Admissible geometrical domains and graphic statics to evaluate constitutive elements of structural robustness

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Abstract

Structural robustness is a matter of insensitivity to local failure, which is linked to the ability of force redistribution. This paper explores how geometrical solution domains directly attached to form and force diagrams can characterize load path alternativeness and provide qualitative, yet relevant indicators of constitutive elements of structural robustness.

Since most methods proposed today for assessing the robustness of structures are based on probabilistic approaches, they are of limited interest for the design phase. Of the few approaches based on a deterministic formulation, all provide a type of survey that is based on an in-depth analysis of the structure once it has been designed, according to specific scenarios. A central challenge is therefore to manage the issue of robustness earlier during the design process, or to be able to interact with a structural model in order to amend its robustness. Geometric solution domains are well suited for early design exploration since they only build on abstract load path networks. This paper explores how indicators linked to the area of these domains can summarize the ability of structures to redistribute forces. Based on case studies, this quantification is then compared with indices of deterministic and energetic criteria currently proposed in the literature.

1. Introduction

Most methods proposed today for assessing the robustness of structures are based on probabilistic approaches [1]. Of the few that have adopted a deterministic formulation, all provide a type of survey that is based on an in-depth analysis of the structure once it has been designed, according to specific scenarios. A central challenge in structural design is to manage the issue of robustness earlier in the design process, or to be able to interact with a model of the future structure in order to adjust features of robustness. This paper contains an overview of a geometrical approach for evaluating constitutive elements of structural robustness. This research is linked to modelling methods and the analysis and refinement of structural networks using almost exclusively geometrical tools, even if they are implemented by means of computers and dynamic geometry. This is a way of simplifying analyses and making them more visual, enabling the designer to interact with the structure during the early stages of its design.

The paper begins with a literature review of methods to assess robustness issues and explains how they have been interpreted in the context of the geometrical approaches taken. It then introduces the geometrical methods that provide the origin of this geometrical approach to robustness. Two case studies are then presented to highlight the major features of the geometrical results when considering undamaged and damaged structures. Elements of deterministic and energetic approaches are explored and compared with the geometrical assessment. The paper finally provides conclusions and future recommendations with regard to the advocated approach.

2. The issue of robustness in the literature

A series of methods are proposed in the literature to characterize robustness [1][3]. Four major approaches can be identified: risk-based, probabilistic, deterministic and energetic approaches. Risk-based approaches and probabilistic approaches are adopted by specialists and require very specific methods. Probabilistic approaches can be evaluated by a reliability-based index linked to redundancy that compares the probability of collapses as in Frangopol & Curley [4]:

$$RI = \frac{P_{f(damaged)} - P_{f(intact)}}{P_{f(intact)}} \quad (1)$$

Risk-based approaches are based on a comparison of direct and indirect risks [5]:

$$I_{rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \quad (2)$$

They are said to be of limited practical interest [3]. This is certainly the case at the design stage. Deterministic and energetic approaches provide indicators produced by structural analyses. They will be used below as references in the assessment of two case studies.

2.1. Deterministic approaches

A deterministic approach is proposed by Frangopol and Curley [4] as the application of a reserve strength factor based on the *Residual Influence Factor* used in the offshore industry. It compares the structural capacity of intact and damaged structures where an element has been completely damaged. A simple way of appropriating this approach is to compare the load capacity of damaged and intact structures according to chosen scenarios:

$$R = \frac{L_{intact}}{L_{intact} - L_{damaged}} \quad (3)$$

2.2. Energetic approaches

Energetic approaches classically consist in calculating the deformation energy (work of failure) of a structure led to collapse [6]. It consists in integrating the space below the curve that characterizes the strain-stress relationship of the structure up to the collapse point. The energetic approach considered in this paper is slightly different and is adapted from the deterministic approach in Starossek and Haberland [7]:

$$R = 1 - max_j\ (E_{r,j}/E_{f,k})$$

where $E_{r,j}$ is the energy released by the initial failure of an element j and available for the damage of the next structural element k, and $E_{f,k}$ is the energy required for the next structural element k to fail. The appropriation of the method in this study consisted in dividing the structure into its elements. The most fragile element is researched as being the one that reduced stiffness the most. For a model made of bars, the stiffness matrix is calculated and $K\,x = f$ is solved. The deformation energy of the system is $1/2\ x^T K\ x$. The structural elements i considered in the scenarios are removed and $K_i\ x_i = f$ is calculated. The difference in deformation energy in each structural element is calculated with the deformation energy before and after the element is removed.

3. Geometrical domains of available equilibriums

The originality of the approach depicted in this paper mainly lies in its use of graphical representations of solution spaces within reciprocal diagrams. The following sub-sections introduce these two concepts.

3.1. Maxwell's reciprocal diagrams

Reciprocal diagrams, as formalized by Maxwell [8], are networks of connected segments satisfying the following rules (Fig.1): (a) each segment in a diagram is related to one unique "reciprocal" segment in the other diagram such that the difference in angle between them is always the same (a common practice is to choose an angle equal to zero so that both segments of a same pair are parallel); (b) all the segments that connect one point in a diagram have reciprocal segments that form a closed polygon in the other diagram.

