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ON THE EFFECTS OF CONJECTURES IN A SYMMETRIC STRATEGIC SETTING

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Abstract

This paper deals with the effect of conjectures in a strategic setting. To do this it focuses on the so-called Conjectural Variation Equilibrium (CVE). According to this concept, each agent chooses his most favorable action taking into account that rival strategies are a conjectured function of his own strategy. In the existing literature, a central role is played by the comparison between the CVE and the Nash Equilibrium (NE). The purpose of such a comparison is to appraise the impact of non zero conjectures on agents' behaviors. The existing results suggest that it is not possible to know, in advance, the consequences of non zero conjectures on behaviors. Our aim is: i) to identify situations where it is indeed possible, a priori, to know which kind of non cooperative concept Pareto dominates the other, ii) to provide out the corresponding theoretical explanations. The economic situations can be divided into two families, depending on whether they admit a stable Nash equilibrium and an interior Pareto solution (family 1) or not (family 2). Within each family it is shown that the sign of the externalities (positive or negative effect of the rival actions on a player's payoffs) together with the properties of conjectures (their sign and their absolute value) : i) indicates how to rank the action levels associated with the NE and the CVE, ii) allows one to predict which kind of behavior leads the players to the most favorable outcome. It turns out that

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the qualitative results prevailing for family 1 are reversed for family 2. This classification is useful in that outcomes and payoffs need not be calculated to assess the impact of conjectures on players' payoffs; the only relevant pieces of information are the sign of second order derivatives of the payoff function and the properties of conjectures, i.e. the description of the game. We then study in which kind of game reasonable conjectures, i.e. consistent conjectures, belongs to the set of conjectures that produces superior outcomes.

Keywords: Conjectural Variations, Nash Equilibria, Consistency.
JEL Classification: C72, D43, D62, E61, H41

1 Introduction

In order to describe the decision process of agents interacting non co-operatively, economists borrow many equilibrium concepts from Game Theory. The most popular equilibrium concept is the Nash Equilibrium, i.e. the outcome that arises when each agent, taking as given the other agents' actions, selects the action that optimizes his payoff. It is fair to say, however, that many empirical studies have rejected this non co-operative concept as a correct way to describe observed behaviors. For instance Slade 1995 [17] surveys some empirical findings concerning several oligopoly industries. It turns out that observed decisions seem to be more collusive than the theoretical prediction given by the Nash equilibrium. There are also some doubts on the ability of the traditional model of private provision of public goods, where givers behave has Nash players, to explain real world examples such as charities and voluntary organizations (Bordignon 1989 [5], Bordignon 1994 [6], Sugden 1982 [18]). For these situations, economists have to find out other theoretical explanations for social interactions.

Another non co-operative equilibrium concept is the so-called Conjectural Variation Equilibrium (CVE). According to this concept, agents behave as follows: each agent chooses his most favorable action taking into account that rival strategies are a conjectured function of his own strategy. This idea dates back to Bowley 1924 [1] and Frisch [10]. The expression "conjectural variation of agent i" refers to the reactions induced by a small variation of agent i's strategy, as perceived by this agent. In the earliest descriptions, the conjectures were left completely exogenous, a characteristic that very soon appeared to undermine the theoretical grounds of the CVE. Indeed, under the CVE the outcome of the game is configured by the conjectures, which means conversely that almost any observed behavior can be described as a CVE with an appropriate choice of conjectures. In other words, it has been realized that the CVE with arbitrary conjectures provides a theory of economic behaviors that is not refutable. Some refinements of the concept have then been proposed to overcome this issue; the conjectures became endogenous. For instance Laitner 1980 [14], Bresnahan 1981 [4], Perry 1982 [16], Kamien and Schwartz 1983 [12] and Boyer and Moreaux 1983 [3] have studied the idea of Consistent Conjectural Variation Equilibrium (CCVE). According to this concept conjectures are required to be *consistent* in the sense that the best reply functions obtained under those conjectures must correspond, to some extent, to the conjectured reply functions. Arguing on the idea of reaction, other researchers have proposed to see the Conjectural Equilibrium outcome as the steady state of an implicit dynamic game (Worthington 1969 [21], Dockner 1992 [9], and so on). Meanwhile many papers have made use of one form or the other of the CVE to predict the outcome of non cooperative behaviors in Industrial Economics, in International Economics, in Macroeconomics, in Public Economics and in Regional Economics. A considerable amount of empirical and econometric works also exists that evaluates what conjectures look like in particular game theoretic situations.

In this literature an important role is played by the comparison between the CVE (or the CCVE) and the Nash Equilibrium (NE). The purpose of such a comparison is obvious. The NE assumes each player behaves as if rival strategies were constant, independent of his own actions. Put differently, each player conjectures zero reaction to his decisions. It is natural to use this concept as a benchmark case from which the impact of non zero conjectures on agents' behaviors can be appraised. For example in a symmetric Cournot Duopoly, under linear costs and linear inverse demand, the consistent conjectural variation is -1, the quantities brought onto the market correspond to Bertrand's outcome and the NE Pareto dominates the CCVE (from the point of view of the two firms). Should one infer that conjectural variations, and in particular, consistent ones, enhance competition and reduce the players' payoffs? Such a conclusion would be generally wrong. Indeed, looking at other numerous comparisons it turns out that sometimes agents are better off under the NE, sometimes they are better off under the CVE. It seems difficult to obtain a clear idea on how behaviors and payoffs are modified when agents make non zero conjectures. Some authors even claim that it is impossible to know, a priori, which kind of behavior yields higher payoffs. We argue in the present paper that this assertion is incorrect. More precisely our aim is: i) to identify situations where it is possible, a priori, to know which kind of non cooperative concept Pareto dominates the other, ii) to provide out the corresponding theoretical explanations.

Our analysis is restricted to the case where players have symmetric payoff functions and identical strategy spaces. This restriction has three advantages. Firstly it allows simple calculations and clear intuitions. Secondly, outside the symmetric setting, equilibria are generally not symmetric. As a consequence CVE and NE can hardly be compared: for instance some players might be better-off under the CVE while the others would be worse-off. Thirdly, and more importantly, we feel that the question we ask in this paper, that of the effects of conjectures in strategic situations, cannot be optimally challenged in non symmetric contexts. In symmetric settings any difference between CVE and NE solely stems from the conjectures. By contrast, in asymmetric worlds any divergence from the NE may occur for two reasons: players hold some conjectures and the conjectures themselves are asymmetric!

The paper is organized as follows. Section 2 presents a symmetric two-player game theoretical framework and establishes two theorems. Briefly speaking, the economic situations can be divided into two families, depending on whether they admit a stable Nash equilibrium and an interior Pareto solution (family 1) or not (family 2). Within each family it is shown that the sign of the externalities (positive or negative effect of the rival actions on a player's payoffs) together with the properties of conjectures (their sign and their absolute value) : i) indicates how to rank the action levels associated with the NE and the CVE, ii) allows one to predict which kind of behavior leads the players to the most favorable outcome. It turns out that the qualitative results prevailing for family 1 are reversed for family 2. This classification is useful in that outcomes and payoffs need not be calculated to assess the impact of conjectures on players' payoffs; the only pertinent pieces of information are the kind of externalities, the properties of conjectures and the sign of second order derivatives of the payoff function i.e. the description of the game. Still in two-player games, Section 3 indicates how our results can be related to the more specific concept of consistent conjectural variation equilibrium (CCVE). In our view, this concept deserves a separate treatment. As explained above, since almost every outcome can be supported as a CVE with an appropriate choice of conjectures, there is no surprise (and no merit) to find some conjectures such that the corresponding CVE Pareto dominates the NE. A more interesting question is: in which kind of game endogenous conjectures, here consistent conjectures, belongs to the set of conjectures that produces superior outcomes? It is shown that the answer to this question depends again entirely on the second order cross derivative of the payoff functions, (when we deal with constant conjectural variations). Section 4 illustrates the theorems with some well known economic examples. Section 5 discusses the definition of consistent conjectural variation for many players, provides an extension of the main theorems for more than two players and illustrates the results with two examples: an economy with a macro-consumption externality, and the private provision of a public good. Finally, Section VI concludes.

