).

Fig. 5.20 summarizes the values of the WV RCS reported in the literature for different frequency bands for both measurements and simulations. The RCS of the WV evaluated in this work correspond to the lowest estimates of the RCS obtained in measurement trials for L, S and C bands, predicting lower RCS for VHF and UHF frequency bands. The RCS values provided in a form of dBsm/m in Gilson (1992) are transformed to dBsm by multiplying them by the width of the antenna pattern (in azimuth) considered in the present work, which is equal to 60 m at the distance of about 1 km.

5.2.4 Impact of the noise on the accuracy of the WV strength estimation

A very low RCS of the WV limits the possibility of the radar detection of the WV on large distances. Fig. 5.21 shows the SNR estimated with Eq. (3.7) for different combinations of the emitted power, P_{tx} , and the range, r . The value of -85 dBsm is taken as the WV RCS. The other parameters of the radar are taken from Table 3.2. For a distance of 1050 m and an emitted power of 600 W, the SNR is approximately equal to -18 dB. Therefore, for the considered geometry and radar parameters, the detection of the WV is not possible.

To quantitatively evaluate the limits of the detectability of the WV as well

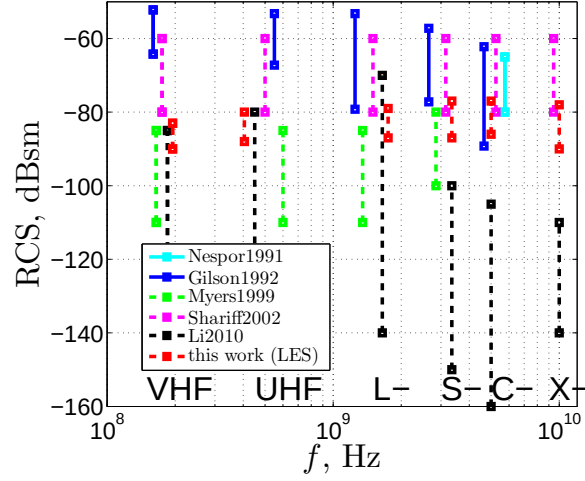


Figure 5.20: Reported WV RCS: dashed lines - simulations; solid lines - measurements. Square markers roughly represent maximum and minimum values of RCS reported in the corresponding source.

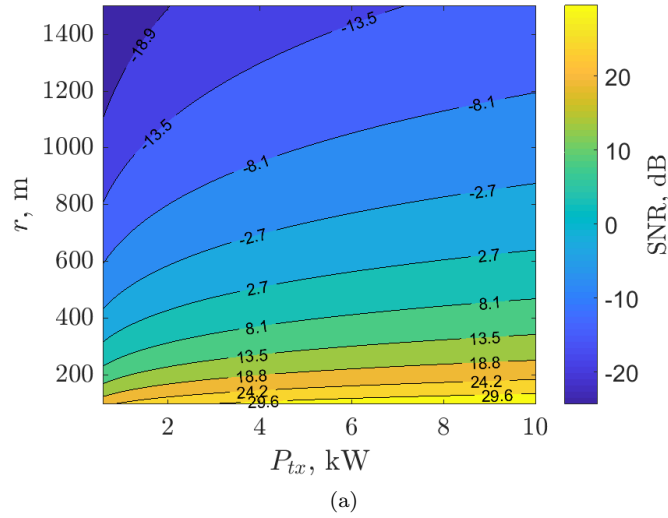


Figure 5.21: Dependence of the SNR on the distance between the radar and the WV and on the emitted power estimated for the WV RCS of -85 dBsm. The lowest limit of the emitted power range is 600 W.

as the dependence of the accuracy of the WV strength estimation on the SNR, the method described in Section 4.6.1 is applied to the simulated spectra with the emitted power adjusted so as to reach a given level of the SNR. For each value of the SNR, the accuracy of the WV estimation is evaluated with Eq. (5.1)

as NRSME. Fig. 5.22 provides the accuracy of the WV strength estimates for different SNR levels. For all LES cases considered, a good accuracy of the

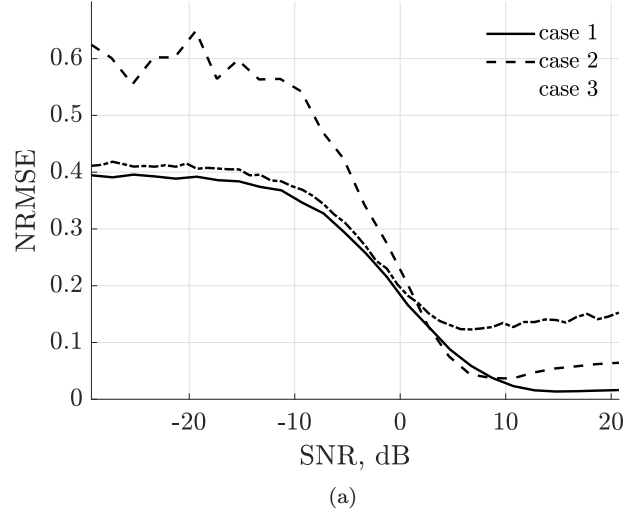


Figure 5.22: Dependence of the accuracy of the WV circulation on the SNR.

WV circulation estimation is reached with about 10 dB of SNR. Note that this value of the SNR is calculated for one pulse. The Doppler peak to noise floor ratio is higher due to the Doppler processing gain. So, to obtain a 10 dB SNR at the distance of 1050 m, it is necessary to emit around 380 kW peak power. Otherwise, if the emitter power is less than 1 kW, the distance, at which an accurate estimation of the WV strength is possible with the methods considered, is reduced to a few hundreds meters (see Fig. 5.21).

5.3 Retrieval of wake vortex parameters in the presence of rain

This section presents results of the estimation of the WV circulation under rainy conditions. The estimation is based on the method described in Section 4.6.2. The locations of the WV cores are detected as the local maxima of the Doppler spectra width within a radar scan. Detected cores of the WV are then tracked in time to evaluate the descending velocity, $\bar{V}_c(t)$, and the distance between the WV cores, $\bar{b}(t)$. Finally, the WV circulation is calculated with Eq. (4.24) under the assumption of a low stratification level of the atmosphere, which is normally observed under rainy conditions.

Section 5.3.1 presents a qualitative comparison of the radar signatures simulated in this work and the WV signatures measured by Thales in the framework of the SESAR 12.02.02 Joint Undertaking. In Section 5.3.2, several examples of the simulated WV's RCS are given. Section 5.3.3 provides the results of the WV circulation retrieval based on the estimation of the WV descent velocity.

5.3.1 Qualitative comparison of the WV radar signatures simulated and measured with an X-band radar

Hereafter, the WV radar signatures extracted from the radar measurements (provided by Thales within the framework of SESAR 12.02.02 JU) are given and then the results of the simulation are provided for the similar geometrical configuration of the measurements.

Radar signatures measured by an X-band radar in the presence of rain

Radar measurements of the WV were made by Thales at Charles de Gaulle airport in 2015 with an X-band phased array radar. The example of the Doppler signatures of the WV provided in this section corresponds to the landing of Airbus A380-841 in Charles de Gaulle airport on January 28, 2015. The wingspan of this airplane is 79.75 m and the length is 72.72 m.

The radar was located at the point with coordinates of 49.0174°N, 2.5781°E; the distance from the radar to the runway was 1126 m; the height of the glide at the point corresponding to the intersection of the glide with the radar beam aimed at north is 76 m; the glide slope is 3°. The radar performs scans from 0 to 360° in azimuth with the antenna beam switching between 4 angles in elevation. The beam of the antenna is switched to a new direction in elevation (and in azimuth due to the mechanical rotation of the antenna) after each 25.6 ms, which is equal to the duration of one train of the emitted pulses. The velocity resolution is approximately of 0.6 m/s.

Examples of the time series of the signal scattered by the wake vortex are shown in Fig. 5.23. The duration of the scattered burst is 25.6 ms (512 pulses). In Fig. 5.24, the Doppler spectra for signals scattered from the left and right WV and also some area outside of the WV are shown. The location of the mean value of the Doppler spectrum is shifted from 0 due to the non-zero radial velocities of the raindrops following the local wind field induced by the WV. Doppler spectra calculated for an azimuth angle of about 14°, obtained with 4 elevation angles of the antenna beam, are provided in Fig. 5.25. The zero level of the DS corresponds to the mean level of the noise.

