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# ADAPTIVE LINEARIZING CONTROL OF NON-ISOTHERMAL REACTORS

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Abstract. This paper deals with the control of the temperature in non-isothermal stirred tank reactors involving M ($M \geq 1$) chemical reactions. The proposed control algorithm is an input-output linearization controller which accounts for the well-known process nonlinearities. The control scheme is also adaptive in order to deal with the parameter uncertainty : it is shown how to include explicit on-line estimates of the M activation energies within the adaptive linearizing controller. The performance of the control law is illustrated in simulation.

Keywords : adaptive control, nonlinear systems, control applications, chemical industry, CSTR

## 1. INTRODUCTION

The dynamics and control of exothermic reactors have been extensively studied over the past decades (e.g. Aris, 1969, Uppal et al, 1974, 1976; Ray, 1981). One important feature of exothermic reactors is their strongly nonlinear dynamics : beside the presence of the classical bilinear terms, they exhibit e.g. exponential nonlinearities due to Arrhenius law's dependence of the temperature. One consequence of the nonlinear dynamical structure of the process is the possible existence of multiple equilibrium points, i.e. of different sets of state variables for one set of input variables at steady-state. It is well-known that, for instance, for a reactor with one exothermic reaction, there may be up to three equilibrium points for the same set of inputs. Moreover, each of these points may present different stability characteristics, some being stable and others being unstable.

It is obvious that, because of the presence of nonlinearities and of potentially unstable characteristics, the control of exothermic reactors requires a careful design. A commonly used approach for the control of nonlinear systems is to consider them as time-varying linear systems and to use black-box linear approximate models to implement an adaptive control law. Several applications of linear adaptive controllers to exothermic reactor are reported in the literature (e.g. Harris et al., 1980; Hallager and Jorgensen, 1983). However, since these control schemes are based on a linearized tangent model of the process dynamics around one defined set point, only local closed loop stability properties can be emphasized which may no longer hold if the process is "moving away" from that particular set point.

On the other hand, since the underlying process is nonlinear, one can expect improved control characteristics and performance by taking advantage of the nonlinear structure of the process. Recent developments in nonlinear system theory and its application to automatic control have given rise to linearizing control schemes based on differential geometry concepts (e.g. Isidori, 1989). In Hoo and Kantor (1985) and Limquesco and Kantor (1990), a full state feedback linearization solution is presented for an exothermic reactor with one reaction. However, the computation of full state linearization controllers may be very much involved for systems of order larger than two.

In this paper, an input-output linearization control algorithm is proposed which does not require the full linearization of the process dynamics and whose computation remains rather simple. Moreover, it is general in the sense that it is applicable to any exothermic CSTR (Continuous Stirred Tank Reactors) involving M reactions ($M \geq 1$).

Since feedback linearization techniques are based on an exact cancellation of the process nonlinearities, they may be very sensitive to modelling uncertainties. One solution consists of adding integral action to the linearizing controller (e.g. Kravaris.and Chung, 1987). Here an adaptive linearizing control scheme is preferred since the estimation procedure already includes an integral action while giving extra potentially useful information about the uncertain process parameters. This approach is in line with different works on adaptive control of nonlinear systems (e.g. Dochain and Bastin, 1984; Taylor et al, 1989; Sastry and Isidori, 1989; Bastin and Dochain, 1990). It is also worth noting that there has been recently some interesting attempts to include the structural system nonlinearities into an adaptive controller with application to a *single exothermic reaction* (M = 1) process (Agarwal and Seborg, 1987; Slotine and Ydstie, 1989). However, in their paper, Agarwal and Seborg assume that the activation energy is known. In this paper, it is shown how to deal with the exponential Arhenius nonlinearity so as to estimate the activation energy when it is unknown (or at least not determined with certainty).

The objective of this paper is therefore to indicate how to introduce the well-known nonlinearities of exothermic processes into a nonlinear controller design which is also capable to deal with the model uncertainties in an adaptive way. The paper is organized as follows. Section 2 introduces the dynamical model of non-isothermal stirred tank reactors. Section 3 is concerned with the design of the adaptive linearizing controller. Section 4 presents simulation results.

## 2. DYNAMICAL MODEL OF STIRRED TANK REACTORS

We can define a chemical process as a set of M reactions in which N components (reactants and/or products) intervene. In the following, we shall concentrate on non-isothermal chemical processes which take place in stirred tank reactors (STR). It is obvious that these reactions can be formalized into a reaction network, as it is now illustrated.

