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Bayesian Analysis of Lamb Survival Using Monte Carlo Numerical Integration with Importance Sampling

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ABSTRACT: Approximate and exact Bayesian analyses of survival from birth to weaning measured as an "all or none" trait were conducted on 2,554 Rambouillet lambs using an asymptotic normal approximation and Monte Carlo numerical integration with importance sampling, respectively. A linear logistic model was used to assess the effects of year, age of dam, sex of lamb, and type of birth on the survival probability. A least squares analysis of the data, ignoring their discrete nature, was also performed. The Bayesian analyses were compared by plotting the marginal posterior distributions and by constructing 95% highest-posterior-density regions for some parameters of interest. The analyses were repeated for a reduced data set consisting of 300 observations selected at random from the original file. For all practical purposes, the Bayesian and non-Bayesian analyses yielded identical results despite their different interpretations. Also, the asymptotic normal approximations to the true posterior distributions were excellent. Undoubtedly, this is because the likelihood functions contained a large amount of information about the parameters. Four-year-old ewes produced lambs with greater survival rates than either younger or older ewes. Female and male lambs had similar rates, and single-born lambs had a 10% higher survival rate than multiple-born lambs.

Key Words: Sheep, Lamb Survival, Bayesian Theory, Monte Carlo Method, Least Squares

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Introduction

The best predictor, or conditional mean of breeding values given the data, maximizes expected genetic progress after one round of selection among all possible selection criteria when a fixed number of animals is selected, and it has minimum mean squared error of prediction (Cochran, 1951; Henderson, 1973; Fernando and Gianola, 1986). Finding the conditional mean requires knowledge of location and dispersion parameters that should be viewed as nuisance parameters. Classical statistical theory fails to deal adequately with these nuisances when their values are unknown (Box and Tiao, 1973; Gianola and Fernando, 1986).

Bayesians, however, have an unambiguous solution. Finding the best predictor entails computation of the posterior mean which, under most models, requires high-dimensional integration with respect to the nuisance parameters. So far, animal breeding applications of the Bayesian approach (e.g., Gianola and Foulley, 1982, 1983; Höschele, 1986) have employed normal approximations; this is so because of the difficulty in carrying out integrations analytically. Foulley (1987) and Cantet et al. (1992) discussed limitations of these approximations and suggested numerical integration (Kloek and van Dijk, 1978; Zellner and Rossi, 1984) to test their value. With advances in computing, Monte Carlo integration with importance sampling (Hammersley and Handscomb, 1979) is now feasible, so exact posterior distributions can be found in some models.

These techniques were applied to study survival from birth to weaning in Rambouillet lambs. The main objectives were 1) to construct the exact posterior distribution of parameters of a logistic model for survival using importance sampling, 2) to compare these results with those obtained with a normal approximation, 3) to assess effects of sample size on the quality of the approximation, and 4) to contrast results of a Bayesian analysis with those of a least squares analysis of binomial data.

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Materials and Methods

Data. Records were from lambs born from 1982 to 1991 in the University of Illinois Rambouillet flock maintained at the Dixon Springs Agricultural Center (latitude 37°26′N; longitude 88°40′W). Each year, lambing was in March and April. At lambing, ewes and lambs were separated from other ewes and placed in individual pens for a period of 2 to 3 d. Shortly after birth, lambs' navel cords were disinfected with an iodine solution. Lambs were identified individually and birth date, sex, birth weight, and type of birth were recorded. Starting at approximately 7 d of age, lambs had ad libitum access to a creep diet containing 17% CP. To prevent enterotoxemia, lambs were vaccinated subcutaneously at approximately 30 d of age using a commercial vaccine. Lambs were weaned at 56 ± 4 d of age. Other information, such as age of dam, was obtained from records maintained on the Rambouillet flock. Additional information on this flock has been reported by Waldron and Thomas (1992).

Lamb survival was defined from birth to weaning and was scored as 0 if the lamb died during this period, or 1 otherwise. Data were available on 2,554 lambs and the average survival rate was 81.9%. Because only a few lambs were born as triplet, only two types of birth classes were defined, single and multiple. Age of dam was age at lambing in years and the five classes represented were 2, 3, 4, 5, and ≥ 6. The number of observations and survival rate by year, sex, type of birth, and age of dam at lambing are presented in Table 1.

