



Prediction of the conducted emissions in the supraharmonic range in low-voltage grid

Catholic University of Louvain

Ing. Muharemi Erzen

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Jury

Chairwoman	Prof. Aude Simar (UCLouvain)
Supervisor	Prof. Emmanuel De Jaeger (UCLouvain)
Secretary	Prof. Claude Oestges (UCLouvain)
Members	Prof. Marc Bekemans (UCLouvain)
	Prof. Jos Knockaert (Ghent University)
	Prof. Jan Meyer (TU Dresden)

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الصالحة

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Abstract

In recent years, power electronics have become increasingly widespread. This evolution has, in turn, given rise to various electrical challenges—among them, the issue of grid stability. Traditionally, the grid has been powered by large synchronous machines that help maintain frequency stability. However, with time, more and more converters—i.e., power electronic devices composed of semiconductor switches that convert electrical energy between DC and AC in various forms—are appearing in the low-voltage grid, which is the focus of this thesis.

The growing presence of converters is fundamentally changing the nature of the grid, shifting it from a classical passive system to a more active one. In parallel, the grid impedance is also evolving, moving from a well-modeled resistive-inductive structure toward a more capacitive behavior. This change raises questions about the reference impedances currently used in standards, which are not well adapted to today's—and even less to tomorrow's—grids. Another important concern is the increasing complexity of assessing the control-related issues that arise as the number of synchronous machines decreases. This work focuses on one particular aspect of these developments: power quality, and more specifically, the harmonic content of current and voltage in the low-voltage grid.

While the main focus of this thesis lies in the harmonic domain, the analysis concentrates on a specific frequency range known as the supraharmonic (SH) range, i.e., from 2 to 150 kHz. Nevertheless, some aspects related to low-frequency harmonics are also addressed. Regarding the supraharmonic range, it is important to highlight that an increasing number of converters operate within it, and that this range is also used for other purposes—such as Power Line Communication (PLC). Despite the widespread use of this frequency band, there is still a lack of standardization, leading to a somewhat ambiguous situation.

This thesis aims to investigate the issue of high-frequency harmonic production, referred to as supraharmonics. It begins with a comprehensive state-of-the-art review, emphasizing the need to distinguish among converter topologies and control structures that are increasingly common in low-voltage grids. Once these have been identified, the second part of the thesis critiques the type of models typically used—first by exploring localized modeling approaches, and then by discussing the challenges posed by frequency aliasing and spectral leakage.

Subsequently, the modeling of three types of converters is presented: the boost converter, the interleaved boost converter, and a conventional inverter topology. The modeling is primarily based on the double Fourier series (DFS) framework but also introduces a novel approach called Analytically Generated Signals (AGS), the foundations of which

are discussed in the final chapter dedicated to deeper insights into the supraharmonic phenomenon.

Chapter 4 investigates the importance of accounting for phase shifts in carrier signals. Then, in Chapter 5, answers are provided to several questions and observations that have not yet found satisfactory explanations. This final chapter explores, in particular, the high-frequency emissions of power electronic devices, including the possibility of voltage variations—despite theoretical assumptions of constant voltage—at the converter terminals.

Finally, several topics are addressed in the appendices, which are relatively theoretical in nature. In order, they include: the presentation of measurement methods for supraharmonics; the introduction of theoretical concepts related to the frequency-domain analysis; an appendix dedicated to two developments based on Maxwell’s equations; the complete development of an interruption function using Bessel functions; and, finally, the connection between the field of electrical engineering and biology. The latter appendix was discussed with an artificial intelligence tool to ensure the relevance of the analogies made and to provide appropriate references.

In summary, this work addresses several key questions and provides a robust foundation for the modeling of power electronic converters in the context of high-frequency emissions (supraharmonics), as well as for the study of the methodologies underpinning such models.

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Introduction

The domain of power quality in the frequency range of supraharmonic, i.e. 2-150 kHz, is gaining interest as time goes on. Indeed, this is merely due to the context on the grid. More and more converters, i.e. power electronic devices, are connected to the grid and the range of frequency at which their switches works is part of the supraharmonic range (depending on the power rating). These converters can be end-user equipment such as laptop chargers and such common devices.

The fact that this range is now used extensively enhance some questions and potential issues that needs to be addressed. Classical home devices used to have topologies including natural switching devices as full-bridge diode rectifier to realise the AC-DC conversion but self-commutating topologies is experiencing a fast grow in their use inducing the shift of the main emissions from below 2 kHz towards the supraharmonic range [1]. In the same time the power network (trivially named the grid) is used to convey information, i.e. which is called the power line communication (PLC), these information are in the range of supraharmonic. This latter fact induces an inherent issue being the trade off between allowing the emission of specified magnitude in that frequency range per frequency and in the same time not interfering with the messages that are transferred. Of course, a bad understanding of this trade off would lead to a chaotic situation with respect to the transmission of these information.

The issue related to the PLC has been extensively investigated [2]. The conclusions are that even though the emission are in the same range, i.e. supraharmonic, it is not observed that the issue is caused by the emission of the end user equipment but rather it appears to come from the electromagnetic filter present at the input of each of these end-user equipment. Even though these EMI filters are designed for frequency greater than 150 kHz they play a crucial role in the phenomenon that are observed in the supraharmonic range in the low-voltage rid (LV). Indeed, these filters contains capacitances that provide a path of low impedance for these high frequency signals.

The determination of the expected emissions is the core of this work and requires the use of tools. Such tools can be either numerical or analytical. The numerical tool in this work will be a software, namely Matlab Simulink and the analytical one the double Fourier series. These tools have their advantages and their disadvantages that can be easily summarized as the computation capacity needed for the numerical method but the rather easily obtained results and the complex development of the analytical expression but that doesn't require any great computation capacity [3].

Objectives and Report Structure

The aim of this thesis will be, based on previous works [1][2][4][3], to address some specific points. As this is explained in [1] many aspects are present in this field, one could cite the measurements, the modelling, the choice of the software to employ, the analytical determination and all the fancy mathematics needed. In this thesis the main focus is placed on the modelling of the supraharmmonic sources based on both numerical and analytical methods with experimental validations and on the prediction of the expected emissions in various situations. Nevertheless, chapter 2 will cover the other aspect by giving a brief discussion, among others, on the propagation of voltage and current, some specifications about the acquired data. This thesis will, thus, follow the next pattern.

First the determination of the context of this thesis will be acquired through chapter 1. The aim of this first chapter is to answer to some fundamental questions

1. Which converters are the main emitters in the supraharmmonic range
2. Which topologies can be observed to be mainly used
3. What are the control strategies employed
4. What kind of modelling is employed

Then, as previously mentioned, chapter 2 will address the key aspects that must be considered when studying high-frequency harmonic emissions. This includes, among other things, the propagation of voltage and current waves, as well as the inherent limitations related to signal and data analysis. Subsequently, to address the main focus of this thesis, it will be presented mathematical tools such as double Fourier series [5] and state-space averaging [6].

Afterwards, in chapter 3 the presented mathematical tools will be used in order to assess what should be emitted by one converter alone, this will be called its individual emission in this thesis and it refers to the emission of a converter when no other converter are connected. The complete circuit is therefore only an ideal voltage source and this converter. Bolstered of the experience of determining individual emissions by using the double Fourier series, another method is developed called the analytically generated signals. Other aspects such as the impact of the EMI filter or the impact of the parasitic interline capacitance are also studied.

A study is proposed in chapter 4 which investigates the impact of having two converters in parallel in different scenarios. Chapter 5 is then presented as a way to deepen the reflection by addressing several specific aspects, such as the impact of variations in the DC-side voltage of devices like active power factor correctors—although the discussion naturally extends to all power electronic devices whose function includes the regulation of a DC voltage. Furthermore, it explores the influence of harmonics present on the modulating wave in the context of supraharmmonic emissions. This chapter serves as an in-depth continuation of the analysis, and also acts as a transition towards the annexes, which are intended as an extension and interconnection of various reflections—mathematical, physical, and even biological in nature.

List of Publications

The research conducted within the framework of this doctoral thesis has resulted in the following publications:

- E. Muharemi, E. De Jaeger, and J. Knockaert, “Supraharmonic in low-voltage distribution grids. Analysis of the specific case of the interleaved boost converter,” *IET Conference Proceedings*, vol. 2023, no. 6, pp. 2958–2962, 2023, doi: 10.1049/icp.2023.0955.
- E. Muharemi, E. De Jaeger, and J. Knockaert, “Investigation of the relationship between power of a boost-type converter used as an active power factor corrector and its supraharmonic emissions,” *IET Conference Proceedings*, vol. 2025, no. 14, pp. 1748–1752, 2025, doi: 10.1049/icp.2025.1914.
- E. Muharemi, E. De Jaeger, and J. Knockaert, “Double Fourier Series Applied to Supraharmonic Determination: The Specific Cases of a Boost and an Interleaved Boost Converter Used as Active Power Factor Correctors,” in *Proceedings of the 2025 IEEE Kiel PowerTech*, pp. 1–5, 2025, doi: 10.1109/PowerTech59965.2025.11180361.

List of Acronyms

F_{sw}	Switching frequency.
I_{in}	Current at the input of the converter under study.
I_{rip}	Supraharmonic in current.
L	Classical inductance, defined as the derivative of the magnetic flux with respect to current.
L_{b1}	Boost inductance of branch 1.
L_{b2}	Boost inductance of branch 2.
L_b	Boost inductance.
T_{sw}	Switching period.
V_{DC}	Voltage on the load capacitance.
V_{ac}	Voltage on the grid upstream of the grid impedance.
V_{grid}	Voltage on the grid upstream of the grid impedance, V_{grid} and V_{ac} are used interchangeably.
V_{in}	Voltage at the terminals of the converter under study.
V_{rip}	Supraharmonic in voltage.
Z_{in0}	In the absence of further specification, Z_{in} and Z_{in0} are used interchangeably, both referring to the impedance $(0.4 + j0.35) \Omega$ at 50 [Hz] considered as the impedance of the grid as a whole.
i	In the absence of further specification, the electrical current is denoted by i .
AC	Alternating current.
AGS	Analytically generated signals.
APFC	Active power factor correction.
CHS	Current harmonic source.
DC	Direct current.
DFS	Double fourier series.

EMI	Electromagnetic interference.
EV	Electric vehicle.
FBDR	Full-bridge diode rectifier.
IBC	Interleaved boost converter.
MPPT	Maximum power point tracking.
NPC	Neutral point clamped.
PFC	Power factor correction.
PLC	Power line communication.
PV	Photovoltaic.
QS	Quadratic sum.
RMS	Root mean square.
SH	Supraharmonic.
TF	Transfer function.
VHS	Voltage harmonic source.

Chapter 1

General Introduction : State-Of-The-Art

In this chapter, a comprehensive state-of-the-art review will be conducted on converters present in the low-voltage network, specifically focusing on those which, due to their large numbers, can be considered as the primary expected emitters of supraharmonics. Additionally, the most commonly employed control mechanisms for these converters will be identified. This endeavor aims to closely align the understanding with the practical realities observed in the low-voltage network. The information provided here is sourced from [7, 8],[9],[10],[11]. This thesis will also be situated in relation to major existing studies [1, 2, 4, 12, 13] in the field of power electronics converter modeling regarding supraharmonic emissions.

1.1 Energetic Context

1.1.1 Energy mix in Belgium

From table 1.1 it is seen that 17 to 21 % of the total energy was coming from renewable energy sources (RES) in 2020 and in 2021 a tiny decrease is observed in the upper value as 17.5 to 18.9 % was at the time provided by RES. Nevertheless, the increase is again observable in 2022 with 19.5 to 22.2 % coming from RES. It is also observed that in 2021 the energy coming from nuclear power plant, 48.1-52.4% has been drastically raised with respect to 2020 where it represented 32.8-40.3%.

From this table it is basically understood that the RES is an important part of the energy mix of Belgium and should be considered in the next as these RES use converters in order to provide that energy to the grid. It is observed that more than 6% of the total energy mix came only from solar energy and the photovoltaic panels have converters and, more especially, inverters in their circuit. A first output is thus the interest that exists to study inverters, at least if the switching frequency corresponds to the range of SH. This will be discussed.

Nevertheless, some nuances need to be brought. This work has as context the LV grid

Table 1.1: Evolution of the energy generation mix in percentage in Belgium 2020-2025

	2020	2021	2022	2023	2024	2025
Nuclear	32.8-40.3	48.1-52.4	41.8-47.3	41.3	42.2	34
Gas	28.2-34.7	22.8-24.7	23.8-26.9	25.2	17.6	19
Off-shore	6.7-8.3	6.7-7.3	6.6-7.5	10.5	10	19 Wind Total
On-shore	4.1-5	4-4.3	4.4-5	8.2	7.9	19 Wind Total
Solar	4.3-5.2	4.7-5.1	6.4-7.3	9.5	11.9	15
Biogas	2-2.5	2.1-2.2	2.1-2.4	2.6	3	13 Biogas + Others
Others	3.1-3.9	3.5-3.8	3.2-3.6	2.7	7.5	13 Biogas + Others

and thus the numbers presented [7] that took as well into account big plants of PV panels than small rooftop PV panels need to be nuanced. It explains why the absolute values are not taken as references but instead the global trend that it describes is considered.

1.1.2 Projection of the energy demand

The energy demand is constantly rising with always more devices connected on the grid, this tendency leads to a consumption of around 87 TWh/year . The energy mix has been shown but another aspect is to be considered, indeed, the situation is not fixed and the demand rises. The prediction shows an increase towards at least 110 TWh/year for 2050 and the worse predictions even assess a future demand of around 170 TWh/year which is approximately two times the current demand.

In this context, it is easily understood that the loads will also provide more emission in the frequency range of interest but also the energy provider that makes up the inverters of PV panels or of the wind turbine. This context enhances the need of a comprehensive state-of-the-art of the situation. This section is intended to address this need, at least sufficiently, to determine the interesting emitters to further study.

1.1.3 Electric vehicle, global trend

A fast expansion of the number of electric vehicle is observed as seen in table 5.1 passing from 40 851 in 2021 to 138 749 in 2023. This indicates clearly a global trend towards more consumption of electricity passing from fossil energy towards electrical energy. The rising is even greater for 2023/2022 with respect to 2022/2021 passing from 75.4% of increase towards 93.6%.

Table 1.2: Evolution of the car fleet in Belgium 2021-2023

	2021	2022	2023	2022/2021	2023/2022
Parc de véhicules au 1 août (y compris motocyclettes)	7.751.422	7.796.877	7.910.513	+0.6%	+1.5%
Voitures particulières	5.927.912	5.947.479	6.030.700	+0.3%	+1.4%
-à essence	2.951.770	3.021.102	3.096.253	+2.3%	+2.5%
-diesel	2.623.556	2.424.932	2.201.549	-7.6%	-9.2%
-à gaz	15.999	17.740	20.373	+10,9%	+14,8%
-à l'électricité	40.851	71.651	138.749	+75,4%	+93,6%
-hybride	258.916	375.107	537.817	+44,9%	+43,4%
-non précisé	36.820	36.947	35.959	+0,3%	-2,7%
-Autobus et autocars	16.628	16.557	16.449	-0,4%	-0,7%

Even though in relative value the number of electric vehicle seems to take more place it is still equivalent to only about 4% of the number of vehicle running on gasoline. Nevertheless, by taking into account the hybrid vehicle the total number reaches more than 670 000 vehicles which represents then more than 10% of the total number of personal vehicles. The European policies lead to believe that until 2030 to 2035 a great part of the total number of personal vehicles will be electric or hybrid.

1.1.4 Photovoltaic : power capacity installed

On 2019 and on 2026 the power capacity installed of photovoltaic panels was respectively of 5 and 11.7 GW. This evolution evidences a clear and sustained transition of the energy mix toward a substantially greater proportion of renewable energy sources. The value to be expected for the future has been evaluated and the majority of the scenarios evaluated expect to reach 20 GW in 2040 and 30 GW in 2050. These values should not be taken in an absolute sense, but rather as indicative of a general trend, which clearly points towards an increasing presence of photovoltaic panels—and consequently, a growing number of inverters.

As stated previously the table 1.1 does not distinguish between great power plant and small local energy production and thus it could be inconsistent to based the decision to study PV inverters based on the previous discussion. In order to discriminate between PV panel that are not of interest in this thesis and the other [14] provides crucial information among others it describes the proportion of PV panels for power rating categories. The information are summarized in table 1.3. As the aim is to investigate LV grid the only interesting point is $< 10kW$ which in Belgium correspond to 62.89% of the total installed capacity in 2017.

Table 1.3: Installed PV capacities and their power rating in 2017

2017	Austria	Belgium	France	Deutschland
Total Installed PV (MW)	1250	3846	8091	43000
< 10kW	62%	62.89%	15.9%	14.2%
< 100kW	32%	17.71%	18.6%	52.4%
< 500kW	2%		13.8%	66.5%
> 500kW	4%	19.39%	51.7%	33.5%

1.1.5 Heat pumps

The electric demand for heating is an important question. Even though for the moment the share of electric heat pumps for heat demand is small it is expected in the scenarios to reach great values such as 8.6% in 2030 and even 40 to 90 % in 2050. Basically, until now the acquired information leads to consider that there are some main applications that uses converters and that are expected to be in numbers in the low-voltage grid. PV panels, electric vehicle chargers and heat pumps has been just presented.

1.1.6 Conclusion on the main emitters

In this section important information related to the energy context in Belgium have been provided, mainly coming from [7] but also from other sources [8][14]. Next to the previous subsections it is logically determined that the interesting application to further investigate are the PV panels, the electric vehicle chargers and the heat pumps. Even though this choice is funded it could be discussed, the aim was to be able to distinguish between all the possible applications to be studied as the number of application is great. In the next the converters associated to each application will be studied without talking specifically about the associated application.

From a scientific literature point of view the decision of taking these application is also strongly supported, among others [1] and [15] talked about the mains emitters and cited the inverters and active power factor corrector circuit which are part of the determined applications. Also in [2][16][17] this topic was discussed and clearly report a prominent importance of the PV panels inverters and the electric vehicle chargers as sources of SH while studying also the impact of these emissions on the power line communication (PLC). Other references could be cited [4][12][18] and more recently a paper [15] from an IEEE Task Force making a summary of the state-of-the-art did also refers to PV panels, EV chargers and heat pumps as being an important part of the SH sources and by the way did also cite the LV grid as being the place where most of the disturbance sources are present.

1.2 Topologies

This section is dedicated to the determination of the topologies to further investigate in the scope of this thesis, each of the determined applications do have specific kind of converters and the interest lies on the determination of the topology of these converters.

Each subsection will be composed of a presentation of the complete electrical circuit related to the application, then based on the literature on the topic a choice is done. The criterion for this choice is the popularity of the converter and the probability of its use on the low-voltage grid.

1.2.1 PV panels

The complete structure of a typical PV panel [19] is presented in Fig.1.1. It is, basically, observed the presence of the source of energy, i.e. DG source, which is followed by a DC/DC converter. A capacitance is then present and constitute the energy storage element of the DC side. The DC/DC converter is controlled in order to maintain the voltage on the capacitance at a certain level depending on the algorithm that regulates the produced power on the grid side. Next to the capacitance a DC/AC converter is present. As the aim is not to further investigate each part of the presented electrical complete circuit no list of the present variables is developed.

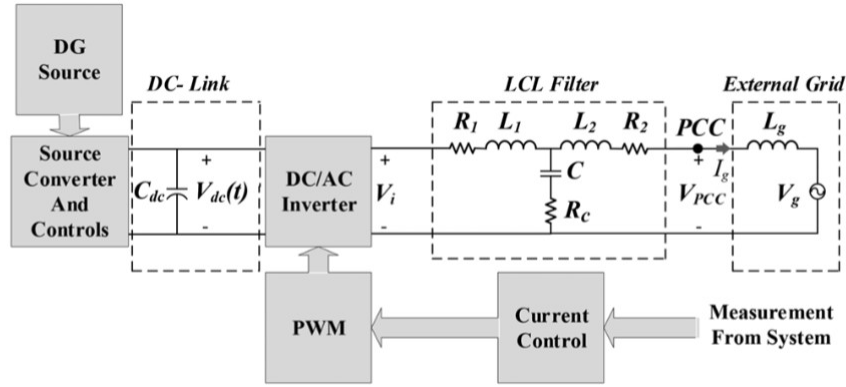


Figure 1.1: Typical PV panel complete electrical circuit

Afterwards, the inverter topology needs to be chosen. This is done by first examining the available topologies and then selecting the most commonly used one based on the literature. In [20] it is stated that transformer-less topologies are preferred due to their advantages such as great efficiency, less weight and less cost even though the current, noted i , of common mode is present with comparison of topologies including transformer either high or low frequency. In [21] a classification, i.e. Fig. 1.2, of the different topologies related to the transformer-less case is presented. This gives a good idea of the great number of possible topologies as each sub-categories contains itself many topologies.

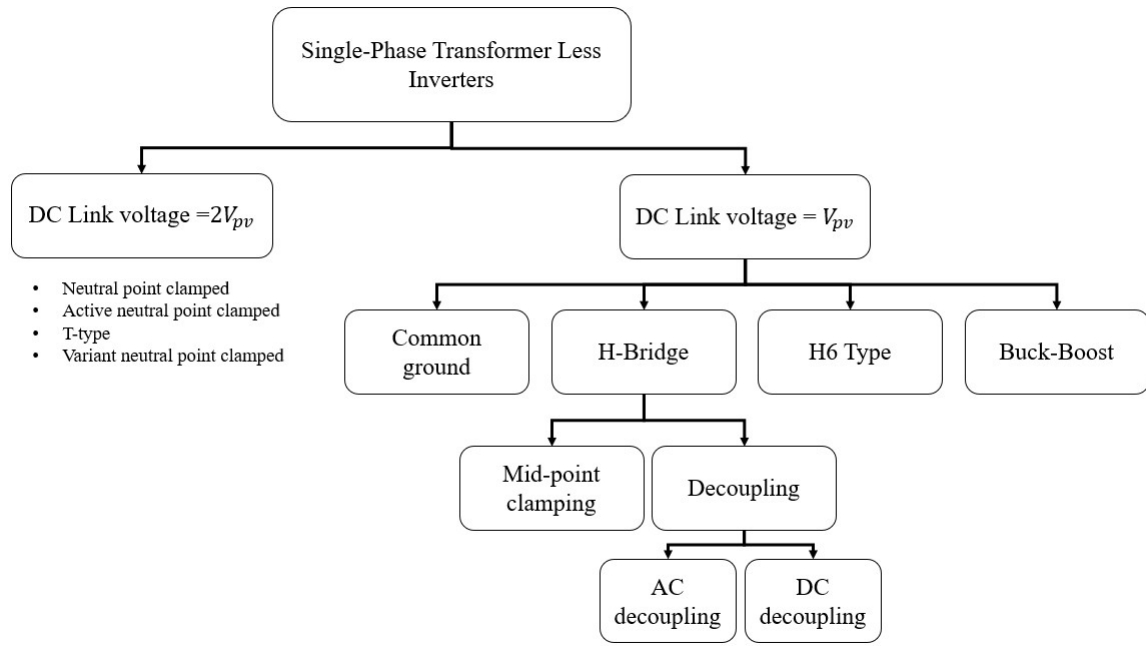


Figure 1.2: Transformer less inverter topologies

In order to provide a realistic framework, one of the criteria considered is the topology most commonly used in industry. This allows to make a distinction between the topologies and finally chose the ones of interest. The mainly used topologies are the neutral point clamped (NPC), flying capacitor and cascaded H-Bridge (CHB) [20][22][23]. These three topologies are multi-level inverters (MLI), i.e. more than two levels of voltage at the output. MLI have first been introduced in 1975 and NPC-MLI is considered as the first MLI topology [22].

1.2.2 Electric vehicle chargers

The classical electrical complete structure is shown in Fig. 1.3. Fundamentally, it consists of an AC/DC converter, i.e. a rectifier, connected to the grid, a DC/DC converter and finally a load which could be represented by a resistor in parallel with a capacitance. This should be understood as a classical complete structure and peculiar aspect are discussed hereafter.

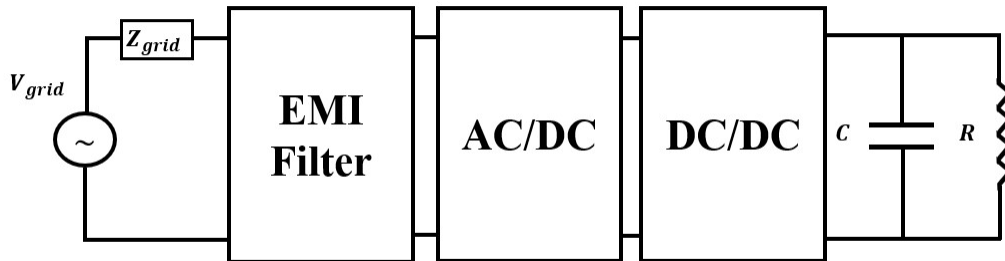


Figure 1.3: EV charger typical complete circuit

As it has been shown in [1] only the grid-connected converter is responsible for the significant disturbances produced on the grid. However, the qualitative explanation of this phenomenon is provided in this thesis in Chap. 5. Based on that knowledge only the DC/DC converter should be chosen when the AC/DC converter is a full-bridge diode rectifier. Consequently, the most used DC/DC converters have been investigated and based on [24][25][26][27][28][29] the classical boost and interleaved boost converter (IBC) have been found to be the most used converters. The DC/DC converter is often referred as the PFC circuit [25][26][30] This is easily understood as these converters are well known and can be easily designed and controlled due to the acquired knowledge on these topologies. In [28] the classical circuit of an on-board EV charger is found to be the one shown in Fig. 1.4 which seems to validate the reported mainly used topology for the DC/DC converter.

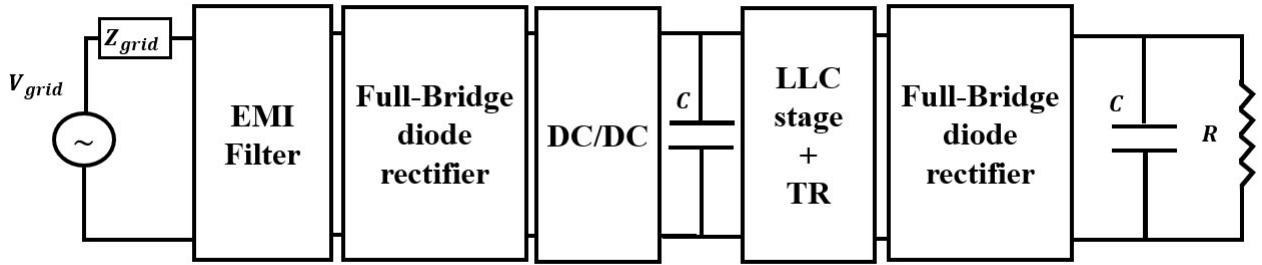


Figure 1.4: EV charger typical complete circuit

Indeed, in Fig. 1.4 the IBC is used. Note that TR stands for transformer. One of the interesting feature of the IBC is that it allows to reduce the emitted SH as only the double of the switching frequency remains as disturbances emitted on the grid. This statement will also be demonstrated in the next section by analytical as well as numerical results.

An other aspect that should be mentioned is the power level, indeed, depending on the power level the charger has more or less chance to be present in the LV grid. Actually, three levels of power do exist. Table 1.4 provides a clear division that depends on the power level and induces the use of one or three phases[30].

Table 1.4: EV Power Level

Power Level Types	Charger Location	Typical Use	Energy Supply Interface	Expected Power Level	Charging Time	Vehicle Technology
Level 1 (Opportunity) 120 Vac (US) 230 Vac (EU)	On-board 1-phase	Charging at home or office	Convenience outlet	1.4kW (12A) 1.9kW (20A)	4–11 hours 11–36 hours	PHEVs (5-15kWh) EVs (16-50kWh)
Level 2 (Primary) 240 Vac (US) 400 Vac (EU)	On-board 1- or 3- phase	Charging at private or public outlets	Dedicated EVSE	4kW (17A) 8kW (32 A) 19.2kW (80A)	1–4 hours 2–6 hours 2–3 hours	PHEVs (5-15 kWh) EVs (16-30kWh) EVs (3-50kWh)
Level 3 (Fast) (208-600 Vac or Vdc)	Off-board 3-phase	Commercial, analogous to a filling station	Dedicated EVSE	50kW 100kW	0.4–1 hour 0.2–0.5 hour	EVs (20-50kWh)

In the scope of this thesis the level one and two are considered as the LV grid is considered. Level one that is up to 1.9 kW and level two up to 19.2 kW. Based on this idea and on Table 1.5 we can chose among all the possibilities the conventional PFC (full-bridge diode rectifier and Boost converter) and the interleaved boost PFC.

Table 1.5: Comparison Of PFC Topologies

Parameters	Conventional Boost PFC	Interleaved Boost PFC	Bridgeless Boost PFC	Totem-pole PFC	Semi Bridgeless Boost PFC	Bridgeless Interleaved Boost PFC
No. of switches	1	2	2	4	2	4
No. of diodes	5	6	2	0	4	4
No. of inductors - size	1-L	2-L/2	2-L	1-L	2-L	4-L
Inductor's location	DC side	DC side	AC side	AC side	AC side	AC side
Max no. of devices in the conduction path	3	3	2	2	3	4
Power rating	<1 kW	<3 kW	<2 kW	>6 kW	<4 kW	>5 kW
EMI emission	High	Medium	Medium	Medium	Medium	Low
Efficiency	Less	Medium	Medium	High	High	High
Cost	Less	Medium	Medium	Medium	High	Very high

This selection could be discussed. Indeed, we can see that Bridgeless Boost PFC and Semi Bridgeless Boost PFC are also used for smaller power than 5 kW but we need to focus on a small number of topologies and the most widely used topology is the Interleaved Boost PFC as reported in the literature. Also, the Conventional Boost PFC has been widely studied and is much cheaper and easily implemented for further modelling.

In order to make a link with realistic situations Table 1.6 is provided. The references provided in the Table are related to [26] the interested reader could refer to it. The IBC appears in two, i.e. the second-gen. Volt and the Delta-Q Technologies of the cases.

Table 1.6: Real Cases of Electric Vehicles and Their PFC

Onboard Topology	Front-end Converter	Battery Interface	Input Voltage (V)	Power (kW)	Output Voltage (V)	Efficiency
Second-gen Volt [10]	Interleaved boost PFC	Phase-shift DC-DC converter	85 to 265	3.65	200 to 430	93 to 95 %
Hyundai vehicle to the grid [9]	Bidirectional boost PFC	Dual-active bridge	85 to 265	7.2	350	>90 %
Tesla Model 3 [12]	Active rectifier	LLC resonant converter	120 V	7.5/11.5	350 V	-
Multifunctional OBC [13]	Full-bridge converter	Bidirectional buck-boost	110	2	250 to 430	96.1 %
Delta-Q technologies [11]	Interleaved boost PFC	Isolated full-bridge DC-DC	85 to 265	3.3	200 to 450	94 %
Ref. [15]	Interleaved bridgeless totem-pole PFC	Bidirectional LLC converter	120	6.6	240 to 420	95.7 %
Ref. [16]	Bridgeless Cuk PFC + Ripple compensator (Single stage)		115	1	300	91

1.2.3 Heat pumps

Actually, regarding the heat pumps the topologies does not require distinct converters than the electric vehicle charger. Even though the application is different the electrical complete circuit can be assimilated to Fig. 1.3 the only difference is that it does contain at the back-end an other converter being an inverter, i.e. DC/AC converter. Nevertheless, it has been stated that only the grid-tied converter does produce, significantly, the emitted disturbances [1]. Consequently, there is no significant difference from an electrical point of view between the heat pumps and the electric vehicle chargers complete circuit.

It should be seen as an opportunity to be able to have results being both valid for EV chargers as for heat pumps. The investigation of the EV chargers will thus have a great importance. However, The complete circuit of the heat pumps is shown in Fig.1.5.

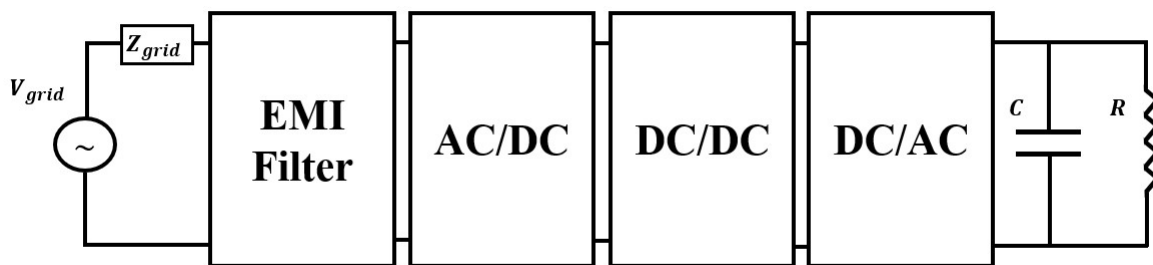


Figure 1.5: Typical Heat Pumps Complete Circuit

1.3 Control

The aim of this section is to provide a clear idea of the control structure which is function of the converter under consideration. Two control structure will thus be determined, one related to the PV inverter and the other one to the EV chargers and heat pumps.

1.3.1 PV panels

Two converters are present in the complete circuit of the PV panel. One DC/DC converter that has as purpose to control the voltage on the DC link in order to be able to send the power to the grid and the other one being an AC/DC converter. These two converters are controlled but the focus will mainly be on the control of the AC/DC converter for the same reason that leads to study only the PFC in the electric vehicle chargers. Indeed, only the grid-tied converter has been determined to be the significant emitters of these disturbances, i.e. SHs.

This fact has been confirmed for application such as PV panel, compact fluorescent lamps (CFLs), wind turbine [1] and also for heat pumps [31]. A Matlab-Simulink model has been developed to investigate such structures from the perspective of SH, in the case of a 5-level inverter with sinusoidal pulse width modulation (SPWM) in open loop control.

The same result has been found, i.e. the presence in the frequency spectrum of the disturbances of the inverter and not of the other converter (the boost converter in this case). It does, basically, mean that only the inverter needs to be studied in its control. Based on number of papers focusing on controlling the inverter switches, it is commonly accepted that the best way to control the output current and limiting the THD is by using the voltage-oriented control [32, 33, 23, 34, 35, 36, 37, 38, 39] strategy which consists in having a cascaded control with an outer controller ensuring constant DC-link voltage and an inner controller ensuring the needed switching pattern in order to get the desired current at the grid side. The latter being based on the DC voltage error, it is worth noting that there is possibility to exchange reactive power with the grid.

The cascaded method is mainly used. The investigations of the different searchers were on the current controller, when almost all researches used a PI controller for the DC voltage control [34] every research tried to get better results by taking different current controller. Note that in order to be able to use the PI controller without getting steady state error in controlling AC signals, it is needed to pass the current into the dq (direct and quadrature) frame which is a synchronous frame rotating at the grid frequency thus bringing constant current value.

The most studied current controller consists of proportional resonant (PR) controller, classical PI controller, current hysteresis controller (CHC), linear quadratic regulator (LQR) etc... [32, 34, 35, 36, 37, 38] Many methods do exist to control the current in the control structure of the inverter. The aim of these methods is to control the power injected to the grid and to limit the power quality issue like harmonics production that has to be limited, a classical target for THD is 5 percents in voltage with specifics value for each harmonic order as specified by EN 50160 [40] or IEEE-519 [41] limitations. In this chapter we will briefly introduce the mainly used methods to get in touch with the way the inverter is actually working we will then see if the change of current controller does have a significant effect on THD produced in output current (AC side) based on the literature.

1.3.2 EV chargers

Knowing the global structure of the on-board charger, allows to understand that the control will take place in the DC/DC converter used for power factor correction (PFC) and in the battery interface converter. The control of this DC/DC converter has two goals as cited in [1]: maintain the DC voltage and ensure that the current has an acceptable sinusoidal waveform and is in phase with the voltage on the grid side (power factor correction).

The DC/DC converter used for PFC could be controlled by many techniques but the most common one seems to be the average current control [1][26][42]. The DC/DC converter can be controlled by PI controller, Fig.1.6 [26] shows the way interleaved PFC and phase shifted full-bridge converter can be controlled. As the converter chosen is the PFC the control structure for further investigation is chosen to be (a).

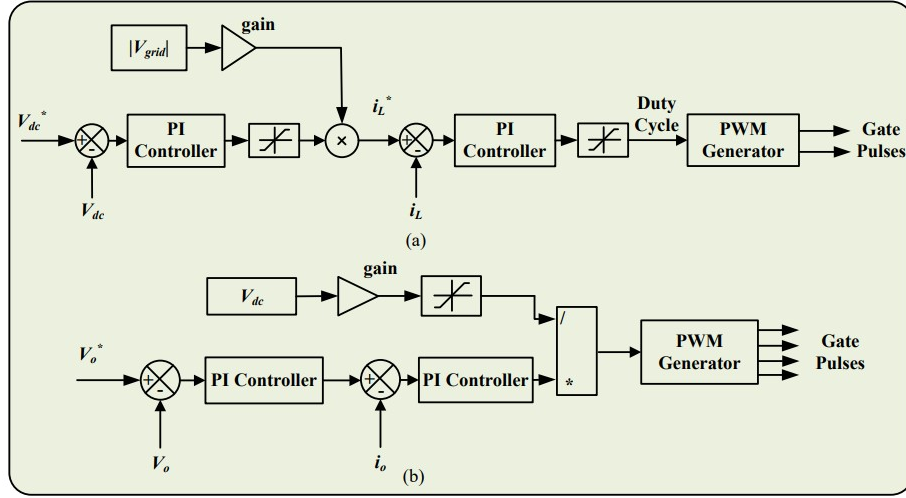


Figure 1.6: Control circuit of onboard charger: (a) PFC and (b) Phase-shifted full-bridge converter

1.4 Thesis Contribution

This section is divided into two parts. The first part is devoted to placing the research into perspective through the establishment of a State-Of-The-Art on the topic of supra-harmonics. This overview provides the necessary framework to position the present work within the existing body of literature and to clearly articulate its specific contributions in the subsequent subsection.

1.4.1 State-Of-The-Art : Supraharmonics

In order to establish the context within which the present research has been conducted and to clarify the specific contributions of this work, a selection of relevant publications is presented below, together with their respective years and principal contributions.

The historical background may be traced back to 1953, when the double Fourier series was first introduced [5]. Initially, this development was motivated by applications in telecommunications. It was not until 1975 that the transition occurred from telecommunications to the modeling of power electronic converters through the application of the double Fourier series [43]. Shortly thereafter, in 1977, the doctoral thesis of Ćuk was published [6], introducing the state-space averaging method, which would subsequently become a fundamental tool in the modeling of power electronic systems.

Advancing in time, several reference textbooks in the field of power electronics were published. In 2007, the book *Fundamentals of Power Electronics* was released [44], in which state-space averaging and small-signal linearization are presented as foundational modeling techniques for power electronic converters. These methods are also employed in the present work. Another reference text, *Power Electronics: A First Course*, was published in 2012 [45] and further contributed to the consolidation of modeling methodologies in the field.

Parallel to the publication of these reference books, early investigations addressing what would later be termed supraharmonics began to emerge, although the terminology had not yet been formally introduced. A 2010 publication examined how the frequency composition of a low-voltage network in Sweden was influenced by the switching on and off of LED lighting systems [46]. This study already noted that power line communication (PLC), typically operating around 40 kHz, appeared to be affected in amplitude by the presence of LED lighting.

In 2013, the term “supraharmonics” was introduced for the first time [47]. In the same year, the doctoral thesis of Sarah Rönnerberg was published [2], providing an extensive investigation of supraharmonic emissions, with particular emphasis on LED lighting systems. This work introduced the distinction between primary and secondary emissions, terminology that would subsequently become central in distinguishing emissions intrinsic to a given power electronic device from emissions originating from external sources and propagating toward it.

In 2014, Jan Meyer and other experts highlighted the growing necessity of addressing the supraharmonic issue [48]. Their work emphasized that systematic investigations into the phenomenon had already begun several years earlier.

In 2016, Jan Meyer *et al.* published a study on supraharmonic emissions from electric vehicles [49]. During this period, numerous works emerged aiming to improve the understanding of supraharmonic phenomena, primarily through measurement-based approaches. These studies began to report emission reduction effects associated with the increasing number of connected devices. In 2019, a master’s thesis [50] investigated several aspects of supraharmonics, including low-voltage cable modeling at high frequencies and supraharmonic propagation. In the same year, a study applying the double Fourier series to model an active power factor corrector (APFC) was published [51].

Subsequently, a series of doctoral dissertations contributed to further consolidation and deepening of knowledge in the field of supraharmonics. In 2019, the doctoral thesis of Amaripadath was published [12], presenting extensive measurement campaigns that confirmed previously observed phenomena, such as emission reduction when multiple devices are connected simultaneously—specifically, a reduction in inverter emissions when an electric vehicle charger was also connected. This thesis also investigated propagation as a function of cable impedance and further developed the concepts of primary and secondary emissions. In 2020, the doctoral thesis of Darmawardana [4] proposed an optimization-based modeling framework for inverter supraharmonic emissions. In 2021, another doctoral dissertation [1] provided an in-depth analytical study based on the double Fourier series for modeling power electronic converters, focusing primarily on inverters and APFCs. Analytical models were proposed and validated against experimental results. Interest in modeling approaches based on the double Fourier series continued to grow, alongside ongoing experimental investigations.

In 2022, one doctoral thesis [17] addressed the non-linear nature of emission summation phenomena. An experimental study demonstrated that, under the investigated conditions, the total emission level ceased to increase and began to decrease beyond the connection of more than 60 LED lamps. A stochastic model was also proposed to study propagation phenomena. In 2023, the doctoral thesis of Sutaria [52] presented a similar investiga-

tion, reaching comparable conclusions but with a different threshold value: whereas the emission decrease occurred beyond 60 lamps in [17], it was observed beyond 20 lamps in [52]. Beyond this specific analysis of non-linear emission summation, the work further elaborated the concepts of induced primary and induced secondary emissions, thereby extending the framework by considering not only the coexistence of primary and secondary emissions but also their mutual interactions.

Finally, recent review papers [15] and [47] have contributed to structuring the field by providing comprehensive overviews of the rapid growth of research interest in supraharmonics. The number of annual publications increased from fewer than ten in 2014 to approximately forty in 2020, before stabilizing thereafter. Notably, [15] emphasizes that, despite the significant increase in research activity, the modeling of power electronic converters remains comparatively underrepresented.

These contextual elements provide a comprehensive overview of the evolution of research on supraharmonics and enable the positioning of the present work within this scientific landscape. The specific contributions of this thesis are detailed in the following subsection.

1.4.2 Thesis Positionning

Even though the interest in the issue related to supraharmonic emissions is increasing the lack of an accurate model for the sources of emission is still present [15]. However, various studies have been conducted on the modeling of power electronic converters with respect to their supraharmonic emissions. It is noteworthy that several distinct approaches have emerged. In the seminal works within this field, it is observed that [4] employs two methodologies. The first, referred to as the "white box" approach, involves the numerical modeling of every detail of the actual circuit under study. The second, termed the "black box" approach, derives a converter model for supraharmonic emission determination based on extensive laboratory measurements.

Additionally, [2] has conducted multiple studies, focusing on measurements to investigate the supraharmonic emissions of various devices, such as LEDs and photovoltaic panels. While [13] did study mainly the supraharmonic emissions of electric vehicle chargers. Their approaches can be categorized as "black box" due to the lack of knowledge about the internal structure, i.e., the topology, of the converters and their control structures.

The works of [12] and [53] can also be classified as "black box," as they are predominantly based on measurements and the determination of practical consequences, as also highlighted in [2], such as the reduction in supraharmonic emissions from a photovoltaic inverter when an electric vehicle charger is connected nearby.

Lastly, the thesis by [1] contributes significantly by considering a "grey box" approach. This method is based on analytical development, allowing for the modeling and prediction of converter emissions using mathematical tools such as the double Fourier series. This thesis bridges the gap between "black box" approaches, which are based on measurements, and "grey box" approaches, which rely on analytical developments. This will initially lead to the validation of grey-box models against laboratory measurements on actual devices. Furthermore, this approach will elucidate phenomena such as the non-production of supraharmonics on the low-voltage network by back-end converters in back-end, front-

end systems. Additionally, this work will enable the determination of the relationship, if any, between the power level of the device and the supraharmmonic emission level. Also, the inherent limitation of the analytical method represented by the double Fourier series are discussed and solutions are provided either by the same analytical tool, i.e. an adaptation of the double Fourier series, or by a new method called the analytical generated signals (AGSs) method.

This work can thus be seen as a new step toward a more global understanding of the issue of supraharmmonic modelling by increasing the knowledge on both the methods used to model supraharmmonic emissions of power electronic devices and the impacting factors of these emissions. Eventually, the contribution of this work can be summarized by three points

- Accurate analytical modelling of typical converters present on low-voltage grid in their supraharmmonic emissions
 - Active power factor corrector circuits are modelled and verified by numerical as well as physical experimentations
 - Classical One-Phase Inverter is modelled and verified by numerical experimentations
- Analytical method for power electronic devices modelling are studied
 - The double Fourier series is limited by its inability to take into account certain additional aspects of non-linearities directly related to the principle of operation of the diodes
 - Based on the acquired knowledge of the working principle of controlled power electronic devices a new method is developed, i.e. the Analytically Generated Signals method. It is actually a numerical application of the named 'universal modelling'
- Explanation for interesting observations are provided
 - The impact of the DC value ,for converter having as purpose to maintain a constant DC value, on the supraharmmonic emissions
 - The consequences on supraharmmonic emissions of having pre-existing harmonics present on the grid
 - Qualitative Explanation on the impact of non constant DC voltage
 - Qualitative explanation on the observed emissions in supraharmmonic range being only coming from front-end converter et front-end, back-end systems
 - The impact of EMI on the supraharmmonic emissions
 - The voltage harmonic source (VHS) or current harmonic source (CHS) behaviour in supraharmmonic mainly linked to the ratio between converter and grid impedances

- The supraharmonic mainly observed in between converters and not propagating towards the grid

Chapter 2

Theoretical Background and Scope of Validity

2.1 Scope of Validity

2.1.1 Introduction

The study and modeling of supraharmonic emissions constitute the aim of this work. However, several aspects must first be investigated. Indeed, modeling always relies on assumptions and approximations. Therefore, it is crucial to identify as comprehensively as possible the factors that could lead to a significant discrepancy between the context considered by the model and the actual physical environment in which the modeled device operates. This chapter therefore aims to establish the validity framework for the models developed in the following chapters.

In this chapter, the following points are discussed

- The development leading to the notion of propagation of voltage and current
- The introduction of the notion of dispersive medium and the related consequences
- The introduction of per-unit-length impedances and typical cable models in LV grids
- Limitations related to signal analysis, namely spectral leakage and frequency aliasing
- A general overview of the limitations and approximations considered in this thesis
- An introduction to the double Fourier series
- an introduction to voltage harmonic sources and current harmonic sources along with their intrinsic limitations

2.1.2 Voltage and current propagation

The complexity of the studied field can be divided into two parts. The first is the model of the converter, and the second is the understanding of how emissions propagate and

interact. This subsection is dedicated to the propagation of emissions.

Indeed, it is not only necessary to model the converter accurately, but also to properly understand the nature of the electrical environment in which this converter is connected, which requires an understanding of the physical mechanisms governing emission propagation. Finally, the non-instantaneous propagation of electric and magnetic fields is addressed in C through two approaches: first, through a direct derivation from Maxwell's equations, and second, via an equivalent infinitesimal quadripole, following the telegrapher's formulation.

It is worth noting that the second-order derivation inherent in the final form of the telegrapher's equations leads naturally to their interpretation as a description of wave phenomena. The speed of propagation can be understood as $\frac{1}{\sqrt{LC}}$ which is, of course, equal to c , the speed of light in vacuum.

The interpretation of these equations is essentially, that both voltage and current are functions not only of time but also of space, specifically along the propagation axis z . Indeed, the second-order partial differential equations show that spatial and temporal variations are intrinsically linked through the inductance L and capacitance C per unit length of the transmission line.

It is important to grasp the implications of this spatial dependence. Considering a wave solution of the form $v(z, t) = V_0 \cos(\omega t - \beta z)$, and recalling that $\beta = \frac{2\pi}{\lambda}$, the spatial term βz corresponds to $2\pi \frac{z}{\lambda}$. Therefore, when the propagation distance z is much smaller than the wavelength λ , the spatial variation becomes negligible compared to the temporal one, and the signal can be approximated as purely time-dependent.

This simplification underlies the *lumped-element model*, where voltage and current are assumed to vary only in time, neglecting any spatial distribution. However, the wave equations derived here clearly demonstrate that this approximation fails when the electrical length becomes comparable to the wavelength — hence the necessity of distributed modeling for high-frequency or long-distance applications. A realistic case is considered in section 2.4.

2.1.3 Dispersive medium

Regarding the notion of a dispersive medium, it is first necessary to introduce two concepts, namely the phase velocity and the group velocity. These are expressed as

$$v_p = \frac{\omega}{k} \quad v_g = \frac{d\omega}{dk} \quad (2.1)$$

Where ω, k, v_p and v_g are respectively the angular frequency ($\frac{rad}{s}$), the wave number [$\frac{rad}{m}$], the phase velocity and the group velocity [$\frac{m}{s}$]. In a non-dispersive medium v_p and v_g are identical so that the shape of a given signal is conserved. When the medium becomes dispersive so that these two speeds are no longer the same, the shape of the signal becomes

distorted over space. Depending on whether the signal is going to travel a long distance (l) or not the final shape, i.e. at $x=l$ with l the length of the line, will be more or less distorted. When losses are neglected, such that the medium remains ideal in this sense, only the shape will be distorted and the magnitude of each frequency component is constant. In other words, performing the discrete Fourier transform will only lead to differences in phase when compared with the same signal at $x = 0$.

The fact that $v_p = v_g$ in a non-dispersive medium follows directly from the following derivation

$$v_p = \frac{\omega}{k} \quad (2.2)$$

$$\omega = v_p k \quad (2.3)$$

$$\frac{d\omega}{dk} = v_p + k \frac{dv_p}{dk} \quad (2.4)$$

By recognizing that $v_g = \frac{d\omega}{dk}$

$$v_g = v_p + k \frac{dv_p}{dk} \quad (2.5)$$

It can thus be concluded that the condition for a medium to be non-dispersive, i.e. $v_p = v_g$, is to have $\frac{dv_p}{dk} = 0$.

In appendix C, fields in vacuum are considered and the intrinsic assumption is that the speed of EM waves, regardless of the frequency, is c , the speed of light. It is accurate for vacuum, however, regarding media guiding EM waves, such as classical cables, used for transmitting energy towards the different consumers in the electrical grid, it is no longer that simple. When considering a real cable, nevertheless still ideal (no losses), the speed at which waves propagate depends on the frequency considered.

The well-known equation relating the speed of the wave with the permittivity and permeability still holds but needs to be adapted to no longer provide the constant c . In a dispersive medium, the speed is

$$v(\omega) = \frac{1}{\sqrt{\mu(\omega)\varepsilon(\omega)}} \quad (2.6)$$

When considering the analysis of the transmission line, which is the medium in which the considered high frequency emissions will propagate, it is important to note the following points.

- Transverse electromagnetic waves (TEM) are of interest
- A dispersive medium has two root causes

- Losses, corresponding to a non-zero resistance and conductance (R and G) which are not 0
- A frequency dependence of the permittivity and permeability

It is worth noting that TEM waves are related to the non-lossy character of the medium. However, small losses do not tremendously change the TEM nature of the waves giving rise to the notion of quasi-TEM waves.

By introducing the notion of propagation factor γ and denoting the wave number k by β , one can write

$$\gamma = \alpha + j\beta \quad (2.7)$$

In terms of circuit-theory elements, this can be written as

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (2.8)$$

It is possible to further explain the link between the non-lossy and the non-dispersive character of this medium. Indeed, the propagation factor is only $\gamma = j\beta = \omega\sqrt{LC}$ when the medium does not experience losses. In that case the relation $v_p = \frac{\omega}{\beta}$ is just $v_p = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$ and thus the dependence of the phase velocity on frequency vanishes. However, it is also possible to obtain the same result when the condition $\frac{R}{L} = \frac{G}{C}$ is met as long as the per-unit-length inductance and capacitance are not functions of frequency. The latter is the second root cause previously identified for a dispersive medium.

As long as a transmission line is under investigation, the small losses associated with the homogeneous character of the line prevent a significant dispersive character of the medium from occurring [54].

2.2 Per-unit-length impedances

Hereafter, some classical knowledge on the definition and values of per-unit-length impedances is provided in order to define the propagation velocity of the emissions. Indeed, it is a logical continuation of the previous theoretical parts which have successively described

- Voltage and currents are actually waves that propagate
- A certain velocity is related to this propagation which depends on the permittivity and permeability of the medium
- The non-linear variation of the permittivity and permeability of the medium with frequency causes a dispersive character of the medium
- Losses also induce a dispersive character in the medium when the relation $\frac{R}{L} = \frac{G}{C}$ is not satisfied

Two Conductors with random sections

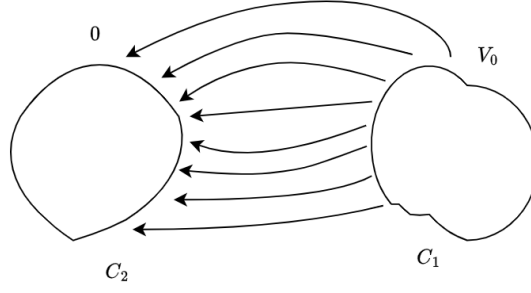


Figure 2.1: Two conductors with random cross section that are at voltages V_0 for the conductor C_1 and V_0 for C_2

When considering the determination of impedances for a transmission line, one crucial fact must be understood. Even though the equations that will define the per-unit-length inductance and capacitance are the same as for low frequency cases, where the voltage can be considered identical at every point in space, it is not possible to multiply the per-unit-length equations by the length of the considered line to obtain the total inductance or capacitance. Indeed, the equations are only defined for an infinitesimal length dz in the direction of propagation.

Nevertheless, these per-unit-length impedances provide the information needed for a more accurate determination of the speed of propagation of the emissions and thus a more accurate determination of the validity of considering that a lumped model is still applicable in the frequency range considered in this work.

First, the global equations for capacitance and inductance determination are provided and then applied to two specific cases, which are the coaxial and two-wire cables. The global equations are provided based on two conductors without specific geometry and being at voltage 0 and V_0 ; this situation is represented in Fig. 2.1.

$$\vec{E} = -\nabla_t \Phi e^{-j\beta z} \quad (2.9)$$

$$V_0 = \int_{C_1}^{C_2} \nabla_t \Phi \cdot d\vec{l} \quad (2.10)$$

$$I_0 = \int_C Y \vec{a}_z \times (-\nabla_t \Phi) \cdot d\vec{l} \quad (2.11)$$

$$C = \frac{Q}{V_0} = \frac{\int_C \rho_s dl}{\int_{C_1}^{C_2} \nabla_t \Phi \cdot d\vec{l}} \quad (2.12)$$

$$L = \frac{\phi}{I_0} = \frac{\int_S \vec{B} \cdot d\vec{S}}{\int_C Y \vec{a}_z \times (-\nabla_t \Phi) \cdot d\vec{l}} \quad (2.13)$$

With

- $\vec{E}$ is the electric field in the transverse plane (perpendicular to the direction of propagation (z))
- $\vec{B}$ is the magnetic field in the transverse plane
- ∇_t is the vector which contains only the two components of the gradient related to the transverse plane
- Φ is the potential vector and is defined based on the electric field $\vec{E}$
- C_1 and C_2 are the two conductors without peculiar sections but homogeneous in the propagation direction (z)
- C is the capacitance
- Q is the charge per unit length
- ϕ is the magnetic flux
- I_0 is the current based on the admittance noted Y

2.2.1 Coaxial cable

The application to the particular case of a coaxial cable is hereafter presented, with a and b respectively the inner and outer conductor radii.

The capacitance is determined through the application of 2.12

$$C = \frac{Q}{V_0} = \frac{\int_C \rho_s dl}{\int_{C_1}^{C_2} \nabla_t \Phi \cdot \vec{dl}} = \frac{2\pi\epsilon}{\ln(b/a)}$$

The inductance is determined through the application of 2.13.

$$L = \frac{\phi}{I_0} = \frac{\int_S \vec{B} \cdot d\vec{S}}{\int_C Y \vec{a}_z \times (-\nabla_t \Phi) \cdot \vec{dl}} = \frac{\mu \ln(b/a)}{2\pi}$$

2.2.2 Two-wire cable

The application to the particular case of a two-wire cable is now investigated, with D the distance between the two wires and d the diameter of each wire.

The capacitance is determined through the same equation, which is Eq. 2.12.

$$C = \frac{Q}{V_0} = \frac{\int_C \rho_s dl}{\int_{C_1}^{C_2} \nabla_t \Phi \cdot d\vec{l}}$$

$$= \frac{\pi \epsilon}{\ln \left[\frac{D + \sqrt{D^2 - d^2}}{d} \right]}$$

The inductance is determined through the same fundamental equation, which is Eq. 2.13.

$$L = \frac{\phi}{I_0} = \frac{\int_S \vec{B} \cdot d\vec{S}}{\int_C Y \vec{a}_z \times (-\nabla_t \Phi) \cdot d\vec{l}}$$

$$= \frac{\mu}{\pi} \ln \left[\frac{D + \sqrt{D^2 - d^2}}{d} \right]$$

2.2.3 Classical low-voltage cables and per-unit-length impedances

The classical cables present in the low-voltage grid are first presented in Table 2.1 based on [50], and then Table 2.2 gives the inductance and resistance per unit length based on [55].

Cable	Type	Cross section (mm ²)
Overhead	BAXB power cable	3 × 95 + 1 × 54.6 + 2 × 16
	BAXB connection	4 × 16 4 × 25
Underground	EAXeVB power cable	4 × 95 4 × 150
	EXVB connection	4 × 16 4 × 25 4 × 35
	XVB connection	2 × 1.5 4 × 1.5 3 × 2.5 4 × 2.5 4 × 6

Table 2.1: Typical cable in low-voltage grid and their cross-sections

It is possible to compare the values of Table 2.2 with the application of the previously cited equation for inductance and the classical Pouillet equation for resistance, which gives for the case XVB 1.5 mm².

$$R_{20C} \left[\frac{\Omega}{km} \right] = \frac{l}{\rho S} = \frac{1000 [m]}{57e6 1.5e^{-6} \left[\frac{Sm^2}{m} \right]} = 11.7 \left[\frac{\Omega}{km} \right]$$

$$L \left[\frac{mH}{km} \right] = l \frac{\mu}{\pi} \ln \left[\frac{D + \sqrt{D^2 - d^2}}{d} \right] = \frac{14\pi 10^{-7}}{\pi} \ln \left[\frac{0.0033 + \sqrt{0.0033^2 - 0.003^2}}{0.003} \right] = 0.177 \left[\frac{mH}{km} \right]$$

Cable	Cross section (mm ²)	Resistance at 20°C [$\frac{\Omega}{km}$]	Inductance [$\frac{mH}{km}$]
EXVB	4x16	1.15	0.264
	4x25	0.727	0.265
	4x35	0.524	0.257
XVB	2x1.5	12.1	0.323
	4x1.5	12.1	0.356
	3x2.5	7.41	0.332
	4x2.5	7.41	0.332
	4x6	3.08	0.294

Table 2.2: Electrical characteristics of typical low-voltage cables

2.3 Signal Analysis : Limitations

Two main limitations will be discussed hereafter, i.e. frequency aliasing and spectral leakage. Both should be well understood in order to avoid issues in interpretation. Indeed, the obtained results are a function of these two limitations and the capability of excluding their non-realistic impact.

Comparison of, for example, results coming from numerical simulations and real experiments could present differences related to these phenomena. Having different sampling frequencies, which in the simulation correspond to the inverse of the time step, would induce, if badly processed, different results even for exactly the same setup.

Frequency Aliasing

First, the definition of frequency aliasing or frequency folding is the overlapping of discrete Fourier transform components. Indeed, knowing that the signal under consideration is sampled leads to a repeating pattern, see Appendix B, with corresponding frequencies being separated in the frequency domain by multiples of the sampling frequency. For example, a signal with maximum frequency 1000 Hz sampled at 3000 Hz will have the corresponding 1000 Hz component at 1000 Hz, -1000 Hz, and at 2000 Hz, 4000 Hz. Basically, this is equivalent to introducing the Nyquist condition, which states that the sampling frequency should be greater than or at least equal to two times the maximum frequency present in the analysed signal.

Throughout this thesis, results will be presented and, if no precision is added, the results coming from numerical simulations were not obtained using an anti-aliasing filter. However, even though this filter is not used, it will be shown that no significant impact can be seen on the magnitude spectrum. It is easily explained by the fact that the sampling frequency, being in this thesis the inverse of the time step (in simulation), is taken to be very high. The time step of the simulation is chosen as 5×10^{-8} s, which is equivalent to a sampling frequency of 20 MHz, based on [45], which states that the time step of the simulation should be at least 100 times smaller than the one related to the switching frequency. This value means, by remembering the previous example, that the maximum frequency of the supraharmonic range, i.e. 150 kHz, would experience frequency folding with components at 19.85 MHz.

Let us consider Fig. 2.2

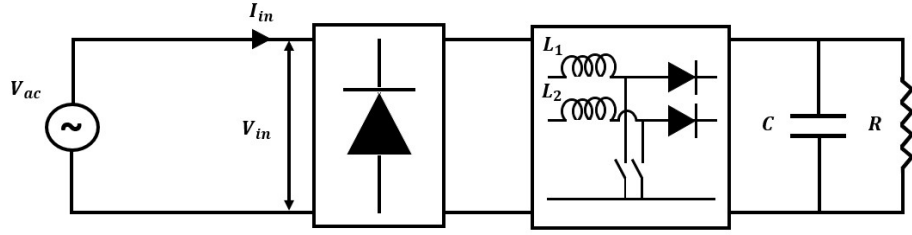


Figure 2.2: Electrical schematic consisting of an ideal voltage source followed by a full-bridge diode rectifier, an interleaved boost converter, and an RC load, used in numerical simulations

As the magnitude decreases with the frequency considered and by looking at the magnitude spectrum in Fig. 2.3 of a numerical simulation of the circuit represented in Fig. 2.2, one can easily see that even the fourth carrier harmonic, as only even carrier harmonics remain (further chapters will go into these aspects in more detail), is 92.15 dB μ A, while the second carrier harmonic was around 102.5 dB μ A. The fourth carrier harmonic is 3.3 times smaller than the second carrier harmonic. The carrier harmonic corresponding to 19.85 MHz is the 794th carrier harmonic, which is totally negligible.

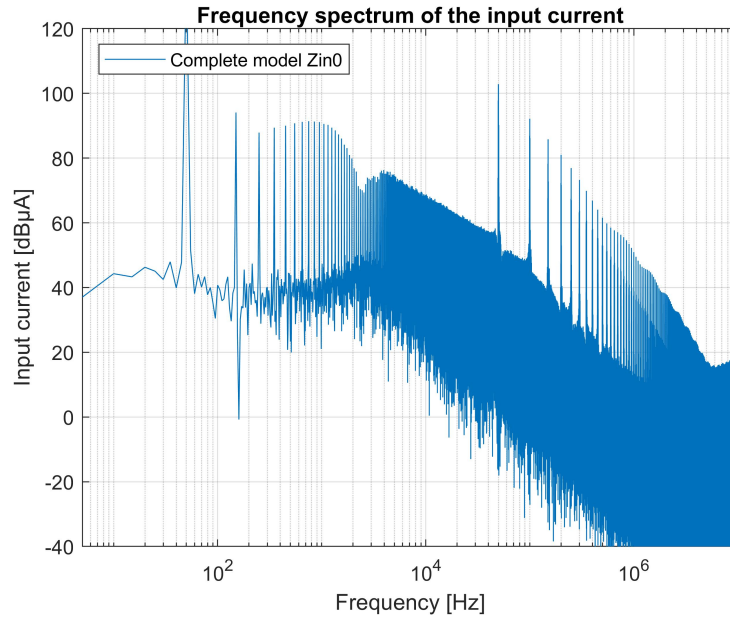


Figure 2.3: Magnitude spectrum of the input current of an interleaved boost converter in a simplified circuit consisting of an ideal voltage source, no grid impedance leading to the legend Z_{in0} , a full-bridge diode rectifier, and the interleaved boost converter

Spectral Leakage

The following information is mainly coming from [56]. When considering limitations in signal analysis, one should look at spectral leakage. Indeed, when performing a discrete Fourier transform, one needs to choose a starting time and an end time, which basically defines the window. The signal contained in this window is considered as periodic; this is inherent to the discrete Fourier transform, so that by repeating this signal in the time domain from $-\infty$ to ∞ , one obtains the signal considered by the mathematical tool used.

The solution to avoid the additional spread of harmonics coming from spectral leakage is to use, for example, a Hamming window, which is shown in Fig. 2.4, along with its magnitude spectrum, to multiply the initial signal. Basically, it ensures that the beginning and the end of the considered signal are zero so that it can effectively be repeated without discontinuity from $-\infty$ to ∞ .

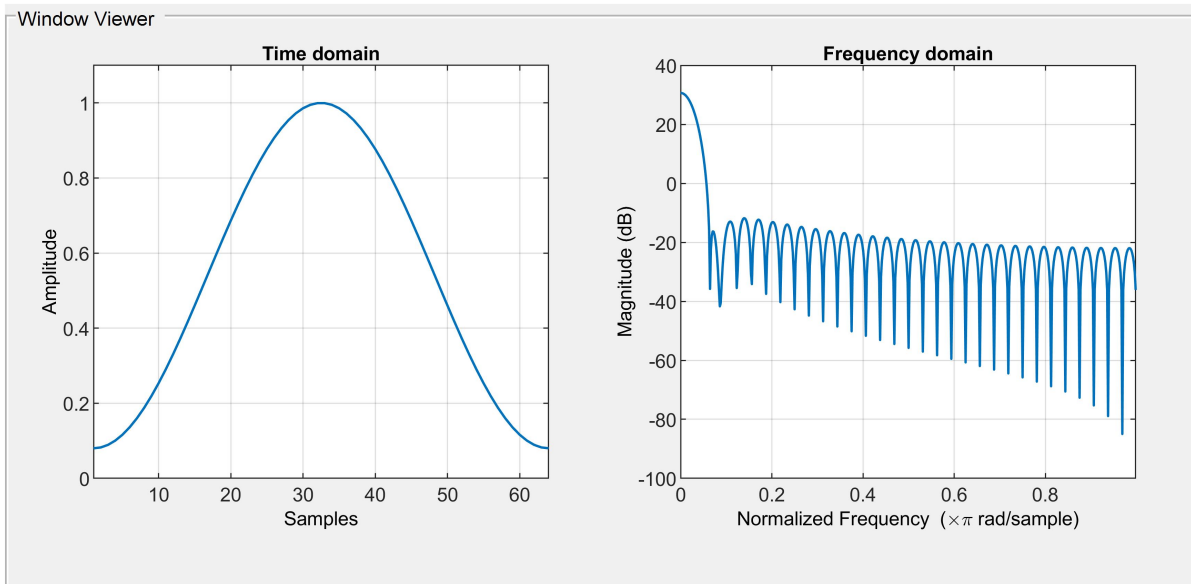


Figure 2.4: Hamming window with 64 samples

As presented in the theoretical background (Appendix B), multiplying a signal in the frequency domain induces a convolution in the time domain and, respectively, multiplying a signal in the time domain induces a convolution in the frequency domain. So, the presented solution has as a consequence to increase the width of the peaks depending upon the considered Hamming length in time. Indeed, this trade-off can also be seen in the famous Heisenberg uncertainty principle, see Eq. 2.14.

$$\Delta x \Delta p \geq \frac{h}{4\pi} \quad (2.14)$$

Basically, the larger the signal in the time domain, the smaller the interval between peaks in the frequency domain, and inversely. In quantum mechanics, physicists would say: the less I know where a particle is, the more I know about its momentum. In the electrical engineering field, it is stated that the greater the portion of the signal I have, the less the

uncertainty on the energy (proportional to the frequency) of this signal, and inversely, see Eq. 2.15. Actually, it is even possible to directly provide the uncertainty principle applied to signal analysis, see Eq. 2.16, with the link between time and frequency noted respectively as t and ω [57].

$$\Delta t \Delta E \geq \frac{h}{4\pi} \quad (2.15)$$

$$\int_{-\infty}^{\infty} t^2 |f(t)|^2 dt \int_{-\infty}^{\infty} \omega^2 |F(\omega)|^2 d\omega \geq \frac{1}{16\pi^2} \quad (2.16)$$

In Fig. 2.5, the input current coming from numerical experimentation of the circuit presented in Fig. 2.2 is shown before and after applying the Hamming window to it. However, it should be noted that this Hamming window does not induce significant differences when computing the discrete Fourier transform of a signal that is constituted of a bunch of sine and cosine functions that fit an integer number of cycles inside the considered window. It is, thus, easy to see if a magnitude spectrum coming from a discrete Fourier transform computation is affected by spectral leakage. Indeed, if this is the case, by first applying the Hamming window as in Fig. 2.5 (right) and then computing the discrete Fourier transform, one would obtain a magnitude spectrum with a reduced level of interharmonics and clear, but wider, peaks.

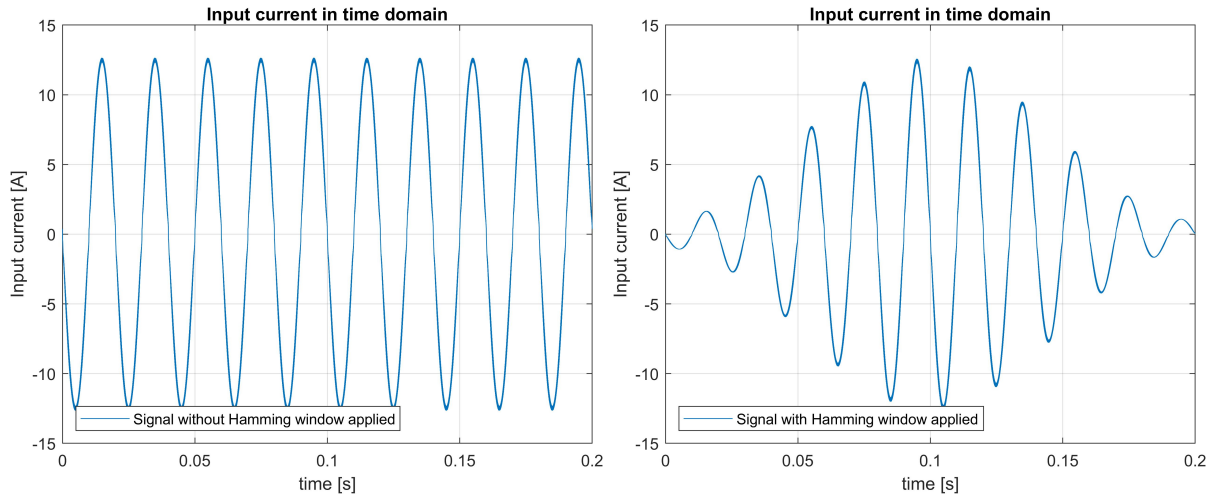


Figure 2.5: Input current in the time domain, with and without applying a Hamming window

Nevertheless, one could argue that in numerical experimentation the supply is ideal so that no small or large frequency variations of the supply are experienced over time, meaning that taking 200 ms does not cause major spectral leakage. However, it should be noted that in reality the supply is not ideal and has small continuous frequency variations, but even without considering this aspect, and even in numerical experimentation, one should be careful. Indeed, having interharmonics with a non-negligible magnitude is possible and could cause, over this range of 200 ms, a non-integer number of cycles. For example,

having a 52.5 Hz component in the signal would cause spectral leakage over 200 ms because the number of cycles contained is 10.5. However, one would not even be aware of that issue because of the frequency resolution being linked to the fundamental frequency and the number of cycles of that frequency contained in the window. Basically, having 10 cycles at 50 Hz results in a 5 Hz resolution magnitude spectrum, so that no frequency with a non-integer number of cycles in the window could be correctly represented in the magnitude spectrum.

This issue is taken into consideration in [4], where a filter is proposed, i.e. the Butterworth filter, which is a second-order filter with 3 dB attenuation at 3 kHz and at 1 MHz. This filter manages the two previously cited issues, frequency aliasing (or folding) and spectral leakage. Indeed, it attenuates the low-frequency (fundamental and harmonics) components as well as the high ones. It is, however, worth noting that in the case of numerical experimentation, where the time step (the inverse of the equivalent sampling frequency) is small enough, no significant frequency folding is expected. Also, as the supply is ideal and does not experience small frequency variations, no interharmonics are expected and thus no spectral leakage should be observed. A comparison is provided in Fig. 2.6 in order to provide a realistic example of the non-impacting spectral leakage phenomenon in numerical experimentation. The circuit considered is again the one in Fig. 2.2. The comparison shows the magnitude spectrum without and with the Hamming window applied to the input current; these two signals are actually presented in Fig. 2.5, on the left and right parts of the figure, respectively.

