

**ANALYSIS OF GRAVITY AND ROTATION
MEASUREMENTS USING PROBABILISTIC INFERENCE
OF PLANETARY INTERIOR FOR THE DESIGN OF
GEODESIC EXPERIMENTS: APPLICATION TO ICY
MOONS**

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To the city of Brussels.

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SUMMARY

This work addresses the internal structure characterization of outer solar system icy satellites, emphasizing the gravity science (GS) experiment design for future missions. We identify that traditional mission planning, driven by engineering constraints, is often limiting and necessitates iterative redesigns to meet scientific objectives. To overcome this, we apply a novel "reverse mapping" approach focusing on desired scientific outcomes—specifically key interior parameter uncertainties—to determine required instrument performance and trajectory requirements. Our methodology combines the well established Precise Orbit Determination (POD) and Bayesian inference techniques.

We validate this strategy through three distinct case studies. First, we validate our POD pipeline by replicating Cassini's Titan gravity field results. Second, regarding Juno's Io flybys, we highlight the limitations of designs not optimized for specific science objectives, showing that, given the nominal experiment design, the radiometric data alone are insufficient to constrain the moon's interior. Third, for the Uranus Orbiter and Probe (UOP), we demonstrate that a well-designed tour significantly enhances science return. We show that while the nominal X-band tour cannot resolve high-degree zonal harmonics ($J_6 - J_{10}$) precisely, achieving these objectives is possible with an upgraded Ka-band radio link and specific polar trajectory optimization during orbit insertion.

A primary contribution of this thesis is applying the "reverse mapping" framework to the major Uranian satellites: Miranda, Ariel, Umbriel, Titania, and Oberon. By simulating thousands of interior models, we construct "lookup tables" mapping geodetic measurement precisions—specifically Moment of Inertia, tidal Love number k_2 , libration amplitude, and obliquity—to key interior parameter uncertainties. A crucial finding is that while determining ice shell thickness and rock-to-ice mass ratio is feasible with proposed UOP architectures, detecting subsurface oceans remains challenging due to difficulty measuring the tidal Love number (k_2) with limited flyby data. For instance, we show that determining Ariel's ocean thickness with 30% uncertainty requires measurement precisions exceeding current strategies.

To address orbital remote sensing limitations, we investigate a complementary in situ approach using a 6-Degrees-of-Freedom (6DoF) motion sensor, the PIONEERS instrument. We demonstrate that a high-performance surface sensor can measure tidal and centrifugal accelerations precisely enough to distinguish differentiated from ocean-bearing interiors for the five major Uranian moons.

Overall, this research establishes a quantitative methodology for science-driven mission design, ensuring that future ocean world exploration is optimized to address fundamental questions regarding habitability and internal dynamics.

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INTRODUCTION

THE exploration of the outer solar system has prompted a paradigm shift in planetary science, redefining distant icy satellites from geologically inert bodies into dynamic ocean worlds (see e.g. [1–3]). This shift began with the reconnaissance of the Voyager missions (1979-1989), which provided the first indications of geologic complexity in the Jovian, Saturnian, Uranian, and Neptunian systems. At Jupiter, Voyager 1 images showed Europa’s young surface, evidenced by a lack of large impact craters and a complex network of linear features [1, 4, 5]. At Ganymede, Voyager 1 revealed a dichotomy between ancient, dark, heavily cratered terrain and younger, bright, grooved terrain [1, 4, 5]. At Saturn, Voyager 1 observed Titan’s thick, hazy atmosphere, composed mainly of nitrogen with methane as a minor constituent [1, 5]. Voyager 2’s subsequent encounter with Enceladus showed bright, smooth terrains indicative of recent resurfacing [1, 6]. Voyager 2 also provided the first, and to date only, close reconnaissance of the ice giants. At Uranus in 1986, it discovered new moons and dark rings, measured the planet’s highly tilted magnetic field [7], and revealed geological complexity on Miranda (e.g., large cliffs and coronae), suggesting past endogenic activity [8]. At Neptune in 1989, Voyager 2 observed the Great Dark Spot, indicative of dynamic weather systems [9]. Its flyby of Triton revealed a geologically young surface with active geyser-like plumes, providing evidence of cryovolcanism [10], possibly driven by tidal forces from its capture [9].

These initial observations were foundational for the Galileo orbiter mission at Jupiter (1995-2003) [11]. Galileo provided strong evidence for Europa’s subsurface ocean via its magnetometer investigation [12, 13]. Perturbations responding to Jupiter’s rotating magnetic field were interpreted as an induced magnetic field, requiring a global, conductive layer consistent with a salty liquid water ocean beneath the ice shell. The inferred conductivity was consistent with salinity comparable to Earth’s oceans [12, 13]. Galileo’s imaging complemented this finding, revealing diverse terrain types indicative of a dynamic surface geology [1, 14, 15]. Galileo also detected Europa’s ionosphere via radio occultation [5, 16]. At Ganymede, Galileo also detected an induced magnetic field indicative of a subsurface ocean, but it also made the discovery of an intrinsic, self-generated magnetic field [1, 17, 18]. Combined gravity and magnetic data suggested a three-layer structure for Ganymede: a metallic core, a silicate mantle, and a thick outer ice shell potentially containing

the liquid layer [2, 19]. Galileo's long-term observations also highlighted the extremes within the Jovian system. Io was confirmed as the solar system's most volcanically active body, with over 200 active volcanic centers and high eruption temperatures (1,300–2,000 K) indicative of ultramafic lavas [2, 20–22]. The mission monitored diverse features and rapid surface changes, confirming Io's high resurfacing rate [21, 23]. In contrast, Callisto showed one of the oldest (≈ 4 Gyr) and most heavily cratered surfaces [24, 25]. Its lack of global tectonics indicated minimal internal activity [26], though magnetic induction data suggested a possible subsurface ocean [12, 13]. Callisto thus serves as a benchmark for less active icy body evolution [25].

At Saturn, the Cassini mission, complemented by the Huygens probe's landing on Titan in 2005 [27], provided new insights into the system. Cassini's RADAR instrument discovered lakes and seas of liquid methane and ethane, primarily in polar regions, including Kraken Mare ($> 400,000 \text{ km}^2$) [1, 28]. These findings supported hypotheses of a methane cycle analogous to Earth's hydrological cycle [2], with observations suggesting seasonal changes [29]. Extensive equatorial dune fields of organic solids were also mapped [1]. Radio science measurements further inferred a subsurface liquid water ocean beneath Titan's icy crust [5, 30]. A key discovery of the Cassini mission was the identification of active plumes erupting from fractures (tiger stripes) near Enceladus's south pole, the primary source of Saturn's E ring [2, 31]. Direct sampling showed the plumes contain salts, complex organic compounds [32], and molecular hydrogen (H_2) [33]. The detection of H_2 and silica nanoparticles provides strong evidence for ongoing, hot ($> 50^\circ \text{ C}$) water-rock interactions at the ocean-core interface [33, 34]. Cassini gravity data also indicated a global subsurface ocean [1, 35].

The persistence of liquid water and geological activity is primarily enabled by tidal heating, where orbital energy is dissipated as heat within a satellite's interior [2, 3]. This mechanism, exemplified by the volcanism on Io [3], is driven by gravitational interactions that maintain forced eccentricity, often through mean-motion resonances (e.g., the Laplace resonance or the Enceladus-Dione 2:1 resonance) [2, 3]. In addition to tidal heating, chemical energy from water-rock interactions [33] and surface oxidants from radiolysis [36] contribute to the potential habitability of these ocean worlds.

The inferred presence of habitable environments within Europa, Ganymede, Callisto, Titan, and Enceladus, due to tidal heating and geochemical processes, has changed the understanding of habitability [37]. These satellites are now prime targets, shifting the scientific objective from discovery to characterization. Key unresolved questions focus on determining interior properties, such as ice shell thickness and structure, ocean depth and composition, and the processes at the ocean-rock interface, including potential hydrothermal activity (see e.g. [38]). Understanding ocean-surface exchange mechanisms also remains a priority.

Answering these questions requires dedicated geophysical investigation, motivating the current and next generation of outer planet missions. NASA's Europa Clipper and ESA's JUICE (JUpiter ICy moons Explorer) missions are currently en route to the Jovian system [39–41]. Europa Clipper will conduct approximately 50 close flybys to characterize Europa's habitability using its Gravity/Radio Science Investigation

(G/RSI), radar (REASON), and other instruments [40, 42]. JUICE will perform flybys of Europa and Callisto before orbiting Ganymede, using radar (RIME), altimetry (GALA), and gravity/radio science (3GM), among other instruments, to characterize the icy moons [39, 41]. NASA's Dragonfly mission to Titan is also under development [43, 44].

Future mission planning has also prioritized ocean world exploration [44, 45]. ESA's Voyage 2050 plan identified Moons of the Giant Planets as a key theme, prioritizing habitability and biosignature searches at targets like Enceladus and Titan [45]. An Enceladus orbiter and lander has been recommended for the L4 slot, with Mimas as a secondary target [46, 47]. Concurrently, the U.S. National Academies' Decadal Strategy 2023-2032 [44] prioritized the Uranus Orbiter and Probe (UOP) as its highest-priority Flagship to characterize the Uranian system, including its moons. The Decadal Survey also ranked an Enceladus Orbilander as the second-priority Flagship [44]. These plans show that characterizing icy moon interiors remains a central goal, reliant on geophysical measurements to constrain internal composition and processes.

Characterizing the interiors of ocean worlds is challenging, as these regions are inaccessible to direct observation. Our knowledge thus relies on interpreting remotely sensed geophysical data. Progress is fundamentally linked to designing missions capable of acquiring these measurements with sufficient precision.

While multiple geophysical methods exist, gravity science experiments are crucial for constraining bulk interior structure. In the conventional framework of a gravity science experiment, mission design is often an iterative process between engineering and scientific objectives. An experiment configuration is defined, and the expected scientific return is subsequently assessed. This approach can be inefficient and does not guarantee that high-priority science questions will be answered, which can be an impediment to optimizing scientific return.

This thesis presents an alternative "reverse mapping" approach to mission design. This methodology uses probabilistic inference to translate a desired scientific outcome (e.g., confirming a subsurface ocean) into specific performance requirements for the mission's instruments and observation strategy. This provides a quantitative link between the scientific goal and the mission design. We apply this approach to gravity and rotation measurements, which are powerful techniques for constraining the interior structure of planetary bodies and represent a key research area at the Royal Observatory of Belgium.

The work is organized as follows. Chapter 2 reviews the theoretical principles of gravity and rotation science, precise orbit determination, and the probabilistic methods used. Chapter 3 applies these principles in several case studies to demonstrate the conventional workflow for gravity science experiments and to highlight its limitations. In Chapter 4, we introduce the "reverse mapping" framework. We detail the development of "lookup tables" for the Uranian moons, using the upcoming Uranus Orbiter and Probe (UOP) mission as a test case. We show how this tool can be used to define requirements for a geophysical experiment and to rapidly evaluate the scientific impact of mission design trade-offs. Finally, because some parameters are difficult to resolve with orbital data, Chapter 5 investigates

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a complementary in-situ technology, assessing the potential for a 6-DoF motion sensor to provide independent constraints. In summary, this thesis develops and validates a methodology for the informed design of planetary missions, establishing a quantitative relationship between scientific goals and mission requirements to ensure future missions can effectively address their primary scientific objectives.

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2

THEORETICAL BACKGROUND

2.1. INTRODUCTION

CHARACTERIZING the interiors of celestial bodies presents a formidable challenge in planetary science, given that direct sampling is frequently impractical. Consequently, our understanding of the internal structure, composition, and dynamics of planets and their satellites must rely heavily on indirect methods. In this work, we focus on the interpretation of geophysical data acquired by remote sensing from spacecraft missions. This chapter establishes the fundamental theoretical framework for the subsequent analyses in this thesis. We will introduce the key geophysical quantities that are sensitive to interior properties, describe the primary measurement techniques used to observe these quantities, and outline the analytical and computational methodologies we employ to translate these observations into meaningful constraints on internal structure models.

Several primary geophysical observables provide crucial insights into planetary interiors, including the external gravitational field, the body's rotational dynamics, magnetic field measurements, and seismic activity (where measurable). In this thesis, we focus specifically on the interpretation of gravity and rotation data. These observables, acquired through remote sensing from spacecraft missions, have proven very effective in providing crucial constraints on the internal structure and dynamics of planetary bodies. A body's gravitational field, which arises from its total mass and internal distribution, governs the motion of nearby objects and thus reflects the internal density structure. We can leverage variations in this field, both static and time-dependent, to probe the degree of differentiation, the presence of large-scale density anomalies, and the body's deformation under external tidal forces, which reveals its internal rheology (material properties like elasticity and viscosity). In parallel, a body's rotational state—including the precise orientation of its spin axis and subtle deviations from uniform rotation like physical librations—depends on by its moments of inertia and the mechanical coupling between internal layers, which govern how it responds to external torques. These rotational variations are known to be particularly sensitive to the existence and properties of fluid layers, such as subsurface oceans or molten cores, which can effectively decouple the rotation of different interior shells.

However, the interpretation of gravity and rotation measurements to infer specific interior properties (e.g., core size, ocean depth, shell thickness, or layer densities and viscosities) does not provide direct and explicit information, and dedicated inversion techniques must be employed. The relationship between the observables and the interior structure parameters is often complex and non-unique. To address this inverse problem, we utilize probabilistic approaches, specifically Bayesian inference combined with Markov Chain Monte Carlo (MCMC) methods. This approach represents a highly effective technique, combining a probabilistic framework with the computational efficiency of MCMC algorithms to allow for the systematic exploration of a vast parameter space of interior models. This, in turn, enables the identification of models consistent with the available data and provides a comprehensive quantification of the uncertainties on the inferred parameters.

This chapter is organized as follows. In [Sec. 2.2](#), we provide an in-depth exploration of gravity science principles, including the representation of gravitational

potentials with spherical harmonics and the relationship between model parameters and interior properties. This section also details the methodology of Precise Orbit Determination (Sec. 2.2.2), covering the underlying estimation problem and specific techniques such as multi-arc solutions and covariance analysis. In Sec. 2.3, we describe the rotational dynamics of icy satellites, introducing the standard model for orientation and the theories of longitudinal librations and spin-axis obliquity, including Cassini states. We also introduce the landmark tracking technique (Sec. 2.3.3) used for measuring rotational states, detailing its measurement principle and parameter estimation via Bundle Block Adjustment. Finally, Sec. 2.4 presents the probabilistic methods for interior structure inversion, focusing on Bayesian inference and MCMC techniques that use the affine-invariant ensemble sampler, and concludes by reviewing examples of their application in previous studies.

2.2. GRAVITY SCIENCE

The study of planetary interiors relies heavily on the precise measurement and analysis of external gravitational fields. Every planetary body generates a gravity field that is a function of its mass and internal distribution, and this field dictates the trajectories of nearby objects, such as orbiting spacecraft. By precisely tracking the perturbations in a spacecraft's orbit—often through Doppler shifts in radio signals—we can map the external gravitational potential. This potential map, in turn, allows us to infer fundamental properties of the body, including its total mass, the distribution of that mass, and its overall interior structure.

2.2.1. MODELING GRAVITY

To translate observed spacecraft trajectory perturbations into a quantitative map of a planet's gravitational field, we utilize a mathematical framework centered on a spherical harmonic expansion of the external potential. This representation allows for the decomposition of the complex field into a sum of orthogonal basis functions, where each function corresponds to a different spatial scale. Importantly, this single formalism is capable of accommodating both the static and time-varying components of the gravity field. The subsequent subsections are dedicated to detailing this mathematical approach and discussing the geophysical significance of its key parameters.

2.2.1.1. STATIC GRAVITY FIELD

The external gravitational potential, U , originating from a planetary body, is classically expressed using a spherical harmonic expansion. This mathematical representation describes the potential at a point defined by radius r , latitude ϕ , and longitude λ in a body-fixed coordinate system [1, 2]:

$$U = \frac{GM}{r} + \frac{GM}{r} \sum_{l=1}^{\infty} \sum_{m=0}^n \left(\frac{R_e}{r} \right)^n \bar{P}_{lm}(\sin \phi_{lat}) [\bar{C}_{lm} \cos(m\lambda) + \bar{S}_{lm} \sin(m\lambda)] \quad (2.1)$$

where GM is the gravitational constant times the body's mass, R_e is its reference equatorial radius, $\bar{P}_{lm}$ are the fully normalized associated Legendre functions, and

$\bar{C}_{lm}$ and $\bar{S}_{lm}$ are the dimensionless, normalized spherical harmonic coefficients quantifying the field's departure from spherical symmetry.

The physical meaning of these coefficients is well-established. The degree-0 term provides the total mass, while the degree-1 coefficients are related to the offset between the coordinate system origin and the body's center of mass (CM), and are thus typically zero [1, 3–5]:

$$\begin{aligned} C_{10} &= \frac{Z_{cm}}{R_e} \\ C_{11} &= \frac{X_{cm}}{R_e} \\ S_{11} &= \frac{Y_{cm}}{R_e} \end{aligned} \quad (2.2)$$

where (X_{cm}, Y_{cm}, Z_{cm}) are the coordinates of the CM. The second-degree coefficients depend on the body's overall mass distribution and are directly linked to its principal moments of inertia (A, B, C) [1, 3–5]:

$$\begin{aligned} C_{20} &= \frac{\frac{A+B}{2} - C}{MR_e^2} \\ C_{21} &= \frac{I_{xz}}{MR_e^2} \\ S_{21} &= \frac{I_{yz}}{MR_e^2} \\ C_{22} &= \frac{B - A}{4MR_e^2} \\ S_{22} &= \frac{I_{xy}}{2MR_e^2} \end{aligned} \quad (2.3)$$

where the zonal coefficient $J_2 = -C_{20}$ is a measure of the body's oblateness. The coefficients C_{21} and S_{21} vanish if the rotation axis aligns with a principal axis; non-zero values indicate an offset, $\delta\phi^2 = (C_{21}^2 + S_{21}^2)/J_2^2$, corresponding to a polar wobble [3–5]. The period of this free Eulerian wobble (P_W) for a rigid body is related to the moments of inertia and the spin rate (ω):

$$P_W = \frac{2\pi}{\omega} \frac{C}{\sqrt{(C-A)(C-B)}} \quad (2.4)$$

Furthermore, external gravitational torques induce a precession of the spin axis, the rate of which ($\partial\psi/\partial t$) is related to J_2 and the polar moment of inertia, C [3–5]:

$$\frac{\partial\psi}{\partial t} = -\frac{3}{2} \frac{n^2}{\omega} \left(\frac{MR_e^2}{C} \right) J_2 \cos(\varepsilon) \quad (2.5)$$

where n is the mean orbital motion and ε is the obliquity. A measurement of the precession rate and J_2 thereby allows for the determination of the polar moment of inertia factor (C/MR_e^2), a key constraint on the internal density distribution.

The root mean square (RMS) gravity spectrum serves to visualize the power at each harmonic degree and evaluate the quality of the gravity field determination. It is computed from the RMS of the normalized coefficients for a given degree l [1, 6]:

$$M_l = \sqrt{\frac{\sum_{m=0}^l (\bar{C}_{lm}^2 + \bar{S}_{lm}^2)}{2l+1}}. \quad (2.6)$$

The associated error spectrum is computed identically, substituting the uncertainties of the coefficients ($\sigma_{\bar{C}_{lm}}$, $\sigma_{\bar{S}_{lm}}$). A comparison of the error spectrum (noise) to the gravity spectrum (signal) defines the resolution of the measured field, a limit referred to as the degree strength.

2.2.1.2. TIME-VARIABLE GRAVITY FIELD

Beyond the static field, time-variable gravity phenomena offer further insight into planetary interiors. External tidal forces from a parent body or the Sun cause periodic deformations, the nature of which depends on the body's internal properties, particularly its rigidity and density stratification [1, 2]. This tidal response is quantified by dimensionless Love numbers: k for the change in the external potential, h for the radial surface displacement, and l for the tangential displacement [1, 7]. Measurements of these Love numbers are used to constrain the body's internal state. The Love number k_2 in particular, which quantifies the degree-2 change in the external gravitational potential arising from tidal deformation, is sensitive to the internal mass redistribution caused by this deformation. The magnitude of this redistribution, and consequently the value of k_2 , is strongly dependent on the body's internal properties, such as its rigidity and density stratification [1]. Moreover, it is particularly sensitive to the presence and characteristics of fluid layers (e.g., subsurface oceans or liquid cores), as these significantly enhance the body's deformability (see e.g. Van Hoolst, Baland, and Trinh [8]). The same principle applies to gravitational Love numbers of any degree n and order m (k_{nm}), as they quantify the gravitational response to the corresponding components of the tidal forcing potential. While k_2 reflects the global response, higher-degree Love numbers (k_n for $n > 2$) are sensitive to shorter-wavelength deformations, thereby providing complementary constraints that often probe shallower structures or regional variations within the body [9, 10]. Measuring Love numbers across multiple degrees, consequently, allows for a more detailed characterization of features such as an ocean's presence or the core's state. These periodic changes to the potential, reflected in the normalized spherical harmonic coefficients ($\Delta\bar{C}_{nm}$ and $\Delta\bar{S}_{nm}$), are expressed as (e.g., as used by the International Earth Rotation and Reference Systems Service [9, 10]):

$$\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm} = \frac{k_{nm}}{2n+1} \frac{M_j}{M} \left(\frac{R}{r_j}\right)^{n+1} \bar{P}_{nm}(\sin\phi_j) e^{-im\lambda_j} \quad (2.7)$$

where k_{nm} are the gravitational Love numbers of degree n and order m ; M_j is the mass of the perturbing primary; and r_j , λ_j , and ϕ_j are the body-fixed distance, longitude, and latitude of the primary, respectively. Since the total gravitational force

is a primary driver of a nearby spacecraft's dynamics, determining the precise values of both static and time-variable gravity parameters requires a detailed analysis of the spacecraft's trajectory via Precise Orbit Determination.

2

2.2.2. PRECISE ORBIT DETERMINATION FOR PLANETARY SCIENCE

Precise Orbit Determination (POD) represents the foundational technique for radio science experiments, offering an effective means of probing the interior structure of planetary bodies. By precisely tracking a spacecraft's trajectory through a body's gravitational field, subtle perturbations in its motion reveal characteristics of the internal mass distribution, as discussed in Sec. 2.2. The reconstruction of the spacecraft's dynamics is enabled by radiometric measurements acquired from dedicated instrumentation. The fundamental principle of POD involves solving an inverse problem: determining the dynamical model whose propagated trajectory best reproduces the observations. This process allows for the inference of the target body's gravity field and, consequently, for placing constraints on its internal composition. This technique has proven particularly valuable for investigating icy satellites, where gravity data, when combined with shape models from imaging or altimetry, can constrain the thickness of subsurface oceans or distinguish between solid and liquid core states (see e.g. [11–15]).

2.2.2.1. MEASUREMENT SYSTEMS AND OBSERVABLES

The acquisition of high-precision radiometric data necessitates specialized hardware both on the spacecraft and at ground-based facilities. A spacecraft payload typically comprises a telecommunications subsystem with a transponder for receiving uplink signals and coherently retransmitting them on the downlink, often through a high-gain antenna to maximize the signal-to-noise ratio (SNR). On Earth, networks of deep space tracking stations employ large, steerable antennas and highly sensitive receivers to detect these faint signals across vast interplanetary distances. These ground stations are equipped with high-stability frequency standards, such as atomic clocks, which enable the detection of the minute frequency shifts that are indicative of spacecraft motion.

The fundamental radiometric observables in POD are derived from the properties of electromagnetic signals exchanged between a ground station and the spacecraft. The ideal instantaneous geometric distance, or range ρ , between the spacecraft at position r and the instrument at r_I can be expressed as [16]:

$$\rho(t) = [(r - r_I) \cdot (r - r_I)]^{1/2} \quad (2.8)$$

This equation provides the instantaneous geometric distance, neglecting the finite time required for signal propagation between the instrument and the spacecraft. The measurement process involves transmitting a signal, which can be modeled as a sinusoidal tone [16]:

$$s(t) = \text{Re}[A_0 \exp j(\omega_0 t + \phi_0)] \quad (2.9)$$

where A_0 is the amplitude, ω_0 is the frequency, and ϕ_0 is the initial phase. Upon reception at a ground station (GS) from the spacecraft (SC), after a propagation time

τ_{GS} , the signal's form reflects the associated delay, attenuation, and phase shift [16]:

$$s_{GS}(t) = R_e[A_{GS} \exp j(\omega_0 t + \phi_0 - k \cdot (r_{GS}(t) - r_{SC}(t - \tau_{GS})))] \quad (2.10)$$

where k is the wavevector and the term $r_{GS}(t) - r_{SC}(t - \tau_{GS})$ represents the signal path vector. The phase of this received signal is therefore:

$$\Phi(t) = \omega_0 t + \phi_0 - k \cdot (r_{GS}(t) - r_{SC}(t - \tau_{GS})). \quad (2.11)$$

The instantaneous received frequency, $f(t)$, is then obtained by differentiating the phase with respect to time [16]:

$$f(t) = \frac{1}{2\pi} \frac{d\Phi(t)}{dt} = f_0 - \frac{1}{2\pi} k \cdot \frac{d}{dt}(r_{GS}(t) - r_{SC}(t - \tau_{GS})) \quad (2.12)$$

Assuming the wavevector k is constant and collinear with the signal path, and with its magnitude $\|k\| = \omega_0/c = 2\pi f_0/c$ (where c is the speed of light), the dot product $k \cdot \frac{d}{dt}(r_{GS}(t) - r_{SC}(t - \tau_{GS}))$ simplifies. Since k is collinear with the path vector, the dot product becomes the product of their magnitudes multiplied by the time derivative operator acting on the path length. Recognizing that the time derivative of the path length $\rho(t)$ is the range-rate $\frac{d\rho(t)}{dt}$, and substituting $\|k\| = 2\pi f_0/c$, the second term in Eq. (2.12) becomes $\frac{1}{2\pi} (2\pi f_0/c) \frac{d\rho(t)}{dt} = \frac{f_0}{c} \frac{d\rho(t)}{dt}$. This leads to the simplified expression for the received frequency [16]:

$$f(t) = f_0 - \frac{f_0}{c} \frac{d\rho(t)}{dt} \quad (2.13)$$

This relationship demonstrates that the frequency shift, $\Delta f(t) = f(t) - f_0$, is directly proportional to the range-rate $\dot{\rho}(t) = \frac{d\rho(t)}{dt}$, which is the line-of-sight velocity [16]:

$$\frac{\Delta f(t)}{f_0} = -\frac{\dot{\rho}(t)}{c} \quad (2.14)$$

This frequency shift forms the basis of the Doppler observable. The integration of the measured frequency shift over a given time interval $[t_0, t_1]$ allows for the determination of the change in range, $\Delta\rho$, during that period [16]:

$$\Delta\rho(t_0, t_1) = \int_{t_0}^{t_1} \dot{\rho} d\tau = -\frac{c}{f_0} \int_{t_0}^{t_1} \Delta f d\tau. \quad (2.15)$$

Although integrated Doppler data yield the change in range, determining the absolute range requires resolving an inherent ambiguity in the phase measurement. This is conceptually equivalent to counting the total number of wavelengths separating the transmitter and receiver. In practice, this ambiguity is resolved by modulating ranging codes onto the carrier signal [16].

2.2.2.2. NOISE, ERROR MITIGATION, AND CALIBRATION

The precision of orbit determination is ultimately constrained by a range of noise sources and systematic errors intrinsic to the measurement process [2, 16]. These impediments encompass instrumental noise originating from thermal fluctuations within spacecraft transponders and ground receivers, phase instabilities in radio frequency electronics, and mechanical antenna vibrations. In addition to instrumental effects, signal propagation through neutral media, notably Earth's troposphere, introduces path delays, while propagation through charged media such as the ionosphere and interplanetary plasma results in frequency-dependent delays and signal bending. Furthermore, unmodeled or inadequately modeled spacecraft-specific forces, including outgassing, thermal radiation imbalances, and uncertainties in antenna phase center locations, can generate spurious accelerations that contaminate the gravitational signal of interest [2, 16].

Consequently, accurate trajectory reconstruction necessitates the precise modeling, estimation, and calibration of these error sources. The calibration of charged-particle effects, for instance, is accomplished using multi-frequency links, whereas tropospheric delays are often corrected using data from ground-based meteorological sensors or dedicated instruments like water vapor radiometers. These calibrated measurements are subsequently processed alongside comprehensive dynamical models that account for all known forces, including gravitational perturbations from celestial bodies, solar radiation pressure, thermal forces, and atmospheric drag where applicable. Advanced filtering and estimation techniques are then employed to isolate the gravitational signatures of interest from the remaining noise and unmodeled effects [2, 16].

2.2.2.3. THE ORBIT DETERMINATION ESTIMATION PROBLEM

Fundamentally, precise orbit determination (POD) seeks to estimate a spacecraft's state—typically its position and velocity, along with other relevant model parameters—by optimally fitting a dynamical model to a set of noisy measurements [2, 16]. The process involves relating observations, which are often indirect measurements of the state, to an underlying, generally non-linear, physical model that governs the system's evolution.

The spacecraft's motion is described by a system of first-order differential equations of the form [16]:

$$\dot{X} = F(X, t), \quad X(t_k) \equiv X_k \quad (2.16)$$

where X is the n -dimensional state vector at time t , which contains position and velocity, and can be augmented to include parameters such as the gravity coefficients C_{nm} , S_{nm} or the Love numbers k_{nm} previously discussed. The function F represents the vector field defined by the equations of motion, incorporating all relevant gravitational and non-gravitational forces. The available measurements, Y_i , acquired at discrete times t_i , relate to the state through a measurement model G (e.g., Eq. (2.15)) [16]:

$$Y_i = G(X_i, t_i) + \varepsilon_i, \quad i = 1, \dots, l \quad (2.17)$$

where Y_i is a p -dimensional vector of all observation types at a single epoch t_i and ε_i represents the measurement error, which is typically assumed to be zero-mean random noise.

Given the non-linearity of both the dynamical (F) and measurement (G) models, the estimation problem is solved iteratively by linearizing the system around a nominal reference trajectory, $X^*(t)$. Let $x(t) = X(t) - X^*(t)$ denote the deviation of the true state from the reference state, and let $y_i = Y_i - G(X_i^*, t_i)$ be the observation residual. By neglecting higher-order terms, the linearized dynamics and measurement equations are [16]:

$$\begin{aligned}\dot{x}(t) &= A(t)x(t) \\ y_i &= \tilde{H}_i x_i + \varepsilon_i\end{aligned}\tag{2.18}$$

where $A(t) = \left[\frac{\partial F(t)}{\partial X(t)} \right]^*$ is the dynamics matrix and $\tilde{H}_i = \left[\frac{\partial G}{\partial X} \right]_i^*$ is the measurement sensitivity matrix, with both evaluated along the reference trajectory X^* .

To relate all measurements to the state deviation x_k at a single reference epoch t_k , the state transition matrix (STM), $\Phi(t_i, t_k)$, is introduced. It maps state deviations from t_k to any other time t_i via the relation $x(t_i) = \Phi(t_i, t_k)x(t_k)$. The measurement equations can then be reformulated as [16]:

$$y_i = \tilde{H}_i \Phi(t_i, t_k) x_k + \varepsilon_i\tag{2.19}$$

By stacking all l measurements, a linear system of $m = p \times l$ equations is formed:

$$y = Hx_k + \varepsilon$$

where y is the stacked vector of observation residuals, H is the stacked matrix of partial derivatives $\tilde{H}_i \Phi(t_i, t_k)$, x_k is the state deviation vector at the reference epoch, and ε is the stacked vector of measurement errors.

The estimation problem thus reduces to finding the best estimate, $\hat{x}_k$, for the state deviation x_k . The most common approach is the method of weighted least squares (WLS), which minimizes the cost function [16]:

$$J = (y - Hx_k)^T W (y - Hx_k)\tag{2.20}$$

where W is a weighting matrix, typically chosen as the inverse of the measurement error covariance matrix, $R = \mathcal{E}[\varepsilon \varepsilon^T]$, to produce a minimum variance estimate. The resulting minimum variance linear unbiased estimator for x_k is given by [16]:

$$\hat{x}_k = (H^T R^{-1} H)^{-1} H^T R^{-1} y.\tag{2.21}$$

The uncertainty of this estimate is quantified by its associated covariance matrix [16]:

$$P_k = (H^T R^{-1} H)^{-1}\tag{2.22}$$

If prior information about the state, $\bar{x}_k$, with an associated covariance $\bar{P}_k$, is available, it can be incorporated into the batch estimation process to yield an updated estimate and covariance [16]:

$$\hat{x}_k = (H^T R^{-1} H + \bar{P}_k^{-1})^{-1} (H^T R^{-1} y + \bar{P}_k^{-1} \bar{x}_k)\tag{2.23}$$

$$P_k = (H^T R^{-1} H + \bar{P}_k^{-1})^{-1}. \quad (2.24)$$

Since the entire procedure relies on linearization, it is performed iteratively. The estimated state correction $\hat{x}_k$ is used to update the reference trajectory X^* , the residuals y and matrix H are recomputed, and the WLS estimation is repeated until the process converges. While batch filters like WLS process all data simultaneously, it is worth noting that sequential filters, such as the Kalman filter, assimilate observations incrementally.

2.2.2.4. SOLVING THE ORBIT PROBLEM WITH THE ROB-MONTE LIBRARY

For the resolution of the POD problem, we employ the NASA Jet Propulsion Laboratory's (JPL) Mission analysis, Operations, and Navigation Toolkit Environment (MONTE), a powerful suite for high-fidelity dynamical modeling, measurement processing, and parameter estimation [17]. The core of MONTE's estimation capabilities is its Square-Root Information Filter (SRIF), a numerically robust formulation related to the Kalman filter that ensures stability by propagating the square root of the information matrix [18–20]. For an exhaustive description of MONTE's underlying measurement models, we refer the reader to the seminal work by Moyer [21].

To adapt MONTE's powerful functionalities specifically for radio science research, we developed the dedicated Python library *ROB-MONTE* at the Royal Observatory of Belgium (ROB). *ROB-MONTE* serves as a structured interface to MONTE, facilitating specialized analyses pertinent to radio science, including experiment design and tracking data analysis, within a unified framework. A detailed technical description of this library is provided in [Appendix C](#).

2.2.2.5. MULTI-ARC STRATEGY

The analysis of long observational time spans in POD presents significant challenges, as the accumulation of errors from complex or inadequately modeled non-gravitational forces can degrade the accuracy of a single, continuous trajectory solution [22]. The multi-arc strategy addresses this issue by partitioning the total observation period into shorter, contiguous segments known as "arcs". Within this framework, each arc is characterized by its own set of "local" parameters, which typically include the initial state (position and velocity) and may encompass other parameters to represent arc-specific unmodeled effects, such as empirical accelerations. This approach allows the estimation filter to absorb time-varying modeling errors into these local parameters, thereby preventing their propagation from corrupting the estimation of parameters considered constant over the entire dataset. As this capability was not natively supported by MONTE, the multi-arc approach was specifically implemented in the *ROB-MONTE* library for this work.

Parameters that are assumed to be constant across all arcs—such as planetary gravitational parameters (GM), tracking station coordinates, or other physical constants—are designated as "global" parameters. The estimation process then simultaneously resolves all local parameters for each individual arc along with the single set of global parameters shared among them. This partitioning of the state

vector, x , into local components $x^{(l_k)}$ for each arc k and a global component $x^{(g)}$, imposes a distinct block structure on the sensitivity matrix H [22]:

$$H = \begin{bmatrix} H^{(l_1)} & 0 & \dots & 0 & H^{(g_1)} \\ 0 & H^{(l_2)} & \dots & 0 & H^{(g_2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & H^{(l_N)} & H^{(g_N)} \end{bmatrix} \quad (2.25)$$

Here, $H^{(l_k)}$ contains the partial derivatives of the observations in arc k with respect to that arc's local parameters, while $H^{(g_k)}$ contains the partials with respect to the global parameters. The off-diagonal zero blocks arise because observations in one arc are, by definition, independent of the local parameters of any other arc.

This block-diagonal structure can be exploited for computational efficiency, as it may permit the parallel processing of individual arcs before the information is combined to solve for the global parameters. Ultimately, the multi-arc formulation implemented within *ROB-MONTE* provides enhanced robustness against the deleterious effects of modeling errors over long observational durations.

2.2.2.6. COVARIANCE ANALYSIS

The development of a reliable POD solution requires the careful selection of dynamical and measurement models, optimization of noise characterization, and comprehensive sensitivity studies to define an appropriate set of estimated parameters. Covariance analysis is an indispensable tool in this process, particularly for mission design, as it allows for an assessment of the expected accuracy of the estimated parameters. This analysis evaluates how uncertainties originating from measurements and modeling assumptions propagate into the final parameter estimates. The foundation of this technique lies in the parameter estimation covariance matrix, P_k , introduced in Sec. 2.2.2.3. The information matrix, $H^T R^{-1} H$, quantifies the sensitivity of the measurements to variations in the estimated parameters; by analyzing its inverse (the covariance matrix), one can predict the uncertainty and correlations for each parameter under a variety of scenarios.

Covariance analysis permits mission designers to evaluate critical trade-offs, such as how orbit geometry affects the ability to resolve specific gravity coefficients; the impact of improved Doppler noise or range precision on the solution; the degree to which accuracy improves with longer tracking durations or more frequent measurements; and the consequences of modeling choices, for instance, the effect of estimating versus fixing certain non-gravitational forces.

A key aspect of this analysis is understanding the impact of unestimated or mismodeled parameters. If physical effects are present but not included in the set of estimated parameters, x_k , they can introduce systematic biases and lead to overly optimistic formal uncertainties in the covariance matrix P_k . To address this, "consider" covariance analysis techniques explicitly account for the potential impact of these unestimated "consider" parameters. By treating these parameters as random variables with known a priori uncertainty, their influence can be mathematically incorporated into the computed covariance of the estimated

parameters, thereby providing more realistic error bounds and mitigating the risk of filter divergence. Such analysis helps determine whether it is necessary to estimate potentially problematic parameters or if their effects can be safely managed through alternative strategies like process noise modeling or the use of shorter data arcs. By systematically evaluating error sources, covariance analysis enables the design of robust POD strategies and informs mission planning to ensure scientific objectives can be achieved.

While POD provides crucial constraints on a satellite's interior through its gravitational influence, the study of the satellite's own rotational dynamics offers a distinct yet complementary approach. Examining how a satellite spins and how its spin axis orients in space reveals sensitivities to its internal mass distribution and the potential presence of decoupling layers, such as global oceans, which may not be uniquely resolved through gravity data alone. Consequently, the investigation of the rotational state, specifically through longitudinal librations and spin axis obliquity, constitutes another fundamental pillar in the geophysical exploration of icy satellites.

2.3. ROTATION OF ICY SATELLITES

The rotational dynamics of planetary satellites provide a powerful means of probing their internal structure, offering insights that complement those gained from gravity field measurements, magnetic field observations, and surface geology. Variations in a satellite's spin rate and the orientation of its spin axis are highly sensitive to the distribution of mass within the body and to the presence of internal layers, particularly global subsurface oceans that can mechanically decouple different parts of the interior.

The orientation of a satellite within an inertial reference frame, such as the International Celestial Reference Frame (ICRF), is typically described by a set of time-dependent angles: the right ascension (α) and declination (δ) of the spin pole, and the rotation angle (W) about this axis, which defines the prime meridian's position. A standard model represents these angles as a combination of secular rates (precession) and various periodic terms (nutations and librations). Following the formalism commonly employed in the literature (e.g., [4, 5, 14, 23]), these angles can be expressed as:

$$\alpha(t) = \alpha_0 + \dot{\alpha}_0 t + \sum_{j=1}^q A_j \sin(\dot{\Omega}_j t + \Omega_j) \quad (2.26)$$

$$\delta(t) = \delta_0 + \dot{\delta}_0 t + \sum_{j=1}^q B_j \cos(\dot{\Omega}_j t + \Omega_j) \quad (2.27)$$

$$W(t) = W_0 + \dot{W}_0 t + \sum_{j=1}^q C_j \sin(\dot{\Omega}_j t + \Omega_j) \quad (2.28)$$

where t is the time from a reference epoch (e.g., J2000, defined as 01-JAN-2000 12:00:00); $(\alpha_0, \delta_0, W_0)$ are the constant orientation angles at that epoch; $(\dot{\alpha}_0, \dot{\delta}_0)$ are the secular precession rates of the pole; and $\dot{W}_0$ is the mean rotation rate. The summations represent the nutation and libration series, which are composed of q

terms with amplitudes A_j and B_j (nutation), and C_j (libration), frequencies $\dot{\Omega}_j$, and phases Ω_j . These periodic variations are driven primarily by gravitational torques exerted by the primary body and other celestial perturbers.

The precise determination of the parameters within this rotational model, particularly any deviations from simple synchronous rotation or uniform precession, provides key insights into the satellite's interior. Two aspects of the rotational state that offer especially crucial constraints are longitudinal librations and spin axis obliquity.

2.3.1. LONGITUDINAL LIBRATIONS

Many satellites in the solar system are in a synchronous rotation, maintaining an average orientation where one hemisphere always faces the parent body. However, the combination of a non-spherical figure and an eccentric orbit gives rise to time-varying gravitational torques from the primary. These torques induce small, periodic oscillations in the satellite's rotation rate around its mean synchronous state, which are known as physical librations [8, 24–27].

Librations includes a free libration component, which represents a satellite's natural mode of oscillation around its equilibrium orientation if perturbed and then left [8, 24]. For an entirely solid body, this mode possesses a characteristic frequency, σ_f (or ω_f), determined by the internal mass distribution (specifically related to the principal moments of inertia ratio $(B - A)/C$) and its mean orbital motion (n). A key characteristic is that this natural frequency is typically much lower, and the period much longer, than the orbital frequency for solid bodies, given that the equatorial asymmetry $(B - A)$ is much smaller than the polar moment of inertia C [8, 24]. Although the specific excitation mechanism for free libration is not known, any initial amplitude is expected to be damped relatively quickly to zero by dissipative processes. Consequently, the amplitude of free libration is generally assumed negligible (zero) in models analyzing the current rotational state [8, 24]. It is important to note that if internal liquid layers, such as a subsurface ocean, exist, additional free libration modes can arise from the coupled motions, and some of these may have frequencies closer to the orbital frequency, potentially leading to resonance effects [8, 24].

In contrast to the damped free modes, forced librations are continuously driven oscillations resulting from time-varying gravitational torques. For a satellite in an eccentric orbit, the changing distance and orientation relative to the primary body cause the torque, which depends on the satellite's equatorial asymmetry $(B - A)$, to fluctuate periodically [24, 25]. The most prominent forced oscillation occurs at the satellite's orbital period (frequency n) and is often referred to as the diurnal libration. Smaller forced librations can also occur at harmonics of the orbital frequency or frequencies related to orbital perturbations; however, these are generally less significant for the purpose of probing interior structure.

Given that free libration is typically damped, investigations aiming to probe interior structure via rotational variations focus almost exclusively on the forced diurnal libration (see e.g. Van Hoolst, Baland, and Trinh [26]). This specific component is continuously sustained by the orbital dynamics, making its signal

persistent and measurable over time. Furthermore, its amplitude is highly sensitive to the satellite's internal mass distribution and, crucially, to the presence and properties of internal fluid layers, such as subsurface oceans or liquid cores [25–27]. The measurement of librations, particularly the forced diurnal component, therefore provides a powerful tool for constraining the interior structure of celestial bodies. Both the amplitude and phase of these oscillations, corresponding to specific periodic terms in Eq. (2.28), can help to constrain the interior. By precisely measuring these sustained rotational variations, for instance with techniques such as spacecraft imaging (see subsection 2.3.3 or radio science (see subsection 2.2.1.1), we can place strong constraints on key interior parameters, such as the moment of inertia, ice shell thickness, ocean depth, or the size and state of a planetary core [26, 27].

2.3.1.1. DIURNAL LIBRATIONS IN LONGITUDE

As previously mentioned, the diurnal libration is the most prominent forced libration for a satellite on an eccentric orbit [27]. Its amplitude is highly sensitive to the satellite's interior, providing a fundamentally different response depending on whether the body is entirely solid or possesses a global subsurface ocean that mechanically decouples the outer shell from the deep interior [26, 27]. Here, we present the theoretical models used to describe this key rotational motion for both cases.

Solid Case For an entirely solid, rigid satellite in a synchronous orbit, the amplitude of the forced diurnal libration, g_{solid} , is primarily governed by the satellite's principal moments of inertia ($A < B < C$), its mean motion (n), and its orbital eccentricity (e) [8, 24, 28]. We can express the diurnal libration angle, γ , which represents the deviation from uniform synchronous rotation, as:

$$\gamma = g \sin(M_0 + \dot{M}t) \quad (2.29)$$

where M_0 and $\dot{M}$ are the satellite's mean anomaly at a reference epoch and its time derivative, respectively [29]. The amplitude g_{solid} for a rigid body, approximated up to first order in eccentricity e , is then given by [24]:

$$g_{solid} \approx -6 \frac{(B - A)}{C} \frac{n^2 e}{n^2 - \sigma_f^2} \quad (2.30)$$

where σ_f is the natural frequency of free libration:

$$\sigma_f = n \sqrt{\frac{3(B - A)}{C}} \quad (2.31)$$

In this expression, the term $(B - A)$ reflects the satellite's equatorial asymmetry on which the gravitational torque acts, while C is its rotational inertia resisting this change in rotation [27]. Since the free libration frequency σ_f is generally much smaller than the orbital frequency n for solid bodies, the denominator $(n^2 - \sigma_f^2)$ can

be approximated as n^2 , which simplifies the amplitude to a high degree of accuracy [24]:

$$g_{solid} \approx -6e \frac{(B-A)}{C}. \quad (2.32)$$

Under the assumption of hydrostatic equilibrium, we can relate the moment of inertia difference $(B-A)$ and the polar moment C to the satellite's normalized mean moment of inertia (MOI) I/MR^2 and its degree-2 gravity coefficients (J_2 , C_{22}) using Clairaut's theory and the Radau equation [24, 27]. Consequently, the libration amplitude for a solid body depends primarily on its MOI, which is a key indicator of the degree of internal differentiation [27]. A more differentiated body, having a lower MOI, will exhibit a smaller $(B-A)/C$ ratio and thus a smaller libration amplitude compared to a homogeneous body (for which $\text{MOI} = 0.4$) [27].

Tidal elastic deformations introduce a slight modification to this response. The effective moments of inertia are altered, and the free libration frequency changes, leading to a new expression for the libration amplitude of an elastic solid body (g_{el}) [25]:

$$g_{el} = -6 \frac{k_f - 5k_2/6}{k_f} \frac{(B-A)}{\tilde{C}} \frac{n^2 e}{n^2 - \sigma_{el}^2} \quad (2.33)$$

where k_f is the fluid Love number, k_2 is the potential Love number, $\tilde{C} = C + \frac{4k_2 n^2 R^5}{9G}$ is the tidally-modified polar moment of inertia, and $\sigma_{el} = n \sqrt{\frac{3(B-A)}{\tilde{C}} \frac{k_f - k_2}{k_f}}$ is the elastic free frequency [25]. For the small and medium-sized icy satellites we consider in this study, elasticity typically reduces the libration amplitude by only a small amount (e.g., <1% for the Uranian moons [27], <0.1% for Enceladus [25]) compared to the rigid case. Therefore, observing a libration amplitude that deviates significantly from the expected solid-body value—beyond the minor adjustments accounted for by elasticity—suggests the influence of other mechanisms, such as the presence of an internal decoupling layer, as discussed next.

Ocean Case If a global subsurface ocean is present, it mechanically decouples the outer ice shell from the solid interior (mantle and core) [26]. The external gravitational torque acts mainly on the non-spherical shell, but the shell's rotational response is altered by gravitational and pressure torques at the shell-ocean and ocean-interior interfaces [8, 27, 30]. This results in more complex libration dynamics, involving the coupled motion of the shell and the potentially librating deep interior.

In the ocean case, the libration amplitude of the shell (g_s) can be generally much larger than that of a solid body. This is because the shell's moment of inertia (C_s) is significantly smaller than the satellite's total moment of inertia (C), while the effective torque on the shell is not reduced proportionally [25]. A first-order approximation for the shell libration amplitude, which is valid for larger satellites or thicker shells where resonance effects are less pronounced, is given by [25]:

$$g_s \approx \frac{4eK_3}{C_s n^2} \quad (2.34)$$

where K_3 is the effective diurnal forcing torque on the shell, a term that accounts for planetary gravity and ocean pressure effects on the shell's static and tidal topography [25]. This approximation clearly highlights the inverse relationship between the amplitude and the shell's moment of inertia, C_s [25, 27].

However, the presence of an ocean introduces new free libration modes. One mode involves an out-of-phase libration between the shell and the core, and its period ($\approx 2\pi\sqrt{C_s/K_1}$, where K_1 is an interior coupling strength) can be close to the orbital period, particularly for thin shells. This proximity to resonance can significantly amplify the forced diurnal libration [26, 29]. A more complete approximation that incorporates this resonance effect is therefore [26]:

$$g_s \approx \frac{4eK_3}{C_s(n^2 - \sigma_{ocean}^2)} \quad (2.35)$$

where $\sigma_{ocean}^2 \approx K_1/C_s$ is the square of the relevant free frequency. As an example, this resonance amplification can increase the libration amplitude of Enceladus by a factor of up to 4 for the thinnest shells [26].

Tidal deformations of the elastic shell and interior are of particular importance in the presence of a global liquid layer, much more so than in the solid case. Periodic tides modify the external torque, the interlayer coupling torques, and the effective moments of inertia [8]. The torque exerted by the primary on the shell's periodic tidal bulge tends to counteract the torque on its static shape, reducing the net forcing [8, 29]. This reduction is quantified by the factor $(k_f^j - k_2^j)/k_f^j$ for a given layer j . For icy satellites with oceans, the shell's potential Love number k_2^s can be a large fraction of its fluid Love number k_f^s , leading to a substantial decrease in the effective torque (e.g., by several 10 % up to 90 %) [8]. Furthermore, additional torques arise between the static bulge of one solid layer and the periodic bulge of another [8, 31].

These tidal effects generally act to dampen the large increase in libration amplitude expected from shell-interior decoupling alone, bringing the amplitude closer to the solid-body case, though often still larger [8]. The magnitude of this reduction strongly depends on the satellite's size and shell properties; it can be modest for a small body like Enceladus but quite significant for larger bodies like Io or the Galilean moons, where it can reduce the amplitude by factors of 2 or more [25, 29]. A measurement of the libration amplitude g_s therefore provides constraints on C_s and the coupling terms (K_1, K_3) , which themselves depend on shell thickness (h_s), shell rigidity (μ_s), layer densities ($\rho_s, \rho_o, \rho_{int}$), and core size (R_c) (see Eqs. 45-64 in [8] and discussions in [26, 27, 29]). Although a single libration measurement often leads to degenerate solutions (e.g., an ambiguity between shell thickness and rigidity) due to its dependence on multiple interior parameters, combining it with other geodetic measurements, such as the tidal Love number k_2 [29] or obliquity [27], can help mitigate these degeneracies and refine constraints on the interior structure.

2.3.2. OBLIQUITY

In addition to librations, a satellite's obliquity—the angle ε between its spin axis and orbit normal—provides complementary constraints on its interior structure.

The instantaneous direction of the spin axis in inertial space is given by its right ascension $\alpha(t)$ and declination $\delta(t)$ (see Eqs. (2.26) and (2.27)), from which the obliquity can be calculated using the following spherical trigonometric relation:

$$\cos \varepsilon = \sin \delta \cos J - \cos \delta \sin J \sin(\alpha - N) \quad (2.36)$$

where J and N are the orbital inclination and the longitude of the ascending node of the satellite's orbit with respect to the ICRF at J2000, respectively.

For a synchronously rotating satellite, the orientation of the spin axis results from the interplay between the gravitational torque exerted by the primary planet on the satellite's non-spherical mass distribution and the satellite's physical response to that torque, which is determined by its internal structure and moments of inertia [25, 32]. Over long timescales, this torque, combined with tidal dissipation, drives the spin axis into an equilibrium configuration known as a Cassini state. In this state, the spin axis, the orbit normal, and the normal to the Laplace plane (the mean orbital plane) are coplanar and precess together around the latter [25, 32]. For the classical Cassini state, which assumes a uniform orbital precession, this configuration results in a constant obliquity [31, 32].

The obliquity is a parameter of particular interest because it both influences a satellite's tidal heating and is sensitive to its internal mass distribution. The misalignment between the spin and orbit axes gives rise to obliquity tides, which can generate substantial heat within a satellite's layers, potentially maintaining a subsurface ocean or driving geological activity [31]. Simultaneously, the magnitude of the obliquity is dictated by the body's moments of inertia, its visco-elastic properties, and the presence of any internal fluid layers [30, 32]. A precise measurement of a satellite's obliquity can therefore provide powerful constraints on models of its interior structure.

2.3.2.1. SOLID AND RIGID CASE

For an entirely solid and rigid satellite locked in a classical Cassini state, the constant obliquity ε is derived from the averaged angular momentum equation. This equation, neglecting wobble, balances the change in angular momentum with the averaged gravitational torque exerted by the planet [31]:

$$nC \frac{d\hat{s}}{dt} = n\kappa(\hat{s} \wedge \hat{n}) \quad (2.37)$$

where $\hat{s}$ and $\hat{n}$ are unit vectors along the spin axis and orbit normal, respectively; n is the satellite's mean motion (equal to its rotation rate); C is the polar moment of inertia; and κ is the coupling strength, which is related to the principal moments of inertia ($A < B < C$):

$$\kappa = \frac{3}{2}(C - A)n = \frac{3}{2}MR^2(-C_{20} + 2C_{22})n \quad (2.38)$$

Here, M and R are the satellite's mass and radius, and C_{20} and C_{22} are its degree-two gravity coefficients. For a uniform orbital precession with a rate $\dot{\Omega}$ and inclination i , the obliquity is given by [31]:

$$\varepsilon = -\frac{i\dot{\Omega}}{\omega_f + \dot{\Omega}} \quad (2.39)$$

where $\omega_f = \kappa/C$ is the satellite's free precession frequency.

However, a satellite's orbital precession is often non-uniform due to gravitational perturbations from other bodies. The orbital precession vector N (the projection of $\hat{n}$ onto the Laplace plane) can be expressed as a sum of terms with different frequencies $\dot{\Omega}_j$ and inclination amplitudes i_j [31]:

$$N = \sum_j i_j e^{I(\dot{\Omega}_j t + \gamma_j - \pi/2)} \quad (2.40)$$

where $I = \sqrt{-1}$. The forced spin precession vector S (the projection of $\hat{s}$) is then a summation of corresponding terms:

$$S = \sum_j (i_j + \varepsilon_j) e^{I(\dot{\Omega}_j t + \gamma_j - \pi/2)} \quad (2.41)$$

with the obliquity amplitudes ε_j for each mode given by:

$$\varepsilon_j = -\frac{i_j \dot{\Omega}_j}{\omega_f + \dot{\Omega}_j} \quad (2.42)$$

The instantaneous obliquity, $\varepsilon(t) \approx ||S - N||$, consequently becomes time-dependent. A resonance can occur if the free precession frequency ω_f is close to $-\dot{\Omega}_j$ for any mode j , which would potentially amplify the corresponding obliquity amplitude ε_j [30, 32].

2.3.2.2. SOLID AND ELASTIC CASE

A satellite's rotation is also affected by periodic tidal deformations, since the periodic obliquity tide modifies the effective moment of inertia difference governing the torque and alters the angular momentum expression. The averaged angular momentum equation for an elastic solid body thus becomes [31]:

$$n\tilde{C} \frac{d\hat{s}}{dt} = n\kappa_{el}(\hat{s} \wedge \hat{n}) + n(\tilde{C} - C) \frac{d\hat{n}}{dt} \quad (2.43)$$

where $\kappa_{el} = \frac{3}{2}n((C - A) - k_2 MR^2 q_r)$ is the elastic coupling strength, and $\tilde{C} = C - \frac{1}{2}k_2 MR^2 q_r$ is the tidally modified polar moment of inertia. In these expressions, k_2 is the dynamical potential Love number and $q_r = n^2 R^3/(GM)$. The elastic free precession frequency is $\omega_f^{el} = \kappa_{el}/\tilde{C}$, and the corresponding obliquity amplitudes for the elastic case are [31]:

$$\varepsilon_j^{el} = -\frac{C}{\tilde{C}} \frac{i_j \dot{\Omega}_j}{\omega_f^{el} + \dot{\Omega}_j} \quad (2.44)$$

For small satellites, such as Enceladus or the Uranian moons, the degree-two potential Love number k_2 is typically very small for a solid body. This value quantifies the tidal response, scaling approximately as $k_2 \propto \rho^2 R^2/\mu$, and is small primarily due to the satellite's small radius R (see e.g. Eq. 27 in [31]). For instance, a solid Enceladus is characterized by $k_2 \approx 1.5 \times 10^{-4}$, while solid Uranian moons have

k_2 values up to ≈ 0.0185 . Consequently, elastic effects modify the obliquity by less than 1% compared to the rigid case, given that the tidally modified polar moment of inertia $\tilde{C}$ and the elastic free precession frequency ω_f^{el} both remain very close to their rigid counterparts. However, for larger satellites (e.g., Titan or the Galilean moons) or for bodies possessing substantial melt (e.g., an ocean or magma ocean, which significantly lowers effective rigidity and increases the effective k_2), elastic effects become more significant. Solid estimates for these larger bodies yield k_2 values of ≈ 0.015 (Europa), ≈ 0.067 (Ganymede, Callisto, Titan), and $\approx 0.04 - 0.8$ (Io) [31]. These larger k_2 values can cause considerable changes; for example, elasticity increases the solid free precession period by $\approx 1\%$ for Europa and up to $\approx 46\%$ for Io (assuming low viscosity) [31]. Titan's measured $k_2 \approx 0.6$, which is indicative of an ocean, implies a $\approx 35 - 56\%$ increase in the solid free precession period compared to the rigid case [31]. This effect amplifies the obliquity by bringing the system closer to resonance, a finding that helps reconcile theory with observations. Even in the case of the Uranian moons, the presence of an ocean would increase k_2 up to ≈ 0.07 , leading to potentially larger elastic effects on obliquity (up to $\approx 10 - 15\%$) than in the solid case [27]. Thus, while elasticity appears to be a minor factor for small solid moons, it substantially modifies obliquity predictions for larger satellites or for those with internal liquid layers, owing to their enhanced tidal response.

2.3.2.3. SUBSURFACE OCEAN

The presence of a global subsurface ocean significantly alters a satellite's rotational dynamics by decoupling the outer solid shell from the deeper solid interior (i.e., core or mantle and core) [26, 32]. Assuming the ocean is inviscid and hydrostatic, the total torque on the ocean vanishes, and the system can be described by a set of coupled angular momentum equations for the shell (sh) and the interior (in) [30, 31].

Rigid Shell and Interior For the case of rigid solid layers, the coupled equations projected onto the Laplace plane are given by [31]:

$$\begin{aligned} C_{sh} \frac{dS_{sh}}{dt} &= I\kappa'_{sh}(N - S_{sh}) - IK(S_{in} - S_{sh}) \\ C_{in} \frac{dS_{in}}{dt} &= I\kappa'_{in}(N - S_{in}) + IK(S_{in} - S_{sh}) \end{aligned} \quad (2.45)$$

where S_{sh} and S_{in} are the projected spin vectors, and C_{sh} and C_{in} are the polar moments of inertia of the shell and interior, respectively. The terms κ'_{sh} and κ'_{in} represent the effective external torque strengths on the shell and interior (which are the effective external torque strengths on the shell and interior, accounting for both the planet's direct gravity on the solid layer and the torque transmitted via pressure from the adjacent ocean layer), while K is the internal gravitational and pressure coupling strength between the shell and interior. Explicit expressions for κ'_{sh} , κ'_{in} , and K are provided by Baland, Yseboodt, and Van Hoolst [31] (Eq. 32).

This coupled system possesses two free precession modes, corresponding to in-phase (ω_+) and out-of-phase (ω_-) motion of the shell and interior. The forced

solution yields obliquity amplitudes, $\varepsilon_{j,sh}$ and $\varepsilon_{j,in}$, for each layer and for each orbital forcing frequency $\dot{\Omega}_j$ [31]:

$$\begin{aligned}\varepsilon_{j,sh} &= \frac{i_j \dot{\Omega}_j (K(C_{sh} + C_{in}) - C_{sh} \kappa'_{in} - C_{in} C_{sh} \dot{\Omega}_j)}{C_{in} C_{sh} (\omega_+ + \dot{\Omega}_j) (\omega_- + \dot{\Omega}_j)} \\ \varepsilon_{j,in} &= \frac{i_j \dot{\Omega}_j (K(C_{sh} + C_{in}) - C_{in} \kappa'_{sh} - C_{in} C_{sh} \dot{\Omega}_j)}{C_{in} C_{sh} (\omega_+ + \dot{\Omega}_j) (\omega_- + \dot{\Omega}_j)}\end{aligned}\quad (2.46)$$

Resonances can occur if either ω_+ or ω_- is close to $-\dot{\Omega}_j$ for any forcing mode j , potentially leading to significantly larger obliquities than would be expected in the solid-body case [30, 32].

Elastic Shell and Interior The elastic deformation of the shell and interior further modifies the torques and angular momenta of the system. The external torque strengths, κ'_{sh} and κ'_{in} , and the internal coupling strength, K , must be corrected for the effect of periodic obliquity tides on both the solid layers and the ocean. This correction includes additional internal torques that arise from the interaction between the static bulge of one layer and the periodic tidal bulge of another. Furthermore, the effective polar moments of inertia, $\tilde{C}_{sh}$ and $\tilde{C}_{in}$, are also modified by these tidal effects. The governing equations therefore become [31]:

$$\begin{aligned}\tilde{C}_{sh} \frac{dS_{sh}}{dt} &= I \tilde{\kappa}_{sh} (N - S_{sh}) - I \tilde{\kappa}_{in \rightarrow sh} (S_{in} - S_{sh}) + (\tilde{C}_{sh} - C_{sh}) \frac{dN}{dt} \\ \tilde{C}_{in} \frac{dS_{in}}{dt} &= I \tilde{\kappa}_{in} (N - S_{in}) + I \tilde{\kappa}_{sh \rightarrow in} (S_{in} - S_{sh}) + (\tilde{C}_{in} - C_{in}) \frac{dN}{dt}\end{aligned}\quad (2.47)$$

where the modified coupling strengths ($\tilde{\kappa}_{sh}$, $\tilde{\kappa}_{in}$, $\tilde{\kappa}_{in \rightarrow sh}$, $\tilde{\kappa}_{sh \rightarrow in}$) now fully incorporate these elastic effects (see Baland, Yseboodt, and Van Hoolst [31] (Eqs. 46-49)). The structure of the solution remains similar to that of Eq. (2.46), yielding elastic obliquity amplitudes $\varepsilon_{j,sh}^{el}$ and $\varepsilon_{j,in}^{el}$.

For small satellites, such as Enceladus or the Uranian moons, the effect of elasticity on obliquity amplitudes in the ocean case is found to be relatively minor (typically $< 10 - 15\%$). This is because the relevant k_2 values remain small, even in the presence of an ocean. Specifically, k_2 values for Enceladus, assuming an ocean, range from 0.005 to 0.072 as a function of shell thickness [31]. In the case of the Uranian moons with oceans, the predicted maximum k_2 values are also small, ranging from ≈ 0.004 for Miranda up to ≈ 0.07 for Titania and Oberon. However, for larger satellites such as Titan, where the effective k_2 is significant (e.g., Cassini measurements indicate $k_2 \approx 0.6$ [33]), elasticity can substantially alter the obliquity response compared to the rigid ocean case, an effect that may improve consistency with observational data.

2.3.2.4. CONSTRAINING INTERIOR STRUCTURE WITH OBLIQUITY

Comparing the predicted obliquity for solid versus ocean-bearing models allows for testing the presence of an ocean. A difference arises, as the ocean mechanically

decouples the shell from the interior. The interior, which typically contains the majority of the satellite's mass, dominates the coupled rotational dynamics [30, 31]. Its polar moment of inertia C_{in} is large (though slightly smaller than the total solid-body inertia C), and its intrinsic free precession frequency ω_{in} is often close to the solid-body frequency ω_f . However, since its inertia C_{in} is less than the total inertia C of the equivalent solid body, its response to gravitational forcing tends to be slightly larger. Consequently, and in the absence of resonances, the interior obliquity $\epsilon_{in}(t)$ is often predicted to be slightly larger than the solid-body obliquity $\epsilon_{solid}(t)$ (or the elastic solid obliquity $\epsilon_{el}(t)$) [30]. Conversely, the shell possesses a much smaller inertia ($C_{sh} \ll C_{in}$) and, as such, has an intrinsic free frequency ω_{sh} that can differ significantly from ω_f [30]. While strong internal coupling forces the shell to precess largely with the dominant interior, the detailed dynamics often result in the shell's obliquity amplitude $\epsilon_{sh}(t)$ being smaller than the solid-body value. For instance, in the case of Enceladus, the maximum predicted interior obliquity (ϵ_{in}^{el}) can reach $\sim 6.5 \times 10^{-4}$ degrees [31]. This value is up to $\approx 45\%$ larger than the maximum solid-body obliquity ($\epsilon_{el} \approx 4.5 \times 10^{-4}$). In contrast, the maximum shell obliquity (ϵ_{sh}^{el}) is only $\approx 4 \times 10^{-4}$, about 11% smaller than the solid case. A similar situation applies to Europa: the solid-body obliquity is predicted to be $\approx 0.055^\circ$, whereas models with an ocean predict a shell obliquity ϵ_{sh} in the range of 0.033° – 0.044° (roughly 20–40% smaller) [30]. The interior obliquity ϵ_{in} in this case is predicted to remain close to the solid-body value. Therefore, an obliquity measurement that is significantly different from the predicted solid-body value (either larger or smaller, depending on whether shell or interior dynamics dominate the observable) would favor the presence of an ocean. The required measurement precision is dependent on the specific satellite and its parameters, but it can range from meters (e.g., ≈ 0.2 m precision on Enceladus' pole position might distinguish ocean vs. solid cases [31]) to hundreds of meters at the surface (e.g., ≈ 100 – 200 m precision for Europa [30]).

