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A modified CKKW matrix element merging approach to angular-ordered parton showers

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KEYWORDS: QCD, Monte Carlo.

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1. Introduction

Monte Carlo event generators have been successful in providing a full simulation of the physics at collider experiments and have proven to be invaluable tools in both planning future experiments and analysing data from current experiments. They use a flexible and convenient event-by-event description which allows a range of physics models to be implemented and as such they describe a wide variety of phenomena. In particular, they provide a means, via the parton shower, of evolving from hard scales, where partons are produced in fixed-order perturbation theory, to soft scales where non-perturbative models must be applied.

Besides the simple fixed-order matrix elements which describe the hard processes, the key perturbative element of Monte Carlo event generators is the parton shower. In this phase of the simulation, partons undergo DGLAP evolution from some initial scale,

characteristic of the underlying hard scattering, down to the hadronization scale. In `Herwig++` [1,2] the evolution variable of the parton shower is chosen such that branchings are ordered in their opening angles, accounting for the effects of QCD coherence [3]. This corresponds to resumming next-to-leading logarithm (NLL) terms to all orders in perturbation theory.

While the parton shower accurately simulates soft and collinear radiation, it does not provide a reliable description of hard (high transverse momentum) emissions. In fact, requiring that the regions of phase space into which each *shower progenitor* (the partons which initiate the shower) can emit do not overlap, to avoid double counting, gives rise to a *dead zone*: a region of phase space corresponding to high transverse momentum, wide angle, emissions which none of the showering particles can emit into. Even within the accessible *shower regions* of phase space, the distribution of radiation involves some degree of approximation, since at any given fixed order in perturbation theory, the parton shower effectively approximates the real emission corrections to the hard scattering process by a product of splitting functions and Sudakov form factors, summed over all combinations of branchings which give rise to the same final state. These approximations account for the NLL corrections associated with soft and collinear radiation in the perturbative series.

In order to improve the description in the large transverse momentum region, the parton shower can be combined with exact fixed-order matrix elements. The earliest and simplest means of forming this combination is known as the matrix element correction method [4]. This corrects the hardest emission generated by the parton shower such that it is distributed according to the real single emission matrix element squared. This technique has been successfully applied to important processes in a number of generators [5,6], including `Herwig++` [2].

In recent years more general matrix element-parton shower merging algorithms have been introduced. These combine tree-level matrix elements with parton showers, for a given process, for all parton multiplicities below some maximum N . Hence these algorithms correct all distributions involving up to N external partons, instead of just that of the hardest emission. Several schemes of this type have been developed and successfully implemented in event generators. The most well known of these are the CKKW [7–9], CKKW-L [10], MLM [11] and pseudo-shower [12] methods. All these methods have the same general approach [13,14] whereby the phase space for parton emissions is divided into two regions by a merging scale y_{MS} , defined in some jet measure. Above the merging scale, emissions are described by exact matrix elements while below it emissions are produced by the parton shower.

In this article we present a matrix element merging scheme based on the CKKW algorithm. A fundamental ingredient in the CKKW method is the association of a *pseudo-shower history* to the configurations generated according to the fixed-order matrix elements. Each shower history is constructed by clustering the two most closely separated partons, according to the transverse momentum measure defining the merging scale, until a leading-order parton configuration is obtained. The resulting branchings in the shower history are therefore ordered according to the jet measure, which may not equate to the ordering variable of the parton shower, as is the case for the angular-ordered parton shower of

Herwig++. This discrepancy is understood to give rise to serious problems, in particular it spoils the colour coherence properties of these shower algorithms. Although this was already noted, and an attempt made to address it, in the original CKKW paper, realisations of the method highlight the fact that the colour structure in the events is nevertheless in conflict with that expected on the grounds of colour coherence, moreover, they show that this is not simply an esoteric consideration but a cause of significant practical problems, including a dependence on the unphysical merging scale [12, 13].

We shall present and validate a modified version of the CKKW method, intended to optimise the implementation of these colour coherence effects, by a fully consistent merging of an angular-ordered parton shower with fixed-order matrix elements. The idea behind this method was originally proposed by Nason in Ref. [15]. The central result of that theoretical work is the observation that the parton shower may be formally decomposed in terms of *truncated showers*, *hard emissions*, and *vetoed showers*. Reference [15] advocates that the CKKW algorithm may then best model the coherent emission of radiation by including these truncated showers, consisting of only soft emissions, prior to, and between, the hard emissions in the shower history, thereby rendering it angular-ordered. In the following we will develop the full details necessary for our practical implementation of this idea for the process $e^+e^- \rightarrow \text{hadrons}$ and compare the results of it to LEP data.

This work builds on the infra-structure introduced in Refs. [18, 19] for the **Herwig++** angular-ordered parton shower, where the truncated shower was initially developed for use in the POWHEG [15–17] next-to-leading order matching scheme applied to Drell-Yan vector boson production. This was the first time a full implementation of the truncated shower had been developed¹.

While this work was in production, Ref. [20] appeared on the arXiv, employing similar techniques to those described in the present work for matrix element merging with a transverse-momentum-ordered dipole shower.

The paper is set out as follows. In Sect. 2 we review the original CKKW merging prescription. In Sect. 3 we go on to describe the way in which the angular-ordered parton shower may be decomposed into hard emissions, truncated showers, and vetoed showers. Having introduced the relevant conceptual ingredients we then give a more detailed technical description of our modified CKKW algorithm in Sect. 4. In Sect. 5 we present a validation of our algorithm by comparing to LEP data for $e^+e^- \rightarrow \text{hadrons}$, before giving our conclusions in Sect. 6.

2. CKKW merging

In this section we present an overview and discussion of the original CKKW algorithm for the process $e^+e^- \rightarrow \text{hadrons}$. We first describe the algorithm for the case where the parton shower evolution variable is identical to the merging variable before describing the adaptations which must be made for the **Herwig++** angular-ordered parton shower.

¹An approximate, single emission truncated shower is also described in Refs. [21, 22].

2.1 Transverse-momentum-ordered CKKW merging

The algorithm is simplest if the merging variable is the same as the ordering variable of the parton shower. We therefore consider first the case where we have a single transverse momentum variable q as the parton shower evolution and merging variables.

The basic principle underlying the CKKW approach, is that the distribution of radiation in the region of phase space where all partons are separated by an amount q greater than the *merging scale* q_{MS} , should be given by tree-level matrix elements, while for $q \leq q_{MS}$ it should be given by the parton shower. The algorithm then requires, as input, samples of events of the process with up to N partons in the final state. These input samples are easily obtained using fully automated tree-level event generators such as Madgraph/MadEvent [25,26]². As well as producing the events themselves, for each sample with n partons the generator will provide a finite, tree-level, jet cross section $\sigma_n^{(ME)}(q_{MS})$.

Naïvely, with the input events in hand, one might then consider filling the remaining phase space by selecting events from each sample with n partons, with a probability proportional to $\sigma_n^{(ME)}$, and simply invoking the parton shower on each of the external legs, starting from the scale q_{MS} . However, the merging scale q_{MS} is not a physical parameter and so all distributions of partons should be insensitive to its value. This would certainly not be the case for such a naïve procedure, since the distribution of radiation from the parton shower and the fixed-order matrix elements are known to differ, especially in the regions corresponding to high and low q emissions. The great success of the CKKW algorithm is in its ability to correct for the mismatch at the phase space partition q_{MS} by providing a smooth, physical, interpolation between the matrix element distribution at high q values and that of the parton shower in the low q region.

To illustrate how this works, consider the simplified case of merging only samples of 2- and 3-parton events, with $q \geq q_{MS}$, for $e^+e^- \rightarrow$ hadrons, with a q -ordered parton shower. In general, the parton shower cross section analogous to $\sigma_n^{(ME)}(q_{MS})$, with n partons resolved at the merging scale, may be written as the product of the leading-order cross section together with a set of Sudakov form factors and splitting functions. The product of these splitting functions and the leading-order cross section approximate the exact tree-level n -jet cross section. For the case of three partons this cross section is

$$\sigma_3^{(PS)}(q_{MS}) = \sigma_2 \times 2 [\Delta_q(q_I, q_{MS})]^2 \int_{q_{MS}}^{q_I} dq \alpha_S(q) \Gamma_{q \rightarrow qg}(q) \Delta_g(q, q_{MS}), \quad (2.1)$$

where q_I is the scale at which the parton shower is initiated and $\alpha_S(q) \Gamma_{\tilde{i}\tilde{j} \rightarrow ij}(q)$ is the probability for a parent parton $\tilde{i}\tilde{j}$ to branch into two daughter partons i and j , in the interval $[q, q + dq]$ ³. The overall normalisation factor σ_2 is simply the leading-order cross section. Finally, in Eq. (2.1), $\Delta_{\tilde{i}\tilde{j}}(q, q_{MS})$ is the Sudakov form factor, which can be interpreted as the probability for the parent parton $\tilde{i}\tilde{j}$ to evolve from a scale q down to the scale q_{MS} .

²Currently, computational efficiency limits the total number of final-state particles to around six.

³The dependence on auxiliary splitting variables has been suppressed.

without undergoing a resolvable branching,

$$\Delta_{\tilde{ij}}(q, q_{MS}) = \exp \left[- \sum_{\tilde{ij} \rightarrow ij} \int_{q_{MS}}^q dq' \alpha_S(q') \Gamma_{\tilde{ij} \rightarrow ij}(q') \right]. \quad (2.2)$$

In parton shower (NLL) expansions of the jet cross sections, such as that in Eq. (2.1), the exact tree-level matrix elements are approximated by the product of the leading-order cross section and splitting functions. In order to improve the parton shower with exact tree-level matrix elements, this product should be replaced by the corresponding exact, tree-level jet cross section.

The CKKW merging should not affect the NLL expansion of the jet cross section therefore the NLL expansion of the matrix element contribution should give the result in Eq. (2.1). Since a NLL expansion of the tree-level matrix elements yields a corresponding product of parton shower splitting functions, it is clear that in order to retain the NLL form of Eq. (2.1), the matrix element contribution above q_{MS} should be given by configurations generated according to the tree-level jet cross sections reweighted by appropriate Sudakov and running α_S factors.

In order to determine appropriate reweighting factors for events from the tree-level generator, a pseudo-shower history must be assigned to each event. This shower history interprets the set of external parton momenta as a set of branchings originating from a leading-order configuration. This procedure gives rise to a set of *nodal values*, q_i , for the scales at which each pseudo-branching occurred. These scales provide the arguments for the Sudakov form factors and α_S factors with which the configuration should be reweighted. In the original CKKW publication, this pseudo-shower history is assigned by repeatedly clustering the pair of partons⁴ with the smallest separation according to the jet resolution variable, until only the particles of the leading-order process remain.

In the case being considered, where the evolution variable has been taken to match the merging scale variable, combining the matrix elements with the parton shower is straightforward. The parton shower evolution can be split into two parts: first an evolution from the initial scale down to the merging scale q_{MS} ; then an evolution from the merging scale down to the hadronization scale q_0 . This results in a simple procedure for attaching the parton shower to the reweighted matrix elements, where each external parton produces a shower line evolving from the merging scale.

The full CKKW algorithm then proceeds as follows:

1. a jet multiplicity n is generated with probability

$$P_n = \frac{\sigma_n^{(ME)}(q_{MS})}{\sum_N \sigma_N^{(ME)}(q_{MS})}, \quad (2.3)$$

where all cross sections are evaluated at a fixed strong coupling $\alpha_{S_{ME}}$;

2. a configuration of n parton momenta is generated according to $d\sigma_n^{(ME)}(q_{MS})$;

⁴Only pairs of partons whose flavours correspond to allowed branchings are considered.

