

Electromagnetic forward scattering by large objects

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Contents

Contents	iii
List of Figures	vii
List of Acronyms	xv
1 Introduction	1
1.1 Context of the thesis	1
1.2 Contributions	5
1.3 Outline of the thesis	7
2 Multipoles and inhomogeneous plane waves for 2D scenarios	9
2.1 Introduction	9
2.2 Method of moments for scattering problems	10
2.2.1 <i>Derivation of the radiation integral</i>	10
2.2.2 <i>Solution with the Method of Moments (MoM)</i>	13
2.2.3 <i>Iterative solvers</i>	16
2.2.4 <i>Preconditioning techniques</i>	17
2.3 Multipoles decomposition and the Fast Multipole Method	18
2.3.1 <i>Number of multipoles</i>	21

2.3.2	<i>Integration within a Krylov iteration</i>	21
2.3.3	<i>Computational complexity</i>	25
2.4	Inhomogeneous plane wave decomposition	26
2.4.1	<i>Spectral representation of the Green's function</i>	26
2.4.2	<i>Number of inhomogeneous plane waves</i>	28
2.5	Applications	30
2.5.1	<i>Computation of the scattering by a wind farm</i>	31
2.5.2	<i>Fast computation of the scattered fields</i>	33
2.5.3	<i>Localization of the radar echoes behind a wind farm</i>	36
2.6	Conclusion	44
3	Fast numerical techniques for finite irregular arrays	47
3.1	Introduction	47
3.2	Iterative MBF solver	49
3.2.1	<i>Direct implementation</i>	49
3.2.2	<i>Acceleration with the FMM</i>	51
3.2.3	<i>Complexity comparison</i>	53
3.2.4	<i>Computation of the embedded element patterns</i>	55
3.3	Antenna element	57
3.4	Irregular array on infinite ground plane	59
3.4.1	<i>Validation with a 16 elements array</i>	61
3.4.2	<i>Large antenna arrays</i>	66
3.5	Effect of the ground plane truncation	71
3.5.1	<i>Single element atop a finite ground plane</i>	73
3.5.2	<i>Antenna arrays atop a finite ground plane</i>	78
3.6	Conclusion	79

4	Improved physical optics computation in forward scattering . .	83
4.1	Introduction	83
4.2	Modified physical optics	86
4.2.1	<i>Limitations of the classical PO</i>	86
4.2.2	<i>Update of the currents via an equivalent problem</i>	89
4.2.3	<i>Acceleration through multipole expansion</i>	92
4.3	Scattering by isolated objects	93
4.3.1	<i>Circular section</i>	95
4.3.2	<i>Elongated cylinder</i>	97
4.3.3	<i>Fields behind a scatterer</i>	99
4.3.4	<i>Computational complexity</i>	100
4.4	MPO for multiple-scattering scenarios	105
4.4.1	<i>Problem statement</i>	108
4.4.2	<i>Numerical results</i>	118
4.5	Extension to three-dimensional scatterers	125
4.6	Conclusion	127
4.7	Appendices	130
4.7.1	<i>Explicit derivation of the MPO for TM incidence</i>	130
4.7.2	<i>FMM for the derivatives of the Green's function</i>	131
4.7.3	<i>Analytical singularity extraction</i>	132
5	Impact of the human body at millimeter waves	135
5.1	Introduction	135
5.2	Models for the LOS human blockage	138
5.2.1	<i>Impact of the shape</i>	141
5.2.2	<i>Influence of the scatterer dimensions</i>	142
5.2.3	<i>Influence of the material</i>	143
5.2.4	<i>Influence of the frequency</i>	144

5.2.5 *Influence of the radiation pattern* 146

5.2.6 *Influence of the numerical method* 148

5.2.7 *Comparison between models and experimental data* 151

5.2.8 *Conclusion* 161

5.3 Human blockage in a multipath environment 163

5.3.1 *Beamforming gain in static scenarios* 163

5.3.2 *Dynamic beamforming in a time-varying environment* 169

5.3.3 *Comparison between models and experimental data* 173

5.4 Discussion and conclusion 182

6 Conclusion 185

List of publications 190

Bibliography 193

List of Figures

1.1	Scattering mechanism: the incident field $\mathbf{E}_i$ is perturbed by a scatterer (here a wind turbine) producing a scattered field $\mathbf{E}_s$ that we want to estimate.	2
1.2	Definition of the back-scattering and forward-scattering regions.	3
1.3	Accuracy and complexity of the different numerical methods investigated in this work.	4
2.1	Scattering problem and its representation with equivalent currents radiating in free space, inspired from [1].	11
2.2	Extraction of the singularity of the $G(\mathbf{r}', \mathbf{r})$. The solid line corresponds to $H_0^{(2)}(x')$ and the dashed line corresponds to the integrand of the first integral in eq. (2.24).	15
2.3	Variables associated to the FMM formulation for the interactions between the sub-domains S and S'	19
2.4	Example of a FMM domain decomposition. The scatterer is represented with a black solid line. The FMM groups correspond to circles (gray dashed line) with a diameter D	22
2.5	Near interaction matrix for the problem in Figure 2.4.	22
2.6	Number of (inhomogeneous) plane waves with respect to the sub-domains size and distance. The target error is $\epsilon = 10^{-4}$	29
2.7	Convergence of the FMM iterative solver at 10 GHz for different minimal distances between each wind turbine.	32

2.8	Normalized total power inside the wind farm for two different frequencies. For higher frequencies, the blockage becomes similar to an optical blockage (i.e. the shadow is located right behind the scatterer and aligned with the propagation vector). .	34
2.9	Total power behind the wind farm.	36
2.10	Total power behind the wind farm for different distances.	37
2.11	Layout of the false echoes for radar orientations. The detected targets are represented by the orange ellipses, whose dimensions represent the power of the echo.	38
2.12	Computation time for three different wind farms at three radar frequencies.	43
2.13	Power of the false echoes normalized by the scattered power of an isolated wind turbine. Two different wind farms of 20 wind turbines are investigated for a radar frequency of 2 GHz.	45
3.1	Convergence of the classical GMRES algorithm and the proposed iterative MBF solver. Machine precision is achieved with more iterations.	52
3.2	Curved monopole element on an infinite ground plane. The orange square represents the feed point and the blue square corresponds to the connection between the antenna and the ground plane.	58
3.3	Equivalent dipole element.	59
3.4	Zoom on the mesh near the feeding point.	59
3.5	Dipole element performance, computed with CST Microwave Studio® and the UCLouvain's MoM code.	60
3.6	Positions of the 16-elements array used for the validation of the proposed solver.	62
3.7	Figures 3.7a and 3.7b: self-impedance of the 16-elements array. The dashed line corresponds to the input impedance of the isolated antenna. Figures 3.7c and 3.7d: mutual impedance of antenna 1, located at (0,0).	62
3.8	Directivity and absolute error (eq. (3.29)) of the embedded element patterns of element 9 and element 15 from Figure 3.6. .	64

3.9 Isolated element pattern. 65

3.10 Radiation pattern of the 16-elements array. 65

3.11 Layout and directivity of three antenna arrays made of 256, 512 and 1024 elements. 68

3.12 Comparison of the directivity of the 256 elements radiation pattern with two different excitations. Phase is constant for both cases. 69

3.13 Relative error of the norm of the currents and relative error of the norm of the radiation patterns for the 256 and 512-elements arrays, with respect to the number of MBFs. 70

3.14 Single element atop a PEC ground plane of dimensions $3.66 \times 3.66 \lambda^2$. The red point corresponds to the feed. 73

3.15 Effect of the finite ground plane on the radiation pattern and validation of the approximation in eq. (3.36). The blue and the orange lines corresponds to the azimuth cuts $\phi = 0$ and $\phi = \pi/2$, respectively. 75

3.16 Impact of the phase extraction of the magnetic field radiated by MBF 1. 77

3.17 Minimal sampling rate for different regions around the source antenna. 77

3.18 Two considered irregular arrays of 8 and 64 elements with finite ground plane. 79

3.19 Radiation pattern for the 8-elements array from Figure 3.18a obtained with the method of moments, the infinite ground plane solution and the physical optics. 79

3.20 Radiation pattern for the 64-elements array from Figure 3.18b obtained with the CST Microwave Studio®, the infinite ground plane solution and the physical optics. 80

4.1 Illustration of the Babinet's principle. In the forward direction, the fields scattered by an object (left figure) is equivalent to the fields scattered by an opaque screen (right figure) with the same silhouette as viewed from the source. 84

4.2	(a): initial scattering problem; (b): equivalent problem with PO currents on the illuminated part of the object.	87
4.3	Bistatic RCS of the problem depicted in Figure 4.2, computed with MoM (reference) and the classical PO.	88
4.4	Different steps for updating the PO currents. Magnetic currents (double arrows) exist only on the equivalence surface S' , while the original surface S only supports electric currents (simple arrow).	89
4.5	Cylinder with a circular section under plane wave incidence. The PEC surface is represented by the solid line, while the equivalence surface is represented by the dotted line.	93
4.6	Magnitude of the currents obtained from three different methods for a TM polarization. MPO(2) stands for the solution obtained with the proposed method after 2 iterations. The inset in a zoom around the shadow boundary.	94
4.7	Normalized far-field radiation pattern of the cylinder under TM incidence in the forward region $\phi \in [90^\circ, 270^\circ]$	94
4.8	Magnitude of the currents for the TE polarization.	96
4.9	Normalized far-field radiation pattern of the cylinder under TE incidence in the forward region ($\phi \in [90^\circ, 270^\circ]$).	96
4.10	Cylinder with elongated section under plane wave incidence (TM case).	97
4.11	Current distributions for the problem of Figure 4.10 (TM case).	98
4.12	Normalized far-field radiation pattern of the elongated cylinder ($R = 5\lambda, L = 10\lambda$) under TM incidence in the forward region ($\phi \in [90^\circ, 270^\circ]$).	98
4.13	Normalized far-field radiation pattern of the elongated cylinder ($R = 50\lambda, L = 100\lambda$) under TM incidence in the forward region. For a better visibility, the azimuth observation angle is limited to $\phi \in [165^\circ, 195^\circ]$. The inset is a zoom of the radiation pattern for $\phi \in [180^\circ, 185^\circ]$	99
4.14	Cylinder with elongated section and a concave part under plane wave incidence (TM case).	99

4.15	Normalized far-field radiation pattern of the elongated cylinder with a concave part ($R = 5\lambda, L = 10\lambda$) under TM incidence in the forward region ($\phi \in [90^\circ, 270^\circ]$).	100
4.16	Magnitude of the total electric field computed with MoM (TM case). The scatterer is illustrated in Figure 4.10 and represented with white solid contour. Dashed lines show the positions of the cuts in which fields are displayed in Figure 4.17.	101
4.17	Comparison of the total electric field of the problem described in Figure 4.10 computed by method of moments (MoM), the classical physical optics (PO) and the method proposed in this work (MPO) after two iterations for a TM incidence. The absolute error in a logarithmic scale is also provided for the six cuts drawn in Figure 4.16.	102
4.18	Total CPU time for both MoM-FMM and MPO for different radii of an infinite cylinder with circle section under TM plane wave incidence.	103
4.19	Currents distribution for a cylinder with a circular cross-section and a radius $R = 100\lambda$. The vertical dotted lines correspond to 82° and 105°	105
4.20	Illustration of the shadow radiation on hidden surfaces and comparison with the geometrical visibility testing. The surfaces are represented by the thin black solid lines. The currents are normalized and for a better visibility, null currents are not represented	107
4.21	Currents for the initialization and the first three iterations. They arrows corresponds to the allowed shadow radiation modes defined by Equation (4.23).	110
4.22	Different steps for the computation of the correction currents $\tilde{\mathbf{J}}^B$ with the MPO.	112
4.23	Currents for the first three reflection iterations, corresponding to $\mathbf{J}_0, \mathbf{J}_1, \mathbf{J}_2, \mathbf{J}_3$. The arrows correspond to the allowed shadow radiation modes defined by Equation (4.27).	114
4.24	Different steps for the computation of the new currents $\mathbf{J}_1^A$ after the first reflection iteration using the MPO.	115
4.25	Final currents using eq. (4.30).	116

4.26	Convergence with respect to the number of reflection iterations for three different test cases. For comparison, the convergence of the iterative MoM is represented with dashed lines.	119
4.27	Multiple scattering between four identical infinite cylinders with a circular cross section.	121
4.28	Multiple scattering between four identical infinite cylinders with an elongated cross section.	122
4.29	Currents and corresponding zones on scatterer 3 from the multiple-scattering problem depicted in Figure 4.28.	123
4.30	Multiple scattering between five aligned infinite cylinders. . . .	124
4.31	Multiple scattering between four different infinite cylinders. . .	125
4.32	Three-dimensional PEC scatterer.	126
4.33	Normalized bistatic radiation pattern in the forward region and the error associated to the MPO and PO approach with respect to the MoM reference.	128
5.1	Validation of the approximation of infinite cylinders.	140
5.2	Normalized received power for different model shapes, with respect to the perpendicular walk position depicted in Figure 5.1a. . . .	142
5.3	Attenuation versus the perpendicular distance, for cylinders of different radii, computed with the PO. Dotted lines correspond to the -6 dB level crossing.	143
5.4	Attenuation versus the perpendicular distance, for different materials, at 60 GHz.	145
5.5	Attenuation versus the perpendicular distance for different frequencies.	145
5.6	Far-field radiation pattern of the transmitter and the receiver antenna arrays, with a -3 dB beamwidth of 6°	146
5.7	Comparison of the attenuation for different positions of the transmitter and the human position. The receiver is located at $[2, 0, 0]$ m. The solid lines corresponds to a transmitter and receiver radiation pattern presented in Figure 5.6; the dotted line assumes an omnidirectional radiation pattern at both TX/RX; the dashed line corresponds to an incident plane wave and an omnidirectional receiving array.	147

5.8	Variables useful for the derivation of the shadowing attenuation of a KED (eq. (5.7)).	149
5.9	Received power during the shadowing event, computed with different numerical methods.	151
5.10	Schematic and realistic view of the measurement environment. .	153
5.11	Pair of steering vectors for the transmitting and receiving arrays maximizing the received power. The red dots represent to the (0,0) azimuth/elevation angle corresponding to the LOS direction, with associated -3 dB region around this direction. . . .	155
5.12	Time dependent power-delay profile for the first realization of a "path 2" scenario. The LOS is indicated by the black arrow. The shadowing event is clearly observable around 3.5 s (dashed circle).	156
5.13	Measured attenuation for the scenario "path 2" and the beam pair $(TX,RX) = (11,7)$, with respect to the time, assuming a constant speed of 0.9 ms. The black line is the averaged value of the six realizations of the same experiment and the grey area corresponds to interval defined by one standard deviation around the mean value.	157
5.14	Comparison between measurements and simulation for the "path 2" scenario.	158
5.15	Attenuation for the "path 2" scenario and different transmitter and receiver beam pairs using the KED model. The orange line corresponds to the omnidirectional TX and RX array and is thus identical for each beam pair.	159
5.16	Measured attenuation for the scenario "path 3" and the beam pair $(TX,RX) = (11,7)$, with respect to the time, assuming a constant speed of 0.9 m/s. The black line is the averaged value of the six realizations of the same experiment and the grey area corresponds to interval defined by one standard deviation around the mean value.	160
5.17	Comparison between measurements and simulation for the "path 3" scenario.	161
5.18	Attenuation for the "path 3" scenario and different transmitter and receiver beam pairs using the KED model.	162

5.19	Impact of the environment on the field distribution at for different frequencies.	165
5.20	Comparison of the cumulative distribution function (CDF) of the beamforming gain at 10 GHz for the same environment as in Figure 5.19c for different beamforming strategies.	168
5.21	Scenario of the time-varying environment. The dark gray shapes numbered from 1 to 4 represent the static scatterers and the light gray object represents the human person moving in the $+\hat{y}$ direction (black arrow). The dashed line corresponds to the LOS. The transmitting array is an ULA of 21 omnidirectional elements with a $\lambda/2$ spacing and is located at $(0,0)$. The receiver is a single omnidirectional element.	170
5.22	Comparison of the attenuation using static and dynamic MRT precoder versus time, assuming a walk aligned with the $\hat{y}$ axis and a frequency of 10 GHz. The green(red) parts corresponds to the LOS(NLOS) conditions, defined by the -6 dB dashed line.	174
5.23	Power delay profile of the empty room (no person present) for different beam pairs.	175
5.24	Representation of the room used for the ray-tracing simulations. The transmitter is located at $[0.2, 1.3, 1]$ and is represented by the red "+" sign; the receiver is located at $[4, 1.3, 1]$ and is represented by the red "x" sign. The black lines represent the ray leaving the transmitter and reaching the receiver through multipath propagation (LOS, 1 st and 2 nd order reflections.	177
5.25	Top view of Figure 5.24. Decomposition of the m^{th} ray of the first order (dotted line) and the n^{th} ray of the second order (dashed line). The LOS ray remains unchanged (solid line).	178
5.26	Knife-edge diffraction models for a non-perpendicular walk.	179
5.27	Comparison of the PDP of the empty room, for two beams pairs.	179
5.28	Impact of the multipath and beam switching on the total received power, for different human position: comparison measurements and simulations.	181
5.29	Measured and modeled time-varying PDP for "path 2" and "path 3" scenarios, for the beam combination (11,3).	183

List of acronyms

AWGN	Additive White Gaussian Noise
BF	Beamforming
CAD	Computer-Aided Drawing
CB	Code Book
CDF	Cumulative Distribution Function
CIR	Channel Impulse Response
CPU	Central Processing Unit
CSI	Channel State Information
DBS	Direct-Broadcast Satellite
EEP	Embedded Element Pattern
EFIE	Electric Field Integral Equation
EGT	Equal Gain Transmission
EM	Electromagnetic
EVD	Eigen-Value Decomposition
FFT	Fast Fourier Transform
FMM	Fast Multipole Method
FSPL	Free-Space Path Loss
GMRES	Generalized Minimal Residual
GP	Ground Plane
GPS	Global Positioning System
HPBW	Half-Power Beam Width
IDFT	Inverse Discrete Fourier Transform
IoT	Internet-of-Things
IPO	Iterative Physical Optics
IPW	Inhomogeneous Plane Wave
KED	Knife-Edge Diffraction

LOS	Line-Of-Sight
MBF	Macro Basis Function
MFIE	Magnetic Field Integral Equation
MLFMA	Multi-Level Fast Multipole Algorithm
MLNG	Multi-Level Non-regular Grid
MoM	Method of Moments
MPC	Multipath Component
MPO	Modified Physical Optics
MRT	Maximum Ratio Transmission
NFFT	Non-regular Fast Fourier Transform
NLOS	Non-Line-Of-Sight
NURBS	Non-Uniform Rational Basis Splines
NYU	New-York University
PCS	Personal Communication Satellite
PDP	Power-Delay Profile
PEC	Perfect Electric Conductor
PO	Physical Optics
RAM	Random Access Memory
RCS	Radar Cross Section
RF	Radio Frequency
RPFMA	Ray-Propagation Fast Multipole Algorithm
RWG	Rao Wilton Glisson
RX	Receiver or Receiving
SAI	Sparse Approximate Inverse
SAR	Synthetic Aperture Radar
SATCOM	Satellite Communications
SLL	Sidelobe Level
SMA	SubMiniature version A
SR	Shadow Radiation
SVD	Singular-Value Decomposition
TE	Transverse Electric
TM	Transverse Magnetic
TX	Transmitter or Transmitting
ULA	Uniform Linear Array
UTD	Uniform Theory of Diffraction

1

Introduction

1.1 Context of the thesis

This thesis focuses on numerical methods used for modeling (or predicting) the electromagnetic scattering which describes the interactions between an electromagnetic wave and its environment. This is represented in Figure 1.1 where an antenna radiates a propagating *incident* field E_i towards an object (or scatterer, here represented by a wind turbine). The incident field undergoes different wave phenomena (reflection, diffraction, ...) which perturbs the incident wave. Those perturbations are represented by the *scattered* field E_s which depends on the characteristic of the incident wave (frequency, amplitude, polarization) as well as on the characteristics of the scatterer (geometry, material, orientation with respect to the source, ...). The knowledge of the scattered field enables the complete description of the electromagnetic scattering. This information is particularly helpful in various contexts where some of them are detailed here below.

Most modern communication systems rely on wireless transfer of information between a source (e.g. a base station) and a receiver (e.g. a user's cellphone). The transfer of information is performed by modulating a carrier signal (a high-frequency electromagnetic sinusoidal wave) which

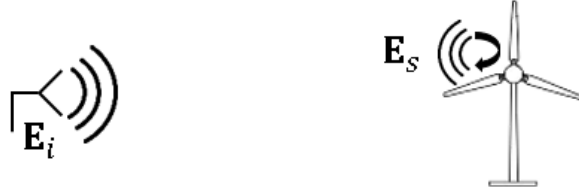


Fig. 1.1 Scattering mechanism: the incident field E_i is perturbed by a scatterer (here a wind turbine) producing a scattered field E_s that we want to estimate.

emanates from a transmitting antenna, propagates in the environment and eventually reaches the receiving antenna. The effect of the communication link is modeled by the *channel impulse response* (CIR) $h(t, \tau)$ of the communication channel, which expresses the received signal $r(t)$ based on the transmitted signal $s(t)$ as:

$$r(t) = \int_{-\infty}^{\infty} h(t, \tau) s(t - \tau) d\tau \quad (1.1)$$

The *a priori* knowledge of the channel is of prime importance in the design of communication systems and protocols, as it enables the prediction of the expected performance and limitations of the system. The knowledge of the field scattered by the environment over the communication channel allows the modeling of the channel impulse response. Based on the knowledge of the scattered field, it is also possible to estimate the power radiated by a base station in a given environment. This information serves different purposes: verifying if the radiated power meets the legal restrictions, estimating the coverage of the base station, ...

The second example for which the knowledge of the scattered field is of prime importance is the estimation of the radar cross section (RCS) of various targets. This quantity estimates the probability of detection of a target by a radar. Indeed, the power P_r available at the radar receiver is computed through the radar equation, defined as [2]

$$P_r \propto P_i \frac{\lambda^2 \sigma}{(4\pi)^3 R^4} \quad (1.2)$$

where P_i is the transmitted power, λ is the wavelength, σ is the RCS and

R is the distance between the radar and the target. One can observe from eq. (1.2) that the received power depends linearly on the RCS, which corresponds to the cross section of an equivalent sphere that would radiate the same power than the actual scatterer. The radar cross section is expressed as an *equivalent* area which usually does not correspond to the physical cross section of the target. This quantity depends on the scatterer geometry and material, on the characteristics of the incident wave (frequency and polarization) as well as on the position of the receiver.

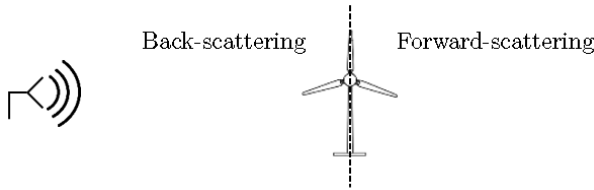


Fig. 1.2 Definition of the back-scattering and forward-scattering regions.

Indeed, the position of the receiver defines two scattering regimes described in Figure 1.2. The first one is called the *back-scattering* region, and corresponds to the situation where the receiver or the point at which the field is measured is located “between” the source and the scatterer. This situation occurs for traffic radar where the transmitter and the receiver are collocated and the wave reflected by a vehicle propagates back towards the radar. This is also the case for earth observation satellites (such as synthetic aperture radars) which measure the electromagnetic field reflected by the earth towards the satellite. The second scattering regime in Figure 1.2 is called the *forward-scattering* region, where in this case the receiver is located “behind” the scatterer. In addition to the study of numerical techniques for large electromagnetic problems, the emphasis of this thesis is on the radiation of scatterers in the forward region. In other words, it considers scenarios where one or various scatterers are located between the transmitter and the receiver. This particular radiation regime is of great interest, as the RCS of a scatterer measured in the forward region can be several orders of magnitude higher than the RCS measured in backscattering [3]. From Babinet’s principle, the RCS in the exact forward direction (i.e. for a bistatic angle equal to 180°) does not depend on the surface but rather on its projected area A on a plane perpendicular to the direction of

propagation. In this particular case, the RCS σ^{180° is expressed as

$$\sigma^{180^\circ} = \frac{4\pi A^2}{\lambda^2} \quad (1.3)$$

However, when the observation angle deviates from the forward direction, the accuracy of the Babinet estimation is much worse [4]. This type of radiation (i.e. with a observation angle close to 180°) is important for radar applications, channel characterization, or electromagnetic compatibility.

The modeling of the scattered field in the forward region is performed using computational electromagnetics. Thanks to the increasing power of modern computers combined with the development of fast numerical methods, it is now possible to solve more and more complex electromagnetic problems. In this work, we investigate different numerical techniques with different accuracy and complexity, as represented in Figure 1.3. Geometries which are described with a high level of detail (such as complex antennas) are commonly simulated using the Method of Moments (MoM) which essentially consists of solving a dense linear system of equations. The major drawback of this method is its computational complexity of $\mathcal{O}(N^3)$. Accelerations of the MoM have thus been developed over the years for the analysis of large electromagnetic problems. Large MoM systems of equations are solved iteratively, where the matrix-vector product at each iteration is reduced using a single-level domain decomposition using multipoles [5] with a complexity in $\mathcal{O}(N^{1.5})$ or a multilevel implementation [6], reducing further the complexity to $\mathcal{O}(N \log N)$. Other decompositions using inhomogeneous plane waves (IPW) [7] or describing the current distribution on the antenna with a limited number of macro-basis functions (MBFs) [8] also aim at reducing the MoM complexity.

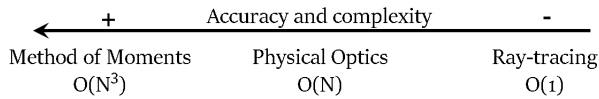


Fig. 1.3 Accuracy and complexity of the different numerical methods investigated in this work.

Despite the different accelerations of the MoM, this approach cannot be applied to arbitrary scatterers, especially when those become very large in terms of wavelength (more than a few dozen). In the particular case where a scatterer can be assumed infinite in a given dimension (e.g. the

most of a wind turbine illuminated by a centimetre-wave [9]), the problem can be reduced to two-dimensions, thus shrinking significantly the number of discretizing functions. Besides those particular cases, scattering from larger structures is obtained through a simplification of the geometry (e.g. represent a person by an ellipsoid [10], or a building with flat surfaces [11]) combined with high-frequency approximations. A well-known technique is the physical optics (PO) approximation which provides a good estimate of the fields backscattered by a smooth surface with a local radius of curvature large compared to the wavelength. In a nutshell, the PO locally assumes an infinite flat surface on the scatterer. It is hence also referred to as "the tangent plane approximation". It thus estimates the surface currents using image theory [12]. By using this approximation, no linear system of equations needs to be solved as for the MoM and it offers a complexity linear to the size of the problem. The PO is also accelerated using analytical integration on sub-domains [13], or by interpolation of the radiation patterns of small groups aggregated in a multilevel fashion, thus accelerating the computation under various illuminations [14]. Another high-frequency technique commonly used for radio propagation simulation is ray-tracing. It can be seen as an asymptotic evaluation of the radiation integral using the stationary phase approximation [15]. This approach considers the received field as a combination of rays emanating from the transmitter and eventually reaching the receiver after a certain number of specular reflections. Ray-tracing is particularly efficient when dealing with flat surfaces in indoor and outdoor scenarios where the number of flat panels is limited. When this number increases (e.g. number of buildings in an urban scenario), the ray-tracing computation may be accelerated with ray-launching techniques (initially developed for the fast computation of the RCS of cavities [16]), or by accelerating the computation of the visibility between the buildings [11]. The conclusion is that it is very important to consider the level of accuracy that is targeted as well as the level of details of the available geometrical models in order to select the appropriate numerical method that meets the accuracy requirements with an affordable computational complexity.

1.2 Contributions

The goal of this thesis is to investigate and develop numerical techniques that can be applied for the efficient evaluation of the scattering by elec-

trically large objects in the forward region. By decreasing level of complexity, it starts from accelerated full-wave methods which can be afforded in terms of memory and computation time under some geometry simplifications or when the sub-domains are the same. Those rapid full-waves methods are applied to the down-range effects of simplified wind turbines and the rapid computation of large irregular arrays. Secondly, the physical optics approximation provides a good estimation of the fields reflected towards the source when the radius of curvature is large with respect to the wavelength; a modification of this method is proposed in order to increase its accuracy in the forward direction, away from the source. Finally, a ray-based method is used for modeling the attenuation caused by a person walking in an indoor environment who crosses the line of sight of a 60 GHz communication link.

More specifically, the contributions of this thesis are listed here below.

- A modified formulation of the physical optics is proposed in order to improve the accuracy of the scattered fields near the forward direction. Starting from the equivalence theorem, equivalent electric and magnetic currents are computed on an equivalent surface encompassing the scatterer. Those radiate in turn on the surface of the scatterer and new currents are computed locally using the classical physical optics formulation. The computation of the equivalent currents is accelerated using a domain decomposition and a multipole representation of the Green's function. The proposed method increases significantly the accuracy within $\pm 30^\circ$ around the forward direction, especially for scatterers including surfaces aligned with the direction of propagation.
- The proposed modified physical optics is particularly attractive in a multiple-scattering context. The modified physical optics is embedded in an iterative process where a given scatterer induces new currents on its surroundings at each iteration. The method starts from the work proposed in [17], where the visibility testing between each pair of facets is avoided thanks to the shadow radiation which naturally creates a shadow on scatterers which are not in direct line of sight as seen from the source.
- A comparative study of various models representing the forward scattering by a person over a 60 GHz link is carried out. It is demonstrated from simulations and measurements that despite the small

wavelength of the signal, a coarse representation of the human body provides a very accurate estimation of the attenuation by a walking person in an office environment. The knife-edge model used in [18] is embedded in a ray-tracing simulator in order to assess the capabilities of beamforming to overcome shadowing. The proposed model is in good agreement with an extensive experimental data set.

- The accelerated full-wave techniques employed throughout the thesis are applied to another type of large electromagnetic problem: the simulation of large irregular arrays. Starting from an MBF representation of the current distribution computed within an iterative solver, the matrix-vector product is accelerated with the help of the fast multipole method. This formulation enables the fast computation of the embedded element patterns of an array made of hundreds of elements within a few minutes. Based on the solution obtained with the fast solver, the effect of a flat finite ground plane is added as a correction term on the radiation pattern of the array. Its impact is rapidly computed with the physical optics and the interpolation of the fields radiated by the MBFs.

1.3 Outline of the thesis

The manuscript is structured as follows.

In Chapter 2, we recall the formulation of two accelerated full-wave methods in a MoM context, namely the fast multipole method (FMM) and an approach based on an inhomogeneous plane wave decomposition presented in [19]. Those methods are compared and used to evaluate the downrange effect of wind turbines on radars. Those problems are good illustrations of the rapid computation of the scattering of large objects in the forward region.

In Chapter 3, the methods previously presented are applied to the simulation of irregular antenna arrays. Even though this problem is not a typical forward scattering application, it uses the same formalism presented in Chapter 2. This is a good illustration the same method can serve very different purposes. This chapter shows how it is possible to combine an accelerated full-wave method for the antennas and an asymptotic physical optics approach for the analysis of the ground plane.

Chapter 4 presents the modified physical optics (MPO) enhancing the accuracy of the scattered fields in the forward region. The presentation of the method is firstly carried out in two dimensions, even though simulation results also show an improvement when considering three dimensional objects. This new approach is combined with shadow radiation iterations for the analysis of multiple-scattering scenarios.

All the investigated numerical methods and model approximations are wrapped up in Chapter 5 for the study of human shadowing at 60 GHz. Due to the electric size of the human (hundreds of wavelengths in height) and the complexity to create a precise model, the validity of different approximations is compared with real-world measurements. This chapter illustrates that the selection of the appropriate model and numerical technique is instrumental in predicting the system performance.

General conclusions and perspectives for future works end this thesis in Chapter 6.

2

Multipoles and inhomogeneous plane waves for 2D scenarios

2.1 Introduction

In many engineering applications, one is interested in modeling the interaction of electromagnetic waves with the environment (or *scatterers*). Those models rely on numerical techniques which enable the prediction of the radar cross section (RCS) of an aircraft [20], the prediction of the coverage by a base station in a given environment [21], or the design of an antenna for a specific application, for example.

Indeed, the prediction of how the fields are perturbed by a scatterer is not a trivial task. Only a few canonical cases are described by a closed form expression. The most famous example is the scattering by a sphere. The analytical RCS of a conducting sphere was proposed by Mie [22] at the beginning of the 20th century. This example is particularly interesting in the radar community [2] as the sphere is used as a reference target whose RCS is equal to its projected area when its diameter is large with respect to the wavelength.

However, for more complex geometries, numerical methods are required for the modeling of the scattering mechanisms, ruled by Maxwell's equations. Among different techniques, the Method of Moments (MoM) has been used for decades in the antenna community. It is particularly convenient for radiation problems, since the solution is expressed as currents on the scatterers which radiate anywhere in three-dimensional space. This method is accurate but requires the solution of a dense linear system of equations, leading to a large computation time as well as large memory space requirements.

This chapter recalls the principles of the method of moments in two-dimensions. It also presents the fundamentals of two acceleration techniques commonly combined with the method of moments. Both approaches rely on an alternative representation of the Green's function, namely a decomposition using multipoles [23, 24, 25] and another decomposition with evanescent and propagating plane waves, called *inhomogeneous plane waves* [26, 7]. Those two methods are extensively employed in the remainder of the thesis, and their performance is compared through different examples at the end of this chapter.

2.2 Method of moments for scattering problems

2.2.1 Derivation of the radiation integral

Let us consider the electromagnetic problem depicted in Figure 2.1a where a perfect electric conductor (PEC) scatterer is illuminated by a source (e.g. a plane wave) or a collection of sources (e.g. electric and/or magnetic dipoles). The incident field impinges on the scatterer, and some part of the incident power is reflected in various directions, depending on the characteristics of the source and of the scatterer. The prediction of the scattered field is in general not trivial and is obtained through the method derived here below. The initial problem in Figure 2.1a can be replaced by an equivalent problem [12], where the scatterer is represented by equivalent currents $\mathbf{J}$ radiating in free space. This is illustrated in Figure 2.1b, where the external sources are set to zero. The scattering problem now consists in determining the currents $\mathbf{J}$ on the equivalence contour and in computing the scattered fields radiated by the currents, through a radiation integral [12]. The latter is derived as follows.

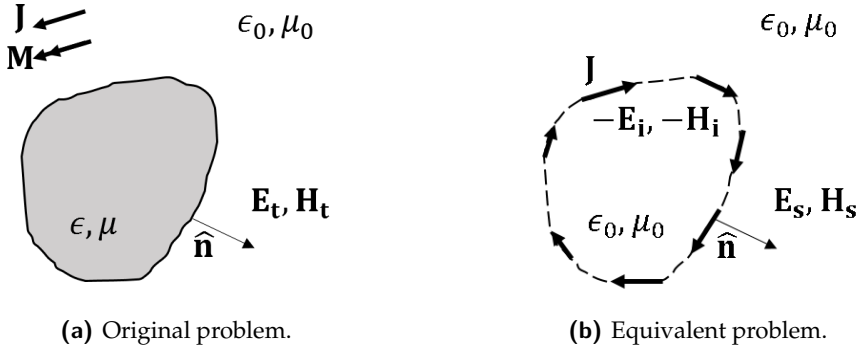


Fig. 2.1 Scattering problem and its representation with equivalent currents radiating in free space, inspired from [1].

Assuming no magnetic sources and a homogeneous medium, the relation between the electric field $\mathbf{E}$ and the surface currents $\mathbf{J}$ is obtained from the three first Maxwell's equations :

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \quad (2.1)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\epsilon\mathbf{E} \quad (2.2)$$

$$\nabla \cdot \mathbf{E} = \frac{q}{\epsilon} \quad (2.3)$$

where $\mathbf{H}$ is the magnetic field, q is the electric charge and μ, ϵ are the electrical parameters of the medium. By taking the curl of eq. (2.1) and using eq. (2.2), one obtains

$$\nabla \times \nabla \times \mathbf{E} - \omega^2\mu\epsilon\mathbf{E} = j\omega\mu\mathbf{J} \quad (2.4)$$

Using eq. (2.3) and the vector identity

$$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2\mathbf{E} \quad (2.5)$$

eq. (2.4) becomes

$$\nabla^2\mathbf{E} + k^2\mathbf{E} = j\omega\mu\mathbf{J} + \frac{1}{\epsilon}\nabla q \quad (2.6)$$

where $k = \omega\sqrt{\mu\epsilon}$.