In Fig. 5.25, the presence of the WV is clearly visible at the distance of about 1200 m. Radar signal backscattered from the raindrops in the atmosphere has a positive Doppler frequency shift around 5 m/s, which indicates the presence of the wind that transfers the raindrops and the WV. For the lowest elevation beam (2°), a strong ground clutter is observed. Radar signals reflected from ground objects, which are located at distances corresponding to the delays at which the WV signature is observed, can mask a weak return from the WV. The presence of a zero-Doppler clutter can result in a bias in the estimates of the Doppler spectrum parameters. To remove zero Doppler clutter a zero-Doppler filter should be applied to the signal before the estimation of the WV parameters.

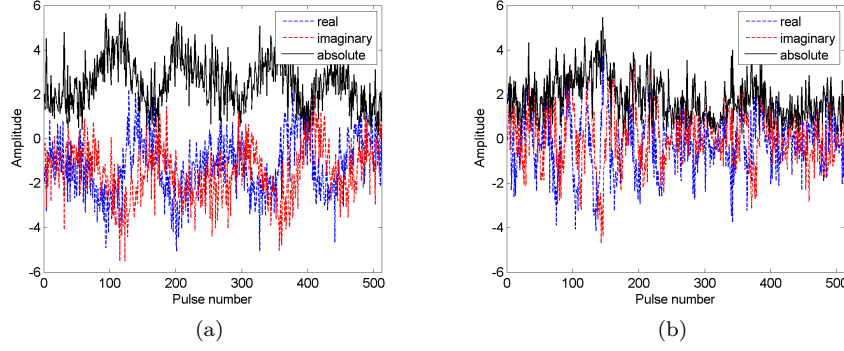


Figure 5.23: Time series of the signal scattered on the (a) left and (b) right wake vortices.

Simulated radar signatures of the WV

The radar simulator developed in this work was run to simulate the detection of the WV under rainy conditions. The parameters of the radar model were chosen to be equal to the parameters of the X-band Thales radar that was used in the trials of the SESAR 12.02.02 JU (Barbaresco et al., 2016) (see Table 3.2). The geometrical configuration of the simulation, nevertheless, differs from (Barbaresco et al., 2016) in order to demonstrate the tracking of the descending WV.

Fig. 5.26 illustrates the geometrical configuration of the simulation: the radar is located at the side of the glide path, at a distance of $r = 1.1$ km from the runway axis; h is the height of the wake vortex core centers with

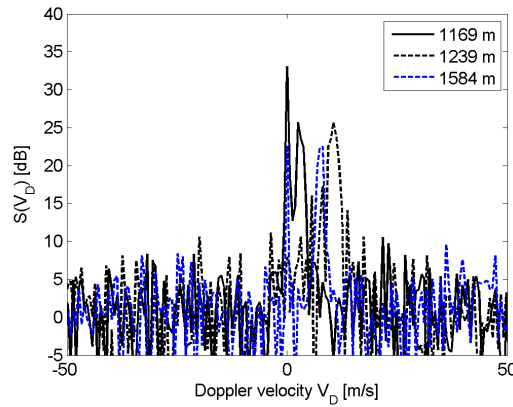


Figure 5.24: Doppler spectra of the signals scattered on the left and right wake vortex cores (1169 m and 1239 m respectively) and from the raindrops in the region of atmosphere outside of the WV (1584 m).

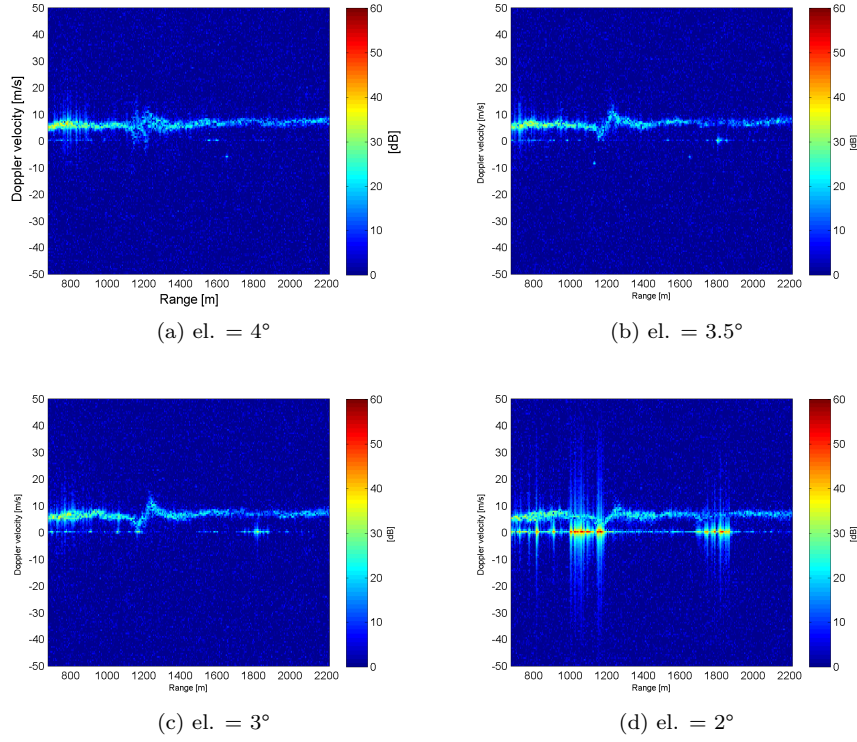


Figure 5.25: Range evolution of the Doppler spectra for 4 elevation angles of the antenna.

respect to the runway; α is the elevation angle of the antenna beam that scans the LES box in elevation to track the WV (antenna beam axis in azimuth is perpendicular to the glide). WVs are descending from 370 m to 280 m. To track the descent of the WV, the radar performs scanning in elevation from 8.8° to 22° with a step of 0.27° . The size of the radar cell in range is 6 m; the half power width of the antenna beam is 1.8° , providing a radar cell width of about 34 m, both in elevation and azimuth, at the range of 1.1 km. Parameters of the radar model are given in Table 3.2.

Fig. 5.27 shows the relative concentration of the 2 mm raindrops in a cross-section of the LES Case 1r (see Table 2.4). The x coordinate of the depicted slice corresponds to the center of the LES box. The borders of the radar cells in the YZ -plane are shown for antenna beam elevation of 17.2° . The simulation presented here was performed for a rain rate of 5 mm/h.

Fig. 5.28a shows the Doppler spectrum calculated with Eq. (3.39) for all radar cells shown in Fig. 5.27. Fig. 5.28b represents the evolution of the Doppler spectrum in range for an elevation angle of antenna beam of 18° . In this work, the positive direction of the radial velocity axis is chosen to be directed towards the radar. In still air surrounding the WV, the mean value and the

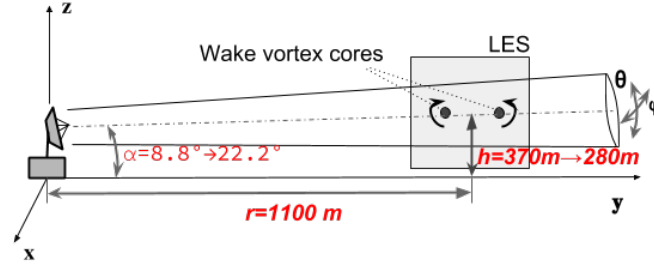


Figure 5.26: Geometrical configuration of the simulation.

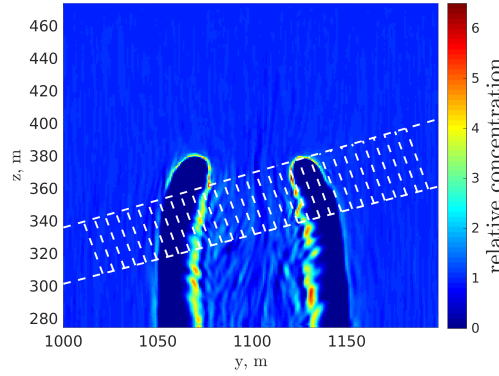


Figure 5.27: Distribution of the relative concentration of the raindrops. Radar cell borders are shown with white dashed lines.

spread of the Doppler spectrum are determined by the mean value and the spread of the falling raindrops terminal velocities respectively. For radar cells that contain the raindrops captured by the WV, the mean Doppler velocity deviates towards the value of the radial velocity of the wind flow induced by the WV. The spread of the spectrum increases (see range 1125 m in Fig. 5.28b). The increase of the spectrum width at range of approximately 1180 m is caused by the raindrops which are located in the same radar cell and are moved in the opposite radial directions. The simulated WV radar signature is in agreement with what was observed with the Thales X-band radar during SESAR 12.02.02 JU. The sharper sine-wave signature of the WV observed in the measurements is due to the low elevation angle compared to the one used in the simulations.