3. external partons are clustered, defining a pseudo-shower history with a set of nodal scales q_i ;
4. the configuration is reweighted by Sudakov and α_S factors: each internal line between two nodes at q_i and q_{i+1} contributes a factor of $\Delta_f(q_i, q_{MS})/\Delta_f(q_{i+1}, q_{MS})$, each external line emanating from a node with scale q_i contributes $\Delta_f(q_i, q_{MS})$, while each node itself contributes $\frac{\alpha_S(q_i)}{\alpha_{SME}}$;
5. the parton shower is invoked on each external parton from a starting scale of q_{MS} .

This scheme is independent of the merging scale to NLL order [7]. We have reweighted configurations such that the NLL three-jet cross section resolved at the merging scale is given by Eq. (2.1). This NLL cancellation of merging scale dependence can be seen by considering the cross section for three jets resolved at the hadronization scale. This cross section is given by the sum of the probability of generating a single emission in the matrix element region and none in the parton shower, together with the probability of generating no emissions in the matrix element region and a single emission in the parton shower region. The cross section is

$$\begin{aligned} \sigma_3^{(PS+ME)}(q_0) &= \bar{\sigma}_3^{(ME)}(q_{MS}) [\Delta_q(q_{MS}, q_0)]^2 \Delta_g(q_{MS}, q_0) \\ &+ \sigma_2 \times 2 [\Delta_q(q_I, q_0)]^2 \int_{q_0}^{q_{MS}} dq \alpha_S(q) \Gamma_{q \rightarrow qg}(q) \Delta_g(q, q_0), \end{aligned} \quad (2.4)$$

where $\bar{\sigma}_3^{(ME)}(q_{MS})$ is the reweighted matrix element contribution for three jets resolved at the merging scale. The first term in Eq. (2.4) corresponds to a single emission above the merging scale followed by parton shower evolution from the merging scale down to the hadronization scale with no resolvable emissions. The second term corresponds to no emissions above the merging scale followed by a single parton shower emission below the merging scale. In the NLL expansion of Eq. (2.4), we replace $\bar{\sigma}_3^{(ME)}(q_{MS})$ by the NLL parton shower approximation in Eq. (2.1). This results in a simplification of Eq. (2.4)

$$\sigma_3(q_0) = \sigma_2 \times 2 [\Delta_q(q_I, q_0)]^2 \int_{q_0}^{q_I} dq \alpha_S(q) \Gamma_{q \rightarrow qg}(q) \Delta_g(q, q_0), \quad (2.5)$$

yielding the expected NLL parton shower cross section for a single resolved emission which is independent of the merging scale.

2.2 Angular-ordered CKKW merging

The merging variable used to define the jet cross sections must regulate both soft and collinear singularities, so it must be a transverse momentum measure. The merging variable in the original CKKW publication is defined in terms of the Durham jet measure [23] for two partons i and j ,

$$y_{\text{dur}_{ij}} = \frac{2 \min(E_i^2, E_j^2)}{s} (1 - \cos \theta_{ij}), \quad (2.6)$$

where $E_{i,j}$ are the energies of the two partons, θ_{ij} is the angle between the two partons and s is the centre-of-mass-energy squared. The merging transverse momentum variable is defined by

$$k_{\perp} = \sqrt{y_{ij}s}. \quad (2.7)$$

The parton shower with which we wish to merge the matrix elements may not be ordered in transverse momentum, in which case the merging variable cannot be chosen to be the same as the evolution variable, as was assumed in Sect. 2.1.

In the **Herwig++** parton shower, splittings are described by the variables $(\tilde{q}, z, \phi)$, where $\tilde{q}$ is an angular-ordered evolution variable, z is the momentum fraction of the emitted parton and ϕ is the azimuthal angle of the branching. The evolution variable is defined by

$$z^2(1-z)^2\tilde{q}^2 = p_{\perp}^2 + (1-z)m_i^2 + zm_j^2 - z(1-z)m_{ij}^2, \quad (2.8)$$

where $p_{\perp}$ is the relative transverse momentum of the branching and $m_{\tilde{ij}}$ and $m_{i,j}$ are the masses of the parent and child partons respectively.

The probability for a branching $\tilde{ij} \rightarrow ij$ in the phase space measure $\tilde{q} \rightarrow \tilde{q} + d\tilde{q}$ is given by

$$d\mathcal{P}_{\tilde{ij} \rightarrow ij}(\tilde{q}, z) = \frac{\alpha_S(p_{\perp})}{2\pi} \frac{d\tilde{q}^2}{\tilde{q}^2} dz P_{\tilde{ij} \rightarrow ij}(z, \tilde{q}), \quad (2.9)$$

where $P_{\tilde{ij} \rightarrow ij}(z, \tilde{q})$ is the corresponding quasi-collinear splitting function [27]. The Sudakov form factor, giving the probability of a parton shower line of flavour $\tilde{ij}$ evolving from a scale $\tilde{q}_1$ down to $\tilde{q}_2$ undergoing no resolvable emissions, is given by

$$\Delta_{\tilde{ij}}(\tilde{q}_1, \tilde{q}_2) = \exp \left[- \sum_{\tilde{ij} \rightarrow ij} \int_{\tilde{q}_2}^{\tilde{q}_1} d\mathcal{P}_{\tilde{ij} \rightarrow ij}(\tilde{q}, z) \right]. \quad (2.10)$$

In order to accommodate the fact that the evolution and merging variables are not identical, the CKKW algorithm must include some additional features to that outlined in Sect. 2.1. Changes must be made to the Sudakov form factors with which the matrix elements are reweighted and the initial conditions with which the parton shower is invoked, furthermore, when the shower is invoked, a veto must be applied to prevent it generating emissions with $k_{\perp}(\tilde{q}, z) > k_{\perp MS}$.

The Sudakov form factor used for the matrix element reweighting, defined in Eq. (2.2), corresponds to the probability of evolving from a scale q down to the hadronization scale with no emissions resolvable at the merging scale. In the case of Sect. 2.1, this was achieved by setting the lower limit on the integral to q_{MS} , however, now this cut, defining what is meant by a *resolvable emission*, must be implemented as a θ -function in the Sudakov form factors used in step 4. The Sudakov form factors for the reweighting are then given by

$$\Delta_{\tilde{ij}}^R(\tilde{q}; k_{\perp MS}) = \exp \left[- \sum_{\tilde{ij} \rightarrow ij} \int_{\tilde{q}_2}^{\tilde{q}} d\mathcal{P}_{\tilde{ij} \rightarrow ij}(\tilde{q}', z) \theta(k_{\perp}(\tilde{q}', z) - k_{\perp MS}) \right]. \quad (2.11)$$

The prescription for constructing the Sudakov weights is then identical to that in Sect. 2.1 except for factors of z in the scale from which each child evolves, which are required for the

angular-ordered evolution. Each intermediate line, connecting branchings at $(\tilde{q}_1, z_1)$ and $(\tilde{q}_2, z_2)$ in the pseudo-shower history, contributes a factor

$$\Delta_{ij}^R(z_1 \tilde{q}_1; k_{\perp MS}) / \Delta_{ij}^R(\tilde{q}_2; k_{\perp MS}). \quad (2.12)$$

Each external line, from a branching at $(\tilde{q}, z)$ in the pseudo-shower history, contributes a factor

$$\Delta_{ij}^R(z \tilde{q}; k_{\perp MS}). \quad (2.13)$$

2.3 Highest multiplicity treatment

The original CKKW publication did not treat the highest multiplicity matrix element contribution any differently to the other multiplicities. In Refs. [9, 12] it was noted that a different treatment of highest multiplicities must be employed in order to fill the phase space in the matrix element region to all orders in α_S . Since computational limits mean that only matrix elements with up to a maximum of N partons can be calculated, the standard approach leads to a maximum of N partons being generated above the merging scale. The parton shower generates to all orders in α_S and therefore we should also let the matrix element region generate to all orders. This can be achieved by allowing the highest multiplicity channel parton shower to generate emissions in the region with $k_{\perp}$ less than that of the lowest transverse momentum of the matrix element emissions, $k_{\perp L}$. This is achieved by changing the scale of the parton shower vetoes and Sudakov form factor cuts from $k_{\perp MS}$ to $k_{\perp L}$.

2.4 Problems with the algorithm

The above procedure is heavily reliant on having an exact mapping between the shower variables and the merging measure $k_{\perp MS}$ so that the parton shower vetoes and Sudakov cuts can be correctly applied. A mapping from the momentum clustered in step 3 to the corresponding shower variables is also required, so that the correct scales for the Sudakov reweighting and initial shower conditions are obtained. In practice obtaining such mappings may be difficult due to the complexity of the shower kinematics.

The initial scale at which the parton shower is invoked is vital to the algorithm. Initiating the parton shower directly from the merging scale would result in a radiation gap, where emissions with transverse momentum less than the merging scale but evolution scale greater than the merging scale are missed. In the angular-ordered shower, this radiation corresponds to soft, wide-angle emissions. The original CKKW publication attempts to resolve this by invoking the parton shower from each external parton at a scale corresponding to the node at which it was ‘created’ in the pseudo-shower history. Although adopting this maximal initial scale helps fill the radiation gap, the extra soft, wide-angle radiation that results, is emitted from the external parton in the pseudo-shower history, rather than the intermediates, as implied by colour coherence [15]. The original CKKW publication argued that this should be a sub-leading effect, however, it will certainly change the colour structure of the configuration, which may cause problems when non-perturbative hadronization models are applied.

The original CKKW algorithm also assumes that the clustering of momentum in step 3 and subsequent mapping to parton shower variables results in a set of emission scales that respect the ordering of the parton shower, *i.e.*

$$\tilde{q}_I > \tilde{q}_1 > \dots > \tilde{q}_n > \tilde{q}_0. \quad (2.14)$$

The clustering scheme of the original algorithm does not guarantee this.

These problems were studied in Ref. [13] and found to result in discontinuities around the merging scale and a false dependence on the merging scale when the parton shower was not ordered in transverse momentum. In Ref. [12], a study of the algorithm with angular and virtuality-ordered parton showers was presented. In that work, a number of *ad hoc* adaptations were applied and tuned in order to achieve a reasonably smooth merging at the parton level, nevertheless, some problems remained at the hadron level. In this article we aim to overcome these problems with a set of well motivated modifications based on the POWHEG shower reorganisation.

3. Shower reorganisation

In the POWHEG next-to-leading-order matching scheme it was shown that a general parton shower may be rearranged such that the hardest emission⁵ can be generated first. The hardest emission can then be generated according to the exact NLO matrix elements such that inclusive observables are distributed according to the NLO cross section while the soft/collinear resummation of the shower is undisturbed. In the POWHEG shower reorganisation, the hardest emission is then dressed by inserting a truncated shower of wide angle, soft emissions prior to it, and a vetoed shower consisting of smaller $p_\perp$, smaller angle emissions, after it.

The CKKW algorithm generates a set of n emissions above the merging scale y_{MS} according to exact tree-level matrix elements up to $\mathcal{O}(\alpha_S^N)$. This defines a set of n hard emissions. In order to reproduce the full shower around this set of hard emissions we employ a generalisation of the POWHEG shower reorganisation. In the following a review of the POWHEG reorganisation is presented followed by its extension to the CKKW case. The notation used relates specifically to that of the Herwig++ shower, however the treatment is independent of the details of the parton shower.