Using the definition of the surface currents $\nabla \cdot \mathbf{J} = -j\omega q$, one finally

2 | Multipoles and inhomogeneous plane waves for 2D scenarios

obtains

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = j\omega\mu \mathbf{J} - \frac{1}{j\omega\epsilon} \nabla(\nabla \cdot \mathbf{J}) \quad (2.7)$$

Let us also introduce the two-dimensional Green's function

$$G(\mathbf{r}, \mathbf{r}') = -\frac{j}{4} H_0^{(2)}(k|\mathbf{r} - \mathbf{r}'|) \quad (2.8)$$

which satisfies

$$\nabla^2 G(\mathbf{r}, \mathbf{r}') + k^2 G(\mathbf{r}, \mathbf{r}') = -\delta(\mathbf{r}, \mathbf{r}') \quad (2.9)$$

where $H_0^{(2)}$ is the Hankel function of the second kind and $\delta(\mathbf{r}, \mathbf{r}')$ is the Dirac function. The Green's function G is the response to a point source. Since Maxwell's equations are linear, the electric field $\mathbf{E}$ radiated by a continuous distribution of current $\mathbf{J}$ is obtained by integration over the volume encompassing the sources. In the case of a PEC material, this reduces to the integral over a surface. Once the currents $\mathbf{J}$ are known, the scattered electric field $\mathbf{E}_s$ in Figure 2.1b is obtained by the radiation integral:

$$\mathbf{E}_s(\mathbf{r}) = -j\omega\mu \iint_S G(\mathbf{r}, \mathbf{r}') \left(\mathbf{J}(\mathbf{r}') + \frac{1}{k^2} \nabla' \nabla' \cdot \mathbf{J}(\mathbf{r}') \right) d\mathbf{r}' \quad (2.10)$$

As $\mathbf{J}(\mathbf{r}')$ depends on the unknown quantity $\mathbf{E}_s$, the boundary condition on the electrical field [12] is used in order to link the currents $\mathbf{J}(\mathbf{r}')$ to the known incident field $\mathbf{E}_i$. Equation (2.10) now reads

$$\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{E}_i(\mathbf{r}) = j\omega\mu \hat{\mathbf{n}} \times \iint_S G(\mathbf{r}, \mathbf{r}') \left(\mathbf{J}(\mathbf{r}') + \frac{1}{k^2} \nabla' \nabla' \cdot \mathbf{J}(\mathbf{r}') \right) d\mathbf{r}' \quad (2.11)$$

where $\hat{\mathbf{n}}$ is the outward normal on the surface. It is possible to express eq. (2.11) in terms of vector potentials, so that the differential operators now act on the observation coordinates, as detailed in [27]. Equation (2.11) then reads

$$\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{E}_i(\mathbf{r}) = j\omega\mu \hat{\mathbf{n}} \times \iint_S \left(\mathbf{I} + \frac{1}{k^2} \nabla \nabla \cdot \right) (\mathbf{J}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}')) d\mathbf{r}' \quad (2.12)$$

A similar derivation can be obtained by enforcing the boundary conditions on the magnetic field, thus leading to the magnetic field integral equation (MFIE) [28].

2.2.2 Solution with the Method of Moments (MoM)

Except for canonical cases, the equivalent currents $\mathbf{J}$ cannot be obtained analytically from eqs. (2.11) and (2.12). Those currents are often solved using the *Method of Moments* (MoM) which approximates eqs. (2.11) and (2.12) as a linear system of equations. Without loss of generality, it consists in determining the unknown $\mathbf{f}$ so that

$$\mathbf{g} = L(\mathbf{f}) \quad (2.13)$$

where L is a linear integro-differential operator (see eqs. (2.11) and (2.12)), $\mathbf{f}$ is the unknown (here, the currents $\mathbf{J}$) and $\mathbf{g}$ is the source vector (here, the incident field $\mathbf{E}_i$). The key idea of the method of moments is to represent the unknown quantity $\mathbf{f}$ as a linear combination of known basis functions $\mathbf{f}_n$ weighted by unknown coefficients i_n , so that

$$\mathbf{f} \simeq \sum_{n=1}^N i_n \mathbf{f}_n \quad (2.14)$$

Since L is linear, eq. (2.13) becomes

$$\mathbf{g} = \sum_{n=1}^N i_n L(\mathbf{f}_n) \quad (2.15)$$

By defining an inner product

$$\langle \mathbf{f}_m, \mathbf{f}_n \rangle = \int \mathbf{f}_m(\mathbf{r}) \cdot \int \mathbf{f}_n(\mathbf{r}') d\mathbf{r}' d\mathbf{r} \quad (2.16)$$

the system of equations in eq. (2.15) now reads

$$\langle \mathbf{f}_m, \mathbf{g} \rangle = \sum_{n=1}^N i_n \langle \mathbf{f}_m, L(\mathbf{f}_n) \rangle \text{ for } m = 1 \dots N \quad (2.17)$$

where it is assumed that the basis and the testing functions are assumed as identical. This procedure is called Galerkin method. The previous equation is written in the matrix form as

$$\mathbf{Z} \mathbf{i} = \mathbf{v} \quad (2.18)$$

where $\mathbf{Z}$ is the so-called *impedance matrix* of dimension $(N \times N)$, where N is the number of basis functions discretizing the scatterer, $\mathbf{v}$ is the *excitation*

2 | Multipoles and inhomogeneous plane waves for 2D scenarios

vector of size $(N \times 1)$ and $\mathbf{i}$ is the vector of the weights of the currents, so that

$$\mathbf{J} \approx \sum_{n=1}^N i_n \mathbf{f}_n \quad (2.19)$$

More explicitly, the entries of the impedance matrix for a TE and TM polarization are given by

$$z_{m,n}^{\text{TM}} = \int_{\mathbf{f}_m} \mathbf{f}_m(\mathbf{r}) \cdot \int_{\mathbf{f}_n} \mathbf{f}_n(\mathbf{r}') H_0^{(2)}(|\mathbf{r}' - \mathbf{r}|) d\mathbf{r}' d\mathbf{r} \quad (2.20)$$

$$\begin{aligned} z_{m,n}^{\text{TE}} = & \int_{\mathbf{f}_m} \mathbf{f}_m(\mathbf{r}) \cdot \int_{\mathbf{f}_n} \mathbf{f}_n(\mathbf{r}') H_0^{(2)}(|\mathbf{r}' - \mathbf{r}|) d\mathbf{r}' d\mathbf{r} \\ & + \frac{1}{k^2} \int_{\mathbf{f}_m} \mathbf{f}_m(\mathbf{r}) \cdot \int_{\mathbf{f}_n} \nabla \nabla \cdot \mathbf{f}_n(\mathbf{r}') H_0^{(2)}(|\mathbf{r}' - \mathbf{r}|) d\mathbf{r}' d\mathbf{r} \end{aligned} \quad (2.21)$$

For two-dimensional problems, the TM polarization corresponds to the situation where the electric field is aligned with the infinite axis, and the TE polarization corresponds to the situation where the magnetic field is aligned with the infinite axis. For both polarizations, the entries of the excitation vector are computed as

$$v_m = \frac{4}{\omega \mu} \int_{\mathbf{f}_m} \mathbf{f}_m \cdot \mathbf{E}_i(\mathbf{r}) d\mathbf{r} \quad (2.22)$$

Singularity extraction

For $m \neq n$, the entries of $\mathbf{Z}$ are directly computed numerically using eqs. (2.20) and (2.21). However, for the *self-term* corresponding to $m = n$, the Green's function $H_0^{(2)}(x)$ diverges when $x \rightarrow 0$. This problem is addressed by operating a singularity extraction [1]. In one dimension, the evaluation of $z_{n,n}$ requires the computation of

$$I = \int_{x_n}^{x_{n+1}} H_0^{(2)}(k|x_0 - x'|) dx' \quad (2.23)$$

$$\begin{aligned} = & \underbrace{\int_{x_n}^{x_{n+1}} H_0^{(2)}(k|x_0 - x'|) + j \frac{2}{\pi} \log \left(\frac{k|x_0 - x'|}{2} \right) dx'}_{\text{non-singular: numerical}} \\ & - j \frac{2}{\pi} \underbrace{\int_{x_n}^{x_{n+1}} \log \left(\frac{k|x_0 - x'|}{2} \right) dx'}_{\text{singular: analytical}} \end{aligned} \quad (2.24)$$

with $x_0 \in [x_n; x_{n+1}]$. This operation corresponds to a singularity extraction, as suggested in [29, 30, 31, 32, 33]. The non-singular part in eq. (2.24) is a smooth function integrated using a numerical quadrature, whereas the singular part is computed analytically, using the indefinite integral given by

$$\int \log \left(\frac{k|x_0 - x'|}{2} \right) dx' = (x' - x_0) \log \left(\frac{k|x_0 - x'|}{2} \right) - x' + C \quad (2.25)$$

Note that the non-singular part in eq. (2.24) provides an undefined value for $x_0 = x'$. In this case, the integrand is evaluated at $x' = x_0 \pm \delta$ with $\delta \rightarrow 0$. This limit is evaluated numerically and is equal to

$$\lim_{\delta \rightarrow 0} H_0^{(2)}(k|\delta|) + j\frac{2}{\pi} \log \left(\frac{k|\delta|}{2} \right) = -0.09187 - j0.25 \quad (2.26)$$

The effect of the singularity extraction is illustrated in Figure 2.2 where the initial integrand has a strong singularity at $x' = 0$ which is properly extracted using eq. (2.24). The same technique is applied when dealing with the singularities of the derivatives of the Green's function involved in eq. (2.21). The derivatives of the Green's function and their singular parts are provided in Section 4.7.3.

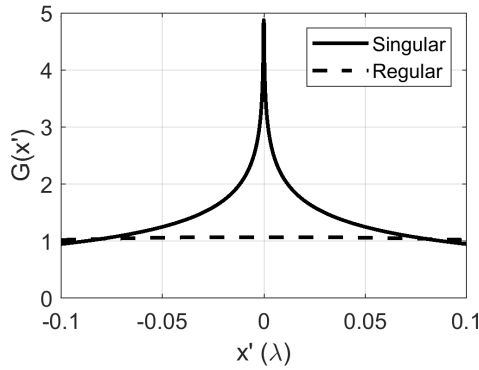


Fig. 2.2 Extraction of the singularity of the $G(\mathbf{r}', \mathbf{r})$. The solid line corresponds to $H_0^{(2)}(x')$ and the dashed line corresponds to the integrand of the first integral in eq. (2.24).

The 2D analytical singularity extraction is more accurate than the Taylor approximation used in [28]. The analytical integration of the singular part is exact. For the regularized non-singular part of the Green's function, the integration error is controlled using an appropriate quadrature rule. A classical Simpson's rule has been selected in this work, as it integrates exactly any polynomial of degree less or equal to 3 [34].

2.2.3 Iterative solvers

Obtaining the coefficients $\mathbf{i}$ in eq. (2.18) requires the computation of the inverse of the impedance matrix $\mathbf{Z}^{-1}$, of dimension $(N \times N)$, where N is the number of basis functions used for the discretization of the scatterer. Using a typical length for a basis function of $\lambda/20$ to $\lambda/10$ [1], the system of equations becomes rapidly very large for a scatterer as small as a few dozens of wavelengths. Besides the memory required for the storage of the impedance matrix (which is $\mathcal{O}(N^2)$), solving the MoM system of equations via the computation of $\mathbf{Z}^{-1}$ also leads to a large computation time (complexity $\mathcal{O}(N^3)$ using Gaussian elimination).

For large systems of equations, iterative solvers are preferred to direct solvers. In particular, Krylov subspace methods have been widely used for solving very large MoM problems. Among them, the generalized minimal residual method (GMRES) has been proposed in [35] for solving non-symmetrical linear systems of equations and has been extensively used for MoM problems. The general principle of Krylov subspace methods is to approximate the solution $\mathbf{i}$ by a vector belonging to a so-called Krylov subspace $\mathcal{K}$, built iteratively as

$$\mathcal{K}_m = \text{span}\{\mathbf{v}, \mathbf{Z}\mathbf{v}, \mathbf{Z}^2\mathbf{v}, \dots, \mathbf{Z}^{m-1}\mathbf{v}\} \quad (2.27)$$

where the vector $\mathbf{Z}^{m-1}\mathbf{v}$ is generated at the m^{th} iteration. The solution is approximated by the vector $\mathbf{i}_m \in \mathcal{K}_m$ which minimizes the norm of the residues, defined as

$$r_m = \|\mathbf{Z}\mathbf{i}_m - \mathbf{v}\| \quad (2.28)$$

Note that the vectors forming the subspace $\mathcal{K}_m$ at the m^{th} iteration may be close to being linearly dependent. An orthogonalization step is thus introduced at each iteration, via a singular value decomposition or an Arnoldi iteration [36], for example. The iterative process is stopped when r_m is smaller than an arbitrary threshold, (e.g. 10^{-4}). Note that the norm of

the residues does not correspond directly to the error on the coefficients $\mathbf{i}$ themselves.

Essentially, at each iteration of the Krylov subspace generation, a matrix-vector product is performed (see eq. (2.27)). The number of operations before convergence is of the order of $\mathcal{O}(N_{\text{iter}}N^2)$ where N_{iter} is the number of iterations. Under the condition that $N_{\text{iter}} \ll N$, the advantage of using an iterative solver versus computing the direct solution in $\mathcal{O}(N^3)$ is quite clear. The reduction of N_{iter} is presented in the next section. On the other hand, the complexity $\mathcal{O}(N^2)$ of the matrix vector product may still be a limiting factor. The complexity of this operation will be reduced with the Fast Multipole Method (FMM), described in Section 2.3.

2.2.4 Preconditioning techniques

The construction of the Krylov subspace $\mathcal{K}$ requires N_{iter} iterations, each of them involving a matrix-vector product as explained before. However it may require a large number of iterations for the iterative process to satisfactorily converge, leading to a long computation time. The number of iterations is linked to the condition number of the impedance matrix, which can be expressed as the ratio between the maximum and minimum eigenvalues of the system of equations [34]. The condition number $\kappa(\mathbf{Z})$ gives a good indicator on the convergence rate of the iterative process as [37]

$$\frac{\|\mathbf{i} - \mathbf{i}_m\|}{\|\mathbf{i}\|} \leq \kappa(\mathbf{Z}) \frac{r_m}{\|\mathbf{v}\|} \quad (2.29)$$

In order to reduce $\kappa(\mathbf{Z})$, a preconditioning matrix $\mathbf{M}$ is employed, so that the original problem is now written as

$$\mathbf{M}\mathbf{Z}\mathbf{i} = \mathbf{M}\mathbf{v} \quad (2.30)$$

where $\kappa(\mathbf{M}\mathbf{Z}) \ll \kappa(\mathbf{Z})$. The selection of an appropriate preconditioner requires $\mathbf{M}$ to be as close as possible to $\mathbf{Z}^{-1}$. This can be achieved by performing algebraic manipulations of the original system of equations. In this case, the preconditioner is more related to the properties of the impedance matrix than to the nature of the physical problem itself.

For algebraic preconditioners, the objective is to find a matrix $\mathbf{M}$ which is computed within reasonable time while improving the convergence rate of the iterative process. The block-diagonal preconditioner is a simple

sparse preconditioner whose non-zero elements are located near the matrix diagonal. This preconditioner exhibits good performance for problems with disconnected sub-domains, while it is quite inefficient for isolated scatterers. An improved version of this preconditioner is the shielded-block preconditioner proposed in [38] which is based on the close interactions between sub-domains. Another preconditioner used in this thesis is the sparse approximate inverse (SAI) preconditioner [39]. It has the following advantages:

- since it is based on a sparse matrix, $\mathbf{M}$ is computed rapidly
- it is well combined with the fast matrix-product enabled by the FMM formulation (presented in the next Section 2.3).

The sparse preconditioning matrix $\mathbf{M}$ is obtained by minimizing the Frobenius norm

$$\|\mathbf{I} - \mathbf{MZ}\|_F^2 \quad (2.31)$$

where the sparsity pattern of $\mathbf{M}$ is defined *a priori*, based on the largest entries of $\mathbf{Z}$ or the closest interactions.

2.3 Multipoles decomposition and the Fast Multipole Method

As stated in Section 2.2.3, the solution of large MoM systems of equations is obtained as a vector belonging to a Krylov subspace. The construction of the latter involves a matrix-vector product at each iteration with a complexity $\mathcal{O}(N^2)$. This operation becomes costly for large problems, so that one is looking for an acceleration of this matrix-vector product. Besides, the computation of the impedance matrix itself becomes cumbersome in terms of computation time and memory space required for the matrix storage. Both the reduction of the complexity of the matrix-vector product and the accelerated computation of $\mathbf{Z}$ are addressed using the Fast Multipole Method (FMM), which is based on a multipole decomposition of the Green's function and the diagonalization of the operator relating the source and testing sub-domains. On one hand, the computation of the reaction integrals between two sub-domains is expressed in a compact form so that the full impedance matrix is never computed explicitly. On the other hand,

this formulation accelerates the matrix-vector products by computing intermediate quantities, as detailed in Sections 2.3.2 and 2.3.3.

Let us consider two sub-domains S and S' illustrated in Figure 2.3. For illustration, the sub-domains are disconnected and discretized with N square basis functions identical for S and S' . The reaction integral I between the source and the test domains involved in eqs. (2.20) and (2.21) is defined as

$$I = \int_S \int_{S'} \mathbf{I}_t(\mathbf{r}) \cdot \mathbf{I}_s(\mathbf{r}') G(k|\mathbf{r}' - \mathbf{r}|) d\mathbf{r} d\mathbf{r}' \quad (2.32)$$

where $\mathbf{I}_t(\mathbf{r})$ and $\mathbf{I}_s(\mathbf{r}')$ are the current distributions on the test group S and the source group S' , respectively. The vectors $\mathbf{r}' \in S$ and $\mathbf{r} \in S'$ correspond to the position of the basis and testing functions on both domains and $G(k|\mathbf{r}' - \mathbf{r}|)$ is the two-dimensional Green's function for the Helmholtz equation (eq. (2.8)).

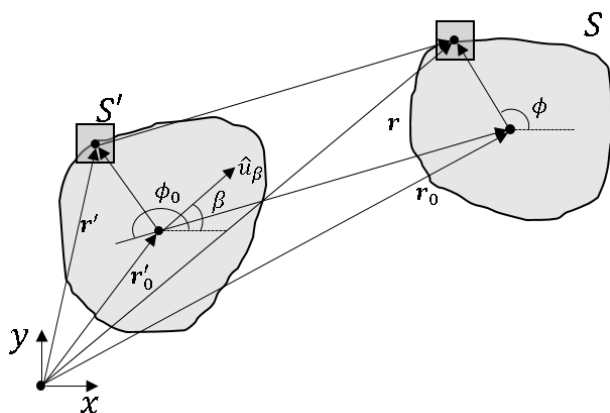


Fig. 2.3 Variables associated to the FMM formulation for the interactions between the sub-domains S and S' .

The key idea of the Fast Multipole Method (FMM) introduced in the 90's [40, 25] is to compute the interactions between *groups* of basis and testing functions, instead of individual pairs of a single basis and a single testing function. Mathematically, the grouping of currents sources is obtained thanks to the addition theorem for the cylindrical waves [34], which expresses the field radiated at $\mathbf{r}$ by a source located at $\mathbf{r}'_0$ as if the source was

2 | Multipoles and inhomogeneous plane waves for 2D scenarios

located at $\mathbf{r}_0$. This is expressed mathematically as

$$H_0^{(2)}(k|\mathbf{r}'_0 - \mathbf{r}|) = \sum_{l=0}^{\infty} H_l^{(2)}(k|\mathbf{r}'_0 - \mathbf{r}_0|) \cos(l(\phi - \phi_0)) \epsilon(l) J_l(k|\mathbf{r}_0 - \mathbf{r}|) \quad (2.33)$$

$$\epsilon(l) = \begin{cases} 1 & \text{if } l = 0 \\ 2 & \text{if } l > 0 \end{cases} \quad (2.34)$$

where the cosine term in eq. (2.33) is due to the summation starting at $l = 0$ instead of $l = -\infty$. Using the integral representation of the Bessel's function, the last terms in eq. (2.33) are expressed as

$$\cos(l(\phi - \phi_0)) J_l(k|\mathbf{r}_0 - \mathbf{r}|) = \frac{(-j)^l}{2\pi} \int_0^{2\pi} \cos(l(\beta - \phi_0)) e^{jk\hat{\mathbf{u}}_\beta \cdot (\mathbf{r} - \mathbf{r}_0)} d\beta \quad (2.35)$$

By combining eqs. (2.33) and (2.35), the Green's function between an offset source $\mathbf{r}'$ and a testing point $\mathbf{r}$ is expressed as

$$H_0^{(2)}(k|\mathbf{r}' - \mathbf{r}|) = \frac{1}{2\pi} \int_0^{2\pi} \underbrace{e^{-jk\hat{\mathbf{u}}_\beta \cdot (\mathbf{r} - \mathbf{r}_0)}}_{\text{disaggregation}} d\beta \quad (2.36)$$

$$\underbrace{\sum_{l=0}^L c(l) \cos(l(\beta - \phi_0)) H_l^{(2)}(k|\mathbf{r}'_0 - \mathbf{r}_0|)}_{\text{translation}} \cdot \quad (2.37)$$

$$\underbrace{e^{jk\hat{\mathbf{u}}_\beta \cdot (\mathbf{r}' - \mathbf{r}'_0)}}_{\text{aggregation}} d\beta \quad (2.38)$$

where

$$c(l) = \begin{cases} 1 & \text{if } l = 0 \\ 2(-j)^l & \text{if } l > 0 \end{cases} \quad (2.39)$$

In the above formulation, the interaction between $\mathbf{r}'$ and $\mathbf{r}$ is performed in three stages: first, an *aggregation* of the fields from the source $\mathbf{r}'$ to a nearby central point $\mathbf{r}'_0$; the *translation* step then propagates the information from $\mathbf{r}'_0$ towards a point $\mathbf{r}_0$ near the test point $\mathbf{r}$; the field is finally tested on $\mathbf{r}$ during the *disaggregation*. Instead of computing the interactions between every pair of source and testing functions, the multipole expansion computes the interaction between the centers of the groups containing the source functions and the testing functions ($\mathbf{r}'_0$ and $\mathbf{r}_0$, respectively). The advantage is

that the translation operator is identical for all the source and testing functions in the corresponding groups. Note that the translation operator can be efficiently computed with a recursion formula [41] using

$$H_l^{(2)}(x) = 2H_{l-1}^{(2)}(x)\frac{l-1}{2} - H_{l-2}^{(2)}(x) \quad (2.40)$$

2.3.1 Number of multipoles

One can observe that the infinite sum in eq. (2.33) has been truncated to a finite number L of multipoles in eq. (2.37) as the translation operator series is convergent. In [42], the number L of terms is derived as

$$L = kD + C(kD)^{1/3} \quad (2.41)$$

where D is the diameter of the smallest circle encompassing each sub-domain individually, and C is a value depending on the desired accuracy. In terms of γ , the number of digits of accuracy, the above formula has been refined in [43] as

$$L = kD + 1.8\gamma^{2/3}(kD)^{1/3} \quad (2.42)$$

For small values of kD , the authors in [44] propose a new estimator of the number of multipoles which also depends on the minimal distance between the centers of two groups. Note that the Hankel function $H_n^{(2)}(x)$ diverges for a fixed x and an increasing n . For this reason, the number of multipoles is limited to the value L obtained with eq. (2.41) or eq. (2.42) and $k|\mathbf{r}_0 - \mathbf{r}'_0|$, the minimal distance between the center of two groups [1].

2.3.2 Integration within a Krylov iteration

Let us consider the scattering problem depicted in Figure 2.4. The contour is discretized by using N pulse basis functions grouped in M sub-domains numbered $1 \cdots M$, with each sub-domain containing $P \approx N/M$ basis functions. All the sub-domains have the same side length d and are contained in a circle of diameter $D = \sqrt{2}d$. Sub-domains whose circles do not intersect are considered as *far interactions*; otherwise, they are considered as *near interactions*. The latter condition is satisfied for interactions within sub-domains and for interactions between directly neighboring sub-domains. The near interactions for this particular problem are represented in a ma-

trix form in Figure 2.5, where the gray entries correspond to the *near interactions*.

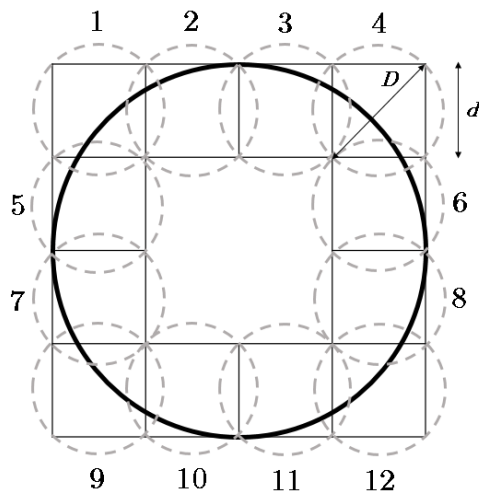


Fig. 2.4 Example of a FMM domain decomposition. The scatterer is represented with a black solid line. The FMM groups correspond to circles (gray dashed line) with a diameter D .

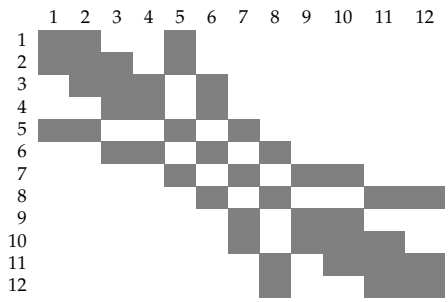


Fig. 2.5 Near interaction matrix for the problem in Figure 2.4.

As stated in Section 2.2.3, a matrix vector product is performed at each iteration of the iterative solver. Using the FMM formulation for the problem described in Figure 2.4, this product is explicitly written as

$$\mathbf{Z}\mathbf{v} \approx \begin{pmatrix} \mathbf{Z}_{1,1} & \mathbf{Z}_{1,2} & \tilde{\mathbf{P}}_1^T \tilde{\mathbf{T}}_{1,3} \tilde{\mathbf{P}}_3 & \cdots & \tilde{\mathbf{P}}_1^T \tilde{\mathbf{T}}_{1,12} \tilde{\mathbf{P}}_{12} \\ \mathbf{Z}_{2,1} & \mathbf{Z}_{2,2} & \mathbf{Z}_{2,3} & \cdots & \tilde{\mathbf{P}}_2^T \tilde{\mathbf{T}}_{2,12} \tilde{\mathbf{P}}_{12} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{P}}_{12}^T \tilde{\mathbf{T}}_{12,1} \tilde{\mathbf{P}}_1 & \tilde{\mathbf{P}}_{12}^T \tilde{\mathbf{T}}_{12,2} \tilde{\mathbf{P}}_2 & \tilde{\mathbf{P}}_{12}^T \tilde{\mathbf{T}}_{12,3} \tilde{\mathbf{P}}_3 & \cdots & \mathbf{Z}_{12,12} \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \\ \mathbf{v}_{12} \end{pmatrix} \quad (2.43)$$

where the near interactions entries $(\mathbf{Z}_{1,1}, \mathbf{Z}_{1,2}, \dots)$ correspond to the gray cells in Figure 2.5, $\tilde{\mathbf{P}}$ is the radiation pattern of the basis functions of the sub-domains and $\tilde{\mathbf{T}}$ is the diagonal translation operator between the center of the sub-domains.. It is obvious that the matrix in eq. (2.43) is never computed explicitly, as that would not bring any advantage regarding the computational cost of the matrix vector product. Instead, $\mathbf{Z}$ is decomposed in two matrices $\mathbf{Z}^{\text{near}}$ and $\mathbf{Z}^{\text{far}}$ (with $\mathbf{Z} = \mathbf{Z}^{\text{near}} + \mathbf{Z}^{\text{far}}$), where $\mathbf{Z}^{\text{near}}$ has non-zero entries for the near-interactions, and the $\mathbf{Z}^{\text{far}}$ elements are given as

$$\mathbf{Z}_{m,n}^{\text{far}} = \tilde{\mathbf{P}}_m^T \tilde{\mathbf{T}}_{m,n} \tilde{\mathbf{P}}_n \quad (2.44)$$

$$= \underbrace{\begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}}_{(P \times N_\alpha)} \underbrace{\begin{pmatrix} \bullet & & \\ & \bullet & \\ & & \bullet \end{pmatrix}}_{N_\alpha \times N_\alpha} \underbrace{\begin{pmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{pmatrix}}_{N_\alpha \times P} \quad (2.45)$$

where N_α is the number of sample directions on the unit circle. The matrix-vector product is equivalently written as

$$\mathbf{Z}\mathbf{v} = \mathbf{Z}^{\text{near}}\mathbf{v} + \mathbf{Z}^{\text{far}}\mathbf{v} \quad (2.46)$$

The product of the sparse matrix $\mathbf{Z}^{\text{near}}$ with the excitation vector $\mathbf{v}$ is computed classically. For the far interactions, the aggregation step $\tilde{\mathbf{P}}_i \mathbf{v}_i$ is performed on each sub-domain and is stored in memory. For a given sub-domain, the aggregated fields from all the other far domains are translated with the diagonalized matrix, before disaggregation. The algorithm of this fast matrix-vector multiplication is presented in algorithm 1, using

2 | Multipoles and inhomogeneous plane waves for 2D scenarios

a block-diagonal preconditioner $\mathbf{M}$ whose entries are

$$\begin{aligned} \mathbf{M}_{i,i} &= \mathbf{Z}_{i,i}^{-1} \\ \mathbf{M}_{i,j \neq i} &= \mathbf{0} \end{aligned} \quad (2.47)$$

```

input : Near interaction matrix  $\mathbf{Z}^{\text{near}}$ , FMM operators  $\tilde{\mathbf{P}}$  and  $\tilde{\mathbf{T}}$ ,
        excitation vector  $\mathbf{v}$ 
output: Rapid computation of  $\mathbf{c} = (\mathbf{M}\mathbf{Z})(\mathbf{M}\mathbf{v})$ 
1 Compute the preconditioned excitation vector  $\mathbf{v}^P$ 
2 for  $i = 1 \dots M$  do
3   |  $\mathbf{v}_i^P = \mathbf{M}_{i,i} \mathbf{v}_i$ 
4 end
5 Compute and store the aggregated fields  $\mathbf{V}$ 
6 for  $i = 1 \dots M$  do
7   |  $\mathbf{V}_i = \tilde{\mathbf{P}}_i \mathbf{v}_i^P$ 
8 end
9 Compute the translation and the disaggregation
10 for  $i = 1 \dots M$  do
11   |  $\mathbf{B} = \mathbf{0}$ ;
12   |  $\mathbf{c}_i = \mathbf{0}$ ;
13   | for  $j = 1 \dots M$  do
14     | if  $\mathbf{Z}_{i,j}$  is a far group then
15       | |  $\mathbf{B} = \mathbf{B} + \tilde{\mathbf{T}}_{i,j} \mathbf{V}_j$ ;
16     | else
17       | |  $\mathbf{c}_i = \mathbf{c}_i + \mathbf{Z}_{i,j}^{\text{near}} \mathbf{v}_j^P$ ;
18     | end
19   | end
20   |  $\mathbf{c}_i = \mathbf{c}_i + \tilde{\mathbf{P}}_i^T \mathbf{B}$ ;
21 end
22 Apply the block-diagonal preconditioner "on-the-fly"
23 for  $i = 1 \dots M$  do
24   |  $\mathbf{c}_i = \mathbf{M}_{i,i} \mathbf{c}_i$ 
25 end

```

Algorithm 1: Fast computation of the matrix vector product using the FMM formulation and a block-diagonal preconditioner.

2.3.3 Computational complexity

The computational complexity of the fast matrix-vector product presented in algorithm 1 is investigated. Using this algorithm, the computational cost of the aggregation step T_1 , the translation step T_2 and the disaggregation T_3 are

$$T_1 = C_1 MP^2 \quad (2.48)$$

$$T_2 = C_2 M^2 P \quad (2.49)$$

$$T_3 = C_3 MP^2 \quad (2.50)$$

where C_1, C_2, C_3 are platform and language dependent. In the above expressions, it is assumed that the number of sampled directions N_α on the unit circle depends on the size of the groups [42], so that $N_\alpha = P$. The computational cost associated with the multiplication of the near interactions blocks is neglected as the number of near groups is significantly smaller than the far interaction groups (see Figure 2.5). The total computational cost of the matrix-vector product is thus

$$T = C_2 M^2 P + (C_1 + C_3) MP^2 \quad (2.51)$$

$$T = C_2 MN + (C_1 + C_3) \frac{N^2}{M} \quad (2.52)$$

where the equality $P = N/M$ was used. The value of M^{opt} which minimizes T is then expressed as

$$\frac{\partial}{\partial M} T = 0 \quad (2.53)$$

$$M^{\text{opt}} = \sqrt{\frac{C_1 + C_3}{C_2}} N^{0.5} \quad (2.54)$$

In other words, the number of groups M should be equal to the number of basis functions P in each group. The total computation time is finally expressed as

$$T^{\text{opt}} = 2\sqrt{C_2}\sqrt{C_1 + C_3}N^{1.5} \quad (2.55)$$

$$= \mathcal{O}(N^{1.5}) \quad (2.56)$$

The decomposition of the radiated and received fields in multipoles, combined with the diagonalization of the translation operator has reduced

2 | Multipoles and inhomogeneous plane waves for 2D scenarios

the complexity of the matrix-vector product from $\mathcal{O}(N^2)$ to $\mathcal{O}(N^{1.5})$. Besides the reduction of the computational complexity, the FMM formulation also dramatically reduces the memory requirements. As stated before, the impedance matrix in eq. (2.43) is never computed explicitly, as it may not fit in the RAM¹. Instead, the radiation patterns of the M groups for the aggregation and disaggregation steps are computed and stored. The same applies for the diagonal translation operator which is stored as a vector containing the diagonal values of $\tilde{\mathbf{T}}$. The memory requirements Mem^{tot} for the pre-computations in eq. (2.43) are

$$\text{Mem}^{\text{tot}} = MN + M \left(\frac{N}{M} - B \right)^2 + B^2 M^2 \quad (2.57)$$

where it was assumed that $N_\alpha = M$ and B is the mean number of close neighbors per sub-domain (i.e. the mean number of brute-force blocks per line of the FMM matrix). Using a single box buffer, this number is close to 9. By using the optimal group size $M = \sqrt{N}$ for the matrix-vector product, the memory space is

$$\text{Mem}^{\text{tot}} = N^{1.5} + N^{0.5} \left(N^{0.5} + B \right)^2 + B^2 N \quad (2.58)$$

$$= \mathcal{O}(N^{1.5}) \quad (2.59)$$

The gain in memory is the same as the gain in the computation time for the matrix vector product and is also reduced from $\mathcal{O}(N^2)$ to $\mathcal{O}(N^{1.5})$.

2.4 Inhomogeneous plane wave decomposition for distant sub-domains

2.4.1 Spectral representation of the Green's function

For scattering problems for which the testing sub-domain is visible from the source sub-domain through a relatively small angular sector, the representation of the scattered field using a decomposition into inhomogeneous plane waves is particularly efficient. This condition is met for sub-domains that are small with respect to the distance between them. The method enabling the fast computation of the reaction integral using the decomposi-

¹For example, a MoM problem of $25 \cdot 10^3$ unknowns requires approximately 10 GB of memory for complex numbers encoded on 64 bits.

tion of the Green's function into homogeneous plane waves is fully described in [26, 7, 19] and is summarized here below.

The method relies on the spectral representation of the two-dimensional Green's function

$$G(\rho) = \frac{H_0^{(2)}(k\rho)}{4j} = \int_{-\infty}^{\infty} \frac{e^{-jk_x x} e^{-jk_y |y|}}{4j\pi k_y} dk_x \quad (2.60)$$

with $\rho = \sqrt{x^2 + y^2}$ and $k_y = \sqrt{k^2 - k_x^2}$. The integral in eq. (2.60) is efficiently computed using the steepest-descent path, which expresses the spectral component as

$$k_x = k_{x\mathcal{R}} + j \frac{k_{x\mathcal{R}}}{\sqrt{1 + \left(\frac{k_{x\mathcal{R}}}{k}\right)^2}} \quad (2.61)$$

$$k_y = k - j \frac{k_{x\mathcal{R}}^2}{k \sqrt{1 + \left(\frac{k_{x\mathcal{R}}}{k}\right)^2}} \quad (2.62)$$

where the index $\mathcal{R}$ corresponds to the real part of the spectral coordinates. Using this integration path, the integral in eq. (2.60) is approximated by

$$\frac{H_0^{(2)}(k\rho)}{4j} \simeq \sum_{l=1}^L \underbrace{\frac{\Delta k_{x\mathcal{R}}}{4\pi j k_y^l} \left(1 + j \frac{k^3}{(k^2 + k_{x\mathcal{R}}^l)^{3/2}}\right)}_{d^l} e^{-j(k_x^l x + k_y^l y)} \quad (2.63)$$

where L is the number of samples in the complex plane. By aligning the y axis with the along the center of the source and testing groups, by choosing the reference of the coordinate system as the center of the source groups and by noting Δy the distance between the center of two sub-domains, the previous equation now reads

$$\frac{H_0^{(2)}(k\rho)}{4j} \simeq \sum_{l=1}^L \underbrace{d_l e^{-j k_y^l \Delta y}}_{D_l} e^{-j(k_x^l (x-x') + k_y^l (y-y'))} \quad (2.64)$$

where the primed variables correspond to the source domain, and the non-primed variables to the test domain. In this formulation, k_x and k_y are complex, so that the Green's function is formulated as a sum of propa-

gating and evanescent waves in the direction of propagation. As for the multipole decomposition, the inhomogeneous plane waves representation produces a separable formulation of the Green's function, where the FMM formulation uses only propagating plane waves.