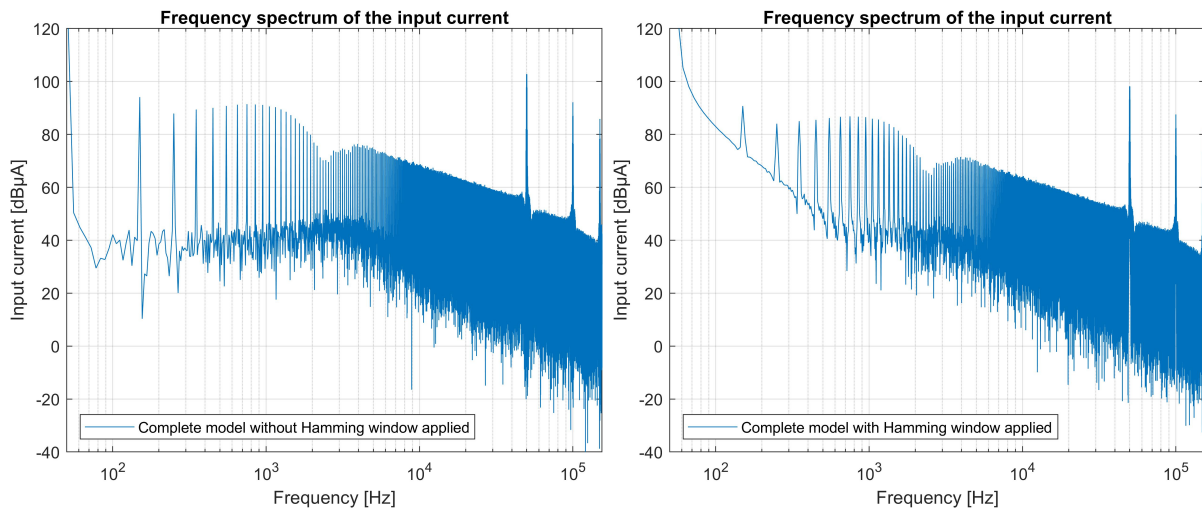


Figure 2.6: Magnitude spectrum of the input current of an interleaved boost converter, with and without applying a Hamming window. The complete circuit is constituted of an ideal voltage source followed by a full-bridge diode rectifier (before which the input current is measured), the interleaved boost converter, and an RC load

This figure could be badly interpreted; indeed, on the right of the figure one could see a kind of spreading of low-frequency harmonics, but one should not forget that the plot uses a logarithmic x-axis. Considering the magnitude spectrum with a non-logarithmic x-axis, see Fig. 2.7, allows one to see that no significant spreading is induced by this Hamming window. Rather, when a Hamming window is applied, a global decrease is observed, which could be explained in two ways. First, it could be due to the fact

that peaks are made wider and smaller in magnitude. The latter is again linked to this duality between convolution and simple multiplication in the time and frequency domains, which has been extensively described in Appendix B. A second way of understanding this result is by considering that previously present spectral leakage has been efficiently removed by this windowing, i.e. by applying a Hamming window. However, based on the discussion presented previously, spectral leakage will be considered negligible as a first-order approximation for the remainder of this work. It may nevertheless constitute a possible source of discrepancy when comparing analytical or numerical results with experimental measurements.

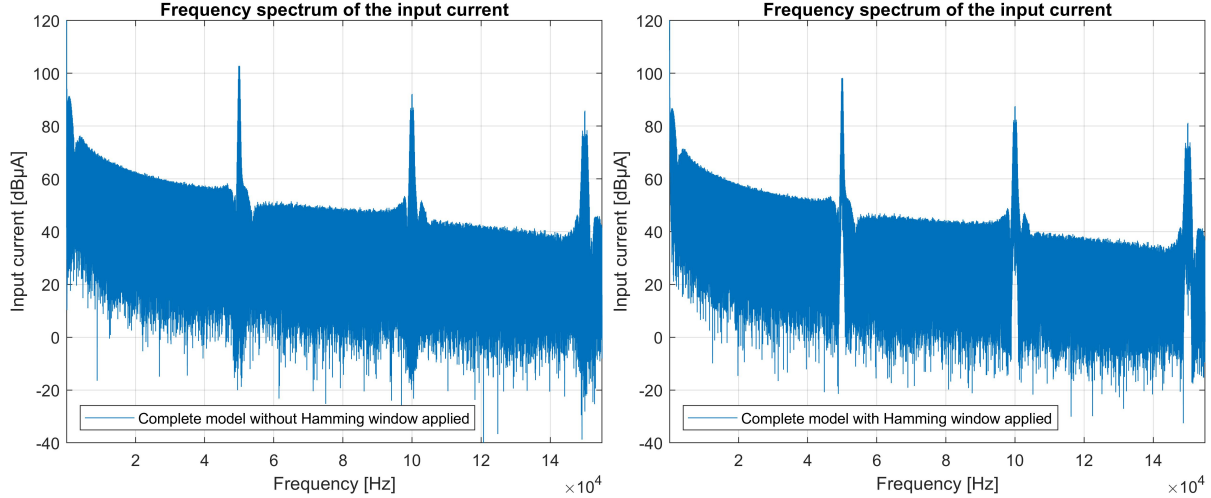


Figure 2.7: Magnitude spectrum of the input current of an interleaved boost converter, with and without applying a Hamming window. The complete circuit is constituted of an ideal voltage source followed by a full-bridge diode rectifier (before which the input current is measured), the interleaved boost converter, and an RC load

2.4 Assumptions and limitations

The previously cited points, i.e. the transmission line theory, the dispersive characteristic of the medium along with the issues related to signal analysis leads to some conclusions. First, the accuracy of a model of converter is intrinsically related to the frequency to be modelled and the distance considered between interacting converters. Second, the values obtained by data post processing are highly sensitive to the two presented limitations of signal analysis, i.e. frequency aliasing and spectral leakage. Regarding the difference that could arise between the theoretical predicted values and the measured one they could also be explained by other elements such as the preexisting condition on the grid, i.e. pre-existing harmonic in voltage, unbalanced grid, variable power emission and consumption during 24h and of course other converters emitting the same or near of the same frequency as the one to be modelled.

Each element of complexity could be considered but a trade off between complexity and accuracy exists. The aim is to model as accurately as possible with assumptions and approximations the emission of one device and eventually predict interactions between devices.

It is worth noting that, the model of the emissions of the converter itself is accurately described by lumped model as the device does not have a dimension which provide a $\frac{l}{\lambda}$ which would be significant, assuming a device of 0.1m and a frequency of 150 kHz with as an approximation a $v_p = c$, one obtains $\frac{l}{\lambda} = 0.00005[/math>.$

It is possible to consider more realistic cases, taking lineic inductance and capacitance related to cables mainly used in LV grid such as EXVB as reported in [50] leads to $v_p \approx \frac{2c}{3}$. Other references regarding the parameters of the cable in low-voltage grid are [58],[59],[60],[61] and [62]. This allows to have an idea of the wavelength associated to the range 2-150 kHz which is thus $\frac{2c}{3f} = 100[km]$ for 2kHz and $\frac{2c}{3f} \approx 1.3[km]$ for 150 kHz. One possible methodology would be to consider the model on $\frac{1}{1000}$ of the wavelength. In the case of the smaller frequency of the range, this self imposed methodology lead to 1 km of line which is of the same order of magnitude as what is expected to be present on the LV grid. Also, regarding the dispersive aspect of the medium it is not directly able to impact the emission process itself but will be of tremendous importance when considering interactions. Actually, the dispersive aspect in the range of frequency and for the purpose of interest which is the LV distribution grid the imaginary part of the propagation coefficient is not expected to experience significant non linear relation with the frequency. As mainly transverse electromagnetic waves are present on that kind of medium.

Eventually, this subsection on the validity of the model has as conclusion that the use of the lumped model is an approximation which is valid, with respect to the ratio of the dimension of the device with respect to the wavelength whatever the frequency is in the range of the SH, for the emission process itself. However, regarding the propagation it is limited to nearby converters, with the definition of nearby being function of the frequency, and the interactions should consider the distance between the converters and the characteristic of the medium to accurately predict the result of n parallel converters with m potentially different cable lengths and characteristics. Nevertheless, the work presented here is mainly focused on the modelling of converters in their emissions which, as explained, does not seem to suffer from this peculiar aspect.

It is interesting to state the assumptions and approximations regarding the models that are developed in this thesis.

Regarding (mainly) the grid side

- Lumped model is used meaning that the current and voltage instantaneously propagate in the whole line and thus these signals are only function of time.
- No dispersive medium has been considered meaning that every frequency does propagates with the same speed, which means in the case of lumped model infinitely fast.
- In this first development, no impedance has been taken into account in the grid.
- No previously existing harmonics are present on the grid in the range of those studied here, i.e. supraharmonics (2-150 kHz)

Regarding (mainly) the converter

- The capacitance is considered to bring ideal constant voltage on the DC side, the validity of this assumption is, indeed, linked to the size of the capacitance.
- The switching process is assumed to be instantaneous, since the transient time is roughly an order of magnitude shorter than the period associated with the highest frequency under consideration (150 kHz)[44]. In addition, quantum effects are not taken into account.
- The inductance is the prominent part, at high frequency, in the impedance of the boost converter.
- No frequency dependent inductance is considered, only one value is taken for the whole frequency spectrum.
- No adaptation was considered for the variation of the passive elements depending upon the temperature.

2.5 Double Fourier Series Presentation

As the validity context has been stated, it is now interesting to present one tool that will be extensively used in this work, i.e. the double Fourier series. It is worth noting that this thesis investigates only converters using pulse width modulation as control strategy.

Actually, the concept of double Fourier series can be first found as coming from the telecommunication domain in [5]. Then this method was used by Bowes and Bird [43] in order to model the emissions of a PWM switched phase leg. Subsequently, an extensive study has been carried out using this double Fourier series for the characterization of the emission of electronic devices of power [3]. More recently, the double Fourier series was used to model power electronic converters such as inverters and active power factor corrector circuits in their high-frequency emissions [1].

This method is quite simple and basically states that a signal which is not linear but periodic and based on the intersection of two periodic signals, a carrier and a modulating signal, can be described using linear tools such as the Fourier series, but to the extent of taking into account an adaptation. The latter being that the dimension of the Fourier series need to be increased and thus be a two-dimensional Fourier series.

This notion of extending the number of considered dimensions is particularly compelling, as it enables the mathematical treatment of signals exhibiting strong nonlinear behavior. Indeed, by projecting such signals into higher-dimensional spaces, one can legitimately apply linear analytical tools—such as the Fourier series—even in contexts where linearity is not inherently present. This dimensional extension is not intrinsically tied to the Fourier series alone; it also emerges in other mathematical constructs. For instance, the scalar product, when extended to an infinite number of dimensions, gives rise to the inner product. Similarly, the concept of Hilbert space, which plays a foundational role in the formalism of quantum mechanics, relies on this very principle of infinite dimensionality [63]. Both of these examples are explored in greater depth in Appendix B, which is devoted to the presentation of key theoretical preliminaries.

Regarding the main interest of this thesis, i.e. modelling of power electronic devices using as modulation strategy the pulse width modulation (PWM), the two dimensions Fourier series is sufficient.

In [3] the complete derivation of the double Fourier series is provided starting from the simple Fourier series and considering that the classical one-dimensional Fourier series is just a peculiar case of the two-dimensional Fourier series. The derivation is further extended to three dimensions in Appendix B and could, of course, be extended to even higher dimensions.

Considering the classical one-dimensional Fourier series that can be written as follows, where $f(t)$ is a periodic signal, a_0 is the average value, a_m and b_m are the Fourier coefficients :

$$f(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m\omega t + b_m \sin m\omega t)$$

Noting that in the cases of interest in this work, i.e., the signals associated with the PWM strategy, the function $f(t)$ can be expressed as a function that depends on two signals, x and y , corresponding respectively to the carrier and the modulating wave. With ω_c , ϕ_c and ω_m , ϕ_m denote the frequencies and initial phases of the carrier and modulating waves, respectively.

$$\begin{aligned} x &= \omega_c t + \phi_c \\ y &= \omega_m t + \phi_m \end{aligned}$$

Therefore, the function $f(t)$ can be written as $f(x, y)$. The one-dimensional Fourier series can then be understood as a description of $f(x, y)$ but for $y = y_0$ with y_0 a particular value of y . Thus, $f(x, y_0)$ can be rewritten as

$$f(x, y_0) = \frac{a_0(y_0)}{2} + \sum_{m=1}^{\infty} (a_m(y_0) \cos mx + b_m(y_0) \sin mx)$$

In the following the pulse width modulation strategy is illustrated along with the representation in the double Fourier series domain of the carrier and the modulating signals and finally the resulting signal, which is actually the switching function, is also shown.

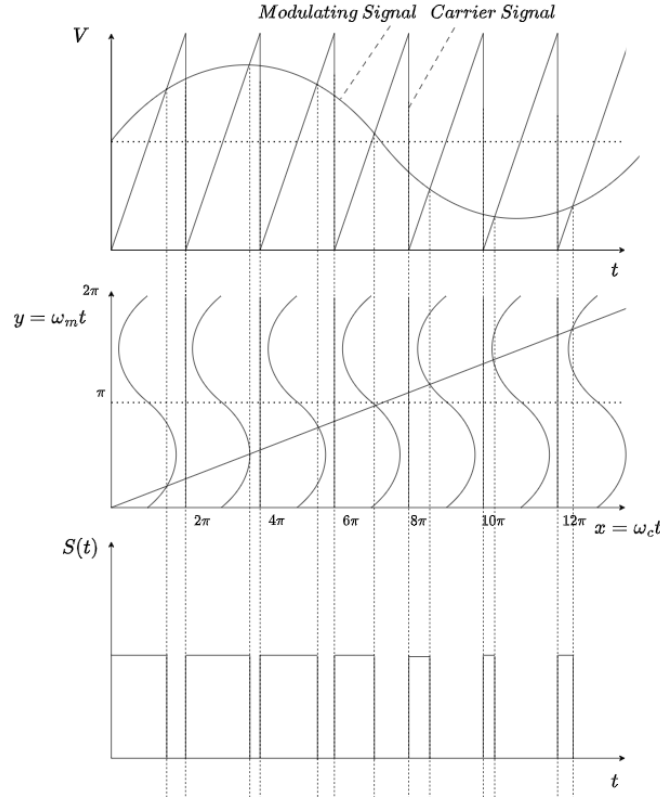


Figure 2.8: Double Fourier series representation of a pulse-width modulation strategy with a sawtooth carrier

Ultimately, this leads to the concept of the switching function, denoted by $S(t)$, which is the final rectangular signal with variable width presented just before. Actually, this switching function will be the interesting feature for emission determination. The analytical expression of this function is derived in 2.17.

Essentially, applying the double Fourier series allows for the determination of the Fourier coefficients, A and B , of the switching function. This process involves evaluating a double integral with carefully defined bounds.

$$A + jB = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{j(mx+ny)} dx dy \quad (2.17)$$

The double Fourier series captures the interactions between two signals: the carrier and the modulating wave, as used in PWM modulation. The bounds of the integrals are determined by these signals, with the outer bounds of the integrals corresponding to the carrier signal and the inner bounds to the modulating wave.

2.6 Introduction to the Theoretical Concept of Voltage and Current Harmonic Sources

This chapter is devoted to the characterization of a converter's behavior, as a voltage harmonic source or a current harmonic source, respectively denoted VHS and CHS. These two notions will be further explained. The objective is to facilitate a deeper understanding of the converter's response to varying operating conditions, which are predominantly linked to the characteristics of electrical networks, and more specifically, to fluctuations in their associated impedance.

Accordingly, this chapter establishes the theoretical foundations of a specific concept and its associated definitions, thereby enabling their rigorous use throughout the remainder of this work.

In essence, following the presentation of the assumptions and limitations underpinning the model developed herein, and based on the method previously introduced—namely, the double Fourier series—this section introduces a characterization framework that enables the simplification of how a converter's reaction to variations in grid impedance is described.

The characterization is based on the sustained and significant presence of supraharmonic emissions at the terminals of the converter, whether in voltage or current.

In practice, neither of these behaviors is purely exhibited; instead, a spectrum of intermediate behaviors can be observed. However, depending on the impedance ratio between the electrical network and that presented by the converter, it becomes possible to classify the emissions as being predominantly maintained in voltage or in current. This leads to the respective nomenclature of Voltage Harmonic Source (VHS) and Current Harmonic Source (CHS).

An illustrative example is provided below to represent the voltages and currents under consideration.

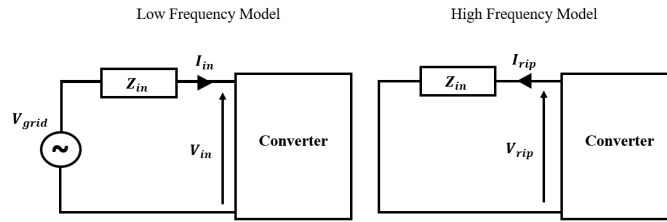


Figure 2.9: Representation of the low-frequency and high-frequency model of any converter with V_{grid} , Z_{in} , I_{in} , V_{in} , I_{rip} and V_{rip} being respectively the low-frequency (50 Hz) voltage, the grid impedance, the input current and voltage (50 Hz), and the ripple current and voltage (at the switching frequency), which actually represent the emission itself

The following is based on the assumption that the emissions are not significantly affected by the considered variation in grid impedance. This is equivalent to assuming that, at 50

Hz, the impedance variation is not substantial enough to noticeably alter the converter's input voltage, denoted as V_{in} . The conditions for characterization as a Voltage Harmonic Source (VHS) or a Current Harmonic Source (CHS) are listed below.

Voltage Harmonic Source, VHS

$$V_{rip}|_{Z_{in}} \approx V_{rip}|_{\frac{Z_{in}}{N}} \approx V_{rip}|_{NZ_{in}}$$

Current Harmonic Source, CHS

$$I_{rip}|_{Z_{in}} \approx I_{rip}|_{\frac{Z_{in}}{N}} \approx I_{rip}|_{NZ_{in}}$$

With N being an arbitrarily chosen constant.

However, although this notion is valuable in that it facilitates the prediction of a converter's behavior with respect to its expected emissions into the grid, it remains a simplification that results in a problematic loss of frequency-specific visibility. Indeed, passive elements such as inductors and capacitors, which are embedded within converters and can also serve to model the grid's impedance, are disregarded in this approach. This leads to the omission of resonance phenomena. Thus, while the model may be accurate in describing emissions at the converter's switching terminals, the actual emissions observed at the converter's interface with the grid are inherently shaped by these passive components and their potential to form series and/or parallel resonances.

As such, even once this characterization has been carried out, it remains necessary to identify the critical frequencies in order to avoid any problematic situations that could lead to damage within the grid or, more simply, to a significant amplification of emissions initially considered negligible.

This can be achieved by analyzing the converter's topology in conjunction with an appropriate modeling of the network at the frequency of interest, which corresponds to the switching frequency. Let us recall that all of this must also take into account the previously mentioned limitations, particularly regarding the dispersive nature of the propagation medium and the invalidity of lumped-element modeling once the wavelength associated with the frequency under consideration becomes comparable to or smaller than the physical length of the line or network. For the interested reader, a more in-depth analysis is provided later in this work, specifically in the section dedicated to the investigation of the impact of line parasitic capacitance for a particular converter case, as detailed in Section 3.2.3.

Chapter 3

Converter Modelling

3.1 Introduction

In keeping with the logical progression of the previously discussed sections, the present chapter introduces the application of theoretical tools to the modeling of converters with regard to their high-frequency emissions, namely supraharmonics. Initially, the energy context and the rationale for the widespread deployment of power converters—particularly in low-voltage networks—were outlined. Subsequently, Chapter 2.1 established that the modeling approach is based on a set of assumptions and approximations and introduced the primary analytical framework upon which the forthcoming analysis relies, namely the double Fourier series. This led to a discussion of a specific characterization methodology addressing the distinction between Voltage Harmonic Sources (VHS) and Current Harmonic Sources (CHS). The application of this modeling approach is thus successively presented for the Boost converter, followed by the Interleaved Boost Converter, and finally the Single-Phase Inverter.

3.2 Analysis of the Boost Converter

Among all the well-known converters, the boost is classically used and studied in the engineering field. Regarding the applications in which boost converters can be used, one could cite, among others, PV panels [64, 65, 66] and electric vehicle chargers [25, 49, 67, 68]. Even though the main interest has been shown to be the interleaved boost converter the author has as idea that the starting point should always be the easiest one. Indeed, the whole methodology of this thesis will follow this logic, developing new understandings requires that the classical, basic knowledge, is well acquired.

The present section presents numerical and analytical results. The latter being obtained through the development of the previously cited tool, i.e. the double Fourier series (DFS). First, a brief presentation of the logic behind this method is provided. Afterwards, DFS will be used to obtain the expression of the Fourier coefficients of a given signal. Based on this expression and on classical circuit theory the input current is derived, for the case of zero grid impedance.

Subsequently, two successive modifications are introduced into the analytical development in order to consider a realistic aspect, i.e. the non-zero grid impedance. The complexity of the studied field emphasizes the necessity of using the present methodology, i.e., adding realistic aspects one at a time and understanding the impact of each before incorporating the next. Without this methodology, the reader may misinterpret the results due to the non-trivial interactions between the realistic aspects. Indeed, beginning by considering an ideal voltage source, for single-phase high-frequency load modelling, as the grid is common practice [69]. However, here the grid impedance is merely represented by the cable impedance which is a simplification, even though it has been shown to be crucial for SH propagation [70] and allows a detailed model of single-phase emitter to be developed. Regarding the topic of grid modelling in SH range one should note that this remains an active research topic and several works have found that the current reference impedance for grid modelling in the SH range is significantly greater than the measured one [13][53][71][72][73].

3.2.1 Modelling of The Boost Converter with Active Power Factor Correction Function

The topology of the boost with DC voltage source is shown in Fig. 3.1.

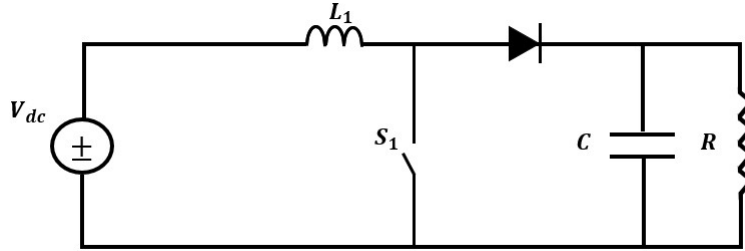


Figure 3.1: Electrical representation of the boost converter

In order to study any converter, it is necessary to be able to control it; therefore, for the numerical study, the control should be well defined. The control considered in this thesis, namely average current control with pulse-width modulation (PWM) as the control strategy, is the same as that used in [1], which was itself developed based on [44].

3.2.2 The double Fourier series applied to the boost converter

First, the boost converter is used as an active power factor corrector (APFC), and the circuit in which it is studied is shown in Fig. 3.2.

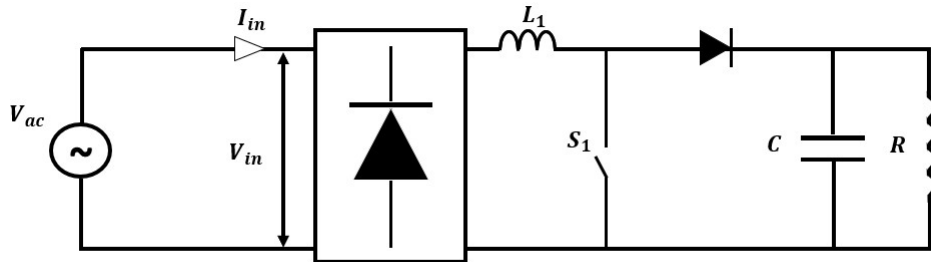


Figure 3.2: Complete circuit under study: an AC voltage source followed by a full-bridge diode rectifier and a boost converter operating as an APFC

The double Fourier series is used to determine the switching function, $S(t)$, which alternates between 1 and 0 depending on time. This time dependence conceals the underlying dependence on two distinct signals, thereby justifying the use of the double Fourier series. These two signals are the modulating wave and the carrier signal, as used in PWM (Pulse Width Modulation). The intersection of these signals triggers the transition of the switching function from 1 to 0. For a sawtooth carrier, the transition from 0 to 1 occurs when the carrier completes a full switching period.

In the case of an APFC, the boost converter has two primary objectives: maintaining a constant voltage on the DC side and achieving a power factor as close as possible to 1.

The modulating wave represents the required duty cycle. For the boost converter and the circuit shown in Fig. 3.2, the modulating wave, $M(t)$, can be easily derived, noting that D is the duty cycle, V_c is the voltage across the load capacitance, V_{in} refers to the rectified input voltage, and ϕ_0 is the initial phase of the input voltage. One should also note that magnitudes are denoted by a hat.

$$\begin{aligned} V_c(t) &= \frac{V_{in}(t)}{1 - D(t)} \\ 1 - D(t) &= \frac{V_{in}(t)}{V_c(t)} \\ D(t) &= 1 - \frac{V_{in}(t)}{V_c(t)} \end{aligned}$$

Noting that the input voltage is, actually, the rectified one

$$V_{in}(t) = |\hat{V}_{in} \sin(\omega t + \phi_0)|$$

$$D(t) = 1 - \frac{\hat{V}_{in}}{V_c(t)} |\sin(\omega t + \phi_0)| = M(t)$$

Considering that the initial phase of the input voltage is $\phi_0 = 0$, and defining $y = \omega_g t$, with ω_g being the grid angular frequency, the modulating wave is therefore expressed as one minus the rectified grid voltage. This leads to the division of Equation 2.17 into two parts, as the modulating wave is defined by Equations 3.1 and 3.2. It is important to note that r represents the ratio between the peak value of the grid voltage and the DC voltage. Hereafter, m denotes the carrier harmonic order and n the grid harmonic order, implying that $m = 1$, $n = 2$ corresponds to the second sideband harmonic in the first carrier harmonic band.

$$M(t) = 1 - r \sin(y) \quad \text{for } 0 < y < \pi \quad (3.1)$$

$$M(t) = 1 + r \sin(y) \quad \text{for } \pi < y < 2\pi \quad (3.2)$$

Finally, Equ. 2.17 becomes

$$A + jB = \frac{1}{2\pi^2} \left(\int_0^\pi \int_0^{2\pi(1-r\sin(y))} e^{j(mx+ny)} dx dy + \int_\pi^{2\pi} \int_0^{2\pi(1+r\sin(y))} e^{j(mx+ny)} dx dy \right)$$

Hereafter, the full development leading to the determination of these coefficients is provided. For clarity, the subscript mn is omitted throughout the derivation.

$$A + jB = \frac{1}{2\pi^2} \left(\int_0^\pi \int_0^{2\pi(1-r\sin(y))} e^{j(mx+ny)} dx dy + \int_\pi^{2\pi} \int_0^{2\pi(1+r\sin(y))} e^{j(mx+ny)} dx dy \right) \quad (3.3)$$

$$\begin{aligned} A_1 &= \int_0^{2\pi(1-r\sin(y))} e^{j(mx+ny)} dx dy \\ &= \left[\frac{e^{j(mx+ny)}}{jm} \right]_0^{2\pi(1-r\sin(y))} \\ &= \frac{1}{jm} \left(e^{j(m(2\pi(1-r\sin(y)))+ny)} - e^{jny} \right) \\ &= \frac{1}{jm} \left(e^{j(-m2\pi r\sin(y))} e^{jm2\pi} e^{jny} - e^{jny} \right) \\ A_2 &= \int_0^{2\pi(1+r\sin(y))} e^{j(mx+ny)} dx dy \\ &= \frac{1}{jm} \left(e^{j(m(2\pi(1+r\sin(y)))+ny)} - e^{jny} \right) \\ &= \frac{1}{jm} \left(e^{j(m2\pi r\sin(y))} e^{jm2\pi} e^{jny} - e^{jny} \right) \end{aligned}$$

By changing the bounds of the second integral for A_2 , i.e. $\int_\pi^{2\pi} A_2 dy$

to $\int_0^\pi A_2 dy$ then one can rewrite A_2 as

$$A_2 = \frac{1}{jm} \left(e^{j(m2\pi r\sin(y-\pi))} e^{jm2\pi} e^{jn(y-\pi)} - e^{jn(y-\pi)} \right)$$

Each impacted element can be rewritten using trigonometric identities

$$\sin(y - \pi) = \sin(y)\cos(\pi) - \cos(y)\sin(\pi) = -\sin(y)$$

$$\begin{aligned} e^{jn(y-\pi)} &= \cos(ny - n\pi) + j\sin(ny - n\pi) \\ &= \cos(ny)\cos(n\pi) + \sin(ny)\sin(n\pi) + j\sin(ny)\cos(n\pi) - j\cos(n\pi)\sin(n\pi) \\ &= \cos(ny)(-1)^n + j\sin(ny)(-1)^n \\ &= (-1)^n e^{jny} \end{aligned}$$

$$\begin{aligned} A_1 &= \frac{1}{jm} \left(e^{j(-m2\pi r \sin(y))} e^{m2\pi} e^{jny} - e^{jny} \right) \\ A_2 &= (-1)^n \frac{1}{jm} \left(e^{j(-m2\pi r \sin(y))} e^{m2\pi} e^{jny} - e^{jny} \right) \end{aligned}$$

For n even $A_1 = A_2$ and as the bounds have been changed to be the same

$$A_1 + A_2 = \int_0^\pi \frac{2}{jm} \left(e^{j(-m2\pi r \sin(y))} e^{m2\pi} e^{jny} - e^{jny} \right) dy$$

Also, it is obvious that $e^{jm2\pi} = 1$

Moreover $\int_0^\pi e^{jny} dy = (-1)^n$ and as n is even $e^{jny} = 1$ thus $\int_0^\pi e^{jny} dy = 0$

$$A + jB = \frac{2}{jm2\pi^2} \int_0^\pi e^{j(-m2\pi r \sin(y) + ny)} dy \quad (3.4)$$

$$\begin{aligned} &= \frac{2}{m2\pi^2} \int_0^\pi \left(-j\cos(-m2\pi r \sin(y) + ny) dy + \sin(-m2\pi r \sin(y) + ny) \right) dy \\ &= \frac{1}{m\pi^2} \int_0^\pi \sin(-m2\pi r \sin(y) + ny) dy - \frac{j}{m\pi^2} \int_0^\pi \cos(-m2\pi r \sin(y) + ny) dy \end{aligned}$$

$$A = \frac{1}{m\pi^2} \int_0^\pi \sin(-m2\pi r \sin(y) + ny) dy \quad (3.5)$$

$$B = -\frac{1}{m\pi^2} \int_0^\pi \cos(-m2\pi r \sin(y) + ny) dy \quad (3.6)$$

Once these coefficients are determined, one can derive a simpler expression

$$S_{rip}(t) = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} X_{mn} \sin(mx + ny + \beta_{mn}) \quad (3.7)$$

With

$$\begin{aligned} X_{mn} &= \sqrt{A_{mn}^2 + B_{mn}^2} \\ \beta_{mn} &= \text{atan}\left(\frac{A_{mn}}{B_{mn}}\right) \end{aligned}$$

Based on A and B coefficients it is possible to go further in order to obtain an expression for the carrier harmonics

$$A = \frac{1}{m\pi^2} \int_0^\pi \sin(-m2\pi r \sin(y) + ny) dy$$

$$B = -\frac{1}{m\pi^2} \int_0^\pi \cos(-m2\pi r \sin(y) + ny) dy$$

Using bessel functions with the next known relations

$$\sin(\xi \sin(\theta')) = 2 \sum_{k=0}^{\infty} J_{2k+1}(\xi) \sin([2k+1]\theta') \quad (3.8)$$

$$\cos(\xi \sin(\theta')) = J_0(\xi) + 2 \sum_{k=1}^{\infty} J_{2k}(\xi) \cos(2k\theta') \quad (3.9)$$

It is possible to rewrite the coefficients A and B when considering $n = 0$, i.e. only the carrier harmonics. Subsequently, A is defined as

$$A = \frac{1}{m\pi^2} \int_0^\pi \sin(-m2\pi r \sin(y)) dy = \frac{2}{m\pi^2} \sum_{k=0}^{\infty} J_{2k+1}(-m2\pi r) \int_0^\pi \sin([2k+1]y) dy \quad (3.10)$$

$$= \frac{2}{m\pi^2} \sum_{k=0}^{\infty} J_{2k+1}(-m2\pi r) \left[\frac{-\cos([2k+1]y)}{2k+1} \right]_0^\pi \quad (3.11)$$

$$= \frac{4}{m\pi^2} J_1(-m2\pi r) + \frac{2}{m\pi^2} \sum_{k=1}^{\infty} J_{2k+1}(-m2\pi r) \left[\frac{-\cos([2k+1]y)}{2k+1} \right]_0^\pi \quad (3.12)$$

$$= \frac{4}{m\pi^2} J_1(-m2\pi r) + \frac{4}{3m\pi^2} J_3(-m2\pi r) + \frac{2}{m\pi^2} \sum_{k=2}^{\infty} J_{2k+1}(-m2\pi r) \left[\frac{-\cos([2k+1]y)}{2k+1} \right]_0^\pi \quad (3.13)$$

Eventually A can be expressed in terms of bessel functions as

$$A = \sum_{k=0}^{\infty} \frac{4}{[2k+1]m\pi^2} J_{2k+1}(-m2\pi r) \quad (3.14)$$

Afterwards B is also developed in terms of bessel functions

$$B = \frac{1}{m\pi} - \frac{1}{m\pi^2} \int_0^{\pi} \cos(m2\pi r \sin(y)) dy = \frac{1}{m\pi} - \frac{1}{m\pi^2} \left(\int_0^{\pi} J_0(m2\pi r) dy \right. \\ \left. + \sum_{k=1}^{\infty} J_{2k+1}(m2\pi r) \int_0^{\pi} \cos(2ky) dy \right) \quad (3.15)$$

$$= \frac{1}{m\pi} - \frac{1}{m\pi^2} \left(\int_0^{\pi} J_0(m2\pi r) dy \right. \\ \left. + \sum_{k=1}^{\infty} J_{2k+1}(m2\pi r) \left[\frac{\sin(2ky)}{2k} \right]_0^{\pi} \right) \quad (3.16)$$

$$= \frac{1}{m\pi} - \frac{1}{m\pi^2} \left(\int_0^{\pi} J_0(m2\pi r) dy + 0 \right) \quad (3.17)$$

$$= \frac{1}{m\pi} - \frac{1}{m\pi^2} \left(J_0(m2\pi r) \int_0^{\pi} dy \right) \\ = \frac{1}{m\pi} - \frac{1}{m\pi} J_0(m2\pi r)$$

Eventually B can be expressed in terms of bessel functions as

$$B = \frac{1}{m\pi} - \frac{1}{m\pi} J_0(m2\pi r) \quad (3.18)$$

Eventually, X_{m0} can be expressed using the new expressions for A and B as follows:

$$X_{m0} = \sqrt{\left(\frac{4}{2k+1} \sum_{k=0}^{\infty} J_{2k+1}(-m2\pi r) \left(\frac{1}{m\pi^2} \right) \right)^2 + \left(\frac{1}{m\pi} - \frac{1}{m\pi} J_0(m2\pi r) \right)^2} \quad (3.19)$$

Even though the ripple component is now expressed, the average value is still not present in the provided expression of the switching function. As stated, the switching function is directly linked to the modulating wave, which itself is equivalent to the desired duty cycle, via the carrier. This interaction between the modulating wave and the carrier explains why high-frequency components are present, while the average value is only a representation of the ideal desired duty-cycle function, i.e., the modulating wave function. Basically, the average value at each time corresponds to the ideal duty cycle at that time. One should remember that, based on the representation of any signal via the classical Fourier series, it is possible to obtain the average part of the function through integration over a period.

Subsequently, circuit theory is employed to extract the Fourier components of the rectified current.

$$V_{inrect}(t) - L \frac{di(t)}{dt} = (1 - S(t))V_{DC} \quad (3.20)$$

$$i(t) = \frac{1}{L} \int (V_{inrect}(t) - V_{DC})dt - \frac{V_{DC}}{L} \int S(t)dt \quad (3.21)$$

Assuming no high frequency present on the input voltage and a voltage almost constant on DC side one could neglect, when high frequency are of interest, the integral containing these two elements leading to

$$i_{rip}(t) = \frac{V_{DC}}{L} \int S_{rip}(t)dt \quad (3.22)$$

Passing from time domain towards frequency domain leads to

$$i_{rip}(\omega) = \frac{V_{DC}}{j\omega L} S_{rip}(\omega) \quad (3.23)$$

Eventually, one could write

$$i_{rip}(\omega) = \frac{V_{rip}(\omega)}{j\omega L} \quad (3.24)$$

From Equ. 3.24 it is easily understood that the frequency composition of the input current, i.e. the current at the input of the boost converter, is a function of the frequency spectrum of the voltage at that point divided by the impedance of the inductance. From Equ. 3.5 and 3.6 one can see that there exists a direct link between the order of the harmonic considered and its magnitude. Also the current is expressed in Equ. 3.24 as divided by the frequency, those two information together leads to a simple but important conclusion. The slope of the current is -40dB/decade, indeed, the denominator can be expressed for the first and the tenth carrier harmonic as

$$i_{rip\ m1}(\omega) = \frac{V_{DC}}{jL} \frac{S_{rip\ m1}}{1\omega_1} \quad (3.25)$$

$$i_{rip\ m10}(\omega) = \frac{V_{DC}}{jL} \frac{S_{rip\ m1}}{10\omega_1 10} \quad (3.26)$$

$$\begin{aligned} 20 \log_{10} \left(\frac{i_{rip\ m1}}{i_{rip\ m10}} \right) &= 20 \log_{10}(100) \\ &= 40 \text{ dB} \end{aligned}$$

As the voltage spectrum itself is dependent on the order of the carrier harmonic and therefore experiences a decrease of 20 dB/decade, the current that is voltage and impedance dependent thus decreases with a slope of 40 dB/decade.

A deeper explanation will be presented in chapter 5 when all the studied converters are presented, this chapter is dedicated to the deepening of the understanding.

As a summary of the previously developed material

- The frequency spectrum of the current is a function of both the frequency spectrum of the voltage at that point and the impedance between this voltage and the grid.
- The frequency spectrum of the voltage is assumed to be, in high frequency, assumed to originate only from the switching process.
- The frequency spectrum of the voltage is only a function of the voltage on the DC side, which acts as a gain, and on the frequency spectrum of the switching function.
- This expression is valid only for a grid which does not contain initially high frequency harmonics.
- Last but not least, based on the DFS it has been shown that it is possible to obtain analytically an expression of the input current that contains the carrier harmonics and their sidebands.

3.2.2.1 Insight regarding the understanding of the analytical expression

In the following, an interpretation of the circuit as a whole is presented. This is based on the superposition of two other circuits.

First, let's remember that the circuit is an active power factor corrector (APFC) meaning that the voltage and current at the converter input are in phase and have sinusoidal waveforms. This fact ensures that, due to the control, the converter is seen from the grid's perspective as a resistance but not only. Indeed, the analytical development shows that a ripple current exists and this does not inherently originate from the grid but from the operation of the converter.

The converter is therefore also a high-frequency source, the question of the nature of this source (voltage or current source) is easily determined when an ideal DC voltage, i.e. a constant voltage, is considered. Indeed, across the switch the voltage is forced to be 0 V or V_{DC} [V] (in the case of an ideal switch) which means that the converter behaves as a high-frequency voltage source. The analysis of the case where V_{DC} is not constant is presented in Chapter 5.

Regarding the analysis of the boost converter, the application of this superposition of models is represented in Fig. 3.3

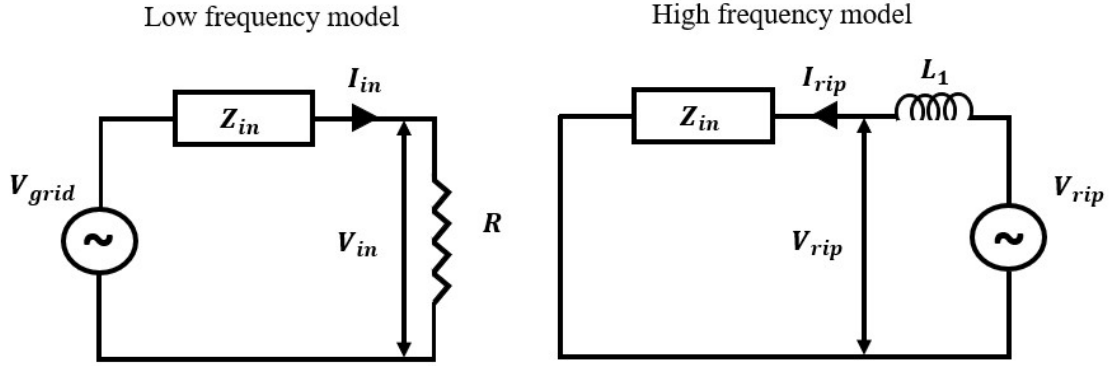


Figure 3.3: Representation of the low and high frequency model that describes both situations with Z_{in} previously considered as 0

Currently, Z_{in} has been assumed to be zero, resulting in a model that constitutes an approximation, as previously described in the section on grid-related approximations. The validity of this simplification should be assessed when aiming to predict real emissions.

A representation of the impedance of the low and high frequency models are respectively shown in Fig. 3.4 left and right.

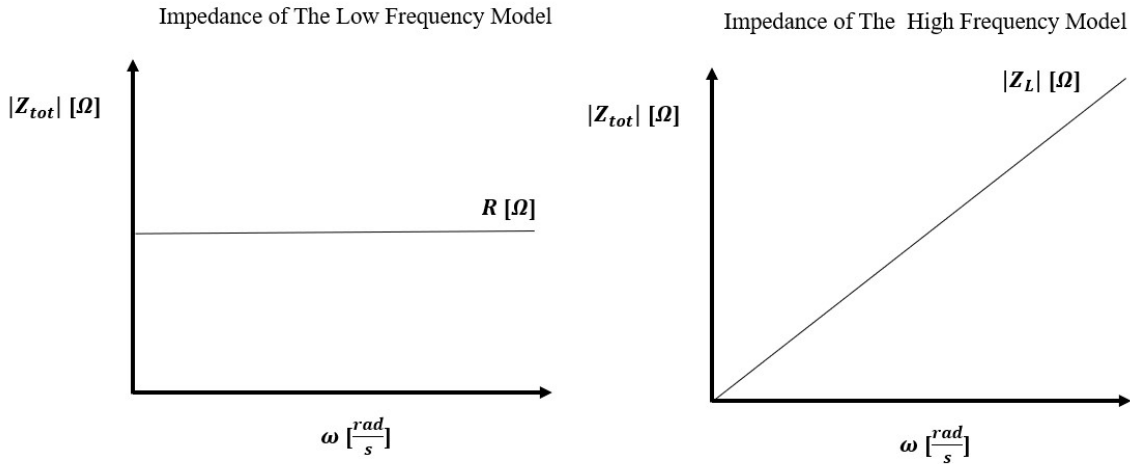


Figure 3.4: Representation of the impedance of the low and high frequency model, noted Z_{tot} and being R and ωL_{b1} for respectively the low and high frequency model, that describes both situations with the grid impedance considered as 0

Even though this model is really simple and does not, as a first step, consider the impedance of the grid. It nevertheless allows to better understand the ongoing process. The switches generate a rectangular signal in voltage, for more details refer to chapter 5,

which is then applied to the only impedance seen, when no grid impedance is considered, meaning the inductance of the boost converter which presents a linear impedance with respect to the frequency.

From the previous discussion one could even easily predict the shape of the current from the classical equation of the voltage across an inductance denoted by L

$$L \frac{di}{dt} = V \quad (3.27)$$

The integration of the voltage applied across the boost inductance divided by the inductance gives the current. It means that the current shape directly depends on the width of the different rectangular signals contained within one half-cycle of the grid period, as a full bridge diode rectifier is present before the boost.

Obviously, an inverter and an APFC will not present the same pattern of switching function, i.e. succession of rectangular shapes with variable width. One could see that the current will exhibit a triangular shape with varying geometries, indeed for a duty-cycle of 0.9 for a given switching period (T_{sw}) the current will experience a rise during $0.9T_{sw}$ and a decrease during only $0.1T_{sw}$ which is, geometrically speaking, an almost right-angle triangle. Under the assumption of non-varying voltage of the grid during T_{sw} one could even prove that the current decrease to reach the value that the current has before it starts rising. The demonstration is developed and presented hereafter with D the duty-cycle for a given moment t .

$$i(0) = 0 \quad (3.28)$$

$$i(DT_{sw}) = i(0) + \frac{di(0)}{dt} DT_{sw} = i(0) + \frac{V_L}{L} DT_{sw} \quad (3.29)$$

$$i(T_{sw}) = i(DT_{sw}) + \frac{V_L}{L} (1 - D)T_{sw} \quad (3.30)$$

$$i(T_{sw}) = i(0) + \frac{V_L}{L} DT_{sw} + \frac{V_L}{L} (1 - D)T_{sw} \quad (3.31)$$

$$i(T_{sw}) = i(0) + \frac{V_{grid}(0)}{L} DT_{sw} + \frac{V_{grid}(0) - V_{DC}}{L} (1 - D)T_{sw}$$

Knowing that V_{DC} is actually the output voltage of the boost converter which can therefore be expressed as $\frac{V_{grid}(0)}{1-D}$ one may therefore write

$$i(T_{sw}) = i(0) + \frac{V_{grid}(0)}{L} DT_{sw} - \frac{V_{grid}(0)}{L} DT_{sw} + \frac{V_{grid}(0)}{L} T_{sw} - \frac{V_{grid}(0)}{L} T_{sw} \quad (3.32)$$

$$i(T_{sw}) = i(0) \quad (3.33)$$

However, this approximation leads to an error because of the non-constant grid voltage during T_{sw} . This error is a function of two parameters. First, the switching frequency,

the greater it is, the smaller the switching period so that the voltage has less time to vary. The second variable is time itself as the duty cycles vary in time, when an APFC is considered, the current has a sinusoidal shape, which inherently includes segments of the signal where the derivative is greater (the variation over time), e.g. when the sinusoidal waveform crosses zero its derivative is at its maximum meaning that at that time the approximation is less valid.

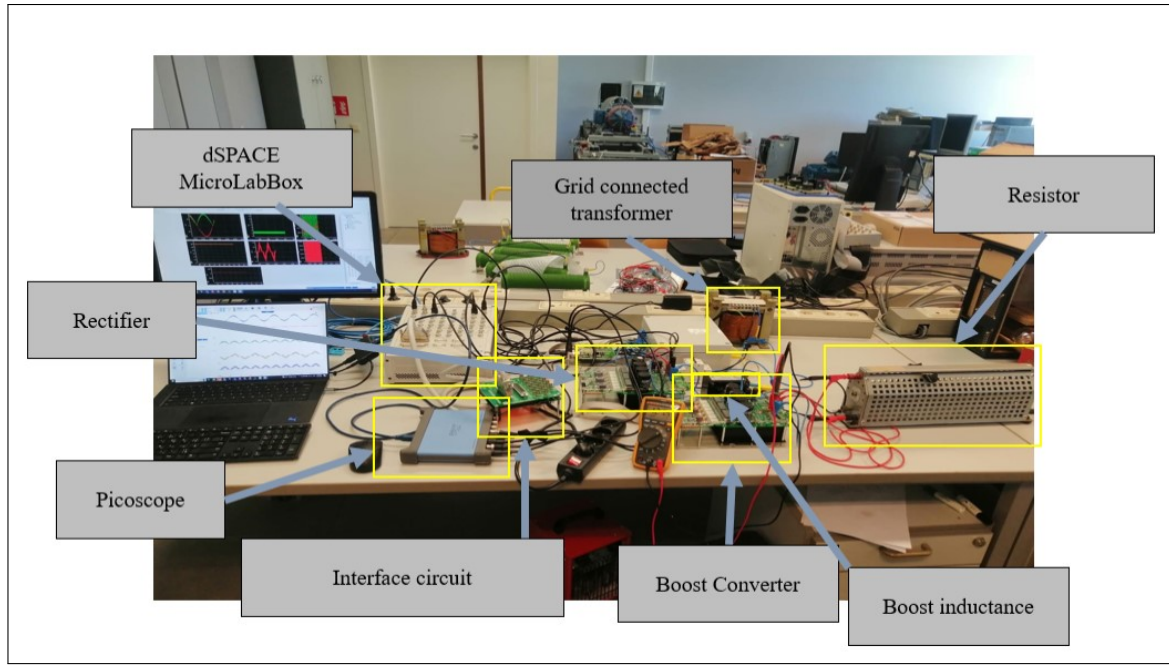
3.2.2.2 Validation of the DFS model

The model that has been developed is able to predict emissions of a classical boost converter, without forgetting that this is a model and intrinsically relies on approximations and assumptions that have been previously cited. The aim is therefore to investigate whether or not the predicted current harmonic values are in accordance with the results that are obtained through numerical and physical experimentation. The circuit under study has been presented in Fig. 3.2. However, a crucial difference exists with the previously cited electrical schematic. The investigated circuit contains a transformer which is intended to model the grid and provide around 60 V_{RMS} , even though accurate when considering the modelling of the converter itself it does introduce filtering and thus the results can not verify the model when directly connected to the grid. Indeed, it is equivalent to being fed by a small grid as the perturbations coming from other devices will see a great impedance on the path leading to the studied device. Regarding the practical aspects, the numerical experiments were conducted using MATLAB/Simulink, while the physical experiments were carried out at the laboratory of Ghent University.

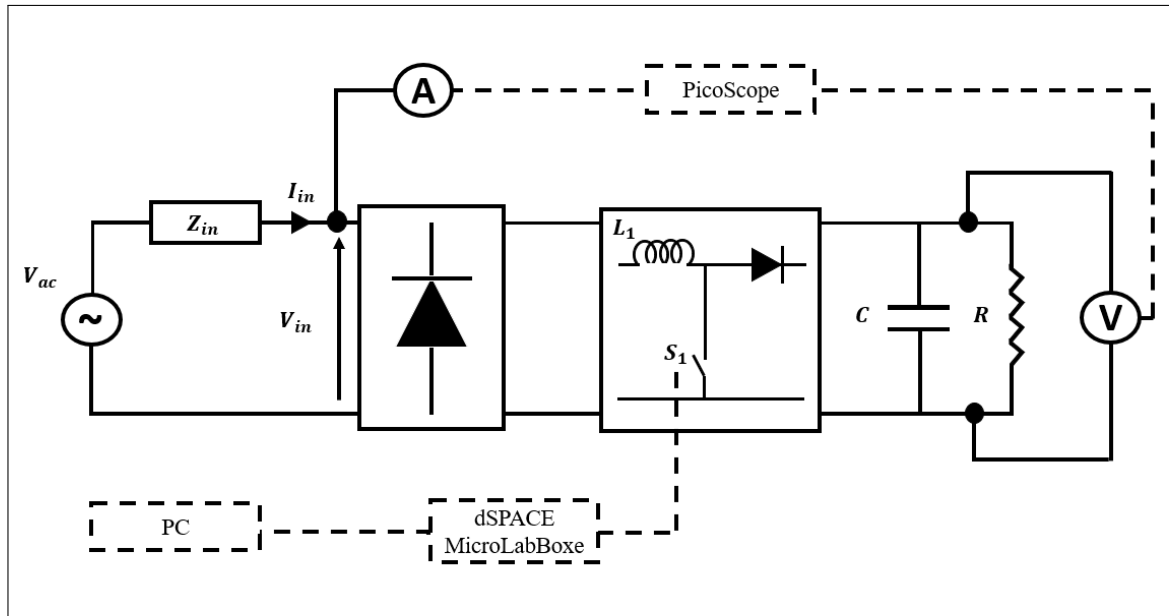
The material used for the experimentation is briefly described in table 3.1 and a picture of the laboratory environment along with an electrical schematic are respectively provided in Fig.3.5 (a) and (b).

Table 3.1: List of equipments needed for physical experimentation reproduction

Name	Function	Value	Origin
Boost Converter	APFC	/	/
Boost Inductance	Energy Storage	10 mH	/
Resistor	Energy dissipation	800-1600 Ω	/
Grid Connected Transformer	Galvanic Isolation	Provide 60 V_{RMS}	/
dSPACE MicroLabBox	Generate Control Signals	/	dspace
Full-Bridge Diode rectifier	Rectify	/	/
PicoScope	Data Acquisition	/	/



(a)



(b)

Figure 3.5: (a) is the picture and (b) is the schematic of the experiment conducted at the UGent laboratory in order to validate a boost model of supraharmonic emission. The dashed lines represent signal measurements, i.e. with voltmeter and ammeter noted respectively V and A , and control of the switch, noted S_1 . PicoScope is the device used for signal acquisition and dSPACE MicroLabBox generates the switching pulses. V_{ac} is the voltage at the secondary of the transformer and provides $60 V_{RMS}$, V_{in} and I_{in} are the voltage and current at the full bridge diode rectifier (FBDR) and is connected to a boost converter consisting of a $10 mH$ inductor and a switch characterized by a switching frequency of $5 kHz$. The load is made of a capacitor of $260 \mu F$ and a resistor denoted by R varying between 800 and 1600Ω

Many operating points have been investigated but only the extreme cases are presented. The values are summarized in the next table. First, a zoom on the first two bands is presented to emphasize the accuracy of the theoretical prediction of the theoretical model. Afterwards, a global view from 50 Hz to 155 kHz, on the magnitude spectrum is provided in order to observe the band-related variation in emissions.

Table 3.2: Investigated operating points for the classical boost converter

Cases	$R[\Omega]$	$V_{DC}[V]$	$P_{device}[W]$
1	800	300	113
2	1600	600	225

First and second carrier harmonic bands

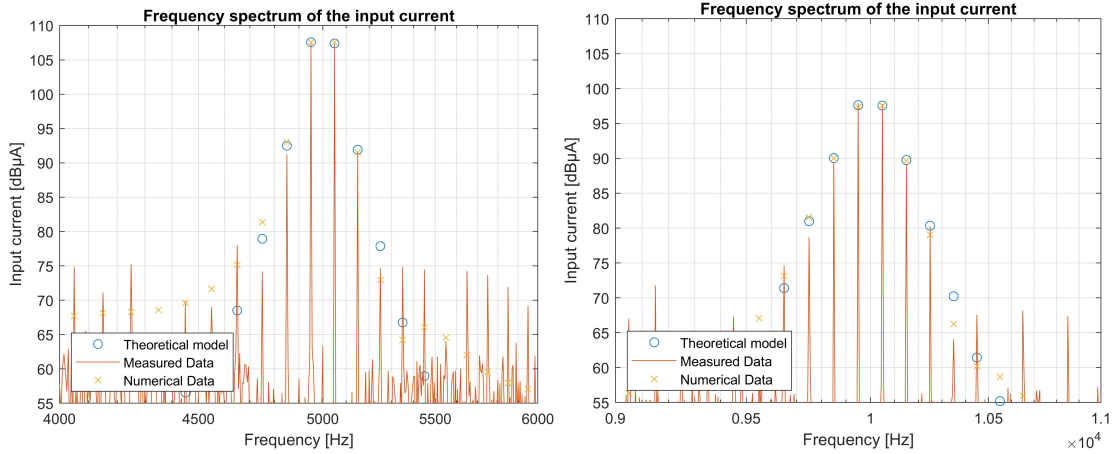


Figure 3.6: Zoom on the first and second band in the frequency spectrum of the input current of a classical boost converter for $R_{Load} = 800\Omega$ and $V_{DC} = 300V$ with $F_{switch} = 5 kHz$. Three methods are represented, the laboratory measurements, the numerical experimentation and the theoretical prediction which has been developed previously

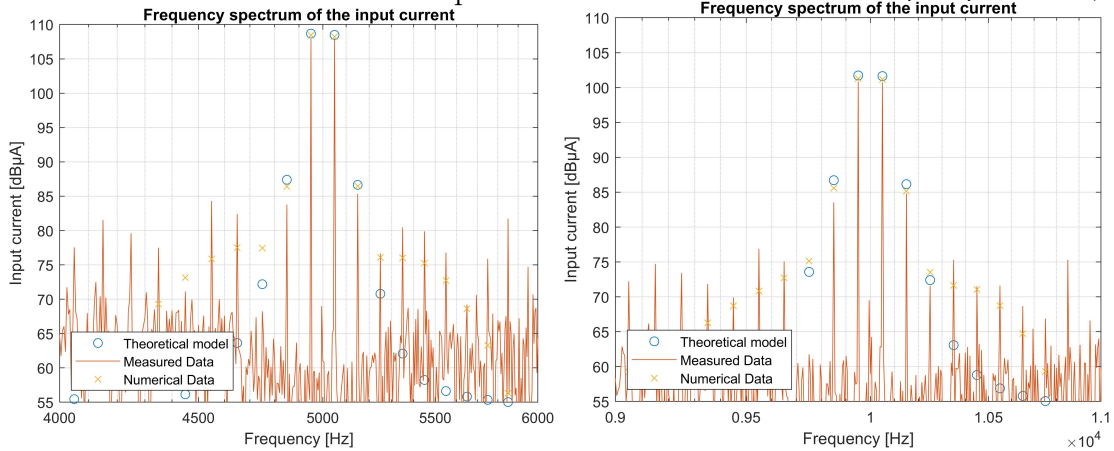


Figure 3.7: Zoom on the first and second band in the frequency spectrum of the input current of a classical boost converter for $R_{Load} = 1600\Omega$ and $V_{DC} = 600V$ with $F_{switch} = 5 kHz$. Three methods are represented, the laboratory measurements, the numerical experimentation and the theoretical prediction which has been developed previously

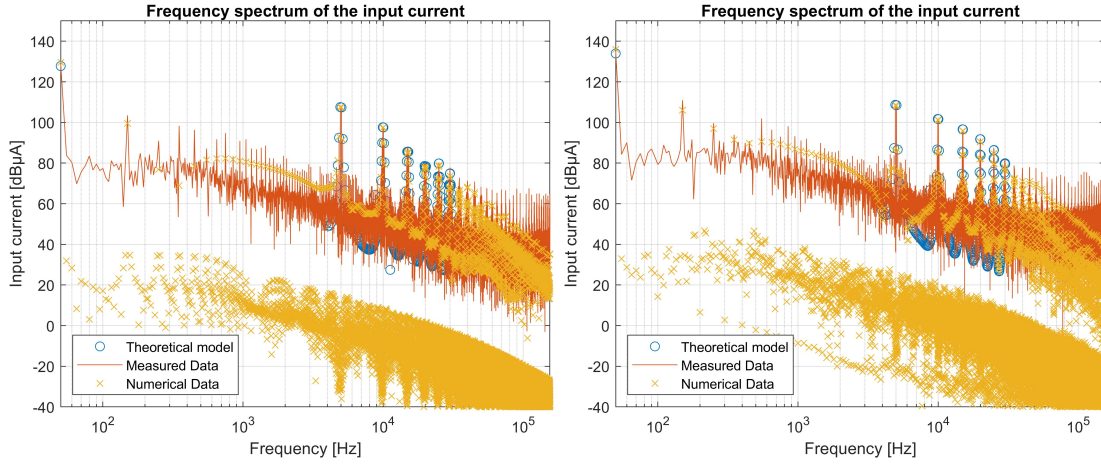


Figure 3.8: Frequency spectrum of the input current of a classical boost converter for $R_{Load} = 800$ (left) 1600 (right) Ω and $V_{DC} = 300$ (left) 600 (right) V with $F_{switch} = 5 \text{ kHz}$. Three methods are represented, the laboratory measurements, the numerical experimentation and the theoretical prediction which has been developed previously

These results demonstrate the ability of a theoretical method to predict emissions of classical converters with a level of accuracy comparable to that achieved using numerical simulations with small time steps (e.g., $5 \cdot 10^{-7} \text{ [s]}$). This time step, which may legitimately be a subject of discussion due to the increased computational demand it entails, is based on the approach developed in [45].

Regarding the analytical method, unlike numerical methods, it requires minimal computational resources and produces results within seconds. In contrast, numerical simulations often take several minutes to generate data, which then require additional post-processing.

While physical measurements remain the fastest and most accurate approach, they demand precise control over the measurement process and do not inherently provide predictive capabilities.

Here, for the first time, the concept of quadratic sum (QS) is introduced. This tool provides a single value for each harmonic band, greatly facilitating the comparison of methods. Indeed, comparing the amplitude spectrum on a frequency-by-frequency basis is not always straightforward. The definition of the quadratic sum is briefly introduced here and will be described in more detail, including its motivations and physical justifications, in the subsequent section dedicated to the analysis of the IBC converter. X can represent either the current or the voltage and in the following definition eight components are considered in the QS. It should be noted that the first harmonic considered is the one at the frequency $F_{sw} - 350 \text{ [Hz]}$ and the last at $F_{sw} + 350 \text{ [Hz]}$ in steps of 100 [Hz] .

$$X_{QS} = \sqrt{\sum_{n=1}^{n=8} X_n^2} \quad (3.34)$$

Now that this definition has been established, it is possible to present the results in the

form of graphs comparing the three methods, namely the numerical, theoretical, and experimental approaches, for the same operating points.

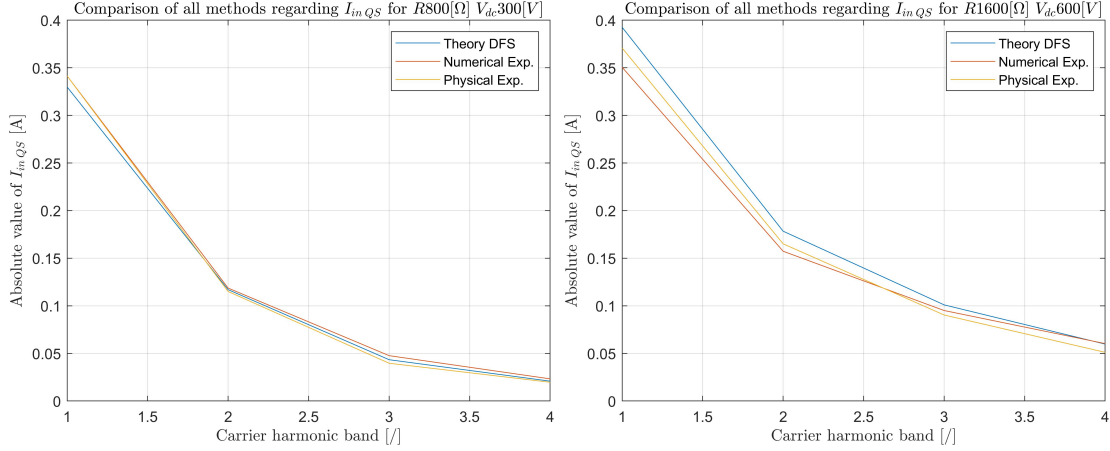


Figure 3.9: Comparison of the input current in quadratic sum (QS) between the three methods, i.e. numerical, theoretical (DFS) and experimental for an interleaved boost converter used as an active power factor corrector with $V_{DC} = [300 \ 600][V]$ and $R = [800 \ 1600][\Omega]$

A substantive analysis is however necessary and reveals several points that require attention. When considering Case 1 pretty much all the theoretically predicted points are close to the physically and numerically measured ones. Indeed, the greater the distance from the carrier harmonic, the larger the difference becomes as it is more subject to complex interactions with the pre-existing grid which is not considered in the analytical development. Obviously, having a perturbation of $40 \text{ dB}\mu\text{A}$ on an emission of $100 \text{ dB}\mu\text{A}$ is less seen than on an emission of $50 \text{ dB}\mu\text{A}$. In the first case the emission passes from $100 \text{ dB}\mu\text{A}$ to $100.009 \text{ dB}\mu\text{A}$ while in the second case the emission pass from $50 \text{ dB}\mu\text{A}$ to $52.39 \text{ dB}\mu\text{A}$.

Another point is the interesting decrease of sideband harmonics for both first and second band while the carrier harmonics at least remain at the same level, i.e. at 4.95 and 5.05 kHz , or they even increase as in the second (slightly), third and fourth band. In order to provide a satisfactory explanation to this phenomenon a development is proposed hereafter and will provide further insight regarding the relation between emissions in SH and DC voltage, thus the power, of the considered device.

The explanation for the nearly constant emission for the first carrier harmonic irrespective of the DC voltage within the range $300\text{-}600 \text{ V}$ is provided by the fact that the voltage ripple has two parts. First the DC voltage which should, if it is the only impacting factor, induce a rise of the emissions. The second factor is the ripple part of the switching function whose magnitude is X_{mn} , noting that when only the carrier harmonics are considered, $n = 0$, and thus Equ. 3.19 is relevant. The only remaining variable in Equ. 3.19 is r which is roughly defined as $\frac{\sqrt{2}V_{grid \text{ RMS}}}{V_{DC}}$.

However, Bessel functions are not linear with respect to r , the variation of the emissions for a given range of V_{DC} and for a given $V_{grid \text{ peak}}$, which is noted $V_{ac \text{ peak RMS}}$ in graphs,

is hardly directly predictable. It is, nevertheless, possible to numerically plot the Bessel functions as function of r , as done in Fig. 3.10 and even to highlight the portion of the curve associated to the range considered here, i.e. from $V_{DC} = 300$ to $V_{DC} = 600$.

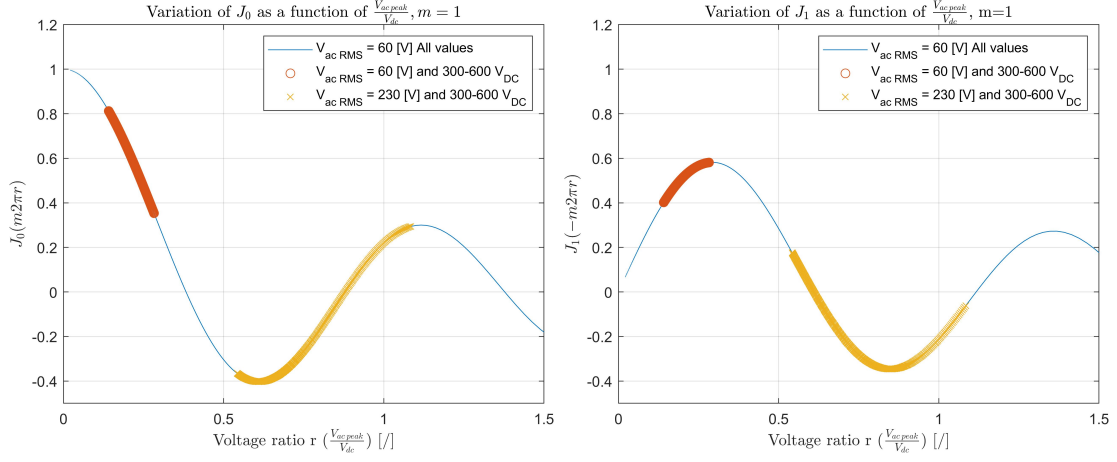


Figure 3.10: Bessel functions of orders 0 and 1 as functions of r , the ratio of the grid peak voltage to the DC voltage, with the portions corresponding to $V_{DC} = 300 - 600$ and for $V_{grid\ RMS} = 60$ or 230 [V]

Based on Equ. 3.19 it is possible to first explain mathematically why an increase in the device power does not affect the supraharmonic emissions. For the first carrier harmonic and for the specific case that has been studied, i.e. V_{DC} between 300 and 600 [V] and for a grid RMS voltage of 60-65 V_{RMS} , the following development is presented

Considering only $k = 0$ which is an approximation but that can be evaluated as bringing less than 10 percent error in the value of J_1

$$\begin{aligned}
X_{m0} &= \sqrt{\left(\frac{4}{2k+1} \sum_{k=0}^{\infty} J_{2k+1}(-m2\pi r) \left(\frac{1}{m\pi^2}\right)\right)^2 + \left(\frac{1}{m\pi} - \frac{1}{m\pi} J_0(m2\pi r)\right)^2} \\
&= \sqrt{\left(\frac{4}{m\pi^2} J_1(-m2\pi r)\right)^2 + \left(\frac{1}{m\pi} - \frac{1}{m\pi} J_0(m2\pi r)\right)^2} \tag{3.35}
\end{aligned}$$

Let the ratio r corresponding to, for the case with $V_{DC}=300$ [V], as r_0 and the one of 600 [V] as r_1

$$= \sqrt{\left(\frac{4}{m\pi^2} J_1(-m2\pi r_0)\right)^2 + \left(\frac{1}{m\pi} - \frac{1}{m\pi} J_0(m2\pi r_0)\right)^2} \tag{3.36}$$

Recalling that only $m = 1$ is considered, at least for this explanation, and that r_1 is $= \frac{r_0}{2}$

$$= \sqrt{\left(\frac{4}{\pi^2} J_1(-2\pi \frac{r_0}{2})\right)^2 + \left(\frac{1}{\pi} - \frac{1}{\pi} J_0(2\pi \frac{r_0}{2})\right)^2} \tag{3.37}$$

$$= \sqrt{\frac{4}{2.25\pi^2} \left(J_1(-2\pi r_0)\right)^2 + \frac{1}{9\pi^2} \left(1 - J_0(2\pi r_0)\right)^2} \tag{3.38}$$

If X_{m0} for respectively $V_{DC} = 300$ and 600 [V] is defined as X_{m00} and X_{m01}

$$X_{m00} = 0.3193$$

$$X_{m01} = 0.1762$$

$$\frac{X_{m01}}{X_{m00}} = 0.55$$

Subsequently, since V_{rip} has been defined as $V_{DC} X_{mn}$ one can finally write

$$\begin{aligned}
V_{rip} &= V_{DC} X_{mn} \\
V_{rip0} &= V_{DC300} X_{m00} \\
V_{rip1} &= V_{DC600} X_{m01} \\
\frac{V_{rip1}}{V_{rip0}} &= \frac{V_{DC600} X_{m01}}{V_{DC300} X_{m00}} \\
\frac{V_{rip1}}{V_{rip0}} &= 2 \cdot 0.55 = 1.1
\end{aligned}$$

The fact that the first carrier harmonic for the specific values given in Table 3.4 is constant was the purpose of the previous development and has been explained. However, some interesting points still remain. First, the analytical development based on Bessel functions does not consider variations of resistance on DC side meaning that the emissions are not a function of the power, as on DC side the power is $P = \frac{V_{DC}^2}{R}$, but only a function of r meaning that only the ratio of the grid peak voltage to the DC voltage is relevant in understanding the variation of emissions. This is a crucial point, as some works based on measurements [13][47][74] have concluded that the emissions are proportional to the DC voltage which is not always the case as shown before. Even when emissions vary with the DC voltage it is non-linearly related to the variation of the DC voltage. Indeed, the voltage ripple has two contributions, the DC voltage itself and X_{mn} factor, the final variation in the voltage ripple is therefore the product of the variation in V_{DC} and that of X_{mn} .

The second interesting point can be seen in Fig. 3.10 where the variation of Bessel functions of order 0 and 1 as a function of r has been presented. It is seen that depending upon the voltage of the grid the same variation of V_{DC} does not represent the same variation of r and thus of values of J_0 and J_1 which constitute X_{mn} . The conclusion is that depending upon the voltage of the grid the same variation of V_{DC} will or will not impact the emissions in SH. For example, for a grid voltage of 230 [V_{RMS}] the factor by which the V_{DC} is multiplied will not be compensated by the variation of J_0 and J_1 , even worse than that, the variation will be enhanced and the final emission will be seen as being four times larger meaning 12 dB μ A greater than the initial emission. This is confirmed in Fig. 3.11.

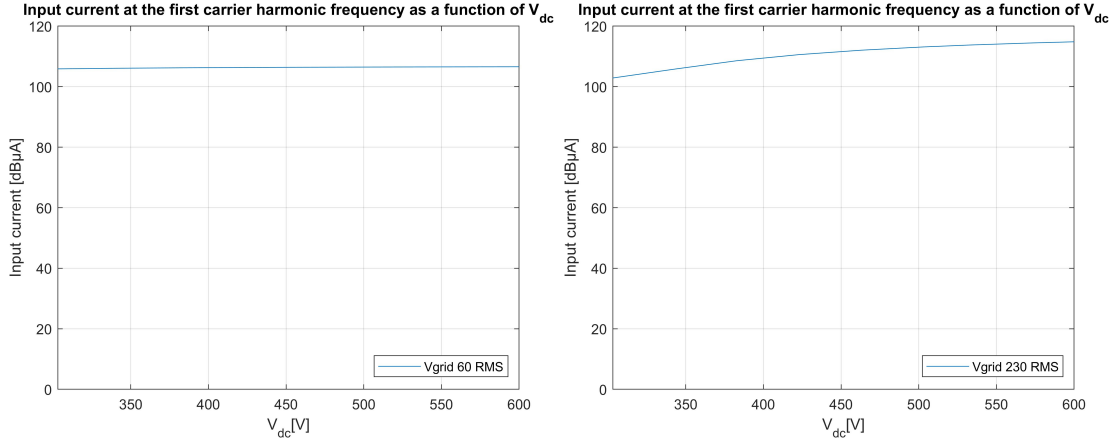


Figure 3.11: First carrier harmonic value of the input current as a function of the DC voltage and depending on the grid voltage (RMS) with the case of $V_{grid\ RMS}= 60\ [V]$ on the left and $V_{grid\ RMS}= 230\ [V]$ on the right

In Fig.3.11 right the input current starts at $103\ dB\mu A$ and ends at $115\ dB\mu A$ which validates the $12\ dB\mu A$ stated before meaning four times larger emission. In that context, one could badly interpret these results as proving the proportional relationship between the power of the device and the emission. Indeed, V_{DC} 2 times greater for a same resistance means four times larger power.