2 First results for two player games

Consider two players interacting in the following symmetric game. They pick their strategies in a compact convex subset of R

$$e_i \in E = [\underline{E}, \overline{E}] \subset R \quad i = 1, 2$$

in order to maximize their payoff function which is assumed to be twice continuously differentiable

$$V^i(e_i, e_j) \in C^2(E^2), \quad i = 1, 2 \ .$$

By convention, the first variable of a player's payoff function is the strategy of this player. We consider symmetric payoffs, i.e.:

$$V^2(e_2, e_1) = V^1(e_2, e_1) = V(e_2, e_1).$$

As indicated by the last equality only one function needs to be considered, the first argument being sufficient to indicate the player associated with the payoff. For instance, in the duopoly model studied in Section 4.3, the payoff functions are:

$$\begin{aligned} V^1(e_1, e_2) &= (\alpha - \beta(e_1 + e_2))e_1 - ce_1 \\ V^2(e_2, e_1) &= (\alpha - \beta(e_2 + e_1))e_2 - ce_2 \ . \end{aligned}$$

Clearly, only one function $V(x, y)$ is needed to describe the payoffs:

$$V(x, y) = (\alpha - \beta(x + y))x - cx \ .$$

In order to assess the impact of conjectures on decisions it is necessary to compare three possible scenarii: i) agents behave non cooperatively taking rival decisions as given, ii) agents decide non cooperatively given some conjectures on rival strategies, iii) decisions are taken by a benevolent authority who is in charge with the interest of both agents. Relating the outcomes of these different scenarii we can learn how conjectures distort the individual decisions and what are the consequences in terms of efficiency.

We shall work throughout this paper with interior symmetric solutions only. Clearly, this choice discards presumably interesting cases; it however fits well with a large body of the literature where, to our knowledge, outcomes turn out ot be interior.

In the first scenario, denote (e^N, e^N) a symmetric Nash equilibrium. A necessary condition satisfied by any interior symmetric Nash equilibrium is:

$$V_1(e^N, e^N) = 0. \tag{1}$$

where subscript 1 is used to indicate the partial derivative of the payoff function with respect to the first argument.

Turning to the second scenario, players have conjectures about the rival behavior. The conjectural variation formed by player i about the reaction of player j to a local variation of i's strategy around e_i is:

$$r_j(e_i) = r(e_i), \quad i = 1, 2, \quad i \neq j.$$

The r.h.s. of this last equality clearly indicates that, in our framework, players are endowed with the same variational conjectures. Let us denote (e^c, e^c) a symmetric conjectural variational equilibrium. A necessary condition satisfied by any interior symmetric conjectural variation equilibrium is (see Bowley 1924 [1], Bresnahan 1981 [4]):

$$V_1(e^c, e^c) + V_2(e^c, e^c) r(e^c) = 0. \tag{2}$$

In the last scenario, the pair (e^p, e^p) denotes a symmetric Pareto optimum. In this symmetric context, it makes sense to focus on the utilitarian criterion to derive a Paretian outcome. In other words the Pareto optimum under consideration will be the one that optimizes the sum of the payoffs $V(e_1, e_2) + V(e_2, e_1)$. A necessary condition satisfied by any interior symmetric Pareto optimum for this utilitarian criterion is:

$$V_1(e^p, e^p) + V_2(e^p, e^p) = 0. \tag{3}$$

Observe that (3) is a particular case of a conjectural equilibrium, with $r(e) = 1$, whereas (1) obtains for $r(e) = 0$.

The following definitions will be useful in the discussion to follow. Let:

- $G^N(e) = V_1(e, e)$ stand for the marginal effect of e_i on agent i's payoff, evaluated at a symmetric outcome,
- $G^c(e) = V_1(e, e) + V_2(e, e)r(e)$ be the marginal effect of e_i on agent i's payoff given a conjectured relation $e_j = r(e_i)$ between strategies, still for a symmetric outcome,
- $G^p(e) = V_1(e, e) + V_2(e, e)$ be the marginal effect of e_i on the utilitarian criterion,
- and let $W(e) = V(e, e)$.

We have:

i) $G^c(e) - G^N(e) = V_2(e,e)r(e)$.

ii) $G^p(e) - G^c(e) = V_2(e,e)(1 - r(e))$.

iii) $G^p(e) - G^N(e) = V_2(e,e)$

iv) $W'(e) = G^p(e)$. Therefore, any solution (e^p, e^p) of (3) which is a global maximum is such that

$$W(e^p) \;=\; V(e^p, e^p) \;\geq\; V(e,e) \;=\; W(e),\ \forall e \ .$$

2.1 Main Results

As we now proceed to show, the impact of conjectures on the outcome of the game is heavily driven by the sign of the function:

$$V_{11}(e,e) + V_{12}(e,e) \ ,$$

as well as the sign of the externality V_2, and the position of the conjectural variation $r(e)$ with respect to the values 0 and 1.

Technically the importance of some of the above elements can be grasped easily. Take a given constant conjecture r. The conjectural variational equilibrium e^c and the corresponding payoffs are parametrized by r,

$$e^c = e^c(r) \quad \text{and} \quad \widetilde{V}(r) = V(e^c(r), e^c(r)).$$

A small change in the conjecture r will have marginal impacts on equilibrium $(e^c)'(r)$ and payoffs as well:

$$\widetilde{V}'(r) = (V_1(e^c(r), e^c(r)) + V_2(e^c(r), e^c(r)))(e^c)' \ .$$

From (2) with r constant, one obtains by differentiating with respect to r:

$$(e^c)' = -\frac{V_2}{(V_{11} + V_{12}) + (V_{12} + V_{22})r}$$

so,

$$(e^c)'\big|_{r=0} = -\frac{V_2}{(V_{11} + V_{12})}$$

which makes clear that locally, starting from a Nash (zero) conjecture, the sign of $(e^c)'$, hence the sign of $\widetilde{V}'$, depends on the sign of externalities and the sign of $V_{11}+V_{12}$. Note also that the sign of $(V_1(e^c(r), e^c(r))+V_2(e^c(r), e^c(r)))$ is related to the zero of function $G^p(e)$.

With these technical informations in mind, consider the following assumptions:

(**H1**) There exists a unique interior Nash equilibrium in E^2 and

$$V_{11}(e,e) + V_{12}(e,e) \leq 0 \qquad \forall e \in E. \tag{4}$$

Alternatively the reverse inequality will also prove useful:

(**H1'**) There exists a unique interior Nash equilibrium in E^2 and

$$V_{11}(e,e) + V_{12}(e,e) \geq 0 \qquad \forall e \in E. \tag{5}$$

(**H2**) There exists at most one zero of the function G^p in the interior of E.