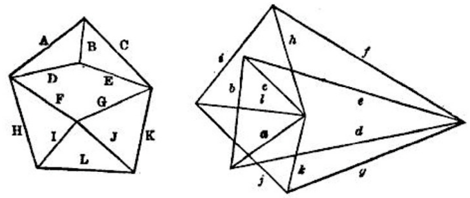


Fig.1. Two reciprocal diagrams [8]. Pairs of segments are tagged with the same letter

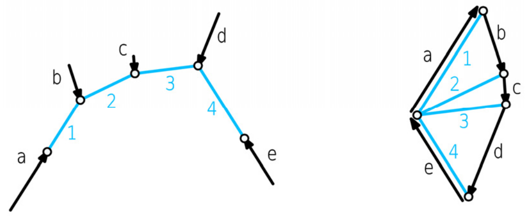


Fig.2. A form diagram on the left and its reciprocal force diagram on the right

Graphic statics [9][10][11] builds on two reciprocal diagrams: a "form diagram" that represents the geometry of a network of bars in compression and tension, and a "force diagram" in which the length of every segment is equal to the force magnitude of its reciprocal segment (a bar or an applied load) in the form diagram (Fig.2). The reciprocal rule ensures that the form diagram is in static equilibrium. Indeed, every closed polygon in the force diagram ensures that the vectorial sum of the forces acting on a specific point in the form diagram is zero.

3.2. Graphical solution spaces

Constraint-Based Graphic Statics [2][12] is a recent development in graphic statics. If both diagrams are built parametrically, geometric constraints can be applied on every node in order to control the range of possible equilibriums. For instance, the position of a node in the form diagram can be limited by constraining this point inside a bounding box (Fig.3, left). Furthermore, the force magnitude of a bar can be limited in the force diagram by compelling one extremity of a segment to remain inside a circle of a given radius (equal to the scaled maximum magnitude) and centered on the other extremity (Fig.3, circle in the force diagram). Since the only variables are nodes in two planes, these geometric constraints can be computationally propagated to any other node defining its parameterization. As a result, every node in the form diagram and in the force diagram will be restricted inside a graphical region that is equal to its solution space, *i.e.* the set of all positions for which no constraint applied on the diagram is violated (Fig.3).

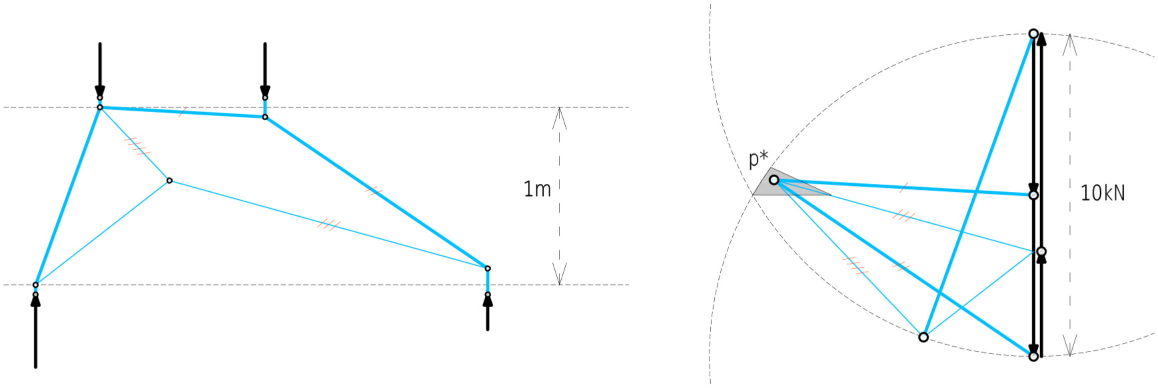


Fig.3. The shaded area in the force diagram (right) is the solution space of the node p such that the strut-and-tie network (left) is not higher than 1 meter and the magnitudes in the bars are below 10kN.*

4. Geometrical approach to robustness

Robustness is defined as "insensitivity to local failure" [7]. This definition emphasizes the capacity of force redistribution in a structure. In other words, robustness correlates with the possibility of finding alternative load paths in a structure.

Under certain conditions, the geometrical domains presented above are a convenient tool for exploring the possible redistributions of forces in a strut-and-tie model or a model made of thrust lines, and hence for characterizing its robustness. The first condition is the necessary aptitude of the structure to develop a plastic redistribution of forces so that the lower bound theorem of plastic design can be applied. Such behavior is commonly assumed for steel frames, concrete frames, arches and shear walls, masonry structures, timber with screw or threaded rods.

4.1. Presentation and methods

According to the dimensional analysis of robustness proposed by Knoll & Vogel [14][15], five dimensions of robustness are likely to concern the design more directly: strength, second line of defense, multiple load paths and redundancy, stiffness considerations and post-buckling strength.