Models. First, a fixed linear model was used to screen effects to be included in the final models. A least squares analysis of binomial records was carried out with the GLM procedure of SAS (1985). The model included fixed effects of year of birth (10 levels), age of dam (five levels), sex of lamb (two levels), type of birth (two levels), and all possible two-factor interactions. The only significant ($P < .01$) effects were the main effects of year of birth, age of dam, and type of birth. Therefore, it was concluded that a main-effects model was adequate. Although not significant, the effect of sex was retained for the logistic model. Because correlations between records of related animals were ignored in this study and observations were discrete, these significance tests are only approximate, and estimates of parameters are not fully efficient. However, because of the low heritabilities and repeatabilities of lamb survival (Shelton and Menzies, 1970; Long et al., 1989; Bunge et al., 1990), it is unlikely that the degree of distortion caused by ignoring random effects is high.

Data were also modeled as outcomes of a sequence of independent Bernoulli random variables ($y_i = 0$ if the lamb died and $y_i = 1$ otherwise, for lamb i) with success probabilities P_{i1} (the survival probability of lamb i), which depended on the above effects. The data were arranged into an s × 2 contingency table where the s rows represented combinations of levels of explanatory variables included in the model (i.e., year of birth-age of dam-sex of lamb-type of birth subclasses). In the i^{th} row (i = 1, 2, . . . , s) there are $n_{i.}$ observations, each of which can be assigned to one of two mutually exclusive and exhaustive categories of response: survival or death. If n_{i1} is the number of lambs surviving, there are $n_{i.} - n_{i1}$ lambs dying. Using a logistic model, the probabilities of survival and death for row i are expressed as follows:

$$P_{i1} = 1/(1 + e^{-\mu_i}) \text{ and } P_{i2} = e^{-\mu_i}/(1 + e^{-\mu_i}) = 1 - P_{i1} \quad [1]$$

The expression $\mu_i = \ln[P_{i1}/(1 - P_{i1})]$ is called a logit, which can take values between plus and minus infinity. The logit μ_i was related to the parameters of interest using the linear model

$$\{\mu_i\} = \mathbf{X}\beta \quad [2]$$

with

$$\mu_i = Y_j + A_k + S_l + T_m \quad [3]$$

and Y_j = effect of year of birth j (j = 1, 2, . . . , 10; with 1 = 1982, 2 = 1983, . . . , 10 = 1991), A_k = effect of the age of dam k (k = 1, 2, . . . , 5; with 1 = 2 yr old, 2 = 3 yr old, etc.), S_l = effect of sex of lamb l (l = 1, 2 for males and females, respectively), and T_m = effect of the type of birth m (m = 1, 2 for singles and multiples, respectively).

The parameterization in Equation [3] does not produce a full-rank incidence matrix, because **X** has 19 columns, but only 16 are linearly independent. A full rank parameterization was obtained by deleting columns pertaining to A_3, S_2, and T_2 in **X**. The parameter vector β of order 16 × 1 was then as follows:

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \cdot \\ \cdot \\ \cdot \\ \beta_{10} \\ \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \beta_{15} \\ \beta_{16} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_1 + \mathbf{A}_3 + \mathbf{S}_2 + \mathbf{T}_2 \\ \mathbf{Y}_2 + \mathbf{A}_3 + \mathbf{S}_2 + \mathbf{T}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{Y}_{10} + \mathbf{A}_3 + \mathbf{S}_2 + \mathbf{T}_2 \\ \mathbf{A}_1 - \mathbf{A}_3 \\ \mathbf{A}_2 - \mathbf{A}_3 \\ \mathbf{A}_4 - \mathbf{A}_3 \\ \mathbf{A}_5 - \mathbf{A}_3 \\ \mathbf{S}_1 - \mathbf{S}_2 \\ \mathbf{T}_1 - \mathbf{T}_2 \end{bmatrix} \quad [4]$$

With this choice, the parameters indicated as Y_1, Y_2, . . . , Y_{10} pertain to year effects on the logistic scale of male lambs, born as singles to 4-yr-old ewes. The age of dam parameters $A_k - A_3$ (k = 1, 2, 4, 5) are now effects of different age classes contrasted with the third age class (4-yr-old dams). Similarly, the parameter for sex of lamb corresponds to the difference

Table 1. Number of observations and survival rate (%) by year, sex of lamb, type of birth, and age of dam at lambing