For the moment, we shall merely note that property (4) implies that $G^N(e)$ is a decreasing function; on the contrary (5) implies that $G^N(e)$ is an increasing function.

Moreover, if there exists a unique Nash equilibrium in a symmetric game, this equilibrium must be symmetric[1]. A consequence of (**H1**) or (**H1'**) is therefore the existence of a unique $e^N \in E$ such that $G^N(e^N) = 0$. We shall come back to these assumptions in Section 2.2.2.

Two main general theorems are developed. They compare Nash, conjectural variation and Pareto strategies and also the corresponding payoffs. The results of the comparisons depends on the sign of externalities and conjectures. These Theorems 1 and 2 are associated respectively with (4) and (5). We then proceed to a discussion of the assumptions and the results.

Theorem 1 *Assume (**H1**) and (**H2**) are satisfied. Then for the (symmetric) Nash equilibrium, any symmetric conjectural variation equilibrium and the (symmetric) Pareto optimum (if it exists):*
Case 1 *If $V_2 > 0$ (positive externality) and $r(e) > 0$ (positive conjectural variation), then:*
Either $r(e) \leq 1$, and then

$$e^N \leq e^c \leq e^p \tag{6}$$

and

$$V(e^N, e^N) \leq V(e^c, e^c) \ ; \tag{7}$$

Or $r(e) > 1$, and then:

$$e^N \leq e^p \leq e^c \tag{8}$$

and it is not possible to order the payoffs associated with the outcomes.
Case 2 *If $V_2 < 0$ (negative externality) and $r(e) > 0$ (positive conjectural*

[1] For if (e_1, e_2) is a Nash equilibrium, then so is (e_2, e_1) from the symmetry assumptions on V^1 and V^2. If $e_1 \neq e_2$, this makes two distinct Nash equilibria.

variation), then:
Either $r(e) \leq 1$, *and then*

$$e^p \leq e^c \leq e^N$$
$$V(e^N, e^N) \leq V(e^c, e^c) \ ;$$

Or $r(e) > 1$, *and then* $e^c \leq e^p \leq e^N$, *and it is not possible to order the payoffs associated with the outcomes.*
Case 3 *If* $V_2 > 0$ *and* $r(e) < 0$ *then*

$$e^c \leq e^N \leq e^p$$
$$V(e^c, e^c) \leq V(e^N, e^N) \ .$$

Case 4 *If* $V_2 < 0$ *and* $r(e) < 0$, *then*

$$e^p \leq e^N \leq e^c$$
$$V(e^c, e^c) \leq V(e^N, e^N) \ .$$

Case 1 is illustrated in Figure 1, and Case 4 is illustrated in Figure 3, with payoff functions taken from the examples of Sections 4.4 and 4.3, respectively.
Proof. **Case 1:** The assumptions of positive externalities and positive conjectures imply $G^c(e) - G^N(e) = V_2(e,e)r(e) > 0$.
If $r(e) \leq 1$ then $G^p(e) - G^c(e) \geq 0$ and since $r(e) > 0$, then $G^c(e) - G^N(e) > 0$. Therefore:

$$G^N(e) < G^c(e) \leq G^p(e) \qquad \forall e \in E \ . \tag{9}$$

This inequality, the assumption that $G^N(e)$ is a decreasing function, and (**H2**) clearly imply that (6) holds for any conjectural equilibrium e^c.

If there exists e^{p_1} solution of $G^p(e) = 0$ in E, then $G^p(e) \geq 0$ if $e \leq e^{p_1}$. If $G^p(e) \neq 0$ in E, then $G^p(e) > 0$ in E. In the latter case $G^p(e)$ can not be negative because it is in contradiction with (9). This implies that $W(e)$ is an increasing function for all $e \leq e^{p_1}$ or for all $e \in E$, and because $e^N \leq e^c$, $V(e^N, e^N) \leq V(e^c, e^c)$, so (7) holds.

If $r(e) \geq 1$ then $G^p(e) - G^N(e) = V_2(e) > 0$ et $G^p(e) - G^c(e) < 0$, then

$$G^N(e) < G^p(e) < G^c(e) \qquad \forall e \in E$$

so (8) holds but no comparison is possible between the payoff functions. We show this through an example. Consider the Public Goods model, with payoff function: $V(x,y) = (x+y)^{3/4}(1-x)^{1/4}$ (we suppose individual income

$I = 1$). The results for this model are recalled in § 4.4. In particular, constant CVE $e^c(r)$ can be easily computed. Figure 1 shows the functions G^N, G^p and G^c. Figure 2 shows the shape of the payoff function V computed in $x = e^c(r)$, $y = e^c(r)$ as a function of r, that we call $f(r)$. It is clear that when $0 < r \leq 1$ the payoff function is larger than the payoff function computed at the Nash equilibrium ($r = 0$). When $r > 1$ it depends on the value of r. In the example for $r = 2$, $f(0) = V(e^N, e^N) < f(2) = V(e^c(2), e^c(2))$, but for $r = 5$, $f(0) > f(5) = V(e^c(5), e^c(5))$.

The proofs of the three other cases are similar and left to the reader. ∎

The next theorem makes use of assumptions (**H1'**) and (**H2**). Taken simultaneously these assumptions point out a situation where there exists no interior Pareto solution. Before turning to the results, it is worth understanding what are the implications of the fact that G^N is an increasing function on E (**H1'**) and that there exists at most one zero for the function G^p on E (**H2**). Take for instance a situation of positive externalities and positive conjectures. In such case $G^p > G^N$. Since G^N is increasing, the zero e^* of G^p is necessarily located at the left hand side of e^N. At e^* the function G^p is necessarily upward slopping; otherwise G^p becomes negative between e^* and e^N, and since G^N takes positive values on the interval $\left]e^N, \overline{E}\right]$ so does G^p: a consequence would be that G^p has a second zero on the interval $\left]e^*, \overline{E}\right]$ which contradicts (**H2**). The inspection of the other possible cases leads to the same conclusion: G^p is negative on the left hand side of e^*, and it is positive on the right hand side. This means that e^* is a minimum for $V(e, e)$ and not a maximum.

Theorem 2 *Assume (**H1'**) and (**H2**) are satisfied. Then for the (symmetric) Nash equilibrium and any symmetric conjectural variation equilibrium, we have:*

Case 1 *If $V_2 > 0$ (positive externality) and $r(e) > 0$ (positive conjectural variation): then*

$$e^N \geq e^c \tag{10}$$

and if $r(e) \leq 1$ then

$$V(e^N, e^N) \geq V(e^c, e^c) \ . \tag{11}$$

If $r(e) > 1$, it is not possible to order the payoffs associated with the outcomes.

Case 2 *If $V_2 < 0$ (negative externality) and $r > 0$ (positive conjectural variation), then*

$$e^c \geq e^N \ ,$$

and if $r(e) \leq 1$ then

$$V(e^N, e^N) \geq V(e^c, e^c) \ .$$

If $r(e) > 1$, it is not possible to order the payoffs associated with the outcomes.