Of the 16 strategies proposed by Knoll and Vogel, not all of them are applicable simultaneously. Some of them are related to the ductility of the structure or its constitutive elements, making a link with the theorems of plastic design that provide the scope of application of the approach presented in this paper. Others are specific to the erection of the structure, its life and maintenance, or disruptive elements to be implemented.

Implementing the dimensions related to robustness during the design – geometry and dimensioning – means (inter)acting with the design, with keystones mainly associated with designer's experience. Features linked to strength and the redistribution of forces are likely to be modelled with load paths, strut-and-tie models or thrust-line models, all belonging to a geometrical thinking.

The key idea is to associate a load path made of struts, ties and/or thrust lines to the structure. In the context of constraint-based graphic statics, allowable stresses and spatial limits are likely to be represented by geometrical constraints applied to this load path. The extent to which a node of this load path is free to move can then be seen as a measure of the model's capacity to redistribute loads. The assumption will be that the integral (in the mathematical meaning of a sum) of relevant characterizing domains represents an interactive measure of the total level of robustness.

5. Application to study cases

The analysis of the geometrical domains characterizing two structures is here performed for different scenarios of integrity: the whole intact structure, variations due to damages and variations of design geometry. The first set of scenarios referred to a comparison of the capacity of redistribution between the undamaged structure and damaged structures according to several scenarios. The second series, implementing geometrical variations, showed the influence of design choices on the capacity to redistribute loads and hence robustness. Here, the extent of geometrical domains was understood to be a constitutive dimension of the robustness. The two study cases analyze (1) a concrete shear wall with openings and (2) the *Ponte della Musica* in Rome, Italy.

5.1. Case study 1: Concrete shear wall

The first case study is adapted from the classic example developed by Schlaich, Schäfer and Jennewein [16]. The original example is a shear wall resting on its two extreme sides, including a square opening near the left support and loaded by a vertical punctual force at around 3/5 of its length. In this analysis, in order to complicate this almost trivial matter and turning it into a real structural issue, the square opening became a door at the bottom of the structure and a new rectangular opening was added to the upper part of the structure. The loading scheme included seven punctual forces of varying magnitude.

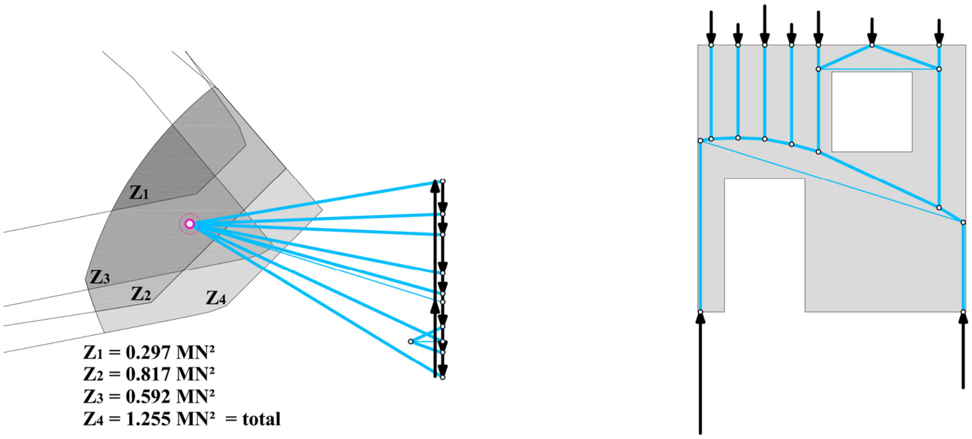


Fig.4. Shear wall and geometrical domains of the highlighted pole: damage of the column-support (Z_3), damage of the support to the right of the shear wall (Z_1), and displacement of the reactions under the wall in different parts of the wall (Z_2, Z_4). The scenario corresponding to the reaction forces as represented in the right-hand figure is linked to Z_2 for a domain of 0,817 MN².

Struts and ties are modelled inside a non-Bernouilli shear wall (Fig.4, right). The analysis of the structure as presented here is partial since it only considers a few characteristic typologies out of all the possible strut-and-tie models. The final global domain would be the sum of all the possible sub-domains for a given set of strut-and-tie models.

In this case, the set comprises the following structural mechanisms: vertical struts supported by a tied arch (Fig.4), inverted arch (Fig.5a), tree branching system (Fig.5b) and fan-like suspended load path (Fig.5c). The study of these various domains helps the modification of the structural configuration. For instance, new load paths can be allowed by adding reinforcement, which will consequently increase robustness.