Year	Total no. of lambs	Sex of lamb		Type of birth		Age of dam, yr				
		Male	Female	Single	Multiple	2	3	4	5	6+
1982	79 (79.8)[a]	35 (77.1)	44 (81.8)	29 (93.1)	50 (72.0)	18 (77.8)	21 (71.4)	29 (86.2)	3 (100.0)	8 (75.0)
1983	251 (80.1)	142 (73.9)	109 (88.0)	147 (87.8)	104 (69.2)	187 (79.1)	22 (81.8)	11 (72.7)	24 (91.7)	7 (71.4)
1984	194 (81.4)	94 (79.8)	100 (83.0)	102 (85.3)	92 (77.2)	26 (69.2)	131 (83.2)	16 (87.5)	5 (100.0)	16 (75.0)
1985	287 (82.9)	146 (81.5)	141 (84.4)	74 (85.1)	213 (82.1)	73 (80.8)	40 (85.0)	145 (84.8)	11 (81.8)	18 (72.2)
1986	284 (85.9)	143 (88.1)	141 (83.7)	94 (89.4)	190 (84.2)	59 (81.4)	61 (90.2)	28 (89.3)	113 (86.7)	23 (78.3)
1987	272 (84.6)	149 (83.9)	123 (85.4)	92 (88.0)	180 (82.8)	51 (82.4)	56 (82.1)	48 (89.6)	22 (90.9)	95 (83.2)
1988	297 (83.5)	155 (83.2)	142 (83.8)	111 (86.5)	186 (81.7)	61 (83.6)	77 (79.2)	54 (88.9)	43 (83.7)	62 (83.9)
1989	329 (84.2)	176 (86.4)	153 (81.7)	100 (90.0)	229 (81.7)	70 (82.7)	53 (84.9)	91 (90.1)	37 (81.1)	78 (79.5)
1990	279 (78.9)	140 (78.6)	139 (79.1)	117 (86.3)	162 (73.5)	54 (79.6)	58 (81.0)	41 (78.0)	66 (78.8)	60 (76.7)
1991	282 (75.2)	146 (78.8)	136 (71.3)	97 (84.5)	185 (70.3)	61 (68.9)	60 (78.3)	74 (86.5)	31 (61.3)	56 (71.4)
Totals	2,554 (81.9)	1,326 (81.7)	1,228 (82.1)	963 (87.2)	1,591 (78.6)	660 (79.2)	579 (82.4)	537 (86.4)	355 (82.8)	423 (78.7)

[a]Numbers in parentheses represent percentage of survival.

Table 2. Approximate and exact posterior means and standard errors, and 95% highest-posterior-density regions (HPD) for age of dam (A), sex (S), and type of birth (T) contrasts given in the logistic scale using two different sample sizes

Parameter	n = 2,554				n = 300			
	Mean ± SE		95% HPD		Mean ± SE		95% HPD	
	Approximate	Exact	Approximate	Exact	Approximate	Exact	Approximate	Exact
$A_1 - A_3$	−.815 ± .173	−.811 ± .174	[−1.155; −.475]	[−1.172; −.467]	−.231 ± .479	−.236 ± .515	[−1.171; .708]	[−1.193; .746]
$A_2 - A_3$	−.436 ± .177	−.430 ± .178	[−.784; −.089]	[−.805; −.082]	−.336 ± .492	−.357 ± .509	[−1.301; .629]	[−1.314; .634]
$A_4 - A_3$	−.364 ± .201	−.355 ± .203	[−.758; .031]	[−.767; .057]	−.513 ± .517	−.540 ± .548	[−1.526; .500]	[−1.523; .537]
$A_5 - A_3$	−.615 ± .181	−.612 ± .183	[−.971; −.260]	[−.963; −.260]	−.120 ± .529	−.108 ± .546	[−1.157; .916]	[−1.205; .977]
$S_1 - S_2$	.037 ± .105	.038 ± .105	[−.168; .243]	[−.171; .241]	−.115 ± .312	−.130 ± .329	[−.727; .496]	[−.775; .541]
$T_1 - T_2$	−.807 ± .123	−.811 ± .125	[−1.047; −.566]	[−1.060; −.581]	−.515 ± .345	−.552 ± .355	[−1.192; .162]	[−1.236; .105]

between males and females, whereas that for type of birth pertains to the difference between single- and multiple-born lambs.