### *Example : A Non-Isothermal STR*

Let us consider a reactor characterized by the following two sequential reactions :

$$A \rightarrow bB$$
$$B \rightarrow dD$$

In the above reaction network, for simplicity and without loss of generality, we have normalized the stoichiometric coefficients in each reaction with respect to the reactants A and B respectively. Let us assume that both reactions are non-isothermal, i.e. characterized by a nonzero heat of reaction.

### *Dynamical Model of Stirred Tank Reactors*

The dynamical model of a STR chemical process is readily obtained from mass and energy balances and can be written in the following matrix form :

$$\frac{dC}{dt} = \frac{F}{V}(C_{in} - C) + K\varphi \qquad (1.a)$$

$$\frac{dT}{dt} = \frac{F}{V}(T_{in} - T) - \frac{1}{\rho C_p}\Delta H^T \varphi - Q \qquad (1.b)$$

In equations (1), C is the reactant concentration vector (mol/l) ($\dim(C) = N$), $C_{in}$ is the influent reactant concentration vector (mol/l) ($\dim(C_{in}) = N$), $\varphi$ is the reaction rate vector (kg/m$^3$/sec) ($\dim(\varphi) = M$), K is the stoichiometric coefficient matrix ($\dim(K) = NxM$), F is the influent flow rate (m$^3$/s), V is the reactor volume (m$^3$), T is the temperature (K), $T_{in}$ is the influent temperature (K), $\rho$ is the density (kg/m$^3$), $C_p$ is the specific heat (kJ/kg/K), $\Delta H$ is a vector which contains the heat of each reaction (kJ/kmole) ($\dim(\Delta H) = M$) :

$$\Delta H^T = [\Delta H_1, \Delta H_2, \ldots, \Delta H_N]$$

and Q is the cooling heat removal rate (K/sec). Q is often written as follows (e.g. Aris, 1969) :

$$Q = \frac{hA_c(T - T_c)}{\rho C_p V} \qquad (2)$$

where h is the overall heat transfer coefficient (W/m$^2$/K), $A_c$ is the heat transfer area (m$^2$) and $T_c$ is the coolant temperature (K).

In equations (1), the reaction rate is, generally speaking, a nonlinear function of the state, i.e. of the concentrations C and of the temperature T, and can be formalized as follows :

$$\varphi = \kappa(C)E(T) \qquad (3)$$

where $\kappa(C)$ is a MxM diagonal matrix and E is a M-dimensional vector. In the important particular case when the dependence of the process kinetics obeys to the Arrhenius law, the reaction rate vector $\varphi$ specializes as follows :

$$\varphi = \begin{bmatrix} k_{01}\alpha_1(C) & 0 & \cdots & 0 \\ 0 & k_{02}\alpha_2(C) & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & 0 \\ 0 & 0 & \cdots & k_{0M}\alpha_M(C) \end{bmatrix} \begin{bmatrix} e^{-E_1/RT} \\ e^{-E_2/RT} \\ \vdots \\ e^{-E_M/RT} \end{bmatrix}$$

where $k_{0i}$ (i = 1 to M) the reaction rate constant of reaction i, $E_i$ (i = 1 to M) is the activation energy of reaction i, R is the gas constant and $\alpha_i(C)$ is a (generally nonlinear) function of the concentration of the reactants intervening in reaction i. For simplicity reasons, the rest of the paper will concentrate on Arrhenius type kinetics (although the ideas developed below can easily be extended to other forms of kinetic models).

### *Example : Non-Isothermal STR (continued)*

Assume that the kinetics of the first reaction of the above non-isothermal reactor is of second order with respect to A and that the second reaction is of first order with respect to B. Then the kinetics (with Arrhenius temperature dependence) of each reaction is equal to :

$$\varphi_1 = k_{01}C_A^2 e^{-E_1/RT}$$
$$\varphi_2 = k_{02}C_B e^{-E_2/RT}$$

It is clear that the dynamics of the process will be described by the model (1)(2)(3) via the following definitions :

$$C = \begin{bmatrix} C_A \\ C_B \\ C_D \end{bmatrix},\ C_{in} = \begin{bmatrix} C_{A,in} \\ 0 \\ 0 \end{bmatrix},\ K = \begin{bmatrix} -1 & 0 \\ b & -1 \\ 0 & d \end{bmatrix},\ \Delta H = \begin{bmatrix} \Delta H_1 \\ \Delta H_2 \end{bmatrix}$$

$$\kappa = \begin{bmatrix} k_{01}C_A^2 & 0 \\ 0 & k_{02}C_B \end{bmatrix},\ E = \begin{bmatrix} e^{-E_1/RT} \\ e^{-E_2/RT} \end{bmatrix}$$