The authors of [19] propose a harmonic decomposition of the radiation patterns of the source and testing groups, so that the reaction integral I of eq. (2.32) induced by the currents $\mathbf{i}$ on the source sub-domain now reads

$$I = \sum_n \beta_n \sum_l D_l (-jk_{o,l})^n \sum_{n'} \beta_{n'}(\mathbf{i}_l^n) (jk_{o,l})^{n'} \quad (2.65)$$

where $\beta_{n'}(\mathbf{i}_l^n)$ is the Fourier transform of the radiation pattern on the source sub-domain and β_n is the Fourier transform of the radiation pattern on the testing group using the testing currents distribution. The variable k_o is a normalized wave number which depends on the angle between the sub-domains and the complex values k_x, k_y corresponding to the L inhomogeneous plane waves. All the details regarding those variables are found in [19]. The parameter L corresponds to the number of plane waves required to reach a chosen error level.

2.4.2 Number of inhomogeneous plane waves

The sampling period Δ_σ over the spectral domain for a desired error level ϵ is expressed for the two-dimensional case in [19] as

$$\Delta_\sigma = \frac{2\pi}{\sqrt{\pi \frac{x}{\lambda} \frac{x}{y}} + \sqrt{\pi \frac{x}{\lambda} \frac{x}{y} - 2 \log \frac{\epsilon}{2}}} \frac{1}{\sqrt{\pi \frac{1}{\lambda y}}} \quad (2.66)$$

where x is the width of the sub-domain and y is the distance between sub-domains. The number L of plane waves required for reaching the targeted error is extrapolated from [19] and is approximated as

$$L \approx 12 \Delta_\sigma^{-2.22} \quad (2.67)$$

Similar derivation in three dimensions can be found in [45]. A large sampling period Δ_σ leads to a small number L of plane waves. From eq. (2.66), this condition is fulfilled when

- the dimensions of the domains x/λ is small

- the distance between the domains is large
- one sub-domain is visible from another one through a small angular sector.

If those conditions are met, the number of inhomogeneous plane waves required for the accurate evaluation of the reaction integral may become much smaller than the number of plane waves required by the multipole decomposition. For comparison, the number of plane waves required by both methods is illustrated in Figure 2.6 for the same accuracy $\epsilon = 10^{-4}$, as a function of the frequency and for circular sub-domains of radius $R = 2$ m. The distance between sub-domains is expressed as the angle from which the sub-domains are visible from each other (IPW 1° , IPW 2° and IPW 4° in Figure 2.6).

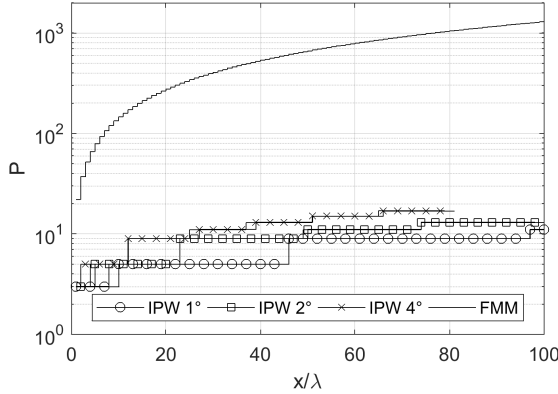


Fig. 2.6 Number of (inhomogeneous) planes waves with respect to the sub-domains size and distance. The target error is $\epsilon = 10^{-4}$.

As a comparison, the number of plane waves required by a multipole representation is also plotted with a black solid line on the same figure. Using this formulation, the number of plane waves used in the translation operator is not impacted by the distance between the sub-domains. However, it grows very fast with the electrical size of the sub-domain. Note that from eq. (2.66), the number of inhomogeneous plane waves also increases with the sub-domain dimensions. However, in certain cases, the number of inhomogeneous plane waves can be significantly smaller than the number of

plane waves in the FMM formulation. This is illustrated in Figure 2.6, the number of multipoles is obtained with eq. (2.42). In the proposed example, the number of inhomogeneous plane waves is significantly smaller than the number of plane waves required by the FMM approach. As suggested in [19], when the sub-domains are distant, the IPW approach outperforms the fast multipole method. Note that for sub-domains visible through a small angular sector, the matrix-vector product involved in a FMM context can be reduced to a complexity $\mathcal{O}(N^{4/3})$ using the Ray-Propagation Fast Multipole Algorithm (RPFMA) proposed in [46], which truncates the translation operator around the direction of propagation between the sub-domains.

2.5 Applications

The acceleration of the radiation integrals enabled by the FMM and the inhomogeneous plane wave decomposition are employed for analyzing the impact of wind turbines on radar systems, with an emphasis in the forward region. Due to their large dimensions and the rotation of their blades, they generate significant interference on radar signals used for weather prediction [47, 48, 49] and on telecommunication systems [50], among others. Due to their large electrical size, the number of unknowns required for an accurate representation of the geometry can reach tens of millions of basis functions. Moreover, the instantaneous silhouette of the wind turbine as observed by the radar is dictated by three degrees of freedom (blade position, orientation with respect to the radar and the pitch of the blades) thus leading to a large number of combinations.

The geometry complexity of a wind turbine is reduced in this work under some hypotheses :

- The radar return from the blades is neglected. It is thus assumed that the radar return is only due to the isolated mast. The signal power reflected by the blades varies within orders of magnitude depending on their orientation with respect to the radar [51], which is not the case for the mast. For this reason, we make the assumption that, on average (i.e. for all the orientations of the wind turbine with respect to the radar), the radar return is only due to the isolated mast.
- The wind turbine is considered as an infinite cylinder. As the geometry is strongly elongated in the vertical direction, it seems reasonable

to make this assumption. This has been validated in [9], where the error on the scattered fields induced by the infinite cylinder assumption is less than 1%.

- A wind turbine is made of PEC material. This is a good approximation as wind turbines are usually supported by cylindrical or conical tubular steel towers.

2.5.1 Computation of the scattering by a wind farm

The first example consists of the rapid computation of the currents on the mast of various wind turbines within a wind farm. Once those currents are known, the scattered field is obtained through the radiation integral in eq. (2.10). Using the above-mentioned simplifications, the original problem now corresponds to the scattering of N_{WT} irregularly disposed circles under a plane wave illumination. The electric field of the plane wave is aligned with the infinite axis of the cylinders (TM incidence). The wind turbines are represented by circles of radius $R = 2$ m, which is representative for the mast of a typical wind turbine.

The computation of the currents is accelerated with the FMM. It is quite natural to consider each wind turbine as an isolated FMM group. By considering identical wind turbines, only one radiation pattern $\mathbf{P}_{dis}$ is computed for the disaggregation operator using eq. (2.36). The aggregation operator is simply the complex conjugate of the latter $\mathbf{P}_{agr} = \mathbf{P}_{dis}^*$ when using Galerkin's testing. The $N_{WT}(N_{WT} - 1)$ translation vectors are computed and stored in memory using the formulation in eq. (2.37). Under the assumption of identical wind turbines, a single self-impedance term has to be computed and stored. Since the sub-domains are identical and isolated, the block-diagonal preconditioner is a good choice when the distance between them is not too small. The preconditioning matrix $\mathbf{M}$ is thus given by eq. (2.47). Note that the preconditioning is applied "on-the-fly" as outlined in algorithm 1 line 23 in order to avoid an explicit matrix multiplication between $\mathbf{M}$ and $\mathbf{Z}$ which would remove the computational savings of the method.

The convergence of the iterative solver is compared for different minimal distances between the wind turbines, at different frequencies. Using the block diagonal preconditioner, the initial currents on the wind turbines are equal to the currents on an isolated element. When the distance between the sub-domains is increased, the mutual coupling is reduced, so

that the currents on an isolated wind turbine are not much impacted by the presence of neighboring wind turbines. This explains why the number of iterations is smaller when the minimal distance increases, as observed in Figure 2.7.

The computation time as well as the required memory are given in Table 2.1. For a given excitation, the currents on the eight wind turbines are computed in a few dozens of seconds, including the mutual coupling. As the sub-domains are identical, the required memory space is very small with respect to the brute-force approach (approximately a ratio 1/50). For a given frequency, the computation time is smaller for distant wind turbines, as the number of iterations is reduced (see Figure 2.7). The computation time increases at higher frequencies, as the number of unknowns increases.

Note that the memory required for the FMM is proportional to N^2 . In this particular case, this value is mostly driven by the unique self-impedance matrix which grows as N^2 , whereas the memory for the brute-force impedance matrix grows as $(NN_{WT})^2$.

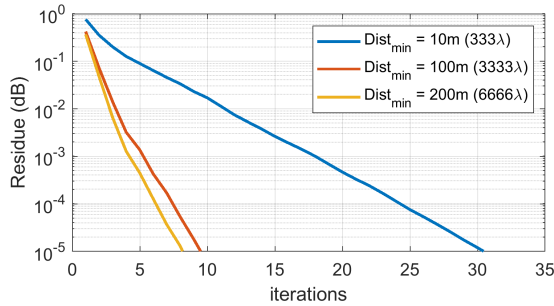


Fig. 2.7 Convergence of the FMM iterative solver at 10 GHz for different minimal distances between each wind turbine.

The normalized signal power inside the wind farm is represented² in Figures 2.8a and 2.8b for a carrier frequency of 2 GHz and 10 GHz, respectively. The spatial sampling is regular and taken every 0.1 m. From this result, one can observe that for the highest frequency, the total field behind

²For illustration purposes, the distance between wind turbines is chosen very small (10 m), which is obviously not a configuration met in practice.

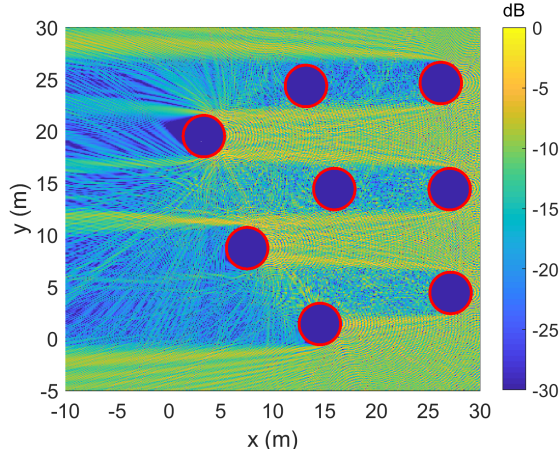
Table 2.1 Summary of the computation time and memory requirement for the iterative solver accelerated with FMM, for the scenario represented in Figure 2.8b. The value with a "*" is an estimated value. The iterative process is stopped when the norm of the residues is smaller than 10^{-5} .

Freq. (GHz)	NN_{WT}	time (s)			$M_{FMM}(GB)$	$M_{BF}(GB)$
		10 m	100 m	200 m		
2	$1 \cdot 10^4$	6	2	2	0.04	1.5
6	$3 \cdot 10^4$	89	28	25	0.36	13.5
10	$5 \cdot 10^4$	357	121	111	1.01	37.5*

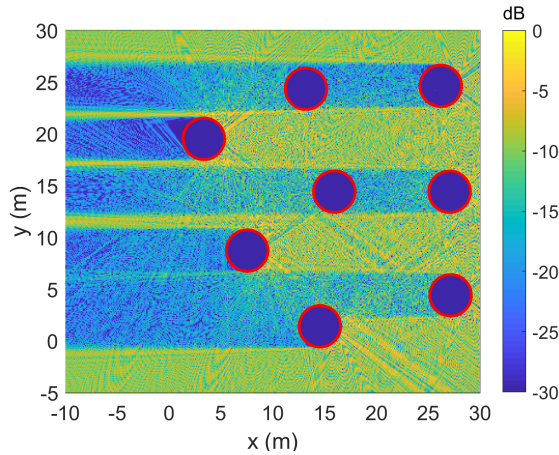
a wind turbine follows a quasi-optical blockage, where a strong attenuation is observed behind each wind turbine. The solution of the iterative solver combined with the FMM is very accurate, as it respects the boundary conditions for a PEC scatterer. From the extinction theorem [12], by setting the excitation (in this case, the incident plane wave) to zero, the fields radiated by the equivalent currents inside the wind turbines should have the same amplitude and the opposite phase of the incident field. For a PEC material, the sum of the scattered fields and the incident fields should be equal to zero. In the examples in Figure 2.8, the amplitude of the total field is 60 dB lower than the amplitude of the incident plane wave, thus validating the accuracy of the proposed method.

2.5.2 Fast computation of the scattered fields

As observed in Figure 2.8, the presence of wind turbines creates a shadow in the direction of propagation of the incident wave. The amount of power available behind a wind farm is a quantity of interest for radar applications, as it gives an insight on the probability of detection of a target located behind a wind farm [52]. The direct computation of the scattered field requires the computation of a radiation integral (similar to eq. (2.32)) for each current basis function located on a wind turbine and each position for which the field is estimated (testing with Dirac function). The complexity is thus of the order of $\mathcal{O}(NN_sN_{WT})$, with N_s the number of locations for which the scattered field is evaluated. This leads to an excessive computation time when estimating the fields over a large area.



(a) 2 GHz.



(b) 10 GHz.

Fig. 2.8 Normalized total power inside the wind farm for two different frequencies. For higher frequencies, the blockage becomes similar to an optical blockage (i.e. the shadow is located right behind the scatterer and aligned with the propagation vector).

This computation is accelerated with the help of the inhomogeneous plane wave decomposition. The sources are naturally decomposed in groups corresponding to each individual wind turbine. The testing domain is decomposed into square groups whose dimensions are typically of the order of the dimensions of the source groups. The reaction integral of eq. (2.65) is then computed for each pair of source and testing domains. For each wind turbine, the harmonic coefficients $\beta_{n'}(\mathbf{i}_i'')$ of the radiation pattern of the source domain are computed. When considering identical testing sub-domains, a unique set of harmonics β_n is computed. As suggested in eq. (2.66), when the test domain is located far away from the source, a very small number of inhomogeneous plane waves is required for an accurate representation of the scattered field. Given that this number decreases when the distance between sub-domains increases, the fast method in [19] is a good candidate for the computation of the field over a large region away from the wind farm.

However, when one is interested in estimating the fields near the wind turbines (or inside the wind farm, for example), the inhomogeneous plane wave representation would require a very large number of plane waves. In this particular case, it is a better choice to use the FMM, as the number of multipoles only depends on the group size and is independent from the distance between sub-domains. As for the IPW, the FMM testing domain is divided into identical square groups, so that a single disaggregation operator is computed with eq. (2.36).

The total field behind a wind farm made of ten wind turbines with a minimal distance of 100 m is represented in Figures 2.9a and 2.9b for a plane wave excitation propagation towards $-\hat{x}$ at 2 GHz and 6 GHz. For a better visibility, the positions of the wind turbines are illustrated in Figure 2.9c. The shadows behind the wind farm are clearly visible at both frequencies, with a deeper shadow at higher frequencies. The total fields have been obtained with the IPW approach with testing domains of dimensions $10 \times 10 \text{ m}^2$ with a uniform spatial sampling every 0.5 m. The targeted error is $\epsilon = 10^{-4}$ and requires less than 100 inhomogeneous plane waves for computing the interaction between any pair of wind turbine and testing group.

For a better visibility, the total power is plotted at different x positions in Figure 2.10 for both frequencies. For a distance of 100 m, the transmitted signal is attenuated up to 20 dB at 6 GHz, whereas the attenuation is limited to 10 dB at 2 GHz. The shadowing induced by the wind turbines is

still clearly visible at 1 km behind the wind farm. After 5 km, some fluctuations of the total power are visible, where the oscillations are more rapid for higher frequencies. Note that the slope of the attenuation with respect to the y position is steeper at higher frequencies. This illustrates the quasi-optical obstruction when the scatterer is large with respect to the wavelength, as already observed in Figure 2.8.

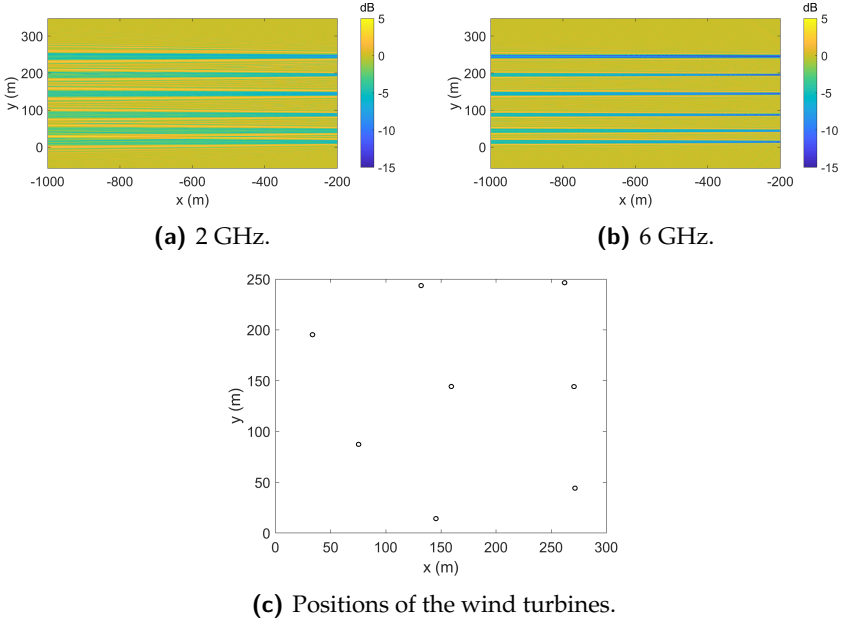


Fig. 2.9 Total power behind the wind farm.

2.5.3 Localization of the radar echoes behind a wind farm

The third illustration consists of the estimation of the false radar echoes in the presence of a wind farm. The multiple reflections between the wind turbines increase the radar clutter, leading to false target detection in the region behind the wind farm [53, 47, 48]. In the radar map, echoes appear behind the wind farm, as a result of the longer distance by each bounce between pairs of wind turbines. The erroneous targets are spread in the azimuth range due to the radiation pattern of the radar and its 360° rotation. Due to the rotation of the radar, a target is previously detected though the sidelobes of the radar with a small power before entering in the main

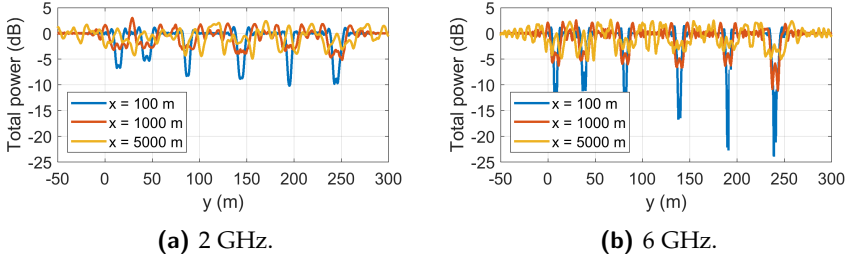


Fig. 2.10 Total power behind the wind farm for different distances.

beam; the power reaches its maximal value when the radar is pointing directly towards the target and decreases when the target leaves the main beam.

This phenomenon is illustrated in Figure 2.11 for different time instants corresponding to different radar orientations. As an example, only three wind turbines are considered here. In Figure 2.11a only the wind turbine located at $\mathbf{r}_1$ is visible through the radar main beam. The detected echoes are the first order echo of the first wind turbine as well as the third order echoes corresponding to the following multiples bounces, with a smaller power:

- radar $\rightarrow$ WT₁ $\rightarrow$ WT₂ $\rightarrow$ WT₁ $\rightarrow$ radar
- radar $\rightarrow$ WT₁ $\rightarrow$ WT₃ $\rightarrow$ WT₁ $\rightarrow$ radar

The other reflections exists with a very smaller associated power and are therefore neglected. For example, even though the path radar $\rightarrow$ WT₁ $\rightarrow$ WT₃ $\rightarrow$ radar is only a second-order reflection, WT₁ is illuminated through a side lobe and the signal received from WT₃ reaches the radar through a sidelobe as well. In Figure 2.11b, the three wind turbines are illuminated by the radar. As a consequence, more multipath components reach the receiver with increasing radial distances and decreasing power. The width of the echoes in the azimuth range depends on the radar beam width, while the width in the radial direction is related to the bandwidth of the transmitted signal. Note that in this work, we do not consider the wave interactions between the wind turbines and other scatterers located in the sky (e.g. precipitation cells or aircraft). These interactions also create false echoes, as outlined in [54].

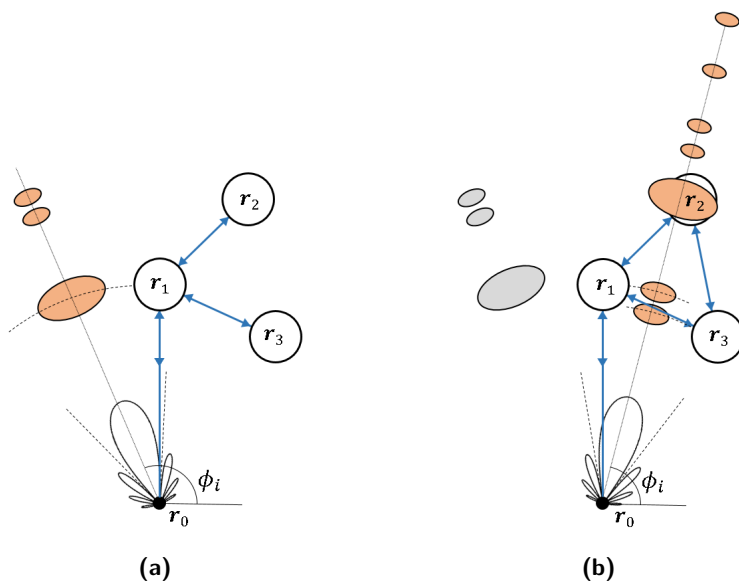


Fig. 2.11 Layout of the false echoes for radar orientations. The detected targets are represented by the orange ellipses, whose dimensions represent the power of the echo.

The radar echoes induced by the multiple reflections are modeled using a full-wave approach, where the currents induced by each reflection are computed using the method of moments in two dimensions. This standard MoM formulation gives information on the field scattered by the whole wind farm but does not provide any insight on the power associated to individual multipath components. Hence, the classical method of moments formulation cannot model the location and strength of the false radar echoes.

In order to model those, each multipath component (or "bounce") is computed successively. The method is detailed in algorithm 2 for a maximum of three bounces, although it can be trivially extended to n bounces. The update of the currents on a given sub-domain can be seen as the update process involved in a stationary iterative process such as the Gauss-Siedel algorithm [37], where the currents at the considered sub-domain at the iteration $i + 1$ depends on the currents on the neighboring sub-domains at the iteration i .

The computation of the first order excitation vector (line 4) for a given radar orientation ϕ_i on the wind turbine j is expressed as

$$v_{i,j}^1[n] = C j k \eta \int_{\text{WT}_j} G(\mathbf{r}_n, \mathbf{r}_0) FF(\phi_i, \hat{\mathbf{r}}_n) \cdot \mathbf{f}_n d\mathbf{r}_n \quad (2.68)$$

where G is the two-dimensional Green's function in free space, $\mathbf{r}_n$ is the position of the n^{th} basis function of the j^{th} wind turbine, $\mathbf{r}_0$ is the position of the radar, $\mathbf{f}_n$ is the pulse basis function used for the discretization. The constant C depend on the radar properties (transmitted power, gain, losses ...). Note that for a given wind turbine, the excitation vector varies with respect to the orientation through the radiation pattern FF . The radiation pattern of the radar is represented by a circular aperture with a -3 dB angle of 1° . This is a typical value from weather radar manufacturers. The currents $\mathbf{i}_{i,j}^1$ are then computed through the solution of the MoM system of equations

$$\mathbf{Z}_{\text{WT}} \mathbf{i}_{i,j}^1 = \mathbf{v}_{i,j}^1 \quad (2.69)$$

As the wind turbines are represented as infinite cylinders, the computation of $\mathbf{Z}_{\text{WT}}^{-1}$ can be performed explicitly and kept in the RAM. Each wind turbine is typically discretized using less than 10^4 basis functions in the X band and thus requires less than 1 GB of memory space. The computations of the currents in lines 5, 10 and 15 thus require only a

```

input : Position of the radar, positions of the wind turbines
output: Location and power of the echoes

1 for  $i = 1 \dots N_\phi$  do
2   // for each radar orientation
3   for  $j = 1 \dots N_{WT}$  do
4     Compute the field  $\mathbf{v}_{i,j}^1$  radiated by the radar on  $WT_j$ 
5     Compute the currents  $\mathbf{i}_{i,j}^1$  on  $WT_j$ 
6     Compute the power of the echo  $P_{i,j}^1(\mathbf{i}_j^1)$ 
7     Compute the position of the echo  $\mathbf{x}_{i,j}^1$ 
8     for  $k = 1 \dots N_{WT}$ , with  $k \neq j$  do
9       Compute the field  $\mathbf{v}_{i,j,k}^2(\mathbf{i}_j^1)$  radiated by  $WT_j$  on  $WT_k$ 
10      Compute the currents  $\mathbf{i}_{i,j,k}^2$  on  $WT_k$ 
11      Compute the power of the echo  $P_{i,j,k}^2(\mathbf{i}_{j,k}^2)$ 
12      Store the position of the echo  $\mathbf{x}_{i,j,k}^2$ 
13      for  $l = 1 \dots N_{WT}$ , with  $l \neq k$  do
14        Compute the field  $\mathbf{v}_{i,j,k,l}^3(\mathbf{i}_{j,k}^2)$  radiated by  $WT_k$  on  $WT_l$ 
15        Compute the currents  $\mathbf{i}_{i,j,k,l}^3$  on  $WT_l$ 
16        Compute the power of the echo  $P_{i,j,k,l}^3(\mathbf{i}_{j,k,l}^3)$ 
17        Store the position of the echo  $\mathbf{x}_{i,j,k,l}^3$ 
18      end
19    end
20  end
21 end

```

Algorithm 2: Computation of the position and the power of the echoes up to the third order.

matrix-vector product.

Using the method of moments formalism, the currents of reflection orders 2 and 3 are obtained as

$$\begin{aligned}\mathbf{i}_{i,j,k}^2 &= \mathbf{Z}_{\text{WT}}^{-1} \mathbf{Z}_{j,k} \mathbf{Z}_{\text{WT}}^{-1} \mathbf{v}_{i,j}^1 \\ &= \mathbf{Z}_{\text{WT}}^{-1} \mathbf{Z}_{j,k} \mathbf{i}_{i,j}^1\end{aligned}\quad (2.70)$$

$$\begin{aligned}\mathbf{i}_{i,j,k,l}^3 &= \mathbf{Z}_{\text{WT}}^{-1} \mathbf{Z}_{k,l} \mathbf{Z}_{\text{WT}}^{-1} \mathbf{Z}_{j,k} \mathbf{Z}_{\text{WT}}^{-1} \mathbf{v}_{i,j}^1 \\ &= \mathbf{Z}_{\text{WT}}^{-1} \mathbf{Z}_{k,l} \mathbf{i}_{i,j,k}^2\end{aligned}\quad (2.71)$$

The norm of the currents for a given sub-domain gets smaller at each iteration, as they correspond to multiple bounces whose effects are reduced when the number of reflections increases. This algorithm differentiates from a typical iterative solver, where at each iteration, the currents are obtained with the left multiplication with $\mathbf{Z}_{\text{WT}}^{-1}$.

The position of the echoes are expressed in cylindrical coordinates (ρ, θ) as

$$\mathbf{x}_{i,j}^1 = (|\mathbf{r}_0 - \mathbf{r}_j|, \phi_i) \quad (2.72)$$

$$\mathbf{x}_{i,j,k}^2 = \left(\frac{|\mathbf{r}_0 - \mathbf{r}_j| + |\mathbf{r}_j - \mathbf{r}_k| + |\mathbf{r}_k - \mathbf{r}_0|}{2}, \phi_i \right) \quad (2.73)$$

$$\mathbf{x}_{i,j,k,l}^3 = \left(\frac{|\mathbf{r}_0 - \mathbf{r}_j| + |\mathbf{r}_j - \mathbf{r}_k| + |\mathbf{r}_k - \mathbf{r}_l| + |\mathbf{r}_l - \mathbf{r}_0|}{2}, \phi_i \right) \quad (2.74)$$

and are proportional to the additional path length traveled by the different bounces. The power P computed in Algorithm 2, lines 6, 11 and 16 is computed using the radiation integral in eq. (2.10) with the corresponding currents $\mathbf{i}^1, \mathbf{i}^2, \mathbf{i}^3$ and the gain of the antenna depending on the angle or arrival of each signal. It is explicitly expressed as

$$P_{i,j}^1 = \left| -j\omega\mu \int_{\text{WT}_j} \mathbf{i}_{i,j}^1 G(|\mathbf{r}_0 - \mathbf{r}_j|) FF(\psi_j) d\mathbf{r}_j \right|^2 \quad (2.75)$$

$$P_{i,j,k}^2 = \left| -j\omega\mu \int_{\text{WT}_j} \mathbf{i}_{i,j,k}^2 G(|\mathbf{r}_0 - \mathbf{r}_k|) FF(\psi_k) d\mathbf{r}_k \right|^2 \quad (2.76)$$

$$P_{i,j,k,l}^3 = \left| -j\omega\mu \int_{\text{WT}_j} \mathbf{i}_{i,j,k,l}^3 G(|\mathbf{r}_0 - \mathbf{r}_l|) FF(\psi_l) d\mathbf{r}_l \right|^2 \quad (2.77)$$

2 | Multipoles and inhomogeneous plane waves for 2D scenarios

The most costly operations in the proposed approach are the computation of the reaction integrals of lines 9 and 14 in algorithm 2. Since the wind turbines are visible from each other in a small angular sector, the IPW formulation is particularly well suited to this application. The currents $\mathbf{i}_j^{n+1}$ on wind turbine j induced by the currents $\mathbf{i}_i^n$ on wind turbine i are obtained similarly to eq. (2.65) as

$$\mathbf{i}_j^{n+1} = \mathbf{Z}_{\text{WT}}^{-1} \sum_n \beta_n \sum_p D_p(-jk_{\circ,p})^n \sum_{n'} \beta_{n'}(\mathbf{i}_i^n)(jk_{\circ,p})^{n'} \quad (2.78)$$

As a comparison, the total computation time for the IPW approach is compared to the time required for the computation of the multiple scattering using an FMM formulation. This is represented in Figure 2.12, by considering up to three reflections inside three wind farms of 10, 25 and 50 wind turbines, for 2, 6 and 10 GHz radar frequencies. The computation time corresponds to a single excitation, i.e. a single radar orientation. It is empirically modeled through the following expressions:

$$T_{\text{FMM}} = C_{\text{FMM}} N_{\text{WT}}^3 f^{1.85} \quad (2.79)$$

$$T_{\text{IPW}} = C_{\text{IPW}} N_{\text{WT}}^3 f^{1.1} \quad (2.80)$$

$$\frac{T_{\text{FMM}}}{T_{\text{IPW}}} \approx 2.2 f^{0.75} \quad (2.81)$$

where C_{FMM} and C_{IPW} are specific to the considered example and depend on the size of the wind turbine, the minimal distance between them and the level of accuracy to be achieved. For this example, $C_{\text{FMM}} = 4.8 \cdot 10^{-3}$ s and $C_{\text{IPW}} = 2.2 \cdot 10^{-3}$ s, computed on an Intel i7-4790 at 3.6 GHz CPU workstation with 32 GB of RAM. The factor N_{WT}^3 comes from the three reflections inside the wind farm and does not impact the gain on the computation time in eq. (2.81) which depends on the frequency, expressed in GHz. The complexity is mostly driven by the factor N_{WT}^3 and may result in a very long computation time. However, so paths can be discarded *a priori* (for example the path radar $\rightarrow$ WT₁ $\rightarrow$ WT₃ $\rightarrow$ radar in Figure 2.11a) depending on the detection threshold of the radar, its orientation and the position of wind turbines. The IPW formulation enables a quasi-linear dependence on the total computation with respect to the frequency. This method outperforms the FMM for all the considered radar bands while providing the same accuracy. Note that the FMM complexity is essentially the same as the theoretical $\mathcal{O}(N^{1.5})$. This optimal complexity is achieved if the number

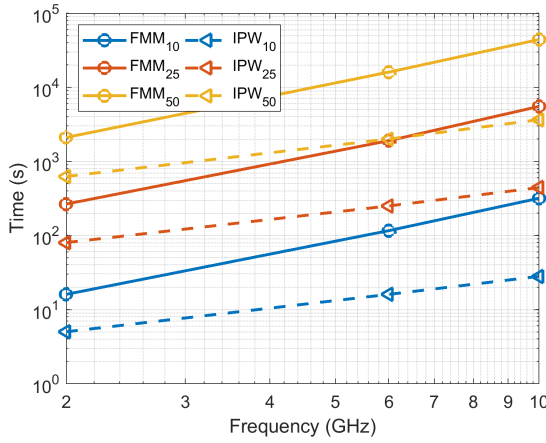


Fig. 2.12 Computation time for three different wind farms at three radar frequencies.

of wind turbines is equal to the number of basis functions representing a single wind turbine, which is not the case here.

The position and the power of the false echoes are illustrated in Figure 2.13 for two different parks with 20 wind turbines and a radar located 10 km south from the wind farm, for a frequency of 2 GHz. The wind turbines are represented by circles with a radius $R = 2$ m with a minimal distance of 200 m. The received power is normalized to the power scattered by an isolated wind turbine aligned with the radar main beam. The effect of the radiation pattern is clearly visible in Figure 2.13b where the received power is spread in azimuth. In the radial direction, echoes are detected inside and beyond the wind farm, with a reduced received power for the second and third order reflections. The width of the radar bins depends on the bandwidth of the radar and is set to a typical value of 100 m. The dynamic range for three reflections is 100 dB. For this reason, echoes from high-order reflections are not accounted for, as they would likely fall below the radar detection threshold. In the proposed examples, the false targets extend up to a few kilometers behind the wind farm, whereas measurements in [47, 48, 55] show that the false targets appear up to tens of kilometers beyond the wind farm. Those discrepancies may come from the ground clutter or higher order multipath components which are not

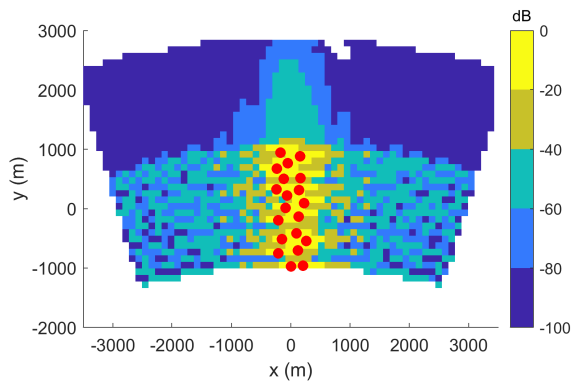
included in our model.

2.6 Conclusion

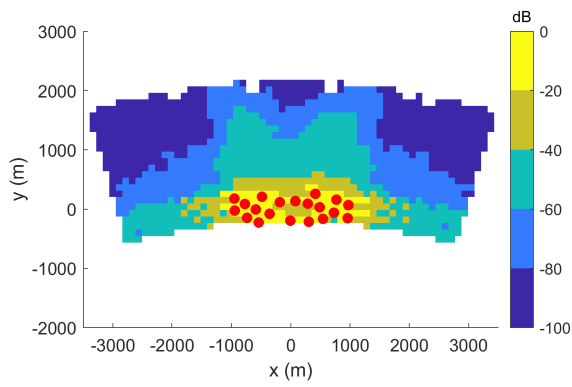
For large electromagnetic problems, the limitations of the MoM are twofold. On one hand, a large linear system of equations has to be solved, where a classical Gaussian elimination would lead to an excessive computation time in $\mathcal{O}(N^3)$. The solution time is reduced with the help of an iterative solver, which essentially performs a matrix-vector product of complexity $\mathcal{O}(N^2)$ at each iteration. The convergence of the solver is improved using a preconditioning matrix which reduces the condition number of the linear system of equations. On the other hand, the MoM system of equations requires a memory space proportional to N^2 , which limits the applicability of the method to relatively small problems.