The third interesting point lies in the intuitive predictability of the variation of emissions based on Fig. 3.10, indeed, when analyzing Equ. 3.19 it is quickly understood that an increase in V_{DC} will be compensated in the case where the grid voltage is $60\ [V_{RMS}]$ as J_0 is rising and J_1 is decreasing.

However, it is worth noting that only the case for 'm = 1', meaning for the first carrier harmonic, has been analysed. The corresponding graphical representations for the other carrier harmonics are presented hereafter.

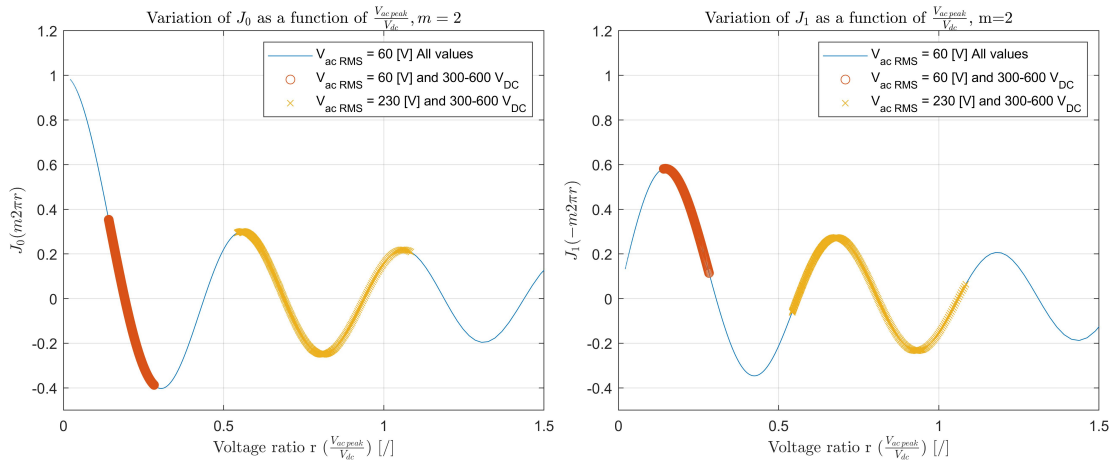


Figure 3.12: Bessel functions, for the second carrier harmonic ($m=2$), of order 0 and 1 as a function of r , the ratio between the grid peak voltage to the DC voltage, with the portion corresponding to $V_{DC} = 300 - 600$ and for $V_{grid\ RMS} = 60\ or\ 230\ [V]$

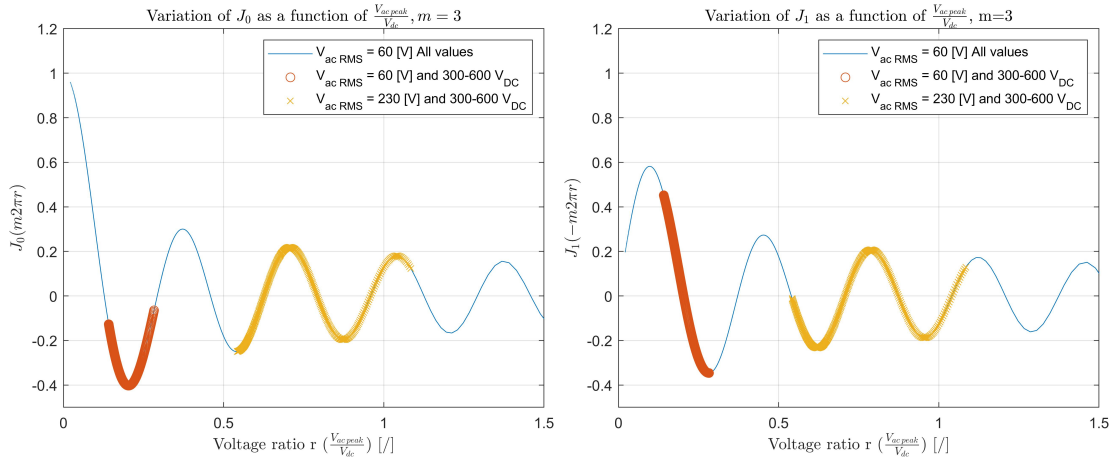


Figure 3.13: Bessel functions, for the third carrier harmonic ($m=3$), of order 0 and 1 as a function of r , the ratio between the grid peak voltage and the DC voltage, with the portion corresponding to $V_{DC} = 300 - 600$ and for $V_{grid\ RMS} = 60$ and 230 [V]

For $m = 3$, J_0 and J_1 have more or less the same value, there is no concern regarding J_1 as this term is squared in Equ. 3.19. Thus, for $V_{DC} = 300V$ and $V_{DC} = 600V$ a prediction would be that for that carrier harmonic the emission would be at least 2 times greater, This is confirmed below

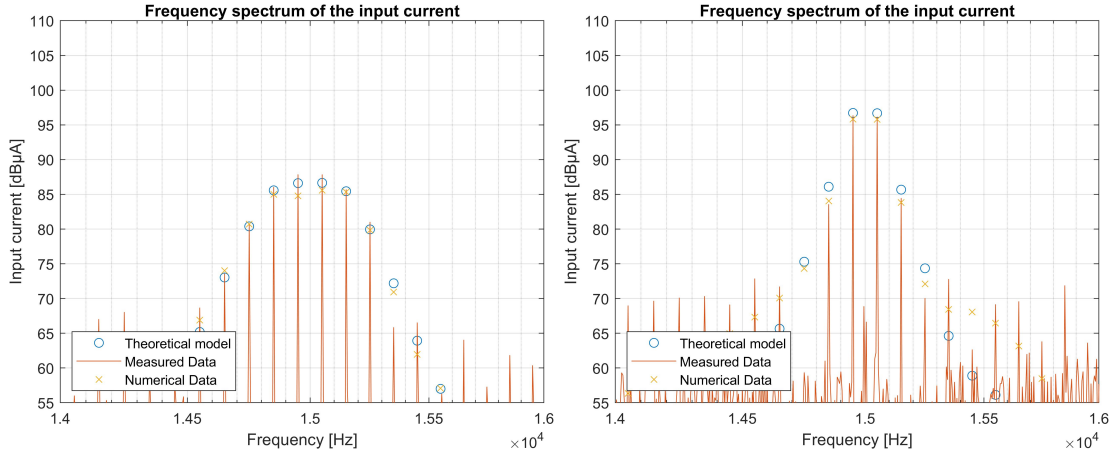


Figure 3.14: Third carrier harmonic band of the input current depending upon the grid voltage (RMS) with the case of $V_{DC} = 300V$ on the left and $V_{DC} = 600V$ on the right

The third carrier harmonic has value $87.9\ dB\mu A$ when $V_{DC} = 300V$ while for $V_{DC} = 600V$ the value rises to $96.4\ dB\mu A$. This increase is higher than the $6\ dB\mu A$ predicted but can be explained by the approximation of constant value for J_0 and J_1 . Nevertheless, this approach allows to predict the behaviour of these emissions based on global information being the DC voltage of the device and the grid voltage at the point of connection.

After these developments and verification, the interested reader would begin to see a crucial point. Equ. 3.19 has only r as variable, also the voltage ripple is defined as $V_{DC}X_{mn}$ meaning that, actually, two parameters are enough to predict the voltage ripple. These are the grid voltage, i.e. the voltage at the input of the full-bridge diode rectifier

on the grid side (as the grid impedance is first considered as 0 this is directly equal to the grid voltage), and the DC voltage. This link between supraharmonic emissions and DC voltage is also one of the points present on the conclusion of [13] which studied EV chargers in which the kind of topology presented here, i.e. APFCs, are widely used. Coming back to our observation and analytical development, one can see that based on the knowledge of the impedance on which this voltage is applied, one can determine the current. The first idea is, thus, to understand that when the approximations that have been highlighted previously are taken into account, the production of SH of a converter is directly linked to the modulating wave.

What is referred to as voltage SH, or essentially voltage ripple, when the only source of harmonics is the converter itself, is simply the decomposition of the switching function into a Fourier series, scaled by a factor equal to the DC voltage. This point is crucial because it provides a global summary of the understanding of the high-frequency (HF) emissions from converters, regardless of the specific type of converter, primarily based on the understanding of its working principle. One can therefore determine the SH produced in voltage based on the Heaviside functions and the PWM working principle. The in-depth exploration of these concepts, as well as the final understanding, will be addressed in the chapter dedicated to the intrinsic comprehension of emissions, i.e., Chapter 5.

3.2.3 Consideration of parasitic elements

Parasitic capacitance between lines

A graph, Fig.3.3, has previously been introduced in order to give an idea of the equivalent circuit of the studied converter along with the high frequency grid in which the switching process takes place.

A further step is now investigated, i.e. taking into account some parasitic elements and evaluating their impact on the model depending on their values. The impact of the inter-line capacitance is now investigated, first, an electrical schematic is shown in Fig. 3.15 to represent the studied situation. The determination of the values to consider is not trivial as the inter-line capacitance, i.e. $C_{para. line}$, is not provided by manufacturers. However, some works [50], [75] investigated this question and report values for XVB 6², 10² and 15² respectively 84.6 [nF] and 51.9 [nF] per kilometer of line.

As the inter-line capacitances, $C_{para. line}$, are of the order of magnitude of the [nF] it is important to consider also the typical values of parasitic inter-turn capacitances, i.e. $C_{para. Ind.}$.

Regarding the typical values of the inductances, L_b , along with the parasitic capacitance, $C_{para. Ind.}$, of these inductors in active power factor correction circuits one can find values between 0.3 [mH] and 1 [mH] for power ratings between 65 and 150 [W] [76],[77], [78] and 10 to 150 [pF] for the parasitic capacitance [79], [80],[81].

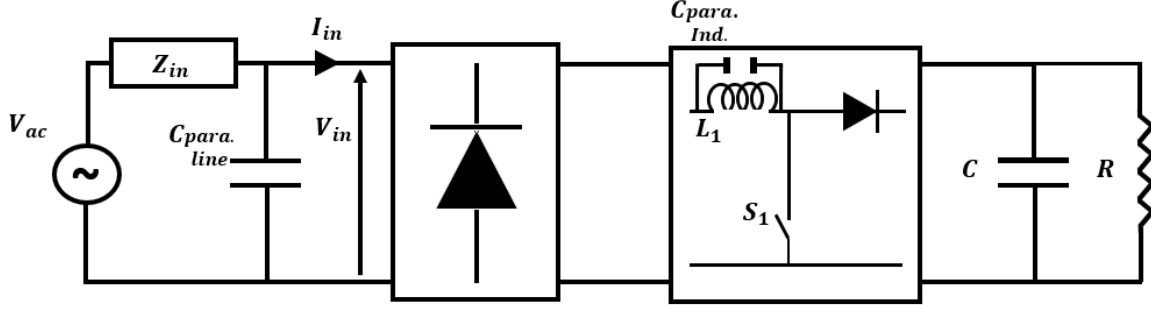


Figure 3.15: Electrical schematic of the boost converter used as APFC with a simplified grid being an ideal voltage source (V_{ac}) connected to an impedance (Z_{in}) which is the resistance and inductance of the low-voltage cable. The peculiarity lies in the consideration of the parasitic capacitances, i.e. the capacitance between lines and inter-turn noted respectively $C_{para. line}$ and $C_{para. Ind.}$.

This newly considered parasitic capacitance, $C_{para. line}$, provides a low-impedance path at high frequencies. However, the high frequency modeling of the cable is complex as it, theoretically, induces the need for a distributed model. As the development made in C.1 established, the voltages and currents actually propagate and thus depending on the frequency under consideration and the characteristics of the cable along with the cable length the lumped model will be a reasonable approximation or not.

Indeed, if one considers typical values of propagation velocity in LV cable that has been provided in section 2.4 then the wavelengths corresponding to the frequencies 50 [Hz], 2 [kHz] and 150 [kHz] are respectively about 4000 [km], 100 [km] and 1.33 [km].

The next investigation is thus restricted to the investigation of the Bode diagram for a frequency range and a cable length that are reasonable and thus of 1 [km] of line and a frequency range up to 2 [kHz].

The context being stated, it is possible to further investigate the physical impact of this capacitance, that actually, acts as a low-impedance path and thus as a filter. Indeed, the harmonic current, i.e. I_{rip} in 3.16, is, depending upon the value of $C_{para. Line}$, more or less flows through the capacitive branch. The new representation of the HF model along with the qualitative reactance of the passive elements and the total impedance seen by the HF emitter which is, actually, the switch, is provided in Fig. 3.16.

High frequency model

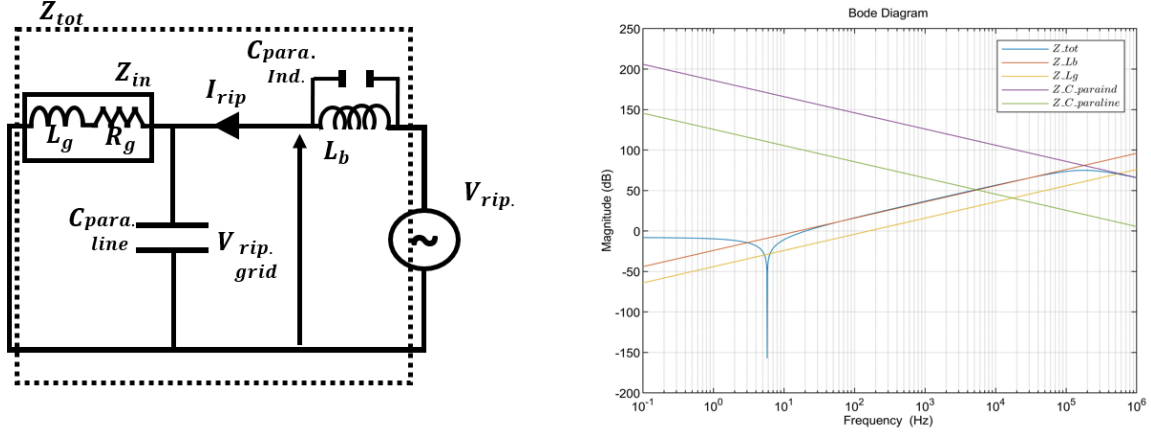


Figure 3.16: Electrical schematic of the high frequency model of the converter, on the left, along with the graph of the reactances and total impedance, i.e. Z_{tot} , on the right

Circuit theory states that for a capacitor, C , and an inductor, L , the parallel resonance, as does the series resonance, exhibits the same factor, i.e. $\omega C - \frac{1}{\omega L}$, but for the parallel resonance this factor is at the denominator which induces that the impedance tends (in theory) to infinity for $\omega = \frac{1}{\sqrt{L_{in}C_{para}}} [\frac{rad}{s}]$.

In the case presented in this section, two parallel resonances and one series resonance are present and generated by the passive elements, i.e. $C_{para. Ind.}$, L_b , $C_{para. Line}$ and L_g . The right part of Fig. 3.16 illustrates the variation of these impedances with respect to the frequency with the intersections indicating the resonance frequencies corresponding to the frequencies at which the magnitude of the total impedance reaches infinity or 0 in the ideal cases, i.e. without any resistance in that path, which is an approximation.

The resonance frequencies are easily seen for the parallel resonances but less apparent for the series resonance. Hereafter, the development that leads to the determination of these frequencies is provided.

$$\begin{aligned}
 Z_{tot}(\omega) &= \left(\frac{1}{\frac{1}{j\omega L_g + R_g} + j\omega C_{para. Line}} \right) + \left(\frac{1}{\frac{1}{j\omega L_b} + j\omega C_{para. Ind.}} \right) \\
 &= \frac{j\omega L_g + R_g}{1 - \omega^2 L_g C_{para. Line} + j\omega R_g C_{para. Line}} + \frac{j\omega L_b}{1 - \omega^2 L_b C_{para. Ind.}}
 \end{aligned}$$

Based on this last equation and by first neglecting the resistive part of Z_{in} , i.e. R_g , one can find that $Z_{tot}(\omega)$ tends to infinity for

$$\omega|_{Z_{tot}=\infty} = \frac{1}{\sqrt{L_g C_{para. Line}}}$$

$$\omega|_{Z_{tot}=\infty} = \frac{1}{\sqrt{L_b C_{para. Ind.}}}$$

Regarding the frequency at which the $Z_{tot}(\omega)$ tends to 0 one can find it below

$$\omega|_{Z_{tot}=0} = \sqrt{\frac{1}{L_g C_{para. Line}} + \frac{1}{L_b C_{para. Line}}}$$

The interesting information to extract from all the developments presented so far is that depending upon the values of the passive elements and the working principle of the emitter, a greater or smaller number of issues are expected to occur.

One can understand that the subject is complex and contains a lot of interactions, but one way of being able to analyze and predict issues is to restrict the complexity to a simpler model, which is, actually, what has been done here. This leads mainly to two points to consider.

- The emitter characteristics and the factors affecting these emissions
- The grid impedance and the potential related issues

Regarding the first point, the previous sections explained that the emissions were mainly a function of the ratio between the grid voltage, i.e. the voltage at the terminals of the full-bridge diode rectifier (FBDR) and noted V_{in} , and the voltage across C in Fig. 3.15, often called V_{DC} , at 50 [Hz] that is not significantly affected by the passive elements considered in this section. Indeed, the grid inductance noted L_g , is taken into account and at that frequency $C_{para. Line}$ has a reactance of $\frac{1}{2\pi 50 C_{para. Line}} \approx 40[k\Omega]$. Eventually, the first point, which is related to the emission itself can be considered as invariable with respect to the passive elements of the circuits, as long as the 50 [Hz] voltage V_{in} at the terminals of the FBDR is not significantly impacted.

Subsequently, the second point is further investigated and is equivalent to considering the final result on the grid of having a generation of high frequency harmonics by the converter, named V_{rip} in Fig. 3.16 and a certain global circuit, with impedance Z_{tot} , which is composed of the converter circuit upstream to the switch, i.e. in the case of a boost it is merely the boost inductance, and the grid circuit itself. Eventually, in a simplified case this leads us back to Fig. 3.16 left.

Two situations can thus be distinguished

- Study of the issues related only to the grid impedance excluding the resonances related to the passive elements of the converter itself
- Study of all possible issues and thus considering Z_{tot} of Fig. 3.16

In the first situation, the notion of VHS or CHS allows us to easily answer the question of the situations that represent a risk for the grid, i.e. high level of supraharmonics in current and/or in voltage. However, it leaves a blind spot which is the one related to resonances induced by the passive elements of the converter itself and the grid ones. This blind spot is not present in the second situation which actually leads to the consideration of two more resonances, one parallel between the boost inductance, i.e. L_1 , and the capacitance related to this boost inductance, i.e. $C_{para. Ind.}$, and one series due to both the boost inductance and the line parasitic capacitance, i.e. $C_{para. Line.}$. All these resonance frequencies are provided along with the conditions under which they do not fall within the supraharmonic range.

Regarding the first situation, if the converter is determined as being a voltage harmonic source (VHS) or current harmonic source (CHS) at the switching frequency then different conditions for issues to happen are expected. In the case VHS, the series resonances will lead to high level of emitted current and if it acts more like a CHS then the parallel resonances are to be considered.

However, When considering active power factor correction (APFC), and for the specific case of the boost converter the behaviour is easily determined as CHS due to the ratio of the boost impedance to the grid impedance which is at the frequency of emission, i.e. generally higher than 5 [kHz] as low power devices are mainly considered, in favour of the boost converter [53][71][72][73].

In order to not have the resonance in the supraharmonic range one condition can be expressed, i.e. for parallel resonances

$$\begin{aligned} L_g C_{para. Line} &< 1.126e - 12 \\ L_b C_{para. Ind.} &< 1.126e - 12 \end{aligned}$$

Regarding series resonance

$$\sqrt{\frac{L_g + L_b}{L_b L_g C_L}} > 94.2e6$$

This idea of considering the parasitic capacitance even though it is negligible when the value of the associated capacitance is small, i.e. in the range of nF, has two benefits.

First, it ensures that non-negligible effects are investigated and even values can be identified for the parasitic capacitance above which it is no longer accurate to neglect it. Taking

the maximum frequency of the SH range (150 kHz) and one value of 1 [mH] for the inductance of the boost leads to a minimum value of around 1 nF for a parasitic capacitance to no longer be considered negligible for the SH range.

The second benefit is that it allows the introduction of a reflection on the electromagnetic filters (EMIs) present for these kinds of converters. Even though designed for frequencies above 150 kHz, these filters have a non-negligible impact regarding the SH range and should be considered for converter alone and even more for interactions.

This last point leads to two subsequent sections, one on this chapter subsection 3.2.4 and one on the chapter 4 dedicated to interactions.

3.2.4 EMI Filters

EMI filters are intended to control emissions above 150 kHz and are widely distributed throughout the grid within the different devices. As they represent a low-impedance path for high frequency emissions due to their capacitance it is crucial to study them. These filters are designed based on both a resonance frequency and a desired attenuation.

Regarding the resonance frequency it is deduced from the attenuation specified in CISPR32 [82] for the first carrier band after 150 kHz, named f_d , and if one band is at 150 kHz then this is the frequency to take into account as f_d in the next equation.

$$f_c = \frac{f_d}{10^{\frac{Att_{req}}{40}}} \quad (3.39)$$

With Att_{req} being defined, in [83] which is itself based on [84], as

$$Att_{req}[dB] = v_{in}(f_d)[dB\mu V] - limit_{CISPR32}(fd)[dB\mu V] + margin[dB] \quad (3.40)$$

Then, based on the known values of CISPR32 at 150 kHz of 66 dB μ V and 56 dB μ V at 500 kHz with a decreasing curve which is linear with the logarithm of the frequency one can, when the voltage at f_d , is known—either from numerical simulations or from the analytical developments presented in this chapter—the required attenuation can be determined by assuming a resistive grid impedance of 100 Ω .

The aim of this subsection is to investigate the importance of this filter on the individual emission of the converter. A first idea would be to say that as it is designed for frequencies above 150 kHz it should not be crucial at the switching frequencies of a few kilohertz or a few tens of kilohertz, one simple example is sufficient to see the limitation of this idea.

Consider an APFC circuit which is designed with an EMI filter with $C_f = 2.14e^{-6}[F]$ and $L_f = 0.63e^{-3}[H]$ is obtained from the previously developed methodology. With real values coming from numerical experimentation of an IBC with $F_{sw} = 25kHz$ one can obtain 4.3 kHz as f_c . By taking into account the inductance inherently present in the boost one would eventually have a f_c of 2.7 kHz. It is, thus, not valid to consider that EMI filter could be neglected when considering prediction of emissions of devices which

contain EMI filters. This leads to the idea of a tool which, basically, is designed to avoid power quality issues related to harmonics while itself causing at another frequency the same kind of issues. One can find in Appendix E an extension, to biological science, of the idea of a technical solution leading to the same kind of issues that it was first designed to correct.

Regarding the specific case of the boost converter, it is possible to investigate the impact of the EMI filter. First, the presented methodology is applied and allows the determination of the L and C values of a classical LC filter. These values are $L_f = 17.8e^{-6} [H]$ and $C_f = 2.14e^{-6} [F]$. Other values can, indeed, be chosen. Anyway, the results are presented hereafter, but first two figures are presented, Fig. 3.17 and Fig. 3.18 which represent, respectively, the studied circuit and the Bode diagram of the total impedance seen by the switch S_1 .

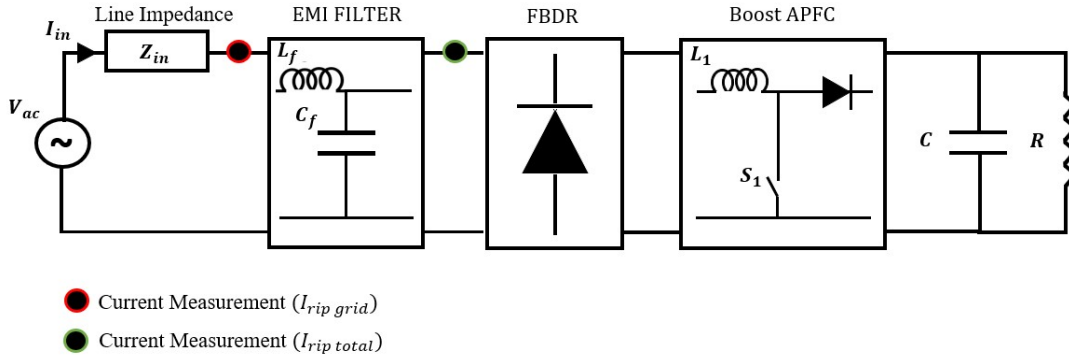


Figure 3.17: Schematic of the studied circuit, i.e. an ideal voltage source connected in series with an impedance being the line impedance after which the EMI filter (classical LC) is connected before having the classical structure meaning the FBDR, APFC and finally the load. $I_{rip\ grid}$ and $I_{rip\ total}$ respectively represent the current at the terminals of the EMI filter on grid side and the total current emitted at the terminals of the FBDR

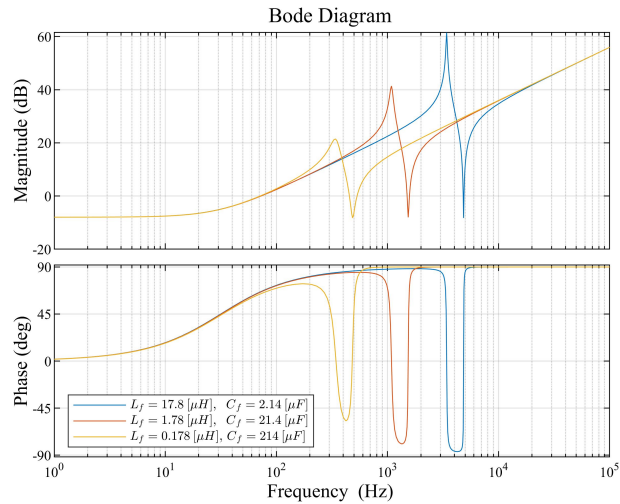


Figure 3.18: Bode diagram of the impedance seen by the switch S_1 as a function of the EMI filter passive elements values

In Fig. 3.19 the comparison, when the EMI filter is connected, between the numerical experimentation and the analytical development regarding the magnitude spectrum of $I_{rip\ total}$ is provided.

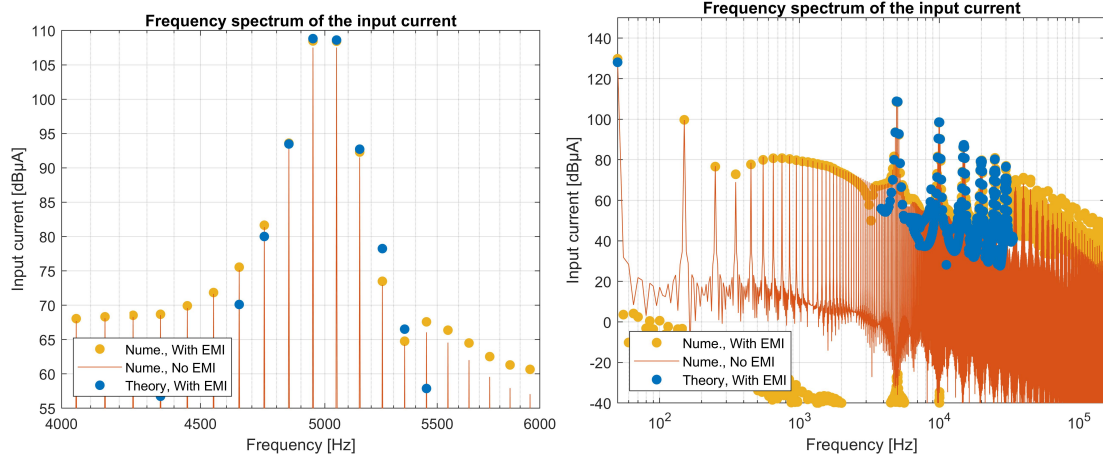


Figure 3.19: Non significant impact of EMI filter on the total emissions, $I_{rip\ total}$, of an APFC converter, here a boost converter

The magnitude spectrum does not exhibit significant variations on the first four carrier harmonics even though slightly greater emissions are observed but the main difference is for the higher order carrier harmonics where the emissions are of an order of magnitude higher than that observed without EMI. Before going further in the explanation of this result the same comparison is carried out for $I_{rip\ grid}$.

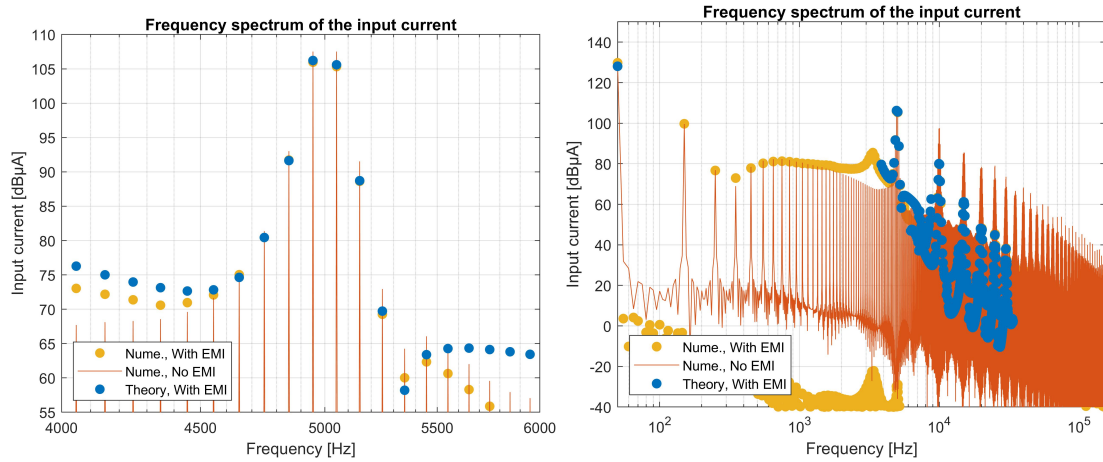


Figure 3.20: Significant impact of EMI filter on the emissions seen on the grid, $I_{rip\ grid}$, of an APFC converter, here a boost converter

The analysis of Fig. 3.20 leads to a totally different observation. Indeed, no significant differences were seen in the first carrier harmonic of Fig. 3.19 while in Fig. 3.19 there exist a clear decrease in the observed ripple in current for $I_{rip\ grid}$. This decrease is observed to be proportional to the considered frequency and thus to the carrier harmonic band. This is, obviously, linked to the value of the capacitance (which itself induce a certain value of the

filter inductance). When the impedance represented by the sum of the impedance of the filter inductance and the impedance named Z_{in} starts being greater than the impedance associated to the filter capacitance the ripple current starts decreasing on the grid and, by the conservation of charges, increasing in the filter capacitance branch.

Actually, the results could be separated in two parts. First, the harmonics generated by the ideal voltage source. Second the harmonics generated by the APFC. While considering the first part, it is easily understood that when the frequency increase the impedance associated to the filter capacitance decrease and thus present a so called low-path impedance. This explains why the emissions seen on the grid have a peak around 3.5 kHz which correspond to the series resonance computed as $\frac{1}{2\pi\sqrt{(L_{in}+L_f)C_f}} = 3.24[kHz]$. The same peak, but negative, is observed in Fig. 3.19 as these harmonics did have their origin from the grid and are pulled by the low-path impedance presented by the capacitance.

Regarding the second part, if the impedance of the circuit is looked with the point of view of the switch, then the same filter capacitance starts presenting this low-path impedance after the first carrier harmonic and by the conservation of charge the carrier harmonics even though almost the same for the five first bands on Fig. 3.19 are decreased in magnitude on the grid side as the grid impedance, i.e. the impedance of Z_{in} in series with Z_{L_f} , is presenting a higher impedance path.

Subsequently to the presentation of these results, the impact of taking different values for C_f and L_f is investigated. Actually, the logic presented for the design of the EMI filter is clear and based on one simple equation which is $f_c = \frac{1}{2\pi\sqrt{L_f C_f}}$. Thus, two cases are next considered to see the potentially favorable impact of a more reasoned design regarding the mitigation of supraharmonics emitted on the grid in parallel with the classical role of the EMI filter. The first case consider a C_f multiplied by a factor 10 while the second case consider a factor of 100. The filter inductance is, indeed, adapted to the chosen values of C_f . First the magnitude spectrum of $I_{rip\ total}$ is shown and followed by the one of $I_{rip\ grid}$

Case 1 $C_f = 10C_{fnom}$ ($I_{rip\ total}$)

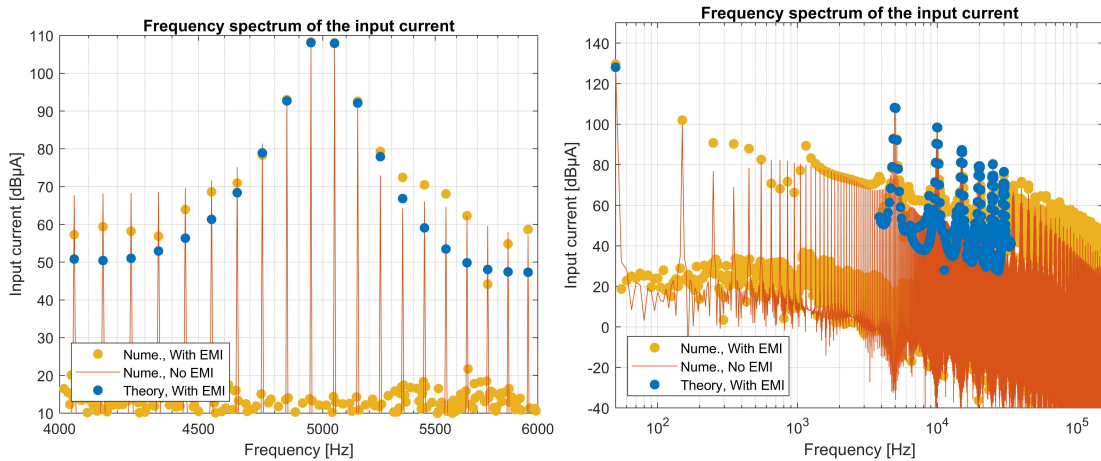


Figure 3.21: Non significant impact of EMI filter on the total emissions, with $C_f = 10C_{fnom}$, of an APFC converter, here a boost converter

As for the case with $C_f = C_{fnom}$ the total ripple is not impacted by the EMI filter, indeed, the total impedance seen by the switch is not significantly impacted by the filter for the different carrier harmonic frequencies. Also the series resonance is displaced around 1 kHz explaining this decrease of $I_{rip total}$ around one kHz as these emissions are coming from the grid and are trapped at a smaller frequency by the series resonance at frequency

$$f = \frac{1}{2\pi\sqrt{(L_{in} + \frac{L_{fnom}}{10})10C_{fnom}}} \approx 1[kHz]$$

Case 1 $C_f = 10C_{fnom}$ ($I_{rip grid}$)

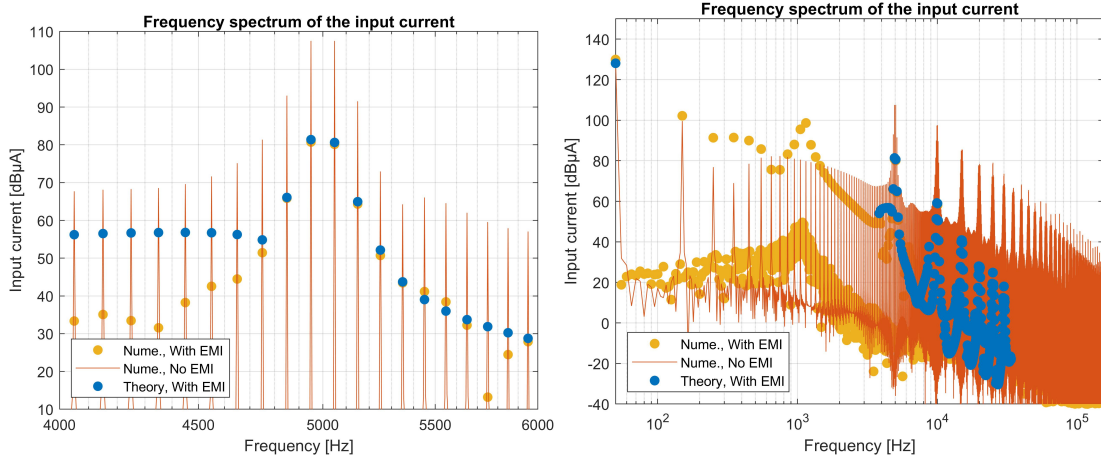


Figure 3.22: Significant impact of EMI filter on the emissions seen on the grid of an APFC converter, here a boost converter

Fig. 3.22 shows a reduction in the emissions observed on the grid that is significantly greater than that obtained with the nominal EMI filter. This further highlights the non-neutral impact of the EMI filter and the possibility of designing EMI filters to provide a first solution to the issue of supraharmonic propagation. Of course, the capacitor should be appropriately sized to fulfil this additional function. An interesting result highlighted here is the relationship between increasing the capacitance by a factor of x , which decreases the corresponding capacitive reactance by the same factor, and the resulting reduction in the current observed on the grid.

The reduction is observed to be approximately $30 \text{ dB}\mu\text{A}$ for the first frequency band compared with the case without EMI filtering. Subsequently, the reduction between successive emission bands is approximately $20 \text{ dB}\mu\text{A}$ with the EMI filter, whereas it is only about $10 \text{ dB}\mu\text{A}$ without EMI filtering. The results for the second case are presented without further comment, as the observations made for Case 1 remain valid.

Case 2 $C_f = 100C_{fnom}$ ($I_{rip total}$)

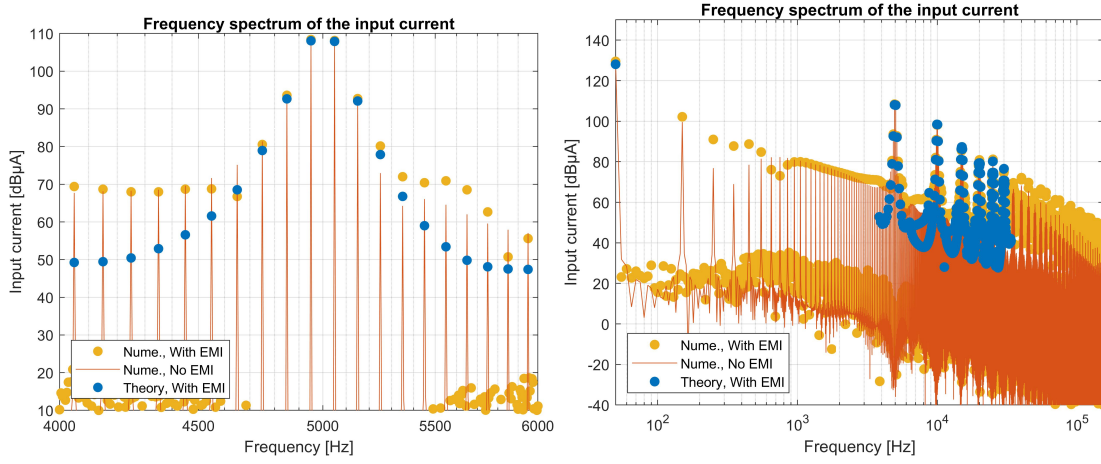


Figure 3.23: Non significant impact of EMI filter on the total emissions, with $C_f = 100C_{fnom}$, of an APFC converter, here a boost converter

Case 2 $C_f = 100C_{fnom}$ ($I_{rip\ grid}$)

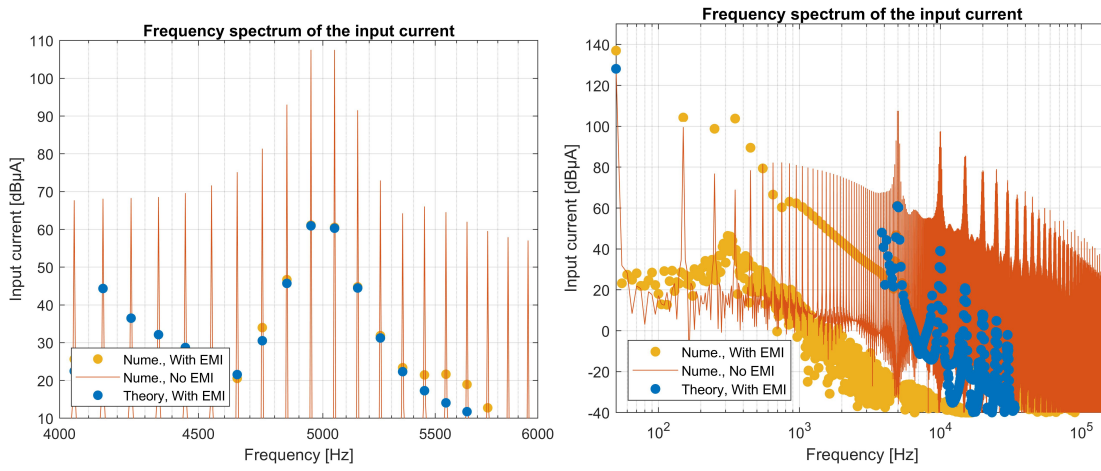


Figure 3.24: Significant impact of EMI filter on the emissions seen on the grid, with $C_f = 100C_{fnom}$, of an APFC converter, here a boost converter

The possibility of reducing the impact of the SH emissions of a converter, here a boost converter used as an APFC, on the grid through the design of the EMI filter has been demonstrated. However, although the EMI filter has been designed to also mitigate emissions within the SH range, it is worth noting that this also affects the proportion of primary and secondary emissions. The concepts of primary and secondary emissions were introduced in [85] and further developed in [86]. The underlying principle is that, as the capacitance increases, its impedance decreases (for a given frequency), and the emissions generated by the converter are therefore more readily diverted through the low-impedance path provided by this capacitance. At the same time, this low-impedance path also exists for other emitting devices, and a proportion of the secondary emissions is therefore expected to flow through the filter capacitor.

Let the converter for which the EMI filter has been designed be referred to as converter 1, while the devices responsible for the secondary emissions are collectively referred to as converters 2. The emissions flowing through the filter capacitor therefore originate from

both converter 1 and converters 2. The magnitude of the secondary emissions (from converters 2) mainly depends on the distance between these devices and the filter capacitor, as this distance determines the equivalent line inductance.

Eventually, it can readily be understood that SH emissions are highly likely to propagate between interconnected converters in low-voltage grids, thereby reducing the amount of SH current injected into the grid. This phenomenon has been reported in numerous studies. Among others, this phenomenon has been reported in [2],[13],[16],[17] and [53]. Another point that may lead to significant issues is the possible resonance between the EMI filter capacitance and the distribution transformer inductance when its resonance frequency lies close to the emission frequency of a power electronic device. Indeed, this issue was discussed in [87], where it was suggested that it would become increasingly significant in future low-voltage grids.

3.2.5 Analytically generated signals

Based on the operating principles of the APFC and PWM, it is possible to derive the expected switching function. This consists of determining the switching instants at which the switches are either closed or open and then applying circuit theory to reconstruct the expected signals, namely the input current of the boost converter in the present case.

Although the material presented in this section constitutes a concrete application, it actually stems from a much broader, and even universal, concept. It should be emphasized, however, that this reasoning applies to converters that intermittently impose a constant voltage at their terminals, which may or may not contain harmonic components of varying amplitudes, and that are assumed to be connected to an AC electrical grid.

Indeed, the current or voltage at the terminals of a converter can be modeled in a given operating context through a method whose underlying principle is universal. In essence, the switching function constitutes the only equation required to determine the behaviour of the converter, regardless of the application under consideration.

This method and its underlying conceptual framework are developed in greater detail in Section 5.8.

The aim is to demonstrate that it is possible, to a certain extent, to reproduce the logic of numerical simulation without the associated constraint of long computation times required to obtain the desired signals. Indeed, this section shows that, based on the switching instants and circuit theory, the signals can be generated and analysed before being compared with those obtained using the previously presented methods. Results obtained using three different approaches, namely numerical simulations, DFS analysis, and experimental measurements, have already been presented. A fourth approach, referred to as the analytically generated signals (AGS) method, is now introduced.

Regarding the operating principle of the APFC, it is now well established that the duty-cycle expression depends on both the grid voltage and the DC-side voltage. This dependence is directly reflected in the modulating wave, which ultimately represents the time evolution of the duty cycle. The modulating wave is given by $M(t) = 1 - \frac{V_{in}}{V_{DC}}$, where V_{in} denotes the input voltage. However, since the APFC is supplied through a full-bridge

diode rectifier (FBDR), and denoting by V_{grid} the voltage on the AC side of the FBDR, the modulating wave can be rewritten as

$$M(t) = 1 - \widehat{V_{grid}} \sin(\omega_g t) \quad \text{for } 0 < \omega t < \pi \quad (3.41)$$

$$M(t) = 1 + \widehat{V_{grid}} \sin(\omega_g t) \quad \text{for } \pi < \omega t < 2\pi \quad (3.42)$$

It is important to note that the duty cycle is updated at the switching frequency. Consequently, it is not continuous but consists of discrete steps, each having a duration equal to one switching period. Based on this observation, the modulating wave can be directly expressed as the duty cycle by simply adapting the notation, leading to

$$D(kT_s) = 1 - \widehat{V_{grid}} \sin(\omega_g kT_s) \quad \text{for } 0 < \omega_g kT_s < \pi \quad (3.43)$$

$$D(kT_s) = 1 + \widehat{V_{grid}} \sin(\omega_g kT_s) \quad \text{for } \pi < \omega_g kT_s < 2\pi \quad (3.44)$$

Once the vector containing the duty cycle for each time step, each of duration T_s (the switching period), has been obtained, it can be used to reconstruct the current waveform. Considering that the converter under investigation is a conventional boost converter, the current through the boost inductance is simply obtained by adding a positive or negative current increment to the initial current, according to the fundamental inductor equation. Let V_L , i_D and $i_{D'}$ respectively denote the voltage across the inductance and the inductor current when the switch is closed (D) and open (D')

$$L \frac{di}{dt} = V_L \quad (3.45)$$

$$DT_s \frac{di}{dt} = DT_s \frac{V_L}{L} = \Delta i_D \quad (3.46)$$

$$D'T_s \frac{di}{dt} = D'T_s \frac{V_L}{L} = \Delta i_{D'} \quad (3.47)$$

The current at the end of the first switching period is therefore simply equal to the initial current i_0 , to which the two current increments i_D and $i_{D'}$ are added. Consequently, the current can be expressed for $k = 1 \dots n$ where $n = \frac{T_g}{2T_s}$ assuming that the switching frequency is an integer multiple of twice the grid frequency.

$$i(kT_s) = i((k-1)T_s) + \Delta i_{D_k} + \Delta i_{(D_k)'} \quad (3.48)$$

Following the presentation of this new method, referred to as the Analytically Generated Signal (AGS) method, a comparison is carried out with the numerical, theoretical, and experimental approaches for selected operating conditions. First, the results obtained for

the boost converter with $R = 800[\Omega]$ and $V_{DC} = 300[V]$ are compared with those obtained using the AGS method.

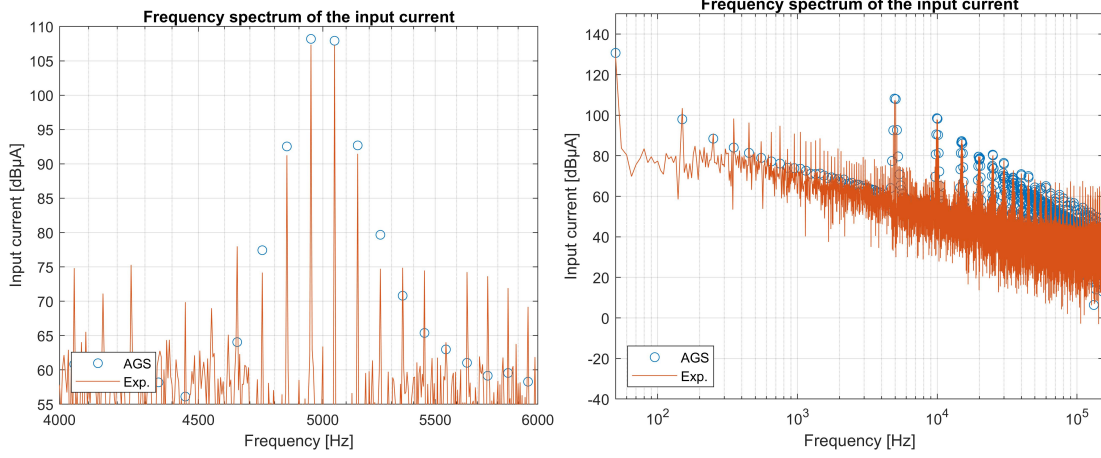


Figure 3.25: Validation of the new method, AGS, via comparison with the experimental results for a boost converter used as APFC and with parameters being $R = 800[\Omega]$ and $V_{DC} = 300[V]$

Here, validity is defined as the ability to accurately reproduce the expected supraharmonic emissions. Fig. 3.25 clearly demonstrates the validity of this new method, namely AGS. However, the comparison with the experimental results shows that, although the first ten carrier harmonic bands are accurately predicted by the AGS method, discrepancies appear at higher frequencies, i.e., above 50 kHz. Furthermore, in the AGS method, the low-frequency harmonics originate solely from the concatenation of the reconstructed current waveform to generate the ten cycles required for the FFT. Consequently, differences are observed when these results are compared with laboratory measurements. Finally, a comparison is performed between the four developed approaches: the theoretical (DFS), numerical, experimental, and AGS methods.

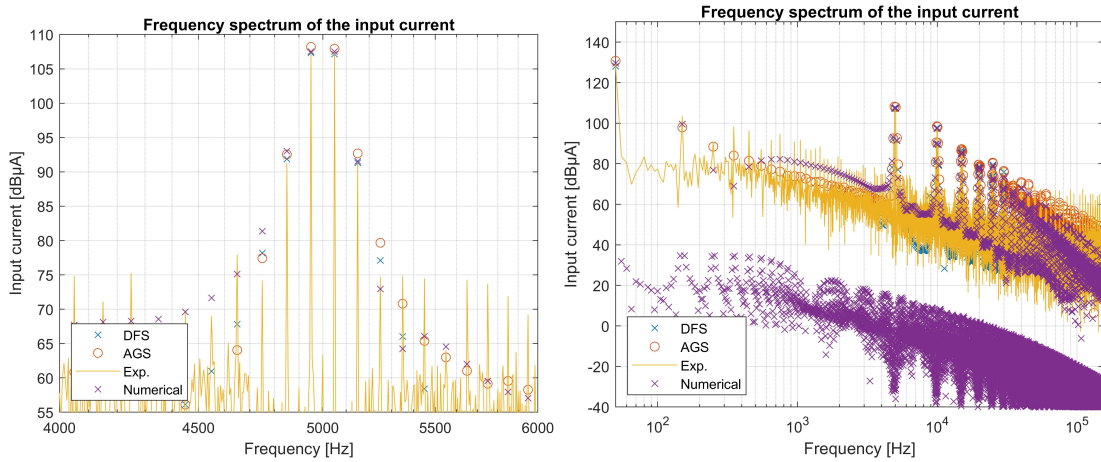


Figure 3.26: Comparison between theoretical, numerical, experimental and AGS method for a boost converter used as APFC and with parameters being $R = 800[\Omega]$ and $V_{DC} = 300[V]$

Compared with the DFS approach, this new method provides a much more intuitive way of determining the supraharmonic emissions of a converter. Moreover, it requires only a limited number of parameters while achieving a level of accuracy comparable to that of the DFS approach. For a boost converter operating as an APFC, the required information consists of the DC voltage, the grid voltage, the boost inductance (provided that it is significantly larger than the equivalent grid inductance), and, when necessary, an estimate of the grid inductance if it cannot be neglected with respect to the boost inductance.

However, the method is highly sensitive to the value of the total inductance implemented in the model and, consequently, to the accuracy with which this parameter is known. By contrast, the DC voltage is generally less critical, as it remains close to its nominal value. Another major influencing factor, previously discussed, is the grid voltage at the point of connection. This voltage depends on the voltage drop across the equivalent impedance between the input of the FBDR and the supplying transformer.

3.3 Analysis of the Interleaved Boost Converter

One particular converter, the interleaved boost converter (IBC), has been identified in chapter 1 as being widely used. Characterizing the behaviour of the IBC in the supraharmonic range is therefore of particular interest. The IBC is considered within a simplified electrical grid consisting of an ideal voltage source, a line impedance (denoted Z_{in}), a full-bridge diode rectifier (FBDR), and the IBC followed by an RC load. The simplified schematic of the considered circuit is shown in Fig. 3.27. This chapter focuses on one specific aspect, namely the possibility of determining whether the IBC behaves as a voltage harmonic source (VHS) or a current harmonic source (CHS). In other words, the objective is to determine whether the converter tends to maintain its voltage or its current when the line impedance, i.e. Z_{in} , varies.

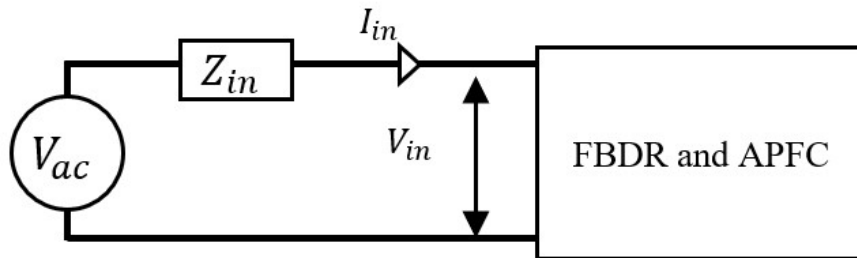


Figure 3.27: Simplified representation of the full circuit containing an ideal voltage source, a line impedance (named Z_{in}), a full-bridge diode rectifier (FBDR) and the interleaved boost converter(IBC)

Since the investigation of supraharmonics focuses on frequency bands containing side-band harmonics, it is useful to reduce each band to a single representative value that can be directly compared. This motivates the introduction of the quadratic sum (QS). The quadratic sum is obtained through an aggregation process, following the principles

described in [88] and [89]. Furthermore, in [4], the author aggregates the components around the first carrier harmonic. This is, however, a matter of choice, as the aggregation could equally be performed around the second carrier harmonic. Nevertheless, the higher the carrier harmonic order, the lower the magnitude of the associated sideband harmonics. Consequently, considering the sidebands around the first carrier harmonic is equivalent to analysing the dominant supraharmonic components.

The interested reader is referred to [4] for a comprehensive discussion of the motivation behind the aggregation process. Among other considerations, it is based on the observation that both the frequency and the amplitude of the components may vary over a few milliseconds, highlighting the need for a more macroscopic indicator. The aggregated value proposed in [4] is defined as

$$X_{QS} = \sqrt{\sum_{f=f_c-395}^{f=f_c+400} X_n^2} \quad (3.49)$$

In the present study, the quadratic sum is defined as

$$X_{QS} = \sqrt{\sum_{n=1}^{n=8} X_n^2} \quad (3.50)$$

where n denotes the index of the sideband harmonics around the first carrier harmonic. In this study, the first component is located at $(F_{sw} - 50 - 300) [Hz]$, where F_{sw} denotes the switching frequency. For an IBC operating at 25 kHz, this expression becomes $(50\,000 - 50 - 300) [Hz] = 49\,650 [Hz]$, with $(50\,000 - 50) [Hz]$ corresponding to the first carrier harmonic observed on the AC side due to the switching process of the FBDR. The remaining components are then taken every 100 Hz up to $(F_{sw} - 50 - 300 + 700) [Hz]$. Consequently, the quadratic sum is computed from the components located at $f_n = [49\,650, 49\,750, 49\,850, 49\,950, 50\,050, 50\,150, 50\,250, 50\,350] [Hz]$.

The objective is first to analyse the circuit shown in Fig. 3.28, although it differs from the circuit presented in Fig. 3.27. A step-by-step approach is adopted. The main differences are that the circuit of Fig. 3.28 is supplied by an ideal DC voltage source and does not include any impedance between the source and the converter, whereas the final circuit is supplied by an AC voltage source through a line impedance, denoted Z_{in} .

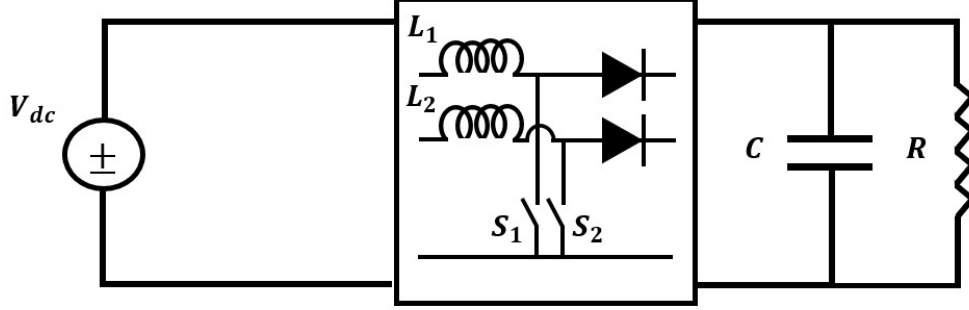


Figure 3.28: Schematic of the interleaved boost converter (IBC) supplied by an ideal voltage source and connected to an RC load

The topology and control structure of this converter allow the suppression of half of the emitted carrier harmonics. The converter contains two switches (S_1 and S_2), both controlled using pulse-width modulation (PWM). This cancellation results from the fact that the carrier signals used to generate the switching functions are phase-shifted by 180° with respect to one another. The choice of this converter for the present study is motivated by its widespread use in numerous applications, including electric vehicle chargers, heat pumps, and many grid-connected household DC appliances operating in low-voltage (LV) distribution networks [26, 28, 30, 31].

3.3.1 Control of the interleaved boost converter

This section is mainly based on the work presented in [90]. Two complete control structures are investigated. Both structures consist of two control loops: an outer voltage loop and an inner current loop. The active power factor correction (APFC) controller has two primary objectives. The first is to regulate the DC voltage through the outer voltage loop. The second is to ensure that the input current remains sinusoidal and in phase with the AC-side voltage [1][44][45]. Since the inner current loop has a significantly higher bandwidth than the outer voltage loop, it is responsible for achieving the second objective. Based on simulations performed with the two control structures presented hereafter, interesting results regarding the behaviour of the converter in the first carrier harmonic band will be highlighted.

The conventional approach for designing the control structure of such a nonlinear system consists of two successive steps [44][45]:

1. State-Space Averaging (SSA),
2. Small-Signal Linearization (SSL).

These two steps allow the derivation of the small-signal transfer functions (TFs) of the system. Once these TFs have been obtained, the controller parameters can be designed using conventional linear control techniques, such as Bode diagram analysis, to tune the proportional-integral (PI) or proportional-integral-derivative (PID) controllers.

Complete control structures for interleaved boost converters can be found in [91, 92]. However, these studies assume that applying the SSA and SSL procedures developed for a conventional boost converter is sufficient to derive the small-signal transfer functions of the IBC. One of the first objectives of the present work is therefore to derive these transfer functions explicitly for the interleaved boost converter. The resulting transfer functions are then compared with those commonly adopted in the literature [45, 91, 92], where the boost converter transfer functions are directly employed.

3.3.1.1 Control Structure 1

This control structure was originally developed in [1] based on the methodology proposed in [44]. The complete derivation is provided in Appendix C of [1]. In the present subsection, only the main features of the control structure are summarized. The corresponding control schematic is shown in Fig. 3.29.

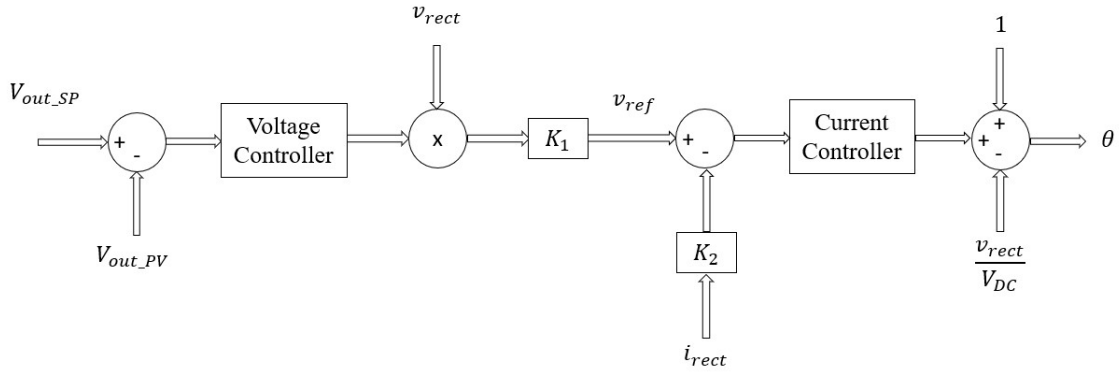


Figure 3.29: Complete control structure 1. K_1 is a constant gain, whereas K_2 is a transfer function. $V_{out,SP}$ and $V_{out,PV}$ denote the setpoint and the measured value of the DC output voltage, respectively. Similarly, v_{rect} and i_{rect} represent the rectified voltage and current measured at the output of the full-bridge diode rectifier (FBDR), i.e., on the DC side of the rectifier.

The previously cited references also define several conditions that must be satisfied for the controller to operate as intended. These assumptions are summarized below.

1. For the inner current loop (IL), the cut-off frequency must be significantly higher than twice the grid frequency while remaining well below the frequency of the first carrier harmonic.
2. For the outer voltage loop (OL), the cut-off frequency must remain significantly lower than twice the grid frequency.

These two requirements are satisfied by appropriately tuning the controller parameters. More specifically, the inner-loop controller gains (K_i) are selected to satisfy the first condition, whereas the outer-loop controller parameters (K_o and I_o) are adjusted to satisfy the second.

3.3.1.2 Control Structure 2

For the second control structure, the transfer functions (TFs) are derived explicitly using the state-space averaging (SSA) and small-signal linearization (SSL) methodologies. The main steps of this derivation are presented in the following subsections. The transfer functions of interest are $\frac{\hat{i}_L}{d}$ and $\frac{\widehat{v_{out}}}{i_L}$.

The corresponding control structure is shown in Fig. 3.30.

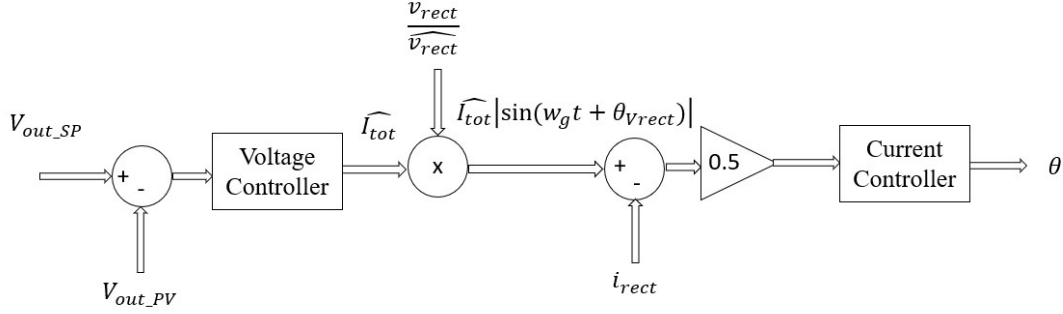


Figure 3.30: Complete control structure 2

It should be emphasized that the transfer functions are derived for the circuit shown in Fig. 3.28, where the IBC is supplied by a constant input voltage. In the actual APFC application investigated in this work, however, the converter is supplied by the rectified grid voltage, which varies continuously over the grid period.

Following the approach proposed in [45], it is assumed that the transfer functions obtained through SSA and SSL are not significantly affected by the variation of the rectified input voltage during a single grid half-cycle. This assumption is adopted throughout the present work and is subsequently verified.

3.3.2 State-Space Averaging

Before deriving the transfer functions, the mathematical framework underlying the state-space representation, upon which the SSA methodology is based, is briefly introduced.

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \quad (3.51)$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} \quad (3.52)$$

This set of equations constitutes the state-space representation of the system, where the bold symbols denote matrices. In its present form, it describes a linear time-invariant (LTI) system. Each switching configuration of the converter can be represented by a set of linear differential equations. The time-invariant assumption means that the system dynamics do not change with time, so that identical inputs always produce identical responses.

In Eq. 3.51, $\mathbf{x}$ denotes the state vector, whose components correspond to the energy-storage elements of the system, namely the inductor currents and capacitor voltages. For the IBC considered in this work, the state vector is defined as

$$\mathbf{x} = [i_{L1} \quad i_{L2} \quad v_C]^T.$$

The output and input vectors are denoted by $\mathbf{y}$ and $\mathbf{u}$, respectively. Furthermore, the direct transmission matrix $\mathbf{D}$ is assumed to be zero, as a non-zero $\mathbf{D}$ would imply an instantaneous influence of the input on the output, which is generally not physically realistic for electrical systems [93]. Equations 3.51 and 3.52 therefore constitute the mathematical foundation of the state-space modelling approach.

The differential equations associated with the different operating configurations of the converter must first be derived. It is worth noting that this study is limited to the IBC operating with an average duty cycle D greater than 0.5. Under this condition, only three switching states occur:

Mode 1: S_1 is ON and S_2 is OFF (25% of the switching period)

Mode 2: S_1 is OFF and S_2 is ON (25% of the switching period)

Mode 3: both S_1 and S_2 are ON (50% of the switching period)

The switching function associated with S_1 is equal to 1 whenever the controller output, i.e. the modulating wave, is greater than the sawtooth carrier signal. The same principle is applied to S_2 , except that its carrier signal is phase-shifted by 180° with respect to that of S_1 .

An example of the analytical development is provided below for Mode 1.

$$V_{in} - i_{L1}r_L - L\frac{di_{L1}}{dt} = 0 \quad (3.53)$$

$$V_{in} - i_{L2}r_L - L\frac{di_{L2}}{dt} = V_c \quad (3.54)$$

$$R(i_{L2} - C\frac{dV_c}{dt}) = V_c \quad (3.55)$$

$$\frac{di_{L1}}{dt} = \frac{V_{in}}{L} + i_{L1}\left(\frac{-r_L}{L}\right) = \dot{i}_{L1} = \dot{x}_1 \quad (3.56)$$

$$\frac{di_{L2}}{dt} = \frac{V_{in}}{L} + i_{L2}\left(\frac{-r_L}{L}\right) - \frac{V_c}{L} = \dot{i}_{L2} = \dot{x}_2 \quad (3.57)$$

$$\frac{dV_c}{dt} = i_{L2}\frac{1}{C} + V_c\left(\frac{-1}{RC}\right) = \dot{V}_c = \dot{x}_3 \quad (3.58)$$

$$\begin{bmatrix} \frac{-r_L}{L} & 0 & 0 \\ 0 & \frac{-r_L}{L} & \frac{-1}{RC} \\ 0 & \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \frac{1}{L} \\ 0 \end{bmatrix} V_{in} = \begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{V}_c \end{bmatrix} \quad (3.59)$$

The same procedure is then applied to Modes 2 and 3.

3.3.3 Small-Signal Linearization

In Eq. 3.59, the first matrix corresponds to A_1 , followed by the state vector x , the input matrix B_1 , the input vector u , and finally the state derivative vector $\dot{x}_1$. Repeating the same derivation for Modes 2 and 3 leads to

$$A_1x + B_1u = \dot{x}_1 \quad (3.60)$$

$$A_2x + B_2u = \dot{x}_2 \quad (3.61)$$

$$A_3x + B_3u = \dot{x}_3 \quad (3.62)$$

The state-space averaging method introduced in [6], together with the small-signal linearization procedure described in [94], leads to the analytical development presented below.

3.3.3.1 Small-Signal Linearization : Analytical Development

First,

$$\dot{x} = x((1 - D)(A_1 + A_2) + (2D - 1)A_3) + V_{in}((1 - D)(B_1 + B_2) + (2D - 1)B_3) \quad (3.63)$$

With $D = D + \tilde{d}$, $x = X + \tilde{x}$, $V_{in} = V_{in} + \tilde{V}_{in}$, $\dot{x} = \dot{X} + \tilde{\dot{x}}$

$$\begin{aligned} \tilde{\dot{x}} = & (X + \tilde{x})((1 - (D + \tilde{d}))(A_1 + A_2) + (2(D + \tilde{d}) - 1)A_3) + V_{in}((1 - (D + \tilde{d}))(B_1 + B_2) \\ & + (2(D + \tilde{d}) - 1)B_3) \end{aligned} \quad (3.64)$$

If $\tilde{\dot{x}}$, $\tilde{d}$, $\tilde{x} = 0$ then

$$0 = X((1 - D)(A_1 + A_2) + (2D - 1)A_3) + V_{in}((1 - D)(B_1 + B_2) + (2D - 1)B_3) \quad (3.65)$$

Which gives

$$\tilde{\dot{x}} = X\tilde{d}(-(A_1 + A_2) + 2A_3) + \tilde{x}((1 - D)(A_1 + A_2) + (2D - 1)A_3) + V_{in}\tilde{d}(-(B_1 + B_2) + 2B_3) \quad (3.66)$$

By defining

$$A = ((1 - D)(A_1 + A_2) + (2D - 1)A_3) \quad (3.67)$$

$$A_x = -(A_1 + A_2) + 2A_3 \quad (3.68)$$

$$B_x = -(B_1 + B_2) + 2B_3 \quad (3.69)$$

It follows that

$$\tilde{\dot{x}} = X\tilde{d}A_x + \tilde{x}A + V_{in}\tilde{d}B_x \quad (3.70)$$

$$(sI - A)\tilde{x} = \tilde{d}(A_xX + V_{in}B_x) \quad (3.71)$$

The final expression of the small-signal variation of the state variables is therefore

$$\tilde{x} = \tilde{d}(A_x X + V_{in} B_x) \frac{1}{(sI - A)} \quad (3.72)$$

Based on Equ. 3.72 it is possible to extract the next transfer functions

$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_{in}(\frac{2}{LC} - \frac{s}{RCD'^2})}{s^2 + \frac{s}{RC} + \frac{2D'^2}{LC}} \quad (3.73)$$

$$\frac{\tilde{i}_{Ltot}}{\tilde{d}} = \frac{2V_{in}(s + \frac{2}{RC})}{D'L(s^2 + \frac{s}{RC} + \frac{2D'^2}{LC})} \quad (3.74)$$

The derived transfer functions are now analysed to assess the validity of the assumption that they remain independent of the input voltage magnitude. Indeed, they have been derived using an equivalent circuit supplied by a constant DC voltage source, whereas they are ultimately intended to be used for an APFC supplied by an AC voltage source. The corresponding Bode diagrams are shown in Fig. 3.31 for V_{in}

$$V_{in} = [230, 200, 170, 140, 110, 80, 50, 20, 0.1] \text{ V}$$

The responses remain almost identical over the entire investigated voltage range, thereby confirming the conclusion of [45] that transfer functions derived under DC operating conditions remain valid for AC applications such as active power factor correctors.

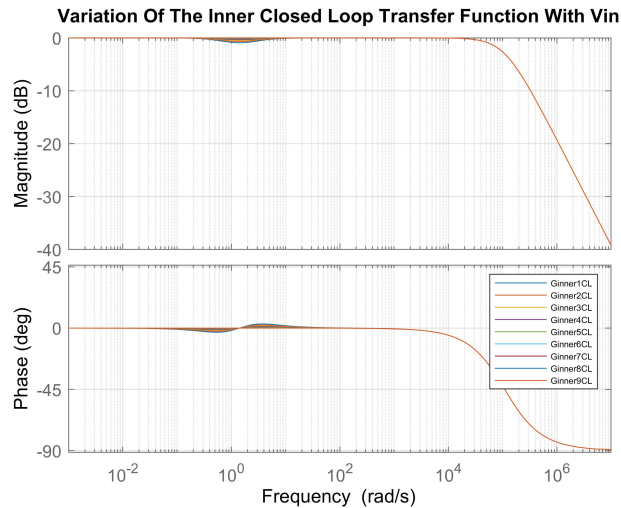


Figure 3.31: Bode diagram for varying V_{in}

In addition, the transfer functions have been validated for small duty-cycle variations of approximately 0.5%.