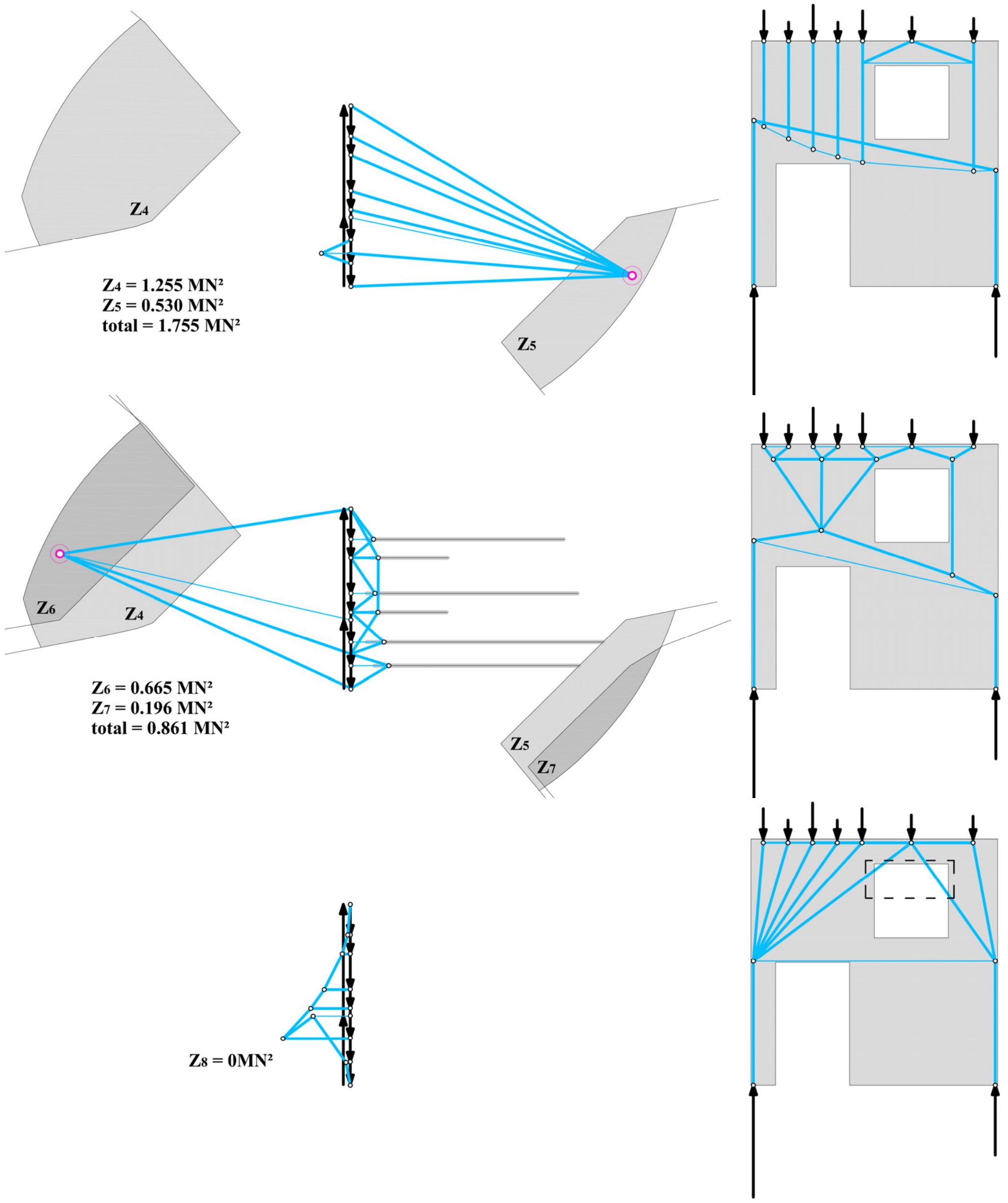


Fig.5. From top to bottom: a, b and c: domains resulting from various load-paths.

The analysis of the basic structural behavior – a funicular compressed load path – has also been tested with an increased door in order to simulate changes of symmetry of the shear wall (Fig.6). This can be considered as a design variant for the position of the edge of the door.

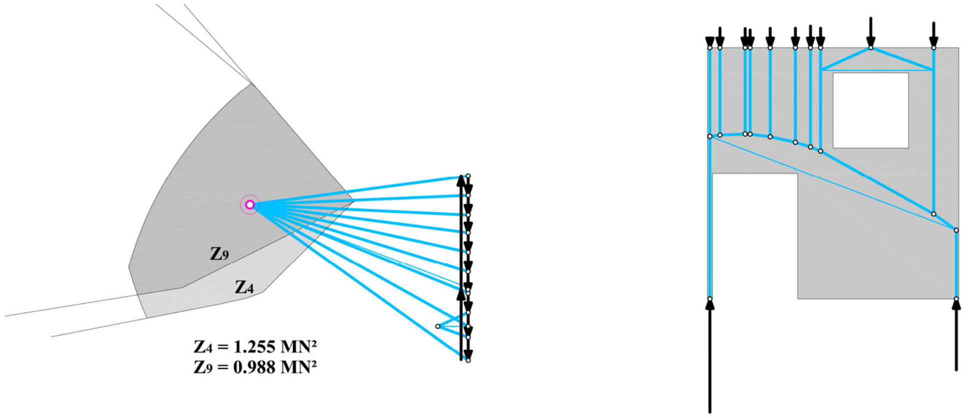


Fig.6. Shear wall and geometrical domain corresponding to the pole of the discharging arch when the initial column is replaced by a thin column (see form diagram of Fig.4 for comparison).