Inferences. In Bayesian analysis (Box and Tiao, 1973), the density function of the observations or likelihood, $p(\mathbf{y}|\beta)$, is combined with the prior density of the parameters, $p(\beta)$, to yield the following posterior density:

$$p(\beta|\mathbf{y}) \propto p(\mathbf{y}|\beta) \quad p(\beta) \qquad [5]$$

Note that Equation [5] is a 16-variate posterior density.

As noted earlier, conditionally on β, the observations $\mathbf{y}$ were assumed independent, following a product binomial distribution. Thus, the likelihood function can be written as follows:

$$p(\mathbf{y}|\beta) \propto \prod_{i=1}^{s} P_{i1}^{n_{i1}}(1 - P_{i1})^{n_i - n_{i1}} \qquad [6]$$

In this study, it was assumed prior knowledge about β was vague, and the uniform prior suggested by Box and Tiao (1973) was as follows:

$$p(\beta) \propto \text{Constant} \qquad [7]$$

Therefore, the posterior distribution was proportional to the likelihood function. The log of the posterior distribution, $L(\beta)$, was as follows:

$$L(\beta) = \sum_{i=1}^{s} n_{i1}\ln(P_{i1}) + (n_{i.} - n_{i1}) \ln (1 - P_{i1}) + \text{Constant} \qquad [8]$$

In this study, the marginal posterior distributions of certain parameters (β_{11} to β_{16} in Equation [4]) were of interest, and for these parameters 95% highest-posterior-density (**HPD**) regions were constructed. To study the effect of sample size, Bayesian analyses were carried out both with the full (n = 2,554) and with a reduced data set (n = 300) randomly chosen from the full data set.

Two different approaches were used in the Bayesian analysis. In the first one, the log-posterior $L(\beta)$ in Equation [8], was expanded in a second-order Taylor series about its mode (Lindley, 1965). This corresponded to using a multivariate normal approximation, instead of the exact posterior. The mean vector of this normal distribution is equal to the posterior mode, and the covariance matrix is equal to the inverse of the negative Hessian of $L(\beta)$, evaluated at the mode. Because the approximate joint posterior is multivariate normal, all approximate marginal posterior distributions of β_i (i = 1, 2, . . . , 16) are also normal. The approximate Bayesian analysis was then carried out using normal distribution theory. The approximate marginal 95% HPD regions are given as follows:

$$[\hat{\beta}_i - 1.96\ s_i;\ \hat{\beta}_i + 1.96\ s_i] \quad i = 1, 2, \ldots, 16 \qquad [9]$$

where the $\hat{\beta}_i$ are the components of the posterior mode, and the s_i are the square roots of the diagonal elements of the covariance matrix. It is important to point out that this Bayesian analysis is similar, mechanically, to maximum likelihood, but the interpretation of results is different. In maximum likelihood, the distribution of the estimator is approximated by a multivariate normal distribution with estimated mean $\hat{\beta}$ and estimated covariance matrix as before, and confidence statements are made about β, which is assumed fixed but unknown. In the Bayesian analysis, one thinks of β as having a probability distribution, and makes probability statements about it. In our case, one arrives at the same inferences because, as noted in Equation [8], the log posterior density is strictly proportional to the log likelihood.

To compute the posterior mode, $L(\beta)$ was differentiated with respect to β and the vector of first derivatives was equated to zero. The resulting system of equations was not explicit in β, so the Newton-Raphson algorithm (Dählquist and Björck, 1974) was used to arrive at $\hat{\beta}$ and the posterior covariance matrix. According to Judge et al. (1982), properties of $L(\beta)$ for the logistic cumulative distribution function guarantee that this procedure converges to the global maximum for any starting vector. As shown by Gianola and Foulley (1982, 1983), the negative Hessian resembles a coefficient matrix in an iterative reweighted least squares analysis.

In the second approach, importance sampling, a Monte Carlo method, was used to conduct a Bayesian analysis based on the exact marginal posterior distributions; see Hammersley and Handscomb (1964) and Geweke (1989) for details. Briefly, this is a numerical integration technique that consists of generating independently and identically distributed random vectors $\beta^{(i)}$ (i = 1, 2, . . . , m) from a probability density function $I(\beta)$, the importance density, which is chosen to match closely the posterior density $p(\beta|\mathbf{y})$. Then, weights, $w_i = p[\beta^{(i)}|\mathbf{y}]/I[\beta^{(i)}]$, are calculated. If $g(\beta)$ is a function of β whose posterior expectation,

$$E[g(\beta)] = \int g(\beta) \quad p(\beta|\mathbf{y})d\beta \qquad [10]$$

we want to approximate, we can use the quantity

$$\frac{1}{m} \sum_{i=1}^{m} g[\beta^{(i)}] \ w_i \qquad [11]$$

as an unbiased estimator of the integral in Equation [10]. Marginal posterior densities are estimated from the importance samples as described later.

