

Polarisation-Asymmetry Correlation in Allowed β -Decay: a Probe for Right-Handed Currents

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Abstract

The sensitivity of polarisation-asymmetry correlation experiments to charged currents of right-handed chirality contributing to allowed β -decay is considered in the most general context possible, independently of any type of approximation nor of any specific model for physics beyond the Standard Model of the electroweak interactions. Results are then particularised to general Left-Right Symmetric Models, and experimental prospects offered by mirror nuclei are assessed explicitly on general grounds. In order of decreasing interest, the cases of ^{17}F , ^{41}Sc and ^{25}Al are the most attractive, providing sensitivities better or comparable to allowed pure Gamow-Teller transitions, with the advantage however, that recoil order corrections are smaller in the case of super-allowed decays.

1 Introduction

Even though the Standard Model (SM) remains unchallenged by an impressive body of precision electroweak measurements, over the years the general consensus has been that there must exist new physics lurking behind the horizon of the related problems of the origin of mass and the chirality structure of the electroweak interactions. Any embodiment of such new physics would manifest itself through deviations from the predictions of the SM for specific observables and production mechanisms, be it at low, intermediate or high energies.

In the case of semi-leptonic weak interactions at low energies, it is essentially only the radioactive nucleus which is available as a laboratory for the search of new physics through β -decay[1]. In particular, it has recently been emphasized[2] that when compared to asymmetry measurements, the *relative* longitudinal polarisation of β particles emitted in directions parallel or antiparallel to the polarisation vector of an oriented nucleus presents an enhanced sensitivity to a hypothetical right-handed charged current contribution. Indeed, one among other attractive extensions of the SM is obtained by enlarging the electroweak sector of the latter model to the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in the context of so-called Left-Right Symmetric Models (LRSM)¹. In such a case, the new physics to be found beyond the SM stems from the existence of additional charged and neutral gauge W' and Z' and physical Higgs bosons, as well as a host of additional phenomenological parameters leading to massive neutrinos and the ensuing leptonic flavour mixing, to new sources of CP violation originating in the Higgs as well as in the Yukawa sectors, and to other physical effects of interest (see for example Refs.[4, 5]). In spite of their great appeal, such extensions do not provide an understanding for the origin of mass nor for the chirality structure of the fundamental electroweak interactions—now enlarged to include right-handed ones—, even though parity invariance may *effectively* be restored in certain classes of such models for processes characterised by momenta transfers much larger than the masses of the new gauge bosons W' and Z' . Nevertheless, the structure of LRSM typically arises as a low-energy effective theory for most grand unified theories—supersymmetric or not—whose gauge symmetry breaking pattern includes the gauge group $SO(10)$.

In this note, the sensitivity offered by the relative longitudinal polarisation measurement mentioned above is considered[2] in the case of allowed β -decays. Specific results are established for mirror nuclei in the context of *general* LRSM, following the discussion of Ref.[6] which analysed the sensitivity offered by *asymmetry* measurements in the context of so-called Manifest Left-Right Symmetric Models (MLRSM) which are constructed such that at tree-level gauge coupling constants and flavour mixing matrices be identical for both chirality sectors of the theory. In fact, two such polarisation-asymmetry correlation experiments have already yielded results in the cases of the allowed pure Gamow-Teller (GT) decays of ^{107}In [7] and ^{12}N [8], for which the sensitivity to possible right-handed charged current contributions must *a priori* be among the largest attainable, owing to the Gamow-Teller character of these transitions and their large asymmetry parameter.

¹For reviews and references to the original literature, see for example Refs.[3, 4, 5],

The outline of the note is as follows. Sect.2 addresses the longitudinal polarisation-asymmetry correlation measurement in the general case of allowed β -decays assuming only vector and axial contributions to the charged current interaction. In Sect.3, these results are particularised to *general* Left-Right Symmetric Models. The case of mirror nuclei is then analysed in Sect.4, in order to identify possible attractive candidates for this type of measurements beyond the cases of allowed pure GT decays. This is done on general grounds, without paying attention to the necessary considerations concerning the technical feasibility either of such experiments or of the required degree of nuclear polarisation, which would have to be addressed on a case by case basis. Further results of interest are presented in an Appendix, while the note ends with some general conclusions.

2 Allowed β -decay and (V, A) charged currents

The expressions for physical observables of relevance to allowed β -decay in the case of the most general four-fermi effective interaction are available from Ref.[9]. In this note, *only vector (V) and axial (A) a priori complex four-fermi coupling coefficients are considered*, associated to pure vector and axial charged current contributions to β -decay. *Under this specific restriction* and ignoring recoil order corrections², when only the $\beta^\mp$ particle is observed the corresponding distribution is then given by the spin density matrix[9]

$$\frac{d^3W}{dE d^2\Omega} = W_0(E) \xi \left\{ 1 + \beta J A(\hat{p} \cdot \hat{J}) + \vec{\sigma} \cdot \left[\beta G \hat{p} + J \frac{\gamma_z}{\gamma} N' \hat{J} + J(1 - \frac{\gamma_z}{\gamma}) N' (\hat{p} \cdot \hat{J}) \hat{p} \mp J \frac{\alpha Z}{\gamma} A(\hat{J} \times \hat{p}) \right] \right\}, \quad (1)$$

with the following notation. The parameter $(-1 \leq J \leq +1)$ stands for the degree of nuclear polarisation of the oriented nucleus, while the normalised vector $\hat{J}$ represents the direction of nuclear polarisation. Of course, β is the velocity of the emitted $\beta^\mp$ particle—normalised to the speed of light in vacuum—, with γ the associated relativistic dilatation factor ($\gamma = 1/\sqrt{1 - \beta^2}$), while E , $\hat{p}$ and $\vec{\sigma}$ are the $\beta^\mp$ particle total energy, normalised momentum and polarisation (spin) quantum operator, respectively. Coulomb corrections[9] induce a dependence on the fine structure constant α , while γ_z is defined as ($\gamma_z = \sqrt{1 - (\alpha Z)^2}$) where Z is the charge of the *daughter* nucleus. The coefficients $W_0(E)$, ξ , A , G and N' are given in the Appendix for β -transitions of initial nuclear spin J and final nuclear spin J' in terms of the effective four-fermi complex coupling coefficients C_V , C'_V , C_A and C'_A introduced in Ref.[9]. Finally, the choice of sign in front of the last term in the above expression corresponds to whether it is an electron or a positron which is emitted; throughout the note, whenever such a choice is

²In the case of super-allowed decays such as those of mirror nuclei, these corrections are small and such an approximation is certainly justified in the present context which aims at identifying potential candidates for relative longitudinal polarisation-asymmetry correlation experiments.

indicated, the upper sign always corresponds to electron emission, and the lower sign to positron emission.

Since the purpose of the present analysis is the identification of attractive candidates for polarisation-asymmetry correlation experiments besides allowed pure GT decays, let us consider the ideal situation in which $\beta^\mp$ particles of a specific energy E —or velocity β —and initial momentum direction $\hat{p}$ are observed and for which it is *exactly* their *longitudinal polarisation* which is measured. All other things being equal, different nuclei may then be compared on an equal footing under these identical *idealised* experimental conditions. Obviously on a practical level, other considerations would also have to be addressed, such as the finite energy and angular acceptance of any experimental set-up leading to additional contributions from transverse spin components as well, the spin precession in magnetic fields of spectrometers or polarimeters, and more importantly, the feasibility of the production and of the polarisation of a given nucleus. As mentioned in the Introduction, such issues are not tackled in this note, since they can only be considered on a case by case basis once a potential candidate is identified.

Under the stated idealised conditions, and given a nucleus of polarisation J , the experimental asymmetry of the β -decay distribution is of the form

$$N_\beta(J) = N_0 \left[1 + \beta J A(\hat{p} \cdot \hat{J}) \right] , \quad (2)$$

where N_0 represents the normalised source activity. Similarly, from the spin density matrix in (1), the *longitudinal* polarisation of the produced $\beta^\mp$ particle is then given by the expression

$$P_L(J) = \frac{\beta G + J N'(\hat{p} \cdot \hat{J})}{1 + \beta J A(\hat{p} \cdot \hat{J})} . \quad (3)$$

Following the suggestion of Ref.[2], let us consider *relative* measurements of such observables for different values of the degree J of the polarisation of the oriented nucleus, without changing the direction $\hat{J}$ of its orientation. The obvious advantage of such measurements is that they are much less sensitive to systematic effects than are absolute measurements of asymmetries or polarisations. Therefore, given two different degrees J_1 and J_2 of nuclear polarisation and *identical* normalised source activities N_0 , the relative *experimental* asymmetry $A_{\text{exp}}(J_2, J_1)$ may be defined according to

$$1 - A_{\text{exp}}(J_2, J_1) = \frac{N_\beta(J_2)}{N_\beta(J_1)} = \frac{1 + \beta J_2 A(\hat{p} \cdot \hat{J})}{1 + \beta J_1 A(\hat{p} \cdot \hat{J})} , \quad (4)$$

while the relative longitudinal polarisation is simply

$$R(J_2, J_1) = \frac{P_L(J_2)}{P_L(J_1)} = \frac{1}{1 - A_{\text{exp}}(J_2, J_1)} \frac{\beta G + J_2 N'(\hat{p} \cdot \hat{J})}{\beta G + J_1 N'(\hat{p} \cdot \hat{J})} . \quad (5)$$

Since the purpose of the present discussion is the identification of potential attractive candidates for this type of measurement, all other things being equal, it proves simpler to consider henceforth only two specific situations with regards to nuclear polarisation. The first is obtained when the reference nuclear polarisation J_1 is vanishing,

($J_1 = 0$), and when ($J_2 = -J$). The second[2] corresponds to a situation in which the $\beta^\mp$ longitudinal polarisation is considered for opposite directions of nuclear polarisation, namely for ($J_2 = -J_1 = -J$).

In the first instance, the experimental asymmetry is given by

$$A_{\text{exp}}(-J, 0) = \beta J A(\hat{p} \cdot \hat{J}) \quad , \quad (6)$$

which upon substitution in the expression for the relative longitudinal polarisation $R(-J, 0)$ leads to the result,

$$R(-J, 0) = \frac{1}{\beta^2} \frac{1}{1 - A_{\text{exp}}(-J, 0)} \left[\beta^2 - A_{\text{exp}}(-J, 0) \frac{\xi(\xi N')}{(\xi A)(\xi G)} \right] \quad . \quad (7)$$

Similarly in the second instance when ($(J_2, J_1) = (-J, J)$), one has

$$A_{\text{exp}}(-J, J) = \frac{2\beta J A(\hat{p} \cdot \hat{J})}{1 + \beta J A(\hat{p} \cdot \hat{J})} \quad , \quad (8)$$

or equivalently,

$$\beta J A(\hat{p} \cdot \hat{J}) = \frac{A_{\text{exp}}(-J, J)}{2 - A_{\text{exp}}(-J, J)} \quad . \quad (9)$$

When substituted in the definition for $R(-J, J)$, one then derives the expression for the relative longitudinal polarisation in this case,

$$R(-J, J) = \frac{1}{1 - A_{\text{exp}}(-J, J)} \left[1 - 2 \frac{1}{1 + \beta^2 \frac{2 - A_{\text{exp}}(-J, J)}{A_{\text{exp}}(-J, J)} \frac{(\xi A)(\xi G)}{\xi(\xi N')}} \right] \quad . \quad (10)$$

Note that it is the same quantity

$$\frac{\xi(\xi N')}{(\xi A)(\xi G)} \quad , \quad (11)$$

depending on the underlying physics, which appears in the expressions for $R(-J, 0)$ and $R(-J, J)$. Since this combination of parameters takes the value unity in the SM (see the Appendix), any deviation of the ratio (11) from unity must stem from some new physics³ beyond the SM. Let us thus introduce the quantity Δ defined by⁴

$$\Delta \equiv \frac{1}{4} \left[\frac{\xi(\xi N')}{(\xi A)(\xi G)} - 1 \right] = \frac{1}{4} \frac{\xi(\xi N') - (\xi A)(\xi G)}{(\xi A)(\xi G)} \quad , \quad (12)$$

which thus characterises the new physics beyond the SM which may be probed through relative longitudinal polarisation-asymmetry correlation experiments.