Should a resonance occur, the obliquity amplitude for one or more modes ($\epsilon_{j,sh}, \epsilon_{j,in}$) can be significantly amplified. This amplification, reflecting the dynamic response of the ocean rather than a simple quasi-static adjustment, can lead to much larger instantaneous obliquities than would be expected in the solid-body case. Consequently, the observation of such a large obliquity would constitute strong evidence for a subsurface ocean. Tyler [34] identified a key mechanism for such amplification: obliquity tides can resonantly excite large-amplitude Rossby-Haurwitz waves within the ocean. These resonances are often highly sensitive to the presence and specific properties of the liquid layer, particularly its thickness (h) and dissipation characteristics (Q), which define the ocean's location in the parameter space governing tidal response and heating [34, 35]. Furthermore, feedback mechanisms between tidal heating, potential freezing/melting, and the ocean's parameters might even trap the ocean in such resonant states, making them potentially stable configurations [34, 35]. For instance, modeling work suggests that Ganymede's obliquity might be resonantly enhanced due to its interior structure, while Europa's configuration makes a strong obliquity-driven Rossby wave resonance less likely, although other resonances might be relevant [30, 34].

Once the existence of an ocean has been inferred, the magnitude of the obliquity (or its time variation, if multiple measurements are available) can be used to constrain interior parameters [30]. In the non-resonant regime, the obliquity depends primarily on moment of inertia ratios (e.g., $(C_{sh} - A_{sh})/C_{sh}$), which are linked to the shell thickness, layer densities, and core size [31]. The sensitivity, however, is often limited, necessitating high-precision measurements to effectively constrain parameters such as shell thickness [24]. In resonant or near-resonant cases, the obliquity becomes highly sensitive to the parameters that determine the free frequencies ω_+ and ω_- . This offers the potential for tighter constraints, although the possibility that multiple interior models could fit the data may still exist [30].

2.3.3. LANDMARK TRACKING FOR ROTATIONAL STATE DETERMINATION

Landmark tracking, enabled by advancements in imaging systems and photogrammetric methods, has proven to be a highly effective technique for determining the rotational state of solar system bodies, thereby complementing gravity and other geophysical investigations. We can consider this technique a specialized application of optical navigation (OpNav), which, in a broader sense, employs images of celestial bodies to determine a spacecraft's trajectory and orientation [36]. In particular, landmark tracking focuses on measuring the positions of distinct surface features—or landmarks—in a sequence of images to precisely constrain the target body's rotational parameters, including its spin axis orientation, rotation rate, precession, nutation, and librations (see, e.g., [14, 37–39]).

2.3.3.1. THE MEASUREMENT PRINCIPLE

The core principle of landmark tracking involves the precise measurement of recognizable surface features within images captured by a spacecraft's camera [36]. These features, which are often impact craters, boulders, or distinct albedo patterns, are used to build a Control Point Network (CPN). The constituent points of this network are then tracked across multiple images acquired from varying viewpoints and at different times [14, 36, 40]. From each image, we derive the fundamental observable: the 2D coordinate (typically sample s and line l) of the landmark's centroid in the camera's focal plane [36].

To relate these 2D image measurements to the 3D geometry of the observation, we employ a detailed model of the camera's projection. A common approach is to model the camera using a pinhole approximation, which is then combined with calibration parameters that account for optical distortions. First, we transform the apparent direction vector A^I from the camera to the landmark (corrected for light-time and stellar aberration) in an inertial frame (I) into the camera's local coordinate system (C). This camera frame is often defined with the L-axis along the optical axis and the M and N axes spanning the focal plane. The transformation utilizes the camera's attitude, or orientation, matrix, C [36]:

$$A^C = CA^I. \quad (2.48)$$

The vector A^C in camera coordinates then projects onto an ideal focal plane at coordinates (x, y) relative to the optical axis (L-axis, or A_3^C). For a simple gnomonic

projection with a focal length f , this relationship is given by [36]:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{f}{A_3^C} \begin{pmatrix} A_1^C \\ A_2^C \end{pmatrix}. \quad (2.49)$$

Real optical systems, however, introduce distortions that must be modeled. These are typically treated as corrections $(\Delta x, \Delta y)$ added to the ideal coordinates (x, y) to obtain the corrected coordinates (x', y') . A common model includes radial distortion (parameter ε_1) and tangential distortion arising from focal plane tilt (parameters $\varepsilon_2, \varepsilon_3$) [36]:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x r^2 & x y & x^2 \\ y r^2 & y^2 & x y \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix} \quad (2.50)$$

where $r^2 = x^2 + y^2$. Finally, these corrected focal plane coordinates (x', y') are mapped to the discrete pixel coordinates (sample s , line l) measured on the detector (e.g., a CCD or CMOS sensor). This mapping uses scaling factors (K_x, K_y) , cross-terms (K_{xy}, K_{yx}) to account for pixel non-squareness or rotation, and the pixel coordinates of the optical axis (s_0, l_0) [36]:

$$\begin{pmatrix} s \\ l \end{pmatrix} = \begin{pmatrix} K_x & K_{xy} \\ K_{yx} & K_y \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} s_0 \\ l_0 \end{pmatrix}. \quad (2.51)$$

This set of equations constitutes the complete measurement model, linking the 3D geometry—encompassing camera position, attitude, and landmark location—to the 2D image observable (s, l) .

2.3.3.2. PARAMETER ESTIMATION VIA BUNDLE ADJUSTMENT

The determination of a body's rotational parameters is achieved through their estimation within a larger framework that simultaneously refines the landmark positions and the camera states for all observations [14, 36]. The most common technique we employ for this purpose is a Bundle Block Adjustment (BBA), a powerful photogrammetric method rooted in least-squares estimation [14]. In practice, the BBA minimizes the sum of squared differences between the observed landmark pixel coordinates (s_{obs}, l_{obs}) across all images and those predicted (s_{pred}, l_{pred}) by the measurement model detailed in Eq. (2.51) [14, 36]. This minimization is performed with respect to a large state vector that typically includes [14, 36, 39]:

- The 3D coordinates of all landmarks in the CPN, usually defined in a body-fixed reference frame.
- The position and orientation (or pose) of the camera at the time each image was acquired.
- Parameters defining the body's rotation model, such as the orientation of the spin pole (α_0, δ_0) , the rotation rate $(\dot{W}_0)$, precession/nutation terms, and libration amplitudes (e.g., g_s), as formulated in Eqs. (2.26) to (2.28).

- Camera calibration parameters (focal length, distortion coefficients, principal point (s_0, l_0) , etc.) if they are not sufficiently well-known a priori.

The estimation process is iterative, progressively refining the reference values for all parameters until the residuals reach a minimum and the solution converges [36, 39]. It is crucial that the rotation model is defined with respect to an inertial reference frame, such as J2000/ICRF, to properly decouple the satellite's rotational kinematics from its orbital motion and any errors in the spacecraft's trajectory [39]. Ultimately, the accuracy of the determined rotational parameters is contingent on several factors, including the number, geometric distribution, and measurement precision of the landmarks; the observational baseline (i.e., the time span and variation in viewpoint); the accuracy of the camera calibration; and the fidelity of the underlying spacecraft trajectory and body ephemeris [39].

2.3.3.3. EVOLUTION AND APPLICATIONS OF LANDMARK TRACKING TECHNOLOGIES

Landmark tracking technologies have evolved considerably, from manual techniques and simple correlation algorithms [40] to the advanced, often machine learning (ML)-based, systems in use today that enable robust feature detection under variable illumination, resolution, and geometric distortions. When integrated with onboard processing, these advancements can support near-autonomous navigation by facilitating real-time corrections to rotational or shape models (see, e.g., [41, 42]). Instrumentation has co-evolved; notably, the Advanced Pointing Imaging Camera (APIC) concept combines a gimballed platform with high-resolution sensors and optics designed to simultaneously capture planetary surfaces and background star fields, thereby embedding inertial referencing directly into the imaging hardware and enhancing optical navigation precision [43]. The importance of landmark tracking is now evident across all mission phases, from the scientific interpretation of rotational parameters relevant to internal structure to operational autonomy.

These capabilities have been successfully validated in diverse mission environments. For Phobos, inertial-frame corrections applied to Viking and Mars Express imagery resolved its forced libration, which helped to constrain its internal structure [38]. On Enceladus, Park *et al.* [14] employed a control point network derived from stereophotoclinometry (SPC) processing of Cassini images to determine its forced libration amplitude. Similarly, landmark tracking was central to guiding operations for the OSIRIS-REx mission at the asteroid Bennu and was used extensively by the Dawn mission at Vesta and Ceres [39, 44]. An instrument like the APIC concept, or similar advanced cameras, is envisioned for future missions where precise landmark tracking will be essential for probing rotational states, characterizing tidal responses, and supporting high-resolution navigation [43, 45]. Despite this progress, challenges remain, such as the need for improved performance under harsh lighting conditions, the fusion of diverse datasets, and enhanced algorithmic scalability for dynamically complex bodies.

2.4. INFERRING INTERIOR STRUCTURE WITH MARKOV CHAIN MONTE CARLO METHODS

Geophysical and geodetic measurements, such as a body's gravity field coefficients ($\tilde{C}_{nm}, \tilde{S}_{nm}$), rotational state (e.g., Mean Moment of Inertia factor I/MR^2 , libration amplitude g), and tidal response (Love numbers k_n, h_n, l_n), do not provide direct and explicit information about the interior structure of a planetary body. The translation of these observables into specific properties of internal layers—such as core radius, ocean thickness, or layer densities and rheologies—represents a complex inverse problem. To address this challenge, advanced methods must be developed. Bayesian inference using the Markov Chain Monte Carlo (MCMC) approach represents a highly effective technique for addressing such inverse problems [46]. It allows for a robust exploration of the parameter space of possible interior models to identify those that are consistent with a given set of measurements and their accompanying uncertainties.

2.4.1. BAYESIAN INFERENCE FRAMEWORK

To constrain the interior structure parameters from geophysical data, we employ a Bayesian inference framework analogous to that defined by Cascioli *et al.* [47]. This approach combines prior knowledge about the interior structure parameters, $\mathbf{p}$, with the likelihood of obtaining a set of measurements, $\mathbf{d}_{obs}$, for any given interior model. This relationship is formally expressed by Bayes' theorem [46]:

$$P(\mathbf{p}|\mathbf{d}_{obs}) \propto P(\mathbf{d}_{obs}|\mathbf{p})P(\mathbf{p}). \quad (2.52)$$

Here, $P(\mathbf{p})$ represents the prior probability distribution for the parameters, which reflects existing constraints from physical principles or previous studies. The likelihood function, $P(\mathbf{d}_{obs}|\mathbf{p})$, quantifies the agreement between the observables predicted by a forward model, $\mathbf{d}_{pred}(\mathbf{p})$, and the measured data, $\mathbf{d}_{obs}$. Our objective is thus to determine the posterior probability distribution, $P(\mathbf{p}|\mathbf{d}_{obs})$, which represents the updated state of knowledge of the parameters once the observational constraints have been incorporated.

We assume that the measurement errors in the observables are Gaussian and can be described by a covariance matrix $\mathbf{C}$. The likelihood function can then be formulated as [46–49]:

$$L(\mathbf{p}) = P(\mathbf{d}_{obs}|\mathbf{p}) \propto \exp\left(-\frac{1}{2}(\mathbf{d}_{obs} - \mathbf{d}_{pred}(\mathbf{p}))^T \mathbf{C}^{-1}(\mathbf{d}_{obs} - \mathbf{d}_{pred}(\mathbf{p}))\right) \quad (2.53)$$

The observables vector $\mathbf{d}_{obs}$ contains the measured quantities, such as GM , C/MR^2 , gravity coefficients, and tidal and rotational parameters, while $\mathbf{C}$ includes their associated uncertainties and any correlations. To compute the predicted observables $\mathbf{d}_{pred}(\mathbf{p})$, we utilize a forward model for the interior structure defined by $\mathbf{p}$, which involves calculating, for instance, hydrostatic equilibrium figures, tidal response, and moments of inertia.

Since a direct analytical solution for the posterior distribution $P(\mathbf{p}|\mathbf{d}_{obs})$ is generally intractable, we use a computational approach. Specifically, we employ an MCMC

method to generate a chain of samples $\{\mathbf{p}_i\}$ that converges to the target posterior distribution. For this work, we adopt the affine-invariant ensemble sampler algorithm proposed by Goodman and Weare [50], which has proven to be particularly effective for geophysical inverse problems where parameters are often correlated.

This algorithm, which is implemented in the publicly available Python package *emcee* [51], utilizes an ensemble of K "walkers," each representing a state $\mathbf{p}_k$ in the N -dimensional parameter space. New candidate positions are generated using a "stretch move," where the position of one walker $\mathbf{p}_k$ is updated based on the position of another randomly selected walker $\mathbf{p}_j$ from the ensemble [51]:

$$\mathbf{p}' = \mathbf{p}_j + Z(\mathbf{p}_k - \mathbf{p}_j) \quad (2.54)$$

In this expression, Z is a random variable drawn from a specific scale distribution $g(z)$, typically $g(z) \propto 1/\sqrt{z}$ over a defined interval $[1/a, a]$ where $a = 2$ [51]. This proposal mechanism is affine-invariant, meaning its performance is not degraded by linear transformations between parameters, which makes it a robust choice requiring minimal hyperparameter tuning [51].

The proposed move from $\mathbf{p}_k$ to $\mathbf{p}'$ is accepted stochastically with a probability q that ensures the resulting chain correctly samples the target distribution [51]:

$$q = \min\left(1, z^{N-1} \frac{P(\mathbf{p}'|\mathbf{d}_{obs})}{P(\mathbf{p}_k|\mathbf{d}_{obs})}\right) \quad (2.55)$$

where z is the specific value drawn for the random variable Z .

2.4.2. THE *geoMCMC* LIBRARY

To facilitate the application of this MCMC technique to planetary geophysical investigations, the *geoMCMC* Python library has been developed as part of this research. Built upon the *emcee* implementation of the Goodman & Weare sampler, *geoMCMC* provides a tailored framework for coupling forward models of planetary interiors (e.g., for calculating mass, moment of inertia, and Love numbers) with the affine-invariant sampling algorithm. The library is designed to streamline the setup and execution of Bayesian inversions to constrain interior structure parameters from geodetic and geophysical observations. Further technical details regarding the *geoMCMC* library are provided in [Appendix D](#).

The ensemble of samples generated by the MCMC process provides an effective map of the posterior probability distribution $P(\mathbf{p}|\mathbf{d}_{obs})$. A detailed analysis of these samples allows for the robust determination of the most probable values and credible intervals (uncertainties) for each interior parameter (e.g., core radius, ocean density, shell viscosity) and reveals potential correlations between them, thereby yielding comprehensive constraints on the internal structure of the investigated planetary body.

2.4.3. APPLICATIONS IN PLANETARY INTERIOR MODELING

The Bayesian MCMC framework, particularly with affine-invariant ensemble samplers, has become a powerful and standard tool for constraining the interior structure of

planetary bodies by integrating diverse geophysical datasets. Its applications span a wide range of Solar System targets, leveraging data from numerous missions. For instance, early applications included studies like Rivoldini *et al.* [52], who used geodetic data (mass, moment of inertia, and tidal Love number k_2) from radio science measurements within an MCMC framework to constrain Mars' interior structure, inferring properties such as the liquid state, size, and sulfur content of its core. Similarly, Matsuyama *et al.* [53] inferred the Moon's interior structure using a parallel tempering MCMC algorithm constrained by mass, moment of inertia, and tidal Love numbers (k_2 and h_2) derived from GRAIL, LLR, and LOLA data. For the Saturnian system, analyses of Cassini radio tracking data for Titan by Goossens *et al.* [12] and along with modeling studies for Enceladus such as Beuthe, Rivoldini, and Trinh [54] and refined estimates by Park *et al.* [14] have employed MCMC to explore interior models consistent with gravity, shape, and libration data, thereby constraining their hydrosphere properties.

Beyond the analysis of existing data, MCMC inversion also serves as a critical tool for designing future geophysical investigations and quantitatively assessing their potential scientific return. By using synthetic datasets based on predicted instrument performance and mission scenarios, researchers can optimize measurement strategies and establish robust science requirements. For example, preparatory studies for future Enceladus orbiters have used MCMC simulations to predict the constraints achievable on its core, ocean, and shell properties [48, 49]. Likewise, Cascioli *et al.* [47] employed MCMC with simulated Europa Clipper data to show that incorporating the full gravity field spectrum alongside mass and moment of inertia significantly enhances the ability to constrain Europa's hydrosphere thickness. Such simulation-based MCMC analyses are fundamental for optimizing mission design and validating the scientific capabilities of gravity investigations before launch.

Collectively, these examples underscore the central role of MCMC methods in planetary geophysics. Whether interpreting acquired measurements or designing future experiments, this probabilistic framework enables the robust integration of diverse geophysical datasets and the rigorous exploration of complex interior structure models, ultimately providing quantitative constraints on otherwise inaccessible planetary properties.

2.5. CONCLUSIONS

This chapter reviewed the theoretical groundwork for the geophysical investigation of planetary bodies, focusing specifically on the methodologies relevant to icy satellites. We have reviewed the fundamental principles that link observable geophysical quantities to internal structure and dynamics. As detailed in Sec. 2.2, the external gravity field—described by spherical harmonics and the tidal response quantified by Love numbers—is a primary probe of a body's mass distribution and rheology. Precise Orbit Determination (POD) of orbiting spacecraft, which relies on radiometric tracking data, provides the means to measure this gravity field (Sec. 2.2.2). We also discussed how analyzing a body's rotational state (Sec. 2.3) yields complementary constraints. In particular, longitudinal librations (Sec. 2.3.1)

and the spin-axis obliquity (Sec. 2.3.2) show distinct sensitivities to the moments of inertia and the decoupling of internal layers; these rotational dynamics are typically measured through landmark tracking with spacecraft imagery (Sec. 2.3.3).

To translate these diverse geophysical measurements into physical constraints on interior parameters, one must solve an inverse problem. For this purpose, we introduced the state-of-the-art computational framework of Bayesian inference coupled with Markov Chain Monte Carlo (MCMC) methods (Sec. 2.4). This framework enables the robust integration of multiple data types—such as gravity coefficients, Love numbers, and libration amplitudes—and provides not only best-fit models but also a rigorous characterization of parameter uncertainties and correlations, which is essential for addressing the inherent non-uniqueness of the inverse problem.

In summary, the suite of tools reviewed in this chapter—spanning gravity field theory, rotational dynamics, POD, landmark tracking, and MCMC inversion—forms the foundation for the work presented in this PhD thesis. Building upon this theoretical basis, the following chapter will shift to practical applications. We will present numerical simulations that leverage these techniques to evaluate and optimize the design of gravity science experiments. These case studies will assess the achievable accuracy in key geophysical parameters under realistic mission constraints, thus bridging the gap between theoretical models and scientific return.

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3

GRAVITY SCIENCE CASE STUDIES

Parts of this chapter have been presented at the Galilean Moons Workshop held in Marseille on Oct 24 – 26, 2023 by Filice et al. (How well can we measure Io's librations, tides and obliquity from the upcoming Juno flybys?) and at Uranus Flagship: Investigations and Instruments for Cross-Discipline Science Workshop 2023 (July 25–27, 2023, Pasadena, California/Virtual) by E. Mazarico et al. (Gravity science with the Uranus orbiter).

3.1. INTRODUCTION

FOLLOWING the theoretical framework established in [Chapter 2](#), this chapter applies these principles to three distinct case studies in planetary science. We utilize the *ROB-MONTE* software library, developed as part of this research, to demonstrate the typical workflow of a gravity science investigation, moving from performance verification and scientific assessment to detailed experiment optimization.

The chapter is organized as follows. We first conduct an analysis of the Cassini-Titan radio science data in [Sec. 3.2](#). The primary objective here is to validate the *ROB-MONTE* library by replicating published gravity field results, thereby establishing a performance benchmark for the subsequent investigations. Second, in [Sec. 3.3](#), we assess the potential for ancillary science from an ongoing mission by investigating the Juno encounters with Io. This case study addresses whether radiometric data from Juno's flybys could, on their own, provide meaningful constraints on Io's interior structure. The final investigation, presented in [Sec. 3.4](#), concerns the Uranus Orbiter and Probe (UOP) mission, which is currently in its preliminary design phase. In light of this, our analysis shifts from assessing a predefined trajectory to performing a focused optimization. We explore how strategic tour design can significantly enhance the determination of Uranus's deep interior structure, a key scientific objective. A comparison of our findings with results from the planetary science team at NASA's Goddard Space Flight Center offers an additional layer of validation for our approach.

These three studies—spanning validation, assessment, and optimization—collectively exemplify the conventional workflow of a gravity science investigation. The Io and UOP analyses yield valuable scientific insights for current and future missions, while the Cassini study provides a crucial methodological benchmark. This combined demonstration provides the necessary context for the novel, science-driven design framework introduced in [Chapter 4](#).

3.2. CASSINI RADIO SCIENCE EXPERIMENT AT TITAN

The Cassini Radio Science Subsystem (RSS) experiment conducted at Titan stands as a cornerstone case study in the geodetic investigation of icy satellites and provides a clear illustration of the methods, challenges, and scientific potential inherent in gravity science [\[1\]](#). Throughout its mission, the experiment yielded progressively refined constraints on Titan's interior structure. The primary scientific objective of the multi-flyby campaign was to characterize Titan's static gravity field and, crucially, to detect its time-variable tidal response to Saturn's gravitational pull [\[2, 3\]](#). This tidal response, which is quantified by the degree-2 Love number k_2 , serves as a direct probe of the interior's deformability and provided the first strong evidence for a global subsurface liquid water ocean. Subsequent analyses of the complete Cassini dataset, in addition to refining the gravity field to higher degrees, have also yielded noteworthy, divergent estimates for the k_2 Love number; these differing k_2 values have, in turn, led to different interpretations of Titan's interior. Whereas the analysis by Durante *et al.* [\[4\]](#) yielded a relatively high Love number ($k_2 \approx 0.62$), suggesting a

high-density ocean or a partially viscous deep interior, a re-analysis by Goossens *et al.* [5] found a significantly lower value ($k_2 \approx 0.38$). They argue this lower value is more consistent with a water-or-ammonia-based ocean, attributing the discrepancy to differences in the underlying tidal models used.

The experiment utilized a set of ten flybys, with nine of these being dedicated gravity science passes conducted between February 27, 2006 (T11), and August 10, 2016 (T122) [4, 5]. Given this extensive time interval, a multi-arc data analysis strategy is required, as detailed in Sec. 2.2.2.5. In such an approach, each flyby is treated as an independent orbital arc for which local parameters, like the spacecraft state vector, are estimated. These arcs are then integrated into a global solution to solve for common parameters, such as the gravity field coefficients and the tidal Love number. A fundamental challenge in these experiments is the robust modeling of all non-gravitational forces acting on the spacecraft, as any unmodeled acceleration directly corrupts the gravity signal and can bias the estimated scientific parameters. For Cassini, these forces included solar radiation pressure, anisotropic thermal emissions from the onboard Radioisotope Thermoelectric Generators (RTGs), and atmospheric drag during low-altitude encounters with Titan's dense atmosphere [2–5]. Consequently, an accurate estimation of formal errors requires a comprehensive covariance analysis that accounts not only for measurement noise but also for the uncertainties in these force models and in the ephemerides. The Cassini-Titan investigation is therefore a complete example of a gravity science experiment at an icy moon, encompassing the full range of analytical complexities—from multi-arc processing to precise force modeling—as discussed in Chapter 2.

This section is dedicated to the validation of the simulation and covariance analysis pipeline developed within the *ROB-MONTE* library, which we presented in Sec. 2.2.2.4. The analysis herein does not aim to produce new scientific interpretations of Titan's interior; rather, the well-documented Cassini-Titan gravity experiment serves as an ideal benchmark case. We achieve this validation by conducting a full numerical simulation of the experiment to demonstrate that our *ROB-MONTE* library can successfully reproduce the published formal errors from the real data analyses. Accordingly, the subsequent subsections will detail the simulation settings and present the validation results, demonstrating how our simulation reproduces the formal errors of the estimated gravity field parameters reported by Durante *et al.* [4] and Goossens *et al.* [5].

3.2.1. SIMULATION SETTINGS

A rigorous analysis of the publicly available RSS observational data is hampered by the fact that a detailed spacecraft macromodel is not publicly available. In its absence, we employ a simplified cannonball model for our simulation, where the spacecraft is represented by a sphere with a radius equal to its largest dimension, excluding the magnetometer boom. Although this model is sufficient for a self-consistent simulation, it would be inadequate for analyzing real data, where precise calculations of accelerations from SRP and atmospheric drag are essential. Recreating a high-fidelity macromodel would be a substantial undertaking, tangential to the primary objective of this work. Given that our goal is to validate the data

processing and covariance analysis pipeline, we therefore chose a full numerical simulation of the experiment as a more direct and controlled approach.

To circumvent the limitation of the simplified model, we generated synthetic two/three-way Doppler observables (X-band uplink; X or Ka-band downlink) for the ten Titan flybys using the *ROB-MONTE* library, with measurement epochs identical to the real RSS data. We perform the validation of the library against two distinct reference solutions. The primary setup emulates the analysis of Durante *et al.* [4], a task which is facilitated by the use of the same core POD software, MONTE [6]. However, our simulation framework also permits a self-consistent validation against the results of Goossens *et al.* [5]. Although they employed a different software and inversion strategy, we can perform a comparable covariance analysis by generating synthetic data based on their specific dynamical models and demonstrating that our pipeline successfully recovers the expected formal errors.

The experiment's design, which we summarize in Table 3.1, is characterized by a varied set of flyby geometries [4, 5]. During nine dedicated passes, the spacecraft's High-Gain Antenna (HGA) was oriented to enable a direct-to-Earth radio link. A tenth pass (T110) was an opportunistic measurement taken with the Low-Gain Antenna (LGA), which proved scientifically valuable by providing coverage over Titan's northern high latitudes. The flyby altitudes ranged from $\approx 1300\text{km}$ to $\approx 3600\text{km}$, and the encounters were strategically distributed throughout Titan's orbit to ensure a broad sampling of its mean anomaly. This combination of spatial and temporal coverage was essential for decorrelating the static gravity field coefficients and for accurately resolving the time-variable tidal signal [4, 5].

We process the simulated data using a multi-arc square root information filter, as detailed in Sec. 2.2.2.4. The data weighting strategy differs depending on which reference solution is being replicated. For the Durante *et al.* [4] scenario, we use a 60 s integration time and weigh the data by noise levels ranging from 0.017 mms^{-1} to 0.068 mms^{-1} . For the Goossens *et al.* [5] scenario, we adopt a 10 s integration time and weigh the data using their final calibrated noise values, which were derived from a Variance Component Estimation (VCE) technique. As shown in Table 3.1, these noise values, when scaled to a 60 s integration time for comparison, range from approximately 0.043 mms^{-1} to 0.083 mms^{-1} .

The solution parameters, detailed in Table 3.2, are categorized as either local or global. Both reference analyses estimate local parameters for each arc, including the spacecraft state, an SRP scaling factor, and atmospheric drag coefficients for the two lowest altitude flybys. The apriori constraints on these parameters, however, differ significantly. For instance, while Durante *et al.* [4] constrained the Cassini state vector, Goossens *et al.* [5] left it unconstrained. A particularly notable difference lies in the local estimation of Titan's ephemeris: Durante *et al.* [4] applied tight constraints of 5 m, 50 m and 500 m in the radial, transverse, and normal directions, respectively, whereas Goossens *et al.* [5] used much looser constraints of 5 km on position and 100 ms^{-1} on velocity. Global parameters, which are solved for using data from all ten flybys, include Titan's GM, the gravity field up to degree and order 5, the tidal Love number k_2 , and RTG-induced accelerations.

The simulation parameters and estimation strategies detailed above provide the

complete framework for validating the *ROB-MONTE* library against the two reference solutions. The following section presents and discusses the results from these simulations, focusing on the direct comparison of our retrieved covariance analyses with the published findings.

Table 3.1.: Titan flybys dedicated to the Cassini RSS experiment and the different Doppler noise levels used in the reference analyses, Durante *et al.* [4] and Goossens *et al.* [5]. Values used by Goossens *et al.* [5] are scaled to an integration time of 60 s for comparison.

Flyby	Epoch	Altitude [km]	Doppler noise [7] [mm s ⁻¹]	Doppler noise [5] [mm s ⁻¹]
T011	27 Feb. 2006	1812	0.017	0.043
T022	28 Dec. 2006	1297	0.024	0.054
T033	29 June 2007	1933	0.028	0.064
T045	31 July 2008	1614	0.068	0.083
T068	20 May 2010	1397	0.025	0.050
T074	18 Feb. 2011	3651	0.063	0.083
T089	17 Feb. 2013	1978	0.033	0.043
T099	06 Mar. 2014	1500	0.022	0.046
T110	16 Mar. 2015	2274	0.064	0.075
T122	10 Aug. 2016	1698	0.061	0.080

Table 3.2.: Estimated parameters and apriori uncertainties for the two simulation cases. The nominal values for GM and k_2 are shown for the Durante *et al.* [4] / Goossens *et al.* [5] cases, respectively. The SPK used for the spacecraft and Titan's initial state are the latest release from the navigation team associated with the RSS data in the NASA Planetary Data System (PDS)*. Titan's position and the acceleration of the Radioisotope Thermoelectric Generators (RTGs) are expressed in the Radial, Normal and Transverse (R,T,N) body-fixed frame.

Parameter	Value	apriori uncertainty [4]	apriori uncertainty [5]	Type
Cassini State	SPK	2 km, 2 cm/s	Unconstrained	Local
Titan position	SPK	5 m (R), 50 m (T), 500 m (N)	5 km (pos), 100 m/s (vel)	Local
SRP scaling factor	1	0.25	0.10	Local
Atmospheric drag coefficient	2.40	0.14	0.06	Local**
RTGs acceleration	3e-9 m/s ²	1.4e-13 m/s ²	0.001 (scaling factor)	Global
GM (km ³ /s ²)	8978.1383 / 8978.1269	Unconstrained	Unconstrained	Global
C_{Im}, S_{Im} with $l = 2, \dots, 5$; $m = 0, \dots, l$	See respective papers	Unconstrained	Unconstrained	Global
Love Number k_2	0.616 / 0.375	Unconstrained	Unconstrained	Global

* https://pds-atmospheres.nmsu.edu/data_and_services/atmospheres_data/Cassini/inst-rss.html.

** Applied only to flyby T022 and T068.

3.2.2. RESULTS AND CONCLUSIONS

In Fig. 3.1, we present the results of our covariance analysis for the simulated gravity field, showing the RMS gravity spectrum of the estimated coefficients, as defined by Eq. (2.6), compared against the two reference solutions.

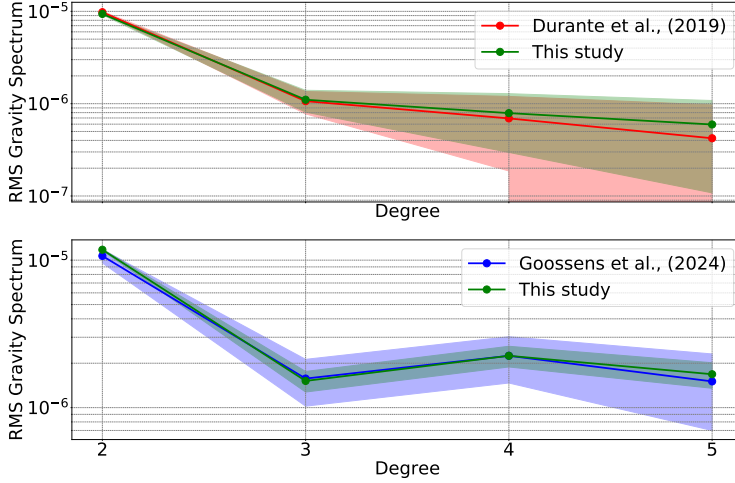


Figure 3.1.: Estimated Titan's Root Mean Square (RMS) gravity spectrum. Solid lines represent the RMS spectrum, while shaded areas show 1σ formal uncertainty bands. (Top Panel) Comparison with Durante *et al.* [4], showing excellent agreement in both the spectrum and uncertainties. (Bottom Panel) Comparison with Goossens *et al.* [5], showing agreement in the estimated spectrum but smaller formal errors in this study, likely due to differences in estimation methodology (e.g., Variance Component Estimation used in [5] but not here).

The top panel of Fig. 3.1 compares our simulation (green) with the results of Durante *et al.* [4] (red). The close alignment of the central values of the two spectra confirms that our estimation process correctly converges to the true gravity field. A critical aspect of validating an orbit determination pipeline is assessing the correctness of its calculated uncertainties. In this context, we distinguish between the "true error"—the difference between our estimated parameters and the known nominal values used to generate the synthetic observables—and the "formal error," which is the statistical 1σ uncertainty derived from the covariance analysis. For a robust solution, the true error must be statistically consistent with, and ideally encompassed by, its formal error. As the figure shows, our estimated 1σ uncertainty band (shaded green area) is extremely close to that of the reference solution (difference at the percentage level for all degrees), demonstrating that our pipeline correctly models the full set of uncertainties. Furthermore, we find that the true errors (the difference between the green and red lines) fall well within our estimated

1σ formal error band. This is a crucial validation result, confirming that the formal errors produced by the *ROB-MONTE* library are both precise and accurate, and provide a realistic measure of the true uncertainty.

In the bottom panel of Fig. 3.1, we show the comparison with the Goossens *et al.* [5] solution (blue). While our estimation again converges correctly to the true values of the gravity coefficients, we find a notable discrepancy in the formal errors. Our estimated uncertainties are approximately 5.4 times smaller for degree 2 and 1.4 times smaller for the higher degrees. We attribute this difference to our estimation approach: while we used their calibrated data noise, we did not employ the VCE technique in the inversion, which would have scaled the covariance matrix from which the formal errors are derived (see the detailed discussion in [5]).

Collectively, these results successfully validate the *ROB-MONTE* simulation and covariance analysis pipeline. The ability to accurately reproduce the formal errors from a complex, multi-arc gravity experiment like the Cassini investigation at Titan provides high confidence in the library's application to future mission design and data analysis, which we will explore in the subsequent chapters.

3.3. CONSTRAINING IO'S INTERIOR STRUCTURE

The extreme volcanic activity of Io, a direct consequence of intense tidal heating primarily driven by Jupiter's gravity and sustained by orbital resonances with Europa and Ganymede, makes it a key target for geophysical investigation. A primary scientific objective is to determine whether this intense internal heating is sufficient to maintain a global subsurface magma ocean (see e.g., [8–12]). Making such a determination is challenging and requires precise geodetic measurements. Recent multi-mission data analyses by Park *et al.* [13] yielded a crucial constraint on Io's interior through the measurement of its tidal Love number, $k_2 = 0.125 \pm 0.047$ (1σ). This measurement provides strong evidence against a shallow global magma ocean, indicating consistency with a mostly solid mantle interior. Notably, the achieved 1σ precision satisfies the 0.05 theoretical target accuracy that was previously identified as necessary to distinguish between such interior models based on k_2 [14]. While this result addresses the shallow ocean hypothesis, a further refinement of our understanding of Io's structure—such as characterizing mantle properties or distinguishing between a solid interior and a potential deep magma ocean—still relies on maintaining or improving the precision of k_2 , in combination with potential measurements of its rotational state. Theoretical studies had already established clear measurement requirements to definitively address the shallow ocean hypothesis using rotational dynamics as well: target accuracies included sub-300 m precision for the libration amplitude [15] and sub-meter precision for obliquity (equivalent to a few tens of milli-arcseconds or 0.16 m at the surface) [16]. These targets remain relevant benchmarks for achieving a more detailed characterization of Io's interior, beyond the constraints provided by the current k_2 measurement.

The objective of this study, conducted prior to the publication of the results by Park *et al.* [13], is to quantitatively assess the feasibility of achieving these demanding measurement accuracies using only data from the two planned Juno flybys of Io. To

this end, we perform a comprehensive covariance analysis utilizing the *ROB-MONTE* library, as described in [Sec. 2.2.2.4](#). We evaluate if the planned radiometric tracking, when supplemented by a hypothetical optical imaging campaign with the JunoCam instrument [17], could successfully constrain Io's interior. In contrast to the multi-mission, multi-decade approach of Park *et al.* [13], our work focuses on the scientific potential of a standalone, Juno-only dataset, allowing for a direct assessment of the inherent limitations of a two-flyby observing geometry.

3

3.3.1. MEASURING LIBRATIONS, TIDES AND OBLIQUITY WITH JUNO

Within the framework of its extended mission, NASA's Juno spacecraft—in orbit around Jupiter since 2016 [18]—executed two close flybys of Io, the innermost Galilean moon, with the specific objective of investigating its internal structure and dynamics [13]. The collection of precise Doppler observables for the Juno Gravity Science Experiment relies upon a dual-frequency radio link, which combines Ka-band (32 GHz) and X-band (8.4 GHz) transmissions between the spacecraft and NASA's Deep Space Network (DSN) [7, 19, 20]. Throughout a typical gravity pass, the spacecraft's High-Gain Antenna (HGA) is maintained in a continuous Earth-pointing attitude.

Additionally, the spacecraft payload includes JunoCam, a visible-light camera whose primary function is public outreach [17]. Despite this, JunoCam could in principle provide valuable optical measurements for constraining Io's rotational state when the satellite is situated within its field of view. As we previously discussed in [Sec. 2.3.3](#), such optical navigation and stereophotogrammetry techniques are instrumental for refining a body's orientation, spin rate, and forced librations. The operational mode of JunoCam is that of a "push-broom" camera, which generates images as the spacecraft's spin—a consequence of its spin-stabilized attitude control [18]—sweeps the sensor across the target. Its shutter time of 3.2 ms is composed of 100 individual exposures, each lasting 3.2 s [17].

The characteristics of the two gravity science passes are summarized in [Table 3.3](#). Both flybys were executed at comparable Closest Approach (CA) altitudes of approximately 1500 km and featured a favorable edge-on configuration with β angles near 90°. Such a geometry is particularly advantageous for radiometric measurements, as it maximizes the Doppler sensitivity to gravitational perturbations. The resultant ground tracks, depicted in [Fig. 3.2](#), yielded substantial hemispherical coverage, with *PJ57* targeting Io's northern hemisphere and *PJ58* its southern. Furthermore, the large SEP angles detailed in [Table 3.3](#) confirm that both encounters occurred far from solar conjunction, a geometry with minimal interference from solar plasma on the radiometric link.

Table 3.3.: Characteristic of the Juno's flybys of Io at time of closest approach.

PJ#	Epoch	Height [km]	Relative velocity [km/s]	Latitude [deg]	Longitude [deg]	SEP [deg]	Mean Anomaly [deg]	β [deg]
57	30-DEC-2023 08:37:09 ET	1500.02	30.05	63.75	−94.61	117.17	249.37	93.05
58	03-FEB-2024 17:49:44 ET	1500.09	30.10	−31.80	−36.89	83.16	276.04	92.91

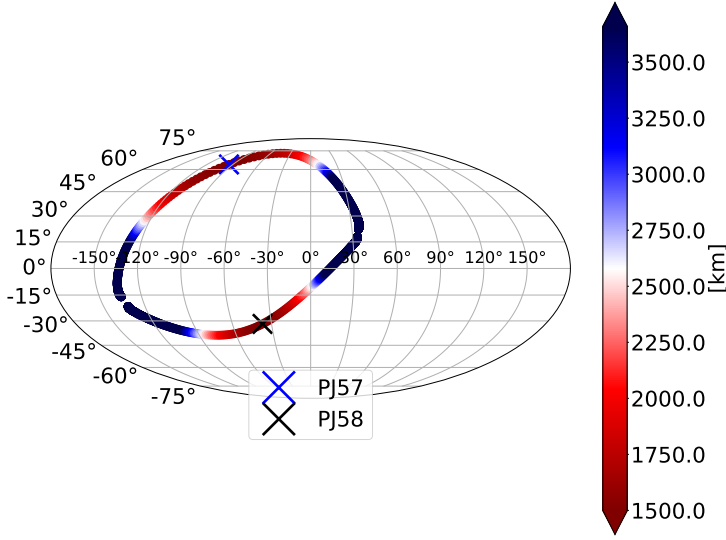


Figure 3.2.: Ground tracks of the Juno spacecraft during its close flybys of Io (*PJ57* and *PJ58*), plotted using a Mollweide projection in Io's body-fixed frame. The tracks cover ± 30 minutes around the closest approach point (marked by 'x'). The color scale indicates the spacecraft's altitude above Io's surface in kilometers.

A critical requirement for the determination of the libration amplitude and tidal response is the adequate temporal sampling of Io's orbital phase. Given Io's 1:1 spin-orbit resonance, both its tidal and libration signals exhibit a periodicity matching its orbital period of approximately 1.77 d. However, the temporal coverage provided by the Juno flybys, illustrated in Fig. 3.3 relative to the modeled signals from Eqs. (2.7) and (2.30), presents a notable limitation. The two encounters, *PJ57* and *PJ58*, transpired at similar Io mean anomalies of 249.37° and 276.04° , respectively. Consequently, this restricted sampling, combined with CA distances for all other orbits exceeding 10000 km that limit the opportunities for additional sensitive radiometric or optical observations of Io, thereby posing a significant challenge to the comprehensive characterization of these periodic phenomena.

3.3.2. COVARIANCE ANALYSIS SETUP

A covariance analysis is conducted to determine the expected uncertainties on Io's libration amplitude g , obliquity ε , and tidal Love number k_2 from simulated Juno data. The primary objective of this investigation is to assess whether combining the planned radiometric measurements with a dedicated, albeit hypothetical, optical observation campaign could achieve the target accuracies required to constrain Io's interior structure, as detailed in Sec. 3.3. This section, therefore, describes the

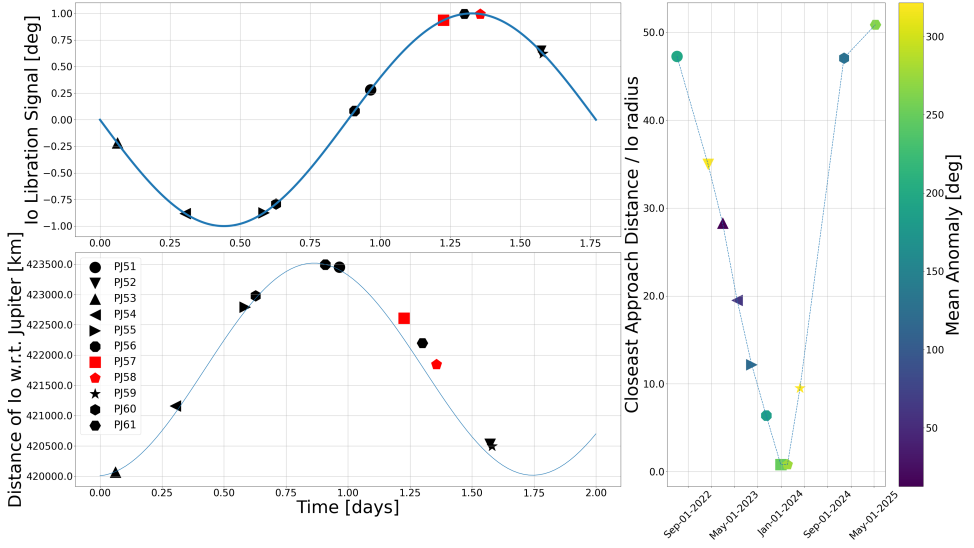


Figure 3.3.: Temporal sampling challenges for Io's libration and tidal signals during Juno's extended mission. Left Panels: The top panel shows Io's predicted longitudinal libration signal (in degrees), and the bottom panel shows the Io-Jupiter distance (proxy for tidal forcing signal), both over one Io orbital period ($T \approx 1.77$ d). Symbols indicate the timing of Juno's perijoves (PJ57 and PJ58) relative to Io's orbital phase, highlighting that the two close flybys (PJ57 - red square, PJ58 - red star) occur at similar mean anomalies. Right Panel: Closest approach distance between Juno and Io (normalized by Io's radius equal to 1821.49 km) for each perijove, color-coded by Io's mean anomaly. This illustrates the limited opportunities for close observations across Io's full orbit.

simulation parameters for both planned and hypothetical measurements, the filter configuration, and the subsequent sensitivity analyses.

We simulate the planned gravity science experiment using two/three-way Doppler data with direct-to-Earth Ka/Ka and X/X radio links, for which we assume nominal noise levels of $10 \mu\text{ms}^{-1}$ and 0.1 mms^{-1} , respectively, at a 60 s integration time [7, 19, 20]. While the nominal tracking window is a conservative two hours per flyby (± 1 h centered on closest approach), a sensitivity analysis was performed by varying this from one to ten hours and by testing a degraded link scenario (X/Ka and X/X only), where the X/Ka link is assumed to have the same noise level as the X/X.

In order to investigate the potential benefits of a multi-instrument approach, we simulate a notional optical campaign, given that none is officially planned for this specific purpose with JunoCam. It is important to note that for this covariance analysis, which is designed to explore theoretical sensitivities and establish requirements, we assume ideal conditions whereby the necessary landmarks can be observed. This simplification consequently allows us to assess the potential

contribution of optical data without being limited a priori by the specific, complex geometric constraints of just two flybys. The notional simulated strategy under this assumption involves four images per flyby (two ingress, two egress), capturing 10 distinct, randomly distributed landmarks hypothetically visible within the camera's field of view. The optical datasets utilized in past planetary science missions provide valuable context for this approach. For instance, Konopliv *et al.* [21] analyzed optical data from the Dawn mission at Ceres, where 300 to 400 pictures per week were acquired during high-altitude and low-altitude mapping orbits (HAMO-LAMO), with approximately 3 to 30 landmarks observed per picture. A similar approach is planned for the Europa Clipper mission, which aims to acquire around 20 pictures per flyby with approximately 6 landmarks per picture [22]. Oberst and Schuster [23], in contrast, analyzed optical observations of Io utilizing data from the Galileo and Voyager missions, where Galileo captured 402 landmarks across 26 pictures and Voyager captured 158 landmarks in 40 pictures. Informed by these precedent studies, we tested a range of scenarios by varying the number of pictures per flyby from 2 to 20, and the number of observed landmarks from 5 to 50. The effect of different Control Point Networks (CPNs) was also explored by confining the randomly distributed landmarks to either the equatorial region (latitudes between $\pm 20^\circ$) or the polar regions (latitudes poleward of $\pm 70^\circ$), as shown in Fig. 3.4. The optical measurements are simulated by modeling JunoCam as a pinhole camera with no distortion, as detailed in Sec. 2.3.3.1, assuming a nominal surface feature detection uncertainty of 2 pixels [17]. However, because JunoCam is a "push-broom" sensor, significant image processing is required to construct a final image from the raw data strips. Advanced photogrammetric techniques, such as super-resolution, sub-pixel feature extraction, and deconvolution algorithms, can substantially improve the effective resolution and centroiding precision of surface features (see e.g., [24–27]). To account for the potential gains from such processing, we conduct a sensitivity analysis on the optical data noise, varying the uncertainty from a best-case 0.1 pixels to a more pessimistic 3 pixels.

To ensure consistency, all simulations employ identical dynamical models. The planetary ephemerides and gravity parameters utilized in this study are from the de440 solution, whereas the Jupiter system ephemerides and gravity parameters are from the jup380s solution. These are consistent with the predicted orbit of Juno used in this study (spk_ref_211112_251021_211112). Io's GM and quadrupole gravity coefficients (J_2 and C_{22}) are set equal to the measured coefficients from the Galileo mission [28]. We use the IAU rotation models for Jupiter and Io [29]. The librations of Io are modeled as a forced oscillation with amplitude g at the orbital frequency, while the Love number k_2 models Io's tidal response to Jupiter and the main Jovian satellites (see Eqs. (2.7) and (2.30)). We set both amplitudes to zero in our simulations, as no prior measurements of Io's libration or tidal response exist. This is a valid approach for a covariance analysis, because the resulting uncertainties are determined by the partial derivatives of the measurements, which for these parameters are independent of the nominal value used. Io's obliquity ε is derived from the spin axis right ascension α and declination δ predicted by the IAU rotation model, using Eq. (2.36).

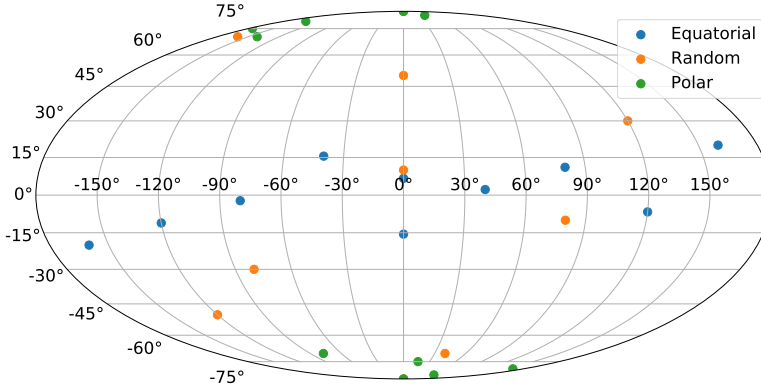


Figure 3.4.: The three different Control Point Networks (CPNs) tested in the sensitivity analysis, shown in Io's body-fixed frame using a Mollweide projection. Each CPN represents a different strategy for distributing simulated surface landmarks observed in optical data: Equatorial (landmarks concentrated between $\pm 20^\circ$ latitude), Random (landmarks randomly distributed across the surface), and Polar (landmarks concentrated poleward of $\pm 70^\circ$ latitude).

The data are processed using a multi-arc orbit determination and covariance analysis filter as described in [Sec. 2.2.2](#). For each arc, the spacecraft state (position and velocity) and Io's GM are estimated with loose a priori uncertainties (1 km and 1 ms^{-1} , and $100\text{ km}^3\text{ s}^{-2}$, respectively) so as not to strongly constrain the solution. The geodetic parameters of interest—libration amplitude g , tidal Love number k_2 , and obliquity ε —are estimated as unconstrained global parameters. To ensure representative uncertainties, it is essential to account for correlations between parameters. As the simulated tidal model directly influences the degree-two gravity field, the zonal coefficient J_2 and tesseral coefficients C_{22} and S_{22} are always co-estimated with a loose a priori uncertainty of 1. When Io's pole orientation (α , δ) is estimated to determine obliquity, the gravity coefficients C_{21} and S_{21} , which are a direct representation of the pole's orientation in the gravity field (see [Sec. 2.2.1.1](#)), are not estimated. For the tidal response, only the real part of the Love number is estimated. We estimate a single, order-independent Love number ($k_2 = k_{20} = k_{21} = k_{22}$) because the sparse temporal and spatial sampling from only two flybys makes resolving individual order-dependent components of the tidal response impractical, a challenge noted in similar studies (e.g., Cappuccio *et al.* [30]). The reference epoch for estimated rotational angles is set to the start of the first flyby (PJ57) tracking window.

In order to determine a more realistic budget for the expected uncertainties, this nominal filter setup is augmented with additional error sources. We explore the effect of uncertainties in the higher degree and order gravity coefficients. To this end, synthetic gravity coefficients for degrees and orders from 3 to 5 are calculated by scaling the estimated gravity field of Titan [4]. This is done using the classical

scaling factor (see e.g. [31, 32])

$$SF = \left(\frac{GM_1}{GM_2} \right)^2 \left(\frac{R_2}{R_1} \right)^4 \quad (3.1)$$

where the subscripts 1 and 2 refer to the reference and scaled body masses M and radii R , respectively. Given that the limited surface coverage from only two flybys cannot provide sensitivity to these higher degrees, we constrain them by applying a tight a priori value derived from Kaula's rule, $K/(l(l+1))$, with $K_{Io} = SF * K_{Titan} = SF * 10^{-5}$ (where K_{Titan} is from [4]). Uncertainty in non-gravitational forces is modeled by fitting a solar radiation pressure scaling factor, using a conservative cannonball model with a 20 m reference radius and a 15% a priori uncertainty. To account for uncertainty in Io's ephemeris, its initial state is estimated on an arc-wise basis with an a priori uncertainty of 500 m in the radial, transverse, and normal (R, T, N) directions. Finally, uncertainties in landmark position and camera pointing are accounted for by estimating the landmark positions in the body-fixed frame (1 km a priori uncertainty in x, y, z) and the camera pointing orientation ($2''$ a priori uncertainty per camera frame axis). While Table 3.4 summarizes the comprehensive set of all parameters considered in this study, the specific subset estimated in any given analysis is a sub-combination of this list and will be explicitly stated when the corresponding results are discussed.

Table 3.4.: Estimated parameters and apriori uncertainties.

Parameter	Nominal Value	apriori Uncertainty	Type
Juno State (pos, vel)	SPK	1 km, 1 m s ⁻¹	Local
Io State (R, T, N)	Ephemeris	500 m	Local
SRP Scaling Factor	1.0	15 %	Local
Landmark Positions (L,M,N)	Random	1 km	Local
Camera Pointing (3 axes)	S/C Attitude (to keep Io in FoV)	$2''$	Local
GM	5959.91 km ³ s ⁻² [28]	100 km ³ s ⁻²	Global
J_2, C_{22} ($\times 10^{-6}$)	1859.5, 558.8 [28]	1.0	Global
C_{21}, S_{21}, S_{22}	0	1.0 (or fixed)	Global
C_{lm} (with $l = 3, 4, 5$; $m = 0, \dots, l$)	Scaled from Titan [4]	Kaula's Rule	Global
g	0	Unconstrained	Global
k_2	0	Unconstrained	Global
ϵ (α, δ)	IAU	Unconstrained	Global

3.3.3. NOMINAL PERFORMANCE ANALYSIS

To establish a performance baseline, we first analyze an optimistic filter configuration that uses a nominal observation strategy. This strategy consists of two hours of radiometric tracking and four optical images per flyby. In this baseline analysis, we test three distinct scenarios by solving for the key geodetic parameters. These scenarios co-estimate only the spacecraft state, Io's GM, and the degree-two gravity field (J_2, C_{22}, S_{22} , and also C_{21} and S_{21} when the frame is not estimated), while intentionally excluding the more comprehensive error sources that are discussed in

Sec. 3.3.2. The three scenarios solve for: (1) tides and Io's obliquity ($k_2 + \epsilon$), (2) tides and libration ($k_2 + g$), and (3) all geodetic parameters simultaneously ($k_2 + \epsilon + g$). The results, summarized in Table 3.5, demonstrate the significant challenge of constraining these geodetic parameters with the planned Juno flybys.

Table 3.5.: Combined estimated 1σ uncertainties for Io's obliquity (ϵ), tidal Love number (k_2), and libration amplitude (g). The values for ϵ and g are in meters. The radio column uses only radiometric data, radio+optical combines radiometric and optical data with the nominal Ka-band link, and radio(X)+optical uses a degraded X-band link.

Case	Parameter	Data Type		
		radio	radio+optical	radio(X)+optical
$k_2 + \epsilon$	σ_ϵ (m)	4407.87	479.85	501.50
	σ_{k_2}	0.66	0.64	5.25
	σ_g (m)	–	–	–
$k_2 + \epsilon + g$	σ_ϵ (m)	4408.00	743.72	754.48
	σ_{k_2}	0.66	0.64	5.25
	σ_g (m)	317929.11	1780.25	1855.38
$k_2 + g$	σ_ϵ (m)	–	–	–
	σ_{k_2}	0.67	0.67	5.41
	σ_g (m)	317929.11	976.60	976.64

Using radiometric data alone proves insufficient to constrain any of the parameters to their target accuracies. For instance, when solving for tides and frame parameters, the resulting uncertainty on the obliquity is 4408m and on the Love number k_2 is 0.66, which are orders of magnitude worse than the required 0.16m and 0.05 precision, respectively. A similar outcome is observed when solving for tides and libration, yielding a libration uncertainty of ≈ 318 km, far from the sub-300m precision needed. This result is not unexpected; radio science experiments relying solely on Doppler data from a few flybys are known to be ill-suited for determining a body's rotational state, which typically requires the long, continuous tracking arcs of an orbiting mission (see e.g., [4, 33–38]). The primary limitation in our case is the sparse temporal sampling from only two flybys, which fail to cover a sufficient fraction of Io's 1.77d orbital period. Because both the tidal and libration signals have periodicities locked to this orbital period, it is challenging to distinguish their signatures from the static gravitational field without more comprehensive phase coverage.

As expected, incorporating simulated optical data alongside the Ka-band Doppler measurements significantly improves the determination of the rotational parameters. In the $k_2 + \epsilon$ case, for instance, the obliquity uncertainty improves by almost an order of magnitude to 480m. However, this value is still far from the target precision required for meaningful geophysical constraints. A similar improvement is seen in

the $k_2 + g$ case, where the libration uncertainty is reduced to 977 m, though this remains more than three times larger than the target uncertainty. The Love number k_2 uncertainty, in contrast, sees no meaningful improvement and remains at ≈ 0.64 and ≈ 0.67 in the respective cases, as the optical data provide no direct sensitivity to the gravity field. Furthermore, when all parameters are estimated simultaneously ($k_2 + \varepsilon + g$), correlations between them degrade the overall solution, increasing the obliquity uncertainty to 744 m and the libration uncertainty to 1780 m. We also find that a degraded radio link (X-band) leads to a substantial loss of precision for the Love number k_2 , with its uncertainty increasing to 5.25. The rotation parameters, being driven by optical data, are only marginally affected (e.g., the libration uncertainty increases from 1780 m to 1855 m). It is therefore clear that even under the most optimistic assumptions of this baseline analysis, the combination of the planned radiometric tracking and a nominal optical campaign is insufficient to meet the scientific requirements for constraining Io's interior.

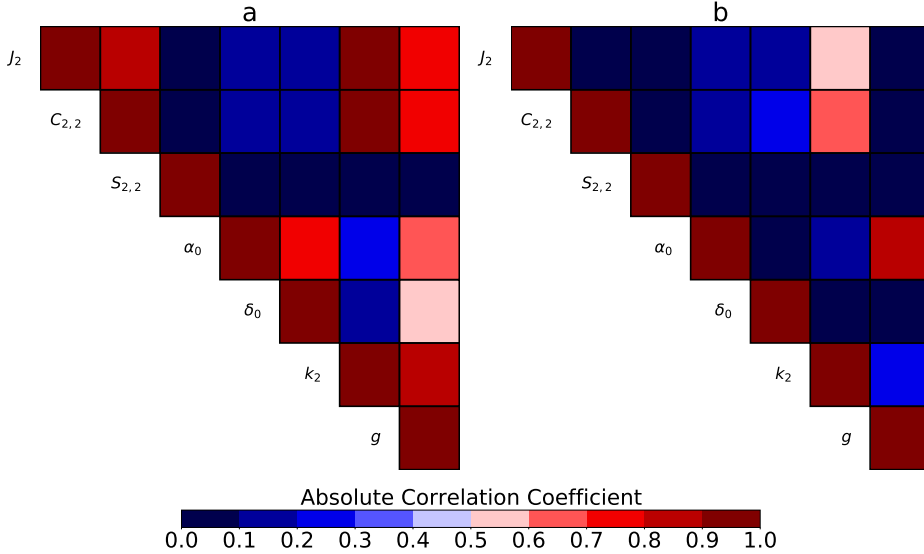


Figure 3.5.: Absolute correlation coefficient matrix for the estimated global parameters in the $k_2 + \varepsilon + g$ scenario. Panel (a) shows the results for the radiometric-only dataset, highlighting the strong correlations (coefficients near 1.0) between the rotational parameters (α_0 , δ_0 , g) and the gravity/tidal parameters. Panel (b) shows the results for the combined radiometric and optical dataset, demonstrating the significant decorrelation of the rotational parameters from the gravity field.

The parameter correlation matrix for the $k_2 + \varepsilon + g$ case, shown in Fig. 3.5, provides further insight into these results. With radiometric-only data (panel a), we observe extremely high correlations (absolute coefficients approaching 1.0), particularly between the libration amplitude and the gravity and tidal parameters

(J_2 , C_{22} , S_{22} , k_2). For example, the correlation between the libration amplitude (g , last column) and J_2 (first row) is ≈ 0.8 , and the correlation between g and k_2 nearly 1.0 (second to last row). This indicates that the filter cannot mathematically distinguish the small gravitational signature of a rotational variation from that of the static gravity field or the tidal response using only Doppler data, which directly explains the poor resulting uncertainties. Introducing optical data (panel b) fundamentally changes the structure of the correlation matrix. Since optical data are directly sensitive to the body's orientation, they effectively break the strong correlations between the libration amplitude and the gravity coefficients. For instance, the correlation between g and J_2 decreases from nearly 0.8 to less than 0.1, and similarly does the correlation between g and C_{22} . This decorrelation is the primary reason for the significant improvement observed in the libration uncertainty. However, as previously mentioned, optical data have no direct sensitivity to the gravity field and consequently cannot break the correlations that persist between the gravity-related parameters. The Love number k_2 thus remains highly correlated with the gravity field coefficients; in fact, its correlation with J_2 only decreases from approximately 0.9 to 0.6, while its correlation with C_{22} drops from approximately 0.9 to 0.7. This persistent high correlation explains why adding optical data offers no improvement for the Love number determination and, ultimately, why the overall solution remains far from the target accuracies.

3.3.4. SENSITIVITY ANALYSIS

Our preceding analysis demonstrated that the nominal observation strategy is insufficient to achieve the target accuracies, even under simplified conditions. We now systematically investigate the sensitivity of the solution to variations in the observational strategy and to the inclusion of a more realistic parameter set. For clarity, we present the results as a relative improvement factor, F , defined as the ratio of a reference uncertainty to the new uncertainty ($F = \sigma_{ref} g / \sigma$). An improvement factor greater than 1 therefore indicates a better solution. Our reference case is the $k_2 + \varepsilon + g$ scenario using the nominal radiometric and optical dataset, which yielded baseline uncertainties of 977 m for obliquity, 0.64 for k_2 , and 1780 m for libration. To meet the scientific targets, improvement factors of approximately 4650 for obliquity, 13 for k_2 , and 6 for libration are thus required. While the seemingly unreachable improvement factor for obliquity would make the $k_2 + g$ case a more logical reference, we use the combined $k_2 + \varepsilon + g$ case to consistently investigate the effects of all variations on all parameters of interest. This choice is justified because the k_2 uncertainty is nearly identical between the two cases, and the conclusions for libration can be readily scaled; its required improvement factor would be a more attainable 3 in the $k_2 + g$ case, rather than 6 (see Table 3.5). The following subsections explore the impact of the optical and radiometric observation strategies before we assess the effect of a more realistic filter configuration that accounts for key model uncertainties.

3.3.4.1. IMPACT OF OPTICAL OBSERVATION STRATEGY

The performance of the combined-data solution is strongly dependent on the quality and characteristics of the hypothetical optical campaign. We therefore investigate the sensitivity of our results to three key factors: the landmark detection noise, the geographic distribution of the landmarks, and the total quantity of optical data.

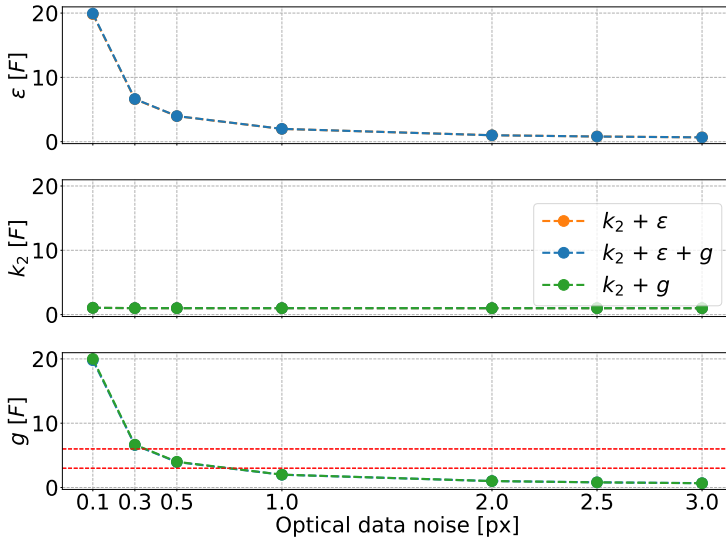


Figure 3.6.: Sensitivity of estimated parameter precision to optical data noise for simulated Juno flybys of Io. The plot shows improvement factor (F) for obliquity (ϵ), tidal Love number (k_2), and libration amplitude (g) as a function of the assumed landmark detection uncertainty in pixels (px). $F > 1$ indicates improvement relative to the reference case (nominal 2-pixel noise). Different colored lines represent different parameter estimation scenarios, combining radiometric and optical data: estimating k_2 and ϵ only ($k_2 + \epsilon$), estimating k_2 , ϵ , and g simultaneously ($k_2 + \epsilon + g$), and estimating k_2 and g only ($k_2 + g$). The red dashed lines in the bottom panel indicate the required improvement factors ($F = 3$ and $F = 6$) to meet the scientific target for libration amplitude under different scenarios.