Case 3 *If $V_2 > 0$ and $r < 0$ then*

$$e^c \geq e^N$$

and

$$V(e^c, e^c) \geq V(e^N, e^N) \ .$$

Case 4 *If $V_2 < 0$ and $r < 0$, then*

$$e^c \leq e^N$$

and

$$V(e^c, e^c) \geq V(e^N, e^N) \ .$$

Proof. **Case 1:** From the assumptions of positive externalities and positive conjecture we have that $G^c(e) - G^N(e) = V_2(e,e)r(e) > 0$. This inequality implies that $e^c < e^N$. What can we learn about the payoffs? As observed above, $G^p > G^N$, has an increasing slope at the point e^* where $G^p(e^*) = 0$, is negative at the left hand side of this point and positive at the right hand side. Thus the unique zero e^* of G^p is a minimum of $W(e)$, which means that the Pareto solution is not interior. If $r(e) \leq 1$ then $G^p(e) - G^c(e) \geq 0$; e^c and e^N are located on the right hand side of e^*, i.e. on the segment where an increase of e induces an increase of the players' payoffs: $V(e^c, e^c) \leq V(e^N, e^N)$. If $r(e) > 1$, then $G^p(e) - G^c(e) < 0$. Therefore e^* lies between e^c and e^N: it is not possible to order the payoffs.

The proofs of the three other cases are similar. ■

2.2 Discussion

2.2.1 Intuitions underlying Theorems 1 and 2

When a typical agent forms conjectures, he no longer considers rival decisions as externalities. He behaves as if there were an existing functional relation between his actions and the externalities. For simplicity take the case of constant conjectural variations, $r(e) = r$, and let us consider successively some cases of Theorems 1 and 2.

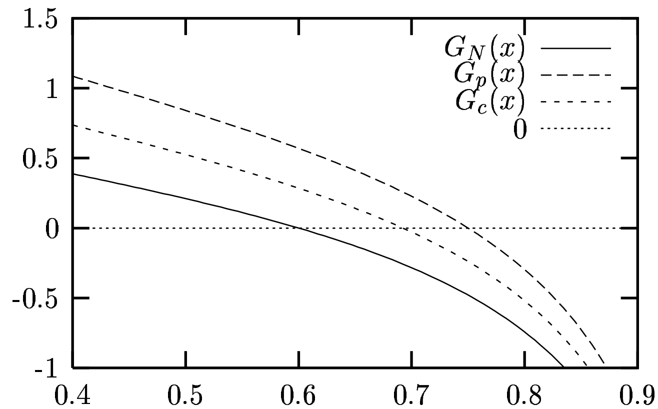

Figure 1: Public Goods, with $V(x,y) = (x+y)^{3/4}(1-x)^{1/4}$, $r(e) = 1/2$

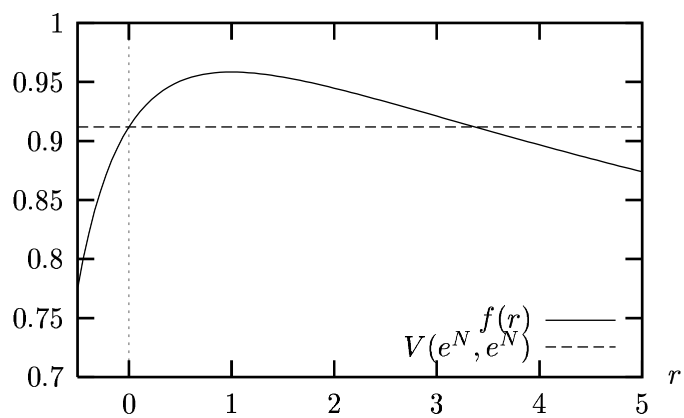

Figure 2: Public Goods, as a function of r, $-0.5 < r < 5$

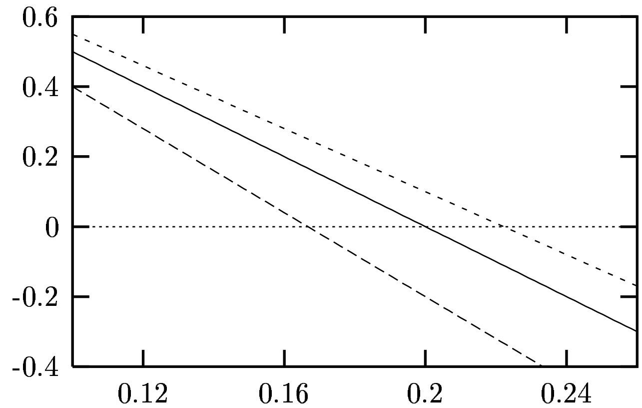

Figure 3: Duopoly with linear price, quadratic convex cost and $r(e) = -1/2$

First take case 2 of Theorem 1, negative externalities and positive conjectures, and remind the first order condition attached to the symmetric conjectural equilibrium:

$$V_1(e^c, e^c) = -rV_2(e^c, e^c). \tag{12}$$

With $r > 0$, when agent i modifies his action he exerts a direct marginal effect on his payoff, $V_1(.,.)$, and a conjectured indirect marginal effect on the negative externality $-rV_2(.,.)$. This "externality" can be interpreted here as a cost induced by the action e_i. Since the externality is negative, the analogy with a cost naturally arises. As usual, the optimal choice e^c, evaluated at a symmetric outcome, is the one that realizes the equality between the marginal benefit $V_1(e, e)$ and the marginal cost $-rV_2(e, e)$. An alternative interpretation would be to say that agent i bears a ratio of the externality he exerts to the second player, that is to say he internalizes r percent of the externality he generates.

This is better illustrated by a drawing of the functions $V_1(e, e)$ and $-rV_2(e, e)$.

In Figure 4, the decreasing curve represents the direct marginal effect of agent i's strategy computed at a symmetric outcome; the increasing curves represent the conjectured indirect marginal effects of i's action for three different value for the conjecture r, more precisely $r = 0.5,\ 0.75$ and 0.99. As r increases the upward slopping curves switch towards the upper left corner. The symmetric conjectural equilibrium decision e^c is the point on the x-axis for which the increasing and the decreasing curves intersect. The

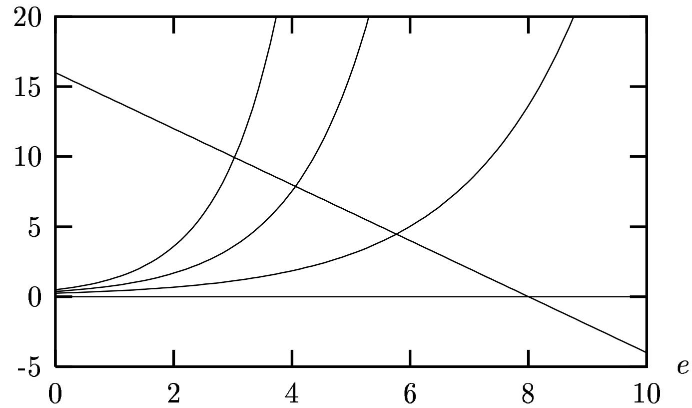

Figure 4: Theorem 1, Case 2.

symmetric Nash decision, that obtains for $r = 0$, corresponds to the point e^N such that $V_1(e^N, e^N) = 0$. The higher the conjecture r, the lower e^c. When $0 < r < 1$, agent i partially internalizes the externality; he under-estimates the negative effect of his decision on the social interest. However as r increases towards unity the inefficiency of non cooperative decisions vanishes. Social effects of decisions are perfectly taken into account when $r = 1$, and Pareto efficiency obtains. When $r > 1$, agent i over-estimates the costs he produces on the other player. Consequently as r increases from unity, the loss of efficiency increases and eventually may end up in a Pareto deterioration as compared to the Nash equilibrium.