In order to assess the sensitivity of these measurements to such new physics, it is necessary to compare the values taken by the relative polarisations $R(-J, 0)$ and

³Note that this remark does not account for possible recoil order corrections to the relative longitudinal polarisation, which are ignored in this note.

⁴The numerical factor of a quarter is for later convenience.

$R(-J, J)$ to their values obtained simply by setting ($\Delta = 0$) in (7) and (10) without modification of the *experimental* asymmetries A_{exp} . This leads to, respectively,

$$R_0(-J, 0) = \frac{1}{\beta^2} \frac{\beta^2 - A_{\text{exp}}(-J, 0)}{1 - A_{\text{exp}}(-J, 0)} \quad , \quad (13)$$

and

$$R_0(-J, J) = \frac{1}{1 - A_{\text{exp}}(-J, J)} \frac{\beta^2 [2 - A_{\text{exp}}(-J, J)] - A_{\text{exp}}(-J, J)}{\beta^2 [2 - A_{\text{exp}}(-J, J)] + A_{\text{exp}}(-J, J)} \quad . \quad (14)$$

Let us emphasize that these expressions *are not those which one may derive in the SM* for the relative longitudinal polarisations, since they are obtained simply by setting only the quantity Δ to zero in the results for $R(-J, 0)$ and $R(-J, J)$ without assuming that the asymmetry parameter A is given by its value A_0 in the SM. Indeed, the expressions for $R_0(-J, 0)$ and $R_0(-J, J)$ still involve the *experimental* asymmetries $A_{\text{exp}}(-J, 0)$ and $A_{\text{exp}}(-J, J)$ which may differ from the values predicted by the SM if new physics does contribute to the asymmetry parameter A , as made explicit in (6) and (8). Nevertheless, it is obviously useful[2] to express the relative longitudinal polarisation in terms of the *directly observable* experimental asymmetries $A_{\text{exp}}(-J, 0)$ and $A_{\text{exp}}(-J, J)$.

Given the quantities $R_0(-J, 0)$ and $R_0(-J, J)$, any genuine physical deviation from the SM contributing to $R(-J, 0)$ and $R(-J, J)$ would be manifested through a value different from unity for the ratios,

$$\frac{R(-J, 0)}{R_0(-J, 0)} \quad , \quad \frac{R(-J, J)}{R_0(-J, J)} \quad . \quad (15)$$

In terms of the quantities introduced in the forthcoming definitions, it is straightforward to establish the following *exact* results, valid independently of whether the parameter Δ is small in comparison to unity or not. One finds,

$$\frac{R(-J, 0)}{R_0(-J, 0)} = 1 - k(-J, 0) \Delta \quad , \quad (16)$$

with the factor $k(-J, 0)$ given by

$$k(-J, 0) = 4 \frac{A_{\text{exp}}(-J, 0)}{\beta^2 - A_{\text{exp}}(-J, 0)} \quad . \quad (17)$$

Similarly for $R(-J, J)$, one has

$$\frac{R(-J, J)}{R_0(-J, J)} = 1 - k(-J, J) \frac{\Delta}{1 + 4 \frac{A_{\text{exp}}(-J, J)}{\beta^2 [2 - A_{\text{exp}}(-J, J)] + A_{\text{exp}}(-J, J)} \Delta} \quad , \quad (18)$$

with the factor $k(-J, J)$ defined by

$$k(-J, J) = 8 \frac{\beta^2 A_{\text{exp}}(-J, J) [2 - A_{\text{exp}}(-J, J)]}{\beta^4 [2 - A_{\text{exp}}(-J, J)]^2 - A_{\text{exp}}^2(-J, J)} \quad . \quad (19)$$

Note that under the considered idealised situation in which it is exactly the longitudinal $\beta^\mp$ polarisation which is measured, both the values of $R_0(-J, 0)$ or $R_0(-J, J)$ and of the factors $k(-J, 0)$ or $k(-J, J)$ only depend[2] on the corresponding *observed experimental* asymmetry A_{exp} and on the value of β^2 . Moreover, the factors $k(-J, 0)$ and $k(-J, J)$ offer an enhanced[2] sensitivity to any deviation from the value unity expected in the SM for the corresponding ratio R/R_0 of relative longitudinal polarisations. Indeed, the factor appropriate to the first case, namely $k(-J, 0)$, diverges⁵ as the experimental asymmetry $A_{\text{exp}}(-J, 0)$ approaches the value

$$A_{\text{exp}}^{(0)}(-J, 0) = \beta^2 . \quad (20)$$

Similarly, the enhancement factor appropriate to the second case, namely $k(-J, J)$, diverges as the experimental asymmetry $A_{\text{exp}}(-J, J)$ approaches the value

$$A_{\text{exp}}^{(0)}(-J, J) = \frac{2\beta^2}{1+\beta^2} , \quad (21)$$

while the quantity multiplied by $k(-J, J)$ in (18) then reduces to

$$\frac{\Delta}{1+2\Delta} . \quad (22)$$

Note that in either case, the optimal experimental asymmetry $A_{\text{exp}}^{(0)}$ corresponds to a degree of nuclear polarisation J and a choice of β such that

$$A J(\hat{p}.\hat{J}) = \beta . \quad (23)$$

In other words, for a given nucleus, namely a given asymmetry parameter A , the optimal sensitivity to a possible contribution from right-handed currents is achieved for values of the *effective* degree of nuclear polarisation,

$$\mathcal{P} = |J(\hat{p}.\hat{J})| , \quad (24)$$

and of the $\beta^\mp$ particle velocity β such that

$$|A \mathcal{P}| = \beta , \quad (25)$$

the choice of sign for $J(\hat{p}.\hat{J})$ being such that the experimental asymmetries A_{exp} as defined in this note be positive.

These conclusions correspond to the advertised[2] sensitivity of this type of measurement to right-handed currents: the closer the experimental asymmetry A_{exp} to

⁵This divergence does not entail a loss of physical significance of the results, but follows simply from the fact that the quantity $R_0(-J, 0)$ vanishes as the experimental asymmetry $A_{\text{exp}}(-J, 0)$ approaches the value $A_{\text{exp}}^{(0)}(-J, 0)$, while at the same time the product $R_0(-J, 0) k(-J, 0)$ remains finite as it should since $R(-J, 0)$ is finite under all circumstances. In other words, the significance of the divergence is that when the experimental asymmetry $A_{\text{exp}}(-J, 0)$ is optimised at the value $A_{\text{exp}}^{(0)}(-J, 0)$, the contribution of $(-R_0(-J, 0) k(-J, 0) \Delta)$ to $R(-J, 0)$ becomes increasingly larger than that of $R_0(-J, 0)$. Of course, the same comments also apply to the case of $R(-J, J)$ which follows.

$A_{\text{exp}}^{(0)}$, namely the closer the choice of values of $(\beta, \mathcal{P})$ to the optimal situation such that $(\beta = |A\mathcal{P}|)$, the larger the sensitivity to a possible deviation ($\Delta \neq 0$) from the SM. Since values of β for which the decay count rate is the largest are typically close to the maximal value of unity⁶, clearly the best sensitivity requires both the largest possible effective degree of nuclear polarisation $\mathcal{P}$ and the largest possible asymmetry parameter $|A|$. In particular, since the asymmetry parameter $|A|$ for allowed pure GT transitions of nuclear spin sequence ($J' = J - 1$) is maximal in the SM (see the Appendix), such β -decays are certainly among the best candidates to probe for right-handed currents through longitudinal polarisation-asymmetry correlation experiments. This is the case for example for the ^{107}In [7] and ^{12}N [8] nuclei.

Given the expressions for ξ , A , G and N' listed in the Appendix, the quantity Δ may easily be related to the underlying effective coupling coefficients C_V , C'_V , C_A and C'_A . It proves useful to introduce the notation,

$$a_L = M_F^2 |C_V + C'_V|^2 + M_{GT}^2 |C_A + C'_A|^2 \quad , \quad (26)$$

$$a_R = M_F^2 |C_V - C'_V|^2 + M_{GT}^2 |C_A - C'_A|^2 \quad , \quad (27)$$

$$b_L = \mp M_{GT}^2 \lambda_{J'J} |C_A + C'_A|^2 - 2\delta_{J'J} M_F M_{GT} \sqrt{\frac{J}{J+1}} \text{Re} \left((C_V + C'_V)(C_A^* + C_A'^*) \right) \quad , \quad (28)$$

$$b_R = \mp M_{GT}^2 \lambda_{J'J} |C_A - C'_A|^2 - 2\delta_{J'J} M_F M_{GT} \sqrt{\frac{J}{J+1}} \text{Re} \left((C_V - C'_V)(C_A^* - C_A'^*) \right) \quad , \quad (29)$$

in terms of which one then derives the *exact* expression

$$\Delta = \frac{1}{2} \frac{a_L b_R + a_R b_L}{(a_L - a_R)(b_L - b_R)} \quad . \quad (30)$$

Since the numerator of the result in (30) involves precisely the differences $(C_V - C'_V)$ and $(C_A - C'_A)$ which vanish for purely left-handed couplings as is the case in the SM, it is clear that the quantity Δ —probed through relative longitudinal polarisation-asymmetry correlation measurements—is indeed sensitive to right-handed charged current contributions to allowed β -decay.