Note that the dynamical model of the process (1) can be rewritten in the following more compact form :

$$\frac{dx}{dt} = -\frac{F}{V}x + \tilde{K}\varphi + U \qquad (4)$$

with :

$$x = \begin{bmatrix} T \\ C \end{bmatrix} \qquad (5.a)$$

$$\tilde{K} = \begin{bmatrix} -\Delta H^T/\rho C_p \\ K \end{bmatrix} \qquad (5.b)$$

$$U = \begin{bmatrix} FT_{in}/V - Q \\ FC_{in}/V \end{bmatrix} \qquad (5.c)$$

## 3. ADAPTIVE LINEARIZING CONTROL OF NON-ISOTHERMAL STIRRED TANK REACTORS

### 3.1. Formulation of the Control Problem

Let us consider the problem of controlling the temperature T at a prescribed value $T^*$ by acting on the cooling heat removal rate Q under the following conditions :

A1. M state variables are available for on-line measurement (i.e. the temperature T and M-1 concentrations); moreover, assume that the MxM submatrix $\bar{K}_1$ (constituted by the M lines of $\bar{K}$ corresponding to the measured variables) is full rank;
A2. the flow rate F, the volume V and the cooling heat removal rate Q are known, either by measurement or by choice of the user;
A3. the stoichiometric coefficient matrix K is known;
A4. the density $\rho$, the specific heat $C_p$ and the heats of reaction $\Delta H$ are known;
A5. the kinetics constants $k_{0i}$ (i = 1 ,M) are known but the activation energies $E_i$ (i = 1, M) are unknown (or, let's say, largely uncertain).

With respect to the above control problem, equation (1.b) constitutes the output dynamical equation and contains the input Q (therefore it is straightforward to note that the relative degree of the process (e.g. Isidori, 1989) is equal to one). This equation will be used to synthetize the adaptive linearizing control law.

***Remark*** : note that, under the above assumptions, the N-M unmeasured concentrations can be estimated on-line by using an asymptotic observer (Dochain et al., 1991; Bastin and Dochain, 1990) which offers the advantage of being independent of the reaction kinetics (which are here assumed to be (at least partially) unknwon and for which theoretical stability and convergence properties can be emphasized.

### 3.2. Zero Dynamics

It is a well-known result (e.g. Isidori, 1989) that, if the zero dynamics of the process is asymptotically stable, input-output linearization may result in a stable closed loop process behaviour. It is straightforward to note that here the zero dynamics corresponds to equation (1.a) in isothermal conditions. Therefore the analysis of the zero dynamics reduces to the analysis of the corresponding isothermal STR (1.a). For instance, if the kinetics are all of first order, then the stability of the zero dynamics is readily deduced from the poles of (1.a) for a fixed value of the temperature T. In case of nonlinear kinetics, sufficient stability conditions can be deduced from the poles of the linearized tangent model (e.g. Bastin and Dochain, 1990). For instance, in the above non-isothermal example, the state matrix of the linearized tangent model of the zero dynamics is equal to :

$$\bar{A} = \begin{bmatrix} -\frac{F}{V} - 2k_{01}\bar{C}_A & 0 & 0 \\ 2bk_{01}\bar{C}_A & -\frac{F}{V} - k_{02}\bar{C}_B & 0 \\ 0 & dk_{02}\bar{C}_B & -\frac{F}{V} \end{bmatrix}$$

where $\bar{C}_A$ and $\bar{C}_B$ hold for the steady-state values of $C_A$ and $C_B$. It is then straightforward to check that, for any strictly positive value of F/V, the poles of the linearized tangent model of the zero dynamics are negative and that therefore the zero dynamics are asymptotically stable.