What happens if the conjecture is negative (and externalities are still negative: case 4 of Theorem 1)? The right hand side of (12) changes its sign. It is as if the agents make a mistake and consider the negative externality as an additional benefit instead of a cost: negative conjectures modify behaviors in the wrong direction. This case is depicted in Figure 5.

The three negative curves correspond to the conjectured indirect marginal effects of i's action for three different negative values for the conjecture r, $r = -0.5$, -0.75 and -0.99. As $|r|$ increases the curves switch downwards and the conjectural equilibrium decision e^c increases, leaving the agents worse-off (remember that the direction of Pareto amelioration is the one where e^c decreases). Cases 1 and 3 can be discussed in much the same way as cases 2 and 4. Readers are invited to recover the intuitions.

It remains to deliver the intuition for Theorem 2. Consider a case of positive externalities and positive conjectures (case 2 of Theorem 2). Re-

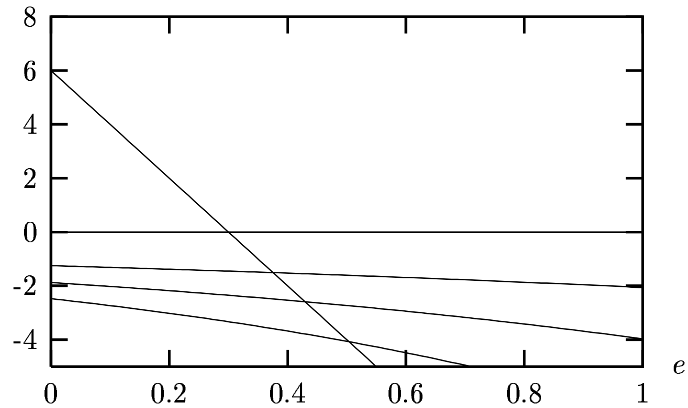

Figure 5: Theorem 1, Case 4.

member that under (**H1'**) we are dealing with situations where the decision e^* such that:

$$V_1(e^*, e^*) = -V_2(e^*, e^*).$$

gives a social minimum instead of a social maximum (actually there is no interior symmetric Pareto solution). This case is depicted in Figure 6 below.

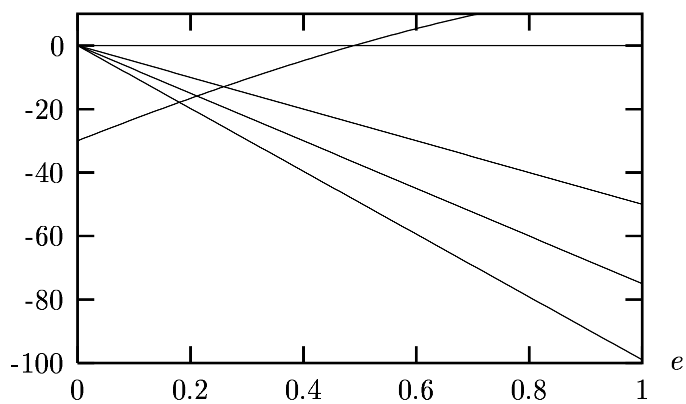

Figure 6: Theorem 2, Case 1.

The increasing curve plots the marginal direct benefit $V_1(e, e)$ of action e_i for agent i evaluated at a symmetric outcome; the decreasing curves shows the conjectured marginal indirect benefit $-rV_2(e, e)$ (with a negative sign) of

i's action for three different value for the conjecture r (as before $r = 0.5,\ 0.75$ and 0.99). As r increases the decreasing curves switch towards the lower left corner and the conjectural equilibrium decision e^c decreases and tends to, for $r = 1$, a social minimum, causing a Pareto-deterioration.

2.2.2 Assumptions on the payoff functions

A number of remarks are in order about (**H1**), (**H1'**) and (**H2**).

In this discussion we shall assume that $V_{11} < 0$ in E^2 (i.e. the concavity of V as a function of e_1) since this is natural in most economic situations.

First, observe that conditions (4) and (5) are related to the slope of the Nash best responses of players. Indeed, the best response of player 1 given implicitly by $V_1(e_1, e_2) = 0$ satisfies $e_1'(e_2) = -V_{12}(e_1(e_2), e_2) / V_{11}(e_1(e_2), e_2)$ in E^2. If (4) holds, and with $V_{11} < 0$, then $-V_{12}/V_{11} \leq 1$. Conversely, if (5) holds, then $-V_{12}/V_{11} \geq 1$. In other words, (**H1**) is equivalent to requiring that there exists a unique interior Nash equilibrium, and that the slope at the Nash best responses is smaller than 1. Similarly (**H1'**) amounts to require that there exists a unique interior Nash equilibrium, and that the slope at the Nash best response is larger than 1.

An implication of the above remarks is that under (**H1'**) the Nash equilibrium is unstable, a quite undesirable property. Under (**H1**), if the game features strategic complementarity ($V_{12} > 0$), i.e. when the Nash best responses are upward sloping, then the Nash equilibrium under consideration is stable. When the game otherwise features strategic substitutability ($V_{12} < 0$), in other words when the Nash best responses are downward sloping, the Nash equilibrium under (**H1**) is stable iff $V_{11} < V_{12}$.

A sufficient condition to have (**H2**) is to require $G^p(e)$ to be a decreasing function in E, or equivalently, that $W(e) = V(e, e)$ be a concave function, that is: $(G^p)'(e) = V_{11} + 2V_{12} + V_{22} < 0$.

Observe that $W(e) = V(e, e)$ is a concave function for symmetric oligopoly with decreasing and concave inverse demand and convex costs, and also for public goods problems (see § 4). We do not mean however that the payoff function corresponding to those games are concave; only the function of one variable obtained when those payoffs functions are evaluated along a symmetric outcome is concave.

If $V(e_1, e_2)$ is a concave function in E^2 (as a function of the two variables) then $G^N(e)$, $G^c(e)$ (when r is constant) and $G^p(e)$ are decreasing functions in E. But unfortunately this is not generally the case in economic applications

2.2.3 Implication of the results

Concerning the theorems, note that the sign of the externalities is irrelevant to compare the payoffs corresponding to scenarii i) and ii). Indeed, from Theorem 1, if (**H1**) and (**H2**) hold, then whatever the sign of externalities:

$$\text{if} : 0 < r \leq 1 \quad \text{then } V(e^N, e^N) \leq V(e^c, e^c)$$

$$\text{if} : r < 0 \qquad \text{then } V(e^c, e^c) \leq V(e^N, e^N) \ .$$

In other words: **When the slopes of the Nash best reply functions are less than unity, positive but less than 1 conjectural variations lead to a Pareto-improvement. Negative conjectural variations lead to a Pareto-worsening.**

On the other hand, if (**H1'**) and (**H2**) hold, i.e. if the Nash equilibrium is unstable and there is no solution, the results are reversed; whatever the sign of externalities:

$$\text{if} : 0 < r \leq 1 \quad \text{then } V(e^N, e^N) \geq V(e^c, e^c)$$

$$\text{if} : r < 0 \qquad \text{then } V(e^c, e^c) \geq V(e^N, e^N).$$

In other words: **When the slopes of the Nash best reply functions are larger than unity, positive but less than 1 conjectural variations lead to a Pareto-worsening. Negative conjectural variations lead to a Pareto-improvement.**

Finally, note that assumption (**H2**) is essential in order to obtain a ranking of Pareto outcomes and conjectural equilibria. For instance, in Theorem 1, Case 1, without this property, it could be that $e^p \leq e^c$ for some Pareto optimum e^p and some CVE e^c and comparing the payoffs would not be possible.