After some experimentation, a multivariate-t density with 6 df, centered at the posterior mode and with covariance matrix equal to the observed information matrix, was used as importance density, and sample size was m = 20,000. However, "effective sample size" as defined by Ritter (1992) was much lower. To explain, consider the weights

$$w_i = p[\beta^{(i)} | \mathbf{y}]/I[\beta^{(i)}], \quad i = 1, 2, \ldots, m. \quad [12]$$

If the tails of the importance density $I(\beta)$ decline faster than the tails of $p(\beta | \mathbf{y})$, some of the weights can be extremely large. This implies that a few weights will dominate the rest in this analysis, thus reducing the effective size of the sample. Ritter (1992) introduced the concept of "effective sample size", m_{eff}, as follows:

$$m_{eff} = \max \left\{ w_i / \sum_{j=1}^{m} w_j \right\}^{-1} \quad i = 1, 2, \ldots, m \quad [13]$$

Clearly, if the importance density is identical to the posterior, then m_{eff} = m. For both the full and the

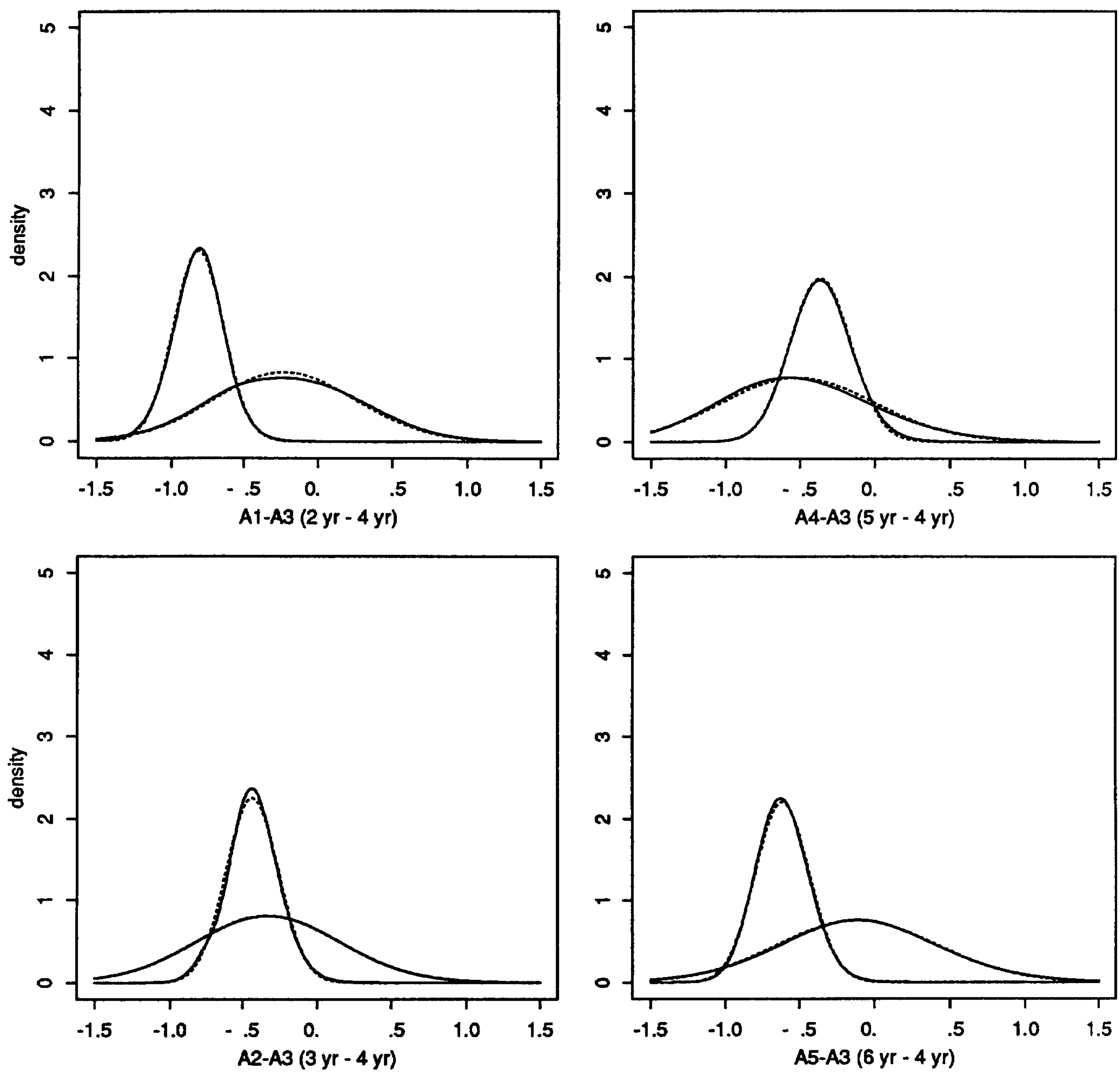

Figure 1. Plots of marginal posterior densities of age-of-dam contrasts using the full and reduced data sets in the logistic scale (solid line for exact distribution; broken line for approximate distribution; smaller dispersion for n = 2,554; larger dispersion for n = 300).