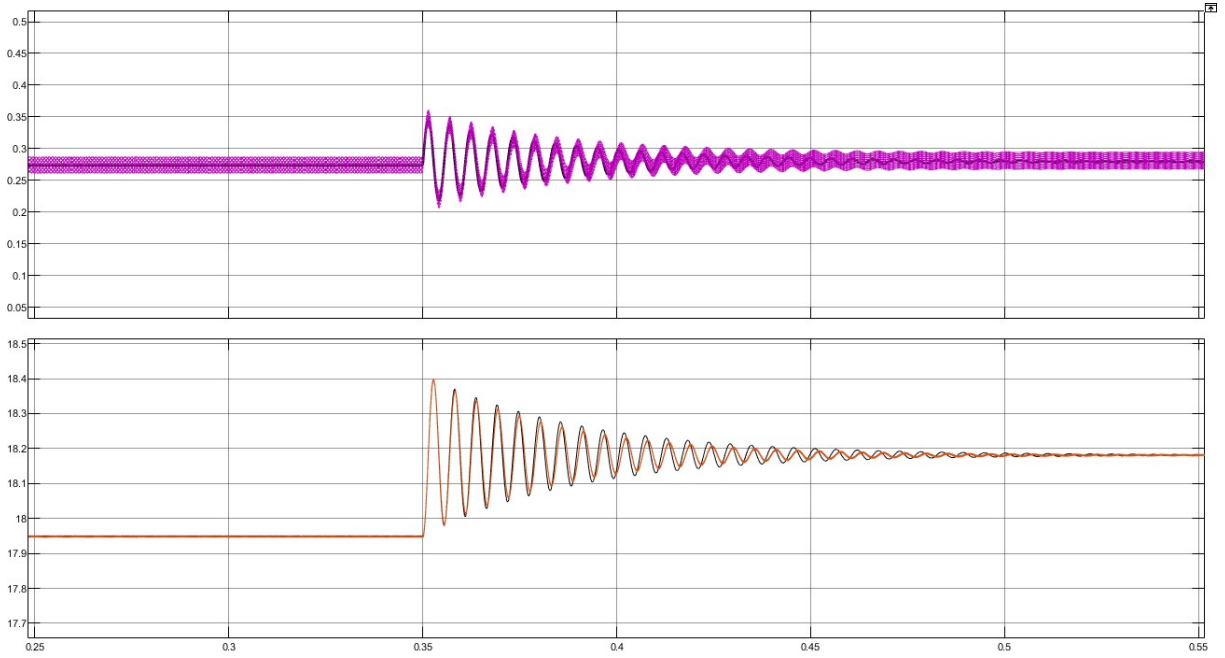


Figure 3.32: Comparison between analytical expression of the TFs (in black) and numerical results (purple (top) and orange (bottom)) for a change in duty cycle of 0.5 percent in current (top) and voltage (bottom)

3.3.4 Results

3.3.5 Numerical Results

After presenting the control structures, the core question addressed in this chapter can now be discussed: is it possible to characterize a converter as a voltage harmonic source (VHS) or a current harmonic source (CHS) in the supraharmonic range, particularly around the first carrier harmonic band?

One possible way to investigate this question is through numerical simulations. The results presented in this chapter were obtained using MATLAB Simulink. The implemented circuit is the one presented in Fig. 3.27. Based on Fig. 3.28, an intuitive understanding of the expected results can be obtained. Indeed, if the converter maintains the quadratic sum of the voltage over the eight nearest components around the first carrier harmonic, as defined in Equ. 3.50, then a variation in the grid impedance (Z_{in}) would logically induce a corresponding variation in the current, also expressed as a quadratic sum. This can simply be regarded as an extension of Ohm's law to quadratic-sum quantities.

The second case corresponds to a converter that maintains the quadratic sum of the current over the eight nearest components around the first carrier harmonic. In this case, a variation in the grid impedance (Z_{in}) directly affects the quadratic sum of the voltage associated with the first carrier harmonic band. The grid impedance values considered in these simulations are given by

$$Z_{in} = [\frac{Z_{in0}}{2.5}; \frac{Z_{in0}}{2}; \frac{Z_{in0}}{1.5}; Z_{in0}; 1.5Z_{in0}; 2Z_{in0}; 2.5Z_{in0}]$$

With $Z_{in0} = (0.4 + j0.35) \frac{\Omega}{km}$ [95], it is worth noting that in [96] and [97], the reference impedance considered for LV grid modelling is $(0.4 + j0.25)\Omega$.

The numerical experiments were performed using several parameters, including inductors, capacitors, resistors, among others. These values are summarized in Table 3.3.

	Control Str. 2	Control Str. 1
$V_{in}[V]$	230	230
$R_g[\Omega]$	0.4	0.4
$L_g[mH]$	1.11	1.11
$L_{b1}[mH]$	2.3	2.3
$L_{b2}[mH]$	2.3	2.3
$C[mF]$	5	5
$R[\Omega]$	200	200
$F_{sw}[kHz]$	25	25
<i>Out.Contr.Ko</i>	3.5e-4	1
<i>Out.Contr.Io</i>	0.01	5
<i>Inn.Contr.Ki</i>	0.35	0.65
<i>Inn.Contr.Ii</i>	1	/

Table 3.3: Simulation parameters used for the numerical experiments

The results presented in Fig. 3.33, expressed as quadratic sums, show both the input voltage and input current as a function of the grid impedance value (Z_{in}). The considered range of Z_{in} corresponds to the values previously defined and is expressed relative to $Z_{in0} = (0.4 + j0.35)\Omega$.

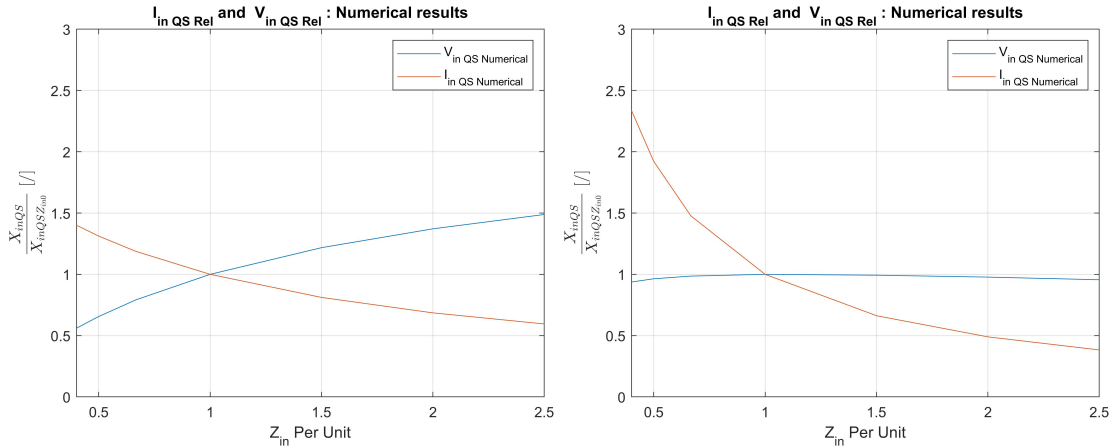


Figure 3.33: Quadratic sum of the voltage and current around 50 kHz as a function of the grid impedance (Z_{in}) relative to Z_{in0} , with on the left control 1 and on the right control 2.

Although the characterization was initially defined for pure voltage or pure current harmonic sources, Fig. 3.33 (left) reveals some ambiguous situations. The voltage and current variations are almost symmetric with respect to an imaginary straight line located at $y = 1$.

In Fig. 3.33 (right), the behaviour is much clearer. Indeed, the variation of the quadratic sum of the voltage for the first carrier harmonic frequency band is approximately 5%, whereas the quadratic sum of the current varies between 240% and 40% of its value for $Z_{in} = Z_{in0}$.

The observed difference can only be attributed to the differences in the control implementations, namely the controller parameters and the control structures. Indeed, the passive components have identical values in both circuits.

Regarding the control structures, the difference may not appear significant at first glance, since both systems rely on cascaded control structures. The only difference is the presence of a feed-forward term in control structure 1, which is not implemented in control structure 2. Moreover, a PI controller is used in the inner loop of control structure 2, whereas only a proportional gain is used in control structure 1.

Intuitively, the observed variation could mainly originate from the controller parameters, which directly influence the bandwidths of the inner and outer closed loops.

This first part provided an analysis of the problem from a numerical experimentation perspective. Ultimately, a converter can be characterized as either a voltage harmonic source or a current harmonic source depending on the considered operating context, including passive component values, controller parameters, control structures, among other factors.

An analytical development is subsequently carried out in order to establish a comparison with the numerical results.

3.3.6 Analytical Results: First Development

This section is dedicated to the presentation of the analytical development carried out to determine the frequency components of the input current, namely $I_{in}(t)$ in Fig. 3.27.

The IBC contains two switches, which leads to the presence of two switching functions, $S_1(t)$ and $S_2(t)$. These switching functions are generated by comparing the same modulating signal with two carrier signals phase-shifted by 180° .

First, the analytical expression of $S_1(t)$ is derived, based on [1], by adapting Eq. 3.75. Subsequently, the expression of $S_2(t)$ is derived, allowing the complete analytical description of the system. Consequently, the final expression of the input current can be determined.

$$A_{mn} + jB_{mn} = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{j(mx+ny)} dx dy \quad (3.75)$$

Equation 3.75 defines the DFS coefficients, which contain information regarding the mag-

nitude and phase of each frequency component present in the signal.

It should be noted that m , referred to as the carrier index, and n , referred to as the baseband index, define the frequency of each spectral component, namely $m\omega_c + n\omega_b$, where ω_c and ω_b correspond respectively to the carrier frequency and the modulating signal frequency.

The term $m\omega_c + n\omega_b$ represents the frequency of the n^{th} baseband harmonic located around the m^{th} carrier harmonic. For instance, if $m = 1$ and $n = 0$, the considered frequency is:

$$m\omega_c + n\omega_b = \omega_c$$

which corresponds to the first carrier harmonic. Conversely, when both m and n are non-zero, the considered frequency corresponds to the n^{th} sideband harmonic around the m^{th} carrier harmonic.

It is essential to note that the variables x and y appear in Eq. 3.75. This results from the fact that the switching function $S_1(t)$ can be rewritten as $S_1(x, y)$, where x and y are periodic variables with a period of 2π and are related to the two signals involved in PWM modulation, namely the modulating signal and the carrier signal.

The definitions of x and y are given below.

$$x = \omega_c t \tag{3.76}$$

$$y = \omega_g t \tag{3.77}$$

In this development the initial phase of the modulating wave is assumed as being nul. By isolating t in Equ. 3.77 and replacing it in Equ. 3.76 the expression becomes

$$x_c = \omega_c / \omega_g y \tag{3.78}$$

Noting that in the case of the IBC used as APFC the modulating wave, i.e. $\theta(t)$ the duty cycle, is a rectified sine wave which involves two expressions per period.

$$\theta(t) = 1 - r \sin(\omega_g t + \phi) \quad 0 < t < T_m/2 \tag{3.79}$$

$$\theta(t) = 1 + r \sin(\omega_g t + \phi) \quad T_m/2 < t < T_m \tag{3.80}$$

With r and ϕ define as

$$r = \frac{\sqrt{2}V_{AC}}{V_{DC}} \sqrt{1 + \left(\frac{L\omega_g}{R_{eq}}\right)^2} \quad (3.81)$$

$$\phi = \text{atg}\left(\frac{-L\omega_g}{R_{eq}}\right) \quad (3.82)$$

Adapting Equ. 3.75 by chosing the bounds as being Equ. 3.79 and 3.80 gives

$$A_{mn} + jB_{mn} = \frac{1}{2\pi^2} \left(\int_{\phi}^{\pi+\phi} \int_0^{2\pi(1-r\sin(y))} e^{j(mx+ny)} dx dy + \int_{\pi+\phi}^{2\pi+\phi} \int_0^{2\pi(1+r\sin(y))} e^{j(mx+ny)} dx dy \right) \quad (3.83)$$

$$A_{mn} + jB_{mn} = \frac{1}{j2\pi^2 m} \left(\int_{\phi}^{\pi+\phi} (e^{j(-m2\pi r\sin(y)+ny)} - e^{jny}) dy + \int_{\pi+\phi}^{2\pi+\phi} (e^{j(m2\pi r\sin(y)+ny)} - e^{jny}) dy \right) \quad (3.84)$$

By substituting

$$\begin{aligned} m2\pi r\sin(y) &= b \\ ny &= c \end{aligned}$$

One obtains Equ. 3.84 describes as

$$A_{mn} + jB_{mn} = \frac{1}{j2\pi^2 m} \left(\int_{\phi}^{\pi+\phi} (e^{j(-b+c)} - e^{jc}) dy + \int_{\pi+\phi}^{2\pi+\phi} (e^{j(b+c)} - e^{jc}) dy \right) \quad (3.85)$$

It is also possible to change the bounds of the second integral to reduce the complexity of this expression. Indeed, choosing ϕ to $\pi + \phi$ as bounds instead of $\pi + \phi$ to $2\pi + \phi$ leads to

$$A_{mn} + jB_{mn} = \frac{1}{j2\pi^2 m} \left(\int_{\phi}^{\pi+\phi} (e^{j(-b+c)} - e^{jc}) dy + \int_{\phi}^{\pi+\phi} (e^{j(-b+c-n\pi)} - e^{j(c-n\pi)}) dy \right) \quad (3.86)$$

$$A_{mn} + jB_{mn} = \frac{1}{j2\pi^2 m} \int_{\phi}^{\pi+\phi} (e^{j(-b+c)} - e^{jc} + e^{j(-b+c-n\pi)} - e^{j(c-n\pi)}) dy \quad (3.87)$$

With

$$e^{-j(n\pi)} = \cos(-n\pi) = (-1)^n$$

$$A_{mn} + jB_{mn} = \frac{1}{j2\pi^2m} \int_{\phi}^{\pi+\phi} (e^{j(-b+c)} - e^{jc} + (-1)^n e^{j(-b+c)} - (-1)^n e^{jc}) dy \quad (3.88)$$

$$A_{mn} + jB_{mn} = \frac{1}{j2\pi^2m} \int_{\phi}^{\pi+\phi} (e^{j(-b+c)} - e^{jc})(1 + (-1)^n) dy \quad (3.89)$$

So for n even

$$A_{mn} + jB_{mn} = \frac{1}{j\pi^2m} \int_{\phi}^{\pi+\phi} (e^{j(-b+c)} - e^{jc}) dy \quad (3.90)$$

$$A_{mn} + jB_{mn} = \frac{1}{j\pi^2m} \int_{\phi}^{\pi+\phi} (e^{j(-2\pi r \sin(y)+ny)} - e^{jny}) dy \quad (3.91)$$

Also considering ϕ near of 0

$$\int_{\phi}^{\pi+\phi} e^{jny} dy = \frac{1}{n} (e^{jn\pi} - 1) = \frac{1}{n} ((-1)^n - 1) = 0 \quad (For n even)$$

Eventually, the expression of the DFS coefficient turns out to be

$$A_{mn} + jB_{mn} = \frac{1}{j\pi^2m} \int_{\phi}^{\pi+\phi} e^{j(-2\pi r \sin(y)+ny)} dy \quad (3.92)$$

$$A_{mn} + jB_{mn} = \frac{1}{j\pi^2m} \int_{\phi}^{\pi+\phi} \left(\cos(-m2\pi r \sin(y) + ny) + j \sin(-m2\pi r \sin(y) + ny) \right) dy \quad (3.93)$$

$$A_{mn} + jB_{mn} = \frac{1}{j\pi^2m} \left(\int_{\phi}^{\pi+\phi} \cos(-m2\pi r \sin(y) + ny) + \int_{\phi}^{\pi+\phi} j \sin(-m2\pi r \sin(y) + ny) dy \right) \quad (3.94)$$

Which can be rewritten as

$$A_{mn} = \frac{1}{\pi^2m} \int_{\phi}^{\pi+\phi} \sin(-m2\pi r \sin(y) + ny) dy \quad (3.95)$$

$$B_{mn} = \frac{-1}{\pi^2m} \int_{\phi}^{\pi+\phi} \cos(-m2\pi r \sin(y) + ny) dy \quad (3.96)$$

The switching function S_1 can be rewritten, based on the representation of the switching function [3] and on the previously determined coefficient, as

$$S_1(t) = \theta(t) + \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} \left(A_{mn} \cos((m\omega_c + n\omega_g)t) + B_{mn} \sin((m\omega_c + n\omega_g)t) \right) \quad (3.97)$$

Equ. 3.97 can be rewritten as

$$S_1(t) = \theta(t) + \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} \left(X_{mn} \sin((m\omega_c + n\omega_g)t + \phi_{mn}) \right) \quad (3.98)$$

With

$$X_{mn} = \sqrt{A_{mn}^2 + B_{mn}^2}$$

$$\phi_{mn} = \text{atg}\left(\frac{A_{mn}}{B_{mn}}\right)$$

The DFS method has been applied for the determination of the first switching function, i.e. $S_1(t)$. For the second one, $S_2(t)$, and as it is known that only the carrier is phase shifted of 180° between $S_1(t)$ and $S_2(t)$ a first assumption is to assume that only x has to be adapted such that

$$x = \omega_c t + \pi \quad (3.99)$$

Which leads to

$$S_2(t) = \theta(t) + \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} \left(X_{mn} \sin((m\omega_c + n\omega_g)t + m\pi + \phi_{mn}) \right) \quad (3.100)$$

Nevertheless, even though $S_1(t)$ and $S_2(t)$ are determined it is still needed to get the final expression of the input current in order to obtain the frequency spectrum in magnitude. This is carried out through the next classical circuit theory development.

$$S(t) = S_{average} + S_{ripple} \quad (3.101)$$

$$V_{tr1} = V_{rect} - L_{b1} \frac{di_1}{dt} ; V_{tr2} = V_{rect} - L_{b2} \frac{di_2}{dt} \quad (3.102)$$

$$V_{tr1} = (1 - S_1(t))V_{dc} ; V_{tr2} = (1 - S_2(t))V_{dc} \quad (3.103)$$

Applying Equ. 3.103 into Equ. 3.102 leads to

$$L_{b1} \frac{di_1}{dt} = V_{rect} - V_{tr1} = V_{rect} - (1 - S_1(t))V_{dc} = V_{rect} - (1 - (S_{average} + S_{ripple}))V_{dc} \quad (3.104)$$

With

$$i_1 = i_{1average} + i_{1ripple} ; i_2 = i_{2average} + i_{2ripple}$$

Wich allows to distinguish between average and ripple part of Equ. 3.104

$$L_{b1} \frac{di_1}{dt} = V_{rect} - (1 - S_{1average})V_{dc} \quad (3.105)$$

Noting that

$$i_{tot} = i_1 + i_2 \text{ and } i_{1average} + i_{2average} = \frac{V_{rect}}{R_{eq}}$$

And as it is symmetric

$$i_{1average} = i_{2average} = \frac{V_{rect}}{2R_{eq}} \text{ with } R_{eq} = \frac{V_{acRMS}^2}{P_{device}}$$

It leads to Equ. 3.105 being rewritten as

$$\frac{L_{b1}}{2R_{eq}} \frac{dV_{rect}}{dt} = V_{rect} - (1 - S_{1average})V_{dc} \quad (3.106)$$

$$S_{1average} = 1 + \frac{L_{b1}}{2R_{eq}V_{dc}} \frac{dV_{rect}}{dt} - \frac{V_{rect}}{V_{dc}} \quad (3.107)$$

$$S_{1average} = 1 + \frac{\sqrt{2}V_{grid}}{V_{dc}} \left(\frac{L_{b1}}{4R_{eq}} \frac{\omega_g \sin(2\omega_g t)}{|\sin(\omega_g t)|} - |\sin(\omega_g t)| \right) \quad (3.108)$$

With

$$\frac{dV_{rect}}{dt} = \frac{\sqrt{2}V_{grid}\omega_g \sin(2\omega_g t)}{2|\sin(\omega_g t)|}$$

As it is symmetric $S_{1average} = S_{2average}$, regarding the ripple part of the current it is possible to extract it from 3.104. Eventually, this ripple part of the current is

$$L_2 \frac{di_{2ripple}}{dt} = S_{2ripple}V_{dc} \quad (3.109)$$

$$S_2(t) = \theta(t) + \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} \left(X_{mn} \sin((m\omega_c + n\omega_g)t + m\pi + \phi_{mn}) \right) \quad (3.110)$$

$$S_{2ripple}(t) = \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} \left(X_{mn} \sin((m\omega_c + n\omega_g)t + m\pi + \phi_{mn}) \right) \quad (3.111)$$

$$S_{2ripple}(t) = \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} X_{mn} \sin(\alpha + m\pi) \quad (3.112)$$

With

$$\alpha = (m\omega_c + n\omega_g)t + \phi_{mn}$$

The final expression of the ripple on current due to the second branch is basically determined through the use of 3.112 in 3.109.

$$i_{2ripple} = \frac{V_{dc}}{L_2} \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} \frac{X_{mn}}{(m\omega_g + n\omega_c)} \sin(\alpha - \frac{\pi}{2} + m\pi) \quad (3.113)$$

Noting $\alpha - \frac{\pi}{2} = \alpha_2$ allows to express $\sin(\alpha - \frac{\pi}{2} + m\pi)$ as $\sin(\alpha_2 + m\pi)$ which for m even is merely $\sin(\alpha_2)$ and for m odd $-\sin(\alpha_2)$. This can be understood as $\sin(\alpha_2 + m\pi) = (-1)^m \sin(\alpha_2)$. Thus the analytical expression of the ripple in current present due to the second branch is

$$i_{2ripple} = \frac{V_{dc}}{L_2} \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} (-1)^m \frac{X_{mn}}{(m\omega_g + n\omega_c)} \sin(\alpha_2) \quad (3.114)$$

$$i_{tot} = i_1 + i_2 = i_{1average} + i_{2average} + i_{1ripple} + i_{2ripple} = \frac{V_{rect}}{R_{eq}} + i_{1ripple} + i_{2ripple} \quad (3.115)$$

With

$$i_{1ripple} = \frac{V_{dc}}{L_{b1}} \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} \frac{X_{mn}}{(m\omega_g + n\omega_c)} \sin(\alpha_2)$$

Ultimately, the final analytical expression of the total rectified current of an interleaved boost converter used as an active power factor corrector is

$$i_{tot} = \frac{V_{rect}}{R_{eq}} + \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ \text{meven}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} 2 \frac{X_{mn}}{(m\omega_g + n\omega_c)} \sin(\alpha_2) \quad (\text{For } L_{b1} = L_{b2}) \quad (3.116)$$

From Eq. 3.116, it can be observed that, compared with the classical boost converter, the IBC only contains even carrier harmonics, whose magnitude is twice as large. In the following, a more intuitive development, based on diagrams and classical circuit theory equations, is presented in order to provide a clearer understanding of these conclusions.

This development is based on diagrams representing the instantaneous value of the switching function and the corresponding current evolution. Along with these diagrams, the derivative of the current is analytically expressed and is, as expected, a function of the inductance values, as well as the grid and DC voltages. First, the boost converter is represented in Fig. 3.34 in order to establish a reference case for comparison with the IBC presented subsequently.

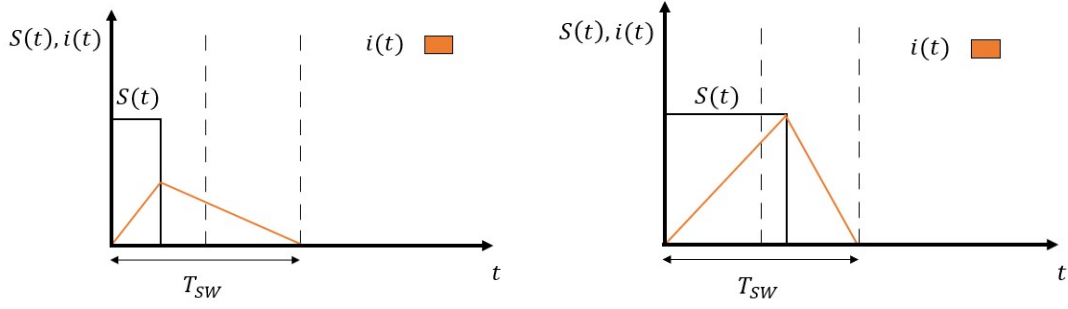


Figure 3.34: Boost : Representation of the qualitative evolution of the current as a function of time along with its related switching function instantaneous value

The behaviour is straightforward, and the current follows two linear slopes given by:

$$\begin{aligned} \frac{di(t)}{dt} &= \frac{V_{ac}(t)}{L_{b1}} & \text{for } 0 < t < T_{sw}D(t), \\ \frac{di(t)}{dt} &= \frac{V_{ac}(t) - V_{DC}}{L_{b1}} & \text{for } T_{sw}D(t) < t < T_{sw} \end{aligned}$$

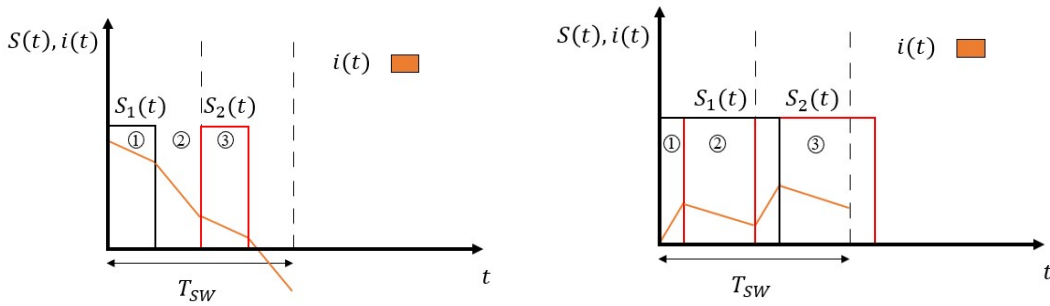


Figure 3.35: IBC: Qualitative representation of the current evolution as a function of time and its corresponding switching function (instantaneous value) for the case $2V_{ac,peak} < V_{DC}$

The IBC exhibits a slightly more complex behaviour. Indeed, for the total current, denoted as $i(t)$, to have a positive slope, it is not sufficient for only one switch to be closed; both switches must be closed simultaneously. Furthermore, if the duty cycle is lower than 50%, the current only exhibits negative slopes. Therefore, for the case where $2V_{ac,peak} < V_{DC}$, it can be deduced that the duty cycle must always satisfy $D > 0.5$.

The three possible switching states, along with their respective equations describing the slope of the total current for $D > 0.5$, are described below.

$$\begin{aligned}
\text{State 1} \quad \frac{di(t)}{dt} &= V_{ac}(t) \frac{(L_{b2} + L_{b1})}{L_{b1}L_{b2}} & \text{for } t_0 < t < T_{sw}(D - 0.5) \\
& & \text{and } \frac{T_{sw}}{2} < t < \frac{T_{sw}}{2} + T_{sw}(D - 0.5) \\
\text{State 2} \quad \frac{di(t)}{dt} &= V_{ac}(t) \frac{(L_{b2} + L_{b1})}{L_{b1}L_{b2}} - \frac{V_{DC}}{L_{b2}} & \text{for } T_{sw}(D - 0.5) < t < \frac{T_{sw}}{2} \\
\text{State 3} \quad \frac{di(t)}{dt} &= V_{ac}(t) \frac{(L_{b2} + L_{b1})}{L_{b1}L_{b2}} - \frac{V_{DC}}{L_{b1}} & \text{for } DT_{sw} < t < T_{sw}
\end{aligned}$$

If $L_{b1} = L_{b2} = L$, then

$$\begin{aligned}
\text{State 1} \quad \frac{di(t)}{dt} &= V_{ac}(t) \frac{2}{L} \\
\text{State 2} \quad \frac{di(t)}{dt} &= V_{ac}(t) \frac{2}{L} - \frac{V_{DC}}{L} \\
\text{State 3} \quad \frac{di(t)}{dt} &= V_{ac}(t) \frac{2}{L} - \frac{V_{DC}}{L}
\end{aligned}$$

It follows from these equations, together with the condition $2V_{ac\ peak} < V_{DC}$, that the slope of the total current during States 2 and 3 is always lower than $-V_{ac\ peak}(t) \frac{2}{L}$, which can be expressed as $i_{slope} < -V_{ac\ peak}(t) \frac{2}{L}$.

Regarding the two previous conclusions, the cancellation of half of the carrier harmonics can clearly be observed in Fig. 3.35 (right). Indeed, the period becomes half of the original one, and consequently the first remaining carrier harmonic appears at the frequency of the second carrier harmonic of the classical boost converter. Also, the current slope during State 1 is twice as large because two inductors are connected in parallel instead of a single inductor as in the classical boost converter. Consequently, for the same conduction interval, the classical boost converter exhibits only half of the current increase.

These two analytical conclusions are further investigated by means of numerical simulations. Indeed, these two conclusions can be verified. Figures 3.36 and 3.37 respectively present a comparison between the classical boost converter and the IBC, using the same control structure, over the complete magnitude spectrum of the input current and a zoom around the first carrier harmonic to verify the analytically predicted doubling of the magnitude.

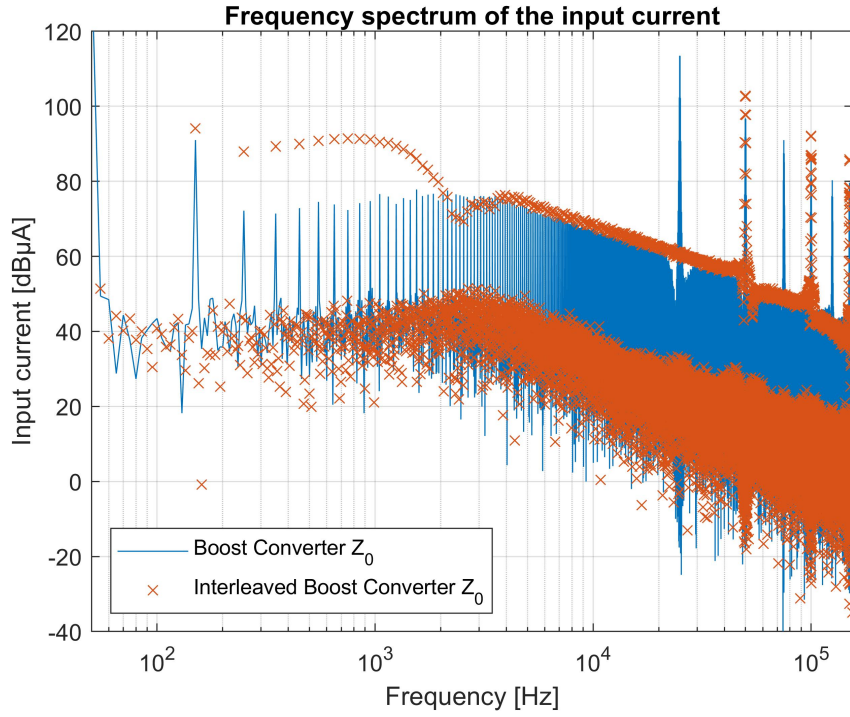


Figure 3.36: Comparison of the magnitude spectrum of the input current i_{in} between boost and IBC for $Z_{in} = 0$

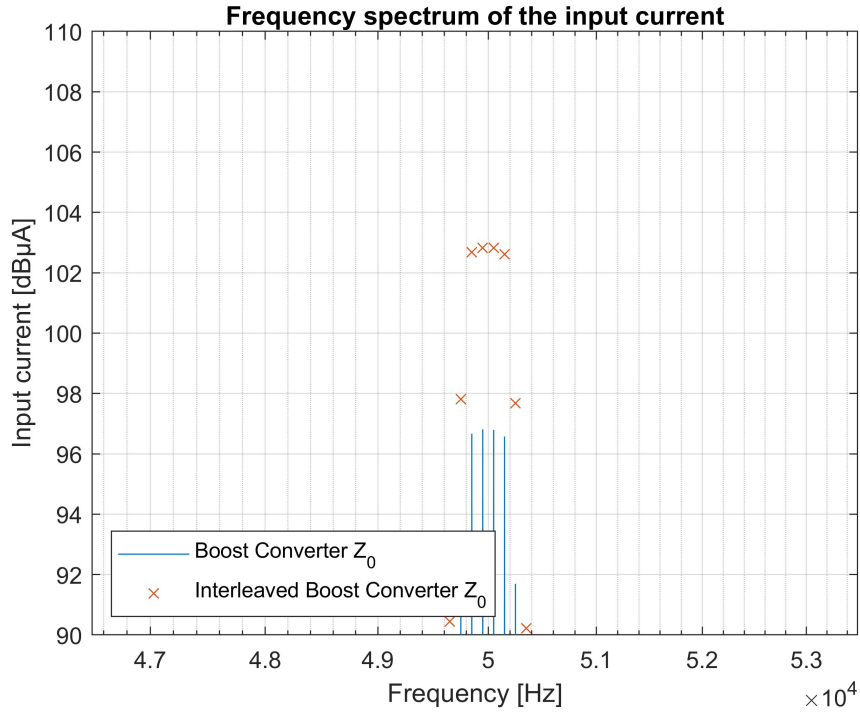


Figure 3.37: Comparison of the magnitude spectrum of the input current i_{in} between boost and IBC for $Z_{in}=0$, Zoom at 50 kHz

Since the two conclusions drawn from the analytical development have been verified, a comparison between the analytical predictions and the numerical simulations for the input

current components around the first carrier harmonic is presented in Fig. 3.38. Nevertheless, Fig. 3.38 only provides a zoomed view illustrating the close agreement between the analytical predictions and the numerical simulations. A more global comparison over the entire frequency range is therefore presented in Fig. 3.39.

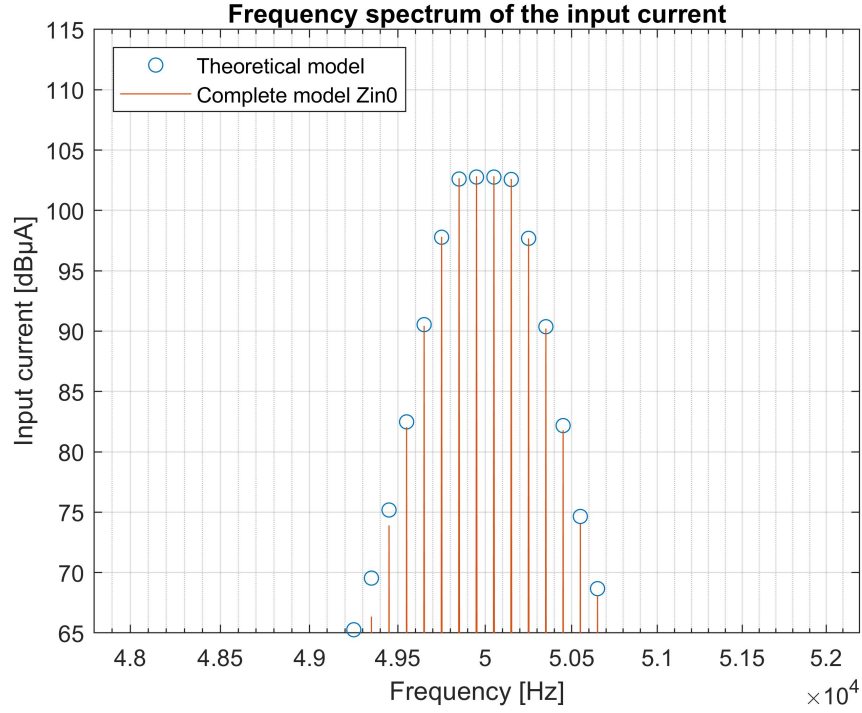


Figure 3.38: Comparison in the magnitude spectrum of the input current i_{in} for $Z_{in}=0$ between theoretical prediction and numerical results (Zoom)

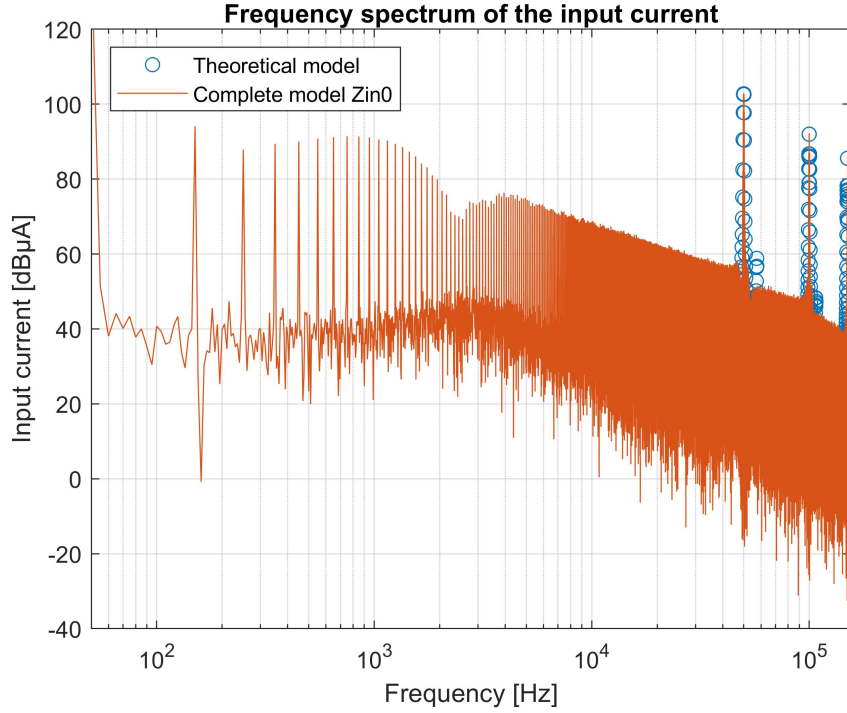


Figure 3.39: Comparison in the magnitude spectrum of the input current i_{in} for $Z_{in}=0$ between theoretical prediction and numerical results (0-150 kHz)

Although an excellent agreement between the analytical predictions and the numerical simulations has been demonstrated, it is important to recall that these results correspond to the case $Z_{in} = 0$. Therefore, the current analytical development is still unable to determine whether the converter behaves as a VHS or a CHS. An adaptation is therefore required and is presented in the following subsection.

3.3.7 Analytical Results : Second development

Since the previous analytical development cannot determine whether the IBC behaves as a VHS or a CHS, an adapted formulation is presented in this subsection. First, an assumption is introduced to enable this adaptation. The assumption considered here is that the input impedance Z_{in} can be transferred to the rectified side without requiring any additional modification of the analytical development.

$$L_{b1} \frac{di_1}{dt} = V_{rect} - V_{tr1} - L \frac{di_{recttot}}{dt} \quad (3.117)$$

$$L_{b2} \frac{di_1}{dt} = V_{rect} - V_{tr2} - L \frac{di_{recttot}}{dt} \quad (3.118)$$

$$L_{b1} \frac{di_1}{dt} = V_{rect} - V_{tr1} - L \frac{d(i_1 + i_2)}{dt} \quad (3.119)$$

$$L_{b2} \frac{di_1}{dt} = V_{rect} - V_{tr2} - L \frac{d(i_1 + i_2)}{dt} \quad (3.120)$$

$$(L_{b1} + L) \frac{di_1}{dt} = V_{rect} - V_{tr1} - L \frac{di_2}{dt} \quad (3.121)$$

$$(L_{b2} + L) \frac{di_2}{dt} = V_{rect} - V_{tr2} - L \frac{di_1}{dt} \quad (3.122)$$

By separating the equations into their average and ripple components, the average equations become

$$(L_{b1} + L) \frac{di_{1average}}{dt} = V_{rect} - (1 - S_{1average})V_{dc} - L \frac{di_{2average}}{dt} \quad (3.123)$$

$$(L_{b2} + L) \frac{di_{2average}}{dt} = V_{rect} - (1 - S_{2average})V_{dc} - L \frac{di_{1average}}{dt} \quad (3.124)$$

While the ripple equations become

$$(L_{b1} + L) \frac{di_{1ripple}}{dt} = (1 - S_{1ripple})V_{dc} - L \frac{di_{2ripple}}{dt} \quad (3.125)$$

$$(L_{b2} + L) \frac{di_{2ripple}}{dt} = (1 - S_{2ripple})V_{dc} - L \frac{di_{1ripple}}{dt} \quad (3.126)$$

Finally, by assuming that $L_{b1} = L_{b2}$, isolating $i_{1ripple}$ and $i_{2ripple}$, and substituting each expression into the other equation, the analytical expression given in Equ. 3.127 is obtained. This expression explicitly accounts for the presence of the input impedance.

$$i_{tot} = \frac{V_{rect}}{R_{eq}} + \frac{V_{dc}L_{b1}}{L_{b1}^2 + 2L_{b1}L} \sum_{\substack{m=2 \\ meven}}^{\infty} \sum_{\substack{n=-\infty \\ neven}}^{\infty} 2 \frac{X_{mn}}{(m\omega_g + n\omega_c)} \sin(\alpha_2) \quad (For \ L_{b1} = L_{b2}) \quad (3.127)$$

This new analytical expression leads to a magnitude spectrum that explicitly depends on the input impedance Z_{in} . The corresponding theoretical predictions can now be compared with the numerical simulations. First, the comparison focuses on the sidebands around the first carrier harmonic for the worst-case scenario, namely Z_{in25} , which corresponds to

an equivalent line length of 2.5 km, as illustrated in Fig. 3.40. Subsequently, the complete magnitude spectrum is presented in Fig. 3.41. Finally, Fig. 3.42 compares this result with the case corresponding to Z_{in0} , i.e. without any additional grid impedance.

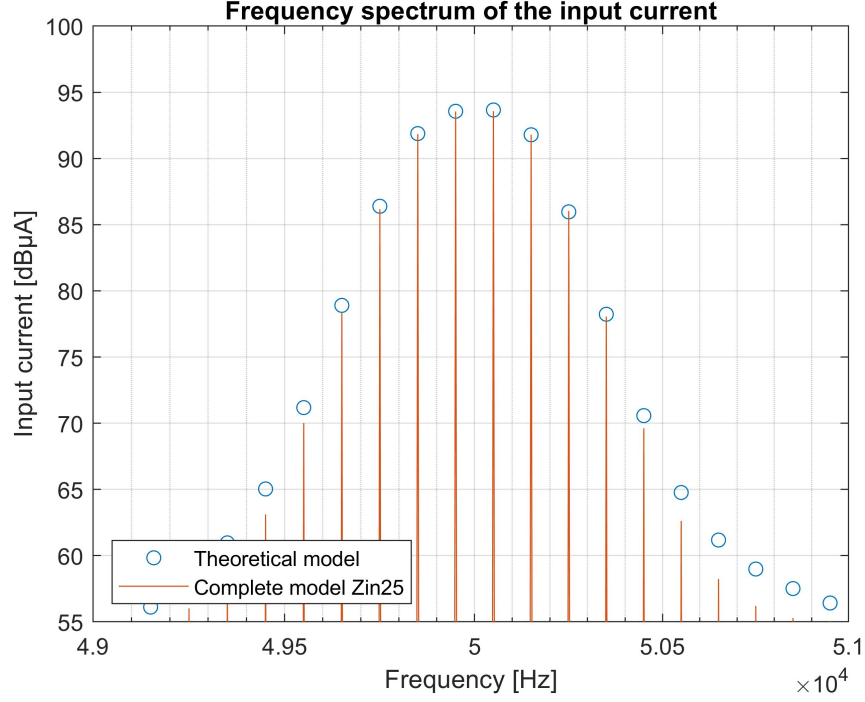


Figure 3.40: Comparison between the adapted theoretical results and numerical ones in magnitude spectrum of the input current, i.e. i_{in} , for Z_{in25} (Zoom)

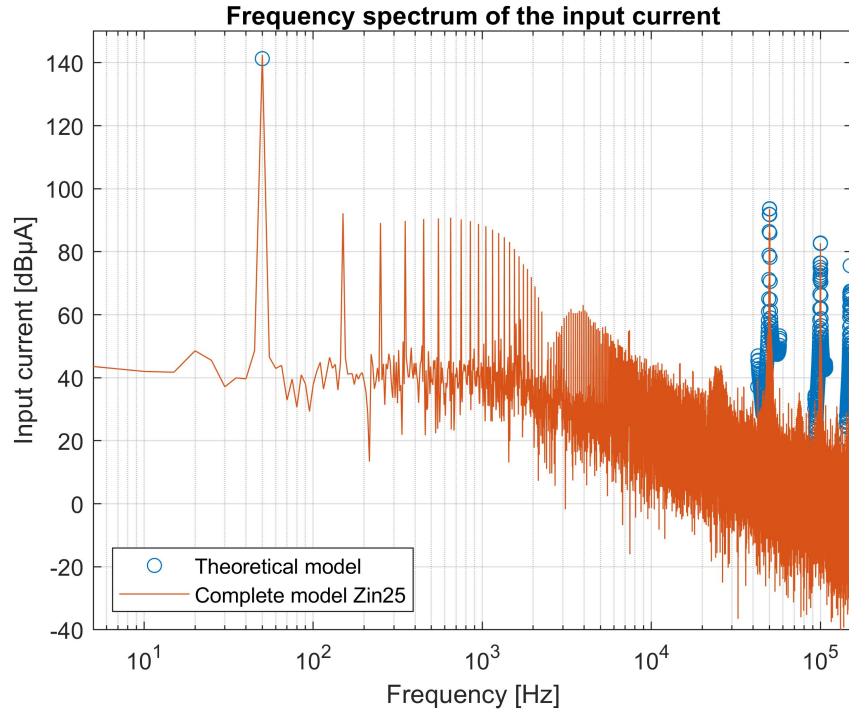


Figure 3.41: Comparison between the adapted theoretical results and numerical ones in magnitude spectrum of the input current, i.e. i_{in} , for Z_{in25}

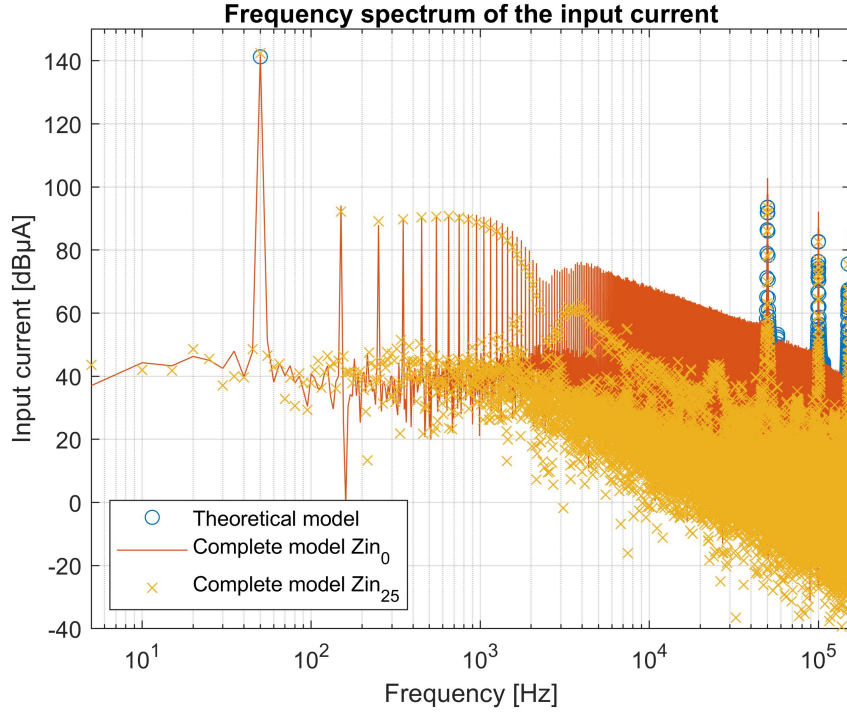


Figure 3.42: Comparison between the adapted theoretical results, in blue Z_{in25} , and numerical ones, in red $Z_{in25} = 0$ and yellow Z_{in25} , in magnitude spectrum of the input current, i.e. i_{in}

Fig. 3.40 shows that the corrections introduced by the second development are capable of following the variation of the input current magnitude spectrum with near-ideal accuracy.

Fig. 3.41 illustrates the validity of the previously developed analytical model over the entire frequency range of interest (0–150 kHz), provided that the limitations introduced in Section 3 are neglected in this first approximation.

To illustrate the differences arising as Z_{in} increases, Fig. 3.43 presents the quadratic sum of the first carrier harmonic sideband obtained from both the adapted analytical expression and the numerical simulation. Figure 3.44 then presents the corresponding relative error, taking the numerical simulation as the reference.

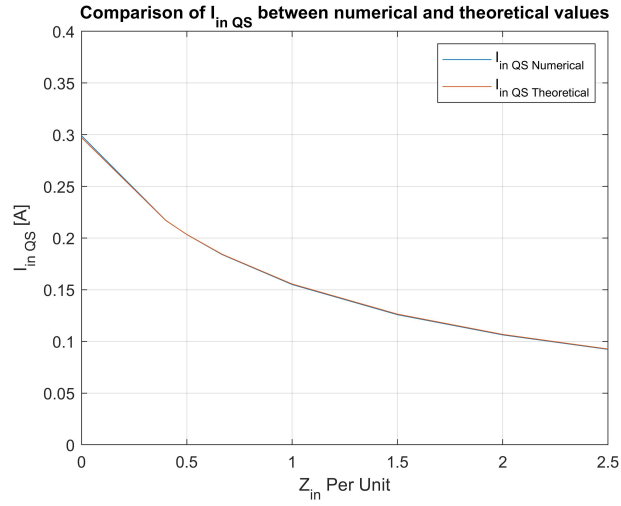


Figure 3.43: Absolute value in quadratic sum of the input current i_{in} of both numerical (blue) and analytical results (red)

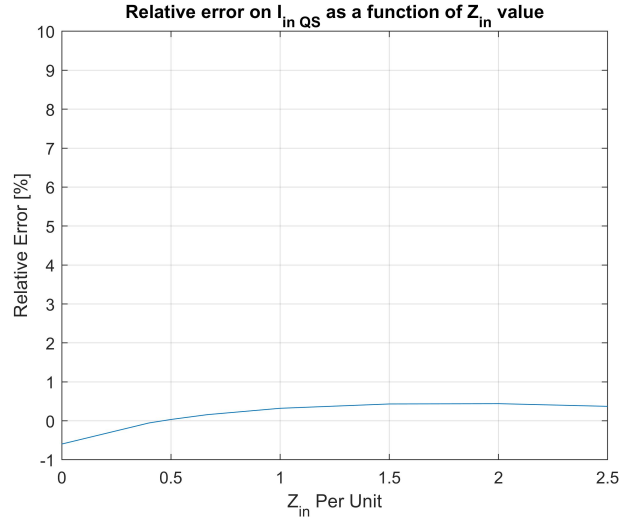


Figure 3.44: Relative error between the predicted quadratic sum value (analytical) and the numerical one as a function of the input impedance value in per unit (with respect to Z_{in0})

Based on Figs. 3.43 and 3.44, it is possible to further investigate the characterization of the studied APFC converter as a voltage harmonic source (VHS) or a current harmonic source (CHS). Indeed, for a purely VHS behaviour, increasing the input impedance Z_{in} from 1 to 2.5 p.u. would ideally reduce the current by a factor of 2.5. However, this is not observed, as the quadratic-sum current decreases from 0.15 A to approximately 0.08 A, whereas a value of 0.06 A would be expected for an ideal VHS.

Besides investigating whether the IBC used as an APFC behaves as a VHS or a CHS, this study also evaluates the ability of the adapted DFS development to reproduce the numerical results. This capability is first illustrated by the comparison of the two curves in Fig. 3.43 and is further quantified through the relative error shown in Fig. 3.44.

3.3.8 Results : experimental comparison

The DFS method has been shown to consistently predict the amplitudes of the frequency components associated with the switching process, namely the carrier harmonic and its associated sideband harmonics, in the input current of the studied interleaved boost converter. These predictions have been successfully validated against numerical simulations performed using Matlab Simulink. The objective of this part of the study is now to extend this comparison to experimental measurements. Indeed, the studied IBC topology and its associated control strategy have been successfully implemented in the laboratory, allowing measurements of the input current, among other quantities. It is worth noting that the comparison is first carried out for the input current around the first and second carrier harmonic bands before being extended to the complete frequency range, from 50 Hz to 150 kHz. Finally, the comparison is performed for a boost inductance of $L_{Boost} = 10^{-3}$ H and a switching frequency of $F_{sw} = 5 \times 10^3$ Hz using the two operating points listed in Table 3.4.

Table 3.4: Investigated operating points for the interleaved boost converter

Cases	$R[\Omega]$	$V_{DC}[V]$	$P_{device}[W]$
1	800	300	113
2	1600	600	225

3.3.8.1 Case 1

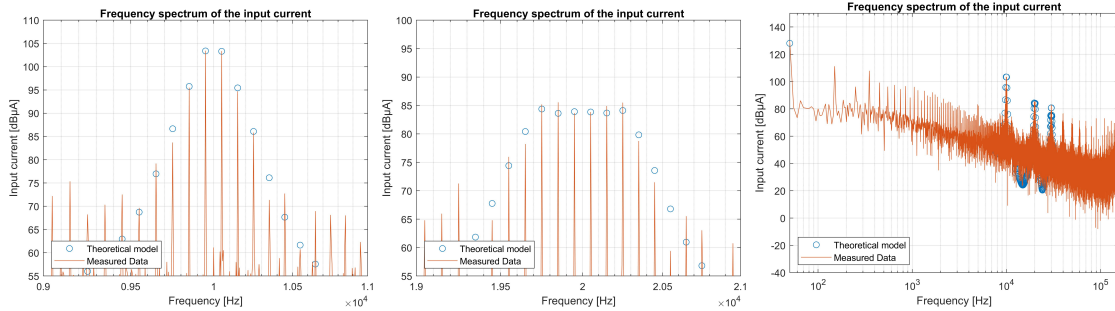


Figure 3.45: Comparison of the experimental measurement and the theoretical prediction, zoom on the first and second band and full range of the input current magnitude spectrum for an interleaved boost converter used as an active power factor corrector with $V_{DC} = 300[V]$ and $R = 800[\Omega]$

3.3.8.2 Case 2

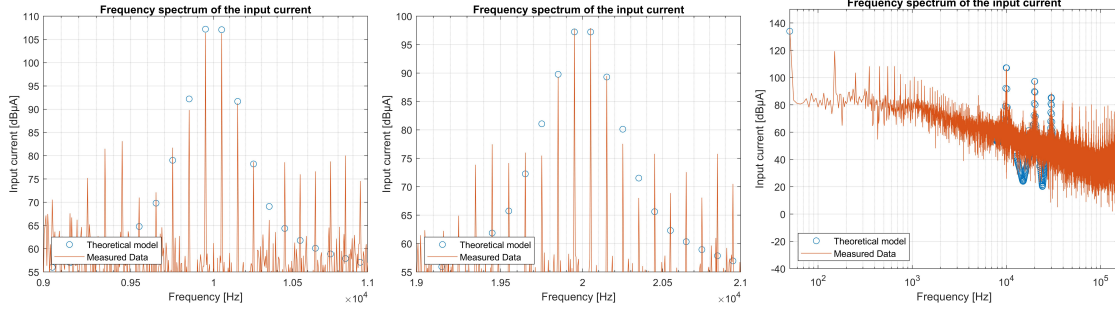


Figure 3.46: Comparison of the experimental measurement and the theoretical prediction, zoom on the first and second band and full range of the input current magnitude spectrum for an interleaved boost converter used as an active power factor corrector with $V_{DC} = 600[V]$ and $R = 1600[\Omega]$

3.4 Analysis of the Inverter

Another converter that is widely used in low-voltage (LV) grids is the inverter. This converter has been extensively described in Chapter 1. Given its widespread use, it is therefore highly relevant to develop an analytical model of this type of converter. This chapter is dedicated to the modelling of the supraharmonic (SH) emissions of the inverter. First, the classical inverter topology considered throughout this chapter is presented.

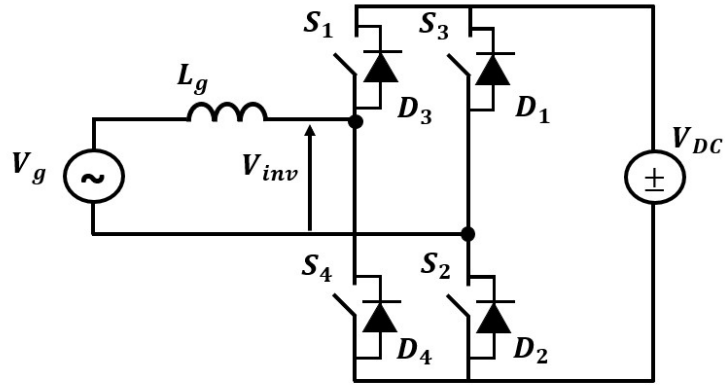


Figure 3.47: Electrical schematic of a classical one phase inverter with the grid impedance simplified as an inductance

Regarding the modelling methodology, the previously introduced analytically generated signals (AGS) method will be applied. The AGS predictions will then be compared with the corresponding numerical results. Afterwards, the operating principle of an inverter used for active power injection is analysed in order to derive several fundamental conclusions. These conclusions provide the basis for understanding the behaviour of inverter emissions under varying operating conditions.

3.4.1 Analytically generated signals for inverter modelling : Introduction and general case

As in the case of the classical boost converter, the AGS method requires the analytical expression of the modulating wave. Here, since the inverter is assumed to inject only active power, the voltage at its input can be considered as a sinusoidal signal of amplitude V_{DC} and phase $\phi_g - \frac{\pi}{2}$, where ϕ_g denotes the phase of the grid voltage.

The modulating wave is simply this sinusoidal signal and is expressed as $V_m = \frac{V_{grid\ peak}}{V_{DC}} \sin(\omega_g t + \phi_g - \frac{\pi}{2})$. This signal is generated using a conventional phase-locked loop (PLL), illustrated in Fig. 3.48. The PLL consists of three stages. The input signal V corresponds to the grid voltage. The first block generates a signal containing both low-frequency and high-frequency components. The low-frequency component contains the phase difference between V and $\tilde{V}$, whereas the high-frequency component contains the sum of their phases. The second block is a first-order low-pass filter characterized by the controller gains K_p and K_i . Finally, the last block generates the synchronized signal $\tilde{V}$.

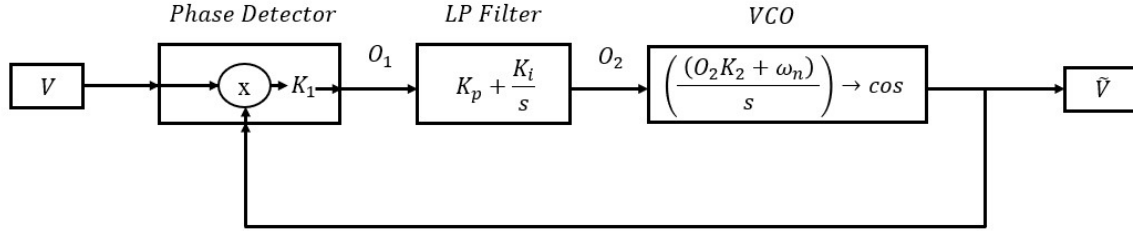


Figure 3.48: Phase-locked loop block diagram

Once the modulating wave is known, the duty cycle can be determined at every instant t . The same methodology as that used for the previously modelled APFC converter is then applied. First, the inverter is considered in islanded operation, i.e. disconnected from the grid. However, the switches are controlled as if the inverter were connected to the grid and operating to inject the maximum active power. The voltage spectrum obtained from numerical simulations can then be compared with the prediction provided by the AGS method. It is worth noting that the switching frequency, denoted F_{sw} , is kept constant at 10 kHz throughout this study and that V_{grid} always denotes the peak grid voltage. Two operating points are considered. In the first case, $V_{grid} = 100$ V and $V_{DC} = 120$ V, whereas in the second case $V_{grid} = 400$ V and $V_{DC} = 120$ V. The results for the first operating point are presented in Figs. 3.49 and 3.50.

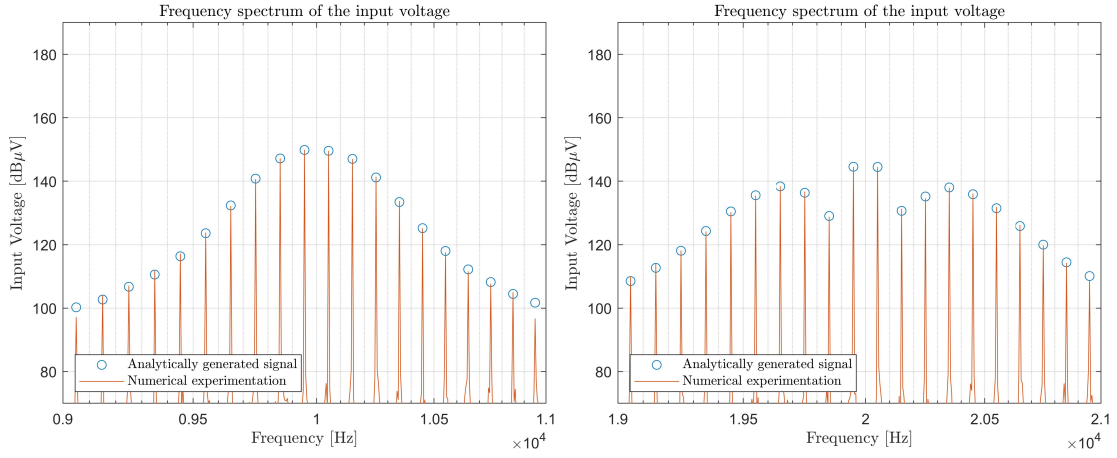


Figure 3.49: Comparison between the numerical experimentation and the analytically generated signal (AGS) method for $V_{grid} = 100$ V and $V_{DC} = 120$ V. Voltage spectrum at the input of a disconnected classical inverter. Zoom on the first (left) and second (right) carrier bands

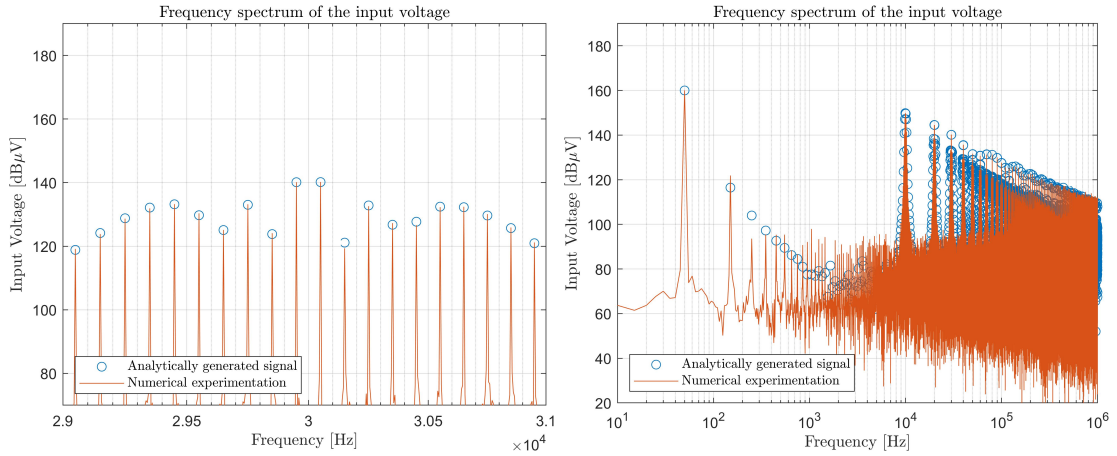


Figure 3.50: Comparison between the numerical experimentation and the analytically generated signal (AGS) method for $V_{grid} = 100$ V and $V_{DC} = 120$ V. Voltage spectrum at the input of a disconnected classical inverter. Zoom on the third (left) and all (right) carrier bands

The AGS method accurately predicts the expected voltage supraharmonics. However, it should be noted that, at this stage, the inverter is not delivering any power since it is disconnected from the grid.

In principle, neglecting the presence of the antiparallel diodes of the switches and/or considering a sufficiently low grid inductance, the obtained spectrum should not significantly change when the inverter is connected to the grid.

The results for the case where V_{grid} and V_{DC} are respectively equal to 400 V and 120 V are presented in Figs. 3.51 and 3.52.

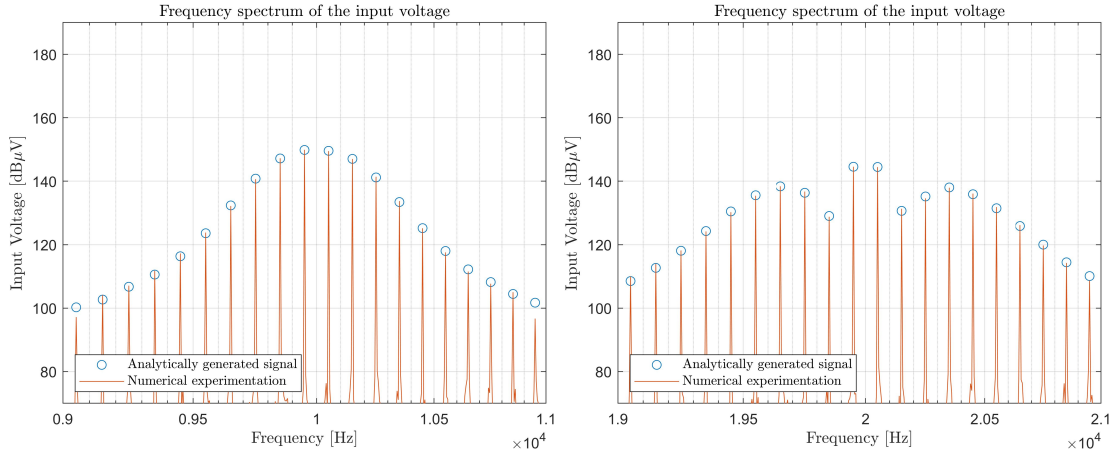


Figure 3.51: Comparison between the numerical experimentation and the analytically generated signal (AGS) method for $V_{grid} = 400$ V and $V_{DC} = 120$ V. Voltage spectrum at the input of a disconnected classical inverter. Zoom on the first (left) and second (right) carrier bands

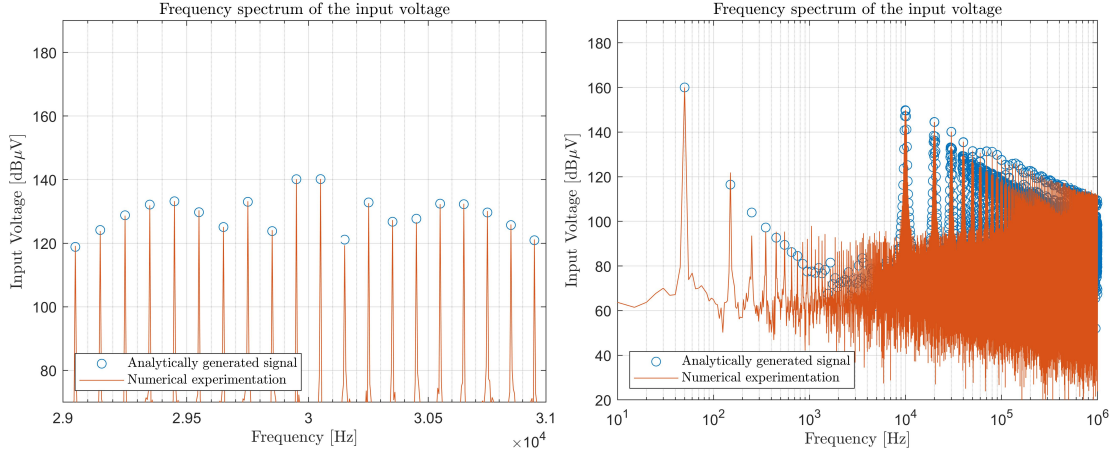


Figure 3.52: Comparison between the numerical experimentation and the analytically generated signal (AGS) method for $V_{grid} = 400$ V and $V_{DC} = 120$ V. Voltage spectrum at the input of a disconnected classical inverter. Zoom on the third (left) and all (right) carrier bands

These last results also demonstrate the ability of the method to handle significant variations in the critical parameters of inverters, namely the grid voltage and the DC voltage.

Nevertheless, the objective is to analyse the inverter operation when delivering power to the grid and to investigate whether the proposed method remains accurate for the prediction of supraharmonic emissions.

Therefore, the case where the inverter is directly connected to the grid is considered next, with only the line impedance being taken into account. The corresponding circuit is therefore described by Fig. 3.47.

As in the previously presented cases, the zooms on the first, second, and third carrier

bands are shown together with the complete spectrum ranging from 1 Hz to 1 MHz.

The considered parameters are $V_{DC} = 120$ V and $V_{grid} = 100$ V.

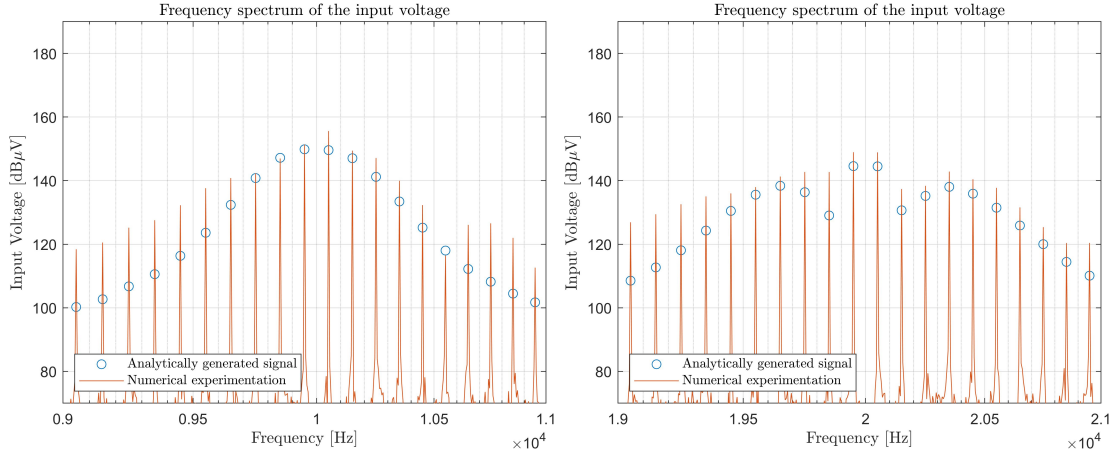


Figure 3.53: Comparison between the numerical experiment and the analytically generated signal (AGS) method for $V_{grid} = 100$ V and $V_{DC} = 120$ V. Voltage spectrum at the input of a grid-connected classical inverter through a simple line impedance model consisting of a series inductance and resistance. Zoom on the first (left) and second (right) carrier bands.

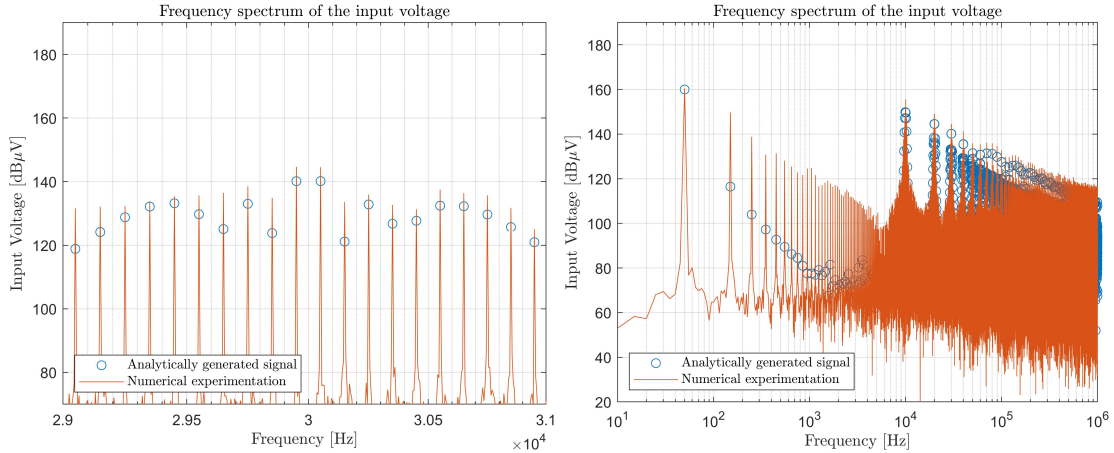


Figure 3.54: Comparison between the numerical experiment and the analytically generated signal (AGS) method for $V_{grid} = 100$ V and $V_{DC} = 120$ V. Voltage spectrum at the input of a grid-connected classical inverter through a simple line impedance model consisting of a series inductance and resistance. Zoom on the third (left) and all (right) carrier bands.

Figs. 3.53 and 3.54 show a significant reduction in the accuracy of the AGS method when the inverter is connected to a line impedance modelled as a 1 mH inductance in series with a 0.4 Ω resistance.

This behaviour is caused by a specific nonlinear process. Initially, it was assumed that the magnitude spectrum of the inverter voltage would remain unchanged when connected

to the grid if ideal switches were considered, i.e., switches without antiparallel diodes and with no switching delay. Under this assumption, the control strategy would be the only mechanism influencing the DC voltage transfer towards the grid.

However, in the real converter, antiparallel diodes are present, introducing an additional nonlinear behaviour and providing another path through which the DC voltage can be transferred to the grid.

When the switches receive an opening command while current is flowing, the inductance prevents an instantaneous change in current, meaning that the current cannot immediately transition from its previous value to zero. Consequently, the current is forced to flow through the antiparallel diode, which directly connects the DC voltage to the grid, either with the same or opposite polarity depending on the current direction at the switching instant.

This nonlinear behaviour results from the physical operating principle of the converter. Indeed, the initial AGS formulation for the inverter did not include this additional nonlinear mechanism, which significantly modifies the input voltage spectrum and introduces increasing errors for higher carrier harmonic bands.

For the first carrier band, the component at 10 050 Hz has an absolute magnitude almost twice as large as the theoretical prediction, although three out of the four neighbouring components are in close agreement with the theoretical results.

For higher carrier harmonic bands, the discrepancy becomes larger and affects a greater number of harmonics. Therefore, in these frequency ranges, the AGS prediction can only be considered as an approximation of the expected emissions.

Considering AGS as an approximation tool, one could introduce a correction factor determined from the grid characteristics, passive component values, switching frequency, and switch properties in order to reduce the discrepancy between AGS predictions and the actual physical supraharmonic emissions.

Although the physical origin of this discrepancy has been identified, namely the presence of antiparallel diodes and the grid inductance, the resulting consequences must also be analysed.

Two main effects must be considered. The inductance value is the main parameter influencing both effects, which are described below and illustrated directly in the time domain.