5.3.2 RCS of the wake vortex in the presence of rain

Fig. 5.29 provides the RCS estimated with Eq. (3.38) for each radar cell in one radar scan in elevation from 8° to 22° . The low relative concentration of the raindrops in the WV cores and below (see Figs. 2.19 – 2.20) causes a decrease of the RCS in this area. However, due to the relatively large size of the radar resolution volume, the RCS does not change significantly among the adjusted

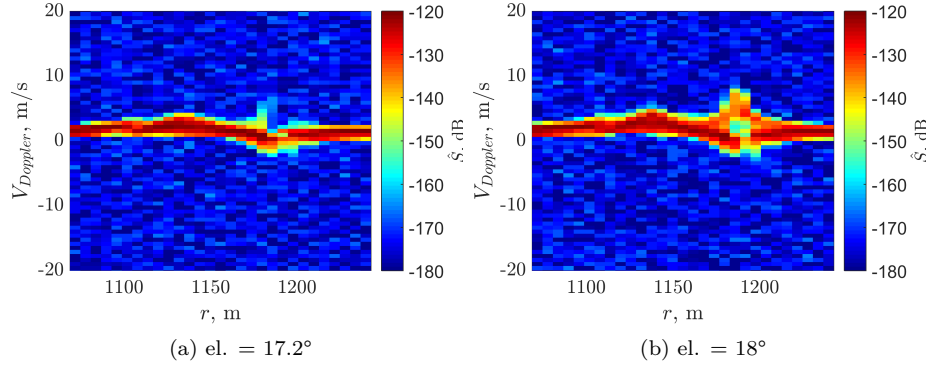


Figure 5.28: Doppler spectrum change in range simulated with LES Case 1r for the time 104 s ($t^* = 2.6$).

radar resolution volumes.

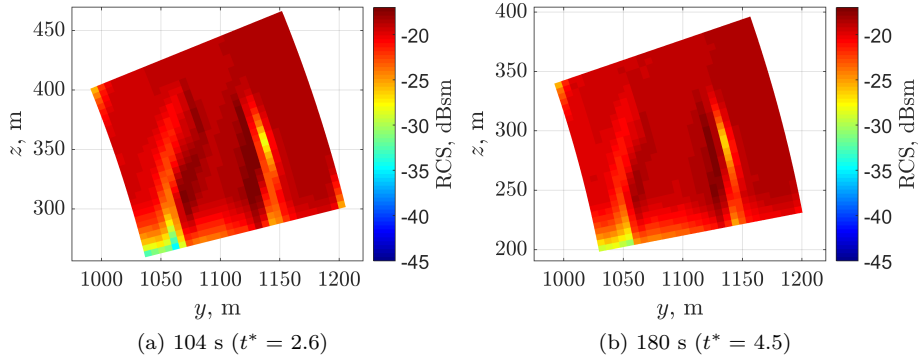


Figure 5.29: RCS of the radar resolution volumes within one vertical scan simulated for Case 1r.

An enhanced level of the RCS is observed in the areas with increased relative concentration of the raindrops. The results shown in Fig. 5.29 are obtained for a rain rate of 5 mm/h. Fig. 5.30 shows the raindrop's size distribution weighted by the RCS of the raindrops. The main contributors to the RCS shown in Fig. 5.29 are raindrops having a diameter of 2 mm. For low rain rates, the main contributors to the RCS are the raindrops with diameters of about 1 mm, while for higher rain rates, the contribution from bigger raindrops becomes more significant (see Figs. 3.9c and 3.9d respectively).

The low values of the RCS at the limit of the scan area are due to the finite size of the LES box. As shown in Section 3.2.4, the simulated LES cases can provide a fair estimate of the WV RCS that allows the estimation of the radar power budget in the X band.

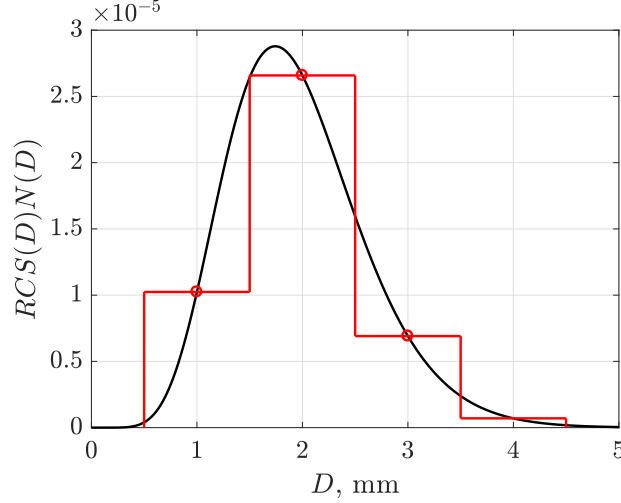


Figure 5.30: Raindrop's size distribution (for a rain rate of 5 mm/h) weighted by the raindrops RCS; red curves represent the approximation of the corresponding dependencies by the raindrops diameters available from the LES simulations.

5.3.3 Estimation of the wake vortex circulation

The strength of the WV can be evaluated indirectly, using the relation between the circulation and the descent velocity of the WV, which is determined by Eq. (4.24). The WV's descent velocity is calculated using the coordinates of the WV cores estimated at each scan. To estimate the location of the WV cores, the Doppler spectrum width is calculated for all radar resolution volumes observed during one scan of the LES box in vertical direction from 8° to 22° in elevation. Fig. 5.31 shows the distribution of the Doppler spectra width in one radar scan at $t^* = 2.6$ for Case 1r. Maximum values of the width provides the estimates of the left and right WV cores location. These values of the DS width (depicted with black circles in Fig. 5.31) are extracted from each vertical scan of the LES frame and then are used to estimate the WV descent velocity, $\bar{V}_c(t)$, and the distance between the WV cores, $\bar{b}(t)$.

Fig. 5.32 provides the results of the WV position estimation for LES frames in the interval from 100 s to 180 s. The absence of noise in the estimation is explained by the high value of the SNR and relatively large volume of the radar cell that filters out the small deviations of the WV position due to the turbulence.

In order to qualitatively check the accuracy of the described method, the circulation available from LES is plotted against the circulation estimated with Eq. (4.24) (Fig. 5.33). The circulation is calculated as an average of the circulation estimated for the left and right wake vortices for both, Γ_{radar} and Γ_{tot} . The estimated values of the circulation are in a good agreement with the true values calculated from the LES wind velocity field. Tracking the descent

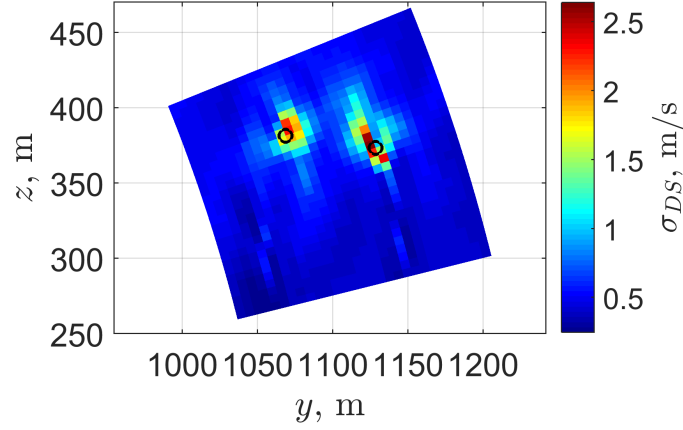


Figure 5.31: Doppler spectrum width estimated for each radar cell. Black circles depict the local maxima of the function.