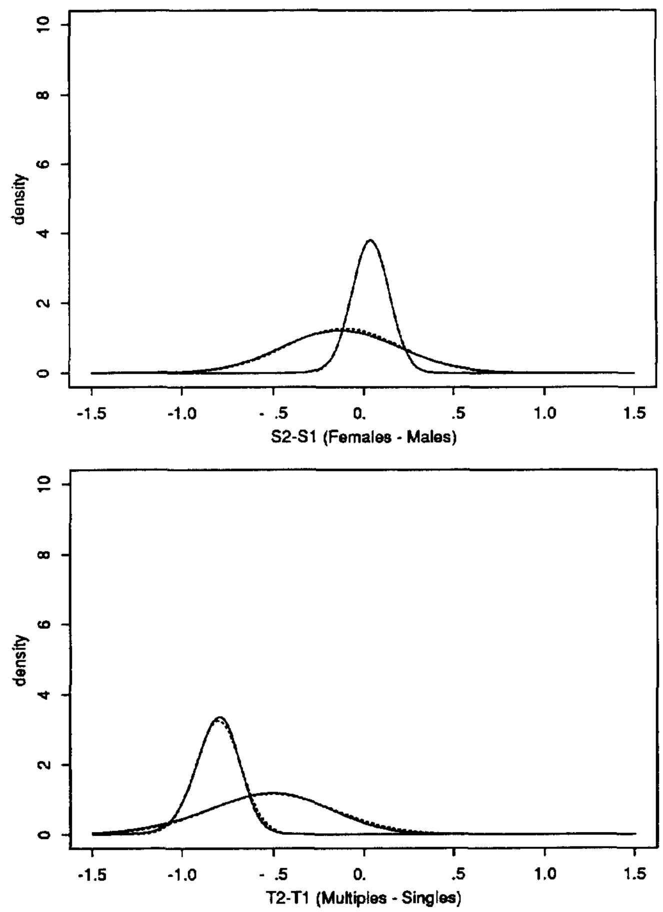

Figure 2. Plots of marginal posterior densities for sex (top) and type of birth (bottom) contrasts using the full and reduced data sets in the logistic scale (solid line for exact distribution; broken line for approximate distribution; smaller dispersion for n = 2,554; larger dispersion for n = 300).

reduced data sets, we observed approximately 2,000 effective observations in samples of 20,000.

To fit the marginal densities and to compute 95% HPD regions from the importance sample, 2,000 parameter vectors were randomly selected from the importance sample of 20,000 using the importance weights as the sampling probabilities. A log-spline density estimation program (Kooperberg and Stone, 1991) was used to fit the marginal densities over fine grids (step size = .001). From these density approximations, 95% HPD regions were obtained by selecting the smallest region of highest density values that generated ≥ 95% of the integral over a sufficiently large interval (−2.5, + 2.5) on the logit scale.

The importance sampling was carried out using the S language (Becker et al., 1988) in conjunction with a collection of FORTRAN subroutines.