### 3.3. Model Reference Linearizing Control Scheme

Assume that we wish to have the following stable first order closed loop dynamics :

$$\frac{de}{dt} + \lambda e = 0 \qquad \lambda \geq 0 \tag{6}$$

where e is the control error :

$$e = T^* - T \tag{7}$$

and $T^*$ is the reference signal of the temperature T. The control law can be readily obtained by combining equations (1.b) and (6) :

$$Q = -\lambda(T^* - T) - \frac{dT^*}{dt} + \frac{F}{V}(T_{in} - T) - \frac{1}{\rho C_p}\Delta H^T \varphi \tag{8}$$

More precisely, if equation (2) is used and h is considered as the control input, equation (8) takes the following form :

$$h = \rho C_p V \frac{-\lambda(T^* - T) - \frac{dT^*}{dt} + g(T,C)}{A_c(T - T_c)} \tag{9}$$

$$\text{with } g(T,C) = \frac{F}{V}(T_{in} - T) - \frac{1}{\rho C_p}\Delta H^T \varphi$$

Equation (8) (or (9)) is the Model Reference Linearizing Control Law (see also Bastin and Dochain, 1990).

### 3.4. Adaptive Model Reference Linearizing Control

In section 3.1, it has been assumed that some parameters of the model (here, the activation energies $E_i$) are unknown. The above linearizing control law (8) (or (9)) will then implemented with estimated values of $E_i$ :

$$Q = -\lambda(T^* - T) - \frac{dT^*}{dt} + \frac{F}{V}(T_{in} - T) - \frac{1}{\rho C_p}\Delta H^T \hat{\varphi} \tag{10}$$

with $\hat{\varphi} = \varphi(E_i)$, or :

$$h = \rho C_p V \frac{-\lambda(T^* - T) - \frac{dT^*}{dt} + \hat{g}(T,C)}{A_c(T - T_c)} \tag{11}$$

$$\text{with } \hat{g}(T,C) = \frac{F}{V}(T_{in} - T) - \frac{1}{\rho C_p}\Delta H^T \varphi(\hat{E}_i)$$

Since the system equations (1)(3) are *not linear in the parameters* $E_i$, some transformations are required so as to be able to use some linear regression estimation method. First of all, let us consider the dynamical equations of the M measured variables (the temperature T and the M-1 concentrations) and define the following state partition :

$$x^T = [x_1 \; x_2]$$

where $x_1$ and $x_2$ contain the M measured and N-M unmeasured state variables, respectively. The dynamics of $x_1$ is clearly described by the following equations :

$$\frac{dx_1}{dt} = -\frac{F}{V}x_1 + \bar{K}_1\varphi + U_1 \tag{12}$$

Since $\bar{K}_1$ is full rank, it can be upper triangularized by using the Gauss method. This defines a state transformation :

$$\xi = Px_1 \tag{13}$$

and the system equations can be rewritten with the new coordinates $\xi$ as follows :

$$\frac{d\xi}{dt} = -\frac{F}{V}\xi + K_T\varphi + U_T \qquad (14)$$

where $K_T = P\tilde{K}_1$ (upper triangular) and $U_T = PU_1$. Let us now discretize the above equations by using a first order Euler approximation (with $\Delta t$ the sampling period and t the time index) :

$$\xi_{t+1} = \xi_t - \frac{F_t\Delta t}{V}\xi_t + \Delta t K_T\varphi_t + \Delta t U_{Tt} \qquad (15)$$

Let us rewrite the last equation (corresponding to $\xi_M$) by isolating the kinetic term on the right hand side :

$$\xi_{Mt+1} - \xi_{Mt} + \frac{F_t\Delta t}{V}\xi_{Mt} - \Delta t U_{Tt} = \varphi_{Mt}$$
$$= k_{0M}\,\alpha_M(C_t)e^{-E_M/RT_t}$$

Let us now take the logarithm of both sides of the above equation (which, from assumption A5, contains only one unknown, i.e. $E_M$) :

$$\log\left\{\xi_{Mt+1} - \xi_{Mt} + \frac{F_t\Delta t}{V}\xi_{Mt} - \Delta t U_{Tt}\right\}$$
$$= \log\{k_{0M}\,\alpha_M(C_t)\Delta t\} - \frac{1}{RT_t}E_M \qquad (16)$$

or, in a classical linear regression form :

$$y_{Mt} = \varphi_t E_M \qquad (17)$$

with :

$$y_{Mt} = \log\left\{\frac{k_{0M}\,\alpha_M(C_t)}{\frac{\xi_{Mt+1} - \xi_{Mt}}{\Delta t} + \frac{F_t}{V}\xi_{Mt} - U_{Tt}}\right\}$$