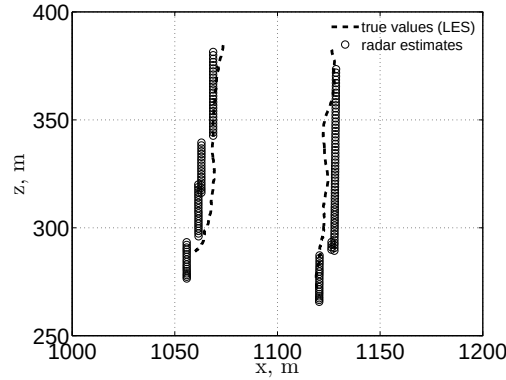


Figure 5.32: Tracking of the WV coordinates.

velocity, e.g. with a Kalman filter, can improve the accuracy of the estimation, especially when the SNR is low.

5.3.4 Backscattering in the W band

Due to the short wavelengths used by radars operating in W band (a few mm), a very narrow antenna beam can be achieved with a relatively small antenna. This enables high resolution radar measurements of the raindrops concentration and raindrops radial velocity influenced by the WVs. The radar simulator was run to simulate the detection of the WVs under rainy conditions (rain rate of 5 mm/h) in the W band. The same radar parameters as in (Seliga and Mead, 2009) were used (see Table 3.3).

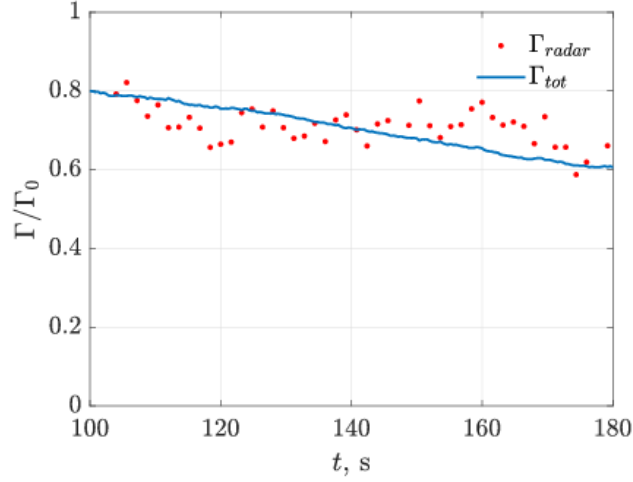


Figure 5.33: WV circulation: estimated and available from LES.

In the W band, the raindrops RCS is calculated with Mie series (Eq. (3.23)). The raindrop's size distribution weighted by the raindrops RCS is shown in Fig. 5.34 for a rain rate of 5 mm/h. As seen from the figure, small rain-

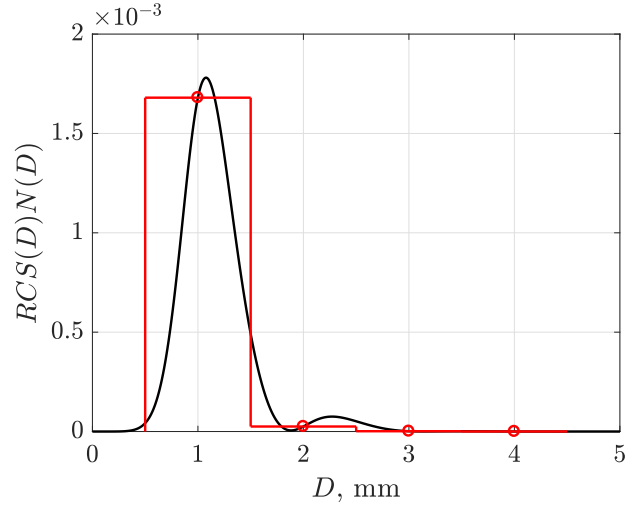


Figure 5.34: Raindrop's size distribution (for a rain rate of 5 mm/h) weighted by the raindrops RCS in the W band; red curves represent the approximation of the corresponding dependencies by the raindrops diameters available from the LES simulations.

drops having diameters of about 1 mm are the main contributors to the total RCS of the radar resolution volume. Configuration C (Section 3.2.1) was used to simulate the backscattered signal's Doppler spectrum. Fig. 5.35 presents

distributions of the RCS and the mean Doppler velocity estimated for radar resolution volumes in a vertical radar scan. The high range and angular resolution allows the detection of the areas with a very low raindrops concentration, which can be used for the WVs identification and tracking. A high resolution of the measured Doppler velocity can be further used to evaluate the WVs' strength. Results shown in Fig. 5.35 qualitatively agree with the results obtained by (Seliga and Mead, 2009) (Fig. 5.36). Note that the direction of the positive radial velocity used in (Seliga and Mead, 2009) is inverse to the direction used in this thesis. However, 1-m radar resolution is about of the LES grid resolution (0.78 m). This can limit the simulations accuracy.

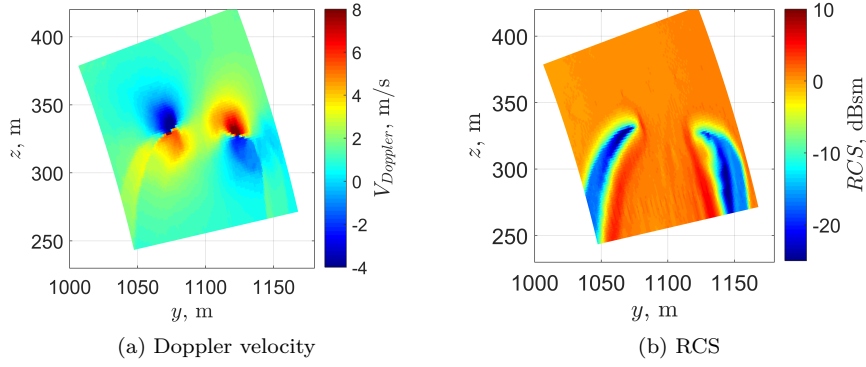


Figure 5.35: Distribution of the Doppler velocity and the RCS in a simulated scan of a W-band radar.

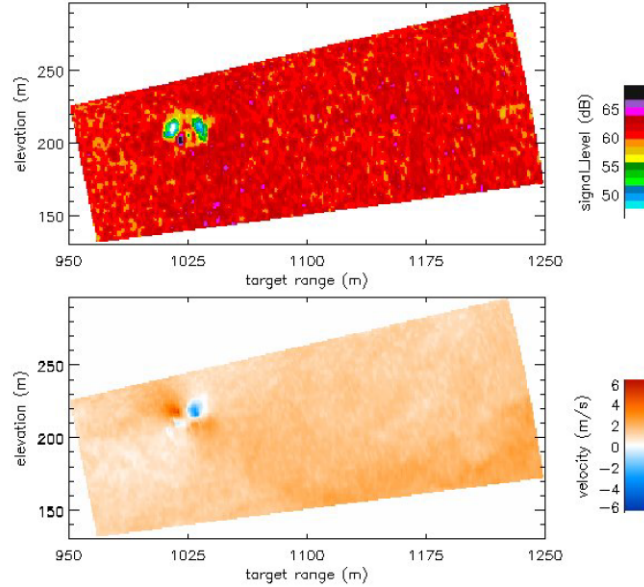


Figure 5.36: Distribution of the Doppler velocity and the RCS in a scan of a W-band radar measured by Seliga and Mead (2009).

5.4 Conclusions

This chapter presents results obtained with a new model for the simulation of the backscattering of a radar signal by WVs in clear air and in the presence of rain. The radar simulator uses, as reference fields, fully developed turbulent WV from LES, evolving in various stratified atmospheres. The LES data serve as inputs for the simulation of the backscattered radar signal, based on the Bragg scattering by the turbulent field created by the WV. The simulation of the radar signal backscattered by the WV, and the associated turbulence, is used for the calculation of the Doppler spectrum, and two models for the retrieval of the circulation are tested.

It is shown that measurements of the turbulence made by a radar in the X band can provide an accurate estimate of the energy dissipation rate and therefore the strength of the turbulence. However for low turbulence intensities, the estimates can be inaccurate due to the fact that the Bragg wavelength of the X-band radar enters the dissipative range of the turbulence spectrum. The simulator confirms the difficulties for a clear-air X-band radar to retrieve accurate EDR values, in particular for EDR values smaller than $0.15 \text{ m}^2/3\text{s}^{-1}$. The sensitivity of the EDR retrieval by clear-air X-band radar can be estimated by using the simulator developed.

The Doppler spectrum components containing the useful information on the WV are located about 20 dB below the peak of the spectrum. Therefore, the circulation can be estimated fairly accurately if the level of the backscattered signal is large enough above the detection threshold of the radar. Both methods considered can provide an accurate estimate of the WV strength. However, they both rely on the Doppler spectrum width estimates, meaning that these methods are very sensitive to the SNR.