Results and Discussion

Exact and Approximate Posterior Distributions and Highest-Posterior-Density Regions. Plots of the approximate and exact marginal posterior distributions of the age of dam, sex of lamb, and type of birth contrasts obtained with both data sets are depicted in Figures 1 and 2. Approximate and exact posterior means (as estimated by the importance sampling) and posterior standard errors for these parameters and HPD regions obtained with both sample sizes are presented in Table 2 in the logistic scale and in Table 3 in the proportional scale. For the full data set (n = 2,554), the exact and the approximate marginal posterior densities agreed almost perfectly. However, the tails of the exact densities were slightly thicker than those of the approximate densities; for n = 300, the match between the exact and approximate densities was slightly less perfect. As expected, Figures 1 and 2 also show the larger posterior dispersion (more uncertainty) in the smaller data set.

Posterior means and standard errors obtained with both Bayesian analyses were similar (Table 2). However, agreement between exact and approximate results was better for the larger sample size. In all cases, posterior standard errors obtained with the approximation understated the true posterior dispersion. This was particularly evident in the smaller data

Table 3. Approximate and exact posterior means and standard errors, and 95% highest-posterior-density regions (HPD) for age of dam (A), sex (S), and type of birth (T) contrasts given in the proportional scale using two different sample sizes

	n = 2,554				n = 300			
	Mean ± SE		95% HPD		Mean ± SE		95% HPD	
Parameter	Approximate	Exact	Approximate	Exact	Approximate	Exact	Approximate	Exact
$A_1 - A_3$	−.101 ± .015	−.101 ± .015	[−.161; −.052]	[−.165; −.051]	−.023 ± .036	−.023 ± .038	[−.165; .048]	[−.169; .050]
$A_2 - A_3$	−.047 ± .015	−.046 ± .015	[−.096; −.008]	[−.100; −.008]	−.035 ± .037	−.037 ± .038	[−.191; .044]	[−.193; .045]
$A_4 - A_3$	−.038 ± .017	−.037 ± .017	[−.092; .003]	[−.094; .005]	−.057 ± .038	−.060 ± .040	[−.239; .037]	[−.239; .039]
$A_5 - A_3$	−.017 ± .015	−.070 ± .015	[−.127; −.026]	[−.126; −.026]	−.011 ± .039	−.010 ± .040	[−.162; .058]	[−.171; .060]
$S_1 - S_2$	.003 ± .009	.003 ± .009	[−.016; .020]	[−.017; .020]	−.011 ± .025	−.012 ± .026	[−.087; .037]	[−.095; .039]
$T_1 - T_2$	−.100 ± .011	−.101 ± .011	[−.141; −.064]	[−.144; −.066]	−.057 ± .027	−.062 ± .028	[−.169; .014]	[−.177; .009]

set; however, the largest discrepancy was only 7% ($A_1 - A_3$ contrast). Examination of the 95% HPD regions reveals that both exact and approximate procedures yielded, for all practical purposes, the same results. As expected, the analysis with a smaller sample size gave wider intervals, and the overlap between exact and approximate HPD regions was not as perfect as the one obtained with the full data set. Again, the approximate analysis understated the true 95% HPD intervals. Comparisons of exact and approximate Bayesian posterior means, standard errors, and 95% HPD regions in the proportional scale (Table 3) also reveal a remarkable agreement. A good match between 95% HPD regions and 95% confidence intervals was observed.

There was remarkable agreement between results obtained with the Bayesian analysis in the proportional scale (Table 3) and those found with the standard least squares analysis (Table 4). However, conceptually, Bayesian and classical analyses have distinct interpretations. A Bayesian analysis provides probability distributions of the parameters, whereas a classical one does not. Therefore, a 95% HPD region for an unknown parameter means that the region contains the parameter with 95% probability, given the observed data sample. In contrast, a 95% confidence interval (random) means that, over repeated sampling, the interval contains the parameter 95% of the times.

In the following discussion on factors affecting lamb survival, only the results from the full data set are taken into consideration.