$$\varphi_t = \frac{1}{RT_t}$$

Equation (17) is linear-in-the-parameter $E_M$. We can proceed similarly for the other M-1 equations by isolating the $j^{th}$ kinetic term in the $j^{th}$ (j =1 to M-1) equation and then applying the logarithmic transformation :

$$\log\left\{\xi_{jt+1} - \xi_{jt} + \frac{F_t\Delta t}{V}\xi_{jt} - \Delta t U_{Tjt} - \Delta t\sum_{i=j+1}^{M} k_{Tji}\varphi_{it}\right\}$$
$$= \log\{k_{0j}\,\alpha_j(C_t)\Delta t\} - \frac{1}{RT_t}E_j \qquad j = 1 \text{ to } M-1 \qquad (18)$$

Equation (18) can be rewriiten in a more linear regression form :

$$y_{jt} = \varphi_t E_j \qquad (19)$$

$$\text{with } y_{jt} = \log\left\{\frac{k_{0j}\,\alpha_j(C_t)}{\frac{\xi_{jt+1} - \xi_{jt}}{\Delta t} + \frac{F_t}{V}\xi_{jt} - U_{Tjt} - \sum_{i=j+1}^{M} k_{Tji}\varphi_{it}}\right\}$$

Note that now each $j^{th}$ (j = 1 to M) equation is linear-in-the parameter $E_j$. For the on-line estimation, we can proceed backward from the last equation to the first one at each time t. Each $j^{th}$ unknown parameter $E_j$ is estimated by using a linear regression estimation method based on the $j^{th}$ equation and incorporating the values of the i = j+1 to M already estimated parameters (appearing in the left hand side of (18)), e.g. a recursive least squares estimation scheme (e.g. Bastin and Dochain, 1990; Shah and Cluett, 1991) :

$$\hat{E}_{jt+1} = \hat{E}_{jt} + \Gamma_t\varphi_t[y_t - \varphi_t\hat{E}_{jt}] \qquad (20.a)$$

$$\Gamma_t = \frac{\Gamma_t}{\gamma}\left(1 - \frac{\Gamma_t\varphi_t^2}{\gamma + \Gamma_t\varphi_t^2}\right) \qquad \Gamma_0 > 0,\ 0 < \gamma \le 1 \qquad (20.b)$$

where $\Gamma_t$ is the estimator gain and $\gamma$ is a forgetting factor.

The combination of equation (10) (or (11)) with the estimation scheme (20) based on equations (16)(18) constitutes the Adaptive Model Reference Linearizing Control Law.

***Remark*** : for simplicity reasons, the estimation has concentrated here on the activation energies. But it is obvious that the above representation (16)(18) can be used to also estimate on-line the kinetics constants $k_{0i}$ (i = 1, M). The main problem consists of being able to guarantee the persistence of excitation (e.g. Bastin and Dochain, 1990) of the regressor vector $[\log(\alpha_j(C_t)\Delta t, 1/RT_t]^T$ which may not be a trivial question because of the nonlinear characteristics of the process.

***Example : Non-Isothermal STR (continued)***

Assume that T and $C_A$ are available for on-line measurement, i.e. $x_1 = [T \quad C_A]^T$. The matrix $\tilde{K}_1$ :

$$\tilde{K}_1 = \begin{bmatrix} \Delta H_1 & \Delta H_2 \\ -1 & 0 \end{bmatrix}$$

is full rank if $\Delta H_2 \neq 0$. The state transformation which upper triangularize $\tilde{K}_1$ is defined as follows :

$$\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = P\begin{bmatrix} T \\ C_A \end{bmatrix} = \begin{bmatrix} 1/\Delta H_1 & 0 \\ 1/\Delta H_1 & 1 \end{bmatrix}\begin{bmatrix} T \\ C_A \end{bmatrix}$$

In discrete-time and after the application of the above state transformation and the logarithmic transformation, the discrete-time equations used for the on-line estimation of the unknown activation energies $E_1$ and $E_2$ are written in terms of the physical variables T and $C_A$ as follows :

$$\log\left\{\frac{\Delta H_1 k_{01} C_{At}^2}{\rho C_p(\psi_{1t} + \psi_{2t})}\right\} = \frac{1}{RT_t}E_1 \qquad (21.a)$$

$$\log\left\{\frac{\Delta H_2 k_{02} C_{Bt}}{\rho C_p(\psi_{1t} + \psi_{3t}\Delta H_1)}\right\} = \frac{1}{RT_t}E_2 \qquad (21.b)$$

with :

$$\psi_{1t} = \frac{T_{t+1} - T_t}{\Delta t} - \frac{F_t}{V}(T_{in} - T_t) + Q_t$$

$$\psi_{2t} = \frac{\Delta H_2}{\rho C_p}k_{02}C_{Bt}e^{-E_2/RT_t}$$

$$\psi_{3t} = \frac{C_{At+1} - C_{At}}{\Delta t} - \frac{F_t}{V}(C_{Ain} - C_{At})$$