Note also that in terms of the quantities a_L , a_R , b_L and b_R introduced above, the asymmetry parameter A reads,

$$A = \frac{b_L - b_R}{a_L + a_R} \quad . \quad (31)$$

When deviations of the coefficients C_V , C'_V , C_A and C'_A from their values in the SM are small, it is justified to consider a first order expansion of Δ in the quantities

⁶Small, albeit vanishing values of β are of course possible also, in which case the optimal sensitivity is achieved for small effective nuclear polarisations $\mathcal{P}$. However, $\beta^\mp$ count rates decrease as β approaches zero, thus leading to a loss in statistics for any precision measurement.

a_R and b_R , leading to

$$\Delta \simeq \frac{1}{2} \left[\frac{a_R}{(a_L)_0} + \frac{b_R}{(b_L)_0} \right] , \quad (32)$$

where $(a_L)_0$ and $(b_L)_0$ are the values of a_L and b_L in the SM, respectively,

$$(a_L)_0 = 4 |C_V^{(0)}|^2 M_F^2 [1 + \lambda^2] , \quad (33)$$

and

$$(b_L)_0 = 4 |C_V^{(0)}|^2 M_F^2 \left[\mp \lambda^2 \lambda_{J'J} - 2 \delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} \right] . \quad (34)$$

Here, $C_V^{(0)}$ is the value of the coefficient C_V in the SM, while λ is the ratio

$$\lambda = \frac{g_A}{g_V} \frac{M_{GT}}{M_F} , \quad (35)$$

g_V and g_A being the nucleon vector and axial couplings, respectively (see the Appendix for further details).

Another situation of particular interest is that of allowed pure GT transitions, for which one simply finds,

$$A_{|GT} = A_{0|GT} \frac{|C_A + C'_A|^2 - |C_A - C'_A|^2}{|C_A + C'_A|^2 + |C_A - C'_A|^2} , \quad (36)$$

$A_{0|GT}$ being the asymmetry parameter for allowed pure GT decays in the SM (see the Appendix), as well as

$$\Delta_{|GT} = \frac{1}{4} \left[\left(\frac{A_{0|GT}}{A_{|GT}} \right)^2 - 1 \right] = \frac{|C_A - C'_A|^2}{|C_A + C'_A|^2} \frac{1}{\left[1 - \frac{|C_A - C'_A|^2}{|C_A + C'_A|^2} \right]^2} . \quad (37)$$

These expressions are independent of the nucleus involved and of the specifics of the underlying new physics which would be leading to ($\Delta_{|GT} \neq 0$). Thus in such a case, the parameter Δ indeed provides a *direct* measure for right-handed current contributions to the coupling of the leptonic charged current to the hadronic axial charged current in nuclear β -decay. On the other hand, note that even though the quantity Δ is in fact related to the asymmetry parameter A in the case of allowed pure GT transitions, a measurement of the relative longitudinal polarisation-asymmetry correlation is potentially far more sensitive to contributions from right-handed currents than is a measurement of the asymmetry parameter A itself. Indeed, the former sensitivity to the ratio $(|C_A - C'_A|^2 / |C_A + C'_A|^2)$ is characterised by the enhancement factors $k(-J, 0)$ and $k(-J, J)$ which *a priori* may reach quite large values by appropriate choices of β and of the effective degree of nuclear polarisation $\mathcal{P}$. On the other hand, the sensitivity of the asymmetry parameter A to the same ratio is essentially characterised by a fixed enhancement factor of two only, as follows from (36).

Before concluding this general discussion, let us address one last issue. As was already pointed out previously, since in most cases the value of β is not much different from unity in the energy domain where the $\beta^\mp$ count rate is maximal, for a

given effective degree of nuclear polarisation $\mathcal{P}$ the enhancement factors $k(-J, 0)$ and $k(-J, J)$ are the largest for experimental asymmetries $A_{\text{exp}}(-J, 0)$ and $A_{\text{exp}}(-J, J)$ as close to the value unity as possible. However, this also implies that the count rate of $\beta^\mp$ particles associated to the direction of nuclear polarisation for which the sensitivity to Δ is the largest, is also the smaller the closer the experimental asymmetry to the value unity. Indeed, this count rate at the optimal sensitivity such that $(|A\mathcal{P}| = \beta)$ is proportional to $(1 - \beta^2)$. Nevertheless, it is possible to show that the loss in count rate is compensated for by the gain in sensitivity; for either configuration of nuclear polarisations considered in this note, the figure of merit characterising the precision with which a deviation from the value unity for the ratio R/R_0 may be established experimentally is indeed optimal for the previously given value $A_{\text{exp}}^{(0)}$ of the experimental asymmetry A_{exp} at which the corresponding enhancement factor k is the largest.

3 General Left-Right Symmetric Models

The results of the previous section are valid quite generally for allowed decays, since the only assumptions made so far are that the effective Hamiltonian for β -decay receives contributions from vector and axial currents only, with arbitrary complex coupling coefficients, and that recoil order corrections to relative longitudinal polarisations are negligible. Let us now particularise the discussion to *general* Left-Right Symmetric Models, based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in the electroweak sector. In as far as semi-leptonic charged weak interactions are concerned, contributions to the β -decay process in such models follow from charged gauge boson and Higgs exchanges. However, charged Higgs exchanges—when contributing⁷—shall be ignored in the present discussion, assuming that they are suppressed through small coupling constants and large masses. This effectively leaves only charged gauge boson exchanges, namely those of the ordinary gauge boson W of mass[10]

$$M_1 = 80.22 \pm 0.26 \text{ GeV}/c^2, \quad (38)$$

and of the hypothetical heavy charged gauge boson W' of mass M_2 . Thus, since scalar and pseudoscalar charged Higgs exchange contributions are ignored, within the framework of LRSM β -decay processes indeed receive contributions from vector and axial couplings only, namely from left- and right-handed fermionic gauge currents.

However, the propagating gauge bosons W and W' are not necessarily those which couple to fermions of definite chirality. Indeed, the physical charged gauge bosons W and W' and the charged gauge bosons associated to the underlying gauge group $SU(2)_L \times SU(2)_R$ which thus couple to currents of specific chirality, are related to one another through the mixing matrix[4],

$$\begin{aligned} W_L^+ &= \cos \zeta W_1^+ + \sin \zeta W_2^+, \\ W_R^+ &= e^{i\omega} \left[-\sin \zeta W_1^+ + \cos \zeta W_2^+ \right], \end{aligned} \quad (39)$$

⁷Charged Higgs exchanges do not contribute in the case of allowed pure GT transitions.

or equivalently

$$\begin{aligned} W_1^+ &= \cos \zeta W_L^+ - e^{-i\omega} \sin \zeta W_R^+ , \\ W_2^+ &= \sin \zeta W_L^+ + e^{-i\omega} \cos \zeta W_R^+ . \end{aligned} \quad (40)$$

Here, W_L and W_R denote the fundamental gauge bosons coupling to the fermionic currents of left- and right-handed chirality, respectively, while W_1 and W_2 denote the physical mass eigenstate gauge bosons of masses M_1 and M_2 , respectively. The parameter ζ is a mixing angle⁸ for charged gauge bosons, while the parameter ω determines a CP violating phase originating from complex vacuum expectation values in the Higgs sector. These quantities are constrained phenomenologically in certain classes of LRSM[4, 5].

In addition, the coupling strength of the gauge bosons W_L and W_R to the fundamental fermions is specified by the gauge coupling constants g_L and g_R , respectively. Again, the ratio g_R/g_L is constrained phenomenologically[11].

Finally, the coupling of the charged gauge bosons W_L and W_R to quarks and leptons also involves different Cabibbo-Kobayashi-Maskawa (CKM) flavour mixing matrices in each chirality sector. Since in the hadronic sector, only the up and down quarks couple to the β -decay process, the relevant CKM matrix elements are denoted

$$V_{ud}^L , \quad V_{ud}^R , \quad (41)$$

for the left- and right-handed sectors, respectively. Similarly in the leptonic sector, *a priori* the emitted electron or positron may be produced together with a mass eigenstate neutrino ν_i , with an amplitude determined by leptonic CKM matrix elements denoted as

$$U_{ie}^L , \quad U_{ie}^R . \quad (42)$$

Generally, the ratios

$$v_{ud} = \frac{V_{ud}^R}{V_{ud}^L} , \quad v_{ie} = \frac{U_{ie}^R}{U_{ie}^L} , \quad (43)$$

are arbitrary complex numbers, related to the underlying complex Yukawa couplings and Higgs vacuum expectation values, thus potentially leading to new CP violating processes in their own right. Indeed, even though it is always possible by an appropriate choice of phases of the fermionic fields to fix the quark as well as the leptonic CKM matrix elements $V_{ud}^{(L,R)}$ and $U_{ie}^{(L,R)}$ either in the left- or in the right-handed sectors to be real—as is the case for the ordinary Cabibbo angle—, this is not possible for both sectors simultaneously. Assuming that either v_{ud} or v_{ie} or both be real, would imply particular restrictions on the class of LRSM being considered. Here again, there exist[4, 5] certain phenomenological constraints on these quark and lepton CKM matrix elements.

The above description thus provides the complete set of parameters required for the application of the discussion of Sect.2 to *general* LRSM, no assumption nor approximation whatsoever as to the definition of such models being implied at this stage. It

⁸Our choice of sign for ζ is opposite to that made in Ref.[5] but agrees with that of Ref.[4].

proves useful to introduce the following combinations of these quantities,

$$t = \tan \zeta \quad , \quad \delta = \frac{M_1^2}{M_2^2} \quad , \quad r = \frac{g_R}{g_L} \quad , \quad (44)$$

$$v_u = \frac{|V_{ud}^R|^2}{|V_{ud}^L|^2} = |v_{ud}|^2 \quad , \quad v_e = \frac{\sum_i' |U_{ie}^R|^2}{\sum_i' |U_{ie}^L|^2} \quad , \quad (45)$$

and finally

$$\eta_0 = \frac{1}{M_1^2} \left(\frac{g_L^2}{8} \right) \cos^2 \zeta = \frac{1}{M_1^2} \left(\frac{g_L^2}{8} \right) \frac{1}{1+t^2} \quad . \quad (46)$$

In particular, the symbol $\sum_i'$ appearing in the definition of the quantity v_e in a notation to be detailed presently stems from the following fact. Since the neutrino produced in the β -decay process is not observed, any measurement involves a sum over all neutrino mass eigenstates whose production is not forbidden kinematically. Therefore, assuming that all neutrinos produced in the process have a mass sufficiently small in order not to induce a significant distortion of the $\beta^\mp$ energy distribution, one need only sum the corresponding decay spectra over all such mass eigenstate neutrinos ν_i without accounting for a modification in phase-space factors. In other words, the symbol $\sum_i'$ stands for a sum over all neutrinos ν_i whose production is not kinematically suppressed. In particular, note that when all mass eigenstate neutrinos do participate in the process, each of the sums

$$\sum_i' |U_{ie}^L|^2 \quad , \quad \sum_i' |U_{ie}^R|^2 \quad , \quad (47)$$

then reduces to the value unity, owing to the unitarity of the corresponding leptonic CKM flavour mixing matrix[4], in which case one simply has ($v_e = 1$).

So-called Manifest Left-Right Symmetric Models (MLRSM) are those LRSM such that the gauge coupling constants g_R and g_L and the quark and lepton CKM matrices in the left- and right-handed sectors are equal, and such that the CP violating phase ω vanishes. Namely, these MLRSM are such that except for the different masses for the charged W and W' and neutral Z and Z' gauge bosons, the sectors of left- and right-handed chirality are indistinguishable, and no CP violation originates in these models except for the two ordinary Kobayashi-Maskawa phases appearing in quark and lepton CKM matrices associated to three generations of quarks and leptons. In particular, parity invariance is then effectively restored in these MLRSM for processes of high momenta transfers. The present purely experimental lower limit on the mass M_2 of a charged heavy gauge boson W' in the context of these MLRSM is[12],

$$M_2 > 652 \text{ GeV}/c^2 \quad (95\% \text{ C.L.}) \quad . \quad (48)$$

However, let us emphasize here again that the restrictions leading to MLRSM are not assumed in this note; our discussion applies to the most general LRSM possible.