The geometrical analysis shows a sensitive reduction of the possible variations of the pole in the case of a damaged support (Z_3 only in Fig.4) corresponding to 0,592 MN² compared to an undamaged arrangement (Z_4 in Fig.4). The geometrical Z_4 domain characterizes the possibility of finding variations in the drawing of the load path directing the forces to the support.

If for instance the left column disappears, the domain moves. Similarly, if the width of the left column is modified (Fig.6), the possible base geometrical domain of Fig.4 is displaced. The domain will be reduced in comparison to the base scenario Z_4. It is therefore a worse option for a robustness-oriented design. Diagrams may also show the cases where there is no solution according to the design constraints, or where the existence of a solution requires an increase in the magnitude of forces (constrained by the circular boundaries) and therefore a revision of the dimensioning.

In summary, a correlation can be shown between the aptitude of the structure to redistribute forces – comprised as an indicator of some constitutive dimensions of the robustness – and a geometrical characterization of the permitted variations of the position of nodes constituting a strut-and-tie modelling of the structure.

5.2. Case study 2: Ponte della Musica

The *Ponte della Musica* in Rome (Fig.7) was built in 2011 in Italy by the architect Kit Powell-Williams and engineers C. Lotti & Associati and BuroHappold. It is a hybrid typology between a steel arch bridge and a bow-string bridge with a clear span of 130m. The hangers are made of rigid steel profiles moving forwards towards the longitudinal central symmetrical axis of the structure. The bridge is used as a footbridge, but is likely to be used to carry buses and trams as well. The exercise here consists in simulating the possible redistributions of forces in the arch according to different support conditions. Finally it compares these redistributions with those allowed when some of the hangers sustaining the deck are damaged.

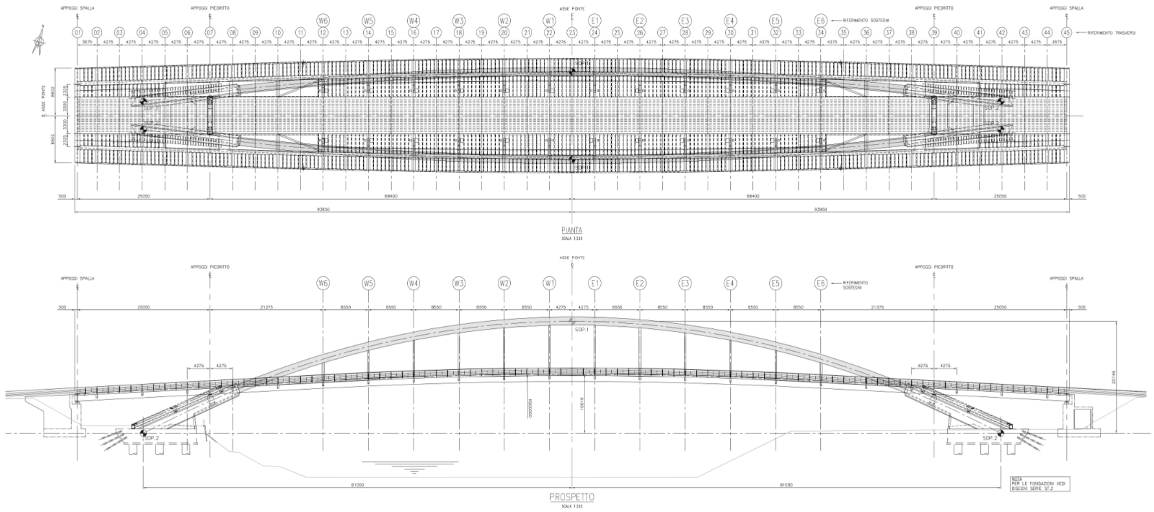


Fig.7. Ponte della Musica, Rome 2011, Powell-Williams, Lotti & Associati and Buro Happold

The bridge's bending strength is first analyzed. Bending forces are modelled with thrust lines. Magnitudes of bending forces are equal to the product of the axial compression forces by their eccentricity to the center of gravity of each section of the arch (Fig.8). Allowable eccentricities of thrust lines – i.e. variations of bending forces – are likely to be represented as a geometrical domain. Using graphic statics, the geometry of a thrust line is actually defined by a single point in the force diagram. The domain of this point consequently informs all the possible configurations of bending strength. Other domains are then generated for altered structures in which hangers are damaged. In the first instance, the structural collaboration between hangers and the arch is neglected.

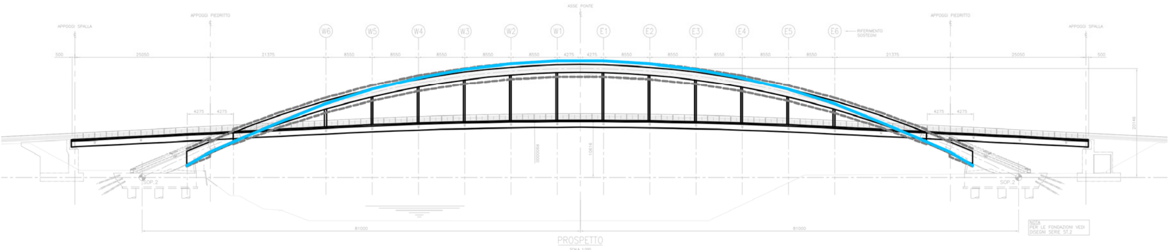


Fig.8. Ponte della Musica: bending strength shown in grey, with a possible thrust line (in blue) if bending forces in the support are limited.