The RCS of the WV evaluated in this work corresponds to the lowest estimates of the RCS obtained in measurement trials for L, S and C bands, and predicts a lower RCS for VHF and UHF frequency bands. A very low RCS of the WV limits the possibility of the radar detection of the WV on long distances. The analysis of the WV's RCS in clear air shows that, in order to allow accurate detection of the WV, the distances between the radar and the zone of monitoring should be a few hundreds meters at most, for the emitted power used in this work. So, to obtain a 10 dB SNR at the distance of 1050 m, it is necessary to emit around 380 kW peak power.

The platform developed and the models/methods tested, constitute a useful tool for the optimization of the radar configuration and parameters, and also for the improvement of the link budget. It could also be used, alongside with other algorithms, for a benchmark of methods for the estimation of WV circulations in clear air. The radar simulator developed provides a useful tool for the optimization of the algorithms dedicated to the detection/classification of the WV in the radar signal.

Chapter 6

Conclusions and perspectives

Monitoring of the wake vortices (WVs) with a radar sensor is still a challenging task with many open problems that should be addressed. To support the development of the radar-based sensor for monitoring of the wake vortices in the proximity of the airports, the present study has the following objectives. First, it was aimed at the simulation of the radar signatures of the WVs and also WVs' radar cross-section under clear atmospheric and rainy conditions. Second, simulations of the backscattered radar signals are used to retrieve the parameters of the WVs, its strength and location, and to evaluate the accuracy of this radar-based estimates and the limits of detectability.

6.1 Summary

Two weather conditions have been considered in the present work. In clear air, the aircraft WVs create cascades of turbulent energy similar to the clear atmosphere turbulence. Fluctuation of the refractive index in the turbulent medium causes a Bragg scattering of the electromagnetic waves emitted by the radar. In the presence of rain, the radar signatures of the WVs are caused by the raindrops carried by the local wind field induced by the WVs. The following steps were performed to simulate and analyze the radar signatures of the wake vortices and retrieve their parameters.

1. LES provided by the TFL group of UCLouvain is used to perform an electrical characterization of the scattering medium as it is described in Chapter 3. The local sub-grid energy dissipation rate is used to simulate the refractive index spectrum at the scales of the Bragg wavelength. It has been shown that the presence of the WVs extends the inertial subrange of the turbulence spectrum, making possible backscattering of the X-band EM waves from the refractive index irregularities. The obtained spectra have been further used to simulate the backscattered signal time series. In the presence of rain, the scattering of the EM waves on the raindrops

dominates over scattering on the irregularities of the refractive index. The LES provides information on the concentration of the raindrops of different sizes as well as on their velocities. This information then has been used to simulate the scattering of EM waves on the raindrops as on the dielectric spherical particles.

2. Simulation of the backscattered signal based on the use of the LES simulations of the turbulent stratified atmosphere with and without the WVs has been performed as described in Chapter 3. A special attention was given to the fact that, for the X-band radar wavelength, the Bragg length scale lies within a transition region between the inertial subrange and dissipative range of the atmospheric turbulence spectrum. In this region, turbulence transforms into heat due to the viscosity forces. This leads to a significant decrease of the backscattered signal power. Nevertheless, it has been shown that the presence of the WVs increases the intensity of the turbulence; it results in a significantly higher level of C_n^2 within the WV system comparing to the clear air turbulence and therefore a higher level of the backscattered EM power. For both spectral closure forms considered: Tatarskii's cut-off and Hill's bump, the level of the refractive index spectrum at the Bragg length scale has been found to be of the same order as in the inertial subrange.

In the presence of rain, for the X-band radar used in this work, LES provide the raindrops concentration in the WVs relative to still air, allowing a direct synthesis of the Doppler spectrum of the backscattered radar signal. It has been shown that the LES with several raindrop diameter classes provides flexibility to simulate different intensities of the rain rate with a reasonable accuracy.

For all weather conditions, the LES allows estimation of the WVs RCS which is used to evaluate the SNR. This enables the analysis of the detectability of the WVs under different weather conditions and optimization of the radar parameters and geometrical configurations of the measurements.

3. Analysis of the backscattered signal based on its Doppler spectrum estimation is performed in Chapter 4. It is shown that different parts of the wake vortex system (the one induced by the WVs turbulent field and the mean value of the induced wind field itself) are responsible for corresponding parts of the Doppler spectra. It is shown how the estimated Doppler spectrum and its moments can be used then to estimate the parameters of the turbulent atmosphere (the energy dissipation rate) and the parameters of the wake vortex: its strength and location.
4. Results obtained with a developed simulator are provided and analyzed in Chapter 5. The simulation of the radar signal backscattered by the WVs, and the associated turbulence, are used for the calculation of the Doppler spectrum, and two models for the retrieval of the circulation have been tested.

It is shown that measurements of the turbulence made by a radar in the X band can provide an accurate estimate of the energy dissipation rate and therefore the strength of the turbulence. However for low turbulence intensities, the estimates can be inaccurate due to the fact that the Bragg wavelength of the X-band radar enters the dissipative range of the turbulence spectrum.

It is shown that the Doppler spectrum components containing the useful information on the WVs are located about 20 dB below the peak of the spectrum. Therefore, the circulation can be estimated fairly accurately if the level of the backscattered signal is large enough above the detection threshold of the radar. It has been shown that both methods considered can provide an accurate estimate of the WVs strength. However, they both rely on the Doppler spectrum width estimates, meaning that these methods are very sensitive to the SNR.

The RCS of the WVs evaluated in this work corresponds to the lowest estimates of the RCS obtained in measurement trials for L, S and C bands, and predicts a lower RCS for VHF and UHF frequency bands. A very low RCS of the WVs limits the possibility of the radar detection of the WVs on long distances. The analysis of the WVs' RCS in clear air showed that, in order to allow accurate detection of the WVs, the distances between the radar and the zone of monitoring should be a few hundreds meters, for the emitted power used in this work. So, to obtain a 10 dB SNR at the distance of about 1 km, it is necessary to emit around 380 kW peak power when an X-band radar is used.

6.2 Main achievements

The platform developed and the models/methods tested, constitute a useful tool for the optimization of the radar configuration and parameters, and also for the improvement of the link budget. It could also be used, alongside with other algorithms, for a benchmark of methods for the estimation of WVs circulations in clear air. The radar simulator developed provides a useful tool for the optimization of the algorithms dedicated to the detection/classification of the WVs in the radar signal. The following main results were achieved in the present study.

1. Electrical characterization of the scattering medium based on the large eddy simulation was preformed.
2. A simulator of the radar signal backscattered on the WVs has been developed. It allows simulation of the time series of the signal backscattered in clear air as well as the direct synthesis of the backscattered signal's Doppler spectrum in the presence of rain.
3. A methodology for estimation of the wake vortices RCS has been implemented, which enables the analysis of the detectability of the WVs under different weather conditions.

4. The radar signatures of the WVs were analyzed to establish the relation between the different parts of the Doppler signatures of the WVs and the physical phenomenon causing those contributions.
5. The methodology for retrieval of the atmospheric turbulence strength was verified with using the LES.
6. Two different methods were implemented to analyze the wake vortex strength. It has been shown that the WVs strength can be accurately retrieved from the radar measurements.

6.3 Perspectives

Considering a growing demand in air traffic capacity, especially at large hub airports, it is most likely that the radar will be employed as one of the main sensor aimed at monitoring the WVs, atmospheric turbulence strength, as well as the wind conditions at the airport. Taking that into account, the following steps can be undertaken to support the development of the radar-based sensor.

1. How to improve the circulation calculation in order to get rid of the evaluation of V_{min} .
2. Developing new methods to estimate the circulation of the WVs from the Doppler spectrum and benchmark them with the developed simulator.
3. Use more diameters of the raindrops in the LES simulation to perform the analysis of the WVs signatures for the higher radar frequencies, such as a W band.
4. Simulations of the WVs for earlier moments in time (before the moment when the equilibrium between background turbulence and the WVs is reached) can be studied to investigate the possibility of the EM scattering on the spiral structure of the WVs.
5. More geometrical setups should be analyzed and new types of radar signals for WVs measurements can be considered.
6. Estimation of the WV radar detection in foggy conditions.
7. Evaluation of the possible advantages of using the bistatic radar.
8. Radar simulator can be used to generate the training data set for the developing of the WVs classification/detection algorithms.