Given this description of weak charged current interactions in general LRSM, it is straightforward to determine the corresponding four-fermi effective Hamiltonian[4, 5] at the quark-lepton level relevant to β -decay, which is thus of the form,

$$\begin{aligned}
H_{\text{eff}}^{\text{quark}} &= \\
&= \eta_{LL} \bar{\psi}_u \gamma_\mu (1 - \gamma_5) \psi_d \bar{\psi}_e \gamma^\mu (1 - \gamma_5) \psi_{\nu_e} + \eta_{LR} \bar{\psi}_u \gamma_\mu (1 - \gamma_5) \psi_d \bar{\psi}_e \gamma^\mu (1 + \gamma_5) \psi_{\nu_e} + \\
&+ \eta_{RL} \bar{\psi}_u \gamma_\mu (1 + \gamma_5) \psi_d \bar{\psi}_e \gamma^\mu (1 - \gamma_5) \psi_{\nu_e} + \eta_{RR} \bar{\psi}_u \gamma_\mu (1 + \gamma_5) \psi_d \bar{\psi}_e \gamma^\mu (1 + \gamma_5) \psi_{\nu_e} , \tag{49}
\end{aligned}$$

ψ denoting the ordinary Dirac spinors for quarks and leptons. The coefficients η_{LL} , η_{LR} , η_{RL} and η_{RR} are given by

$$\begin{aligned}
\eta_{LL} &= \eta_0 v_{LL} (1 + \delta t^2) , \\
\eta_{LR} &= -\eta_0 v_{LR} r t e^{-i\omega} (1 - \delta) , \\
\eta_{RL} &= -\eta_0 v_{RL} r t e^{i\omega} (1 - \delta) , \\
\eta_{RR} &= \eta_0 v_{RR} r^2 (t^2 + \delta) , \tag{50}
\end{aligned}$$

with the notation

$$v_{LL} = V_{ud}^L U_{ie}^{L*} , \quad v_{LR} = V_{ud}^L U_{ie}^{R*} , \quad v_{RL} = V_{ud}^R U_{ie}^{L*} , \quad v_{RR} = V_{ud}^R U_{ie}^{R*} . \tag{51}$$

In the particular case of MLRSM, these expressions agree of course with those derived in Refs.[13, 14].

At the nucleon level, the effective four-fermi Hamiltonian is defined in terms of the couplings C_V , C'_V , C_A and C'_A introduced in the Appendix. Given the relations above, in LRSM these coupling coefficients are thus determined to be

$$\begin{aligned}
C_V &= g_V \left[\eta_{LL} + \eta_{LR} + \eta_{RL} + \eta_{RR} \right] , \\
C'_V &= g_V \left[\eta_{LL} - \eta_{LR} + \eta_{RL} - \eta_{RR} \right] , \\
C_A &= g_A \left[\eta_{LL} - \eta_{LR} - \eta_{RL} + \eta_{RR} \right] , \\
C'_A &= g_A \left[\eta_{LL} + \eta_{LR} - \eta_{RL} - \eta_{RR} \right] . \tag{52}
\end{aligned}$$

With the help of the results listed in the Appendix, it is then possible to compute the expression of any observable of interest. In particular, the asymmetry parameter A and the quantity Δ as defined in (30) are given by, respectively,

$$A = \frac{\mp \lambda^2 \lambda_{J'J} \left[Z_+ - X_+ \right] - 2\delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} \left[T + Y \right]}{\left[Z_- + X_- \right] + \lambda^2 \left[Z_+ + X_+ \right]} , \tag{53}$$

and

$$\Delta = \frac{1}{2} \frac{1}{\left[(Z_- - X_-) + \lambda^2(Z_+ - X_+) \right] \left[\mp \lambda^2 \lambda_{J'J} (Z_+ - X_+) - 2\delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} (T + Y) \right]} \times$$

$$\times \left\{ \mp \lambda^2 \lambda_{J'J} [(X_+ Z_- + X_- Z_+) + 2\lambda^2 X_+ Z_+] - \right.$$

$$\left. - 2\delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} [(X_- T - Z_- Y) + \lambda^2 (X_+ T - Z_+ Y)] \right\} . \quad (54)$$

In these expressions, the parameter λ is defined in (35), while the quantities $X_\pm$, Y , $Z_\pm$ and T are given by,

$$X_+ = v_e r^2 t^2 (1 - \delta)^2 + v_u v_e r^4 (t^2 + \delta)^2 + 2 \operatorname{Re} (v_{ud} e^{i\omega}) v_e r^3 t (1 - \delta) (t^2 + \delta) , \quad (55)$$

$$X_- = v_e r^2 t^2 (1 - \delta)^2 + v_u v_e r^4 (t^2 + \delta)^2 - 2 \operatorname{Re} (v_{ud} e^{i\omega}) v_e r^3 t (1 - \delta) (t^2 + \delta) , \quad (56)$$

$$Y = v_e r^2 t^2 (1 - \delta)^2 - v_u v_e r^4 (t^2 + \delta)^2 , \quad (57)$$

$$Z_+ = (1 + t^2 \delta)^2 + v_u r^2 t^2 (1 - \delta)^2 + 2 \operatorname{Re} (v_{ud} e^{i\omega}) r t (1 - \delta) (1 + t^2 \delta) , \quad (58)$$

$$Z_- = (1 + t^2 \delta)^2 + v_u r^2 t^2 (1 - \delta)^2 - 2 \operatorname{Re} (v_{ud} e^{i\omega}) r t (1 - \delta) (1 + t^2 \delta) , \quad (59)$$

$$T = (1 + t^2 \delta)^2 - v_u r^2 t^2 (1 - \delta)^2 . \quad (60)$$

In particular for allowed pure GT transitions, one simply has

$$A_{|GT} = A_{0|GT} \frac{Z_+ - X_+}{Z_+ + X_+} , \quad \Delta_{|GT} = \frac{1}{4} \left[\left(\frac{A_{0|GT}}{A_{|GT}} \right)^2 - 1 \right] = \frac{X_+ Z_+}{[Z_+ - X_+]^2} , \quad (61)$$

$A_{0|GT}$ being the asymmetry parameter in the SM for allowed pure GT decays, given in (112) in the Appendix.

Another particular case of interest is obtained when the mixing angle ζ vanishes identically, ($\zeta = 0$), for which one finds,

$$A_{|\zeta=0} = A_0 \frac{1 - v_u v_e r^4 \delta^2}{1 + v_u v_e r^4 \delta^2} , \quad \Delta_{|\zeta=0} = \frac{v_u v_e r^4 \delta^2}{[1 - v_u v_e r^4 \delta^2]^2} , \quad (62)$$

A_0 being the asymmetry parameter in the SM for arbitrary allowed β -decays, given in (111) in the Appendix. Note that in this particular case, the following identity happens to be satisfied, independently of whether the allowed transition is pure GT or not,

$$\Delta_{|\zeta=0} = \frac{1}{4} \left[\left(\frac{A_0}{A_{|\zeta=0}} \right)^2 - 1 \right] . \quad (63)$$

Let us recall that these results, and in particular the general ones in (53) and (54), are of application in the *most general* LRSM possible, and do not assume that the

parameters δ nor $(\tan \zeta)$ are small compared to the value unity. The expressions given in (53) and (54) for A and Δ are *exact*, no approximation whatsoever being implied at this stage (except for the fact that recoil order corrections and possible charged Higgs contributions to the ratio R/R_0 are neglected in the present discussion).

In order to gain some more insight into these general results, let us now consider them in the particular case that no CP violation originates either from the parameter ω nor from the ratio v_{ud} . Namely, let us assume that the former parameter takes one of the two values ($\omega = 0$) or ($\omega = \pi$), and that the ratio v_{ud} is a real number, in which case

$$v_{ud} e^{i\omega} = \epsilon \sqrt{v_u} \quad , \quad \epsilon = \pm 1 \quad . \quad (64)$$

Under these circumstances, it proves useful to define the quantities

$$x = \sqrt{v_u v_e} r^2 (\delta + t^2) - \epsilon \sqrt{v_e} r t (1 - \delta) \quad , \quad (65)$$

and

$$y = \sqrt{v_u v_e} r^2 (\delta + t^2) + \epsilon \sqrt{v_e} r t (1 - \delta) \quad , \quad (66)$$

as well as

$$\bar{x} = \delta t^2 - \epsilon \sqrt{v_u} r t (1 - \delta) \quad , \quad (67)$$

and

$$\bar{y} = \delta t^2 + \epsilon \sqrt{v_u} r t (1 - \delta) \quad . \quad (68)$$

Indeed, one then observes that⁹

$$X_+^{(\text{CP})} = y^2 \quad , \quad X_-^{(\text{CP})} = x^2 \quad , \quad Y^{(\text{CP})} = -xy \quad , \quad (69)$$

as well as

$$Z_+^{(\text{CP})} = (1 + \bar{y})^2 \quad , \quad Z_-^{(\text{CP})} = (1 + \bar{x})^2 \quad , \quad T^{(\text{CP})} = (1 + \bar{x})(1 + \bar{y}) \quad . \quad (70)$$

Therefore, under the assumptions stated above concerning ω and v_{ud} , the asymmetry parameter A and the quantity Δ reduce to, respectively,

$$A^{(\text{CP})} = \frac{\mp \lambda^2 \lambda_{J'J} [(1 + \bar{y})^2 - y^2] - 2\delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} [(1 + \bar{x})(1 + \bar{y}) - xy]}{[(1 + \bar{x})^2 + x^2] + \lambda^2 [(1 + \bar{y})^2 + y^2]} \quad , \quad (71)$$

and

$$\begin{aligned} \Delta^{(\text{CP})} = & \frac{1}{2} \left[(1 + \bar{x})^2 - x^2 + \lambda^2 ((1 + \bar{y})^2 - y^2) \right] \times \\ & \times \frac{1}{\left[\mp \lambda^2 \lambda_{J'J} ((1 + \bar{y})^2 - y^2) - 2\delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} ((1 + \bar{x})(1 + \bar{y}) - xy) \right]} \times \\ & \times \left\{ \mp \lambda^2 \lambda_{J'J} \left[x^2 (1 + \bar{y})^2 + y^2 (1 + \bar{x})^2 + 2\lambda^2 y^2 (1 + \bar{y})^2 \right] - \right. \\ & \left. - 2\delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} \left[x(1 + \bar{y}) + y(1 + \bar{x}) \right] \left[x(1 + \bar{x}) + \lambda^2 y(1 + \bar{y}) \right] \right\} \quad . \quad (72) \end{aligned}$$

⁹The upper-script (CP) stands for the fact that the expressions in the remainder of this section are valid only when no CP violation originates either from ω nor from v_{ud} .