First, the current ripple is directly related to the inductance value. Second, the validity of the ideal switching function assumption is also affected by the inductance value.

The latter effect is related to the duration during which the current maintains the same direction. Indeed, when the inverter voltage reference, namely

$$V_m = \frac{V_{grid\ peak}}{V_{DC}} \sin(\omega_g t + \phi_g - \frac{\pi}{2}),$$

changes polarity, the current starts following this variation but with a time constant determined by the grid inductance and resistance.

The first effect increases when the inductance value decreases, whereas the second effect decreases, and vice versa.

This behaviour is illustrated in Fig. 3.55, where the nominal inductance value is compared with a case using an inductance ten times smaller.

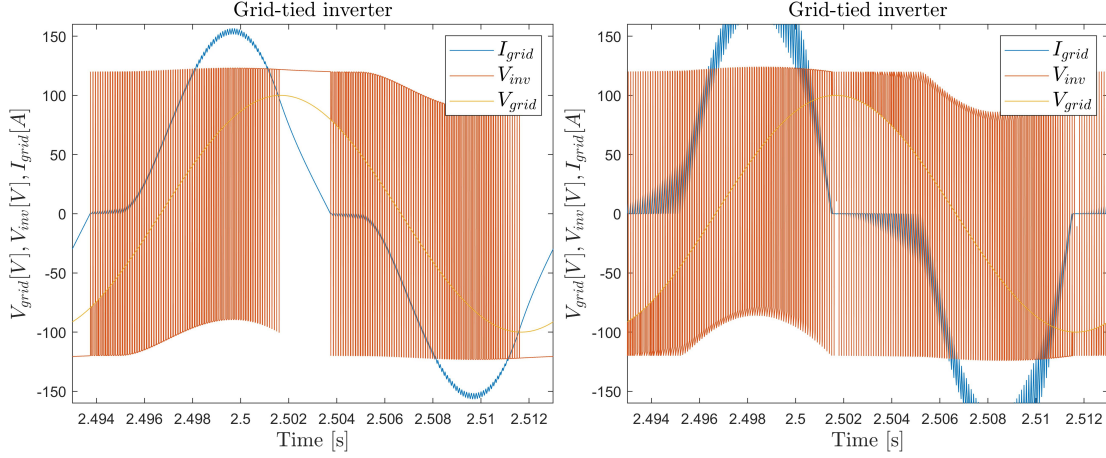


Figure 3.55: Identification of the error source between the numerical experiment and the analytically generated signal method for $V_{grid} = 100$ V and $V_{DC} = 120$ V. Two cases are considered: a grid inductance of 1 mH (left) and 0.1 mH (right). Increasing the inductance reduces the voltage drop at the inverter input but increases the duration during which the switching function assumption is no longer valid.

3.4.2 Analytically Generated Signals for Inverters: Adaptation

The main advantage of the AGS method is its ability to provide accurate predictions of supraharmmonic emissions within a reduced computational time. However, depending on the converter under study, adaptations may be required.

It has been demonstrated that including the grid impedance, here represented by 1 km of line impedance corresponding to 1.11 mH and 0.4Ω , as described in the APFC modelling section, introduces significant errors.

In this subsection, an adaptation of the AGS method is developed in order to account for the presence of grid inductance. Initially, only the inductive component is considered.

The required adaptation originates from the fact that the antiparallel diode connects the DC voltage to the grid when the switches are turned off. By considering the following four operating conditions:

- Inverter current > 0 and Ideal Inverter Voltage > 0
- Inverter current < 0 and Ideal Inverter Voltage > 0
- Inverter current > 0 and Ideal Inverter Voltage < 0
- Inverter current < 0 and Ideal Inverter Voltage < 0

Based on these operating conditions, the required AGS adaptations can be developed. The resulting predictions are then available in both the time and frequency domains.

The following sections provide a comparison between the inverter current and voltage obtained with the original and adapted AGS methods.

Frequency-Domain Comparison

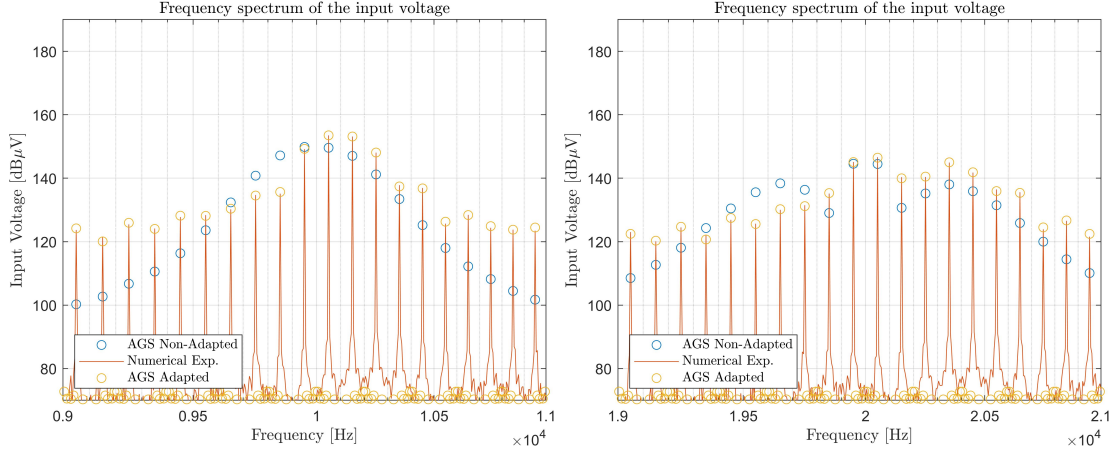


Figure 3.56: Comparison between the numerical experiment and the adapted version of the analytically generated signal (AGS) method for $V_{grid} = 100$ V and $V_{DC} = 120$ V, of the voltage spectrum at the input of a grid connected, through a simple line impedance model consisting of an inductance, classical inverter. Zoom on the first (left) and second (right) carrier bands

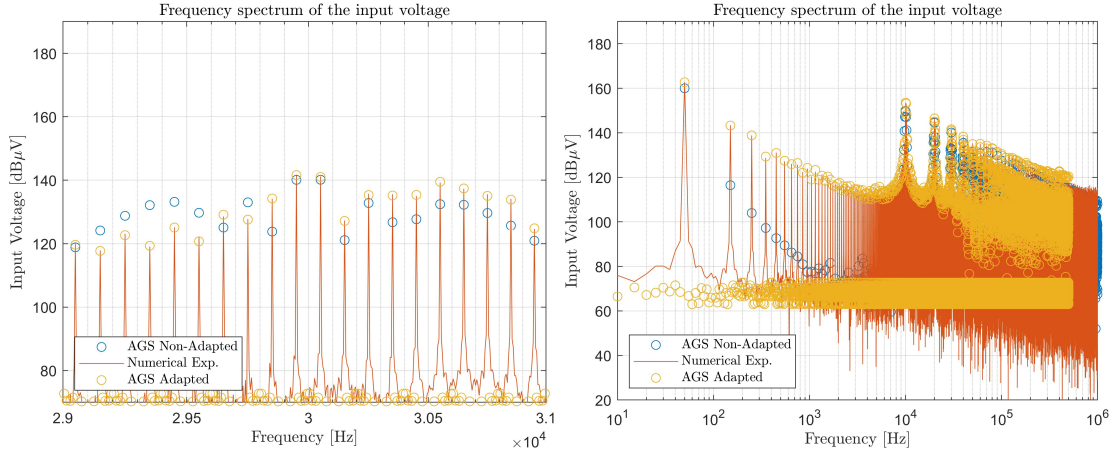


Figure 3.57: Comparison between the numerical experiment and the adapted version of the analytically generated signal (AGS) method for $V_{grid} = 100$ V and $V_{DC} = 120$ V, of the voltage spectrum at the input of a grid connected, through a simple line impedance model consisting of an inductance, classical inverter. Zoom on the third (left) and all (right) carrier bands

Time-Domain Comparison

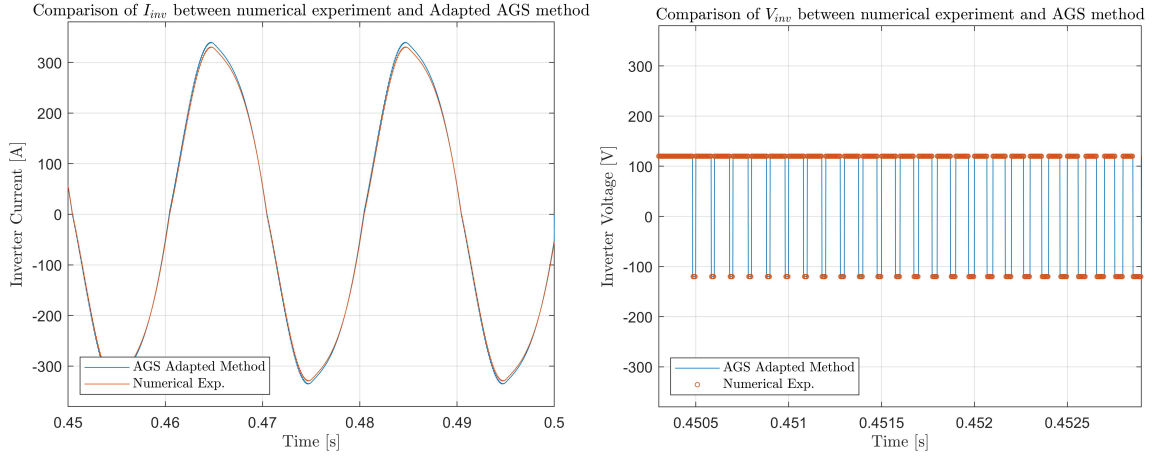


Figure 3.58: Comparison of the current and voltage for $V_{grid} = 100V$ and $V_{DC} = 120V$, between numerical experimentation and the adapted version of the analytically generated signal method (AGS) of a grid connected, through a simple line impedance model constituted of an inductance, classical inverter

The AGS method is thus able to accurately predict the emissions of a grid-connected inverter controlled by a PLL. However, the previous adaptation only considers the presence of an inductive grid impedance. Hereafter, the results are presented for a line impedance including both an inductance and a resistance, i.e. 1 mH and 0.4 Ω , respectively.

Frequency-Domain Comparison

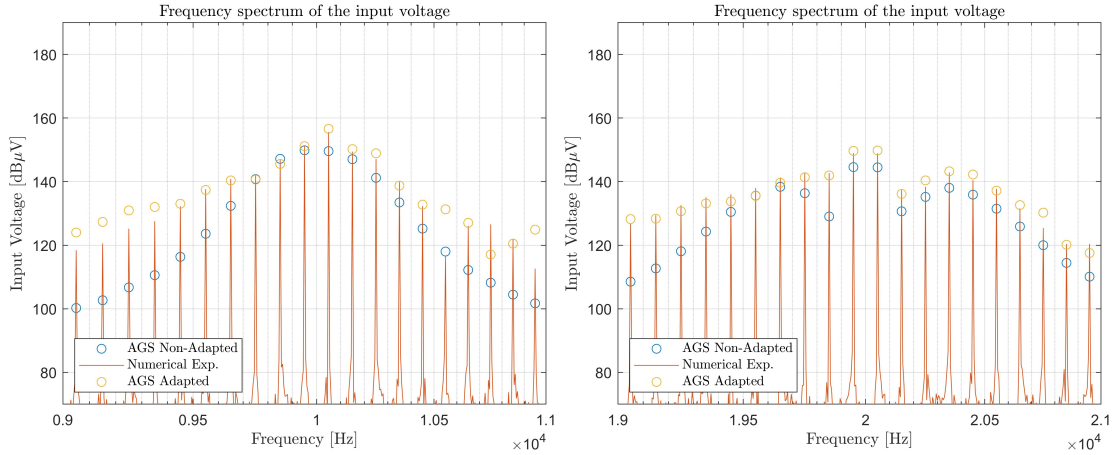


Figure 3.59: Comparison for $V_{grid} = 100V$ and $V_{DC} = 120V$, between numerical experimentation and the adapted version of the analytically generated signal method (AGS), of the voltage spectrum at the input of a grid connected, through a simple line impedance model constituted of an inductance in series with a resistance, classical inverter. Zoom on the first (left) and second (right) carrier band

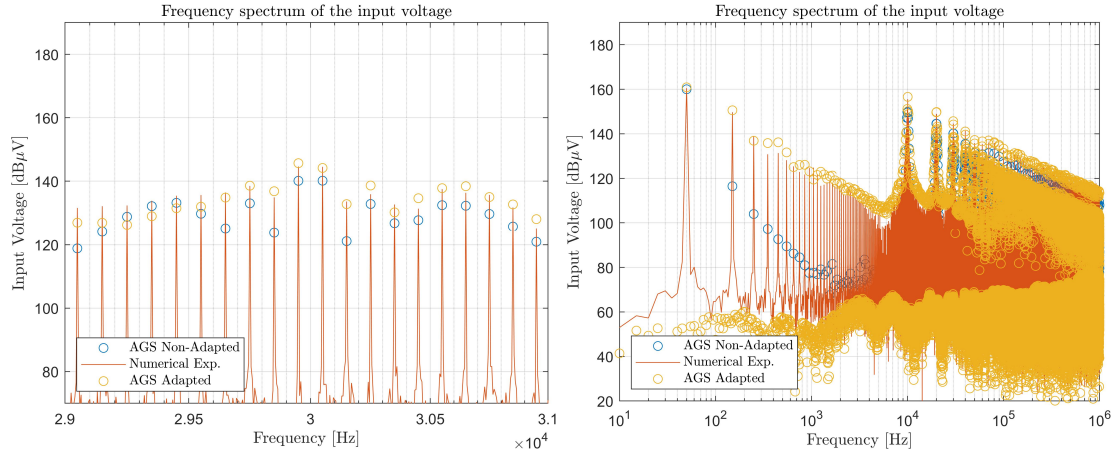


Figure 3.60: Comparison for $V_{grid} = 100V$ and $V_{DC} = 120V$, between numerical experimentation and the adapted version of the analytically generated signal method (AGS), of the voltage spectrum at the input of a grid connected, through a simple line impedance model constituted of an inductance in series with a resistance, classical inverter. Zoom on the third (left) and all (right) carrier bands

An interesting feature of the adapted AGS method is its capability to predict the transient response when the inverter is connected to the grid. However, it is worth noting that this transient response is based on an ideal PLL, i.e. assuming instantaneous phase detection and modulating-wave generation.

Another interesting aspect concerns the time–frequency characterization of the inverter SH emissions. Indeed, since some nonlinear and uncontrollable processes occur, such as diode switching, the magnitude spectrum may vary from one fundamental grid cycle to the next. Consequently, the key requirement for the applicability of the double Fourier series, namely signal periodicity, could be partially or even completely lost. Fig. 3.61 shows the time–frequency analysis of the inverter voltage over a duration of 8.2 s, with a frequency resolution of 50 Hz and a time resolution of 0.02 s.

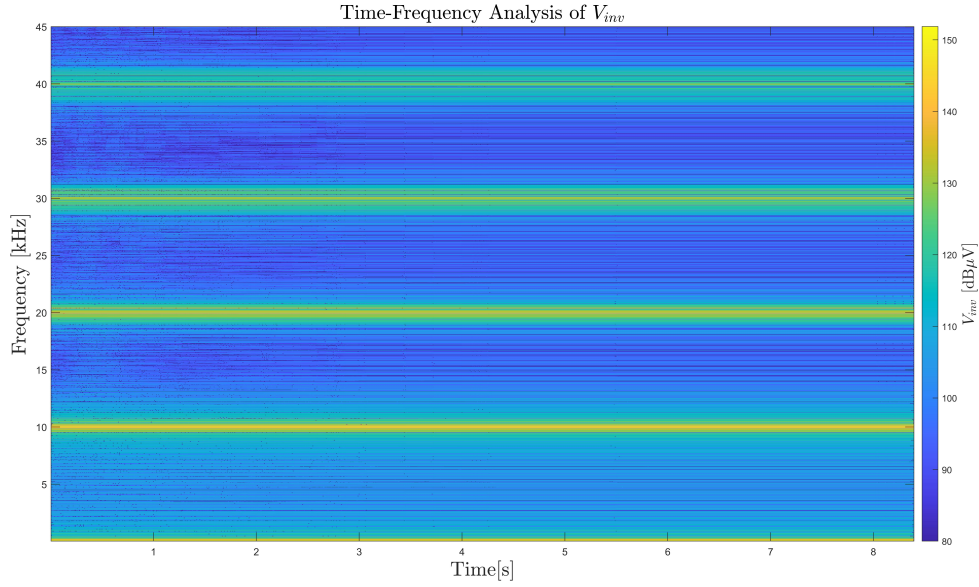


Figure 3.61: Time-Frequency analysis of a classical inverter voltage with connection to a grid with as only impedance an inductance of 1 [mH] in series with a resistance of 0.4 [Ω]

Basically, the emissions remain constant over time, meaning that the periodicity is not significantly affected. Consequently, the emissions predicted by the adapted AGS method are accurate, at least over the analysed time interval. Since steady state is reached, it is even reasonable to suggest that the prediction remains valid for any duration, provided that the operating conditions remain unchanged.

Therefore, the condition required for the applicability of the double Fourier series is satisfied. However, since the double Fourier series considers the ideal behaviour of the controlled converter, it would not be possible, using this analytical tool alone, to obtain results that match the numerical simulations as accurately as those provided by the adapted AGS method. The AGS method is not only easier to understand but also essential when studying converters involving nonlinear and uncontrollable processes.

The idea of extending the dimensionality of the double Fourier series originates from considering the nonlinear and uncontrollable phenomena associated with antiparallel diodes. Indeed, it is possible to conceive of these diodes as contributing additional signals that interfere with the two original ones, thereby modifying the rectangular switching signal to varying degrees. The result presented in Fig. 3.61 thus acquires an additional layer of significance, in the sense that the periodicity of the supraharmonic emissions remains unaffected. Consequently, it becomes plausible to consider that the sum of the signals representing the influence of the antiparallel diodes on the switches also possesses a periodic nature, and can therefore be incorporated within the framework of an extended double Fourier series.

A deeper exploration of this concept is presented in Appendix B.

3.4.3 Double Fourier Series : Application to the case of a classical inverter

This subsection provides a more detailed and well-founded discussion on the inherent limitation of the double Fourier series regarding the modelling of SH emitted by a certain kind of converters. Indeed, it has been previously stated that when non-linear and non-controllable processes take place then the DFS starts to provide predictions which do not fit anymore with the results that could be obtained through numerical experimentation. The explanation has been provided and is merely summarized as being a logical consequence of a method which does not consider that something else than the PWM signals could affect the way the DC voltage is presented to the grid. The numerical simulations account for these phenomena.

Hereafter, the development needed for being able to predict the emission of a classical inverter is provided along with the needed adaptation linked to the way the PWM is managed.

3.4.3.1 The PWM signals generation

There is no unique way to implement PWM., the idea is just to compare two signals in order to provide switching signal(s). In the case of a classical inverter which aims to inject active power into the grid, the main point for the inverter voltage is to be 90° phase shifted with respect to the grid voltage. This phase-shift being directly related to the assumption of an inductive grid. It is, also, possible to have a modulation of the desired peak voltage at the input of the inverter. This last point leads to the definition of a variable noted 'Q' which can be defined as $Q = \frac{V_{grid\ peak}}{V_{DC}}$. Indeed, the DC voltage can, theoretically, be any value.

Hereafter the classical way of performing PWM is illustrated in time domain along with the way it would be represented in the domain of the double Fourier series. The figure is then followed by the equation providing the Fourier components of the switching function that this peculiar PWM would provide.

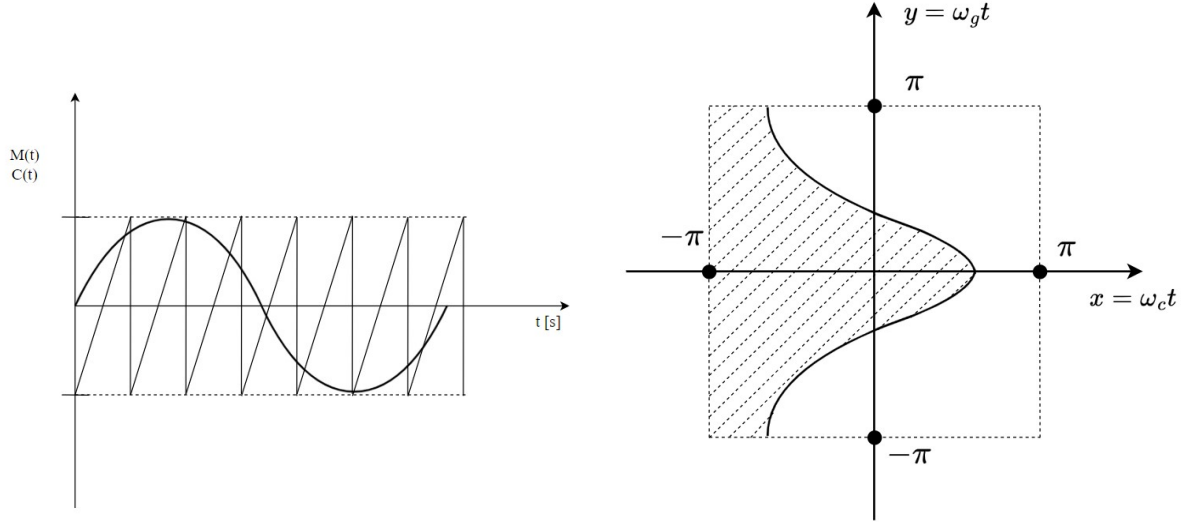


Figure 3.62: Classical PWM for sinusoidal modulating wave and sawtooth carrier both in time domain and in the double Fourier series domain

$$A + jB = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi Q \cos(y)} e^{j(mx+ny)} dx dy \quad (3.128)$$

Regarding the way the PWM has been managed in this work one would refer to Fig. 3.63. It is worth noting that the double Fourier series domain is here represented for only the first switching function whereas there are two switching functions, $S_1(t)$ and $S_2(t)$. The corresponding representation for the second switching function is provided below, and the equation describing the Fourier components is coming from the sum of three integrals.

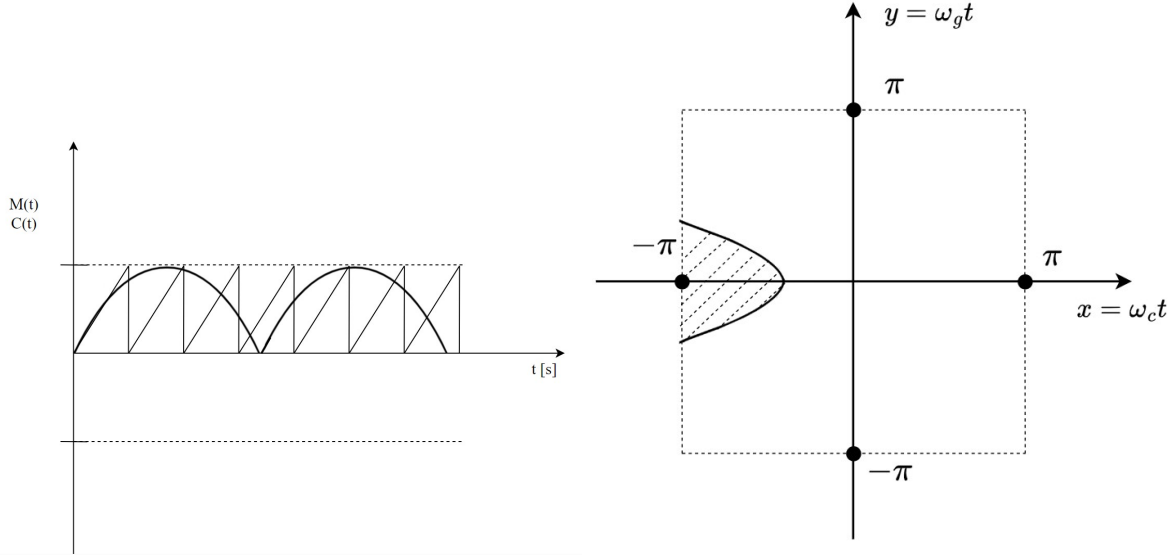


Figure 3.63: Peculiar PWM with sawtooth carrier for providing in average a sinusoidal inverter voltage, an illustration both in time domain (Left) and in the double Fourier series domain (Right) are provided, however the double Fourier series domain is only represented for one of the two needed switching function, i.e. $S_1(t)$

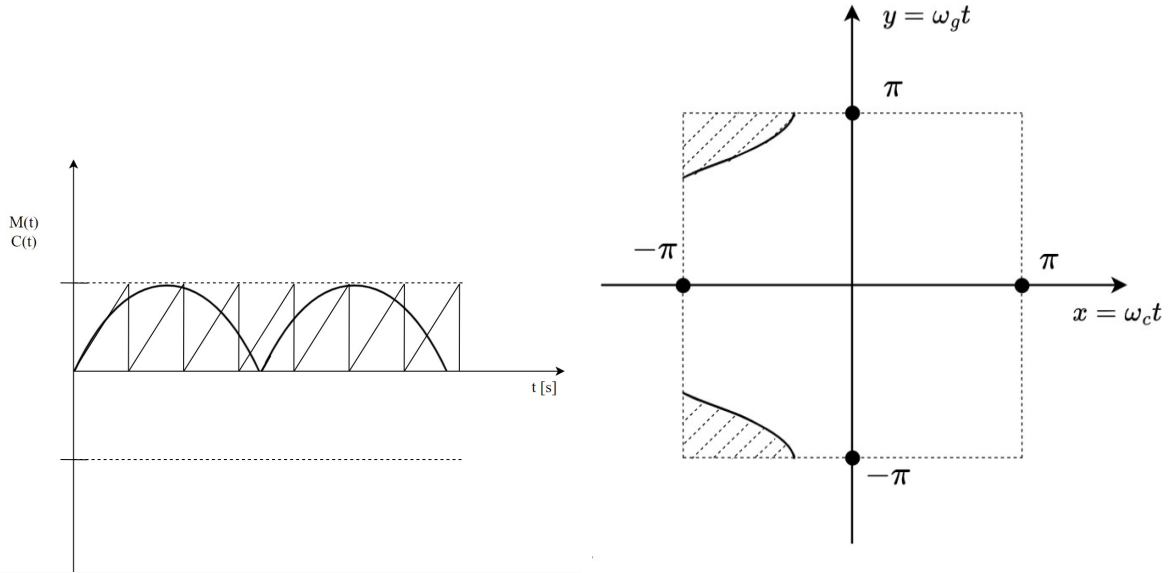


Figure 3.64: Peculiar PWM with sawtooth carrier for providing in average a sinusoidal inverter voltage, an illustration both in time domain (Left) and in the double Fourier series domain (Right) are provided, however the double Fourier series domain is only represented for one of the two needed switching function, i.e. $S_2(t)$

In fact, the functions represented on the left of Fig. 3.63 and 3.64 do represent the modulating wave as two sine functions added with one phase-shifted by π radians. The negative part has been intentionally omitted from the representation. However, since the carrier only exists on the positive part of the y axis this is not impacting. The same

carrier is, thus, used for both $S_1(t)$ and $S_2(t)$ and only the modulating wave is phase-shifted. However as the modulating wave amplitude is now equal to the height of the sawtooth carrier the bound of the integral is not anymore $\pi Q \cos(y)$ but $2\pi Q \cos(y)$ and as the sawtooth carrier has been shifted from the previous classical starting point at -1 to 0 this also shifts the minimum x upper bound from 0 to $-\pi$ and thus finally leads to the upper bound being $2\pi Q \cos(y) - \pi$.

The three previously mentioned integrals arise from the fact that three dashed surfaces have to be considered. One in Fig. 3.63 right and two in Fig. 3.64 right. This leads to the following equations.

For $S_1(t)$

$$A_1 + jB_1 = \frac{1}{2\pi^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\pi}^{2\pi Q \cos(y) - \pi} e^{j(mx+ny)} dx dy \quad (3.129)$$

For $S_2(t)$

$$A_{21} + jB_{21} = \frac{1}{2\pi^2} \int_{-\pi}^{-\frac{\pi}{2}} \int_{-\pi}^{2\pi Q \cos(y-\pi) - \pi} e^{j(mx+ny)} dx dy \quad (3.130)$$

$$A_{22} + jB_{22} = \frac{1}{2\pi^2} \int_{\frac{\pi}{2}}^{\pi} \int_{-\pi}^{2\pi Q \cos(y-\pi) - \pi} e^{j(mx+ny)} dx dy \quad (3.131)$$

3.4.3.2 Inverter voltage expression using the DFS

Based on the Fourier components previously derived using the DFS, and adapted to the used PWM, one can derive the complete expression of the inverter voltage.

$$\begin{aligned} V_{inv,rip}(t) &= V_{DC}S_1(t) + (-V_{DC})S_2(t) \\ &= V_{DC}(S_1(t) - S_2(t)) \end{aligned} \quad (3.132)$$

$$S_1(t) = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} X_{mn1} \sin((m\omega_{sw} + n\omega_g)t + \psi_{mn1}) \quad (3.133)$$

$$\begin{aligned} S_2(t) &= \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} X_{mn21} \sin((m\omega_{sw} + n\omega_g)t + \psi_{mn21}) \\ &\quad + \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} X_{mn22} \sin((m\omega_{sw} + n\omega_g)t + \psi_{mn22}) \end{aligned} \quad (3.134)$$

3.4.4 Double Fourier Series : Analytical determination using Bessel functions

By using simple identities involving Bessel functions, it is possible to finally obtain the complete equation of the input voltage of a classical one phase inverter. Regarding the

previously determined integrals expressing the double Fourier series applied to the determination of $A_1 + jB_1$ and $A_{21} + jB_{21} + A_{22} + jB_{22}$ the complete analytical development leading to the expression of these elements has been performed and is presented in Appendix E. However, the aim here is only to provide the result of this derivation.

It is worth noting that these elements only depend on the indices m and n which respectively represent the order of the carrier harmonic and the sideband harmonic under consideration. These coefficients are therefore denoted. Eventually, the expression of these coefficients are provided hereafter $S_1(t) : A_1(m, n) + jB_1(m, n)$ and for $S_2(t) : A_{21}(m, n) + jB_{21}(m, n) + A_{22}(m, n) + jB_{22}(m, n)$.

Finally, the expressions of these coefficients are provided below.

$$\begin{aligned}
& A_{21}(m, n) + jB_{21}(m, n) + A_{22}(m, n) + jB_{22}(m, n) \\
&= \frac{(-1)^m}{2\pi^2 jmn} J_0(-m2\pi Q) \sin(-n\frac{\pi}{2}) + \frac{(-1)^m}{2\pi^2 jm} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\sin(-\frac{\pi}{2}(n+k))}{n+k} \right. \\
&\quad \left. - \frac{\sin(-\pi(n+k))}{n+k} + \frac{\sin(-\frac{\pi}{2}(n-k))}{n-k} - \frac{\sin(-\pi(n-k))}{n-k} \right) + \frac{(-1)^m}{2\pi^2 jm} \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) \\
&\quad J_k(-m2\pi Q) \left(\frac{-\cos(-\frac{\pi}{2}(n+k))}{n+k} + \frac{\cos(-\pi(n+k))}{n+k} - \frac{\cos(-\frac{\pi}{2}(n-k))}{n-k} + \frac{\cos(-\pi(n-k))}{n-k} \right) \\
&\quad + \frac{(-1)^m}{2\pi^2 mn} J_0(-m2\pi Q) (-\cos(-n\frac{\pi}{2}) + \cos(-n\pi)) + \frac{(-1)^m}{2\pi^2 m} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \\
&\quad \left(-\frac{\cos(-\frac{\pi}{2}(n+k))}{n+k} + \frac{\cos(-\pi(n+k))}{n+k} - \frac{\cos(-\frac{\pi}{2}(n-k))}{n-k} + \frac{\cos(-\pi(n-k))}{n-k} \right) + \frac{(-1)^m}{2\pi^2 m} \\
&\quad \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\sin(-\frac{\pi}{2}(n+k))}{n+k} - \frac{\sin(-\pi(n+k))}{n+k} + \frac{\sin(-\frac{\pi}{2}(n-k))}{n-k} - \right. \\
&\quad \left. \frac{\sin(-\pi(n-k))}{n-k} \right) + \frac{(-1)^m}{2\pi^2 jmn} J_0(-m2\pi Q) (-\sin(n\frac{\pi}{2})) + \frac{(-1)^m}{2\pi^2 jm} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \\
&\quad \left(\frac{\sin(\pi(n+k))}{n+k} - \frac{\sin(\frac{\pi}{2}(n+k))}{n+k} + \frac{\sin(\pi(n-k))}{n-k} - \frac{\sin(\frac{\pi}{2}(n-k))}{n-k} \right) + \frac{(-1)^m}{2\pi^2 jm} \sum_{k=1 \text{ odd}}^{\infty} \\
&\quad \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\cos(\frac{\pi}{2}(n+k))}{n+k} - \frac{\cos(\pi(n+k))}{n+k} + \frac{\cos(\frac{\pi}{2}(n-k))}{n-k} - \frac{\cos(\pi(n-k))}{n-k} \right) \\
&\quad + \frac{(-1)^m}{2\pi^2 mn} J_0(-m2\pi Q) (-\cos(n\pi) + \cos(n\frac{\pi}{2})) + \frac{(-1)^m}{2\pi^2 m} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \\
&\quad \left(-\frac{\cos(\pi(n+k))}{n+k} + \frac{\cos(\frac{\pi}{2}(n+k))}{n+k} - \frac{\cos(\pi(n-k))}{n-k} + \frac{\cos(\frac{\pi}{2}(n-k))}{n-k} \right) + \frac{(-1)^m}{2\pi^2 m} \sum_{k=1 \text{ odd}}^{\infty} \\
&\quad \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\sin(\pi(n+k))}{n+k} - \frac{\sin(\frac{\pi}{2}(n+k))}{n+k} + \frac{\sin(\pi(n-k))}{n-k} - \frac{\sin(\frac{\pi}{2}(n-k))}{n-k} \right) \\
&\quad + \frac{(-1)^m}{2\pi^2 jmn} \left(\sin(n\frac{-\pi}{2}) - j\cos(n\frac{-\pi}{2}) + j\cos(-n\pi) - \sin(n\frac{\pi}{2}) + j\cos(n\frac{\pi}{2}) - j\cos(n\pi) \right)
\end{aligned}$$

3.5 Conclusion

This chapter comprehensively investigated the capability of the considered method, the Double Fourier Series (DFS), to address the determination of supraharmonic (SH) emissions through converter modeling. Initially, the context of validity was presented, providing the foundation for further investigation into the modeling of power electronic devices. Subsequently, the first studied converter was introduced, and the complete mathematical development associated with the use of DFS was elaborated.

The boost converter has been the subject of an extensive study concerning emissions in the supraharmonic (SH) range. Additionally, the influencing factors have been investigated. Among others, the presence of the EMI filter was studied in its impact on the emissions seen on both sides of the EMI filter which were found to depend on the relative value of the EMI inductance with respect to the grid and boost inductance values. However, as the boost inductance is in the studied case much greater than the EMI and the grid line inductance the SH voltage components were significantly reduced at the input of the converter. Also, the parasitic capacitance between lines of the one phase APFC is not expected to produce significant variations of the emissions in the studied range which is of maximum 1500 [pF]. The accuracy and capability of the method to align with numerical as well as physical experiments have been demonstrated. Subsequently, a comprehensive analysis using Bessel functions was developed to address the issue of the relationship between DC voltage and SH emissions, given that no correlation was found between the power of the device and SH emissions.

Regarding the Interleaved Boost Converter (IBC), simulations and theoretical analyses have been conducted. While each method individually offers insights into characterizing the behavior as VHS or CHS, a comparison between the two provides greater confidence in the conclusions drawn. An analysis of the obtained results can be performed by comparing the outcomes from numerical simulations with those derived from theoretical developments.

The results obtained for the IBC, both theoretical and numerical results indicate a varying input current with grid impedance, i.e., Z_{in} . This behaviour had already been predicted prior to obtaining these results and was associated with the behavior of a constant voltage source. However, this behaviour is not as pronounced as in the case of control structure 2.

Regarding the simulation results, they support the theoretical findings and reinforce the notion that the observed behaviour at the carrier harmonic frequency could be due to the controller parameters, which influence the bandwidth of the inner and outer loops. Indeed, it has been shown that for the control, named 2 in this chapter, a near-ideal VHS behaviour has been observed. The only significant difference with the control named 1 is the controller parameter.

The controller parameters are important and have been studied for their impact on the magnitude spectrum for the same control structure, i.e., control structure 1. It has been shown that for a well-designed control, the analytical development presented in this work is able to predict with high accuracy the numerical as well as real emissions of an APFC. An interesting conclusion is that for an inner controller gain that is less than the well-

designed value, the magnitude spectrum is not significantly affected—this is obviously the case for a reasonable reduction of that gain—while for a higher inner-loop controller gain, the emissions start to decrease as the control bandwidth increases with the inner gain and begins to interpret the SH emissions as part of the control error to cancel out.

It is worth noting that some works have been carried out on the determination of the kind of behaviour, VHS or CHS, of power electronic devices under specific operating conditions [16][13]. It is understood that mainly the ratio between the grid impedance and the device self impedance influence this behaviour. Basically, having a high converter impedance with respect to the grid impedance leads to having a CHS behaviour, which has been observed to be the case in [13] for seven electric vehicle chargers on eight which covered in 2020 92 percent of the Netherlands market, while in the opposite case the VHS is prevalent.

Subsequently, the capability of the developed analytical model to predict the magnitude spectrum of an APFC converter in a simplified grid, here the interleaved boost converter, has been demonstrated. However, even though the numerical and theoretical results are nearly identical, with less than 1 percent error, this is under the assumption that the limitations presented in the first part of this chapter can be neglected. It was thus crucial to compare with real cases, as has been done. The laboratory experiments have demonstrated a close alignment with the theoretically predicted values, exhibiting a difference of the same order of magnitude as those obtained from numerical experimentation. Eventually, the limitations and approximations that have been presented can be considered as negligible (at least in the conditions in which the experiments were conducted) leading to analytical and numerical results that are very close to the experimental ones.

Regarding the analytical tools that are available for SH prediction, a new method has been developed, the analytically generated signals method (AGS). This method, basically, is an extension and numerical application of the idea of a switching-function-based modelling method which is further developed in Chap. 5. The AGS method has been compared with DFS, numerical and physical experimentations. The results demonstrated a high level of agreement and the method did provide a high level of accuracy of the same order as the results coming from the DFS method.

Afterwards, in this chapter a classical one-phase inverter has been studied based on DFS and an inherent limitation was observed regarding the use of this method. Indeed, some phenomena named non-linear and non-controllable behaviour were observed in classical inverter signals leading to a Fourier decomposition which no longer aligned with the predictions obtained using the DFS. It turns out that the newly developed method, i.e. AGS, is able to predict these phenomena and to adapt the results with respect to them. The AGS method is therefore capable not only of handling the same situations as the DFS but also those for which the DFS exhibits inherent limitations.

Chapter 4

Interaction analysis

This chapter builds upon the conclusions drawn in section 3.3. It is based on the assumption that the converters are sufficiently far apart. Although this notion may appear somewhat qualitative at first, it is intentionally introduced without a quantitative definition at this stage. It simply refers to a separation distance large enough for all interaction mechanisms, except those that constitute the main focus of this study, to be neglected.

This assumption establishes a conceptual framework in which interference phenomena are limited to the superposition of emissions. In other words, the analysis focuses exclusively on the interaction resulting from the combination of the emitted supraharmonics, while all additional coupling mechanisms are neglected in a first approximation.

First, several configurations are analysed in order to identify the conditions under which interactions occur. Subsequently, the study focuses on the resulting emissions at the PCC produced by two converters connected in parallel and operating without significant mutual interaction, while considering a phase shift between their carrier signals.

Indeed, a phase shift between the carrier signals is expected to introduce a corresponding phase shift in the emitted supraharmonics, which may result in partial or even complete cancellation depending on the operating conditions. Among the influencing factors, the number of connected converters plays a significant role [52].

The same analytical model developed in Chapter 3 is employed throughout this chapter. Numerical simulations are also performed in order to compare both approaches and determine the conditions leading to interactions in the supraharmonic frequency range for two converters connected in parallel.

Under the initial assumption that no significant interaction exists between sufficiently distant converters, connecting two converters in parallel with their carrier signals in phase simply results in a doubling of the emitted current harmonics. In other words, the current frequency components add constructively at the PCC.

One of the main objectives of this chapter is therefore to determine analytically the resulting emissions at the PCC when a known phase shift exists between the carrier signals of the two converters.

Demonstrating this capability would also confirm that, under simplified conditions—namely neglecting the transformer impedance and modelling the grid as an ideal voltage source connected to the converters through a line impedance—it is possible to analytically predict the resulting supraharmonic emissions produced by two identically controlled converters operating with a prescribed carrier phase shift.

Predicting the resulting emissions produced by two converters connected in parallel in such a simplified grid constitutes a first step towards the analysis of more complex electrical networks, while avoiding the computational burden associated with numerical simulations.

A schematic of the basic idea of this work is shown in Fig. 4.1, with FBDR standing for Full-Bridge Diode Rectifier.

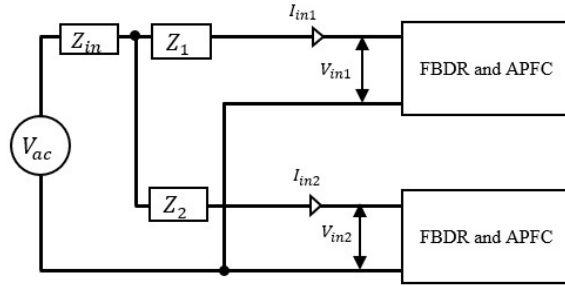


Figure 4.1: Schematic representation of two interleaved boost converters operating as active power factor correctors connected in parallel and separated by a given distance represented by the impedances Z_1 and Z_2 . The grid is modelled as an ideal voltage source connected through a line impedance. The input voltage (V_{in1}) and current (I_{in1}) of the first converter are also indicated.

Fig. 4.1 shows the case for separated converters. The impact of having the converters nearby is also studied. Actually, it is equivalent to considering the specific (and general) case shown in Fig.4.1 with Z_1 and Z_2 being near to zero. Eventually, the schematic is shown in 4.2.

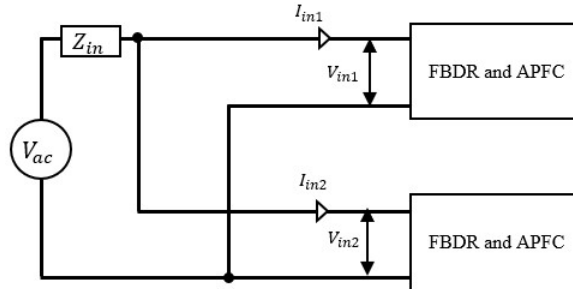


Figure 4.2: Schematic representation of two interleaved boost converters operating as active power factor correctors connected in parallel and located in close proximity ($Z_1 = Z_2 = 0$). The grid is modelled as an ideal voltage source connected through a line impedance. The input voltage (V_{in2}) and current (I_{in2}) of the second converter are also indicated.

Throughout this chapter, the impedances Z_{in} , Z_1 , and Z_2 are expressed in per unit, using the impedance of 1 km of distribution line as the reference value. This reference impedance, denoted Z_{in0} , is equal to $(0.4 + j0.35) \Omega$ [95].

The objective is to evaluate the discrepancy between the analytical prediction, which was originally developed for a single converter, and the numerical results obtained for two converters connected in parallel. If the assumption of weak interaction between distant converters holds, only minor discrepancies are expected.

All the tested contexts are summarized in tables 4.1, 4.2 and 4.3.

Table 4.1: Case 1 without EMI and without phase shift between carriers

Case 1	Distance from the ideal voltage source [km]	Distance between converters [km]
1.1	0,4	0,4
1.2	0,3	0,2
1.3	0,4	0

Table 4.2: Case 2 with EMI and without phase shift between carriers

Case 2	Distance from the ideal voltage source [km]	Distance between converters [km]
2.1	0,4	0,4
2.2	0,3	0,2
2.3	0,4	0

Table 4.3: Case 3 without EMI and with 30° phase shift between carriers

Case 3	Distance from the ideal voltage source [km]	Distance between converters [km]
3.1	0,4	0,4
3.2	0,3	0,2
3.3	0,4	0

This study will induce a better understanding of the limitation of the analytical method developed in section 3.3 as a function of the different considered distances, i.e. from the ideal voltage source and between converters. Analytical results will be compared with numerical ones. The final meaning of this comparison is that, it is possible under certain conditions, which will be presented next to the comparisons of the results, to determine the final emission on the PCC regarding the peculiar case of two converters in parallel.

Actually, the magnitude for each sideband harmonic around the carrier harmonics will be derived from the combination of the double Fourier series and the classical circuit theory as extensively described in section 3.3.

The analytical approach assumes that each converter behaves independently, meaning that the presence of the second converter does not modify the individual emissions of the first.

Consequently, if the emission spectrum obtained numerically for one converter agrees with the analytical prediction, it can be concluded that no significant interaction takes place.

Since the analytical development provides both the magnitude and the phase of every frequency component, the resulting emissions at the PCC can be directly obtained by applying the superposition principle whenever the converters can be considered non-interacting.

4.1 Results

This section presents the analytical and numerical results and analyses their agreement in order to assess the capability of the proposed analytical approach to predict the resulting emissions at the PCC for two converters connected in parallel.

The comparison first considers the analytical development established for a single converter and the numerical results obtained for two converters operating in parallel. Four different configurations are investigated by considering the presence or absence of an EMI filter together with the phase shift between the carrier signals. The corresponding electrical circuit is shown in Fig. 4.1.

4.1.1 Magnitude

The values of the passive components, together with the switching frequency used in the following numerical investigations, are summarized in Table 4.4. Here, L_b , L_1 , L_2 , and L_{TR} denote the boost inductance, the inductances associated with Z_1 and Z_2 , and the transformer-path inductance, respectively. Since L_1 and L_2 are assumed to be identical throughout this study, only L_1 is reported in the table.

Table 4.4: Characterization of the considered study of parallel converters

	L_b [mH]	L_1 [mH]	L_{TR} [mH]	F_{sw} [kHz]
Case 1.1 - 2.1 - 3.1	10	0.22	0.22	5
Case 1.2 - 2.2 - 3.2	10	0.11	0.22	5
Case 1.3 - 2.3 - 3.3	10	0	0.44	5

For Case 1, three different configurations are considered, as summarized in Table 4.1.

Case 1.1 - 1.2 - 1.3

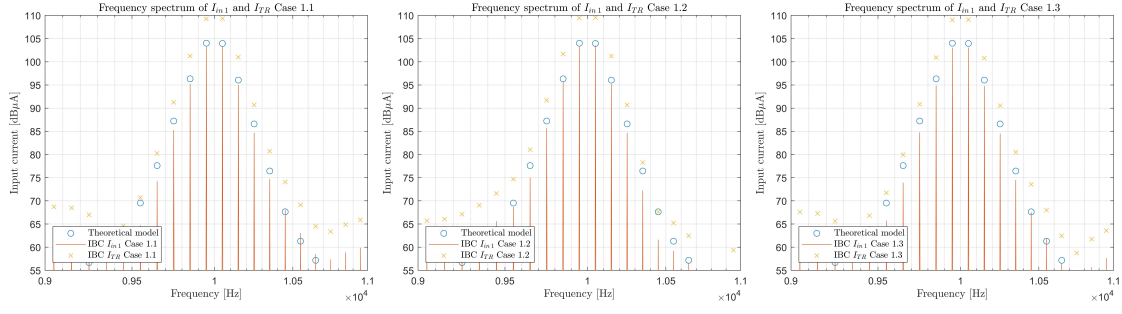


Figure 4.3: Comparison between the analytical and numerical results for the current magnitudes. I_{in1} and I_{TR} denote the current at the input of Converter 1 and the current at the PCC, respectively. Zoom on the first carrier sideband for Cases 1.1 (left), 1.2 (centre), and 1.3 (right)

Case 2.1 - 2.2 - 2.3

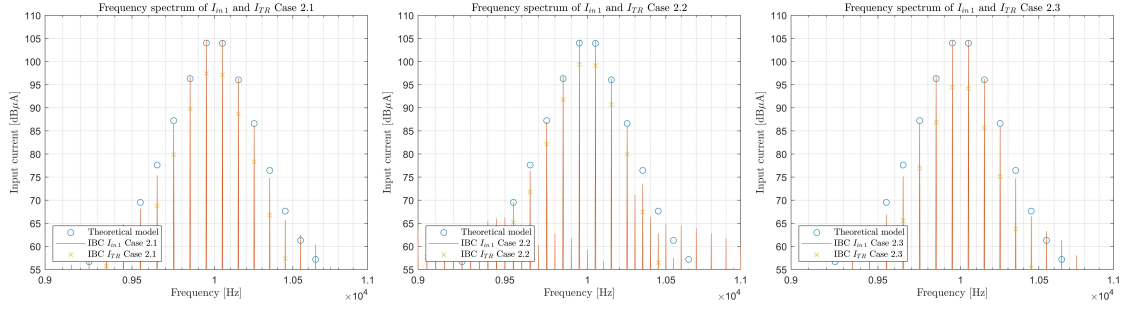


Figure 4.4: Comparison between the analytical and numerical results for the current magnitudes. I_{in1} and I_{TR} denote the current at the input of Converter 1 and the current at the PCC, respectively. Zoom on the first carrier sideband for Cases 2.1 (left), 2.2 (centre), and 2.3 (right)

Case 3.1 - 3.2 - 3.3

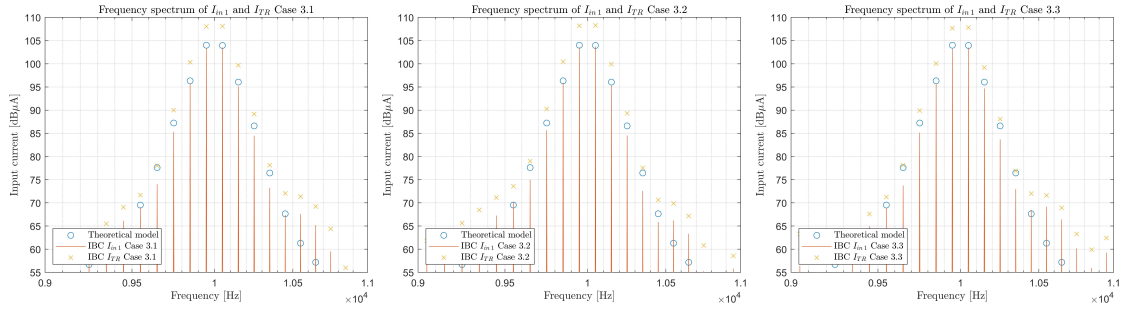


Figure 4.5: Comparison between the analytical and numerical results for the current magnitudes. I_{in1} and I_{TR} denote the current at the input of Converter 1 and the current at the PCC, respectively. Zoom on the first carrier sideband for Cases 3.1 (left), 3.2 (centre), and 3.3 (right)

These results show that connecting two converters in parallel without a phase shift between their carrier signals and without an EMI filter, as illustrated in Fig. 4.3, does

not modify the individual emissions compared with the single-converter case. Likewise, introducing a phase shift between the carrier signals of the two converters, while still neglecting the EMI filter (Fig. 4.5), leads to the same conclusion.

In contrast, the configuration including the EMI filter without a phase shift between the carrier signals is the only one among the investigated cases that results in a reduction of the current I_{TR} measured at the PCC.

A qualitative explanation of these observations is presented below. For the sake of clarity, the electrical circuit is simplified in order to avoid unnecessarily cumbersome equations and diagrams. The simplified circuit considered throughout the following analysis is shown in Fig. 4.6.

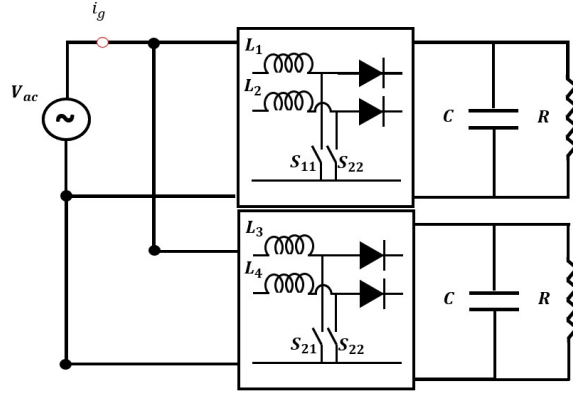


Figure 4.6: IBC in parallel : Simplified circuit for qualitative analysis of the effect of phase-shift between carriers (PS_{BC})

It should be recalled that the two converters are identical interleaved boost converters operating with the same switching frequency and control structure. Since the analysis is restricted to the supraharmonic frequency range, the inductive components dominate the network impedance. Consequently, only the inductances are retained in the following qualitative discussion.

The passive elements are therefore reduced to the converter inductances, denoted L_1 and L_2 for the first converter, and L_3 and L_4 for the second converter.

To avoid any ambiguity in the following discussion, two different phase shifts are defined. The phase shift between the two branches of a single converter is denoted by PS_C , whereas the phase shift between the carrier signals of the two converters is denoted by PS_{BC} .

Supraharmonic emissions can essentially be interpreted as voltage and current ripple components. When $PS_{BC} = 0^\circ$, the slope of the current flowing through the PCC, denoted $i_g(t)$, is doubled during both the rising and falling intervals.

As PS_{BC} increases, the well-known phenomenon of wave cancellation progressively appears. However, the maximum cancellation occurs for $PS_{BC} = 90^\circ$. This behaviour results from the fact that each converter already contains two interleaved branches operating with an internal phase shift of $PS_C = 180^\circ$.

Considering the particular case where $PS_{BC} = 90^\circ$, the following expressions describe the current slope over one half of the switching period:

$$\begin{aligned}\frac{di_g}{dt} &= \frac{V_{ac}(L_1 + L_2)}{L_1 L_2} + \frac{V_{ac}}{L_4} + \frac{V_{ac} - V_{DC2}}{L_3} \\ \frac{di_g}{dt} &= \frac{V_{ac}}{L_1} + \frac{V_{ac} - V_{DC1}}{L_2} + \frac{V_{ac}}{L_4} + \frac{V_{ac} - V_{DC2}}{L_3} \\ \frac{di_g}{dt} &= \frac{V_{ac}}{L_1} + \frac{V_{ac} - V_{DC1}}{L_2} + \frac{V_{ac}(L_3 + L_4)}{L_3 L_4} \\ \frac{di_g}{dt} &= \frac{V_{ac}}{L_1} + \frac{V_{ac} - V_{DC1}}{L_2} + \frac{V_{ac}}{L_4} + \frac{V_{ac} - V_{DC2}}{L_3}\end{aligned}$$

These expressions can be simplified by assuming that $V_{DC1} = V_{DC2}$ and $L_1 = L_2 = L_3 = L_4 = L$, which gives

$$\begin{aligned}\frac{di_g}{dt} &= \frac{4V_{ac}}{L} - \frac{V_{DC}}{L} \\ \frac{di_g}{dt} &= \frac{4V_{ac}}{L} - \frac{2V_{DC}}{L} \\ \frac{di_g}{dt} &= \frac{4V_{ac}}{L} - \frac{V_{DC}}{L} \\ \frac{di_g}{dt} &= \frac{4V_{ac}}{L} - \frac{2V_{DC}}{L}\end{aligned}$$

This behaviour is illustrated in Fig. 4.7 for $V_{ac} = \frac{3}{4}V_{ac\,peak}$ and $V_{DC} = 2V_{ac\,peak}$. Extending the previous qualitative explanation regarding the cancellation of odd carrier harmonics within a single IBC, it can be concluded that connecting two converters in parallel with a phase shift PS_{BC} of 90° , i.e. carrier 1.1 at 0° , carrier 1.2 at 180° , carrier 2.1 at 90° , and carrier 2.2 at 270° , leads to the cancellation of half of the remaining carrier harmonics.

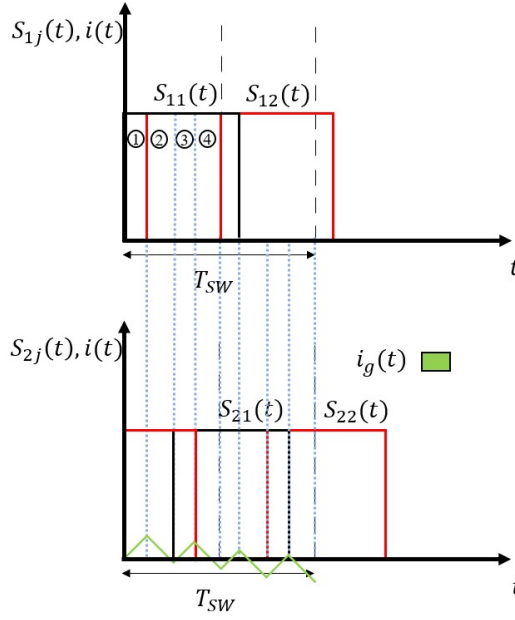


Figure 4.7: IBC in parallel : Representation of the qualitative evolution of the total current noted $i_g(t)$ as a function of time along with its related switching function (instantaneous value) for the case $2V_{acpeak} < V_{DC}$

The considered situation corresponds to a particular case, since both converters operate with the same DC voltage and use identical control structures and passive-element values. Nevertheless, the general equations have been provided and allow one to conclude that the harmonic spectrum resulting from parallel converters is not straightforward. It is, naturally, the frequency-domain representation of the time-domain current waveform, which can be constructed directly from the previously presented general equations.

Furthermore, two parameters that influence the individual emissions have deliberately been neglected in order to provide a simplified explanation of the final emissions produced by two converters connected in parallel. Indeed, it has been shown throughout the previous chapters that the emissions of APFC converters depend on both the ratio $\frac{V_{acpeak}}{V_{DC}}$ and the value and spectral composition of V_{DC} . Since no impedance has been considered between the ideal voltage source, which can be regarded as an ideal transformer, and the two converters, no variation in V_{acpeak} can occur.

However, this approximation is well justified because the inductance representing the transformer corresponds only to the line inductance, i.e. $1.11 \left[\frac{mH}{km} \right]$, which is negligible compared with the $10 [mH]$ boost inductances used in the IBC. Consequently, the individual emissions are not expected to be significantly affected by the presence of the line inductance. Moreover, the voltage drop across the line inductance remains small because the ratio $r = \frac{V_{acpeak}}{V_{DC}}$ is governed by the 50 Hz fundamental component. Regarding the assumption of a constant DC voltage, it is equivalent to considering a sufficiently large DC-link capacitance. Since the converter has been designed accordingly, no significant variation of the DC voltage is expected.

It is clearly observed that the superposition of the currents emitted by the two converters

leads to the cancellation of half of the supraharmonic components at the PCC. Not only does each IBC individually cancel the odd carrier harmonic bands as a consequence of the internal phase shift $PS_C = 180^\circ$, but the additional phase shift between the converters ($PS_{BC} = 90^\circ$) also cancels half of the remaining carrier harmonic bands. This result is particularly interesting from the perspective of distribution system operators, as it may contribute to improved power-quality management. It should, however, be emphasized that this conclusion is valid under the assumption of symmetric electrical distances, corresponding in this study to $Z_1 = Z_2 = 0 \text{ } [\Omega]$. More generally, this condition requires that the impedances separating the converters be approximately identical.

4.1.2 Phase

Regarding the phase analysis, the same converter topology, namely the IBC, is further investigated but with a different switching frequency. The switching frequency is now set to $25 \text{ } [kHz]$. This modification does not affect the previous analysis or the conclusions drawn regarding the cancellation of carrier harmonics, since this cancellation mechanism is independent of the considered switching frequency and is only related to the phase opposition between the switching signals.

The main objective is to compare the numerical and analytical phases associated with each frequency component. The absolute values, together with the relative differences between the analytical and numerical results, are presented. The following results therefore provide a numerical validation of the DFS method as a tool capable of determining not only the magnitude but also the phase of each harmonic component. Moreover, consistently with the previous conclusions, this DFS analysis combined with numerical experimentation provides further confirmation of the intuitive explanation of the PS_{BC} effect, namely the cancellation resulting from phase opposition.

Case 1

Case 1.1

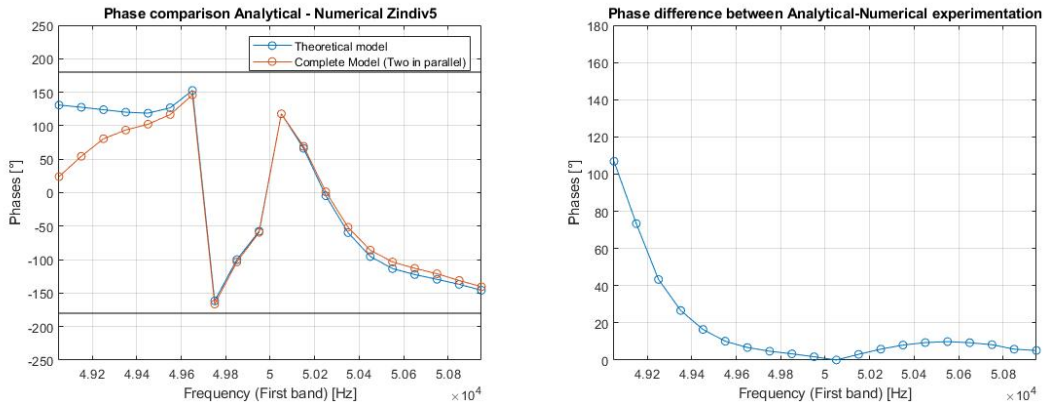


Figure 4.8: Comparison between numerical and analytical phases (between -180° and 180°) of each sideband harmonic around the first carrier harmonic, i.e. around $50 \text{ } kHz$ in this case, for two converters connected in parallel and far from each other with $Z_{in} = \frac{Z_{in0}}{5}$

Case 1.2

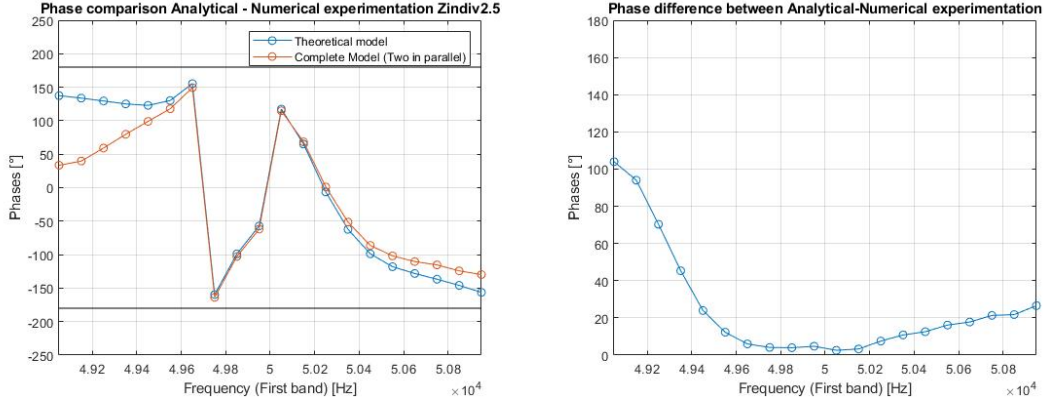


Figure 4.9: Comparison between numerical and analytical phases (between -180° and 180°) of each sideband harmonic around the first carrier harmonic, i.e. around 50 kHz in this case, for two converters connected in parallel and far from each other with $Z_{in} = \frac{Z_{in0}}{2.5}$

The first observation is the small difference between the phases obtained from the analytical expression and those obtained from numerical simulations around the first carrier harmonic. Indeed, in the frequency range between $49\,600$ and $50\,400\text{ Hz}$, the maximum phase difference remains below 10° . It should be noted that the quadratic sum (QS) is computed from the eight closest frequency components around the first carrier harmonic and therefore corresponds to the frequency range $(49\,650 - 50\,350)\text{ Hz}$.

Consequently, in this context, predicting the interaction between converters and, ultimately, the resulting QS at the PCC appears realistic. Indeed, for both $\frac{Z_{in0}}{5}$ and $\frac{Z_{in0}}{2.5}$, the discrepancies remain limited for both magnitudes (approximately 5% in the QS calculation) and phases (maximum deviation of 10°) over the complete frequency range considered for the QS computation.

Case 2

Case 2.1

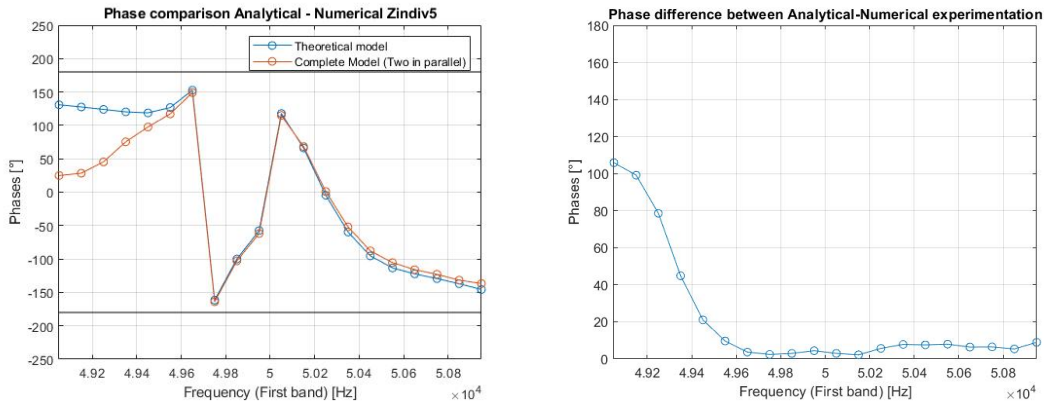


Figure 4.10: Comparison between numerical and analytical phases (between -180° and 180°) of each sideband harmonic around the first carrier harmonic, i.e. around 50 kHz in this case, for two converters connected in parallel and located close to each other with $Z_{in} = \frac{Z_{in0}}{5}$

Case 2.2

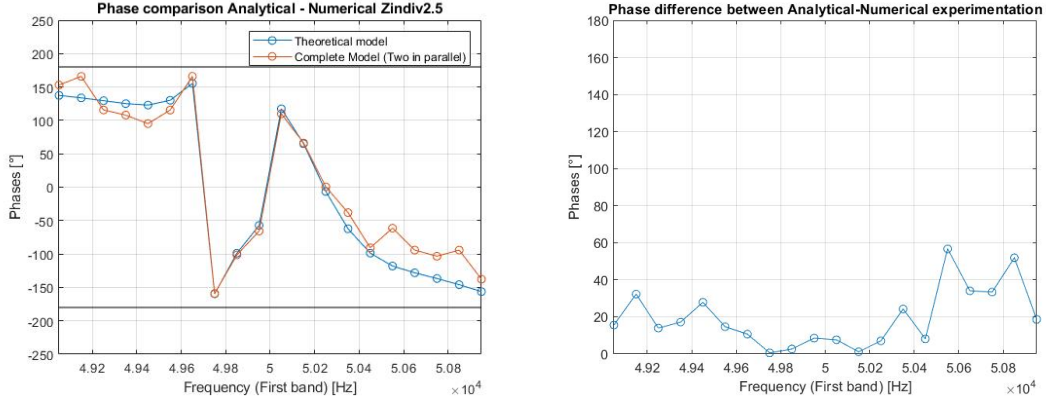


Figure 4.11: Comparison between numerical and analytical phase (between -180° and 180°) of each sideband harmonic around the first carrier harmonic, i.e. around 50 kHz in this case, for two converter in parallel and nearby with $Z_{in} = \frac{Z_{in0}}{2.5}$

The comparison presented in Fig. 4.11 confirms the tendency already observed in the previous cases. When the two converters are connected in close proximity, the agreement between the analytical and numerical phase values remains satisfactory over most of the considered sideband harmonics. Nevertheless, the relative phase error becomes larger than in the case of separated converters, reaching approximately 25° for the component located at 50 350 Hz. This increase confirms that the interaction between converters first manifests itself through a modification of the phase relationship between the emitted supraharmonics before producing more significant discrepancies in magnitude.

Although the increase in phase error remains moderate, it highlights the limitation of the analytical model developed for a single isolated converter. Indeed, the analytical formulation assumes that the switching behaviour of each converter is independent of the neighbouring converter, an assumption that progressively loses its validity as the electrical distance between converters decreases. Nevertheless, the observed discrepancies remain sufficiently limited to preserve the physical interpretation of the emission mechanisms and to justify the use of the analytical model as a first-order prediction tool.

In order to predict the resulting emission on the PCC when intentional phase shifts are introduced between converters, the analytical formulation must therefore be extended. The objective is to account explicitly for a phase displacement of the carrier signals while preserving the analytical structure previously derived using the Double Fourier Series. The first investigated case consists of introducing a 90° phase shift on both carriers of the second interleaved boost converter.

More precisely, the carriers of the second converter, denoted CA2.1 and CA2.2, are shifted respectively by $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, whereas the first converter keeps the conventional carrier positions of 0 and π . The following analytical development establishes the corresponding switching functions and derives the resulting ripple current expressions. The purpose is to determine analytically the phase evolution of every sideband harmonic and, ultimately, to predict the superposition of the emissions generated by the two converters on the PCC.

4.2 Analytical Development

4.2.1 Adaptation of the Analytical Development for Phase Shift Consideration

The analytical developments established in section 3.3 are now extended to account for a phase shift between the carriers of two converters operating in parallel. Following the same methodology as previously employed and inspired by the developments presented in [1], new switching functions are obtained by introducing the desired carrier phase displacement directly into the Fourier representation. This approach preserves the mathematical framework of the Double Fourier Series while allowing the influence of the carrier phase to be investigated analytically.

The second converter is considered to operate with a carrier phase shift of $\frac{\pi}{2}$ for CA2.1 and $\frac{3\pi}{2}$ for CA2.2. Meanwhile, the first converter keeps its original carrier positions of 0 and π , corresponding to the classical interleaved operation. Under these assumptions, the modified switching functions are expressed as follows.

$$S_{CA2.1}(t) = \theta(t) + \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} X_{mn} \left(\sin((m\omega_c + n\omega_m)t + \frac{m\pi}{2} + n(\phi + \theta_0 + \phi_{mn})) \right) \quad (4.1)$$

$$S_{CA2.2}(t) = \theta(t) + \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} X_{mn} \left(\sin((m\omega_c + n\omega_m)t + \frac{m3\pi}{2} + n(\phi + \theta_0 + \phi_{mn})) \right) \quad (4.2)$$

Knowing that for the interleaved boost converter the next relation holds

$$L_1 \frac{di_{1rip}}{dt} = S_{CA2.1}(t) V_{dc}$$

Also, having

$$S_{CA2.1ripple}(t) = \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} X_{mn} \left(\sin((m\omega_c + n\omega_m)t + \frac{m\pi}{2} + n(\phi + \theta_0 + \phi_{mn})) \right) \quad (4.3)$$

$i_{1ripple}(t)$ can be expressed as

$$i_{1ripple}(t) = \frac{V_{dc}}{L_1} \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \left(\sin((m\omega_c + n\omega_m)t + \frac{m\pi}{2} + n(\phi + \theta_0 + \phi_{mn} - \frac{\pi}{2})) \right) \quad (4.4)$$

Defining α_2 as

$$\alpha_2 = (m\omega_c + n\omega_m)t + n(\phi + \theta_0 + \phi_{mn} - \frac{\pi}{2}) \quad (4.5)$$

Using this substitution in 4.6 leads to

$$i_{1ripple}(t) = \frac{V_{dc}}{L_1} \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2 + \frac{m\pi}{2}) \quad (4.6)$$

Afterwards, the same logic is applied for $i_{2ripple}(t)$

$$i_{2ripple}(t) = \frac{V_{dc}}{L_2} \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ \text{neven}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2 + \frac{m3\pi}{2}) \quad (4.7)$$

For $i_{1ripple}(t)$

Looking at $\sin(\alpha_2 + \frac{m\pi}{2})$ leads to

$$\sin(\alpha_2 + \frac{m\pi}{2}) = \sin(\alpha_2)\cos(\frac{m\pi}{2}) + \cos(\alpha_2)\sin(\frac{m\pi}{2})$$

For m even $\sin(\alpha_2 + \frac{m\pi}{2}) = \sin(\alpha_2)$ for $m = 4k$ with $k = 0, 1, \dots, \infty$

$$\sin(\alpha_2 + \frac{m\pi}{2}) = -\sin(\alpha_2) \text{ for } m = 4k + 2$$

For m odd $\sin(\alpha_2 + \frac{m\pi}{2}) = \cos(\alpha_2)$ for $m = 4k + 1$

$$\sin(\alpha_2 + \frac{m\pi}{2}) = -\cos(\alpha_2) \text{ for } m = 4k + 3$$

For $i_{2ripple}(t)$

Looking at $\sin(\alpha_2 + \frac{m3\pi}{2})$ leads to

$$\sin(\alpha_2 + \frac{m3\pi}{2}) = \sin(\alpha_2)\cos(\frac{m3\pi}{2}) + \cos(\alpha_2)\sin(\frac{m3\pi}{2})$$

For m even $\sin(\alpha_2 + \frac{m3\pi}{2}) = \sin(\alpha_2)$ for $m = 4k$

$$\sin(\alpha_2 + \frac{m3\pi}{2}) = -\sin(\alpha_2) \text{ for } m = 4k + 2$$

For m odd $\sin(\alpha_2 + \frac{m3\pi}{2}) = \cos(\alpha_2)$ for $m = 4k + 3$

$$\sin(\alpha_2 + \frac{m3\pi}{2}) = -\cos(\alpha_2) \text{ for } m = 4k + 1$$

For $i_{ripple\text{tot}}$

Knowing that the global ripple is the sum of $i_{1ripple}(t)$ and $i_{2ripple}(t)$

For m even $i_{ripple\text{tot}}$ is

$$i_{ripple\text{tot}}(t) = \frac{V_{dc}}{L_1} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2 + \frac{m\pi}{2}) + \frac{V_{dc}}{L_2} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2 + \frac{m3\pi}{2}) \quad (4.8)$$

Knowing that $L_1 = L_2 = L$

$$i_{ripple\text{tot}}(t) = 2 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2) (-1)^{\frac{m3\pi}{2}} \quad (4.9)$$

For $m = 4k$ with $k = 0, 1, \dots, \infty$

$$i_{ripple\text{tot}}(t) = 2 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2) \quad (4.10)$$

For $m = 4k+2$

$$i_{ripple\text{tot}}(t) = 2 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} (-\sin(\alpha_2)) = 2 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2 + \pi) \quad (4.11)$$

For m odd $i_{ripple\text{tot}}$ is

$$i_{ripple\text{tot}}(t) = \frac{V_{dc}}{L_1} \sum_{\substack{m=1 \\ \text{odd}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2 + \frac{m\pi}{2}) + \frac{V_{dc}}{L_2} \sum_{\substack{m=1 \\ \text{odd}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2 + \frac{m3\pi}{2}) \quad (4.12)$$

For $m = 4k+1$

$$i_{ripple\text{tot}}(t) = \frac{V_{dc}}{L_1} \sum_{\substack{m=1 \\ \text{odd}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \cos(\alpha_2) + \frac{V_{dc}}{L_2} \sum_{\substack{m=1 \\ \text{odd}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} - \cos(\alpha_2) = 0 \quad (4.13)$$

For $m = 4k+3$

$$i_{ripple\,tot}(t) = \frac{V_{dc}}{L_1} \sum_{\substack{m=1 \\ modd}}^{\infty} \sum_{\substack{n=-\infty \\ neven}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} - \cos(\alpha_2) + \frac{V_{dc}}{L_2} \sum_{\substack{m=1 \\ modd}}^{\infty} \sum_{\substack{n=-\infty \\ neven}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \cos(\alpha_2) = 0 \quad (4.14)$$

It is immediately apparent from these expressions that the previous conclusion coming from the development made in section 3.3 is still valid, i.e. cancellation of the odd switching frequency harmonics. Another conclusion emerges, as the final result on the PCC is the sum of both emissions of each converter in parallel and noting that for $m=4k+2$ the sign changes with respect to the classical case of section 3.3 it is expected a cancellation on the PCC of the emissions related to $m=4k+2$ which represent half of the remaining frequency harmonics still emitted. This is verified in the following results.