List of publications

Journals:

1. **D. Kovalev**, D. Vanhoenacker-Janvier, P. Billuart, M. Duponcheel and G. Winckelmans, “Modeling of Wake Vortex Radar Detection in Clear Air Using Large Eddy Simulation”. - under review, *Journal of Atmospheric and Oceanic Technology*, AMS, 2018.
2. Oude Nijhuis, A., L. Thobois, F. Barbaresco, S. De Haan, A. Dolfi-Bouteyre, **D. Kovalev**, O. Krasnov, D. Vanhoenacker-Janvier, R. Wilson, and A. Yarovoy, “Wind hazard and turbulence monitoring at airports with lidar, radar and Mode-S downlinks: The UFO Project”. *Bull. Amer. Meteor. Soc.*, 2018, <https://doi.org/10.1175/BAMS-D-15-00295.1>

Conferences:

1. **D. A. Kovalev**, D. Vanhoenacker-Janvier, “Estimation of Wake Vortices Radar Cross-Section in Clear Air Using Large Eddy Simulations”, - submitted to Radar 2019 Conference, Toulon, 2019.
2. **D. Kovalev** and D. Vanhoenacker-Janvier, “Validation of electromagnetic wind radar simulator based on LES with scanning X-band radar measurements and meteorological data”, *38th Conference on Radar Meteorology*, Chicago, available online at <https://ams.confex.com/ams/38RADAR/meetingapp.cgi/Paper/320370>, 2017
3. **D. Kovalev**, D. Vanhoenacker-Janvier, R. Wilson and F. Barbaresco, “Electromagnetic wind radar simulator validation using meteorological data and a zenith X-band radar”, 2016 *European Radar Conference (EuRAD)*, London, 2016, pp. 121-124.

Workshops:

1. A.C.P. Oude Nijhuis, L.P. Thobois, F. Barbaresco, S. De Haan, A. Dolfi-Bouteyre, **D. Kovalev**, O.A. Krasnov, D. Vanhoenacker-Janvier, R. Wilson and A.G. Yarovoy, Monitoring of wind hazards and turbulence at airports with lidar and radar sensors and mode-S downlinks: The

UFO Project, *Poster at SESAR Innovation Days*. Technical University of Delft, The Netherlands, 2016

Bibliography

- Adler, R., Fano, R., and Chu, L. (1968). *Electromagnetic Energy Transmission and Radiation*. MIT Press Classics. MIT Press.
- Al-Sakka, H., Weil, A., Le Gac, C., Ney, R., Chardenal, L., Vinson, J.-P., Barthès, L., and Dupont, E. (2009). CURIE : a low power X-band, low atmospheric boundary layer Doppler radar. *Meteorologische Zeitschrift*, 18(3):267–276.
- Atlas, D. and Ulbrich, C. W. (1974). The physical basis for attenuation-rainfall relationships and the measurement of rainfall parameters by combined attenuation and radar methods. *J. Rech. Atmos.*, 209(8):275–298.
- Barbaresco, F. (2012). Radar/lidar sensors for wind & wake-vortex monitoring on airport: First results of SESAR P12.2.2XP0 trials campaign at Paris CDG airport. *Wakenet Conference*.
- Barbaresco, F., Brion, V., and Jeannin, N. (2016). Radar wake-vortices cross-section/Doppler signature characterisation based on simulation and field tests trials. *IET Radar, Sonar & Navigation*, 10(1):82–96.
- Barbaresco, F., Brovelli, P., Currier, P., Garrouste, O., Klein, M., Juge, P., Ricci, Y., and Schneider, J. (2012). Radar sensors for wind & wake-vortex monitoring on airport: first results of SESAR P12. 2.2 XP0 trials campaign at Paris CDG Airport. In *proceeding of the seventh European conference on radar in meteorology and hydrology (ERAD 2012)*, Toulouse.
- Barbaresco, F., Jeantet, A., and Meier, U. (2007). Wake vortex detection & monitoring by X-band Doppler radar : Paris Orly radar campaign results. In *2007 IET International Conference on Radar Systems*, pages 1–5.
- Barbaresco, F., Juge, P., Bruchec, P., Canal, D., Klein, M., Maintoux, J., Orlandi, F., Rahatoka, C., Ricci, Y., and Schneider, J. (2015). Eddy Dissipation Rate (EDR) retrieval with ultra-fast high range resolution Electronic-Scanning X-band airport radar: Results of European FP7 UFO Toulouse Airport trials. In *2015 European Radar Conference (EuRAD)*, pages 145–148.
- Barbaresco, F., Wasselin, J. P., Jeantet, A., and Meier, U. (2008). Wake Vortex Profiling by Doppler X-band radar : Orly trials at Initial Take-Off & ILS interception critical areas. In *2008 IEEE Radar Conference*, pages 1–6.

- Blackman, R. B. and Tukey, J. W. (1958). The Measurement of Power Spectra from the Point of View of Communications Engineering — Part I. *Bell System Technical Journal*, 37(1):185–282.
- Brewster, K. A. and Zrnić, D. S. (1986). Comparison of Eddy Dissipation Rates from Spatial Spectra of Doppler Velocities and Doppler Spectrum Widths. *Journal of Atmospheric and Oceanic Technology*, 3(3):440–452.
- Bringi, V. N. and Chandrasekar, V. (2001). *Polarimetric Doppler Weather Radar: Principles and Applications*. Cambridge University Press.
- Brookner, E. and Cynamon, J. J. (2000). Book Review : Tracking and Kalman Filtering Made Easy. *Simulation*, 75(3):170.
- Burnham, D., Gorstein, M., Hallock, J., Kodis, R., and McWilliams, I. (1971). Aircraft wake vortex sensing systems. Technical report, John A Volpe National Transportation Systems Center Cambridge MA.
- Campbell, W., Chadwick, R., Earnshaw, K., and Moran, K. (1980). Low elevation angle wind measurements by FM-CW radar. In *19th Conference on Radar Meteorology, American Meteorological Society*, pages 722–726.
- Chadwick, R. B., Moran, K. P., Campbell, W. C., Earnshaw, K. B., and Detman, T. R. (1981). Development of a clear air radar to detect meteorological hazards at airports. Technical Report AFGL -TR-81-0268, NOAA/ERL/Wave Propagation Laboratory, Boulder, Colorado.
- Cheng, J., Tittsworth, J., Gallo, W., and Awad, A. (2016). The Development of Wake Turbulence Recategorization in the United States. In *8th AIAA Atmospheric and Space Environments Conference*.
- Chu, Y.-H. (2002). Beam Broadening Effect on Oblique MST Radar Doppler Spectrum. *Journal of Atmospheric and Oceanic Technology*, 19(12):1955–1967.
- Cottet, G. and Koumoutsakos, P. (2000). *Vortex Methods, Theory and Practise*. Cambridge University Press.
- Crow, S. C. (1970). Stability Theory for a Pair of Trailing Vortices. *AIAA journal*, 8:2172–2179.
- Davis, J. I. (1966). Consideration of Atmospheric Turbulence in Laser Systems Design. *Appl. Opt.*, 5(1):139–147.
- De Visscher, I. (2012). *Interaction of wake vortices with the atmosphere and the ground: advanced numerical simulation and operational modeling*. Ecole polytechnique de Louvain, doctoral thesis.
- De Visscher, I., Bricteux, L., and Winkelmann, G. (2013a). Aircraft Vortices in Stably Stratified and Weakly Turbulent Atmospheres : Simulation and Modeling. *AIAA journal*, 51:551–556.