In particular for allowed pure GT transitions, these results simplify to

$$A_{|_{GT}}^{(\text{CP})} = A_{0|_{GT}} \frac{(1 + \bar{y})^2 - y^2}{(1 + \bar{y})^2 + y^2} , \quad (73)$$

as well as

$$\Delta_{|_{GT}}^{(\text{CP})} = \frac{y^2(1 + \bar{y})^2}{[(1 + \bar{y})^2 - y^2]^2} = \frac{1}{4} \left[\left(\frac{A_{0|_{GT}}}{A_{|_{GT}}^{(\text{CP})}} \right)^2 - 1 \right] , \quad (74)$$

$A_{0|_{GT}}$ being the value of the asymmetry parameter in the SM for allowed pure GT β -decays.

The above expressions generalise those obtained in Ref.[2] in the case of MLRSM for which ($r = 1$), ($v_u = 1 = v_e$) and ($\omega = 0$) and in the limit that both δ and ($t = \tan \zeta$) are much smaller than unity. In contradistinction, the results derived in this note are valid for *arbitrary* LRSM parameters, independently of such or any other approximations (except for recoil order corrections and possible charged Higgs contributions to the ratio R/R_0 which are ignored in the present discussion).

Nevertheless, to conclude let us consider the limit in which both δ and ($t = \tan \zeta$) are indeed much smaller than unity, still under the assumption that the parameter ($v_{ud}e^{i\omega}$) is real. Given the definitions,

$$\tilde{\delta} = \sqrt{v_u v_e} r^2 \delta , \quad \tilde{t} = \epsilon \sqrt{v_e} r t , \quad (75)$$

to first order in the quantities δ and ($t = \tan \zeta$), one then finds

$$x \simeq \tilde{\delta} - \tilde{t} , \quad y \simeq \tilde{\delta} + \tilde{t} , \quad (76)$$

as well as

$$\bar{x} \simeq -\sqrt{\frac{v_u}{v_e}} \tilde{t} , \quad \bar{y} \simeq +\sqrt{\frac{v_u}{v_e}} \tilde{t} . \quad (77)$$

In other words, within the approximation that ($\delta \ll 1$) and ($\tan \zeta \ll 1$), both $A^{(\text{CP})}$ and $\Delta^{(\text{CP})}$ are determined by quadratic expressions in terms of the parameters $\tilde{\delta}$ and $\tilde{t}$. One then has,

$$A_{|\delta, t \ll 1}^{(\text{CP})} \simeq \frac{\mp \lambda^2 \lambda_{J'J} [(1 + \bar{y})^2 - y^2] - 2\delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} [(1 + \bar{x})(1 + \bar{y}) - xy]}{[(1 + \bar{x})^2 + x^2] + \lambda^2 [(1 + \bar{y})^2 + y^2]} , \quad (78)$$

as well as

$$\Delta_{|\delta, t \ll 1}^{(\text{CP})} \simeq \frac{1}{2} \left\{ \frac{\mp \lambda^2 \lambda_{J'J} y^2 - 2\delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} xy}{\mp \lambda^2 \lambda_{J'J} - 2\delta_{J'J} \lambda \sqrt{\frac{J}{J+1}}} + \frac{x^2 + \lambda^2 y^2}{1 + \lambda^2} \right\} , \quad (79)$$

with x , y , $\bar{x}$ and $\bar{y}$ now given in (76) and (77), provided that ($\delta \ll 1$) and ($\tan \zeta \ll 1$). Within these approximations and under the assumptions that ($\omega = 0$) and that v_{ud} is real, the expression (79) for $\Delta_{|\delta, t \ll 1}^{(\text{CP})}$ in terms of the parameters x and y coincides precisely with the one following from Ref.[2] within the context of MLRSM. In other

words, given the approximations ($\delta \ll 1$) and ($\tan \zeta \ll 1$) and the restriction that $(v_{ud}e^{i\omega})$ is real, the result obtained[2] for $\Delta_{|\delta, t \ll 1}^{(\text{CP})}$ in the context of MLRSM remains valid for general LRSM provided the parameters δ and ($t = \tan \zeta$) are simply replaced by the parameters $\tilde{\delta}$ and $\tilde{t}$ defined in (75), respectively.

The quadratic form obtained in (78) for the asymmetry parameter $A_{|\delta, t \ll 1}^{(\text{CP})}$ in terms of the parameters (v_u/v_e) , $\tilde{\delta}$ and $\tilde{t}$ has been analysed in Ref.[6] already, albeit in the context of MLRSM, namely when $(v_u = 1 = v_e)$, ($\tilde{\delta} = \delta$) and ($\tilde{t} = t$). For the quantity $\Delta_{|\delta, t \ll 1}^{(\text{CP})}$ in (79), the corresponding quadratic form is characterised by the relation

$$\Delta_{|\delta, t \ll 1}^{(\text{CP})} \simeq \tilde{\delta}^2 + 2 \Delta_{\tilde{t}\tilde{\delta}} \tilde{t}\tilde{\delta} + \Delta_{\tilde{t}\tilde{t}} \tilde{t}^2, \quad (80)$$

with coefficients $\Delta_{\tilde{t}\tilde{\delta}}$ and $\Delta_{\tilde{t}\tilde{t}}$ which may easily be determined from (79). Therefore, any experimental upper limit or value Δ_0 for the quantity Δ would determine an elliptic or hyperbolic (exclusion) contour in the plane $(\tilde{t}, \tilde{\delta})$, under the conditions for which the result in (79) is applicable. In particular, the slope of this contour at $(\tilde{t} = 0)$, corresponding to a vanishing mixing angle ($\zeta = 0$), is simply given by the coefficient $(-\Delta_{\tilde{t}\tilde{\delta}})$, namely,

$$\left(\frac{d\tilde{\delta}}{d\tilde{t}} \right)_{|\zeta=0} = - \Delta_{\tilde{t}\tilde{\delta}}. \quad (81)$$

On the other hand, the hyperbolic or elliptic character of the contour plot is simply determined from the sign of the quantity

$$\Delta_{\tilde{t}\tilde{t}} - \Delta_{\tilde{t}\tilde{\delta}}^2. \quad (82)$$

When this sign is positive, the contour is elliptic; when it is negative, the contour is hyperbolic; and when this quantity vanishes identically, the contour is a straight line. The latter instance applies in particular to allowed pure GT decays, since $\Delta_{|\delta, t \ll 1}^{(\text{CP})}$ then reduces precisely to $(y^2 \simeq (\tilde{\delta} + \tilde{t})^2)$.

4 The case for mirror nuclei

Let us now apply the results of the previous general developments to specific super-allowed β -decays, namely mirror nuclei. The interest of this choice lies with the fact that the initial and daughter nuclei being members of a single isospin multiplet, one may be quite confident in the determination of recoil order corrections[15] to the ratio R/R_0 for such nuclei using both experimental information and theoretical considerations such as CVC and PCAC.

The list of mirror nuclei considered here[6] is given in Table 1, with in the second column the spin sequence (J, J') . Since for mirror nuclei one has $(J' = J)$, only the common value of J is indicated. Table 1 also includes the two allowed pure GT transitions of ^{107}In and ^{12}N , for comparison. The third and fourth columns of the

Table give the values for the quantities λ and A_0 introduced previously, as evaluated¹⁰ in Ref.[6]. The fifth column gives the end-point total energy E_0 , thus including the electron (positron) rest-mass m , while the sixth column lists the corresponding value β_0 of the velocity of the $\beta^\mp$ particle. The meaning of the last two columns of Table 1 is detailed below. Note that except for the first two entries, namely the neutron and ^3H , all nuclei listed in Table 1 decay by positron emission.

In order to assess the potentiality of a given nucleus as to its sensitivity to right-handed current contributions through a relative longitudinal polarisation-asymmetry correlation measurement, given the ideal experimental conditions assumed in this note—namely a measurement of exactly the longitudinal polarisation at a specific energy E and momentum direction $\hat{p}$ of the $\beta^\mp$ particle—, the following strategy is applied. The ratio R/R_0 is determined experimentally with a certain precision, and is established either not to differ from the value unity by more than that precision ϵ_0 , or to differ from the value unity by a value ϵ_0 . In other words, in either case one may write¹¹

$$\left| 1 - \frac{R}{R_0} \right| = |k \Delta| \leq \epsilon_0, \quad (83)$$

where k is one of the enhancement factors $k(-J, 0)$ or $k(-J, J)$ depending on whether it is $R(-J, 0)/R_0(-J, 0)$ or $R(-J, J)/R_0(-J, J)$ which is measured. In the general case, the quantity Δ given in (54) is a rather complicated function of the LRSM parameters δ , t , $\text{Re}(v_{ud}e^{i\omega})$, v_u , v_e and r , and as such a characterisation of the potentiality of a given nucleus in terms of the quantity Δ may not be very indicative of the new physics it would imply. Rather, it seems more efficient to characterise this potentiality in terms of the mass range for the W' mass M_2 one may hope to reach with this type of experiment. For this purpose, it is appropriate to consider the expression of the quantity Δ for a vanishing mixing angle ζ , namely,

$$\Delta_{|\zeta=0} = \frac{\tilde{\delta}^2}{[1 - \tilde{\delta}^2]^2}, \quad \tilde{\delta}^2 = v_u v_e r^4 \delta^2 = v_u v_e r^4 \frac{M_1^4}{M_2^4}, \quad (84)$$

which is valid in the most general LRSM possible (see (62)). Since $\tilde{\delta}^2$ is positive, one always has,

$$\tilde{\delta}^2 \leq \frac{\delta^2}{[1 - \delta^2]^2} = \Delta_{|\zeta=0}, \quad (85)$$

which upon substitution of the upper bound or value for Δ given in (83), leads to the following lower limit on M_2 ,

$$M_2 \geq |r| (v_u v_e)^{1/4} M_{\min}, \quad (86)$$

¹⁰The data for the neutron are from Ref.[10], while a change of sign for the parameter λ [16] as compared to the one given in Ref.[6], and the ensuing modification of the value for the asymmetry parameter A_0 , are effected in the case of ^3H . The sign of λ relative to that of the neutron for all cases listed in Table 1 agrees with the results given in Ref.[17] on basis of the shell model.

¹¹In the case of the ratio $R(-J, J)/R_0(-J, J)$, strictly speaking the expression in (83) in fact ignores an additional correction factor dependent on Δ , β and $A_{\text{exp}}(-J, J)$, given in (18). However, for the purpose of the discussion of the present section, any correction brought about by that factor may safely be ignored, since Δ is certainly small in comparison to the value unity.

where the mass-reach $M_{\min}$ is defined as,

$$M_{\min} = \left(\frac{|k|}{\epsilon_0} \right)^{1/4} M_1, \quad (87)$$

($M_1 = 80.22 \text{ GeV}/c^2$) being the mass of the ordinary gauge boson W . Therefore, the potentiality of a given nucleus is to be characterised in terms of the quantity $M_{\min}$, evaluated for either of the two measurements considered here, namely corresponding to the ratios $R(-J, 0)/R_0(-J, 0)$ or $R(-J, J)/R_0(-J, J)$. Note that $M_{\min}$ represents precisely the lower bound to be obtained for M_2 in the context of MLRSM, in the limit that δ is much smaller than the value unity. For general LRSM however, this is not the case and the potential lower bound on M_2 is obtained by multiplying the mass-reach $M_{\min}$ by the factor $(|r|(v_u v_e)^{1/4})$.