First, Equ. 4.10 does not exhibit any change from the original case presented in section 3.3 for $m=4k$ but Equ. 4.11 shows that π appears in the argument of the sine wave for $m=4k+2$. These results are verified respectively in Fig. 4.12 and Fig. 4.13.

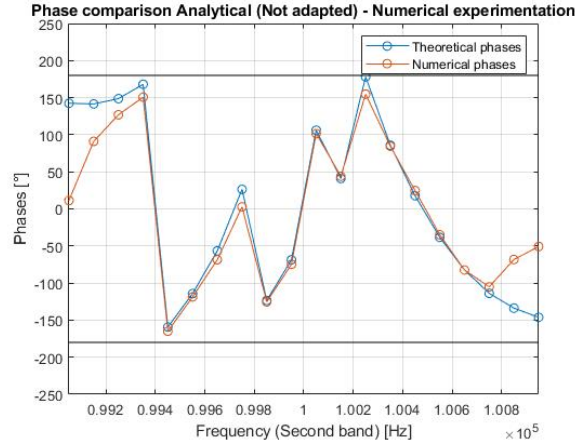


Figure 4.12: Phase comparison, for each sideband harmonic, between numerical and theoretical values (sideband of the second carrier frequency harmonic)

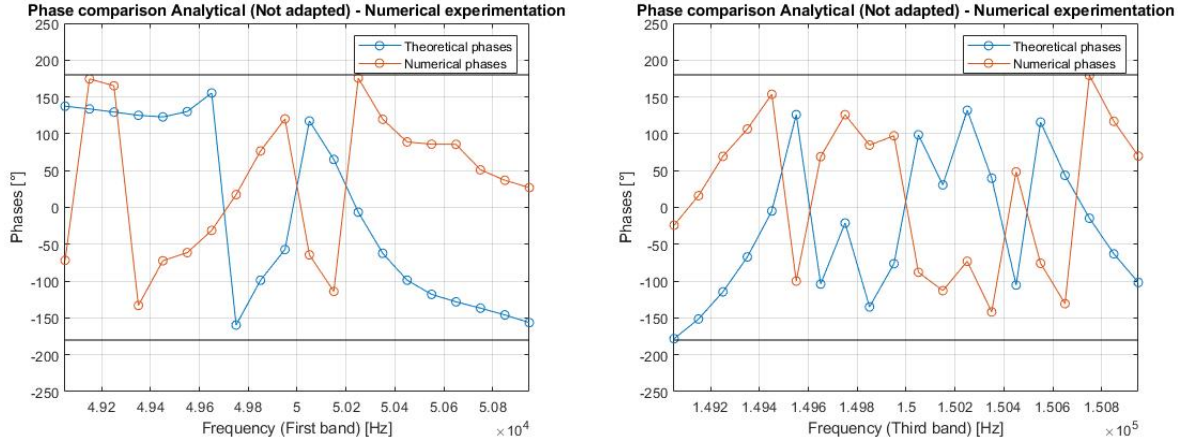


Figure 4.13: Phase comparison, for each sideband harmonic around the first (left) and third (right) carrier frequency harmonic, between numerical and theoretical values

Eventually, based on Fig. 4.12 and 4.13 the analytical conclusions are verified. Indeed, the second band represents actually the fourth carrier harmonic, i.e. $m = 4$. For that carrier frequency harmonic the phases of the sideband harmonics are in excellent agreement as those predicted by the analytical development. In Fig. 4.13 it can be observed a phase shift of 180° also predicted by the analytical development for $m=4k+2$.

An interesting point is the possibility to directly compare the phases between the first converter, i.e. not phase shifted, and the second, i.e. 90° phase shifted. Indeed, the theoretical phases are determined in Fig. 4.12 and 4.13 with the non-adapted analytical expression as this expression provides the same phase values as the numerical simulation.

As far as the assessment of the global emission on the PCC for two converters in parallel is concerned, this determination can be carried out based on the analytical development of this work. Basically, by knowing that the converters are the same with the same control it is possible to state that the expression of their current will be the same except for the cases linked to Equ. 4.11, i.e. $m=4k+2$. The final current is, as a first approximation, by neglecting the line impedance

For $m = 4k$

$$i_{\text{ripple} \text{ tot}}(t) = 2 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2) + 2 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2) \quad (4.15)$$

$$i_{\text{ripple} \text{ tot}}(t) = 4 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2) \quad (4.16)$$

For $m = 4k+2$

$$i_{\text{ripple} \text{ tot}}(t) = 2 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} - \sin(\alpha_2) + 2 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ \text{even}}}^{\infty} \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2) \quad (4.17)$$

$$= 0 \quad (4.18)$$

These last results show, for a phase shift in the carriers of an interleaved converter of 90° with respect to the same converter placed in parallel, a cancellation of the switching harmonics for $m=4k+2$ (2-6-10 etc. . .). Mathematically this analytical development demonstrate the possibility of considering phases in the carrier of converters. Fig. 4.14 and Fig. 4.16 show a cancellation, as predicted, of the $m = 2$ and $m = 6$ sidebands while Fig. 4.17 illustrate the constructive addition of the magnitudes that happens in the second band, i.e. $m = 4$.

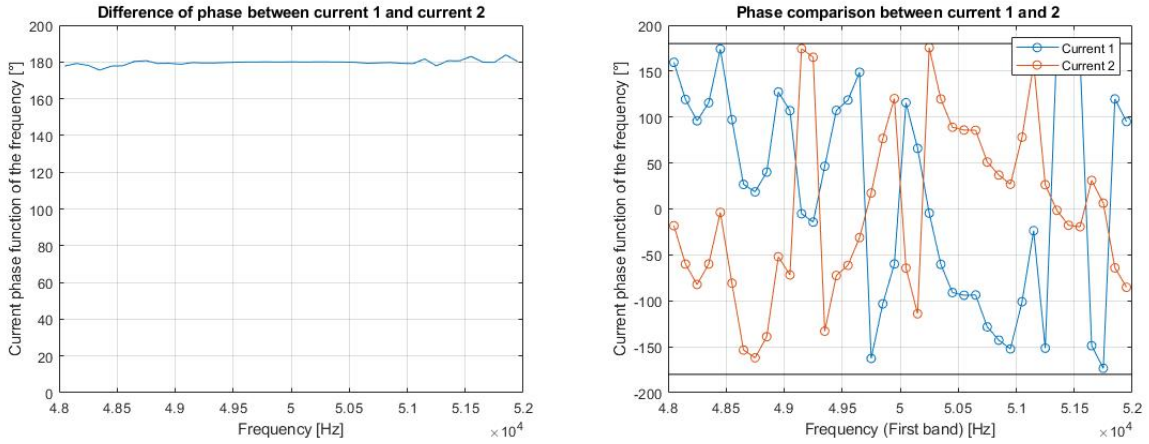


Figure 4.14: Phase comparison, for each sideband harmonic around the first carrier frequency harmonic, between the input current of converter 1 and of converter 2. The phase difference (left) and the absolute value (right) are illustrated

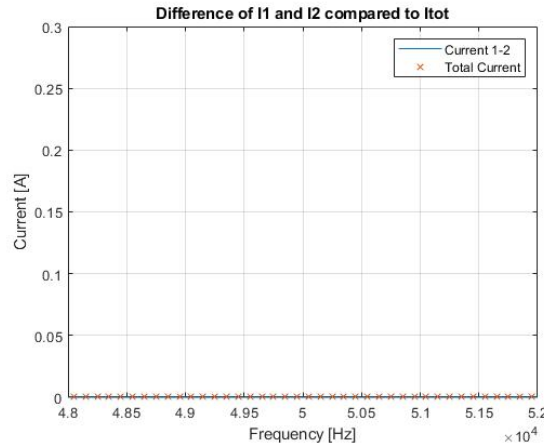


Figure 4.15: Absolute value of the input current on the sideband of the first carrier frequency harmonic

The same difference in phase can be observed as predicted for the third sideband. This is shown afterwards in Fig. 4.16

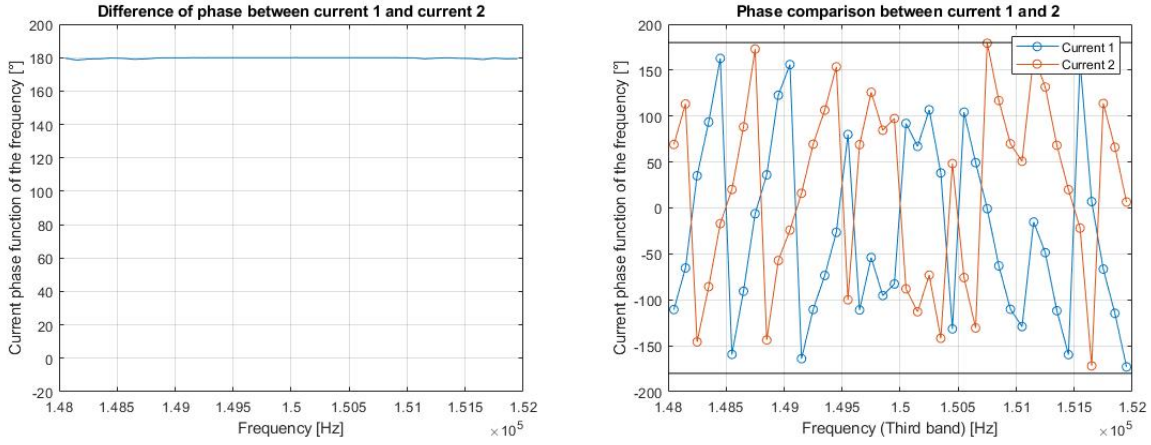


Figure 4.16: Comparison of the phase of each sideband harmonic around the third carrier harmonic between the currents of converters 1 and 2

the bands corresponding to $m=4k+2$

Finally, the complete cancellation of the bands corresponding to $m=4k+2$ has been demonstrated. The second band, i.e. $m=4$, is illustrated in Fig. 4.17.

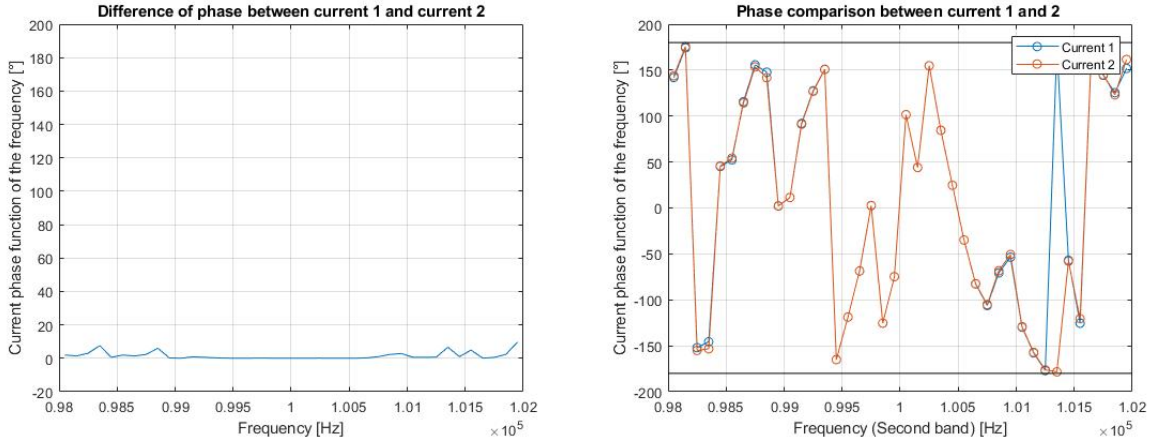


Figure 4.17: Comparison of the phase of each sideband harmonic around the second carrier harmonic between the currents of converters 1 and 2

The analytically predicted behaviour, namely the phase opposition for the third carrier band ($m = 6$), as previously observed for the first carrier band ($m = 2$), and the identical phases for the second carrier band ($m = 4$), is validated in Fig. 4.14 and Fig. 4.16. The left-hand side of these figures shows the phase difference between the currents of converters 1 and 2, clearly highlighting the 0° and 180° phase shifts, corresponding respectively to currents that are in phase and in phase opposition.

Since each frequency component is sinusoidal, two components of identical magnitude that are in phase opposition naturally cancel each other. Consequently, the corresponding frequency components disappear from the magnitude spectrum of the total input current, i.e. the current flowing at the PCC.

4.2.2 Analytical Determination of the Unrectified Current

Although the predicted phase relationships have been obtained analytically and confirmed through numerical experimentation, it remains both important and valuable to derive directly the expression of the input current from the analytical expression of the rectified current. Such a development makes it possible to determine not only the phase of each harmonic around the carrier frequency, but also its magnitude. To achieve this objective, an additional analytical development is required and is presented in this subsection.

The purpose of this development is to derive the expression of the input ripple current (before the rectifier) from the previously obtained expression of the rectified ripple current (after the rectifier). For a single converter, whose carrier is phase-shifted by 90° , the expression of the rectified ripple current has previously been determined as

For $m=4k$

$$i_{ripple\,tot}(t) = 2 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ m\,even}}^{\infty} \sum_{\substack{n=-\infty \\ n\,even}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2) \quad (4.19)$$

Taking the action of the rectifier into account leads to

$$i_{in\,ripple\,tot}(t) = \frac{4}{\pi} \sum_{\substack{h=1 \\ h\,odd}}^{\infty} \frac{1}{h} \sin(h\omega_m t + h\theta_0) * 2 \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ m\,even}}^{\infty} \sum_{\substack{n=-\infty \\ n\,even}}^{\infty} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2) \quad (4.20)$$

$$i_{in\,ripple\,tot}(t) = \frac{8}{\pi} \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ m\,even}}^{\infty} \sum_{\substack{n=-\infty \\ n\,even}}^{\infty} \sum_{\substack{h=1 \\ h\,odd}}^{\infty} \frac{1}{h} \sin(h\omega_m t + h\theta_0) \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2) \quad (4.21)$$

$$i_{in\,ripple\,tot}(t) = \frac{8}{\pi} \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ m\,even}}^{\infty} \sum_{\substack{n=-\infty \\ n\,even}}^{\infty} \sum_{\substack{h=1 \\ h\,odd}}^{\infty} \frac{1}{h} \frac{X_{mn}}{(m\omega_c + n\omega_m)} \sin(\alpha_2) \sin(h\omega_m t + h\theta_0) \quad (4.22)$$

Let

$$\begin{aligned} A &= \sin(\alpha_2) \sin(h\omega_m t + h\theta_0) \\ &= \frac{1}{2} [\cos(\alpha_2 - h\omega_m t - h\theta_0) - \cos(\alpha_2 + h\omega_m t + h\theta_0)] \end{aligned} \quad (4.23)$$

Since $\alpha_2 = \alpha - \frac{\pi}{2}$, the previous expression can be rewritten as

$$\begin{aligned}
A &= \frac{1}{2} \left[\cos \left(\alpha - \frac{\pi}{2} - h\omega_m t - h\theta_0 \right) - \cos \left(\alpha - \frac{\pi}{2} + h\omega_m t + h\theta_0 \right) \right] \\
&= \frac{1}{2} [\sin(\alpha - h\omega_m t - h\theta_0) - \sin(\alpha + h\omega_m t + h\theta_0)] \\
&= \frac{1}{2}(A.1 - A.2)
\end{aligned} \tag{4.24}$$

A.1

$$A.1 = \sin(\alpha - h\omega_m t - h\theta_0) = \sin((m\omega_c + n\omega_m)t + n(\phi + \theta_0) + \phi_{mn} - h\omega_m t - h\theta_0) \tag{4.25}$$

$$= \sin((m\omega_c + (n - h)\omega_m)t + n\phi + (n - h)\theta_0 + \phi_{mn}) \tag{4.26}$$

Introducing the change of variable $p = n - h$, i.e. $n = p + h$, leads to

$$A.1 = \sin((m\omega_c + p\omega_m)t + (p + h)\phi + p\theta_0 + \phi_{mn}) \tag{4.27}$$

Defining

$$a = (m\omega_c + p\omega_m)t, \quad b = (p + h)\phi + p\theta_0 + \phi_{mn},$$

the trigonometric identity

$$\sin(a + b) = \sin(a) \cos(b) + \cos(a) \sin(b)$$

can be applied, yielding

$$A.1 = \sin((m\omega_c + p\omega_m)t) \cos((p + h)\phi + p\theta_0 + \phi_{mn}) + \cos((m\omega_c + p\omega_m)t) \sin((p + h)\phi + p\theta_0 + \phi_{mn}) \tag{4.28}$$

A.2

$$A.2 = \sin(\alpha + h\omega_m t + h\theta_0) = \sin((m\omega_c + n\omega_m)t + n(\phi + \theta_0) + \phi_{mn} + h\omega_m t + h\theta_0) \tag{4.29}$$

$$= \sin((m\omega_c + (n + h)\omega_m)t + n\phi + (n + h)\theta_0 + \phi_{mn}) \tag{4.30}$$

Introducing the change of variable $p = n + h$, i.e. $n = p - h$, leads to

$$A.2 = \sin((m\omega_c + p\omega_m)t + (p - h)\phi + p\theta_0 + \phi_{mn}) \tag{4.31}$$

Similarly, by defining

$$a = (m\omega_c + p\omega_m)t, \quad b = (p - h)\phi + p\theta_0 + \phi_{mn},$$

the same trigonometric identity gives

$$A.2 = \sin((m\omega_c + p\omega_m)t)\cos((p-h)\phi + p\theta_0 + \phi_{mn}) + \cos((m\omega_c + p\omega_m)t)\sin((p-h)\phi + p\theta_0 + \phi_{mn}) \quad (4.32)$$

Combining the expressions of A.1 and A.2 finally yields

$$A = \frac{1}{2}(A.1 - A.2) \quad (4.33)$$

$$= \frac{1}{2} \left(\sin((m\omega_c + p\omega_m)t)\cos((p+h)\phi + p\theta_0 + \phi_{mn}) + \cos((m\omega_c + p\omega_m)t)\sin((p+h)\phi + p\theta_0 + \phi_{mn}) \right. \quad (4.34)$$

$$\left. - \left(\sin((m\omega_c + p\omega_m)t)\cos((p-h)\phi + p\theta_0 + \phi_{mn}) + \cos((m\omega_c + p\omega_m)t)\sin((p-h)\phi + p\theta_0 + \phi_{mn}) \right) \right) \\ = \frac{1}{2} \left(\sin(a)\cos((p+h)\phi + p\theta_0 + \phi_{m(p+h)}) + \cos(a)\sin((p+h)\phi + p\theta_0 + \phi_{m(p+h)}) \right. \quad (4.35)$$

$$\left. - \sin(a)\cos((p-h)\phi + p\theta_0 + \phi_{m(p-h)}) - \cos(a)\sin((p-h)\phi + p\theta_0 + \phi_{m(p-h)}) \right) \\ = \frac{1}{2} \left(\cos(a)(\sin((p+h)\phi + p\theta_0 + \phi_{m(p+h)}) - \sin((p-h)\phi + p\theta_0 + \phi_{m(p-h)})) \right. \quad (4.36)$$

$$\left. + \sin(a)(\cos((p+h)\phi + p\theta_0 + \phi_{m(p+h)}) - \cos((p-h)\phi + p\theta_0 + \phi_{m(p-h)})) \right)$$

Afterwards, Eq. 4.36 is substituted into Eq. 4.22, which eventually leads to the final expression of the input current:

$$i_{in\ ripple\ tot} = \frac{4}{\pi} \frac{V_{dc}}{L} \sum_{\substack{m=2 \\ m\ even}}^{\infty} \sum_{\substack{n=-\infty \\ n\ even}}^{\infty} \sum_{\substack{h=1 \\ h\ odd}}^{\infty} \frac{1}{h} \left(\cos(a) \left(\frac{X_{mn}}{(m\omega_c + (p+h)\omega_m)} \sin((p+h)\phi + p\theta_0 + \phi_{m(p+h)}) \right. \right. \quad (4.37)$$

$$\left. - \frac{X_{mn}}{(m\omega_c + (p-h)\omega_m)} \sin((p-h)\phi + p\theta_0 + \phi_{m(p-h)}) \right) \\ + \sin(a) \left(\frac{X_{mn}}{(m\omega_c + (p+h)\omega_m)} \cos((p+h)\phi + p\theta_0 + \phi_{m(p+h)}) \right. \\ \left. - \frac{X_{mn}}{(m\omega_c + (p-h)\omega_m)} \cos((p-h)\phi + p\theta_0 + \phi_{m(p-h)}) \right) \right)$$

This final expression represents the complete analytical equation describing the input current of a converter while taking into account the presence of a full-bridge diode rectifier.

Three summations appear in the expression: the first is associated with the carrier harmonic index (m), the second with the sideband harmonic index (n), and the third with the harmonic content introduced by the full-bridge diode rectifier (h).

Based on this expression, it is possible to numerically determine both the magnitude and the phase of every harmonic defined by the indices m , n , and h . Since the previous conclusions were concerned only with the overall behaviour at the PCC, this complete expression was not required. It becomes essential, however, when an accurate description of the magnitude and phase of every individual frequency component at the PCC is sought.

4.3 Conclusion

This chapter provided a solid foundation, although representing only a first step, for the investigation of the complex domain of converter interactions. First, the conclusions drawn in Chapter 3, which established the necessary analytical modelling framework, were revisited and served as the basis for studying interaction phenomena.

Several configurations involving two converters connected in parallel were first investigated by considering both the presence of an EMI filter and different phase shifts between the carrier signals. In addition, various physical distances between the converters were analysed. These investigations led to the conclusion that no significant differences are expected in the emissions measured at the PCC as long as no EMI filter is included. This result highlights the possibility of employing simplified analytical models, such as those considering only a single converter, when EMI filters are absent.

However, the presence of an EMI filter required an adaptation of the previously developed analytical framework in order to reduce the discrepancy between the analytical predictions and the numerical results.

Subsequently, the observed phenomena were explained and illustrated through graphical representations of the analytical developments. It was shown that, even when considering only two converters sharing the same topology and control strategy, the resulting emissions strongly depend on several parameters, such as the values of the passive components, thereby leading to non-trivial interaction behaviours.

The study of the phase of the different frequency components further reinforced confidence in the capability of the developed analytical framework to accurately predict both magnitudes and phases. The latter constitute the key element governing interaction phenomena. Additional analytical developments were then carried out, leading to specific conclusions regarding the behaviour of the different carrier harmonic bands. These theoretical conclusions were subsequently confirmed by graphical representations that exhibited excellent agreement with the numerical experimentations.

Finally, for the sake of completeness, a final analytical derivation accounting for the presence of the full-bridge diode rectifier was presented. This development also clarified how the previously simplified equations can still be used to extract the fundamental conclusions regarding converter interactions while the complete expression allows a detailed determination of the magnitude and phase of every individual harmonic.

Chapter 5

Investigation of the Nature of Supraharmonic Emissions and Their Influencing Factors

5.1 Introduction

The aim of this chapter is to provide a deeper insight into the essence of the present work, namely signal analysis. The previous chapters were mainly devoted to predicting the magnitude of the frequency components appearing in voltage and current waveforms. In this chapter, the focus shifts to the signals themselves, independently of their physical nature, i.e. whether they represent voltages or currents. The following subsections successively discuss

- The switching function
- The effect of a non-constant DC voltage
- The impact of harmonic and supraharmonic components in the modulating wave on the double Fourier series
- The impact of the grid impedance on the supraharmonic emissions of the boost converter
- The distribution of supraharmonic emissions as a function of the grid impedance
- The fundamental nature of supraharmonic emissions
- A switching-function-based model

5.2 The switching function

The switching function is a fundamental analytical tool. It makes it possible to derive analytical developments and, ultimately, to predict the behaviour of power electronic

converters. These predictions naturally depend on the assumptions and approximations introduced throughout the derivations. Consequently, the agreement between analytical predictions and experimental measurements should not be interpreted as an intrinsic measure of the quality of the method itself. Rather, it reflects the extent to which the adopted assumptions and approximations accurately represent the physical system under consideration.

Let us first recall the concept of the switching function for pulse-width modulation (PWM). It consists of a sequence of rectangular pulses with an amplitude equal to one. The width of each pulse varies according to the PWM process, while the switching instants are determined by comparing two signals, commonly referred to as the carrier and the modulating wave, although any pair of comparison signals could be considered.

The switching function can be analysed both in the time and frequency domains. While its time-domain description is straightforward, its frequency-domain representation is of particular interest in the present work. A rectangular pulse can easily be expressed using Heaviside functions. Let $R(t)$ denote a rectangular pulse equal to one for $0 \leq t \leq T$ and zero elsewhere.

$$R(\omega) = \left[\pi\delta(\omega) + \frac{1}{j\omega} \right] - \left[\pi\delta(\omega) + \frac{1}{j\omega} \right] e^{-j\omega T} \quad (5.1)$$

Its time-domain representation together with its magnitude spectrum is shown in Fig. 5.1.

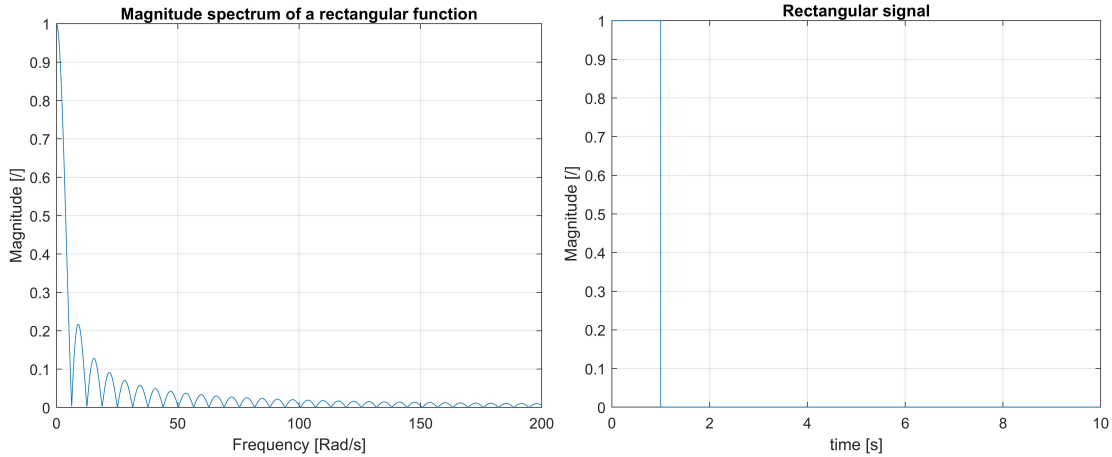


Figure 5.1: Rectangular function in the time domain together with its magnitude spectrum

The magnitude of the supraharmonic emissions is intrinsically related to this elementary rectangular function. Indeed, the PWM control signal is nothing more than a succession of rectangular pulses with varying widths, each pulse contributing to the overall spectral content of the switching function.

For a boost converter designed to regulate its DC-link voltage, the voltage across the switch can be written as

$$V_{switch}(t) = V_{DC}(t)S(t),$$

where $S(t)$ denotes the switching function, i.e. the sequence of rectangular pulses generated by the PWM strategy.

Once this interpretation is established, several important conclusions naturally follow. These are developed in the following sections.

5.3 Analysis of the Effect of a Non-Constant DC Voltage

In Chapter 3.2, the following equation was derived:

$$V_{in rect}(t) - L \frac{di(t)}{dt} = (1 - S(t))V_{DC}.$$

This expression assumes that the DC-link voltage remains essentially constant and therefore contains no significant harmonic components. It is now interesting to investigate the effect of introducing a perturbation, namely a harmonic component superimposed on the DC voltage. The previous equation therefore becomes

$$V_{in rect}(t) - L \frac{di(t)}{dt} = (1 - S(t))V_{DC}(t). \quad (5.2)$$

Assuming, for the moment, that the rectified input voltage does not contain harmonic components yields

$$-L \frac{di_{rip}(t)}{dt} = (1 - S(t))V_{DC}(t), \quad (5.3)$$

$$L \frac{di_{rip}(t)}{dt} = -V_{DC}(t) + S_{avg}(t)V_{DC}(t) + S_{rip}(t)V_{DC}(t). \quad (5.4)$$

The ripple voltage, denoted $V_{rip}(t)$, is therefore represented by the right-hand side of Equ. 5.4. Its Fourier-domain representation, $V_{rip}(\omega)$, is obtained by writing

$$V_{rip}(t) = -V_{DC}(t) + S_{avg}(t)V_{DC}(t) + S_{rip}(t)V_{DC}(t), \quad (5.5)$$

$$V_{rip}(\omega) = -(V_{DC} + V_{DC}(\omega)) + \frac{1}{2\pi} \left[S_{avg}(\omega) * (V_{DC} + V_{DC}(\omega)) \right] + \frac{1}{2\pi} \left[S_{rip}(\omega) * (V_{DC} + V_{DC}(\omega)) \right]. \quad (5.6)$$

Since neither the constant component of $V_{DC}(t)$ nor the average component of the switching function contributes directly to the ripple frequencies under investigation, these terms can be omitted when analysing the ripple spectrum $V_{rip}(\omega)$.

The comparison between this more general formulation and the simplified case presented in Chapter 3.2 is provided below. The notations $V_{rip V_{DC} Cst}$ and $V_{rip V_{DC} NCst}$ respectively denote the ripple voltage obtained under the assumptions of a constant DC voltage and a non-constant DC voltage.

$$V_{rip V_{DC} Cst}(\omega) = \frac{1}{2\pi} [S_{rip}(\omega) * V_{DC}] \quad (5.7)$$

$$\begin{aligned} V_{rip V_{DC} NCst}(\omega) = & -V_{DC}(\omega) + \frac{1}{2\pi} [S_{avg}(\omega) * V_{DC}(\omega)] + \frac{1}{2\pi} [S_{rip}(\omega) * V_{DC}(\omega)] \\ & + \frac{1}{2\pi} [S_{rip}(\omega) * V_{DC}] \end{aligned} \quad (5.8)$$

Taking V_{DC} as a pure constant allows one to move from Equ. 5.8 to Equ. 5.7. In this case, the DC voltage merely scales the initial spectral composition of the switching function, which is itself entirely dictated by the control objective, namely generating a signal as close as possible to the modulating wave.

The spectral composition of the voltage across the switch is therefore composed of

- The modulation of the average value of the switching function by the spectral composition of the DC voltage.
- The modulation of the spectral composition of the switching function by the spectral composition of the DC voltage.
- The spectral composition of the switching function scaled by the average value of the DC voltage.
- The spectral composition of the rectified voltage, which is preserved in the spectral composition of the voltage across the switch.

Two important conclusions can readily be extracted from the previously derived relations. They also provide a qualitative explanation for the experimentally observed absence of supraharmonic emissions from back-end converters in front-end/back-end converter systems [1].

- Using a sufficiently large DC-side capacitance minimizes the voltage ripple across the capacitor and therefore reduces the spectral content of the DC voltage.
- The presence of an active converter on the DC side, as in front-end/back-end systems, does not significantly affect the grid provided that the DC-link capacitance is sufficiently large.

5.4 Supraharmonics on the Modulating Wave: An Analytical Development

5.4.1 Introduction

The objective of this section is to introduce an additional level of complexity into the analytical development by considering the presence of pre-existing harmonics on the grid. Previous studies have shown that these harmonics can significantly affect the primary emissions of photovoltaic inverters, LED lamps, and electric vehicle chargers [98][99][100][101]. The proposed approach consists of incorporating these harmonics directly into one of the two variables of the double Fourier series. An alternative approach could also be considered. As discussed in Appendix B, every additional harmonic may be interpreted as an additional dimension of the Fourier series. Consequently, considering n pre-existing harmonics would require extending the original double Fourier series into an $(n + 2)$ -dimensional Fourier series. Although this idea is conceptually interesting, no further analytical development is carried out in the present work.

The objective is to derive the analytical expression of the input current of an active power factor correction (APFC) converter. The converter is controlled using pulse-width modulation (PWM) with a sawtooth carrier, which allows the use of the double Fourier series formulation. Ultimately, the objective is to derive the Fourier coefficients, which constitute the fundamental quantities required to determine the magnitude spectrum of the input current.

First, the original expression of the Fourier coefficients is recalled. The proposed modification is then introduced before presenting the complete analytical development leading to the adapted expression of the Fourier coefficients.

5.4.2 Analytical Development: Increasing the Complexity

5.4.2.1 First Part: Derivation of the Fourier Coefficients

Initial expression without considering oscillations on the modulating wave

$$A_{mn} + jB_{mn} = \frac{1}{2\pi^2} \left(\int_0^\pi \int_0^{2\pi(1-r\sin(y))} e^{j(mx+ny)} dx dy + \int_\pi^{2\pi} \int_0^{2\pi(1+r\sin(y))} e^{j(mx+ny)} dx dy \right) \quad (5.9)$$

Adapted expression of the Fourier coefficients accounting for oscillations on the modulating wave

$$A_{mn} + jB_{mn} = \frac{1}{2\pi^2} \left(\int_0^\pi \int_0^{2\pi(1-r\sin(y)+V_{rip})} e^{j(mx+ny)} dx dy + \int_\pi^{2\pi} \int_0^{2\pi(1+r\sin(y)+V_{rip})} e^{j(mx+ny)} dx dy \right) \quad (5.10)$$

where

$$V_{rip} = \hat{V}_{rip} \sin((m\omega_c + n\omega_g)t + \sigma_{I_{mn}} + \sigma_{Z_{L_{Boost}}}) \quad (5.11)$$

Defining A_1 as

$$A_1 = \int_0^{2\pi(1-r\sin(y)+V_{rip})} e^{j(mx+ny)} dx = \left[\frac{e^{j(mx+ny)}}{jm} \right]_0^{2\pi(1-r\sin(y)+V_{rip})} \quad (5.12)$$

$$= \frac{1}{jm} (e^{j(m2\pi(1-r\sin(y)+V_{rip})+ny)} - e^{jny}) \quad (5.13)$$

$$= \frac{1}{jm} (e^{jm2\pi} e^{-jm2\pi r\sin(y)} e^{jm2\pi V_{rip}} e^{jny} - e^{jny}) \quad (5.14)$$

Knowing that

$$e^{jm2\pi} = \cos(m2\pi) + j\sin(m2\pi) = \cos(m2\pi) = 1 \quad (5.15)$$

and

$$\begin{aligned} \int_0^\pi e^{jny} dy &= \left[\frac{e^{jny}}{jn} \right]_0^\pi = \frac{1}{jn} (e^{jn\pi} - 1) \\ &= \frac{1}{jn} ((-1)^n - 1) \end{aligned} \quad (5.16)$$

As n is even

$$= 0 \quad (5.17)$$

A_1 therefore becomes

$$A_1 = \frac{1}{jm} (e^{-jm2\pi r\sin(y)} e^{jm2\pi V_{rip}} e^{jny}) = \frac{1}{jm} (e^{j(-m2\pi r\sin(y)+ny)} e^{jm2\pi V_{rip}}) \quad (5.18)$$

The same procedure is then applied to the quantity denoted B_1 .

$$B_1 = \int_0^{2\pi(1+r\sin(y)+V_{rip})} e^{j(mx+ny)} dx = \left[\frac{e^{j(mx+ny)}}{jm} \right]_0^{2\pi(1+r\sin(y)+V_{rip})} \quad (5.19)$$

$$= \frac{1}{jm} (e^{j(m2\pi(1+r\sin(y)+V_{rip})+ny)} - e^{jny}) \quad (5.20)$$

$$= \frac{1}{jm} (e^{jm2\pi} e^{jm2\pi r\sin(y)} e^{jm2\pi V_{rip}} e^{jny} - e^{jny}) \quad (5.21)$$

As previously mentioned $\int_0^\pi e^{jny} dy = 0$ and actually $\int_0^\pi e^{jny} dy = 0 = \int_\pi^{2\pi} e^{jny} dy = 0$ thus for n even

$$B_1 = \frac{1}{jm} (e^{jm2\pi r\sin(y)} e^{jm2\pi V_{rip}} e^{jny}) = \frac{1}{jm} (e^{j(m2\pi r\sin(y)+ny)} e^{jm2\pi V_{rip}}) \quad (5.22)$$

Changing the bounds of the integral from $\int_\pi^{2\pi}$ towards $\int_0^\pi$ leads to

$$= \frac{1}{jm} (e^{j(m2\pi r\sin(y-\pi)+n(y-\pi))} e^{jm2\pi V_{rip}}) \quad (5.23)$$

Consequently, B_1 becomes

$$B_1 = \frac{1}{jm} (e^{j(-m2\pi r\sin(y)+n(y))} e^{jm2\pi V_{rip}}) \quad (5.24)$$

Eventually, Equ.2 is expressed as

$$A_{mn} + jB_{mn} = \frac{1}{2\pi^2} \int_0^\pi (A_1 + B_1) dy = \frac{1}{j\pi^2 m} \int_0^\pi e^{j(-m2\pi r\sin(y)+n(y))} e^{jm2\pi V_{rip}} dy \quad (5.25)$$

$$= \frac{1}{j\pi^2 m} \int_0^\pi e^{j(m2\pi(-r\sin(y)+V_{rip})+n(y))} dy \quad (5.26)$$

$$= \frac{1}{j\pi^2 m} \int_0^\pi \cos\left(m2\pi(-r\sin(y) + V_{rip}) + n(y)\right) \quad (5.27)$$

$$+ j\sin\left(m2\pi(-r\sin(y) + V_{rip}) + n(y)\right) dy$$

$$= \frac{1}{\pi^2 m} \int_0^\pi \sin\left(m2\pi(-r\sin(y) + V_{rip}) + n(y)\right) \quad (5.28)$$

$$- j\cos\left(m2\pi(-r\sin(y) + V_{rip}) + n(y)\right) dy$$

The Fourier coefficients can therefore be expressed as

$$A_{mn} = \frac{1}{m\pi^2} \int_0^\pi \sin\left(m2\pi(-r\sin(y) + V_{rip}) + ny\right) dy \quad (5.29)$$

$$B_{mn} = -\frac{1}{m\pi^2} \int_0^\pi \cos\left(m2\pi(-r\sin(y) + V_{rip}) + ny\right) dy \quad (5.30)$$

The expression of the Fourier coefficients is now rewritten and further generalized by introducing a possible phase shift, denoted ϕ , in the modulating wave.

Also, by defining

$$p_1 = \sigma_{I_{mn}} + \sigma_{Z_{L_{Boost}}} \quad (5.31)$$

$$p_2 = p_1 - \phi \left(\frac{m\omega_c}{\omega_g} - n \right) \quad (5.32)$$

the expressions of the Fourier coefficients can be rewritten as

$$A_{mn} = \frac{1}{m\pi^2} \int_\phi^{\pi+\phi} \sin\left(-m2\pi\left(-(\hat{V}_{rip} \sin(y(\frac{m\omega_c}{\omega_g} + n) + p_2)) + r \sin(y)\right) + ny\right) dy \quad (5.33)$$

$$B_{mn} = -\frac{1}{m\pi^2} \int_\phi^{\pi+\phi} \cos\left(-m2\pi\left(-(\hat{V}_{rip} \sin(y(\frac{m\omega_c}{\omega_g} + n) + p_2)) + r \sin(y)\right) + ny\right) dy \quad (5.34)$$

If an additional ripple is considered on the rectified voltage, with a prescribed magnitude and no phase shift, then $p_2 = 0$ and the Fourier coefficients simply become

$$A_{mn} = \frac{1}{m\pi^2} \int_\phi^{\pi+\phi} \sin\left(-m2\pi\left(-(\hat{V}_{rip} \sin(y(\frac{m\omega_c}{\omega_g} + n))) + r \sin(y)\right) + ny\right) dy \quad (5.35)$$

$$B_{mn} = -\frac{1}{m\pi^2} \int_\phi^{\pi+\phi} \cos\left(-m2\pi\left(-(\hat{V}_{rip} \sin(y(\frac{m\omega_c}{\omega_g} + n))) + r \sin(y)\right) + ny\right) dy \quad (5.36)$$

Adapting the theoretical model so that the Fourier coefficients are expressed as in Equ. 5.35 and Equ. 5.36 makes it possible to assess the expected variations resulting from an additional voltage ripple superimposed on the rectified voltage. One particular case is considered in the following, namely an additional ripple of 10 V peak with a frequency of $\omega_{add} = 2\pi 100 [\frac{rad}{s}]$. In practice, introducing the indices m and n is not mandatory for this particular perturbation. Consequently, the additional ripple can simply be written

$$V_{rip} = \hat{V}_{rip} \sin\left(\frac{y}{\omega_g} \omega_{add}\right) \quad (5.37)$$

The predictions obtained with the adapted DFS are now compared with the numerical results. The analysis first focuses on the rectified voltage before investigating the resulting input current.

Rectified voltage : not adapted

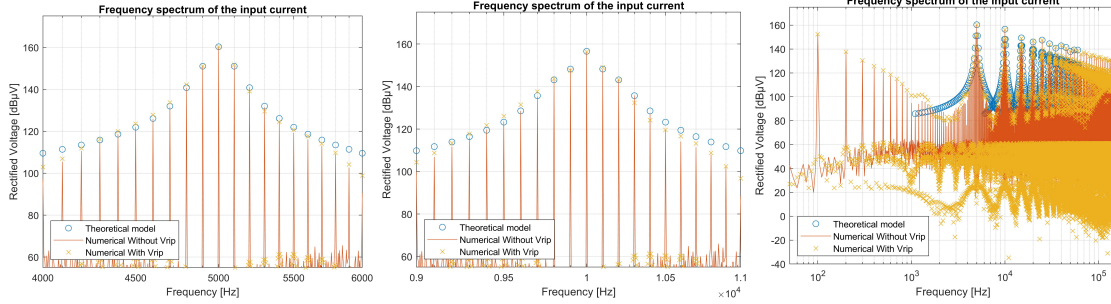


Figure 5.2: Comparison of the numerical results and the theoretical prediction, zoom on the first and second band and full range of the rectified voltage magnitude spectrum for an interleaved boost converter used as an active power factor corrector with $V_{DC} = 300V$ and $R = 1600\Omega$ with an added ripple on rectified voltage of 10 V with frequency 100 Hz

The introduction of an additional 10 V, 100 Hz ripple on the rectified voltage does not significantly modify the previously obtained results, although it does introduce small variations. Based on the previous analytical development, the proposed adaptation of the analytical model, which accounts for this additional harmonic, is expected to reduce the remaining discrepancies resulting from the introduction of the rectified voltage ripple.

Rectified voltage : adapted

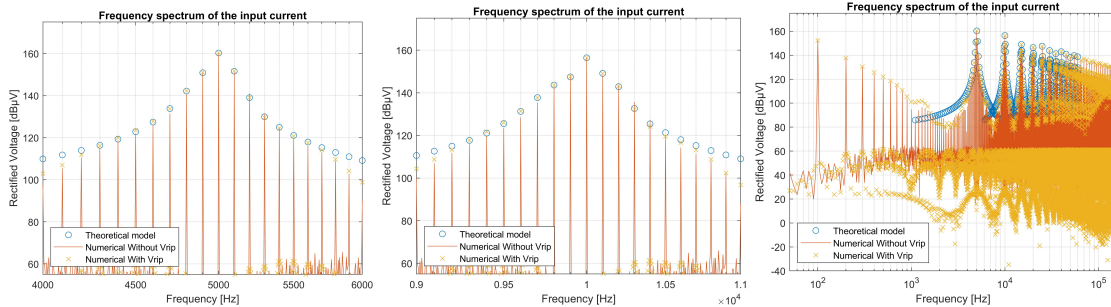


Figure 5.3: Comparison of the numerical results and the theoretical prediction, zoom on the first and second band and full range of the rectified voltage magnitude spectrum for an interleaved boost converter used as an active power factor corrector with $V_{DC} = 300V$ and $R = 1600\Omega$ with an added ripple on rectified voltage of 10 V with frequency 100 Hz

Even though the impact of the low-frequency ripple (100 Hz) remains limited, the adapted analytical model correctly reproduces the expected variations. This can be interpreted as the equivalent consideration of a modified modulating wave, as discussed in the previous subsection.

Afterwards, the same analysis is carried out for the input current and leads to the same conclusions. The adapted analytical model accurately captures the variations introduced by the additional ripple on the rectified voltage and follows the numerical results more closely than the original formulation. As the observed behaviour is identical to that obtained for the rectified voltage, no further discussion is required.

Input current : not adapted

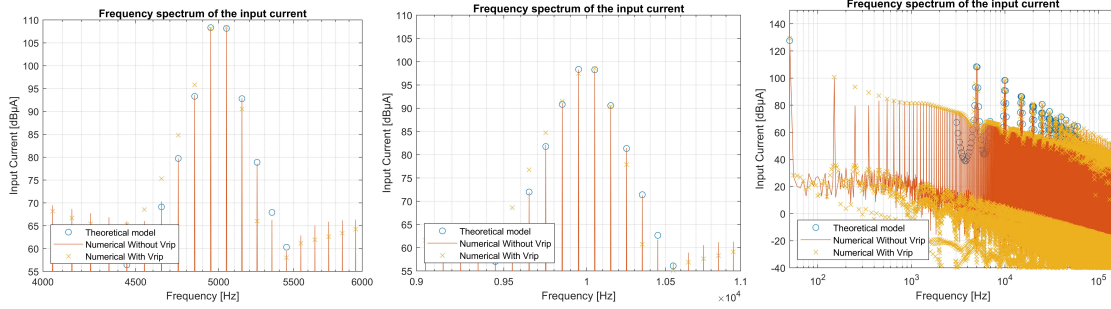


Figure 5.4: Comparison between the numerical results and the theoretical prediction, showing a zoom on the first and second carrier harmonic bands together with the full-range magnitude spectrum of the input current for an interleaved boost converter used as an active power factor corrector with $V_{DC} = 300$ V and $R = 1600$ Ω , considering an additional 10 V, 100 Hz ripple superimposed on the rectified voltage

Input current : adapted

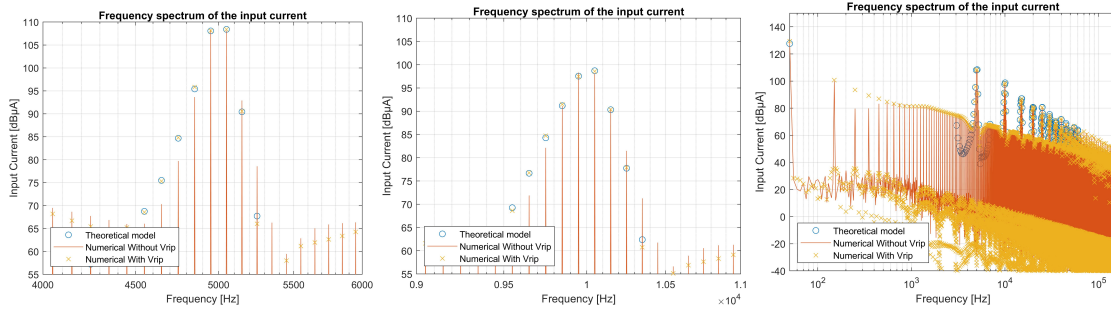


Figure 5.5: Comparison between the numerical results and the theoretical prediction, with a zoom on the first and second carrier harmonic bands and on the full-range magnitude spectrum of the input current for an interleaved boost converter used as an active power factor corrector with $V_{DC} = 300$ V and $R = 1600$ Ω , considering an additional 10 V, 100 Hz ripple on the rectified voltage

In the logical continuation of the analysis performed on the voltage, Fig. 5.4 and Fig. 5.5 demonstrate the capability of the adapted DFS to accurately follow the expected results regarding the emission of SHs under harmonic-polluted conditions.

5.4.2.2 Consideration of Usually Present Odd Harmonics

Another case is examined hereafter. Although the additional 10 V ripple at 100 Hz on the rectified voltage could originate from the 50 Hz fundamental and/or the 150 Hz

harmonic, a more realistic scenario consists in considering the odd harmonics commonly present on the grid voltage. The following results correspond to a case where the voltage total harmonic distortion (THD_v) is 8%, with the third, fifth, seventh, eleventh and thirteenth harmonics having magnitudes of 4, 5, 4, 2.5 and 2% of the fundamental voltage, respectively. It is worth noting that a phase angle of 0° with respect to the fundamental is assumed for each of these harmonics.

Voltage across the switch

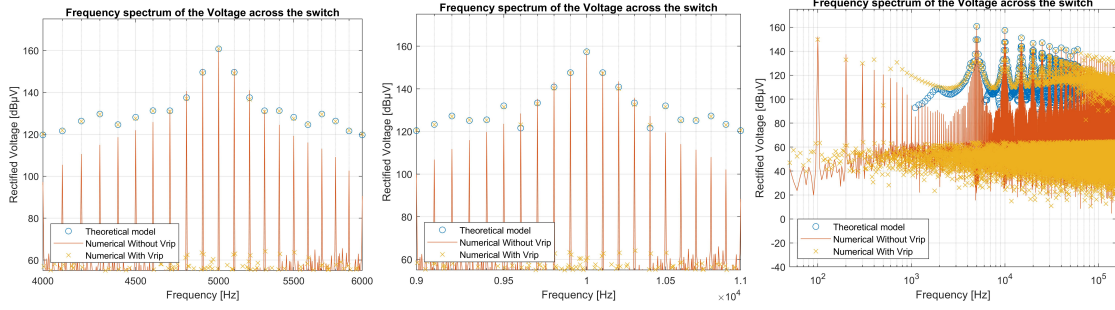


Figure 5.6: Comparison between the numerical results and the theoretical prediction of the supraharmonic emissions of a classical boost converter controlled by PI controllers and using the conventional pulse-width modulation strategy. The main parameters are $V_{DC} = 300$ V, $R = 800$ Ω , $L_{Boost} = 10$ mH, and $C_{Load} = 260$ μ F. The rectified voltage contains the typical odd harmonics (3rd, 5th, 7th, 11th, and 13th) with amplitudes equal to 4%, 5%, 4%, 2.5%, and 2% of the fundamental voltage, respectively

By applying the necessary adaptations, the capability of the DFS method to accurately predict supraharmonic emissions under realistic harmonic pollution has been demonstrated. This robustness also paves the way for extending the analytical development to more complex operating conditions, such as highly fluctuating supply voltages.

5.5 Impact of the Grid Impedance on the Supraharmonic Emissions of a Boost Converter Used as an APFC

5.5.1 Introduction

This section investigates the impact of the grid impedance on the supraharmonic emissions of a boost converter used as an APFC. Although the complete grid impedance is considered, particular attention is paid to its inductive component, as it introduces an additional nonlinear behaviour in the full-bridge diode rectifier (FBDR). First, the circuit under consideration is presented in Fig. 5.7.

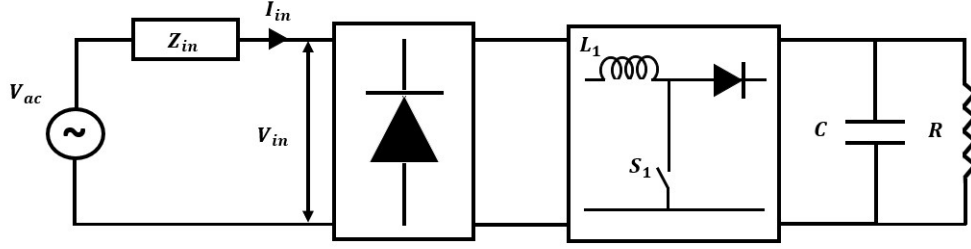


Figure 5.7: Complete circuit under study: AC voltage source followed by the grid impedance, denoted Z_{in} , a full-bridge diode rectifier, and a boost converter used as an APFC

The additional non-linear behaviour originates from the well-known relation $L \frac{di}{dt} = V_L$. The inductance maintains the current flowing through a diode even when the ideal FBDR would naturally stop conducting. As a result, the input voltage on the AC side of the FBDR exhibits discontinuities. Consequently, the voltage is no longer an ideal sinusoidal waveform but instead contains an extended time interval during which the input voltage remains equal to zero.

It is worth noting that this effect has two practical consequences that should be taken into account. First, although the fundamental frequency remains 50 Hz, the presence of discontinuities within each period requires an adaptation of the fundamental waveform considered in the DFS. Second, the time interval during which these discontinuities occur within the fundamental period should also be explicitly considered in the DFS.

In the following, the results are first presented without adaptation and then with the proposed adaptation in order to verify that the analytical modification effectively reduces the error associated with the presence of grid inductance, which introduces this additional non-linear behaviour.

5.5.2 Results without DFS adaptation

First, the signals under consideration are presented. The input voltage and the input current on the AC side of the FBDR are shown in Fig. 5.8.

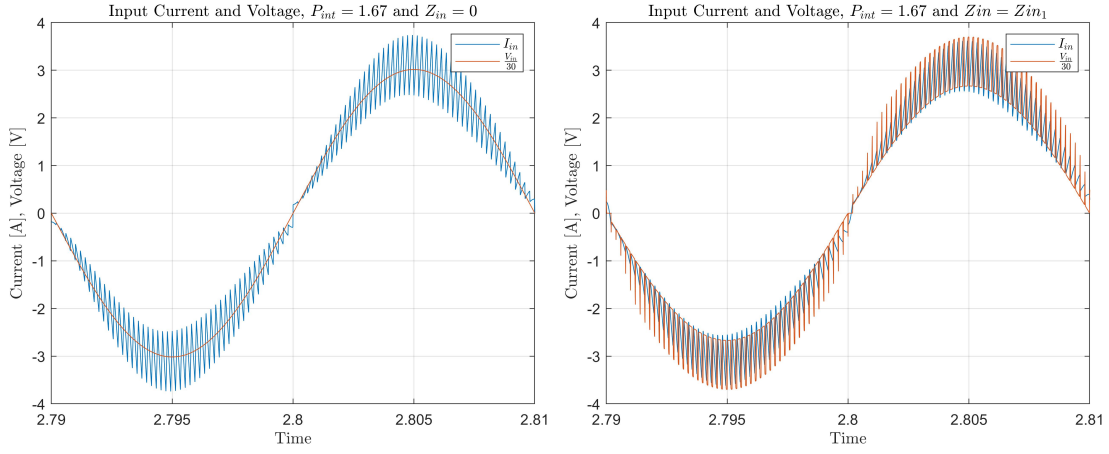


Figure 5.8: Numerical results: input voltage (V_{in}) and input current (I_{in}) on the AC side of the FBDR for an inner controller (pure gain) parameter $P_{int} = 1.67$, with a grid impedance of $0 \text{ } [\Omega]$ (left) and $Z_{in1} = j\omega 1.1 \times 10^{-3} + 0.4 \text{ } [\Omega]$ (right).

Even though the time interval during which the voltage remains equal to zero is relatively short, it still modifies the modulating wave considered in the DFS and therefore affects the analytical prediction. The magnitude spectrum of the input current is shown below for both cases without any adaptation of the DFS.

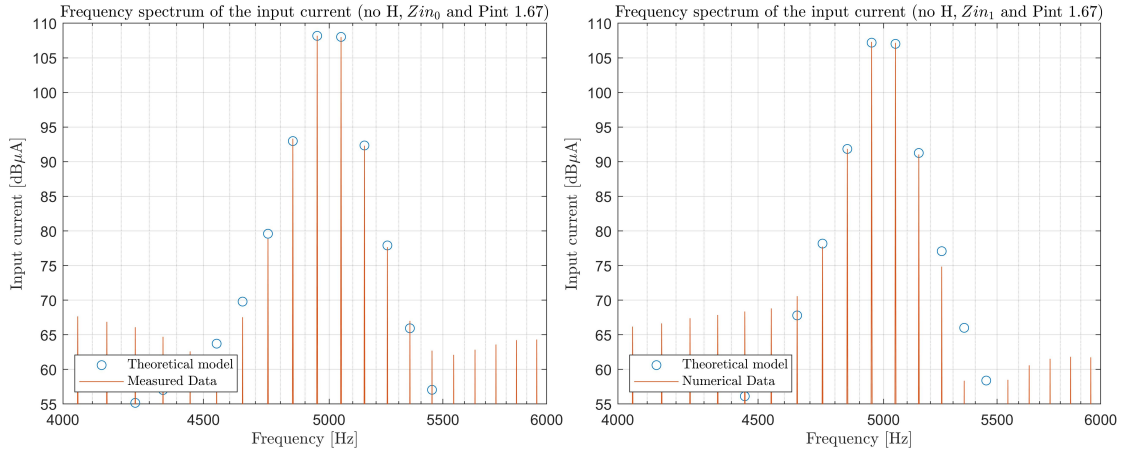


Figure 5.9: Numerical results: magnitude spectrum of the input current (I_{in}) on the AC side of the FBDR for an inner controller (pure gain) parameter $P_{int} = 1.67$, with a grid impedance of $0 \text{ } [\Omega]$ (left) and $Z_{in1} = j\omega 1.1 \times 10^{-3} + 0.4 \text{ } [\Omega]$ (right).

Although the error remains relatively small, it strongly depends on the grid inductance. Larger inductance values would therefore further increase the discrepancy between the analytical prediction and the numerical results. Nevertheless, the carrier harmonics, i.e. the centre harmonics at 4950 Hz and 5050 Hz, still follow the variations with high accuracy when the grid impedance is taken into account. Indeed, the dominant effect of the grid impedance on these components is not the additional non-linear behaviour but rather the increase of the impedance at the corresponding frequencies due to its inductive part. In contrast, the additional non-linear behaviour mainly affects the sideband harmonics.

5.5.3 Results with DFS adaptation

The previous results show that the error associated with the sideband harmonics increases when the grid impedance is non-zero. The analytical adaptation proposed to account for this behaviour is now introduced together with the corresponding results.

The required adaptation consists in accounting for the time interval during which the input voltage of the FBDR remains equal to zero. Since this voltage constitutes the modulating wave used in the DFS, this modified waveform must be explicitly considered in the analytical development.

Two approaches may be considered. The first consists in reconstructing the input voltage by individually introducing all its harmonic components so that the analytical waveform matches the numerical one as closely as possible. However, this approach would require an infinite number of harmonics, since every harmonic present on the AC side is transferred to the rectified side, where each component is further decomposed into an infinite series of $+n50$ Hz and $-n50$ Hz components, with n being an odd integer ranging from $-\infty$ to ∞ due to the rectification process. The second approach directly modifies the integration bounds of the DFS so that the modulating wave is considered only over its effective conduction interval. Although both approaches are theoretically equivalent, the second one is adopted here because of its practical implementation.

The representations of the modulating wave before and after introducing the grid impedance are presented below to illustrate the influence of the grid inductance on the waveform.

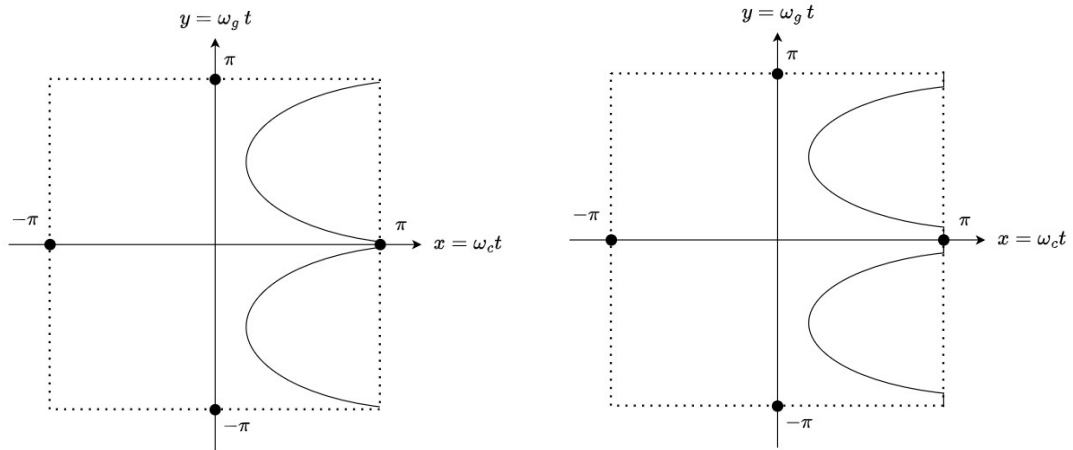


Figure 5.10: DFS x - y plane with $x = \omega_c t$ and $y = \omega_g t$. Comparison of the modulating wave for $Z_{in} = 0$ (left) and $Z_{in} = Z_{in1}$ (right), with $Z_{in1} = j\omega 1.1 \times 10^{-3} + 0.4 [\Omega]$.

Two modifications are taken into account. The first concerns the time interval during which the input voltage remains equal to zero, which strongly depends on the grid inductance and therefore increases the influence of the grid impedance on the accuracy of the analytical prediction. The second concerns the slight modification of the fundamental frequency. Since the waveform contains a discontinuous interval, its effective period becomes

shorter, resulting in a slightly higher fundamental frequency. The magnitude spectrum of the input current obtained without and with the proposed adaptation is presented in Fig. 5.11.

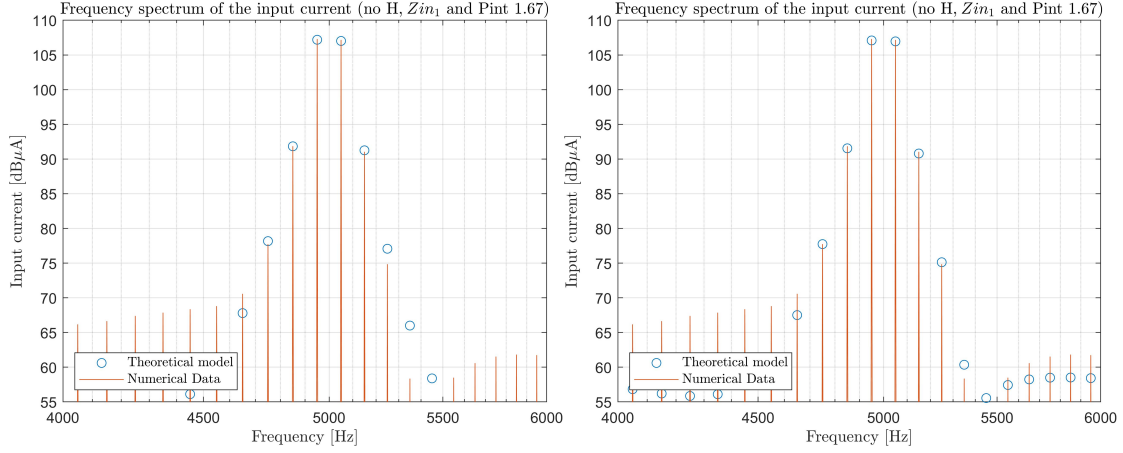


Figure 5.11: Numerical results: magnitude spectrum of the input current (I_{in}) on the AC side of the FBDR before (left) and after (right) applying the proposed DFS adaptation, for $P_{int} = 1.67$ and $Z_{in1} = j\omega 1.1 \times 10^{-3} + 0.4 [\Omega]$.

For the sideband harmonics located close to the carrier harmonic, i.e. near the centre of each band, the proposed adaptations significantly reduce the prediction error. However, the method was not designed to accurately reproduce the harmonics located farther from the centre of the sidebands, which still exhibit noticeable discrepancies with the numerical results. This behaviour is observed both with and without grid impedance, indicating that it originates from another phenomenon. Among the contributing factors, the controller design has a significant influence on these distant sideband harmonics. The larger the controller bandwidth, the smaller the sideband harmonics become, although the SH emissions are then progressively attenuated by the control itself, which is its intended function. Nevertheless, the analytical method accurately predicts the carrier harmonics and the neighbouring sideband harmonics, which correspond to the highest emission peaks. It is worth noting that these accurately predicted components are typically one to two orders of magnitude larger than the sideband harmonics located far from the centre of the bands, which are less accurately predicted.

5.6 Consideration of the Grid Impedance Variation

Several aspects have already been investigated, including the effect of EMI filters, the presence of harmonics in the grid voltage, and the influence of carrier phase shifts between converters connected in parallel. It is also possible to investigate the impact of grid impedance variations on the final SH emissions produced by the studied converter. First, the relationship between variations of the transformer inductance and the resulting SH emissions is analysed. The following discussion is restricted to periodic steady-state operation, i.e. once the transient response has vanished. The transient behaviour naturally introduces additional complexity and is therefore beyond the scope of this work.

Before extending the discussion, it is useful to recall that the analytical expression of the SH voltage across the transistor of an APFC converter was shown to depend on two main contributions. The first is the DC voltage, which is assumed to be constant in the present analysis. The second is the magnitude of the Fourier coefficients of the switching function, denoted X_{mn} .

The second part can be affected by several phenomena, although the mechanism through which X_{mn} is modified remains the same. Indeed, the switching function is merely an analytical description of the intersections between the modulating and carrier waves. The modulating wave is generated by the converter control, based on the DC voltage reference and the power factor correction objective. Therefore, the control is the only mechanism through which X_{mn} can be affected. Within the control, the rectified voltage is directly used and therefore contains harmonics, although these are normally filtered out. In the present case, a feed-forward term is also employed, which directly contains the rectified voltage.

Based on the previous explanation, it can be concluded that the first source of emissions, namely the generated supraharmmonic voltage across the transistor, remains essentially unchanged as long as the rectified voltage does not exhibit significant anomalies.

However, all the results presented throughout this work assume that the inductances in the circuit remain constant. The case in which the transformer inductance suddenly increases is investigated hereafter, together with the resulting consequences on the supraharmmonic emissions. Let us first consider the simplified circuit shown in Fig. 5.12, which only includes the inductances, since at supraharmmonic frequencies the inductive impedances constitute the dominant part of the total impedance, provided that the skin effect is neglected, which is one of the assumptions made in this work.

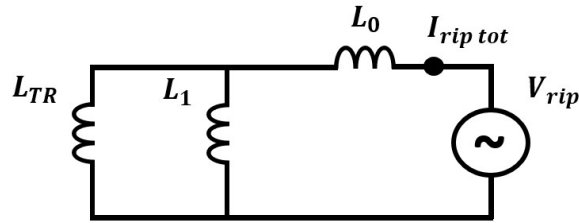


Figure 5.12: Simplified representation of the emitter connected to three inductances: its own inductance, L_0 , the transformer represented by its inductance L_{TR} , and the remainder of the grid represented by L_1 . This simplified model is motivated by the frequency range under investigation (supraharmronics) and by the assumption of negligible skin effect.

The total ripple current, $I_{rip\ tot}$, is indeed affected. This impact can be quantified by considering the relative values of the inductances L_1 and L_{TR} with respect to L_0 .

The impact of a variation of the transformer inductance is readily understood. Indeed, the total impedance seen by the emitting converter changes. The magnitude of this variation depends on the relative values of the different inductive elements. As a result, one can conclude that, depending on the considered context, the SH current emission may remain

more or less close to its initial value. The SH voltage, on the other hand, is assumed to remain constant according to the assumptions and approximations made regarding both the grid voltage and the DC voltage.

For the current emission to remain almost constant, two main conditions have to be fulfilled.

1. The variation of the transformer inductance should not induce a significant variation of the grid voltage.
2. The equivalent inductance of the remainder of the circuit, modelled as L_1 in Fig. 5.12, should be small compared with the transformer inductance. Under this condition, simple circuit theory shows that if $\frac{L_{TR}}{L_1} = 10$, the variation of the current I_1 when the transformer inductance increases towards infinity is only about 10% of its nominal value.

To provide a clearer picture of the expected variations, several realistic examples are presented in the following figures together with tables containing all the necessary parameters.

Table 5.1: Passive element values, switching frequency, and nominal supraharmonic voltage considered in the study.

	L_0 [mH]	L_1 [mH]	L_{TR} [mH]	F_{sw} [kHz]	$V_{tot\ nom}$ [V]
Case 1	10	2	1	5	80
Case 2	10	2	2	5	80
Case 3	10	2	5	5	80
Case 4	10	2	10	5	80

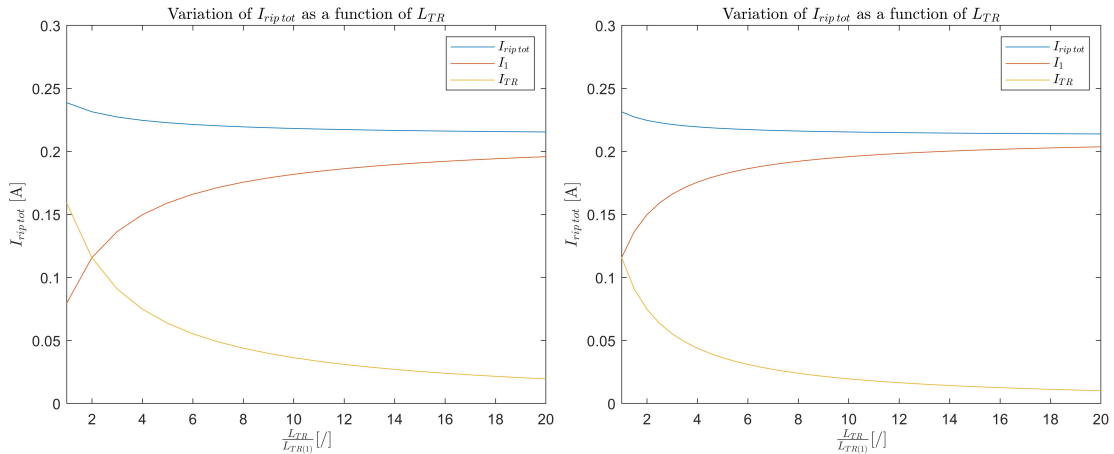


Figure 5.13: Case 1 (left) and Case 2 (right): variation of the total current (blue), branch 1 current (red), and branch 2 current (yellow) for nominal transformer inductances of 1 mH (left) and 2 mH (right).