- De Visscher, I., Bricteux, L., and Winckelmans, G. (2013b). Aircraft Vortices in Stably Stratified and Weakly Turbulent Atmospheres : Simulation and Modeling. *AIAA journal*, 51:551–556.
- Deardorff, J. W. (1970). A numerical study of three-dimensional turbulent channel flow at large Reynolds numbers. *Journal of Fluid Mechanics*, 41(2):453–480.
- della Faille, R. and van Swieten, B. (2009). Détection par radar des tourbillons de sillage d’avions. Technical report.
- DGAC (2016). DSNA AIC FRANCE A 03/16 (2016) - Implementation of the RECAT-EU wake turbulence separation scheme at Paris-Charles de Gaulle, Paris-Le Bourget and Pontoise - Cormeilles-en-Vexin airports from March 22nd 2016. Technical report.
- DLR website (Accessed January 26, 2019). Deutsches Zentrum für Luft- und Raumfahrt. https://www.dlr.de/rm/desktopdefault.aspx/tabid-3836/8930_read-16475.
- Doviak, R. J. and Zrnić, D. S. (1984). *Doppler Radar and Weather Observations*. Academic Press, Orlando, FL.
- Easterbrook, C. C. and Joss, W. W. (1971). The Utility of Doppler Radar in the Study of Aircraft Wing-Tip Vortices. In Olsen, J. H., Goldburg, A., and Rogers, M., editors, *Aircraft Wake Turbulence and Its Detection*, pages 97–112, Boston, MA. Springer US.
- Elghobashi, S. (1994). On predicting particle-laden turbulent flows. *Applied Scientific Research*, 52(4):309–329.
- EUROCONTROL (2013). European proposal for revised wake turbulence categorisation and separation minima on approach and departure, “RECAT-EU” - Safety Case Report. Technical report.
- Frehlich, R. (1992). Laser scintillation measurements of the temperature spectrum in the atmospheric surface layer. *Journal of the Atmospheric Sciences*, 49(16):1494–1509.
- Frehlich, R. and Sharman, R. (2005). Maximum likelihood estimates of vortex parameters from simulated coherent doppler lidar data. *Journal of Atmospheric and Oceanic Technology*, 22(2):117–130.
- Gilson, W. (1992). “Radar measurements of aircraft wakes”, Project report AAW-1 1, Lincoln Laboratory. Technical report.
- Hallock, J. N. (1991). Aircraft wake vortices: An assessment of the current situation. Technical report, John A Volpe National Transportation Systems Center Cambridge MA.
- Hanson, J. M. and Marcotte, F. J. (1997). Aircraft wake vortex detection using continuous-wave radar. 18.

- HARMONIE (2016). Hirlam homepage. http://hirlam.org/index.php/component/docman/cat_view/114-modeland-system-documentation?Itemid=70.
- Hayes, M. H. (1996). *Statistical Digital Signal Processing and Modeling*. John Wiley & Sons, Inc., New York, NY, USA, 1st edition.
- Hill, R. J. (1978). Spectra of fluctuations in refractivity, temperature, humidity, and the temperature-humidity cospectrum in the inertial and dissipation ranges. *Radio Sci.*, 13:953–961.
- Hill, R. J. and Clifford, S. F. (1978). Modified spectrum of atmospheric temperature fluctuations and its application to optical propagation. *J. Opt. Soc. Am.*, 68(7):892–899.
- Hocking, W. (1983). On the extraction of atmospheric turbulence parameters from radar backscatter doppler spectra—i. theory. *Journal of Atmospheric and Terrestrial Physics*, 45(2):89 – 102.
- Hocking, W. K. (1985). Measurement of turbulent energy dissipation rates in the middle atmosphere by radar techniques: A review. *Radio Science*, 20(6):1403–1422.
- Holzäpfel, F., Gerz, T., Köpp, F., Stumpf, E., Harris, M., Young, R. I., and Dolfi-Bouteyre, A. (2003). Strategies for circulation evaluation of aircraft wake vortices measured by lidar. *Journal of Atmospheric and Oceanic Technology*, 20(8):1183–1195.
- Huffaker, R. M., Jelalian, A. V., Keene, W. H., and Sonnenschein, C. M. (1971). Application of laser doppler systems to vortex measurement and detection. In Olsen, J. H., Goldburg, A., and Rogers, M., editors, *Aircraft Wake Turbulence and Its Detection*, pages 113–124, Boston, MA. Springer US.
- Iannuzzelli, R. J., Schemm, C. E., Marcotte, F. J., Manzi, L. P., Gilreath, H. E., Hanson, J. M., Frostbutter, D. A., Hughes, A. S., Bric, A. D., Kershner, D. L., and McKenzie, L. E. (1998). Aircraft wake detection using bistatic radar : Analysis of experimental results.
- IEEE (2003). Ieee standard letter designations for radar-frequency bands. *IEEE Std 521-2002 (Revision of IEEE Std 521-1984)*, pages 1–3.
- ITU-R (2005). Specific attenuation model for rain for use in prediction methods.
- ITU-R (2017). The radio refractive index: its formula and refractivity data.
- Jeanmart, H. and Winkelmanns, G. (2007). Investigation of eddy-viscosity models modified using discrete filters: A simplified “regularized variational multiscale model” and an “enhanced field model”. *Phys. Fluids*, 19(5):055110–055126.

- Jiang, H., Sano, M., and Sekine, M. (1997). Weibull raindrop-size distribution and its application to rain attenuation. *IEEE Proceedings - Microwaves, Antennas and Propagation*, 144(3):197–200.
- Kolmogorov, A. N. (1941). Dissipation of energy in locally isotropic turbulence. *Doklady Akademii Nauk SSSR*, 32:16.
- Köpp, F. (1994). Doppler lidar investigation of wake vortex transport between closely spaced parallel runways. *AIAA journal*, 32(4):805–810.
- Köpp, F., Rahm, S., Smakikho, I., Dolfi, A., Cariou, J.-P., and Harris, M. (2005). Comparison of wake-vortex parameters measured by pulsed and continuous-wave lidars. *Journal of aircraft*, 42(4):916–923.
- Koumoutsakos, P. (2005). Multiscale flow simulations using particles. *Annual Review of Fluid Mechanics*, 37(1):457–487.
- Kovalev, D., Vanhoenacker-Janvier, D., Wilson, R., and Barbaresco, F. (2016). Electromagnetic wind radar simulator validation using meteorological data and a zenith x-band radar. In *2016 European Radar Conference (EuRAD)*, pages 121–124.
- Kovalev, D. A., Vanhoenacker-Janvier, D., Billuart, P., Duponcheel, M., and Winckelmans, G. (2018). Modeling of wake vortex radar detection in clear air using Large Eddy Simulation. *under review, Journal of Atmospheric and Oceanic Technology*.
- Krause, S. (2003). *Aircraft Safety: Accident Investigations, Analyses, & Applications, Second Edition*. McGraw-Hill Education.
- Lanchester, F. W. (1908). *Aerodynamics: constituting the first volume of a complete work on aerial flight*, volume 1. D. Van Nostrand.
- Li, J. (2010). Study on the radar signatures of aircraft wake vortices.
- Li, J., Wang, T., Liu, Z., and Wang, X. (2016). Circulation retrieval of wake vortex in fog with a side-looking scanning radar. *IEEE Transactions on Aerospace and Electronic Systems*, 52(5):2242–2254.
- Li, J., Wang, X., and Wang, T. (2010). Scattering mechanism of aircraft wake vortices generated in clear air. In *2010 IEEE Radar Conference*, pages 117–122.
- Li, J., Wang, X., Wang, T., Liu, J., Gao, H., and Chandrasekar, V. (2017). Circulation retrieval of wake vortex under rainy condition with a vertically pointing radar. *IEEE Transactions on Aerospace and Electronic Systems*, 53(4):1893–1906.
- Lilly, D. K. (1967). In *The representation of small-scale turbulence in numerical simulation experiments.*, pages 195–210, Thomas J. Watson Res. Center, Yorktown Heights, N.Y.