As shown in Fig. 3.6, reducing the optical data noise has a pronounced impact on the determination of the rotational parameters. Improving the landmark detection precision from the nominal 2 pixels to just 0.3 pixels yields an improvement factor of approximately 7 for both libration amplitude and obliquity, which is sufficient to meet the target uncertainty for libration. A lower noise level of 0.1 pixels would further increase the improvement factor to nearly 20. The geographic distribution of landmarks also impacts the expected uncertainties (Fig. 3.7). Consistent with

previous work (e.g., Park *et al.* [22]), we find that concentrating landmarks in the equatorial region provides the best constraint on the libration amplitude, yielding an improvement factor of ≈ 1.25 over a random distribution. In contrast, obliquity shows a clear increasing trend as landmarks are moved from the poles toward the equator. The factor is at its minimum for a polar distribution, where it degrades to approximately 0.7. This suggests that determining the pole position benefits from data that captures the body's rotation across a wider range of latitudes, especially lower ones, to get a more robust result. As expected, and consistent with our correlation analysis, these variations in the optical data strategy have no effect on the determination of the Love number k_2 , for which the improvement factor remains close to 1.0 in all cases. This confirms that while optimizing the optical data strategy is critical for constraining the rotational parameters, it offers no benefit for the detection of tides.

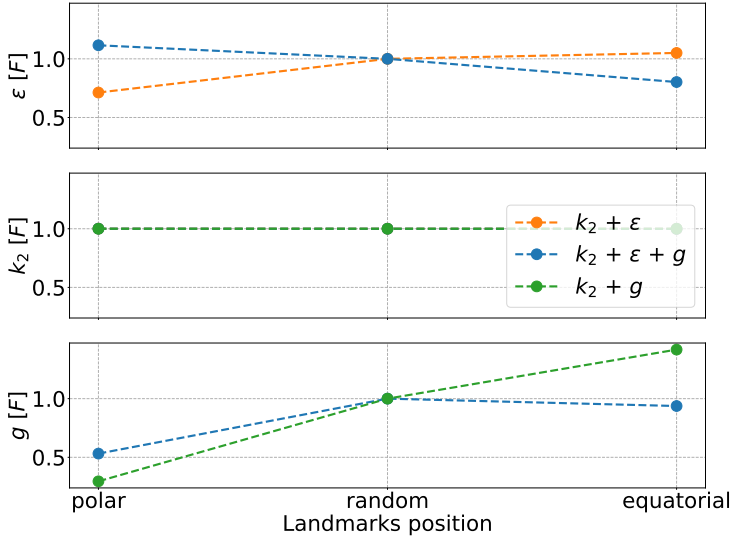
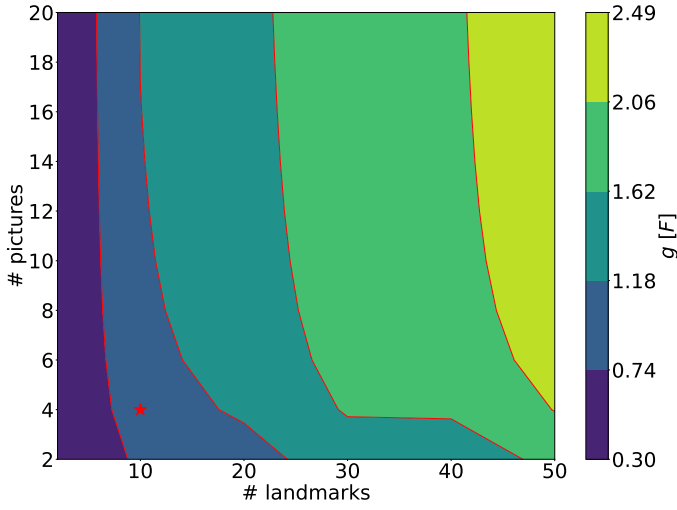
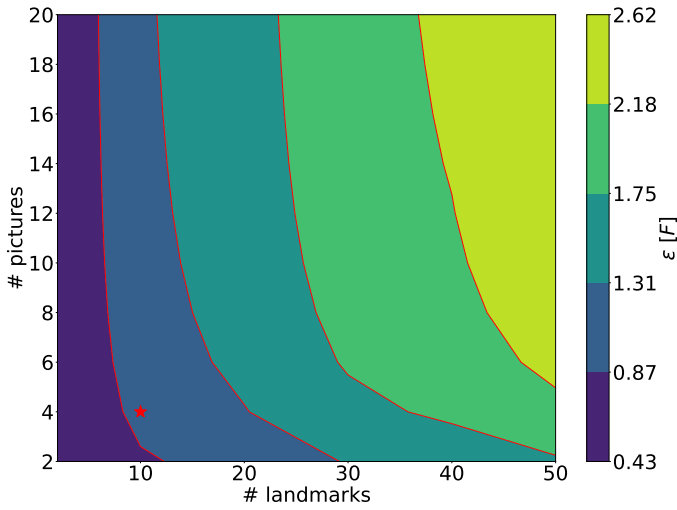


Figure 3.7.: Sensitivity of estimated parameter precision to the spatial distribution of landmarks in the simulated Control Point Network (CPN) for Juno flybys of Io. The plot shows improvement factor (F) for obliquity (ϵ), tidal Love number (k_2), and libration amplitude (g) for three different CPN strategies: landmarks concentrated near the poles ('polar'), randomly distributed ('random'), or concentrated near the equator ('equatorial'). $F > 1$ indicates improvement relative to the reference case ('random' distribution). Different colored lines represent different parameter estimation scenarios combining radiometric and optical data as defined in Fig. 3.6.

The total volume of optical data, defined by the number of images per flyby and the number of landmarks per image, also influences the expected uncertainties on



(a)



(b)

Figure 3.8.: Sensitivity of estimated parameter precision to the volume of optical data for simulated Juno flybys of Io. The contour plots show the improvement factor (F) relative to the nominal case ($F = 1$ at the red star) for (a) the diurnal libration amplitude (g) and (b) the obliquity (ε), as a function of the number of simulated optical images acquired per flyby ('# pictures') and the number of distinct landmarks observed in each image ('# landmarks'). $F > 1$ indicates improvement. The nominal case assumes 4 pictures and 10 landmarks. The analysis uses the $k_2 + \varepsilon + g$ estimation scenario combining radiometric and optical data.

the rotation parameters. As illustrated in Fig. 3.8, increasing either the number of images or the number of landmarks per image leads to a general improvement in the determination of both libration and obliquity. However, even when increasing the data volume to an operationally demanding scenario of 20 images with 50 landmarks each, the maximum improvement factor achieved is only approximately 2.5 for both parameters. While this represents a significant enhancement over the baseline, it remains insufficient to meet the required improvement factor of 6 (3 if ϵ is not estimated) for libration. We therefore conclude that for a mission profile involving only a limited number of flybys, such as the two Juno encounters with Io discussed here, even an expanded optical campaign—well beyond what might be operationally feasible for such brief encounters—cannot bridge the gap to the target accuracies for the rotational parameters. This highlights the constraints imposed by sparse geometric and temporal coverage, a situation distinct from dedicated multi-flyby missions like Europa Clipper, which plans 49 flybys specifically designed to accumulate the necessary data over time [35].

3.3.4.2. INFLUENCE OF DOPPLER TRACKING TIME

The duration of the Doppler tracking window also influences the solution, albeit to a lesser extent than the optical observation strategy. Our analysis confirms that gravity-related parameters are primarily sensitive to the radiometric data, with the optical data providing no additional constraining power for these parameters, as discussed in Sec. 3.4.2. While increasing the length of the tracking pass is a common strategy in orbit determination to better resolve the spacecraft state and its correlations with other estimated parameters (e.g., [34, 39]), our results show that this provides only a modest benefit in this case. As shown in Fig. 3.9, extending the tracking time from the nominal 2 hours to 10 hours per flyby yields the most significant gain for the Love number k_2 , with an improvement factor of approximately 1.2. In contrast, the rotational parameters, which are primarily constrained by the optical data, see a negligible improvement. This is analogous to our previous finding that the Love number k_2 was insensitive to changes in the optical strategy. We conclude that while longer tracking is always preferable, the resulting gains are marginal because the solution is approaching the fundamental sensitivity limit imposed by the fixed number and geometry of the two flybys.

3.3.4.3. A MORE REALISTIC ESTIMATION SETUP

We now evaluate the impact of a more realistic filtering approach by incorporating the additional error sources previously detailed in Sec. 3.3.2 and summarized in Table 3.6. We first assess the impact coming from uncertainties in Io's higher-degree gravity field. Given the limited surface coverage from only two encounters, it is impossible to constrain gravity coefficients beyond degree and order two from the simulated data alone. Indeed, if these terms were fitted without a strong a priori constraint, their formal errors would become extremely large, completely jeopardizing the entire solution. It is noteworthy to mention that a possible strategy to mitigate this and improve the overall orbit determination is to include tracking data acquired not only during the closest approach (C/A), when the gravity signal

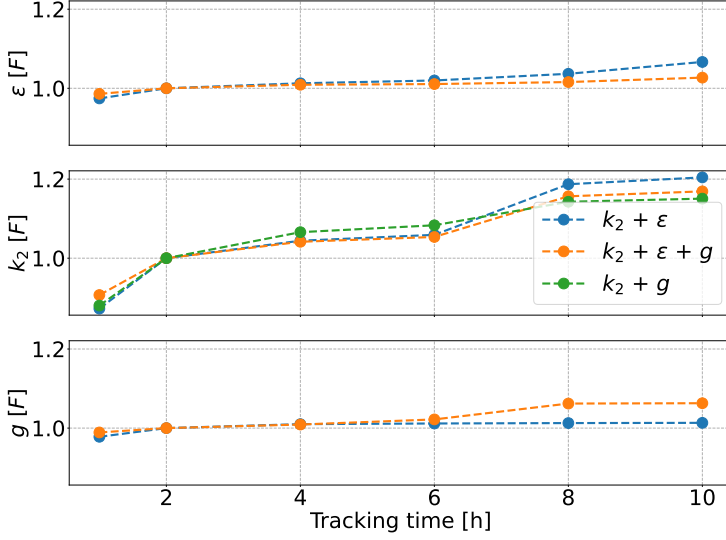


Figure 3.9.: Sensitivity of estimated parameter precision to Doppler tracking time for simulated Juno flybys of Io. The plot shows the improvement factor (F), defined as the ratio of the reference uncertainty to the current uncertainty ($F > 1$ indicates improvement), for obliquity (ϵ), tidal Love number (k_2), and libration amplitude (g) as a function of the tracking duration per flyby (in hours). Different colored lines represent different parameter estimation scenarios combining radiometric and optical data as defined in Fig. 3.6. The reference case for $F = 1$ is the scenario using 2 hours of tracking time.

is strongest, but also during periods significantly before and after the flyby (see e.g. Cappuccio *et al.* [39]). Extending the observational baseline in this way provides better constraints on the spacecraft's trajectory (state vector), which helps to decrease the correlation between the estimated state and the gravity coefficients being estimated. Despite potential benefits from such strategies, for this specific analysis assessing the fundamental limitations of the limited number of flybys, we account for the effect of higher-degree terms by including them in the filter as constrained parameters, using the tight a priori constraint derived from Kaula's rule as defined in Sec. 3.3.2. This strategy, however, relies on the optimistic assumption that each higher-degree coefficient is already known to the level imposed by the Kaula's rule constraint. Consequently, the resulting degradation of the solution should be considered a lower bound. As shown in Fig. 3.10, including terms up to degree and order 5 degrades the solution for the Love number k_2 by a factor of approximately 2.5 (i.e., an improvement factor of 0.4). This degradation highlights the strong correlation between the tidal signal and the static gravity field, and suggests that any unmodeled or mismodeled higher-degree gravity terms would

likely corrupt the solution for k_2 even further.

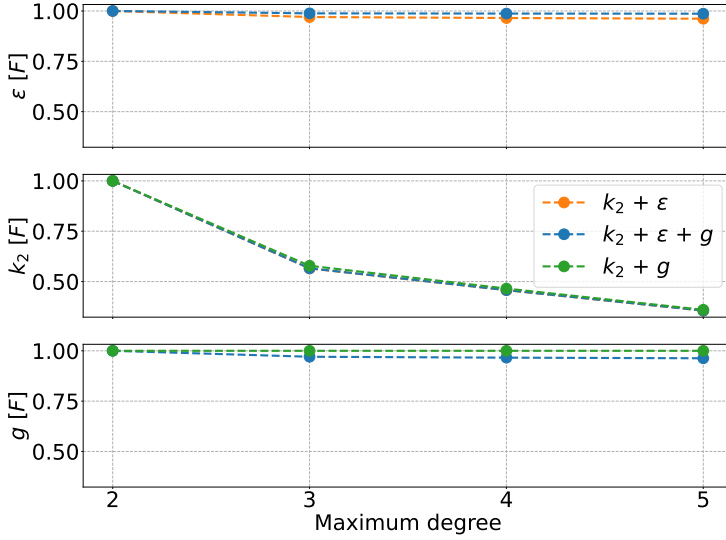


Figure 3.10.: Sensitivity of estimated parameter precision to the complexity of the estimated gravity field model for simulated Juno flybys of Io. The plot shows the improvement factor (F) for obliquity (ϵ), tidal Love number (k_2), and libration amplitude (g) as a function of the maximum degree of the spherical harmonic gravity field estimated in the simulation (e.g., "3" means estimating up to degree and order 3×3). Different colored lines represent different parameter estimation scenarios combining radiometric and optical data as defined in Fig. 3.6. The reference case for $F = 1$ is the scenario estimating only up to degree 2 (J_2 , C_{22} , S_{22}).

Finally, we assess the cumulative impact of all major error sources considered. As shown in Fig. 3.11, including uncertainties in the solar radiation pressure (SRP) has a limited impact given the considered uncertainty of 15%. Io's ephemeris error (Ephem), instead, degrades the solution for the Love number k_2 by a factor of $\approx 1.5 - 2$ (an improvement factor of $\approx 0.5 - 0.6$), since these error directly corrupt the spacecraft trajectory from which the gravity information is derived. Conversely, and as expected, uncertainties in landmark positions and camera pointing have the largest effect on the rotational parameters, degrading the obliquity and libration solutions by a similar amount (improvement factor of ≈ 0.86). This final, most realistic case demonstrates that while the Love number and obliquity targets remain out of reach, the libration amplitude target is achievable, but only under specific and optimistic conditions. The libration uncertainty saw a maximum potential improvement factor of ≈ 6.6 by assuming an optical noise level of 0.3 pixels, and concentrating landmarks in the equatorial region provides a further improvement of ≈ 1.25 . This gain is only modestly reduced by a degradation factor of ≈ 1.16

($F \approx 0.86$) coming from uncertainties in the optical parameters. However, when all error sources are included simultaneously, the cumulative degradation renders the scientific targets for all observables unachievable, reinforcing the conclusion that the experiment is not feasible as currently designed.

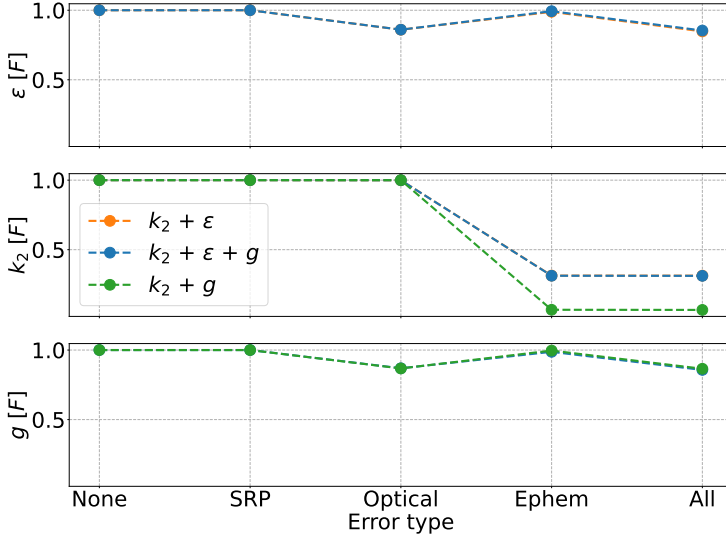


Figure 3.11.: Impact of including additional error sources in the estimation process on parameter precision for simulated Juno flybys of Io. The plot shows the improvement factor (F) for obliquity (ϵ), tidal Love number (k_2), and libration amplitude (g). The x-axis represents different sets of estimated parameters beyond the baseline (geodetic parameters + spacecraft state + GM + degree-2 gravity): 'None' (baseline), 'SRP' (adds Solar Radiation Pressure scaling factor), 'Optical' (adds landmark positions and camera pointing), 'Ephem' (adds Io state estimation), and 'All' (includes all error sources simultaneously). Different colored lines represent different geodetic parameter estimation scenarios combining radiometric and optical data as defined in Fig. 3.6. The reference case for $F = 1$ is the 'None' scenario.

3.3.5. CONCLUSIONS

In this analysis, we investigated the feasibility of using data from the two Juno flybys to constrain Io's interior structure, an objective that extended beyond the nominal goals of the mission. Our study reveals this to be a particularly challenging endeavor, with only a hypothetical possibility of determining one of the three geodetic parameters investigated. The simulations confirm that the radiometric dataset alone is insufficient to achieve the target accuracies for Io's libration amplitude (g), tidal

Love number (k_2), or obliquity (ϵ). The fundamental limitations imposed by the two-flyby geometry—namely the sparse temporal and spatial coverage—preclude the decorrelation of the small tidal and rotational signals from the static gravity field using only Doppler data. Consequently, the inclusion of a hypothetical optical campaign is essential, as it substantially improves the determination of the rotational parameters by breaking the strong correlations between the libration amplitude and the gravity field. Nevertheless, even with this combined dataset, the scientific objectives remain largely unattainable. The target accuracies for k_2 and obliquity are not met under any realistic scenario; the former is insensitive to optical data, while the latter would require an improvement of several orders of magnitude. The measurement of the libration amplitude, therefore, emerges as the only potentially feasible objective. Our sensitivity analysis indicates that achieving the required sub-300 m precision is critically contingent on a combination of optimal conditions: a landmark detection noise of 0.3 pixels, which assumes advanced image processing, and an observation strategy focused on the equatorial region. While these conditions could potentially overcome the degradation from realistic model errors, they represent a best-case scenario that would be operationally demanding. We, therefore, conclude that while the Juno flybys provided a valuable opportunity, they are insufficient to robustly constrain Io's interior structure, leaving only a marginal chance of success for the libration measurement.

The findings of Park *et al.* [13], published subsequent to our analysis, provide an empirical validation of our conclusions. Their successful determination of Io's tidal response, $k_2 = 0.125 \pm 0.047$, which ruled out a shallow magma ocean, was achieved by combining data from the two Juno flybys with a much larger dataset incorporating five Galileo flybys and long-term astrometric observations. The necessity of this multi-mission, multi-decade dataset confirms the conclusion from our simulations that the two Juno flybys alone are insufficient to constrain the Love number k_2 to the required precision of 0.05, given our best-case estimate (disregarding all error sources) was ≈ 0.64 . The sub-300 m libration target for this study was based on the work of Van Hoolst *et al.* [15], who predicted a clear distinction between a solid body (266–275 m) and a shallow magma ocean (> 500 m). The MCMC analysis by Park *et al.* [13], constrained by their measured k_2 , now predicts a libration amplitude of 250–268 m for a solid-body Io, consistent with these earlier models. However, their work also reveals that a deep magma ocean would produce a libration of 261–371 m, a range that significantly overlaps with the solid-body case. This implies that while our 300 m target was appropriate for testing the shallow magma ocean hypothesis, it is, however, insufficient to distinguish between the currently viable interior models.

This analysis also highlights the inherent limitations of assessing a mission with a fixed trajectory. In the subsequent case study, we will explore how these limitations can be addressed through the strategic design of a future mission's flight path.

3.4. URANUS ORBITER AND PROBE

Flagship missions such as Cassini-Huygens at Saturn and Juno at Jupiter have fundamentally advanced our understanding of giant planet systems, having utilised

precise radio tracking data to unravel the complex interior structures of these distant worlds [1, 18]. In a similar vein, our current knowledge of the Uranian system stems almost entirely from the Voyager 2 flyby on 24 January 1986. This single encounter provided the first precise measurements of the masses and radii of Uranus and its major moons, detected the planet's complex magnetic field, determined its interior rotation rate, and captured pioneering images of the system [40, 41]. These images, revealing dynamic atmospheric weather and the surprisingly diverse geology of the moons, laid the essential groundwork for future exploration. Building upon this legacy, the Uranus Orbiter and Probe (UOP) mission was designated the highest-priority new flagship mission in the 2023-2032 Planetary Science and Astrobiology Decadal Survey [42, 43]. NASA is currently evaluating the UOP mission concept, a proposal recognized for its potential to yield high-impact scientific discoveries while operating within a low-risk and cost-effective framework. The mission is designed to address fundamental questions concerning the origin of Uranus, its ongoing physical processes, its internal structure, and the potential habitability of its moons [43].

The proposed mission architecture consists of an orbiter and an atmospheric probe. Following its arrival at Uranus, the spacecraft will initiate the Uranus Orbit Insertion (UOI) phase, during which the probe will be detached to descend into the planet's atmosphere [43]. Concurrently, the orbiter will execute maneuvers to reduce its initially high orbital inclination, transitioning into the equatorial plane of the system. Once this equatorialization is complete, the orbiter will commence its primary Science phase, undertaking a scientifically optimised tour of the Uranian system for approximately 2 years.

A critical component of the UOP instrument suite is the radio telecommunications system, which is expected to include a 3.1 m High-Gain Antenna (HGA) and an UltraStable Oscillator (USO) to enable Gravity Science (GS) experiments [43], as we explained in Sec. 2.2.2. The provisional payload is comprehensive, featuring a magnetometer, a thermal IR camera, multiple spectrometers, a Narrow Angle Camera and a Wide Angle Camera [43].

As detailed in its Science Traceability Matrix (STM), the UOP mission sets forth scientific objectives that span the entire Uranian system [43]. For Uranus itself, key goals relating to its interior structure involve determining its bulk composition and depth dependence, investigating whether it possesses discrete layers or a fuzzy core, and understanding how these characteristics relate to its formation and unique axial tilt. A GS experiment is central to achieving these objectives, requiring measurements of the zonal gravity field coefficients up to at least degree 8 (J_8), similar to what was accomplished for Jupiter by the Juno mission [4, 18–20]. The higher-degree zonal harmonics are the primary signature of deep atmospheric dynamics; resolving them is essential for distinguishing the gravitational signal of these flows from that of the static interior structure [7, 44]. Furthermore, the mission aims to determine the true rotation rate of Uranus, assess its rotational uniformity, and probe the depth of its winds through a combination of gravity field measurements and planet shape determination [43]. Of particular relevance to this thesis, a primary set of Gravity Science (GS) objectives has also been defined for the large Uranian moons, forming

the central focus of the investigation we present in [Chapter 4](#).

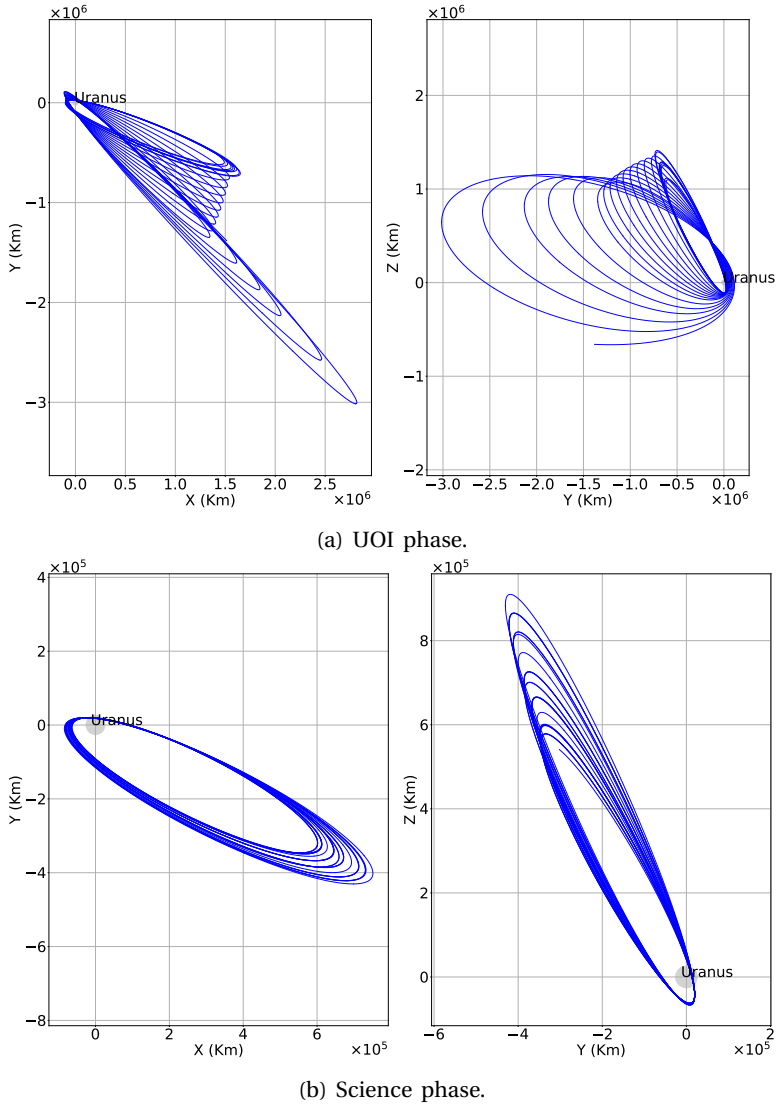


Figure 3.12.: Nominal trajectory for the Uranus Orbiter and Probe (UOP) mission relative to Uranus, shown in the EME2000 inertial reference frame. (a) The initial Uranus Orbit Insertion (UIO) phase (2045-2047), showing the spacecraft reducing its inclination over 13 orbits. (b) The subsequent equatorial Science phase (2048-2049), showing 24 orbits with closer periapses used for scientific observations. Plots show projections onto the X-Y and Y-Z planes.

3.4.1. NOMINAL TOUR AND GRAVITY SCIENCE CONFIGURATION

The nominal UOP mission trajectory we evaluate, based on the design presented by Simon, Nimmo, and Anderson [43], is divided into two distinct phases, which are illustrated in Figs. 3.12 and 3.13. The initial phase is the Uranus Orbit Insertion (UOI) from 2045 to 2047, which spans 13 orbits. Throughout this period, the spacecraft systematically reduces its high orbital inclination to transition towards the equatorial plane, with periapsis altitudes evolving from approximately $5R_U$ down to about $1.7R_U$, where Uranus's mean radius R_U is 25362 km [29]. The ground tracks during the UOI phase, shown in Fig. 3.12a, reflect this progression towards a near-equatorial configuration. Following this, the mission enters its equatorial science phase from 2048 to 2049, consisting of 24 orbits. This second phase is characterized by much closer periapsis passages, with altitudes generally ranging between 22000 km and 30000 km, and ground tracks that remain tightly confined to low Uranian latitudes, typically between -1° and -5° (see Fig. 3.13b).

For the gravity science simulations, we assume standard X-band Doppler tracking, adopting a noise level of 0.1 mms^{-1} at a 60 s integration time (rescaled to a 10 s count time). These data are collected during 8 h tracking windows centered on each periapsis passage. In addition to this baseline, we also investigate the potential improvement offered by a Ka-band radio link, for which we assume a noise level of 0.01 mms^{-1} . We analyze the simulated Doppler measurements using a covariance analysis to assess the expected formal error on the Uranus zonal gravity field coefficients and the planet's Love number k_2 , imposing no a priori constraints on these parameters. The full set of estimated parameters in our simulation includes the spacecraft state (position and velocity with a priori sigmas of 1 km and 1 m/s, respectively), Uranus's gravitational parameter GM (with a $100 \text{ km}^3 \text{ s}^{-2}$ a priori uncertainty), and the planet's orientation parameters, for which we adopt the current uncertainties from Jacobson [45] as a priori sigmas for the pole orientation angles (α , δ), their rates ($\dot{\alpha}$, $\dot{\delta}$), and the rotation rate ($\dot{W}$).

3.4.2. EXPECTED PERFORMANCE OF THE NOMINAL GS EXPERIMENT

To assess the capability of the nominal tour to determine Uranus's zonal gravity field, we performed a series of simulations using the *ROB-MONTE* library (see Sec. 2.2.2.4). Our results, shown in Fig. 3.14, indicate that relying on X-band tracking presents significant limitations. In a simulation estimating zonal harmonics only up to degree 5 (Fig. 3.14a), the combined data from both mission phases yields a marginal improvement in the uncertainties for J_2 and J_4 over current knowledge from Jacobson [45], while J_3 remains poorly constrained. This result, however, might be overly optimistic as it neglects the correlations and systematic effects from higher-degree gravity terms. Indeed, when we perform a more comprehensive assessment by extending the estimation to degree 10 (Fig. 3.14b), the predicted uncertainties for coefficients J_2 through J_5 increase by approximately two orders of magnitude. For instance, the uncertainty for J_2 degrades from $\approx 5 \times 10^{-8}$ in the degree-5 case to $\approx 5 \times 10^{-7}$ in the degree-10 case, while the uncertainty for J_5 increases from $\approx 4 \times 10^{-7}$ to $\approx 1 \times 10^{-5}$. The formal errors for the higher-degree terms ($J_6 - J_{10}$) are very large, falling far short of the mission's target uncertainty of 1×10^{-6} .

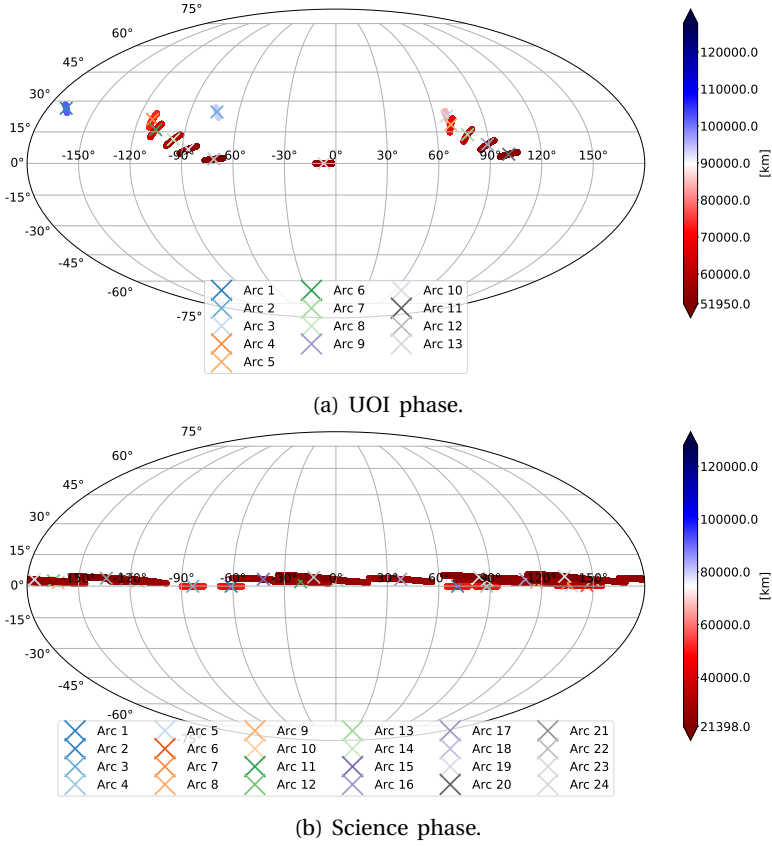
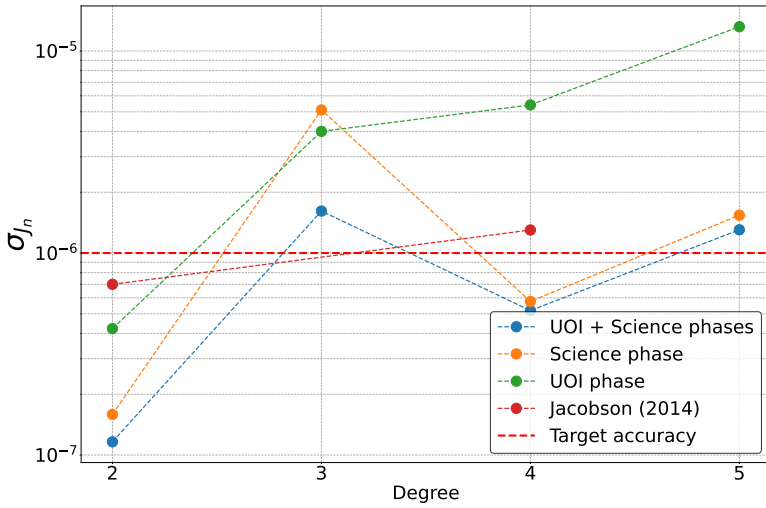


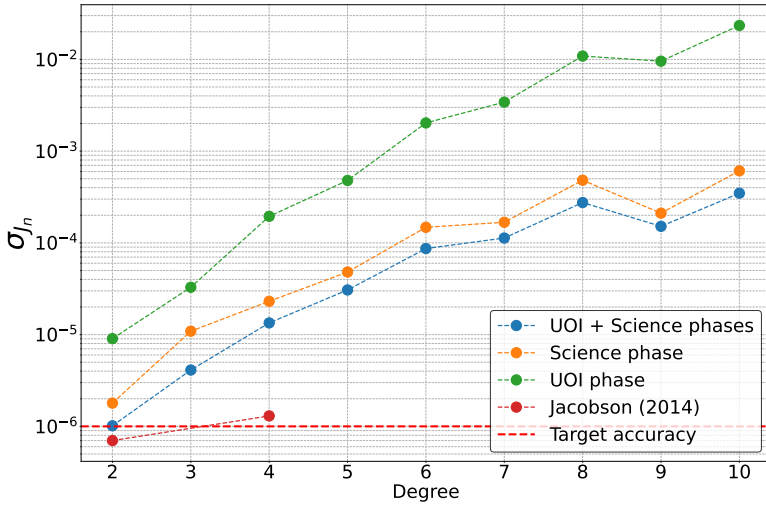
Figure 3.13.: Ground tracks for the nominal Uranus Orbiter and Probe (UOP) trajectory relative to Uranus's body-fixed frame, showing coverage during ± 2 h hours around each periapsis. (a) Ground tracks during the Uranus Orbit Insertion (UOI) phase (13 orbits). (b) Ground tracks during the equatorial Science phase (24 orbits). Colors along the tracks indicate spacecraft altitude above Uranus in kilometers. Different symbols/colors in the legend distinguish individual orbital arcs.

[43].

Given these limitations, we also evaluated the potential benefits of employing a Ka-band radio link for the nominal tour. As illustrated in Fig. 3.15, this upgrade leads to a substantial decrease in the predicted uncertainties. In the degree-5 estimation (Fig. 3.15a), the combined-phase solution delivers uncertainties for J_2 through J_5 that are not only significantly better than current knowledge but also meet the 10^{-6} target for every coefficient. Specifically, the J_2 uncertainty of $\approx 5 \times 10^{-9}$ is an order of magnitude better than the X-band case, with other coefficients resolved at the $\approx 5 \times 10^{-8}$ level. When extending the estimation to degree 10 (Fig. 3.15b),

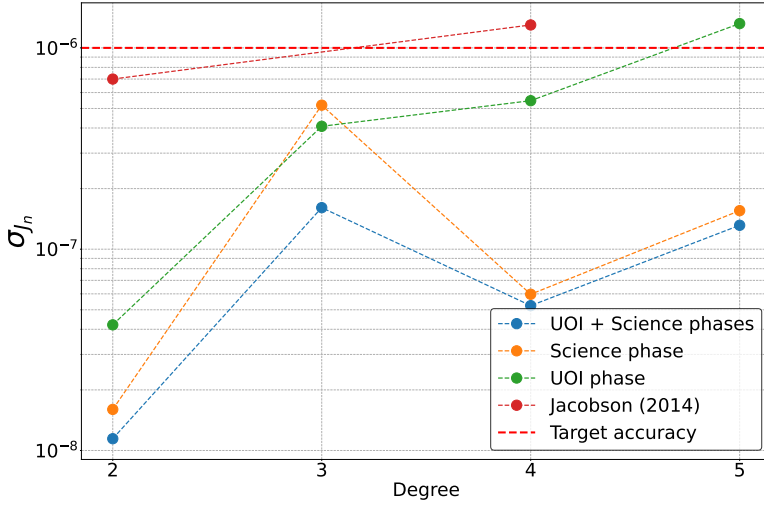


(a)

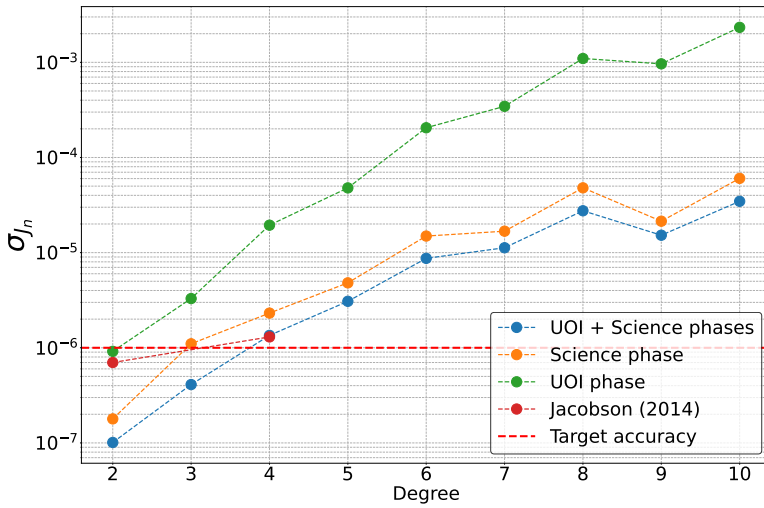


(b)

Figure 3.14.: Predicted 1σ formal uncertainties (σ_{J_n}) for Uranus's zonal gravity coefficients (J_n) achievable with the nominal Uranus Orbiter and Probe (UOP) tour, assuming X-band Doppler tracking. Results are shown for different data subsets: Uranus Orbit Insertion (UOI) phase only, Science phase only, and the combined UOI + Science phases. Current knowledge from Jacobson [45] and the mission's target accuracy (1×10^{-6}) are shown for comparison. (a) Results when estimating coefficients up to degree 5. (b) Results when estimating coefficients up to degree 10, demonstrating increased uncertainties due to correlations when fitting higher-degree terms.



(a)



(b)

Figure 3.15.: Predicted 1σ formal uncertainties (σ_{J_n}) for Uranus's zonal gravity coefficients (J_n) achievable with the nominal Uranus Orbiter and Probe (UOP) tour, assuming Ka-band Doppler tracking. Lines and comparison data are as described in Fig. 3.14. (a) Results when estimating coefficients up to degree 5, showing significant improvement over X-band and meeting the target for $J_2 - J_5$. (b) Results when estimating coefficients up to degree 10, showing uncertainties approaching or meeting the target accuracy even for higher-degree terms like $J_8 - J_{10}$.

the uncertainties for $J_2 - J_5$ remain below the target accuracy, and higher-degree terms like J_8 ($\approx 6 \times 10^{-6}$) and J_{10} ($\approx 8 \times 10^{-6}$) become much better constrained. This demonstrates that while the nominal tour geometry has inherent limitations, upgrading the tracking system to Ka-band makes the UOP science goals for the gravity field appear achievable. The determination of Uranus's tidal Love number k_2 also benefits markedly, with its predicted precision improving from ≈ 0.1 with X-band to ≈ 0.01 with Ka-band. These results highlight a fundamental trade-off in our analysis: estimating to a lower degree may yield smaller formal errors for the included terms but risks under-representing the true uncertainty by ignoring the influence of higher-order effects. Conversely, estimating to a higher degree provides a more globally robust, albeit potentially larger, uncertainty for all terms. It is therefore possible to conclude that with X-band tracking, only zonal harmonics up to degrees 4-5 are likely measurable with the required precision. In contrast, with Ka-band tracking, achieving a similar precision for coefficients up to $J_8 - J_{10}$ appears feasible, which would enable the mission to begin addressing its core science objectives.

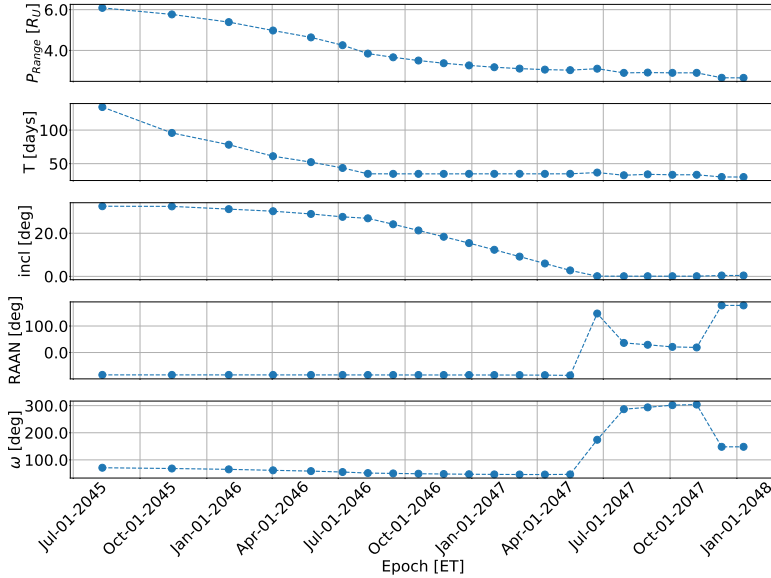


Figure 3.16.: Evolution of key osculating orbital elements for the nominal Uranus Orbiter and Probe (UOP) trajectory during the Uranus Orbit Insertion (UOI) phase (2045-2047), plotted at each pericenter passage. The coordinate elements are listed in order from top to bottom, as follows: the periapsis distance w.r.t. Uranus center of mass P_{range} ; orbital periods T ; the orbital inclination i ; right ascension of the ascending node $RAAN$; and the argument of perigee ω .

3.4.3. OPTIMIZING THE TRAJECTORY FOR ENHANCED SCIENCE RETURN

Although Ka-band tracking significantly improves the performance of the Uranus gravity science experiment, our analysis indicates that the nominal trajectory may still be insufficient to meet the mission requirements, in particular for the highest-degree zonal harmonics. Therefore, in order to further enhance the scientific return, we investigate the impact of simple trajectory tweaks. This investigation focuses specifically on modifications to the initial UOI phase, given that the subsequent science phase is constrained to a near-equatorial orbit to facilitate the tour of the Uranian satellites. We assume X-band tracking, as a Ka-band system is not guaranteed, and combine data from the adjusted UOI phase with those from the nominal equatorial science phase.

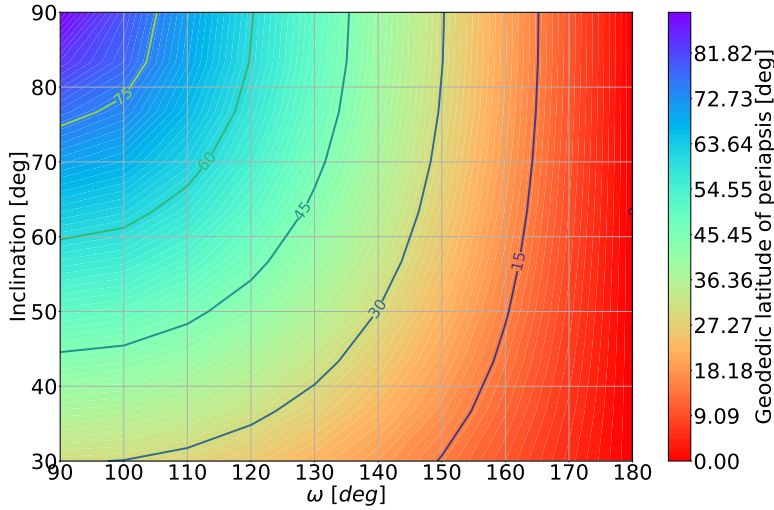


Figure 3.17.: Relationship between the initial orbital inclination (at the start of the Uranus Orbit Insertion (UOI) phase), the argument of periapsis (ω , kept constant during UOI), and the resulting geodetic latitude of the first pericenter for the modified UOP trajectories explored in the optimization study. This map illustrates how varying these two parameters allows targeting different latitudes to enhance gravity science return.

The orbital elements of interest for the nominal UOI trajectory are presented in Fig. 3.16. In designing the adjusted UOI phase, several of the original orbital parameters were held constant, specifically the number of orbits and the initial (≈ 120 d) and final (≈ 34 d) orbital periods. The primary objective of these trajectory modifications was to vary the geodetic latitude of the pericenters. Consequently, maximizing the latitudinal coverage of these pericenters is essential for resolving the full zonal spectrum, an observation strategy exemplified by the Juno mission at Jupiter, whose polar orbit was specifically designed to sample all latitudes for a precise determination of the gravity field (see e.g. [44]). By varying the pericenter

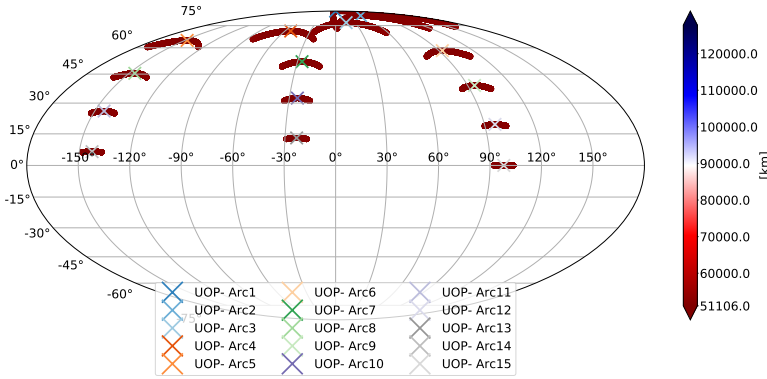
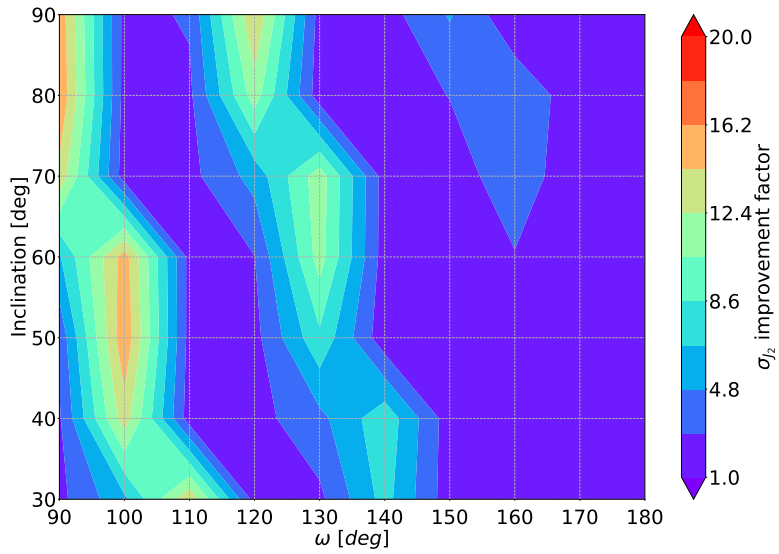


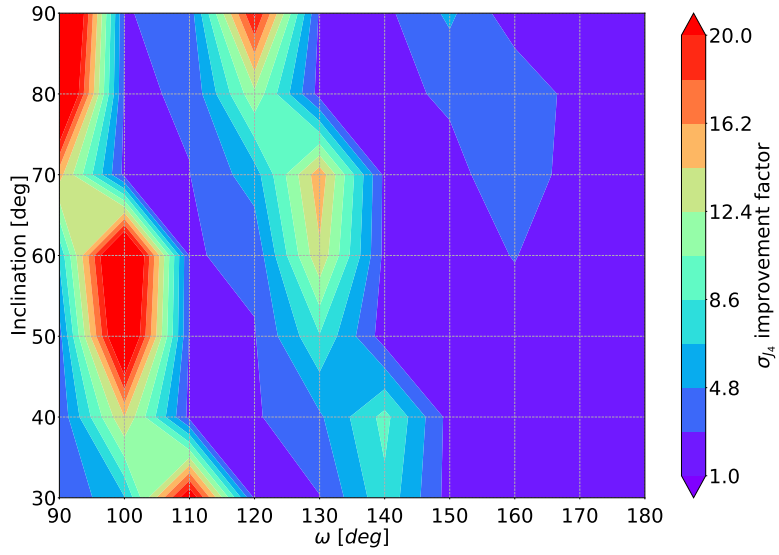
Figure 3.18.: Example ground track coverage for one of the optimized Uranus Orbiter Insertion (UOI) phase trajectories explored in this study. This specific trajectory starts with an initial inclination of 90° (polar insertion) and maintains a constant argument of periapsis (ω) of 90° throughout the UOI phase. The plot shows ground tracks relative to Uranus's body-fixed frame during ± 2 h hours around each periapsis. Colors indicate spacecraft altitude; legend labels distinguish individual orbital arcs. Compare with the nominal UOI coverage in Fig. 3.13a.

latitude, the modified orbits can sample a wider latitudinal range, thereby enhancing the gravity science investigation. This was achieved by systematically varying two key orbital elements during the UOI phase. First, the orbital inclination of the initial pericenter was varied in a range from 30° , which is the nominal UOI starting inclination, to 90° for a polar orbit insertion. From this starting value, the inclination was then set to decrease linearly over all subsequent pericenters until the spacecraft reached an equatorial orbit. Second, we varied the argument of periapsis (ω) from 90° to 180° . Unlike the inclination, the value of ω was kept constant for all pericenters within a single modified trajectory sequence. The resulting geodetic latitude of periapsis, as a function of these two varied orbital elements (initial inclination and ω), is illustrated in Fig. 3.17. Through the use of these ranges for inclination and ω , the modified orbits remain broadly similar to the original nominal tour in terms of their orientation within the inertial EME2000 reference system. A key assumption for these modified orbits was that all periapses would occur at a geodetic height of $3R_U$. This altitude was selected as it represents the lowest achieved by the nominal trajectory, and Doppler measurements become increasingly sensitive to the planet's gravity field at closer pericenter distances. However, practical mission constraints for a safe passage inward of the rings would need to be validated for such close approaches, as noted in the mission STM [43]. An example of a specific UOI phase, characterized by an initial inclination of 90° and an argument of perigee of 90° , is shown in Fig. 3.18.

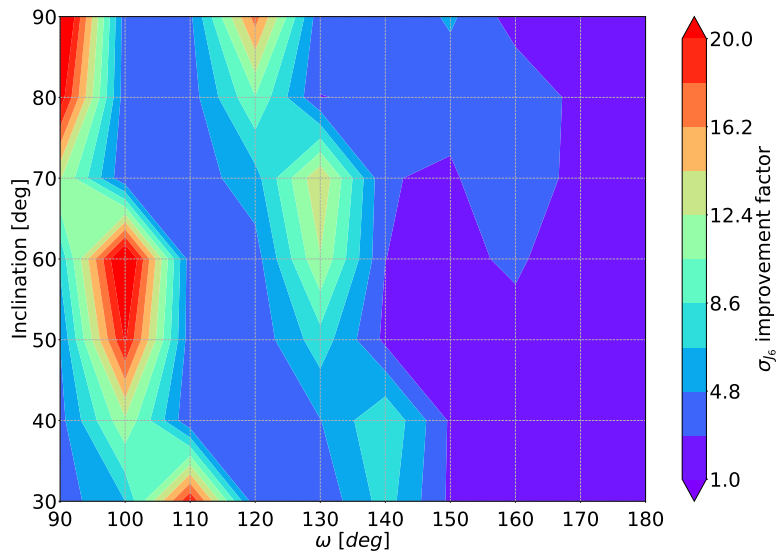
The impact of these UOI trajectory adjustments on the determination of individual gravity field parameters is presented in terms of an improvement factor, as previously



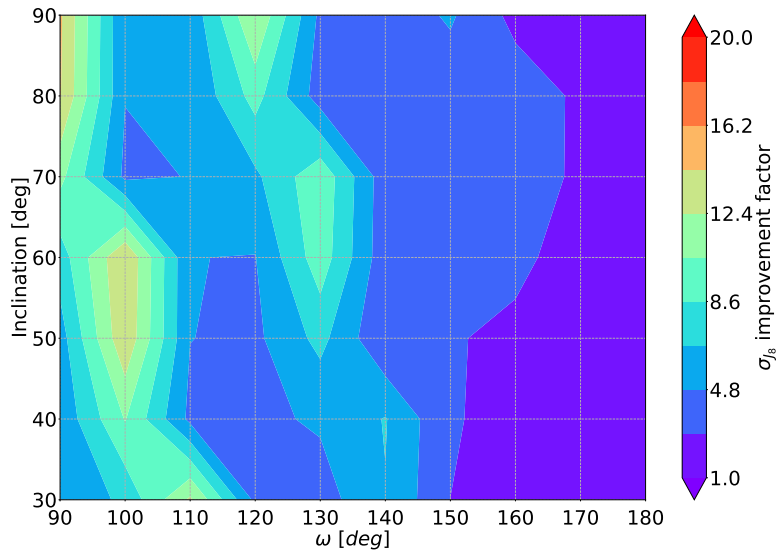
(a) J_2 .



(b) J_4 .



(c) J_6 .



(d) J_8 .

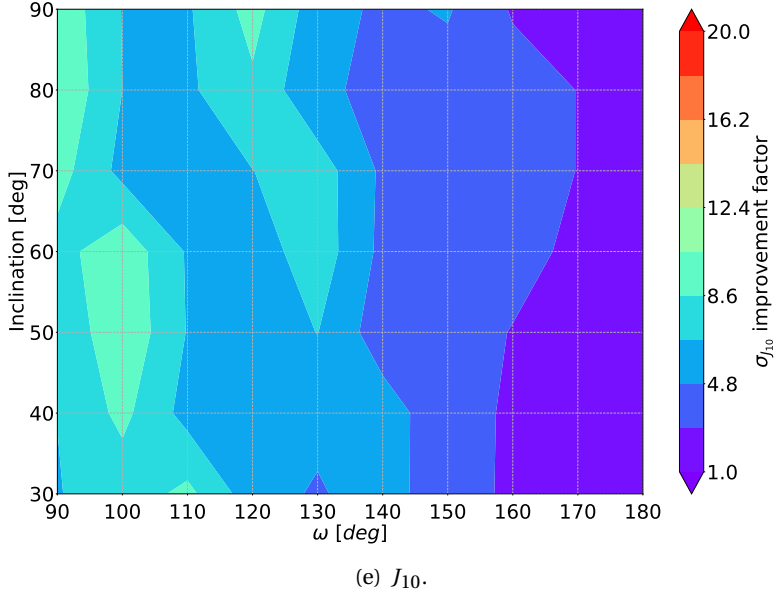


Figure 3.19.: Improvement factor for the estimated uncertainty of Uranus's even zonal gravity coefficients (J_2 , J_4 , J_6 , J_8 , J_{10}) achieved by optimizing the Uranus Orbit Insertion (UOI) phase trajectory. The improvement factor (color scale) is calculated as the ratio of the uncertainty from the nominal UOI phase to the uncertainty from a modified UOI (defined by initial inclination and ω), assuming X-band tracking. Higher values indicate greater improvement (smaller uncertainty) compared to the nominal tour. These maps show regions in the parameter space (initial UOI inclination vs. ω) that significantly enhance the determination of specific zonal harmonics, particularly when targeting mid-to-high latitudes. The odd zonal harmonics (not shown here), generally follow similar improvement trends as their adjacent even harmonics.

used in Sec. 3.3.4. We define this factor as the ratio of the nominal expected uncertainty (from the nominal UOI phase) to the expected uncertainty from the modified UOI trajectory. Thus, an improvement factor of 10 signifies that the adjusted trajectory yields a tenfold smaller (i.e., better) formal error for a given parameter. The improvement factor plots for the even zonal harmonics, presented in Fig. 3.19, consistently show two main areas of high improvement within the tested parameter space. The first area, which generally yields the best results, is found for ω between 90° and 120° . Within this region, a sub-area with initial UOI inclinations between 40° and 60° is particularly effective for the lower-degree even harmonics, resulting in improvement factors for J_2 and J_4 that exceed 15 (see Fig. 3.19a and Fig. 3.19b). Another sub-region, at higher inclinations between 70° and 90° , also shows some improvement, although it is generally less pronounced for the lower-degree terms in this ω range. A second main area of notable improvement occurs for ω values between approximately 110° and 150° , and this region is most effective for higher initial UOI inclinations, typically in the range of 70° to 90° . In this second zone, the improvement factors for coefficients such as J_6 and J_8 can reach values greater than 15, although these are generally smaller than the maximum improvements found in the first area (where $\omega < 120^\circ$). For instance, while J_6 can be improved by a factor of over 20 in the first area, the improvement in the second area is typically around 15, as shown in Fig. 3.19c. This indicates that UOI phase designs with periapses in the $\omega \approx 90^\circ$ to 120° range, combined with initial inclinations of either 50° to 60° or 80° to 90° , allow for a significant enhancement of the gravity field solution. These specific orbital parameters are optimal because they place the pericenters at mid-to-high geodetic latitudes of approximately 50° and 70° , respectively (see Fig. 3.17). As the gravitational signatures of the higher-degree zonal harmonics are stronger at these latitudes, this targeted sampling enables a more robust determination of the gravity field. The odd zonal harmonics (not shown here) also benefit from these trajectory modifications, and their improvement maps generally follow similar trends to those of their adjacent even harmonics (e.g., J_3 exhibits a similar trend to J_2).

From this parametric study, we identified the optimal UOI phase geometry, which corresponds to an initial UOI inclination of 90° and $\omega = 90^\circ$. This configuration was selected as it yields the minimum total RMS error of the entire gravity spectrum up to degree 10 (as defined by Eq. (2.6)). Fig. 3.20 shows this RMS value as a function of the UOI initial inclination and argument of periapsis (ω), with the objective being to identify the (initial inclination, ω) pair that minimizes this total RMS error. The regions of lowest RMS error (darkest blue/purple colors) align with the zones identified from the individual improvement factor plots. A broad region of low RMS values is found for initial inclinations around 40° to 50° with $\omega \approx 90^\circ$ to 100° . Additionally, another very narrow zone of minimum RMS values is observed at high initial inclinations (80° - 90°) and an ω of approximately 90° . The absolute minimum RMS is indicated by the star marker in this second zone. However, given the broader area of comparable RMS values, an initial UOI inclination of approximately 45° combined with an ω around 95° also represents a highly favorable scenario for minimizing the total gravity spectrum RMS error.

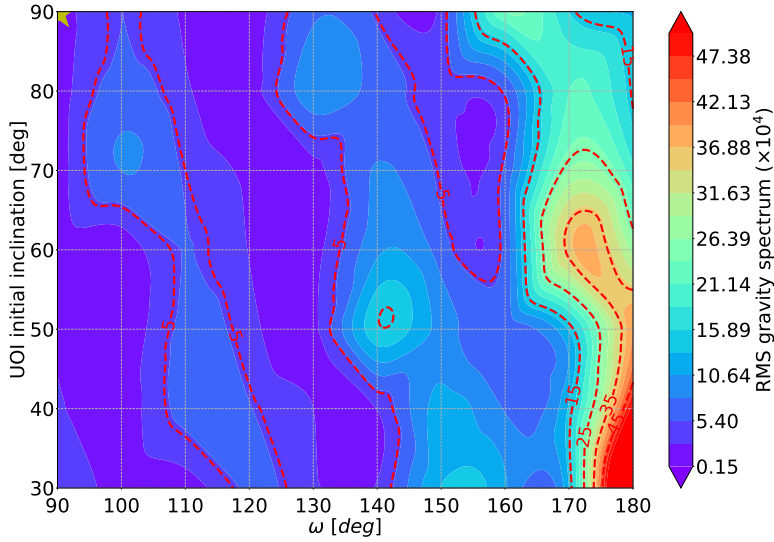


Figure 3.20.: Total Root Mean Square (RMS) error of the gravity spectrum (up to degree 10) as a function of the Uranus Orbit Insertion (UOI) phase initial inclination and argument of periapsis (ω) for the optimized trajectory study. Each point on this map represents the total RMS error calculated by combining simulated X-band Doppler data from a modified UOI phase (defined by the initial inclination and ω on the axes) with data from the nominal equatorial Science phase. Lower RMS values (cooler colors) indicate a better overall gravity field determination. The yellow star marks the trajectory configuration (initial inclination = 90° , $\omega = 90^\circ$) yielding the absolute minimum RMS error in this study.

The overall benefit of such an optimized UOI phase, when combined with the equatorial science phase, is clearly demonstrated in Fig. 3.21, which illustrates an X-band tracking scenario. The new, adjusted UOI phase combined with the equatorial science phase (green line) exhibits lower formal errors for the zonal harmonics when compared to the nominal solution (blue line). The "Science Phase" alone (orange line), which comprises only equatorial orbits, performs the poorest, thereby emphasizing the crucial role of inclined orbits during the UOI phase for a robust determination of the gravity zonal coefficients. With the adjusted UOI phase (green line), the X-band uncertainties for J_2 improve to $\approx 6 \times 10^{-8}$, J_3 to $\approx 2 \times 10^{-7}$, and J_4 to $\approx 4 \times 10^{-7}$. While this represents a significant improvement over the nominal combined solution, it also highlights that for X-band, even an optimized UOI phase may not suffice for all high-degree science goals. However, when these trajectory optimizations are coupled with Ka-band tracking, achieving the target precision for high-degree zonal harmonics appears possible, given the potential improvements discussed in Sec. 3.4.2. These findings suggest that a combination of strategic trajectory optimization during the UOI phase and the inclusion of a

Ka-band tracking system is crucial for the UOP mission to meet its gravity science objectives for Uranus.

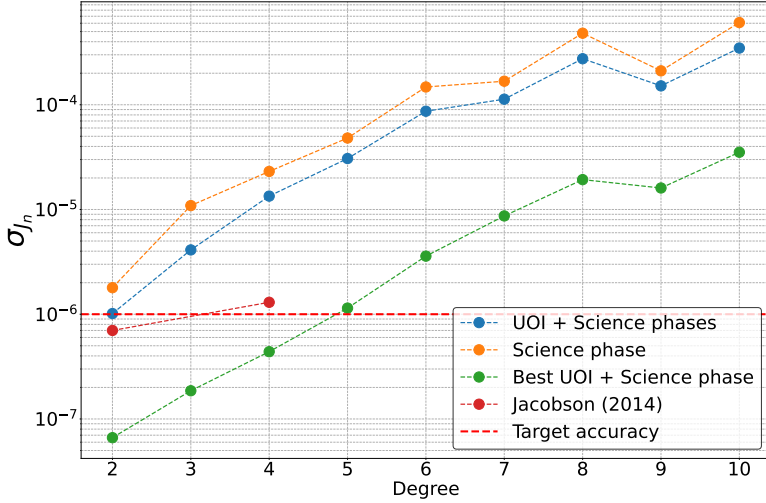


Figure 3.21.: Comparison of predicted 1σ formal uncertainties (σ_{J_n}) for Uranus's zonal coefficients (J_n) using X-band tracking, highlighting the benefit of trajectory optimization. Lines show: the nominal UOI + Science phases combined (blue), the nominal Science phase alone (orange), and the Best UOI + Science phase (green), where "Best UOI" refers to the optimized trajectory identified in Fig. 3.20 (initial inclination = 90° , $\omega = 90^\circ$). Current knowledge [45] and the target accuracy are shown for reference. The optimized trajectory significantly improves precision compared to the nominal tour, especially for lower-degree harmonics.

3.4.4. VALIDATION OF THE ROB-MONTE LIBRARY

This comprehensive simulation work for the UOP mission also served to validate the *ROB-MONTE* library introduced in Sec. 2.2.2.4. To this end, we performed a detailed comparative analysis of the Uranus GS experiment simulation, which involved a step-by-step comparison between the OD solutions from our analysis pipeline and those produced by the Planetary Geology, Geophysics & Geochemistry Laboratory at NASA Goddard Space Flight Center (henceforth Goddard team) using their established analysis pipeline. The specific case chosen for this validation was the estimation of Uranus's gravity field using data solely from the Science phase of the nominal tour, assuming only X-band tracking and the filter configuration detailed in Sec. 3.4.1. Table 3.6 summarizes the results of this comparison, listing the a priori sigmas and the resulting formal errors (1σ) for key geodetic parameters and for the state vector of the first analyzed arc.

As shown in Table 3.6, the estimated formal errors from both the ROB and

Table 3.6.: Comparison of the estimated formal error (1σ) solutions of the ROB and Goddard teams.