3.1 POWHEG reorganisation

A general Herwig++ parton shower, evolving from a parton of flavour $i\bar{j}$ and initial scale $\tilde{q}_I$, can be represented by a generating functional $S_{i\bar{j}}(\tilde{q}_I)$. The evolution of the parton shower may be expressed by the recursive equation,

$$S_{i\bar{j}}(\tilde{q}_I) = \Delta_{i\bar{j}}(\tilde{q}_I, \tilde{q}_0) S_{i\bar{j}}(\tilde{q}_0) + \int_{\tilde{q}_0}^{\tilde{q}_I} \Delta_{i\bar{j}}(\tilde{q}_I, \tilde{q}) d\mathcal{P}_{i\bar{j} \rightarrow i\bar{j}}(\tilde{q}, z) S_i(z\tilde{q}) S_j((1-z)\tilde{q}). \quad (3.1)$$

The first term in Eq. (3.1) is a no emission term corresponding to the probability of evolving from the initial scale to hadronization scale with no resolvable emissions generated.

⁵The hardest emission here refers to the emission with highest transverse momentum.

The second term represents the probability of producing at least one emission, with the first generated at a scale $\tilde{q}$ and further showers evolving down to the hadronization scale.

It is possible to expand Eq. (3.1) to explicitly show the hardest emission of the shower. The hardest emission is described by the shower variables $(\tilde{q}_h, z_h, \phi_h)$ and has an associated transverse momentum $p_{\perp_h}$. The shower may produce any number of other emissions before the hardest emission and any number of emissions after it but all of these must have $p_{\perp} < p_{\perp_h}$. The shower can therefore be written as

$$S_{ij}^{\sim}(\tilde{q}_I) = \Delta_{ij}^{\sim}(\tilde{q}_I, \tilde{q}_0) S_{ij}^{\sim}(\tilde{q}_0) + \int_{\tilde{q}_0}^{\tilde{q}_I} \bar{S}_{ij}^T(\tilde{q}_I, \tilde{q}_h; p_{\perp_h}) d\mathcal{P}_{ij \rightarrow i\bar{j}}^{\sim}(\tilde{q}_h, z_h) \bar{S}_i^V(z_h \tilde{q}_h; p_{\perp_h}) \bar{S}_j^V((1 - z_h) \tilde{q}_h; p_{\perp_h}) \quad (3.2)$$

where $\bar{S}^T$ refers to a *truncated shower* and $\bar{S}^V$ refers to a *vetoed shower*. The truncated shower in Eq. (3.2) is responsible for evolving from the initial scale down to the scale of the hardest emission producing any number of emissions with transverse momentum less than $p_{\perp_h}$. The emissions within the truncated shower correspond to soft, wide angle gluon emissions, and so do not change the flavour of the shower line. The vetoed shower evolves from the scale of the hardest emission down to the hadronization scale also generating only emissions with transverse momentum less than $p_{\perp_h}$, its evolution is defined by

$$\begin{aligned} \bar{S}_{ij}^V(\tilde{q}_h; p_{\perp_h}) &= \Delta_{ij}^{\sim}(\tilde{q}_h, \tilde{q}_0) S_{ij}^{\sim}(\tilde{q}_0) + \int_{\tilde{q}_0}^{\tilde{q}_h} \Delta_{ij}^{\sim}(\tilde{q}_h, \tilde{q}) d\mathcal{P}_{ij \rightarrow i\bar{j}}^{\sim}(\tilde{q}, z) \theta(p_{\perp_h} - p_{\perp}(\tilde{q}, z)) \\ &\quad \times \bar{S}_i^V(z \tilde{q}; p_{\perp_h}) \bar{S}_j^V((1 - z) \tilde{q}; p_{\perp_h}). \end{aligned} \quad (3.3)$$

The recursive equation describing the evolution of the truncated shower is given by

$$\begin{aligned} \bar{S}_{ij}^T(\tilde{q}_I, \tilde{q}_h; p_{\perp_h}) &= \Delta_{ij}^{\sim}(\tilde{q}_I, \tilde{q}_h) S_{ij}^{\sim}(\tilde{q}_h) + \int_{\tilde{q}_h}^{\tilde{q}_I} \Delta_{ij}^{\sim}(\tilde{q}_I, \tilde{q}) d\mathcal{P}_{ij \rightarrow i\bar{j}}^{\sim}(\tilde{q}, z) \theta(p_{\perp_h} - p_{\perp}(\tilde{q}, z)) \\ &\quad \times \bar{S}_{ij}^T(z \tilde{q}, \tilde{q}_h; p_{\perp_h}) \bar{S}_g^V((1 - z) \tilde{q}; p_{\perp_h}). \end{aligned} \quad (3.4)$$

The Sudakov form factors and splitting functions appearing in the Eqs. (3.4, 3.3) are identical to those in the standard shower equation of Eq. (3.1) with the exception that the splitting functions in both new showers have an additional θ -function. This θ -function guarantees that no emissions with transverse momentum greater than that of the hardest emission are generated. Standard Monte Carlo techniques require that the splitting functions of a parton shower match those appearing in the Sudakov form factors. The introduction of the θ -functions mean that this is not the case for the vetoed and truncated showers in Eq. (3.2), we highlight this in our notation with a bar.

In order to make the truncated and vetoed showers suitable for a Monte Carlo treatment, the original POWHEG publication [15] splits the Sudakov form factor appearing in Eqs. (3.4, 3.3) into two parts according to

$$\Delta_f(z_i \tilde{q}_i, \tilde{q}_{i+1}) = \Delta_f^V(z_i \tilde{q}_i, \tilde{q}_{i+1}; p_{\perp_h}) \bar{\Delta}_f^R(z_i \tilde{q}_i, \tilde{q}_{i+1}; p_{\perp_h}). \quad (3.5)$$

Here, Δ_f^V refers to a *vetoed Sudakov* in which the exponent contains a θ -function, which matches that in the splitting function of Eqs.(3.4, 3.3) and is given by

$$\Delta_{ij}^V(z_i \tilde{q}_i, \tilde{q}_{i+1}; p_{\perp h}) = \exp \left[- \sum_{\tilde{ij} \rightarrow ij} \int_{\tilde{q}_{i+1}}^{z_i \tilde{q}_i} d\mathcal{P}_{\tilde{ij} \rightarrow ij}(\tilde{q}, z) \theta(p_{\perp h} - p_{\perp}(\tilde{q}, z)) \right]. \quad (3.6)$$

The other factor, $\bar{\Delta}_f^R$, contains the opposite θ -function and is referred to as a *remnant Sudakov* given by

$$\bar{\Delta}_{ij}^R(z_i \tilde{q}_i, \tilde{q}_{i+1}; p_{\perp h}) = \exp \left[- \int_{\tilde{q}_{i+1}}^{z_i \tilde{q}_i} \sum_{\tilde{ij} \rightarrow ij} d\mathcal{P}_{\tilde{ij} \rightarrow ij}(\tilde{q}, z) \theta(p_{\perp}(\tilde{q}, z) - p_{\perp h}) \right]. \quad (3.7)$$

The combination of the splitting functions in Eqs. (3.4, 3.3) and the vetoed Sudakov form factors result in a parton shower that may be generated with standard vetoes allowing only emissions with $p_{\perp} < p_{\perp h}$, however, the presence of the remnant Sudakov form factors appears to spoil this picture. On the contrary, it turns out that the seemingly awkward remnant factors have a key role to play in formalising how to generate the hardest emission first. In Ref. [15] it was proven that the hardest emission in the shower is generated along the hardest line, the line for which all $z_i > \frac{1}{2}$ and, moreover, all non-soft emissions preceding it give rise to subleading collinear contributions. This means that the truncated shower may be considered as comprising solely of soft gluon emissions and that z_i can be effectively replaced by one in all of the associated remnant factors. It is also shown that the θ -function in the remnant Sudakov form factor exponent is zero for scales less than $\tilde{q}_h$ and so the replacement $z_i \rightarrow 1$ also holds for the vetoed showers. The net result of these replacements is that the product of all remnant Sudakov form factors combine as a single remnant Sudakov factor:

$$\Delta_{ij}^R(\tilde{q}_I, \tilde{q}_0; p_{\perp h}) = \exp \left[- \int_{\tilde{q}_0}^{\tilde{q}_I} d\mathcal{P}_{\tilde{ij} \rightarrow ij}(\tilde{q}, z) \theta(p_{\perp}(\tilde{q}, z) - p_{\perp h}) \right]. \quad (3.8)$$

The POWHEG treatment which has been outlined, results in a reorganisation of the shower such that the hardest emission may be generated first. The Monte Carlo interpretation of this reorganisation is:

1. the hardest emission (q_h, z_h, ϕ_h) is generated according to the appropriate splitting function reweighted with the remnant Sudakov factor of Eq. (3.8);
2. a truncated shower, allowing only non-flavour-changing emissions with $p_{\perp} < p_{\perp h}$ is initiated, evolving the shower from $\tilde{q}_I$ down to $\tilde{q}_h$;
3. the hardest emission is forced with shower variables (q_h, z_h, ϕ_h) ;
4. showers with a veto, allowing only emissions with $p_{\perp} < p_{\perp h}$, evolve all external lines down to the hadronization scale.

A shower generated in this way should differ from the standard shower by only sub-leading terms.

The vetoed shower is defined by

$$S_{ij}^V(\tilde{q}_h; p_{\perp_h}) = \Delta_{ij}^V(\tilde{q}_h, \tilde{q}_0) S_{ij}^V(\tilde{q}_0) + \int_{\tilde{q}_0}^{\tilde{q}_h} \Delta_{ij}^V(\tilde{q}_h, \tilde{q}) d\mathcal{P}_{\tilde{ij} \rightarrow ij}(\tilde{q}, z) \theta(p_{\perp_h} - p_{\perp}(\tilde{q}, z)) \quad (3.9) \\ \times S_i^V(z\tilde{q}; p_{\perp_h}) S_j^V((1-z)\tilde{q}; p_{\perp_h}).$$

and corresponds to a standard shower with vetoes applied such that only emissions with $p_{\perp} < p_{\perp_h}$ are generated. The truncated shower is defined by

$$S_i^T(\tilde{q}_I, \tilde{q}_h; p_{\perp_h}) = \Delta_i^V(\tilde{q}_I, \tilde{q}_h) S_{ij}^V(\tilde{q}_0) + \int_{\tilde{q}_h}^{\tilde{q}_I} \Delta_i^V(\tilde{q}_I, \tilde{q}) d\mathcal{P}_{i \rightarrow ig}(\tilde{q}, z) \theta(p_{\perp_h} - p_{\perp}(\tilde{q}, z)) \quad (3.10) \\ \times S_i^T(z\tilde{q}, \tilde{q}_h; p_{\perp_h}) S_g^V((1-z)\tilde{q}; p_{\perp_h}).$$

and corresponds to a standard vetoed parton shower line, constrained not to produce any flavour changing emissions, that is stopped once the truncated line has evolved down to the scale $\tilde{q}_h$.

3.2 CKKW shower reorganisation

In the POWHEG treatment reviewed in Sect. 3.1 a single hardest emission is separated such that it may be corrected with matrix elements. In the CKKW algorithm we aim to improve the parton shower with tree-level matrix elements for all parton multiplicities resolved at the merging scale $k_{\perp_{MS}}$. We perform a reorganisation of the parton shower, analogous to the POWHEG reorganisation, splitting the shower into two parts: a *hard shower* describing emissions resolved above the merging scale; and another shower producing the rest of the shower emissions around this hard shower. The hard shower can then be generated according to the tree-level matrix elements as required by the CKKW algorithm.