This chapter has presented two approaches which reduce both the computation time and the memory requirements. Both methods are based on a modified representation of the Green's function. The multipole decomposition allows one to express the field radiated by a point source as if it was radiated by another source located nearby. When multiple sources are considered, this decomposition represents the radiation by each individual source as if it was radiated by an equivalent source located in the center of the group. On the other hand, the inhomogeneous plane wave decomposition represents the Green's function in the spectral domain with a limited and small number of plane waves. For the two methods, the reduction of the computation time is obtained thanks to the decomposition of the Green's function in plane waves, which are separable with respect to the source and the observation point. The interactions are then computed between *groups* of basis functions, instead of individual interactions between each pair of elementary functions.

The advantages of both approaches have been illustrated through different examples. By an appropriate selection of the size of the sub-domains, it has been demonstrated that the complexity of the FMM can be reduced to $\mathcal{O}(N^{1.5})$. However, the number of terms in the decomposition required for a targeted error level becomes very large when the size of the sub-domain grows. In the particular case where the sub-domains are distant and visible from each other through a small angle, the reaction integral can be approximated by a very small number of plane waves. Under this



(a)



(b)

Fig. 2.13 Power of the false echoes normalized by the scattered power of an isolated wind turbine. Two different wind farms of 20 wind turbines are investigated for a radar frequency of 2 GHz.

condition, the IPW approach outperforms the FMM, as illustrated by the computation of the multiple scattering within a wind farm. The complexity can be further reduced by decomposing each wind turbine in different sub-domains in order to reach the optimal complexity. However, it would require the computation of many different radiation patterns, whereas a single radiation pattern has to be computed once when the sub-domains are identical.

Besides MoM problems, the accelerated computation of radiation integrals between sub-domains is of great interest for non full-wave methods. The methods presented in the chapter are also of great help in Chapter 4, which is focused on the forward scattering mechanisms using asymptotic methods.

3

Fast numerical techniques for finite irregular arrays

3.1 Introduction

SOME modern communication systems require sophisticated radiation patterns. For example, communications satellites (COM-SAT, or SATCOM) or navigation satellites (e.g. GPS) may look for an isoflux radiation pattern [56], in order to transmit the same power to different users over a large area. Combined with beamforming techniques, a unique phased array is able to transmit a signal through multiple beams covering different earth areas [57]. For earth observation applications, synthetic aperture radar (SAR) satellites are employed and require a fine control on the radiation pattern. In particular, for the main lobe beam width which defines the swath width, the side lobes level (SLL), as well as the cancellation of parasitic sources (e.g. the nadir return [58]).

Such complex radiation patterns are obtained with the theory of irregular arrays (also named sparse, or aperiodic arrays) initiated in the 60's [59, 60]. Besides the control of the aperture tapering, the irregular positioning of the antennas adds more degrees of freedom in order to reduce the SLL, for example. The advantage is that, for a given targeted gain, irregular

arrays require fewer elements than regular arrays [61]. This is particularly convenient for space applications, where the weight and cost of the overall system are minimized. On top of that, an irregular positioning suppresses the grating lobes which appear in the radiation pattern of a periodic array when the inter-element spacing is larger than $\lambda/2$. Those unwanted side lobes cannot be afforded, as they create ambiguities in localization applications and increase the interferences from other RF sources.

However, the mathematical techniques required for the synthesis of irregular arrays are more complex than for their regular counterpart, which can rely on effective approximations (e.g. the theory of infinite uniform arrays, including the effect of the truncation [62, 63]). One major concern is the evaluation of the mutual coupling in the synthesis procedure [64], which drastically increases the computational complexity. Although its impact in the main lobe is small, the mutual coupling must be taken into account when the position and the amplitude of the side lobes are critical. This is the case for SAR satellites radiation patterns, or when the beam of a phased array is steered in different directions. The effect of the mutual coupling is included in the *embedded element pattern* (EEP), which corresponds to the radiation pattern of the array when a single element is excited. For large antenna arrays, the computation of the EEPs is time consuming. Moreover, those have to be recomputed at each new position of the elements in the optimization process [64, 65].

The goal of this chapter is to provide efficient numerical techniques which enable the fast computation of the embedded element patterns including the mutual coupling for large irregular arrays. This is achieved thanks to the representation of the antenna currents using macro-basis functions (MBFs), which are computed within a Krylov iteration, as proposed in [66], where in our case, the set of MBFs is assumed as identical for all the antennas [67]. This approach is combined with the FMM presented in Chapter 2, thus accelerating the computation of the augmented Krylov subspace. The method can reach any level of accuracy up to machine precision by allowing more iterations (i.e. reducing the stopping criteria of the iterative solver). However, in a synthesis context (e.g. by optimizing the position of the elements), it is suggested to speed up the computation of the EEPs at each iteration by using a set of MBFs identical for each excitation. Those MBFs are obtained by exciting a central element. The proposed MBF formulation is also convenient for evaluating the effect of the ground plane truncation, which has a non-negligible impact on the side lobes for

relatively small arrays. This study is carried out using an asymptotic evaluation of the currents on the ground plane at the end of this chapter.

The structure of this chapter is as follows: firstly, the iterative method based on MBFs is presented, as well as its acceleration using the FMM. The method is then validated with a full-wave solver for a small 16-elements irregular array under the infinite ground plane (GP) approximation. After validation, the method is used for the analysis of large antenna arrays and the fast computation of all the embedded elements patterns. Finally, the effect of the ground plane truncation is taken into account using the physical optics approximation for a fast computation of the currents on the GP. Those are induced by the fields radiated by the active elements and computed efficiently using an interpolating technique similar to [68, 69] and by taking advantage of the MBFs formulation.

This chapter may seem a little bit out of the scope of the thesis, as it does not treat a forward scattering application. However, antennas play an important role in radar or propagation applications. In this context, antennas are studied here using the same numerical tools presented in Chapter 2 that are used in the remaining of the thesis. Contents of this chapter are inspired from previously published works [67, 70, 71, 72].

3.2 Iterative MBF solver

3.2.1 Direct implementation

Macro-basis functions (MBFs) [8] approaches aim at reducing the size of the initial problem, so that the system of equations can be solved through a direct inversion, or with an LU decomposition. The general idea of this method is to represent the current distribution of all the elements as a linear combination of a the M columns of the matrix $\mathbf{K}$ corresponding to M MBFs, with $M \ll N$ ($\mathbf{K}$ is of dimensions $(N \times M)$). A reduced linear system of equations is thus written

$$\mathbf{Z}' \mathbf{i}' = \mathbf{v}' \quad (3.1)$$

By using the 3D EFIE, RWG basis functions and a Galerkin testing (i.e. the MBFs of the basis functions are the same as the testing functions), the

different terms in eq. (3.1) are expressed as

$$\mathbf{Z}'_{m,n} = \mathbf{K}_n^H \mathbf{Z}_{m,n} \mathbf{K}_m \quad (3.2)$$

$$\mathbf{i}'_n = \mathbf{K}_n^H \mathbf{i}_n \quad (3.3)$$

$$\mathbf{v}'_n = \mathbf{K}_n^H \mathbf{v}_n \quad (3.4)$$

where $\mathbf{i}'_n$ corresponds to the M coefficients of each MBFs associated to the element n . The current coefficients are then obtained as

$$\mathbf{i}_n = \mathbf{K}_n \mathbf{i}'_n \quad (3.5)$$

There are different ways of selecting an appropriate set of MBFs $\mathbf{K}$. A commonly used approach is to consider a set of MBFs for each element that includes the isolated current distribution (often called the "primary" MBF) as well as the currents induced on the element when another element is radiating ("secondary" MBFs). The primary MBF has a strong impact on the isolated element solution, whereas the secondary MBFs include the effect of the mutual coupling [73, 74]. This approach gives good results for arrays of disconnected elements [75]. For each antenna, the set of MBFs (i.e. 1 primary MBF and $(N_a - 1)$ secondary MBFs) is orthogonalized, using a singular value decomposition, for example.

In this work, another approach is used, where the generation of the MBFs is embedded within a Krylov iteration, presented in Section 2.2.3. At each iteration, a new generating vector of size $(NN_a \times 1)$ is added to the Krylov subspace. The MBFs are selected as the segment of this new generating vector, after orthogonalization of the $(N \times N_a)$ matrix obtained after the reshaping of the new generating vector. As all the elements composing the array are identical, it is assumed that the set of MBFs is common for all the antennas, i.e. $\mathbf{K}_n = \mathbf{K}$, for $n : 1 \cdots N_a$. The pseudo-code of the method is presented in algorithm 3.

When the number of antennas grows, the system of equations in eq. (3.1) may become hard to solve. For this reason, the number of MBFs is limited to $M \ll N$. The impact of the number M of MBFs will be investigated in Section 3.4.2. It is important to emphasize that any accuracy can be achieved by reducing the stopping criteria ϵ , at a price of a higher number of iterations. For a given accuracy, the proposed iterative MBF approach requires a smaller number of iterations than the classical GMRES solver, as observed in Figure 3.1. However, this is compensated by a

higher number of operations for the iterative MBF solver than for the classical GMRES (mostly driven by the computational savings enabled by the Arnoldi iteration for the latter). Hence, the MBF approach does not enable any significant improvement in terms of absolute computation time. Yet, for a given excitation and a targeted error level, the iterative MBF solver provides a set of macro-basis functions which are re-used for any other excitation, thus enabling significant computational savings. This will be covered in Section 3.4.

input : Impedance matrix $\mathbf{Z}$, excitation vector $\mathbf{v}$, stopping criterion ϵ output: Currents coefficients $\mathbf{i}$ <pre> 1 Initialization 2 $\mathbf{i} = \mathbf{0}$; 3 $\mathbf{r} = \mathbf{v}$; 4 Start the iteration 5 while $\mathbf{r} > \epsilon$ do 6 Compute the generating vector of size $(NN_a \times 1)$ 7 $\mathbf{v}_g = \mathbf{Z} \mathbf{v}$; 8 Resize the generating vector in a matrix form 9 $\mathbf{V}_g = \text{resize}(\mathbf{v}, N, N_a)$; 10 Orthogonalize the columns of $\mathbf{V}_g$ 11 $\tilde{\mathbf{V}}_g = \text{SVD}(\mathbf{V}_g)$; 12 Take the MBFs are the M first sing. vectors 13 $\mathbf{K} = \tilde{\mathbf{V}}_g(:, 1 \cdots M)$; 14 Build the reduced system using eqs. (3.2) to (3.4) 15 Compute the MBFs coefficients with eq. (3.1) 16 Update the solution and the residue 17 $\mathbf{i}_n = \mathbf{i}_n + \mathbf{K} \mathbf{i}'_n$; 18 $\mathbf{r} = \mathbf{r} - \mathbf{Z} \mathbf{i}$; 19 Set the new excitation to the residue 20 $\mathbf{v}_g = \mathbf{r}$ 21 end </pre>

Algorithm 3: Proposed iterative solver combined with MBFs.

3.2.2 Acceleration with the FMM

As pointed out in [66], the matrix products involved in eq. (3.2) and line 7 in algorithm 3 contribute significantly to the overall computation time. Furthermore, for very large arrays, solving the reduced system of equations in

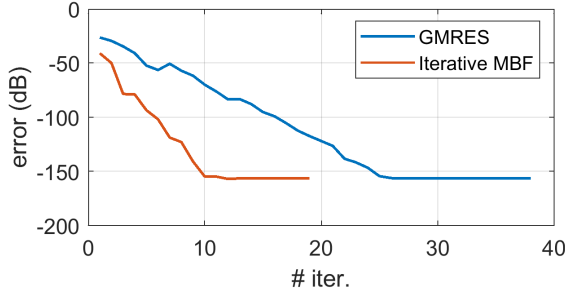


Fig. 3.1 Convergence of the classical GMRES algorithm and the proposed iterative MBF solver. Machine precision is achieved with more iterations.

eq. (3.1) is still challenging. Under the assumption of a unique $\mathbf{K}$ for all the antennas, the FMM is a very attractive for tackling the growing complexity associated to very large antenna arrays.

The FMM formulation in three dimensions is briefly explained here, as it is very similar to the method presented in Section 2.3. Let us consider two identical antennas each contained within a sphere of radius R . If the distance between the antennas' center $|\mathbf{r}_1 - \mathbf{r}_2| < 2R$, the mutual impedance matrix $\mathbf{Z}'_{1,2}$ is written as

$$\mathbf{Z}'_{1,2} = \mathbf{P}^H(\hat{\mathbf{u}}) \mathbf{T}_{1,2}(\hat{\mathbf{u}}) \mathbf{P}(\hat{\mathbf{u}}) \quad (3.6)$$

where $\mathbf{P}$ corresponds to the radiation patterns of the MBFs representing the current distribution, of size $(M \times N_\phi)$, where M is the number of MBFs and N_ϕ is the number of directions $\hat{\mathbf{u}}$ on the unit sphere. The translation operator $\mathbf{T}$ from antenna 1 to antenna 2 is expressed as

$$\mathbf{T}_{1,2}(\hat{\mathbf{u}}) = \sum_{l=0}^L j^l (2l+1) h_l^{(2)}(k|\mathbf{r}_{1,2}|) P_l(\hat{\mathbf{u}} \cdot \hat{\mathbf{r}}_{1,2}) \quad (3.7)$$

where $h_l^{(2)}$ is the spherical Hankel function of the second kind, P_l is the Legendre polynomial of order l and L is the number of multipoles used in the expansion of the Green's function. The number of multipoles is obtained using the same rule as for the two-dimensional case (see Section 2.3.1), where $D = \sqrt{3}d$ instead of $D = \sqrt{2}d$ in two dimensions.

It is worth noticing that, as the elements are identical, there is a single

matrix $\mathbf{P}$ that is computed and stored. The translator operator $\mathbf{T}$ is a diagonal matrix and is therefore stored in the RAM as a vector for more memory efficiency.

In order to improve the convergence of the iterative solver, a block diagonal preconditioning matrix $\mathbf{M}$ is applied to the original MoM system of equations which now reads

$$\mathbf{M} \mathbf{Z} \mathbf{i} = \mathbf{M} \mathbf{v} \quad (3.8)$$

where $\mathbf{M}$ is a matrix whose diagonal blocks correspond to the inverse of the self-impedance matrix $\mathbf{Z}_{\text{self}}$ of a single element

$$\mathbf{M}_{m,n} = \begin{cases} \mathbf{Z}_{\text{self}}^{-1} & \text{for } m = n \\ \mathbf{0} & \text{for } m \neq n \end{cases} \quad (3.9)$$

The reduced impedance matrix in eq. (3.2) including the preconditioner and the FMM formulation in eq. (3.6) now reads

$$\mathbf{Z}'_{m,n} = \begin{cases} \mathbf{I} & \text{for } m = n \\ \mathbf{K}^H \mathbf{Z}_{\text{self}}^{-1} \mathbf{P}^H \mathbf{T}_{m,n} \mathbf{P} \mathbf{K} & \text{for } m \neq n \end{cases} \quad (3.10)$$

For diagonal blocks, the matrix is the identity matrix $\mathbf{I}$. This is responsible for the improved convergence of the iterative process. The identity matrix is obtained for the diagonal elements thanks to the block diagonal preconditioner and the orthogonalization of the MBFs.

3.2.3 Complexity comparison

The computational complexity is compared for the direct approach of Section 3.2.1 and the MBF-FMM implementation Section 3.2.2. As the FMM implementation does not aim at reducing the number of iterations, the complexity is computed in regard to the number of operations involved in a single iteration. This comparison is focused on the computationally most intense operation, namely the computation of the reduced impedance matrix $\mathbf{Z}'$. In the case of large antennas arrays, it is assumed that the $\mathbf{Z}$ matrix is not explicitly computed at any time as it would not fit in the RAM. In the direct approach, the $\mathbf{Z}_{m,n}$ blocks required in eq. (3.2) are computed on the fly at each iteration for each pair of antennas. The algorithm of the fast

matrix-vector product is the same as algorithm 1, except that it is applied to the reduced system of equations and that the self-term is unique when all the sub-domains are identical.

For the direct approach, the computation of $\mathbf{Z}_{m,n}$ is equal to $T_1 = C_1 N^2$, where the product $\mathbf{K}_n^H \mathbf{Z}_{m,n} \mathbf{K}_m$ has a cost $T_2 = C_2 MN(M + N)$. Those two operations are then repeated for the $N_a(N_a - 1) \approx N_a^2$ off-diagonal blocks, thus leading to a total time T_{direct}

$$T_{\text{direct}} = N_a^2 (T_1 + T_2) \quad (3.11)$$

$$T_{\text{direct}} = N_a^2 \left(C_1 N^2 + C_2 MN(M + N) \right) \quad (3.12)$$

For large arrays $N_a \gg 1$ and under the assumption $M \ll N$, eq. (3.11) is approximated by

$$T_{\text{direct}} \approx (NN_a)^2 (C_1 + C_2 M) \quad (3.13)$$

where C_1 and C_2 are platform and software dependent. The first term in eq. (3.13) corresponds to the filling time of $\mathbf{Z}$ with a "brute-force" MoM; the second term corresponds to the extra time spent in the computation of $\mathbf{Z}'$. It is important to consider C_1 and C_2 as constants of different nature, as C_1 corresponds to the computation of reaction integrals and C_2 to products and sums between complex numbers. In our case, $C_2/C_1 \approx 10^{-5}$, such that the total computation time for the direct method is mostly driven by the computation of $\mathbf{Z}$.

For the FMM implementation, only the diagonal matrix $\mathbf{T}_{m,n}$ changes for different pairs of antennas. Therefore, the intermediate quantities $\mathbf{K}^H \mathbf{Z}_{\text{self}}^{-1} \mathbf{P}^H$ of size $(M \times N_\phi)$ and $\mathbf{PK}$ of size $(N_\phi \times M)$ are computed and stored at each iteration, where N_ϕ is the number of sampling point of the radiation patterns on the unit sphere (grows with the electrical size of the antenna). The time T_3 spent for the computation of those intermediate products is equal to

$$T_3 = C_2 \left(MN^2 + 2MNN_\phi \right) \quad (3.14)$$

$$T_3 \approx C_2 MN^2 \quad (3.15)$$

The product of the two previous intermediate results with a given translation vector has a cost

$$T_4 = C_2 M^2 N_\phi \quad (3.16)$$

The total computational cost for the FMM approach T_{FMM} is then

obtained as

$$T_{\text{FMM}} = T_3 + N_a^2 T_4 \quad (3.17)$$

$$T_{\text{FMM}} \approx C_2 M \left(N^2 + M N_\phi N_a^2 \right) \quad (3.18)$$

The gain using the FMM formulation is finally written as

$$\frac{T_{\text{direct}}}{T_{\text{FMM}}} \approx 10^5 \cdot \frac{N^2}{M^2 N_\phi} \gg 1 \quad (3.19)$$

3.2.4 Computation of the embedded element patterns

When analyzing antenna arrays, the embedded element patterns (EEP) [76, 77, 78] are quantities of interest, e.g. for beamforming or array thinning [79] applications. The EEP represents the radiation pattern of a given element in the presence of the neighboring inactive antennas. The EEP of the i^{th} element is obtained as

$$F_{\text{EEP}}^i(\hat{\mathbf{u}}) = -j\omega\mu \iint \sum_{j=1}^{N_a} \mathbf{i}_j^i e^{jk\hat{\mathbf{u}} \cdot \mathbf{r}_j'} dS \quad (3.20)$$

where the sum is performed on the all the radiating elements of the array, $\hat{\mathbf{u}}$ is the direction of radiation and $\mathbf{r}_j'$ is the position of the basis functions of the j^{th} antenna. The current $\mathbf{i}_j^i$ corresponds to the current on antenna j when the antenna i is excited while the other $N_a - 1$ antennas are inactive and terminated by a Z_g matching load. The computation of the N_a EEPs thus requires to solve for each i^{th} antenna the MoM system of equations:

$$\mathbf{Z} \begin{pmatrix} \mathbf{i}_1^i \\ \vdots \\ \mathbf{i}_i^i \\ \vdots \\ \mathbf{i}_{N_a}^i \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \vdots \\ \mathbf{v} \\ \vdots \\ \mathbf{0} \end{pmatrix} \quad (3.21)$$

where $\mathbf{v}$ is a vector of zeros except for the feed port. The computation of all the EEPs is very costly for large antenna arrays since the explicit computation of $\mathbf{Z}^{-1}$ is time-consuming and does not usually fit in the RAM. The complexity associated with the computation of the EEPs is tackled by

taking benefit from the MBF formulation.

To this end, it is also assumed that the set of MBFs computed with a single element excited can be reused for any other individual excitation in eq. (3.21). The unique set of MBFs is rapidly obtained using the iterative MBF solver accelerated with the FMM, as presented in Section 3.2.2. More accuracy is achieved by selecting a desired error level: the smaller the error, the larger the number of iterations, thus generating a larger number of MBFs. With this approach, the computation of the embedded element patterns is significantly accelerated, at the price of a reduced accuracy. However, the antenna designer is free to increase the accuracy on the EEP through the stopping criteria ϵ if a higher computation time can be afforded.

Under this assumption, the reduced system of equations in eq. (3.21) is thus written as

$$\underbrace{\mathbf{K}^H \mathbf{Z} \mathbf{K}}_{\mathbf{Z}'} \underbrace{\begin{pmatrix} \mathbf{K}^H \mathbf{i}_1^i \\ \vdots \\ \mathbf{K}^H \mathbf{i}_i^i \\ \vdots \\ \mathbf{K}^H \mathbf{i}_{N_a}^i \end{pmatrix}}_{\mathbf{i}'} = \underbrace{\begin{pmatrix} \mathbf{0} \\ \vdots \\ \mathbf{K}^H \mathbf{v} \\ \vdots \\ \mathbf{0} \end{pmatrix}}_{\mathbf{v}'} \quad (3.22)$$

The reduced matrix $\mathbf{Z}'$ is inverted and stored only once for all the EEPs. The coefficients of the MBFs for each of the N_a antennas and each of the N_a excitations are then rapidly computed through reduced matrix-vector products. The computational cost, assuming a Gauss-Jordan elimination, shrinks from $\mathcal{O}(N^3 N_a^3)$ to $\mathcal{O}(M^3 N_a^3)$. The radiation patterns $F_{\text{MBF}}^m(\hat{\mathbf{u}})$ of each MBF are also computed and stored at a low computational cost (since the current distribution on each antenna is represented by the same set of MBFs) and are expressed as

$$F_{\text{MBF}}^m(\hat{\mathbf{u}}) = -j\omega\mu \iint \mathbf{K}(m) e^{jk\hat{\mathbf{u}} \cdot \mathbf{r}'} dS \quad (3.23)$$

where $\mathbf{K}(m)$ corresponds to the m^{th} MBF. The EEP of the i^{th} element now reads

$$F_{\text{EEP}}^i(\hat{\mathbf{u}}) = \sum_j^{N_a} \sum_m^M \mathbf{i}_j'^i(m) F_{\text{MBF}}^m(\hat{\mathbf{u}}) e^{jk\hat{\mathbf{u}} \cdot \mathbf{r}_j^o} \quad (3.24)$$

where $\mathbf{r}_j^o$ is the vector from the origin towards the center of element j . By

interchanging the summations, the embedded element pattern for each antenna is written as

$$F_{\text{EEP}}^i(\hat{\mathbf{u}}) = \sum_m^M F_{\text{MBF}}^m(\hat{\mathbf{u}}) \underbrace{\sum_j^{N_a} \mathbf{i}'_j(m) e^{j\mathbf{k} \cdot \hat{\mathbf{u}} \cdot \mathbf{r}_j^0}}_{\text{IDFT}} \quad (3.25)$$

The second summation corresponding to the array factor has the form of an inverse discrete Fourier transform (IDFT). For regular arrays, this formulation enables the fast computation of the second term in eq. (3.25) using the fast-Fourier transform (FFT) algorithm, thus reducing the complexity of this operation from $\mathcal{O}(N_a^2)$ to $\mathcal{O}(N_a \log N_a)$. When the inter-element spacing is not uniform, the standard FFT formulation does not hold anymore. A modification of the FFT for non regular arrays (NFFT) is thus implemented for the fast computation of the array factor, as proposed in [69, 80].

Since the elements are not large compared to the wavelength, the radiation patterns F_{MBF} are band limited functions of $\hat{\mathbf{u}}$. For this reason, the radiation patterns of the MBFs are sampled in a few directions fulfilling the Nyquist's criteria. Those are rapidly interpolated using FFT before summation between all the elements, as suggested in [14].

3.3 Antenna element

The iterative MBF-FMM solver presented in Section 3.2 is applied for the design of an array of identical metallic elements devoted to a Synthetic Aperture Radar (SAR) satellite application. As the power transmitted with SAR antennas can be very high, the use of fully metallic radiating elements can alleviate this problem thanks to their better thermal conductivity. However, it is not easy to find purely metallic elements with patterns as wide and smooth as those of antennas printed on dielectric material. This challenge becomes more critical when significant bandwidth is required.

The fully metallic element designed by Sumit Karki¹ has a relative frequency of 3% around 11 GHz, suitable for SAR applications [81] in terms of beamwidth [82] and axial ratio. Leverage is found in the limited scan

¹Formerly associated with UCLouvain; currently associated with Aerospacelab, 1348 Louvain-la-Neuve, Belgium.

range of SAR antennas onboard satellites [81]. This means that elements with substantial gain can be used, i.e. of the order of 8 to 9 dB, as compared with 6 or 7 dB for microstrip antennas.

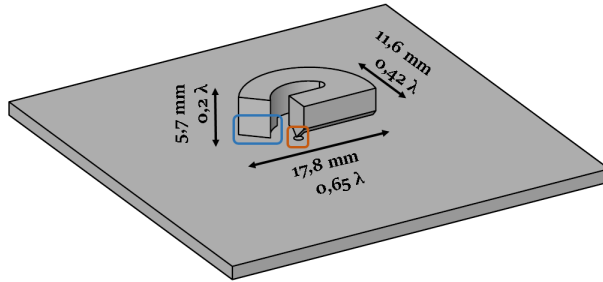


Fig. 3.2 Curved monopole element on an infinite ground plane. The orange square represents the feed point and the blue square corresponds to the connection between the antenna and the ground plane.

The antenna element is shown in Figure 3.2 atop a ground plane. The antenna is a monopole that is strongly curved and aligned horizontally to the ground plane, to radiate in the broadside direction. The tip of the monopole is grounded to drain the horizontal current, which helps to improve the circular polarization purity, and also to improve the mechanical support. The element can be fed directly in the center with a SMA connector going through the metallic ground plane.

The performance of the antenna element is assessed assuming that the antenna element is connected to an infinite metallic ground plane. Under this assumption, the effect of the latter is modeled using the image method. The monopole element is then replaced by an equivalent dipole element as shown in Figure 3.3. Since the monopole is matched to a 50Ω impedance, a 100Ω series impedance is connected to the generator of the dipole. The excitation is represented by a rooftop basis function (see Figure 3.4a) between the two monopoles and two hybrid basis function (half-rooftop and half-triangular) connecting the feed point to both monopoles (see Figure 3.4). The rest of the structure is meshed with Rao-Wilton-Glisson (RWG) basis functions [83]. The mesh of the complete dipole element is obtained with GMSH software [84].

The reflection coefficient S_{11} at the input of the studied element obtained with CST Microwave Studio® and the MoM code developed at

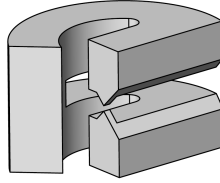


Fig. 3.3 Equivalent dipole element.



(a) One rooftop basis function.



(b) Two hybrid basis functions.

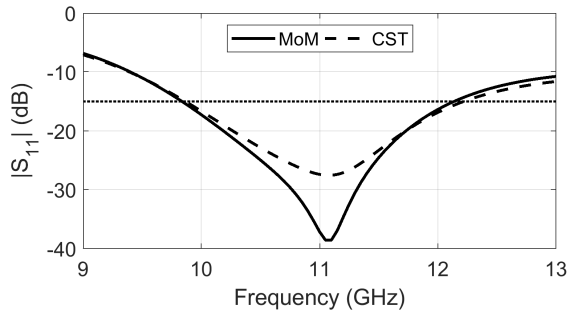
Fig. 3.4 Zoom on the mesh near the feeding point.

UCLouvain is represented in Figure 3.5a for a frequency sweep of 4 GHz around the central frequency. One can observe that the results obtained with both software are in good agreement, where discrepancies are due to the different meshing techniques between CST Microwave Studio® and GMSH, in particular near the feed. The element atop an infinite ground plane has a -15 dB bandwidth of 16 %. Figure 3.5b shows the directivity for the isolated element in the $\phi = 0^\circ$ and $\phi = 90^\circ$ cuts. The element radiates at broadside with a 30° half-power beamwidth and a directivity of 8.4 dB. Both the reflection coefficient and the radiation pattern obtained with our MoM code are in good agreement with CST Microwave Studio®. These results for a single element give us confidence on the accuracy of the software used for the study of larger problems.

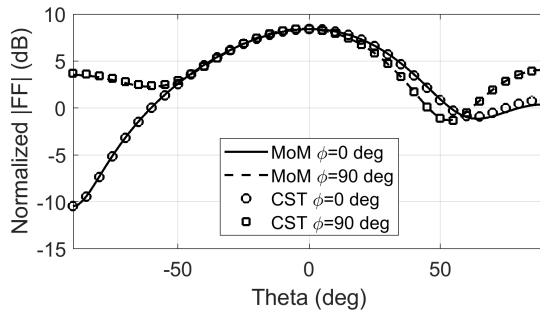
3.4 Irregular array on infinite ground plane

The radiating element presented in Section 3.3 is now included in an irregular array. In this section, the infinite ground plane approximation is used, so that the element considered is the equivalent dipole presented in Figure 3.3.

The position of the antennas is generated randomly on a rectangular surface with a ratio of 4 between the width and the height [58]. Antennas



(a) Reflection coefficient S_{11} . The dotted line corresponds to the -15 dB limit.



(b) Directivity of the element in the $\phi = 0^\circ$ and $\phi = 90^\circ$ cuts.

Fig. 3.5 Dipole element performance, computed with CST Microwave Studio[®] and the UCLouvain's MoM code.

are positioned one by one, under the constraint that the distance between the new candidate and all the elements already populating the array is larger than a predefined threshold. This threshold corresponds to the minimal distance between the centers of two spheres encompassing the element, and which do not intersect. Under this condition, the interactions between all the pairs of antennas are compressed using the FMM. For the considered element, this minimal distance D_{FMM} is thus equal to

$$D_{\text{FMM}} = 25 \text{ mm} \approx 0.92\lambda \quad (3.26)$$

Note that this distance is very close to the minimal physical distance $D_{\text{phys}} = 0.81\lambda$ from which two adjacent elements do not overlap. This value comes from the dimensions of the element dimensions itself (see Figure 3.2). As $D_{\text{phys}} > \lambda/2$, using this antenna in a regular array would create unwanted grating lobes; those are suppressed in aperiodic arrays, as it will be illustrated in the following section. For neighboring antennas, the distance between basis functions can be as small as 0.1λ ; the mutual coupling thus cannot be neglected, as is the case in [85].

3.4.1 Validation with a 16 elements array

The proposed solver is validated with a relatively small irregular array of 16 elements. Each antenna is discretized by 1245 basis functions. The full impedance matrix is approximately of dimensions $(20 \cdot 10^3 \times 20 \cdot 10^3)$ for which is it still possible to fill explicitly and invert the impedance matrix. The 16 elements are randomly positioned in a square, with a minimal distance D_{FMM} between each element, as represented in Figure 3.6. The parameters of the FMM are $L = 9$ and $N_\phi = 18$.

Self and mutual impedance

The first parameter of interest is the input impedance of each element. This value is important as a mismatch between the generator impedance and the load can induce significant loss of transmitted power. As stated in Section 3.3, the reference impedance is 100Ω . The self-impedance of the 16 elements and the mutual impedance of the central antenna are represented in Section 3.4.1 for the real and the imaginary part. The MBF-FMM solver shows a very good agreement with both the self and the mutual impedance, within 0.1 dB.

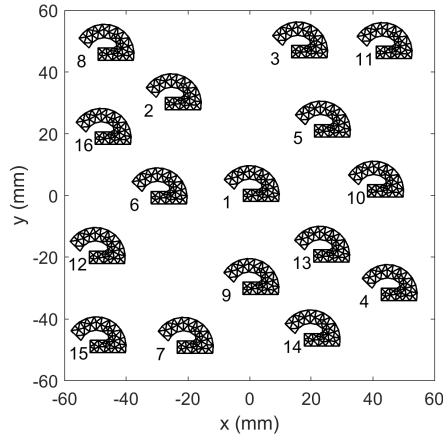
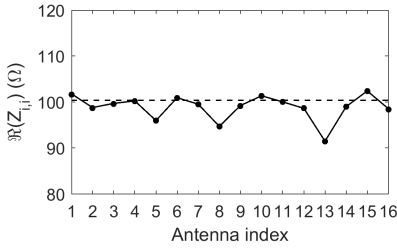
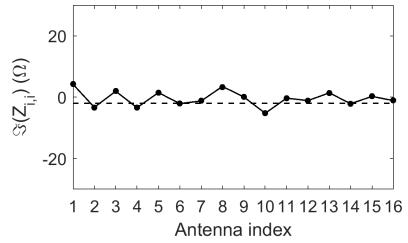


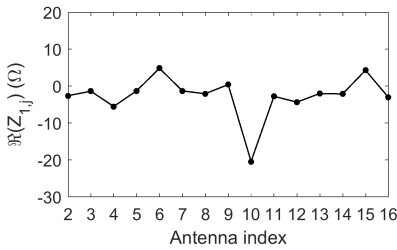
Fig. 3.6 Positions of the 16-elements array used for the validation of the proposed solver.



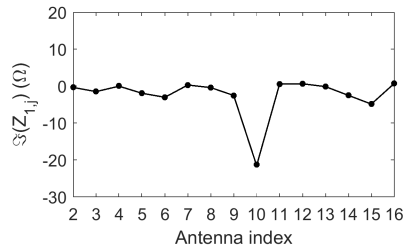
(a) Self-impedance: real part.



(b) Self-impedance: imaginary part.



(c) Mutual impedance: real part.



(d) Mutual impedance: imaginary part.

Fig. 3.7 Figures 3.7a and 3.7b: self-impedance of the 16-elements array. The dashed line corresponds to the input impedance of the isolated antenna. Figures 3.7c and 3.7d: mutual impedance of antenna 1, located at (0,0).

Embedded element patterns

The embedded element patterns are rapidly computed using the formulation in Section 3.2.4. A set $\mathbf{K}$ of 20 MBFs is obtained by exciting the antenna element 1 located at the center of the array (see Figure 3.6). Under the assumption that $\mathbf{K}$ is identical for each antenna, it has been observed that the MBFs obtained by exciting a central element enable a more accurate modeling of the mutual coupling between all the pairs of antennas. Exciting an element near the edge of the array (e.g. element 15) does not enable such an accuracy for all the EEPs.

The EEPs of the 9th and the 15th elements obtained with the fast MBF-based method are represented in the uv -plane in Figures 3.8a and 3.8c, where

$$u = \sin \theta \cos \phi \quad (3.27)$$

$$v = \sin \theta \sin \phi \quad (3.28)$$

with θ and ϕ corresponding to the elevation and azimuth angles, respectively. The absolute error with respect to the brute-force MoM solution obtained by solving eq. (3.21) is represented in Figures 3.8b and 3.8d. The error metric is computed on the normalized radiation patterns, as

$$\text{error} = 10 \log_{10} |F_{\text{EEP}} - F_{\text{BF}}|^2 \quad (3.29)$$

For a central element (Figure 3.8a), the fast computation of the EEP is very accurate, with a maximal absolute error level below -30 dB. For the 15th element which is not fully surrounded by neighboring antennas, the chosen set of MBFs induces a larger absolute error of the embedded element pattern, which is still smaller than -30 dB for any direction. A larger error is expected for elements located near the edge of the array, as the MBFs are obtained by exciting a central element. Nevertheless, this reduced accuracy can be afforded, as the ratio between the number of central elements and the number of edge elements is larger than 1 when the number of antennas increases. The contribution of the edge elements is thus smaller than for the central ones.

As a comparison, the isolated element pattern is represented in Figure 3.9. One can observe that the smooth isolated pattern is significantly affected by the mutual coupling when the element is embedded in the array. This justifies the importance to include the mutual coupling in the

array design.