3 Consistent conjectures (two players)

Among all conjectural equilibria, we are more interested in those which are consistent. In order to apply Theorems 1 and 2 to such endogenous conjectures, it is necessary to know their sign. Requiring the reaction function for player i (implicitly defined in (2)) coincides with the conjectural variation that player j makes about player i (and conversely). See Bresnahan 1981 [4].

For a symmetric game, with symmetric conjectures, this is equivalent to requiring that the conjectural variation satisfies the following equation in a neighborhood of the equilibrium, when x is equal to the best response given implicitly by Equation (2):

$$\begin{aligned} 0 &= V_{11}(x,y)r(y) + V_{12}(x,y) \\ &\quad +(V_{21}(x,y)r(y) + V_{22}(x,y))r(x) \\ &\quad +V_2(x,y)r'(x)r(y) \ . \end{aligned} \tag{13}$$

For a *constant* variational conjecture, Equation (13) becomes a quadratic equation in the variable r:

$$V_{12}(x,y)r^2 + (V_{11}(x,y) + V_{22}(x,y))r + V_{12}(x,y) = 0 \ . \tag{14}$$

The two solutions r_1 and r_2 of this equation have the same sign:

$$r_1 r_2 = 1 \qquad \text{and} \qquad sig(r_1 + r_2) = sig(-\frac{V_{11} + V_{22}}{V_{12}}) \ .$$

In several models, the payoff function has a quadratic form: see the duopoly model and the model of competition between regions in the illustrations below. In such cases, the V_{ij} are constant parameters and in general $V_{11} < 0$ and $V_{22} \leq 0$ to ensure the existence of a Nash equilibrium. Therefore, **the sign of every consistent conjecture r is given by the sign of V_{12}.** For example if $V_{12} < 0$ then consistent conjectural variations are negative. In addition, $V_{11} + V_{12} < 0$ so Theorem 1 case 1 or 2 apply (provided, of course, that the others assumptions of Theorem 1 are satisfied and consistent conjectural variation exists).

4 Illustrations (two players)

The examples of this section illustrate some cases of Theorems 1 and 2.

4.1 A model of competition between regions

In this example, under appropriate assumptions on parameters, (**H1**) and (**H2**) are satisfied. There exist two consistent conjectural variation equilibria, and one of them gives a Pareto-improvement over the Nash equilibrium.

This is a model of competition between regions due to Wildasin 1991 [20]. Two regions $i = 1, 2$, are located around the same watershed and get benefit from it. The representative agent in each region has a utility function

$U(x_i, s_i) = x_i + s_i$, where x_i is a private consumption good and s_i an index of the water quality.

The production technology for the water quality is given by $s_i = f(e_i, e_j)$ where $e_i \in [0, \overline{E}]$ are the public expenses in region i. The budget constraint is given by $x_i + e_i = y_i$, y_i an exogenous revenue.

Plugging $f(e_i, e_j)$ and the budget constraint in the utility function $U(x_i, s_i)$, the problem of the benevolent planner in region i is:

$$\max_{e_i} V(e_i, e_j), \text{ with } \quad V(e_i, e_j) = y_i - e_i + f(e_i, e_j) \ .$$

If we consider a quadratic formulation $f(e_i, e_j) = -a_1 e_i^2 - a_2 e_j^2 + a_3 e_i e_j + a_4 e_i + a_5 e_j + a_6$, with $a_k > 0$[2] for all k, then $V(e_i, e_j)$ is also a quadratic function with $V_{12} > 0$, $V_{11} < 0$, $V_{22} < 0$. The marginal externality is: $V_2(e_i, e_j) = -2a_2 e_j + a_3 e_i + a_5$.

Under mild conditions on the model parameters (**H1**) and (**H2**) are satisfied. This is the case for instance if $a_3 < a_1$, $1 < a_4$, and $\frac{1-a_4}{a_3 - 2a_1} < \overline{E}$.

To characterize the conjectural equilibrium with a constant conjecture r let us compute the first and second order conditions; given that $\partial e_j / \partial e_i = r$:

$$\frac{\partial V}{\partial e_i}(e_i, e_j) = -1 - 2a_1 e_i - 2a_2 e_j r + a_3 e_j + a_3 e_i r + a_4 + a_5 r$$

$$\frac{\partial V^2}{\partial e_i^2}(e_i, e_j) = -2(a_2 r^2 - a_3 r + a_1) \ .$$

So, for instance, $a_3 \leq 2\sqrt{a_1 a_2}$ implies $\frac{\partial V^2}{\partial e_i^2}(e_i, e_j) < 0$ for any r, and there is a maximum at:

$$e^c \ = \ \frac{1 - a_4 - r a_5}{a_3(1 + r) - 2a_1 - 2r a_2} \ ,$$

which is the conjectural equilibrium with a constant conjecture r.
As $V_{12} > 0$, from § 3 every consistent constant conjectural variations, if any exist, must be positive. It is easy to see that the CCVE must be:

$$r \ = \ \frac{1}{a_3}\left(a_1 + a_2 \pm \sqrt{(a_1 + a_2)^2 - a_3^2}\right) \ . \tag{15}$$

Consistent conjectural variations exist when $a_1 + a_2 \geq 2a_3$. If $a_1 + a_2 = 2a_3$, $r = 1$ and the Pareto solution obtains. If the inequality is strict, there exist two distinct and positive consistent conjectural variations: one

[2]In particular, $a_3 > 0$ captures the case of strategic complementarity; strategic substitutability would be modeled when $a_3 < 0$.

of them is larger than 1, and the other one is smaller. Whatever the sign of the externality, the conjectural equilibrium for the conjecture $0 < r < 1$ gives a higher payoff than the Nash equilibrium; agents under-internalize the externalities. With the conjecture $r > 1$, agents over-internalize the externalities and their behaviors might result in a lower payoff than the one of the Nash equilibrium. When $V_2(e_i, e_j) > 0$ (public underspending), Theorem 1, Case 1 holds. Should $V_2(e_i, e_j) < 0$ (public overspending)? Then Theorem 1, Case 2 holds.

Remark If $a_3 > 2\sqrt{a_1 a_2}$, the conjectural equilibrium is a maximum when

$$r < \frac{a_3 - \sqrt{a_3^2 - 4a_1 a_2}}{2a_2} \text{ or } r > \frac{a_3 + \sqrt{a_3^2 - 4a_1 a_2}}{2a_2}.$$

In order to know if constant consistent CVE exist, it must be verified if r given by (15) belongs to one of these sets.

4.2 The search model of Diamond

This model proposed in Diamond 1982 [8] is a macroeconomic model with aggregate demand externalities where assumption (**H1'**) and (**H2**) are simultaneously satisfied. Constant CVE exist for positive and negative values of r, but there are no constant *consistent* CVE.

The setup of the model is the following. Two symmetric agents have payoff functions

$$V(e_i, e_j) = \alpha\, e_i\, e_j - C(e_i) \ ,$$

where e_i is player i' search intensity, $C(e_i)$ is a convex cost function of search, $e_i\, e_j$ is the probability of finding a trading partner (thus, $E = [0, 1]$), and α is the gain when a partner is found. We shall assume that

$$C(e_i) = \frac{c}{2} e_i^2 + b e_i \ .$$

The parameters α, b and c are all strictly positive. The payoff is then symmetric and concave with respect to player i's strategy. The externality is $V_2 = \alpha e_i > 0$.