The result of the analysis (Fig.9) shows a domain equal to 4,50 MN² (Z_1) for the pole of the thrust line, defining the extent of possible geometries for the load path in the case of symmetrical loading. In the case of damage applied to four central hangers (Z_2), this domain is reduced by 30 % to 3,15 MN², but still allows multiple load paths – and in the case of damage applied to three lateral hangers (Z_3), this domain is reduced by 38 % to 2,77 MN². Under asymmetrical loadings, the intact structure (Z_4) has the ability to redistribute load paths to an equivalent of 2,78 MN² (62 % of the symmetrical reference), *i.e.* less than the reference maximum symmetrical loading. Where the four central hangers are damaged (Z_5), the domain becomes 1,73 MN² (62 % of the asymmetrical reference and 38 % of the symmetrical reference) and in the case of damage applied to three lateral hangers (Z_6), this domain is 2,76 MN² (99 % of the asymmetrical reference and 61 % of the symmetrical reference).

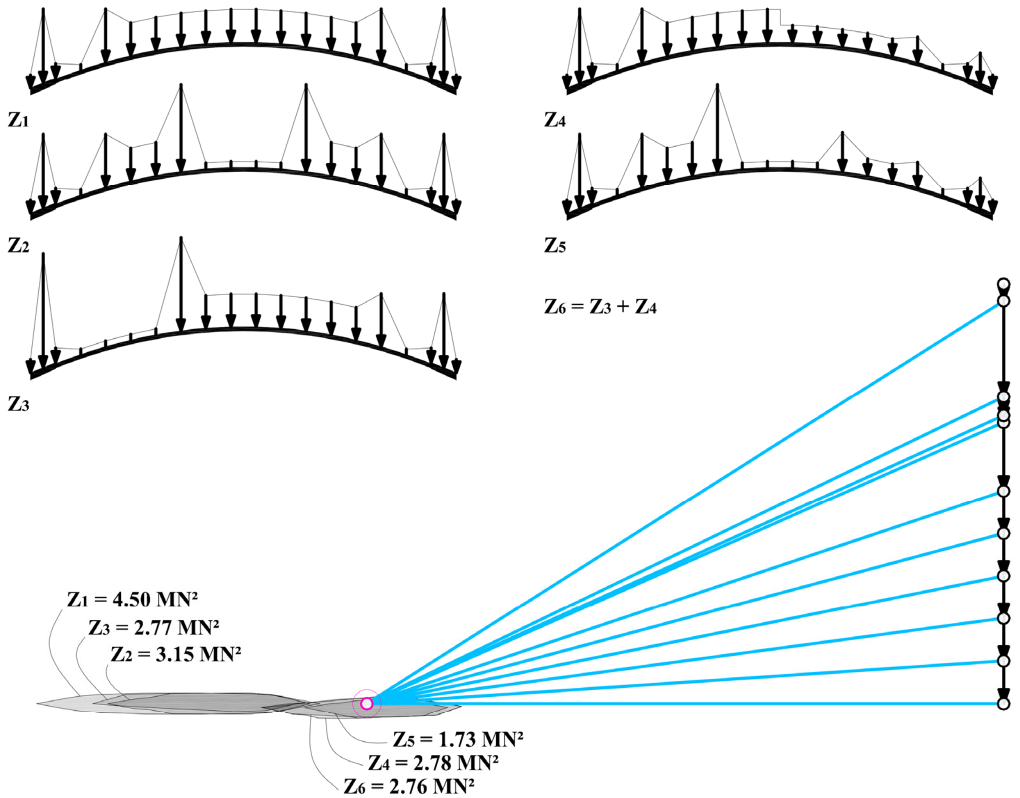


Fig.9. Ponte della Musica, domains for different loadings and levels of damage: intact with symmetrical load (Z_1) and with asymmetrical load (Z_4); 4 central hangers removed with symmetrical load (Z_2) and with asymmetrical load (Z_5); 3 lateral hangers removed with symmetrical load (Z_3) and asymmetrical load (Z_6).

The slightly different position of this asymmetrical domain demonstrates the ability of the structure to redistribute bending forces on both sides of the arch, enabling the reduction of the domain to be less significant.