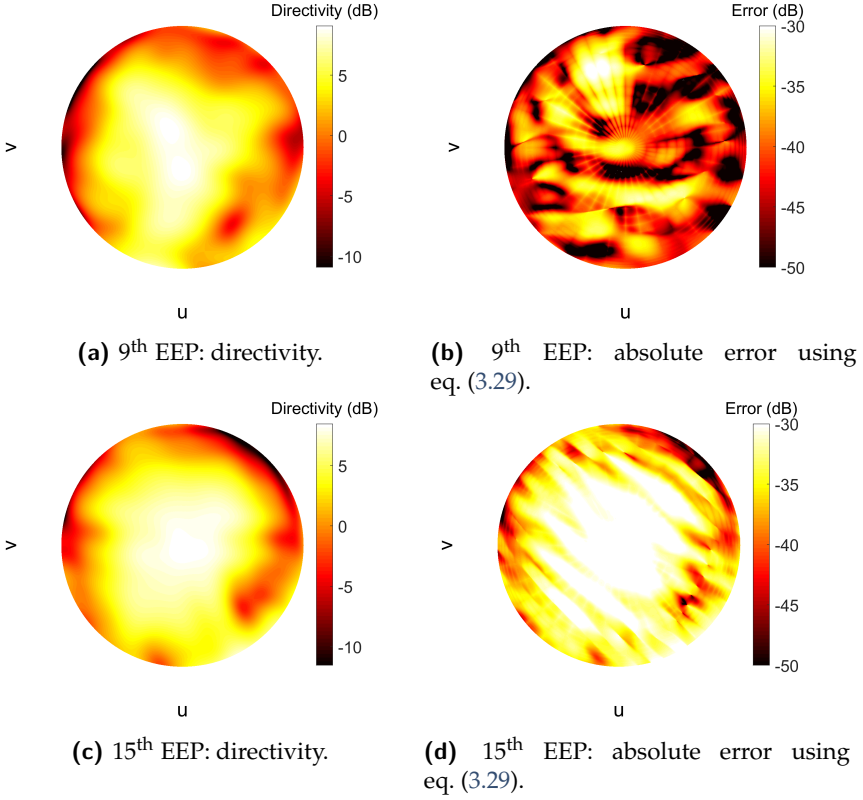


Fig. 3.8 Directivity and absolute error (eq. (3.29)) of the embedded element patterns of element 9 and element 15 from Figure 3.6.

Array radiation pattern

Once the embedded element patterns have been computed and stored, the array radiation pattern F_{tot} for any excitation is obtained as a linear combination of the N_a embedded element patterns as

$$F_{\text{tot}}(\hat{\mathbf{u}}) = \sum_i^{N_a} c_i F_{\text{EEP}}^i(\hat{\mathbf{u}}) \quad (3.30)$$

where c_i is the complex coefficient representing the amplitude and the phase of the signal fed to antenna i . For a uniform excitation (i.e. $\mathbf{c} = \mathbf{1}$),

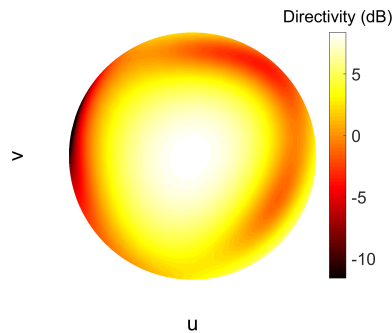


Fig. 3.9 Isolated element pattern.

the array radiates at broadside. The resulting radiation pattern obtained with eq. (3.30) is illustrated in Figure 3.10a, as well as the associated absolute error with respect to the brute-force solution in Figure 3.10b. The norm of the absolute complex error is smaller than -50 dB for all elevation and azimuth angles.

As observed in Figure 3.10a, the irregular layout of the 16 antennas suppresses the grating lobes that would appear in a regular array with an antenna spacing larger than $\lambda/2$. With a uniform excitation, the first sidelobes are located 19° away from broadside, with a SLL of -9.8 dB.

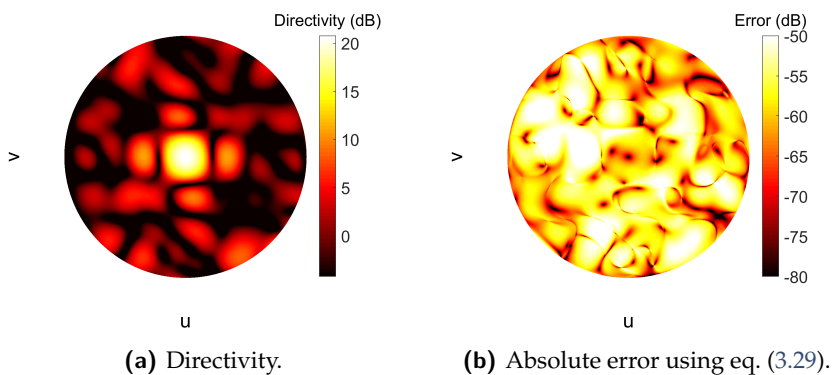


Fig. 3.10 Radiation pattern of the 16-elements array.

3.4.2 Large antenna arrays

Array performance

The performance of the methods presented in Section 3.2 and validated in Section 3.4.1 is now analyzed for three different arrays of 256, 512 and 1024 elements. The position of the antennas is randomly disposed on a rectangular surface whose dimensions ratio is equal to 4, under the constraint that the minimal distance between each pair of elements is larger than D_{FMM} , defined in eq. (3.26). This condition is convenient from a numerical point of view, as the computation of the interactions between different subdomains are all accelerated thanks to the FMM. Anyway, it is not possible to reduce significantly that minimal distance in our case, as the antenna would touch each other, thus modifying significantly the performances of the element itself. The three array layouts and their corresponding radiation patterns in the uv -plane are illustrated in Figure 3.11. The excitation is uniform for both the amplitude and the phase over the full aperture. The directivity in function of the uv coordinates is presented over a 35 dB dynamic range. The main array characteristics are summarized in Table 3.1. The directivity of the array grows linearly with the number of elements and can be approximated by

$$D_{\text{array}} \approx D_{\text{elem}} + 10 \log N_a \quad (3.31)$$

where D_{elem} is the directivity of the isolated antenna element (8.4 dB in this case). The sidelobe level (SLL) of each considered array is close to -13 dB when considering a uniform element excitation. This value is close to the SLL of the continuous aperture, as explained in [86]. A power of -13 dB is accepted for SAR applications [58, 87]. The SLL can be reduced by optimizing the antennas position (for example using compressive sensing [88], or with an iterative optimization using a convex relaxation method [64]) or by applying amplitude tapering on each antenna element. As an example of the effect of amplitude tapering, the directivity of the 256 elements array is illustrated in Figure 3.12 for which each element is weighted by a Gaussian function $G_w(x_a, y_a)$ defined by

$$G_w(x_a, y_a) = \exp \left(-\frac{x_a^2}{2\sigma_x} - \frac{y_a^2}{2\sigma_y} \right) \quad (3.32)$$

where (x_a, y_a) are the coordinates of each element and $(\sigma_x, \sigma_y) = (120, 30)$. One can observe on Figure 3.12 the reduced SLL (-20 dB) at a price of a

Table 3.1 Array properties for different number of elements.

Property	units	256 elem.	512 elem.	1024 elem.
# basis functions	N. A.	318,720	537,440	1,274,880
Directivity	(dB)	32.6	35.7	38.6
SLL	(dB)	-12.25	-12.9	-12.9
Area	(m ²)	0.21	0.45	0.9
A_e	(%)	51.7	48.5	48.2
HPBW- u	(°)	5.4	4.1	2.8
HPBW- v	(°)	1.5	0.9	0.7
Density	(mm ² · elem ⁻¹)	816	888	880

slightly reduced gain (31.8 dB) and a larger HPBW. Although amplitude tapering enables to reduce the SLL, it also increases significantly the hardware complexity, cost and weight of the system.

Note that the aperture efficiency A_e is constant for the three considered arrays. This is not surprising as when the number of elements increases, the increased physical area is compensated by the increased directivity, as highlighted by eq. (3.31).

Impact of the number of MBFs

The algorithm of the proposed solver described in Section 3.2 enables the control on the error currents using an iterative solver (GMRES) minimizing the residual error. At each new iteration, a new set of MBFs is created thus reducing the norm of the residue. Using this approach and its acceleration with the FMM, the common set of MBFs for all the elements is different for every excitation. In other words, this iterative procedure should be restarted for each of the N_a antenna in order to compute the embedded element patterns. However, it has been shown in Section 3.2.4 that the computation of the EEPs is greatly accelerated by using the MBF formulation of the element currents. For this reason, we make the additional hypothesis that the set of MBFs can be kept constant for any excitation.

In practice, for any irregular array composed of the same antenna element, the MBFs are computed and stored once for all, by solving eq. (3.1) for a "reference" array of a few tens of elements. The number of elements should not be too small in order to form a good basis. In particular, the minimal distance between the elements in the reference array should be

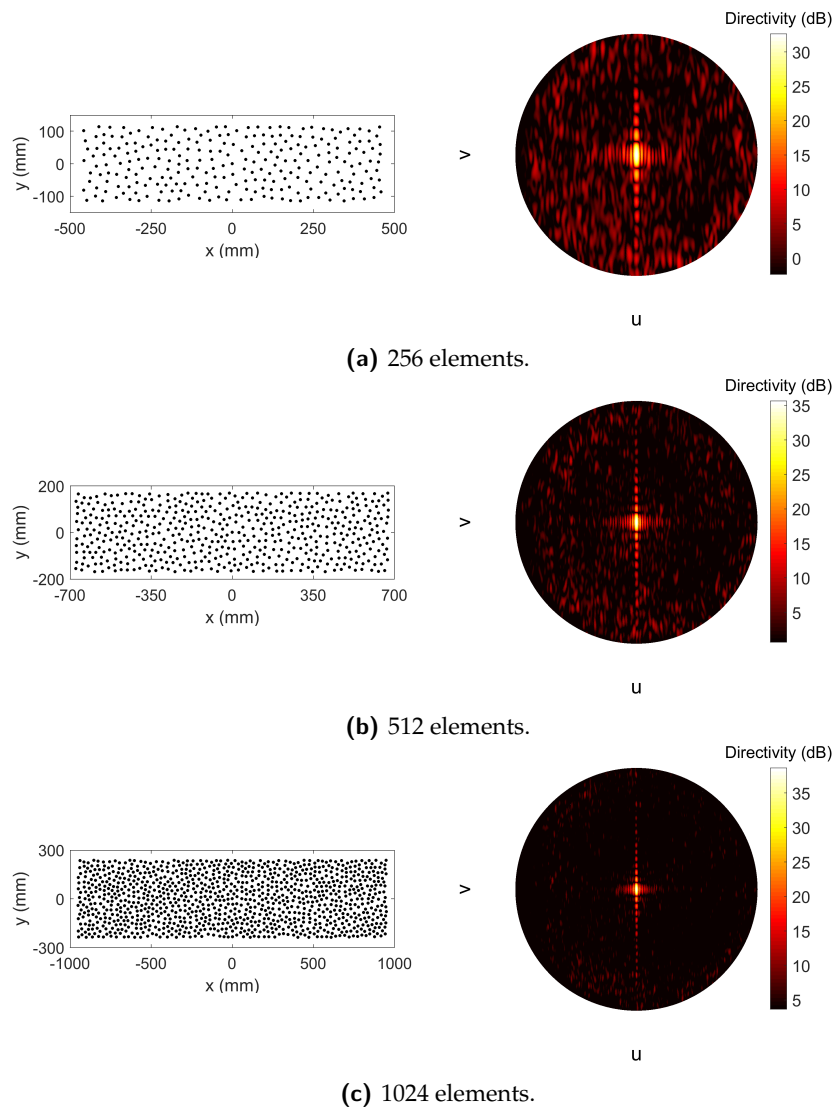


Fig. 3.11 Layout and directivity of three antenna arrays made of 256, 512 and 1024 elements.

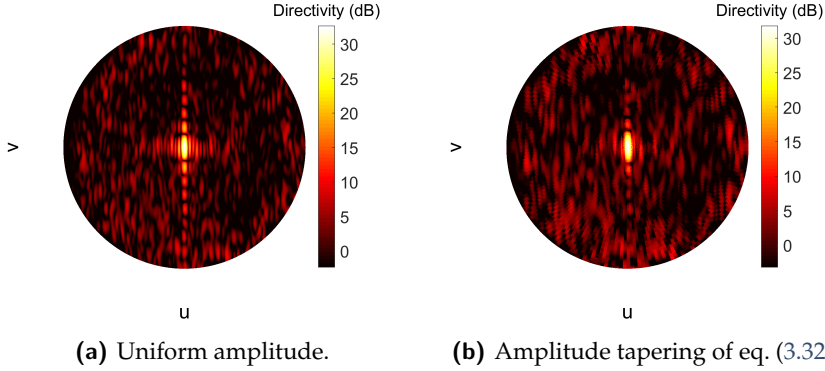


Fig. 3.12 Comparison of the directivity of the 256 elements radiation pattern with two different excitations. Phase is constant for both cases.

close to that of the array under study, in order to model the mutual coupling. As suggested in Section 3.4.1, the MBFs are obtained by solving the reduced system of equations by exciting a single central element while the others are terminated by a matched load.

The relative error of the norm of the currents as well as the relative error of the norm of the radiation patterns are plotted in Figure 3.13 by varying the number of MBFs for the 256 and 512-elements arrays of the previous section. The MBFs are computed through the solution a reference problem of 10×10 irregularly placed elements with a minimal distance of D_{FMM} and by exciting a central element. The "exact" solution for both arrays is computed using the proposed iterative solver, where the iterative process is stopped when the residual norm is below 10^{-8} . One can observe from Figure 3.13 that using the same reference set of MBFs for both the 256-elements and the 512-elements arrays leads to a similar error on the currents as well as the radiated pattern, where the error on the patterns is globally lower than the error on the currents. The typical error value of -30 dB is achieved in both cases with a number of MBFs close to $10 \ll N$. Much higher accuracy on the radiation pattern can be obtained by increasing the number of MBFs, which can be afforded for the computation of arrays with less than ≈ 500 elements on a typical workstation.

Computation time

The total computation time is analyzed for different array sizes. All the computations are performed on a desktop computer with 32 GB of RAM

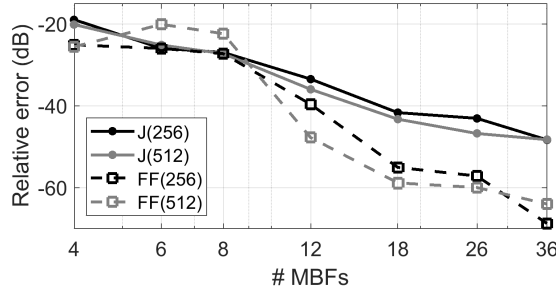


Fig. 3.13 Relative error of the norm of the currents and relative error of the norm of the radiation patterns for the 256 and 512-elements arrays, with respect to the number of MBFs.

and a Intel Core i7-4790 3.6 GHz CPU. As explained in the previous paragraph, the MBFs are computed through the solution of a problem of 100 elements irregularly disposed and a central excitation. The preparation stage includes the computation of the impedance matrix of a single element, the preconditioner, the FMM operators (i.e. the radiation patterns of the element and the translation operator) and the MBFs through the iterative solver. This preparation phase is computed in 83 s using 74 MB of memory. Although performed quickly, we emphasize that this procedure is performed just once for a specific antenna element. All the above mentioned quantities are saved on a hard disk and fit in the memory of any workstation.

For each of the three arrays previously considered (i.e. 256, 512, 1024-elements arrays from Figure 3.11), the computation time and the memory requirements are summarized in Table 3.2 by considering a common set of 12 MBFs. The "FMM op." stands for the computation of the radiation patterns of the transmitting and receiving groups (the former being the complex conjugate of the latter, as all the elements are identical), as well as the translation operator. The "matrix filling" corresponds to the construction of $\mathbf{Z}'$ using eq. (3.10). This operation is accelerated with the help of the FMM formulation, as all the non-diagonal terms are expressed in a compressed form. The coefficients of the MBFs are then computed for all the N_a elements. The associated computation time corresponds to the computation of $\mathbf{Z}'^{-1}$. This explains why the memory requirements for both the matrix filling and the computation of the MBFs coefficients are identical.

Table 3.2 Computation time and memory requirements for the analysis of 256, 512 and 1024 elements arrays using 12 different MBFs.

N_a	Time (s)			Memory (GB)		
	256	512	1024	256	512	1024
FMM op.	12	23	75	0.3	1.3	5.1
Matrix filling	7	34	177	0.1	0.6	2.2
MBFs coeff.	4	35	250	0.1	0.6	2.2
EEPs	18	71	276	0.2	0.3	0.6
Total	41	163	778	0.7	2.8	10.1

Finally, the N_a EEPs are computed and accelerated using the NFFT algorithm [89, 90].

The computation of all the EEPs of arrays composed of a thousand elements is performed in approximately 10 min and requires 10 GB of memory, which can be afforded on most of the typical workstations. The computation complexity T_{total} and the memory usage Mem_{total} are both of $\mathcal{O}(N_a^2)$ and are expressed as

$$T_{\text{total}} (\text{s}) = 7.4 \cdot 10^{-4} N_a^2 \quad (3.33)$$

$$Mem_{\text{total}} (\text{MB}) = 9.6 \cdot 10^{-6} N_a^2 \quad (3.34)$$

where the constants are platform and software dependent.

3.5 Effect of the ground plane truncation

The ground plane plays an important role on antenna and array characteristics. Being part of the radiating element, physical properties of the ground plane have an impact on the radiation pattern, input impedance, etc. The presence of the ground plane allows undesired surface waves which increase the mutual coupling. Those surface waves decay slowly with the distance [91]. As presented in the previous sections, including the mutual coupling effects in large arrays with direct methods leads to large computation time and memory requirements; this is all the more true when a finite ground plane is considered.

When the ground plane is electrically very large, it is sometimes assumed as infinite. Under this assumption, its effect can be modeled by

removing it and adding image currents of the antenna under the ground plane with the image method [92]. This approach was chosen in Section 3.4. Although the effect of the truncation of a ground plane supporting a single antenna has been widely studied [93, 94], its impact on large antenna arrays is not well covered in the literature. This is because the ground plane size grows with the number of elements composing the array, thus leading to very large EM problems.

Integral equation solvers are usually not an option for the analysis of large ground planes. Techniques such as the Method of Moments are restricted to lower frequency bands, as in [95] where the effect of a car on a mounted antenna is modeled with the MoM at 80 MHz. In [96], the MoM is also used for the analysis of microstrip antennas on a finite ground plane at 2.5 GHz and requires hours of computation. The effect of the ground plane truncation can be modeled with asymptotic methods, in order to reduce the complexity of the problem and the computation time. In [94], the fields diffracted by the ground plane borders are computed with the help of the uniform theory of diffraction (UTD) for different ground plane thicknesses. UTD is also used in [93] for the study of aperture antennas connected to square and circular ground planes.

The PO approximation is well suited to the analysis of finite ground planes and was first used for microstrip antennas in [97]. The effectiveness of this asymptotic approach has been validated in [98] with a full-wave solution for finite dielectric slabs. This method is combined with MoM in [39], where currents on the antenna are computed with MoM, while the current on the surrounding structure is approximated with the PO and the effect of the dielectric ground is modeled with the image theory.

The method proposed in this section is also a hybrid MoM/PO approach, as physical optics estimates quite accurately the perturbation on the radiation pattern caused by the ground plane [99]. The currents on the antennas are computed with the MoM under the infinite ground plane assumption and using the MBF formulation proposed in Section 3.2. Those MBFs radiate a magnetic field inducing currents on the physical ground plane, which are computed with the PO. This reduces drastically the memory requirements and computation time, while providing a good estimation of the total radiated fields.

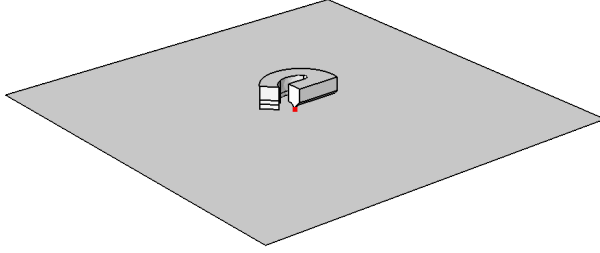


Fig. 3.14 Single element atop a PEC ground plane of dimensions $3.66 \times 3.66 \lambda^2$. The red point corresponds to the feed.

3.5.1 Single element atop a finite ground plane

Let us first consider the effect of a finite ground plane on a single element, represented in Figure 3.14, where the antenna element presented in Section 3.3 is located at the center of a square PEC ground plane of area $14.4 \lambda^2$. By denoting respectively $\mathbf{i}_a$ and $\mathbf{i}_g$ the coefficients of the surface currents on the antenna and the ground plane, the method of moments linear system of equations is written as

$$\begin{bmatrix} \mathbf{Z}_{a,a} & \mathbf{Z}_{a,g} \\ \mathbf{Z}_{g,a} & \mathbf{Z}_{g,g} \end{bmatrix} \begin{bmatrix} \mathbf{i}_a \\ \mathbf{i}_g \end{bmatrix} = \begin{bmatrix} \mathbf{v}_a \\ \mathbf{0} \end{bmatrix} \quad (3.35)$$

where the impedance matrix $\mathbf{Z}$ is composed of the self-terms for both the antenna and the ground plane ($\mathbf{Z}_{a,a}$ and $\mathbf{Z}_{g,g}$, respectively) and the mutual impedance matrices between the antenna and the ground plane ($\mathbf{Z}_{a,g}$ and $\mathbf{Z}_{g,a}$). The excitation vector $\mathbf{v}_a = (0, \dots, 0, 1, 0, \dots, 0)^T$ is the excitation vector containing all zero values, except at the index of the source port (red dot in Figure 3.14).

Infinite ground plane approximation

The MoM system of equations in eq. (3.35) becomes difficult to solve when an antenna array is considered or when the size of the ground plane increases. On the other hand, when the ground plane is very large, it is well-known from the image theory that its effect can be easily modeled by solving a MoM problem with the mirror image of the antenna, as it was done in the previous sections. In order to inherit from the fast solver developed in Section 3.2, it is assumed that the truncation of the ground plane

will not affect the currents on the top part of the antenna computed under the infinite ground plane hypothesis. The system of equations in eq. (3.35) is thus simplified as

$$\begin{bmatrix} \mathbf{Z}_{a,a} & \mathbf{0} \\ \mathbf{Z}_{g,a} & \mathbf{Z}_{g,g} \end{bmatrix} \begin{bmatrix} \mathbf{i}_a \\ \tilde{\mathbf{i}}_g \end{bmatrix} = \begin{bmatrix} \mathbf{v}_a \\ \mathbf{0} \end{bmatrix} \quad (3.36)$$

The approximated ground plane currents $\tilde{\mathbf{i}}_g$ are then obtained as

$$\tilde{\mathbf{i}}_g = -\mathbf{Z}_{g,g}^{-1} \mathbf{Z}'_{g,a} \mathbf{i}'_a \quad (3.37)$$

where the primed quantities use the MBF formalism which enables us to write the reduced mutual impedance matrix and the MBFs coefficients as

$$\mathbf{Z}'_{g,a} = \mathbf{Z}_{g,a} \mathbf{K} \quad (3.38)$$

$$\mathbf{i}'_a = \mathbf{K}^H \mathbf{i}_a \quad (3.39)$$

The far-field radiation pattern of the antenna and the ground plane is then obtained as the sum of the radiation patterns of both the top antenna and the ground plane. The comparison of the radiation patterns is provided in Figure 3.15. The truncation of the ground plane generates ripples of the order of 1 dB in the main beam. Larger differences are observed for larger elevation angles. The effect of the finite ground plane is even more dominant when its size is smaller with respect to the wavelength. This figure also validates the approximation that the ground plane truncation does not influence the antenna currents (eq. (3.36)). Indeed, the solid line and the dotted markers are in very good agreement, with an absolute error level below -40 dB.

Asymptotic ground plane currents

As the area of the ground plane grows with the number of antennas N_a , filling and inverting the self-impedance matrix of the ground plane $\mathbf{Z}_{g,g}$ may lead to large computation time and memory requirements when computing eq. (3.37). Those two problems are overcome by computing the currents on the ground plane with the help of the physical optics approximation, which provides a good approximation of the currents for flat surfaces. The PO approach is also employed in [39] where it is used for the computation of the currents on a vehicle in the vicinity of an antenna. The advantage of using the PO is that no linear system of equations needs to be

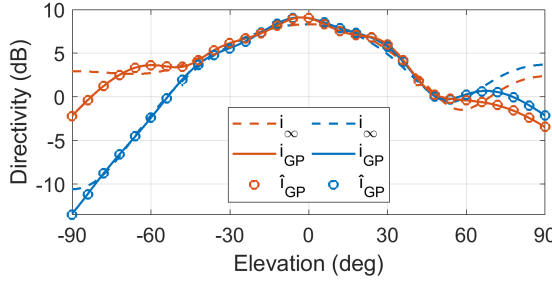


Fig. 3.15 Effect of the finite ground plane on the radiation pattern and validation of the approximation in eq. (3.36). The blue and the orange lines corresponds to the azimuth cuts $\phi = 0$ and $\phi = \pi/2$, respectively.

built and solved. Assuming a PEC ground plane, the surface currents are thus approximated by

$$\mathbf{i}_g \approx \mathbf{i}_g^{\text{PO}} \quad (3.40)$$

$$\mathbf{i}_g^{\text{PO}} = 2 \hat{\mathbf{n}} \times \mathbf{H} \quad (3.41)$$

where $\hat{\mathbf{n}}$ is the normal vector of the ground plane (constant in the case of a flat surface) and $\mathbf{H}$ is the magnetic field radiated by the currents $\mathbf{i}_a$ on the antenna. In the general case where multiple antennas form an array, the total magnetic field $\mathbf{H}$ is obtained as

$$\mathbf{H} = -j\omega\epsilon \sum_{n=1}^{N_a} \int_{\text{GP}} \mathbf{f}_g \cdot \int_{S_n} G(\mathbf{r}, \mathbf{r}') \nabla' \times (\mathbf{i}_{a,n} \cdot \mathbf{f}_a) d\mathbf{r}' d\mathbf{r} \quad (3.42)$$

where $G(\mathbf{r}, \mathbf{r}')$ is the three-dimensional Green's function, $\mathbf{f}_a$ is the set of discretizing RWGs [83] basis functions representing each antenna, $\mathbf{f}_g$ are the testing functions of the ground plane (orthogonal rooftop testing functions aligned on the plane containing the ground plane), $\mathbf{r}$ is the coordinates of the ground plane and $\mathbf{r}'$ the coordinates of the n^{th} antenna. The singularity of G is properly treated using an analytical singularity extraction, similar to the two-dimensional scenario presented in Section 2.2.

Tabulation of the magnetic field

The physical optics provides a fast approximation of the currents $\mathbf{i}_g$ on the ground plane, once the total magnetic field radiated by the antennas

is known. However, for a large number of antennas and a large ground plane, testing the contribution of each antenna through eq. (3.42) is time-consuming. This procedure is accelerated thanks to the MBF representation of the surface currents. Indeed, the contribution of each antenna in the total magnetic field is expressed as the sum of the magnetic field radiated by each MBF $\mathbf{K}_m$ multiplied by the proper coefficient $\mathbf{i}_n^m$ as

$$\mathbf{H} \approx -j\omega\epsilon \sum_{n=1}^{N_a} \sum_{m=1}^M \mathbf{i}_n^m \underbrace{\int_{\text{GP}} \mathbf{f}_g \cdot \int_{S_n} G(\mathbf{r}, \mathbf{r}') \nabla' \times (\mathbf{K}_m \cdot \mathbf{f}_a) d\mathbf{r}' d\mathbf{r}}_{\text{tabulated}} \quad (3.43)$$

This process is further accelerated by tabulating the magnetic field radiated by each MBF. It is then interpolated at the location of the ground plane basis functions for each antenna. As the distance between a given antenna and a ground plane basis function can be as large as a hundred of wavelengths, the storage and the computation of the tabulated values while respecting the Nyquist's sampling criteria is cumbersome. For this reason, a phase extraction is performed before evaluating the tabulated magnetic field, as suggested in [68]. The tabulation of the magnetic field associated to the m^{th} MBF is performed on the quantity $\mathbf{H}_m^*$ defined as

$$\mathbf{H}_m^* = \mathbf{H}_m \exp(jk|\mathbf{r}'_0 - \mathbf{r}|) \quad (3.44)$$

where $\mathbf{r}'_0$ is the position of the antenna feed point and $\mathbf{r}$ is the position of a testing function on the ground plane. The effect of the phase extraction is illustrated in Figure 3.16, where the phase variation of $\mathbf{H}_1^*$ is much smoother than for $\mathbf{H}_1$ and thus allows a coarser sampling. As distance from an antenna increases, a coarser sampling is allowed in order to recover correctly the magnetic field after interpolation and phase correction. This is illustrated on Figure 3.17 where the minimal sampling rate increases with the distance from the antenna. Close to the antenna, the spatial sampling is very fine because of the rapid variations of the magnetic field near the feed point. The spatial sampling may become much finer than the regular sampling near the feed point in order to capture those rapid variations before interpolation. Irregular sampling rates are also used in [68] after a change of variable for the distance and tabulation is performed on concentric circles around the source.

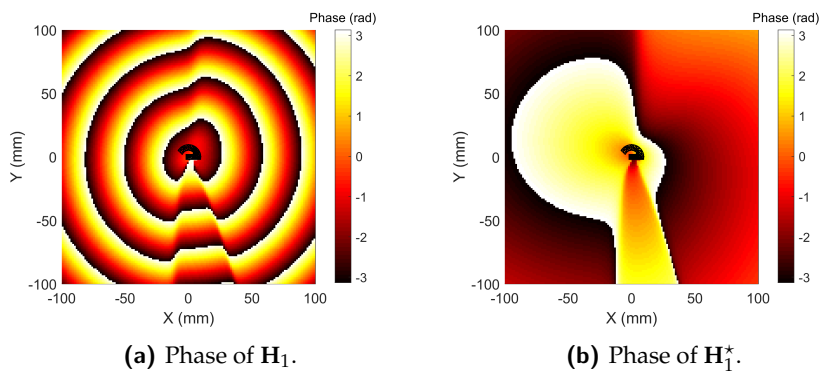


Fig. 3.16 Impact of the phase extraction of the magnetic field radiated by MBF 1.

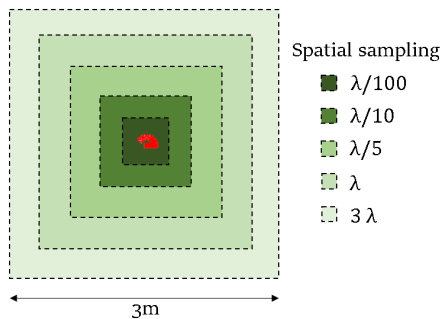


Fig. 3.17 Minimal sampling rate for different regions around the source antenna.

3.5.2 Antenna arrays atop a finite ground plane

The effect of the ground plane truncation is now evaluated for two irregular antenna arrays of 8 and 64 elements, represented in Figure 3.18. The ground planes are discretized using approximately $9 \cdot 10^3$ and $8.5 \cdot 10^4$ basis functions, respectively. All the elements are uniformly fed in amplitude and phase for a radiation at broadside.

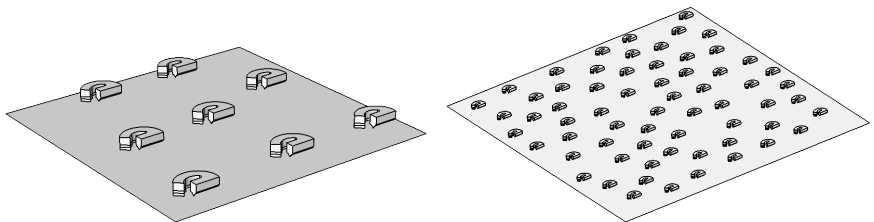
The radiation pattern of the 8-elements array in the two azimuthal cuts $\phi = 0$ and $\phi = \pi/2$ is illustrated in Figure 3.19. The ripples in the main beam induced by the ground plane truncation are much less visible than for the case of the single element illustrated in Figure 3.15. The absolute difference in the main beam between the PO and the exact solution is smaller than 0.1 dB and is also in good agreement with the infinite ground plane solution. For larger elevation angles, the infinite ground plane solution deviates from the MoM reference and the PO solution reduces this error. This example illustrates the importance of including the ground plane truncation effects, as large errors on the radiation pattern are observed.

The radiation pattern for the 64-elements array is represented in Figure 3.20. For this example, the brute-force solution cannot be obtained with UCLouvain's MoM code. The reference solution² is obtained with CST Microwave Studio®. By comparing the blue lines and dashed black lines in Figures 3.20a and 3.20b, one can observe that the effect of the ground plane truncation is significantly smaller than for the 8-elements array. This behavior is expected, as for large arrays, the number of radiating elements which "see" a large ground plane (i.e. central elements) is proportionally larger than for small arrays. The PO solution improves the accuracy of the infinite ground plane solution, whereas for large elevation angles, the PO solution and the infinite ground plane solution deviate from the reference solution. However, it is not expected that the PO solution provides a high accuracy in this region, as it does not model the diffracting currents on the border of the ground plane.

Regarding the computation time, once the magnetic field has been computed and stored for each MBF in the different sampling regions illustrated in Figure 3.17, the interpolation and the phase correction are computed in 11 s and 101 s for the 8-elements and the 64-elements arrays, respectively. The computation time of the currents themselves (see eq. (3.41)) is negligible compared to the interpolation of the fields, as this operation only

²Kindly provided by Sumit Karki

requires a cross product between the normal of the ground plane and the magnetic field.



(a) 8 elements atop a finite ground plane of dimensions $4.84 \times 4.84 \lambda^2$. (b) 64 elements atop a finite ground plane of dimensions $14.66 \times 14.6 \lambda^2$.

Fig. 3.18 Two considered irregular arrays of 8 and 64 elements with finite ground plane.

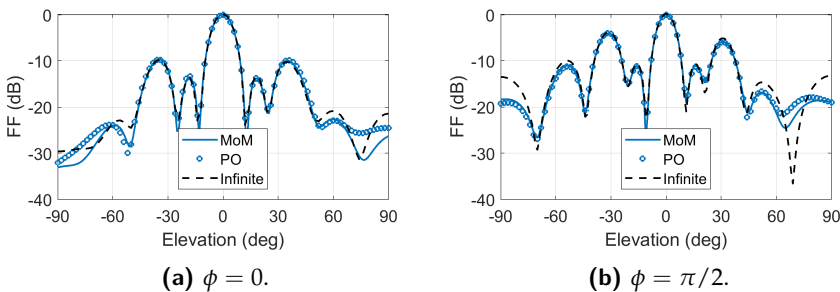


Fig. 3.19 Radiation pattern for the 8-elements array from Figure 3.18a obtained with the method of moments, the infinite ground plane solution and the physical optics.

3.6 Conclusion

This chapter presented numerical methods for the analysis of another type of large electromagnetic problem: large irregular antenna arrays. This study has been carried out using accelerated full-wave methods. On one hand, the complexity is reduced by representing the currents as a linear combination of known current distributions, called macro basis functions. This formulation allows one to reduce the size of the initial method of moments system of equations by a factor N^2/M^2 , which corresponds to two to

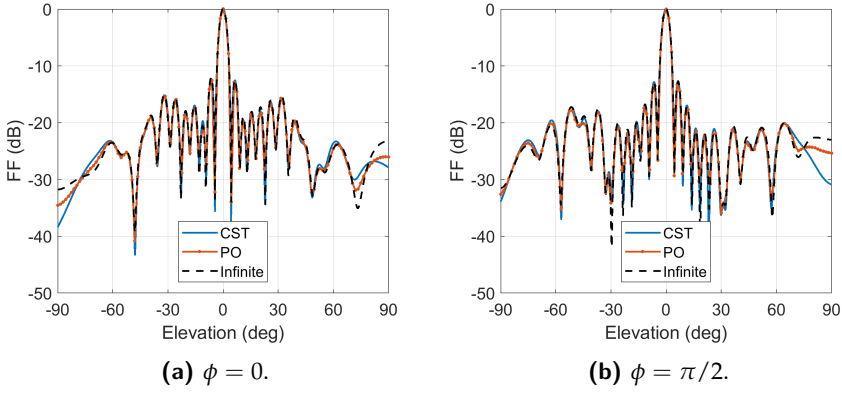


Fig. 3.20 Radiation pattern for the 64-elements array from Figure 3.18b obtained with the CST Microwave Studio®, the infinite ground plane solution and the physical optics.

four orders of magnitude in terms of memory requirements. The computation time is further reduced with the help of the fast multipole method, which enables the reduction of the computation time by five orders of magnitude for the considered examples.

The MBF formulation becomes particularly effective when one is interested in computing the embedded element patterns of large arrays. Previous versions of this work were restricted to a single excitation [67, 70]. Although a single problem was already solved quickly, this computation time had to be multiplied by the number of different excitations. This issue has been addressed by considering that the MBFs obtained by solving a representative reference problem can be used for any excitation. This hypothesis has been validated and provides an error below -30 dB for the EEP of any element composing the array. The total computation time is now proportional to the square of the number of elements, multiplied by a constant of the order of dozens of microseconds. An antenna array of 1024 elements represented by more than 1 million basis functions was fully analyzed (including the 1024 embedded element patterns) within less than 15 minutes on a standard workstation. Allowing more iterations for a given configuration enables a very high accuracy, as illustrated in Figure 3.1.