In a first step, equation (21.b) is used for the estimation of $E_2$ and in a second step, equation (21.a)(which

contains $E_2$) is used for the estimation of $E_1$.

## 4. SIMULATION RESULTS

The non-isothermal stirred tank reactor example presented above has been simulated under the following conditions (based on data from Kravaris and Chung, 1987) :

$b = d = 1$, $k_{01} = 4$ l/mol/sec, $k_{02} = 172.2$ sec$^{-1}$,
$E_1 = 2.09 \times 10^4$ kJ/kmol, $E_2 = 4.18 \times 10^4$ kJ/kmol,
$\Delta H_1 = -4.18 \times 10^4$ kJ/kmol, $\Delta H_2 = -8.36 \times 10^4$ kJ/kmol,
$T_{in} = 300$ K, $R = 8.3143$ kJ/kmol/K,
$h = 1.3$ W/m$^2$/K, $A_c = 170$ m$^2$, $T_c = 300$K,
$F = 0.1$ m$^3$/sec, $V = 10$ m$^3$,
$C_{A,in} = 10$ mol/l, $\rho = 1000$ kg/m$^3$, $C_p = 1$ kJ/kg/K

The temperature T and the concentration of the reactant A, $C_A$, are assumed to be measured on-line. We also assume that the concentration of the reactant B, $C_B$, is not measured on-line : its value is given by an asymptotic observer (for a general approach on the asymptotic observer in chemical reactors, see Dochain et al., 1991). The asymptotic observer in our example is derived as follows.

Let us define the auxiliary variable z :

$$z = T - \frac{\Delta H_1 + \Delta H_2}{\rho C_p} C_A - \frac{\Delta H_2}{\rho C_p} C_B$$

Let us calculate the time derivative of z by using equations (1) :

$$\frac{dz}{dt} = -\frac{F}{V} z + \frac{F}{V}\left(T_{in} - \frac{\Delta H_1 + \Delta H_2}{\rho C_p} C_{Ain}\right) - Q$$

This equation is independent of the kinetics (which have been assumed to be unknown (assumption A5)). Under assumptions A2-A4, the above equation can be used to compute z and therefore to implement the following "asymptotic observer" :

$$\frac{d\hat{z}}{dt} = -\frac{F}{V}\hat{z} + \frac{F}{V}\left(T_{in} - \frac{\Delta H_1 + \Delta H_2}{\rho C_p} C_{Ain}\right) - Q$$

$$\hat{C}_B = \frac{\rho C_p}{\Delta H_2}(T - \hat{z}) + \frac{\Delta H_1 + \Delta H_2}{\Delta H_2} C_A$$

The above observer has been called "asymptotic" because the estimation error in z :

$$\tilde{z} = z - \hat{z}$$

(and in consequence in $C_B$ in our example) obeys to the following equation :

$$\frac{d\tilde{z}}{dt} = -\frac{F}{V}\tilde{z}$$

which is asymptotically stable if F/V is positive.

The initial values of T, $C_A$, $C_B$ and $C_D$ considered in the simulation correspond to an open-loop unstable steady state :

$T(0) = 507$ K, $C_A(0) = 1.716$ mol/l
$C_B(0) = 4.474$ mol/l, $C_D(0) = 0.3811$ mol/l

The reference set point T* has been set to 507K. In the simulation, the overall heat transfer coefficient h has been considered as the control input. The controller design parameter $\lambda$ has been set to 0.08. The estimates of $E_1$, $E_2$ and $C_B$ have been initially set to the following (incorrect) values :

$$\hat{E}_1(0) = \hat{E}_2(0) = 10^4 \text{ kJ/kmol}, \quad \hat{C}_B(0) = 3.2 \text{ mol/l}$$