Clearly, the evaluation of the mass-reach $M_{\min}$ requires the value of the enhancement factor k , which in turn needs the values for the $\beta^\mp$ particle velocity β and the experimental asymmetry A_{exp} . The choice of value for β is conditioned by the necessity of high statistics measurements, which makes it is preferable to work at the maximum of the energy distribution $W_0(E)$. When ignoring the Coulomb correction represented by the Fermi function $F(\pm Z, E)$, this maximum is reached at an energy E_{max} whose value is given in (96) of the Appendix in terms of the end-point total energy E_0 . The values of E_{max} as well as the associated value for the velocity β_{max} are listed in the last two columns of Table 1. Thus, β_{max} is the value at which we choose to evaluate the enhancement factors $k(-J, 0)$ and $k(-J, J)$. Accounting for Coulomb corrections through Fermi's function $F(\pm Z, E)$ would not modify the value for β_{max} significantly, since β -decay energy spectra are typically rather smooth around their maximum and the effect of the shift in β_{max} due to Fermi's function is small for small values of Z , as is the case for the nuclei considered here. In any case, from the practical experimental point of view, this issue is rather academic since any energy acceptance is always of finite resolution.

As shown in Sect.2, the enhancement factors $k(-J, 0)$ and $k(-J, J)$ are the largest when the experimental asymmetries $A_{\text{exp}}(-J, 0)$ and $A_{\text{exp}}(-J, J)$ approach the values β^2 and $2\beta^2/(1 + \beta^2)$, respectively. Given the value β_{max} chosen here, these optimal experimental asymmetries $A_{\text{exp}}^{(0)}$ are listed in the last two columns of Table 2. Note that with the exception of ^3H , these values are rather large and thus imply the requirement of the largest degree of nuclear polarisation possible, as is indeed to be expected.

The other data in Table 2 give on the one hand, the coefficients $\Delta_{\tilde{t}\tilde{\delta}}$ and $\Delta_{\tilde{t}\tilde{t}}$ defining the quadratic form in (80) which determines the quantity $\Delta_{|\delta, t \ll 1}^{(\text{CP})}$ for values of δ and ($t = \tan \zeta$) small compared to the value unity and when $(v_{ud} e^{i\omega})$ is real, and on the other hand, the type of curve so obtained, “E”, “H” and “L” standing for an elliptic, hyperbolic or linear curve, respectively. In particular, the coefficient $(-\Delta_{\tilde{t}\tilde{\delta}})$ determines the slope $(d\tilde{\delta}/d\tilde{t})|_{\zeta=0}$ at ($\zeta = 0$) of the dependence $\tilde{\delta}(\tilde{t})$ determined by $\Delta_{|\delta, t \ll 1}^{(\text{CP})}$ when both δ and ($t = \tan \zeta$) are small in comparison to the value unity; this slope thus characterises the sensitivity of the measurement of Δ to values of ζ different from zero when δ is small compared to unity. Incidentally, note that the data listed in Tables 1 and 2 are independent both of the experimental precision ϵ_0 and of the experimental

asymmetries $A_{\text{exp}}(-J, 0)$ and $A_{\text{exp}}(-J, J)$ —namely the attainable degree of nuclear polarisation J —, to which we now turn.

The experimental asymmetries $A_{\text{exp}}(-J, 0)$ and $A_{\text{exp}}(-J, J)$ are determined in terms of the quantity $(\beta J A(\hat{p}, \hat{J}))$. Besides the quantity β whose value has now been specified to be β_{max} , one also requires the asymmetry parameter A and the *effective* degree of nuclear polarisation ($\mathcal{P} = |J(\hat{p}, \hat{J})|$). For the sake of the present evaluation, it is obviously justified to approximate the former quantity by the asymmetry parameter A_0 in the SM, which is listed[6] in Table 1. Indeed, even though this approximation ignores possible right-handed contributions, the latter are certainly small and may effectively be accounted for through a small rescaling of the degree of nuclear polarisation J . Finally, the effective degree of nuclear polarisation is characterised by the quantity $(\mathcal{P} = |J(\hat{p}, \hat{J})|)$, with the relative directions of $\hat{J}$ and $\hat{p}$ chosen such that the experimental asymmetries $A_{\text{exp}}(-J, 0)$ and $A_{\text{exp}}(-J, J)$ as defined in this note are *positive*.

For the comparison of the potentiality offered by the nuclei considered here, the following values for the experimental precision ϵ_0 on the measurement of R/R_0 and for the effective degree of nuclear polarisation $\mathcal{P}$ are used,

$$\epsilon_0 = 0.01 \quad , \quad \mathcal{P} = 0.80 \quad , \quad (88)$$

corresponding in fact to rather stringent experimental requirements and achievements. The corresponding results for the enhancement factor k , the experimental asymmetry A_{exp} and the mass-reach M_{min} are listed in Table 3, for both types of configurations of nuclear polarisation considered in this note.

The values of M_{min} in Table 3 reveal that among mirror nuclei, those with the best prospects with regards to our purpose are ^{17}F , ^{41}Sc and ^{25}Al , in order of decreasing potentiality. These nuclei are also those for which the enhancement factors are the largest, and for which the asymmetry parameter A_0 is closest to the maximal value of unity attained for allowed pure GT transitions. Indeed, these three mirror nuclei compete well with the two examples of the latter type of decay, namely ^{107}In and ^{12}N . In fact, there are three factors—related to one another—which concur to explain the distinguished role of these three mirror nuclei: a large asymmetry parameter A_0 , allowing an experimental asymmetry A_{exp} close to its optimal value $A_{\text{exp}}^{(0)}$, hence a large enhancement factor.

In spite of the large effective degree of nuclear polarisation ($\mathcal{P} = 0.80$) assumed here, experimental asymmetries A_{exp} are still less than their optimal values $A_{\text{exp}}^{(0)}$ for all nuclei considered, given the present choice for β , namely ($\beta = \beta_{\text{max}}$). The values for $A_{\text{exp}}^{(0)}$ are quite close to unity—except for ^3H —simply because the values for β_{max} are also quite close to the maximal value of unity. Indeed, as was pointed out in Sect.2, the optimal sensitivity is achieved for values of β and of the effective degree of nuclear polarisation $\mathcal{P}$ such that $(\beta = |\mathcal{A}\mathcal{P}|)$, which is not possible for asymmetry parameters $|A|$ less than unity and an effective nuclear polarisation ($\mathcal{P} = 0.80$), once the value of β is specified to be β_{max} . Therefore for all nuclei considered, given this value for β , an increased sensitivity to right-handed charged currents requires a larger enhancement factor, hence a larger effective degree of nuclear polarisation $\mathcal{P}$. Independently

of the technical feasibility of the production and polarisation of the mirror nuclei ^{17}F , ^{41}Sc and ^{25}Al —as well as of ^{107}In [7] and ^{12}N [8]—with sufficient yields and degrees of polarisation, it thus appears that a mass-reach $M_{\min}$ of the order of 600 GeV/c² is the ultimate limit attainable using relative longitudinal polarisation-asymmetry correlation measurements in allowed nuclear β -decays. Note that all these cases correspond to positron emitters, for which well established precision polarimetry techniques are readily available[7, 8].

However, let us remark that this conclusion which is established on quite general grounds leaves open two possible types of loopholes. On the one hand, there exist specific mirror nuclei for which efficient polarisation techniques are becoming available, possibly reaching the ideal degree of polarisation of 100%. The above analysis has then to be reconsidered separately for such particular cases. On the other hand, but then at the cost of a loss of statistics, there also remains the possibility to work at values of β smaller than $\beta_{\max}$, in order to reach more easily the optimal sensitivity attained when ($\beta = |A\mathcal{P}|$) given a certain effective degree of nuclear polarisation $\mathcal{P}$ achieved in practice.

The first possibility is realised for example in the cases of ^{21}Na [18] and ^{37}K [19]. To illustrate the point, Table 4 lists the same information as Table 3 for a precision ($\epsilon_0 = 0.01$) but for an effective degree of nuclear polarisation taking the maximal value possible ($\mathcal{P} = 1.00$). Note that ^{17}F , ^{41}Sc and ^{25}Al then still remain the favorite cases among mirror nuclei, but now with a different order of interest. This is due to the choice of the $\beta^\mp$ particle energy at $E_{\max}$, which is such that for the first two cases the experimental asymmetries A_{exp} are *larger* than their optimal values $A_{\text{exp}}^{(0)}$. Indeed, the enhancement factors $k(-J, 0)$ and $k(-J, J)$ are now negative for ^{17}F and ^{41}Sc , whereas those for ^{25}Al remain positive but have become large. The same applies also to the pure GT transitions of ^{107}In and ^{12}N . In other words, in these specific cases, a choice of β slightly less than $\beta_{\max}$ may lead to large enhancement factors indeed, at no significant loss in statistics. Nevertheless, this assumes the maximal possible effective degree of nuclear polarisation ($\mathcal{P} = 1.00$), quite a unique experimental circumstance. For example, even though the case of ^{12}N may appear from Table 4 to be the most attractive with a mass-reach of 1.7 TeV/c², an effective nuclear polarisation of ($\mathcal{P} \simeq 0.15$) only is obtained[8] in practice.

Therefore, given a technically achievable effective degree of nuclear polarisation $\mathcal{P}$ for a specific mirror nucleus, the other avenue open towards large enhancement factors is to consider working at values of β lower than $\beta_{\max}$. The ensuing loss in statistics has then to be weighted against the possibly important gain in sensitivity, but such an evaluation is possible only on a case by case basis in contradistinction to the general considerations of this note. However, as was remarked previously, the β particle count rate for the direction of nuclear polarisation offering the greatest sensitivity to right-handed currents is proportional to $(1 - \beta^2)$ when the optimal choice of values for $(\beta, \mathcal{P})$ such that ($\beta = |A\mathcal{P}|$) is made. Thus, even though the overall statistics may decrease by choosing a value for β smaller than $\beta_{\max}$ in order to achieve a sensitivity closer to the optimal situation, the relative statistics measured for the direction of nuclear polarisation most sensitive to the sought-for physical effect will increase.

One particular case which is to be distinguished from that point of view is that

of ${}^3\text{H}$, with a maximum value of the β^- particle velocity at ($\beta_0 = 0.2626$), in spite of the rather small asymmetry parameter of ($A_0 = -0.09405$). Since the corresponding value of ($\beta_{\text{max}} = 0.1208$) is quite small, working at a value of β such that the optimal configuration ($\beta = |A\mathcal{P}|$) is achieved should not lead to any significant loss of statistics, provided an effective degree of nuclear polarisation of at least ($\mathcal{P} = 0.80$) can be achieved. For example, given an effective nuclear polarisation ($0.80 \leq \mathcal{P} \leq 1.00$), the optimal choice for β lies in the interval ($0.0752 \leq \beta \leq 0.0941$), which is not much less than the value ($\beta_{\text{max}} = 0.1208$). Under such circumstances, given sufficient energy resolution¹², quite large enhancement factors may be expected, possibly opening up a mass-reach in the TeV/ c^2 region. Nevertheless, such instances of relative longitudinal polarisation-asymmetry correlation measurements can be assessed on a case by case basis only.