6. Comparison of deterministic and energetic approaches to robustness

6.1. Deterministic approach: Ponte della Musica

A simulation is undertaken for the six previous cases in order to obtain the ratio of maximal service loadings that would lead to the failure of the structure. The structure is modelled as an arch for a maximum bending strength of 32,2 MN.m (considering the axial compression force in the arch) in steel tubes and 24,9 to 40,6 MN.m in the concrete bases of the arch (depending on the section and its reinforcement). In a real scenario, redistributions between the bending forces in the supports and those in the central part of the arch are allowed. For that reason, progressive variations of stiffness in the supports are introduced (brackets in the table below) to optimize the distributions of maximal bending forces between arch and supports. To be consistent in the analysis and with real occurrences in the structural behavior, a slight asymmetry is introduced to the *symmetrical* loading applied on the symmetrical structure. The reason is due to the fact that in an arch under symmetrical loading, the structural behavior based on compression forces is dominant and maximizes the strength capacity far beyond asymmetrical scenarios, where bending forces are governing. This ideal symmetrical scenario is likely to be challenged by geometrical defects, asymmetrical settings or temperature loading. The results are given below:

Multiplying coefficient	Symmetrical Q	Asymmetrical Q
Intact	6,15	1,79 (15)
4 central hangers damaged	2,93	1,23 (29)
	1,71 (8)	1,40 (49)

The factor for the symmetrical loading still remains large, leading to a singular comparison between the different cases. In the asymmetrical case, if the 31 % reduction from the four central hangers damaged to the undamaged structure is taken as a temporary reference, it corresponds to a reduction of 30 % in the geometrical approach (symmetrical loads). However results from these analyses depend on scenarios that varies according to the yielding hypothesis. The comparison between the various indices obtained in the geometrical and deterministic approaches therefore appears to be further questionable.

6.2. Energetic approach: *Ponte della Musica*

The energetic approach used here is a stiffness-based measure of robustness expressed in energies computed as described above in paragraph 2.2. The cases and the energies obtained for maximal service loadings are:

Energies [MJ]	Symmetrical Q	Asymmetrical Q
Intact	0,540	0,437
4 central hangers damaged	0,663	0,543
3 lateral hangers damaged	1,827	1,936

The analysis reveals magnitudes that are larger in the case of asymmetrical damage: it is likely to happen since this case presents deformations that become far larger than in symmetrical scenarios and it directly impacts the result of the analysis that is based simultaneously on forces and deformations.

If forces in the yielding region are to be considered (using the multiplier coefficient from the deterministic approach), energies obtained for yielding loading (as computed in 6.1) become:

Energies [MJ]	Symmetrical Q	Asymmetrical Q
Intact	20,42	1,40
4 central hangers damaged	5,70	0,82
3 lateral hangers damaged	5,34	3,79

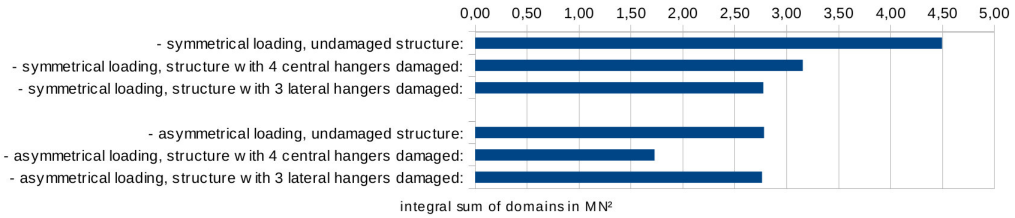
In both cases, magnitude of asymmetrical damaged structures are larger than symmetrical scenarios, which leads to question the relevance of this kind of indicator.

During the service phase, the total asymmetric loading is half the symmetric total loading. For a similar total loading, energies in the asymmetric case would be greater. As has been seen in 6.1, the damage occurs earlier if loading is asymmetrical, so the work is reduced. A comparison between the undamaged and damaged bridges shows that the energy of deformation is greater in the latter case, since there would be greater deformations between the start point and the point where the failure occurs. Again, it is observed that both measures – the geometrical domain approach and the energetic approach – are not directly correlated, as they are not correlated to the deterministic approach shown in 6.1. Nevertheless, a similar evolution between different measures can be observed, sometime inverted but still existing, at variable scales. The impact of the asymmetrical loading is observed for the two approaches, but to a rather different extent. Indeed, one analysis refers to the complete structure (energetic approach) and the other to the extent of the possible redistribution of load paths. Similarly, the parallel evolution of the indicators in both approaches cannot be sustained further or more closely.

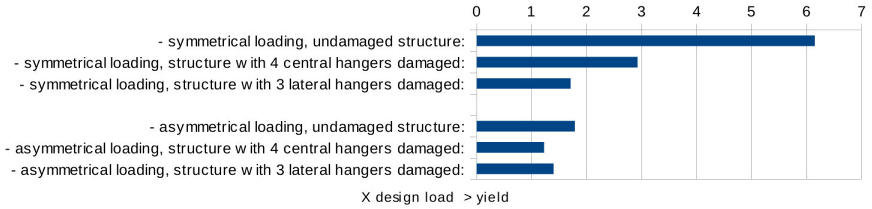
7. Comparison through direct results and indices