The effect of the ground plane was also studied and accelerated. We have shown how the results obtained quickly under the infinite ground plane solution are reused for analyzing the effect of the ground plane trun-

cation. The hypothesis that the ground plane truncation does not modify the previously computed currents has been firstly validated for a single element. In this case, the impact of the truncation is quite visible in the main beam as well as for all the elevation angles. When considering arrays, the ground plane currents are computed using the physical optics approximation. The computation of the magnetic field radiated by the elements is accelerated by interpolating the magnetic field radiated by each MBF. This interpolation is performed on tabulated values on a square grid with a non-uniform sampling rate, which reduces when the distance from the antenna increases, thanks to a prior phase extraction.

Results from an 8 elements irregular array have shown that the perturbation of the truncation is less visible than for the single element. Nevertheless, the asymptotic contribution of the ground plane reduces the error with the MoM reference when compared to the error due to the infinite ground plane approximation. It is expected that the ground plane truncation effect is less significant for large antenna arrays, for which the number of central elements grows faster than the number of elements located near the border of the ground plane. This has been highlighted with a 64-elements array where the reference solution was obtained from a commercial software.

Fast and accurate numerical methods as the one presented in this chapter are necessary for the development of advanced antenna systems devoted to satellite applications, for example. Usage of metallic elements as presented in this chapter is well suited to increase the downlink power [100]. Advanced phased arrays enable complex multi-beams pattern, for Direct-Broadcast Satellites (DBS) and Personal Communication Satellites (PCS) [101]. Finally, including the mutual coupling in the array design enable de prediction of the side lobes for a SAR in spotlight mode, for example. Note that the proposed element is circularly polarized. This characteristic is very common for SATCOM, even though SARs usually work with orthogonal linear polarizations. However, SARs using a circularly polarized signal are currently studied for the detection of ships [102], for example.

4

Improved physical optics computation near the forward scattering region

4.1 Introduction

FORWARD scattering plays a significant role in many wireless applications. For example, it strongly impacts the power transmitted or received by an antenna located in the vicinity of a large structure (such as cars or planes) [103]; it may also be viewed as nearly canceling the incident field when blockage takes place, for instance shadowing by humans in millimeter-wave indoor communications [104]. Forward-scattering radars are another relevant example which exploits the enhanced radar cross section (RCS) of a target when the angle between the transmitter and the receiver is close to 180 degrees [105]. In these conditions, the RCS is often computed using Babinet's principle [4], where the scattered fields in the forward direction (i.e. for a bistatic angle equal to 180 degrees) correspond to the fields radiated by an aperture having the same silhouette as the considered target (see Figure 4.1); however, this method

does no longer hold when the bistatic angle significantly deviates from 180 degrees.

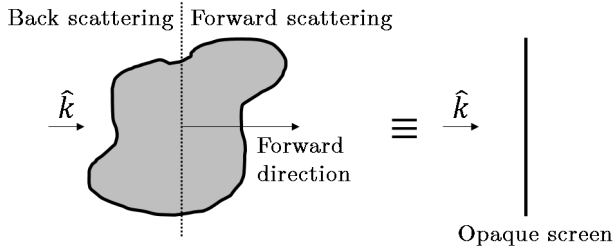


Fig. 4.1 Illustration of the Babinet's principle. In the forward direction, the fields scattered by an object (left figure) is equivalent to the fields scattered by an opaque screen (right figure) with the same silhouette as viewed from the source.

Therefore, RCS predictions are usually obtained with the help of different numerical methods offering various levels of complexity and accuracy. Among them, the Method of Moments (MoM) provides a very high accuracy, with an error level driven by the mesh size, at the expense of high memory and computational complexities (respectively, $\mathcal{O}(N^2)$ and $\mathcal{O}(N^3)$ for direct solvers, with N the number of discretizing functions). To overcome those limitations, one may employ asymptotic methods approximating acceptably the scattered fields when the electrical size of the target is very large. Among the asymptotic methods, the Physical Optics (PO) approximation [106] produces a moderate error level. This method describes quite precisely a variety of scattering mechanisms, while offering a complexity that grows linearly with the electrical surface of the object.

While PO provides a satisfactory estimation of the fields at an observation point located near the specular direction, as well as for a bistatic angle $\phi = 180^\circ$, the accuracy on the fields deteriorates significantly in the forward region [107, 108, 109], defined by bistatic angles $\phi \in [90^\circ, 270^\circ]$. The reason is that PO delivers a good approximation of the equivalent currents on the illuminated part of the surface but lacks accuracy near the shadow boundary and in the non-illuminated region of the scatterer where it assumes zero currents. A better estimation of the currents in those regions is thus required for a more accurate evaluation of the fields in the forward region.

In the literature, PO currents on the shadowed parts of objects are obtained using the Iterative Physical Optics (IPO) method, initially developed for resonances in open cavities [110]. In a nutshell, the initial PO currents (computed via the classical formulation) act as new sources inducing in turn currents on parts of the object visible from the domains where those currents are flowing. This has been extended to general scenarios involving resonances, such as concave objects [111, 112]: ships [113], tanks [114], or antennas mounted on large platforms [103]. The IPO is also used when different objects interact with each other, so that an object in the forward region of another one induces currents on the shadow part after reflection on the surface of the former [17, 115]. An explicit visibility testing can be avoided thanks to the shadow radiation introduced by Ufimtsev, where objects in front of other objects naturally compensate the initial incident field and creates a "shadow" on the shadowed object. This approach implemented in [17, 112] enables a more accurate estimation of the currents than an explicit visibility testing between each facet of multiple objects. In other words, currents on the shadowed surfaces computed with IPO correspond to the currents induced by a reflection on surrounding objects (if any) but do not model the currents due to the diffraction by the scatterer itself. In particular, IPO cannot provide currents in the non-illuminated part of the object when the studied scenario involves a single scatterer.

Unlike the above-mentioned works, the method proposed here does not require reflections on any other nearby entities to estimate the currents on the unlit part of an isolated scatterer. To the authors' best knowledge, such capabilities are not yet available in the literature. Starting from the total fields obtained with the classical PO, electric and magnetic currents are computed on a surface enclosing the scatterer by invoking the equivalence theorem [12, Sec. 3.5]. By superposition, those currents in turn radiate magnetic fields which induce new currents computed with the classical PO on the whole object. We emphasize that the proposed method, as an improved version of the classical physical optics, does not truly compete with the method of moments in terms of accuracy.

As for the classical PO formulation, the proposed method is a current-based approach: once the equivalent currents representing the scattering problem have been computed, the fields scattered in any direction of observation are computed easily. This can for instance be done very efficiently using the method described in [14]. This is not the case for other asymptotic approaches, such as the Uniform Theory of Diffraction (UTD). Using

this ray-based method, the diffracted field at a particular observation point in the shadow region depends on the incident field where the ray hits the surface and on the point from which the ray leaves the surface towards the observation point, thus requiring a new computation for every pair of source and observation points [116, 117].

This chapter is structured as follows: Section 4.2 illustrates the limitations of the classical PO formulation based solely on electrical currents for perfect electric conductors (PEC). Those limitations are overcome with the new proposed method using electric and magnetic equivalent currents, accelerated with the Fast Multipole Method (FMM). Results for isolated (i.e. in free space, with no multiple scattering) PEC bodies in two dimensions are given in Section 4.3, where the scattered fields are compared for the MoM reference solution, the classical PO method and the proposed approach. The multiple-scattering effects taking place when various scatterers are present is then modeled in Section 4.4 by combining our method within an iterative physical optics solution. Finally, the extension of the method for the three-dimensional case is proposed as a perspective for future works in Section 4.5 and general conclusions are drawn in Section 4.6.

4.2 Modified physical optics

4.2.1 Limitations of the classical PO

Physical Optics is a current-based asymptotic method with a $\mathcal{O}(N)$ complexity, where N is the number of discretizing functions representing an object surface (this number also depends on the need to accurately capture the oscillations of the surface fields). It is therefore very suitable for the prediction of scattering by electrically large structures, where full-wave methods fail due to their computational complexity. The PO method replaces the physical object by equivalent currents $\mathbf{J}_{\text{PO}}$ radiating in free space. Assuming a perfect electric conductor (PEC), those currents are expressed as

$$\mathbf{J}_{\text{PO}} = \begin{cases} 2 \hat{\mathbf{n}} \times \mathbf{H}_i & \text{on the "lit" region} \\ 0 & \text{on the "unlit" region} \end{cases} \quad (4.1)$$

with $\hat{\mathbf{n}}$, the outward normal of the surface and $\mathbf{H}_i$ the incident magnetic field on the object surface. Note that the PO approximation is exact for the case of an infinite PEC plate with a normal incidence. The "lit" region encompasses the part of the surface which is in line-of-sight with the source,

while the “unlit” region is the complementary part. In the case of a convex object, those regions are defined for a point $\mathbf{r}$ belonging to the surface S as

$$\mathbf{r} \in S : \begin{cases} \hat{\mathbf{k}}_i \cdot \hat{\mathbf{n}}(\mathbf{r}) \leq 0 & \text{defines the “lit” region} \\ \hat{\mathbf{k}}_i \cdot \hat{\mathbf{n}}(\mathbf{r}) > 0 & \text{defines the “unlit” region} \end{cases} \quad (4.2)$$

where $\hat{\mathbf{k}}_i$ is the wave vector of the incident field. For concave objects, or when many objects are involved, different techniques allow the determination of the “lit” region, such as geometrical visibility [118, 119], or shadow radiation [17, 115].

In order to illustrate the validity and the limitations of eq. (4.1), consider the problem depicted in Figure 4.2. The initial problem consists of a PEC smooth scatterer (Figure 4.2a) complying with the above-mentioned criteria and an incident wave (taken here as a plane wave, although this is not mandatory) propagating from right to left. The equivalent PO problem is presented on Figure 4.2b, where the PO currents are computed on the lit region according to eq. (4.1).

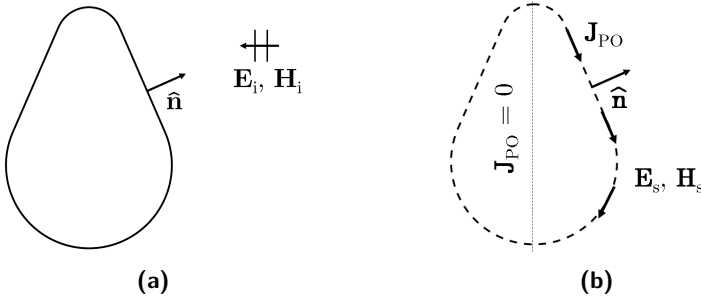


Fig. 4.2 (a): initial scattering problem; (b): equivalent problem with PO currents on the illuminated part of the object.

The bistatic RCS of the considered object is shown in Figure 4.3, where the PO solution is compared to a full-wave MoM reference. One can observe that in the vicinity of the specular region (around 45°), the PO solution is in good agreement with the exact result. The approximation is also valid in the vicinity of the forward direction (observation angle close to 180°). Besides those two aforementioned regions, the PO approximation in its classical formulation does not hold, providing an inaccurate estimation of the fields when the observation angle ϕ lies in the forward region,

roughly defined as $\phi \in [90^\circ, 270^\circ]$. Similar effects have been observed in previous works [120, 107, 121].

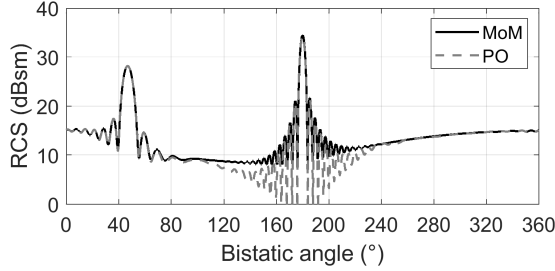


Fig. 4.3 Bistatic RCS of the problem depicted in Figure 4.2, computed with MoM (reference) and the classical PO.

These discrepancies arise from the fact that the PO does not accurately represent the currents near the shadow boundary [121] and assumes as null the currents on the backside of the scatterer. Different approaches have been proposed for the modeling of those currents on convex objects. Fock's theory [122] proposes an asymptotic representation of the currents near the boundary between the lit and the shadowed region, within a region of length $d = (\lambda\rho^2/\pi)^{1/3}$, where ρ is the radius of curvature at the boundary [122]. Although providing a very accurate solution for simple geometries, this approach does not enable a uniform description of the currents for arbitrary convex shapes [123]. An improvement of the PO formulation is proposed in [124, 125], where a new normal vector is defined, which depends on the incident and observation angles, and thus requires a new computation for every observation point.

In this work, our goal is to provide a good approximation of the currents in the shadow boundary, as well as in the deep shadowed part of the surface, in order to improve the accuracy of the fields computed behind the scatterer. The method derives surface currents flowing on the surface of the scatterer. The solution depends only upon the incident field, as opposed to other techniques that require the *a priori* knowledge of the scattering direction.

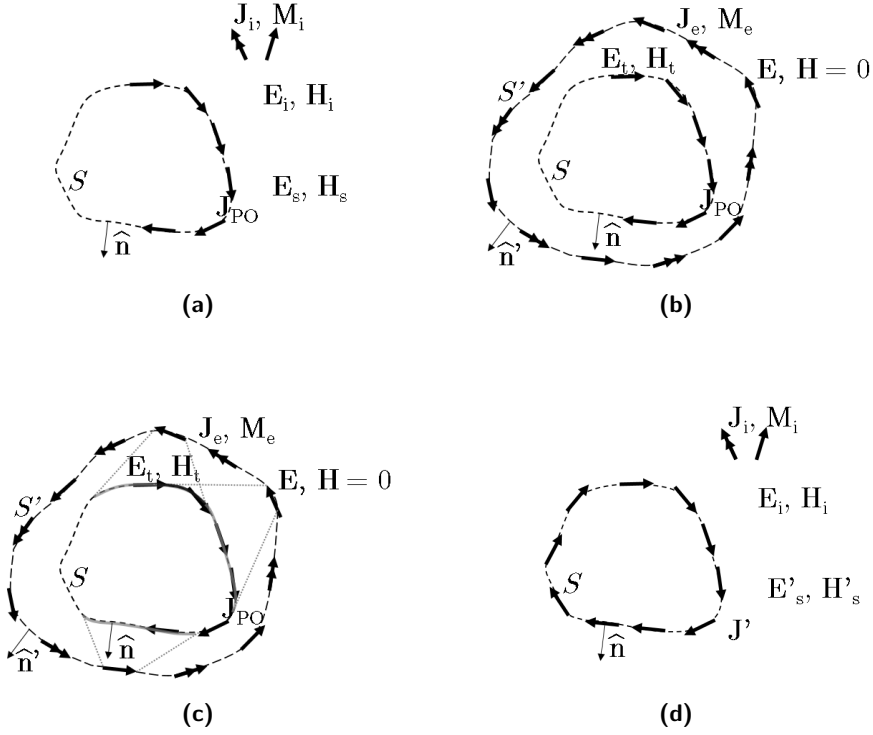


Fig. 4.4 Different steps for updating the PO currents. Magnetic currents (double arrows) exist only on the equivalence surface S' , while the original surface S only supports electric currents (simple arrow).

4.2.2 Update of the currents via an equivalent problem

Let us consider a PEC convex body bounded by a surface S and with sources $(\mathbf{J}_i, \mathbf{M}_i)$ located outside S and radiating the incident fields, as depicted in Figure 4.4a. As a first guess, the total PO fields $\mathbf{E}_t$ and $\mathbf{H}_t$ are obtained via the classical PO approximation, and are the sum of the incident fields $(\mathbf{E}_i, \mathbf{H}_i)$ and the fields radiated by the PO currents (see Figure 4.4a). It is possible to derive an internal equivalent problem [12, Sec. 3.5], so that the total PO fields inside the equivalence surface S' are exactly described by new equivalent electric and magnetic currents $\mathbf{J}_e$ and $\mathbf{M}_e$ (resp.) flowing on S' . Outside S' , the fields are set as null (see Figure 4.4b), so that the

equivalent currents are computed as

$$\mathbf{J}_e = \hat{\mathbf{n}}' \times (0 - \mathbf{H}_t) \quad (4.3)$$

$$-\mathbf{M}_e = \hat{\mathbf{n}}' \times (0 - \mathbf{E}_t) \quad (4.4)$$

Note that the equivalent surface S' can be chosen arbitrarily. In practice, the surface S' is a magnified version of S as it will be discussed later on. The electric and magnetic fields $\mathbf{E}_e, \mathbf{H}_e$ radiated by the equivalent currents defined above are computed as follows [1]

$$\mathbf{E}_e = -j\omega\mu \left(\mathbf{A} + \frac{1}{k^2} \nabla \nabla \cdot \mathbf{A} \right) - \nabla \times \mathbf{F} \quad (4.5)$$

$$\mathbf{H}_e = \nabla \times \mathbf{A} - j\omega\epsilon \left(\mathbf{F} + \frac{1}{k^2} \nabla \nabla \cdot \mathbf{F} \right) \quad (4.6)$$

where $\mathbf{A}$ and $\mathbf{F}$ are respectively defined by

$$\mathbf{A} = \int_{S'} \mathbf{J}_e G(\mathbf{r}, \mathbf{r}') dS' \quad (4.7)$$

$$\mathbf{F} = \int_{S'} \mathbf{M}_e G(\mathbf{r}, \mathbf{r}') dS' \quad (4.8)$$

In eq. (4.5), eq. (4.6), μ is the vacuum permeability, ϵ is the vacuum permittivity, j is the imaginary unit, $\omega = 2\pi f$, where f is the frequency, $k = 2\pi/\lambda$ is the wavenumber, $\mathbf{r}$ is the observation point coordinate, $\mathbf{r}'$ is the source coordinate and belongs to S' and $G(\mathbf{r}, \mathbf{r}')$ is the 2D Green's function. Note that the differential operators apply on the observation coordinates. Full derivation for a TM incident wave is provided in Section 4.7.1.

The current distribution can be represented as a weighted sum of basis functions $\mathbf{f}^i$ with weights j_e^i and m_e^i , so that eq. (4.3) and eq. (4.4) now read

$$\mathbf{J}_e = \sum_i j_e^i \mathbf{f}^i \quad (4.9)$$

$$\mathbf{M}_e = \sum_i m_e^i \mathbf{f}^i \quad (4.10)$$

Among different types of basis functions, pulse basis functions are used in this work and are defined as

$$\mathbf{f}_i(l) = \begin{cases} 1 & l^i \leq l \leq l^{i+1} \\ 0 & \text{elsewhere} \end{cases} \quad (4.11)$$

where l corresponds to a coordinate along the object boundary.

The new electric currents $\mathbf{J}'$ on the object are then computed as follows: each elementary electric and magnetic current $\mathbf{J}_e^i$ and $\mathbf{M}_e^i$ associated with basis function i on S' , radiate a magnetic field through eq. (4.6) on $S^{\text{vis}(i)}$, defined as the portion of S which is visible from the source basis function i . This process is illustrated in Figure 4.4c, where one can observe that $S^{\text{vis}(i)} \cap S^{\text{vis}(i+1)} \neq \emptyset$. The modified electric currents $\mathbf{J}'$ on $S^{\text{vis}(i)}$ are then computed as

$$\mathbf{J}'^{\text{vis}(i)} = 2 \hat{\mathbf{n}} \times \mathbf{H}_e^{\text{vis}(i)} \quad (4.12)$$

where $\mathbf{H}_e$ stands for the magnetic field eq. (4.6) re-radiated by the equivalent currents. The contributions from each portion of S' are added up, thus providing modified electric currents on the whole surface of the object (Figure 4.4d). Redistributing the incident source on an equivalence surface enclosing the scatterer allows one to illuminate the latter from every angle, thereby also illuminating the shadowed zone. Note that this process can be repeated iteratively as discussed in Section 4.3.2, where a very small number of iterations (typically two) is considered.

Another iterative technique based on the physical optics and the Neumann series is proposed in [126, 127]. It starts from the magnetic field integral equation (MFIE) written as [27]

$$\underbrace{2 \hat{\mathbf{n}} \times \mathbf{H}_i}_{\mathbf{J}_0} = \mathbf{J}(\mathbf{r}') - 2 \hat{\mathbf{n}} \times \int_S \mathbf{J}(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') d\mathbf{r}' \quad (4.13)$$

where the term $\mathbf{J}_0$ in eq. (4.13) is similar to the PO currents defined in eq. (4.1), except that the currents are defined everywhere on the scatterer (i.e. not only on the illuminated part of the scatterer) and $\mathbf{J}(\mathbf{r}')$ are the unknown currents on the scatterer. Equation (4.13) can be written in a matrix form as

$$\mathbf{J}_0 = \mathbf{J}(\mathbf{I} - \mathbf{B}) \quad (4.14)$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{B}$ is the integro-differential operator in eq. (4.13). The unknown currents $\mathbf{J}$ are obtained as

$$\mathbf{J} = (\mathbf{I} - \mathbf{B})^{-1} \mathbf{J}_0 \quad (4.15)$$

$$\mathbf{J} \approx (\mathbf{I} + \mathbf{B} + \mathbf{B}^2 + \mathbf{B}^3 + \dots) \mathbf{J}_0 \quad (4.16)$$

where the currents are approximated in eq. (4.16) using the Neumann se-

ries. Note that it is not granted that this series converges and the convergence depends on the $\mathbf{B}$ through the geometry of the scatterer. At each iteration, the currents are projected on the scatterer itself, whereas the method proposed in this work uses an equivalent surface.

In [115], electric and magnetic currents are also used for Physical Optics calculations. Modified PO electric and magnetic currents extend over the whole structure, regardless of the visible or shadowed regions, producing nearly zero fields in the deep shadow behind the scatterer. This approach bypasses any visibility testing procedure for isolated objects or multiple-scattering scenarios. This is different from our approach, for which electric and magnetic currents are located on an equivalence surface surrounding the object and illuminate the shadowed part of it.

4.2.3 Acceleration through multipole expansion

Obtaining the equivalent currents requires the computation of the total fields from the initial PO problem (eqs. (4.5) and (4.6)). This operation is the bottleneck of the proposed method; if it is performed naively, the computational complexity is $\mathcal{O}(MN)$, with (M, N) the number of basis functions on S and S' , respectively. The number of operations is large, since the standard PO approximation is valid for objects conforming to $kR \gg 1$. This difficulty is addressed with the help of the Fast Multipole Method (FFM) [25]. This method and its multilevel implementation (Multi-Level Fast Multipole Algorithm, MLFMA [128]), are usually employed in a MoM context for reducing the computation time and the memory requirement by speeding up matrix-vector products, enabling the analysis of scattering by large structures [6, 129, 130].

In this work, the two-dimensional FMM presented in Chapter 2 is used for the rapid computation of fields radiated on S' by the electrical currents on S . To this end, the contours S and S' are divided into smaller sub-domains, for which the radiation patterns and the translation operators are computed and stored. The reader may refer to Section 2.3 for more details on the rapid computation of the reaction integrals between sub-domains using the multipole decomposition. For near groups, an analytical singularity extraction is performed on the Green's function and its derivatives (see Section 4.7.3). The fast computation of eqs. (4.5) and (4.6) also requires the partial derivatives of the aggregation operator. Those are provided in Section 4.7.2.

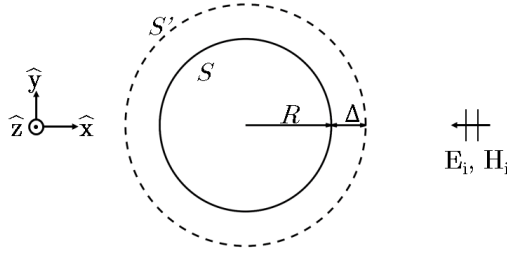


Fig. 4.5 Cylinder with a circular section under plane wave incidence. The PEC surface is represented by the solid line, while the equivalence surface is represented by the dotted line.

4.3 Scattering by isolated objects

This section presents the results obtained via the exact solution (analytical or via a full-wave MoM simulation), the classical PO approximation and the proposed method for various test cases. For each example, a single object is considered; the presence of multiple objects is studied in the next Section 4.4. The given examples consist of infinitely long cylinders with different convex contours, except for an attempt for a concave contour. The examples are thus considered as two-dimensional problems. Nevertheless, the proposed method can be generalized to three-dimensional problems, as presented in Section 4.5. As stated before, the equivalence surface is chosen so that the distance between S and S' is fixed and equal to the parameter Δ . The value of this parameter will be further discussed. The excitation comes from a plane wave, although the method is not restricted to this type of excitation. Except for the cylinder with a circular cross-section (Section 4.3.1), the incident electric field is aligned with the cylinder axis (TM case). Surfaces are discretized with pulse basis functions of maximal length $\lambda/8$ for the PO and the proposed method. For the MoM solution, the surface is sampled at $\lambda/15$. Except for the cylinder with a circular section which admits an analytical solution, other reference solutions are computed with the MoM code.

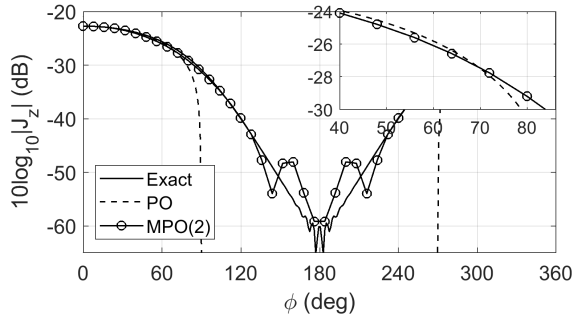


Fig. 4.6 Magnitude of the currents obtained from three different methods for a TM polarization. MPO(2) stands for the solution obtained with the proposed method after 2 iterations. The inset in a zoom around the shadow boundary.

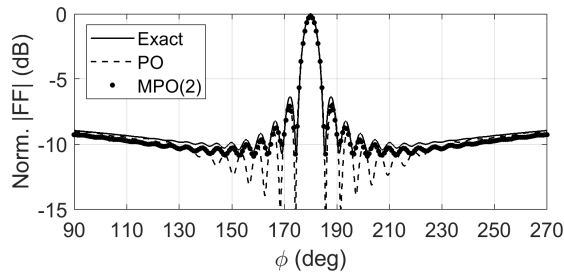


Fig. 4.7 Normalized far-field radiation pattern of the cylinder under TM incidence in the forward region $\phi \in [90^\circ, 270^\circ]$.

4.3.1 Circular section

TM polarization

As a first example, we consider the cylinder with a circular section of radius $R = 5\lambda$ represented in Figure 4.5, with the electric field polarization parallel to the cylinder axis $+\hat{z}$ (TM case) propagating towards $-\hat{x}$. The equivalence surface S' is also represented and has the same shape as the cylinder, with a radius $R' = R + \Delta$, with $\Delta = \lambda/10$. The impact of this parameter will be discussed below. This canonical problem admits an analytical solution for the surface currents, given by [12, Sec. 5.9]

$$J_z = \frac{-2}{\omega\mu\pi R} \sum_{n=-\infty}^{\infty} \frac{j^{-n} e^{jn\phi}}{H_n^{(2)}(kR)} \quad (4.17)$$

Comparison between the exact series solution from eq. (4.17), the standard PO formulation and the proposed approach (named MPO in the following figures) is displayed on Figure 4.6. It is observed that the MPO solution provides a good approximation of the currents near the shadow boundary [122] (boundary between the lit and the unlit regions), where the PO solution starts to deteriorate. Note that for parts of the scatterer located in the deep lit region, the PO provides a slightly better approximation of the MoM currents (inset of Figure 4.6), although simulation results show that this does not have a significant impact on the accuracy of the scattered fields.

The normalized far-field radiation pattern is illustrated in Figure 4.7. In the exact backscattering direction ($\phi = 0^\circ$), PO (as well as MPO) matches with the exact solution [120]. Fictitious zeros appearing with the PO solution around the forward region (periodically located at $\phi = \pi \pm n(\lambda/(2R))$) are reduced by the proposed method.

TE polarization

Let us now consider the same problem of Figure 4.5 with the electric field aligned with $-\hat{y}$ and still propagating towards $-\hat{x}$ (TE case), with the same dimension parameters. The analytical solution for the surface currents is given as [12, Sec. 5.9]

$$J_\phi = \frac{2j}{\omega\mu\pi R} \sum_{n=-\infty}^{\infty} \frac{j^{-n} e^{jn\phi}}{H_n^{(2)'}(kR)} \quad (4.18)$$

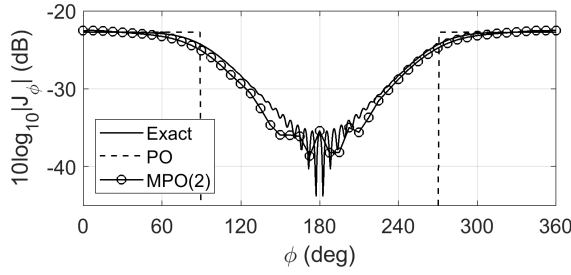


Fig. 4.8 Magnitude of the currents for the TE polarization.

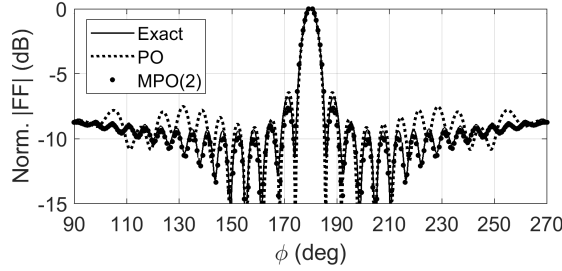


Fig. 4.9 Normalized far-field radiation pattern of the cylinder under TE incidence in the forward region ($\phi \in [90^\circ, 270^\circ]$).

From Figure 4.8, one can observe that the modified physical optics approach smoothens the abrupt current transition between the lit and unlit regions, as obtained with the classical PO, which assumes the magnitude of the currents as constant on the illuminated region. The comparison between radiation patterns is presented in Figure 4.9, where the contribution of the proposed method is quite obvious in the forward region.

Impact of the parameter Δ

The proposed method relies on a single parameter Δ which corresponds to the distance between the object and the surface on which the intermediate equivalent currents are computed. The authors observed that the smaller is Δ , the better the approximation of the currents. Nevertheless, one should ensure that, on one hand, at least one basis function of S is visible by each discretizing function on S' . That implies that if S and S' are too close to each other, the spatial sampling of the surface should be increased, thus increasing the computation time and the memory required for the com-

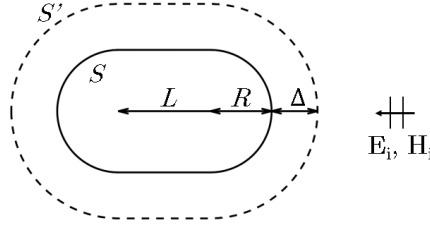


Fig. 4.10 Cylinder with elongated section under plane wave incidence (TM case).

putation of the near-field interactions. On the other hand, when a pair of basis functions on S and S' are very close, despite singularity extraction, a larger number of integration points is required for the computation of the equivalent currents due to the singular behavior of the Green's function for small $\mathbf{k}|\mathbf{r} - \mathbf{r}'|$ (see Section 4.2.3). As a rule of thumb, choosing the distance parameter as $\Delta \approx \text{size } l$ of basis functions is a good trade-off between accuracy and computational complexity.

4.3.2 Elongated cylinder

The second example is illustrated in Figure 4.10. The section of this cylinder corresponds to two half discs of radius $R = 5\lambda$ connected by a rectangle of length $L = 10\lambda$ and width $2R$. This is an example of interest since the initial PO currents are the same as in the problem analyzed in Section 4.3.1. Indeed, PO currents are assumed null if $\hat{\mathbf{n}} \cdot \hat{\mathbf{k}}_i = 0$ (i.e. the surface is parallel to the incident propagation vector). The surface currents are displayed on Figure 4.11. The modified PO solution approximates quite accurately the currents on the flat part of the object, within a 0.1 dB absolute error, whereas the classical PO assumes null currents on that part. Results for the MPO at iteration 0 and iteration 2 are compared. One can observe that for iteration 0, the proposed approach induces an abrupt change in the current distribution, located at the transition between the flat surface and the shadowed part. Those currents spurts are damped after an additional iteration, the MPO thus providing a good approximation of the exact solution, even for small current intensities. The number of additional iterations is fixed to 2 (hence, a total of three iterations, including the initial evaluation), which does not impact significantly the computation time, as discussed in Section 4.3.4. Since the method is based on the tangent-plane approximation

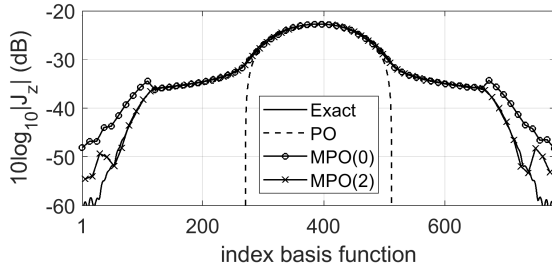


Fig. 4.11 Current distributions for the problem of Figure 4.10 (TM case).

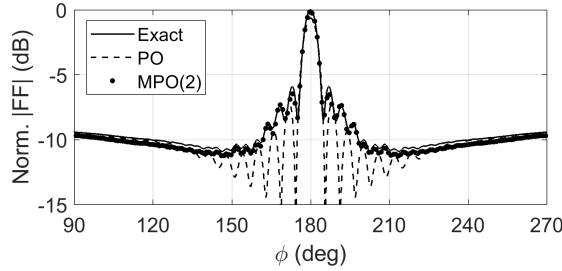


Fig. 4.12 Normalized far-field radiation pattern of the elongated cylinder ($R = 5\lambda$, $L = 10\lambda$) under TM incidence in the forward region ($\phi \in [90^\circ, 270^\circ]$).

(the same applies for classical PO), it is not expected that more iterations yield convergence towards the exact solution.

The radiation pattern corresponding to this problem in the forward region is provided in Figure 4.12. The fields radiated by the PO solution are the same as those presented in Figure 4.7, since the PO current distribution is the same for both examples. One can conclude that the proposed approach efficiently deals with curvature discontinuities [131] and nose-on illuminations for strongly elongated bodies, where standard PO fails [132]. The radiation pattern of a magnified version of the problem ($R = 50\lambda$, $L = 100\lambda$) is also provided in Figure 4.13. This shows that the proposed PO approach also contributes to a better estimation of the scattered fields for larger problems. Due to its large dimensions, the reference solution of this problem has been computed with a MoM-FMM approach.

A variation on this problem is illustrated in Figure 4.14, where a con-

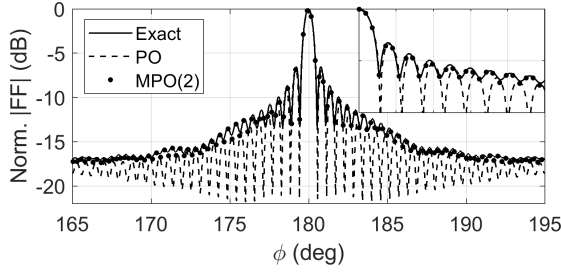


Fig. 4.13 Normalized far-field radiation pattern of the elongated cylinder ($R = 50\lambda$, $L = 100\lambda$) under TM incidence in the forward region. For a better visibility, the azimuth observation angle is limited to $\phi \in [165^\circ, 195^\circ]$. The inset is a zoom of the radiation pattern for $\phi \in [180^\circ, 185^\circ]$.

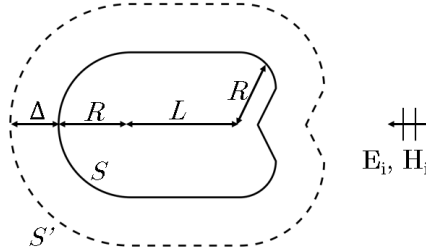


Fig. 4.14 Cylinder with elongated section and a concave part under plane wave incidence (TM case).

cave corner has been added to the half-circle facing the incident wave direction. The angle of the corner is chosen large enough in order to reduce the effect of higher-order reflections. The comparison of the radiation pattern for this example is provided in Figure 4.15, with a noticeable improvement on the accuracy of the scattered fields in the forward region, despite the little concave part in the contour, while classical PO performs better in the backward region.