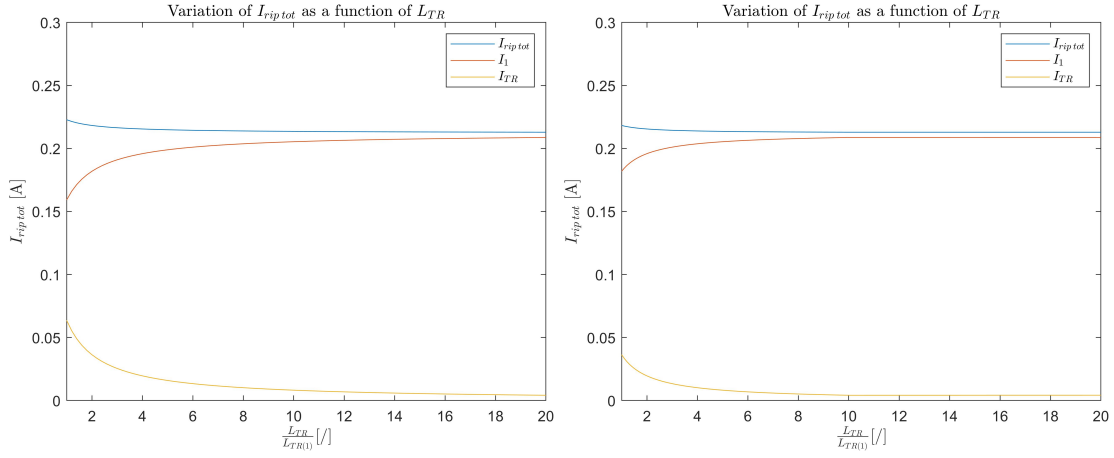


Figure 5.14: Case 3 (left) and Case 4 (right): variation of the total current (blue), branch 1 current (red), and branch 2 current (yellow) for nominal transformer inductances of 5 mH (left) and 10 mH (right).

These results simply indicate that, under the two previously stated assumptions, the current emissions can be analysed using classical circuit theory, even though the initial generation of these supraharmonic currents originates from controlled nonlinear power electronic converters. Indeed, once the converter behaviour has been characterised, the nonlinear source can ultimately be represented by an equivalent voltage source, in the case of an APFC whose objective is to maintain a constant DC voltage. Consequently, the sharing of supraharmonic currents between the two branches, which may represent another converter (or several converters) and the transformer branch, i.e. ultimately the grid, follows the classical laws of circuit theory. Similar observations were also reported in [53].

5.7 Nature of the Supraharmonic Source

In Chapter 3, a method was introduced to model a power electronic converter using two sub-models. The first describes the low-frequency dynamics (50 Hz), whereas the second represents the high-frequency dynamics (SH). Conceptually, it is interesting to decouple these two sub-models and analyse them in a pseudo-isolated manner. Although interactions between them still exist, they remain implicitly embedded in the modelling.

However, it may appear surprising that the frequency composition of the SH current is not directly related to the instantaneous grid voltage. Nevertheless, it still depends on its peak value, although in a less explicit manner, as previously explained. Indeed, the high-frequency sub-model contains a voltage source that alternates between V_{DC} and 0 depending on the switching instant.

From a frequency-domain perspective, this behaviour can be interpreted by decomposing the voltage source of the high-frequency sub-model into a Fourier series over the period of the rectified voltage. This decomposition is performed using the double Fourier series. Consequently, the switching function contains all the information describing the switching state transitions of the voltage source. It is therefore possible to interpret the converter

using the equivalent high-frequency sub-model introduced in Chapter 3. This interpretation is also fully consistent with the analytical development presented in Chapter 3, which showed that, when only SH currents are of interest, the remaining excitation reduces to $S_{ripple}(t)V_{DC}$. Nevertheless, this interpretation must also be validated numerically. Therefore, comparisons are carried out between the complete IBC model, including its control, and a simplified system reproducing only the equivalent high-frequency voltage source.

First, the equivalent circuit is shown in Fig. 5.15. Two equivalent representations are provided. The first directly represents the source as a rectangular waveform taking the values V_{DC} or 0, whereas the second reproduces the physical implementation, i.e. a DC source controlled by two switches. Both circuits are strictly equivalent.

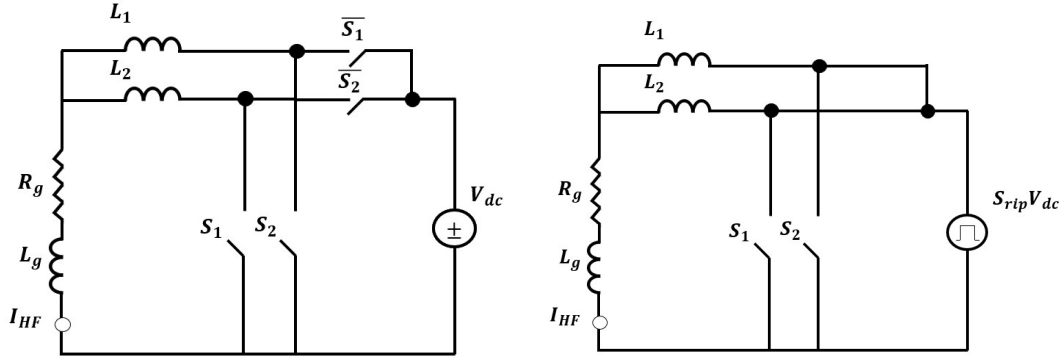


Figure 5.15: Equivalent high-frequency model of the supraharmonic emissions of an interleaved boost converter used as an active power factor corrector. The left-hand side shows the physical implementation, whereas the right-hand side presents the equivalent representation that directly reflects the analytical expression derived for the equivalent high-frequency voltage source.

It is worth noting that, in the actual circuit, current cannot flow from the DC source towards the remainder of the circuit. Consequently, the high-frequency model is only equivalent in the sense that it reproduces the same current variation, although with the opposite slope. In other words, whenever the current decreases in the real circuit, it increases in the equivalent high-frequency model. The magnitude of the variation remains identical, which is the feature of interest.

The switches connected to the DC source are introduced to reproduce the behaviour of the real converter as closely as possible. In the actual converter, the DC source causes the current to decrease only when switches S_1 or S_2 are open. Therefore, the equivalent model uses the complementary switching signals, namely not S_1 and not S_2 . Consequently, when S_1 is ON, the DC source is disconnected and the current decreases, whereas when S_1 is OFF, the DC source is connected and the current increases. This arrangement removes the low-frequency source while preserving both the DC source and the same high-frequency behaviour.

Finally, the current spectrum is analysed using the theoretical model, the complete numerical model including both the controller and the low-frequency source, and the equivalent high-frequency model.

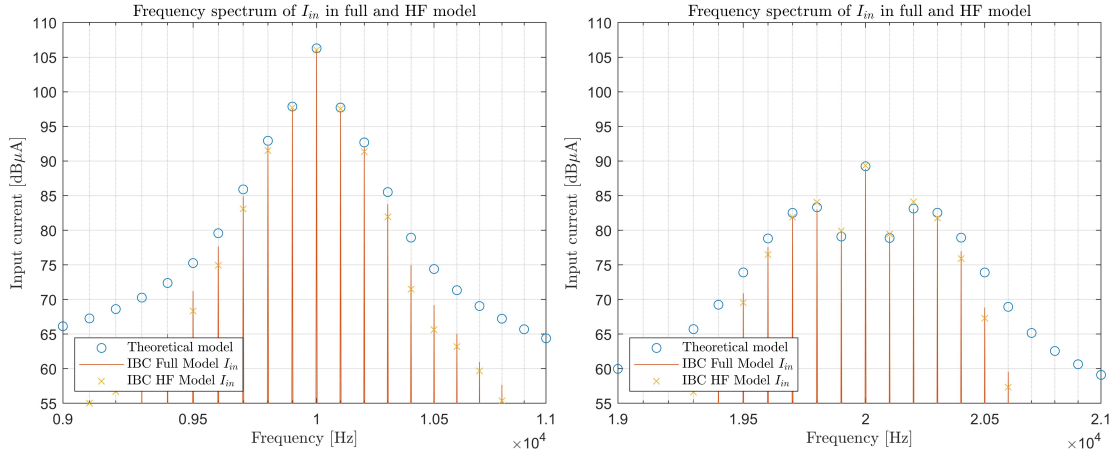


Figure 5.16: Comparison of the supraharmonic current spectrum obtained using the theoretical model, the complete numerical model, and the equivalent high-frequency numerical model.

These results could be reproduced for different values of the passive components or switching frequencies. However, the objective here is to provide a physical interpretation of the nature of the SH source. It is shown that the low-frequency source is only required to determine the switching instants. Once these instants have been obtained analytically, no low-frequency source is required. Indeed, the switching function can be used directly to drive the switches of the equivalent high-frequency model while producing nearly identical SH emissions.

This observation highlights the ability of simplified numerical models to accurately predict SH emissions without requiring a complete converter model. More importantly, it provides additional insight into the physical nature of the SH source, whose mathematical description was previously derived analytically. For converters supplied by a constant DC voltage, the converter can ultimately be represented as a supraharmonic voltage source whose characteristics are entirely contained in the switching function. Multiplied by the DC voltage, this switching function defines the equivalent SH voltage source.

5.8 Switching-Function-Based Model

5.8.1 Introduction

The idea behind this modelling approach naturally follows from the developments presented in this chapter, namely that the high-frequency emissions, referred to as supraharmonics, can ultimately be reduced to one key function: the switching function.

In the first section of this chapter, the switching function was described in both the time and frequency domains using the well-known Heaviside function. Indeed, a succession of variable-width rectangular pulses, i.e. the switching function, can be analytically represented by combining Heaviside functions. It is therefore straightforward to understand that, by summing a sufficient number of Heaviside functions, one can reconstruct one complete period of the switching function. This concept is illustrated in Fig. 5.17.

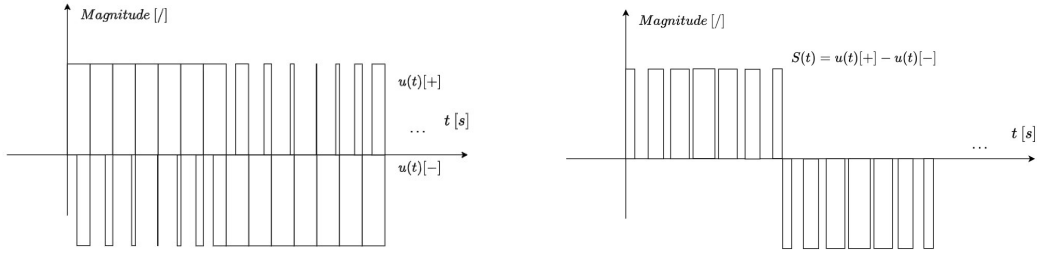


Figure 5.17: Representation of the positive ($u(t)^{+}$) and negative ($u(t)^{-}$) Heaviside functions (Left) and their sum (Right), which forms the switching function $S(t)$.

The entire concept of obtaining a universal model for supraharmmonic emissions relies on the description of one period of the switching function. As the switching function represents the control signal of the switch, it can be directly associated with the physical consequences of each switching event. In the case considered here, namely a power electronic converter connected in parallel with a sufficiently large capacitor so that the DC voltage can be assumed constant, each switching event initially produces a variation of the voltage across the switch. This voltage variation constitutes the very origin of what is referred to as a supraharmmonic.

Consequently, from a purely theoretical point of view, the model presented here is not restricted to power electronic converters. Instead of describing the voltage across a switch, it could equally describe another physical quantity, such as pressure. Indeed, a clear analogy exists between these two concepts. Voltage is expressed as energy per unit charge, whereas pressure can be expressed as energy per unit volume. By considering a unit charge for voltage and a unit volume for pressure, both quantities are ultimately related to the same concept of energy. Therefore, the concept of the switching function can be extended to any physical system driven by a control signal. As long as this control signal is periodic, it can be represented by a Fourier series and, in the particular case considered in this work, by a double Fourier series since it depends on two independent periodic signals.

5.8.2 Expressing One Period of the Switching Function

Taking into account the following definitions

- $t_0 = 0$;
- j denotes the switching period under consideration;
- D is the vector containing the duty cycle associated with each switching period j ;
- T_s is the switching period.

The general expression of the switching function can then be written as

$$S(jT_s) = \sum_{j=0}^{N-1} \left(u(t - jT_s) - u(t - D(j+1)T_s - jT_s) \right) \quad (5.38)$$

Once the switching function $S(jT_s)$ has been obtained, the discrete Fourier transform (DFT) can be applied to determine the complex coefficients from which both the magnitude and the phase of each frequency component can be extracted. Applying the DFT to the switching function leads to Equ. 5.39, where k denotes the index of the frequency component and $\frac{2\pi}{t_{tot}}$ is the frequency resolution, with t_{tot} being the total observation time. It is worth noting that the Nyquist criterion imposes that the maximum observable frequency must not exceed half the sampling frequency.

$$S_k = \left(\sum_{j=0}^{N-1} S_j e^{-i2\pi \frac{jk}{N}} \right) \frac{1}{N} \quad (5.39)$$

Eventually, for a classical boost converter operating with negligible pre-existing grid harmonics and assuming a constant DC voltage, the magnitude spectrum of the voltage across the switch is simply given by Equ. 5.40 and illustrated in Fig. 5.18.

$$V_{ripple\ switch}(\omega) = \frac{1}{2\pi} [S(\omega) * V_{DC}] \quad (5.40)$$

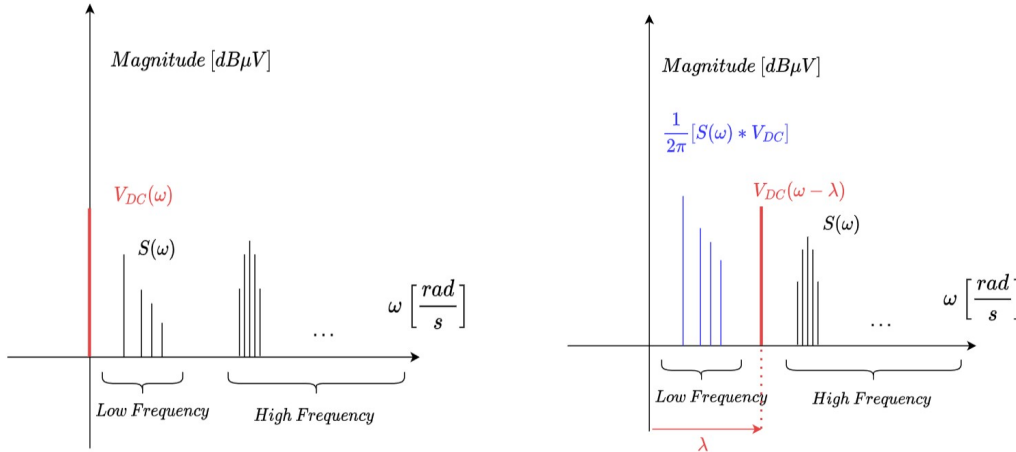


Figure 5.18: Magnitude spectra expressed in $dB\mu V$ of the DC voltage and the switching function before convolution (Left) and the resulting voltage across the switch after convolution (Right).

5.8.3 Application to an Active Power Factor Corrector Circuit: The Particular Case of the Boost Converter

The objective of this subsection is to apply the previously developed concept to a specific converter topology. The boost converter operating as an active power factor corrector (APFC) is considered. First, the analytical expression of the duty cycle, denoted by D , is introduced. Subsequently, the expression of the voltage across the switch is derived from the switching function. Finally, the predictions of the proposed universal model are compared with numerical simulations.

Equation 5.38 is employed with the duty-cycle vector D expressed as

$$D(k) = 1 - \frac{V_{ac\,peak}}{V_{DC}} \sin(\omega_g t(k) + \phi_0) \quad (5.41)$$

where

- $V_{ac\,peak}$ is the peak value of the grid voltage;
- V_{DC} is the DC output voltage of the boost converter;
- ω_g is the grid angular frequency;
- ϕ_0 is the phase angle of the grid voltage.

Assuming that no pre-existing grid harmonics are present and that the DC voltage remains constant, the ripple voltage across the switch is directly given by Equ. 5.40.

Finally, the comparison between the proposed universal model and the numerical simulations for a boost converter operating as an APFC is presented in Fig. 5.19.

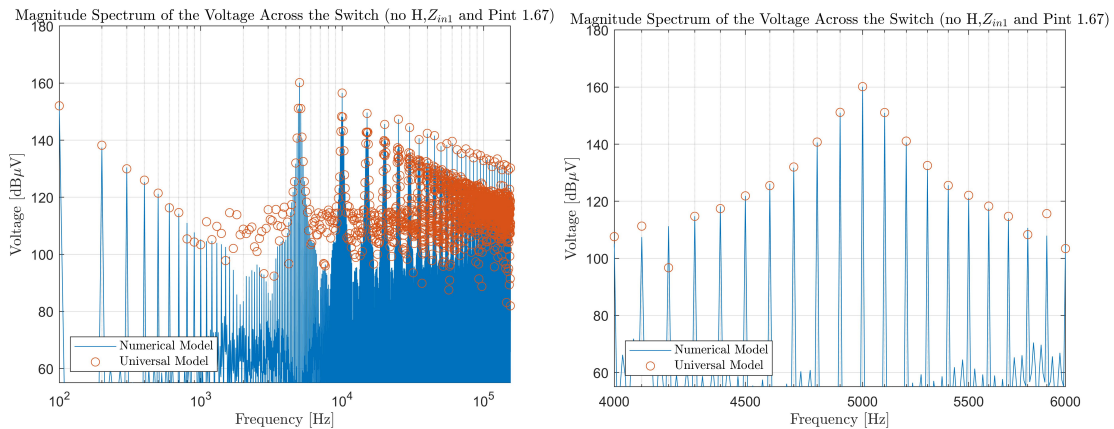


Figure 5.19: Comparison of the magnitude spectrum of the voltage across the switch obtained with the universal model and the numerical model for a boost converter used as an active power factor corrector circuit. All frequency bands (Left) and first frequency band only (Right).

5.8.4 Application to Classical Full-Bridge Single-Phase Inverter Topology

Regarding the classical single-phase inverter topology, the same methodology as the one presented in the previous subsection is followed.

The duty cycle, denoted by D , is expressed as

$$D(k) = \frac{V_{ac\ peak}}{V_{DC}} \sin(\omega_g t(k) + \phi_0 + \phi_1) \quad (5.42)$$

where

- $V_{ac\ peak}$ is the peak value of the grid voltage;
- V_{DC} is the DC voltage;
- ω_g is the grid angular frequency;
- ϕ_0 is the phase angle of the grid voltage;
- ϕ_1 is the phase angle introduced by the active and reactive power injection control.

Based on the knowledge of the duty-cycle expression, it is then possible to obtain the switching function, which can be expressed as a sum of Heaviside functions according to Equ. 5.38.

Finally, the comparison between the predicted high-frequency emissions at the point of common coupling (PCC) obtained using the universal modelling concept and the numerical full-model simulation for the particular case of a classical single-phase inverter is presented in Fig. 5.20.

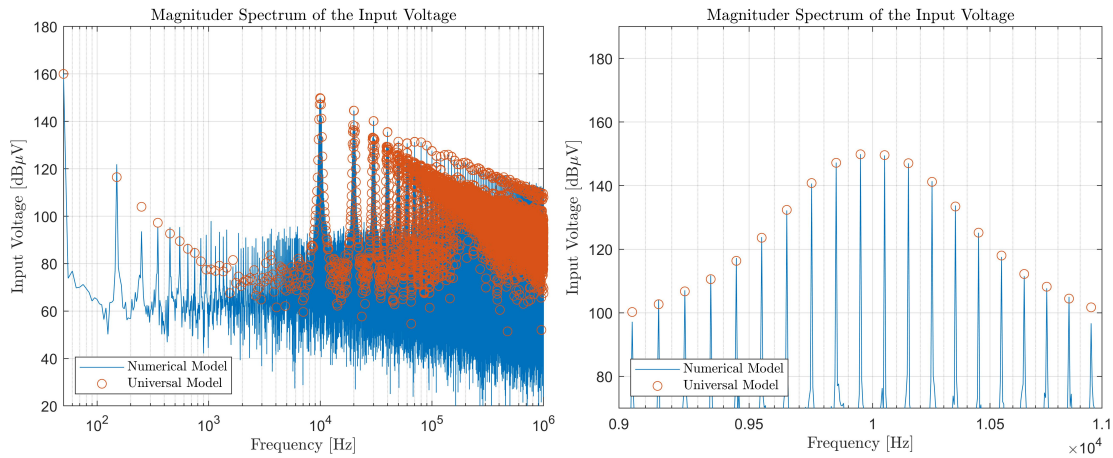


Figure 5.20: Comparison of the magnitude spectrum of the voltage at the point of common coupling obtained using the universal model and the numerical model for the particular case of a classical full-bridge single-phase inverter. All frequency bands (Left) and first frequency band only (Right).

5.8.5 Conclusion on Switching-Function-Based Modelling

In this section, a concept referred to as universal modelling has been developed. The objective was to provide a global perspective on the main topic of this thesis, namely the prediction of high-frequency emissions (supraharmonics). The introduced concept is an analytical development based on the fundamental idea that the switching function is the key element for supraharmonic prediction.

Once the significant importance of the switching function had been established, it was possible to derive its analytical expression based on Heaviside functions. This eventually led to the determination, in both the time and frequency domains, of the voltage across the switch.

Subsequently, the concept of universal modelling was applied to both APFC and inverter circuits. The obtained results were then compared in the frequency domain for both systems, leading to accurate predictions of the voltage across the switch(es).

However, the concept developed here is not limited to power electronic systems and can be extended to any controlled and periodic system.

The universal modelling concept developed here is general and does not initially consider practical aspects such as the grid impedance, which can modify high-frequency emissions in both magnitude and phase. In order to provide accurate predictions even when such initially neglected practical aspects significantly influence the emissions, a numerical extension of the universal modelling concept has been developed and previously presented. This approach is referred to as analytically generated signals (AGS).

Chapter 6

Conclusion

In this thesis, two main points were of interest:

- The methods for supraharmmonic modelling
- The study of typical converters present in LV grids

Both aspects were deeply analysed and conclusions were provided throughout the thesis. The aim here is to provide a comprehensive summary in order to highlight the key outcomes of this work. To achieve this objective, this chapter is divided into two sections: the conclusions regarding the modelling methods and the conclusions regarding power electronic device emissions.

6.1 On the Methods

In the literature, several approaches can be found to address the modelling of supraharmmonic emissions [1],[2],[4] or [12]. However, analytical methods have not been extensively investigated [1], [3], [5], even though all studies agree on the interest and potential of such approaches. Indeed, the time required for complete circuit simulations (numerical experiments) is generally greater than several minutes or tens of minutes depending on the circuit complexity and simulation time step, while the analytical models proposed in this thesis address the same issue within a few seconds. It is worth noting that after simulation, the generated data still require post-processing.

Therefore, having an analytical method that is both accurate and computationally efficient is of significant value. This objective has been achieved in this thesis for two types of converters: active power factor correctors (APFCs) and classical single-phase inverters. The results obtained for APFCs were compared with experimental measurements and showed high accuracy, with the analytical predictions and measured emissions exhibiting close agreement in terms of magnitude.

However, while studying converters that maintain a constant DC voltage, the author identified the possibility of developing a more general method for predicting supraharmmonic emissions generated by power electronic devices. This led to the development of the ana-

lytically generated signal (AGS) method. The principle is to obtain the modulating wave from the operating principle of the converter and, based on this information, regenerate the signals through a more or less complex algorithm in order to reproduce the signals that would have been obtained through long-duration simulations. Once again, the time required to execute the complete algorithm and perform the post-processing was approximately 2 seconds for a classical boost converter, while maintaining the same accuracy as the analytically rigorous double Fourier series method.

After extending the DFS method and developing the AGS method, several limitations were identified. The objective here is to briefly summarize them. When APFCs are considered, it was demonstrated and explained throughout this thesis that the low-frequency harmonics are affected by the presence of grid impedance. This is mainly due to the inductive component of the grid impedance, which modifies the grid voltage by introducing intervals during which the voltage becomes equal to zero and therefore deviates from a purely sinusoidal waveform. By considering the impact on the grid voltage, it was possible to adapt the DFS method and bring the main results closer to the numerical predictions. Regarding inverters, the control is directly applied to the switches, creating an equivalent behaviour to a full-bridge diode rectifier with variable frequency. This leads to the conclusion that a similar phenomenon to that observed for APFCs occurs, but with different consequences. A summary of these consequences is provided below:

- For inverters, the grid inductance induces modifications of the supraharmonic emission magnitudes.
- For APFCs, the grid inductance induces modifications of the low-frequency (LF) harmonic magnitudes.

However, regarding the specific case of inverters, even though the DFS method was strongly limited by the presence of grid inductance, the error remained limited when the grid inductance corresponded to a line length equal to or smaller than 1 km. Conversely, the AGS method did not encounter any difficulty in considering this non-linear and unpredictable behaviour of the inverter. The corresponding results were presented, and the conclusions regarding the modelling methods were established.

6.2 On the Power Electronic Devices Emissions

While the modelling methods were essential to obtain the presented results, once the operating conditions were such that the methods did not present fundamental limitations, other points of interest emerged, such as the impact of EMI filters on the final supraharmonic emissions or the voltage harmonic source and current harmonic source behaviour of power electronic devices. Several additional questions were addressed in this thesis, and a non-exhaustive list is presented below:

- Impact of the EMI filter

EMI filters, which in the present study correspond to differential-mode filters, are typically designed to attenuate emissions above 150 kHz. However, despite this design objective, they cannot be considered neutral within the supraharmonic fre-

quency range. This has been demonstrated in the present thesis.

A concise conclusion can be formulated as follows: increasing the filter capacitance while decreasing the filter inductance leads to the capture of dominant emissions at lower frequencies. Such a design modification could therefore be considered as a means of achieving local emission management. In this context, the role of EMI filters should be reconsidered—not only as devices intended to reduce emissions above 150 kHz, but also as a first line of protection against the propagation of supraharmmonic disturbances into low-voltage networks.

- Impact of having V_{DC} not constant

The DC voltage constitutes the supply voltage across which the switching device operates, opening and closing in order to generate the desired function, commonly referred to as the modulating wave.

The underlying idea is that if the DC voltage is not perfectly constant, its non-zero spectral content must be considered. From a frequency-domain perspective, the DC voltage can no longer be regarded as a purely constant component, but rather as a signal exhibiting its own spectral distribution. Consequently, the interaction between the switching function and a non-ideal DC voltage results in convolution operations in the frequency domain.

This convolution mechanism leads to the transfer of high-frequency components initially present in the DC voltage towards the grid side. In other words, high-frequency disturbances contained in the DC-link voltage can propagate towards the grid due to the modulation process itself.

- Impact of supraharmmonic and low-frequency harmonics present on the grid

An additional question addressed in this work concerns the possible impact of low-frequency harmonics, such as those commonly observed at 150, 250, 350, and 550 Hz, on emissions within the supraharmmonic frequency range.

The main conclusion is that the dominant emissions, which appear as two principal components due to the modulation induced by the diode bridge, can be affected by the presence of these low-frequency harmonics. However, the secondary emission components are more significantly impacted as the considered frequency moves further away from the principal emission frequency.

It should nevertheless be emphasized that, in the present analysis, the low-frequency harmonics were assumed to have a zero phase shift with respect to the fundamental grid voltage component. Consequently, an important aspect has not been fully investigated and is briefly discussed here. Depending on the phase relationship between the low-frequency harmonics and the fundamental component, transient conduction phenomena may occur. This means that the presence of harmonics in the grid voltage may temporarily force some diodes into conduction, independently of the intrinsic operating conditions of the connected device.

Such phase-dependent conduction effects may introduce additional transient mech-

anisms influencing supraharmonic emission patterns and therefore require further investigation.

- Link between V_{DC} , device power rating and supraharmonic emissions

It has been mathematically demonstrated that a relationship exists between the emission amplitude and the ratio between the low-voltage grid amplitude and the DC voltage amplitude. However, this relationship is inherently non-linear and can be described through the consideration of specific terms of a Bessel function expansion. Further details are provided in the section dedicated to the development of this analytical framework.

Finally, it has been demonstrated that no direct relationship exists between the rated power of a device and the amplitude of its supraharmonic emissions. This result constitutes one of the major contributions of the present thesis.

- Invariance of the emissions with the active power of the controlled inverter
- ...

The most important outputs of this thesis regarding the emissions of power electronic devices (APFCs and inverters) can be summarized as follows:

- The emissions are mainly a function of the ratio between V_{DC} and the grid voltage amplitude.
- No trivial prediction can be made regarding the variation of emissions with the variation of the ratio between V_{DC} and the grid voltage, as this relationship is governed by non-linear functions.
- Supraharmonics are most likely to propagate between devices through the low-impedance path created by EMI filters.
- The type of EMI filter strongly impacts the contribution of secondary emissions, as LCL filters significantly reduce these components.
- Non-linear and unpredictable behaviours can emerge from converter control strategies. These behaviours do not significantly affect the low-frequency operation but can have a substantial impact on supraharmonic emissions.
- Grid impedance affects high-frequency harmonics in inverter systems.
- Grid impedance affects low-frequency harmonics in APFC systems.

6.3 Perspectives

The work accomplished in this thesis can be considered as a step towards a deeper understanding of high-frequency harmonics in low-voltage electrical networks. The main contributions of this research have already been presented. The purpose of this section is therefore to draw the necessary conclusions in order to identify the fundamental aspects that should be addressed in future research devoted to this topic.

Accordingly, this section aims to outline the next steps required to extend and consolidate the understanding of supraharmonic phenomena and, more generally, of electromagnetic compatibility (EMC), which is inherently influenced by these high-frequency disturbances.

Several aspects appear to be particularly important for future investigations.

First, the study of the propagation of high-frequency emissions represents a major research direction. Such an investigation should build upon the theoretical framework developed in this thesis while considering that lumped-parameter modelling approaches become insufficient at higher frequencies. As the frequency increases and the propagation distance becomes significant compared with the wavelength, distributed-parameter models become necessary. This requirement is directly related to the equations presented in the chapter dedicated to the validity domain of the modelling assumptions.

Second, particular attention should be given to the evolving nature of electrical networks. The increasing number of connected devices results in a growing presence of EMI filters within the network. These filters cannot be considered neutral with respect to supraharmonic phenomena. Their influence has been investigated in this thesis, and the corresponding conclusions have been summarized in the previous section. Nevertheless, further systematic studies are required in order to better quantify their collective and interactive effects at the network level.

In addition, the development of modelling methodologies suitable for converters exhibiting non-constant emission frequencies represents an important research direction. Many modern power electronic converters operate with variable switching frequencies, introducing additional complexity in both analytical and numerical modelling. Robust and adaptable modelling frameworks are therefore essential to address these systems.

Finally, although the experimental measurements performed in this thesis showed good agreement with both analytical and numerical predictions, systematic emission measurements on widely commercialized converters would provide valuable additional insight. Such investigations would enable comparative analyses and contribute to a more comprehensive assessment of real-world supraharmonic emission levels.

Appendix A

Measurement Methods

Throughout this chapter, the interested reader will find a presentation of the measurement methods that can be used to measure signals within the frequency range of interest, i.e. the supraharmonic range. Some specific aspects are also discussed, such as the limitations inherent to signal analysis. Indeed, one should be aware of the limitations associated with the tools employed. However, these aspects, together with the corresponding theoretical background, are presented in Appendix B. More precisely, two major concerns are investigated: frequency aliasing and spectral leakage.

A.1 Introduction

Even though no normative method currently exists, numerous studies have investigated this topic. Indeed, Appendix B of the IEC 61000-4-7 standard presents a measurement method dedicated to the frequency range from 2 to 9 kHz [88]. Furthermore, IEC 61000-4-30 and CISPR 16-2 also consider this topic [89], [102]. However, these methods are informative and lead to differences depending on the selected approach.

This absence of a normative method represents a critical issue. Indeed, in order to objectively assess the importance of supraharmonics, it is first necessary to be able to measure them in a reproducible manner. In [103], a discussion regarding the conditions required for reproducible measurements can be found, including the importance of characterizing the power supply voltage.

A significant amount of research has been carried out, each study highlighting the lack of standardization and attempting to compare the different classical informative methods [104], [105], [106]. Before discussing some results from these studies, the classical informative methods are presented hereafter.

A.1.1 IEC 61000-4-7

This standard deals with measurements up to 9 kHz, and the corresponding method can be found in Appendix B.

The underlying principle is the same as the one used for frequencies below 2 kHz, i.e.

applying a discrete Fourier transform (DFT) to a signal over an interval of 200 ms, leading to a frequency resolution of 5 Hz. The final result is obtained by grouping the frequency components into 50 Hz subgroups.

The main difference concerns the bandwidth of the considered groups, which is 200 Hz for this method. This approach could theoretically be extended to the entire frequency range considered in this thesis (up to 150 kHz).

A.1.2 IEC 61000-4-30

The measurement method can be found in Appendix C and is briefly described here.

It is dedicated to measurements within the frequency range of interest, i.e. 2–150 kHz. The principle is the following: ten periods of the signal are recorded and divided into 32 intervals of 0.5 ms each. These intervals are then processed using a DFT. Finally, the maximum, minimum, and average values of the root mean square (RMS) voltage are recorded for each frequency component.

A.1.3 CISPR 16-2-1

This standard provides a measurement method for conducted disturbance phenomena within the frequency range from 9 kHz to 30 MHz.

It is based on the use of specific detectors (defined in CISPR 16-1). Three types of detectors exist, and the selection of one detector over another depends on the expected type of emission, i.e. narrowband or broadband.

- Average Current Detector: Narrowband
- Quasi-Peak Detector: Narrowband – Broadband
- Peak Detector: Narrowband – Broadband

Basically, the methodology presented in [102] and illustrated in Fig. A.1 aims at minimizing the measurement time required to determine whether the equipment under test (EUT) complies with the limitations imposed by the standards.

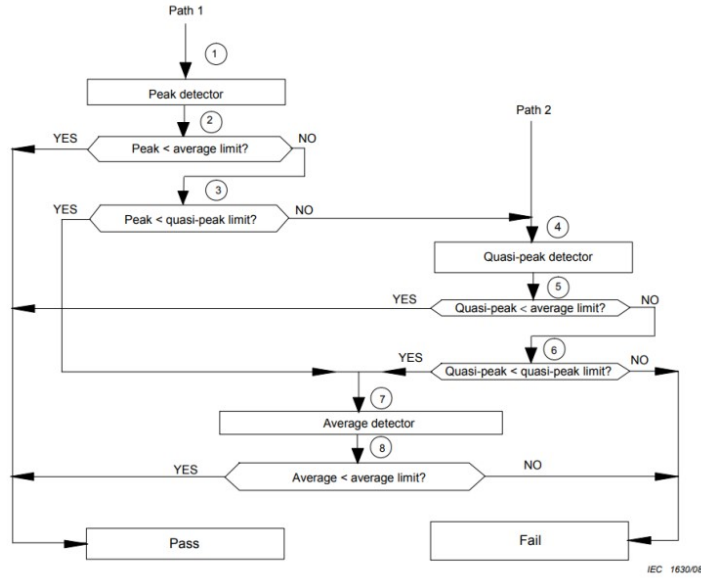


Figure A.1: Decision tree for optimizing speed of conducted disturbance measurements with peak, quasi-peak and average detectors

Receivers and detectors incorporated into these measurement systems are described in CISPR 16-1.

It can therefore be concluded that no normative method currently exists, leading to difficulties such as variations in the results depending on the selected measurement method. This statement is supported, among others, by [1], [4], and [107].

A.1.4 Chosen method for experimental setup

The classical measurement methods for supraharmonics have been presented, and some relevant results are discussed hereafter. Basically, one method can be selected over another depending on its limitations, both in terms of measurement time and the required investment cost.

In this thesis, the selected method is based on two criteria. First, the ease of implementation is considered. Second, the accuracy of the obtained results is evaluated with respect to a reference method. The chosen reference corresponds to the value obtained using the method presented in [102], i.e. the CISPR 16-2 method. Indeed, this method is used for the definition of compatibility levels in IEC 61000-2-2 [106], [107].

The first criterion is easily satisfied by the method presented in IEC 61000-4-7, requiring only a measurement device with a sufficient sampling rate and specific conditions regarding the analysed signal, such as a window of 10 cycles at 50 Hz (200 ms), resulting in a frequency resolution of 5 Hz.

Indeed, particular attention must be given to the risks associated with spectral leakage and frequency aliasing (or folding), which are discussed in greater detail in the following section.

The second criterion is also fulfilled by the method presented in IEC 61000-4-7 [106], as illustrated in Fig. A.2 from [106].

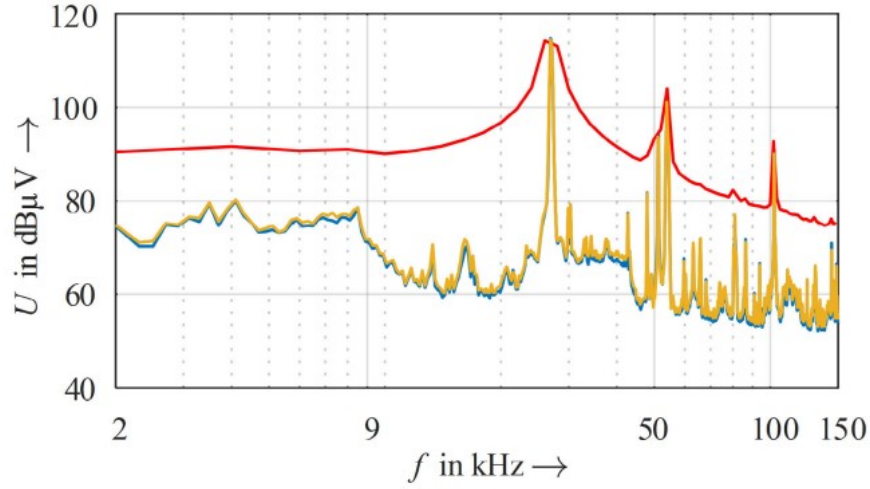


Figure A.2: Comparison of three measurement methods, IEC 61000-4-7 (Blue), IEC 61000-4-30 (Red) and CISPR 16-2 (Yellow) on a laboratory measured signal

The state of the art presented in this section, together with the two previously mentioned criteria, highlights the relevance of the method described in IEC 61000-4-7.

Additionally, as mentioned in [4], most high-frequency measurements reported in the literature rely on this IEC 61000-4-7 method. Among the studies applying this approach, one can cite [108], [109], [110], [111], and [112].

Considering all the presented information, the author selected the measurement method defined in IEC 61000-4-7 for further experimental investigations.

Appendix B

Theoretical Background

B.1 Signals and Systems

B.1.1 Introduction

The material presented here can mainly be found in [93, 56]. This first appendix presents some fundamental elements of signal analysis. It is useful for providing a complete understanding of the limitations presented in Chapter 2. Indeed, it provides a clear illustration of the aliasing issue, which can potentially lead to frequency folding. Furthermore, spectral leakage is defined and its consequences are highlighted.

B.1.2 Fourier series and transform

B.1.2.1 Fundamental idea

In 1807, Jean-Baptiste Joseph Fourier presented his work in which the fundamental concept for frequency analysis was introduced. Indeed, he stated that any periodic signal can be represented as the sum of a set of sinusoidal functions with appropriate magnitudes, phases, and frequencies. The resulting equations are the Fourier series and the Fourier transform, which are respectively used for periodic and non-periodic signals. It should be noted that the Fourier transform can be interpreted as considering a periodic signal with an infinitely large period.

The Fourier series is simply expressed as a sum of cosine and sine functions with magnitudes depending on the considered frequency. The fundamental equation of the Fourier series is presented in Equ. B.1

$$f(t) = \sum_{k=0}^{\infty} (A_k \cos(k\omega t) + B_k \sin(k\omega t)) \quad (\text{B.1})$$

With ω being the fundamental frequency and $k\omega$ representing the set of corresponding harmonics. Basically, this equation states that a periodic function $f(t)$ is equivalent to

the sum of an infinite number of sinusoidal functions. It should also be noted that for $\omega = 0$, a constant value is obtained, which corresponds to the mean value of the signal.

Another notation can be found using the complex form, as presented in Equ. B.2.

$$f(t) = \sum_{k=0}^{\infty} c_k e^{jk\omega t} \quad (\text{B.2})$$

B.1.3 Frequency analysis

Based on the idea introduced by Fourier, many fields have exploited the possibility of describing any function as a sum of sinusoidal functions. Fourier series have been extended to classical mechanics as well as quantum mechanics, but they are also found in the original field for which they were developed, namely heat propagation. Other fields, such as engineering, have been significantly influenced by this concept and extensively use the equations presented hereafter. Indeed, in electrical engineering, it is common practice to consider the frequency response of a system frequency by frequency. Many aspects could be further discussed; however, no deeper explanation is provided at this point.

Let a signal be defined in the frequency domain. The time-domain signal is denoted $f(t)$ and the frequency-domain signal is denoted $F(\omega)$. The definition provided in Equ. B.2 contains the Fourier coefficients, denoted c_k , which can be expressed as

$$c_k = \frac{1}{T} \int_0^T f(t) e^{-jk\omega t} dt \quad (\text{B.3})$$

Considering that $f(t) = \lim_{T \rightarrow \infty} \hat{f}(t)$, one can obtain the expression of $F(\omega_k)$ as follows

$$\begin{aligned} F(\omega_k) &= T c_k \\ k\omega &= \omega_k \\ T &= \frac{2\pi}{\delta\omega} \\ \hat{f}(t) &= \sum_{k=0}^{\infty} c_k e^{jk\omega t} \\ &= \sum_{k=0}^{\infty} F(\omega_k) e^{j\omega_k t} \frac{\delta\omega_k}{2\pi} \\ F(\omega_k) &= \int_0^T \hat{f}(t) e^{-j\omega_k t} dt \end{aligned} \quad (\text{B.4})$$

Indeed, when T tends towards infinity, ω_k and $\delta\omega$ tend towards ω and $d\omega$, respectively, and the final expression becomes

$$F(\omega) = \int_0^{\infty} f(t)e^{-j\omega t} dt \quad (\text{B.5})$$

Eventually, the final expression of a signal in the frequency domain is obtained as the integral of the time-domain signal multiplied by the complex exponential term, which contains a cosine and sine component at frequency ω . However, it should be noted that the expression in Equ. B.5 corresponds to the one-sided Fourier transform.

B.1.4 Sampling theory and frequency folding

The Fourier series has been presented, and the expression of a signal in the frequency domain has also been introduced. However, in practice, analysed signals are not continuous. Indeed, continuous signals would require an infinite number of samples and would consequently provide infinite resolution in both the frequency and time domains. In practice, one must consider the limited sampling frequency and therefore the limited frequency resolution.

First, let us understand how a sampled signal is obtained. A continuous signal is composed of an infinite number of points; however, only a finite number of these points can be considered due to technological limitations. The way these points are selected can be mathematically described as follows. Let us consider the Dirac impulse function, denoted $\delta(t)$. Its definition implies that it exists only at $t = 0$, meaning that before and after this point, the function is equal to zero. The area of this function is defined as $\int_{-\infty}^{\infty} \delta dt = 1$. One could even be more restrictive by considering only the interval between 0^- and 0^+ . Nevertheless, this function, the Dirac impulse, is used to construct the discrete-time signal required for sampling. Once the Dirac impulse is understood, the Dirac comb can be introduced as presented in Equ. B.6.

$$\delta_{Ts}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kTs) \quad (\text{B.6})$$

To obtain a sampled signal, one only needs to consider the following equation

$$f_s(t) = f(t)\delta_{Ts}(t) \quad (\text{B.7})$$

As previously demonstrated, the expression of a signal in the frequency domain is obtained by applying Equ. B.5. By recalling that a time-domain signal can be expressed as in Equ. B.2, one can determine the Fourier coefficients of the Dirac comb. Basically, the Fourier coefficients of $\delta_{Ts}(t)$ can be obtained using Equ. B.3 and are finally found to be equal to $1/T_s$. Consequently, the sampled signal in the frequency domain is expressed as

$$\begin{aligned}
F_s(\omega) &= \mathcal{F}[f(t)\delta_{Ts}(t)] \\
&= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \mathcal{F}[f(t)e^{jk\omega_s t}] \\
&= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} F(\omega - k\omega_s)
\end{aligned} \tag{B.8}$$

B.1.4.1 Frequency aliasing

Having Equ. B.8 allows the understanding of the issue related to frequency aliasing. Indeed, the frequency composition of a signal is repeated every $k\omega_s$, which means that for a particular value of ω_s , a critical phenomenon can occur, namely frequency folding. This specific value of ω_s should be at least two times greater than the maximum frequency contained in the signal in order to avoid frequency-domain overlapping. This condition leads to the well-known Nyquist frequency, defined as $\omega_N = \frac{\omega_s}{2}$, where the latter must be greater than or equal to the maximum frequency present in the signal.

B.1.4.2 Spectral leakage

Spectral leakage is one of the crucial limitations in digital signal processing. Together with frequency aliasing, spectral leakage increases the risk of misinterpreting the results of a discrete Fourier transform. The principle of spectral leakage can be explained by the inherent operating principle of the discrete Fourier transform. The latter considers, instead of a portion of the signal with an infinite number of zeros added before and after the considered segment, a signal that is periodically repeated in the time domain from $-\infty$ to ∞ . This assumption makes the computation possible, as it limits the number of sinusoidal functions required to reconstruct the signal in the time domain, in contrast with the discrete-time Fourier transform, which requires an infinite decomposition into sinusoidal functions.

Knowing that the considered signal is assumed to repeat itself exactly without limitation, it is easily understood that applying the discrete Fourier transform to a portion of a signal that does not contain an integer number of cycles for a given frequency induces discontinuities in the reconstructed signal. However, this issue can be moderated by considering that the consequences depend on the considered frequency. Indeed, in many signals, a decreasing magnitude can be observed as the frequency increases. Consequently, a non-integer number of cycles at a high frequency component will generally have less impact than a non-integer number of cycles at a low-frequency component. Nevertheless, each case remains specific and should be analysed individually.

B.1.4.3 Fourier transform properties

The Fourier transform is linear, meaning that the superposition principle applies. This property is particularly useful when explaining the consequences of discontinuities. Indeed, the superposition property can be mathematically expressed as

$$\mathcal{F}[a + b] = \mathcal{F}[a] + \mathcal{F}[b] \quad (\text{B.9})$$

If a signal noted $f(t) = \cos(\omega_0 t)$ is considered. Its fourier transform is expressed as $\mathcal{F}[f(t)] = \mathcal{F}[\cos(\omega_0 t)]$ and is equal to $\pi(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$ basically this trivial results express that an infinite support signal in time with an infinite energy in the sense of signal processing, i.e. $\int_{-\infty}^{\infty} |f(t)| dt = \infty$, induces in the frequency domain two dirac impulse functions with location being in $-\omega_0$ and ω_0 .

The goal of this section is to show how spectral leakage impact the frequency domain representation of a signal that contains discontinuities. Indeed, in the numerical tools used to perform such kind of frequency analysis, the discrete Fourier transform (DFT) is widely used. It is worth noting that this DFT considers a repetition of a portion of signal from $t = -\infty$ towards $t = \infty$ meaning it considers the signal periodic. It is thus easily understood that by taking a number of cycle that is not an integer one (for each frequency present on the analysed signal) induces a repetition of that portion of signal that will not fit and will presents discontinuities.

Hereafter, three situations will be presented along with their analytical frequency spectrum expression

- 1) Discrete signal with one discontinuity

$$f_2(t) = f_s(t)(1 - u(t - T) + u(t - (T + \alpha)))$$

- 2) Discrete signal with periodic discontinuities

$$f_3(t) = f_s(t)(1 - u(t - n(T + \alpha) + \alpha) + u(t - n(T + \alpha)))$$

- 1) Discrete signal with one discontinuity

$$\mathcal{F}[f_2(t)] = \mathcal{F}[f_s(t)(1 - u(t - T) + u(t - (T + \alpha)))] \quad (\text{B.10})$$

$$= \mathcal{F}[f_s(t)] - \mathcal{F}[f_s(t)u(t - T)] + \mathcal{F}[f_s(t)u(t - (T + \alpha))] \quad (\text{B.11})$$

$$\mathcal{F}[f_s(t)] = \mathcal{F}[\delta_{T_s}(t)f(t)] = \frac{1}{T_s} \sum_k \mathcal{F}(\omega - k\omega_s)$$

$$\mathcal{F}[f_s(t)u(t - T)] = \frac{1}{2\pi} (\mathcal{F}[f_s(t)] * \mathcal{F}[u(t - T)])$$

$$\mathcal{F}[f_s(t)u(t - (T + \alpha))] = \frac{1}{2\pi} (\mathcal{F}[f_s(t)] * \mathcal{F}[u(t - (T + \alpha))])$$

$$\mathcal{F}[u(t - T)] = [\pi\delta(\omega) + \frac{1}{j\omega}]e^{-j\omega T}$$

$$\mathcal{F}[u(t - (T + \alpha))] = [\pi\delta(\omega) + \frac{1}{j\omega}]e^{-j\omega(T+\alpha)}$$

$$\mathcal{F}[f_s(t)] = \frac{1}{T_s} \sum_k (\pi\delta(\omega + \omega_0 - k\omega_s) + \pi\delta(\omega - \omega_0 - k\omega_s))$$

$$\mathcal{F}[f_2(t)] = \mathcal{F}[f_s(t)] - \frac{1}{2\pi} (\mathcal{F}[f_s(t)] * \mathcal{F}[u(t - T)]) + \frac{1}{2\pi} (\mathcal{F}[f_s(t)] * \mathcal{F}[u(t - (T + \alpha))]) \quad (\text{B.12})$$

$$\begin{aligned} &= \mathcal{F}[f_s(t)] - \frac{1}{2\pi} \frac{1}{T_s} \sum_k (\pi\delta(\omega + \omega_0 - k\omega_s) + \pi\delta(\omega - \omega_0 - k\omega_s)) [\pi\delta(\omega) + \frac{1}{j\omega}] e^{-j\omega T} \\ &\quad + \frac{1}{2\pi} \frac{1}{T_s} \sum_k (\pi\delta(\omega + \omega_0 - k\omega_s) + \pi\delta(\omega - \omega_0 - k\omega_s)) [\pi\delta(\omega) + \frac{1}{j\omega}] e^{-j\omega(T+\alpha)} \end{aligned} \quad (\text{B.13})$$

$$\begin{aligned} &= \mathcal{F}[f_s(t)] - \frac{1}{2\pi} \frac{1}{T_s} \sum_k \left(\pi \frac{e^{-j(\omega + \omega_0 - k\omega_s)T}}{j(\omega + \omega_0 - k\omega_s)} + \pi \frac{e^{-j(\omega - \omega_0 - k\omega_s)T}}{j(\omega - \omega_0 - k\omega_s)} \right) \\ &\quad + \frac{1}{2\pi} \frac{1}{T_s} \sum_k \left(\pi \frac{e^{-j(\omega + \omega_0 - k\omega_s)(T+\alpha)}}{j(\omega + \omega_0 - k\omega_s)} + \pi \frac{e^{-j(\omega - \omega_0 - k\omega_s)(T+\alpha)}}{j(\omega - \omega_0 - k\omega_s)} \right) \end{aligned} \quad (\text{B.14})$$

$$\begin{aligned} &= \mathcal{F}[f_s(t)] + \frac{1}{2T_s} \sum_k \left(\frac{e^{-j(\omega + \omega_0 - k\omega_s)T}}{j(\omega + \omega_0 - k\omega_s)} (e^{-j(\omega + \omega_0 - k\omega_s)\alpha} - 1) \right. \\ &\quad \left. + \frac{1}{2T_s} \sum_k \left(\frac{e^{-j(\omega - \omega_0 - k\omega_s)T}}{j(\omega - \omega_0 - k\omega_s)} (e^{-j(\omega - \omega_0 - k\omega_s)\alpha} - 1) \right) \right) \end{aligned} \quad (\text{B.15})$$

$$\begin{aligned} \mathcal{F}[f_2(t)] &= \frac{1}{T_s} \sum_k (\pi\delta(\omega + \omega_0 - k\omega_s) + \pi\delta(\omega - \omega_0 - k\omega_s)) \\ &\quad + \frac{1}{2T_s} \sum_k \left(\frac{e^{-j(\omega + \omega_0 - k\omega_s)T}}{j(\omega + \omega_0 - k\omega_s)} (e^{-j(\omega + \omega_0 - k\omega_s)\alpha} - 1) \right. \\ &\quad \left. + \frac{1}{2T_s} \sum_k \left(\frac{e^{-j(\omega - \omega_0 - k\omega_s)T}}{j(\omega - \omega_0 - k\omega_s)} (e^{-j(\omega - \omega_0 - k\omega_s)\alpha} - 1) \right) \right) \end{aligned} \quad (\text{B.16})$$

2) Discrete signal with periodic discontinuities

$$\mathcal{F}[f_3(t)] = \mathcal{F}[f_s(t)(1 - u(t - n(T + \alpha) + \alpha) + u(t - n(T + \alpha)))] \quad (\text{B.17})$$

$$= \mathcal{F}[f_s(t)] - \mathcal{F}[f_s(t)u(t - n(T + \alpha) + \alpha)] + \mathcal{F}[f_s(t)u(t - n(T + \alpha))] \quad (\text{B.18})$$

$$\mathcal{F}[f_s(t)] = \mathcal{F}[\delta_{T_s}(t)f(t)] = \frac{1}{T_s} \sum_k \mathcal{F}(\omega - k\omega_s)$$

$$\mathcal{F}[u(t - n(T + \alpha) + \alpha)] = \sum_n [\pi\delta(\omega) + \frac{1}{j\omega}] e^{-j\omega(n(T+\alpha)+\alpha)}$$

$$\mathcal{F}[u(t - n(T + \alpha))] = \sum_n [\pi\delta(\omega) + \frac{1}{j\omega}] e^{-j\omega n(T+\alpha)}$$

$$\mathcal{F}[f_3(t)] = \mathcal{F}[f_s(t)] - \frac{1}{2\pi}(\mathcal{F}[f_s(t)] * \mathcal{F}[u(t - n(T + \alpha) + \alpha)]) + \frac{1}{2\pi}(\mathcal{F}[f_s(t)] * \mathcal{F}[u(t - n(T + \alpha))]) \quad (\text{B.19})$$

$$\begin{aligned} A &= \mathcal{F}[f_s(t)] * \mathcal{F}[u(t - n(T + \alpha) + \alpha)] \\ &= \frac{1}{T_s} \sum_k \sum_n F(\omega - k\omega_s) * \pi\delta(\omega) + \frac{1}{T_s} \sum_k \sum_n F(\omega - k\omega_s) * \frac{e^{-j\omega(n(T+\alpha)-\alpha)}}{j\omega} \\ &= \frac{1}{T_s} \sum_k \sum_n (\pi^2\delta(\omega + \omega_0 - k\omega_s) + \pi^2\delta(\omega - \omega_0 - k\omega_s) \\ &\quad + \pi \frac{e^{-j(\omega+\omega_0-k\omega_s)(n(T+\alpha)-\alpha)}}{j(\omega + \omega_0 - k\omega_s)} + \pi \frac{e^{-j(\omega-\omega_0-k\omega_s)(n(T+\alpha)-\alpha)}}{j(\omega - \omega_0 - k\omega_s)}) \end{aligned}$$

B.2 Dimensional Increase

When working with mathematical objects such as functions or vectors, one implicitly considers a specific number of dimensions. For instance, consider a function f depending solely on a scalar variable x in the Cartesian plane, which can be denoted as $f(x)$.

However, such objects—particularly functions—can more generally be interpreted as elements of infinite-dimensional spaces. This perspective naturally leads to the concept of Hilbert spaces, which provide a rigorous mathematical framework in which such infinite-dimensional entities are studied. Although Hilbert spaces will be revisited in more detail later, it is sufficient at this stage to note that these spaces are equipped with an inner product and characterized by the existence of a countably infinite orthonormal basis.

Once the premise of infinite dimensionality is accepted, it becomes evident that any representation restricted to a finite number of dimensions is, in essence, an approximation. Such an approximation is only valid under the assumption that all other dimensions remain constant or equal to zero, depending on the considered mathematical object. Consequently, a function or a vector

$$f(x), \quad v_1 = \begin{bmatrix} v_{1,1} \\ v_{1,2} \end{bmatrix}$$

Expressed solely as a function of a single variable or as a vector with two components is merely a reduced representation of the actual entity, which should instead be considered as a function or vector depending on an increasing number of dimensions:

$$f(x_0, x_1, \dots, x_n), \quad v_1^{(n)} = \begin{bmatrix} v_{1,1} \\ v_{1,2} \\ \vdots \\ v_{1,n} \end{bmatrix}, \quad \text{where } n \rightarrow \infty.$$

This can be considered as an illustration of how the concept of an inner product naturally emerges from the scalar product. Indeed, when considering two vectors, v_1 and v_2 , each composed of two components, their scalar product corresponds to the sum of the products of their respective components:

$$v_1 \cdot v_2 = v_{1,1}v_{2,1} + v_{1,2}v_{2,2}.$$

In fact, this is equivalent to summing the products of corresponding components in a two-dimensional space. Once again, this idea can be extended. If the vectors are considered to contain an infinite number of components, one is naturally led to the concept of a transform, which represents a transition from discrete to continuous elements. In this context, the transform can be interpreted as an inner product, defined for such vectors as:

$$\langle v_1, v_2 \rangle = \int v_1(x)v_2(x) dx.$$

This inner product can also be represented graphically as

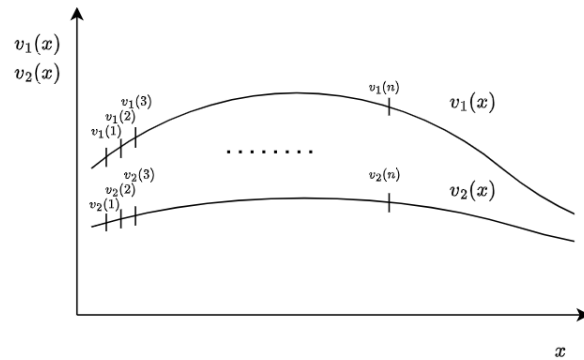


Figure B.1: Representation of the graphical meaning of the extension of the scalar product to infinite dimensions, leading to the inner product between signals $v_1(x)$ and $v_2(x)$, which can be considered as vectors of infinite dimension

This concept can be further extended. Indeed, let us consider the forces acting on a body, regardless of their nature—whether associated with the generation of a gravitational or electrical potential—as components of a vector $\mathbf{v}$. It is then possible to consider that such forces originate from each individual charge, in the case of electrical potentials, or from each mass, in the case of gravitational potentials, existing in the universe. Accordingly, the vector $\mathbf{v}$ can be interpreted as having a number of components tending towards infinity, where each charge or mass contributes an effect represented as one component of this vector.

The concept of infinite dimensionality can also be observed, from a more practical perspective, in quantum physics, where the wave function is in fact a sum of contributions within an infinite-dimensional vector space known as a Hilbert space—a term briefly introduced previously. This application of the concept of infinite dimensions is closely related to the central theme of this work: Fourier series and Fourier transforms. Both frameworks assign a relative contribution to each component. However, although the mathematical formalism is elegant and structurally similar, the interpretation differs significantly. Whereas Fourier series describe the weighted contribution of specific components—namely sinusoidal signals of well-defined frequencies within the corresponding vector space—quantum mechanics employs the same mathematical structure to determine the probability of occurrence of physical events. This probability is obtained as the integral, over a given interval, of the squared modulus of the wave function, denoted $\psi(x)$ and considered as a function of x .

$$\int |\psi(x)|^2 dx$$

However, in most practical cases, the approximations that neglect the infinite-dimensional nature of such vectors remain entirely valid with respect to the intended objective, namely solving concrete problems. Restricting the number of dimensions introduces a simplification that makes the associated problems tractable through human analysis within a reasonable timeframe. In reality, the level of complexity is even greater than what is described here. A fully exhaustive description would require the consideration of nonlinear interactions. Although fundamental, the scope of the present discussion regarding the necessity of extending the notion of finite-dimensional spaces does not yet require the introduction of nonlinear effects or complex, potentially chaotic, responses.

Several emblematic examples of simplification can be cited, which have enabled the resolution of practical problems. One such example is the use of Newtonian mechanics within the framework of non-relativistic dynamics and, of course, its relativistic counterpart developed by Einstein through the formulation of special relativity in 1905 and general relativity in 1915. The validity of the non-relativistic approximation can be justified through the correction factor expressed as:

$$\sqrt{1 - \frac{v^2}{c^2}}$$

where v is the velocity under consideration and c is the speed of light. Ultimately,

physics—and even more broadly the sciences derived from it, such as chemistry and biology—are fundamentally based on approximations.

B.2.1 Towards an extended representation of state spaces and prediction capability

The previously introduced concept of dimensional extension, understood as an extension of the number of variables considered in the representation of a system, suggests a broader and theoretically significant perspective: the possibility of extending predictive capabilities without intrinsic constraints related to the domain under consideration.

Indeed, considering that the state of a system could be represented within an abstract space without restriction regarding its number of dimensions leads to the idea that systems of increasing complexity could, in principle, be described through increasingly comprehensive state representations. Such a framework naturally raises the question of whether prediction could be extended to situations involving an arbitrarily large number of interacting variables.

In reality, the predictive capabilities provided by current physical theories are inherently limited by two fundamental aspects. The first limitation originates from the imperfection of the governing equations themselves. A historical and emblematic example is provided by the transition from Newtonian mechanics to Einstein's theory of relativity, where the former was revealed to be an approximation valid within a restricted domain of application.

The second limitation is related to the finite resolution with which initial conditions can be characterized. Even assuming that the exact governing equations of a system were known, a prediction remaining valid over an arbitrarily extended temporal or spatial horizon would require an infinitely accurate description of the initial state. Consequently, prediction without any limitation in its range appears theoretically inaccessible due to both the incomplete knowledge of the governing laws and the impossibility of determining initial conditions with infinite precision.

Nevertheless, this discussion should be understood as a theoretical exploration rather than as a practical requirement. Indeed, in the overwhelming majority of physical situations encountered, predictions of finite range and limited accuracy are entirely sufficient. The purpose of this reflection is rather to introduce the possibility that an extension of the mathematical representation of systems may provide an increased predictive capability when considering problems involving a larger number of interacting variables.

In this perspective, the concept of state space becomes particularly relevant. A state space can be interpreted as a mathematical representation in which each dimension corresponds to a variable required to characterize the state of a given system. Extending this idea towards spaces of arbitrarily large, or even infinite, dimensionality establishes a conceptual connection with highly abstract mathematical frameworks, such as Hilbert spaces, extensively employed in quantum mechanics for representing systems described by an infinite number of degrees of freedom.

The idea developed here originates from the extension of the classical Fourier series to-

wards multidimensional Fourier representations, as previously discussed in this appendix. However, a complete mathematical development of the implications of such an extension lies beyond the scope of this thesis. Nevertheless, a related conceptual framework can be identified in Waddington's epigenetic landscape theory [113], where the evolution of biological systems, particularly during cellular differentiation, is represented as an evolution within an abstract state space constrained by the underlying biological interactions.

The establishment of an extended predictive capability, potentially independent of disciplinary boundaries, through the representation of systems within state spaces of increasing dimensionality, theoretically tending towards infinite dimensions.

B.3 Extension of the Double Fourier Series to Triple Fourier Series

Following the introduction of the concept of increasing the number of dimensions, which can be interpreted as an extension of the number of factors considered in a mathematical representation, a connection can be established with several concepts encountered in quantum physics.

The purpose of this discussion is to highlight that any representation based on functions can inherently be considered as a simplified description of the actual phenomenon. Such a representation is generally restricted to a finite number of dimensions or contributing factors, whereas the complete description may involve a significantly larger number of variables.

In practice, the appropriate level of complexity is reached when the additional factors that have not been considered are assumed to have a negligible influence on the final result predicted by the established mathematical formulation.

This introduces an intrinsic element of subjectivity. However, the term subjectivity should be understood here as a consequence of individual judgement rather than arbitrariness. It corresponds to the interpretation required to determine whether the neglected contributions remain sufficiently small with respect to the desired accuracy.

Based on this fundamental consideration, a direct application can be introduced. The classical Fourier series is a one-dimensional representation since it describes a function depending on a single variable. In contrast, the double Fourier series employed extensively throughout this thesis constitutes an extension toward higher dimensional representations.

The physical interpretation of the double Fourier series arises from the consideration of two independent periodic variations. Although time remains the fundamental physical variable, the mathematical representation includes two distinct periodic contributions associated with two different signals. This framework naturally appears in the analysis of pulse width modulation (PWM), which constitutes a fundamental operating principle of a large number of power electronic converters.

Since the double Fourier series has already been extensively studied in the literature, and

since the objective of the present section is mainly to introduce the concept of dimensional extension rather than to provide an application-oriented analysis, only the fundamental principles are presented.

The following development extends the double Fourier series formulation toward a triple Fourier series. Ultimately, the interpretation of both approaches remains identical: each additional dimension introduces an additional periodic component, resulting in a more complete representation of the original function. The implications of this extension will be discussed at the end of this section.

$$f(x) = \frac{a_0}{2} + \sum_m (a_m \cos(mx) + b_m \sin(mx)) \quad (\text{B.20})$$

$$\begin{aligned} f(x, y_1) &= \frac{a_0(y_1)}{2} + \sum_m (a_m(y_1) \cos(mx) + b_m(y_1) \sin(mx)) \\ a_m(y) &= \frac{c_{m0}}{2} + \sum_n (c_{mn} \cos(ny) + d_{mn} \sin(ny)) \\ a_m(y, z_1) &= \frac{c_{m0}(z_1)}{2} + \sum_n (c_{mn}(z_1) \cos(ny) + d_{mn}(z_1) \sin(ny)) \\ b_m(y) &= \frac{e_{m0}}{2} + \sum_n (e_{mn} \cos(ny) + f_{mn} \sin(ny)) \\ b_m(y, z_1) &= \frac{e_{m0}(z_1)}{2} + \sum_n (e_{mn}(z_1) \cos(ny) + f_{mn}(z_1) \sin(ny)) \end{aligned}$$

$$\begin{aligned} c_{mn}(z) &= \frac{g_{mn0}}{2} + \sum_p (g_{mnp} \cos(pz) + h_{mnp} \sin(pz)) \\ d_{mn}(z) &= \frac{i_{mn0}}{2} + \sum_p (i_{mnp} \cos(pz) + j_{mnp} \sin(pz)) \\ e_{mn}(z) &= \frac{K_{mn0}}{2} + \sum_p (K_{mnp} \cos(pz) + L_{mnp} \sin(pz)) \\ f_{mn}(z) &= \frac{M_{mn0}}{2} + \sum_p (M_{mnp} \cos(pz) + N_{mnp} \sin(pz)) \end{aligned}$$

$$\begin{aligned} a_m(y, z) &\Rightarrow c_{mn}(z), \quad d_{mn}(z) \Rightarrow g_{mnp}, h_{mnp}, i_{mnp}, j_{mnp} \\ b_m(y, z) &\Rightarrow e_{mn}(z), \quad f_{mn}(z) \Rightarrow K_{mnp}, L_{mnp}, M_{mnp}, N_{mnp} \end{aligned}$$

$$a_m(y, z) = F f(x, y, z) \cos(mx) \, dx$$

$$c_{mn}(z) = \frac{1}{\pi} \int_{-\pi}^{\pi} a_m(y, z) \cos(ny) \, dy = \frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \cos(mx) \cos(ny) \, dx dy$$

$$d_{mn}(z) = \frac{1}{\pi} \int_{-\pi}^{\pi} a_m(y, z) \sin(ny) \, dy = \frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \cos(mx) \sin(ny) \, dx dy$$

$$b_m(y, z) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x, y, z) \sin(mx) \, dx$$

$$e_{mn}(z) = \frac{1}{\pi} \int_{-\pi}^{\pi} b_m(y, z) \cos(ny) \, dy = \frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \sin(mx) \cos(ny) \, dx dy$$

$$f_{mn}(z) = \frac{1}{\pi} \int_{-\pi}^{\pi} b_m(y, z) \sin(ny) \, dy = \frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \sin(mx) \sin(ny) \, dx dy$$

$$\begin{aligned}
g_{mnp} &= \frac{1}{\pi} \int_{-\pi}^{\pi} c_{mn}(z) \cos(pz) dz \\
&= \frac{1}{\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \cos(mx) \cos(ny) \cos(pz) dx dy dz \\
h_{mnp} &= \frac{1}{\pi} \int_{-\pi}^{\pi} c_{mn}(z) \sin(pz) dz \\
&= \frac{1}{\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \cos(mx) \cos(ny) \sin(pz) dx dy dz \\
i_{mnp} &= \frac{1}{\pi} \int_{-\pi}^{\pi} d_{mn}(z) \cos(pz) dz \\
&= \frac{1}{\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \cos(mx) \sin(ny) \cos(pz) dx dy dz \\
j_{mnp} &= \frac{1}{\pi} \int_{-\pi}^{\pi} d_{mn}(z) \sin(pz) dz \\
&= \frac{1}{\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \cos(mx) \sin(ny) \sin(pz) dx dy dz
\end{aligned}$$

$$\begin{aligned}
K_{mnp} &= \frac{1}{\pi} \int_{-\pi}^{\pi} e_{mn}(z) \cos(pz) dz \\
&= \frac{1}{\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \sin(mx) \cos(ny) \cos(pz) dx dy dz \\
L_{mnp} &= \frac{1}{\pi} \int_{-\pi}^{\pi} e_{mn}(z) \sin(pz) dz \\
&= \frac{1}{\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \sin(mx) \cos(ny) \sin(pz) dx dy dz \\
M_{mnp} &= \frac{1}{\pi} \int_{-\pi}^{\pi} f_{mn}(z) \cos(pz) dz \\
&= \frac{1}{\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \sin(mx) \sin(ny) \cos(pz) dx dy dz \\
N_{mnp} &= \frac{1}{\pi} \int_{-\pi}^{\pi} f_{mn}(z) \sin(pz) dz \\
&= \frac{1}{\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \sin(mx) \sin(ny) \sin(pz) dx dy dz
\end{aligned}$$

$$\begin{aligned}
g_{mnp} &= > \cos(mx) \cos(ny) \cos(pz) = \frac{1}{4} (\cos(mx + ny + pz) + \cos(mx - (ny + pz)) \\
&\quad + \cos(mx + ny - pz) + \cos(mx - (ny - pz))) \\
h_{mnp} &= > \cos(mx) \cos(ny) \sin(pz) = \frac{1}{4} (\sin(mx + ny + pz) - \sin(mx - (ny + pz)) \\
&\quad - \sin(mx + ny - pz) - \sin(mx - (ny - pz))) \\
i_{mnp} &= > \cos(mx) \sin(ny) \cos(pz) = \frac{1}{4} (\sin(mx + (ny + pz)) - (\sin(mx - (ny + pz)) \\
&\quad + \sin(mx + ny - pz)) - \sin(mx - (ny - pz))) \\
j_{mnp} &= > \cos(mx) \sin(ny) \sin(pz) = \frac{1}{4} (\cos(mx + ny - pz) + \cos(mx - (ny - pz)) \\
&\quad - \cos(mx + ny + pz) - \cos(mx - (ny + pz)))
\end{aligned}$$

$$\begin{aligned}
K_{mnp} &= > \sin(mx) \cos(ny) \cos(pz) = \frac{1}{4} (\sin(mx + ny + pz) + \sin(mx - (ny + pz)) \\
&\quad + \sin(mx + ny - pz) + \sin(mx - (ny - pz))) \\
L_{mnp} &= > \sin(mx) \cos(ny) \sin(pz) = \frac{1}{4} (\cos(mx - (ny + pz)) - \cos(mx + (ny + pz)) \\
&\quad + \cos(mx - (ny - pz)) - \cos(mx + (ny - pz))) \\
M_{mnp} &= > \sin(mx) \sin(ny) \cos(pz) = \frac{1}{4} (\cos(mx - (ny + pz)) - \cos(mx + (ny + pz)) \\
&\quad + \cos(mx + ny - pz)) - \cos(mx - (ny - pz))) \\
N_{mnp} &= > \sin(mx) \sin(ny) \sin(pz) = \frac{1}{4} (\sin(mx + ny - pz) + \sin(mx - (ny - pz)) \\
&\quad - \sin(mx + ny + pz) - \sin(mx - (ny + pz)))
\end{aligned}$$

$$f(x, y, z) = \frac{a_o(y, z)}{2} + \sum_m (a_m(y, z) \cos(mx) + b_m(y, z) \sin(mx))$$

| — — — |
1

| — — — — — |
2

| — — — — — |
3

(B.21)

$$\begin{aligned}
a_m(y, z) &= \sum_m \frac{c_{m0}(z)}{2} + \sum_m \sum_n (c_{mn}(z) \cos(ny) + d_{mn}(z) \sin(ny)) \\
&= \sum_m \frac{g_{m00}}{4} + \frac{1}{2} \sum_m \sum_p (g_{m0p} \cos(pz) + h_{m0p} \sin(pz)) \\
&\quad + \sum_m \sum_n \left(\frac{g_{mn0}}{2} \cos(ny) \right) + \sum_m \sum_n \sum_p (g_{mnp} \cos(pz) \cos(ny) + h_{mnp} \sin(pz) \cos(ny)) \\
&\quad + \sum_m \sum_n \left(\frac{i_{mn0}}{2} \sin(ny) \right) + \sum_m \sum_n \sum_p (i_{mnp} \cos(pz) \sin(ny) + j_{mnp} \sin(pz) \sin(ny))
\end{aligned}
\tag{B.22}$$

$$\begin{aligned}
b_m(y, z) &= \sum_m \frac{e_{m0}(z)}{2} + \sum_m \sum_n (e_{mn}(z) \cos(ny) + f_{mn}(z) \sin(ny)) \\
&= \sum_m \frac{K_{m00}}{4} + \frac{1}{2} \sum_m \sum_p (K_{m0p} \cos(pz) + L_{m0p} \sin(pz)) \\
&\quad + \sum_m \sum_n \left(\frac{K_{mn0}}{2} \cos(ny) \right) + \sum_m \sum_n \sum_p (K_{mnp} \cos(pz) \cos(ny) + L_{mnp} \sin(pz) \cos(ny)) \\
&\quad + \sum_m \sum_n \left(\frac{M_{mn0}}{2} \sin(ny) \right) + \sum_m \sum_n \sum_p (M_{mnp} \cos(pz) \sin(ny) + N_{mnp} \sin(pz) \sin(ny))
\end{aligned}
\tag{B.23}$$

$$\begin{aligned}
f(x, y, z) = & \frac{1}{2} \left(\frac{g_{000}}{4} + \frac{1}{2} \sum_p (g_{00p} \cos(pz) + h_{00p} \sin(pz)) + \sum_n \frac{1}{2} (g_{0n0} \cos(ny) + i_{0n0} \sin(ny)) \right. \\
& + \sum_n \sum_p (g_{0np} \cos(pz) \cos(ny) + h_{0np} \sin(pz) \cos(ny)) \\
& \left. + \sum_n \sum_p (i_{0np} \cos(pz) \sin(ny) + j_{0np} \sin(pz) \sin(ny)) \right) \\
& + \sum_m \frac{g_{m00}}{4} \cos(mx) + \frac{1}{2} \sum_m \sum_p (g_{m0p} \cos(pz) \cos(mx) + h_{m0p} \sin(pz) \cos(mx)) \\
& + \frac{1}{2} \sum_m \sum_n (g_{mn0} \cos(ny) \cos(mx)) + \sum_m \sum_n \sum_p (g_{mnp} \cos(pz) \cos(ny) \cos(mx) \\
& \quad + h_{mnp} \sin(pz) \cos(ny) \cos(mx)) \\
& + \frac{1}{2} \sum_m \sum_n (i_{mn0} \sin(ny) \cos(mx)) + \sum_m \sum_n \sum_p (i_{mnp} \cos(pz) \sin(ny) \cos(mx) \\
& \quad + j_{mnp} \sin(pz) \sin(ny) \cos(mx)) \\
& + \sum_m \frac{K_{m00}}{4} \sin(mx) + \frac{1}{2} \sum_m \sum_p (K_{m0p} \cos(pz) \sin(mx) + L_{m0p} \sin(pz) \sin(mx)) \\
& + \frac{1}{2} \sum_m \sum_n (K_{mn0} \cos(ny) \sin(mx)) + \sum_m \sum_n \sum_p (K_{mnp} \cos(pz) \cos(ny) \sin(mx) \\
& \quad + L_{mnp} \sin(pz) \cos(ny) \sin(mx)) \\
& + \frac{1}{2} \sum_m \sum_n (M_{mn0} \sin(ny) \sin(mx)) + \sum_m \sum_n \sum_p (M_{mnp} \cos(pz) \sin(ny) \sin(mx) \\
& \quad + N_{mnp} \sin(pz) \sin(ny) \sin(mx))
\end{aligned}$$

(B.24)

Appendix C

Maxwell's equations and interesting derivations

C.1 From Maxwell equations towards the voltage and current propagation equations

The complexity of the studied field can be understood as originating from two main aspects. The first one concerns the accurate modelling of the converter, while the second one relates to the understanding of how the generated emissions propagate and interact with the surrounding network. This subsection is dedicated to the study of the propagation mechanisms of these emissions.