5 Conclusion

In this note, the sensitivity[2] of relative polarisation-asymmetry correlation β -decay experiments to charged weak current interactions of right-handed chirality is considered independently of any specific model for physics beyond the Standard Model of the electroweak interactions. Starting with the general results of Ref.[9] based on a four-fermi effective Hamiltonian for allowed β -decay including arbitrary complex vector and axial coefficients C_V , C'_V , C_A and C'_A only, and ignoring recoil order corrections expected to be small for super-allowed decays, it is shown that this class of experiments is directly sensitive to physics beyond the SM through a certain combination of these four parameters which is characterised by a single quantity Δ given in (30). A non vanishing value for Δ would establish the existence of charged right-handed currents and thus of new physics beyond the SM.

These general considerations are then developed further in the specific case of so-called Left-Right Symmetric Models in their most general form possible, assuming only that possible charged Higgs contributions are negligible. Which combinations of the fundamental parameters of such LRSM are probed through the class of measurements mentioned above is made explicit, in particular in terms of the asymmetry parameter A and the quantity Δ in (53) and (54), respectively. These expressions, which do not involve any simplifying restriction nor approximation whatsoever, are also considered for restricted classes of LRSM in which no CP violation originates from a lack of complete complex phase alignment between the two sectors of opposite chiralities in such theories, neither in the Higgs nor in the Yukawa sectors. In particular, the associated results generalise those obtained previously[2] in the context of so-called Manifest Left-Right Symmetric Models—which provide but one type of a very restricted class of LRSM—under the approximation that both the ratio of the squared masses of light to heavy charged gauge bosons W and W' as well as their mixing angle be much smaller than the value unity.

These general results are then applied specifically to the case of mirror nuclei,

¹²A poor energy resolution dilutes enhancement factors which otherwise could reach very large values.

which offer the advantage that recoil order corrections are more amenable to sufficiently precise evaluation than for other instances of allowed β -decay, since initial and daughter nuclei then belong to the same isospin multiplet. The potentiality of these mirror nuclei as to the sensitivity to contributions from charged currents of right-handed chirality is then characterised in terms of the mass-reach—the precise technical meaning of this notion in the context of general LRSM is defined in Sect.4—for the hypothetical W' charged gauge boson which may be achievable by using each of these nuclei, given a certain experimental precision and degree of nuclear polarisation. The analysis establishes that among mirror decays, the cases of ^{17}F , ^{41}Sc and ^{25}Al , in order of decreasing interest, certainly offer the best prospects, which are comparable to those achieved by on-going experiments using the allowed pure Gamow-Teller transitions of ^{107}In [7] and ^{12}N [8]. Indeed, allowed pure Gamow-Teller decays are expected to provide the best mass-reach possible owing to their maximal asymmetry parameter.

The analysis is performed on quite general grounds, not paying attention to specific circumstances which may apply to a given particular nucleus, nor to the technical feasibility of the production and polarisation of these nuclei. In fact, the sensitivity to right-handed charged current contributions of the type of experiment considered here may become quite large for some special cases, by appropriately choosing to work at a specific value of the $\beta^\mp$ particle energy, given an achievable effective degree of nuclear polarisation. It is then not excluded that some particular mirror nucleus presents the potential to extend the mass-reach of relative longitudinal polarisation-asymmetry correlation measurements into the TeV/c^2 region. One such instance which may be worth pursuing further could be that of the β^- -decay of ^3H , owing to the rather low value of the end-point energy in that case.

This conclusion also opens the prospect that for specific values of the $\beta^\mp$ particle energy and of the nuclear polarisation, other observables in the β -decay of mirror nuclei offer a similarly large sensitivity to other couplings appearing[9] in the effective Hamiltonian for nuclear β -decay, including for example scalar or tensor contributions, as well as time reversal violating effects[19]. Such possibilities certainly deserve to be investigated in detail, along lines similar to those developed here.

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Appendix

In the case of vector (V) and axial (A) contributions only, as assumed in this note, the general four-fermi effective Hamiltonian considered in Ref.[9] at the nucleon level is of the form

$$H_{\text{eff}}^{\text{nucleon}} = \bar{\psi}_p \gamma_\mu \psi_n \bar{\psi}_e (C_V \gamma^\mu - C_V' \gamma^\mu \gamma_5) \psi_{\nu_e} - \bar{\psi}_p \gamma_\mu \gamma_5 \psi_n \bar{\psi}_e (C_A \gamma^\mu \gamma_5 - C_A' \gamma^\mu) \psi_{\nu_e} , \quad (89)$$

where C_V , C_V' , C_A and C_A' are *a priori* arbitrary complex coefficients, and ψ_p , ψ_n , ψ_e and ψ_{ν_e} represent Dirac spinors for the proton, the neutron, the electron and the neutrino of electronic flavour, respectively. Our phase conventions are as follows. The chirality operator γ_5 is defined with a sign such that left-handed couplings are of the form $\gamma_\mu(1 - \gamma_5)$, which is the choice opposite to that taken in Ref.[9]. To account for that difference, changes of sign have been included in the expression in (89) in such a way that the coefficients C_V , C_V' , C_A and C_A' are as defined in Ref.[9].

The quantities $W_0(E)$, ξ , A , G and N' appearing in (1) are then given by the following expressions[9],

$$\xi = M_F^2 [|C_V|^2 + |C_V'|^2] + M_{GT}^2 [|C_A|^2 + |C_A'|^2] , \quad (90)$$

$$\begin{aligned} \xi A &= M_{GT}^2 \lambda_{J'J} [\mp 2 \text{Re} (C_A C_A'^*)] + \\ &+ \delta_{J'J} M_F M_{GT} \sqrt{\frac{J}{J+1}} [- 2 \text{Re} (C_V C_A'^* + C_V' C_A^*)] , \end{aligned} \quad (91)$$

$$\xi G = M_F^2 [\mp 2 \text{Re} (C_V C_V'^*)] + M_{GT}^2 [\mp 2 \text{Re} (C_A C_A'^*)] , \quad (92)$$

$$\begin{aligned} \xi N' &= M_{GT}^2 \lambda_{J'J} [|C_A|^2 + |C_A'|^2] + \\ &+ 2 \delta_{J'J} M_F M_{GT} \sqrt{\frac{J}{J+1}} [\pm \text{Re} (C_V C_A^* + C_V' C_A'^*)] , \end{aligned} \quad (93)$$

and finally

$$W_0(E) = \frac{1}{(2\pi)^4} p E (E_0 - E)^2 F(\pm Z, E) . \quad (94)$$

Here, E_0 is the β -spectrum end-point total energy, p and E the momentum and total energy of the $\beta^\mp$ particle, respectively, and $F(\pm Z, E)$ Fermi's function for Coulomb corrections, Z being the charge of the *daughter* nucleus. For an allowed transition from an initial state of nuclear spin J to a final state of nuclear spin J' , the quantity $\lambda_{J'J}$ is defined by[9]

$$\lambda_{J'J} = \begin{cases} 1 & J \rightarrow J' = J - 1 \\ \frac{1}{J+1} & J \rightarrow J' = J \\ -\frac{J}{J+1} & J \rightarrow J' = J + 1 \end{cases} . \quad (95)$$

And finally, M_F and M_{GT} are the Fermi and Gamow-Teller nucleon matrix elements, respectively.

Ignoring the factor $F(\pm Z, E)$, it is possible to show that the function $W_0(E)$ reaches its maximal value for a total energy $E_{\max}$ given by

$$\frac{E_{\max}}{E_0} = \frac{1}{6} + \rho_E \sin \left\{ \frac{1}{3} \arcsin \left[\frac{1}{\rho_E^3} \left(\frac{1}{2} \left(\frac{m}{E_0} \right)^2 - \frac{1}{27} \right) \right] + \frac{2\pi}{3} \right\} , \quad (96)$$

where

$$\rho_E = \sqrt{\left(\frac{m}{E_0} \right)^2 + \frac{1}{9}} , \quad (97)$$

m being of course the electron (positron) mass.

Note that in terms of the quantities a_L , a_R , b_L and b_R introduced in (26) to (29) of Sect.2, one may also write

$$\xi = \frac{1}{2} (a_L + a_R) , \quad (98)$$

$$\xi A = \frac{1}{2} (b_L - b_R) , \quad (99)$$

$$\xi G = \mp \frac{1}{2} (a_L - a_R) , \quad (100)$$

$$\xi N' = \mp \frac{1}{2} (b_L + b_R) . \quad (101)$$

Moreover, in the particular case of allowed pure GT transitions, one observes that

$$(\xi G)_{|_{GT}} = \frac{1}{\lambda_{J'J}} (\xi A)_{|_{GT}} , \quad (\xi N')_{|_{GT}} = \lambda_{J'J} \xi_{|_{GT}} , \quad (102)$$

thus showing that in this instance only the matrix element $\xi_{|_{GT}}$ and the asymmetry parameter $A_{|_{GT}}$ are relevant to the description of the decay. In particular, relative measurements as those considered in this note are then only dependent on the asymmetry parameter $A_{|_{GT}}$ in the case of allowed pure GT transitions.

In the Standard Model, the coefficients C_V , C'_V , C_A and C'_A are simply determined up to a common factor C_V^0 by the relations,

$$C'_V = C_V , \quad C'_A = C_A , \quad (103)$$

together with

$$C_V = C_V^0 , \quad C_A = \frac{g_A}{g_V} C_V^0 , \quad (104)$$

g_V and g_A being the standard vector and axial couplings of nucleon β -decay, respectively, such that[10]

$$\frac{g_A}{g_V} = -1.2573 \pm 0.0028 . \quad (105)$$

Given the definitions

$$\rho = \frac{g_A}{g_V} < 0 , \quad \lambda = \frac{g_A}{g_V} \frac{M_{GT}}{M_F} = \rho \frac{M_{GT}}{M_F} , \quad (106)$$

one then obtains,

$$\xi_0 = 2 |C_V^0|^2 \left[M_F^2 + \rho^2 M_{GT}^2 \right] = 2 |C_V^0|^2 M_F^2 \left[1 + \lambda^2 \right] , \quad (107)$$

$$\begin{aligned} (\xi A)_0 &= 2 |C_V^0|^2 \left[\mp \rho^2 M_{GT}^2 \lambda_{J'J} - 2 \delta_{J'J} \rho M_F M_{GT} \sqrt{\frac{J}{J+1}} \right] \\ &= 2 |C_V^0|^2 M_F^2 \left[\mp \lambda^2 \lambda_{J'J} - 2 \delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} \right] , \end{aligned} \quad (108)$$

$$(\xi G)_0 = \mp \xi_0 , \quad (109)$$

and finally

$$(\xi N')_0 = \mp (\xi A)_0 . \quad (110)$$

Thus for example, the asymmetry parameter A_0 in the SM simply reduces to

$$A_0 = \frac{1}{1 + \lambda^2} \left[\mp \lambda^2 \lambda_{J'J} - 2 \delta_{J'J} \lambda \sqrt{\frac{J}{J+1}} \right] . \quad (111)$$

In particular, for allowed pure GT transitions this result becomes

$$A_{0|GT} = \mp \lambda_{J'J} , \quad (112)$$

which is thus maximal only for transitions such that $(J' = J - 1)$,

$$A_{0|GT} = \mp 1 , \quad J' = J - 1 . \quad (113)$$

This is the case for example for ^{107}In and ^{12}N .