4.3.3 Fields behind a scatterer

The following example analyzes the field distribution in the vicinity of the scatterer in the forward region. Previous examples focused on the far-field radiation pattern, while the proposed improvement of PO also provides a better estimation of the fields in the close vicinity of the scatterer. This re-

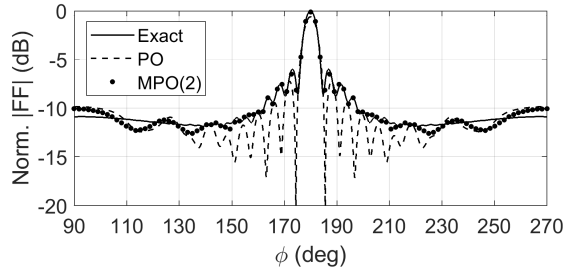


Fig. 4.15 Normalized far-field radiation pattern of the elongated cylinder with a concave part ($R = 5\lambda$, $L = 10\lambda$) under TM incidence in the forward region ($\phi \in [90^\circ, 270^\circ]$).

gion is of particular interest in different applications, such as electromagnetic compatibility, stealth technology, antennas installed on vehicles [103], weather radar blockage by wind turbines [133], etc. The proposed modification of the physical optics method greatly improves the accuracy of the fields in the region behind the object. Let us consider the same example as previously, with the scatterer in Figure 4.10. The total electric field computed with MoM is illustrated in Figure 4.16, where dashed lines represent different cuts in which the total field obtained with PO and the proposed approach are compared in Figure 4.17. The proposed method performs better than PO in the vicinity of the surface, as shown in Figures 4.17a and 4.17b. The poor performance of PO is expected due to the specific geometry of the problem, since the approximation does not hold for grazing incidence. The farther the observation, the better the accuracy of both PO and the proposed method. However, the method described here significantly outperforms the classical PO for any of the presented scenarios, with an absolute error below -30 dB for any observation in the forward region. The error is higher right behind the scatterer, where the object obstructs substantially the incident wave, producing a total field with a very small amplitude.

4.3.4 Computational complexity

The computation time required for the proposed approach is compared with the time required by a MoM-FMM solver. The reference test consists in computing the currents on an infinite cylinder of circular cross-section with a TM plane wave excitation (see Section 4.3.1), for increasing radii.

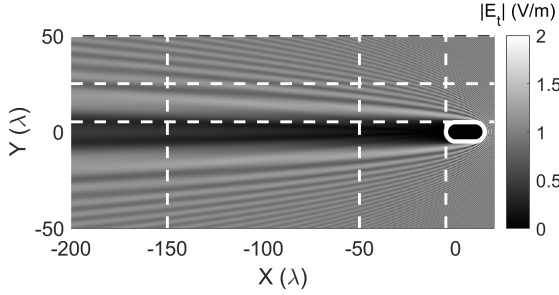


Fig. 4.16 Magnitude of the total electric field computed with MoM (TM case). The scatterer is illustrated in Figure 4.10 and represented with white solid contour. Dashed lines show the positions of the cuts in which fields are displayed in Figure 4.17.

The MoM solution is obtained by solving the electric field integral equation (EFIE), using pulse basis and testing functions, and is accelerated with the FMM for a fair comparison with the method presented in this work. The solution is computed iteratively using the generalized minimal residual error (GMRES) [35], for which the classical PO solution is used as the initial guess. The iterative process is stopped when the norm of the residual is smaller than 10^{-5} . The number of iterations is reduced by means of a sparse approximate inverse (SAI) preconditioner [134], with a nonzero pattern identical to the near-field impedance matrix. The MPO solution is taken after three iterations (i.e. MPO(2)), as presented in the aforementioned examples. For both methods, the size of the FMM groups is chosen optimally with respect to the FMM preparation time. This comprises the time T_{pat} required for the computation of the radiation patterns of the transmitting and receiving groups, the time T_{near} for the brute-force computation of the near-field interactions and the time T_{trans} for the translation operator. Those are computed and stored prior to the iteration process, and thus do not depend on the number of iterations. Considering a problem with N unknowns divided among G groups and H unknowns per group,

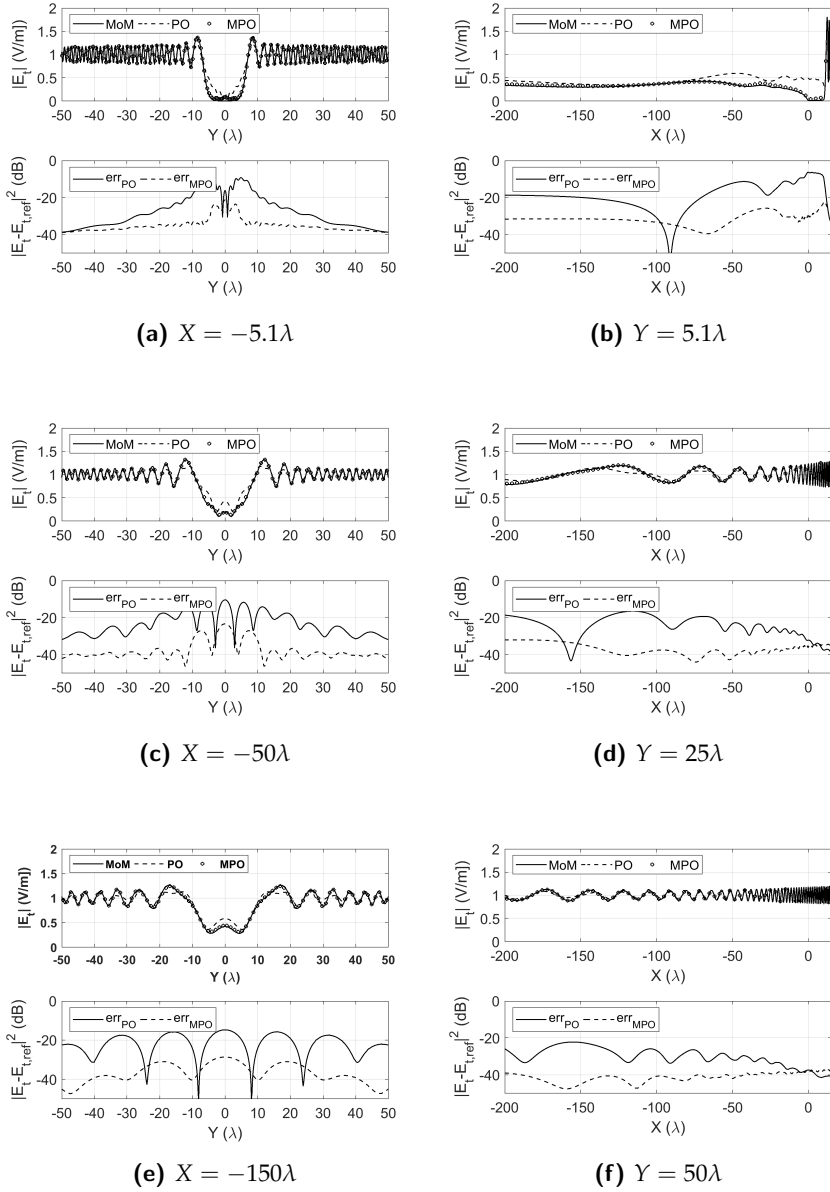


Fig. 4.17 Comparison of the total electric field of the problem described in Figure 4.10 computed by method of moments (MoM), the classical physical optics (PO) and the method proposed in this work (MPO) after two iterations for a TM incidence. The absolute error in a logarithmic scale is also provided for the six cuts drawn in Figure 4.16.

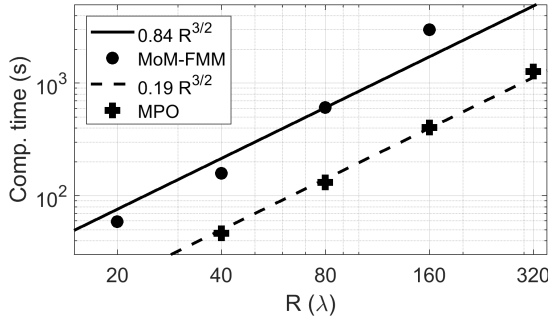


Fig. 4.18 Total CPU time for both MoM-FMM and MPO for different radii of an infinite cylinder with circle section under TM plane wave incidence.

so that $GH = N$, the above-mentioned times are expressed as

$$T_{\text{pat}} = C_1 NH \quad (4.19)$$

$$T_{\text{near}} = C_2 NH \quad (4.20)$$

$$T_{\text{trans}} = C_3 G^2 H = C_3 \frac{N^2}{H} \quad (4.21)$$

where C_1 , C_2 and C_3 are platform and software dependent. The value of H which minimizes the total construction time $T_{\text{constr}} = T_{\text{pat}} + T_{\text{near}} + T_{\text{trans}}$ is

$$H = \sqrt{\frac{C_3}{C_1 + C_2}} N \quad (4.22)$$

which leads to a computational complexity of $\mathcal{O}(N^{3/2})$. For the considered problems and the platform used, the constants C_1 , C_2 and C_3 are of the order 10^{-5} , 10^{-3} and 10^{-4} , respectively. Once those constants are known, the total CPU time for the modified physical optics is estimated using eqs. (4.19) to (4.21), where the time for the three iterations is neglected, as it is marginal compared to the FMM preparation (see Table 4.1). This is not the case for the MoM-FMM for which the number of iterations is not known *a priori* and the time required for reaching convergence is not negligible as compared to the total computation time.

For each considered radius, the maximal mesh element size is $\lambda/15$ for the MoM-FMM and $\lambda/8$ for the proposed approach. Those are typi-

	MoM-FMM	(N_{iter})	MPO	(N_{iter})
T_{pat}	2 s		4 s	
T_{trans}	284 s		217 s	
T_{near}	562 s		185 s	
T_{precond}	40 s		N.A.	
T_{iter}	1.02 s	(2044)	2 s	(3)
T_{sol}	2086 s		6 s	
T_{tot}	2934 s		412 s	

Table 4.1 Comparison of the computation time for the MoM and the MPO, for an infinite cylinder with a circular cross-section and a radius of $R = 160\lambda$.

cal values for the discretizing step for the MoM and the PO. In order to provide a fair comparison, the computation time is expressed with respect to the electrical size of the scatterer rather than the number of unknowns. The computations are performed on Intel i7-4790 at 3.6 GHz CPU workstation with 32 GB of RAM. The computation time versus the cylinder radius is shown in Figure 4.18. One can observe that the theoretical complexity order is met and is the same for both methods, whereas the MPO has a smaller absolute computation time than the MoM-FMM (factor 3.4 to 7.4 depending on the considered radii). The total CPU time T_{tot} includes the computation of the FMM pre-computations (i.e. the radiation patterns of the transmitting/receiving groups and the translation operator), as well as the computation of the near interactions impedance matrices for both methods. Besides, the MoM-FMM solution also requires the building of the preconditioner and converges after an *a priori* unknown number of iterations, usually growing with the electrical size of the object [135]. The proposed modification of the PO is attractive in this respect, since the number of iterations is fixed and limited to a maximum of three, after which no significant improvement on the accuracy has been observed. Those three iterations have a marginal cost (T_{iter} , with $T_{\text{sol}} = T_{\text{iter}} \cdot N_{\text{iter}}$) in terms of computation time, negligible compared to the FMM preparation and near interactions computations. We emphasize that, even when using the classical PO solution as the initial guess, the MoM-FMM solution obtained after three iterations is less accurate than the converged MPO(2). This is observed in Figure 4.19, where the MoM-FMM solution after three iterations starts to deviate from exact solution at 82° , whereas the MPO(2) curve is in

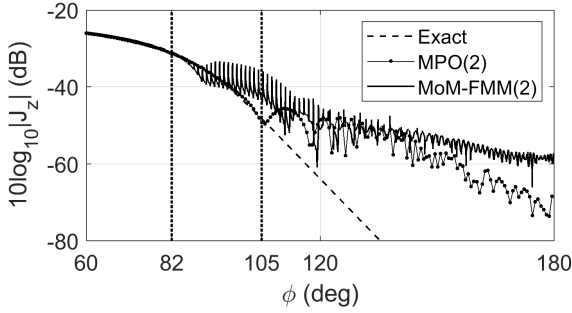


Fig. 4.19 Currents distribution for a cylinder with a circular cross-section and a radius $R = 100\lambda$. The vertical dotted lines correspond to 82° and 105° .

very good agreement with the reference currents up to 105° . In this region, the non-converged MoM-FMM currents are oscillating due to the division of the scatterer in FMM sub-domains and those currents are significantly less accurate than the MPO(2).

Table 4.1 summarizes the detailed CPU time for both methods for a cylinder of radius $R = 160\lambda$. It is observed that the time for one MPO iteration (T_{iter}) is roughly two times larger than for a MoM-FMM iteration; that is because the sources of the magnetic field are both electric and magnetic in the case of MPO, whereas it is purely electric for the MoM-FMM (assuming a PEC reflector).

4.4 Modified iterative physical optics for multiple-scattering scenarios

The proposed modified physical optics method is now employed in a multiple-scattering context. So far, only convex and slightly concave isolated objects have been considered. However, in many practical scenarios, multiple scatterers interact with each other (for example, the wind turbines presented in Section 2.5). The computation of the multiple reflections (or *bounces*) between different objects using the physical optics is commonly carried out using the iterative physical optics (IPO). At each iteration, the updated currents are obtained through a reaction integral using the currents of the previous iteration, including the visibility function of eq. (4.2). The IPO was first introduced in [110] for the asymptotic

evaluation of the surface currents for open cavities, where the number of iterations is related to the expected number of internal reflections. This method has been extended to other more complex and general geometries [111, 112, 113, 114, 103]. The two main difficulties arising when using IPO are the rapid evaluation of the reaction integral within an IPO iteration and the evaluation of the visibility function between different surfaces, both of complexities of the order of $\mathcal{O}(N^2)$.

Regarding the fast computation of the reaction integrals, one may employ the FMM or the IPW decomposition for distant sub-domains, as illustrated through different examples in this thesis (see Chapter 2). Another approach was proposed in [136], where the acceleration is based on a domain decomposition. The field scattered by each sub-domain is tabulated on a non-uniform spherical grid prior to an amplitude and phase extraction. The tested fields are then obtained using an interpolation and a phase re-introduction at the testing positions. Using this two level implementation, the computational complexity is reduced to $\mathcal{O}(N^{1.5})$. This method has been further extended in three-dimensions using a multilevel domain decomposition, thus achieving a computation complexity of $\mathcal{O}(N \log N)$ [137].

Regarding the visibility function determining which testing functions are "visible" by which basis functions, two different approaches are commonly considered. On one hand, one may use a *geometrical* visibility operator. In this case, a pair of basis and testing functions are "visible" if the ray connecting both elementary functions does not intersect any surface. Note that this operation with a complexity in $\mathcal{O}(N^2)$ is not trivial for complex geometries. This problem can be solved using computer vision algorithms, as the Z-buffering [138] or methods based on a convex hull transformation [118], for example. On the other hand, the shadow radiation method introduced by Ufimtsev [139] naturally compensates the incident field on the shadowed parts of the scatterer. This second approach enables a higher accuracy as compared to the geometrical visibility, as outlined in [112, 17]. Those two approaches are illustrated in Figure 4.20, where the initial currents on the scatterer A (Figure 4.20a) create a shadow on scatterer B located in the forward region (see Figure 4.20b). One can clearly observe the diffraction effect due to the scatterer A on scatterer B. For comparison, the initial currents using a geometrical visibility testing are illustrated in Figure 4.20c and abruptly drop to zero on the portion of scatterer B hidden by scatterer A.

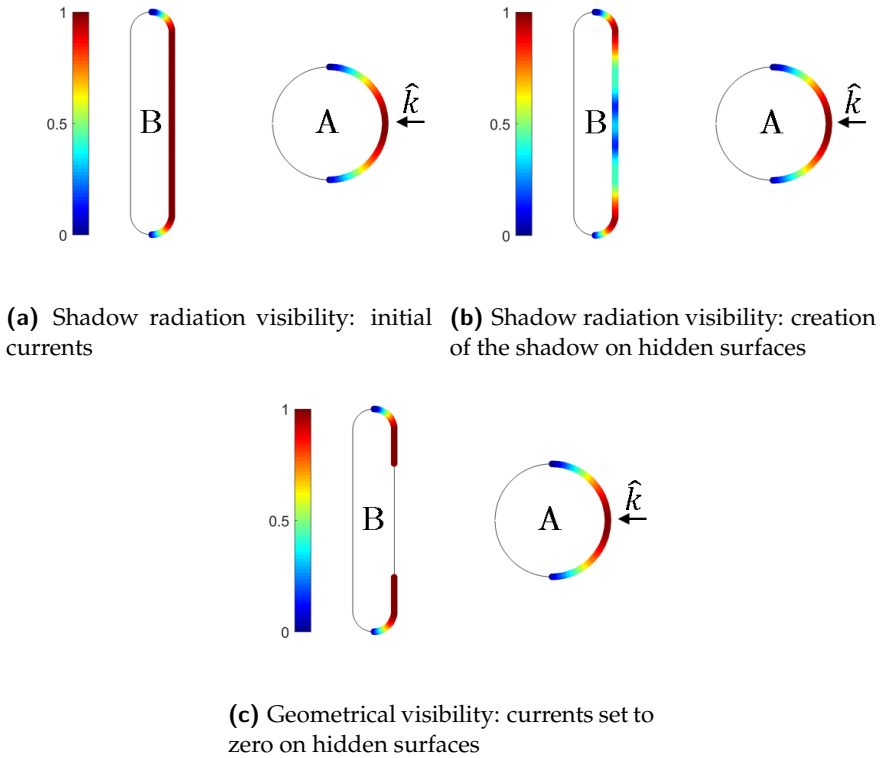


Fig. 4.20 Illustration of the shadow radiation on hidden surfaces and comparison with the geometrical visibility testing. The surfaces are represented by the thin black solid lines. The currents are normalized and for a better visibility, null currents are not represented

For all the iterative physical optics approaches, the converged solution strongly depends on the initial guess obtained using the classical PO formulation from eq. (4.1). The goal of this section is to increase the accuracy of an iterative PO scheme, by using the modified physical optics approach introduced in Section 4.2. The proposed iterative method relies on the shadow radiation mechanism for the determination of the visibility among different scatterers while the acceleration of the reaction integrals is performed using the FMM, where the sub-domains are considered as the entire individual scatterers. At each iteration, the updated PO currents are obtained with the modified physical optics, by computing the equivalent currents on an equivalent contour. The proposed method combines the method presented in [17] with the modified physical optics for isolated objects presented in Sections 4.2 and 4.3. The combination of the modified physical optics and an iterative PO solver increases significantly the accuracy of the scattered fields in the forward region for which the modified physical optics is designed.

4.4.1 Problem statement

Let us consider the problem depicted in Figure 4.21 which consists of four infinite cylinders. This particular case is selected on purpose as it represents simultaneously two extreme cases where i) multiple shadowings occur (for cylinders A, B & C which are aligned with the direction of propagation of the incident wave) ii) some scatterers are at the same distance with respect to the source (cylinders B & D). The radius of curvature of those scatterers should be large enough such that PO is applicable. A TM polarization is considered for all the examples.

In the initialization step, the currents on each scatterer are computed using the MPO as if each scatterer was isolated. This is performed by applying exactly the same procedure as described in Section 4.3, where each scatterer is taken individually. The magnitude of the resulting currents is represented in Figures 4.21a and 4.21b. Since the objects are identical, the magnitude of the currents is the same because of the constant amplitude of the incident plane wave in the direction of propagation. This is obviously not the case for the phase which varies linearly with the traveled distance. Given that the MPO is used, it is not surprising to observe non-zero currents on the back of the scatterers, similarly to what was obtained in Section 4.3. The currents in those regions are equal to zero if the classical physical optics is employed, as illustrated in [17, Fig. 2].

Shadow radiation

Obviously, because they were computed in isolation, the currents on shadowed scatterers B & C are not physical, as the amplitude of the incident field impinging those scatterers should be attenuated by the diffraction in the forward direction of scatterer A (as it was observed in the diffraction by wind turbines in Section 2.5). The attenuation caused by the blockage of scatterer A is modeled using the shadow radiation principle [139], by letting the initial currents on the surface of a given scatterer radiate in the direction of propagation of the incident wave. The fields radiated by those currents tend to cancel the incident field in the forward region. In other words, the shadow radiation occurs for scatterers located behind (with respect to the source) other scatterers. Mathematically, this condition is expressed by a logical operator $\mathcal{V}_{\text{sh}}$ defined as

$$\mathcal{V}_{\text{sh}}(i, j) = \begin{cases} 1 & \text{if } (\mathbf{r}_i^{\circ} - \mathbf{r}_j^{\circ}) \cdot \hat{\mathbf{k}} \geq 0 \\ 0 & \text{if } i = j \\ 0 & \text{otherwise,} \end{cases} \quad (4.23)$$

where $\mathbf{r}_i^{\circ}$ and $\mathbf{r}_j^{\circ}$ are respectively the centers of the source and testing domains corresponding to a full scatterer, and $\hat{\mathbf{k}}$ is the direction of propagation of the excitation (here considered as a plane wave). This operator is represented by black arrows in Figure 4.21 which leave the source domains and reach the testing domains when the shadow radiation condition in eq. (4.23) is met.

The corrected currents including the effect of the shadow radiation on a given scatterer are computed using the MPO, where the total field inside the equivalence surface is due to the contribution of the scatterer itself and the scatterers for which the shadow radiation is allowed according to eq. (4.23). The update of the currents follows the exact same steps as illustrated in Figure 4.4. For example, this operation is detailed for the computation of the corrected MPO currents on scatterer B. As represented in Figure 4.21, the currents $\mathbf{J}_0^{\text{B}}$ on scatterer B are subject to the shadow radiation due to the currents $\mathbf{J}_0^{\text{A}}$ and $\mathbf{J}_0^{\text{D}}$ on scatterers A and D, respectively (see Figure 4.22a). In a way similar to the case of isolated scatterers, the total field inside an equivalent contour S' encompassing scatterer B is equivalent to the field radiated by electric and magnetic equivalent currents $\mathbf{J}_{\text{Equ}}$, $\mathbf{M}_{\text{Equ}}$ located on S' with the external fields set to zero (see Figure 4.22b). Those

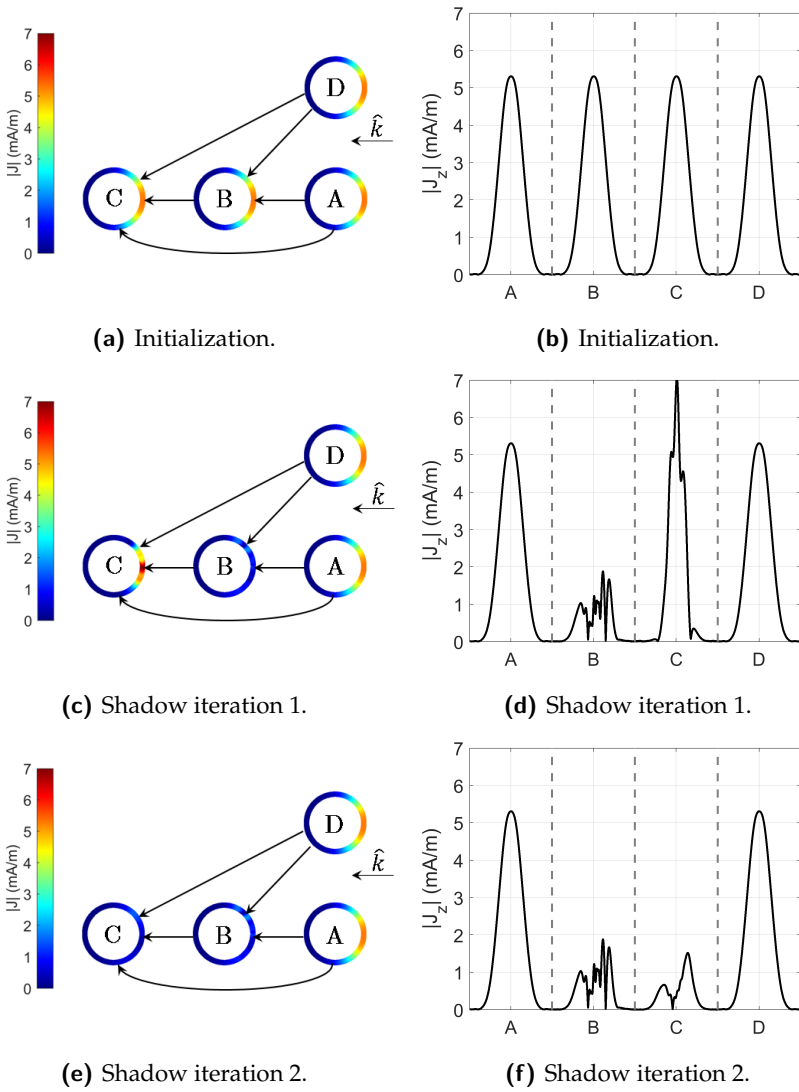


Fig. 4.21 Currents for the initialization and the first three iterations. They arrows corresponds to the allowed shadow radiation modes defined by Equation (4.23).

equivalent currents are expressed as

$$\begin{aligned}\mathbf{J}_{\text{Equ}}^{\text{B}} &= \hat{\mathbf{n}}' \times (\mathbf{0} - \mathbf{H}_t) \\ &= -\hat{\mathbf{n}}' \times \left(\mathbf{H}(\mathbf{J}_0^{\text{A}}) + \mathbf{H}(\mathbf{J}_0^{\text{B}}) + \mathbf{H}(\mathbf{J}_0^{\text{D}}) \right) \Big|_{S'}\end{aligned}\quad (4.24)$$

$$\begin{aligned}\mathbf{M}_{\text{Equ}}^{\text{B}} &= -\hat{\mathbf{n}}' \times (\mathbf{0} - \mathbf{E}_t) \\ &= \hat{\mathbf{n}}' \times \left(\mathbf{E}(\mathbf{J}_0^{\text{A}}) + \mathbf{E}(\mathbf{J}_0^{\text{B}}) + \mathbf{E}(\mathbf{J}_0^{\text{D}}) \right) \Big|_{S'}\end{aligned}\quad (4.25)$$

where the electric and magnetic fields $\mathbf{E}$, $\mathbf{H}$ are computed on S' using eqs. (4.5) and (4.6). The equivalent currents $\mathbf{J}_{\text{Equ}}$ and $\mathbf{M}_{\text{Equ}}$ then radiate a magnetic field on S , which induce a correction current $\tilde{\mathbf{J}}^{\text{B}}$ on scatterer B, computed with the standard physical optics as illustrated in Figure 4.22c. This correction current $\tilde{\mathbf{J}}^{\text{B}}$ in Figure 4.22d is mostly due to the shadow radiation of scatterer A which has a main lobe in the direction of scatterer B. The new currents on scatterer B are then obtained as

$$\mathbf{J}_0^{\text{B}} = \mathbf{J}_0^{\text{B}} + \tilde{\mathbf{J}}^{\text{B}} \quad (4.26)$$

It is clearly visible in Figures 4.21c and 4.21d that the magnitude of the currents on scatterer B has been reduced due to the shadowing of scatterer A and that some perturbation is visible due to the presence of scatterer D.

This process is repeated iteratively for all the sub-domains until the convergence of the shadow radiations. As explained in [17], the number of iterations depends on the number scatterers obscuring a given scatterer. As a rule of thumb, we consider that the number of shadowing iterations is equal to the maximal number of (partial) shadowing, depending on the problem geometry. In this example, scatterer B undergoes two shadowings (from A and D) and scatterer C undergoes three shadowings (from A, B and D). The number of shadow iterations for this problem is thus set to three iterations. This is quite critical, as a single shadow radiation still produces currents with a high amplitude on scatterer C (see Figures 4.21c and 4.21d). Those currents are damped after an additional shadow iteration (see Figures 4.21e and 4.21f).

Reflection radiation

After the convergence of the shadow iterations on all scatterers, the multiple-scattering phenomenon as such takes place. The currents obtained with eq. (4.26) now radiate back towards the source, thus

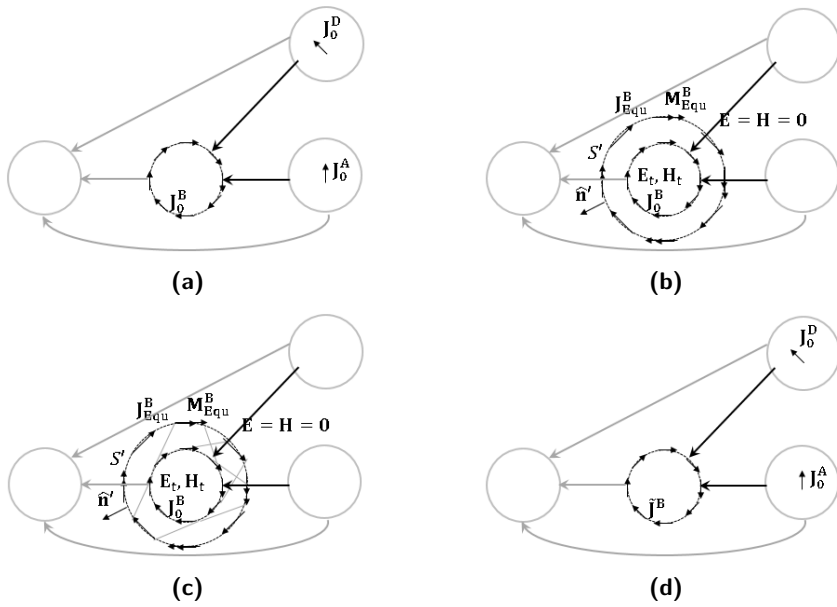


Fig. 4.22 Different steps for the computation of the correction currents $\mathbf{J}^B$ with the MPO.

illuminating the "back" of the scatterers. Note that since the MPO was used as the initial guess, as well as for the correction of currents due to the shadow iterations described above, the currents on the back of the scatterers are already not equal to zero. That would be the case if the standard physical optics was employed, as in [17].

A reflection occurs on a scatterer if it is located between another scatterer and the source. This condition is denoted mathematically with a another logical operator $\mathcal{V}_{\text{ref}}$ defined as

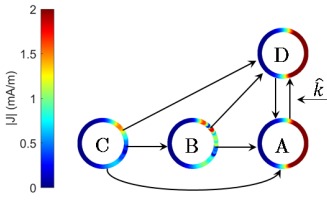
$$\mathcal{V}_{\text{ref}}(i, j) = \begin{cases} 1 & \text{if } (\mathbf{r}_i^{\circ} - \mathbf{r}_j^{\circ}) \cdot \hat{\mathbf{k}} < 0 \\ 0 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad (4.27)$$

For a better understanding of the latter operator, the allowed reflection modes are represented with arrows leaving the source sub-domains and reaching the testing sub-domains in Figure 4.23. As for the shadow radiation, the effects of the reflecting modes are computed using the modified physical optics. This is illustrated by detailing the reflection propagation for the scatterer A. As hinted by eq. (4.27) and the arrows in Figure 4.23, the scatterers B, C and D radiate fields back to scatterer A (see Figure 4.24a). By applying the same method as for the shadow radiations in the previous section, an equivalent contour S' is selected around scatterer A. By setting the fields outside S' to zero (see Figure 4.24b), the fields inside the contour can be described by equivalent currents on S' expressed as

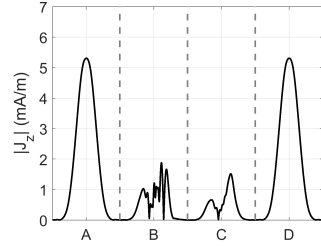
$$\begin{aligned} \mathbf{J}_{\text{Equ}}^{\text{A}} &= \hat{\mathbf{n}}' \times (\mathbf{0} - \mathbf{H}_t) \\ &= -\hat{\mathbf{n}}' \times \left(\mathbf{H}(\mathbf{J}_0^{\text{A}}) + \mathbf{H}(\mathbf{J}_0^{\text{B}}) + \mathbf{H}(\mathbf{J}_0^{\text{C}}) + \mathbf{H}(\mathbf{J}_0^{\text{D}}) \right) \Big|_{S'} \end{aligned} \quad (4.28)$$

$$\begin{aligned} \mathbf{M}_{\text{Equ}}^{\text{A}} &= -\hat{\mathbf{n}}' \times (\mathbf{0} - \mathbf{E}_t) \\ &= \hat{\mathbf{n}}' \times \left(\mathbf{E}(\mathbf{J}_0^{\text{A}}) + \mathbf{E}(\mathbf{J}_0^{\text{B}}) + \mathbf{E}(\mathbf{J}_0^{\text{C}}) + \mathbf{E}(\mathbf{J}_0^{\text{D}}) \right) \Big|_{S'} \end{aligned} \quad (4.29)$$

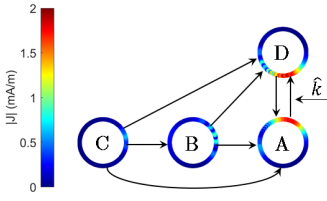
where the electric and magnetic fields $\mathbf{E}$, $\mathbf{H}$ are computed on S' using eqs. (4.5) and (4.6) and $\mathbf{J}_0^{\text{A,B,C,D}}$ are the currents obtained after the convergence of the shadow radiations (see eq. (4.26) for scatterer B). The new currents $\mathbf{J}_1^{\text{A}}$ are then computed with the standard physical optics on each elementary function of scatterer A (see Figure 4.24d). Given that the reflection modes radiate back towards the source, their impact is more visible on



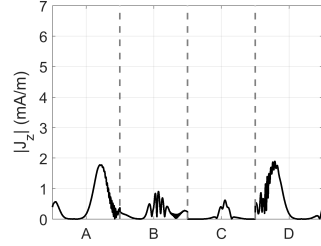
(a) Reflection iteration 0: J_0 .



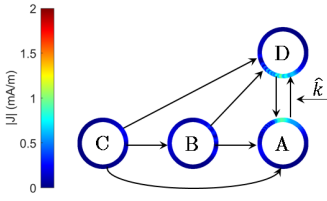
(b) Reflection iteration 0: J_0 .



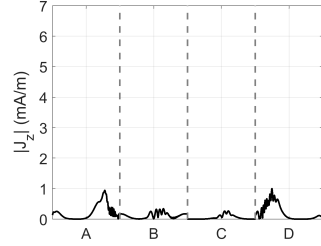
(c) Reflection iteration 1: J_1 .



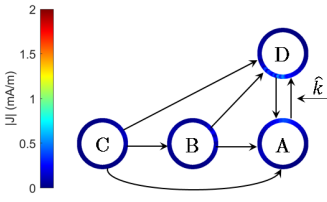
(d) Reflection iteration 1: J_1 .



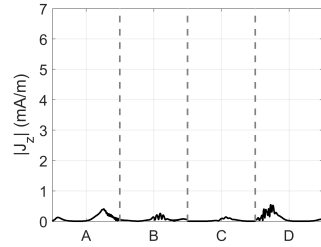
(e) Reflection iteration 2: J_2 .



(f) Reflection iteration 2: J_2 .



(g) Reflection iteration 3: J_3 .



(h) Reflection iteration 3: J_3 .

Fig. 4.23 Currents for the first three reflection iterations, corresponding to J_0, J_1, J_2, J_3 . The arrows correspond to the allowed shadow radiation modes defined by Equation (4.27).

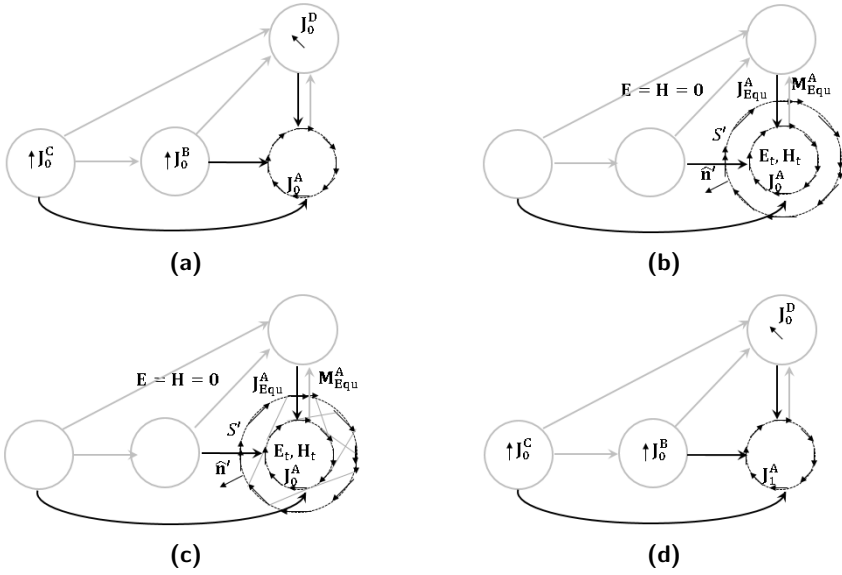


Fig. 4.24 Different steps for the computation of the new currents J_1^A after the first reflection iteration using the MPO.

the left part of scatterer A (which was initially shadowed). In particular, the currents J_0^B and J_0^C in eqs. (4.28) and (4.29) are relatively small, as they were attenuated by the previous shadow radiations. The same procedure is applied to the other sub-domains with their own allowed reflection modes, thus leading to a new set of currents $J_1^{A,B,C,D}$, illustrated in Figures 4.23c and 4.23d.