Parameter name	apriori σ	ROB	Goddard
X	1 km	2.16×10^{-1} km	1.04×10^{-1} km
Y	1 km	7.55×10^{-1} km	5.64×10^{-1} km
Z	1 km	3.19×10^{-1} km	4.79×10^{-1} km
V_x	1 km s^{-1}	$1.53 \times 10^{-5} \text{ km s}^{-1}$	$1.44 \times 10^{-5} \text{ km s}^{-1}$
V_y	1 km s^{-1}	$5.85 \times 10^{-5} \text{ km s}^{-1}$	$1.11 \times 10^{-4} \text{ km s}^{-1}$
V_z	1 km s^{-1}	$1.30 \times 10^{-5} \text{ km s}^{-1}$	$1.39 \times 10^{-5} \text{ km s}^{-1}$
α_0	2.7°	$5.69 \times 10^{-2^\circ}$	$1.17 \times 10^{-2^\circ}$
$\dot{\alpha}$	$3.16 \times 10^{-12} \text{ }^\circ \text{ s}^{-1}$	$3.16 \times 10^{-12} \text{ }^\circ \text{ s}^{-1}$	$3.16 \times 10^{-12} \text{ }^\circ \text{ s}^{-1}$
δ_0	0.7°	$5.84 \times 10^{-2^\circ}$	$1.18 \times 10^{-2^\circ}$
$\dot{\delta}$	$3.16 \times 10^{-12} \text{ }^\circ \text{ s}^{-1}$	$3.16 \times 10^{-12} \text{ }^\circ \text{ s}^{-1}$	$3.16 \times 10^{-12} \text{ }^\circ \text{ s}^{-1}$
$\dot{W}$	Unconstrained	$1.57 \times 10^{-1} \text{ }^\circ \text{ s}^{-1}$	$2.99 \times 10^{-1} \text{ }^\circ \text{ s}^{-1}$
GM	Unconstrained	$9.19 \times 10^{-1} \text{ km}^3 \text{ s}^{-2}$	$5.60 \times 10^{-1} \text{ km}^3 \text{ s}^{-2}$
k_2	Unconstrained	1.55×10^{-1}	1.58×10^{-1}
J_2	Unconstrained	5.40×10^{-7}	1.08×10^{-6}
J_3	Unconstrained	1.52×10^{-6}	3.60×10^{-6}
J_4	Unconstrained	4.83×10^{-6}	1.03×10^{-5}
J_5	Unconstrained	9.15×10^{-6}	1.62×10^{-5}
J_6	Unconstrained	2.33×10^{-5}	5.19×10^{-5}
J_7	Unconstrained	2.54×10^{-5}	4.55×10^{-5}
J_8	Unconstrained	6.03×10^{-5}	1.38×10^{-4}
J_9	Unconstrained	2.67×10^{-5}	4.83×10^{-5}
J_{10}	Unconstrained	6.32×10^{-5}	1.47×10^{-4}

Goddard solutions are of the same order of magnitude across all compared parameters. For instance, the formal error for the spacecraft's X-component of position in the first arc is 2.16×10^{-1} km in our solution compared to 1.04×10^{-1} km in the Goddard solution. A similar level of agreement is observed for Uranus's GM, for which our solution yields $\sigma_{GM} = 9.19 \times 10^{-1} \text{ km}^3 \text{ s}^{-2}$ while the Goddard solution gives $\sigma_{GM} = 5.60 \times 10^{-1} \text{ km}^3 \text{ s}^{-2}$. This consistency extends to the body-fixed orientation parameters (α_0 , δ_0 , $\dot{\alpha}$, $\dot{\delta}$, $\dot{W}$), the tidal Love number k_2 , and the analyzed zonal harmonic coefficients (J_2 through J_{10}). For the zonal harmonics, although our solution consistently yields slightly smaller formal errors (e.g., for J_2 , ROB $\sigma = 5.40 \times 10^{-7}$ versus Goddard $\sigma = 1.08 \times 10^{-6}$), the values remain comparable. These minor discrepancies, which are typically less than a factor of two, are likely attributable to small differences in the adopted measurement error models and data weighting strategies. For example, the Goddard team's simulations incorporate more complex noise models that scale with observational geometry to down-weight data acquired during periods of potentially higher plasma noise, such as near solar conjunctions when the SEP angle is close to 0° . Such differences in the data analysis approach could lead to the slightly more conservative (larger) error estimates in their

solution. Nevertheless, the strong overall agreement, with formal errors consistently falling within the same order of magnitude, provides robust validation for the accuracy and reliability of the analysis tools and methodologies developed and employed in this thesis.

3.4.5. CONCLUSIONS

The investigation into the gravity science capabilities of the Uranus Orbiter and Probe (UOP) mission highlights several factors critical for maximizing scientific return, particularly for Uranus's zonal gravity field and tidal response. Our simulations demonstrate that for the nominal mission tour, relying solely on X-band tracking imposes significant limitations on constraining higher-degree zonal harmonics (J_n) to the precision required by mission objectives. Even when combining the Uranus Orbit Insertion (UOI) and Science phases to improve the solution, the precision of the determined zonal harmonics beyond $J_4 - J_5$ is insufficient when higher-degree terms are included in the inversion. In contrast, employing a Ka-band radio link provides a significant enhancement. With Ka-band, the nominal tour can meet the 1×10^{-6} target uncertainty for zonal harmonics up to J_5 . The solution for higher-degree terms, instead, approaches the required precision, with expected uncertainties for J_8 and J_{10} of approximately 6×10^{-6} and 8×10^{-6} , respectively. Furthermore, the determination of Uranus's Love number k_2 improves substantially with Ka-band, from an uncertainty of approximately 0.1 to 0.01.

We show that further improvements can be achieved through a strategic optimization of the trajectory. This analysis focused on the UOI phase, as we assumed its orbital parameters can be adjusted to enhance latitudinal coverage, whereas the subsequent science phase must remain in a near-equatorial plane to enable the tour of the Uranian satellites. By systematically varying the initial inclination and argument of periapsis, and adopting a consistent lower periapsis altitude of $3R_U$, we found that certain UOI configurations yield substantial improvements in the precision of the zonal harmonic coefficients. For instance, with optimized UOI trajectories, we showed it is possible to improve σ_{J_2} and σ_{J_4} by factors greater than 8–10 for X-band tracking during the UOI phase alone. When these optimized UOI trajectories are combined with the equatorial science phase and Ka-band tracking, our simulations indicate that a precision of approximately 10^{-6} is achievable for J_8 and, in the most favorable scenarios, even for J_{10} .

Further improvements are realized through a strategic optimization of the UOI phase trajectory. We conducted a parametric study by systematically varying the initial UOI inclination (from 30° to 90°) and the argument of periapsis (ω from 90° to 180°), while keeping the periapsis altitude at a constant $3R_U$. The results, analyzed for a combined mission phase (i.e., UOI and science phases) with X-band tracking, reveal that specific UOI configurations yield substantial improvements in the precision of zonal harmonic coefficients, as measured by an improvement factor relative to the nominal tour. The analysis of individual zonal harmonics shows two distinct zones of high improvement. The most effective zone, with improvement factors exceeding 15 for most coefficients, corresponds to pericenter geodetic latitudes between approximately 50° and 90° (achieved with an argument

of periastris, ω , between 90° and 120° and initial inclinations of 50° - 60° or 80° - 90°). A second, slightly less effective zone of improvement also occurs at high latitudes, resulting from an ω between 110° and 150° at higher inclinations of 80° - 90° . An analysis of the total RMS error of the gravity spectrum identified the optimal UOI phase as one placing the initial pericenter directly over the pole (achieved with an initial polar orbit and $\omega \approx 90^\circ$). While these optimized trajectories with X-band tracking reduce the absolute uncertainties for J_8 and J_{10} to approximately 1×10^{-5} , this is still an order of magnitude above the science target. However, these results demonstrate that when such an optimized trajectory is combined with Ka-band tracking, the target precision of approximately 1×10^{-6} for high-degree zonal harmonics like J_8 and J_{10} becomes achievable.

These analyses demonstrate that achieving the gravity science objectives for Uranus, particularly the detailed characterization of its internal structure through high-degree zonal harmonics and tidal response, requires a combined approach. The use of Ka-band tracking is critical, and strategically optimizing the UOI phase trajectory to ensure diverse latitudinal coverage at close altitudes offers significant further enhancements.

Finally, we used this simulation work to validate the *ROB-MONTE* library. A comparative analysis with solutions from the Planetary Geology, Geophysics & Geochemistry Laboratory at NASA Goddard Space Flight Center demonstrated strong agreement, with estimated formal errors from both pipelines falling within the same order of magnitude across key geodetic parameters and spacecraft state vectors. This provides a robust validation for the tools and methodologies employed in this thesis.

3.5. CONCLUSIONS

This chapter presented three case studies to illustrate the practical application of gravity science principles, moving from the verification of performance to scientific assessment and experiment optimization. These analyses not only yielded key insights for both ongoing and future missions but also served to validate the *ROB-MONTE* software library used throughout this thesis.

The investigation started with the Cassini-Titan case study (Sec. 3.2), which verified the *ROB-MONTE* library by successfully replicating published gravity field results from the Cassini RSS experiment, thereby providing confidence in the scientific results of this work. The subsequent Io case study (Sec. 3.3) assessed the feasibility of leveraging an active mission for objectives beyond its primary design. We concluded that the planned Juno flybys are insufficient to robustly constrain Io's interior; a successful libration measurement, for instance, has a low probability and would require supplementing the radiometric data with a hypothetical optical campaign under highly optimistic conditions. This illustrates a key principle: covariance analysis can be used to precisely define the boundaries of a given observation strategy and identify the necessary conditions to achieve a scientific goal. Finally, the Uranus Orbiter and Probe study (Sec. 3.4) demonstrated that while the nominal mission tour with X-band tracking is insufficient for determining Uranus's high-degree zonal harmonics, a combination of improved instrumentation (Ka-band

tracking) and optimized trajectory design can enhance the science return by over an order of magnitude. This finding was further validated by the strong agreement between our results and those from the NASA Goddard pipeline. These case studies collectively demonstrate that the conventional design of a gravity science experiment is a multi-stage process reliant on robustly verified tools, a rigorous assessment of scientific feasibility, and detailed optimization to maximize science return within mission constraints.

However, this conventional workflow, as demonstrated, is fundamentally iterative: an experiment is designed, its performance is simulated, and it is subsequently redesigned to meet scientific objectives. Such a process can be inefficient, especially as mission requirements and constraints evolve. This challenge underscores the need for a more direct, science-driven paradigm. Having illustrated the standard methodology and its limitations, we now turn in the following chapter to a novel "reverse mapping" framework that inverts this process, allowing scientific goals to guide mission design more efficiently from the earliest phases.

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4

INFORMED DESIGN OF A GRAVITY SCIENCE EXPERIMENT FOR THE FUTURE GEOPHYSICAL INVESTIGATION OF THE URANIAN MOONS

This chapter is based on work published in The Planetary Science Journal: V. Filice, G. Cascioli, S. Le Maistre, R.-M. Baland, A. Trinh, E. Mazarico, and S. Goossens. “Informed Design of a Gravity Science Experiment for the Future Geophysical Investigation of the Uranian Moons”. In: The Planetary Science Journal 6.2 (Feb. 2025.). For this thesis, content originally presented as appendices in the publication has been integrated into the main text, adapted for context, and enriched with additional quantitative statements.

4.1. INTRODUCTION

So far, the outer solar system beyond Saturn has never been explored with a dedicated orbital mission. Our current knowledge of Uranus and its major moons, Miranda, Ariel, Umbriel, Titania, and Oberon, mostly comes from Earth-based observations (e.g., Cartwright *et al.* [1]) and in-situ measurements from the Voyager 2 spacecraft, which flew by the Uranian system in January 1986 [2, 3]. A 10-year launch window to Uranus will open in 2029, prompting the scientific community to prioritize a mission to the Uranian system. Consequently, the Planetary Science and Astrobiology Decadal Survey selected the Uranus Orbiter and Probe (UOP) mission concept [4] as the highest priority new NASA Flagship mission for the decade 2023-2032 [5].

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The UOP mission aims to conduct a comprehensive and interdisciplinary study of the Uranian system, exploring its origins, processes, and habitability. In addition to addressing specific science questions about the enigmatic ice giant Uranus, UOP would place particular emphasis on exploring the major satellites of Uranus. While Voyager 2 and Earth-based observations have provided partial insights into these bodies, the lack of a dedicated mission has left many fundamental questions unanswered. Of particular importance, given the newly discovered astrobiological potential of other icy moons, is the question of whether any of the Uranian satellites have or have had habitable conditions. A first-order question to start is to constrain the interior of these moons.

The notional payload of the UOP spacecraft includes a comprehensive suite of instruments as introduced in Sec. 3.4. The 3.1-meter HGA and USO facilitate the execution of GS experiments, while the NAC and WAC enable the characterization of the global shape of the moons and the determination of the distribution and topography of surface features [4].

Radiometric and optical measurements have proven very effective in determining the internal structure and rock-to-ice mass ratio, as well as in detecting liquid layers inside icy satellites as introduced in Chapter 2. Moreover, Doppler tracking data collected by the Radio Science Subsystem (RSS) experiment of the Cassini mission revealed the presence of a global ocean beneath the surface of Titan [6–8], also supported by the large obliquity estimated from synthetic aperture radar images acquired by the Cassini RADAR experiment [9–12]. In parallel, landmark tracking data acquired by the Cassini Imaging Science Subsystem have allowed the estimation of the diurnal libration amplitude of Mimas and Enceladus, also revealing the presence of global subsurface oceans in these worlds [13–16].

Despite their wide use for inference of geophysical properties, remote-sensing GS experiments do not provide direct and explicit information about the interior structure of a body and advanced methods and tools must be developed to retrieve it. Bayesian inference using the MCMC approach, introduced in Sec. 2.4, represents a highly effective technique for addressing complex inverse problems in the field of space geodesy. This method combines the probabilistic framework of Bayesian inference with the computational efficiency of MCMC algorithms to estimate the posterior distributions of model parameters, for a given set of measurements with accompanying uncertainties, thereby providing a comprehensive quantification of

uncertainties (see e.g., Robert and Casella [17]). A number of measurements from previous missions have been analyzed using MCMC in order to constrain interior structures (refer to [Sec. 2.4.3](#) for examples of this approach in both real and simulated data).

In this work, we present an alternative approach in which the scientific objectives of the mission serve as the primary driver for its design. Such an approach offers a significant advantage, as it allows mission designers to secure scientific objectives while making the necessary adjustments. We apply this new approach to the GS experiment of the UOP mission, which aims to address the scientific questions raised by the latest Planetary Science and Astrobiology Decadal Survey regarding the Uranian satellites. The design of this mission is in its preliminary stages, where payloads and observation strategies are still being defined. This makes it an excellent test case. In [Sec. 4.2](#), we explain our working strategy, which consists of the combination of a reverse approach based on the Bayesian inversion technique and of POD inversions, allowing to derive mission requirements (instrument performance and observation strategy) ensuring the achievement of the scientific objectives. In [Sec. 4.3](#), we apply the MCMC approach to build lookup tables for uncertainties in interior parameters (densities, thickness, and rock-ice mass ratio) corresponding to given measurement precisions on geodetic parameters (mean moment of inertia, tidal Love number, obliquity and diurnal libration amplitude), for a range of interior models for each satellite. Readers can access these lookup tables as Machine readable tables (MRTs) (see [Appendix B](#)). By interpolating from the lookup tables, any uncertainty on one interior parameter can be mapped to a set of geodetic parameters precision measurements. In what follows, we will use the terms "look-up tables" and "maps" interchangeably. We illustrate how these lookup tables facilitate a rapid assessment of the precision required in geodetic measurements to achieve the desired level of uncertainty for any interior parameter, and consequently, any interior objective. In [Sec. 4.4](#), we demonstrate a second use of these maps by evaluating different designs for the UOP GS experiment in terms of their ability to meet the scientific goals of the mission. We perform a covariance analysis of the geodetic parameters by simulating synthetic radiometric and optical data, then use the maps developed in [Sec. 4.3](#), and finally propose general guidelines for the future development of the UOP GS experiment. Conclusions are gathered in [Sec. 4.5](#).

4.2. THE MAPPING METHOD: DRIVING MISSION DESIGN THROUGH SCIENCE OBJECTIVES

In the conventional framework of a GS experiment, the mission design results in an experiment configuration. A POD simulation is then conducted to predict the expected geodetic measurement uncertainties. Finally, an inverse technique (such as, but not limited to, MCMC) is employed to assess the degree to which these uncertainties can constrain the interior structure of the investigated body. This traditional approach can be limiting, as it strongly depends on the specific experiment design at hand (e.g., spacecraft orbit, instrument performance) and frequently necessitates iterative redesigns to meet scientific objectives, especially

when confronted with evolving mission constraints (see for example the Europa Clipper case [18]). In light of these limitations, we adopt a 'reverse mapping' approach, scaling up works such as that of Ermakov *et al.* [19]. Rather than starting with experiment designs, we focus on the desired scientific outcomes, namely the uncertainty on key interior parameters. We then map these back to the required geodetic measurements precisions, which depend directly on the mission design.

One objective of the UOP GS experiment, as highlighted in the UOP STM, is to investigate the interior properties of the Uranian satellites. In particular, the objective is to determine their rock-to-ice mass ratio and identify which moons, if any, are ocean worlds. Here we consider Doppler measurements alone or in combination with image landmark tracking data. Other types of measurements, such as the intensity and direction of the magnetic fields, the global shapes or the detection of plumes/activity, could also contribute to the science goals [4], but are outside the scope of this study. As indicated in the introduction, GS experiments carried out in the past on icy moons have demonstrated that it is possible to precisely measure the satellite's static gravity, tidal response, spin axis orientation, and libration amplitudes with multiple spacecraft flybys. These geodetic observations allow the inference of some characteristics of the internal structure of the moons, thereby providing critical insights into their composition, thermal state and formation.

The hydrostatic components of the quadrupole gravity field coefficients J_2 and C_{22} can be used to infer the body's normalized mean moment of inertia (MoI or I/MR^2) as explained in Sec. 2.2. The MoI provides insights into the overall density distribution of a body, indicating the extent of differentiation. When combined with data on the body's overall composition (such as its bulk density deduced from its total mass and volume) and assumption on the hydrosphere density, the MoI can be used to estimate the rocky core density and radius (see e.g., Iess *et al.* [20]). The relation between the hydrostatic contribution to the gravity field and the MoI can be written using the Radau-Darwin approximation (RDA) for synchronously rotating moons as (e.g., Van Hoolst *et al.* [21]):

$$\text{MoI} = \frac{I}{MR^2} = \frac{2}{3} \left(1 - \frac{2}{5} \sqrt{\frac{5}{3\Lambda_{2,0} + 1}} - 1 \right) \quad (4.1)$$

with $\Lambda_{2,0} = 2J_2/5q$ and $q = n^2 R^3/GM$, where J_2 is the zonal degree-two gravity coefficient, n is the mean motion, and R is the body's mean radius. The body's periodic response to tidal forces arising from Uranus's gravity field is described by the Love number k_2 [22]. In the presence of an ocean, the amplitude of k_2 for an icy satellite is primarily driven by the shell thickness, with a secondary influence from its rigidity and viscosity [23]. The rotational state of the body is also influenced by its interior structure as thoroughly discussed in Sec. 2.3 and, specifically for the Uranian satellites, in a companion paper [24]. The obliquity (ϵ) of bodies locked into a Cassini state is a RDA-independent measurement of their differentiation level, as discussed for instance by Margot *et al.* [25] for Mercury. Similarly, the libration amplitude (g_s) provides k_2 -independent constraints on the ice shell thickness and rigidity, as discussed by Van Hoolst, Baland, and Trinh [26] for Enceladus. By combining the determinations of g_s and k_2 , we can expect to resolve the thickness

and rigidity of the ice shell more efficiently than when each observation is used alone [27].

The MCMC approach applied in this study is analogous to that defined by Cascioli *et al.* [28]. Differently from them, the interior is constrained based on the assumption that measurements of the aforementioned geodetic observables —namely, mass, MoI (as derived from the degree-two gravity coefficients), k_2 Love number, libration amplitude, and obliquity— have been obtained. Specifically, we estimate the parameters (densities and thicknesses) of a three-layer interior by minimizing the residuals between the synthetic geodetic measurements and the geodetic observables as computed from dedicated models. Our three-layer interiors comprise a core, a global liquid layer, and an outer ice shell, each with homogeneous densities. We use ranges of interior models which are fairly consistent with the ranges defined in Hussmann, Sohl, and Spohn [29], Bierson and Nimmo [30], and Castillo-Rogez *et al.* [31]. The dedicated models to calculate the geodetic observables are the propagator matrix technique to calculate the k_2 Love number [32] and the models for a satellite with an ocean of Baland and Van Hoolst [33] and Baland, Coyette, and Van Hoolst [34] to compute the libration amplitude and obliquity of the ice shell (we here neglect the small effect of tides on the rotational response). The MoI is calculated by direct integration of the density profile.

Our objective is to assess the expected uncertainties associated with the interior parameters estimation, such as the ice shell thickness (d_s), ocean thickness (d_o) and the rock-to-ice mass ratio, as a function of the geodetic measurement uncertainties (i.e., the precisions on k_2 , g_s , ε , and MoI). Since these uncertainties in interior parameters depend on their own true value, a series of truth models representing diverse interior structures, encompassing both ocean-bearing and oceanless cases, are simulated. To this end, the MCMC setup entailed running 625 simulations per satellite truth model, testing measurement precisions ranging from 100 % to 0.01 % of the signal levels (i.e. of the predicted amplitude of k_2 , g_s , ε , and MoI). The extensive range of truth models allows the construction of a comprehensive “lookup table” for each interior parameter, mapping all possible combinations of geodetic measurement uncertainties to their corresponding uncertainties on interior parameters. Specifically, we define the interior parameter uncertainty as half of the range between the 15.87th and 84.13th percentiles of the posteriori distribution. Each MCMC run is defined by 50 model realizations, or walkers, and the affine-invariant ensemble sampler chain for each walker (i.e., the Markov chain) is set to 100000. In particular, we use the affine-invariant ensemble sampler implemented in the publicly available emcee Python library [35]. We validated this configuration by simulating independently the cases with the most stringent measurement uncertainties and subsequently tested convergence through standard techniques (e.g., chain autocorrelation time). However, as the number of walkers and the chain length required for convergence vary case by case, in order to populate the “lookup tables”, we exclude runs where the simulated true value is not within the 5th to 95th percentile range of the resulting probability distribution of the parameter of interest. This ensures that the mapping is correctly done. An additional constraint is defined when the map is created to determine the geodetic measurements precision starting from an

interior objective in order to reject flat a posteriori distributions whose uncertainty will be non-informative. To this end, we discard solutions where the calculated ratio between the width of the 5th to 95th percentile range and the width of the uniform prior distributions of the interior parameter of interest is higher than 60 %.

The aforementioned “lookup tables” can be used to determine the required geodetic measurement uncertainties following the definition of an interior-related scientific objective, as detailed in [Sec. 4.3](#). They can also be used to quickly assess the performance of an experiment in terms of interior parameters, given the expected uncertainty associated with the geodetic parameters as shown in [Sec. 4.4](#). This mapping approach can therefore offer a significant advantage by enabling mission designers to make necessary trade-offs without sacrificing scientific returns. By initially focusing on desired scientific outcomes and mapping these to the required geodetic measurement uncertainties, designers can easily identify which geodetic measurements require high precision and which can tolerate larger uncertainties. Consequently, they can allocate resources more efficiently to ensure that the most crucial geodetic measurements given a scientific objective receive the necessary attention and investment. Furthermore, by establishing a comprehensive relationship between geodetic measurement quality and scientific objectives, we can maintain flexibility in the mission design. As the mission preparation progresses and new constraints emerge, this approach allows for informed adjustments to be made without jeopardizing the scientific goals.

In the following sections, we apply this approach to the five major Uranian satellites in order to provide useful guidelines for the design and implementation of the UOP gravity science experiment.

4.3. A FORWARD APPLICATION FRAMEWORK: LINKING INTERIOR STRUCTURE TO GEODETIC MEASUREMENT REQUIREMENTS

The five major Uranian satellites are small to medium sized, with radii ranging from 235.8 km for the innermost Miranda to 788.9 km for the second to last Titania [\[29, 36\]](#). Given their relatively small size and mass [\[37\]](#), the signature of the static parts of their gravity fields is weak, compared to that of larger icy satellites, like Titan or Europa, which could hinder a precise determination of their MoI. Compared to those of large icy satellites, their periodic tides are also relatively small (k_2 smaller than 0.01 for the Uranian satellites, see Baland *et al.* [\[24\]](#), against 0.4-0.6 for Titan, see *less et al.* [\[6\]](#), Durante *et al.* [\[7\]](#), and Goossens *et al.* [\[8\]](#)). As the amplitude of the rotation signals not only depends on the size of the satellite, but also on its orbital parameters, there is a chance of obtaining detectable and useful signals for Uranus’s satellites as discussed in Baland *et al.* [\[24\]](#). For instance the libration amplitude of Miranda is expected to range between about 40 m and 90 m for an ocean thickness between 2.5 km and 50 km, against less than 10 m for Ganymede [\[27\]](#). Oberon’s obliquity could be of the order of 0.1° , or even larger in the case of a resonance due to the presence of a relatively thin global ocean, values similar to Titan’s obliquity

(0.3° , Stiles *et al.* [9]).

On the one hand, the MoI, as an individual measurement, is indicative of the degree of differentiation and, provided the precision of the measurement is sufficient, of the thickness of the hydrosphere (see Castillo-Rogez *et al.* [31] for the measurement requirements, revised in Baland *et al.* [24]). A precision of ± 0.005 on the MoI corresponds to no better than 50 km uncertainty on the hydrosphere thickness for all the satellites of Uranus except Miranda for which it maps to 30 km uncertainty. Excluding homogeneous interiors (MoI = 0.4) is easier, but the required uncertainty depends on the actual value.

On the other hand, the tidal Love number and rotation parameters (diurnal libration amplitude and obliquity), considered separately, could be used to assess differentiation and detect the presence of a global subsurface ocean [24]. The measurement requirements identified in Baland *et al.* [24] are summarized in Table 4.1 and commented on here:

4

- *Libration requirements:* Confirming the differentiation or presence of an ocean inside Titania and Oberon from a libration measurement would require tighter precision than for the other three satellites. To detect an ocean, an independent estimation of the MoI is needed to help discriminate between possible interiors. As a general rule, the thicker the ocean, the thinner the shell, the larger the libration, and the easier it is to detect the ocean and perhaps even to constrain the thickness of the shell, given the seemingly inverse relation between libration amplitude and shell thickness.
- *Obliquity requirements:* It would be easier to confirm the differentiation of Miranda, Titania and Oberon, if they were solid bodies, by measuring obliquity than by measuring libration. The opposite is true for Ariel and Umbriel. In the ocean case, the obliquity is close to that of the solid case, because on the long timescales of precession, the body tends to precess as a whole. For some interior structures, however, the presence of the ocean may enhance or reduce the obliquity through resonance between a forcing and a free mode. For Miranda, this may only occur for oceans less than 6 km thick. For the other satellites, a wider range of ocean thicknesses makes resonance possible, but not guaranteed.
- *k_2 requirements:* According to Table 3 in Baland *et al.* [24], a differentiated body can be distinguished from a homogeneous body with a measurement uncertainty on k_2 of about 0.0002 for Miranda, about 0.0035 for Ariel and Umbriel, and about 0.0075 for Titania and Oberon. These levels of precision also apply to the detection of a liquid ocean, since the lowest value of k_2 in the ocean case for the considered range of interior models is almost equal to the value for the homogeneous case. The aforementioned requirements are approximately ten times larger for Miranda and approximately six times larger for the other satellites, when the upper bounds for k_2 in the ocean case are considered.

The aforementioned requirements on the single measurements for tides and rotation are particularly challenging, especially for a mission such as UOP with

Table 4.1.: Geodetic measurement requirements to distinguish between homogeneous and differentiated solid bodies and to constrain the hydrosphere thickness or detect an internal global ocean from their individual use (adapted from Baland *et al.* [24]). MoI requirements correspond to an uncertainty of 50 km on the hydrosphere thickness, except for Miranda (30 km). Libration requirements correspond to the detection of an ocean under an ice shell 150 km thick, except for Miranda (40 km).

(*) requires an independent determination of the MoI at a few percent.

Measurement	Goal	Miranda	Ariel	Umbriel	Titania	Oberon
MoI	Differentiation	(depends on actual value)				
	Hydrosphere thickness	0.005	0.005	0.005	0.005	0.005
Librations	Differentiation	5 m	5 m	5 m	0.5 m	0.25 m
	Ocean detection (*)	20 m	5 m	5 m	0.6 m	0.3 m
Obliquity	Differentiation	8 m	1 m	2 m	30 m	435 m
	Ocean detection	(requires resonance, depends on actual value)				
Love number	Differentiation	0.0002	0.0032	0.0032	0.0077	0.0072
	Ocean detection (conservative)	0.0002	0.0031	0.0028	0.0067	0.0062
	Ocean detection (optimistic)	0.0019	0.0174	0.0165	0.0350	0.0345

multiple science objectives. Reaching similar relative precisions is already particularly ambitious even with a dedicated mission such as Europa Clipper, for which 49 flybys are planned [38]. Furthermore, the determination of time-varying signals, such as tides, is inherently challenging due to the low eccentricity of the Uranian satellites (e.g. Ariel's eccentricity is about thirty times smaller than that of Titan). In this section, we apply the reverse approach described in Sec. 4.2 to the five major Uranian satellites. As previously stated, we combine measurements of the MoI, k_2 Love number, libration amplitude, and obliquity, along with the mass, in order to explore their combined sensitivity to the interior parameters and relax the stringent requirements reported in Table 4.1. In particular, we focus our discussion on the expected probability distribution of the ice shell thickness (d_s), ocean depth (d_o), and rock-to-ice mass ratio

$$rock/ice = \frac{\rho_c R_c^3}{\rho_s (R_s^3 - R_o^3) + \rho_o (R_o^3 - R_c^3)}. \quad (4.2)$$

where the subscripts c , o , and s refer to silicate core, ocean and ice shell quantities, respectively.

Synthetic “true” geodetic observables are computed for a suite of interior models characterized by ocean thickness, d_o , ranging from 0 km to 50 km in increments of 10 km. Based on the interior evolution modeling by Castillo-Rogez *et al.* [31], we restrict our models to relatively thick ice shells, considering oceans not exceeding the 50 km upper limit suggested for Titania and Oberon in their study. The densities for the core, ocean, and ice shell are also adopted from their work and fixed at 3060 kg m^{-3} , 1000 kg m^{-3} , and 950 kg m^{-3} , respectively. The core radius and ice shell

thickness are computed to ensure consistency with the mean satellite radius and mass. Shell and core are assumed elastic, with rigidity fixed at 3.3×10^9 Pa and 50×10^9 Pa, respectively [29]. Rheological properties such as ice shell rigidity and viscosity are not varied explicitly, as their influence on the tidal Love number (k_2) is relatively minor compared to the ice shell thickness. Specifically, the rigidity of the core is sufficiently large that plausible variations would have negligible effects on k_2 . Variations in ice shell rigidity within realistic bounds (approximately 3 GPa to 4 GPa) yield only limited changes to k_2 , typically less than 10 % (see Wu *et al.* [39] for Europa). Concerning viscosity, we employed a simplified Maxwell rheological model, assuming a convective lower ice shell layer viscosity of 10^{14} Pas. Typically, the ice shell is divided into a convective lower layer, close to the melting point of ice with viscosity ranging from 10^{13} Pas to 10^{15} Pas, and an upper conductive layer with significantly higher viscosity due to the steep thermal gradient. Even with a substantial convective layer extending radially up to half of the shell thickness, viscoelastic effects remain modest, resulting in an increase of k_2 of up to 20 % (Fig. 4.1, left panel). For thinner convective layers (approximately 10 % radial extent), these effects are generally below 15 % for all five Uranian satellites considered, as illustrated in Fig. 4.1 (right panel), thus significantly smaller than the impact of variations in shell thickness itself. Moreover, Hussmann, Sohl, and Spohn [29] obtained values for k_2 consistent within a few percent of our elastic-shell results, indicating minimal influence from rheological complexities under typical conditions. Consequently, while the potential rheological impacts on observables (up to $\approx 20\%$ for k_2 under certain viscoelastic assumptions) are acknowledged, the simplification in our rotational and tidal modeling is adopted primarily because these effects are secondary compared to the influence of parameters like ice shell thickness. This simplification is also crucial for computational efficiency, which is essential for generating the extensive lookup tables intended to inform mission design during its preliminary stages. While this approach affects the absolute predictive accuracy compared to more complex rheological models, it preserves the relative sensitivity mapping needed for the lookup tables. Therefore, it does not undermine the robustness of our conclusions regarding the required measurement precision for determining ice shell thickness at this early design phase. Such detailed refinements using more sophisticated models are appropriate for later design stages and are beyond the current scope of demonstrating this specific lookup table methodology.

The uncertainties associated with the synthetic geodetic observables are varied from 100 % to 0.01 % relative to the model true value, with the exception of the mass, for which a larger sigma of 1 % is fixed for every run. This larger mass uncertainty strategy is commonly used to incorporate the inherent uncertainty associated with the interior model unknowns into the inversion process, thus preventing the potential issue of non-convergence (see e.g., Cascioli *et al.* [28] for simulation, Genova *et al.* [40] for real data analysis or Sec. 4.4.2.1 for a dedicated test case applied to Ariel). This approach also yields more conservative results, thereby making it well suited for simulations at the initial stages of mission design. An uncertainty level of 100 % generally means that the estimation of the geodetic observable has failed and that it provides no information about the internal structure. There may, however, be

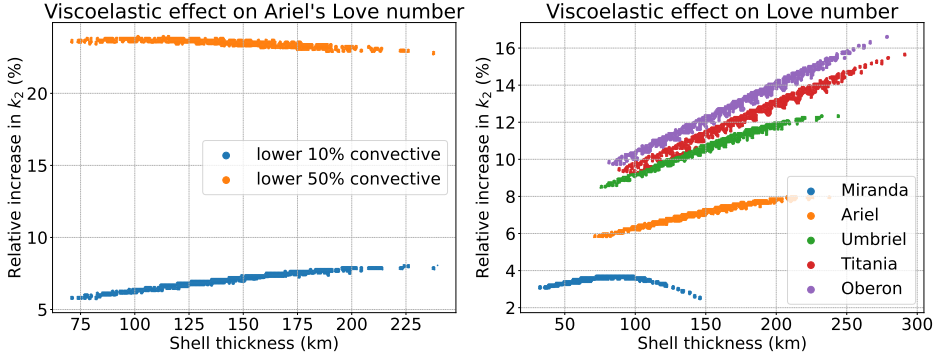


Figure 4.1.: Relative increase of the Love number due to viscoelastic effects for Ariel with two different convective layers (depicted on the left panel, thickness is given in terms of radial extent) and for all five satellites with a thin convective layer (depicted on the right panel, thickness is 10 % in radial extent and viscosity is 10^{14} Pas).

exceptions. For example, given the seemingly inverse relation between k_2 and shell thickness, an individual measurement of k_2 with a precision level above 100 % can, for interiors with thick shells and therefore small k_2 , provide a useful constraint on the thickness of the shell in the form of a lower bound. In some cases, the true value defining the 100 % level is not always the maximum value within the range of model predictions. Instead, it can be observed to be relatively small in comparison to the range as shown, for example, in Fig. 4.2 for the case of Miranda's Love number. Therefore, it should be kept in mind that requiring a measurement at signal level to constrain the interior does not always mean that the measurement is useless when considered in conjunction with other geodetic observables. Moreover, in certain instances, such requirements may still prove challenging, particularly when the absolute parameter values are very small. Specific cases are tested and discussed in detail in Sec. 4.3.1, and it is recommended to take a case-by-case approach in order to define the final instrument requirements.

Given that Ariel and Umbriel on the one hand, and Titania and Oberon on the other hand, have approximately the same size, mass and similar tidal and rotational responses [24], we only discuss the results for Miranda, Ariel, and Titania. The latter two will serve as proxies for Umbriel and Oberon, respectively. We discuss below the needed precision on the geodetic measurements (i.e. on MoI, k_2 , g_s and ϵ) relatively to their true value, as predicted by the forward modeling and reported in Table 4.3 for a case with a 50 km ocean and for a case without. We do not give the true values for all the ocean thicknesses considered here, because the order of magnitude of the geodetic observables remains similar.

All the following results are based on the interpolation of our “lookup tables” that map the geodetic measurements combinations to the interior parameters given a specific interior structure. Such measurements to interior mapping enables very

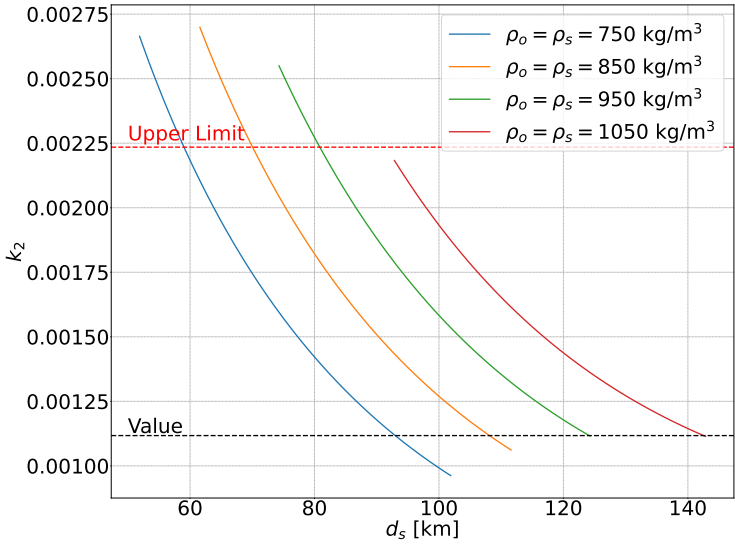


Figure 4.2.: Miranda's k_2 Love Number as a function of the ice shell thickness for a range of ocean and ice shell densities ($\rho_o = \rho_s$). The upper limit in k_2 corresponds to a measurement of 0.015(15), implying a lower limit on ice shell thickness (≈ 60 km).

Table 4.2.: Uniform priors used for interior parameter space sampling.

	Unit	Miranda	Ariel	Umbriel	Titania	Oberon
Ice-I shell density	ρ_s kgm ⁻³	700–1050	700–1050	700–1050	700–1050	700–1050
Water ocean density	ρ_o kgm ⁻³	700–1050	700–1050	700–1050	700–1050	700–1050
Rocky core density	ρ_c kgm ⁻³	2400–3500	2400–3500	2400–3500	2400–3500	2400–3500
Ice-I shell thickness	d_s km	30–150	70–250	70–250	70–290	70–280
Water ocean thickness	d_o km	0–100	0–100	0–100	0–100	0–100

Table 4.3.: Expected geodetic observables for a three-layer interior with and without a global subsurface ocean. Ariel is a proxy for Umbriel and Titania for Oberon.

		Unit	Miranda		Ariel		Titania	
			Oceanless	50 km Ocean	Oceanless	50 km Ocean	Oceanless	50 km Ocean
Moment of Inertia	MoI		0.3409	0.3417	0.3123	0.3125	0.3116	0.3117
Love Number	k_2		4.9×10^{-4}	2.52×10^{-3}	1.72×10^{-3}	2.03×10^{-2}	3.1×10^{-3}	3.99×10^{-2}
Libration amplitude	g_s	m	-48.29	-58.88	-21.46	-29.27	-2.27	-3.19
Obliquity	ϵ	m	84.6	83.6	7.6	7.5	186.9	262.7

rapid assessment of the geodetic measurement precision required to achieve a desired level of uncertainty of any interior parameter and therefore any interior objective. As the cases discussed in this paper in terms of interior parameters determination constitute only a handful among many possibilities (we did not investigate the determination of the individual densities of the different layers for instance), we deliver our lookup tables (6 per satellite, one for each ocean thickness considered here) as Machine Readable Tables (MRTs) (see [Appendix B](#)). Readers can use them to construct their own tool in accordance with their specific requirements in terms of measurements and interior objectives. Note that these maps can also be used in reverse to determine how well interior properties can be constrained given four geodetic measurement uncertainties. This usage is demonstrated and discussed in [Sec. 4.4.2](#).

4

4.3.1. NOMINAL CASE (TRUE OCEAN THICKNESS = 20 km)

We first consider a nominal case for which the “true” thickness of the ocean is fixed at 20 km. Castillo-Rogez *et al.* [31] do not predict an ocean thicker than 30 km for Ariel and Umbriel and 50 km for Titania and Oberon, so a thickness of 20 km can be used as a reference case for the medium-sized satellites. Although Miranda is not thought to harbor a global subsurface ocean, we are nevertheless exploring this possibility here.

[Fig. 4.3](#) shows the combinations of geodetic measurement precisions relative to the model true value necessary to determine the ocean thickness (d_o), the rock-to-ice mass ratio (*rock/ice*), and the ice shell thickness (d_s) within a specified uncertainty level (from 5% to 30%) assuming the satellites have an ocean of 20 km thickness. The rock-to-ice mass ratio uncertainty is calculated from the a posteriori distribution of the core (ρ_c), ocean (ρ_o), and ice shell (ρ_s) densities and of the core (R_c), ocean (R_o) and ice shell (R_s) radii using [Eq. \(4.2\)](#). There are multiple ways to select combinations of geodetic measurement uncertainties that give the same result in terms of interior parameters uncertainties. Here we discuss combinations that are obtained such that the $(\sigma_{\text{MoI}}, \sigma_{k_2}, \sigma_{g_s}, \sigma_\epsilon)$ quadruplet maximizes the Euclidean (L2) norm. In other words, we choose the combinations that require the loosest constraints across all four geodetic measurements.

Ocean thickness For all the satellites, determining the thickness of the ocean to an accuracy of 30% requires a combination of MoI, k_2 and Libration measurements with an uncertainty equal to or less than one percent and of obliquity measurement with a larger uncertainty, which may be at signal level for Miranda. For Ariel, the obliquity measurement is required with an absolute precision of 0.25 m, whereas the MoI, k_2 , and g_s must be determined with absolute precisions of 9.2×10^{-4} , 1.4×10^{-4} , and 0.24 m, respectively. Overall, the ocean thickness is more challenging to determine than the rock-to-ice mass ratio and ice shell thickness (see below). It is worth mentioning that measuring the ocean thickness at the 5% level for Miranda is not feasible without relative uncertainties that are smaller than 10^{-4} (no blue polygon on the left panel of [Fig. 4.3a](#)).

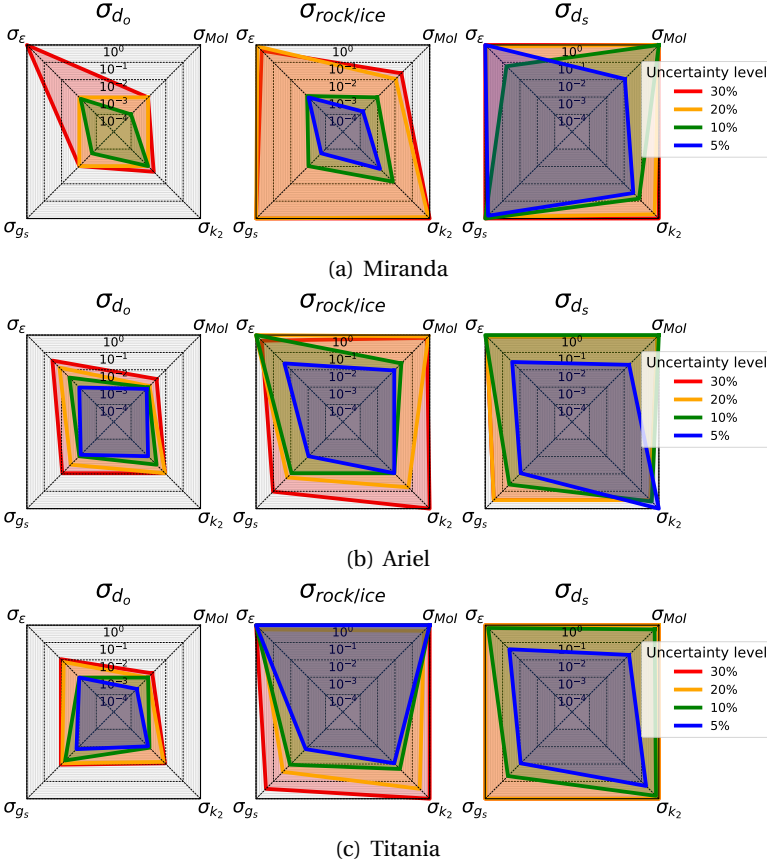


Figure 4.3.: Combinations of measurement relative precisions allowing to measure the ocean thickness (d_o), the rock-to-ice ratio (rock/ice), and the ice shell thickness (d_s) with a given uncertainty level when the satellites have an ocean of 20km depth. The combinations shown are the ones that have the maximum L2-norm (loosest measurement requirements) and that achieve the desired level of uncertainty.

Rock-to-ice mass ratio For Miranda, a relative uncertainty level of 30% on the rock-to-ice mass ratio can be achieved by measuring the MoI with a precision of the order of the percent (so 7.95×10^{-3} in absolute) along with a determination of the k_2 Love Number, obliquity and libration amplitude at the signal level (so 1.5×10^{-3} , 0.015° and $\sim 0.0002^\circ$, respectively). From a dedicated MCMC analysis where only the MoI measurement is considered, we find an uncertainty of 34% on the rock-to-ice mass ratio. Therefore, the 100% required levels on k_2 , obliquity and libration means that these parameters barely contribute to the constraint on the *rock/ice* ratio with an equivalent 30% uncertainty. Ariel (Umbriel) requires

similar measurement requirements to those of Miranda for obliquity and k_2 , both at the signal level (i.e. 7.4m and 1.6×10^{-2} in absolute, respectively). However, the measurement at the percent level of libration (2.8m in absolute) is required instead of the MoI measurement. The same relative precisions are needed for Titania (Oberon), corresponding in absolute to 0.8m on g_s , 0.3 on the MoI, 0.003 on k_2 and 162m on ε . The latitude we gain in determining the MoI for Ariel and Titania comes at the expense of the precision needed in determining g_s (a dedicated MCMC analysis indicates the same trend as discussed for Miranda). To reach an uncertainty of the order of 5% or better on the rock-to-ice mass ratio, it is necessary, for Miranda and Ariel, to determine the four geodetic observables with a precision $\leq 1\%$ of the signal level. In the case of Titania, however, the same precision is achieved with the measurement of obliquity and moment of inertia at the signal level.

Ice shell thickness The ice shell thickness d_s is the less challenging interior parameter to constrain from the geodetic observables. It can be determined with an uncertainty of 30% or 20% by measuring the four observables at the signal level for all satellites. This illustrates how sometimes an observable with 100% uncertainty is still necessary to infer an interior parameter (just because it provides an upper bound for its true value). Generally speaking, MoI and ε are less sensitive to d_s than k_2 and g_s , as expected from theory [24]. For Ariel and Titania, MoI and ε are even not needed to still measure d_s with 10% precision. If one wants to be more precise and get d_s with 5% uncertainty, then MoI and obliquity need to be determined to a few percents. For Miranda, k_2 drives the uncertainty level on d_s , while the libration is the main driver for the determination of d_s for Titania (Oberon). For Ariel (Umbriel), k_2 is not contributing and g_s is the most influencing observable among the other three.

4.3.2. SENSITIVITY TO OCEAN THICKNESS

The results described above are based on a truth model with $d_o = 20\text{km}$. The impact of this assumption on the results has been tested and is discussed Fig. 4.4 for Ariel only (as the behavior is the same for all moons), where we vary the thickness of its ocean from 0km to 50km. As we are interested in the impact of the value of the “true” d_o on the required precisions, we show here the ratio of the latter with respect to their value in the nominal case ($d_o = 20\text{km}$), presented in Fig. 4.3. We see on Fig. 4.4 that the actual ocean thickness (i.e. the “true” d_o in x-axis) mostly impacts the required uncertainties to determine d_o itself (top panel). The thinner the ocean, the more precise the geodetic observables must be to reach a given level of precision on d_o . This is not true for the other two interior parameters, d_s and *rock/ice*, for which the required uncertainties vary less with d_o and do not always increase with it (see mid and bottom panels of Fig. 4.4).

4.3.3. COMBINATIONS FROM A SUBSET OF GEODETIC OBSERVABLES

It is important to note that there exists a multitude of combinations of MoI, k_2 , g_s and ε that allow the determination of an interior parameter below a desired

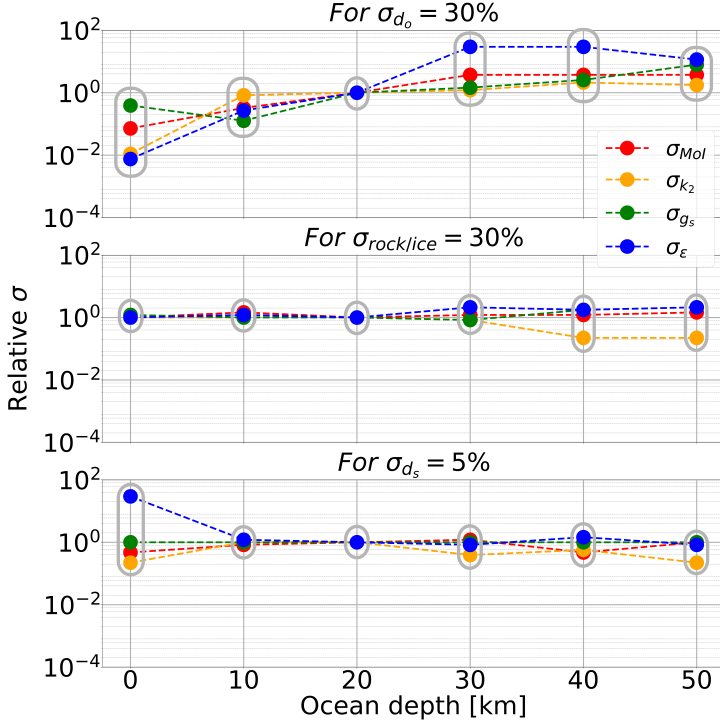


Figure 4.4.: Ocean thickness dependence of required precision quadruplets (marked with gray frames) in Ariel's geodetic observables, relative to our $d_o = 20$ km nominal case. These uncertainties in the MoI, k_2 , g_s , and ϵ are selected such that one can determine the ocean thickness itself (d_o) and the rock-to-ice ratio ($rock/ice$) at the 30 % level, and the ice shell thickness (d_s) at the 5 % level. For the case with no ocean ($d_o = 0$ km), the σ_{MoI} , σ_{k_2} , σ_{g_s} , and σ_{ϵ} are selected such that d_o can be inferred with an uncertainty of 30 km, which makes this case not readily comparable to the others, but nonetheless quite instructive. The quadruplets are the ones with the maximum L2-norm among all possible combinations.

degree of precision. Some of these combinations could be seen (from the mission design point of view) as particularly interesting since they do not require all four geodetic observables to be determined at a precision better than 100 %. Most of the time (but not always, as discussed above for d_s at 20-30 %) this means that one (or more) observables are basically not systematically needed, which can simplify the operations. For instance if the libration is not needed to reach the targeted uncertainty on the interior of one of the satellites (e.g., for d_s of Miranda), then one can use the camera for another purpose. We show some of these particular combinations in Fig. 4.5. Their number depends on the interior parameter and the

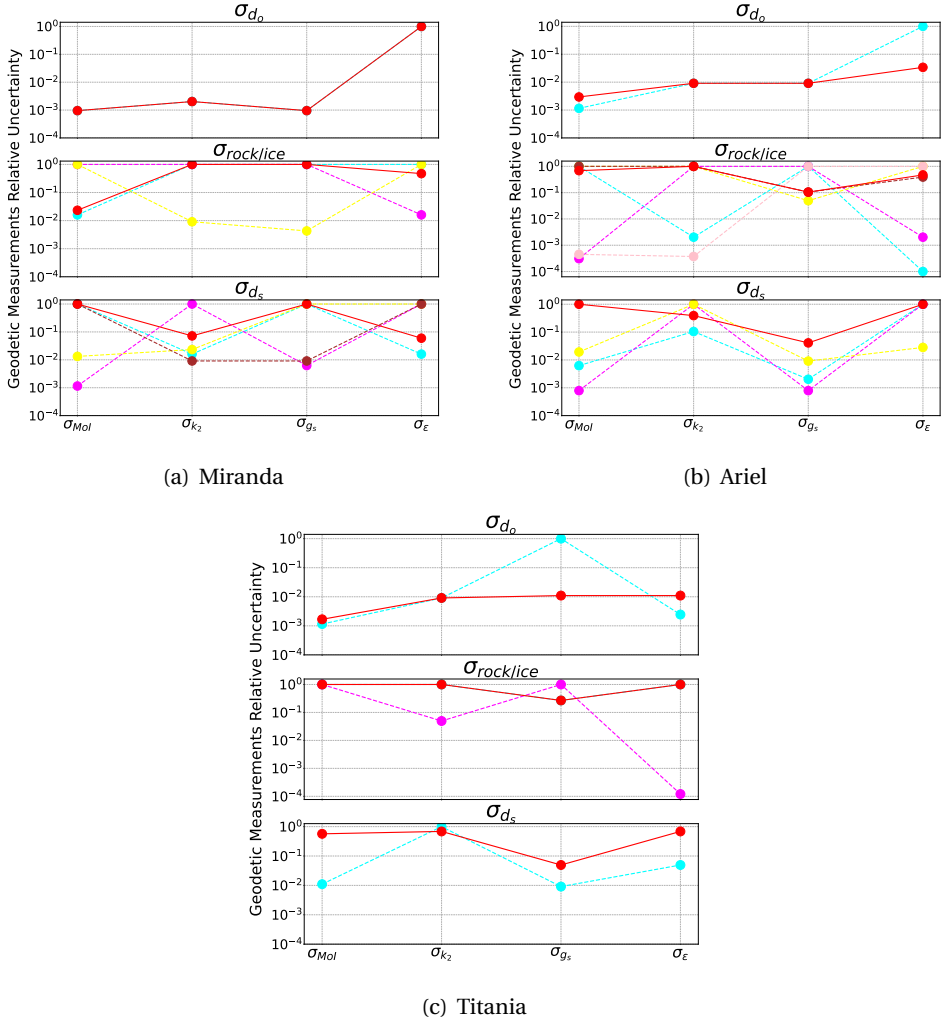


Figure 4.5.: Alternative quadruplets allowing to measure the ocean thickness (d_o) and the rock-to-ice mass ratio (rock/ice) with a 30 % uncertainty level, and the ice shell thickness (d_s) at 5 % when the satellites have an ocean of 20 km depth. These combinations all have one or more geodetic parameters with 100 % uncertainty. Each colored dotted line represents a quadruplet. The red solid lines indicate the solutions presented in Fig. 4.3 for the nominal case.

body considered. For example there are more of these combinations that lead to a 5 % uncertainty estimate of Miranda's ice shell thickness than for Ariel and Titania. In regard to the latter two, we only found one quadruplet that allows to determine

d_o with a precision of 30% with one geodetic parameter determined with 100% uncertainty (e.g., for Titania a relative precision of about 10^{-3} is required for σ_{MoI} and σ_ε , 10^{-2} for σ_{k_2} , whereas libration can be determined to signal level). As it might be expected, relaxing the needed uncertainty on a given observable results in an increase in the precision needed on the others, as clearly shown in Fig. 4.5. It seems also that such a trade-off is “observable-dependent”, meaning that a given observable will preferably compensate for another one specifically. For instance, libration and k_2 tend to compensate each other.

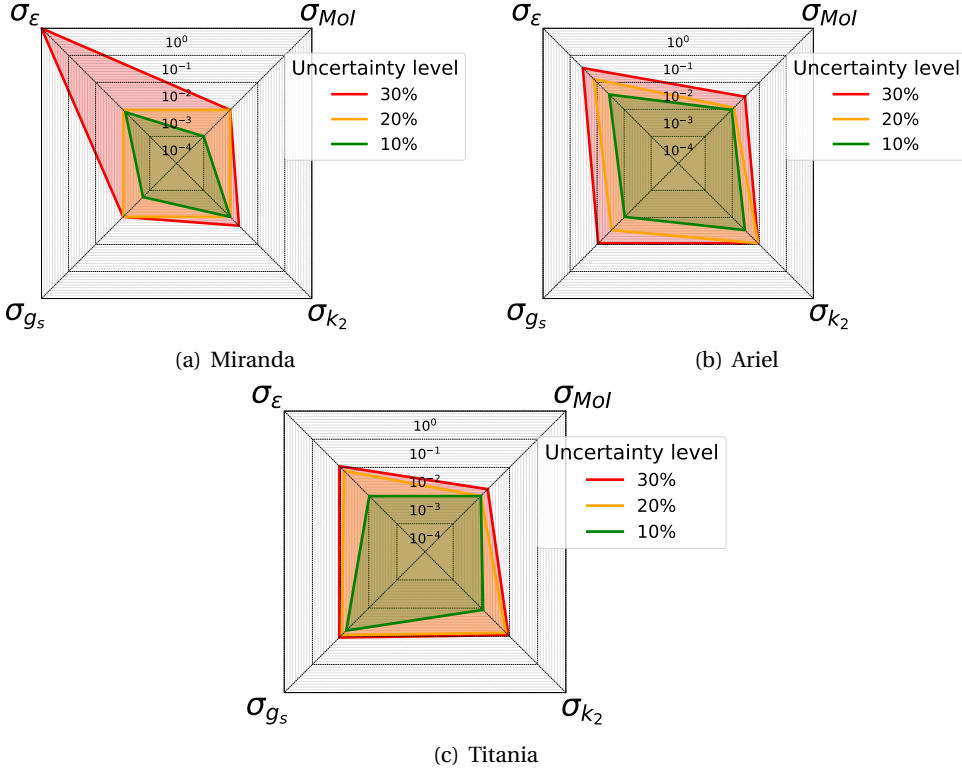


Figure 4.6.: Required relative precisions on MoI, k_2 , g_s , and ε to determine all three interior parameters (d_o , rock/ice, and d_s) with a specified degree of uncertainty for a satellite with an ocean depth of 20 km. The relative precisions are set to be equal for each interior parameter. The combination shown is the one with the maximum L2-norm among all possible combinations.

4.3.4. SIMULTANEOUSLY ACHIEVING DIFFERENT INTERIOR OBJECTIVES

So far, we have been selecting the geodetic measurement precision quadruplets such that one specific interior parameter can be determined with a given

level of uncertainty. We now discuss the case where the interior parameters (d_o , *rock/ice*, d_s) are simultaneously estimated at a same relative level of precision. As previously, our truth model assumes $d_o = 20$ km and we target an uncertainty on all three interior parameters at 30%. Fig. 4.6 shows how challenging it is to precisely measure d_o , *rock/ice* and d_s at once, in particular for Titania where all four geodetic parameters must be determined with 1% uncertainty or better. Even for the case with only 30% uncertainty on d_o , *rock/ice* and d_s , one would need to measure the geodetic parameters of Miranda with the following formal errors: $\sigma_{\text{MoI}} = 1.5 \times 10^{-3}$, $\sigma_{k_2} = 6.5 \times 10^{-6}$, $\sigma_{g_s} = 0.2$ m, $\sigma_\varepsilon = 9$ m. For Ariel (Umbriel), the required uncertainties would be $\sigma_{\text{MoI}} = 5 \times 10^{-4}$, $\sigma_{k_2} = 1.5 \times 10^{-4}$, $\sigma_{g_s} = 0.2$ m, $\sigma_\varepsilon = 4.5$ m, and for Titania (Oberon) we would need $\sigma_{\text{MoI}} = 5 \times 10^{-4}$, $\sigma_{k_2} = 3 \times 10^{-4}$, $\sigma_{g_s} = 3.5 \times 10^{-2}$ m, $\sigma_\varepsilon = 3$ m. This will be almost unachievable with UOP as discussed in the next section and as we can anticipate given the level of precision obtained on Titan's geodetic parameters from 10 Cassini flybys [7, 8]. This means that one might have to decide which of the three interior parameters really needs to be measured with good precision and which may not.

There are very few subsets of geodetic observables where at least one observable is measured at the signal level while allowing us to determine all three interior parameters simultaneously with needed uncertainty without requiring the other observables to be measured with extremely high precision ($\leq 10^{-4}$). Fig. 4.7 illustrates four potential combinations that enable the simultaneous determination of all three interior parameters at either the 30% (top) or at 20% (middle) or at 10% (bottom) level of precision. These combinations are characterized by one geodetic measurement having the loosest requirement, with the additional constraint that the others geodetic measurements should have a relative uncertainty $\geq 10^{-3}$. Reaching the 10% precision level on the three interior parameters of Ariel does not allow to loosen any geodetic observable significantly compared to the 10^{-3} threshold. That level of uncertainty on d_o , *rock/ice*, d_s can even not be reached for Titania and Miranda under these conditions, and the looser 20% level is still not achievable for the latter. Even at 30%, only libration (for Titania) or obliquity (for Ariel) can be relaxed to signal level, requiring then the other three to be measured at the 10^{-3} threshold level. For all other cases, the four geodetic observables must all have a relative precision better than 10% to reach the target uncertainty on d_o , *rock/ice*, d_s .

4.4. A BACKWARD APPLICATION FRAMEWORK: FROM GS EXPERIMENT DESIGN TO INTERIOR STRUCTURE OBJECTIVES

In this section, we analyze the ability of the UOP GS experiment designs to fulfill the mission's scientific objective regarding the interior properties of the Uranian satellites, as outlined in Sec. 4.3. The Precise Orbit Determination (POD) of the spacecraft usually relies on range and range-rate radiometric observations collected by the GS experiment of the mission to precisely reconstruct the spacecraft's

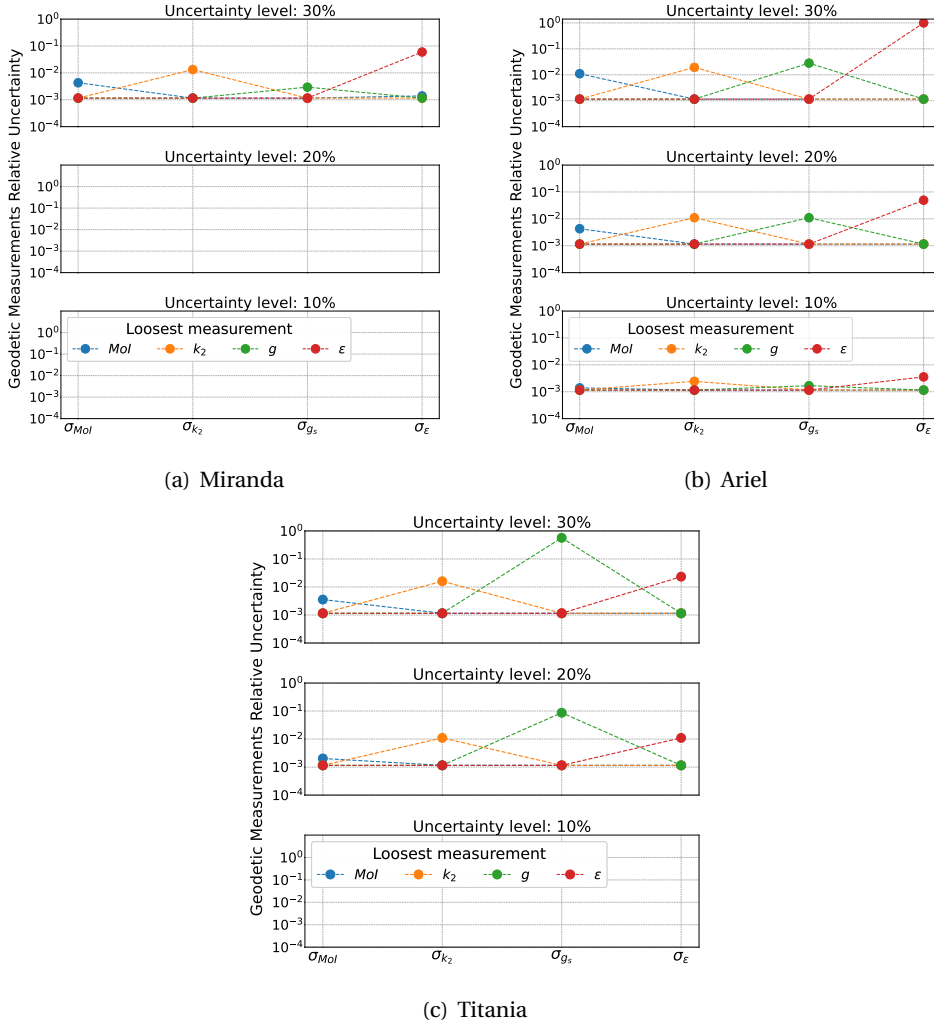


Figure 4.7.: Alternative measurement combinations to determine together the three internal parameters (ocean thickness (d_o), rock-to-ice ratio ($rock/ice$), and ice shell thickness (d_s)) with a specified degree of uncertainty for a satellite with an ocean depth of 20km. These combinations are characterized by one observable having the loosest possible requirement, whereas the others have a requirement greater than 10^{-3} . The absence of lines indicates that no combination has been identified.

trajectory, with the ultimate goal of constraining the interior structure of the body through the precise determination of its gravity field. The interior structure of a planet can also affect its rotation and orientation in space, but these are difficult

to measure using radiometric data alone, especially from flybys (e.g., Durante *et al.* [7] and Mazarico *et al.* [38]). In such a case, and therefore in particular for the UOP mission, landmark tracking from spacecraft image data is required to precisely measure the body's rotational parameters (e.g., Thomas *et al.* [14], Park *et al.* [16], and Konopliv *et al.* [41]).