7.1. Comparison of results

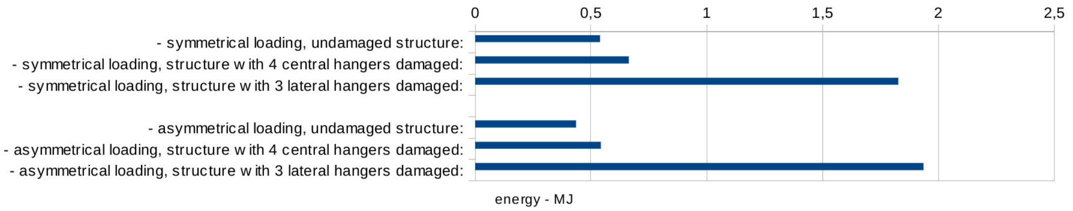
geometric approach



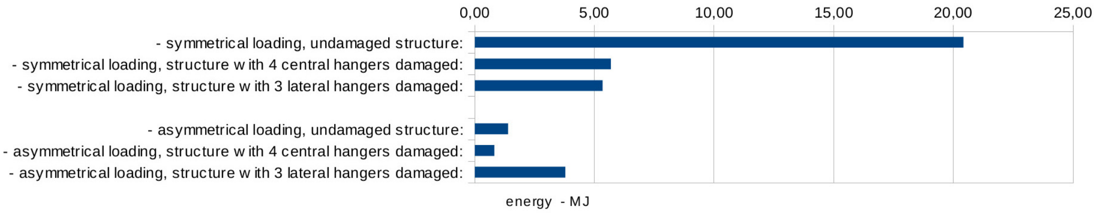
deterministic approach



energetic approach (service state)



energetic approach (yield state)



Four types of values characterizing the robustness are here analyzed: a geometrical approach, a deterministic approach and two energetic approaches, one assessing the service state and the other assessing the state of yielding. These approaches compare very different types of magnitudes. The geometrical approaches compare only forces, redistributing load paths while loadings are maintained; the deterministic approach considers only forces, with load paths unchanged and loadings multiplied to reach yielding; energetic approaches consider forces and displacements, with load paths unchanged, one during service state, and the other with loadings required to reach the yielding state. They produce different indicators whose relevance depends on what dimension exactly has to be investigated.

7.2. Comparison through indices

Two families of indices are proposed in the literature:

Type A: $I[1\ldots\infty]=\dfrac{X_{intact}}{X_{intact}-X_{damaged}}$ according to Frangopol and Curley [4] or Baker, Schubert and Faber [5] where X is the value to be compared. When energies growing with damage are considered, $X_{damaged}-X_{intact}$ replaces the denominator.

Indices of robustness		SLS Geometric	SLS Energetic	Yield limit Energetic	Yield limit Deterministic
Symmetrical loading	4 center damaged	3,35	4,37	1,39	1,91
	3 side damaged	2,61	0,42	1,35	1,39
Asymmetrical loading	4 center damaged	2,64	4,10	2,42	3,20
	3 side damaged	148,81	0,29	-0,58	4,59

Type B: $I[0\ldots1]=1-\dfrac{X_{intact}-X_{damaged}}{X_{intact}}$ according to Frangopol and Curley [4] with X the values to compare. When energies are considered the numerator becomes $X_{damaged}-X_{intact}$.

Indices of robustness		SLS Geometric	SLS Energetic	Yield limit Energetic	Yield limit Deterministic
Symmetrical loading	4 center damaged	0,70	0,77	0,28	0,48
	3 side damaged	0,62	-1,38	0,26	0,28
Asymmetrical loading	4 center damaged	0,62	0,76	0,59	0,69

Proportional indices present the advantage of producing dimensionless magnitudes within a same system of reference. Each index can show how each scope of assessment impacts the measure of the robustness when the structure is damaged. Comparisons between indices therefore prove to be theoretically correct.

Firstly, it can be observed that some calculated magnitudes belong to a forbidden range of values. Indeed, since some deformations become larger under asymmetric damage, the global energy of deformations increases greatly whereas the robustness itself does not. The reason is that the structural behavior moves from a stiff compression behavior towards a bending behaviour that implies wide deformations, but not necessarily high redistributions of forces through a change of geometry.

Secondly, similar evolutions of indices can generally be observed between the various approaches. This variation indicates the relevance of such indices although they still compare different dimensions of structural robustness.

It can be summarized that the various indices proposed above are not interchangeable – and therefore do not need to converge – but when they prove to be similar, their evolution among equal scenarios attest the relevance of the assessments proposed above.

8. Conclusions and perspectives

This paper presents a geometrical approach to evaluate constitutive elements of structural robustness and compares it to other indices from the literature. An analysis applied on two case studies exemplifies its relevance. The geometrical approach proves to be of interest during the design phase since it provides a qualitative summary of possible load path redistributions while no data other than the geometry and magnitudes of a preliminary strut-and-tie network is needed. By enabling scenarios of loading and damage to be compared together, it provides a useful tool for designers, especially during the conceptual stage.

The analysis shows that the set of indices used in the literature is highly heterogeneous since they relate different magnitudes. The geometrical index proposed here is no exception. However it has been shown that, if put in the form of dimensionless indices, they present generally similar variations for a given load and damage condition even though these variations are of different magnitude. This comment highlights an issue for codes that have to propose universal methods to assess robustness.

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