Age of Dam Effects. Lambs born to 4-yr-old ewes had higher survival rates than lambs born to 2-yr-old ewes ($A_1 - A_3$). The results also convey strong evidence supporting that 4-yr-old ewes produce lambs with greater survival rates than either ewes 3 yr old or ≥ 6 yr old. Note that the 95% HPD regions for these contrasts ($A_1 - A_3$, $A_2 - A_3$, and $A_5 - A_3$), both in the logistic (Table 2) and proportional scales (Table 3), do not contain zero. There is some evidence that 4-yr-old ewes produced lambs with greater survival chances than those produced by 5-yr-old ewes; however, the 95% HPD intervals did include zero. Despite their different interpretation, inferences from the least squares analysis on the 0 to 1 scale (Table 4) lead to similar conclusions.

Similar trends on the effects of age of dam on lamb survival were observed by Bunge et al. (1990) in this same Rambouillet flock for the years 1983 through 1985, by Shelton and Menzies (1970) in two Rambouillet flocks in Texas, by Hight and Jury (1970) in Romney and Border Leicester × Romney flocks, and by Dalton et al. (1980) in several New Zealand breeds. However, in a Rambouillet flock under an accelerated lambing system, Gama et al. (1991) found lower survival rates in lambs born to 3-yr-old ewes than in those from dams in other age classes. Long et al. (1989) did not find a significant age of dam effect on lamb survival in a flock composed of Suffolk and Targhee breeds and their crosses. In a study including Suffolks, Hampshires, and a synthetic breed, Hohenboken et al. (1976) did not find statistically significant effects of age of dam on lamb survival. However, lambs were weaned at an average of 136 d of age, or 80 d older than the lambs in our study.

Sex Effects. In our data set, sex had essentially no effect on lamb survival (note that the marginal posterior distribution is concentrated narrowly and almost symmetrically around zero; this sharpness indicates that the parameter was well estimated from the data). The point estimate on the proportional scale is .3% in favor of females with a 95% HPD region stretching from −1.7% to 2%. In the literature, most reports indicate a significant effect of sex on lamb survival; females had higher survival rates than males. Oltenacu and Boylan (1981) reported that females had 1.4% higher survival rate than males in a study that included Finnsheep, Suffolk, Targhee, and a synthetic breed. In another study involving several U.S. purebreds and crossbreds including the Rambouillet, Smith (1977) found survival rates that were 4.3 and 5.6% higher in pure and crossbred females than in males, respectively. Hinch et al. (1986) found significant sex effects on lamb survival from birth to 10 to 12 wk in a Romney flock. However, in data from two Booroola × Romney flocks also analyzed in their study, no sex effects on survival were found.

Type of Birth Effects. Single-born lambs clearly had higher survival rates (10 to 11%) than multiple-born lambs. The 95% HPD region stretched approximately from 6.4 to approximately 14.4%. This result is in close agreement with most reports found in the

Table 4. Least squares estimates, standard errors, and 95% confidence intervals (CI) for age of dam (A), sex (S), and type of birth (T) contrasts using two different sample sizes

	n = 2,554			n = 300		
Parameter	Estimate ± SE	95% CI	*P*-value	Estimate ± SE	95% CI	*P*-value
$A_1 - A_3$	−.112 ± .024	[−.159; −.065]	.0001	−.030 ± .068	[−.163; .103]	.6606
$A_2 - A_3$	−.058 ± .024	[−.105; −.011]	.0176	−.049 ± .072	[−.190; .092]	.4998
$A_4 - A_3$	−.047 ± .028	[−.102; .008]	.0924	−.080 ± .079	[−.235; .075]	.3141
$A_5 - A_3$	−.086 ± .026	[−.137; −.035]	.0008	−.017 ± .076	[−.166; .132]	.8237
$S_1 - S_2$	.006 ± .015	[−.023; .035]	.7077	−.018 ± .046	[−.108; .072]	.1450
$T_1 - T_2$	−.110 ± .016	[−.141; −.078]	.0001	−.071 ± .048	[−.108; .072]	.6997

literature (e.g., Hight and Jury, 1970; Dalton et al., 1980; Gama et al., 1991).

Implications

The results suggest that asymptotic normal approximations to conduct a Bayesian analysis for inferences about fixed effects can yield satisfactory results for some sample sizes frequently encountered in experimental animal breeding. With small sample sizes, for which some parameters may be poorly estimated (e.g., herd-year-season effects), the approximations would be expected to be less satisfactory. However, advances in computer power facilitate the use of numerical integration techniques, needed to implement an exact Bayesian analysis. A Bayesian analysis of survival in Rambouillet lambs revealed that single lambs born to 4-yr-old ewes had the highest survival rates, but there was no difference in survival between sexes.

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