The POD approach refines the spacecraft's dynamical model by comparing synthetic data, derived from both dynamical and observational models, with simulated measurements. A weighted least squares filter is used to improve the knowledge of the spacecraft's initial state vector and other variable parameters in the model, such as the gravity spherical harmonic coefficients. The goal is to minimize the sum of squared residuals between the synthetic data and the simulated measurements. An in-depth description of the POD least-squares technique can be found in Tapley, Schutz, and Born [42], whereas the landmark tracking technique used to process optical data is described in e.g., Park *et al.* [43].

First, we conduct a covariance analysis to assess the ability of radiometric and landmark tracking data from the UOP's satellite tour to infer the geodetic parameter of interest, namely σ_{Mol} , σ_{k_2} , σ_{g_s} and σ_{ϵ} . Full details about our POD simulations are discussed in Sec. 4.4.1. Secondly, the GS experiment designs are evaluated in terms of their capacity to infer the interior properties of the Uranian satellites, using the lookup tables computed in Sec. 4.3.

4.4.1. POD SIMULATION SETTINGS

4.4.1.1. SATELLITE TOURS

Two different tours of the Uranian satellites are evaluated in this study, the Decadal Tour and the Alternative Tour. Their ground tracks are shown in Figs. 4.8 and 4.9.

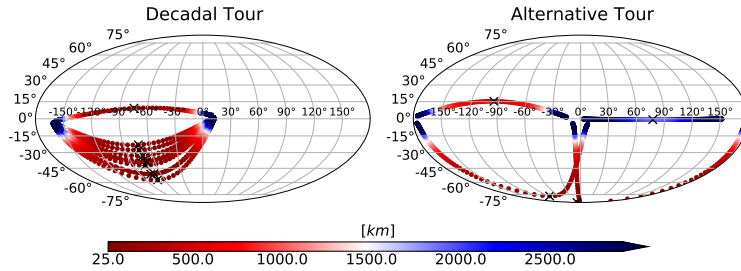


Figure 4.8.: UOP flybys of Ariel (± 30 min around C/A). Colors show the altitude w.r.t. the moon's surface, whereas the crosses show the point of C/A. Only flybys with a C/A distance less than three body radii are shown.

The Decadal Tour, as selected by the Planetary Science and Astrobiology Decadal Survey, is characterized by an initial inclined Uranus capture phase that enables the deployment of a probe designed to enter Uranus's atmosphere and relay scientific data to the orbiter. Thereafter, the spacecraft reaches a near-equatorial orbit

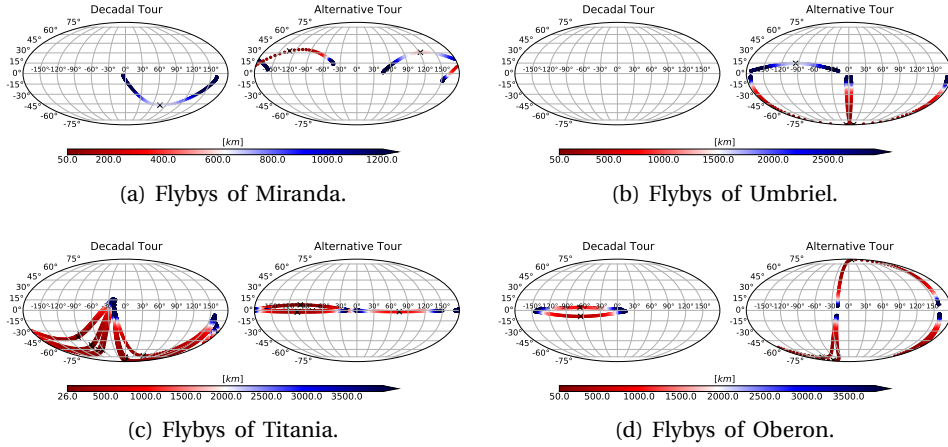


Figure 4.9.: UOP groundtracks (± 30 min around C/A) with respect to the Uranian satellites. Ariel's flybys are shown in Fig. 4.8. Colors show the altitude w.r.t. the moons' surface, whereas the crosses show the point of C/A. Only flybys with a C/A distance less than three body radii are shown.

that allows for a detailed science investigation of the five major moons. The equatorialization phase, which includes fifteen resonant¹ flybys of Titania, and the subsequent non-resonant flybys of the other moons are designed to be nominally ballistic, meaning no fuel is used. This tour is advantageous because it allows for targeting other moons through non-resonant transfers, as Uranus's major moons orbit near its equator. In this scenario, the spacecraft is planned to perform eight far non-resonant flybys of Umbriel, Oberon, and Ariel, followed by eleven close resonant flybys of Ariel to decrease the periapsis altitude relative to Uranus. The finale of this tour is characterized by a last maneuver that will dispose of the spacecraft into Uranus's atmosphere.

However, optimizing the required ΔV to reduce fuel consumption does not necessarily correlate with optimizing flybys for science objectives. Close (< 3 body radii), slow ($< 10 \text{ km s}^{-1}$), and varied geometries are key to properly infer the geophysical characteristics of a body using radiometric data (see e.g., Cassini at Titan [7]). Due to the resonant orbits used, the Decadal Tour performs 12 and 14 close flybys of Ariel and Titania, respectively, all concentrated in the southern hemisphere with latitudes up to -57° and -87° , respectively. The tour concludes with two close equatorial flybys of Oberon and only one far flyby of Miranda in the southern-east hemisphere. The use of resonance orbits not only limits the spatial sampling of the satellites, but also the temporal sampling which is necessary to determine signals at orbital frequencies, such as tides and librations. This temporal sampling is determined by the position of a satellite along its orbit, described by the

¹Two bodies are in orbital resonance if their orbital periods can be expressed as a ratio of two integers.

value of its mean anomaly at the flyby epoch. The mean anomaly sampling of Ariel and Titania shown in Fig. 4.10 is limited to $[111^\circ, 164^\circ]$ and $[117^\circ, 221^\circ]$ respectively.

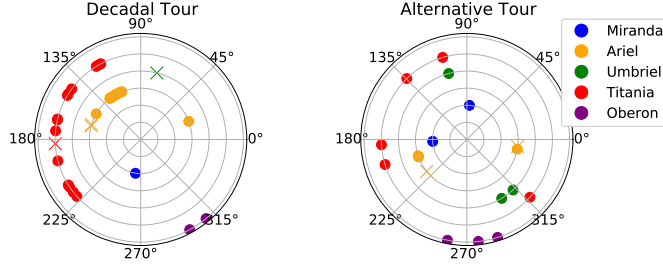


Figure 4.10.: Orbital mean anomalies of the Uranian satellites at UOP closest approach epochs. Dots are for flybys with closest distances less than three body radii, x otherwise.

The Alternative Tour employs an equatorialization maneuver following the second post-capture apoapsis, rather than a resonance orbit with Titania (Amy Simon, personal communication). This necessitates non-resonant transfers between the satellites from the initial phase of the mission, as the spacecraft rapidly aligns with Uranus's equatorial plane. As a consequence, the number of flybys of Ariel and Titania is reduced, while the global coverage of the five moons is enhanced with at least two close flybys per satellite. Additionally, the ground track geometries of Ariel are more diversified, allowing for better overall coverage of the satellite's with low and high latitude flybys. In contrast, the situation is reversed for Titania and Oberon, with high-latitude flybys being replaced by equatorial flybys for Titania and vice versa for Oberon. The Closest Approach (C/A) distances and relative velocities of the best flybys for each moon are comparable between the two tours, namely C/A distances of less than a body radius and relative velocities of approximately 5 km s^{-1} . The Alternative Tour demonstrates improved mean anomaly sampling, particularly for Ariel and Titania, with flybys occurring at $198.55^\circ, 199.92^\circ, 218.55^\circ, 349.04^\circ, 352.00^\circ$; and $106.41^\circ, 134.58^\circ, 183.57^\circ, 197.01^\circ$ and 317.49° respectively. This allows a better temporal sampling of the libration and tidal signals (see Fig. 4.10).

4.4.1.2. SIMULATED MEASUREMENTS

The UOP spacecraft will be equipped with a high-gain antenna and potentially an ultra-stable oscillator that will allow the establishment of a two/three-way coherent link with NASA's Deep Space Network (DSN) ground stations to support Telemetry, Tracking, and Command (TT&C) activities. As with many past planetary missions, the telecommunication subsystem can be used to perform gravity science experiments by collecting Doppler measurements (e.g., Asmar *et al.* [44]). The precision of radiometric measurement depends primarily on propagation noise, including plasma and Earth troposphere noises, as well as thermal and mechanical noises from ground equipment and onboard spacecraft, rather than on the performance of the onboard coherent transponder. Ka-band (32.5–34.0 GHz) is less susceptible to plasma

effects than X-band (7.2–8.4 GHz), making it preferable for scientific measurements. Nevertheless, a dual-link configuration can effectively reduce the impact of charged particle noise by approximately 75 % during superior solar conjunctions, provided that spacecraft tracking is required during this period [45]. The wet component of the troposphere can be calibrated up to 70 % using one or multiple Advanced Water Vapor Radiometers (AWVRs) [46]. At present, only the DSS-25 antenna in the DSN Goldstone complex is equipped with a Ka-Band uplink and an AWVR. By the beginning of the UOP science phase (early 50s), the Canberra and Madrid complexes are expected to also be equipped with at least one ground station with the same capabilities as Goldstone's.

The notional payload of UOP also includes a NAC (Narrow Angle Camera) and a WAC (Wide Angle Camera) that can be used for optical navigation. The latest developments in space image systems involve the use of gimbals for precise pointing and enhanced pixel resolution to obtain high-resolution imaging data for measuring a target's geophysical and geodetic properties. For instance, the Advanced Pointing Imaging Camera (APIC) concept [43] provides a system that comprises a NAC with a pixel resolution of $18\mu\text{rad}$ and 4° field of view. Additionally, the system is equipped with a WAC that has a pixel resolution of $82\mu\text{rad}$ and 18° field of view. The combination of these two cameras, in conjunction with the presence of two gimbals, enables the instrument to be pointed with a pointing accuracy of $2''$, without the need to change the attitude of the spacecraft. Other imaging systems that have flown, such as the Long Range Reconnaissance Imager (LORRI) on the New Horizons Mission [47], or that will fly, such as the Europa Imaging System (EIS) on Europa Clipper [48], provide similar performance characteristics.

The *ROB-MONTE* library (Sec. 2.2.2.4) is used to simulate Doppler and optical data with noise levels that reflect the current state of the art. Specifically, synthetic Doppler data is computed for a coherent two/three-way direct-to-Earth link, with a noise level of 0.1 mms^{-1} and $10\mu\text{ms}^{-1}$ for X/X and Ka/Ka, respectively, at a 60 s integration time [49]. In order to extensively characterize the possible GS experiment, we test independently the X-Band and Ka-Band performances, as well as different tracking times (from 1 h to 6 h around C/A). As is done for radiometric data, optical data are also simulated with a state-of-the-art noise level of $2''$. The impact of the number of landmarks observed per image (5, 10, 30 and 50) and of the number of images per flyby (2, 4, 6 and 8) are investigated.

4.4.1.3. DYNAMICAL MODELS

The spacecraft trajectory is mainly driven by the gravity field of Uranus, and is perturbed by the gravity fields of the Sun, the other planets, and of the satellites during the flybys. For our modeling, we use the zonal field of Uranus and Body Frame (BF) orientation of Jacobson [37, Table 12] and the orbit of the Uranian satellites as defined by the JPL's Uranian satellite ephemeris file URA111². We account for the point-mass gravity of the Sun and planets. Non-gravitational forces also perturb the spacecraft motion. We consider the solar radiation pressure

²https://naif.jpl.nasa.gov/pub/naif/generic_kernels/spk/satellites/ura111.bsp

(SRP) acting on a sphere that encompasses the maximum dimension of the UOP spacecraft.

The external gravity potential of the satellites is classically modeled using a spherical harmonic expansion as defined in Eq. (2.1). The unnormalized second-degree coefficients (J_2 and C_{22}) are computed for a three-layer model with an ocean of 20 km assuming hydrostatic equilibrium, and by inverting Radau Eq. (4.1) (see Table 4.4). The higher-degree coefficients (up to degree 10) are computed following the method of Cascioli *et al.* [28] (and references therein). In this method, a set of spherical harmonic coefficients is generated to simulate a three-layer body consisting of a rocky core, a global liquid layer, and an ice shell. The seafloor and ice shell topography are generated using a power law to define their random spectrum (after the assigned layer thicknesses are set), specifically $P_l = k/l^\beta$, where P is the power spectrum, l is the degree, and k and β are prescribed coefficients. The resulting topography coefficients are then converted to gravity coefficients using Eqs (3a-3b) of Cascioli *et al.* [28]. It is assumed that there is no compensation of the seafloor.

Periodic changes in the potential, as reflected in changes to the normalized spherical harmonic coefficients, i.e., $\Delta\bar{C}_{nm}$ and $\Delta\bar{S}_{nm}$, due to gravitational tides can be modeled as defined by Eq. (2.7). We set $k_{2m} = k_2$ because achieving the minimum spatial and temporal coverage required to measure better than the global average of the second-degree gravitational tide with only a few flybys per satellite is too challenging (see e.g., Cappuccio *et al.* [50]). Moreover we compute the Love number with the method of [32] assuming a spherical body, for which the Love numbers are degenerate with respect to m ($k_{20} = k_{21} = k_{22}$) [19]. The k_2 values are computed for a three-layer model with an ocean thickness of 20 km (see Table 4.4).

The orientation of a satellite's BF is defined by the right ascension (α) and declination (δ) of its spin axis and by the prime meridian location (W) with respect to the ICRF J2000 as shown by Eqs. (2.26) to (2.28). The current IAU recommendations [51] for the Uranian satellites are based on outdated ephemerides and neglect libration and obliquity. Here we use an orientation model that is consistent with the URA111 ephemeris and which includes the libration and obliquity as modeled in Baland *et al.* [24]. This is essential for our study, since we need an orientation model that is consistent with the ephemerides we use and since the satellite orientation parameters are estimated parameters. Similarly to Eqs. (2.26) to (2.28), the three angles are defined as follow:

$$\alpha(t) = \alpha_{2050} \quad (4.3)$$

$$\delta(t) = \delta_{2050} \quad (4.4)$$

$$W(t) = W_0 + \dot{W}t + g_s \sin(M_0 + \dot{M}t) \quad (4.5)$$

Since the obliquity of Uranus's largest satellites varies over longer periods than those of the UOP space mission [24], we set α and δ to their values at 2050, a date representative of the mission period (see Table 4.4). The epoch rotation angle W_0 and mean anomaly M_0 are set at J2000. The libration signal is modeled as defined by Eq. (2.29).

Table 4.4.: Dynamical model parameters for the Uranian satellites. The reference epoch of α_{2050} and δ_{2050} is the mission epoch (01-JAN-2050 12:00:00 UTC). The reference epoch for W_0 and M_0 is J2000.

	J_2	k_2	α_{2050} (°)	δ_{2050} (°)	W_0 (°)	$\dot{W}$ (° d ⁻¹)	g_s (°)	M_0 (°)	$\dot{M}$ (° d ⁻¹)
Miranda	6.74e-03	1.48e-03	78.2590	19.5040	148.70	254.69	-1.27e-02	72.37	254.64
Ariel	1.30e-03	1.56e-02	77.3177	15.1278	23.09	142.84	-2.66e-03	119.76	142.82
Umbriel	4.88e-04	1.55e-02	77.3711	15.1200	71.26	86.87	-3.15e-03	258.30	86.86
Titania	1.01e-04	3.29e-02	77.3605	15.1450	101.58	41.35	-2.16e-04	53.20	41.35
Oberon	4.20e-05	3.15e-02	77.3222	14.8438	172.57	26.74	-1.03e-04	139.73	26.74

4.4.1.4. ESTIMATED PARAMETERS

The Doppler and optical data for each flyby are simulated and processed with a multi-arc approach in order to determine the expected uncertainties (1σ formal errors) of the estimated parameters in our POD solution (see [Sec. 2.2.2.5](#)). This approach enables the inherent errors of the dynamical model, numerical rounding errors of the orbit propagators and orbit maneuvers to be effectively absorbed by localizing errors to their respective intervals. This is accomplished in practice by separation of dynamical parameters into local (e.g., spacecraft initial state) and global parameters (e.g., gravity coefficients).

Table 4.5.: Estimated parameters and associated a priori uncertainties.

Parameter	Type	A priori Uncertainty
UOP state	Local	1 km, 1 ms ⁻¹
SRP scaling factor	Local	0.25
Camera pointing	Global	2''
Landmark position	Global	Unconstrained
GM	Global	Unconstrained
$C_{l,m}, S_{l,m}$	Global	Unconstrained
α	Global	Unconstrained
δ	Global	Unconstrained
$\dot{W}$	Global	Unconstrained
k_2	Global	Unconstrained
g_s	Global	Unconstrained

The estimated parameters and the associated a priori uncertainties used for the UOP gravity science experiment simulations are presented in [Table 4.5](#). It should be noted that the set of estimated parameters vary because some parameters are sometimes kept fixed, when preliminary results show no sensitivity to them, as we discuss in [Sec. 4.4.2](#). The estimated parameter set, however, always includes the

spacecraft state (one per flyby), the satellite GM , a minimal combination of gravity coefficients (J_2 and C_{22} , see below), and one SRP scaling factor per arc. If optical data are simulated, the base set is completed by the camera pointing errors along the camera frame axes and by the landmarks' position (latitude, longitude, and radial coordinates for each landmark). Table 4.6 summarizes the tested parameter sets.

Table 4.6.: Characteristics of the Gravity Science experiments tested in this study. The geodetic parameter sets—including gravity, frame, tides, and libration—are combined with the base set described in Sec. 4.4.1. These parameter sets are tested using multiple measurement strategies and tour configurations, defining the explored parameter space. Each case is evaluated with and without the hydrostatic constraint ($J_2/C_{22} = 10/3$).

(*) Degree and order 3×3 for Miranda, Umbriel, and Oberon; degree and order 4×4 for Ariel and Titania.

Setting	Values
Orbital tour	Decadal; Alternative
Data type	Radiometric; Radiometric+Optical
Radio link	X-Band; Ka-Band
Tracking pass duration	1 h, 2 h, 4 h and 6 h
Pictures per flyby	2, 4, 6 and 8
Landmarks per picture	5, 10, 30 and 50
Gravity field	From J_2 and C_{22} only, to full degree 3/4 (*)
Body-fixed frame	α , δ , $\dot{W}$
Libration	g_s
Tides	k_2

Different combinations of the maximum degree (n) and order (m) of the estimated gravity static coefficients are tested, beginning with only the degree-2 coefficients J_2 and C_{22} up to a maximum full degree and order (3 for Miranda, Ariel and Umbriel, 4 for Ariel and Titania). The impact of applying a hydrostatic constraint ($J_2/C_{22} = 10/3$) on the estimated formal errors is also evaluated. When the satellite body frame orientation with respect to ICRF is estimated (parameters: α , δ , $\dot{W}$), the coefficients C_{21} , S_{21} and S_{22} , are not estimated and set to 0. These coefficients are instead estimated when the frame is fixed. The term “quadrupole” is employed when only J_2 and C_{22} gravity coefficients are estimated. The tidal love number k_2 is estimated through temporal variations in J_2 , C_{22} and S_{22} . The uncertainty in the obliquity is derived from the uncertainties in the determination of the spin axis right ascension α and declination δ from the spherical trigonometric relation defined by Eq. (2.36). The libration amplitude g_s is estimated only if the frame is estimated.

4.4.2. CASE STUDY: ARIEL

As mentioned earlier, the approach presented in Sec. 4.2 can be utilized not only to determine the necessary geodetic measurements uncertainties to achieve a specific scientific objective, as illustrated in Sec. 4.3, but also to rapidly assess the performance of a POD solution in terms of interior parameter estimation. By testing POD performances for just two different orbit tours and a handful of measurement strategies and filter settings (see Table 4.6), we have already performed no less than approximately 15000 simulations per satellite. It is impractical to perform an MCMC simulation for each POD solution, as there are too many of them and the process would be unfeasible time-consuming. The mapping method described in Sec. 4.2, on the other hand, can very quickly assess the performance of each POD solution in its ability to constrain the interior parameters, simply by extracting/interpolating the corresponding interior parameter uncertainties from the lookup table. Here we discuss and apply this reverse approach to Ariel trying to propose requirements on the mission design (instrument and operations) in order to reach scientific objectives. We first validate our mapping method in Sec. 4.4.2.1. Subsequently in Sec. 4.4.2.2, we discuss the results of the covariance analysis with respect to the single measurement requirements shown in Table 4.1 and with respect to the uncertainties on the ocean and ice shell thicknesses and on the rock-to-ice ratio from the combined measurements. A comprehensive sensitivity analysis, which details the effects of various measurement strategies and the influence of the estimated parameter set on the uncertainties associated with geodetic and interior parameters, can be found in Sec. 4.4.2.3. Furthermore, the same appendix provides the interested reader with the results of the sensitivity analysis for Miranda, Umbriel, Titania, and Oberon.

4.4.2.1. MAPPING METHOD VALIDATION

The reliability of our mapping method can be demonstrated by comparing the interior parameter uncertainties interpolated from the lookup table with those resulting from a dedicated MCMC run. We do so for the specific GS experiment design characterized by the Alternative Tour, 4 hours of tracking time centered on the C/A, Ka-band radiometric data and optical data made of 4 pictures per flyby with 10 landmarks per picture. This experiment configuration was found to have sensitivity to all key measurements (MoI, libration, tides, and obliquity), and thus our POD estimated parameter set includes all of them, as well as the degree-two gravity coefficients J_2 and C_{22} . This specific simulation results in the following uncertainties in Ariel's geodetic parameters of interest: $\sigma_{\text{MoI}} = 2.2 \times 10^{-5}$, $\sigma_{\epsilon} = 5.3\text{m}$, $\sigma_{g_s} = 1.8\text{m}$ and $\sigma_{k_2} = 1.6 \times 10^{-2}$. Propagating the latter to the interior parameters by using our mapping method we get: $\sigma_{d_o} = 25.74\text{km}$, $\sigma_{d_s} = 24.56\text{km}$ and $\sigma_{\text{rock/ice}} = 0.27$. As previously stated in Sec. 4.2, the uncertainty of an interior parameter is defined as half of the range between the 15.87th and 84.13th percentile. In order to predict possible posteriori distributions that are flat, the non-informative constraint is not applied in this instance, prior to interpolation.

These interior parameter uncertainties are in fairly good agreement with the 15.87th and 84.13th percentiles ranges reported at the top of Fig. 4.11, which result from a dedicated MCMC run. We can see, in fact, that the highest relative error of

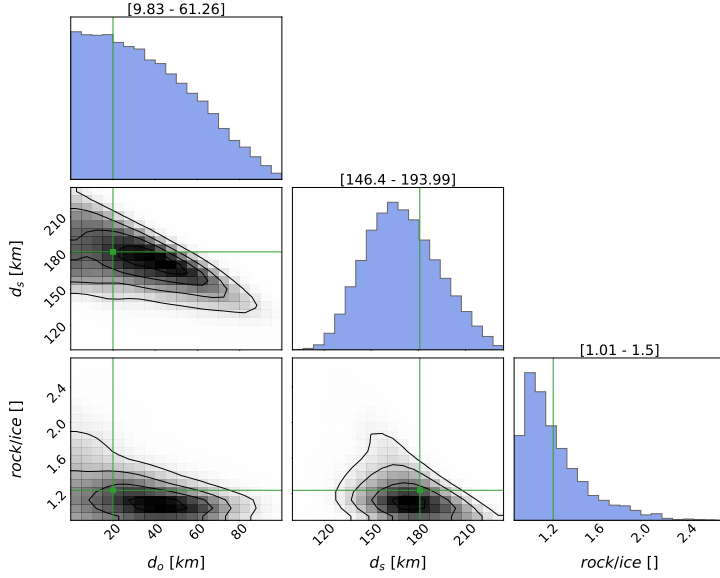
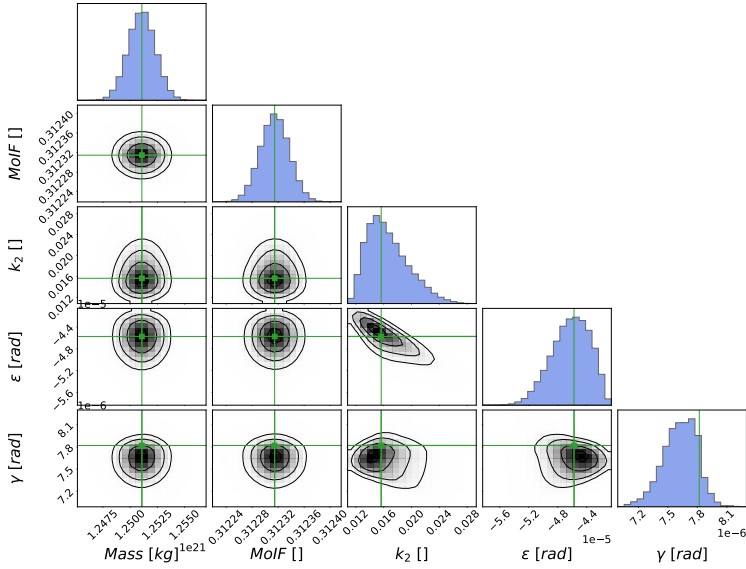


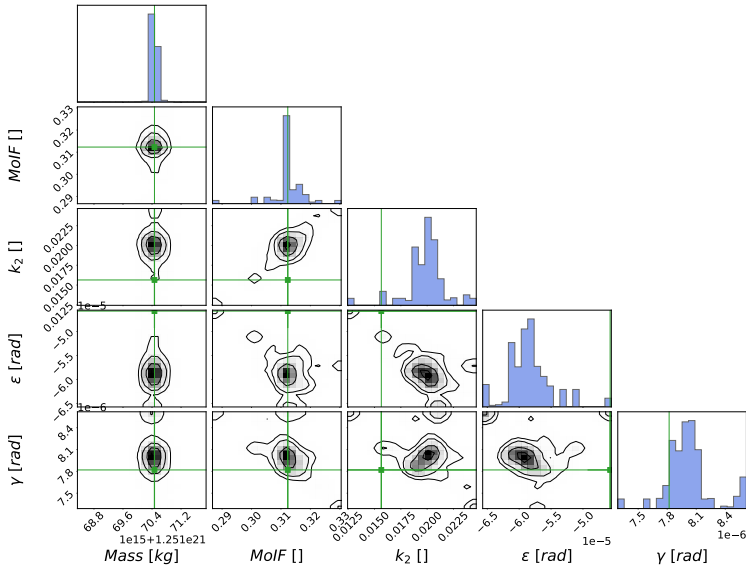
Figure 4.11.: Posterior distributions of Ariel's interior parameters, derived from a dedicated MCMC run (50 walkers, 100000 steps). The uncertainty on the geodetic observables (MoI, k_2 , ε , and g_s) was quantified using covariance analysis. This POD solution was characterized by the Alternative Tour, 4 hours of tracking time around C/A, radiometric (Ka-Band), and optical (4 pictures per flyby, 10 landmarks per picture). The titles show the 15.87th and 84.13th percentiles of the posterior distributions, which correspond to the following uncertainty as defined in Sec. 4.2: $\sigma_{d_o} = 25.72\text{km}$, $\sigma_{d_s} = 23.80\text{km}$, and $\sigma_{rock/ice} = 0.25$. Green points show the truth values.

the mapping method, amounting to $\sim 11\%$, is observed for the *rock/ice* uncertainty. The ocean and ice shell thicknesses, in contrast, show a relative error of 0.1% and 3% , respectively. This gives us confidence in the preliminary evaluations obtained with our mapping method on the POD solutions, which we discuss below. Note that the mapping method results remain an approximation and that we recommend to run a full MCMC once the GS experiment design has been chosen.

A dedicated validation test was performed to explicitly investigate the choice of mass uncertainty fixed to 1% as discussed in Sec. 4.3. Although Ariel's mass can indeed be measured with substantially higher precision (approximately $6 \times 10^{-6}\%$ as per our simulation settings detailed at the beginning of this section), our tests showed that using excessively tight constraints on the mass parameter led to severe model rejection and convergence issues within the MCMC framework. Our simulations clearly illustrated this effect (Fig. 4.12), where the MCMC inversion failed entirely when the formal, very precise mass uncertainty was imposed, but converged



(a)



(b)

Figure 4.12.: A posteriori distribution of Ariel's synthetic observables from the MCMC inversion using 1% of the mass as uncertainty (a) and the expected $\approx 10^{-6}\%$ mass uncertainty from GS simulation (b). Green points and lines show the geodetic observables truth values and uncertainties.

successfully under the more relaxed uncertainty condition of 1%.

This result underscores that, while under idealized conditions, where the interior structure model perfectly mirrors reality, a tighter uncertainty might be justifiable, our adopted model necessarily includes simplifications such as constant density within layers and neglecting mineralogic phase transitions. In such practical scenarios, model errors must be accommodated. A similar strategy was also successfully applied by Genova *et al.* [40], where a relaxed uncertainty for Mercury's bulk density ($\sigma = 0.002\rho$) was employed to account for potential inaccuracies in planetary shape measurements.

Ultimately, our choice of employing a larger uncertainty for mass proved essential in balancing the fitting process across all observables, avoiding overfitting to the mass parameter, and ensuring robust MCMC convergence. This approach effectively integrates unknown model errors and provides conservative yet reliable parameter constraints, thus reinforcing the practical robustness of the chosen uncertainty strategy.

4

4.4.2.2. UOP MISSION REQUIREMENTS FOR INTERIOR DETERMINATION

In a POD solution, the set of estimated parameters significantly impacts the estimated formal errors associated with the investigated parameters. The determination of the appropriate parameter set is contingent upon the data at hand (i.e., the final trajectory, instrumentation, and observation strategy). Consequently, it is not possible to make an informed choice at this stage of the mission design process. Moreover, even when the mission design is finalized, detailed numerical simulations (e.g., perturbative, robustness analysis, etc.) will need to be run for each candidate tour to assess the optimal parameter set, that is, the one that captures all the effects without overparametrizing the solution. Our mapping method provides a solution to this problem during the early stages of the mission design process, allowing for direct testing of the impact of different measurement strategies and estimated parameter sets on the determination of the interior parameters of interest, such as ocean and ice shell thickness and the rock-to-ice ratio, as demonstrated in Sec. 4.4.2.3. Furthermore, a plausible estimated parameter set can be postulated based on an analysis of past GS experiments (e.g., degree and order 5×5 and k_2 of Titan with 10 flybys [8], body fixed frame and libration amplitude from optical measurements [16, 52]). The Decadal Tour provides twelve close flybys (i.e., flybys with a C/A ratio less than three times the body's radius) of Ariel, whereas the Alternative Tour provides only four, but with an improved spatial coverage (see Sec. 4.4.1.1 for details). A sensitivity analysis of the gravity error spectra indicates that the maximum resolvable spherical harmonics degree and order is 3×3 for the Alternative tour or 4×4 for Decadal tour, given our truth gravity field (as defined in Sec. 4.4.1.3). The use of optical data may facilitate the estimation of the satellite's rotational state, thereby enabling the determination of its obliquity and libration amplitude. The determination of tides, however, is a more challenging process due to the necessity of both spatial and temporal coverage. Libration amplitude as well can be more accurately determined with complete temporal coverage.

In order to discuss the expected uncertainties on the geodetic and interior

parameters, we rule out right from the start POD solutions that poorly constrain all the geodetic parameters. In practice, that means that we ignore all solutions where the four geodetic parameters uncertainties (σ_{MoI} , σ_{k_2} , σ_{g_s} and σ_ϵ) are above 100 % of their nominal values (see Sec. 4.4.1.3). From the total of ≈ 25000 simulations for Ariel, approximately 42 % of the solutions satisfy this condition. We discuss here the required measurement strategies for the two tested tours, assuming the availability of Ka/Ka Doppler and optical data and a parameter set comprising libration amplitude, obliquity and gravity field coefficients up to degree and order $3 \times 3/4 \times 4$. Note that considering correlations with higher degrees coefficients (up to 6) does not alter our conclusions as discussed in Sec. 4.4.2.3. Additionally, we will discuss the measurement requirements for an inversion strategy that includes tides, in combination with the aforementioned parameters.

Table 4.7 summarizes the minimum observation strategy for the two investigated tours and for the aforementioned parameter sets that allows to meet the target uncertainties shown in Table 4.1. In order to determine Ariel's hydrosphere thickness at the $\pm 50\text{km}$ level of precision, its MoI must be determined with an uncertainty of $\sigma_{\text{MoI}} \leq 0.005$, which can be achieved using both tours with 1 h of tracking time, 2 pictures per flyby, and 5 landmarks per picture, when estimating the gravity field up to degree and order 3×3 or 4×4 and rotation parameters, but not tides. It is a challenging task to properly measure the k_2 Love Number and achieve the target uncertainty shown in Table 4.1. In fact, the necessity to solve for k_2 demands 6 h of tracking time, 4 pictures per flyby, and 10 landmarks per picture for the Decadal Tour if degree and order 3×3 gravity field coefficients and rotation parameters are estimated. If the gravity of degree and order 4×4 is included, two additional pictures per flyby are required to meet the target uncertainties of both MoI and tides. k_2 can not be estimated to the required level with the Alternative Tour. The greater diversity and variety of flyby geometries, coupled with an improved temporal sampling of the Alternative Tour, permit the determination of the libration amplitude and obliquity with an uncertainty of 1 m and 5 m, respectively, despite the reduced number of flybys (see Sec. 4.4.1.1 for a detailed description of the two tours). Specifically, only 2 pictures per flyby and 5 landmarks per picture are needed instead of 4 and 10 for g_s , while 6 pictures and 5 landmarks are needed instead of 4 and 50 for ϵ .

As illustrated in Table 4.7, the Alternative Tour is unable to adequately constrain Ariel's k_2 Love Number, which significantly limits its capacity to constrain the interior parameters. Consequently, the subsequent discussion focuses on the Decadal Tour unless otherwise stated. There are only a handful of solutions that can constrain the ocean thickness with a relative uncertainty better than 100 %. The best relative uncertainty among the tested cases is 92 % and can be achieved by the observation strategy that maximizes the number of measurements (i.e. 6 h, 8 pictures and 50 landmarks) and fitting gravity field to a maximum degree and order equal to 3. According to Sec. 4.3, d_o should be more challenging to constrain than d_s and the rock-to-ice mass ratio. Indeed, the rock-to-ice mass ratio can be constrained with a relative uncertainty of about 28 % with 6 h Ka tracking around C/A, 4 pictures per flyby and 10 landmarks per picture when the maximum degree and order 3 spherical harmonics are estimated. Two additional pictures are required when the

Table 4.7.: Minimum requirements for radiometric (Ka-Band) and optical measurements to meet the specified target uncertainties for the geodetic parameters of Ariel presented in Table 4.1. The target uncertainty for the k_2 Love Number represents the most optimistic target uncertainty for ocean detection, given that all other targets for k_2 have not been met by any of the tested strategies. The total tracking time around C/A is defined as the tracking time in hours. The number of pictures is the total number of pictures per flyby, whereas the number of landmarks is the number of landmarks per picture. Empty lines means that no solutions are found where the considered geodetic parameters uncertainties (σ_{MoI} , σ_{k_2} , σ_{g_s} and σ_ϵ) are below 100 % of their nominal values (see Sec. 4.4.1.3).

Parameter Set	Decadal Tour			Alternative Tour			
	Track. time [h]	Pictures	Landmarks	Track. time [h]	Pictures	Landmarks	
MoI	$sh3+\epsilon+g_s$	1	2	5	1	2	5
	$sh3+\epsilon+g_s+k_2$	6	4	10			
	$sh4+\epsilon+g_s$	1	2	5	1	2	5
	$sh4+\epsilon+g_s+k_2$	6	6	10			
k_2	$sh3+\epsilon+g_s+k_2$	6	4	10			
	$sh4+\epsilon+g_s+k_2$	6	6	10			
g_s	$sh3+\epsilon+g_s$	1	4	10	1	2	5
	$sh3+\epsilon+g_s+k_2$	6	6	10			
	$sh4+\epsilon+g_s$	4	4	10	1	2	5
	$sh4+\epsilon+g_s+k_2$	6	6	10			
ε	$sh3+\epsilon+g_s$	1	4	50	1	6	5
	$sh3+\epsilon+g_s+k_2$	1	4	50			
	$sh4+\epsilon+g_s$	1	4	50	1	6	5
	$sh4+\epsilon+g_s+k_2$	4	4	50			

considered gravity degree and order are equal to 4. It was not possible to constrain the the rock-to-ice mass ratio with an uncertainty level better than 21 % using any of the tested measurement strategies. We confirm that d_s is the most easily constrained of the interior parameters discussed in this study. Our findings indicate that 6 h Ka-Band tracking, 4 (6) pictures per flyby and 10 landmarks per picture can constrain the ice shell thickness at the 16 % level if gravity of degree and order 3 (4) is included. A similar degree of uncertainty on d_s can be attained by reducing the tracking time and increasing the number of optical measurements, which will result in a less precise determination of k_2 but a more accurate determination of g_s . This is because these two factors offset each other with regard to the determination of d_s (see Sec. 4.3.3). It is noteworthy that a comparable level of uncertainty on *rock/ice* and d_s can be achieved by the Alternative Tour, provided that a

measurement with an uncertainty at the signal level of k_2 can be obtained with another measurement technique, such as the assessment of surface deformations with a 6-degree-of-freedom sensor positioned on the surface [53, 54]. Specifically, with 4h Ka-Band tracking time, 2 pictures per flyby, and 30 landmarks per picture, the rock-to-ice mass ratio can be constrained to 26% and the ice shell thickness to 15%.

We remind to the reader that this is a particular discussion of the mission requirements pertaining to the $sh2/sh3$, k_2 , ε and gs parameter set. The results of a comprehensive sensitivity analysis are presented in [Sec. 4.4.2.3](#).

4.4.2.3. IMPACT OF MEASUREMENT STRATEGIES AND PARAMETER ESTIMATES

Given the early stage of the UOP mission, many different design choices are not finalized yet. This includes the type of radiometric subsystem, the trajectory, and the instrumentation on board the spacecraft. For this reason we explored a wide number of combinations of these assumptions to better understand their effect on the retrieved geophysical observables uncertainty (see [Sec. 4.4.1](#) for a detailed description). The objective is to characterize the general trend with respect to flyby geometries, measurement strategies, and parameter set. It should be noted that the selection of the final parameter set is entirely data-dependent, a known principle within the GS community. The optimal parameter set is the one that yields the best fit to the data, and thus cannot be selected in advance. The aim of testing all these parameter sets is to assess the impact of correlations and identify observation settings that can mitigate them. Here, we provide a qualitative discussion for Ariel, focusing on the expected uncertainties on the key geodetic parameters—the MoI ([Fig. 4.13](#)), the k_2 Love Number ([Fig. 4.14](#)), the libration amplitude g_s ([Fig. 4.15](#)) and obliquity ([Fig. 4.16](#))—and their translation into constraints on interior structure parameters like ocean thickness, rock-to-ice ratio, and ice shell thickness [Figs. 4.21 to 4.23](#). The results of this comprehensive sensitivity analysis, equivalent to [Figs. 4.13 to 4.16](#) and [Figs. 4.21 to 4.23](#), for the other Uranian satellites (Miranda, Umbriel, Titania, and Oberon) are presented in [Appendix A](#). The following analysis is organized by the different aspects of the experiment design.

Impact of Tour Design The choice of orbital tour presents a key trade-off. We remind the reader that a thorough discussion of the two analyzed tours is provided in [Sec. 4.4.1.1](#). The Decadal Tour, with its higher number of flybys, allows for a higher-resolution gravity model, enabling the determination of the gravity field up to degree and order 4×4 , whereas the Alternative Tour only resolves up to 3×3 (see [Fig. 4.17](#)). This improved resolution means the Decadal Tour performs slightly better in the determination of the gravity field and MoI, with its median σ_{MoI} potentially improving by a factor of ~ 2 compared to the Alternative Tour under optimal conditions ([Fig. 4.13](#)). The enhanced resolution of the gravity field also results in a reduction in the correlation between the higher degree and order gravity coefficients and the MoI (or differently with the quadrupole coefficients, see [Eq. \(4.1\)](#)). This is evidenced by the reduced impact on the formal error when higher degrees and orders are estimated; for the Decadal tour, increasing the maximum

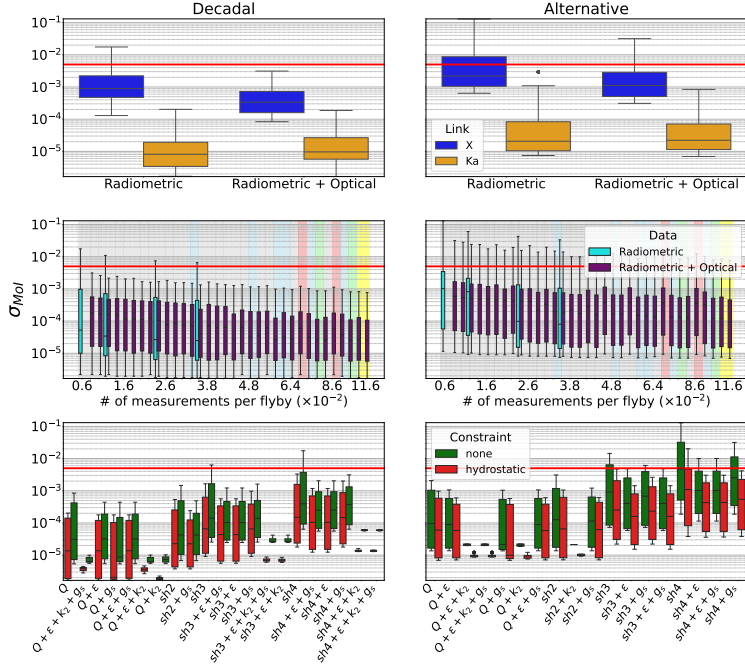


Figure 4.13.: Computed formal errors of Ariel's MoI. The box-and-whisker plot illustrates the conventional statistical measures: the lower boundary of the box represents the 25th percentile, the upper boundary the 75th percentile, and the line within the box the 50th percentile. The lower whiskers extend to a value that is 1.5 times below the 25th percentile, while the upper whiskers extend to a value that is 1.5 times above the 75th percentile. Circles represent data points that fall outside the range of the whiskers. Symbols are used to define the parameter sets. Specifically, the letter Q stands for “quadrupole”, whereas shN means that all gravity coefficients up to maximum degree and order $N \times N$ are estimated. The number of simulated measurements per flyby is equal to $2N_{pictures} * N_{landmarks} + N_{Doppler}$. Given that the incremental step is not constant, colored areas are used to define the step (grey: 0.2, blue: 0.4, orange: 0.6, green: 0.8, yellow: 1.2). The red solid lines represent the target uncertainty (0.005), as outlined in Table 4.1.

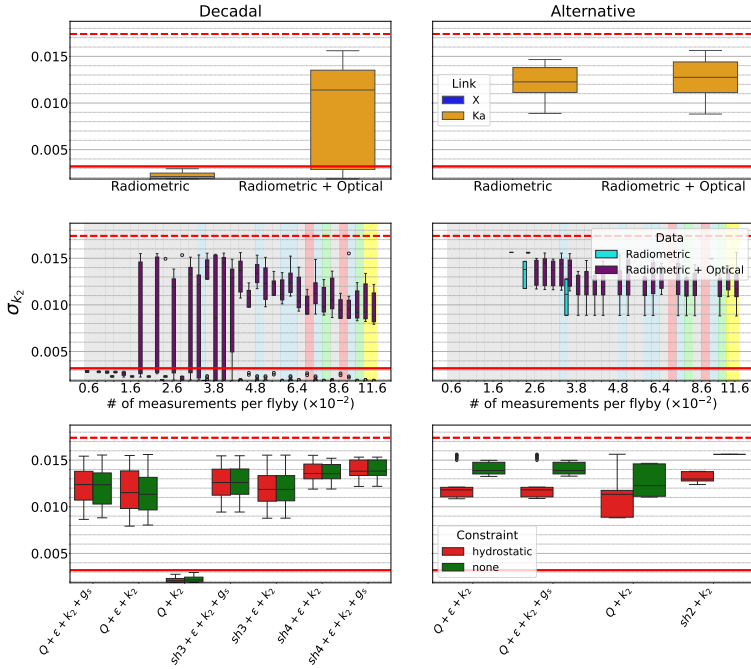


Figure 4.14.: Computed formal errors of Ariel's Love Number (k_2). A detailed explanation of the figure can be found in Fig. 4.13. The red dashed line is the optimistic requirement for ocean detection (0.0174), the red solid line is the requirement to detect differentiation (0.0032) (see Table 4.1).

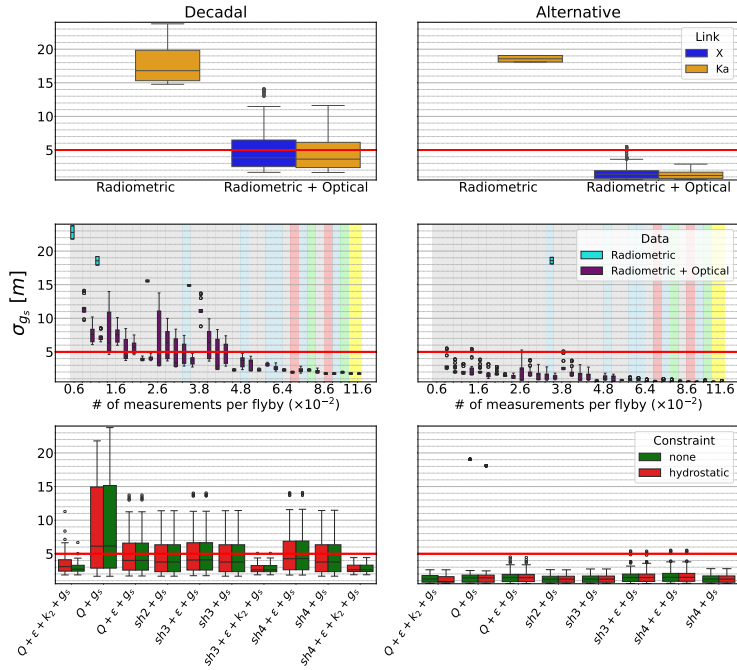


Figure 4.15.: Computed formal errors of Ariel's libration amplitude (g_s). A detailed explanation of the figure can be found in Fig. 4.13. The red solid line represents the target uncertainty (5 m), as outlined in Table 4.1.

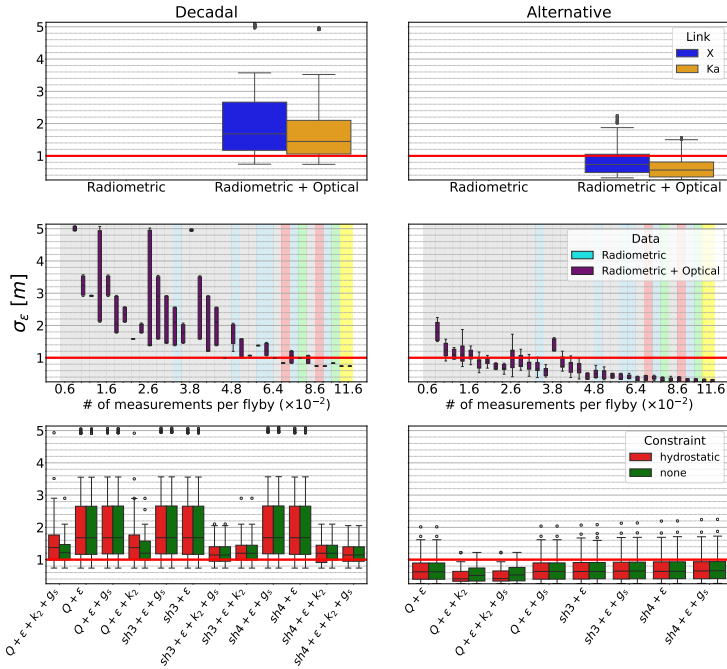


Figure 4.16.: Computed formal errors of Ariel's obliquity (ϵ). A detailed explanation of the figure can be found in Fig. 4.13. The red solid line represents the target uncertainty (1 m), as outlined in Table 4.1.

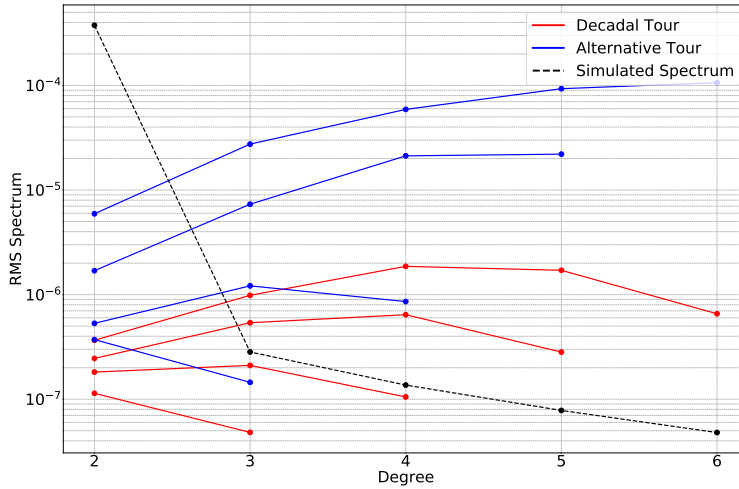


Figure 4.17.: Ariel's Root Mean Square (RMS) gravity and error spectra expected (see Eq. (2.6)) from the GS experiment with the Decadal and Alternative tour trajectories and nominal observation strategy (4 hours of tracking time centered on the C/A, Ka-band radiometric data, and optical data comprising 4 pictures per flyby with 10 landmarks per picture). The simulated gravity field (black dashed line) is computed in accordance with the methodology outlined in Sec. 4.4.1.3.

degree fitted from 3 to 6 degrees σ_{MoI} by less than a factor of 2, compared to nearly an order of magnitude degradation for the Alternative tour under the same conditions (see Fig. 4.20). Conversely, the Alternative Tour's primary strength lies in its varied and complementary flyby geometries. This diversity allows for achieving a more comprehensive surface coverage for landmark tracking, which provides a more robust determination of the rotational parameters, yielding median uncertainties for g_s and ϵ that are roughly 30–50% lower (e.g., ~ 2 m vs ~ 3 m for g_s with Ka-band and optical data) compared to the Decadal tour as shown by Figs. 4.15 and 4.16. This improved rotational solution reduces the correlations between the rotational and gravity parameters across the entire solution space. The effect of this de-correlation is such that, even with fewer flybys, the formal errors for the MoI become broadly comparable between the two tours (Fig. 4.13). For the k_2 Love number, a different factor becomes dominant: the sensitivity is fundamentally limited by the sampling of the moon's mean anomaly (see Fig. 4.9 and Sec. 4.4.1.1). Due to its slightly better coverage of the tidal signal's period, the Alternative tour's k_2 estimation results in formal errors comparable to those from the Decadal tour, with median uncertainties typically clustering around 0.010–0.015 for both tours in combined radiometric and optical scenarios using Ka-band (see Fig. 4.14), once the optimistic outlier case, $Q + k_2$, is excluded. A detailed analysis of this parameter set is given in the following paragraph *Impact of Inversion Strategy*.

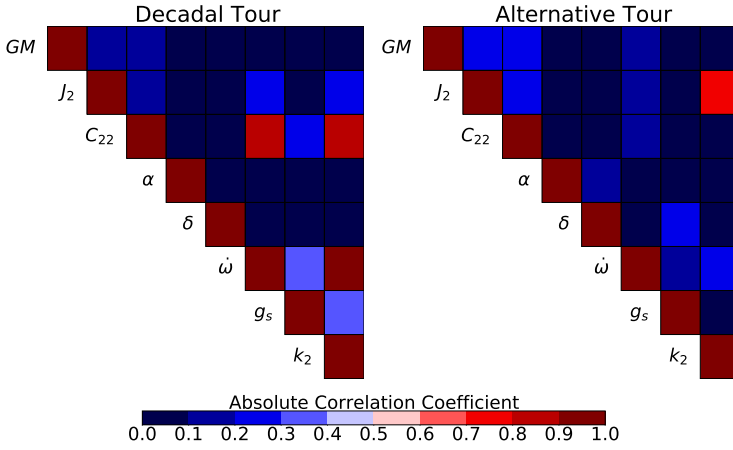


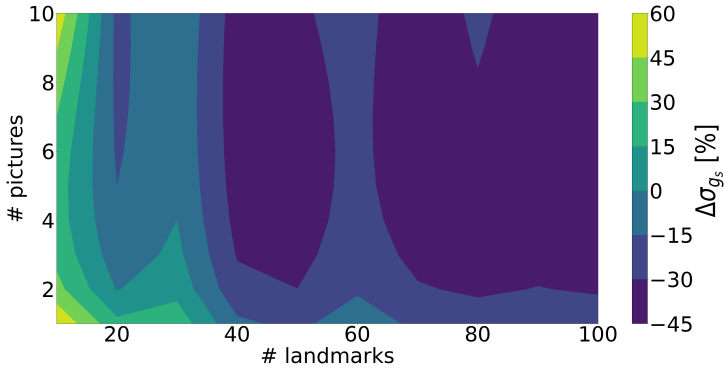
Figure 4.18.: Correlation matrices of the POD solutions using the Decadal and Alternative Tours for the parameter set $Q + \varepsilon + k_2 + g_s$. The measurement settings are characterized by: Ka-Band and 4 hours tracking time, 4 pictures per flyby with 10 landmarks per picture. Hydrostatic constraint is not applied to the presented cases.

Impact of Data and Measurements The quality and type of data are decisive factors in the experiment's success. The inclusion of optical data alongside radiometric measurements is essential for determining the libration amplitude (g_s) and obliquity (ε) to the required precision, reducing median uncertainties by roughly an order of magnitude (e.g., from > 15 m down to $\sim 2-3$ m for g_s with Ka-band data), bringing σ_{g_s} down to the $\sim 2-3$ m level, close to or meeting the 5 m target, and σ_ε down to $\sim 0.5-1.5$ m, approaching or meeting the 1 m target depending on the tour (see Figs. 4.15 and 4.16). Its impact on the gravity parameters is more complex. It should be noted that the degree-two coefficients C_{21} , S_{21} , and S_{22} are fitted when the rotation body-fixed frame is not estimated, as explained in Sec. 4.4.1.4. Indeed, there is an implicit (and indirect) relationship between the gravity field and the (unknown) rotation frame due to correlation effects. By providing a robust estimate of the body's orientation, optical data helps to de-correlate these parameters. However, this trade-off is not straightforward because the enhanced dataset (radio + optical) introduces additional sensitivity, but also an overall uncertainty due to imperfect a priori knowledge of the landmark positions. This is reflected in the results, which show that adding optical data improves the median MoI uncertainty by roughly a factor of 2-3 when using Ka-band (e.g., from $\sim 3 \times 10^{-5}$ down to $\sim 1-2 \times 10^{-5}$) (Fig. 4.13), while σ_{k_2} sees little benefit and, in some Ka-band scenarios, even degrades slightly (e.g., median σ_{k_2} increasing from ~ 0.002 to ~ 0.012), likely due to increased model complexity and correlations introduced when fitting rotational parameters simultaneously, combined with the precision being dominated by the limited sensitivity from the mean anomaly sampling as

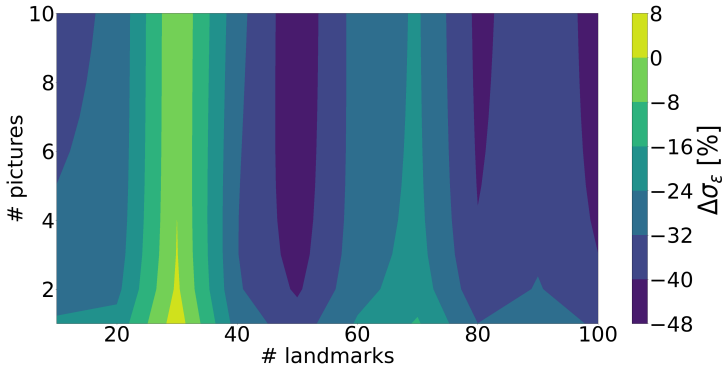
previously discussed (Fig. 4.14). Furthermore, our results show the expected benefit of using the higher-precision Ka-band link over the X-band link, which consistently yields lower uncertainties across all estimated parameters (Figs. 4.13 to 4.16, top panels). For instance, in Fig. 4.13, the MoI formal error with Ka-band data is about two orders of magnitude smaller (e.g., median σ_{MoI} dropping from $\sim 1-2 \times 10^{-3}$ to $\sim 1-3 \times 10^{-5}$) than for the X-band case. For k_2 , the median Ka-band uncertainty (around 0.010–0.015 in combined data scenarios) approaches the expected signal level (true value ~ 0.016), whereas X-band uncertainties are significantly larger (Fig. 4.14). For the rotational parameters, this improvement from the radio link is more limited, since optical data are more sensitive to these parameters. It is worth noting, however, that while the libration amplitude can be measured with radiometric data only (median $\sigma_{g_s} > 15$ m), it does not reach the target precision needed (5 m); the addition of optical data (median $\sigma_{g_s} \sim 2-3$ m) is paramount (Fig. 4.15). Finally, there is a clear trend of improving precision as the number of measurements per flyby increases; for MoI and k_2 , this improvement is mainly driven by the increased volume of radiometric data, whereas for the rotational parameters, the improvement is almost entirely driven by the increased number of optical measurements (Figs. 4.13 to 4.16, middle panels).

To further investigate this point for the rotational parameters, Fig. 4.19 isolates the impact of the optical measurement strategy, showing the trade-off between the number of pictures taken per flyby and the number of landmarks identified per picture, relative to a nominal case (Decadal tour, Ka-Band, 4 hours tracking, 4 pictures, 10 landmarks). For both the libration amplitude and obliquity, increasing the number of pictures and landmarks generally reduces the uncertainty. For the libration amplitude, the greatest improvements (a reduction in formal error, $\Delta\sigma_{g_s}$, of up to 45 %) are seen when both factors are increased, with diminishing returns beyond approximately 6 pictures and 60 landmarks (see Fig. 4.19a). For the obliquity, the number of pictures appears to be the more dominant factor, with significant improvements ($\Delta\sigma_\epsilon$ reaching almost –50 %) up to about 6 pictures, after which the number of landmarks has a more limited effect on the final precision as shown in Fig. 4.19b.

Impact of Inversion Strategy The choices made during the data inversion process also influence the final parameter precision. The complexity of the estimated parameter set has a significant effect; in accordance with least-squares theory, increasing the number of estimated parameters while the dataset size remains fixed can lead to higher correlations between them, which in turn inflates the variance of the solution (see Sec. 2.2.2.3 and, e.g., [42]). Consequently, as more higher-degree gravity terms are estimated, the uncertainties on MoI and k_2 tend to increase, often by factors of $\sim 1.5-3$ when moving from a quadrupole-only fit to a degree 3 or 4 fit (Figs. 4.13 and 4.14); this effect is not visible for the rotational parameters, as the optical data effectively removes their correlation with the gravity field. However, certain simplified parameter sets can produce distinct outliers. For instance, the Decadal Tour exhibits an exceptionally low uncertainty for k_2 when using an optimistic filtering strategy that only solves for the quadrupole field and k_2



(a)



(b)

Figure 4.19.: Percentage variation of the estimated formal error (1σ) of the libration amplitude g_s (panel a) and obliquity ε (panel b) with respect to the nominal case (Decadal tour, Ka-Band, 4 hours tracking, 4 pictures, 10 landmarks) as a function of the number of pictures per flyby and the number of observed landmarks per picture.

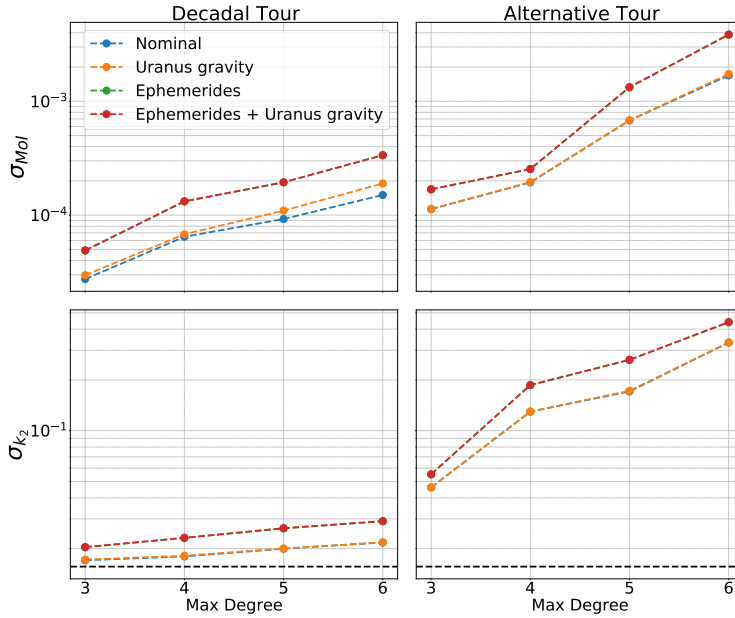


Figure 4.20.: Ariel's expected uncertainty on the MoI and k_2 Love Number as a function of the maximum estimated degree and order gravity field. The observation settings are the same as those of Fig. 4.17. The effect of Ariel's position and/or Uranus gravity errors on the geodetic parameters of interest is tested. The black dashed line in the k_2 panels represents the true value of Ariel's Love Number. Ariel's position is here fitted with an a priori uncertainty of 500 m along the three Cartesian coordinates ('Ephemerides case'), and the influence of the errors on Uranus gravity is done assuming an a priori of 30% of the current uncertainties on GM , J_2 , and J_4 .

itself ($Q + k_2$) (Fig. 4.14). This result, while mathematically valid within the simplified model, is not scientifically feasible. As demonstrated in Fig. 4.17, the Decadal Tour allows measurement of coefficients up to degree and order 4×4 . Therefore, fitting only the quadrupole coefficients represents a significant under-parameterization of the problem, leading to an unrealistically optimistic formal error by neglecting the influence and correlations of the higher-degree terms. Lastly, the application of a physically-motivated hydrostatic constraint consistently helps to reduce the final uncertainty, often by $\sim 10 - 30\%$ compared to the unconstrained case, particularly for the more complex parameter sets (see red vs. green boxes in Figs. 4.13 to 4.16, bottom panels). In the context of least-squares estimation, in fact, applying such a constraint provides a priori information and reduces the dimension of parameter space. Specifically, this stabilization directly improves the estimation of the degree-two coefficients and, in turn, improves the estimation of k_2 being the

amplitude of the periodic variation of the degree-two gravity field as shown in Eq. (2.7).

Robustness to Model Assumptions and External Errors We further test the robustness of our solutions against both model over-parameterization and unmodelled external errors, with results shown in Fig. 4.20. Although the sensitivity analysis in Fig. 4.17 indicates a maximum resolvable degree and order of 3×3 or 4×4 , we examine the impact of fitting a gravity field up to degree and order 6×6 . For the Decadal tour, the formal errors for MoI and k_2 are degraded by less than a factor of 2, while the degradation is up to one order of magnitude for the Alternative tour. This confirms the point made previously that the Decadal tour is more robust to the gravity model complexity. Importantly, for both tours, the estimated MoI precision remains sufficient (errors $< 2 \times 10^{-4}$) to satisfy the scientific requirement of 0.005, even in this over-parameterized case (Fig. 4.20).

We also quantify the impact of assumptions made regarding Uranus's gravity and the moon ephemerides, which were considered perfectly known in our nominal simulations. Fig. 4.20 shows that uncertainty in the Uranus gravity field has a minor effect on our results. However, simultaneously fitting the moon ephemerides with the other parameters degrades the estimated precisions for both MoI and k_2 by a factor of about two. While this degradation is significant, it does not alter our overall conclusions regarding the feasibility of the measurements or the relative performance of the different observation strategies.

Translation to Interior Structure Constraints The 'reverse mapping' method enables direct discussion of the performance of a GS experiment design in terms of the internal parameters, as illustrated in Figs. 4.21 to 4.23 for the ocean thickness, rock-to-ice ratio and ice shell thickness, respectively, and discussed in Sec. 4.4.2. In order to correctly interpolate from the "lookup table," it is necessary to assume an uncertainty at the signal level for the non-estimatable parameters (i.e. the parameters for which the estimated uncertainty is higher than the simulated nominal value are the non-estimatable parameters). This objective can be achieved through the implementation of alternative measurement techniques, such as the determination of k_2 through the assessment of surface deformations with a 6-degree-of-freedom sensor positioned on the surface, as suggested in Sec. 4.4.2.2, or through the measurement of the libration amplitude derived from the analysis of the body's orbital evolution, as illustrated by Lainey *et al.* [55]. It is acknowledged that this is a significant assumption with regard to the evaluation of the GS experiment design. However, the objective of this investigation is to explore trends as a function of the experiment design, rather than to provide final recommendations.