Indeed, accurately modelling the converter is not sufficient; it is also necessary to understand the physical nature of the environment in which the converter is connected. This requires knowledge of the fundamental propagation mechanisms governing electromagnetic fields. Therefore, the development of the equations required to identify the main parameters influencing the non-instantaneous propagation of electric and magnetic fields is presented. The developments presented in this section are mainly based on [114, 115].

First, Maxwell's equations are expressed as

$$\begin{aligned}\nabla \cdot \mathbf{D} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \\ \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\end{aligned}\tag{C.1}$$

where μ_0 and ε_0 are respectively the permeability and permittivity of vacuum.

C.1.1 Origin of the Development from Maxwell's Equations

The derivation presented in this appendix originates from the general formulation of electromagnetic wave propagation based on Maxwell's equations. These equations constitute the most general description of classical electromagnetism and, when applied to waveguide structures under harmonic steady-state conditions, naturally lead to the vector Helmholtz equations governing the electromagnetic field components.

For a wave propagating along the z -axis, the Helmholtz formulation expresses the transverse field components (E_x, E_y, H_x, H_y) as functions of the longitudinal components (E_z, H_z). Consequently, the determination of the complete electromagnetic field inside a waveguide is generally reduced to the determination of the longitudinal field components.

The resulting expressions contain the quantity

$$p^2 = k^2 - \beta^2, \quad (\text{C.2})$$

where $k = \omega\sqrt{\mu\varepsilon}$ denotes the wavenumber of the medium and β is the propagation constant along the guide.

For the vast majority of waveguide modes, namely transverse electric (TE) and transverse magnetic (TM) modes, the longitudinal field components are non-zero and the factor p is also non-zero. Under these conditions, the Helmholtz equations provide a complete and well-defined description of the electromagnetic fields.

The situation is fundamentally different for transverse electromagnetic (TEM) waves. By definition, a TEM wave satisfies

$$E_z = 0, \quad H_z = 0. \quad (\text{C.3})$$

At the same time, TEM propagation requires

$$\beta = k, \quad (\text{C.4})$$

which immediately leads to

$$p^2 = k^2 - \beta^2 = 0. \quad (\text{C.5})$$

Consequently, the classical expressions of the transverse field components obtained from the Helmholtz equations become indeterminate. Indeed, their numerators depend on the longitudinal components E_z and H_z , which vanish for TEM waves, while their denominators contain the factor p^2 , which also vanishes. The resulting mathematical structure therefore takes the indeterminate form

$$\frac{0}{0}, \quad (\text{C.6})$$

preventing the direct application of the general Helmholtz formulation.

This apparent singularity does not imply that TEM waves cannot exist. Instead, it indicates that the Helmholtz formulation is no longer the appropriate mathematical framework for describing this particular propagation mode. The electromagnetic fields must therefore be determined directly from Maxwell's equations by explicitly imposing the intrinsic properties of TEM propagation.

An important consequence immediately follows from this observation. Since TEM propagation requires

$$\beta = k = \omega\sqrt{\mu\varepsilon}, \quad (\text{C.7})$$

the phase velocity remains

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}. \quad (\text{C.8})$$

Therefore, all frequency components propagate with the same phase velocity. Under these conditions, the propagation medium is non-dispersive, which explains why the derivation presented throughout this appendix assumes a constant propagation velocity.

Transmission lines constitute a particular class of waveguides supporting the TEM propagation mode. Although they belong to the broader family of waveguides, their mathematical treatment cannot rely on the conventional Helmholtz formulation because of the intrinsic degeneracy associated with TEM propagation. This is precisely the reason why the following development starts directly from Maxwell's equations rather than from the general waveguide equations. The relationship between Maxwell's equations, waveguide theory, and the particular case of transverse electromagnetic (TEM) propagation is summarized in Fig. C.1.

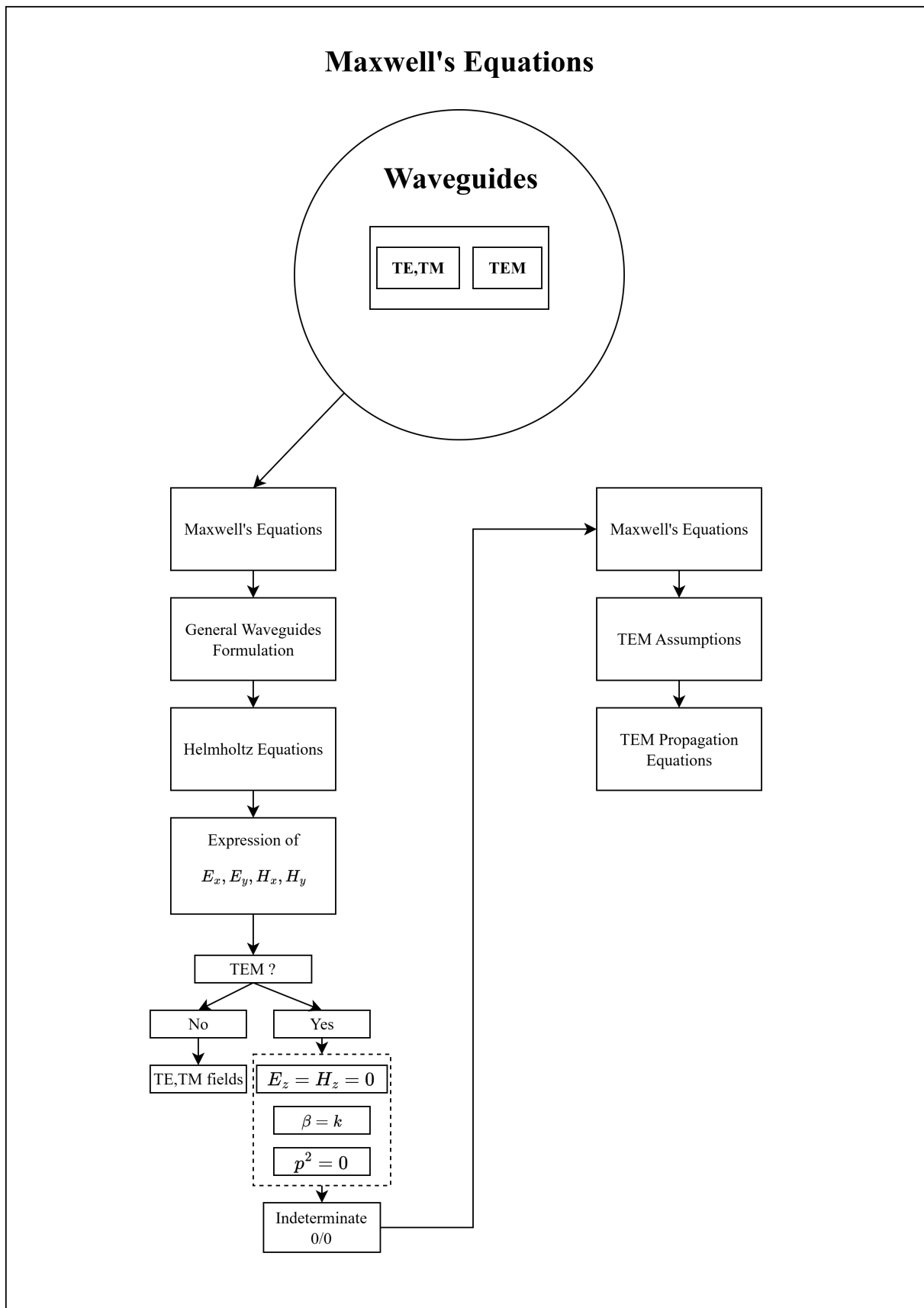


Figure C.1: Conceptual representation of the derivation path from Maxwell's equations to electromagnetic propagation equations, highlighting the specific case of TEM waves

C.1.2 Maxwell's Equations Applied to TEM Wave Propagation

The following development is carried out for transverse electromagnetic (TEM) waves propagating along a specific structure composed of two ideal parallel plates, i.e. plates with conductivity tending towards infinity, connected by a non-ideal element characterized by a resistance R_c . The two plates are separated by a distance b , and the electromagnetic waves are assumed to propagate exclusively along the z direction. A representation of the considered structure is presented hereafter.

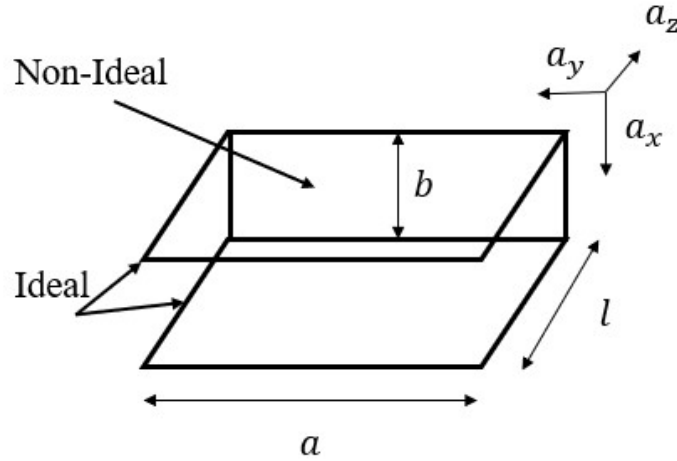


Figure C.2: Representation of two ideal parallel plates connected through a non-ideal resistive element for the study of TEM wave propagation

Obviously, TEM waves imply that both the electric and magnetic fields are transverse to the direction of propagation. In the considered structure, the electric field is oriented along the x direction and the magnetic field along the y direction. Furthermore, since the propagation does not occur in vacuum, the general Maxwell equations can be adapted to the considered medium, leading to the following two relevant equations:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \Rightarrow \frac{d\mathbf{E}_x}{dz} = -j\omega\mu\mathbf{H}_y \quad (\text{C.9})$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \Rightarrow \frac{d\mathbf{H}_y}{dz} = j\omega\varepsilon\mathbf{E}_x \quad (\text{C.10})$$

Assuming that an electric field exists at $z = 0$ and is expressed as $E_0 \sin(\omega t)$, the coupling between the electric and magnetic fields can be used to derive the spatial evolution of both fields. The resulting expressions are obtained in phasor form, with the phase reference chosen such that the phase is zero at $t = 0$.

$$E_x = E_{x0} + \omega E_{x1} + \omega^2 E_{x2} + \dots \quad (\text{C.11})$$

$$B_y = B_{y0} + \omega B_{y1} + \omega^2 B_{y2} + \dots \quad (\text{C.12})$$

E_{x0} , E_{x1} and the subsequent terms represent magnitude coefficients. The first coefficient, E_{x0} , can be expressed as a function of I_0 , the current magnitude, where a denotes the width of the plates.

$$I_0 = aH_y$$

$$E_{x0} = I_0 \frac{R_c}{b}$$

Subsequently, by substituting the electric and magnetic field expansions defined in Equ. C.11 and C.12 into Equ. C.9 and C.10, and by collecting the terms according to their respective order in ω , the following relations are obtained.

$$\frac{dE_{x0}}{dz} = 0 \quad \frac{dE_{x1}}{dz} = -j\mu H_{y0} \quad \frac{dE_{x2}}{dz} = -j\mu H_{y1} \quad \dots \quad (\text{C.13})$$

$$\frac{dH_{y0}}{dz} = 0 \quad \frac{dH_{y1}}{dz} = j\varepsilon E_{x0} \quad \frac{dH_{y2}}{dz} = j\varepsilon E_{x1} \quad \dots \quad (\text{C.14})$$

Since only E_{x0} is imposed at $z = 0$, all the higher-order components E_{xn} for $n \neq 0$ are initially equal to zero. Consequently, integrating the differential relations along the z direction leads to the replacement of the derivatives with the corresponding spatial terms involving z . The resulting set of equations can be rewritten as follows:

$$\frac{dE_{x0}}{dz} = 0 \quad E_{x1} = -j\mu H_{y0}z \quad E_{x2} = -j\mu H_{y1}z \quad \dots \quad (\text{C.15})$$

$$\frac{dH_{y0}}{dz} = 0 \quad H_{y1} = j\varepsilon E_{x0}z \quad H_{y2} = j\varepsilon E_{x1}z \quad \dots \quad (\text{C.16})$$

By retaining only the first-order terms of the expansion in ω and multiplying the resulting equations by ω , the following expressions are obtained:

$$\omega E_{x1} = -j\omega\mu H_{y0}z = -j\omega\mu \frac{I_0}{a}z = -j\beta_0 z \left(\frac{I_0\eta}{a} \right) \quad (\text{C.17})$$

$$\omega H_{y1} = j\omega\varepsilon E_{x0}z = j\omega\varepsilon I_0 \frac{R_c}{b}z = j\beta_0 z \left(I_0 \frac{R_c}{\eta b} \right) \quad (\text{C.18})$$

$$\text{With} \quad \beta_0 = \omega\sqrt{\mu\varepsilon} \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Applying the same approach to all subsequent orders of ω eventually leads to the following expressions for the electric and magnetic fields:

$$E_x = I_0 \frac{R_c}{b} \left(1 - \frac{\eta b}{a R_c} (j\beta_0 z) + \frac{(j\beta_0 z)^2}{2!} - \frac{\eta b}{a R_c} \frac{(j\beta_0 z)^3}{3!} + \dots \right) \quad (\text{C.19})$$

$$H_y = \frac{I_0}{a} \left(1 + \frac{a R_c}{\eta b} (j\beta_0 z) - \frac{(j\beta_0 z)^2}{2!} + \frac{a R_c}{\eta b} \frac{(j\beta_0 z)^3}{3!} - \dots \right) \quad (\text{C.20})$$

Then, using the simple relation $E_x = V/b$, the voltage expression is obtained as

$$V = I_0 R_c \left(1 + \frac{(j\beta_0 z)^2}{2!} + \frac{(j\beta_0 z)^4}{4!} + \dots - \frac{\eta b}{a R_c} \left((j\beta_0 z) + \frac{(j\beta_0 z)^3}{3!} + \dots \right) \right) \quad (\text{C.21})$$

Using the formulas of *cosh* and *sinh* leads to rewriting V as

$$V = I_0 R_c \left(\cosh(j\beta_0 z) - \frac{\eta b}{a R_c} \sinh(j\beta_0 z) \right) \quad (\text{C.22})$$

$$V = \frac{I_0 R_c}{2} \left(e^{j\beta_0 z} + e^{-j\beta_0 z} - \frac{\eta b}{a R_c} (e^{j\beta_0 z} - e^{-j\beta_0 z}) \right) \quad (\text{C.23})$$

$$V = \frac{I_0 R_c}{2} \left(e^{j\beta_0 z} \left(1 - \frac{\eta b}{a R_c} \right) + e^{-j\beta_0 z} \left(1 + \frac{\eta b}{a R_c} \right) \right) \quad (\text{C.24})$$

$$(\text{C.25})$$

Remembering that the development is performed in the phasor domain, the time-dependent term $e^{j\omega t}$, which is implicitly omitted in the phasor representation, must in reality multiply Equ. C.24. The voltage expression can therefore be rewritten as

$$V = \frac{I_0 R_c}{2} \left(e^{j(\omega t + \beta_0 z)} \left(1 - \frac{\eta b}{a R_c} \right) \right) + \frac{I_0 R_c}{2} \left(e^{j(\omega t - \beta_0 z)} \left(1 + \frac{\eta b}{a R_c} \right) \right) \quad (\text{C.26})$$

The interpretation of this equation is that the voltage is not only a function of time but also of space. Nevertheless, this dependency should be properly understood. Remembering that $\beta_0 = \frac{2\pi}{\lambda}$, it follows that $\beta_0 z = 2\pi \frac{z}{\lambda}$. Therefore, when z is small compared with the wavelength, the spatial contribution becomes negligible and only the temporal term ωt remains significant.

It is worth noting that this spatial dependency is always present, regardless of the frequency under consideration. However, when the wavelength associated with the considered frequency is large compared with the characteristic dimension of the studied medium or element, in the direction of propagation of the considered TEM waves, this dependency

can be neglected. The system can then be approximated as a function of time only, leading to the concept of the lumped-element model.

Regarding the main interest of this thesis, i.e. the supraharmonic range, the maximum frequency considered is 150 kHz. In vacuum, this frequency corresponds to a wavelength given by

$$\lambda = \frac{2\pi}{\beta_0} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}} = \frac{c}{f},$$

where c is the speed of light in $[m/s]$ and f is the frequency in $[Hz]$. For $f = 150 [kHz]$, the wavelength is $\lambda = 1998.62 [m]$. Therefore, when considering nearby converters or household devices, the lumped-element model remains an acceptable approximation. However, when studying propagation phenomena over the entire LV grid, this approximation is no longer valid.

Even for a distance of 100 meters, the voltage difference between the initial point at $z = 0$ and the point located at $z = 100$ meters is not negligible and varies with time.

This point should be understood as an inherent limitation of any model established using lumped elements. The non-instantaneous propagation of electromagnetic waves is one aspect that must be considered; however, it is not the only limitation. Indeed, throughout the previous development leading to the final expression of the voltage as a function of both time and space, an additional assumption was introduced: a constant propagation velocity regardless of the frequency of the electromagnetic wave.

In the previous derivation, β_0 , corresponding to the wave number, was considered as only dependent on the frequency through the propagation velocity, which was assumed to be equal to $\sqrt{\mu_0\varepsilon_0} = c [m/s]$. This assumption is valid for propagation in vacuum. However, when propagation occurs in a dispersive medium, the frequency dependence of the propagation velocity must be taken into account, as discussed in the following subsection.

C.1.3 Telegrapher's Equations

It is also possible to describe the same phenomenon and arrive at the same conclusions previously obtained through the direct application of Maxwell's equations, by instead employing the development of the telegrapher's equations. This approach is presented hereafter. The presented matter is mainly based on [114, 115].

Afterwards a schematic is proposed for the infinitesimal two-port network in Fig. C.3 in which the considered elements are L, R, G and C which are respectively the per-unit length inductance, resistance, conductance and capacitance.

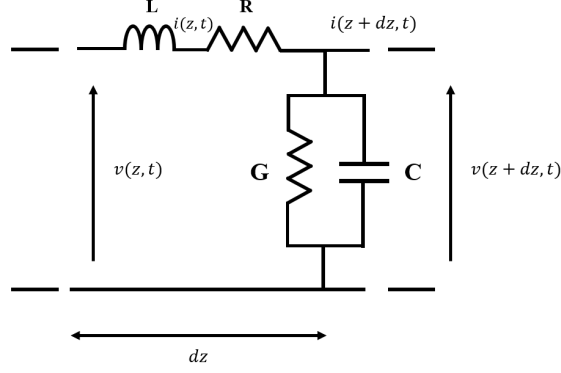


Figure C.3: Simplified electrical diagram of an infinitesimal four-pole network

$$\frac{\partial v}{\partial z} = -Ri - L \frac{\partial i}{\partial t} \quad (\text{C.27})$$

$$\frac{\partial i}{\partial z} = -Gv - C \frac{\partial v}{\partial t} \quad (\text{C.28})$$

These are the fundamental equations allowing for further development which is presented just below

$$\begin{aligned} \frac{\partial}{\partial z} \left(\frac{\partial v}{\partial z} \right) &= \frac{\partial}{\partial z} \left(-Ri - L \frac{\partial i}{\partial t} \right) \\ &= -R \frac{\partial i}{\partial z} - L \frac{\partial}{\partial t} \frac{\partial i}{\partial z} \end{aligned} \quad (\text{C.29})$$

Using Equ. C.27 and C.28 to develop Equ. C.29 leads to

$$\frac{\partial^2 v}{\partial z^2} = RGv + (RC + LG) \frac{\partial v}{\partial t} + LC \frac{\partial^2 v}{\partial t^2} \quad (\text{C.30})$$

$$\frac{\partial^2 i}{\partial z^2} = RGi + (RC + LG) \frac{\partial i}{\partial t} + LC \frac{\partial^2 i}{\partial t^2} \quad (\text{C.31})$$

It is then possible to consider a simplified case, the one without losses, in order to bring the main idea

$$\frac{\partial^2 v}{\partial z^2} = LC \frac{\partial^2 v}{\partial t^2} \quad (\text{C.32})$$

$$\frac{\partial^2 i}{\partial z^2} = LC \frac{\partial^2 i}{\partial t^2} \quad (\text{C.33})$$

C.2 Attenuation of an Electric Wave Inside a Conductor as a Function of the Conductivity and Frequency

This part has as purpose to develop analytically, based on Maxwell's equations, the next conclusion : an electric wave is attenuated when passing inside a conductor and this is dependent upon the considered frequency and the conductivity of the conductor. The work presented here is mainly based on [116].

C.2.1 Analytical Development

Maxwell's equations are

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \quad (\text{C.34})$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (\text{C.35})$$

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{C.36})$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (\text{C.37})$$

With $\mathbf{E}$, $\mathbf{B}$, μ_0 , ε_0 being respectively the electric field, magnetic field, permeability and permittivity of vacuum.

Consider now the two next relations that expressed the Ohm's law and the conservation of charges

$$\mathbf{J}[\frac{c}{s \, m^2}] = \sigma[\frac{c^2 \, s}{kg \, m^3}] \mathbf{E}[\frac{kg \, m}{s^2 \, c}] \quad (\text{C.38})$$

$$\nabla \cdot \mathbf{J} = \frac{\partial \rho}{\partial t} \quad (\text{C.39})$$

With c , s , m and kg being the Coulomb, second, meter and kilogram units. Regarding $\mathbf{J}$ and σ , they stands for the current density and the electrical conductivity.

Lets assume the conservation of charge in sinusoidal mode

$$\nabla \cdot \mathbf{J} - \frac{\partial \rho}{\partial t} = 0 \quad (\text{C.40})$$

$$\sigma \nabla \cdot \mathbf{E} - j\omega \rho = 0 \quad (\text{C.41})$$

$$(\frac{\sigma}{\varepsilon_0} - j\omega) \rho = 0 \quad (\text{C.42})$$

To respect the conservation of charge law when the conductivity tends to infinity ρ tends to 0. Basically, it is equivalent to say that there is a charge density (ρ) of 0 inside an ideal conductor.

Another way of proving it is to consider the fourth Maxwell law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (\text{C.43})$$

$$= \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \quad (\text{C.44})$$

$$= \mu_0 \left(\sigma \mathbf{E} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \quad (\text{C.45})$$

In sinusoidal mode it gives

$$\nabla \times \mathbf{B} = \mu_0 (\sigma \mathbf{E} - j\omega \varepsilon_0 \mathbf{E}) \quad (\text{C.46})$$

Two current are there identified, the conduction current and the displacement current respectively $\sigma \mathbf{E}$ and $-j\omega \varepsilon_0 \mathbf{E}$. Considering that the conductivity is much greater than $\omega \varepsilon_0$ then the expression becomes

$$\nabla \times \mathbf{B} = \mu_0 \sigma \mathbf{E} \quad (\text{C.47})$$

Then to get ride of the magnetic field one can merely take the derivative with respect to time and noting that as the derivative is a scalar operator the order of the two operators, i.e. curl and derivative with respect to time can be changed.

$$\nabla \times \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \sigma \frac{\partial \mathbf{E}}{\partial t} \quad (\text{C.48})$$

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \sigma \frac{\partial \mathbf{E}}{\partial t} \quad (\text{C.49})$$

$$\nabla^2 \mathbf{E} = \mu_0 \sigma \frac{\partial \mathbf{E}}{\partial t} \quad (\text{C.50})$$

Assuming sinusoidal mode it gives

$$\nabla^2 \mathbf{E} = -j\omega \mu_0 \sigma \mathbf{E} \quad (\text{C.51})$$

Looking for a plane wave monochromatic solution then leads to having an electric field which is defined as

$$\mathbf{E} = \mathbf{E}_0 e^{j(kx - \omega t)} \quad (\text{C.52})$$

The factor k is thus determined

$$k^2 = j\omega\mu_0\sigma \quad (C.53)$$

$$k = \frac{1+j}{\sqrt{2}}\sqrt{\omega\mu_0\sigma} = \frac{1+j}{\delta} \quad \text{With } \delta = \frac{\sqrt{2}}{\sqrt{\omega\mu_0\sigma}} \quad (C.54)$$

For a wave being normal to a metal conductor which propagation direction is y just as the polarisation of the electric wave and by replacing k as a function containing δ in the expression of Equ. C.52 one can find

$$\mathbf{E}_y = \mathbf{E}_1 e^{-x/\delta} e^{j(x/\delta - \omega t)} + \mathbf{E}_2 e^{x/\delta} e^{j(-x/\delta - \omega t)} \quad (C.55)$$

This is an interesting result even though the second part related to E_2 is not realistic when x being the width of the conductor becomes too great with respect to δ such that this part of the equation is eliminated and eventually the electric wave is

$$\mathbf{E}_y = \mathbf{E}_1 e^{-x/\delta} e^{j(x/\delta - \omega t)} \quad (C.56)$$

The basic interpretation of this last equation is that the penetration depth inside a conductor is depending upon the frequency of the considered electromagnetic wave and the conductivity of the conductor. For an ideal conductor the conductivity is infinite meaning that the penetration depth δ is 0. This idea of having the annihilation of the electric field inside an ideal conductor is deeply linked with a more fundamental principle which is that in order to have force acting it is necessary to have a non ideal item which can be the conductance in Maxwell's equations but can be extended to biological and psychological sciences as it is done in Appendix E.

Appendix D

Inverter : Analytical Development

D.1 Determination of the Switching Function of a Classical Inverter Using Bessel Functions

The classical inverter shown in Fig. D.1 is studied in this appendix. The spectral magnitudes are determined, i.e. for the switching function $S_1(t) : A_1(m, n) + jB_1(m, n)$ and for the switching function $S_2(t) : A_{21}(m, n) + jB_{21}(m, n) + A_{22}(m, n) + jB_{22}(m, n)$. The validation of the method is both provided by the comparison between the numerical experimentation, which does as a matter of fact contain the control and the model of each element, and the analytical developments. Also, the comparison between the two analytical method has been carried out in order to validate the expression using only Bessel functions.

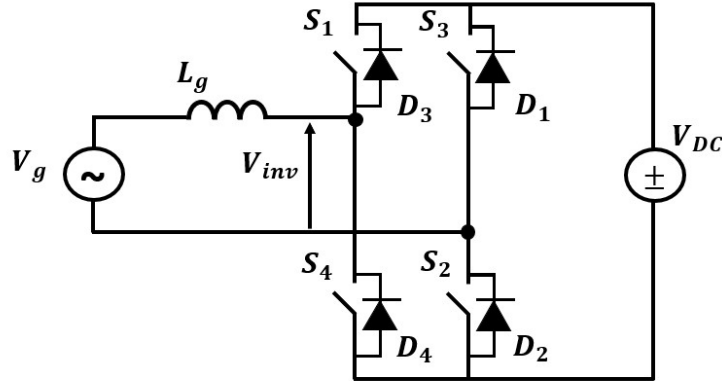


Figure D.1: Electrical schematic of a classical one phase inverter with the grid impedance simplified as an inductance

It is worth noting that using Bessel functions is just a way to get ride of the integrals . All the development is hereafter provided and finally brings the spectral magnitude

components of the switching function which control the switch 3 and 4, i.e. $S_2(t)$. First, $S_2(x, y)$ is logically determined using the classical representation of the double Fourier series adapted to this specific case as it can be seen if Fig. D.2,D.3.

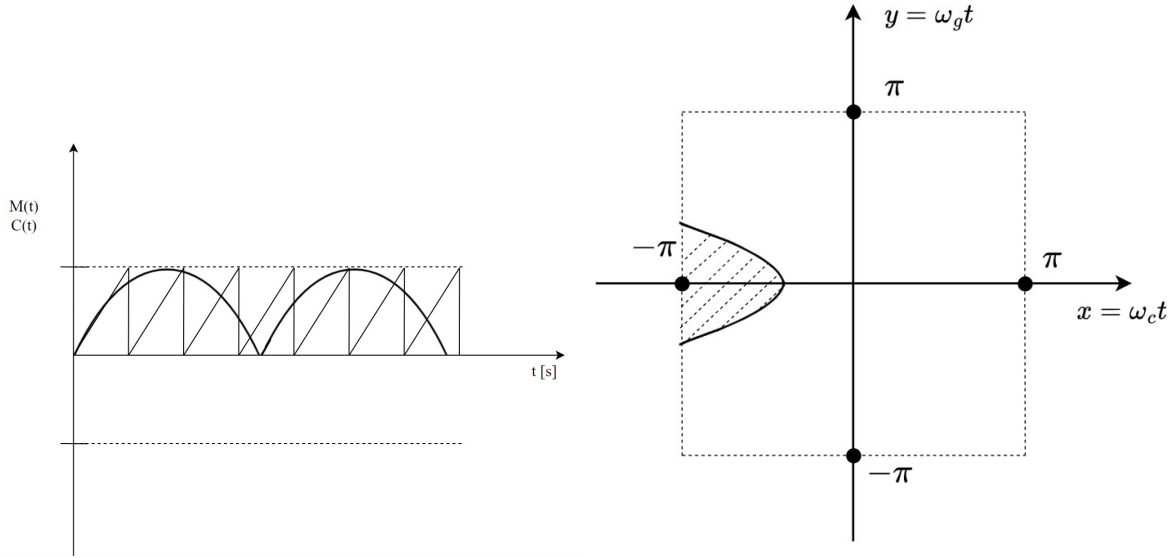


Figure D.2: Peculiar PWM with sawtooth carrier for providing in average a sinusoidal inverter voltage, an illustration both in time domain (Left) and in the double Fourier series domain (Right) are provided, however the double Fourier series domain is only represented for one of the two needed switching function, i.e. $S_1(t)$

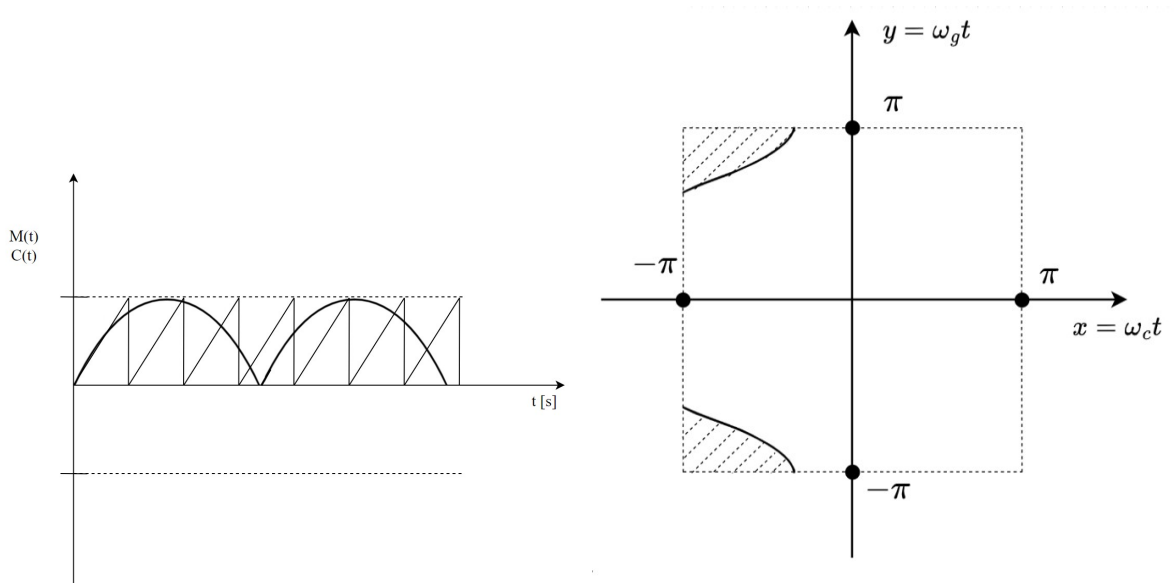


Figure D.3: Peculiar PWM with sawtooth carrier for providing in average a sinusoidal inverter voltage, an illustration both in time domain (Left) and in the double Fourier series domain (Right) are provided, however the double Fourier series domain is only represented for one of the two needed switching function, i.e. $S_2(t)$

$$\begin{aligned}
A_{21}(m, n) + jB_{21}(m, n) + A_{22}(m, n) + jB_{22}(m, n) &= \frac{1}{2\pi^2} \int_{-\pi}^{\frac{-\pi}{2}} \int_{-\pi}^{2\pi Q \cos(y-\pi)-\pi} e^{j(mx+ny)} dx dy \\
&+ \frac{1}{2\pi^2} \int_{\frac{\pi}{2}}^{\pi} \int_{-\pi}^{2\pi Q \cos(y-\pi)-\pi} e^{j(mx+ny)} dx dy
\end{aligned}$$

$$\begin{aligned}
AA(m, n) &= \int_{-\pi}^{2\pi Q \cos(y-\pi)-\pi} e^{j(mx+ny)} dx dy \\
AA(m, n) &= \frac{1}{jm} [e^{j(m2\pi Q \cos(y-\pi)-m\pi+ny)} - e^{j(ny-m\pi)}] \\
&= \frac{1}{jm} [\cos(-m2\pi Q \cos(y) + ny - m\pi) + j \sin(-m2\pi Q \cos(y) + ny - m\pi) \\
&\quad - (\cos(ny - m\pi) + j \sin(ny - m\pi))] \\
&= \frac{(-1)^m}{jm} [\cos(-m2\pi Q \cos(y) + ny) + j \sin(-m2\pi Q \cos(y) + ny) \\
&\quad - (\cos(ny) + j \sin(ny))] \\
&= \frac{(-1)^m}{jm} [AA1 + AA2 + E]
\end{aligned}$$

$$\begin{aligned}
AA1 &= \cos(-m2\pi Q \cos(y) + ny) \\
&= \cos(\xi \cos(\theta) + \delta)
\end{aligned}$$

$$\begin{aligned}
AA2 &= j \sin(-m2\pi Q \cos(y) + ny) \\
&= j \sin(\xi \cos(\theta) + \delta)
\end{aligned}$$

$$E = -(\cos(ny) + j \sin(ny))$$

$$\begin{aligned}
AA1 &= \cos(\xi \cos(\theta)) \cos(\delta) - \sin(\xi \cos(\theta)) \sin(\delta) \\
&= \cos(\delta) \left(J_0(\xi) + 2 \sum_{k=1}^{\infty} \cos(k \frac{\pi}{2}) J_k(\xi) \cos(k\theta) \right) \\
&\quad - \sin(\delta) \left(2 \sum_{k=1}^{\infty} \sin(k \frac{\pi}{2}) J_k(\xi) \cos(k\theta) \right)
\end{aligned}$$

$$\begin{aligned}
&= J_0(\xi)\cos(\delta) + \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2})J_k(\xi)(\cos(\delta + k\theta) + \cos(\delta - k\theta)) \\
&\quad - \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2})J_k(\xi)(\sin(\delta + k\theta) + \sin(\delta - k\theta))
\end{aligned}$$

$$\begin{aligned}
AA2 &= j \left(\sin(\delta)\cos(\xi\cos(\theta)) + \cos(\delta)\sin(\xi\cos(\theta)) \right) \\
&= j \left(\sin(\delta)(J_0(\xi) + 2 \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2})J_k(\xi)\sin(\delta)\cos(k\theta)) \right. \\
&\quad \left. + \cos(\delta)(2 \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2})J_k(\xi)\cos(k\theta)) \right) \\
&= j \left(J_0(\xi)\sin(\delta) + \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2})J_k(\xi)(\sin(\delta + k\theta) + \sin(\delta - k\theta)) \right. \\
&\quad \left. + \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2})J_k(\xi)(\cos(\delta + k\theta) + \cos(\delta - k\theta)) \right)
\end{aligned}$$

$$\begin{aligned} \frac{(-1)^m}{2\pi^2 jm} \int_{-\pi}^{-\frac{\pi}{2}} AA1 dy &= \frac{(-1)^m}{2\pi^2 jm} \int_{-\pi}^{-\frac{\pi}{2}} \left(J_0(\xi) \cos(\delta) + \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(\xi) (\cos(\delta + k\theta) + \cos(\delta - k\theta)) \right. \\ &\quad \left. - \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(\xi) (\sin(\delta + k\theta) + \sin(\delta - k\theta)) \right) dy \end{aligned}$$

$$\begin{aligned} P1.I1.1 &= \frac{(-1)^m}{2\pi^2 jm} \int_{-\pi}^{-\frac{\pi}{2}} \left(J_0(\xi) \cos(\delta) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 jm} \int_{-\pi}^{-\frac{\pi}{2}} \left(J_0(-m2\pi Q) \cos(ny) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 jmn} J_0(-m2\pi Q) \sin(-n\frac{\pi}{2}) \end{aligned}$$

$$\begin{aligned} P1.I1.2 &= \frac{(-1)^m}{2\pi^2 jm} \int_{-\pi}^{-\frac{\pi}{2}} \left(\sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(\xi) (\cos(\delta + k\theta) + \cos(\delta - k\theta)) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 jm} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \int_{-\pi}^{-\frac{\pi}{2}} \left(\cos(y(n+k)) + \cos(y(n-k)) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 jm} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\sin(-\frac{\pi}{2}(n+k))}{n+k} - \frac{\sin(-\pi(n+k))}{n+k} \right. \\ &\quad \left. + \frac{\sin(-\frac{\pi}{2}(n-k))}{n-k} - \frac{\sin(-\pi(n-k))}{n-k} \right) \end{aligned}$$

$$\begin{aligned} P1.I1.3 &= \frac{-(-1)^m}{2\pi^2 jm} \int_{-\pi}^{-\frac{\pi}{2}} \left(\sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(\xi) (\sin(\delta + k\theta) + \sin(\delta - k\theta)) \right) dy \\ &= \frac{-(-1)^m}{2\pi^2 jm} \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \int_{-\pi}^{-\frac{\pi}{2}} \left(\sin(y(n+k)) + \sin(y(n-k)) \right) dy \\ &= \frac{-(-1)^m}{2\pi^2 jm} \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{-\cos(-\frac{\pi}{2}(n+k))}{n+k} + \frac{\cos(-\pi(n+k))}{n+k} \right. \\ &\quad \left. - \frac{\cos(-\frac{\pi}{2}(n-k))}{n-k} + \frac{\cos(-\pi(n-k))}{n-k} \right) \end{aligned}$$

$$\begin{aligned} \frac{(-1)^m}{2\pi^2 jm} \int_{-\pi}^{-\frac{\pi}{2}} AA2 dy &= \frac{(-1)^m}{2\pi^2 jm} \int_{-\pi}^{-\frac{\pi}{2}} j \left(J_0(\xi) \sin(\delta) + \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(\xi) (\sin(\delta + k\theta) \right. \\ &\quad \left. + \sin(\delta - k\theta)) + \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(\xi) (\cos(\delta + k\theta) + \cos(\delta - k\theta)) \right) \end{aligned}$$

$$\begin{aligned} P2.I1.1 &= \frac{(-1)^m}{2\pi^2 m} \int_{-\pi}^{-\frac{\pi}{2}} J_0(\xi) \sin(\delta) dy \\ &= \frac{(-1)^m}{2\pi^2 m} \int_{-\pi}^{-\frac{\pi}{2}} J_0(-m2\pi Q) \sin(ny) dy \\ &= \frac{(-1)^m}{2\pi^2 mn} J_0(-m2\pi Q) \left(-\cos(-n\frac{\pi}{2}) + \cos(-n\pi) \right) \end{aligned}$$

$$\begin{aligned} P2.I1.2 &= \frac{(-1)^m}{2\pi^2 m} \int_{-\pi}^{-\frac{\pi}{2}} \left(\sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(\xi) (\sin(\delta + k\theta) + \sin(\delta - k\theta)) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 m} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \int_{-\pi}^{-\frac{\pi}{2}} (\sin(y(n+k)) + \sin(y(n-k))) dy \\ &= \frac{(-1)^m}{2\pi^2 m} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(-\frac{\cos(-\frac{\pi}{2}(n+k))}{n+k} + \frac{\cos(-\pi(n+k))}{n+k} \right. \\ &\quad \left. - \frac{\cos(-\frac{\pi}{2}(n-k))}{(n-k)} + \frac{\cos(-\pi(n-k))}{n-k} \right) \end{aligned}$$

$$\begin{aligned} P2.I1.3 &= \frac{(-1)^m}{2\pi^2 m} \int_{-\pi}^{-\frac{\pi}{2}} \left(\sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(\xi) (\cos(\delta + k\theta) + \cos(\delta - k\theta)) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 m} \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \int_{-\pi}^{-\frac{\pi}{2}} (\cos(y(n+k)) + \cos(y(n-k))) dy \\ &= \frac{(-1)^m}{2\pi^2 m} \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\sin(-\frac{\pi}{2}(n+k))}{n+k} - \frac{\sin(-\pi(n+k))}{n+k} \right. \\ &\quad \left. + \frac{\sin(-\frac{\pi}{2}(n-k))}{n-k} - \frac{\sin(-\pi(n-k))}{n-k} \right) \end{aligned}$$

$$\begin{aligned} \frac{(-1)^m}{2\pi^2 jm} \int_{\frac{\pi}{2}}^{\pi} AA1 dy &= \frac{(-1)^m}{2\pi^2 jm} \int_{\frac{\pi}{2}}^{\pi} \left(J_0(\xi) \cos(\delta) + \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(\xi) (\cos(\delta + k\theta) \right. \\ &\quad \left. + \cos(\delta - k\theta)) - \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(\xi) (\sin(\delta + k\theta) + \sin(\delta - k\theta)) \right) dy \end{aligned}$$

$$\begin{aligned} P1.I2.1 &= \frac{(-1)^m}{2\pi^2 jm} \int_{\frac{\pi}{2}}^{\pi} \left(J_0(\xi) \cos(\delta) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 jm} \int_{\frac{\pi}{2}}^{\pi} \left(J_0(-m2\pi Q) \cos(ny) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 jmn} J_0(-m2\pi Q) \left(-\sin(n\frac{\pi}{2}) \right) \end{aligned}$$

$$\begin{aligned} P1.I2.2 &= \frac{(-1)^m}{2\pi^2 jm} \int_{\frac{\pi}{2}}^{\pi} \left(\sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(\xi) (\cos(\delta + k\theta) + \cos(\delta - k\theta)) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 jm} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \int_{\frac{\pi}{2}}^{\pi} \left(\cos(y(n+k)) + \cos(y(n-k)) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 jm} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\sin(\pi(n+k))}{n+k} - \frac{\sin(\frac{\pi}{2}(n+k))}{n+k} \right. \\ &\quad \left. + \frac{\sin(\pi(n-k))}{n-k} - \frac{\sin(\frac{\pi}{2}(n-k))}{n-k} \right) \end{aligned}$$

$$\begin{aligned} P1.I2.3 &= \frac{-(-1)^m}{2\pi^2 jm} \int_{\frac{\pi}{2}}^{\pi} \left(\sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(\xi) (\sin(\delta + k\theta) + \sin(\delta - k\theta)) \right) dy \\ &= \frac{-(-1)^m}{2\pi^2 jm} \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \int_{\frac{\pi}{2}}^{\pi} \left(\sin(y(n+k)) + \sin(y(n-k)) \right) dy \\ &= \frac{-(-1)^m}{2\pi^2 jm} \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\cos(\frac{\pi}{2}(n+k))}{n+k} - \frac{\cos(\pi(n+k))}{n+k} \right. \\ &\quad \left. - \frac{\cos(\frac{\pi}{2}(n-k))}{n-k} - \frac{\cos(\pi(n-k))}{n-k} \right) \end{aligned}$$

Part 2 Integral 2

$$\begin{aligned} \frac{(-1)^m}{2\pi^2 jm} \int_{\frac{\pi}{2}}^{\pi} A A_2 dy &= \frac{(-1)^m}{2\pi^2 jm} \int_{\frac{\pi}{2}}^{\pi} j \left(J_0(\xi) \sin(\delta) + \sum_{k=2 \text{ even}}^{\infty} \cos(k \frac{\pi}{2}) J_k(\xi) (\sin(\delta + k\theta) \right. \\ &\quad \left. + \sin(\delta - k\theta)) + \sum_{k=1 \text{ odd}}^{\infty} \sin(k \frac{\pi}{2}) J_k(\xi) (\cos(\delta + k\theta) + \cos(\delta - k\theta)) \right) \end{aligned}$$

$$\begin{aligned} P2.I2.1 &= \frac{(-1)^m}{2\pi^2 m} \int_{\frac{\pi}{2}}^{\pi} J_0(\xi) \sin(\delta) dy \\ &= \frac{(-1)^m}{2\pi^2 m} \int_{\frac{\pi}{2}}^{\pi} J_0(-m2\pi Q) \sin(ny) dy \\ &= \frac{(-1)^m}{2\pi^2 mn} J_0(-m2\pi Q) (-\cos(n\pi) + \cos(n \frac{\pi}{2})) \end{aligned}$$

$$\begin{aligned} P2.I2.2 &= \frac{(-1)^m}{2\pi^2 m} \int_{\frac{\pi}{2}}^{\pi} \left(\sum_{k=2 \text{ even}}^{\infty} \cos(k \frac{\pi}{2}) J_k(\xi) (\sin(\delta + k\theta) + \sin(\delta - k\theta)) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 m} \sum_{k=2 \text{ even}}^{\infty} \cos(k \frac{\pi}{2}) J_k(-m2\pi Q) \int_{\frac{\pi}{2}}^{\pi} (\sin(y(n+k)) + \sin(y(n-k))) dy \\ &= \frac{(-1)^m}{2\pi^2 m} \sum_{k=2 \text{ even}}^{\infty} \cos(k \frac{\pi}{2}) J_k(-m2\pi Q) \left(-\frac{\cos(\pi(n+k))}{n+k} + \frac{\cos(\frac{\pi}{2}(n+k))}{n+k} \right. \\ &\quad \left. - \frac{\cos(\pi(n-k))}{n-k} + \frac{\cos(\frac{\pi}{2}(n-k))}{n-k} \right) \end{aligned}$$

$$\begin{aligned} P2.I2.3 &= \frac{(-1)^m}{2\pi^2 m} \int_{\frac{\pi}{2}}^{\pi} \left(\sum_{k=1 \text{ odd}}^{\infty} \sin(k \frac{\pi}{2}) J_k(\xi) (\cos(\delta + k\theta) + \cos(\delta - k\theta)) \right) dy \\ &= \frac{(-1)^m}{2\pi^2 m} \sum_{k=1 \text{ odd}}^{\infty} \sin(k \frac{\pi}{2}) J_k(-m2\pi Q) \int_{\frac{\pi}{2}}^{\pi} (\cos(y(n+k)) + \cos(y(n-k))) dy \\ &= \frac{(-1)^m}{2\pi^2 m} \sum_{k=1 \text{ odd}}^{\infty} \sin(k \frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\sin(\pi(n+k))}{n+k} - \frac{\sin(\frac{\pi}{2}(n+k))}{n+k} \right. \\ &\quad \left. + \frac{\sin(\pi(n-k))}{n-k} - \frac{\sin(\frac{\pi}{2}(n-k))}{n-k} \right) \end{aligned}$$

Part 3 Integral 1

$$\begin{aligned} \frac{(-1)^m}{2\pi^2 jm} \int_{-\pi}^{-\frac{\pi}{2}} E dy &= \frac{(-1)^m}{2\pi^2 jm} \int_{-\pi}^{-\frac{\pi}{2}} (\cos(ny) + j \sin(ny)) dy \\ &= \frac{(-1)^m}{2\pi^2 jmn} \left(\sin(n \frac{-\pi}{2}) - j \cos(n \frac{-\pi}{2}) + j \cos(-n\pi) \right) \end{aligned}$$

Part 3 Integral 2

$$\begin{aligned}\frac{(-1)^m}{2\pi^2 jm} \int_{\frac{\pi}{2}}^{\pi} E dy &= \frac{-(-1)^m}{2\pi^2 jm} \int_{\frac{\pi}{2}}^{\pi} (\cos(ny) + j\sin(ny)) dy \\ &= \frac{-(-1)^m}{2\pi^2 jmn} \left(-\sin(n\frac{\pi}{2}) + j\cos(n\frac{\pi}{2}) - j\cos(n\pi) \right)\end{aligned}$$

$$\begin{aligned}A_{21}(m, n) + jB_{21}(m, n) + A_{22}(m, n) + jB_{22}(m, n) \\ = \left(P1.I1.1 + P1.I1.2 + P2.I1.1 + P2.I1.2 + P2.I1.3 + P3.1 + P1.I2.1 \right. \\ \left. + P1.I2.2 + P1.I2.3 + P2.I2.1 + P2.I2.2 + P2.I2.3 + P3.I2 \right)\end{aligned}$$

$$\begin{aligned}
& A_{21}(m, n) + jB_{21}(m, n) + A_{22}(m, n) + jB_{22}(m, n) \\
&= \frac{(-1)^m}{2\pi^2 jmn} J_0(-m2\pi Q) \sin(-n\frac{\pi}{2}) + \frac{(-1)^m}{2\pi^2 jm} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\sin(-\frac{\pi}{2}(n+k))}{n+k} \right. \\
&\quad \left. - \frac{\sin(-\pi(n+k))}{n+k} + \frac{\sin(-\frac{\pi}{2}(n-k))}{n-k} - \frac{\sin(-\pi(n-k))}{n-k} \right) + \frac{(-1)^m}{2\pi^2 jm} \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) \\
&\quad J_k(-m2\pi Q) \left(\frac{-\cos(-\frac{\pi}{2}(n+k))}{n+k} + \frac{\cos(-\pi(n+k))}{n+k} - \frac{\cos(-\frac{\pi}{2}(n-k))}{n-k} + \frac{\cos(-\pi(n-k))}{n-k} \right) \\
&\quad + \frac{(-1)^m}{2\pi^2 mn} J_0(-m2\pi Q) (-\cos(-n\frac{\pi}{2}) + \cos(-n\pi)) + \frac{(-1)^m}{2\pi^2 m} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \\
&\quad \left(-\frac{\cos(-\frac{\pi}{2}(n+k))}{n+k} + \frac{\cos(-\pi(n+k))}{n+k} - \frac{\cos(-\frac{\pi}{2}(n-k))}{(n-k)} + \frac{\cos(-\pi(n-k))}{n-k} \right) + \frac{(-1)^m}{2\pi^2 m} \\
&\quad \sum_{k=1 \text{ odd}}^{\infty} \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\sin(-\frac{\pi}{2}(n+k))}{n+k} - \frac{\sin(-\pi(n+k))}{n+k} + \frac{\sin(-\frac{\pi}{2}(n-k))}{n-k} - \right. \\
&\quad \left. \frac{\sin(-\pi(n-k))}{n-k} \right) + \frac{(-1)^m}{2\pi^2 jmn} J_0(-m2\pi Q) (-\sin(n\frac{\pi}{2})) + \frac{(-1)^m}{2\pi^2 jm} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \\
&\quad \left(\frac{\sin(\pi(n+k))}{n+k} - \frac{\sin(\frac{\pi}{2}(n+k))}{n+k} + \frac{\sin(\pi(n-k))}{n-k} - \frac{\sin(\frac{\pi}{2}(n-k))}{n-k} \right) + \frac{(-1)^m}{2\pi^2 jm} \sum_{k=1 \text{ odd}}^{\infty} \\
&\quad \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\cos(\frac{\pi}{2}(n+k))}{n+k} - \frac{\cos(\pi(n+k))}{n+k} + \frac{\cos(\frac{\pi}{2}(n-k))}{n-k} - \frac{\cos(\pi(n-k))}{n-k} \right) \\
&\quad + \frac{(-1)^m}{2\pi^2 mn} J_0(-m2\pi Q) (-\cos(n\pi) + \cos(n\frac{\pi}{2})) + \frac{(-1)^m}{2\pi^2 m} \sum_{k=2 \text{ even}}^{\infty} \cos(k\frac{\pi}{2}) J_k(-m2\pi Q) \\
&\quad \left(-\frac{\cos(\pi(n+k))}{n+k} + \frac{\cos(\frac{\pi}{2}(n+k))}{n+k} - \frac{\cos(\pi(n-k))}{(n-k)} + \frac{\cos(\frac{\pi}{2}(n-k))}{n-k} \right) + \frac{(-1)^m}{2\pi^2 m} \sum_{k=1 \text{ odd}}^{\infty} \\
&\quad \sin(k\frac{\pi}{2}) J_k(-m2\pi Q) \left(\frac{\sin(\pi(n+k))}{n+k} - \frac{\sin(\frac{\pi}{2}(n+k))}{n+k} + \frac{\sin(\pi(n-k))}{n-k} - \frac{\sin(\frac{\pi}{2}(n-k))}{n-k} \right) \\
&\quad + \frac{-(-1)^m}{2\pi^2 jmn} \left(\sin(n\frac{-\pi}{2}) - j\cos(n\frac{-\pi}{2}) + j\cos(-n\pi) - \sin(n\frac{\pi}{2}) + j\cos(n\frac{\pi}{2}) - j\cos(n\pi) \right)
\end{aligned}$$

Appendix E

Interconnections Between Different or Inside Scientific Disciplines

E.1 Introduction to the Idea of Complexity

The study of interconnections between scientific domains is intrinsically compelling, insofar as the ability to relate ideas and concepts not only serves to consolidate knowledge and improve understanding, but also enables the extension of knowledge through carefully selected analogies. Such analogies must be grounded upon a significant degree of similarity between fields while duly acknowledging the potentially profound and persistent differences that may exist among them.

The etymology of the central term in this section, namely "complex", originates from Latin. The word *complexus* is the past participle of *complecti*, itself derived from *com* and *plectere*, meaning respectively "with" and "to entwine".

Consequently, the notion of complexity, understood as the idea of "binding together", acquires its full significance. One cannot conceive of our world, our sciences, or our lives as anything other than manifestations of this concept of complexity.

This leads the author to present hereafter a few examples aimed primarily at illustrating the notion of complexity, a concept extensively studied by Edgar Morin, a contemporary and essential thinker regarding this subject. Among his numerous works, he published in 1990 *Introduction à la pensée complexe* [117].

By establishing links between domains, or between concepts within a given domain, the objective is to pursue what appears to be a fruitful path toward a deeper understanding of natural phenomena. These phenomena constitute the entirety of what is given to us to experience or observe, either in actuality or in potentiality.

A brief reminder may be provided regarding the notion of natural phenomena. Indeed, these phenomena belong to the sensible world described by Plato and Socrates. This

concept is also extensively addressed in the work of the twentieth- and twenty-first-century scientist and Nobel laureate in medicine, Christian de Duve, particularly in his book *Poussière de vie*, published in 1996 [118].

Albert Einstein also encouraged, in his 1936 article *Physics and Reality* [119], the rigorous search for fundamental laws capable of providing the most complete possible explanation of observed phenomena, namely natural phenomena.

The relevance of this profound invitation will become apparent through the presentation of selected examples, chosen to highlight phenomena rooted in disciplines that habitual practice and the necessity of simplification have compelled us to distinguish rigorously. Such distinctions have sometimes generated reflections that do not necessarily correspond to the true nature of the underlying phenomena.

E.2 Passive Filters and Antibiotics : Two Solutions but Two New Issues

E.2.1 Introduction

It may appear unconventional to establish a connection between two domains that seem, at first sight, to be so distant from one another. In reality, this difference is primarily superficial, and it is the responsibility of the scientist to remain aware of the artificial boundaries that have been established between scientific disciplines, even though such boundaries may be justified from a practical perspective.

This section therefore proposes an analogy between electrical engineering, through the presentation of passive filters, and medical research, through the illustration of a specific issue related to the use of antibiotics.

The general objective is to demonstrate that both passive filters and antibiotics represent effective solutions to common problems, while neither of them is exempt from secondary consequences.

E.2.2 Passive Filter

Passive filters are designed to obtain a desired electrical behavior at a specific frequency or within a defined frequency range, which may otherwise be affected by resonance phenomena. In electrical systems, resonances can lead to critical issues in the operation and management of part or all of a power grid, depending on their magnitude.

Their construction is conceptually simple and relies on the arrangement of passive components, namely resistors, capacitors (denoted C), and inductors (denoted L). An illustration is provided in Fig. E.1, showing the classical configurations of both series and parallel passive filters.

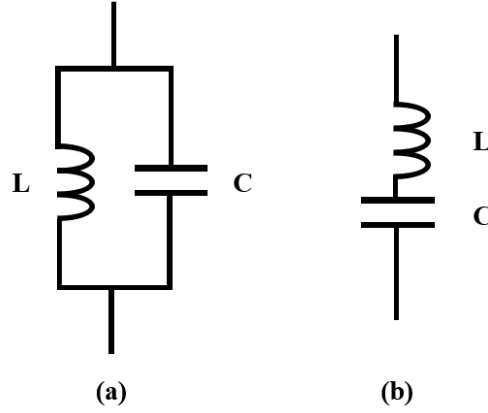


Figure E.1: Electrical schematic of a classical passive filter: the series configuration (a) and the parallel configuration (b)

Then, using the classical impedance expressions, the equation allowing the determination of the resonance frequency, namely ω_{res} or F_{res} , can be obtained. At this frequency, the resulting impedance becomes zero in the case of the series filter, or tends toward infinity in the case of the parallel filter.

$$Z_L = j\omega L \quad | \quad Z_C = \frac{1}{j\omega C}$$

$$Z_{tot} = j\omega L + \frac{1}{j\omega C}$$

If $Z_{tot} = 0$ then

$$\omega_{res} = \frac{1}{\sqrt{LC}}$$

$$F_{res} = \frac{\omega_{res}}{2\pi}$$

In fact, configurations (a) and (b) share the same resonance frequency expression; therefore, no further development is required.

The underlying concept is now established: the objective is to adapt the impedance of a system to a specific frequency or frequency range in order to avoid, more generally, the risks associated with resonances. However, this approach is not without consequences.

Indeed, an electrical network is composed of multiple branches arranged in series and parallel configurations. Considering a simplified case, let us return to the previously illustrated passive filter and assume the existence of an equivalent passive circuit connected in parallel, whether of the series or parallel type. The combination of these two passive circuits in parallel creates an additional resonance frequency at which the total impedance tends toward infinity.

If the initial objective was to design a filter providing zero impedance at a given frequency, the resulting system may nevertheless exhibit another frequency at which the impedance increases significantly and, depending on the operating conditions, potentially very rapidly. If this behavior has not been anticipated, it may negatively affect the operation and management of the electrical grid.

E.2.3 Antibiotics, β -lactamines

In 1928, medicine experienced one of the most significant revolutions in its history: the discovery of penicillin by Alexander Fleming. Penicillin provided unprecedented therapeutic capabilities. Before its discovery, bacterial infections were among the leading causes of mortality, as illustrated by the statement of Dr. Lewis Thomas: “For the most part, for patients with infectious diseases at Boston City Hospital in 1937, there was nothing we could do except put them to bed and give them good nursing care.”

Consequently, penicillin—and more generally, the family of antibiotics based on the β -lactam ring—has become a fundamental component of modern medicine. However, similarly to passive filters, these therapeutic approaches are not exempt from limitations. The following section aims to describe a secondary effect associated with β -lactam antibiotics, which may contribute to explaining the occurrence of certain infections following antibiotic treatment.

The following explanations are based on several scientific publications [120][121][122][123][124].

β -lactamases are enzymes primarily responsible for the hydrolysis of antibiotics belonging to the β -lactam family. Two main categories can be distinguished: ion-independent β -lactamases and ion-dependent metallo- β -lactamases. Regarding the latter, eighteen genes have been identified in bacteria as encoding metallo- β -lactamases (MBLs).

However, particular interest is associated with the protein SNM1C/Artemis. Due to its structural similarity with other β -lactamases, it is classified within this family, although it is not encoded by any of the aforementioned eighteen genes. In reality, its biological function differs significantly from classical β -lactamases, whose primary role is antibiotic degradation. SNM1C/Artemis acts as a nuclease involved in the non-homologous end joining (NHEJ) pathway responsible for the repair of double-stranded DNA breaks, particularly during the recombination of V(D)J gene segments. These segments are essential for adaptive immunity, as they generate the diversity of B and T lymphocytes required for recognition of a broad range of antigens.

The studies underlying this discussion have demonstrated, *in vitro*, that β -lactam antibiotics inhibit SNM1C/Artemis activity. From a clinical perspective, patients treated with β -lactam antibiotics have been observed to present an increased risk of systemic fungal infections, collectively referred to as candidemia, as well as a reduced efficacy of certain immunotherapies.

Consequently, while combating an ongoing infection, one may simultaneously compromise

the immune system's ability to respond effectively to future infectious threats.

E.2.4 Conclusion

The analysis presented here highlights that, although solutions have been developed for certain commonly encountered problems in the fields of electrical engineering and medicine, secondary effects may persist and can exhibit characteristics closely related to the initially addressed issue.

Indeed, whether considering electrical filters designed to mitigate resonance phenomena or antibiotics employed against bacterial infections, both approaches may introduce additional risks—respectively, new resonances or increased susceptibility to certain infections. A scientist familiar with either domain could have formulated such a hypothesis a priori and subsequently investigated it, as demonstrated by the studies previously cited.

This observation therefore justifies the introduction of this appendix. Indeed, complex thinking not only contributes to the consolidation of existing knowledge but may also facilitate the emergence of new interpretations and deeper understanding.

E.3 On The Nature of Forces, Tensions: A Triple Point of View

E.3.1 Introduction

In continuity with the previous developments, this section addresses a fundamental question: the notion of *situational change*. Although physical, medical, and psychological perspectives may initially appear largely independent—and should indeed be understood as distinct regarding both their nature and the methodologies required to investigate them—it has nevertheless been demonstrated that apparently separated domains may be profoundly interconnected.

The focus of the present discussion is therefore the concept of *force*, which will be introduced and supported through theoretical considerations arising from the aforementioned fields. Indeed, any modification of a given situation can be associated with the existence of an interaction capable of inducing change. In what follows, the concept of force will successively be related to electric fields, mechanical pressure, and finally psychological tension as it appears within interpersonal relationships.

All these perspectives converge toward a common objective: to illustrate the potential—and arguably the necessity—of *sublimating the notion of complexity*, understood here as the interconnection of notions, ideas, and concepts in the broader pursuit of understanding.

E.3.2 Electrical engineering

Electric fields are remarkable physical phenomena. They can indeed be understood as the consequence of the presence of electric charges. Maxwell's equations provide a complete

theoretical framework for electromagnetism, while numerous analytical and numerical methods have been developed to enable its practical application.

Let us recall that the electric field is a vector field, meaning that each point in space is associated with a vector quantity describing the force experienced by a unit positive charge placed at that location. Accordingly, the unit of the electric field is newtons per coulomb.

Regarding the central question addressed here—the relationship between force and the (not always intuitive) notion of change or transformation of a situation—it may be stated that any non-zero uncompensated force induces a modification of motion. This principle is expressed through Newton’s second law, $F = ma$, where F represents the force vector, m the mass, and a the resulting acceleration vector.

From the perspective of electrical engineering, particular attention is given to the concept of the *skin effect*, which is discussed in greater detail in Appendix C. The skin effect describes the penetration depth, denoted δ , of an electromagnetic field within a conducting material. It has been demonstrated that this penetration depth decreases with increasing frequency, ω , and conductivity, σ , of the material. Regarding the magnetic permeability, μ , for non-magnetic materials such as copper or aluminium, it can be considered equal to the vacuum permeability, denoted μ_0 .

$$\delta = \frac{\sqrt{2}}{\sqrt{\omega\mu_0\sigma}}$$

In the case of ideal conductors, whose conductivity is assumed to be infinite, the theoretical penetration depth of the electromagnetic field tends toward zero. Extending this reasoning provides an explanation of the Faraday cage phenomenon. Specifically, if one considers a closed structure made of a material with extremely high (or ideally infinite) conductivity, the interior becomes isolated from external electric fields. Consequently, no electric force acts upon charges located inside the structure. However, in the absence of an external force, no induced motion can occur.

It therefore follows that the ideal nature of a conductor—understood as the complete exclusion of an internal electric field—paradoxically prevents the existence of the very driving mechanism responsible for charge motion, namely the electric force.

E.3.3 Biological science

A biological perspective allows this reasoning to be extended through the consideration of a widely studied concept: *compliance*. Compliance describes the variation in volume, V , of a vessel resulting from a variation in internal pressure, P .

$$Compliance = \frac{\Delta V}{\Delta P}$$

This concept provides an interesting parallel with the physical framework previously introduced, highlighting how structural and environmental constraints influence the ability

of a system to undergo internal transformation.

It is therefore possible to establish an analogy between the skin effect discussed in electrical engineering and a hypothetical situation involving infinite compliance in biological systems. This analogy leads to the consideration that a pressure field—representing a force per unit area—shares certain conceptual similarities with the electric field, which represents a force per unit charge.

In this perspective, pressure fields and electric fields may be compared: the former is expressed in newtons per square meter, while the latter is expressed in newtons per coulomb. Both describe spatial distributions associated with forces, whose effects depend on the properties of the surrounding medium and the constraints imposed on the system.

Infinite compliance would imply that an infinitesimal variation in pressure would result in an infinite variation in volume, which is physiologically impossible, just as an electric current of infinite magnitude cannot exist physically. Consequently, in a similar manner to the confinement of the electric field at the surface of an ideal conductor, pressure variations would theoretically remain restricted to the boundary of the vessel rather than propagating within the structure itself.

Once again, the absence of a pressure gradient within the vessel wall would imply the absence of a mechanism capable of generating deformation. Paradoxically, both electrical current and vascular deformation would theoretically tend toward extreme values in the idealized limit, yet they would not manifest in practice because the entire effect becomes confined to a boundary—the surface of the ideal conductor or that of the idealized vessel wall.

E.3.4 Psychological science

Within the field of psychology, the same conceptual framework may be extended to describe *tension* as a condition that can contribute to the modification of an experienced or perceived situation. Interestingly, this hypothesis—rooted in the notion of complexity developed throughout this chapter—finds conceptual and theoretical support in various studies, articles, and scholarly works addressing this subject from different perspectives [125][126][127][128][129][130].

Accordingly, an individual subjected—whether voluntarily or involuntarily—to a given situation may be considered through a framework analogous to the physical systems previously discussed. Indeed, by introducing the notion of *social compliance*, understood as an individual's tendency to conform to or yield to external demands, the individual may be represented as a system subjected to an external influence, similarly to the Faraday cage analogy introduced previously. In this analogy, the incident field corresponds to external constraints originating from other individuals, cultural frameworks, institutions, or broader sociocultural structures.

An individual's inability—or deliberate refusal—to oppose such external influences may result in the absence of internal tension and, consequently, in the disappearance of the internal driving force required to initiate a movement toward a modification of the existing situation. Without such an internal mechanism, transformation becomes unlikely.

Ultimately, by unconditionally accepting external expectations, an individual may progressively lose the capacity to initiate meaningful change. This conceptual reflection can be extended to various relational contexts, including romantic relationships, family dynamics, hierarchical organizations, or any social configuration involving an individual experiencing constraint and an external agent—concrete or abstract—exerting influence. Such an agent may correspond to another person, an institution, a cultural framework, or even a socially established norm.

The notion of *differentiation of self*, introduced by Murray Bowen within the framework of family systems theory [131], provides an interesting illustration of the conceptual idea developed throughout this appendix. Indeed, it may be regarded as another example supporting the unifying perspective proposed across the different scientific disciplines considered.

Before proceeding further, an important clarification should be made regarding the terminology adopted throughout this appendix. From a strictly physical point of view, the term *tension* is not the most appropriate. In reality, the underlying mathematical concept is that of a *force*. The use of the word *tension* should therefore be understood only as a simplified terminology intended to facilitate the exposition of a more fundamental mathematical and physical concept related to the evolution of dynamical systems within their state spaces.

In this framework, the notion of tension is interpreted as the manifestation of a force field. Such a field is represented mathematically by a vector field which, under equilibrium conditions, may be expressed as the gradient of a scalar potential. This interpretation implicitly assumes conditions analogous to those discussed previously for equilibrium systems, namely the existence of a conservative field whose curl vanishes within the considered domain. A rigorous mathematical treatment of this idea would require a precise definition of the considered state space and of the associated physical constraints. However, since the objective of the present appendix is primarily conceptual, the simplified terminology based on the word *tension* will be retained.

Within Bowen's theory, differentiation refers to the capacity of an individual to progressively establish psychological autonomy while remaining emotionally connected to the family system. Conversely, insufficient differentiation results in a persistent emotional coupling between the individual and other family members. Thoughts, emotions, and behaviours therefore remain strongly correlated with those of the family system, limiting the emergence of autonomous patterns of functioning.

This interpretation appears remarkably consistent with the conceptual framework developed throughout the present appendix. Indeed, the persistence of the original relational structure may be interpreted as the consequence of an insufficient driving force capable of modifying the initial state of the relationship. Such a force should not necessarily be interpreted as conflict, but rather as the existence of a sufficient psychological tension allowing the relationship itself to evolve toward a new equilibrium.

From this perspective, the absence of significant differentiation may be interpreted as resulting from a force that was either absent or insufficient to produce a substantial modification of the initial relational configuration. Consequently, the original coupling persists,

leading to strong correlations between the individual's thoughts, emotional responses, behaviours, and those of the surrounding family system. The emergence of psychological autonomy and of an individual pattern of functioning therefore becomes considerably more difficult.

Ultimately, the objective of this appendix has been to illustrate how a common conceptual framework may be identified across several scientific disciplines. This framework provides a coherent interpretation of the interaction between electromagnetic waves and conducting materials, suggests a possible conceptual analogy with biological systems through the notions of pressure and compliance, and finally offers an interpretative perspective for psychological concepts such as Bowen's differentiation of self. Although the underlying mechanisms remain fundamentally different within each discipline, they all appear to share the common idea that the evolution of a system requires the existence of a driving interaction capable of modifying its current state.

A more exhaustive development would require extending the simplified concepts intentionally adopted throughout this appendix. Such an extension would naturally lead toward the mathematical framework introduced in Appendix B, where the notion of increasing dimensionality was presented. Indeed, representing systems within high-dimensional vector spaces—and, in the limiting case, within infinite-dimensional Hilbert spaces—offers a possible mathematical framework for describing increasingly complex systems. If the evolution of such systems were ultimately governed by a sufficiently rich deterministic description, then the identification of a sufficiently large number of variables defining the initial state could, at least theoretically, permit the prediction of their subsequent evolution. Within this interpretation, each additional variable would simply correspond to an additional dimension of the associated state space.

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