In this case (**H2**) is satisfied since G^p is affine. A necessary (second order) condition for the existence of a CVE with a constant conjecture r (for both players) is $2\alpha r < c$; the CVE is then given by:

$$e^c \ = \ \frac{b}{\alpha(1+r) - c} \ .$$

This equilibrium lies in the interval $(0,1)$ if and only if $r > \frac{b+c}{\alpha} - 1$. To summarize, the CVE exists when $\frac{b+c}{\alpha} - 1 < r < c/2\alpha$. In particular for $r = 0$, there exists a Nash equilibrium when $b + c < \alpha$. On the other hand, $V_{11} + V_{22} = \alpha - c$, so $b + c < \alpha$ also implies that Assumption (**H1'**) is satisfied.

However, the condition for this equilibrium to be consistent is $\alpha r^2 - cr + \alpha = 0$, and this equation has real solutions if and only if $c \geq 2\alpha$. This is not compatible with the existence of Nash equilibria.

On the other hand, if $\frac{b+c}{\alpha} - 1 < r < 0$, then by Theorem 2, Case 3, the conjectural equilibrium Pareto-improves over the Nash equilibrium. If $0 < r < c/2\alpha$, by Theorem 2, Case 1, the converse happens.

4.3 Cournot's duopoly

Consider the Cournot's duopoly with linear inverse demand and cost. This very classical example illustrates the case where (**H1**) and (**H2**) are satisfied and there exists an unique consistent constant CVE.

The payoff function has the form:

$$V^i(e_i, e_j) = (\alpha - \beta(e_i + e_j))e_i - ce_i \ .$$

The parameters α, β and c are strictly positive. Then, $V_{11} = -2\beta < 0$, $V_{12} = -\beta < 0$, and $V_{22} = 0$. Accordingly (see § 3), the sign of the consistent conjectures is negative. Actually, it is well known from Bresnahan 1981 [4] that $r = -1$ is the unique constant consistent conjecture in this case.

Here, (**H1**) and (**H2**) are satisfied when $\alpha < c$.

The externality is $V_2(e_i, e_j) = -\beta e_i < 0$. It is easy to check directly that Theorem 1, Case 4 holds since $e^c = (\alpha - c)/4\beta < e^N = (\alpha - c)/3\beta < e^p = (\alpha - c)/2\beta$.

4.4 The Public Good model of Itaya and Dasgupta

In Itaya and Dasgupta 1995 [11] the conjectural variations equilibria were computed in the following model. As in the duopoly case, (**H1**) and (**H2**) are satisfied. Here however, the externality is positive.

Consider an individual i with an income I and a contribution e_i to some public good. Assume that the utility obtained from the consumption of private and public goods has the form:

$$V(e_i, e_j) = (I - e_i)^\alpha (e_i + e_j)^{1-\alpha}$$

where $0 < \alpha < 1/2$.

Itaya and Dasgupta showed that the consistent constant conjectural variation equilibrium is given by:

$$r = -\frac{\alpha}{1-\alpha} < 0 \ , \qquad e^c = (1-2\alpha)I \ .$$

The externality is:

$$V_2(e_i, e_j) = (1-\alpha)(I-e_i)^{\alpha}(e_i+e_j)^{-\alpha} > 0$$

and

$$\begin{aligned} V_{11} + V_{12} \quad = \quad & -\alpha(1-\alpha)(I-e_i)^{\alpha-2}(e_i+e_j)^{1-\alpha} \\ & -3\alpha(1-\alpha)(I-e_i)^{\alpha-1}(e_i+e_j)^{-\alpha} \\ & -2\alpha(1-\alpha)(I-e_i)^{\alpha}(e_i+e_j)^{-1-\alpha} < 0 \end{aligned}$$

This is the situation of Theorem 1, Case 3. Therefore, the consistent conjecture yields lower payoffs than the Nash equilibrium.

5 Many players games

We seek to extend the results to games with several players.
As in Perry 1982 [16], Cornes and Sandler 1984 [7] and Sugden 1985 [19], we consider the games where for each player, the contribution of all other players to the externality is aggregated. The idea is that each player plays against an unique (virtual) player representing the remaining players.

5.1 Modifications of assumptions and results

The payoff functions under consideration are $V(e_i, y_i)$ where $y_i = \sum_{j \neq i} e_j$. The payoff of agent i thus depend on his strategies and the sum of the rival strategies. Each agent conjectures how the other players, as a whole, react to his choices. It is assumed that these conjectures are all identical. i.e. $\frac{dy_i}{de_i} \equiv r(e_i)$, $\forall i$.

Given the form of the payoff functions, marginal external effects read as:

$$\frac{\partial V}{\partial e_j} = \frac{\partial V}{\partial y_i}$$

for all $j \neq i$. We shall use the notations V_e and V_y for the partial derivatives with respect to e and y.

An interior symmetric Nash equilibrium $(e^N, ..., e^N)$ verifies:

$$V_e(e^N, (n-1)e^N) = 0 \ .$$

An interior symmetric conjectural variation equilibrium $(e^c, ..., e^c)$ satisfies:

$$V_e(e^c, (n-1)e^c) + V_y(e^c, (n-1)e^c)r(e^c) = 0$$

Finally, a interior symmetric Pareto outcome $(e^p, ..., e^p)$ must verify:

$$V_e(e^p, (n-1)e^p) + (n-1)V_y(e^p, (n-1)e^p) = 0$$

It is straightforward to extend the study and results established for two players to this symmetric n-player framework.

Indeed, consider, as in Section 2, the functions defined on E:

- $G^N(e) = V_e(e, (n-1)e)$,
- $G^c(e) = V_e(e, (n-1)e) + r(e)V_y(e, (n-1)e)$,
- $G^p(e) = V_e(e, (n-1)e) + (n-1)V_y(e, (n-1)e)$
- and $W(e) = V(e, (n-1)e)$ (we still have $W'(e) = G^p(e)$),

so that e^N, e^c and e^p are solution of $G^N(e^N) = 0$, $G^c(e^c) = 0$ and $G^p(e^p) = 0$, respectively.

Assumption (**H2**) is unchanged. In order to state the new (**H1**) one simply has to replace the assumption

$$V_{11}(e, e) + V_{12}(e, e) \leq 0$$

by

$$V_{ee}(e, (n-1)e) + (n-1)V_{ey}(e, (n-1)e) \leq 0 \ .$$

To obtain the new (**H1'**), simply reverse this above inequality. Then Theorems 1 and 2 hold with "$r(e) \gtreqless 1$" replaced by "$\boldsymbol{r(e) \gtreqless n-1}$". This is because the differences between functions G^N, G^c, G^p have the same form as before, with V_2 replaced by V_y and $(1 - r(e))$ replaced by $(n - 1 - r(e))$. But the sign of the differences $G^N - G^c$, $G^N - G^p$, $G^p - G^c$ are not altered.

5.2 Consistency in the case of many players

Consistency in the case of many players is not extensively studied in the literature. Only a few papers give a definition of constant consistent conjectural variation in a symmetric game: Perry 1982 [16], Cornes and Sandler 1984 [7] and Sugden 1985 [19].

In all this papers consistency is required only in the hyperplane $e_i = e_j$ and evaluated at the equilibrium. It is to be noted that this is not a natural extension of the definition of Bresnahan 1981 [4] for two players.