- Liu, Z. (2013). *Modeling of radar signatures of wake vortices*. Université de Toulouse, doctoral thesis, Toulouse.
- Liu, Z., Jeannin, N., Vincent, F., and Wang, X. (2013). Modeling the radar signature of raindrops in aircraft wake vortices. *Journal of Atmospheric and Oceanic Technology*, 30(3):470–484.
- Lovejoy, S. and Schertzer, D. (2008). Turbulence, raindrops and the $1\frac{1}{2}$ number density law. *New Journal of Physics*, 10(7):075017.
- Mackenzie, A. I. (1997). Measured changes in c-band radar reflectivity of clear air caused by. Technical report.
- Marshall, R. and Mudukutore, A. (1996). Wake vortex radar performance studies and simulated detection of wake vortices by a ka-band radar in fog. Technical report.
- Marshall, R. E., Mudukutore, A., Wissel, V. L. H., and Myers, T. (1997). Three-centimeter doppler radar observations of wingtip-generated wake vortices in clear air, technical report nasa/cr-97-206260. Technical report.
- Marshall, R. E. and Myers, T. J. (1996). Wingtip generated wake vortices as radar target. *IEEE Aerospace and Electronic Systems Magazine*, 11(12):27–30.
- McComb, W. D. (1990). *The Physics of Fluid Turbulence*. Clarendon Press.
- Morris, P. J., McLaughlin, D. K., Gabrielson, T. B., and Boluriaan, S. (2004). The development of a plan for the assessment, improvement and deployment of a radar acoustic sounding system (rass) for wake vortex detection.
- Mudukutore, A., Staton, L., White, J., and Nguyen, D. (1998). Pre-experiment report: Wake vortex ka-band radar performance studies. Technical report.
- Muschinski, A., Sullivan, P. P., Wuertz, D. B., Hill, R. J., Cohn, S. A., Lenschow, D. H., and Doviak, R. J. (1999). First synthesis of wind-profiler signals on the basis of large-eddy simulation data. *Radio Science*, 34(6):1437–1459.
- Myers, T. J. (1997). *Determination of Bragg Scatter in an Aircraft Generated Wake Vortex System for Radar Detection*. Virginia Polytechnic Institute, doctoral thesis, Blacksburg, Virginia.
- Myers, T. J., Scales, W. A., and Marshall, R. E. (1999). Determination of aircraft wake vortex radar cross section due to coherent bragg scatter from mixed atmospheric water vapor. *Radio Science*, 34(1):103–111.
- NASA website (Accessed January 26, 2019). NASA. <https://www.dfrc.nasa.gov/Gallery/Photo/B-727/HTML/ECN-3831.html>.
- Nastrom, G. D. and Eaton, F. D. (1997). Turbulence eddy dissipation rates from radar observations at 5–20 km at white sands missile range, new mexico. *Journal of Geophysical Research: Atmospheres*, 102(D16):19495–19505.

- Nathanson, F., Reilly, J., and Cohen, M. (1999). *Radar Design Principles: Signal Processing and the Environment*. Scitech Pub.
- Neece, R. T., Britt, C. L., White, J. H., Mudukutore, A., Nguyen, C., and Hooper, B. (2005). Wake vortex tracking using a 35 ghz pulsed doppler radar. *5th NASA Integrated Communications, Navigation, and Surveillance (ICNS) Conference and Workshop*.
- Nespor, J., Hudson, B., Stegall, R., and Freedman, J. (1991). Doppler radar detection of vortex hazard indicators. In *Proceedings of the Aircraft Wake Vortices Conference*, Washington, DC.
- Noonkester, V. R. and Richter, J. H. (1980). Fmcw radar sensing of the lower atmosphere. *Radio Science*, 15(2):337–353.
- Noonkester, V. R., Richter, J. H., and Jensen, D. R. (1974). Unique echoes observed by fm-cw radar at a jet airport. Technical Report Tech. Note 2787, Naval Electronics Lab. Center, San Diego, CA. (Information to be identified as tentative and unpublished).
- Olsen, J. (1970). *Aircraft Wake Turbulence and Its Detection: Proceedings of a Symposium on Aircraft Wake Turbulence Held in Seattle, Washington, September 1–3. Sponsored Jointly by the Flight Sciences Laboratory, Boeing Scientific Research Laboratories and the Air Force Office of Scientific Research*. Springer Science & Business Media.
- Ottersten, H. (1969). Atmospheric structure and radar backscattering in clear air. *Radio Science*, 4(12):1179–1193.
- Oude Nijhuis, A. C. P., Thobois, L. P., Barbaresco, F., De Haan, S., Dolfi-Bouteyre, A., Kovalev, D., Krasnov, O. A., Vanhoenacker-Janvier, D., Wilson, R., and Yarovoy, A. G. (2018). Wind hazard and turbulence monitoring at airports with lidar, radar, and mode-s downlinks: The UFO project. *Bulletin of the American Meteorological Society*, 99(11):2275–2293.
- Pope, S. B. (2011). *Turbulent flows*. Cambridge Univ. Press, Cambridge.
- Proctor, F. H. (1987). *The terminal area simulation system*. National Aeronautics and Space Administration, Scientific and Technical Information Branch.
- Richter, J. H. (1969). High resolution tropospheric radar sounding. *Radio Science*, 4(12):1261–1268.
- Richter, J. H., Jensen, D. R., and Phares, M. L. (1971). Scanning fm-cw radar sounder. *Review of Scientific Instruments*, 43(11):1623–1625.
- Rossow, V. J. (1999). Lift-generated vortex wakes of subsonic transport aircraft. *Progress in Aerospace Sciences*, 35(6):507 – 660.
- Rubin, W. L. (2000). Radar Acoustic Detection of Aircraft Wake Vortices. *Journal of Atmospheric and Oceanic Technology*, 17:1058–1065.

- Rubin, W. L. (2005). The generation and detection of sound emitted by aircraft wake vortices in ground effect. *Journal of Atmospheric and Oceanic Technology*, 22(5):543–554.
- Sauvageot, H. (1992). *Radar Meteorology*. Artech House.
- Seliga, T. A. and Mead, J. B. (2009). Meter-scale observations of aircraft wake vortices in precipitation using a high resolution solid-state w-band radar. In *Proc. 34th Conf. Radar Meteorol.*
- SESAR 12.02.02 (2016). Runway wake vortex detection, prediction and decision support tools d000, 00.02.00.
- Shariff, K. (2016). Making aircraft vortices visible to radar by spraying water into the wake. *Journal of Atmospheric and Oceanic Technology*, 33(12):2615–2638.
- Shariff, K. and Wray, A. (2002). Analysis of the radar reflectivity of aircraft vortex wakes. *Journal of Fluid Mechanics*, 463:121–161.
- Shephard, D. J., Kyte, A. P., and Segura, C. A. (1994). Radar wake vortex measurements at f and i band. In *IEEE Colloquium on Radar and Microwave Imaging*, pages 7/1–7/5.
- Smagorinsky, J. (1963). General circulation experiments with the primitive equations. *Monthly Weather Review*, 91(3):99–164.
- Sreenivasan, K. R. (1995). On the universality of the kolmogorov constant. *Physics of Fluids*, 7(11):2778–2784.
- Strauch, R. G. (1979). Applications of meteorological doppler radar for weather surveillance near air terminals. *IEEE Transactions on Geoscience Electronics*, 17(4):105–112.
- Sutton, O. (1953). *Micrometeorology: A Study of Physical Processes in the Lowest Layers of the Earth's Atmosphere*. McGraw-Hill.
- Tatarskii, V. I. (1971). *The effects of the turbulent atmosphere on wave propagation*. Israel Program for Scientific Translations.
- Taylor, G. I. (1935). Statistical theory of turbulence. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 151(873):421–444.
- Taylor, G. I. (1938). The spectrum of turbulence. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 164(919):476–490.
- Thurai, M., Huang, G. J., Bringi, V. N., Randeu, W. L., and Schönhuber, M. (2007). Drop shapes, model comparisons, and calculations of polarimetric radar parameters in rain. *Journal of Atmospheric and Oceanic Technology*, 24(6):1019–1032.

- UFO project (2016). Accessed January 26, 2019. <http://www.ufo-wind-sensors.eu/home>.
- Vanhoenacker-Janvier, D., Djafri, K., and Barbaresco, F. (2012). Model for the calculation of the radar cross section of wake vortices of take-off and landing airplanes. In *2012 9th European Radar Conference*, pages 349–352.
- Vanhoenacker-Janvier, D., Pereira, C., Duponcheel, M., and Oude Nijhuis, A. (2014). *Radar Simulator for wind monitoring, UFO Deliverable: D2110-2*. Université catholique de Louvain-Delft University of Technology.
- White, A. B., Lataitis, R. J., and Lawrence, R. S. (1999). Space and time filtering of remotely sensed velocity turbulence. *Journal of Atmospheric and Oceanic Technology*, 16(12):1967–1972.
- Williamson, J. (1980). *Low storage Runge-Kutta schemes*.
- Winckelmans, G. (2004). *Encyclopedia of Computational Mechanics, Volume 3 Fluid, Chapter 5 Vortex Method*.
- Zeman, O. (1995). The persistence of trailing vortices: A modeling study. *Physics of Fluids*, 7(1):135–143.
- Zhang, Y., Wang, F. Y., and Hardin, J. C. (2003). Spectral characteristics of wake vortex sound during roll-up.