The result of this generalisation of the POWHEG reconstruction is a set of truncated and vetoed showers which fill in the radiation between the hard emissions defined by the hard shower history. In order to see how this works we first consider the next step up from the POWHEG case of a single hard emission, where we have exactly two hard emissions along the hard shower line, generated at scales q_1 and q_2 . One possible configuration of this hard shower line is given in Fig. 1. As was done in formulating the POWHEG scheme, the full parton shower can be constructed around this hard shower line by constructing an equation analogous to Eq. (3.2),

$$S_{ijk}^{(2)}(\tilde{q}_I) = \int_{\tilde{q}_0}^{\tilde{q}_I} \bar{S}_{ijk}^T(\tilde{q}_I, \tilde{q}_1; k_{\perp_{MS}}) d\mathcal{P}_{\tilde{ijk} \rightarrow \tilde{ijk}}(\tilde{q}_1, z_1) \bar{S}_i^V((1-z_1)\tilde{q}_1; k_{\perp_{MS}}) \quad (3.11) \\ \times \int_{\tilde{q}_0}^{\tilde{q}_1} \bar{S}_{jk}^T(z_1\tilde{q}_1, \tilde{q}_2; k_{\perp_{MS}}) d\mathcal{P}_{\tilde{jk} \rightarrow jk}(\tilde{q}_2, z_2) \bar{S}_j^V(z_2\tilde{q}_2; k_{\perp_{MS}}) \bar{S}_k^V((1-z_2)\tilde{q}_2; k_{\perp_{MS}}).$$

The superscript (2) on S denotes that this does not describe a general shower line, but the subset of shower lines with exactly two emissions above the merging scale. Eq. (3.11)

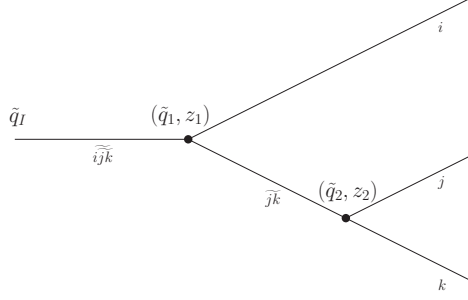


Figure 1: An example of a hard shower line configuration where two emissions are generated above $k_{\perp MS}$.

contains two truncated showers, one containing parton shower emissions with $k_{\perp} < k_{\perp MS}$ before the hard emission $(\tilde{q}_1, z_1)$ and the other containing emissions with $k_{\perp} < k_{\perp MS}$ between the hard emissions at $(\tilde{q}_1, z_1)$ and $(\tilde{q}_2, z_2)$. The truncated and vetoed showers are given by Eq. (3.4) and Eq. (3.3)⁶, respectively.

As in the POWHEG case, the splitting functions and Sudakov form factors for the truncated and vetoed showers in Eq. (3.11) do not match each other and are therefore not suitable for a standard Monte Carlo treatment. However, we can use the same manipulations as in the POWHEG formulation to split the Sudakov form factors into a product of a Sudakov form factor that matches the vetoed splitting functions and a remnant Sudakov form factor, as in Eq. (3.5). The truncated showers contain only soft radiation so again we may set $z_i \rightarrow 1$ in the remnant Sudakov form factor of Eq. (3.7) with only sub-leading differences. The result of this is that the product of remnant Sudakov form factors for a particular truncated or vetoed line combine to give a remnant Sudakov factor. Rather than resulting in a single remnant Sudakov factor as in the POWHEG scheme, we now get a product of remnant Sudakov factors. The product of remnant Sudakov factors for the hard shower configuration of Fig. 1 is given by

$$\begin{aligned} & \frac{\Delta_{\widetilde{ijk}}^R(\tilde{q}_I; k_{\perp MS})}{\Delta_{\widetilde{ijk}}^R(\tilde{q}_1; k_{\perp MS})} \Delta_i^R((1 - z_1)\tilde{q}_1; k_{\perp MS}) \frac{\Delta_{\widetilde{jk}}^R(z_1\tilde{q}_1; k_{\perp MS})}{\Delta_{\widetilde{jk}}^R(\tilde{q}_2; k_{\perp MS})} \\ & \times \Delta_j^R(z_2\tilde{q}_2; k_{\perp MS}) \Delta_k^R((1 - z_2)\tilde{q}_2; k_{\perp MS}), \end{aligned} \quad (3.12)$$

where the remnant Sudakov factor is given by

$$\Delta_{\widetilde{ij}}^R(\tilde{q}; k_{\perp MS}) = \exp \left[- \int_{\tilde{q}_0}^{\tilde{q}} \sum_{\widetilde{ij} \rightarrow ij} d\mathcal{P}_{\widetilde{ij} \rightarrow ij}(\tilde{q}, z) \theta(k_{\perp}(\tilde{q}, z) - k_{\perp MS}) \right]. \quad (3.13)$$

In the CKKW algorithm the hard shower is generated by choosing a jet multiplicity n as described in Sect. 2.1 and generating n parton momenta according to the appropriate jet cross section. A pseudo-shower history and corresponding shower variables are then assigned by applying a clustering algorithm to the n parton momenta, until they are clustered

⁶With the replacement $p_{\perp} \rightarrow k_{\perp}$. The variable $p_{\perp}$ is the shower definition of transverse momentum and $k_{\perp}$ is the merging variable transverse momentum measure.

back to a leading order configuration. The shower reorganisation presented here results in a product of remnant Sudakov factors, with which these hard shower configurations should be reweighted. These remnant Sudakov factors can generally be found from the pseudo-shower history by applying the following prescription:

- each internal line from a branching at (q_1, z_1) to (q_2, z_2) contributes a factor

$$\frac{\Delta_f^R(z_1 \tilde{q}_1; k_{\perp MS})}{\Delta_f^R(\tilde{q}_2; k_{\perp MS})}; \quad (3.14)$$

- each external line from a branching at (q, z) contributes a factor

$$\Delta_f^R(z \tilde{q}; k_{\perp MS}). \quad (3.15)$$

These remnant Sudakov factors match the Sudakov factors in Eq.(2.11) that we argued should be introduced in order to extend the CKKW procedure for transverse momentum showers to the angular-ordered shower.

The parton shower for emissions below the cut is generated by producing truncated and vetoed showers around the hard shower according to the following prescription:

- each internal line from a branching at (q_1, z_1) to (q_2, z_2) results in a truncated shower

$$S_f^T(z_1 \tilde{q}_1, \tilde{q}_2; k_{\perp MS}); \quad (3.16)$$

- each external line from a branching at (q, z) results in a vetoed shower

$$S_f^V(z \tilde{q}; k_{\perp MS}). \quad (3.17)$$

4. The algorithm

In order to implement the procedure described in Sect. 3.2 we employ the strategy of Ref. [18], where the hardest emissions, or set of hard emissions in this case, are interpreted as parton shower emissions. This approach leads to a straightforward implementation of the truncated showers, where a truncated shower, evolving between hard emissions at $(\tilde{q}_1, z_1, \phi_1)$ and $(\tilde{q}_2, z_2, \phi_2)$, is generated by initiating a standard parton shower at $z_1 \tilde{q}_1$ with vetoes allowing only non-flavour-changing emissions with $k_{\perp} < k_{\perp MS}$ and stopping the truncated shower once it has evolved beyond $\tilde{q}_2$, at which point the second hard emission is forced with splitting variables $(\tilde{q}_2, z_2, \phi_2)$. This allows the full shower of truncated showers, hard emissions and vetoed showers to be generated as a single shower evolution from the leading-order configuration. This results in a substantial improvement over earlier CKKW implementations with angular-ordered showers [7, 12], since now the *colour structure* in the event is plainly equivalent to that which the shower would have produced by default *i.e.* it respects colour coherence.

In order to interpret the matrix element emissions as shower emissions, we require an exact mapping from the set of n external parton momentum and assigned pseudo-shower

history to a set of shower splitting variables, $(\tilde{q}, z, \phi)$, describing each emission. Obtaining such a mapping equates to inverting the momentum reconstruction, which is performed at the end of the standard parton shower to translate the set of shower variables into the parton momenta, and is described in Sect. 4.2. Having such a mapping also provides the exact shower variables that are to be used for the Sudakov and α_S reweighting.

The full modified CKKW algorithm is described below.

1. The jet multiplicity n is generated with probability

$$P_n = \frac{\sigma_n(k_{\perp MS})}{\sum_N \sigma_N(k_{\perp MS})}, \quad (4.1)$$

where cross sections are evaluated at a fixed strong coupling $\alpha_{S_{ME}}$.

2. The n external parton momenta are generated according to $d\sigma(k_{\perp MS})$.
3. Pairs of external parton momenta are clustered⁷ down to a leading-order configuration, assigning a pseudo-shower history.
4. The inverse momentum reconstruction is applied to the external momenta and shower history such that a set of shower splitting variables $(\tilde{q}, z, \phi)$ are found, describing $n-2$ hard branchings.
5. The configuration is reweighted to include the Sudakov form factors and running α_S . This corresponds to assigning the configuration a weight W and rejecting the configuration if $W < \mathcal{R}$ ⁸. The weight is constructed from the pseudo-shower history, according to the following prescription:

- each hard emission at $(\tilde{q}, z)$ contributes a running α_S factor

$$\frac{\alpha_S(p_{\perp}(\tilde{q}, z))}{\alpha_{S_{ME}}}; \quad (4.2)$$

- each internal line between hard emissions at $(\tilde{q}_1, z_1)$ to $(\tilde{q}_2, z_2)$ contributes a Sudakov factor

$$\frac{\Delta_f^R(z_1 \tilde{q}_1; k_{\perp MS})}{\Delta_f^R(\tilde{q}_2; k_{\perp MS})}; \quad (4.3)$$

- each external line from a hard emission at $(\tilde{q}, z)$ contributes a Sudakov factor

$$\Delta_f^R(z \tilde{q}; k_{\perp MS}). \quad (4.4)$$

If the configuration is rejected⁹ return to step 1.

⁷The clustering procedure is discussed in Sect. 4.4.

⁸ $\mathcal{R}$ refers to a random number, generated in the interval $[0, 1]$.

⁹This reweighting procedure relies on the weight generated in this step satisfying $W < 1$. The fixed strong coupling used in the matrix elements α_{ME} can be chosen to be large enough that the α_S weight is always less than one. Individual Sudakov form factors are also guaranteed to be less than one while the ratio of Sudakov form factors contributed by intermediate lines must be less than one due to the angular ordering condition $z_i \tilde{q}_i > \tilde{q}_{i+1}$.

6. Parton shower lines are initiated from the leading-order configuration which are to be evolved according to the procedure:

- (a) If a hard emission exists at a lower scale on the shower line, then the shower is evolved as a truncated shower otherwise proceed with step (6c). The truncated showers evolve as the standard parton shower with vetoes allowing only non-flavour-changing-emissions with $k_{\perp} < k_{\perp MS}$. Each truncated emission generates a soft gluon which should be evolved according to step (6c).
- (b) Once the scale of the next hard emission is reached, the hard emission is forced creating two further shower lines, each of which should be evolved according to step (6a).
- (c) Vetoeed showers evolve all external shower lines down to the hadronization scale, with vetoes allowing only emissions with $k_{\perp} < k_{\perp MS}$.

The above scheme is adapted for the highest multiplicity channel, where $n = N$, by the replacement $k_{\perp MS} \rightarrow k_{\perp l}$ in the shower vetoes and Sudakov form factors.

4.1 Shower kinematics

In order to implement the inverse momentum reconstruction and parton shower vetoes, the kinematics of the Herwig++ final-state parton shower must be understood. The full details of this are given in Ref. [2] and we present a review here for completeness and to introduce our notation.

Each external parton from the hard sub-process with momentum p_J is interpreted as a progenitor for a parton shower jet. Each parton shower jet evolves from the initial scale $\tilde{q}_I = \sqrt{s}$ down to the hadronization scale $\tilde{q}_0$ undergoing a series of branchings, each described by the shower variables $(\tilde{q}, z, \phi)$.