The same general trends discussed for the geodetic parameters are evident in the resulting interior constraints. The combination of radiometric and optical data enables a more precise estimation of the rotational parameters and consequently the interior parameters. Indeed, it is the synergy of the previously discussed geodetic parameters that drives the uncertainty of the interior parameter estimation, despite the absence of a solution that can accurately determine the ocean thickness,

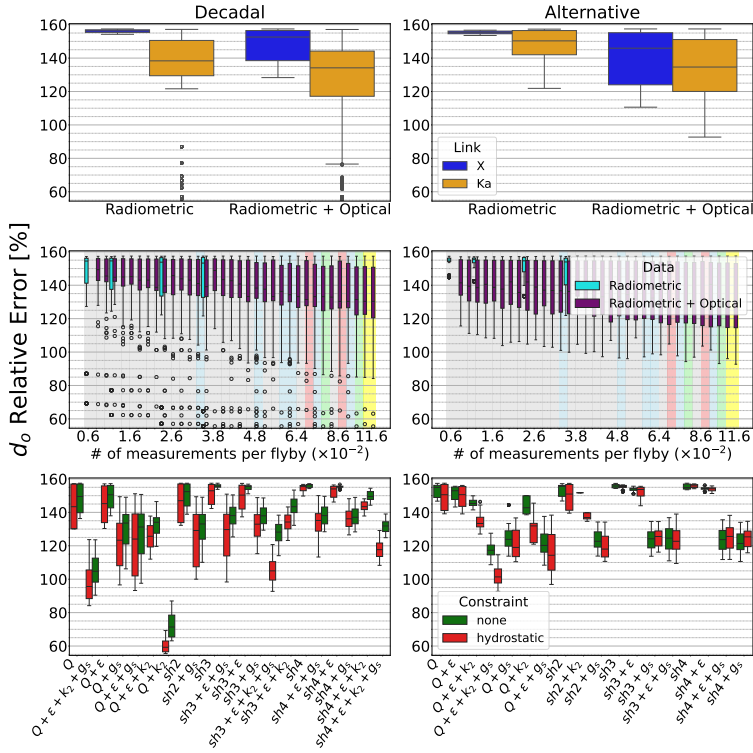


Figure 4.21.: Interpolated uncertainty of Ariel's ocean thickness (d_o). A detailed explanation of the figure can be found in Fig. 4.13.

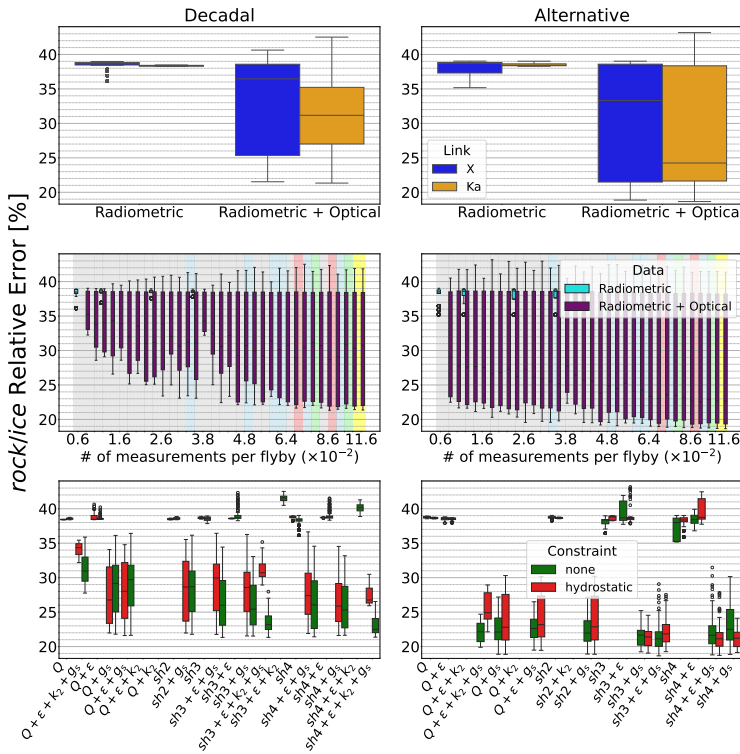


Figure 4.22.: Interpolated uncertainty of Ariel's rock-to-ice ratio. A detailed explanation of the figure can be found in Fig. 4.13.

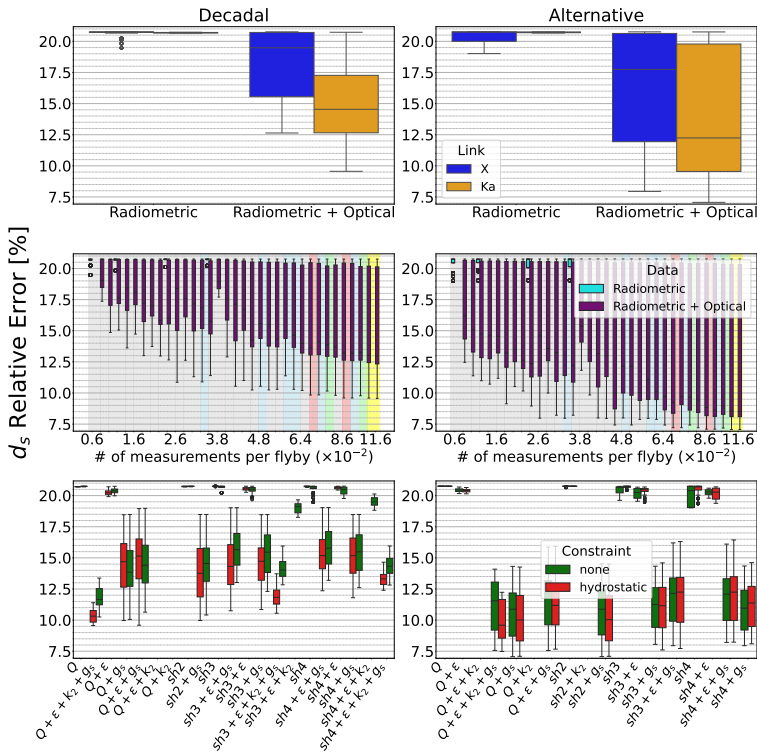


Figure 4.23.: Interpolated uncertainty of Ariel's ice shell thickness (d_s). A detailed explanation of the figure can be found in Fig. 4.13.

d_o . It is also clear that a greater number of measurements per flyby leads to improved precision, underscoring the value of more intensive observation campaigns (Figs. 4.21 to 4.23, middle panels). The determination of the rock-to-ice mass ratio is overall constrained by the necessity of accurately determining the layer densities, which represents a particularly challenging task.

A detailed look at Figs. 4.21 to 4.23 reveals the feasibility of constraining each interior parameter. Confirming its status as the most challenging parameter, the ocean thickness (d_o) uncertainty is very high across almost all scenarios, with most solutions yielding relative errors greater than 100 % (Fig. 4.21). The Decadal Tour does exhibit a distinct outlier for this parameter, represented by the same optimistic and scientifically unfeasible filtering strategy ($Q + k_2$) highlighted in the *Impact of Inversion Strategy* paragraph. This strategy proposes a relative uncertainty from 60 % to 80 % contingent on the applicability of the hydrostatic constraint. In contrast, the ice shell thickness (d_s) is the most precisely determined interior parameter, with optimal scenarios under the Decadal Tour achieving a relative uncertainty of $\approx 15\%$ (Fig. 4.23). The rock-to-ice ratio lies between these two extremes, where the best scenarios under the Decadal Tour can constrain it with a relative uncertainty of $\approx 28\%$ (Fig. 4.22). The necessity of combining multiple precise geodetic measurements is evident from the bottom panels of all three figures. These panels explore scenarios where the uncertainty of one parameter is improved relative to its nominal value, while the others are assumed to be perfectly known, as explained by the interpolation method utilized. Even under this optimistic assumption, it is clear that no single improved measurement robustly constrains the interior, underscoring that only a synergistic combination of measurements can properly resolve Ariel's structure. Finally, despite having fewer flybys, the diversified geometries of the Alternative Tour allow it to achieve a comparable performance in constraining the more achievable interior parameters (d_s and rock/ice ratio), contingent upon the k_2 Love Number being measurable with other techniques. As has been demonstrated, the final design of the tour has a significant effect on the scientific return of the GS experiment. Consequently, it is essential to conduct a thorough evaluation of the future and necessary iterations of the UOP trajectory and/or observation campaign in order to ascertain their impact on the experiment's scientific return.

4.5. CONCLUSIONS

In this paper, we presented a new approach to fold science objectives at the earliest stages of mission design. By establishing a clear relationship between the required precision on the geodetic measurements and the scientific goals of the mission, such an approach would allow the mission designers to make essential, rapid and informed adjustments without compromising scientific outcomes, both at the beginning and during the later stages of the mission's development. This "reverse mapping" approach is noteworthy for its originality and efficiency, which could potentially serve as a point of inspiration for other experiments in planetary science missions to pursue a similar approach. We employed this methodology to develop guidelines for the UOP gravity experiment designs, demonstrating how this inverse

approach can facilitate the definition of measurement requirements subsequent to the identification of scientific objectives, but also rapidly assess the efficacy of geodetic measurements in constraining interior parameters.

Based on a series of truth interiors encompassing both ocean-bearing and oceanless models of the Uranian satellites, we assessed the expected uncertainties associated with key interior parameters (i.e. the ice shell and ocean thicknesses, and the rock-to-ice mass ratio) as a function of the uncertainties on key geodetic measurements (mean moment of inertia, tidal Love number, diurnal libration amplitude, and obliquity). By conducting ~ 600 MCMC simulations for each truth model, we build comprehensive measurements-to-interior maps for each interior parameter. We used these “lookup tables” to determine the required geodetic measurement precisions needed to meet some of the UOP scientific objectives. These maps are provided to the scientific community as MRTs (see [Appendix B](#)).

Assuming that Ariel has a 20 km thick ocean, our results suggest that determining its rock-to-ice ratio with better than 5% accuracy requires measuring all four geodetic parameters (MoI , k_2 , g_s , and ϵ) with 1% accuracy or better. Determining the ocean thickness presents an even more challenging task. In order to determine Ariel’s ocean thickness with a 30% uncertainty level, the absolute precisions required on the MoI , k_2 , g_s , and ϵ are 9.2×10^{-4} , 1.4×10^{-4} , 2.4×10^{-1} m and 2.5×10^{-1} m, respectively. The ice shell thickness (d_s) is comparatively easier to constrain and can be determined with an uncertainty of 20-30% by measuring the tidal Love number and/or libration amplitude at the signal level for all satellites. An enhancement in the determination of the libration amplitude enables a more precise estimation of the ice shell thickness for Ariel, Umbriel, Titania and Oberon, whereas k_2 is the primary influencing factor for Miranda. The actual ocean thickness mainly affects the required precision to determine d_o itself. The thinner the ocean, the more precise the geodetic measurements must be to achieve a given level of uncertainty on d_o . The precisions on the determination of the ice shell thickness and of the rock-to-ice ratio are less affected by the actual ocean thickness. Our study also demonstrated that a multitude of combinations for the measurement precision on MoI , k_2 , g_s , and ϵ can be employed to determine an interior parameter with a desired uncertainty. Some combinations may be particularly advantageous for mission design, as they do not necessitate the measurement of all four geodetic observables with high precision. Relaxing the uncertainty requirement for one observable results in an increased precision requirement for the others. Some observables tend to compensate for each other, specifically g_s and k_2 . When attempting to estimate several interior parameters simultaneously, such as d_o , rock-to-ice ratio, and d_s , it became evident that achieving precise measurements across all parameters is a particularly difficult endeavor. For instance, in the case of Titania, it is necessary to determine all four geodetic parameters with an uncertainty of 1% or less to determine the three interior parameters at the 30% level. It is unlikely that the three parameters can be measured to a 30% level with UOP, given the experience from the ten Cassini flybys of Titan and the resulting level of precision on its geodetic parameters.

A covariance analysis was conducted to confirm the difficulty in constraining the interior parameter of the Uranian satellites. Specifically, we simulated Doppler

and optical data from the UOP mission. Given the limited characterization of the gravity experiment design at this early stage of the mission, we analyzed two orbital tours, various measurement strategies, and inversion settings, resulting in approximately fifteen thousand simulations per moon. Our mapping technique enabled us to rapidly assess the performance of each of these POD simulations in constraining the interior parameters. The discussion focused on the observation strategies required to achieve the desired precision in geodetic measurements for Ariel's hydrosphere thickness, ocean detection, and other internal parameters. To illustrate, we discovered that the most challenging interior parameter to constrain is the ocean thickness d_o , where only few of the tested solutions achieved a relative uncertainty better than 100%. However, the rock-to-ice ratio could be constrained with a relative uncertainty of approximately 28% using 6 h of Ka-band tracking, 4 pictures per flyby, and 10 landmarks per picture with the Decadal Tour. Instead, the ice shell thickness d_s , being the most precisely determined interior parameter, can be constrained with a relative uncertainty of 16% when using 4 pictures per flyby, 10 landmarks per picture, and 6 hour of Ka-Band tracking with the same orbital tour. Nevertheless, diversified flyby geometries, exemplified by the Alternative Tour, demonstrated comparable performance in constraining Ariel's interior properties, despite the reduction in the number of close passes. This is contingent upon the k_2 Love Number being measurable with other techniques. As has been demonstrated, the final design of the tour has a significant effect on the scientific return of the GS experiment. Consequently, it is essential to conduct a thorough evaluation of the future and necessary iterations of the UOP trajectory in order to ascertain their impact on the experiment's scientific return.

These findings highlight the intricate and challenging nature of the measurement requirements for accurately characterizing Ariel's interior structure. However, these are specific observation strategies that are contingent upon particular assumptions, the relevance of which can shift during the course of mission development. A comprehensive sensitivity analysis is provided in [Sec. 4.4.2.3](#), in which the impact of several gravity experiments designed to study the interior of Ariel is assessed in terms of the geodetic parameter uncertainties, as well as in terms of interior parameters. This analysis highlighted several key trade-offs crucial for mission design. We demonstrated that a tour with more flybys (Decadal) performs better for gravity field determination—resolving the field up to degree and order 4×4 versus 3×3 for the Alternative tour and exhibiting greater robustness when fitting higher-degree gravity terms. Conversely, a tour with more diverse geometries (Alternative) is better for constraining rotational parameters, yielding median uncertainties for libration amplitude (g_s) and obliquity (ϵ) that are roughly 30-50% lower compared to the Decadal tour under optimal conditions (e.g., median $\sigma_{g_s} \approx 2 \text{ m/s} \approx 3 \text{ m}$). The inclusion of optical data alongside radiometric measurements was found to be essential for measuring libration and obliquity to required precisions, reducing median uncertainties by roughly an order of magnitude (e.g., bringing median σ_{g_s} from $>15 \text{ m}$ down to $\approx 2 - 3 \text{ m}$ with Ka-band data). Furthermore, the choice of Ka-band over X-band for the radio link provides a significant improvement across parameters, reducing the median MoI formal error by about two orders of magnitude

(e.g., from $\sim 10^{-3}$ down to $\sim 10^{-5}$) and bringing the median k_2 uncertainty down to the signal level ($\approx 0.010 - 0.015$). Ultimately, constraining Ariel's interior requires a synergistic approach; as demonstrated by the mapping method, no single geodetic parameter provides sufficient constraint, and the optimal observation strategy will depend on a careful balance between tour design (number of flybys vs. geometry diversity), data types (radio-only vs. radio+optical), instrument capabilities (X-band vs. Ka-band, optical data volume), and inversion strategy (parameter set complexity, use of constraints) to achieve the desired scientific goals. Furthermore, the results of the aforementioned sensitivity analysis, as applied to the other Uranian satellites (namely Miranda, Umbriel, Titania, and Oberon), are also provided in [Appendix A](#).

Although the “lookup tables” have been developed for gravity experiments at the Uranian satellites, we wish to emphasize that this approach has the potential to be beneficial to a wider range of experiments and missions. Indeed, this approach enables the required precision for geodetic measurements to be determined in line with a given scientific objective, while also facilitating the rapid evaluation of the outcome of a thousand simulations in terms of the mission objectives, as demonstrated in this study.

While this framework proves powerful for designing orbital remote-sensing experiments, our analysis also highlights the inherent challenge in resolving specific parameters, such as the tidal Love number k_2 , when relying solely on limited flyby data. This difficulty consequently suggests the potential value of complementary, in-situ measurement techniques, which could overcome the limitations inherent to orbital observations. This is particularly relevant for multi-body tours that allows only a few encounters per satellite. The following [Chapter 5](#) will therefore explore such an alternative. We will assess how a novel landed instrument, designed to measure surface rotational variations and tidal deformations, could provide independent constraints on key geophysical parameters (including those related to tidal response, such as Love numbers h_2 and k_2), and thereby potentially address the measurement gaps identified for future missions to the Uranian moons.

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5

A 6DoF MOTION SENSOR TO MEASURE ROTATION AND TIDES IN THE SOLAR SYSTEM

This chapter is based on work previously published in the Earth, Planets and Space journal: V. Filice, S. Le Maistre, V. Dehant, T. Van Hoolst, F. Bernauer, R.F. Garcia. PIONEERS: a 6DoF motion sensor to measure rotation and tides in the Solar System. Earth Planets Space 76, 12 (2024). Content from the original publication's appendices has been integrated into the main text and adapted for this chapter, and the specific analysis relating to the Uranian satellites has been added for this thesis.

5.1. INTRODUCTION

As established in the previous Chapter 4, designing gravity science experiments to characterize icy moon interiors presents significant challenges, especially for resolving weak tidal signals that are critical for detecting subsurface oceans. To overcome the limitations inherent in remote sensing from multiple flybys, in-situ measurements from the surface can be a complementary approach. In this chapter, we therefore focus on the potential of measuring rotation changes and tidal deformations using a newly developed 6DoF (Degrees of Freedom) motion sensor, such as the one proposed by the PIONEERS project. PIONEERS, which stands for Planetary Instruments based on Optical technologies for an iNnovative European Exploration using Rotational Seismology, was a project funded by the Horizon 2020 research and innovation program of the European Commission to set the basic scientific requirements of a 6DoF motion sensor and provide an outlook of what science can be expected from the deployment of such an instrument in the Solar System.

5

Two different types of sensors were being developed in the PIONEERS project to address a wide range of applications: a compact model and a high-performance prototype sensor. The first consists of three orthogonally aligned quartz vibrating beam accelerometers based on Micro-Electro-Mechanical Systems (MEMS) technology and three orthogonally aligned Fiber-Optic Gyroscopes (FOGs). FOGs have significant potential to accurately measure rotational motion over a broad frequency spectrum while maintaining a high dynamic range. The operating principle of FOGs is based on the Sagnac effect, as described by Laue [1] and Sagnac [2], which exploits the differences in optical path lengths between two counter-propagating beams within a rotating optical fiber loop. These devices are widely used as rotation rate sensors in gyrocompasses and inertial measurement units, especially in inertial navigation (e.g. Lefèvre [3]). The PIONEERS instrument is based on a specialized FOG tailored for rotational seismology, the blueSeis-3A from iXblue (France), as presented by Bernauer *et al.* [4]. The three FOG sensors measure the absolute rotation rate vector of the instrument in the 0-1000 Hz frequency range allowing to infer both the rotation rate of the planetary body and long and short period variation of this rotation rate [3]. The biases of such sensors are usually very low due to a proper temperature calibration. MEMS are commonly used for acceleration sensing in various applications, including strong-motion and engineering seismology as well as inertial navigation. These devices are characterized by their compact size, typically ranging from a few millimeters to centimeters. They integrate logic circuits and mechanical structures onto a microchip, and provide benefits such as efficient power usage, sleek design, and durability (e.g. the Short-Period (SP) sensors featured on the InSight mission [5]). Specifically, the integrated MEMS sensor is a quartz vibrating beam navigation grade accelerometer, which monitors the deformation of a vibrating beam under the influence of an acceleration [6]. The use of three of these MEMS sensors allow to measure the acceleration vector of the instrument in the same frequency range as the rotation rate measurements. These measurements are calibrated relative to temperature variations as well. Performance targets for the compact model include self-noise levels of $2\mu\text{rad s}^{-1}\text{Hz}^{-1/2}$ and $20\mu\text{m s}^{-2}\text{Hz}^{-1/2}$

within a frequency range of 0.01 Hz to 400 Hz.

The high-performance prototype integrates a Very Broadband Seismometers (VBB) seismometer with an optical readout to measure translational motion, along with a giant fiber optic loop (from 0.5 m to 1 m) for rotation rate sensing. The implementation of a band-pass response with respect to acceleration enables highly sensitive broadband recording, while also mitigating problems related to saturation caused by impulsive disturbances, long-term thermal drifts, and tilt [7]. The VBB used in the high-performance model employs a readout devices approach based on laser interferometry principles to track the seismic mass position (see Berger *et al.* [8] for a detailed comparison of different VBBs). The performance targets for this configuration are self-noise levels of $5 \text{ nrad s}^{-1} \text{ Hz}^{-1/2}$ and $10 \text{ pm s}^{-2} \text{ Hz}^{-1/2}$ across a frequency range of 0.001 Hz to 10 Hz.

The PIONEERS 6DoF concept's novelty lies in its synergistic integration of sensor technologies for planetary science, contrasting with their typical use in navigation or established Earth science applications [4, 9]. For rotation sensing, we leverage Fiber-Optic Gyroscopes (FOGs) as a space-feasible approach, which provides a compact and robust package suited for landers compared to the highly sensitive but bulky terrestrial Ring Laser Gyroscopes (RLGs), although both use the Sagnac effect [9]. The PIONEERS high-performance model, in particular, uses a large fiber loop to boost FOG sensitivity for specific scientific targets [9]. For translational sensing, the compact model utilizes robust, low-resource MEMS, similar to InSight's SP sensors [9]. The high-performance model, however, employs an optically-read VBB using laser interferometry, targeting sensitivity comparable to high-end terrestrial seismometers (like the force-balance STS2) [9]. Integrating these translational and rotational measurements fundamentally distinguishes our concept from standalone 3-component seismometers, as it enables the correction of tilt contamination and facilitates single-station array analyses [9].

The project's objective was the development of two distinct Fiber Optic Gyroscope (FOG) models: a compact version targeting a Technology Readiness Level (TRL) of 6/7 and a high-performance version aiming for TRL 4/5. However, the program was prematurely terminated due to logistical and operational difficulties posed by the COVID-19 pandemic. As a consequence of the project's early conclusion, the planned testing and validation campaign for the high-performance instrument was not carried out. The project's original scope, development milestones, and achievements prior to its termination can be reviewed on the official project webpage [10].

The scientific objectives of the 6DoF PIONEERS instrument were threefold:

1. computing the landing trajectory of a free falling lander;
2. determining the rotational dynamics and tidal deformation of planetary objects;
3. measuring seismic activity from the surface of celestial bodies [9].

In this study we focus on the second objective, and explain and discuss in detail the ability of PIONEERS like instrument to measure the rotational and tidal motions of a celestial body. Such geodetic quantities are of paramount importance for understanding the interior properties of planetary bodies. Both tides and rotation

are sensitive to the interior structure and to the presence of internal liquid layers in particular as explained in [Chapter 2](#). Separately or together, they have been used to detect and characterize a liquid core in the center of planets (e.g. Yoder *et al.* [11], Le Maistre *et al.* [12] for Mars, Margot *et al.* [13], Steinbrügge *et al.* [14], Genova *et al.* [15] for Mercury) or subsurface oceans in icy moons (e.g. Iess *et al.* [16], Baland *et al.* [17], Tajeddine *et al.* [18], Thomas *et al.* [19], and Van Hoolst, Baland, and Trinh [20]). None of these previous discoveries relied on a 6DoF sensor though. They were mainly determined from radiometric observations or from optical data. The accuracy of the radio data is limited by the propagation noise affecting the radio waves, and both radio and optical data accuracies depend on the spacecraft trajectory accuracy when carried out from orbit. Other techniques using star trackers from the surface of bodies (e.g. Le Maistre *et al.* [21]), radar speckle observations from Earth [13], or altimeter crossover measurements from orbit (e.g. Rosat *et al.* [22], Mazarico *et al.* [23], Stark *et al.* [24], and Bertone *et al.* [25]) have been used to measure tides and rotation. When possible, the performance of a PIONEERS like instrument will be compared to the performances of all these alternative techniques in the present chapter.

5

We first present the 6DoF instrument observables that will be used to constrain the interior of planets, namely the rotation rate, centrifugal acceleration, and tidal accelerations. We then assess the contribution to these observables from the interiors of nine bodies in our Solar System that have been identified as future or potential targets for upcoming deep space missions: Dimorphos, Phobos, Europa, Io, Titan, Enceladus, Triton, the Moon, and Mars. This diverse set of objects allows for a broad evaluation of the instrument's performance across the solar system. Reflecting a recent shift in scientific community priorities, detailed in [Chapter 4](#) with the introduction of the Uranus Orbiter and Probe mission, we will also extend this investigation to the Uranian satellites. As established in aforementioned chapter, reliably inferring the interior structure of these moons requires measuring multiple geodetic parameters, including the tidal Love number k_2 and the libration amplitude. A key motivation for this chapter's analysis is to determine if a 6DoF sensor can provide a crucial independent measurement of the ove number k_2 —a parameter shown to be unresolvable for most of the Uranian satellites with the mission's baseline gravity science experiment alone. The resulting ranges of velocities and accelerations for all these bodies are then used to define the precision required from a 6DoF sensor. Finally, we discuss the results and compare the required accuracies with the precision levels of the PIONEERS prototypes, with a special focus on the compelling case of the Uranian moons.

5.2. ROTATION AND TIDE OBSERVABLES OF A 6DOF SENSOR

The angular velocity vector (in a body-fixed frame) of the nine considered celestial objects is generally not uniform because of the presence of free rotational modes or because they undergo time-varying gravitational torques exerted by other solar system bodies (see [Sec. 2.3](#)). The way a planetary body changes its rotation and orientation in response to the external forcing is influenced by its internal structure

and the dynamics of its fluid layers, if any. A fluid core or a subsurface ocean can notably change the rotation of a body since they can rotate with a different rate and around a different axis than the solid layers it interacts with (e.g. Van Hoolst, Baland, and Trinh [26]). Bodies with an atmosphere like the Earth, Mars, Venus or Titan experience rotation variations resulting from angular momentum exchanges between the solid body and the atmosphere. The rotation rate variations are usually either named Length-of-Day (*LOD*) variations for atmospheric planets or longitudinal librations for bodies in spin-orbit resonance (see details on libration and ΔLOD in Van Hoolst [27]). In the rest of the chapter we will always use the word “libration” to denote periodic variations in the rotation, even for Mars where ΔLOD would be more suitable. Note that only one wave is considered in our models as, for synchronous rotators, the libration amplitude at orbital frequency is much larger than the amplitude at harmonic frequencies and is most suitable to infer interior structure properties of the body (see discussion in Le Maistre *et al.* [21] for Phobos and see Van Hoolst, Baland, and Trinh [26] for further details). Librations at a free libration frequency might in principle have an even larger amplitude but would require an unlikely large recent excitation. Since these amplitudes can also not reliably be predicted we do not consider them any further. Librations at frequencies of orbital perturbations can have amplitudes comparable to those of the libration at orbital frequency (see e.g. Yseboodt *et al.* [28] for the case of Mercury, Rambaux, Van Hoolst, and Karatekin [29] for the Galilean satellites), but must be observed over much longer timescales to be estimated because of their longer periods. We here do not further consider those signals. We also restrict ourselves here to longitudinal libration and do not consider other usually smaller time-varying rotation variations such as polar motion or nutation.

The rotation rate, which is an observable of our 6DoF sensor, is the time derivative of Eq. (2.28). It can be expressed as a function of the libration quantities (denoted with subscripts *L*) as follows

$$\dot{W}(t) = \dot{W}_0 \left(1 + \frac{g_L \cos(\Omega_L t)}{2\pi} \right), \quad (5.1)$$

where *t* is the time, Ω_L is the libration frequency and g_L is the libration amplitude expressed in radian, which depends on the interior properties and/or of the dynamics of the atmosphere of the body. We chose the origin of time so that the phase of libration is equal to zero. For bodies in 1:1 spin-orbit resonance such as the Moon, most of the satellites of Jupiter and Saturn and some binary asteroids, Ω_L is equal to the mean rotation frequency $\dot{W}_0$, which is also equal to the orbital frequency $2\pi/T_{orb}$ (T_{orb} is the orbital period).

In addition to the rotation rate, a 6DoF motion sensor can measure the centrifugal acceleration induced by the body's librations, as well as the tidal acceleration generated by the primary body (mother planet, primary asteroid or Sun). The centrifugal acceleration that will be measured by a sensor located on the surface is a translational acceleration whose amplitude depends on the latitude ϕ of the sensor as

$$a_c(t) = \dot{W}(t)^2 r \cos(\phi), \quad (5.2)$$

where r is the distance from the center of the body.

The tidal acceleration a_T that will be measured by the 6DoF motion sensor is the combination of the direct tidal acceleration at the surface a_{dT} , the tidal displacement of the ground described by the Love number h , and the effect of the mass redistribution due to the tides related to Love number k [30] according to $a_T = a_{dT} (1 + h - \frac{3}{2}k)$. The direct tidal acceleration can be estimated evaluating the perturbations on the body's potential due to tides (V_t) and the total gravity potential (V). Manipulating the tidal potential equation truncated at degree two, the variations in the external potential field can be expressed as

$$\left| \frac{V_T}{V} \right| \approx \frac{M_p}{M_b} \left(\frac{r}{d} \right)^3 \Gamma \quad (5.3)$$

where M_p is the mass of the perturbing body, d is the distance between the center of mass of the two bodies, r is the radius of the body and Γ is equal to $6e$ for synchronous bodies or $3/2$ for fast rotators. The ratio between the direct tidal (a_{dT}) and self gravitational ($g = GM_b/r^2$) accelerations equals the corresponding ratio between tidal (V_T) and gravitational potentials V , therefore, using Eq. (5.3), we can write

$$\frac{a_{dT}}{g} = \frac{V_T}{V} = \frac{M_p}{M_b} \left(\frac{r}{d} \right)^3 \Gamma \quad (5.4)$$

from where the expression of a_{dT} can easily derived.

The acceleration as measured by a system on the surface of the body feels the direct tidal attraction a_{dT} , the displacement of the surface related to the Love number h (hV_T/g) and the effect of the mass redistribution due to the tides related to Love number k (additional potential at surface kV_T). The combination $\delta = 1 + h - \frac{3}{2}k$ defines the tidal gravimeter factor that is used to express the total tidal acceleration

$$a_T(t) = \delta \frac{GM_p r}{d(t)^3} \Gamma \quad (5.5)$$

Since d is time-dependent, to evaluate the average amplitude of the tidal signal we use the mean distance of the body with respect to the primary. If the moon's spin axis is tilted relative to its orbital plane, a second tidal force is due to the moon's obliquity. The tidal potential due to obliquity tides is defined by Eq. 2 in Tyler [31] and it is proportional to

$$V_{T_o} \propto \frac{3}{2} \dot{W}^2 r^2 I \quad (5.6)$$

where $\dot{W}$ is the rotation rate and I is the obliquity. Using the same reasoning as above we can express the total tidal acceleration due to obliquity as:

$$a_{T_o} = \delta a_{dT_o} \propto \delta \frac{3}{2} \dot{W}^2 r I. \quad (5.7)$$

Eqs. (5.1), (5.2) and (5.5) describe the observables of the 6DoF motion sensor. Because we are interested in the bodies' interior properties, which affect the libration amplitude g_L in Eqs. (5.1) and (5.2), we rewrite the observables as the difference

between the measured rate and acceleration and those of a body rotating at the constant rate $\dot{W}_0$:

$$\dot{W}_{lib} = \dot{W}_0 \left(\frac{g_L \cos(\Omega_L t)}{2\pi} \right), \quad (5.8)$$

$$a_{c_{lib}} = \dot{W}_0^2 \left(\frac{g_L \cos(\Omega_L t)}{2\pi} \right)^2 r \cos(\phi). \quad (5.9)$$

Following the same procedure, we could have rewritten the tidal observable as $a_T - a_{dT}$, given the fact that the dependence to the interior of the tidal acceleration is carried by the Love numbers (h, k) . However, this does not give any added value and we prefer to use δ as the parameters of interest to infer the interior from tidal accelerations. We thus keep Eq. (5.5) to model that observable.

As explained in Sec. 5.1, both the PIONEERS models feature three FOG sensors that measure rotation rate variations and three MEMS sensors that measure linear and centrifugal accelerations and their short period variations as those induced by seismic events or by the interactions between the lander and the ground. Consequently, the two instruments are able to perform independent measurements of the rotation and the acceleration vectors.

5

5.3. INTERIOR SIGNATURES

The science requirements for the instruments design are defined based on the physical signal one wants to detect. In our case, such a signal corresponds to the signature of the interior in the body's rotation rate and centrifugal acceleration as well as in the tidal acceleration. In this section we propose a method to determine the precision of the 6DoF observables needed to improve our knowledge of the interior of a given body. This corresponds to the measurement precision needed to distinguish between different interior model predictions. For that, we must measure with an error significantly smaller than the signal itself and than the difference between two end member predictions. Model predictions are based on Eqs. (5.5), (5.8) and (5.9) and the ranges of interior signature in the observables are obtained by substituting the expected amplitude of libration in Eqs. (5.8) and (5.9) and that of the expected tidal gravimeter factor in Eq. (5.5) by the minimum $(g_{L_{min}}, \delta_{min})$ and maximum $(g_{L_{max}}, \delta_{max})$ values allowed by interior models. The precision needed for the observables shall thus be smaller than $(g_{L_{max}} - g_{L_{min}}) \partial(Rot) / \partial(g_L)$ for the rotation observables with $Rot \in \{\dot{W}_{lib}; a_{c_{lib}}\}$, and smaller than $(\delta_{max} - \delta_{min}) \partial g_{dT} / \partial \delta$ for the tidal observable. Consequently, the needed precision in the rotation rate measurement can be defined as

$$p_{\dot{W}} = \kappa \dot{W}_0 \left(\frac{g_{L_{max}} - g_{L_{min}}}{2\pi} \right), \quad (5.10)$$

that of the centrifugal acceleration measurement as

$$p_{a_c} = \kappa \dot{W}_0^2 \left(\frac{g_{L_{max}} - g_{L_{min}}}{\pi} \right) r \cos(\phi), \quad (5.11)$$

and that of the tidal acceleration measurements as

$$p_{a_T} = \kappa (\delta_{max} - \delta_{min}) \frac{GM_p r}{d^3} \Gamma, \quad (5.12)$$

where d is the average distance to the perturbing body and κ is a scale factor to be set within $(0, 1)$ given the level of precision wanted.

In instances where the precision levels for librations and tides have accurately been identified, as is the case with the Uranian satellites (see [Sec. 5.5](#)), the required libration amplitude $g_{L_{req}}$ and the combination of required Love numbers $h_{2_{req}} - \frac{3}{2}k_{2_{req}}$ are used in place of the values for $\kappa(g_{L_{max}} - g_{L_{min}})$ and $\kappa(\delta_{max} - \delta_{min})$ respectively.

5.3.1. AMPLITUDE SPECTRAL DENSITY OF A PURE SINE SIGNAL

As explained, the PIONEERS instrument is a combination of a FOG, a MEMS and a VBB whose performances are expressed in the frequency domain. Therefore the above precisions shall be expressed in this domain if we want to compare them to the sensor self-noise level or translate them into requirements for the instruments of future missions.

The Amplitude Spectral Density (ASD) of a pure sine wave is represented in the frequency domain as a single peak at its own frequency, with an amplitude proportional to the strength of the wave. The ASD level of a pure sine wave depends on the associated frequency and bandwidth. Using the approach defined by [32] for signals with a finite bandwidth, we know that the root mean square (RMS) amplitude (or effective amplitude) of a wavelet $f(t)$ can be approximated as (Eq.4.16 in [32])

$$g_{RMS} = S = \sqrt{P \times (f_u - f_l)} \quad (5.13)$$

where P is the power spectral density, f_u and f_l are the upper and lower corner frequencies of the band-passed signal. Assuming a signal with a 1/3 octave bandwidth centered around the frequency f_0 , we have $(f_u - f_l) = 0.23f_0$. Given the fact that $f_0 = 1/T$, where T is the period of the signal, and the total observation duration defined as $t_m = n * T_{orb}$ (with $n \in \mathbb{N}$), we can write:

$$S_{ASD} = \sqrt{P} = g_{RMS} \sqrt{\frac{t_m}{2.3}} \approx S \sqrt{\frac{t_m}{2}}. \quad (5.14)$$

This formula differs from the classical approximation ($S/\sqrt{2}$), by the fact that it relies on the observation duration (t_m), i.e. on the frequency and bandwidth of the signal. [Fig. 5.1](#) shows that our [Eq. \(5.14\)](#) correctly approximates the effective amplitude of a sinusoidal signal while the classical approximation ($S/\sqrt{2}$) does not.

5.4. PHYSICAL AND DYNAMICAL PARAMETERS OF THE SELECTED BODIES

The nine bodies considered in this study have been identified by the scientific community and space agencies as targets for future missions. Besides the five ones

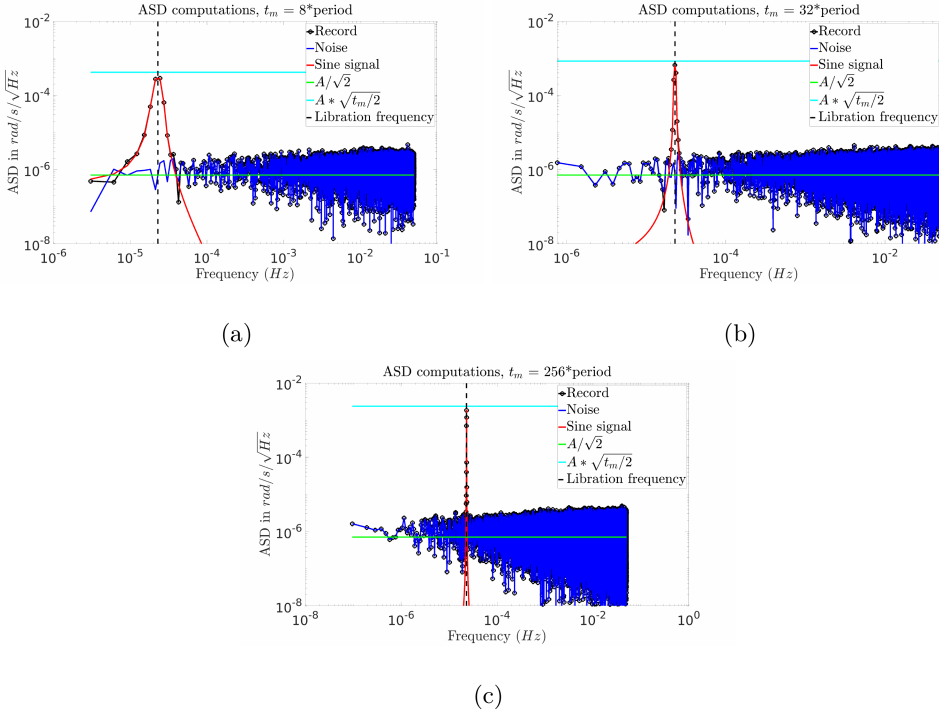


Figure 5.1.: Numerical calculation of the ASD of a pure sine with white noise for different observation duration of eight (a), thirty-two (b) and two hundred fifty six (c) times the target signal period. The traditional approximation (green line), fails to predict the ASD value, while Eq. (5.14) (cyan line) successfully provides the effective amplitude of the signal.

considered in Bernauer *et al.* [9], namely Dimorphos, Phobos, Europa, The Moon and Mars, we also consider here Io, Titan, Enceladus and Triton for the reasons explained below.

An orbiter mission to Io was proposed for the NASA Discovery program, but was not selected [33]. Nevertheless, Io will be studied through a couple of flybys of Juno in 2024 [34] and will most likely be the target of a future mission, as the least known Galilean moon in the Jovian system, after the JUICE and Europa clipper missions are completed. Measurement of Io's tides and rotation variations are key since one can use them to characterize the melt zone and confirm or not the presence of a global magma ocean beneath the surface [35, 36]. Identified as ocean worlds from Cassini's tides and rotation measurements [16, 19], both Titan and Enceladus are attractive bodies that will be visited again in a near future. The NASA Dragonfly lander mission, planned for arrival at Titan in the mid-2030s [37], is a good candidate to host our compact sensor, as is the lander of the proposed Enceladus Orbilander NASA flagship mission, which will be dedicated to search

for chemicals conducive to life in the ocean of Enceladus [38]. Following the new priorities set by the scientific community, this work also includes the major Uranian satellites. The UOP was designated as the highest-priority new flagship mission in the latest decadal survey (see Chapter 4). Investigating the interiors of its major moons—Ariel, Umbriel, Titania, and Oberon—is critical to understanding their geological history, their potential to harbor subsurface oceans, and the overall evolution of ice giant systems, which are a common class of exoplanets [39]. Triton has also always attracted interest from the scientific community, as evidenced by the multiple missions proposed after the only close encounter by Voyager 2 in 1989. The most recent of these was the Neptune Odyssey concept [40]. In the latest decadal survey, the UOP mission was prioritized over Neptune Odyssey [41]. However, Triton’s scientific importance remains undiminished. As a likely captured Kuiper Belt Object and a candidate ocean world with a geologically young surface and active plumes, it is a target of primary importance for understanding the evolution of icy bodies and the potential for habitable environments in the outer Solar System. For these reasons, we retain it in our analysis.

5

Table 5.1.: Physical and dynamical parameters of the selected bodies. References for Dimorphos are Daly *et al.* [42] and Thomas *et al.* [43]. All other radii are taken from Archinal *et al.* [44]. Masses and orbital parameters values are from the [Jet Propulsion Laboratory - Solar System Dynamics website](#).

Body	r [km]	Mass [kg]	Parent body mass [10^{24} kg]	Mean orbital distance d [km]	T_{orb} [day]	$\dot{W}_0$ [rads^{-1}]
Dimorphos	75.5×10^{-3}	4.84×10^9	Didymos: 5.557×10^{-13}	1.20	0.474	1.46×10^{-4}
Phobos	11.08	1.06×10^{16}	Mars: 0.642	9.40×10^3	0.319	2.28×10^{-4}
Europa	1560.80	4.80×10^{22}	Jupiter: 1898.125	6.71×10^5	3.551	4.92×10^{-4}
Io	1821.49	8.93×10^{22}	— " —	4.22×10^5	1.769	4.11×10^{-5}
Titan	2574.73	1.35×10^{23}	Saturn: 568.317	1.22×10^6	15.945	1.10×10^{-4}
Enceladus	252.10	1.08×10^{20}	— " —	2.38×10^5	1.372	5.30×10^{-5}
Miranda	235.8	6.60×10^{19}	Uranus: 86.81	1.299×10^5	1.413	5.14×10^{-5}
Ariel	578.9	1.29×10^{21}	— " —	1.909×10^5	2.520	2.89×10^{-5}
Umbriel	584.7	1.22×10^{21}	— " —	2.66×10^5	4.144	1.75×10^{-5}
Titania	788.9	3.42×10^{21}	— " —	4.363×10^5	8.706	8.35×10^{-6}
Oberon	761.4	2.88×10^{21}	— " —	5.835×10^5	13.463	5.40×10^{-6}
Triton	1352.60	2.14×10^{22}	Neptune: 102.409	3.55×10^5	5.877	1.24×10^{-5}
Moon	1737.40	7.346×10^{22}	Earth: 5.972	3.84×10^5	27.322	2.66×10^{-6}
Mars	3389.50	6.417×10^{24}	Sun: 1988500	1.52 au	686.97	7.09×10^{-5}

We describe here the selected physical and dynamical parameters of the aforementioned bodies that we used for the calculation of the expected rotation and tidal signals and their amplitude levels. Most of these values are taken from the literature and they are gathered in Table 5.1 and Table 5.2. The target accuracies for the libration amplitude and the tidal Love number k_2 of the Uranias satellites, discussed in Sec. 4.3, and summarized in Table 5.3. While only the requirements for the tidal Love number k_2 are detailed in Sec. 4.3, computing the necessary gravimeter factor also requires the Love number h_2 . The relationship between these two Love numbers is dependent on the satellite’s internal structure. As shown by Hussmann, Sohl, and Spohn [45] (Tables 5 and 7), the average factor between h_2 and k_2 for the Uranian satellites is approximately 2.3 for a two-layer interior model and

risers to 2.8 for a three-layer model. For our purposes, we adopted the factor of 2.8, as a three-layer body is consistent with our calculated k_2 requirements. Although this represents an approximation of the satellites' true physical state, it is a valid approach within an order-of-magnitude analysis used in this study to establish the instrument's performance requirements. For Dimorphos and Triton, the values are evaluated and discussed here below.

Table 5.2.: Libration amplitudes and degree two Love numbers of the selected bodies.

For the Moon, we consider the Lunar Laser Ranging (LLR) uncertainty on the libration has the signature of the interior signal to retrieve. For Mars, we consider the uncertainties in the LOD variations at the orbital period (annual terms) as the level of precision to reach in order to further constrain the Mars rotation variations and thereby the planet's atmospheric cycles. The h_2 values have been estimated from several Mars interior models such that they match the range of the measured k_2 values (A. Rivoldini personal communication). [1] Lainey *et al.* [46]; [2] Van Hoolst, Baland, and Trinh [26]; [3] Van Hoolst *et al.* [36]; [4] Thomas *et al.* [19]; [5] Rambaux and Williams [47]; [6] Konopliv *et al.* [48]; [7] Le Maistre *et al.* [21]; [8] Iess *et al.* [16]; [9] Durante *et al.* [49]; [10] Van Hoolst, Baland, and Trinh [20]; [11] Baland, Yseboodt, and Van Hoolst [50]; [12] Konopliv *et al.* [51]; [13] Lemoine *et al.* [52].

Body	g_L [°]		k_2	h_2	
Dimorphos	[7.00,45.00]	This study	$[1.72,2.55] \times 10^{-4}$	$[2.86,4.25] \times 10^{-4}$	This study
Phobos	[1.08,1.10]	[1]	$[0.53,5.30] \times 10^{-4}$	$[0.88,8.80] \times 10^{-4}$	[7]
Europa	$[2.42,3.80] \times 10^{-2}$	[2]	[1.20,1.30]	$[2.3,2.7] \times 10^{-1}$	[2]
Io	$[0.53,3.12] \times 10^{-1}$	[3]	$[5.1,8.3] \times 10^{-2}$	[0.09,2.1]	[3]
Titan	$[4.19,7.69] \times 10^{-3}$	[2]	$[5.49,6.83] \times 10^{-1}$	[1.30,1.70]	[8-9]
Enceladus	$[1.05,1.35] \times 10^{-1}$	[4]	$[1.00,7.00] \times 10^{-2}$	$[0.15,1.9] \times 10^{-1}$	[10-11]
Triton	$[3.2,5.4] \times 10^{-6}$	This study	$[0.1,3.4] \times 10^{-1}$	[0.1,1.2]	This study
Moon	$[4.66,4.67] \times 10^{-3}$	[5]	$[2.22,2.26] \times 10^{-2}$	$[3.64,3.78] \times 10^{-2}$	[12-13]
Mars	$[1.22,3.25] \times 10^{-6}$	[6]	$[1.66,1.82] \times 10^{-1}$	$[3.08,3.38] \times 10^{-1}$	[6]

The asteroid binary system Didymos-Dimorphos is the target of the NASA Double Asteroid Redirection Test (DART) mission. DART is the first demonstration of the kinetic impactor technique in the framework of Planetary Defense techniques [53]. Physical characteristics of Dimorphos have been estimated by Daly *et al.* [42] using the DRACO camera on board the spacecraft. Post impact orbital characteristics are described by Thomas *et al.* [43]. The librations remained unassessed during the DART mission; however, the forthcoming ESA's Hera mission [54] holds the promise of filling this knowledge gap. We therefore rely on the latest predictions of the rotation dynamics. The libration angle of Dimorphos is highly dependent on changes to its velocity, therefore Dimorphos has a different libration state after the DART impact [55]. The libration of the secondary is expected to be close to zero pre-impact and up to a maximum of 7° for the post-impact nominal case. However, from the parametric study realized by Richardson *et al.* [55] in order to overcome the multiple unknowns of the secondary dynamics and shape, as well

as the impact effects, four resonance frequencies have been determined that can be reached by a multitude of combination of semi-axis ratios leading to a chaotic motion or attitude instability. For the majority of semi-axis ratios combinations such as $a/b \geq 1.3$, the maximum libration angle in longitude is about 45° . We therefore choose a libration amplitude g_L range from 6° to 45° taking out the resonance cases since a conservative instrument design will not jeopardize its functioning for higher magnitudes of the measured signal.

Table 5.3.: Required precision in libration amplitude and combination of Love numbers (k_2 , h_2) for detecting a differentiated interior or subsurface ocean in the Uranian satellites, as identified in Table 4.1.

Body	g_L [$^\circ$]		$h_{2req} - \frac{3}{2}k_{2req}$		
	Differentiation	Ocean	Differentiation	Ocean (cons.)	Ocean (opt.)
Miranda	2.12×10^{-5}	8.48×10^{-5}	3.33×10^{-5}	3.33×10^{-5}	3.17×10^{-4}
Ariel	8.64×10^{-3}	8.64×10^{-3}	5.33×10^{-4}	5.17×10^{-4}	2.90×10^{-3}
Umbriel	8.55×10^{-3}	8.55×10^{-3}	5.33×10^{-4}	4.67×10^{-4}	2.75×10^{-3}
Titania	6.34×10^{-7}	7.61×10^{-7}	1.28×10^{-3}	1.12×10^{-3}	5.83×10^{-3}
Oberon	3.28×10^{-7}	3.94×10^{-7}	1.20×10^{-3}	1.03×10^{-3}	5.75×10^{-3}

Given that tides affecting the Didymos system are currently poorly constrained, we compute the tidal Love numbers assuming a deformable, homogeneous and incompressible spherical body as done for the Uranian satellites [30]

$$h = \frac{5}{3}k = \frac{5}{2} \left(1 + \frac{19}{2} \frac{\mu}{g\rho r} \right)^{-1}, \quad (5.15)$$

where ρ is the bulk density of the body, g is the surface gravity, r is the mean radius and μ is the rigidity of the rocks that compose it. Since it is believed that Dimorphos could be a rubble-pile body, we use $\tilde{\mu}_r \gtrsim \sqrt{\mu/\epsilon_Y}$ to compute its rigidity Goldreich and Sari [56]. According to the nominal values of the system as assumed by Daly *et al.* [42], we set $\rho = 2400 \pm 300 \text{ kgm}^{-3}$ and $r = 75.5 \pm 1.25 \text{ m}$. Following Naidu *et al.* [57], the Young's modulus E is fixed at 10^7 Pa , the Poisson's ratio ν equal to 0.25 and the yield strain ϵ_Y equal to 10^{-2} . We calculate the shear modulus μ from the equation of isotropic materials resulting in $\tilde{\mu}_r \approx \sqrt{E/((2(1+\nu))\epsilon_Y)} \approx 6 \text{ kPa}$ and therefore $h \in [2.86 - 4.25] \times 10^{-4}$ and $k \in [1.72 - 2.55] \times 10^{-4}$.

It will be much more challenging to probe the interior of Triton from the measurement of its rotation and tides because both are expected to be very small due to its orbital eccentricity close to zero ($e \sim 10^{-5}$, Jacobson [58]). The libration amplitude g_L of Triton can be expressed as [59]:

$$g_L = \frac{2e}{1 - \frac{1}{3\gamma}}, \quad (5.16)$$

where $\gamma = (B - A)/C$ and ($A < B < C$) are the principal moments of inertia. Actual knowledge of Triton's shape is not accurate enough to constrain its moment of

inertia [60]. We calculate the relative difference between moments of inertia from the secular Love number k_s (a measure of the body-yield to centrifugal deformation in the course of its secular evolution). For this, we employ Eq. 20 from Comstock and Bills [61], which is valid for to homogeneous bodies:

$$\gamma = \frac{45k_s q}{18 + 25k_s q}. \quad (5.17)$$

$q = M_p/M_b(r/a)^3$ is a dimensionless quantity where M_p is the mass of Neptune, M_b and r are the mass and the radius of Triton, respectively, and a is the orbit's semi-major axis. Given that Triton's internal structure and moments of inertia are poorly constrained, we use this relation as a first-order approximation sufficient for estimating the potential range of γ and the resulting libration amplitude for the purpose of defining instrument requirements. Therefore, differently from Comstock and Bills [61], who took the hydrostatic value $k_s = 3/2$, we consider a range of values in agreement with values of other icy satellites with which Triton shares most of its physical characteristics, i.e. $k_s \in [0.9 - 1.5]$ [62]. The resulting range of g_L is $[3.2^\circ - 5.4^\circ] \times 10^{-6}$ (i.e. only at milliarcsecond level) showing that the variations of Triton's rotation rate and acceleration are very small (see Table 5.1). The signatures of the internal structure are then expected to be even smaller, unless the presence of a subsurface ocean would strongly increase the amplitude [26]. In order to calculate Triton's tides, we calculate a set of interior structure models of Triton that satisfy the observed mass and radius. Similarly to other outer Solar System satellites, we assume that Triton has a metallic core, a silicate mantle and an icy shell, with a possible subsurface ocean. We further assume that the interior is hydrostatic (follows the fluid pressure-depth relation). By solving the equation describing the deformation of a self-gravitating, spherically symmetric body [63, 64], we calculate the Love number h_2 and k_2 and we predict $h_2 \in [0.15 - 1.2]$ and $k_2 \in [0.1 - 0.32]$. We assumed that the ice shell, ocean, mantle and core have a uniform density and that they are incompressible.

5.5. PREDICTED SIGNALS LEVELS AND REQUIRED PRECISION

For the representative set of solar system bodies, we computed the interior-related ranges of rotation rates and centrifugal and tidal accelerations using the physical and dynamical parameters provided in Tables 5.1 and 5.2. The difference between the minimum and maximum values of each range corresponds to the signature of the bodies' interior in the observables. For the Uranian satellites, the analysis is instead based on the specific target precisions required to distinguish between a differentiated interior and one possessing a subsurface ocean, as detailed in Table 5.3. The resulting signal levels and requirements for all bodies are shown in Fig. 5.2, with Fig. 5.2a displaying the ranges for the representative set and Fig. 5.2b showing the specific requirements for the Uranian satellites. Libration and tidal periods are considered to be equal to the bodies' orbital period, T_{orb} . The centrifugal accelerations are evaluated at the equator (i.e. with $\phi = 0$ in Eq. (5.2)).

As per the order of magnitude of the differences in the rotation rate, three bodies stand out in Fig. 5.2a: Triton, the Moon and Mars. These are very challenging

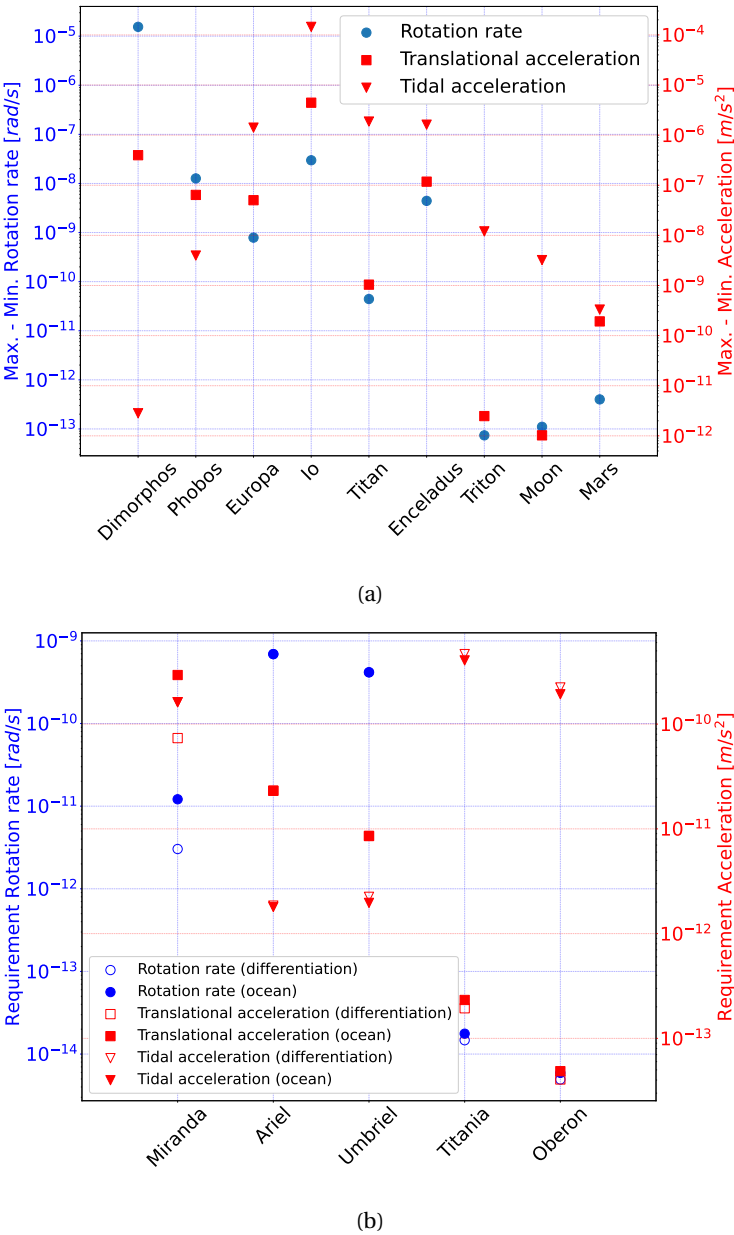


Figure 5.2.: Panel (a) shows the difference between the maximum and minimum expected signal levels for the representative set of bodies, based on the interior structure models in Table 5.2. Panel (b) shows the specific measurement requirements for the Uranian satellites needed to confirm a differentiated interior and one with a subsurface ocean, with target values taken from Table 5.3. For all bodies, the centrifugal acceleration is evaluated at the equator ($\phi = 0$), using the radii from Table 5.1.

targets for a 6DoF sensor since the low eccentricity of Triton leads to a small rotation rate and a small centrifugal acceleration, whereas the rotation rates of Mars and the Moon are already precisely measured. Fig. 5.2a also shows the predicted contribution of the interiors to tidal accelerations. Two bodies can be distinguished by their tidal signal, which is much greater than the others for Io and much smaller than the others for Dimorphos. The former is due to the fact that we account for the effect of a subsurface magma ocean in the calculation of the tidal gravimeter factor (δ) of Io. For Dimorphos, the signal is extremely small due its small size and to the small mass of Didymos. It is worth noting that the ranges computed for the outer system bodies are rough estimates since only limited data are available for most of them (see references in Table 5.1). For all bodies in spin-orbit resonance, we considered the tides related to their non-zero orbital eccentricity. These eccentricity tides are larger than the obliquity tides, which are due to the moon's axial tilt relative to its orbital plane [31], except for Triton, which has a very small eccentricity of 1.6×10^{-5} but a significantly large obliquity of up to 0.7° [65]. As a consequence, Triton's obliquity tides are three times larger than its eccentricity tides. However, since the required measurement precision is the one that allows distinguishing between different interior model predictions, and because both types of tidal accelerations linearly depend on the gravimeter factor (see Eqs. (5.7) and (5.12)), the instrument requirement is roughly the same for both obliquity and eccentricity tidal accelerations.

For the Uranian satellites, the requirements shown in Fig. 5.2b are derived directly from the target values in Table 5.3. Since the requirements for detecting a differentiated interior versus a subsurface ocean are of the same order of magnitude for both libration and tides for each moon, the required signal for differentiation (empty symbols) and ocean worlds (filled symbols) are closely clustered in the figure. The requirements for rotation rate and centrifugal acceleration (circles and squares) exhibit a clear decreasing trend with increasing distance from Uranus. This is a direct consequence of the satellites being in a 1:1 spin-orbit resonance, where their rotation rate (W_0) matches their orbital frequency. In contrast, the tidal acceleration requirement (triangles) does not follow this simple distance-based trend. The higher geophysical requirements for Titania and Oberon (see Table 5.3) effectively counterbalance the weakening of tidal forces at their greater orbital distance, resulting in required tidal signals on the order of 10^{-10} ms^{-2} , which is comparable to that of Miranda and roughly three orders of magnitude larger than those for Ariel and Umbriel ($\approx 10^{-13} \text{ ms}^{-2}$). A comparison between the two panels of Fig. 5.2b reveals that the entire Uranian system is in a difficult measurement regime, comparable to the most challenging targets of the representative set (i.e., Triton, Mars, and the Moon). In fact, the requirements for the Uranian satellites shown in Fig. 5.2b are in a narrow and low range, with rotation rates from $10^{-10} \text{ rads}^{-1}$ to $10^{-15} \text{ rads}^{-1}$ and accelerations from 10^{-10} ms^{-2} to 10^{-14} ms^{-2} .

Given the signal shown in Fig. 5.2, we have computed the precisions needed to infer information about the bodies' interior. Tables 5.4 and 5.6 show the results for the representative set of solar system bodies, while Tables 5.5 and 5.7 detail the requirements for the Uranian satellites. As outlined in Sec. 5.3, the calculation

Table 5.4.: Required measurement precisions in the rotation rate and acceleration for the representative set of bodies, along with the associated 6DoF instrument requirements corresponding to the ASD of these precisions. The mission durations used to compute the requirements (i.e. ASD quantities) corresponds to 10 times the orbital period except for Mars for which we set to be equal to one martian year.

Body	Libration frequency [Hz]	$p_{\dot{W}}$ [rads ⁻¹]	$p_{\dot{W}_{ASD}}$ [rad/s/√Hz]	p_{a_c} [ms ⁻²]	$p_{a_{cASD}}$ [m/s ² /√Hz]	Mission duration [days]
Dimorphos	2.330×10^{-5}	1.55×10^{-6}	7.16×10^{-4}	3.71×10^{-8}	1.72×10^{-5}	5
Phobos	3.628×10^{-5}	1.27×10^{-9}	4.70×10^{-7}	6.40×10^{-9}	2.38×10^{-6}	3
Europa	3.259×10^{-6}	7.87×10^{-11}	9.75×10^{-8}	5.03×10^{-9}	6.23×10^{-6}	35.5
Io	6.542×10^{-6}	2.97×10^{-9}	2.59×10^{-6}	4.44×10^{-7}	3.88×10^{-4}	17.5
Titan	7.259×10^{-7}	4.43×10^{-12}	1.16×10^{-8}	1.04×10^{-10}	2.72×10^{-7}	160
Enceladus	8.436×10^{-6}	4.42×10^{-10}	3.40×10^{-7}	1.18×10^{-8}	9.09×10^{-6}	14
Triton	1.969×10^{-6}	7.40×10^{-15}	1.18×10^{-11}	2.47×10^{-13}	3.95×10^{-10}	60
Moon	4.209×10^{-7}	1.11×10^{-14}	3.80×10^{-11}	1.02×10^{-13}	3.51×10^{-10}	275
Mars	1.128×10^{-5}	4.00×10^{-14}	6.89×10^{-10}	1.92×10^{-11}	3.31×10^{-8}	687

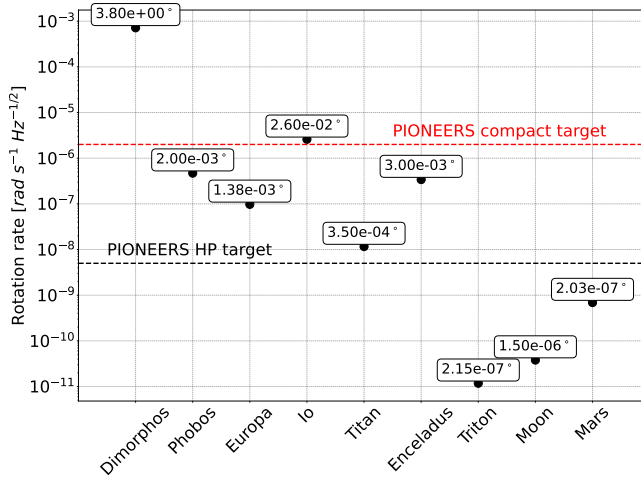
5

Table 5.5.: Required measurement precisions in rotation rate ($p_{\dot{W}}$) and centrifugal acceleration (p_{a_c}) for the Uranian satellites, along with the associated instrument requirements (ASD values). The mission duration corresponds to 10 times the orbital period.

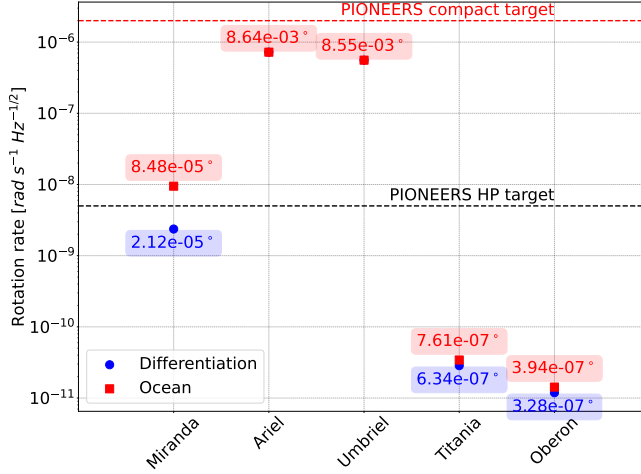
Body	Libration frequency [Hz]	Target	$p_{\dot{W}}$ [rads ⁻¹]	$p_{\dot{W}_{ASD}}$ [rad/s/√Hz]	p_{a_c} [ms ⁻²]	$p_{a_{cASD}}$ [m/s ² /√Hz]	Mission duration [days]
Miranda	8.19×10^{-6}	Differentiation	3.03×10^{-12}	2.37×10^{-9}	7.35×10^{-11}	5.75×10^{-8}	14.1
		Ocean	1.21×10^{-11}	9.47×10^{-9}	2.94×10^{-10}	2.30×10^{-7}	
Ariel	4.59×10^{-6}	Differentiation	6.92×10^{-10}	7.22×10^{-7}	2.31×10^{-11}	2.41×10^{-8}	25.2
		Ocean	6.92×10^{-10}	7.22×10^{-7}	2.31×10^{-11}	2.41×10^{-8}	
Umbriel	2.79×10^{-6}	Differentiation	4.17×10^{-10}	5.58×10^{-7}	8.55×10^{-12}	1.14×10^{-8}	41.4
		Ocean	4.17×10^{-10}	5.58×10^{-7}	8.55×10^{-12}	1.14×10^{-8}	
Titania	1.33×10^{-6}	Differentiation	1.47×10^{-14}	2.85×10^{-11}	1.94×10^{-13}	3.76×10^{-10}	87.1
		Ocean	1.76×10^{-14}	3.42×10^{-11}	2.33×10^{-13}	4.51×10^{-10}	
Oberon	8.60×10^{-7}	Differentiation	4.93×10^{-15}	1.19×10^{-11}	4.05×10^{-14}	9.77×10^{-11}	134.6
		Ocean	5.91×10^{-15}	1.43×10^{-11}	4.86×10^{-14}	1.17×10^{-10}	

method differs for the two groups: for the representative set these values were obtained by setting $\kappa = 0.1$ in Eqs. (5.10) to (5.12) to ensure the measurement precision is an order of magnitude better than the signal variation, whereas for the Uranian satellites, the precision is set equal to the required signal amplitude itself (e.g., $g_{L_{req}}$). Assuming a mission duration t_m 10 times longer than the orbital period of the body, we then compute the ASD of these targeted precisions using Eq. (5.14) in order to convert them into instrument requirements. For Mars, we set a mission duration equal to one orbital period (i.e. one martian year) in order to keep reasonable mission duration.