In the illustrations that follow we restrict attention to the concept of consistency given for the oligopoly in Perry 1982 [16] and for public goods in Cornes and Sandler 1984 [7] and Sugden 1985 [19].

5.3 Illustrations (many players)

5.3.1 Consumption externalities and Kantian behaviors (Laffont 1975)

Laffont 1975 [13] put forward the idea that in some circumstances, agents do not behave as Nash players. On the basis of some real life events, he made the assumption that human nature is capable of non selfish decisions, along the lines suggested by Kant's moral. To be more precise, he postulated that a typical agent assumes that the other agents will act as he does, and he maximizes his utility function under this constraint. As Laffont recognized, this is "equivalent to a special assumption about anticipation of others' behaviour".

We present here a slightly modified version of the model he used to illustrate kantian behaviors. The difference stems from the fact that Laffont considered an economy with a continuum of agent whereas we assume that there are $n \geq 2$ agents. All agents are endowed with the same differentiable, increasing and concave utility function

$$U\left(d_i, e_i, \sum_{j=1}^{n} e_j\right)$$

where d_i and e_i stand for the agent i's consumptions of the commodities D and E respectively. The last argument of the utility function captures a macro-externality of consumption associated to commodity E. Examples of such externalities abound : network externalities, pollution, and so on...

Before playing the game, initial endowments of $(1,1)$ are given to each agent and the price of D is normalized to one. There is a technology that transforms good D into good E according to $e_i = \alpha d_i$. The price of good E will then be $p = 1/\alpha$ and the budget constraint of each agent reads as:

$$d_i + (1/\alpha)\, e_i = 1 + (1/\alpha)\, 1 \ .$$

Plugging the budget constraint and the definition $y_i = \sum_{j \neq i} e_j$ into the utility function, one has:

$$V(e_i, y_i) = U\left(1 + (1/\alpha)\, 1 - (1/\alpha)\, e_i, e_i, e_i + y_i\right) \ .$$

A kantian behavior consists then in maximizing $V(e_i, y_i)$ under the constraint that $y_i = (n-1)e_i$. In other words, the kantian equilibrium is a conjectural equilibrium for the specific conjectural variation $r = (n-1)$. As detailed in Section V the first order condition under such behaviors is typically that of the Pareto solution. This observation led Laffont to advocate a governmental intervention in order to induce people to behave in a kantian way, by means for instance of an information campaign.

We argue that the relevance of such a policy recommendation is strongly questionable. Laffont did not give any further precisions on the properties of the utility function. Actually it is easy to point out relevant economic assumptions on the utility function such as (**H1'**) holds: in such a case kantian behaviors would reduce welfare as compared to Nash behaviors. We will elaborate on the economic policy implications of non-Nash behaviors in the last section of the paper.

5.3.2 Public good

Reconsider the example of Section 4.4 with now many players. The individual payoff function has the form:

$$V(e_i, y_i) = (I - e_i)^{\alpha}(e_i + y_i)^{1-\alpha}.$$

The first order condition for a constant conjectural variation equilibrium gives

$$-\alpha(e_i + y_i) + (1-\alpha)(1+r)(I - e_i) = 0, \qquad (16)$$

so that the symmetric equilibrium is given by:

$$e^c = \frac{I(1-\alpha)(1+r)}{(1-\alpha)(1+r) + n\alpha}.$$

The sign of the second order condition is the same as that of:

$$\begin{aligned}&-((I-e_i)^{\alpha-2}(e_i+y_i)^{1-\alpha}\\&+2(I-e_i)^{\alpha-1}(e_i+y_i)^{-\alpha}(1+r)\\&+(I-e_i)^{\alpha}(e_i+y_i)^{-\alpha-1}(1+r)^2)\ ,\end{aligned}$$

which is negative. Therefore, $r>-1$ ensures the existence of a CVE $e^c>0$. Moreover

$$V_{11}(e_i,y_i)+(n-1)V_{12}(e_i,y_i)=$$
$$-\alpha(1-\alpha)(I-e_i)^{\alpha-2}(e_i+y_i)^{-1-\alpha}(I+y_i)(y_i+n(I-e_i)+e_i)$$

is negative and (**H1**) holds.
We can obtain the following results:

- If the conjectural variations are of the form $r=e_i/y_i$ or $r=e_i/(y_i+e_i)$ i.e. $r=1/(n-1)$ or $r=1/n$ in the symmetric case, (as in Cornes and Sandler 1984 [7]), Theorem 1 Case 3 says that constant conjectural equilibrium Pareto-improves over the Nash equilibrium.

- As far as CCVE are concerned, Sugden 1985 [19] concludes that there do not exist positive constant consistent conjectural variations for publics goods with n players (for the Cobb-Douglas utility function). Therefore, if a constant consistent conjectural equilibrium exists it must be negative; so Case 3 of Theorem 1 holds and CCVE is a Pareto-worsening over Nash.

6 Summary and economic policy implications

The purpose of this paper was to assess the role played by conjectures on agents' behaviors when those agents interact strategically. In a symmetric framework and taking the Nash behavior as a benchmark case, it is shown how the strategies and the payoffs are altered when agents hold non zero conjectures.

The results depend crucially on the payoffs structure. When the payoff function satisfies (**H1**) and (**H2**) (the slope of the Nash best response is less than unity and the Pareto solution is interior) then positive but less than 1 conjectures means that agents correctly understand the nature of their common interest and partially internalize the externalities. As a consequence the conjectural equilibrium: i) lies between the Nash equilibrium and the Pareto solution, ii) and produces a Pareto improvement over the NE. For conjectures equal to unity, the CVE is efficient. For conjectures greater than one the Pareto solution lies between the NE and the CVE; and the CVE might result in lower payoffs than the NE. Still under (**H1**) and (**H2**), negative conjectures can be interpreted as a case where agents misunderstand the nature of their interactions and conjectures modifies behaviors in the direction of Pareto deteriorations. When the payoffs satisfies (**H1'**) and (**H2**) (the Nash equilibrium is unstable and there is no interior symmetric Pareto solution) the results are reversed.

An additional contribution of the paper is to show how those results apply to the interesting case of consistent conjectural variation equilibrium. From an operational point of view, the paper indicates how to relate the sign of any consistent conjecture to the payoffs structure; once the sign of consistent conjectures is known the previous results trivially apply.

Results are then illustrated by mean of well known economic situations, for instance the Cournot duopoly or the standard model of private contributions to a public good.

Along the lines suggested by Laffont 1975 [13], it is tempting to conclude the paper with an investigation of the economic policy implications of Theorems 1 and 2. If one accepts the idea that behaviors are endogenous to the system, then there is a case for public intervention through for instance an information campaign. In modern economies, with sophisticated and widely spread media, such an economic policy would have the advantage to be cheaper than an intervention through a tax system or a repressive policy. It also has the advantage of flexibility since it indicates a direction of Pareto improvements; this is less ambitious but also definitely less demanding than indicating a point of socially efficient decisions. However we do not follow Laffont blindly when he suggests to induce people adopting the kantian ethic without much care of the problem at hand. Indeed the interest of Theorem 2 is to identify situations where such behaviors would settle the economy on a social minimum instead of a maximum. An information campaign that would advocate this ethic without an accurate diagnostic of the economic problem under consideration might result in a large scale disaster. Unfortunately, Kantian behaviors are not a panacea ; public policies in order to change conjectures about others' strategy are confronted with standard information problems.

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