Once all of the shower lines have evolved down to the hadronization scale, the shower evolution is stopped and the momentum of all external and intermediate partons are reconstructed from the shower variables. This is done in the centre-of-mass frame via the Sudakov decomposition, where the momentum of a parton in the shower is written

$$q_i = \alpha_i p_J + \beta_i n_J + q_{\perp i}, \quad (4.5)$$

where the reference vector n_J is taken to be a light-like vector with three-momentum equal to that of the colour partner to the jet progenitor and $q_{\perp}$ is the component of momentum transverse to both p_J and n_J . The momentum fraction z of each branching is defined by

$$z = \frac{\alpha_i}{\alpha_{\tilde{ij}}}, \quad (4.6)$$

the scale of the emission $\tilde{q}$ is defined in Eq. (2.8). The relative transverse momentum of the branching is defined by

$$p_{\perp i} = q_{\perp i} - z q_{\perp \tilde{ij}}. \quad (4.7)$$

The $p_{\perp i}$ vector is written in terms of the azimuthal angle ϕ

$$p_{\perp} = (|p_{\perp}| \cos \phi, |p_{\perp}| \sin \phi, 0; 0). \quad (4.8)$$

The Sudakov variables α_i , β_i and $q_{\perp i}$ are calculated from the shower variables recursively for all partons in the shower jet and the momentum are constructed according to Eq. (4.5).

The momentum reconstruction procedure results in the progenitor momenta q_J gaining some off-shell momentum and leads to the loss of global momentum conservation. Longitudinal boosts are applied to each shower jet to restore momentum conservation while disturbing the internal structure of each jet as little as possible. These reshuffling boosts are defined for each jet by the transformation

$$\left(\vec{q}_J; \sqrt{\vec{q}_J^2 + q_J^2}\right) \rightarrow \left(k\vec{p}_J; \sqrt{k^2\vec{p}_J^2 + q_J^2}\right). \quad (4.9)$$

Momentum conservation is ensured by requiring that the rescaling parameter k satisfies

$$\sum_J \sqrt{k^2\vec{p}_J^2 + q_J^2} = \sqrt{s}. \quad (4.10)$$

4.2 Inverse momentum reconstruction

In the CKKW algorithm a clustering procedure is applied to a configuration of external parton momenta, defining a pseudo-shower history for producing that configuration as a set of branchings from a leading-order parton configuration, which in this case would be $q\bar{q}$. We aim to interpret this branching history as a set of shower branchings such that when this set of branchings are forced, the momentum reconstruction will reconstruct the original parton momenta. In order to map the shower history to a set of shower variables the momentum reconstruction procedure must be inverted; this requires two steps. First, the boost applied to each shower jet in order to conserve global momentum must be found and its inverse applied to the momenta of the shower jet. Second, the resulting momenta are decomposed into the shower variables according to Eq. (4.5).

The momenta of the set of progenitors defined by the branching q'_J correspond to the momenta on the right hand side of Eq. (4.9). The original on-shell progenitors p_J are related to q'_J by

$$p_J = \left(\frac{\vec{q}'_J}{k}; \sqrt{\frac{\vec{q}'_J{}^2}{k^2} + m_J^2}\right), \quad (4.11)$$

where m_J is the on-shell mass of the jet progenitor. The set of on-shell progenitors respect global momentum conservation therefore we can find the boost parameter k by solving

$$\sum_J \sqrt{\frac{\vec{q}'_J{}^2}{k^2} + m_J^2} = \sqrt{s}. \quad (4.12)$$

Once k is found, the reference vector p_J is given by Eq. (4.11); similarly n_J is given by

$$n_J = \left(\frac{\vec{q}'_{\bar{J}}}{k}; \sqrt{\frac{\vec{q}'_{\bar{J}}{}^2}{k^2}}\right), \quad (4.13)$$

where $\bar{J}$ refers to the colour partner jet of the shower jet J . In the reconstruction procedure, the Sudakov parameters of the progenitor partons are set to $\alpha_0 = 1$ and $q_{\perp 0} = 0$.

Furthermore, Eq. (4.5) implies that

$$\beta_0 = \frac{q_J^2 - m_J^2}{2p \cdot n}. \quad (4.14)$$

Since q'_J and q_J are related by a boost, we also have $q_J^2 = q'^2_J$. The momentum of the reconstructed progenitors q_J can then be constructed according to Eq. (4.5). This defines the reshuffling boost as in Eq. (4.9). The boosts for all shower jets can then be calculated, inverted and applied to all momenta in each jet. The momentum can then be decomposed into Sudakov parameters and the shower variables $(\tilde{q}, z, \phi)$ for each branching calculated from Eqs. (4.6–4.8).

4.3 Shower vetoes

The vetoes that are applied to the truncated and vetoed showers and the cuts applied to the remnant Sudakov form factors require a mapping between the shower variables, $(\tilde{q}, z)$ and the merging scale transverse momentum measure $k_\perp$. The merging variable, for an emission $\tilde{i}j \rightarrow ij$, is defined in some jet measure according to, $k_\perp = \sqrt{y_{ij}s}$. We have implemented the merging algorithm with the Durham [23] and LUCLUS [24] jet measures, defined by

$$y_{\text{dur}_{ij}} = \frac{2\min(E_i^2, E_j^2)}{s} (1 - \cos \theta_{ij}), \quad (4.15)$$

$$y_{\text{luc}_{ij}} = \frac{2(E_i E_j)^2}{s(E_i + E_j)^2} (1 - \cos \theta_{ij}). \quad (4.16)$$

In order to implement these vetoes a mapping between the shower variables $(\tilde{q}, z, \phi)$ and y_{ij} , in the chosen jet measure, must be found. The **Herwig++** shower produces off-shell intermediate states and therefore a set of boosts must be applied to each shower line in order to ensure momentum conservation. Since the boosts depend on the full shower history, an exact mapping between the shower variables and the merging variable cannot be found. We use a mapping that is exact for a single shower emission and should give a good approximation for larger numbers of emissions. For clarity in the following, we treat partons as massless while in our implementation parton masses are retained.

In the case of $e^+e^- \rightarrow \text{hadrons}$ we have two shower progenitor partons $q(p_a)$ and $\bar{q}(p_b)$. The momenta of these progenitors, in the centre-of-mass frame, can be written,

$$p_{a,b} = \frac{\sqrt{s}}{2} (0, 0, \pm 1; 1). \quad (4.17)$$

We now consider a single gluon emission along the quark line such that $q(p_a) \rightarrow q(q_1)g(q_2)$. This is the first emission so we have $\alpha_1 = z$ and $\alpha_2 = 1 - z$. The transverse momentum of the emitted partons are given by $q_{\perp 1,2} = \pm p_\perp$. The emitted partons are considered to be external partons and therefore their momenta should be set on-shell. The β parameters in Eq. (4.5) are therefore given by

$$\beta_1 = \frac{p_\perp^2}{zs}, \quad (4.18)$$

$$\beta_2 = \frac{p_\perp^2}{(1-z)s}. \quad (4.19)$$

The vetoes should correspond to vetoes on the reshuffled momenta that have had the boosts, defined in Eq. (4.9), applied to them. We should therefore solve Eq. (4.10) and calculate these boosts before applying Eqs.(4.15, 4.16). The reconstructed progenitor momenta are given by $q_a = q_1 + q_2$ for the quark jet and $q_b = p_b$ for the anti-quark jet. Inserting these momenta into Eq. (4.10) yields the solution

$$k = 1 - \frac{p_\perp^2}{sz(1-z)}, \quad (4.20)$$

for the boost parameter. The reshuffling boost for the quark line is then defined by Eq. (4.9). It follows that the three-vector of the shuffled quark progenitor q'_a should be given by

$$\vec{q}'_a = \frac{\sqrt{s}}{2} \left(1 - \frac{p_\perp^2}{sz(1-z)} \right). \quad (4.21)$$

The expression in Eq.(4.21) is exactly the same as q_a and therefore the boost to be applied to the quark jet is the unit matrix. The shuffled momenta for the emitted partons have now been constructed and we can apply Eqs. (4.15, 4.16) to give expressions for the jet measures used to define the merging scale. These give

$$y_{\text{dur}} = \min \left[z + \frac{p_\perp^2}{sz}, (1-z) + \frac{p_\perp^2}{s(1-z)} \right]^2 \frac{(1 - \cos \theta)}{2}, \quad (4.22)$$

$$y_{\text{luc}} = \left[\frac{(p_\perp^2 + (1-z)^2 s)(p_\perp^2 + z^2 s)}{p_\perp^2 s + s^2 z(1-z)} \right]^2 \frac{(1 - \cos \theta)}{2}, \quad (4.23)$$

where

$$\cos \theta = 1 - \frac{2p_\perp^2 s}{(p_\perp^2 + (1-z)^2 s)(p_\perp^2 + z^2 s)}. \quad (4.24)$$

These mappings allow a transverse momentum measure, $k_\perp = \sqrt{y}s$, to be calculated in the merging variable for each parton shower emission. Parton shower vetoes and Sudakov cuts can then be applied by comparing this measure to the merging scale.

4.4 Clustering scheme

The parton shower decomposition presented in Sect. 3.2 relied on our ability to interpret the series of hard branchings, defined by the matrix element momenta and assigned pseudo-shower history, as a parton shower. The inverse momentum reconstruction procedure ensures that, given an assigned pseudo-shower history, a set of parton shower emissions are found that will exactly reproduce the matrix element momenta.

The assignment of a pseudo-shower history is dependent on the clustering scheme used and this should be carefully chosen to ensure that the assigned history is similar to that which the parton shower would have generated. In particular, Sect. 3.2 assumes that the assigned history is angular-ordered. We therefore assign histories that obey the angular-ordering condition

$$\tilde{q}_i z_i > \tilde{q}_{i+1}, \quad (4.25)$$

for all emissions along all shower lines.

The inverse momentum reconstruction allows us to find the shower variables of all branchings in a particular pseudo-shower history. We can therefore determine whether a history is angular-ordered by following all shower lines outwards from the hard process and explicitly checking that all of the branchings satisfy Eq. (4.25).

We employ a clustering procedure that creates all possible angular-ordered pseudo-shower histories and chooses the one that the shower was most likely to produce. This is achieved by the procedure:

1. all possible shower histories are created by clustering all pairs of partons whose flavours correspond to allowed branchings;
2. non-angular-ordered histories are discarded;
3. the angular-ordered history for which the scalar sum of the transverse momentum of its branchings is smallest is chosen;
4. if no angular-ordered histories could be found, the unordered history with smallest transverse momentum is chosen.

The procedure of always choosing the shower history with smallest transverse momentum is taken rather than a probabilistic choice so that unnatural clusterings are not assigned, for example in a three jet event a gluon collinear to the quark line could be clustered to the anti-quark.

5. Results

In this section we present the results of the implementation of the modified CKKW algorithm for the process $e^+e^- \rightarrow \text{hadrons}$ at a centre-of-mass energy of 91.2 GeV at both parton and hadron level. The parton level results provide a test of the algorithm's ability to provide a smooth merging between the matrix element and parton shower regions of phase space, showing features that may be hidden by the addition of a hadronization model. The hadron level results provide the ultimate test of the algorithm's ability to describe data and in particular are sensitive to the parton colour structure assignment which we expect the modified algorithm to improve with respect to traditional CKKW methods.

A key test of the merging algorithm is its insensitivity to changes in the merging scale and merging variable. The algorithm was implemented with two merging variables: the Durham and LUCUS jet resolution variables. For each merging variable, merging scales of $y_{MS} = 5 \times 10^{-2}$, $y_{MS} = 10^{-2}$ and $y_{MS} = 5 \times 10^{-3}$ were used. Samples of events with all partons separated by $y > y_{MS}$ were generated using MadGraph/MadEvent [25, 26] for the process with up to five partons in the final state¹⁰.

¹⁰The maximum multiplicity for each merging scale was decided according to the phase space available in the matrix element region.