In the case of LRSM, the coefficients C_V , C'_V , C_A and C'_A are given in (52) in terms of the combinations of fundamental parameters of LRSM defined in (43) to (46). In order to list the expressions required for the evaluation of the parameters ξ , A , G and N' , let us also introduce the notation

$$C_N^2 = 2 g_V^2 \eta_0^2 |V_{ud}^L|^2 \sum_i' |U_{ie}^L|^2 , \quad (114)$$

and refer to the relations (55) to (60) in Sect.3 for the definition of the other quantities appearing in the expressions which follow. One then obtains,

$$|C_V|^2 + |C'_V|^2 = C_N^2 \left[Z_- + X_- \right] , \quad (115)$$

$$|C_A|^2 + |C'_A|^2 = \rho^2 C_N^2 \left[Z_+ + X_+ \right] , \quad (116)$$

$$2 \text{Re} \left(C_V C_V'^* \right) = C_N^2 \left[Z_- - X_- \right] , \quad (117)$$

$$2 \operatorname{Re}\left(C_A C_A'^*\right) = \rho^2 C_N^2 \left[Z_+ - X_+ \right] \quad , \quad (118)$$

$$\operatorname{Re}\left(C_V C_A'^* + C_V' C_A^*\right) = \rho C_N^2 \left[T + Y \right] \quad , \quad (119)$$

$$\operatorname{Re}\left(C_V C_A^* + C_V' C_A'^*\right) = \rho C_N^2 \left[T - Y \right] \quad , \quad (120)$$

$$|C_V + C_V'|^2 = 2 C_N^2 Z_- \quad , \quad (121)$$

$$|C_V - C_V'|^2 = 2 C_N^2 X_- \quad , \quad (122)$$

$$|C_A + C_A'|^2 = 2 \rho^2 C_N^2 Z_+ \quad , \quad (123)$$

$$|C_A - C_A'|^2 = 2 \rho^2 C_N^2 X_+ \quad , \quad (124)$$

$$\operatorname{Re}\left(C_V + C_V'\right)\left(C_A^* + C_A'^*\right) = 2 \rho C_N^2 T \quad , \quad (125)$$

$$\operatorname{Re}\left(C_V - C_V'\right)\left(C_A^* - C_A'^*\right) = -2 \rho C_N^2 Y \quad . \quad (126)$$

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Isotope	(J, J')	λ	A_0	E_0 (MeV)	β_0	$E_{\max}$ (MeV)	$\beta_{\max}$
n	1/2	-2.178	-0.1127	1.293	0.9186	0.7580	0.7386
^3H	1/2	-2.095	-0.09405	0.5296	0.2626	0.5148	0.1208
^{11}C	3/2	0.7390	-0.5992	1.472	0.9378	0.8326	0.7895
^{13}N	1/2	0.5552	-0.3330	1.701	0.9538	0.9328	0.8366
^{15}O	1/2	-0.6272	0.7080	2.241	0.9737	1.1815	0.9016
^{17}F	5/2	-1.2860	0.9972	2.259	0.9741	1.1899	0.9031
^{19}Ne	1/2	1.6000	-0.0396	2.751	0.9826	1.4245	0.9334
^{21}Na	3/2	-0.7039	0.8617	3.021	0.9856	1.5549	0.9445
^{23}Mg	3/2	0.5400	-0.5574	3.610	0.9899	1.8419	0.9607
^{25}Al	5/2	-0.8004	0.9362	3.781	0.9908	1.9256	0.9641
^{27}Si	5/2	0.6870	-0.6973	4.361	0.9931	2.2108	0.9729
^{29}P	1/2	-0.5112	0.6060	4.456	0.9934	2.2577	0.9740
^{31}S	1/2	0.5131	-0.3301	4.901	0.9945	2.4774	0.9785
^{33}Cl	3/2	0.2888	-0.3821	5.021	0.9948	2.5368	0.9795
^{35}Ar	3/2	-0.2722	0.4201	5.451	0.9956	2.7497	0.9826
^{37}K	3/2	0.5811	-0.5720	5.641	0.9959	2.8438	0.9837
^{39}Ca	3/2	-0.6424	0.8213	6.001	0.9964	3.0224	0.9856
^{41}Sc	7/2	-1.0664	0.9983	6.121	0.9965	3.0820	0.9862
^{12}N	(1,0)	∞	1.0000	16.891	0.9995	8.4532	0.9982
^{107}In	(9/2,7/2)	∞	1.0000	2.761	0.9827	1.4293	0.9339

Table 1: List of mirror nuclei considered in Sect.4 and their characteristics, including the two allowed pure Gamow-Teller decays of ^{107}In [7] and ^{12}N [8] for comparison. The last column gives the value of β at which the potentiality of each nucleus is evaluated. Further details are given in the text.

Isotope	$\Delta_{\tilde{t}\tilde{\delta}}$	$\Delta_{\tilde{t}\tilde{t}}$	Type	$\beta_{\max}^2$	$2\beta_{\max}^2/(1+\beta_{\max}^2)$
n	2.769	4.887	H	0.5455	0.7059
^3H	3.200	5.771	H	0.0146	0.0288
^{11}C	-0.2647	-0.2358	H	0.6233	0.7680
^{13}N	-0.5003	-0.4718	H	0.6999	0.8235
^{15}O	-0.08476	0.2658	E	0.8129	0.8968
^{17}F	0.2125	0.1786	E	0.8156	0.8984
^{19}Ne	-5.839	-12.12	H	0.8713	0.9312
^{21}Na	-0.09177	0.1538	E	0.8920	0.9429
^{23}Mg	-0.3553	-0.1620	H	0.9230	0.9600
^{25}Al	-0.0499	0.1192	E	0.9296	0.9635
^{27}Si	-0.2451	-0.1314	H	0.9466	0.9726
^{29}P	-0.1788	0.2279	E	0.9488	0.9737
^{31}S	-0.5021	-0.4209	H	0.9575	0.9783
^{33}Cl	-0.4633	-0.08057	H	0.9594	0.9793
^{35}Ar	-0.3982	0.06566	H	0.9655	0.9824
^{37}K	-0.3358	-0.1765	H	0.9677	0.9836
^{39}Ca	-0.1368	0.1423	E	0.9714	0.9855
^{41}Sc	0.09134	0.1184	E	0.9725	0.9861
^{12}N	1.000	1.000	L	0.9963	0.9982
^{107}In	1.000	1.000	L	0.8722	0.9317

Table 2: Coefficients of the quadratic form in (80) characterising the parameter Δ for values of δ and $(\tan \zeta)$ much smaller than unity and for $(v_{ud}e^{i\omega})$ real. The symbols “E”, “H” or “L” stand for whether this quadratic form determines an elliptic, hyperbolic or linear curve in the plane $(\tilde{t}, \tilde{\delta})$, respectively (further details are given in the text). The last two columns give the optimal values $A_{\text{exp}}^{(0)}$ for the experimental asymmetries $A_{\text{exp}}(-J, 0)$ and $A_{\text{exp}}(-J, J)$, respectively, when $(\beta = \beta_{\max})$.

Isotope	$k(-J, 0)$	$A_{\text{exp}}(-J, 0)$	M_{min} (GeV/c ²)	$k(-J, J)$	$A_{\text{exp}}(-J, J)$	M_{min} (GeV/c ²)
n	0.556	0.0666	219	0.991	0.125	253
^3H	6.60	0.0091	407	8.14	0.018	428
^{11}C	6.18	0.378	400	7.69	0.549	422
^{13}N	1.87	0.223	297	2.83	0.365	329
^{15}O	6.76	0.511	409	8.30	0.676	431
^{17}F	30.3	0.720	595	32.2	0.838	604
^{19}Ne	0.141	0.0296	155	0.272	0.0574	183
^{21}Na	10.8	0.651	460	12.5	0.789	477
^{23}Mg	3.46	0.428	346	4.73	0.600	374
^{25}Al	13.9	0.722	490	15.7	0.839	505
^{27}Si	5.38	0.543	386	6.83	0.704	410
^{29}P	3.96	0.472	358	5.29	0.642	385
^{31}S	1.48	0.258	280	2.33	0.411	313
^{33}Cl	1.81	0.299	294	2.77	0.461	327
^{35}Ar	2.08	0.330	305	3.10	0.496	337
^{37}K	3.48	0.450	346	4.75	0.621	374
^{39}Ca	8.00	0.648	427	9.60	0.786	447
^{41}Sc	17.0	0.788	515	18.8	0.881	528
^{12}N	16.2	0.799	509	17.9	0.888	522
^{107}In	23.9	0.747	561	25.7	0.855	571

Table 3: The enhancement factors k , experimental asymmetries A_{exp} and mass-reaches M_{min} for $(\epsilon_0 = 0.01)$, $(\mathcal{P} = 0.80)$ and $(\beta = \beta_{\text{max}})$.

Isotope	$k(-J, 0)$	$A_{\text{exp}}(-J, 0)$	M_{min} (GeV/c ²)	$k(-J, J)$	$A_{\text{exp}}(-J, J)$	M_{min} (GeV/c ²)
n	0.720	0.0832	234	1.25	0.154	268
^3H	14.1	0.0114	491	15.8	0.0225	506
^{11}C	12.6	0.473	478	14.3	0.642	493
^{13}N	2.64	0.279	324	3.78	0.436	354
^{15}O	14.6	0.638	496	16.4	0.779	510
^{17}F	-42.4	0.901	647	-40.3	0.948	639
^{19}Ne	0.177	0.0370	165	0.340	0.0713	194
^{21}Na	41.7	0.814	644	43.6	0.897	652
^{23}Mg	5.53	0.536	389	7.00	0.698	413
^{25}Al	134.0	0.903	863	136.0	0.949	866
^{27}Si	10.1	0.678	452	11.8	0.808	470
^{29}P	6.59	0.590	406	8.12	0.742	428
^{31}S	2.04	0.323	303	3.05	0.488	335
^{33}Cl	2.56	0.374	321	3.68	0.545	351
^{35}Ar	2.99	0.413	334	4.19	0.584	363
^{37}K	5.56	0.563	389	7.03	0.720	413
^{39}Ca	20.0	0.809	536	21.8	0.895	548
^{41}Sc	-329	0.984	1080	-327	0.992	1079
^{12}N	-2187	0.998	1735	-2185	0.999	1734
^{107}In	-60.5	0.934	708	-58.5	0.966	701

Table 4: The enhancement factors k , experimental asymmetries A_{exp} and mass-reaches M_{min} for $(\epsilon_0 = 0.01)$, $(\mathcal{P} = 1.00)$ and $(\beta = \beta_{\text{max}})$.