In Figs. 5.3 to 5.5, these instrument requirements are superimposed with the self-noise levels of the PIONEERS instrument, which are defined for $f > 0.001$ Hz [9]:

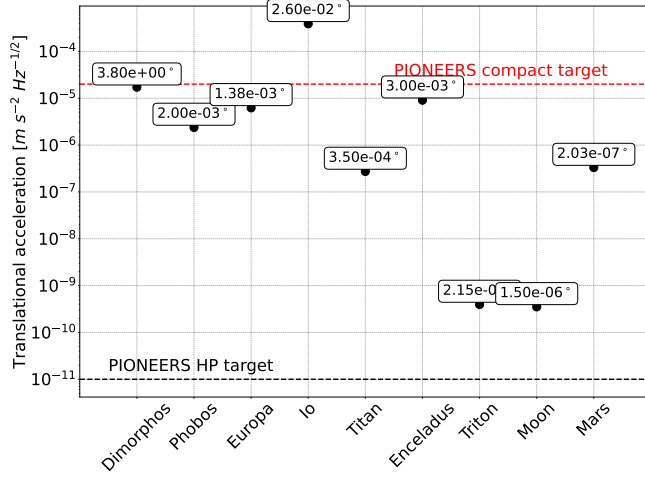


(a)

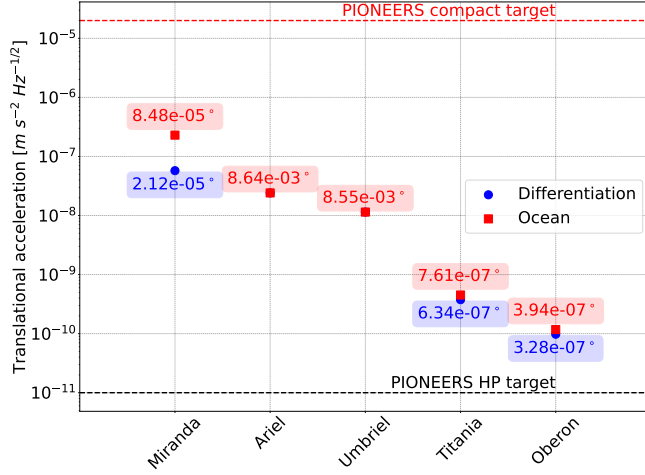


(b)

Figure 5.3.: Required precision in rotation rate measurements, assuming a mission duration of 10 times the libration period for each body (except for Mars, where it is one Martian year). The dashed red and black lines represent the self-noise of the compact and high-performance (HP) PIONEERS instruments, respectively. Panel (a) shows the requirements for the representative set of bodies, with labels indicating the target libration amplitude accuracies, calculated as $\kappa[g_{L_{max}} - g_{L_{min}}]$ with $\kappa = 0.1$. Panel (b) shows the specific measurement requirements for the Uranian satellites needed to confirm a differentiated interior and one with a subsurface ocean. The labels in this panel represent the targeted precision for the libration amplitude from Table 5.3; for Ariel and Umbriel, only one target value is shown since the requirements for differentiation and ocean detection are equal.



(a)



(b)

Figure 5.4.: Required precision in translational acceleration measurements, assuming a mission duration of 10 times the libration period for each body (except for Mars, where it is one Martian year). The dashed red and black lines represent the self-noise of the compact and high-performance (HP) PIONEERS instruments, respectively. The accelerations are evaluated at the equator ($\phi = 0$), using the radii from Table 5.1. Panel (a) shows the requirements for the representative set of bodies, with labels indicating the target libration amplitude accuracies, calculated as $\kappa[g_{L_{max}} - g_{L_{min}}]$ with $\kappa = 0.1$. Panel (b) shows the specific measurement requirements for the Uranian satellites needed to confirm a differentiated interior and one with a subsurface ocean. The labels in this panel represent the targeted precision for the libration amplitude from Table 5.3; for Ariel and Umbriel, only one target value is shown since the requirements for differentiation and ocean detection are equal.

Table 5.6.: Required measurement precisions in the tidal acceleration along with the associated 6DoF instrument requirements corresponding to the ASD of these precision. The mission duration used to compute the requirements (i.e. ASD quantities) corresponds to 10 times the tidal period except for Mars for which we set to be equal to one martian year.

Body	Tidal frequency [Hz]	p_{a_T} [ms ⁻²]	$p_{a_{T,ASD}}$ [m/s ² /√Hz]	Mission duration [d]
Dimorphos	2.330×10^{-5}	2.82×10^{-13}	1.31×10^{-10}	5
Phobos	3.628×10^{-5}	3.96×10^{-10}	1.47×10^{-7}	3
Europa	3.259×10^{-6}	1.41×10^{-7}	1.75×10^{-4}	35
Io	6.542×10^{-6}	1.45×10^{-5}	1.27×10^{-2}	18
Titan	7.258×10^{-7}	1.87×10^{-7}	4.90×10^{-4}	160
Enceladus	8.435×10^{-6}	1.62×10^{-7}	1.25×10^{-4}	14
Triton	1.969×10^{-6}	1.20×10^{-9}	1.91×10^{-6}	60
Moon	4.236×10^{-7}	3.21×10^{-10}	1.10×10^{-6}	275
Mars	1.685×10^{-8}	3.31×10^{-11}	3.59×10^{-7}	687

Table 5.7.: Required measurement precisions in tidal acceleration (p_{a_T}) for the Uranian satellites, along with the associated instrument requirements (ASD values) and mission duration.

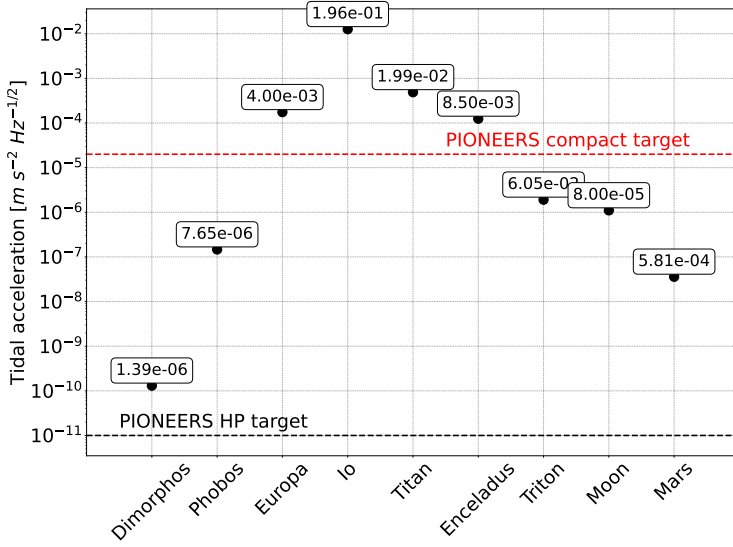
Body	Tidal frequency [Hz]	Target	p_{a_T} [ms ⁻²]	$p_{a_{T,ASD}}$ [m/s ² /√Hz]	Mission duration [days]
Miranda	8.19×10^{-6}	Differentiation	1.62×10^{-10}	1.27×10^{-7}	14.1
		Ocean (cons.)	1.62×10^{-10}	1.27×10^{-7}	
		Ocean (opt.)	1.54×10^{-9}	1.20×10^{-6}	
Ariel	4.59×10^{-6}	Differentiation	1.85×10^{-12}	1.93×10^{-9}	25.2
		Ocean (cons.)	1.79×10^{-12}	1.87×10^{-9}	
		Ocean (opt.)	1.01×10^{-11}	1.05×10^{-8}	
Umbriel	2.79×10^{-6}	Differentiation	2.24×10^{-12}	3.00×10^{-9}	41.4
		Ocean (cons.)	1.96×10^{-12}	2.63×10^{-9}	
		Ocean (opt.)	1.16×10^{-11}	1.55×10^{-8}	
Titania	1.33×10^{-6}	Differentiation	4.66×10^{-10}	9.03×10^{-7}	87.1
		Ocean (cons.)	4.05×10^{-10}	7.86×10^{-7}	
		Ocean (opt.)	2.12×10^{-9}	4.11×10^{-6}	
Oberon	8.60×10^{-7}	Differentiation	2.24×10^{-10}	5.39×10^{-7}	134.6
		Ocean (cons.)	1.93×10^{-10}	4.65×10^{-7}	
		Ocean (opt.)	1.07×10^{-9}	2.58×10^{-6}	

- rotation rate measurements: $2\mu\text{rad/s}/\sqrt{\text{Hz}}$ for the compact version and $5\text{ nrad/s}/\sqrt{\text{Hz}}$ for the high-performance prototype,
- translational (centrifugal and tidal) acceleration measurements: $20\mu\text{m/s}^2/\sqrt{\text{Hz}}$ for the compact version and $10\text{ pm/s}^2/\sqrt{\text{Hz}}$ for the high-performance prototype.

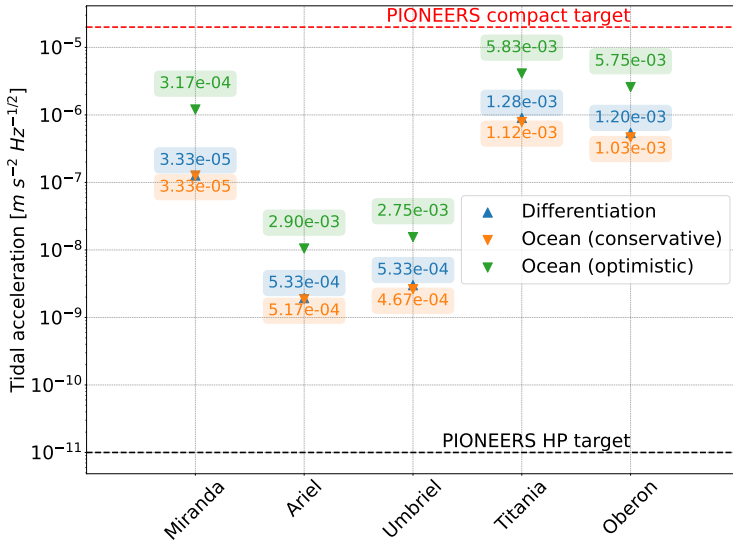
We assume that these noise levels are constant below 0.001 Hz i.e. for libration and tidal frequencies (see [Tables 5.4](#) and [5.6](#)). In fact, thermal variations will introduce a noise at the same frequencies as libration and tides, with amplitudes depending on the environment and on the system in which the sensor is integrated. However, we neglect such thermal noise because it will most probably be shifted in phase with respect to the target signals and can be calibrated using a model derived from ground measurements.

Under the above assumptions (i.e. $\kappa = 0.1$, $t_m = 10 * T_{orb}$ and the noise levels), one can conclude that the compact version of the 6DoF sensor would only be suitable for measuring the rotation rate of Dimorphos and Io ([Fig. 5.3a](#)), while the high-performance (HP) version of the instrument would be good enough to make use of rotation rate measurements also for Phobos, Europa, Titan and Enceladus. For Triton, Mars and the Moon, an even higher-performance sensor would be needed to measure the rotation rate with enough precision to gain information about their interior. For the Uranian satellites ([Fig. 5.3b](#)), none of the rotation rate requirements can be met by the compact version. The HP instrument, however, would be precise enough to measure the libration signals of Miranda, Ariel, and Umbriel (for both ocean and differentiation targets), but not those of Titania and Oberon. Regarding acceleration, one can see in [Fig. 5.4a](#) that the HP version would allow to constrain the interior of the nine bodies by measuring the centrifugal acceleration on their surface and that the compact version would still be suitable for a mission to Io (For Europa and Io in particular, the radiation environment might place enormous challenges on mission and sensors. We assume here that our sensor will be well protected from such aggressive radiations). Similarly, tidal accelerations ([Fig. 5.5a](#)) can be measured with the HP sensor with sufficient accuracy for all selected bodies while the less precise compact model would not meet the required accuracy for Dimorphos, Phobos, Triton, the Moon and Mars. For the Uranian satellites, the compact sensor is not sufficient for any acceleration measurements. With the HP sensor, however, both the centrifugal acceleration requirements ([Fig. 5.5b](#)) and the tidal acceleration requirements ([Fig. 5.5b](#)) can be met for all moons and for both differentiation and ocean targets.

It is finally interesting to note that combining the three observables allows the compact version of the PIONEERS instrument to constrain the interior properties of all the bodies considered in this study except Phobos, Triton, the Moon and Mars. Moreover, as demonstrate for the Uranian satellites in [Chapter 4](#), such a combined analysis would certainly be very powerful to infer the bodies' interior due to the complementarity of these observables (see also Van Hoolst *et al.* [36] for an example of this complementarity for Io). The aforementioned analysis also demonstrated that measuring the k_2 Love number via radiometric tracking is challenging for these moons, and that determining the libration amplitude requires demanding



(a)



(b)

Figure 5.5.: Required precision in tidal acceleration measurements, assuming a mission duration of 10 times the tidal period for each body (except for Mars, where it is one Martian year). The dashed red and black lines represent the self-noise of the compact and high-performance (HP) PIONEERS instruments, respectively. Panel (a) shows the requirements for the representative set of bodies, with labels indicating the target tidal gravimeter factor accuracies, calculated as $\kappa[\delta_{max} - \delta_{min}]$ with $\kappa = 0.1$. Panel (b) shows the specific measurement requirements for the Uranian satellites, with different markers for detecting a differentiated interior versus a subsurface ocean under conservative and optimistic assumptions.

operational requirements for optical measurements (see Sec. 4.4). The results of this study show that the deployment of the PIONEERS HP sensor can effectively address these issues. First, its ability to measure tidal accelerations enables the constraints of tidal Love numbers, a measurement that was previously demonstrated to be challenging with a gravity science experiment. Second, measuring the libration with a 6DoF sensor can eliminate or reduce the requirement for optical data.

The labels (framed numbers) in Figs. 5.3 to 5.5 indicate the target accuracies of the libration amplitudes, expressed in degrees, and that of the tidal gravimetric factors enabling direct comparison with other instrument performances published in recent literature. As previously mentioned, a detailed analysis of the foreseen uncertainties for a gravity science experiment at the the Uranian satellites is presented in Chapter 4. For the representative set, we compare with the performances predicted for the Radio-Science (RS) experiment of the Europa Clipper mission [66], those of a multidisciplinary experiment on board the Hera mission to the Didymos system [54], and those of an experiment combining RS and star-tracker measurements collected from the surface of Phobos [21]. The Europa Clipper RS experiment foresees a determination of the libration amplitude with ~ 400 m (i.e. 0.015°) uncertainty, and a determination of the tidal Love numbers k_2 and h_2 with uncertainties of 0.014 and 0.1, respectively [66], which translates into an uncertainty of 0.12 on δ . The former is 10 times larger than the precision needed (~ 40 m), and that we can reach with both PIONEERS models as shown in Fig. 5.3, to constrain Europa's interior from libration. The latter is two orders of magnitude higher than the 0.004 target precision for δ , easily achievable with PIONEERS. We note that a RS experiment on an orbiter mission will be more precise than a multi flyby mission as Europa Clipper. The ESA JUICE mission [67], which will be inserted in orbit around in Ganymede in 2034, is expected to measure k_2 of Ganymede at the level of 10^{-4} [68] and to reach a measurement accuracy of on h_2 of a few percent [69]. Assuming a typical value of h_2 of 1.3 and an accuracy of 2%, we determine an uncertainty in δ at the 10^{-2} . If we use this value as an order of magnitude of the expected precision of tidal measurements from a spacecraft orbiting an icy moon, we find that it will only outperform the target precision of Io by one order of magnitude. The Dimorphos libration will be estimated by Hera with a 0.02° precision [70] by combining direct-to-Earth RS data with Inter-Satellite radio links, optical data and altimetric data. This is two orders of magnitude better than what we can get from the PIONEERS compact model, but three orders of magnitude higher compared to the rotation rate measurement precision reachable with PIONEERS HP model alone (i.e. without using other instruments). No prediction for the measurement of tides by Hera has been published so far. Finally, the Japanese Martian Moons eXploration (MMX) mission will be put in orbit around Phobos in a couple of years and should determine the Phobos libration amplitude with a 0.02° accuracy [71]. This is 10 times larger than our target accuracy. RS from a lander would meet the required precision for the libration estimate, but would still be unable to measure tides with sufficient precision [21]. At Phobos, the PIONEERS compact/HP model would measure in about 3 d the libration amplitude at the $2 \times 10^{-3^\circ}$ level and the tides with 7×10^{-6} precision, while RS and star-tracker would provide libration with 0.001° uncertainty

and tides with 0.001 uncertainty after a couple of weeks of operation [21], showing the great interest of the PIONEERS instrument.

Mission duration is a key parameter in defining instrument performance. In order to properly measure periodic signals, best is to get observations over a duration t_m larger than one entire period T of the target signal, i.e. $t_m = nT$ with $n > 1$. This is why we computed the ASD of the required precisions assuming the nominal mission duration to be 10 times the period of the rotational and tidal motions. For the sake of inter-body comparison, we kept $n = 10$ for each quantity reported in Tables 5.4 and 5.6 and in Figs. 5.3 to 5.5, except for Mars for which we set $n = 1$ to avoid an unrealistically long mission (~ 20 years). This setting leads to quite stringent duration for a mission at Titan (160 d), at Triton (60 d) or at the Moon (275 d). However, one could think of reducing these missions life time (with $1 < n < 10$) given the margin offered by the instrument performances (i.e. the difference between PIONEERS's self-noise (horizontal lines) and body-dependent required accuracy (black circles) in Figs. 5.3 to 5.5). Indeed, because the needed accuracy, when expressed in ASD, depends on the mission duration according to $\sqrt{t_m/2}$, one can easily identify the best suited t_m that would make the mission as short as possible while guaranteeing the scientific objectives.

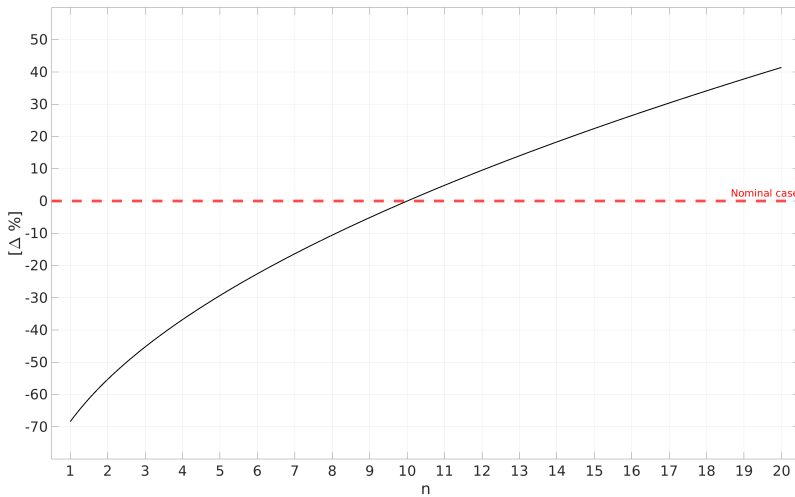


Figure 5.6.: Relative percentage difference of the required precision as a function of the number n of orbital periods during which observations are taken. The variation of the required precision is expressed relatively to the nominal case obtained with $n = 10$. The number of periods defines the mission duration as $t_m = nT_{orb}$, which is used in the ASD conversion factor $\sqrt{t_m/2}$ that is equal for every observables.

Fig. 5.6 shows how much varies the required instrument precision as a function of

Table 5.8.: Minimum number of orbital periods (n_{min}) required to reach the targeted accuracy ($\kappa = 0.1$) for the three observables, depending on the PIONEERS model (compact versus High-Performance). n_{min} is related to the mission duration by $t_{m_{min}} = n_{min} T_{orb}$. The symbol $\emptyset$ indicates that the requirement cannot be met.

Body Observables	n_{min} (Compact)			n_{min} (HP)		
	$\dot{W}$	a_c	a_T	$\dot{W}$	a_c	a_T
Dimorphos	1	$\emptyset$	$\emptyset$	1	1	1
Phobos	$\emptyset$	$\emptyset$	$\emptyset$	1	1	1
Europa	$\emptyset$	$\emptyset$	1	1	$\emptyset$	1
Io	6	$\emptyset$	1	1	1	1
Titan	$\emptyset$	$\emptyset$	1	2	$\emptyset$	1
Enceladus	$\emptyset$	$\emptyset$	1	1	1	1
Triton	$\emptyset$	$\emptyset$	1	$\emptyset$	$\emptyset$	1
Moon	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	1
Mars	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	1

the number n of orbital periods covered by the mission with respect to the nominal case of $n = 10$. For instance, if the mission life time is reduced to its minimum (i.e. $n = 1$), then the precision to achieve is $\sim 70\%$ less stringent than if we were taking measurements over 10 periods ($n = 10$) as considered above. This approach reduces instrument-related challenges and significantly eases lifetime-related constraints in mission design. However, it comes at the cost of only being able to observe for a single orbital period. Generally speaking, Fig. 5.6 shows us that the longer is the mission relatively to the orbital period of the celestial body, the more demanding is the instrument precision needed to probe the interior of a celestial object (note that the three observables will be affected the same way by the mission duration since t_m appears only in the ASD conversion factor). There is a balance between the instrument performance and the mission duration that we can tune to optimize the mission scenario given by the margin of the body-dependent required accuracy w.r.t. the instrument self noise as explained above. For instance in the case of the Moon, the usage of the HP sensor permits to reduce the mission duration to one orbital period since it would measure tidal accelerations with the required precision already after 27 days. Similarly, one could reduce the problematic duration of a mission to Titan or Triton down to one orbital period (i.e. 16 days and 6 days, respectively), and still get information about their interior from tides measurements using the compact sensor or the HP sensor. For Titan, it is also possible to use the HP sensor measuring rotation rate if the mission duration is at least twice the orbital period). For Io, instead, six orbital period is the minimum if we want to infer interior proprieties from rotation rate measurements using the compact model. A similar analysis applied to the Uranian satellites confirms the HP instrument is generally required. The inner moons Ariel and Umbriel could have all science goals met by the HP sensor using any of the three observables within a single orbital period ($n_{min} = 1$). Miranda's acceleration goals are also met in one period with the

HP sensor, but its rotation rate measurements require three periods. The outer moons Titania and Oberon have rotation rate signals too small to be measured even with the HP sensor; however, their interiors can be constrained with acceleration measurements from the HP sensor in a single period. Notably, for these two outer moons, the compact sensor could meet the tidal acceleration requirements in a single period, offering flexibility in mission design. In Tables 5.8 and 5.9, one can see the minimum number of orbital periods (i.e. minimum mission duration) necessary to gain knowledge about the bodies' interior depending on the PIONEERS model used.

Table 5.9.: Minimum number of orbital periods (n_{min}) required to reach the targeted accuracy for the three observables indicated in Table 5.3, depending on the PIONEERS model (Compact versus High-Performance). n_{min} is related to the mission duration by $t_{m_{min}} = n_{min} T_{orb}$. The symbol $\emptyset$ indicates that the requirement cannot be met.

Body	Target Observable	n_{min} (Compact)			n_{min} (HP)		
		$\dot{W}$	a_c	a_T	$\dot{W}$	a_c	a_T
Miranda	Differentiation	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	1	1
	Ocean (cons.)	$\emptyset$	$\emptyset$	$\emptyset$	3	1	1
	Ocean (opt.)			$\emptyset$			1
Ariel	Differentiation	$\emptyset$	$\emptyset$	$\emptyset$	1	1	1
	Ocean (cons.)	$\emptyset$	$\emptyset$	$\emptyset$	1	1	1
	Ocean (opt.)			$\emptyset$			1
Umbriel	Differentiation	$\emptyset$	$\emptyset$	$\emptyset$	1	1	1
	Ocean (cons.)			$\emptyset$			1
	Ocean (opt.)	$\emptyset$	$\emptyset$	$\emptyset$	1	1	1
Titania	Differentiation	$\emptyset$	$\emptyset$	1	$\emptyset$	1	1
	Ocean (cons.)			1		1	1
	Ocean (opt.)	$\emptyset$	$\emptyset$	1	$\emptyset$	1	1
Oberon	Differentiation	$\emptyset$	$\emptyset$	1	$\emptyset$	1	1
	Ocean (cons.)			1		1	1
	Ocean (opt.)	$\emptyset$	$\emptyset$	1	$\emptyset$	1	1

5.5.1. SENSITIVITY ANALYSIS AND LEVELS OF PRECISION

The geophysical signal in the 6DoF observables could be hidden by errors in other quantities defining them, namely $\dot{W}_0$, Ω_L , r and ϕ for the rotation and of G , M_p , r , Γ and d for the tides (see Eqs. (5.5), (5.8) and (5.9)). The time tag errors, which affect all types of data, must also be maintained (by design) smaller than the interior signal. For the real data analysis, all these errors will have to be taken into account when inferring interior properties from the observables. A per-body

rough estimation of these levels can be quantified by computing the sensitivity of the 6DoF observables to the different parameters defining them and compare that of the target-interior parameters (i.e. g_L and δ) with that of the uncertainties/errors in all the other parameters entering in Eqs. (5.5) and (5.9). The relative contribution of each parameter to the 6DoF observables are given below at first order in the Δ quantities:

$$\frac{\Delta \dot{W}}{\dot{W}} \approx \frac{\Delta g_L}{g_L} + \frac{\Delta \dot{W}_0}{\dot{W}_0} - t^2 \Omega_L \Delta \Omega_L - \Omega_L^2 t \Delta t, \quad (5.18)$$

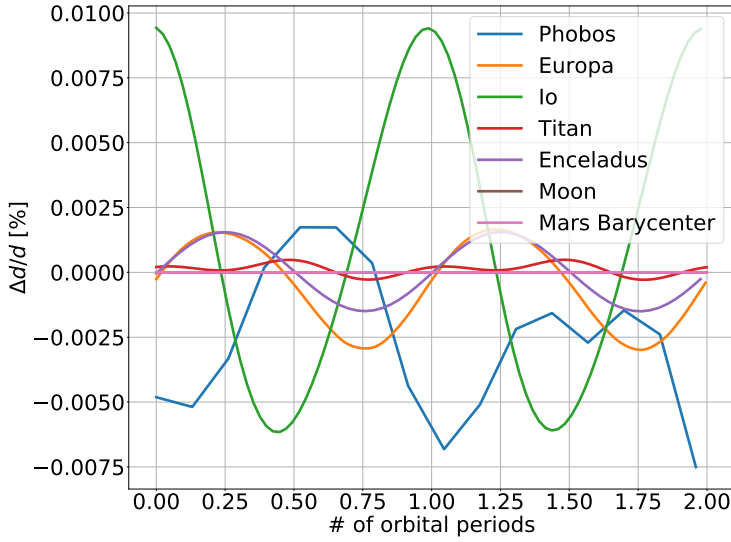
$$\frac{\Delta a_c}{a_c} \approx \frac{2\Delta g_L}{g_L} + \frac{2\Delta \dot{W}_0}{\dot{W}_0} - 2t^2 \Omega_L \Delta \Omega_L - 2\Omega_L^2 t \Delta t + \frac{\Delta r}{r} - \tan(\phi) \Delta \phi, \quad (5.19)$$

$$\frac{\Delta a_T}{a_T} \approx \frac{\Delta \delta}{\delta} + \frac{\Delta M_p}{M_p} + \frac{\Delta r}{r} - \frac{3\Delta d}{d} + \frac{\Delta \Gamma}{\Gamma}. \quad (5.20)$$

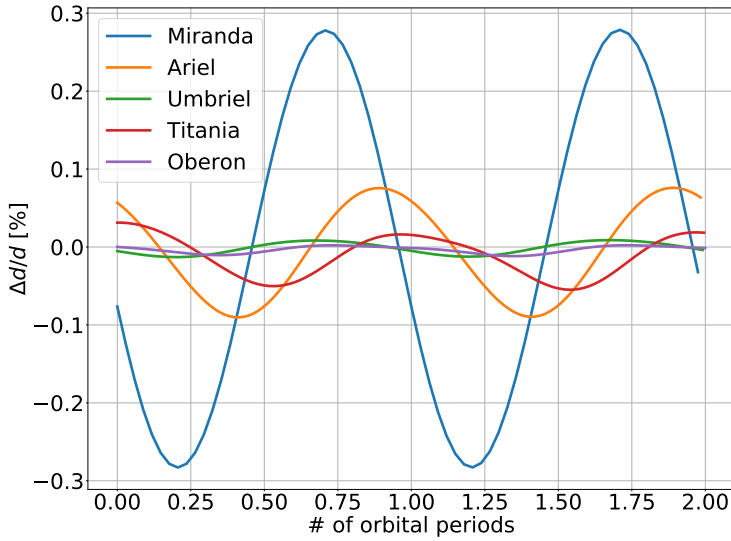
Table 5.10.: $\Delta = 1\sigma$ Uncertainties in bodies' mass and radius from [Jet Propulsion Laboratory - Solar System Dynamics website](#). Values of the Didymos system are from [42].

Body	$\Delta r [km]$	$\Delta M_p [10^{24} kg]$
Dimorphos	0.004	Didymos: 5e-15
Phobos	0.04	Mars: 0.000030
Europa	0.30	Jupiter: 0.088
Io	0.50	— " —
Titan	0.02	Saturn: 0.026
Enceladus	0.20	— " —
Miranda	0.7	Uranus: 0.0040
Ariel	0.6	— " —
Umbriel	2.8	— " —
Titania	1.8	— " —
Oberon	2.6	— " —
Triton	2.40	Neptune: 0.0048
Moon	0.1	Earth: 0.00028
Mars	0.2	Sun: 7e-11

To evaluate each term of the above equation, we use numerical values reported in [Table 5.1](#) for M_p and reference radii r , and the numerical values of their associated variations are taken from [Table 5.10](#), assuming $\Delta = 1\sigma$ of the considered parameter. For the Uranian satellites, the required relative uncertainties for the geophysical parameters of interest, $\Delta g_L/g_L$ and $\Delta \delta/\delta$, must be established. Based on recent interior structure modeling, the relative uncertainty in libration amplitude ($\Delta g_L/g_L$) is estimated by comparing the required precision to distinguish interior models with the expected libration signal calculated by Baland *et al.* [72]. To distinguish a differentiated solid body from a homogeneous one, the required relative uncertainty is substantial, ranging from approximately 26 % for Miranda to 40-50 % for the other four moons, based on the required precisions stated in the text and the reference signals from Table 5 of Baland *et al.* [72].

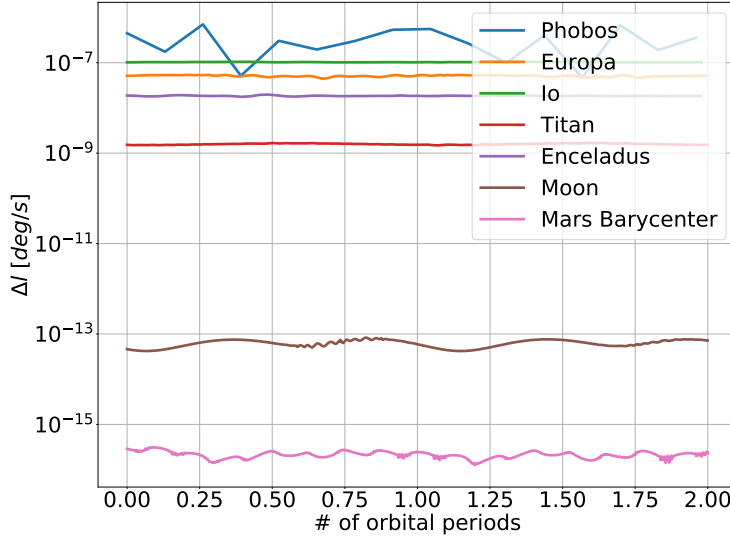


(a)

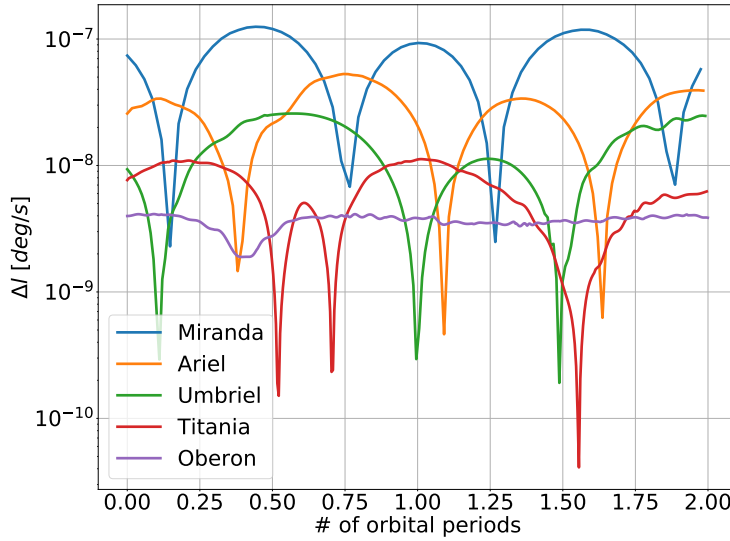


(b)

Figure 5.7.: Estimated relative uncertainty ($\Delta d/d$) in the mean orbital distance (d) for the selected celestial bodies, used in the sensitivity analysis. Panel (a) shows the representative set of bodies, and Panel (b) shows the Uranian satellites. These uncertainties are estimated by comparing recent ephemerides from JPL and IMCCE to assess the current knowledge level, which contributes to the error budget when measuring tidal accelerations.



(a)



(b)

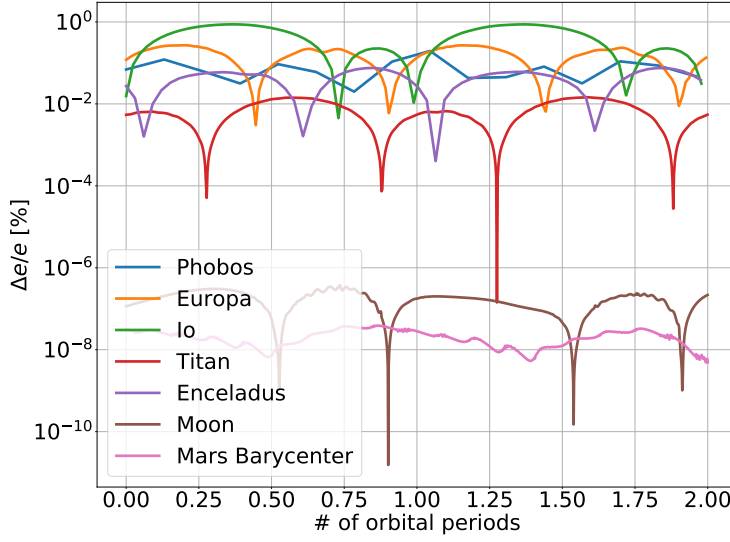
Figure 5.8.: Estimated absolute uncertainty (Δl) in the mean motion (l), equivalent to the orbital frequency, for the selected celestial bodies. For synchronously rotating bodies, this also represents the uncertainty in the mean rotation rate ($\dot{W}_0$) and the primary libration/tidal frequency (Ω_L). Panel (a) shows the representative set, and Panel (b) shows the Uranian satellites. These uncertainties, derived from ephemeris comparison, contribute to the error budget for rotation rate and centrifugal acceleration measurements in the sensitivity analysis.

The requirement for detecting a subsurface ocean is more stringent; using the necessary precision from the text and the expected signals from Figures 6 and 8 of Baland *et al.* [72], the necessary relative uncertainty ranges from approximately 5 % for Miranda to about 17-23 % for the larger satellites. For the tidal signal, the relative uncertainty in the gravimeter factor $\Delta\delta/\delta$ is estimated using the target values from 5.3 and the assumption that $h_2 = 5/3k_2$, with the reference k_2 values extrapolated from Figure 5 of Baland *et al.* [72]. This analysis shows that the tidal requirements are much stricter, with a relative uncertainty on the order of 0.003-0.13 % to detect differentiation or a conservative ocean, and 0.27-0.58 % to detect an optimistic ocean. To encompass all science targets in the following analysis, conservative relative uncertainties for libration are set to approximately 26 % for Miranda and 40-50 % for the other moons, while for the gravimeter factor, the requirements are set to 0.003 % for Miranda and range from 0.3 % to 0.6 % for the larger satellites.

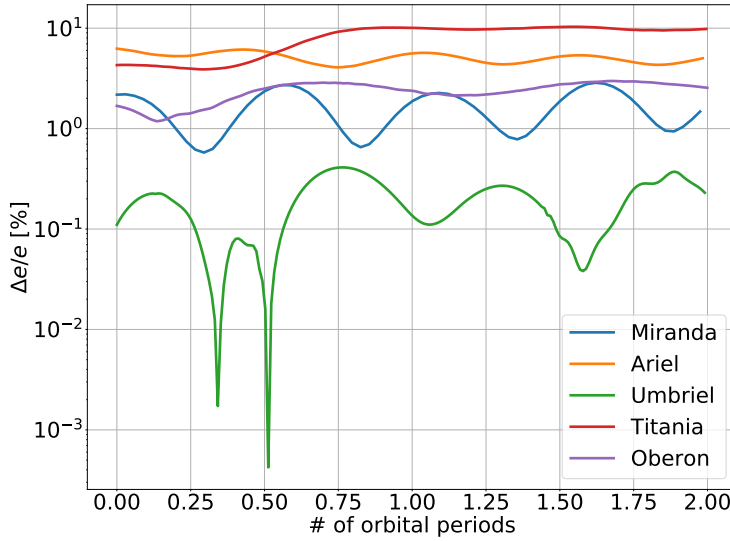
Table 5.11.: Relative contributions to PIONEERS observables.

Body	$\Delta g_L/g_L$	$\Delta \dot{W}_0/\dot{W}_0$	$\Delta r/r$	$\Delta M_p/M_p$	$\Delta d/d$	$6\Delta e/e$	$\Delta\delta/\delta$
Dimorphos	1.4	0.048	5.298	0.07	0.1	~ 0	1.38e-5
Phobos	0.018	0.008	0.361	4.7e-7	0.004	0.452	7.65e-7
Europa	0.44	1e-5	0.019	6.6e-6	0.002	0.923	0.02
Io	2.00	4e-4	0.027	— " —	0.007	2.610	0.98
Titan	0.6	2e-5	0.001	1.9e-6	5e-4	0.042	0.11
Enceladus	0.25	3e-4	0.079	— " —	0.002	0.252	0.0078
Miranda	0.26	2e-5	0.003	4.6e-5	2e-5	0.1	3e-5
Ariel	0.5	1e-5	0.001	— " —	5e-5	0.3	0.006
Umbriel	— " —	1e-5	0.005	— " —	1e-5	0.01	— " —
Titania	— " —	1e-5	0.02	— " —	1e-4	0.5	— " —
Oberon	— " —	1e-5	0.03	— " —	4e-5	0.1	— " —
Triton	0.6	0.14	0.177	1.4e-4	0.009	0.1	0.43
Moon	0.003	~ 0	0.006	2e-7	~ 0	~ 0	8.0e-4
Mars	1.00	~ 0	0.006	7.5e-9	~ 0	~ 0	7.9e-3

Except for Mars, all the bodies considered in this study are in 1:1 spin-orbit resonance, so we have $\dot{W}_0 = \Omega_L = 2\pi/T_{orb} = l$, with l the mean motion. $\Gamma = 3/2$ for fast rotators like Mars, but it becomes variable for synchronous bodies for which $\Gamma = 6e$ and $\Delta\Gamma/\Gamma = 6\Delta e/e$. Therefore, $\dot{W}_0$, Γ and d are all related to the orbit of the bodies. We thus infer the order of magnitude of $\Delta\dot{W}_0$, Δd and Δe for each body from the accuracy of their orbit, which is taken from literature and/or quantified here by comparing two recent ephemeris, one from the IMCCE team and the other from JPL, when both are available. Figs. 5.7 to 5.9 show $\Delta d/d$, Δl and $\Delta e/e$ for each body, respectively. For Triton, there are no ephemeris from IMCCE, so we use the same error as Io for d (i.e. $\Delta d \sim 30$ km) and conservative errors of 0.01 % for its rotation rate and of 0.1 % for its orbital eccentricity. Table 5.11 provides per observable and in percents, the relative contributions/errors of each quantity affecting the 6DoF measurements. Uncertainties in the lander position are usually very small. Fig. 5.10 shows how an error in the lander latitude propagates to the centrifugal acceleration.



(a)



(b)

Figure 5.9.: Estimated relative uncertainty ($\Delta e/e$) in the orbital eccentricity (e) for the selected celestial bodies. Panel (a) shows the representative set, and Panel (b) shows the Uranian satellites. Orbital eccentricity influences the amplitude of tidal forces and librations, particularly for synchronous bodies. These uncertainties, derived from ephemeris comparison, contribute to the error budget in the sensitivity analysis.

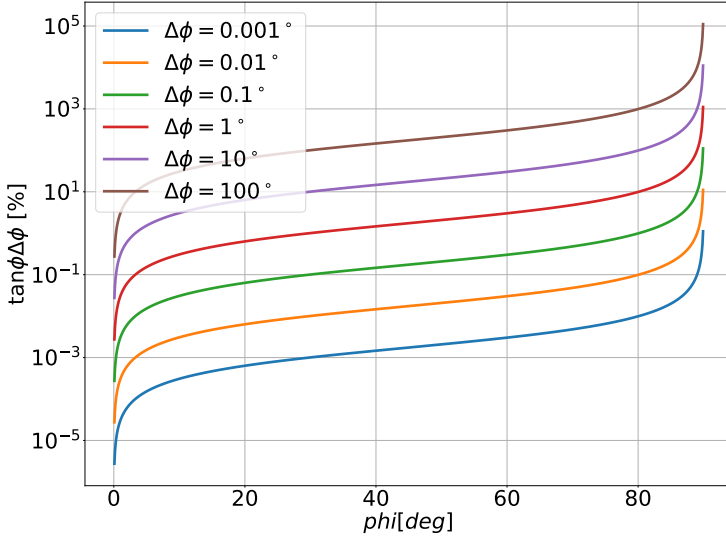


Figure 5.10.: Sensitivity analysis showing the relative contribution of an error in the lander's geodetic latitude ($\Delta\phi$) to the measured centrifugal acceleration (a_c). The y-axis represents the term $\tan\phi\Delta\phi$ from [Sec. 5.5.1](#), indicating the fractional error in a_c caused by a given latitude uncertainty $\Delta\phi$ (in degrees) as a function of the true latitude (ϕ). This analysis helps assess the required lander position knowledge, showing errors are negligible near the equator but become significant at high latitudes.

We see that for low latitude landers, position errors are negligible and that an error lower than 0.001° in the lander latitude should be targeted for high latitude landers. Note that we considered equatorial landers in the chapter, so position error has no influence on a_c in our study. The reference radius, r , is the limiting factor which can surpass the target signal if the topography of the body is poorly known. In particular, current knowledge on Dimorphos shape could prevent us from properly retrieve the libration amplitude from centrifugal acceleration and the tidal gravimeter factor from the tidal acceleration. Nevertheless, the periodic signature of the libration signal in a_c could still be detectable and if not, then one could still determine librations from the rotation rate observable, which is not depending on r . The same reasoning applies to Phobos and the Moon, for which the signature of Δr in a_c is slightly larger than the signature of the interior in the libration amplitudes (see Table 5.11). Note that for Dimorphos, the large uncertainty in r will be reduced by the forthcoming Hera mission. For the Uranian satellites, the uncertainty in the radius is also a limiting factor, with its contribution to the error budget being comparable to or exceeding the target geophysical signal for both centrifugal and tidal accelerations. However, it is expected that the forthcoming UOP mission will reduce this uncertainty. The determination of the tidal gravimeter factor, δ , is the most challenging. It only exceeds the noise coming from the uncertainties in the other parameters for Europa, Io, Titan, Triton and Mars. For the Uranian satellites, the very small target signal for δ is consistently smaller than the errors introduced by uncertainties in the mass of Uranus ($\Delta M_p/M_p \approx 4.6 \times 10^{-5}\%$) and the satellite's radius ($\Delta r/r \approx 0.1 - 0.5\%$), making its detection difficult without improved knowledge of these parameters.

Table 5.12.: Required data timing accuracy to measure libration amplitudes.

Body	Δt [s]
Dimorphos	4.775e-2
Phobos	9.154e-4
Europa	1.992e-3
Io	1.799e-2
Titan	6.238e-4
Enceladus	2.900e-3
Miranda	2.928e-3
Ariel	3.157e-3
Umbriel	1.920e-3
Titania	9.141e-4
Oberon	5.911e-4
Triton	1.629e-3
Moon	1.866e-6
Mars	1.551e-2

An error in the data timing can also hide the target signal. To avoid this, we need the onboard computer to time tag the 6DoF observations with a precision that induces relative errors smaller than $\Delta g_L/g_L$ and smaller than $\Delta \delta/\delta$. As timing

errors multiplied by t in the expressions for the total error on the rotation rate and centrifugal acceleration (see Sec. 5.5.1), we compute Δt when maximum, i.e. at $t = t_m = nT_{orb}$ with $n = 10$ (except for Mars where n is set to 1 as done in the rest of the calculations of the chapter), such that

$$\Delta t < \frac{1}{20\pi\Omega_L} \frac{\Delta g_L}{g_L}, \quad (5.21)$$

and $\Delta t_{Mars} = 10\Delta t$ for the rotation rate observable. For centrifugal acceleration the time tag requirements reported in Table 5.12 must be divided by 2. For tidal acceleration measurements, the timing error is of the order of that of $\Delta d/d$.

5.6. CONCLUSIONS

A 6DoF sensor has recently been proposed to improve the science return of future planetary missions by accurately measuring centrifugal and tidal accelerations and rotation rate experienced by a spacecraft landed on a celestial body [9]. Such an instrument has being prototyped by the iXblue French company within the PIONEERS framework, an Horizon 2020 research and innovation program of the European Commission. The instrument consists in a combination of Fiber-Optic Gyroscopes (FOGs), micro-electro-mechanical systems (MEMS) and a very-broad-band seismometer (VBB), which makes it perfectly well suited to measure the rotation, tides and quakes of a celestial object.

The present study provides details regarding the determination of the rotation and tides of the five bodies considered by Bernauer *et al.* [9], namely Dimorphos, Phobos, Europa, the Moon and Mars, along with four additional bodies also identified as probable target for future missions, namely Io, Titan, Enceladus and Triton and the major Uranian satellites, Miranda, Ariel, Umbriel, Titania and Oberon. For all these bodies, the level of precision in the rotation rate and centrifugal and tidal accelerations necessary to improve our knowledge of their interior structure (or atmosphere dynamics for Mars) has been identified and converted into requirements for a 6DoF sensor. We then discussed the relevance of deploying a 6DoF sensor on each of these bodies, by comparing the required measurement accuracy and the targeted self-noise of the PIONEERS instrument. The former depends on the mission duration because it has to be expressed in Amplitude Spectral Density (ASD) to be compared to the latter. Following Bernauer *et al.* [9], two levels of self-noise for the instrument have been considered: that of a compact instrument capable of measuring linear acceleration spectral density at a level of $10^{-5} \text{ m/s}^2/\sqrt{\text{Hz}}$ and rotation rate spectral density at $10^{-6} \text{ rad/s}/\sqrt{\text{Hz}}$, and that of a high-performance instrument about one thousand times more accurate ($10 \text{ pm s}^{-2}\text{Hz}^{-1/2}$ and $5 \text{ nrad s}^{-1}\text{Hz}^{-1/2}$, for linear acceleration and rotation rate respectively). Assuming for each body a mission operating over 10 orbital periods (except for Mars for which we set to be equal to only one martian year), we found that Phobos, the Moon, Mars, as well as for most measurements of the Uranian satellites, could not be investigated with the PIONEERS compact model, while the others could all have their interior constrained from either the measurement of their

rotation rate and/or centrifugal acceleration and/or tidal acceleration with such a model.

We also found that the PIONEERS high-performance prototype instrument would be suitable for all bodies with regard to acceleration measurements, enabling the detection of differentiation or a subsurface ocean on all five major Uranian moons. For rotation rate, the HP sensor would be sufficient for Phobos, Europa, Titan, Enceladus, and the inner Uranian moons (Miranda, Ariel, and Umbriel), but would still not be precise enough to infer interior constraints for the most challenging targets like Triton, the Moon, Mars, and the outer Uranian moons (Titania and Oberon). Furthermore, the precision of PIONEERS' instruments is comparable or superior to the RS experiments of upcoming Phobos and Europa missions. Precise measurements of Phobos' libration amplitude such as those provided by PIONEERS, coupled with more precise measurements of the degree two gravity field from the upcoming MMX mission, can confirm the heterogeneous interior of the Martian moon, as explained by Le Maistre, Rivoldini, and Rosenblatt [73]. Accurate tidal measurements of Europa will help to determine the thickness and rheology of the ice shell, as well as the density of the ocean. This information is crucial for assessing the Jovian moon's potential for habitability [66]. The case for the Uranian system is particularly compelling; as demonstrated in [Chapter 4](#), a combined set of geodetic measurements is essential to characterize their interiors, but measuring the tidal Love number k_2 and libration is challenging with a gravity science experiment. The HP sensor can directly address these critical gaps by providing an independent measurement of tides and by measuring libration, which can ease or remove the dependency on optical data.

These examples demonstrate the relevance of libration and tidal measurements in advancing our understanding of the origins of our Solar System and the identification of potentially habitable environments within our Solar System, making this instrument worthy of consideration in a payload suite for a future mission. We finally pointed out the balance existing between the instrument performance and the mission duration: the longer the mission is, the more accurate is the instrument precision needed. Given this, we have identified for each body the shortest mission duration allowed by the instrument self-noise that would still achieve the geophysical objectives considered in this chapter.

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6

CONCLUSION

WE began this dissertation to confront a foundational challenge in planetary exploration: the frequent disconnect between a mission's engineering-driven design and the scientific questions it aims to answer. The conventional, iterative planning process, while functional, is often inefficient and does not guarantee that a final mission architecture will be optimal for scientific discovery. In response to this, we have developed and validated a "reverse mapping" framework that inverts the traditional workflow. By employing probabilistic inference to directly link desired scientific outcomes to technical requirements, we have demonstrated a more robust, efficient, and science-driven methodology for designing future missions to the outer solar system.

The key findings of our research provide a comprehensive validation of the developed tools and their direct application to the design of the Uranus Orbiter and Probe (UOP) mission. In [Chapter 3](#), we first established the robustness of our simulation software by replicating published results from the Cassini mission at Titan and assessing the scientific potential of Juno's flybys of Io. We then applied this validated, conventional workflow to a gravity science investigation for Uranus with the Uranus Orbiter Probe (UOP) mission, a prioritized NASA Flagship whose design is currently in its preliminary stages. This analysis revealed that the nominal mission tour is insufficient with standard X-band tracking and that achieving high-priority science goals will require a combination of an upgraded Ka-band radio link and a strategically optimized trajectory. These results highlighted the limitations of the traditional design process and motivated the main contribution of this thesis: the development of our "reverse mapping" framework to plan the UOP mission's investigation of the Uranian moons ([Chapter 4](#)). The resulting "lookup tables" demonstrated the methodology's two critical applications. First, by defining the required precision for geophysical parameters, they established that detecting subsurface oceans is fundamentally more challenging than constraining the ice shells. Second, our covariance analysis of potential UOP mission designs, using Ariel as a primary test case, confirmed this theoretical finding in a practical context: the simulations showed that while the mission can feasibly constrain the ice shell and rock-to-ice ratio, it is unlikely to achieve the precision required to robustly characterize a subsurface ocean. This difficulty stems directly from the challenge of measuring key parameters, such as the tidal Love number k_2 , with the limited number of flybys available in a system-wide tour. By identifying the limitations of orbital remote sensing for parameters like the tidal

Love number our work also highlighted the value of complementary, in-situ measurements (Chapter 5). The analysis showed that a high-performance 6-DoF sensor could successfully measure the required tidal accelerations for all the major Uranian satellites, providing a crucial, independent constraint and reinforcing the value of a synergistic, multi-platform approach.

The scope of this work, while comprehensive, has limitations that present opportunities for future research. The lookup tables we presented are specific to the Uranian system and a gravity science investigation; a crucial next step is to expand this framework to other targets and instrument types, such as magnetometry for ocean detection or spectroscopy for surface composition. Furthermore, the interior structure models we used were simplified for computational efficiency. Future work should incorporate more complex models with variable rheologies and thermal states to refine the fidelity of the scientific predictions. The most impactful future direction, however, would be to evolve the static lookup table concept into a dynamic, collaborative software tool. Such a platform would enable different science and engineering teams to input their expected performance in real-time and immediately see the integrated impact on the mission's ability to achieve its scientific goals, thereby fully realizing the collaborative design paradigm we have proposed.

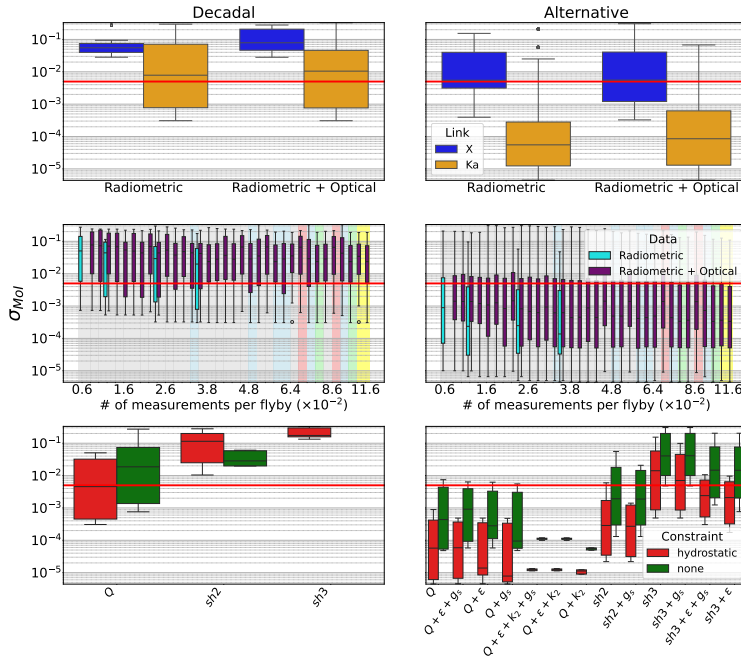
Ultimately, the exploration of ocean worlds is a critical step in our quest to understand the potential for life beyond Earth. The framework we developed in this dissertation provides a more efficient and scientifically focused method for designing the missions that will undertake this endeavor. By ensuring that a mission is designed from the ground up to answer the most fundamental scientific questions, our work offers a tangible pathway to maximizing the scientific return from future ventures into the outer solar system. We intend for this methodology not just to optimize mission design, but to accelerate discovery, bringing us closer to unveiling the secrets hidden beneath the ice of these distant and compelling worlds.



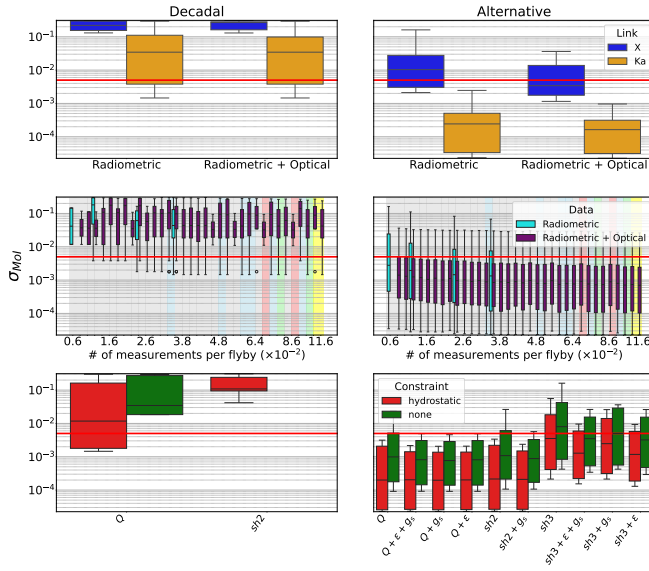
COVARIANCE ANALYSIS RESULTS FOR URANIAN SATELLITES

THIS section contains additional figures extending the covariance analysis investigation presented in [Sec. 4.4.2.3](#), which discusses the impact of measurement strategies and parameter estimation specifically for the Uranian satellite Ariel. Given the early development stage of the UOP mission and the ongoing discussions about mission design, a comprehensive exploration of various measurement scenarios and parameter combinations has been conducted. This appendix extends the analysis presented for Ariel to the other Uranian satellites, namely Miranda, Umbriel, Titania and Oberon, providing detailed figures illustrating the expected uncertainties and correlations among geophysical observables across a range of tested measurement strategies, trajectories, and parameter sets which are defined in [Sec. 4.4.1](#).

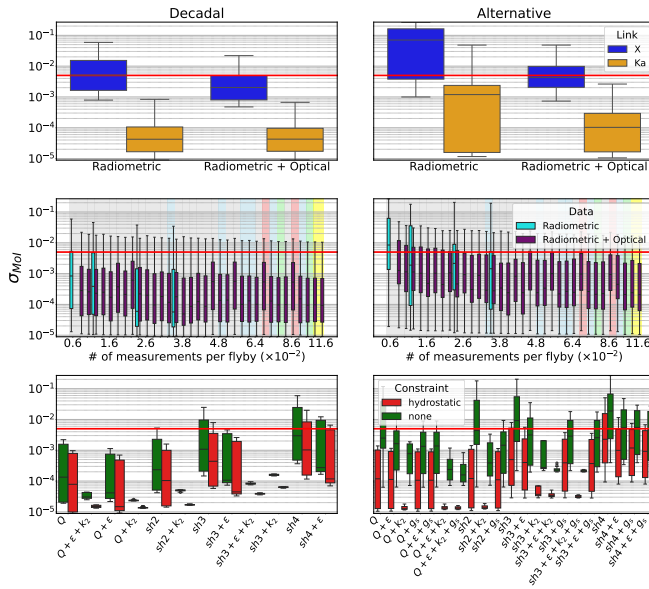
Figure A.1.: MoI computed formal errors. The box-and-whisker plot illustrates the conventional statistical measures: the lower boundary of the box represents the 25th percentile, the upper boundary the 75th percentile, and the line within the box the 50th percentile. The lower whiskers extend to a value that is 1.5 times below the 25th percentile, while the upper whiskers extend to a value that is 1.5 times above the 75th percentile. Circles represent data points that fall outside the range of the whiskers. Symbols are used to define the parameter sets. Specifically, the letter Q stands for “quadrupole”, whereas shN means that all gravity coefficients up to maximum degree and order $N \times N$ are estimated. The number of simulated measurements per flyby is equal to $2N_{pictures} * N_{landmarks} + N_{Doppler}$. Given that the incremental step is not constant, colored areas are used to define the step (grey: 0.2, blue: 0.4, orange: 0.6, green: 0.8, yellow: 1.2). The red solid and dashed lines represents the target uncertainties as outlined in Table 4.1. Empty panels mean that no solution with a formal error less than or equal to the true value was found.



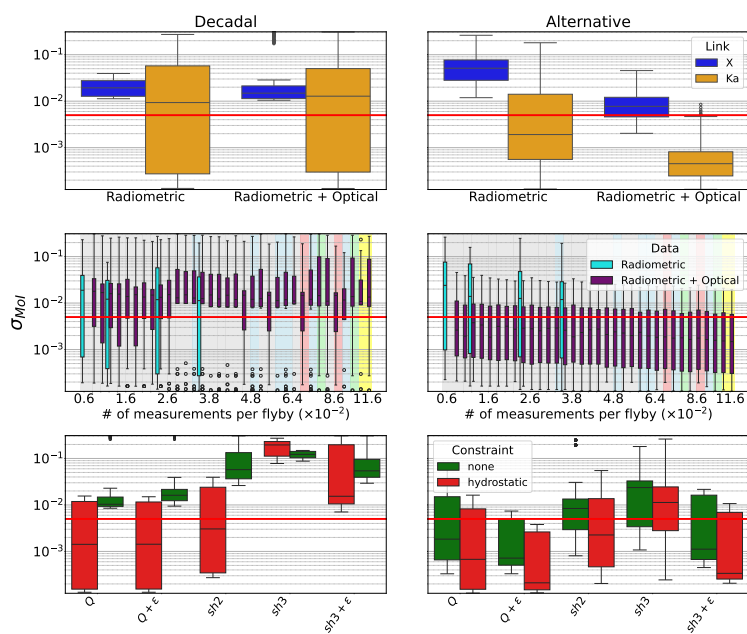
(a) Miranda.



(b) Umbriel.

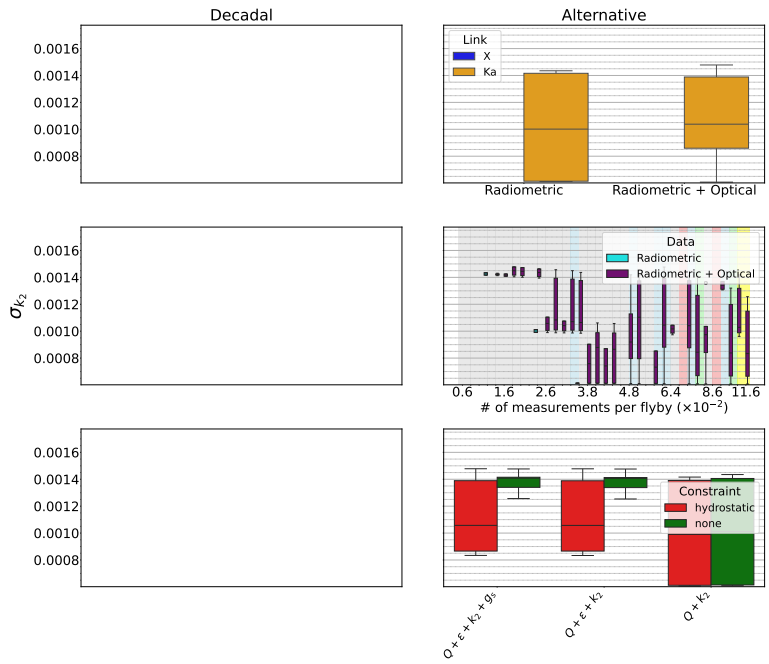


(c) Titania.

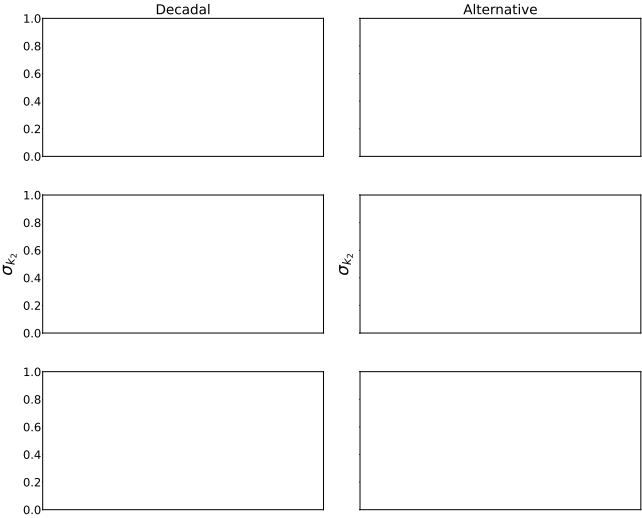


(d) Oberon.