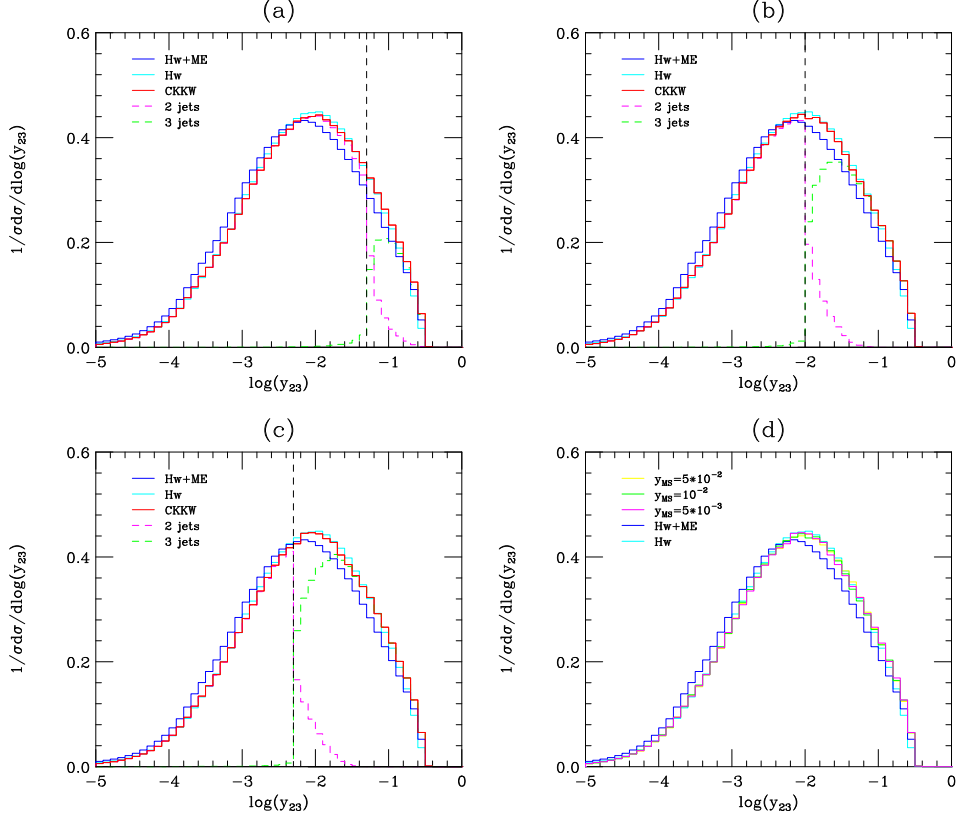


Figure 2: Parton level distributions of the scale at which three jets are resolved in the Durham jet measure for $e^+e^- \rightarrow \text{hadrons}$ at $\sqrt{s} = 91.2 \text{ GeV}$. The red line shows the CKKW distribution with maximum multiplicity set to three, the blue line shows the Herwig++ parton shower distribution with a matrix element correction applied and the cyan line shows the Herwig++ parton shower distribution without the matrix element correction. Plots (a)-(c) show the CKKW distributions with merging scales set to $y_{MS} = 5 \times 10^{-2}$, $y_{MS} = 10^{-2}$ and $y_{MS} = 5 \times 10^{-3}$ in the Durham jet measure. Plot (d) shows the CKKW distributions at all of the merging scale choices on the same plot.

5.1 Parton level results

We present the distributions of the merging variable itself since these should be the most sensitive to problems with the merging procedure. In order to provide a direct comparison to Ref. [13], we first present a systematic look at the algorithm with the maximum multiplicity set to three, so that the matrix element region is responsible for, at most, a single hard emission.

Figure 2 shows distributions of the scale at which three jets are resolved in the Durham jet measure for the three chosen merging scales with the Durham jet measure as the merging variable. Jet analyses were performed with the KtJet package [28]. Each of the merging scale choices exhibit a smooth transition between the two phase space regions, there also appears to be little dependence on the choice of merging scale.

Figure 3 shows the same distributions as Fig. 2 but with the truncated shower switched off. Switching off the truncated shower results in a radiation gap, meaning that emissions

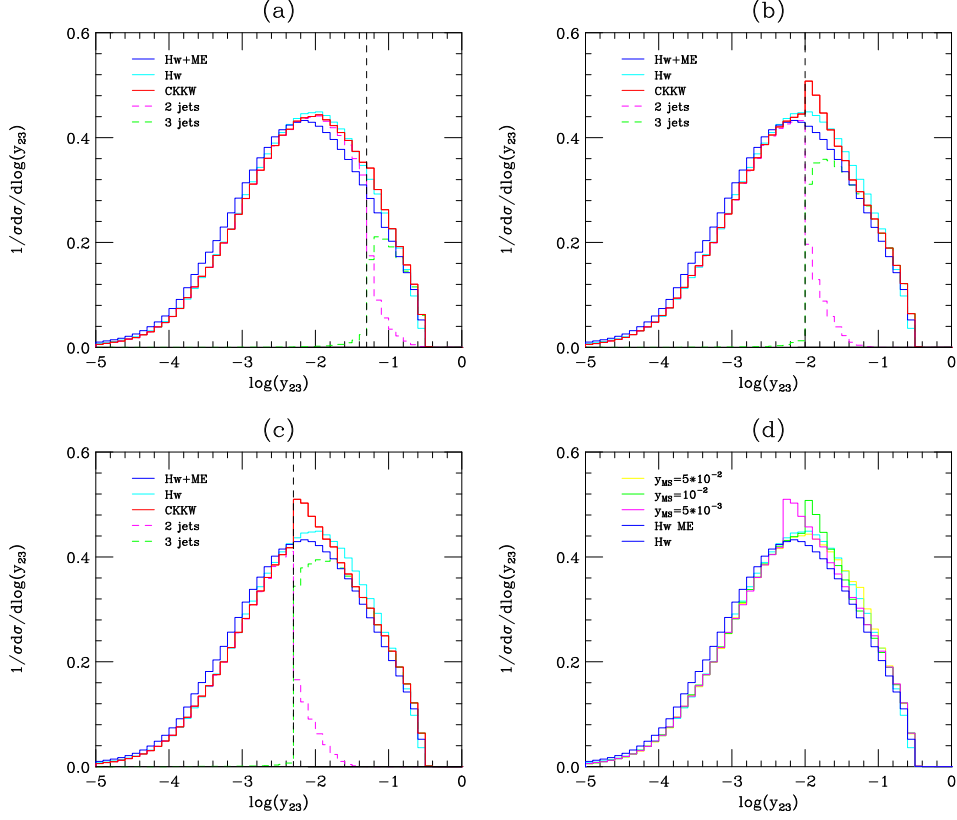


Figure 3: The same distributions as in Fig. 2 but with the truncated shower switched off in the CKKW treatment.

that would be generated at scales greater than that of the hard emission, but with transverse momentum less than that of the hard emission, are never produced. This radiation gap corresponds to a deficit in the amount of soft wide angle emissions produced from the three jet samples. This problem will only become more serious as higher multiplicity contributions are included and underlines the importance of the truncated shower in the merging algorithm.

Figure 4 shows the same distributions as Fig. 2 but with the highest multiplicity treatment switched off. The result of switching off the highest multiplicity treatment is that a maximum of three emissions may be generated in the matrix element region. This violates the all-orders-in- α_S resummation of the parton shower. The result is a suppression of the three-jet channel and a deficit in the hard radiation generated in the three jet channel. In Fig. 4 this presents itself as distributions that are peaked around the merging scale and have a large dependence on the choice of merging scale.

Figures 5 and 6 show the distributions of the scale at which three jets are resolved, for the algorithm with maximum multiplicity set to up to five jets, with the merging algorithm defined in the Durham and LUCCLUS jet measures respectively. As in Fig. 2, all distributions appear to be smooth around the merging scale and to be relatively insensitive to the choice of merging scale.

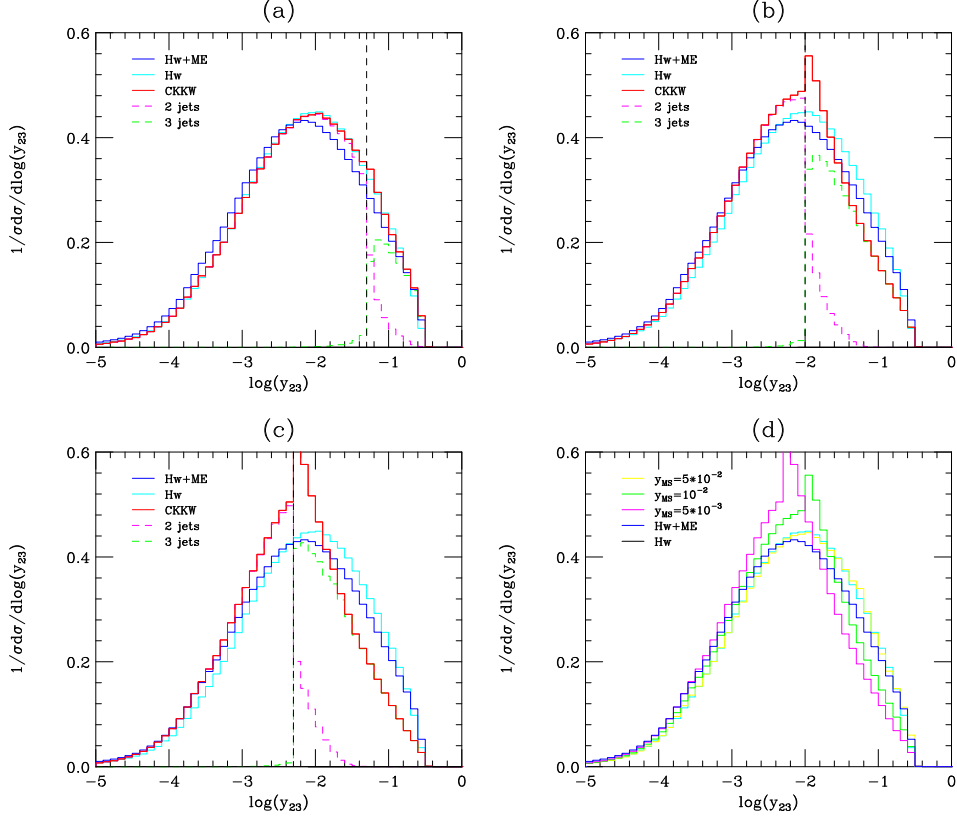


Figure 4: The same distributions of Fig. 2 but with the highest multiplicity treatment switched off in the CKKW treatment.

Since we are now including higher multiplicity channels in our merging algorithm we check the distributions of scales at which higher numbers of jets are resolved. This is done in Fig. 7 for the resolution of four and five jets in the Durham and LUCCLUS jet measures. The merging in these distributions is well behaved.

These distributions demonstrate a degree of insensitivity to the choice of merging scale, which has been varied over an order of magnitude, however there is still some residual dependence on this choice. While the parton shower and merged matrix element treatments formally have the same large logarithm behavior, there are differences between the two. The degree of these differences will directly influence the amount of residual dependence on the merging scale that is observed. In changing the merging scale we are changing the volume of the matrix element phase space region and therefore changing the proportion of parton emissions that are corrected by exact matrix elements.