$E_1$ and $E_2$ have been estimated by using a recursive least squares estimator with a forgetting factor (equal to 0.99 for both estimations)(the estimator gain has been initially set to $10^8$ in both cases). A sampling period $\Delta t$ equal to 10 s has been considered both for estimation and control. Figure 1 shows the closed loop behaviour of the process when a square wave value of the influent reactant concentration, $C_{Ain}$, (Fig.1a) is applied to the reactor. Figure 1b&c show the controlled output T and the control input h; note the ability to compensate the variations of the disturbance $C_{A,in}$ (whose measurement only appear indirectly in the controller via the estimation scheme (see equation (18b)). Note also the good control performance in spite of the uncertainties on $E_1$, $E_2$ and $C_B$ (Fig.2a&b, which also show the convergence of the estimates of $E_1$, $E_2$ and $C_B$ to their true values).

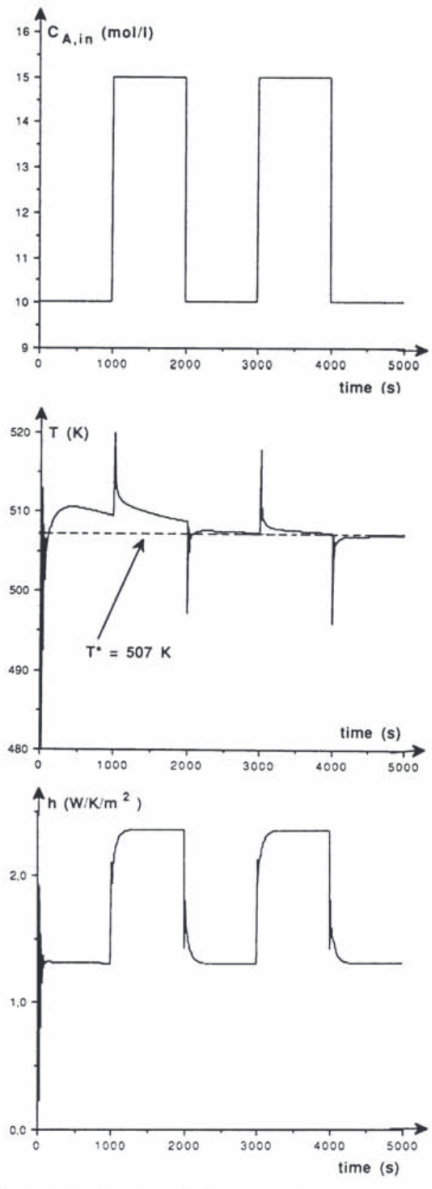


Fig.1. Adaptive linearizing control : control results

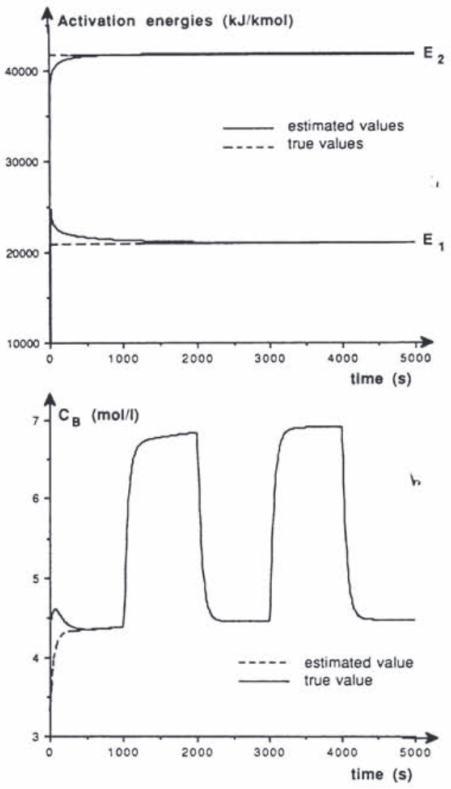


Fig.2. Adaptive linearizing control : estimation results

## 5. CONCLUSIONS

In this paper, it has been shown how to design an adaptive linearizing controller for an non-isothermal process involving M ($\geq$ 1) reactions. Here the particular (although widely encountered) situation of Arrhenius kinetics has been considered. It has been suggested how to include the estimates of the activation energies into an adaptive version of an input-output linearization control scheme. The proposed adaptive linearizing controller can be extended so as to include the estimation of other process parameters or to treat other types of nonlinearities (see e.g. Chen and Bastin (1991) who treat the identifiability of biological kinetics models which are ratios of polynomials (such as the well-known Monod model)).

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