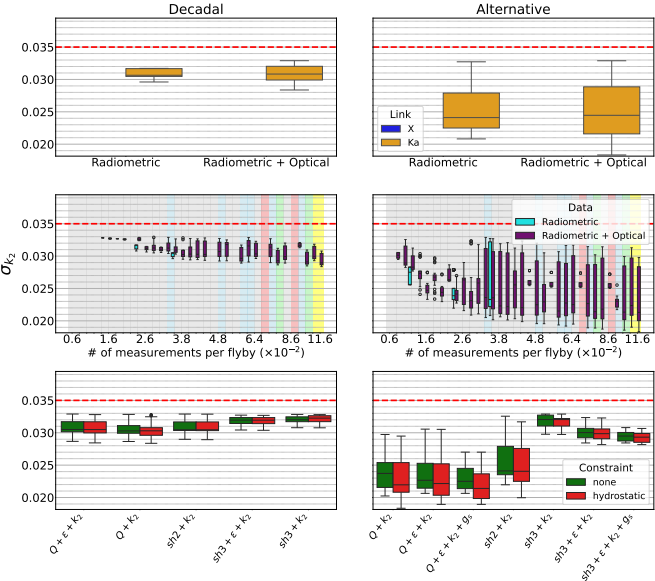
Figure A.2.: Love Number (k_2) computed formal errors. A detailed explanation of the figure can be found in Figure A.1. The red solid and dashed lines represents the target uncertainties as outlined in Table 4.1. Empty panels mean that no solution with a formal error less than or equal to the true value was found.



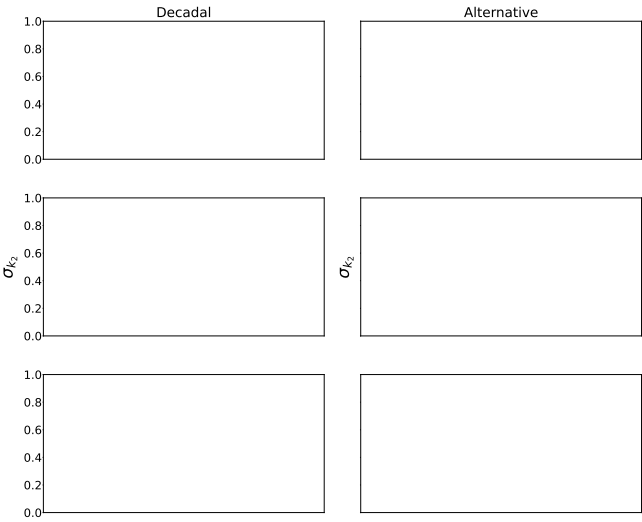
(a) Miranda.



(b) Umbriel.

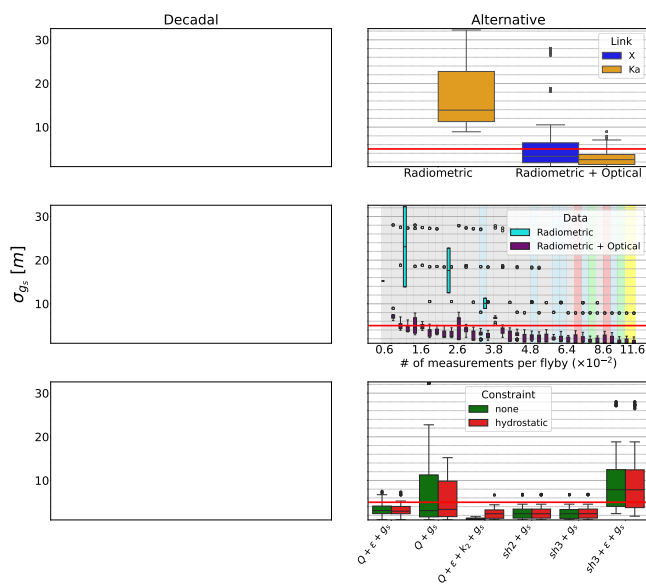


(c) Titania.

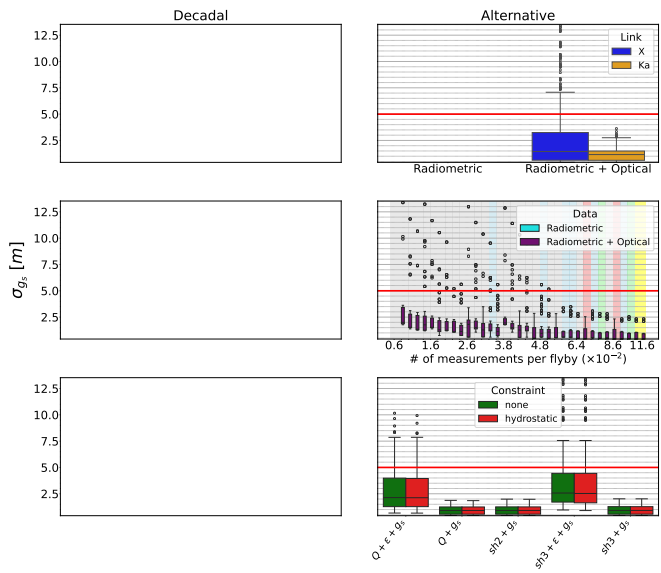


(d) Oberon.

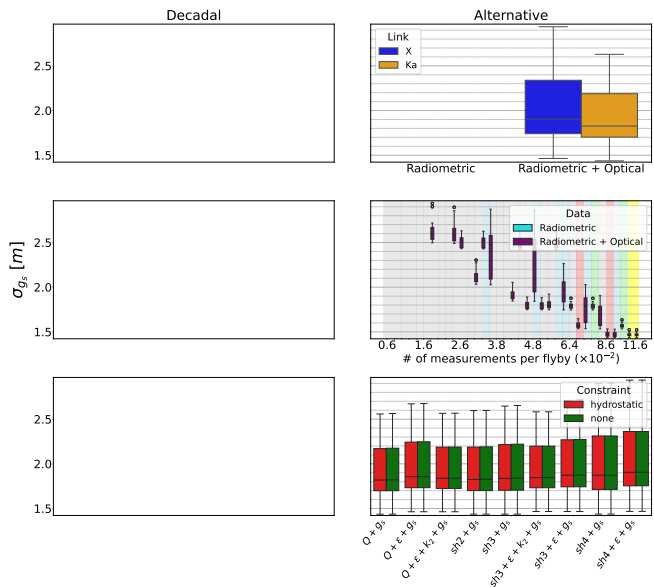
Figure A.3.: Libration amplitude (g_s) computed formal errors. A detailed explanation of the figure can be found in Figure A.1. The red solid and dashed lines represents the target uncertainties as outlined in Table 4.1. Empty panels mean that no solution with a formal error less than or equal to the true value was found.



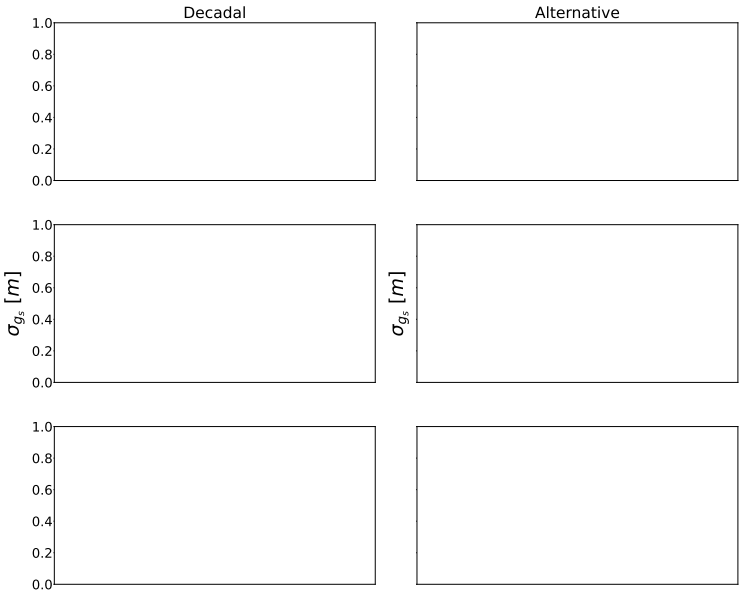
(a) Miranda.



(b) Umbriel.

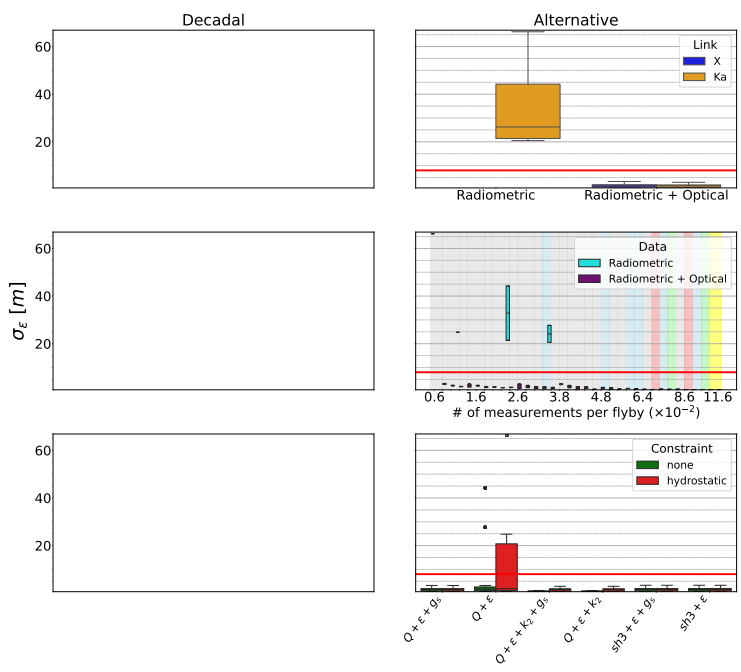


(c) Titania.



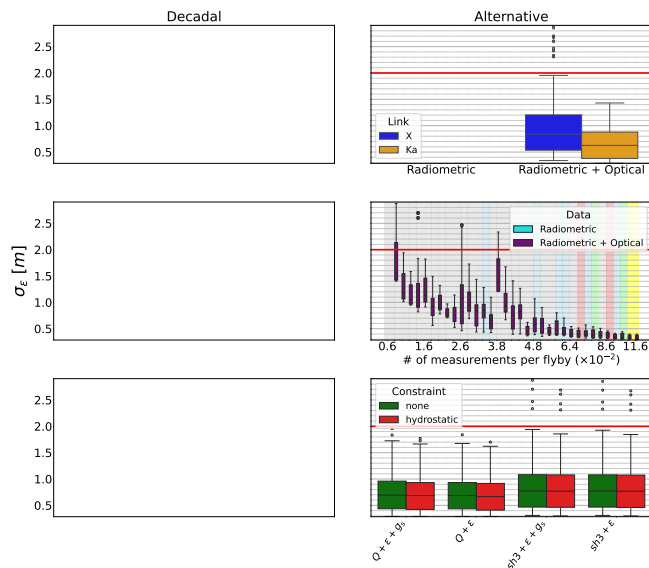
(d) Oberon.

Figure A.4.: Obliquity (ε) computed formal errors. A detailed explanation of the figure can be found in Figure A.1. The red solid and dashed lines represents the target uncertainties as outlined in Table 4.1. Empty panels mean that no solution with a formal error less than or equal to the true value was found.

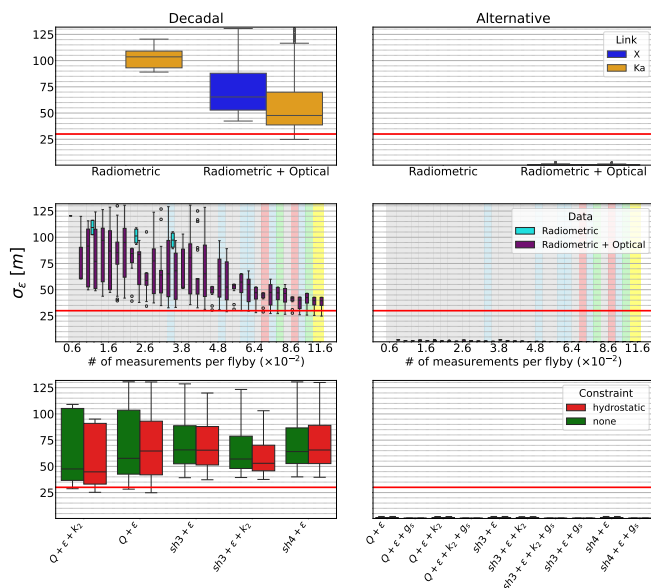


(a) Miranda.

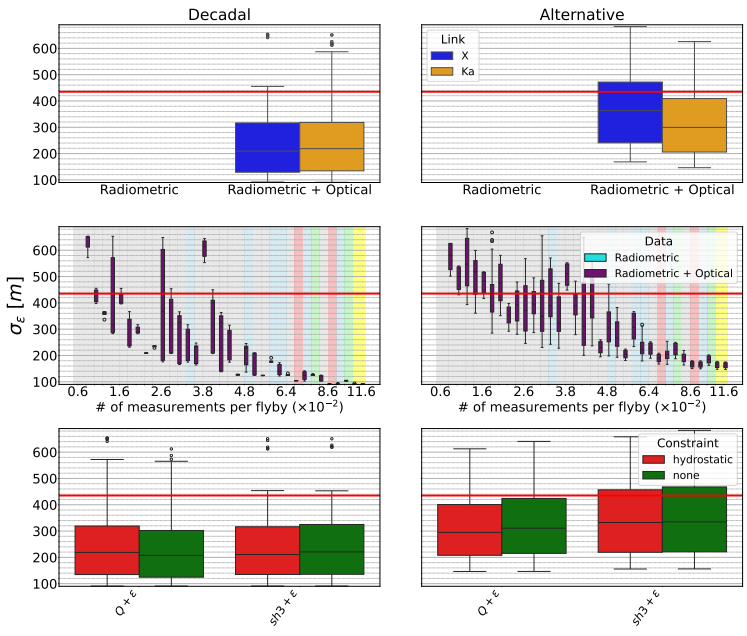
A



(b) Umbriel.

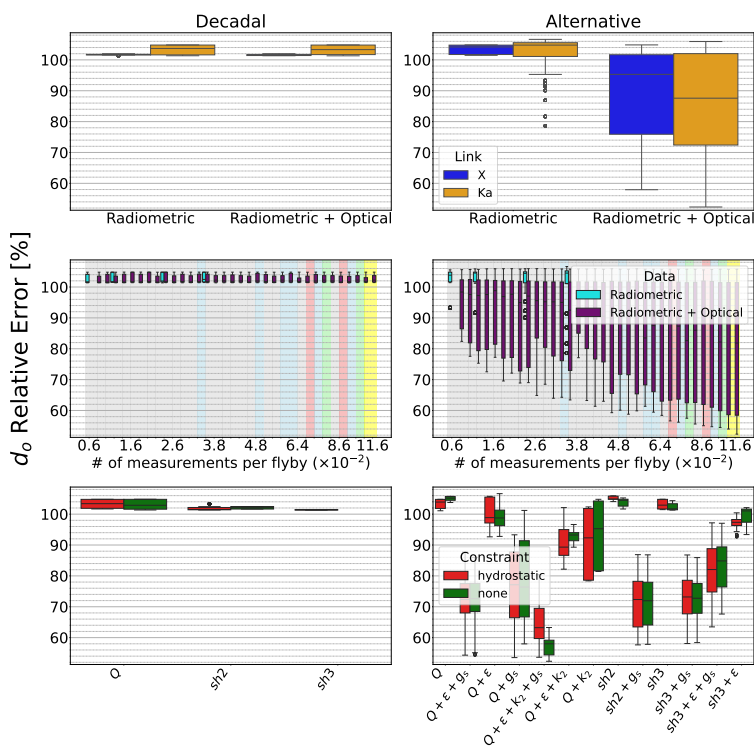


(c) Titania.

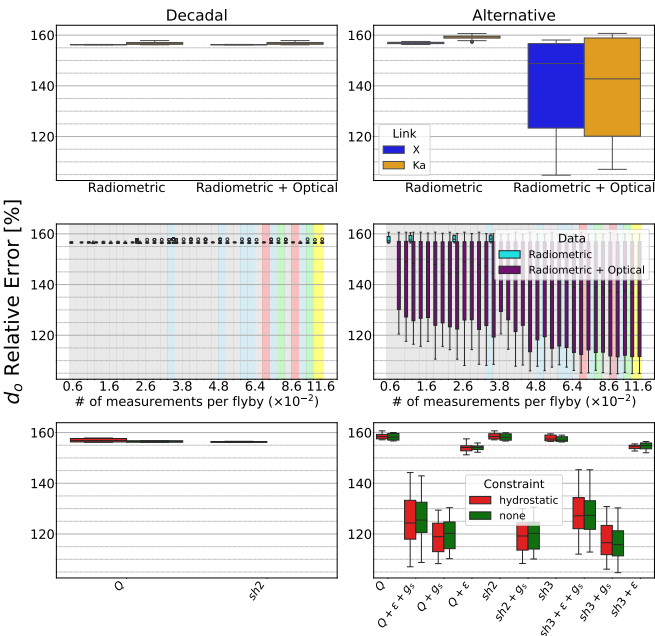


(d) Oberon.

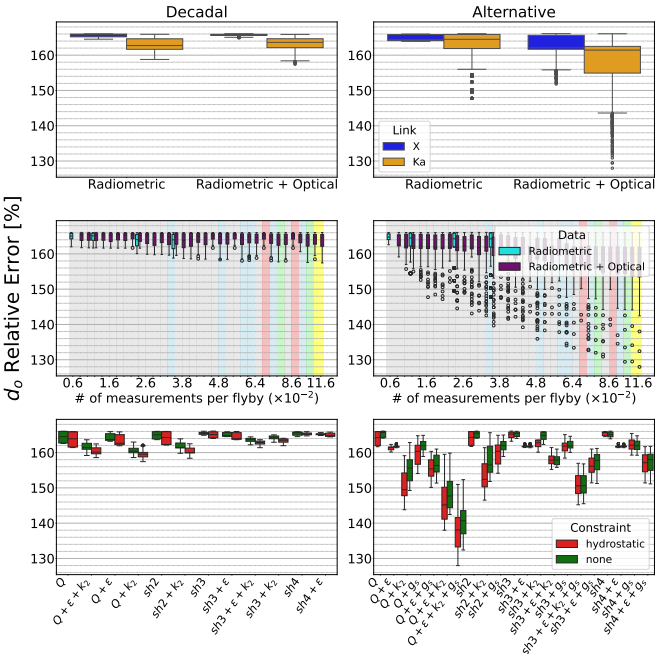
Figure A.5.: Ocean thickness (d_o) interpolated uncertainty. A detailed explanation of the figure can be found in [Figure A.1](#).



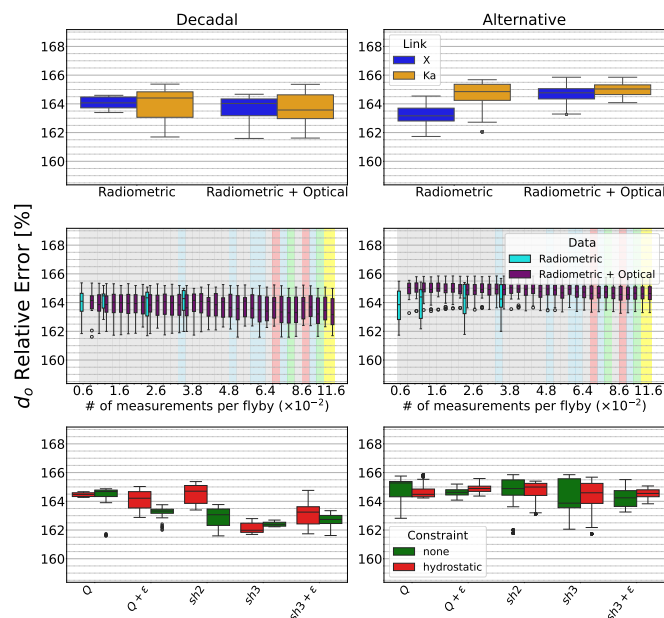
(a) Miranda.



(b) Umbriel.

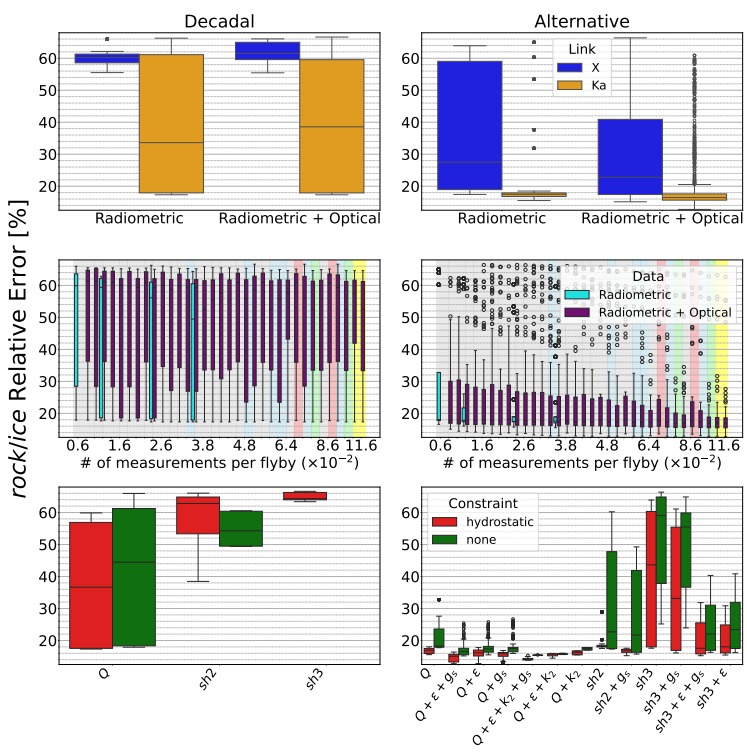


(c) Titania.



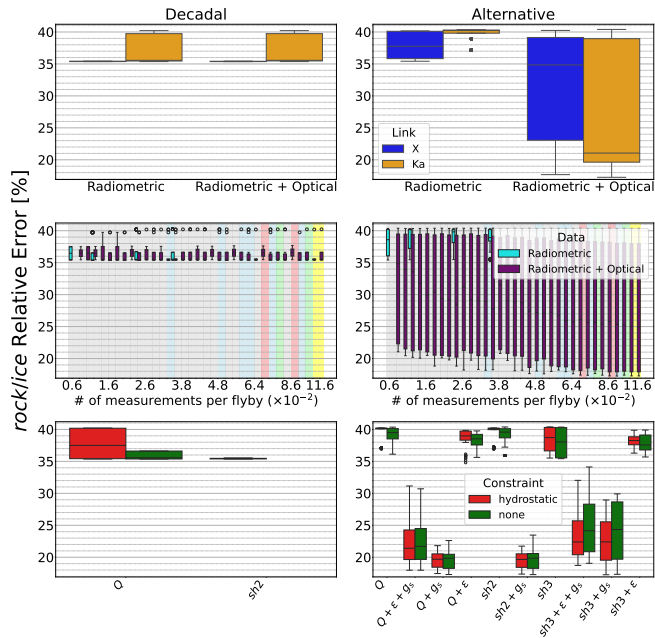
(d) Oberon.

Figure A.6.: rock-to-ice ratio interpolated uncertainty. A detailed explanation of the figure can be found in [Figure A.1.](#)

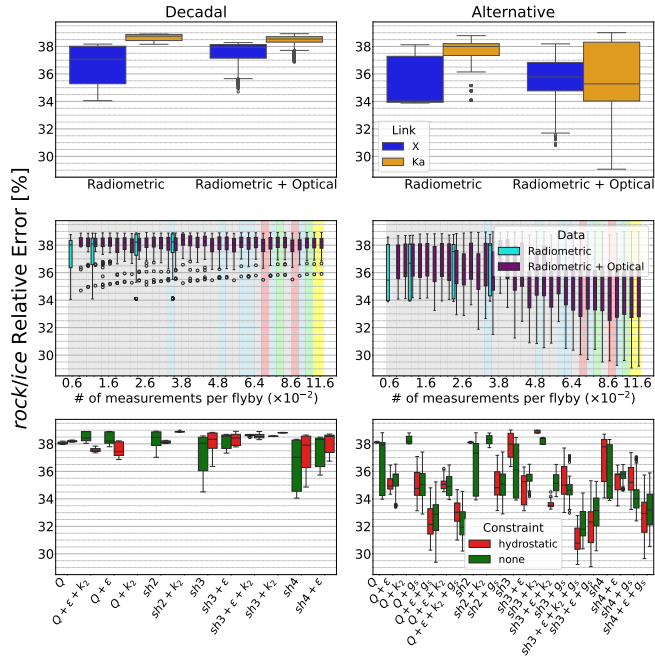


(a) Miranda.

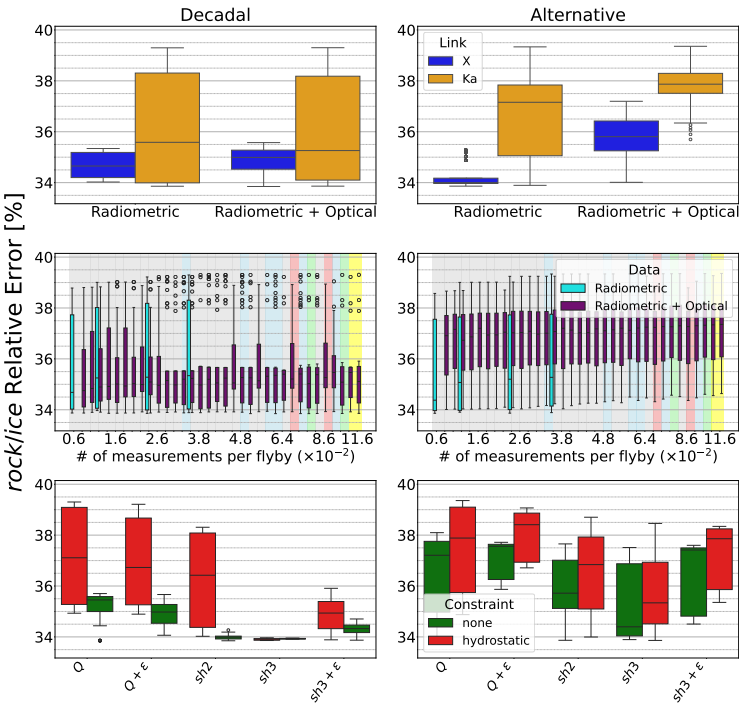
A



(b) Umbriel.

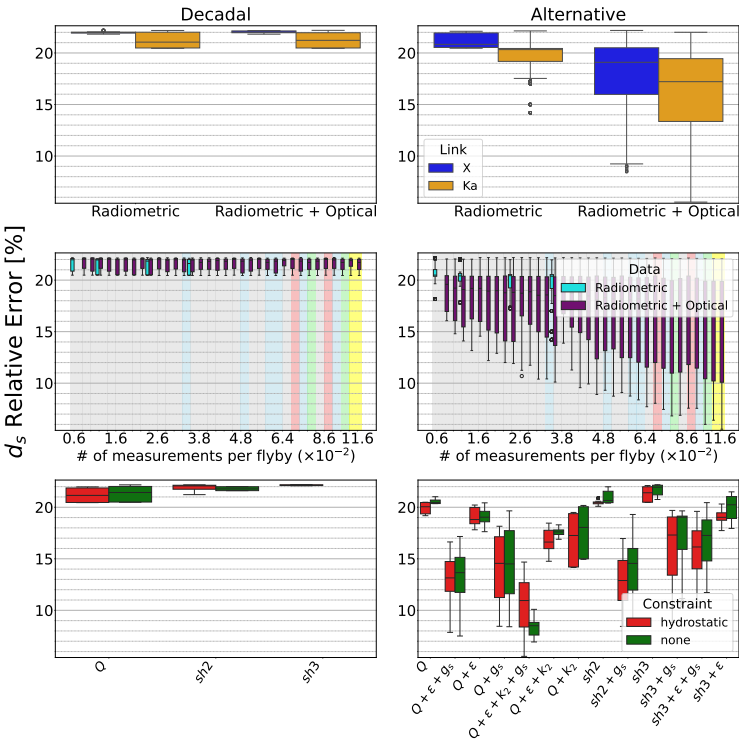


(c) Titania.

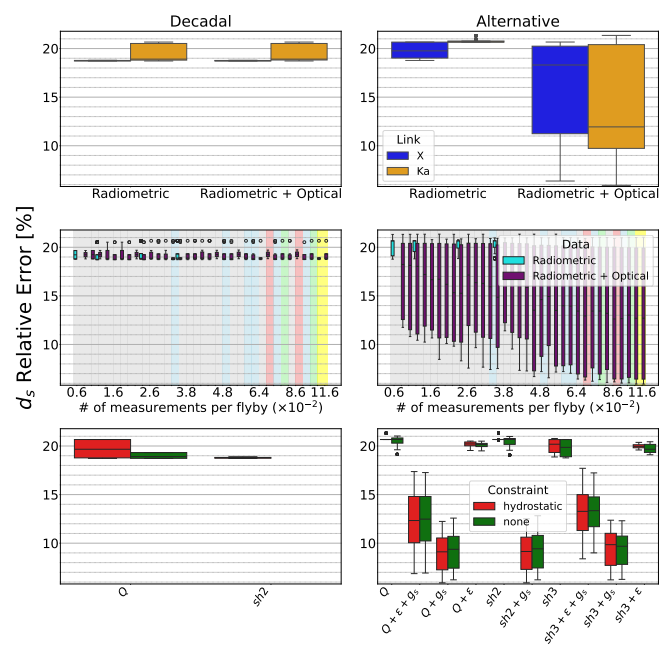


(d) Oberon.

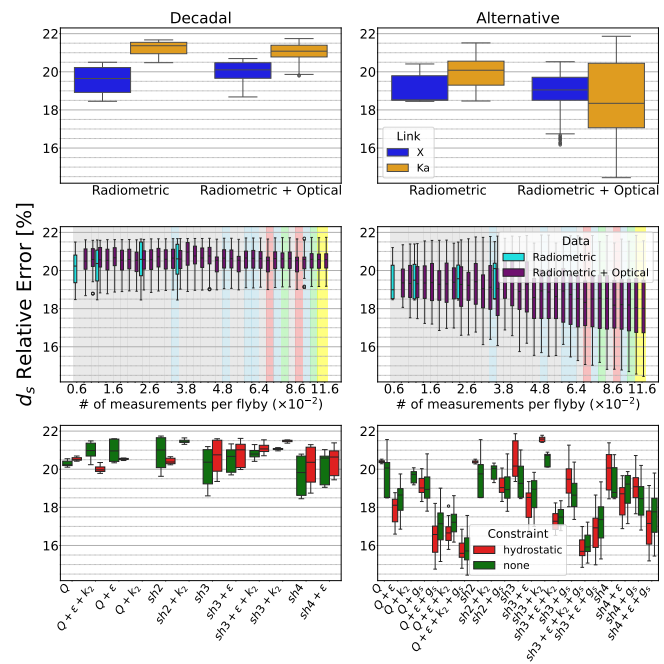
Figure A.7.: Ice shell thickness (d_s) interpolated uncertainty. A detailed explanation of the figure can be found in [Figure 4.13](#).



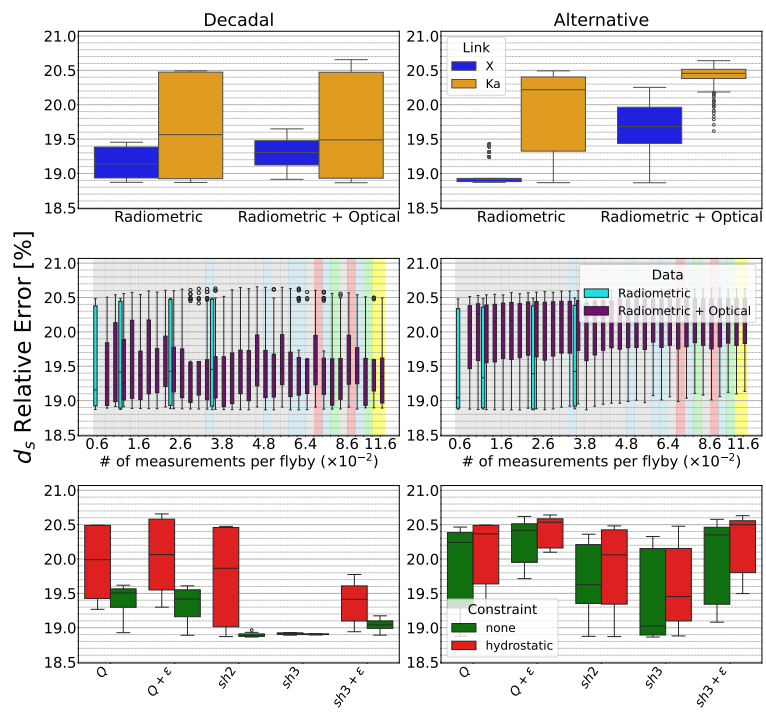
(a) Miranda.



(b) Umbriel.



(c) Titania.



(d) Oberon.

B

LOOKUP TABLES

WE provide the "lookup tables" that we have constructed in order to characterize the Gravity Science (GS) experiment for the future geophysical investigation of the Uranian moons. They can be downloaded from the Planetary Science Journal website¹. The tables in machine-readable format, which can be filtered by satellite ("sat" column) and by interior model via the ocean thickness value ("ocean" column). A summary of the complete lookup tables, illustrating their structure and content, is provided in Table B.1. The columns "sigma-moi", "sigma-k2", "sigma-lib" and "sigma-obl" represent the uncertainty scaling factor utilized to define the observables uncertainty in the MCMC runs. This is calculated as, for instance, $\sigma_{Mol} = Mol * \text{sigma-moi}$. Table B.2 collects the interior truth values of the Uranian satellites, as well as the expected geodetic observables for each interior model. These can be employed to quantify the absolute uncertainty based on the provided uncertainty factors. Each line represents the output of a single MCMC run. The provided data set includes the 5th, 95th, 15.87th, and 84.13th percentile values of the a posteriori distributions of the investigated interior parameters, namely, ρ_c , ρ_o , d_o , ρ_s and d_s . To illustrate, the column containing the 95th percentile value of ocean density (ρ_o) is designated as "95th-rho-o". As the rock-to-ice mass ratio (*rock/ice*) is determined directly from the interior parameter a posteriori distributions, the same percentiles are provided for this ratio, which is henceforth referred to as "ratio". For each interior parameter, including the *rock/ice* ratio, we also provide the ratio between the width of the 5th to 95th percentile range and the width of the uniform prior distributions defined in Table 2 (also referred to as non-informative ratio). For example, for the ocean density ρ_o , it is "infRatio-rho-o".

¹<https://iopscience.iop.org/article/10.3847/PSJ/ada7ef>

Table B.1.: Machine-Readable Format Example Table for Miranda

sat	sigma-moi	sigma-k2	sigma-tilb	sigma-obl	ocean	5th-rho-c	95th-rho-c	5th-rho-o	95th-rho-o	5th-d-o	95th-d-o
Miranda	1	0.1	1	1	0	2653.9632	3468.6963	715.8306	1030.2059	576.4677	16698.1623
Miranda	1	1	1	1	0	2469.0415	3450.6254	717.9793	1033.1759	2147.5867	52578.6124
Miranda	1	0.1	0.1	1	0	2638.5007	3467.5551	715.7617	1029.9643	561.4932	16716.7990
Miranda	1	1	0.1	1	0	2479.5367	3453.6119	717.1165	1032.4058	1834.0822	44263.5632
Miranda	1	0.1	1	0.1	0	2647.1815	3467.5253	714.8648	1029.5303	543.0070	16049.1530

5th-rho-s	95th-rho-s	5th-d-s	95th-d-s	15th-rho-c	84th-rho-c	15th-rho-o	84th-rho-o	15th-d-o	84th-d-o	15th-rho-s	84th-rho-s
723.3713	1040.3274	89576.2042	135877.92	2836.8308	3399.1709	751.4338	988.6579	1853.1596	12501.4425	771.9491	1016.2167
728.8298	1040.3548	61573.9810	126873.75	2607.7915	3342.9144	755.1218	996.4336	6774.4329	41169.6717	785.3860	1016.9020
730.0151	1039.9144	89643.2431	136019.61	2823.0282	3395.5804	750.3411	988.2742	1802.1616	12431.2008	787.9383	1016.0347
729.9063	1037.3352	69386.0180	127515.51	2628.8227	3352.1752	754.2234	993.0543	5724.6623	34655.5708	785.5771	1010.2935
768.9488	1040.4720	94134.0552	136193.53	2832.7598	3395.3854	748.2802	986.1485	1751.7738	11949.5108	834.4036	1018.8537

15th-d-s	5th-ratio	95th-ratio	15th-ratio	84th-ratio	infratio-rho-c	infratio-rho-o	infratio-d-o	infratio-rho-s	infratio-d-s	infratio-ratio
96535.1775	0.2109	0.9866	0.2548	0.8256	0.7407	0.8982	0.1612	0.9056	0.3858	0.2091
73233.8538	0.2285	0.9640	0.2802	0.7876	0.8923	0.9006	0.5043	0.8901	0.5442	0.1983
97323.1442	0.2115	0.9751	0.2539	0.7849	0.7537	0.8977	0.1616	0.8854	0.3865	0.2058
78839.6339	0.2323	0.9621	0.2867	0.7885	0.8855	0.9008	0.4243	0.8784	0.4844	0.1967
102611.1936	0.2103	0.8528	0.2493	0.6542	0.7458	0.8990	0.1551	0.7758	0.3505	0.1732

Table B.2.: Truth interior models with an ocean

	Miranda	Ariel
<i>Three-layer interiors</i>		
Ice-I shell density	ρ_s (kg m ⁻³)	950
Water ocean density	ρ_o (kg m ⁻³)	1000
Rocky core density	ρ_c (kg m ⁻³)	3060
Ice-I shell thickness	h_s (km)	190.70, 180.95, 171.22, 161.5, 151.8
Water ocean thickness	(h_o) (km)	10,20,30,40,50
Rocky core radius	R_c (km)	378.2, 377.95, 377.68, 377.40, 377.1
MOI	0.341, 0.341, 0.3411, 0.3414, 0.3417	0.3123, 0.3123, 0.3123, 0.3124, 0.3125
Umbriel	Titania	Oberon
950	950	950
1000	1000	1000
3060	3060	3060
196.35, 186.61, 176.88, 167.16, 157.46	232.23, 222.48, 212.74, 203.01, 193.29	221.15, 211.4, 201.66, 191.93, 182.21
10,20,30,40,50	10,20,30,40,50	10,20,30,40,50
378.35, 378.09, 377.82, 377.54, 377.24	546.67, 546.42, 546.16, 545.89, 545.61	530.25, 530.00, 529.74, 529.47, 529.19
0.3126, 0.3126, 0.3127, 0.3128	0.3116, 0.3117, 0.3117, 0.3117, 0.3117	0.3117, 0.3117, 0.3117, 0.3118, 0.3118

C

THE ROB-MONTE LIBRARY

THE *ROB-MONTE* library is a specialized Python framework developed at the Royal Observatory of Belgium (ROB) to facilitate advanced radio science investigations. It is built directly on top of the existing Python scripting layer of the NASA Jet Propulsion Laboratory’s (JPL) Mission analysis, Operations, and Navigation Toolkit Environment (MONTE). While MONTE is an exceptionally powerful and advanced toolkit, its core application is the operational navigation of space missions. Although there are significant overlaps between the needs of mission operations and radio science analysis, the workflows and priorities can differ.

The primary purpose of *ROB-MONTE* is to address this by combining MONTE’s powerful functionalities into a framework optimized for scientific applications. It encapsulates complex, multi-step procedures into intuitive, high-level commands that streamline the classic steps of a radio science investigation, from experiment design to data analysis. This allows researchers to focus on the scientific interpretation of results, freeing them from the need to combine MONTE’s base functionalities and debug complex, low-level implementations. By providing this structured and unified environment, *ROB-MONTE* allows to conduct more efficient, reproducible and high-fidelity analyses. The source code is available in the online repository on the GitLab server of ROB¹.

C.1. CORE ARCHITECTURE AND DESIGN PHILOSOPHY

The design philosophy of *ROB-MONTE* is centered on providing a structured and unified interface to MONTE’s powerful functionalities, adapting them for a scientific research context. This is achieved by building upon Monte’s User Interface (UI) system. The MONTE UI is composed of two major pieces: the Data Setup System, used to create and modify objects in the MONTE database (the “lock” and “update” steps, typically handled by “setup scripts”), and the Command System, used to execute operations and produce results (the “run” step, handled by “command scripts”). *ROB-MONTE* adds new functionalities to both layers, abstracting the granular UI commands into a modular Python library that logically organizes tasks common to radio science investigations.

¹<https://gitlab-as.oma.be/monte/monterob>

A key motivation behind this framework is the simplification of complex but common analysis techniques. For instance, multi-arc analysis is a critical method for mitigating the accumulation of dynamical errors in long-duration experiments. In MONTE, setting up a multi-arc estimation requires numerous steps to define propagators, manage reference states, and link parameters across arcs. *ROB-MONTE* automates this process through functions that handle the creation of distinct propagators for each arc and manage their respective state vectors and epochs within a coherent filter setup.

The core architecture is built upon three fundamental modules, as illustrated by the directory structure below. The *orbitDetermination* module is the central component, providing the high-level commands for performing Precise Orbit Determination (POD) and related simulations. Complementing this is the *utils* module, which serves as a collection of powerful utility functions for data extraction, computation, and file manipulation essential for preparing simulations and analyzing results. Finally, the *plots* module offers a set of pre-configured functions for the rapid visualization of common data products in radio science.

```
monterOB/
|-- orbitDetermination/
|   |-- __init__.py
|   |-- podCmd.py
|   |-- simMsrCmd.py
|   |-- fullSimCmd.py
|   |-- utils.py
|-- utils/
|   |-- __init__.py
|   |-- extractODSol.py
|   |-- orbGeometryCalculator.py
|   |-- filesUtils.py
|   |-- constrainClasses.py
|-- plots/
|   |-- __init__.py
|   |-- odPlots.py
|   |-- trajPlots.py
```

This modular architecture creates a “unified framework” that allows a researcher to seamlessly move from simulation setup to data analysis and visualization within a single, coherent environment.

C.2. DETAILED FUNCTIONALITIES

We detail here the functionalities of the library, organized by module.

C.2.1. THE ORBITDETERMINATION MODULE

The *orbitDetermination* module provides science-focused wrapper functions that automate the complex sequence of MONTE UI operations required for a typical POD analy-

sis, automatically handling both single and multi-arc scenarios. These high-level commands are supported by the utility submodule, `orbitDetermination.utils`, which is fundamental for preparing the dynamic models and propagators required by the main commands. This submodule provides a suite of functions to automate the complex setup process:

- **Propagator Configuration (`createPropSetup`):** This is the workhorse function for setting up the simulation's dynamics. It automates the configuration of MONTE's `DivaPropagator` objects, which are responsible for integrating the spacecraft's trajectory. For each arc in an analysis, this function defines the initial state, the force models to be applied (e.g., gravity, solar radiation pressure), and, critically, the list of parameters for which partial derivatives are required. It distinguishes between global parameters (like a planetary body's GM) and local parameters (like an arc-specific solar radiation pressure coefficient), ensuring the correct setup for the filter's estimation process.
- **Multi-Arc utilities:** For multi-arc analyses, several functions work in concert to manage the complex setup:
 - **`getRefStateInfo`:** This function inspects the MONTE database to extract information about the reference state propagators created for each arc. It identifies the arc-specific state parameters and any local parameters, which is essential for correctly linking them in the filter.
 - **`getRefEpoch`:** To ensure a consistent time reference across all arcs in a multi-arc solution, this function determines the earliest start time among all propagators and establishes it as the global reference epoch for the filter.
- **Light-Time correction (`getLtPropTimeInterval`):** This utility calculates the appropriate time intervals for propagation and measurement simulation, accounting for the one-way light time between the spacecraft and Earth. This is critical to ensure that trajectory is propagated for a sufficient duration to simulate measurements according to the provided schedule.

This automation of the setup process is a cornerstone of the *ROB-MONTE* library, providing the flexibility and reliability necessary to configure single- and multi-arc simulations efficiently. The main commands of the module are:

- **Measurement simulation (`simMsr`):** This function manages the creation of any observables natively supported by MONTE or user-defined. The process begins by integrating a “true” trajectory based on the provided dynamical models. Using this trajectory, the function generates an ideal measurement schedule based on a set of user-defined measurement strategies. These strategies dictate all aspects of the simulation, such as the measurement type, tracking cadence, station visibility, etc. It then simulates the raw observables and applies realistic, user-defined noise models to the data, producing a final set of measurements ready for the estimation process.

- **Precise Orbit Determination (POD):** This is the core filtering command that executes the parameter estimation process. It takes a set of simulated or real measurement data and iteratively runs MONTE's Square-Root Information Filter (SRIF). Within its iterative loop, the command integrates the spacecraft trajectory, computes measurement residuals, and updates the estimated parameters. A key feature is its automated convergence check, which assesses the weighted root mean square (WRMS) of the residuals to determine if the solution has converged. The command also includes specialized logic for multi-arc processing, such as copying partial derivatives from arc-specific states to a global reference state, and for specialized analysis techniques, such as accounting for the permanent tides of synchronous satellites. The process concludes by running the filter, and the library also wraps MONTE's native covariance analysis functionality, which can be enabled via the `CovStudy=True` flag to assess the statistical uncertainty of the estimated parameters without solving for their central values.
- **Full simulation workflow (`fullSim`):** This command serves as a high-level coordinator that integrates `simMsr` and POD into a single, seamless function call. It is designed for end-to-end studies, from measurement generation to final parameter estimation or covariance analysis. A critical feature of `fullSim` is the ability to inject intermediate setup modifications between the simulation and estimation steps. This allows a user to, for example, simulate data with one set of dynamical parameters and then estimate a solution with a different set, a common requirement for sensitivity and robustness testing.

C.2.2. THE UTILS MODULE

This module provides the useful tools to interact with MONTE's outputs and perform scientific analysis.

- **Solution extraction (`extractODSol`):** A key function within this submodule, `getSolToDf`, parses MONTE's binary solution files (`sigsolution.boa`) and organizes the results into a Pandas DataFrame. It extracts the estimated value, final uncertainty (sigma), a priori uncertainty, and true value (if available from a simulation) for each parameter in the solution. It also includes functions like `getCorrMat` to extract the full correlation matrix between the estimated parameters, providing crucial insight into parameter dependencies and the robustness of the solution.
- **Trajectory and geometry analysis (`orbGeometryCalculator`):** This package includes functions to find close approach events (`findCloseastApproach`), calculate ground tracks (`groundTrack`), and extract Cartesian trajectory data (`getCartesianTraj`) in any specified reference frame.
- **Constraint Handling (`constrainClasses`):** This file defines the base classes used to create and manage constraint measurements, allowing for the application of any kind of relationship (linear or non-linear) among the estimated parameters. It employs the classical least-squares technique of "pseudo-measurements" to define soft constraints. In this approach, a constraint equation is treated as an additional

measurement that is appended to the observation vector, thereby enriching the design matrix H . The user provides a model that defines the relationship between parameters and its partial derivatives with respect to the estimated parameters, and the library computes the “observable” as required by the MONTE’s framework. The specified sigma for this pseudo-measurement acts as the uncertainty on the right-hand side of the constraint equation, controlling the weight of the constraint in the filter.

C.2.3. THE PLOTS MODULE

This module relies on the data extraction and orbit geometry functionalities of the *utils* module to provide a set of high-level functions for visualizing the results.

- Trajectory plotting (*trajPlots*): This submodule provides functions like `traj2D` and `traj3D` that generate 2D and 3D plots of spacecraft trajectories relative to a central body, or the `groundTrack` to visualize the ground track, offering a quick and easy way to visualize orbits.
- Orbit Determination plots (*odPlots*): This submodule includes functions like `corrMat` to visualize the correlation matrix as a heatmap and `sigmaRatio` to create bar plots showing the improvement of parameter uncertainties (a posteriori vs. a priori sigma), which is a key metric for assessing the strength of a solution.

C.3. PRACTICAL WORKFLOW: A USE-CASE EXAMPLE

To illustrate the practical application of *ROB-MONTE*, we present a simplified workflow based on the MMX (Marian Moons eXploration) mission covariance studies; the complete example is available in the online repository of the source code. The following steps outline a covariance analysis designed to assess the expected accuracy of Deimos’s gravitational parameters from synthetic Doppler data simulated using the mission nominal flybys of Deimos.

1. Simulation setup: The user first defines the parameters of the simulation through a series of configuration files, typically organized into the following directories:
 - Models (*models*): This directory contains scripts that define the fundamental environment models for the simulation. This includes defining the gravity fields, reference frames, body shapes, and spacecraft properties (e.g., solar radiation pressure model).
 - OD setup (*odSetups*): A script in this directory defines the propagation and filter settings of a specific orbit determination problem. This includes specifying the spacecraft, central body, reference frame, force models, and the baseline set of parameters to be estimated (e.g., GM / Deimos).
 - BOA updates (*boaUpdates*): These are scripts that apply specific modifications to the environment or the filter setup for a particular case. For example, a script might add a new parameter, like the J_2 gravity coefficient, to the list of estimated parameters for a specific run.

C

- Measurement strategy (*msrStrategies*): This file defines the simulated tracking schedule. For instance, one might specify Doppler tracking for a duration of ± 4 hours around each close approach and define rules to exclude data during solar occultations.
 - Measurement configuration (*msrConfig*, *msrNoise*): These files specify the details of the radiometric links, such as the stations involved, frequency bands (e.g., X-band), and the noise level to be applied to the simulated data.
2. Coordinating the simulation: A main script is created to run the study. For trade studies and sensitivity analyses, *ROB-MONTE* allows for the automated execution of multiple simulation cases. The user can define a series of configurations, each with a unique combination of the setup files. For example, a user might define cases that vary the tracking duration (2h, 4h, 6h, 8h) and the set of estimated parameters (e.g., *GM only*, *GM+J2*) to study their impact on the solution accuracy.
 3. Executing the workflow: When the main script is run, *ROB-MONTE* handles the entire backend process. For each case, it:
 - Creates a dedicated output directory.
 - Calls MONTE to simulate the measurements according to the specified strategy.
 - Runs the SRIF to perform the covariance analysis.
 - Saves the resulting solution and covariance information in binary `sigsolution.boa` files.
 4. Analysis and visualization: After the simulations are complete, the user employs the *utils* and *plots* modules for analysis.
 - An analysis script uses `extractODSol.getSolToDf` to load the results from all the case directories into a single Pandas DataFrame.
 - This DataFrame can then be used to study trends. For example, the user can plot the percentage uncertainty of Deimos's *GM* as a function of tracking time or the number of flybys, allowing for a quantitative assessment of the experiment's performance.
 - Functions like `odPlots.corrMat` can be used to visualize the correlation between, for example, the J_2 and C_{22} gravity coefficients to check for solution stability.

This workflow demonstrates how *ROB-MONTE* provides a powerful, end-to-end framework for conducting complex radio science studies in a structured and reproducible manner.

C.4. CONCLUSION AND FUTURE DEVELOPMENT

The *ROB-MONTE* library provides a significant advancement for radio science research within the MONTE ecosystem. Its primary advantage lies in its ability to abstract the

complexity of MONTE into a high-level, science-oriented interface. This unified framework accelerates the research lifecycle, from experiment design and sensitivity studies to the final analysis of tracking data, while ensuring that the full fidelity of JPL's state-of-the-art navigation software is leveraged. The modular architecture promotes code reusability and simplifies the process of conducting complex trade studies involving numerous simulation parameters.

The library is currently available only to users at ROB and is designed to be extensible to meet the evolving needs of various research projects within ROB, with the goal of establishing it as a standard internal tool for planetary science investigations using MONTE.

D

THE GEOMCMC LIBRARY

TO facilitate the application of powerful MCMC techniques to planetary science, the ‘geoMCMC’ Python library has been developed as part of this research. The primary aim of this library is to provide a framework around the `emcee` implementation of the Goodman & Weare sampler, specialized for the geophysical investigation of planetary bodies. `geoMCMC` is tailored for coupling forward models of planetary interiors with the affine-invariant sampling algorithm, and is designed to streamline the setup and execution of Bayesian inversions to constrain interior structure parameters based on geodetic and geophysical observations.

The core of the library is its ability to model a planetary body as a series of concentric layers, each with distinct physical properties. From this layered structure, `geoMCMC` can compute a variety of geophysical observables, including the planet’s total mass, moment of inertia factor, the k_2 tidal Love number, gravity field coefficients, libration, and obliquity. By comparing these computed values to observational data, the MCMC sampler efficiently explores the parameter space (e.g., the density and thickness of layers) to map the posterior probability distribution. This analysis allows for a robust determination of the most probable values and credible intervals for each interior parameter, revealing potential correlations and providing comprehensive constraints on the internal structure of the explored planetary body. Further technical details regarding the `geoMCMC` library are provided in the following sections. The source code is available in the online repository on the GitLab server of ROB¹.

D.1. CODE ARCHITECTURE AND DESIGN

The `geoMCMC` library is designed using an Object-Oriented Programming (OOP) paradigm. This approach encapsulates the different conceptual parts of the inversion problem into distinct, interacting classes. This design enhances modularity, making it easier to maintain the code and extend its functionality in the future. The high-level architecture separates the physical forward model, the observational data, the MCMC parameter setup, and the simulation execution into specialized components.

The main modules and their corresponding classes are organized as follows:

¹<https://gitlab-as.oma.be/valeriof/geomcmc>

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- `planet.py`: This is the scientific core of the library. It contains the `Planet` class, which implements the forward model for a planetary body. It takes global properties and a list of layers as input and computes geophysical observables.
- `layerParams.py`: Contains the `LayerParams` class, which is a container for the parameters that will be varied during the MCMC simulation. It manages parameter names, their initial values, and their prior bounds.
- `observables.py`: Contains the `Observables` class, which stores the observational data and their associated uncertainties. This class is used to compare the output of the forward model with real-world measurements.
- `mcmcModel.py`: This module contains the `MCMCmodel` class, which serves as a bridge between the physical model (`Planet`), the parameters (`LayerParams`), and the data (`Observables`). Its primary role is to calculate the log-probability, which is the function that the MCMC sampler seeks to optimize.
- `mcmcSimulation.py`: This module contains the `MCMCsimulation` class, which acts as a high-level wrapper for the `emcee` library. It orchestrates the MCMC run, managing the walkers, backend storage, and retrieval of results.
- `utilities.py`: A collection of procedural helper functions that support the calculations performed in the `Planet` class, such as computing gravity from topography or calculating tidal response functions.

These components interact in a logical sequence: a `Planet` instance is created to represent the physical body. `Observables` are defined based on known data. `LayerParams` specifies which of the planet's properties are to be constrained. These three components are then passed to an `MCMCmodel` instance, which defines the probability landscape. Finally, the `MCMCsimulation` class takes the `MCMCmodel` and runs the sampling process to find the solution.

D.2. CORE COMPONENTS ANALYSIS

This section provides a detailed breakdown of the main classes and methods within the `geoMCMC` library.

'PLANET.PY'

This module contains the classes that define the physical structure of the planetary body.

- Class 'GlobalProperties': A data class to hold global parameters of the planet such as its radius (R), gravitational parameter (GM), and orbital properties.
- Class 'LayerProperties': A data class to hold properties for a single layer, such as density (ρ), thickness (d), and elastic parameters (μ , E , etc.).
- Class 'Planet': The main forward modeling class.

- `__init__(self, global_properties, layers_input)`: The constructor takes an instance of `GlobalProperties` and a list of tuples, where each tuple defines a layer with its name, number, and an instance of `LayerProperties`. It automatically calculates the thickness of the one layer for which `d` is not provided, ensuring the sum of layer thicknesses equals the planet's radius.
- `update_layer_properties(self, layer_params)`: Updates the properties of the planet's layers with new values from a `LayerParams` object. This method is called at every step of the MCMC simulation to evaluate a new point in the parameter space.
- `compute_mass(self)` & `compute_moif(self)`: These methods calculate the total mass and normalized moment of inertia factor (MoI) of the planet by summing the contributions from each spherical shell layer.
- `compute_gravity(self)`: Computes the spherical harmonic coefficients of the gravity field generated by the topography of the layers.
- `compute_k2(self)`: Computes the degree 2 tidal Love number (k_2), a crucial observable for constraining interior structure, particularly the presence of oceans. It can use different methods depending on whether an ocean layer is present.
- `get_<observable>()`: A series of getter methods (e.g., `get_mass()`, `get_k2()`, `get_libration()`) that compute the respective observable if it hasn't been computed yet and then return the value. These are the methods called by the `MCMCmodel` to get the model's output.

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'LAYERPARAMS.PY'

- Class 'LayerParams': Manages the parameters being sampled by the MCMC.
 - `add_layer(self, layer_name, parameters, parameter_bounds)`: Adds a layer and its associated parameters to be explored. `parameters` is a dictionary of initial values, and `parameter_bounds` is a dictionary where each value is a (min, max) tuple defining the prior range for that parameter.
 - `set_params(self, params_list)`: Takes a flat list of new parameter values (as provided by an emcee walker) and updates the internal parameter dictionary. It checks if the new values are within the defined bounds.
 - `is_within_bounds(self, layer_name, param_name, value)`: A utility method to check if a given value for a parameter is within its predefined minimum and maximum bounds.

'OBSERVABLES.PY'

- Class 'Observables': A container for the observational data.
 - `add(self, name, value, sigma)`: Adds an observable quantity, its measured value, and its one-sigma uncertainty.

- `get_vector(self)`: Returns two NumPy arrays: a vector of the observed values and a diagonal covariance matrix constructed from the squared sigmas. This is used to calculate the chi-squared value in the log-likelihood function.

‘MCMCModel.py’

- Class ‘MCMCmodel’: The link between the model, its parameters, and the data.
 - `__init__(self, planet, observables, layer_params)`: The constructor takes instances of the three core components described above. It precomputes the inverse of the covariance matrix for efficient likelihood calculation.
 - `log_probability(self, params, blobs=False)`: This is the primary function passed to the MCMC sampler. It first updates the model with a new set of params. If the params are outside the prior bounds, it returns `-inf`. Otherwise, it calls `log_likelihood()` and returns its value.
 - `log_likelihood(self)`: Calculates the log-likelihood of the model given the data. It does this by fetching the model’s predictions for the observables, calculating the residual between the model and the data, and then computing $-0.5 \times \chi^2$.

‘MCMCSimulation.py’

- Class ‘MCMCsimulation’: Manages the execution of the MCMC sampler.
 - `__init__(self, mcmc_model, nwalkers, ...)`: Takes an MCMCmodel object and the number of walkers for the ensemble sampler. It also handles the setup of the HDF5 backend for saving the simulation state.
 - `run_mcmc(self, chain_length, ...)`: Starts the MCMC simulation. It initializes the `emcee.EnsembleSampler` with the `log_probability` function from the MCMCmodel and runs the sampler for a given number of steps (`chain_length`).
 - `get_samples(self, burn_in_factor=0, flatFlag=False)`: Retrieves the sample chains from the sampler. It allows for discarding a fraction of the initial steps as "burn-in" and can flatten the chains from all walkers into a single array for easy analysis.

D.3. DEPENDENCIES

The `geoMCMC` library relies on several key external Python libraries for its functionality. These dependencies must be installed in the Python environment for the software to run correctly.

- ‘numpy’: This is a fundamental dependency used throughout the library for numerical operations. It is used for handling arrays, performing mathematical calculations (e.g., in `planet.py` and `utilities.py`), and for linear algebra operations like inverting the covariance matrix in `mcmcModel.py`.

- ‘emcee’: This is the core MCMC sampling engine used by the library. The `MCMCsimulation` class is essentially a wrapper around emcee’s `EnsembleSampler` to manage the Bayesian inference process.
- ‘pyshtools’: This library is used for spherical harmonic transformations and analysis. It is used in `utilities.py` to generate random topographic fields for the forward model.
- ‘corner’: This library is used for visualizing the results of the MCMC simulation. The `plot_samples` and `plot_obs` methods in `MCMCsimulation` use `corner` to create the classic "corner plots" that show 1D and 2D projections of the posterior probability distributions.

D.4. USAGE EXAMPLE

This section provides a simple but complete example of how to use the `geoMCMC` library to constrain the properties of a fictional three-layered planetary body.

D.4.1. EXAMPLE SCRIPT

```
import numpy as np
from geoMCMC import (
    Planet,
    GlobalProperties,
    LayerProperties,
    Observables,
    MCMCmodel,
    MCMCsimulation,
    LayerParams,
)

# A.1 Define the planetary body's structure and properties
# -----

# Define the global properties of our fake planet
global_props = GlobalProperties(R=1.5e6) # Radius in meters

# Define the properties of each layer
# We will leave the mantle thickness (d) as None, so it will be calculated automati
core_props = LayerProperties(rho=5500.0, d=7e5)
mantle_props = LayerProperties(rho=3200.0)
crust_props = LayerProperties(rho=2800.0, d=5e4)

# Create the list of layers for the Planet object
layers = [
    ("core", 1, core_props),
    ("mantle", 2, mantle_props),
```

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```

    ("crust", 3, crust_props),
]

# Instantiate the Planet model
planet_model = Planet(global_props, layers)

# A.2 Define the "observed" data
# -----
# For this example, we'll use the model to generate our "true" data
true_mass = planet_model.get_mass()
true_moif = planet_model.get_moif()

# Now, create an Observables object and add the data with some uncertainty
obs = Observables()
obs.add(name="mass", value=true_mass * 1.01, sigma=true_mass * 0.02)
obs.add(name="moif", value=true_moif * 0.99, sigma=true_moif * 0.02)

# A.3 Define the parameters for the MCMC to sample
# -----
# We want to constrain the density of the core and the crust.
layer_params = LayerParams()

# Add core parameters and their bounds (min, max)
layer_params.add_layer(
    layer_name="core",
    parameters={"rho": 5000.0},
    parameter_bounds={"rho": (4000.0, 7000.0)},
)

# Add crust parameters and their bounds
layer_params.add_layer(
    layer_name="crust",
    parameters={"rho": 2500.0},
    parameter_bounds={"rho": (2000.0, 3500.0)},
)

# A.4 Set up and run the MCMC simulation
# -----
# Create the MCMC model object
mcmc_model = MCMCmodel(planet_model, obs, layer_params)

# Create the simulation object
nwalkers = 20
mcmc_sim = MCMCsimulation(mcmc_model, nwalkers=nwalkers)

```

```
# Run the MCMC sampler
chain_length = 1000
mcmc_sim.run_mcmc(chain_length=chain_length, progress=True)

# A.5 Analyze the results
# -----
# Get the samples, discarding the first 30% as burn-in
samples = mcmc_sim.get_samples(burn_in_factor=0.3, flatFlag=True)

# Print the median and 1-sigma uncertainties for each parameter
param_names = mcmc_model.get_params_names()
for i, name in enumerate(param_names):
    mcmc = np.percentile(samples[:, i], [16, 50, 84])
    q = np.diff(mcmc)
    print(f"{name}: {mcmc[1]:.2f} (+{q[1]:.2f} / -{q[0]:.2f})")

# The 'corner' library can be used for plotting
# mcmc_sim.plot_samples(burn_in_factor=0.3);
```

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D.4.2. WALKTHROUGH OF THE EXAMPLE

1. **Define Structure:** First, we define the physical structure of our target body. We set its global radius using `GlobalProperties`. Then, we define three layers (core, mantle, crust) using `LayerProperties`. Note that the mantle's thickness (`d`) is omitted; the `Planet` class will automatically calculate it as $R - d_{\text{core}} - d_{\text{crust}}$. These are used to create a `Planet` instance, our forward model.
2. **Define Observations:** We create an `Observables` object to hold the data we want to match. In a real scenario, these would be measured values. For this test, we use our `planet_model` to generate "true" values for mass and MoI, then add them to the `obs` object with some simulated noise and uncertainty.
3. **Define Sampling Parameters:** We use the `LayerParams` class to specify which model parameters we want the MCMC to investigate. Here, we add the densities (`rho`) of the core and crust. For each parameter, we provide an initial guess and, crucially, a (`min`, `max`) tuple that defines the uniform prior probability distribution for that parameter.
4. **Run Simulation:** We assemble the components into an `MCMCmodel` object. This object encapsulates everything needed to calculate the log-probability for a given set of parameters. We then create an `MCMCsimulation` instance, providing the model and specifying the number of walkers. The `run_mcmc` method is called to start the sampling process for 1000 steps.
5. **Analyze Results:** After the simulation, we use the `get_samples` method to retrieve the results. We discard the first 30% of the chain as the "burn-in" phase, where walkers are still converging on the high-probability region. The resulting `samples`

array contains the posterior probability distribution for our parameters (`core_rho` and `crust_rho`). We then calculate and print the median (50th percentile) and the 16th-84th percentile range (the 1-sigma credible interval) for each parameter. This gives us the most probable value and its uncertainty as constrained by the data.

D.5. FUTURE WORK AND LIMITATIONS

The `geoMCMC` library, in its current form, is considered a beta release. While it provides a robust framework for the geophysical problems it is designed to solve, there are several avenues for future development that would significantly enhance its capabilities and broaden its applicability.

The primary area for expansion is the inclusion of additional forward models. Future work will focus on integrating models for thermal evolution, magnetic field generation, and more complex rheological behaviors. Incorporating these models will allow for the inclusion of a wider range of geophysical observations, such as heat flow data and magnetic field measurements, leading to more comprehensive and tightly constrained models of planetary interiors.

A significant current limitation is that the forward models assume a spherically symmetric body. A major goal for future development is to remove this constraint, allowing for the modeling of laterally varying structures and non-hydrostatic shapes. This will be crucial for accurately modeling bodies like Mars or the Moon, where large-scale asymmetries are a key feature.

Finally, the current implementation only allows for the definition of uniform (box) priors for the MCMC parameters. Future versions will extend this functionality to allow for more flexible prior definitions, including Gaussian, logarithmic, and other forms of probability distributions. This will provide researchers with greater control over the Bayesian inference process, allowing them to incorporate more nuanced prior knowledge into their models.

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