It can be seen in Fig. 5 and even more so in Fig. 6 that when the merging scale is set to its upper value of $y_{MS} = 5 \times 10^{-2}$, the volume of phase space into which the matrix elements are allowed to emit is small. This results in the CKKW distributions being very close to those of the Herwig++ parton shower with no matrix element correction. Lowering the merging scale increases the volume of the phase space region described by exact matrix elements and moves the distributions farther from those of the bare parton shower. Table

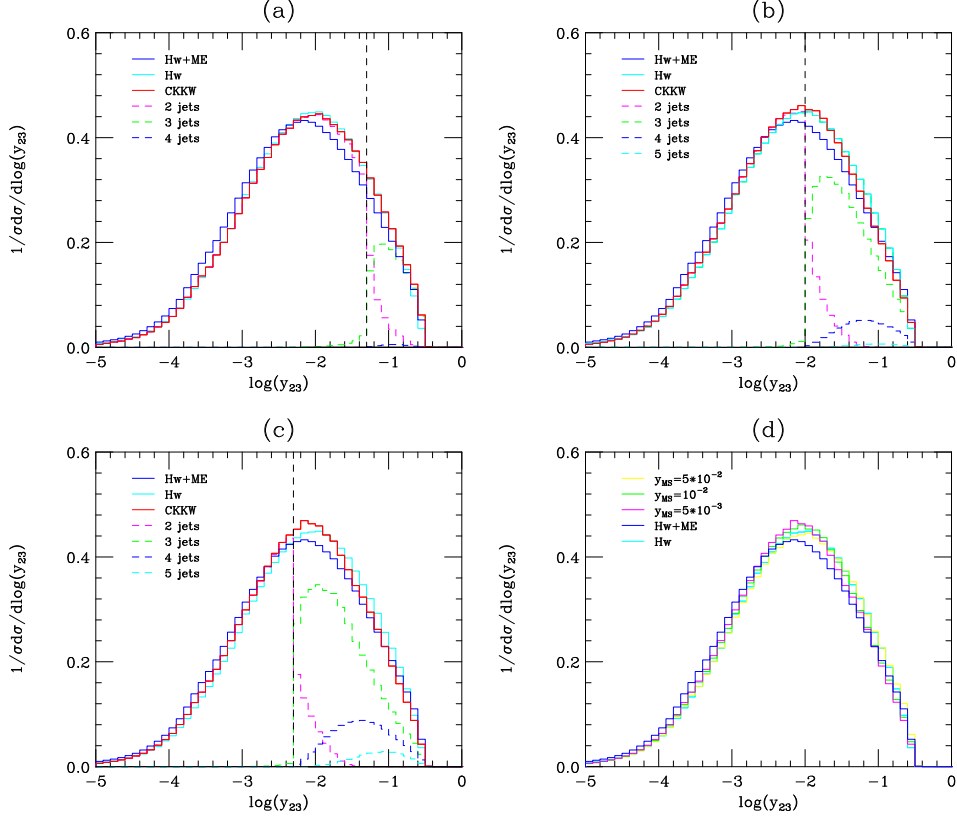


Figure 5: Distributions of the scale at which three jets are resolved in the Durham jet measure. The red line shows the distributions for the CKKW treatment with all multiplicity channels (up to a maximum of five jets) included at a set of merging scale choices in the Durham jet measure.

1 gives the cross sections for the CKKW treatment at different choices of the merging scale and exhibits variation at the 5% level.

y_{MS}	Durham cross section / nb	LUCLUS cross section / nb
5×10^{-2}	38.2	38.6
10^{-2}	36.5	37.1
5×10^{-3}	35.7	35.9

Table 1: Table of cross sections of the process $e^+e^- \rightarrow \text{hadrons}$ for different choices of the merging scale in the Durham and LUCLUS jet measures.

5.2 Hadron level results

We present a comparison of the **Herwig++** CKKW implementation with hadronization switched on to LEP data for a variety of event shapes. It is standard practice to *tune* the free parameters of an event generator to LEP data and this has been done with the default **Herwig++** parton shower with matrix element corrections. Since the CKKW merging algorithm significantly changes the parton shower component of the event generator and in order to provide a fair comparison with default **Herwig++**, a new tune was performed on

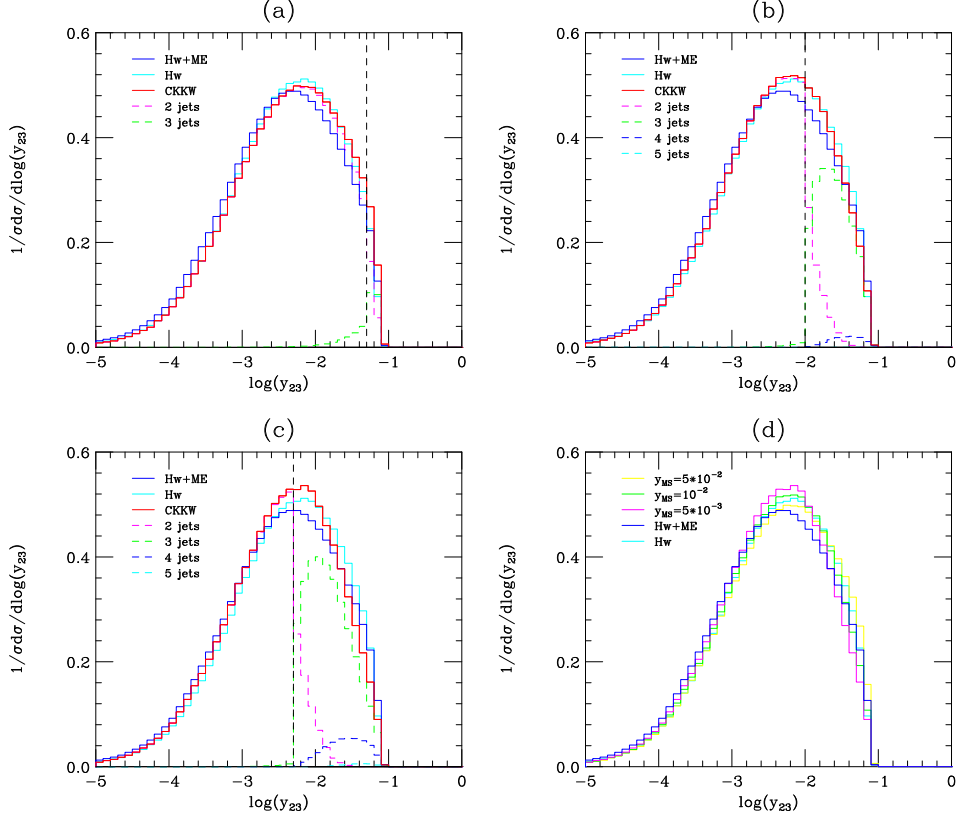


Figure 6: Distributions of the scale at which three jets are resolved in the LUCLUS jet measure. The red line shows the distributions for the CKKW treatment with all multiplicity channels (up to a maximum of five jets) included at a set of merging scale choices in the LUCLUS jet measure.

the free parameters for `Herwig++` with the CKKW algorithm. This tune was performed with the merging scale set to $y_{MS} = 10^{-2}$ in the Durham jet measure. The most notable change to the default `Herwig++` parameter set was a change in the value of the strong coupling at Z mass from a value of $\alpha_S(M_Z) = 0.128$ for `Herwig++` with matrix elements corrections down to $\alpha_S(M_Z) = 0.120$ for `Herwig++` with CKKW. This change brings the parameter closer to its measured value and is indicative of an improved treatment of hard radiation.

Figures 8-10 show distributions of a range of event shape, jet resolution and four-jet observables in comparison to LEP data. The parton level analysis shows that the merging scale choice of $y_{MS} = 5 \times 10^{-2}$ leaves only a very small region of phase space that is corrected by the matrix elements. This very high scale choice will therefore not give the improvement expected in introducing the merging algorithm, we therefore omit this merging scale choice from the hadron level analysis. In each of the figures the red band shows the variation in distributions over the four merging scale choices of $y_{MS} = 10^{-2}$ and $y_{MS} = 5 \times 10^{-3}$ in the Durham and LUCLUS jet measures.

The CKKW distributions (red band) in Figs. 8-10 all demonstrate improved descriptions of the data in comparison to the default `Herwig++` parton shower with matrix element

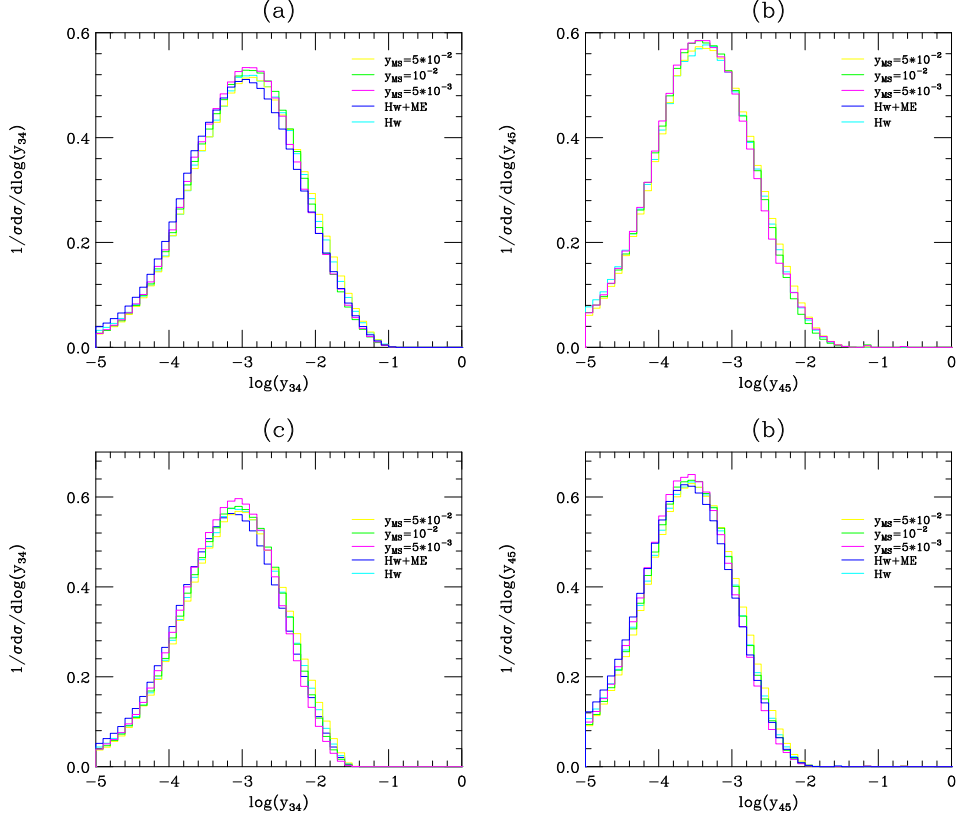


Figure 7: Distributions of the scale at which (a) four and (b) five jets are resolved in the Durham jet measure and the resolution scales for (c) four and (d) five jets in the LUCLUS jet measure.

corrections. In particular the tails of the distributions in Fig. 8, corresponding to hard emissions, and the jet resolution distributions of Fig. 9 with four and five jets are significantly improved as would be expected given the aims of the merging algorithm. The four-jet angle distributions of Fig. 10 are also all improved, with the exception of the α_{34} angle, which was already well described by the default **Herwig++** parton shower. The θ_{NR} distribution provides the most notable improvement in its description of the data in comparison to the default **Herwig++** parton shower.

The width of the red band on the distributions shows that there is some residual dependence on the merging scale however it does not appear to be too serious and is at a similar level to that observed at parton level. This shows that the problems with colour structure, that appear in the standard CKKW algorithm, are not present here and that the truncated shower is working as intended. It should be noted that a fixed set of **Herwig++** shower and hadronization parameters was used for each of the four merging scale choices; the variation would be reduced further if a tune of the parameters was performed for each merging scale choice.

The χ^2 per degree of freedom values for the distributions in Figs. 8-10 are given in Table 2 for the merging scale choice of $y_{MS} = 10^{-2}$ in the Durham jet measure, which was used in the tune. The CKKW values are lower than those of the default **Herwig++** shower

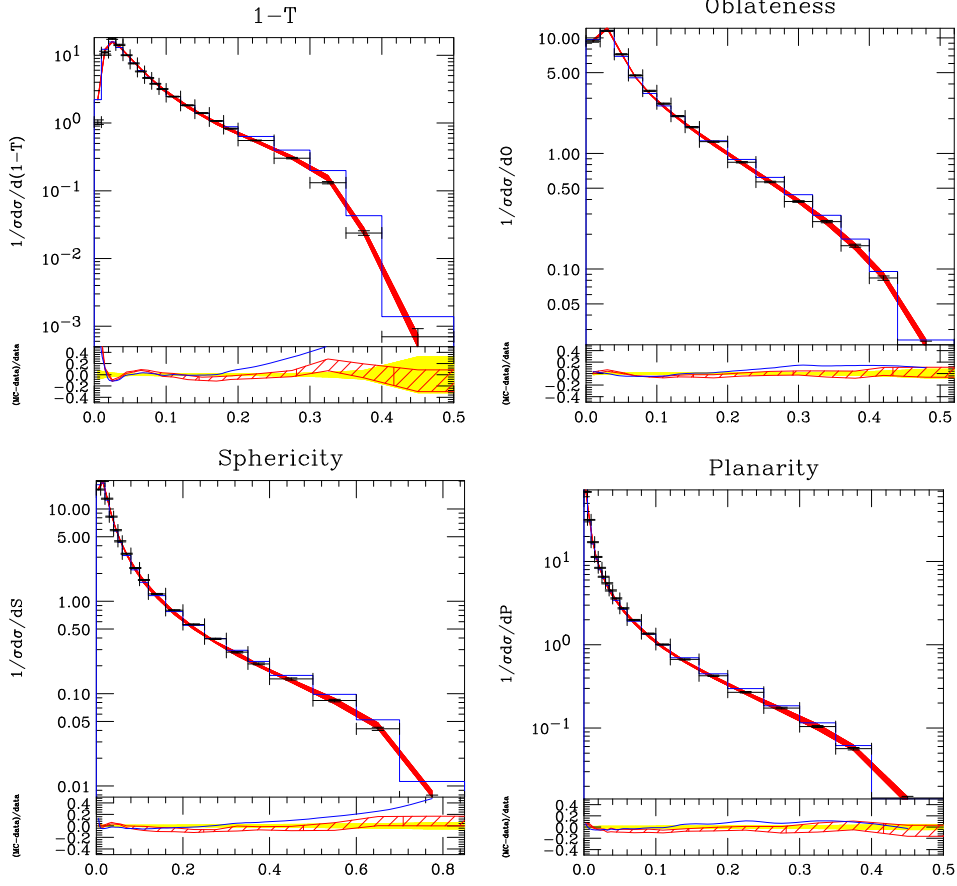


Figure 8: Distributions of the event shape variables thrust, oblateness, sphericity and planarity for $e^+e^- \rightarrow \text{hadrons}$ at a centre-of-mass energy of $\sqrt{s} = 91.2 \text{ GeV}$ in comparison to LEP data (black) [32]. The red band gives the variation of the distributions of the CKKW implementation with merging scales choices of $y_{MS} = 10^{-2}$ and $y_{MS} = 5 \times 10^{-3}$ in the Durham and LUCLUS jet measures. The blue histogram gives the distributions of the default *Herwig++* parton shower with matrix element corrections. The lower panel shows the ratio of the difference between simulation and data to the data in comparison to the error bounds of the data (yellow region).

in all cases except for the α_{34} angle, where the default implementation already gave an accurate description, and in many cases the CKKW values are significantly lower.

6. Conclusions

A modified version of the CKKW algorithm has been implemented in *Herwig++* for the process $e^+e^- \rightarrow \text{hadrons}$. The modified algorithm uses truncated showers in order to provide smooth merging between the *Herwig++* angular-ordered parton shower and a set of transverse-momentum-ordered emissions defined by inverting the *Herwig++* momentum reconstruction procedure on a samples of parton momenta generated according to exact tree-level matrix elements.

The truncated shower was found to result in a smooth merging between parton shower and matrix element regions of phase space with parton level distributions appearing free

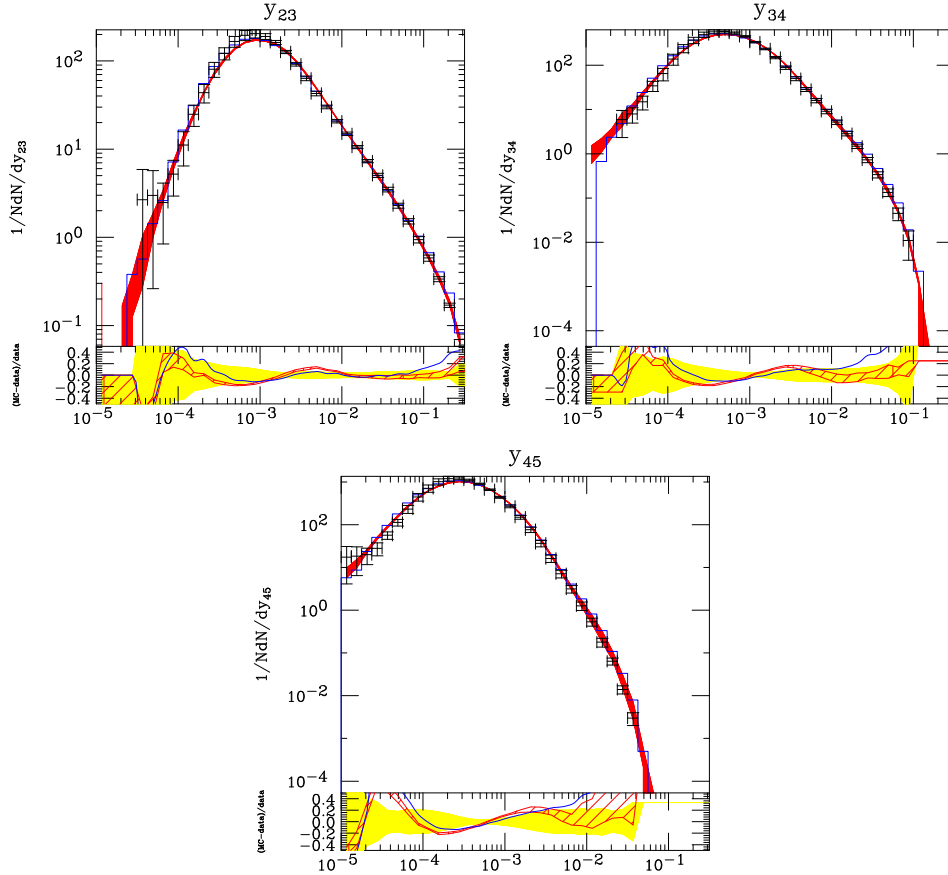


Figure 9: Distributions of the scale at which three, four and five jets are resolved in the Durham jet measure for $e^+e^- \rightarrow \text{hadrons}$ at a centre-of-mass energy of $\sqrt{s} = 91.2 \text{ GeV}$ in comparison to LEP data [33]. The colours of the lines are the same as those in Fig. 8.

of discontinuities around the merging scale and relatively insensitive to changes in the merging scale.

A full tune of the `Herwig++` free parameters was performed for the CKKW implementation with a merging scale of $y_{MS} = 10^{-2}$ in the Durham jet measure. This was found to give a good description of LEP data, demonstrating a significant improvement over the results from the default `Herwig++` parton shower with matrix element corrections applied.

The results show a comparable level of merging scale dependence and agreement with LEP data to that found in Ref. [20], in which a similar CKKW merging approach was performed with a transverse-momentum-ordered dipole shower.

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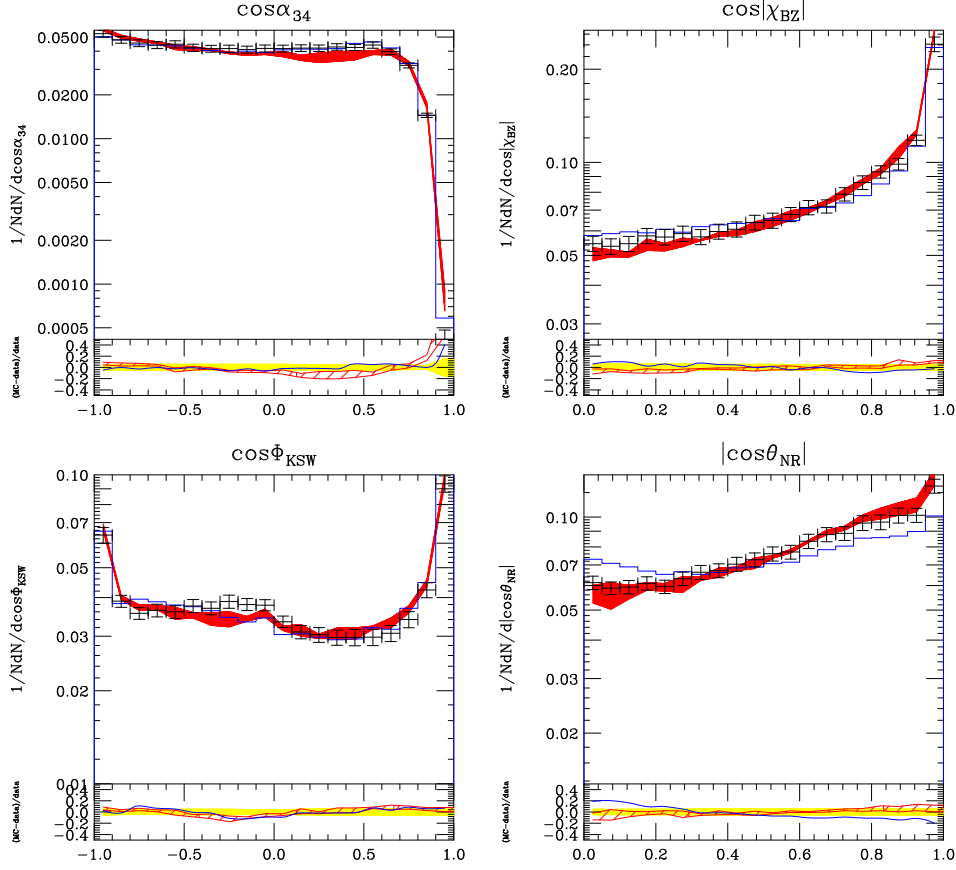


Figure 10: Distributions of four-jet angles for $e^+e^- \rightarrow \text{hadrons}$ at a centre-of-mass energy of $\sqrt{s} = 91.2 \text{ GeV}$ in comparison to LEP data [34]. Figures (a)-(d) give the angle between the lowest energy jets α_{34} , the Bengtsson-Zerwas angle [29] χ_{BZ} , the Korner-Sielsholtz-Willrodt [31] Φ_{KSW} and the Nachtmann-Reiter angle [30] θ_{NR} . The colours of the lines are the same as those in Fig. 8.

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Observable	Hw+ME $\chi^2/\text{d.o.f}$	CKKW $\chi^2/\text{d.o.f}$
Thrust	23.48	10.62
Sphericity	5.638	0.580
Oblateness	2.450	0.339
Planarity	1.249	1.211
y_{23}	2.400	0.867
y_{34}	1.887	1.026
y_{45}	4.571	2.018
$\cos \alpha_{34}$	0.569	3.301
$\cos \chi_{BZ}$	1.002	0.775
$\cos \Phi_{KSW}$	1.469	1.337
$\cos \theta_{NR}$	4.509	0.702

Table 2: A comparison of the χ^2 per degree of freedom for event shape observables in $e^+e^- \rightarrow \text{hadrons}$ with default Herwig++, with matrix element corrections, and the CKKW implementation, with merging scale set to $y_{MS} = 10^{-2}$ in the Durham jet measure.

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