This reflection iteration is then followed by a new series of shadow iterations described in the previous section using the currents J_1 , before performing a reflection iteration providing new currents J_2 , etc. This process is performed iteratively until convergence of the solution and will be illustrated through different examples in Section 4.4.2. The pseudo-code of this algorithm (inspired and modified from [17]) is presented in algorithm 4, where the shadowing iterations are nested within the outer loop corresponding to the reflection iterations. The final currents J are then obtained as

$$J_{\text{MPO}} = \sum_{i=1}^{N_{\text{ref}}} J_i \quad (4.30)$$

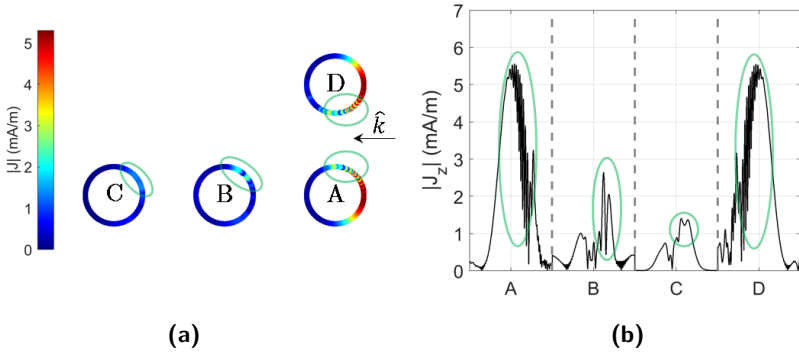


Fig. 4.25 Final currents using eq. (4.30).

The final currents represented in Figure 4.25 correspond to the sum of the outputs of each reflection iteration illustrated in Figure 4.23. In Figure 4.25a, one can observe rapid oscillations in the amplitude of the currents on scatterers A and D: they correspond to the interference between current contributions from different reflections between the portions of those scatterers facing each other. As expected, the magnitude of the currents is smaller on scatterer B and C, where the fields diffracted on scatterer D are clearly observed on scatterer B. Those different zones of interest are highlighted in Figure 4.25.

One should note that the visibility functions $\mathcal{V}_{\text{sh}}$ and $\mathcal{V}_{\text{ref}}$ are computed for entire sub-domains, where classical iterative physical optics computes the visibility for each pair of basis functions [110, 112, 17]. This is because in our method, the MPO is used for the update of the currents. Hence, the fields radiated by the neighboring scatterers during the shadow and reflection iterations are thus evaluated on the equivalent contour encompassing the testing sub-domain. The accuracy enabled by this approach is compared to the solution using a classical physical optics for updating the currents in Section 4.4.2.

Reduction of the computational complexity

In [17], the update of the currents is obtained using the classical PO formulation, which requires computing only the magnetic field radiated by electrical currents using the MFIE. This operation is accelerated using an interpolation of the fields based on a multilevel non-uniform grid, thus achieving a complexity in $\mathcal{O}(N \log N)$.

```

1 Initialization: compute  $\mathbf{J}_1$  on isolated scatterers using the MPO
2 Apply the reflection operator
3 for  $i = 1 \dots N_{ref}$  do
4   if  $i \neq 1$  then
5      $\mathbf{E}_{Equ}, \mathbf{H}_{Equ} = \text{reactionIntegral}(\mathbf{J}_{i-1}, \mathcal{V}_{ref});$ 
6      $\mathbf{J}_i = \text{MPO}(\mathbf{J}_{i-1}, \mathbf{E}_{Equ}, \mathbf{H}_{Equ});$ 
7   end
8    $\mathbf{J}' = \mathbf{J}_i;$ 
9   Apply the shadowing operator
10  for  $l = 1 \dots N_{sh}$  do
11     $\mathbf{E}_{Equ}, \mathbf{H}_{Equ} = \text{reactionIntegral}(\mathbf{J}', \mathcal{V}_{sh});$ 
12     $\mathbf{J}' = \mathbf{J}_i + \text{MPO}(\mathbf{J}', \mathbf{E}_{Equ}, \mathbf{H}_{Equ});$ 
13  end
14   $\mathbf{J}_i = \mathbf{J}'$  corresponds to Figures 4.23d, 4.23f and 4.23h;
15 end
16 Compute the final currents
17  $\mathbf{J}_{final} = \sum_{i=1}^{N_{ref}} \mathbf{J}_i$ 

```

Algorithm 4: Modified iterative physical optics.

In our implementation, the currents are obtained with the modified physical optics, which enables a better accuracy on the currents in the shadow region. It is thus necessary to compute the electric field as well as the magnetic field radiated by a given source sub-domain on the equivalent contours containing the testing domains (see Equations (4.24), (4.25), (4.28) and (4.29)). For a given sub-domain, the computation of the external electric and magnetic fields (lines 5 and 11 in algorithm 4) is accelerated using the FMM, where the FMM groups are taken as the entire sub-domains (as it was done in the example in Section 2.5.1). The FMM operators used for the fast computation of the fields on the equivalent contours are precomputed and stored. Given that only a single-level FMM has been implemented, the computation complexity of this operation is thus $\mathcal{O}(N^{1.5})$.

Once the electric and magnetic fields $\mathbf{E}_{Equ}, \mathbf{H}_{Equ}$ have been computed, the new currents are updated on each sub-domain using the formulation of the modified physical optics from Section 4.2. This operation is also accelerated with the FMM, where two inner iterations are performed (i.e. the solution corresponds to the MPO(2) currents, as for the examples in Section 4.3). The computational complexity of the proposed iterative scheme

does not compete with the multilevel implementation presented in [17]. The reason is that in this work, a single-level FMM is employed, and more reaction integrals are computed, as both electric and magnetic fields are required. Besides, the update of the currents is itself an iterative process, even though the number of inner iterations is limited to two. However, the proposed algorithm enables a significant increase of accuracy on the scattered fields, as presented here below.

4.4.2 Numerical results

Convergence of the iterative MPO

The convergence of the the solution obtained with the MPO and the shadow radiation with respect to the number of reflection iterations is illustrated in Figure 4.26 for three different test cases composed of a various number of identical infinite cylinders of radius $R = 5\lambda$. The error on the currents is computed as

$$\text{Err} = 10 \log_{10} \frac{|\mathbf{J}_{\text{MoM}} - \mathbf{J}_{\text{MPO}}|^2}{|\mathbf{J}_{\text{MoM}}|^2} \quad (4.31)$$

for different number of reflection iterations. One can observe that the test case 1 converges within a single reflection iteration, whereas this number increases for test case 2 & 3. This is explained as in test case 1, all scatterers are aligned with the wave vector, hence the currents after the first series of shadow iterations (as in Figure 4.21) have a relatively low amplitude due to the complete obstruction. When those currents radiate fields in the backscattering direction, they only induce small correction terms on the back of the scatterers. Those do not have a significant impact during the following shadow iterations. For test cases 2 & 3, some pairs of scatterers are visible without a complete shadowing; the currents obtained by the first reflection iterations thus have a larger amplitude so that more iterations are required before convergence of the solution.

For comparison, the convergence of the proposed algorithm is compared to the convergence of a classical iterative MoM using a block-diagonal preconditioner for each of the test cases. For the three considered examples, a smaller error is achieved within less iterations using the iterative MPO. However, a higher accuracy can be obtained with the MoM when more iterations can be afforded.

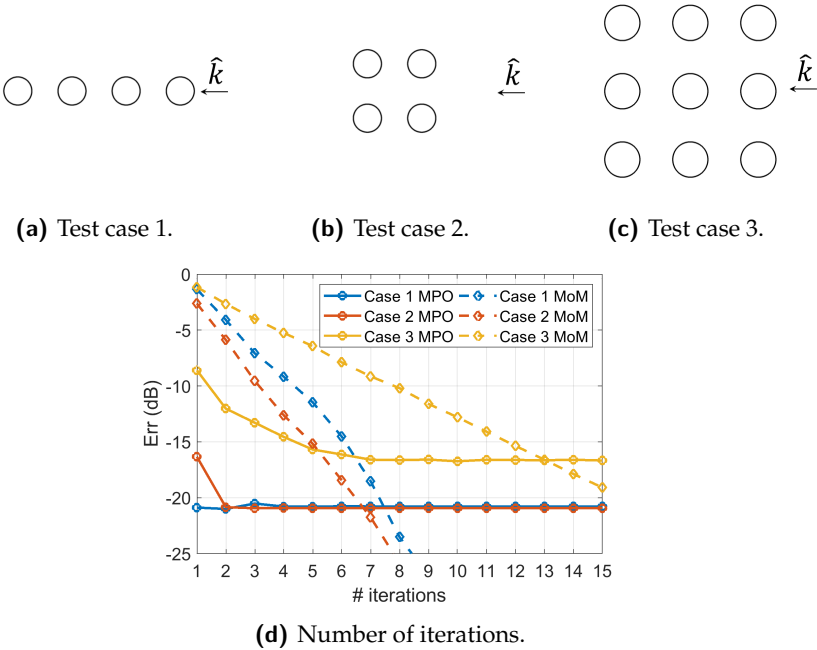


Fig. 4.26 Convergence with respect to the number of reflection iterations for three different test cases. For comparison, the convergence of the iterative MoM is represented with dashed lines.

Accuracy of the iterative MPO

The performance of the modified iterative physical optics is then evaluated by comparing the accuracy of the scattered fields with respect to the reference solution obtained with the method of moments in a multiple-scattering scenario. The obtained solution is also compared to a classical iterative physical optics solution using the shadow radiation as the visibility testing procedure [112, 17]. This approach is represented by the abbreviation IPO-SR, whereas the method proposed in this work is named MPO-SR in the following figures. In all the proposed examples, the excitation is induced by a plane wave propagating towards $-\hat{x}$ and with the electric field aligned with the infinite axis of the cylinders.

The first investigated case consists of four identical infinite cylinders with a circular cross section (radius $R = 5\lambda$). The geometry of the problem is represented in Figure 4.27a. As the multiple scattering between the different domains creates rapid variations on the far-field radiation pattern for a varying bistatic angle, only a small part of the radiation pattern is displayed in Figure 4.27b for more visibility. Although the IPO-SR already provides a fair estimation of the scattered fields, the MPO-SR significantly improves the accuracy, in particular near the forward region. The mean error over the range of bistatic angles is -32 dB for the former and -43 dB for the latter, with the error metric defined by

$$\text{Rel. error} = 10 \log_{10} |f_{\text{MoM}} - \tilde{f}|^2 \text{ (dB)} \quad (4.32)$$

where f_{MoM} corresponds to the fields radiated by the MoM solution and $\tilde{f}$ are the fields radiated by the IPO-SR or the MPO-SR solution. The improvement provided by the modified iterative physical optics is even more visible in the region close to the scatterers, as illustrated in Figures 4.27c and 4.27d. Note that the proposed approach also improves the accuracy of the fields in the backscattering region (i.e. towards $+\hat{x}$).

The gain of using the modified physical optics in an iterative scheme is even more pronounced with scatterers for which the accuracy of the classical PO is limited (i.e. elongated objects or surfaces at grazing incidence, see Section 4.3.2). This is illustrated in Figure 4.28.

The improvement is particularly visible behind the scatterer 3, as it is explained by analyzing the currents on its surface in Figure 4.29. In the region A, the scatterer 3 is not directly illuminated by the incident wave

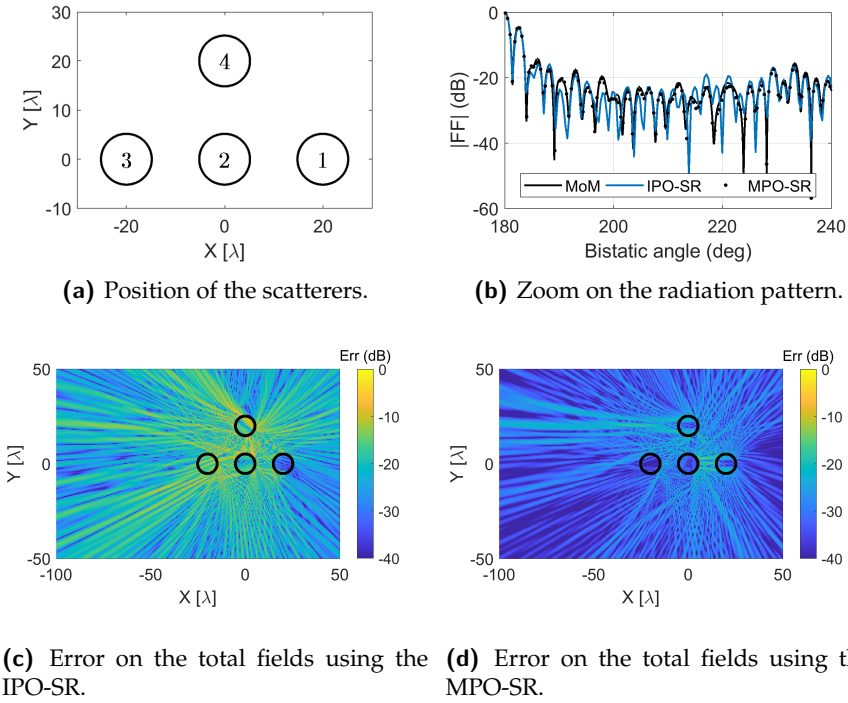


Fig. 4.27 Multiple scattering between four identical infinite cylinders with a circular cross section.

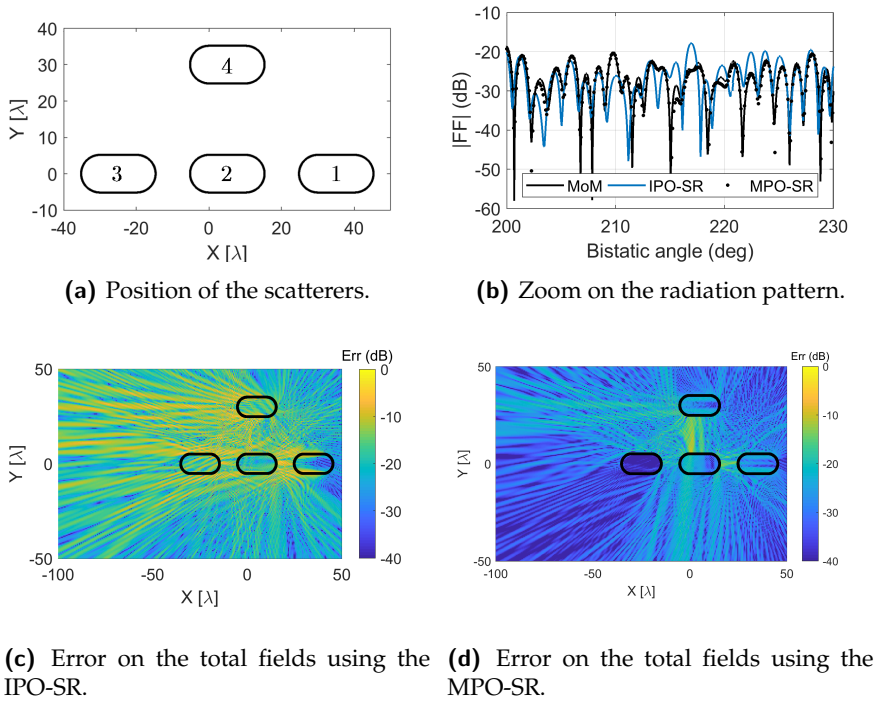
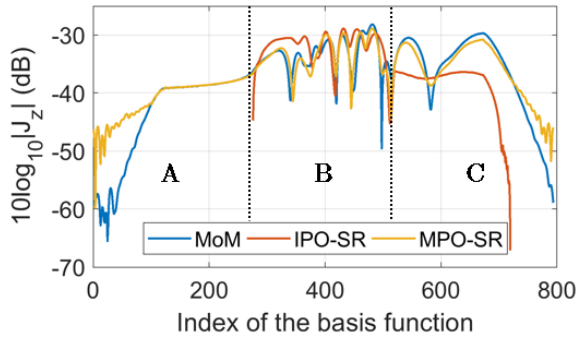
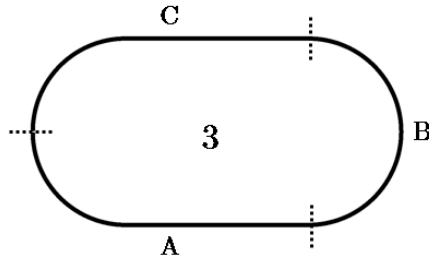


Fig. 4.28 Multiple scattering between four identical infinite cylinders with an elongated cross section.



(a) Currents distribution on scatterer 3.



(b) Corresponding sections of scatterer 3.

Fig. 4.29 Currents and corresponding zones on scatterer 3 from the multiple-scattering problem depicted in Figure 4.28.

neither by a reflection on a neighboring object. For this reason, the IPO-SR currents are equal to zero, whereas the MPO-SR currents are very close to the MoM currents. Large differences are also noticed in the region C for the same reason: the classical PO does not model accurately the currents on surface with grazing incidence and assumes null currents in the non-illuminated part. One can observe that the MPO-SR is also a good approximation of the MoM currents in this region. The accuracy of the IPO-SR is better in the region B facing the source, even though the currents obtained with the modified physical optics are closer to the actual MoM solution.

Another example presented in Figure 4.30 considers five identical scatterers aligned with the incident wave vector. This example is interesting

4 | Improved physical optics computation in forward scattering

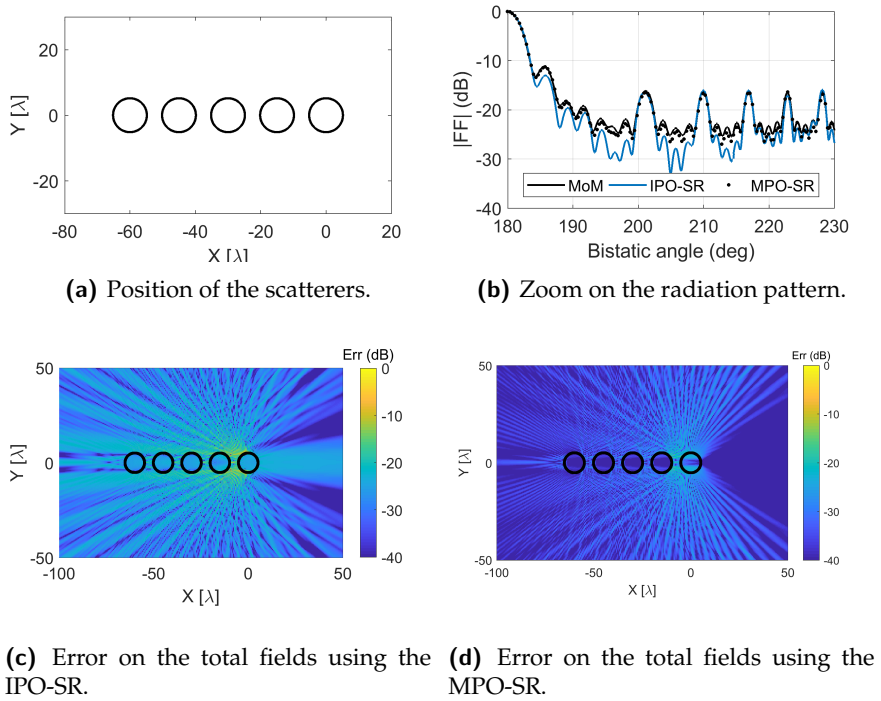


Fig. 4.30 Multiple scattering between five aligned infinite cylinders.

because the first scatterer shadows completely the others. This example cannot be solved using the geometrical visibility method. Indeed, the initial currents on the hidden scatterers would be equal to zero due to the blockage by the first scatterer (see Figure 4.20c). In this case, the shadow radiation approach is particularly useful as it allows one to approximate the currents on the shadowed scatterers quite accurately through successive shadow and reflection radiations.

For the sake of simplicity, the previous examples were composed of the same elements. However, the iterative process works for scenarios composed of different scatterers, as long as the physical optics is a good approximation for the considered geometries (i.e. for smooth surfaces with large radius of curvature with respect to the wavelength). This is illustrated by the results in Figure 4.31, where four different scatterers interact with each other and where partial shadowing occurs. Again, it is quite clear that the combination of the modified physical optics with the shadow radiation

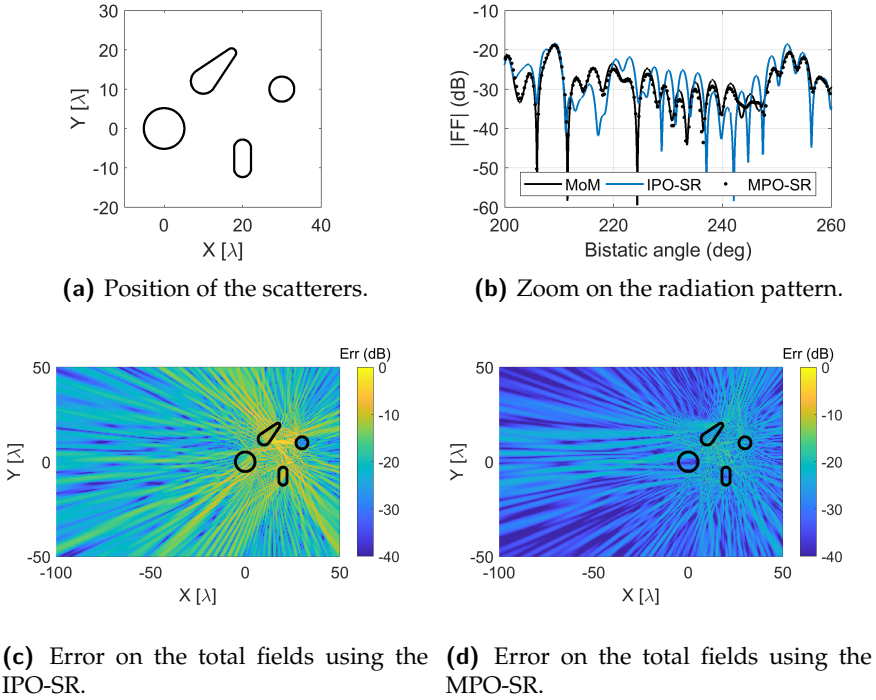


Fig. 4.31 Multiple scattering between four different infinite cylinders.

for modeling the multiple-scattering effects enhances the accuracy of the scattered fields, both in the near-field (Figures 4.31c and 4.31d), and in the far-field (Figure 4.31b).

4.5 Extension to three-dimensional scatterers

So far, the modified physical optics has been presented and validated for two-dimensional scenarios. However, the method relies on three principles which are all valid in two dimensions as well as in three dimensions, namely the physical optics approximation, the equivalence theorem and the fast multipole method for the acceleration of the reaction integrals. The implementation of the MPO in 3D is the generalization of the method presented in Section 4.2, where the major modifications are the following:

- The equivalent currents are computed on a closed equivalence sur-

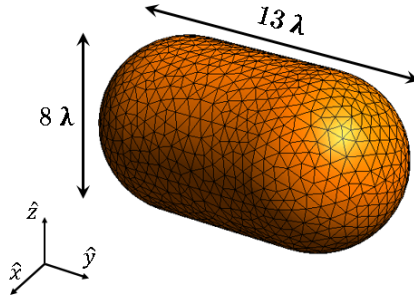


Fig. 4.32 Three-dimensional PEC scatterer.

face encompassing the scattering (instead of an equivalent contour)

- The scalar Green's function G is modified as

$$G = \frac{e^{-jk|\mathbf{r}' - \mathbf{r}|}}{4\pi|\mathbf{r}' - \mathbf{r}|} \quad (4.33)$$

- The surface currents are discretized using triangular basis functions with a constant amplitude

Although three-dimensional scattering problems are commonly discretized with RWG basis functions [83], the computation of the PO currents in Equation (4.1) involves a cross product which is not trivial and is more easily performed using individual basis functions. The scatterer considered for the modified physical optics in three dimensions is represented in Figure 4.32 and consists of a circular cylinder with hemispherical caps. The maximal dimensions of the object are $8 \times 8 \times 13\lambda^3$. The discretizing triangular functions are obtained with the software GMSH [84] and have an average area of approximately $\lambda^2/100$ (the triangular basis functions in Figure 4.32 are magnified for more visibility), so that the scatterer is described by roughly $5 \cdot 10^4$ basis functions. The incident wave is propagating towards $-\hat{y}$ with an electric field aligned with $+\hat{z}$. The equivalence surface on which $\mathbf{J}_e$, $\mathbf{M}_e$ are computed (see eqs. (4.3) and (4.4)) is centered on the scatterer with a distance of $\lambda/10$ between the scatterer surface and the equivalence surface (not represented in Figure 4.32 for clarity).

The normalized radiation pattern of the considered scatterer in the forward region (i.e. for $\theta \in [0 : \pi]$ and $\phi \in [\pi/2; 3\pi/2]$) using the MoM

(obtained with the PUMA-EM¹), the MPO and the classical PO is illustrated in Figures 4.33a, 4.33b and 4.33d, respectively, with their associated error in Figures 4.33c and 4.33e. One can observe that the MPO solution is more accurate than the classical PO solution in the forward region. This is particularly true for elongated objects where the accuracy deteriorates for low incidence angles, as it was outlined in the 2D validation approach.

The proposed 3D example is of interest, as it reveals that the MPO is not restricted to two-dimensional scenarios. However, one should bear in mind that the MPO in its current implementation is quite cumbersome in terms of computation time and memory usage. Since the first approximation of the MPO currents corresponds to the PO solution, it implies that sufficiently large objects with respect to the wavelength should be considered for the PO solution to be valid. Although the computational complexity and the memory usage involved in the computation of the electric and magnetic currents on the equivalence surface have been reduced from $\mathcal{O}(N^2)$ to $\mathcal{O}(N^{1.5})$ thanks to a single-level FMM, the MPO for 3D objects increases significantly the total computational cost as compared to the classical PO or state-of-the-art accelerated full-wave methods. Although the same methods are used for the matrix-vector products in an iterative MoM and for the computation of the equivalent currents in the MPO approach, it is expected that the number of iterations using the latter approach (limited to three iterations) is much smaller than the number of MoM iterations (as it is the case in two dimensions).

4.6 Conclusion

A modified version of the classical physical optics solution has been proposed in this chapter. This method provides a more accurate solution where the classical physical optics fails, namely in the forward region, by finding an approximation of the currents induced on the shadowed part of the object, where the classical PO assumes zero currents. Those currents are obtained by illuminating the shadowed part of the scatterer with equivalent currents flowing on a surface surrounding the object, acting as the source of new incident fields.

Simulation results show that the new approach significantly enhances the accuracy of the estimated scattered fields compared to the classical PO,

¹<http://puma-em.sourceforge.net/>

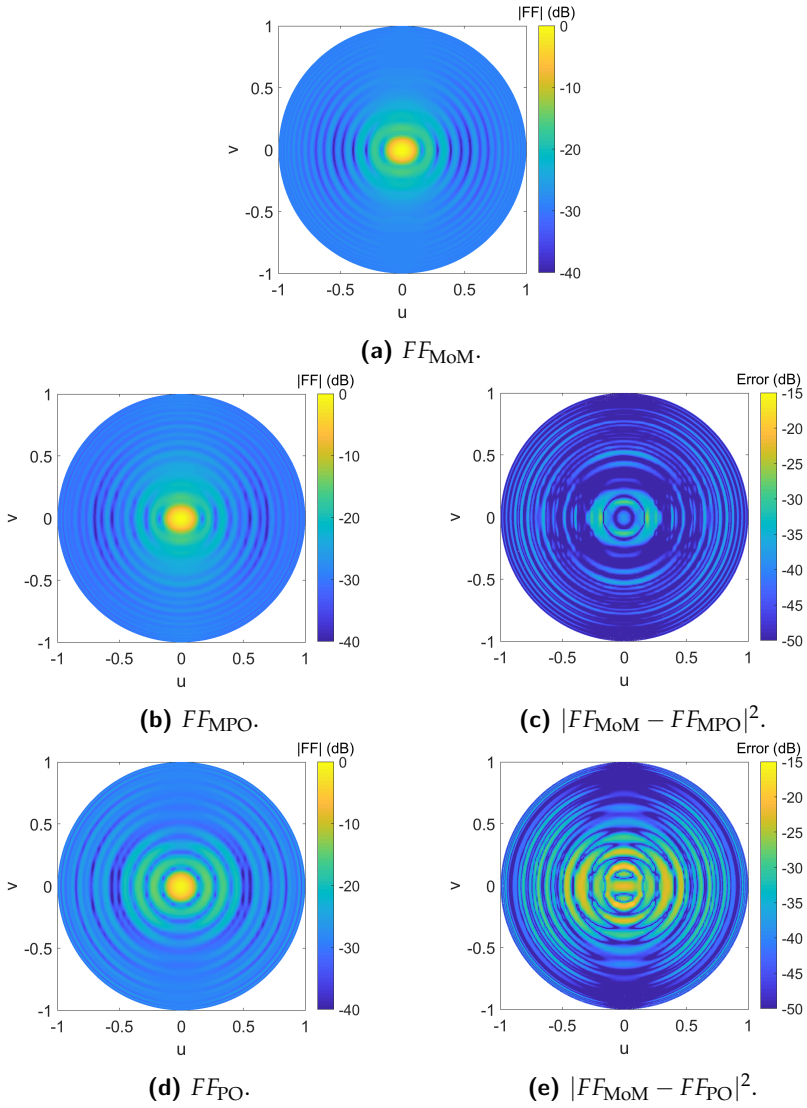


Fig. 4.33 Normalized bistatic radiation pattern in the forward region and the error associated to the MPO and PO approach with respect to the MoM reference.

in particular for observation points located behind the scatterer. Indeed, results displayed on Figure 4.17 show that the absolute error in this region is below -30 dB, which is reduced by 10 to 15 dB with respect to the classical PO. In particular, the method is useful for nose-on illumination problems or when the surface exhibits discontinuities (Section 4.3.2), as well as for a TE polarized incident field, where the classical PO delivers a poor approximation in the non-specular region (Section 4.3.1).

The computation of those equivalent currents is the bottleneck of the algorithm and is thus accelerated in two ways. On one hand, for interactions between basis and testing groups on S and S' which are far from each other, a lower complexity is reached with the help of the single-level FMM, which reduces the complexity from $\mathcal{O}(N^2)$ to $\mathcal{O}(N^{3/2})$, with N being the number of basis and testing functions. For close groups, on the other hand, the integration has to be computed explicitly with eqs. (4.5) and (4.6). These integrals require a large number of integration points due to the singularity of the Green's function for small argument. This constraint is alleviated by performing an analytical singularity extraction without loss of accuracy (Section 4.7.3). Simulation results have shown that, although the asymptotic complexity of this method is the same as for the MoM, smaller absolute CPU times (factor of order 1/5) are achieved with the proposed method for scatterers having the same electrical size.

The proposed PO modification has been applied to multiple-scattering scenarios, where various disconnected objects interact with each other through a given number of "bounces". The effect of multiple objects is modeled using an existing iterative scheme which updates the currents on each sub-domain at a given iteration using the fields radiated by the neighboring sub-domains at the previous iteration. The explicit mutual visibility testing between sub-domains is avoided thanks to the shadow radiation which naturally compensates for the incident field on sub-domains (partially) shadowed by other sub-domains, as proposed in [112, 17]. Results have shown that the proposed modified physical optics is particularly useful for multiple-scattering scenarios, where the MPO significantly increases the accuracy of the scattered fields, as it was demonstrated through different examples. The computational efficiency of the proposed method is limited by the computation of the reaction integrals between sub-domains. In this work, this operation is accelerated with the help of the single-level FMM, enabling an asymptotic complexity in $\mathcal{O}(N^{1.5})$. Other propagation operators should be investigated for reducing the cost of this operation,

such as the inhomogeneous plane wave decomposition presented in Section 2.4, which outperforms the FMM for disconnected and distant subdomains.

Finally, the principles of the MPO have been applied to more realistic three dimensional scenarios. It was shown that for a geometry with flat surfaces and a grazing illumination, the MPO in three dimensions improves the accuracy of the scattered fields in the forward region. However, one should bear in mind that the current implementation with a single level FMM does not enable the simulation of very large objects, due to their high associated computation time and memory required for the storage of the near interaction groups and the FMM operators. Other propagation techniques should be investigated in order to reach a more attractive computation time. Among them, multilevel decomposition methods (MLFMA [128, 20], or MLNG [137], for example) can be considered, as they can lead to computational complexities in $\mathcal{O}(N \log(N))$.

4.7 Appendices

4.7.1 Explicit derivation of the MPO for TM incidence

Let assume a two-dimensional TM problem, where the electric field of the wave illuminating the object is parallel to the axis of the PEC cylinder, as depicted in Figure 4.2. Under these conditions, the initial PO currents on the object are given by:

$$\mathbf{J}_{\text{PO}} = [0, 0, J_{\text{PO}}^z]^T, \quad \mathbf{M}_{\text{PO}} = \mathbf{0} \quad (4.34)$$

The currents $\mathbf{J}_e$ and $\mathbf{M}_e$ are obtained with the help of eqs. (4.3) and (4.4), where $\mathbf{E}_t = \mathbf{E}_i + \mathbf{E}_{\text{PO}}$ and $\mathbf{H}_t = \mathbf{H}_i + \mathbf{H}_{\text{PO}}$. The fields $\mathbf{E}_{\text{PO}}$ and $\mathbf{H}_{\text{PO}}$ radiated by the PO initial currents are computed as:

$$\mathbf{E}_{\text{PO}}^z = -j\omega\mu \int_S J_{\text{PO}}^z g \, dS \quad (4.35)$$

$$\mathbf{H}_{\text{PO}}^x = \int_S J_{\text{PO}}^z \frac{\partial g}{\partial y} \, dS \quad (4.36)$$

$$\mathbf{H}_{\text{PO}}^y = - \int_S J_{\text{PO}}^z \frac{\partial g}{\partial x} \, dS \quad (4.37)$$

The equivalent currents on S' are expressed as:

$$\mathbf{J}_e = [0, 0, J_e^z]^T, \quad \mathbf{M}_e = [M_e^x, M_e^y, 0]^T \quad (4.38)$$

The updated electrical current is obtained with the help of eq. (4.12), where the magnetic field $\mathbf{H}_e$ radiated by $\mathbf{J}_e$ and $\mathbf{M}_e$ is expressed as:

$$\begin{aligned} H_e^x = & - \int_{S'} j\omega\epsilon \left[M_e^x g + \frac{1}{k^2} \left(M_e^x \frac{\partial^2 g}{\partial x^2} + M_e^y \frac{\partial^2 g}{\partial x \partial y} \right) \right] dS' \\ & + \int_{S'} J_e^z \frac{\partial g}{\partial y} dS' \end{aligned} \quad (4.39)$$

$$\begin{aligned} H_e^y = & - \int_{S'} j\omega\epsilon \left[M_e^y g + \frac{1}{k^2} \left(M_e^x \frac{\partial^2 g}{\partial x \partial y} + M_e^y \frac{\partial^2 g}{\partial y^2} \right) \right] dS' \\ & - \int_{S'} J_e^z \frac{\partial g}{\partial x} dS' \end{aligned} \quad (4.40)$$

The partial derivatives involved in the above equations are provided in Section 4.7.3. This example can be derived similarly for the TE problem.

4.7.2 FMM for the derivatives of the Green's function

The multipole decomposition also holds for the derivatives of the Green's function, and are listed here below. Since the differential operators in eqs. (4.5) and (4.6) apply on the observation coordinates, the differentiation operates solely on the receiving pattern.

$$P(x, y, \hat{\alpha}) = \exp(-jk\hat{\alpha} \cdot \mathbf{r}_{mm'}) \quad (4.41)$$

$$\frac{\partial}{\partial x} P(\cdot) = P(\cdot)jk \cos \alpha \quad (4.42)$$

$$\frac{\partial}{\partial y} P(\cdot) = P(\cdot)jk \sin \alpha \quad (4.43)$$

$$\frac{\partial^2}{\partial x^2} P(\cdot) = -P(\cdot)k^2 \cos^2 \alpha \quad (4.44)$$

$$\frac{\partial^2}{\partial y^2} P(\cdot) = -P(\cdot)k^2 \sin^2 \alpha \quad (4.45)$$

$$\frac{\partial^2}{\partial xy} P(\cdot) = -P(\cdot)k^2 \cos \alpha \sin \alpha \quad (4.46)$$

4.7.3 Analytical singularity extraction

We provide here the expression of the Green's function and its partial derivatives as well as their singular part around the origin. Thanks to the analytical singularity integration, the number of sampling points is significantly reduced when the argument of the Green's function gets small. In the following equations, $\rho = |\mathbf{r}' - \mathbf{r}|$, with $\mathbf{r} = x \hat{\mathbf{x}} + y \hat{\mathbf{y}}$ and $\mathbf{r}' = x' \hat{\mathbf{x}} + y' \hat{\mathbf{y}}$.

$$G(x) = \frac{-j}{4} H_0^{(2)}(k\rho) \quad (4.47)$$

$$G_s(x) = \frac{-j}{2\pi} \log(k\rho/2) \quad (4.48)$$

$$\frac{\partial}{\partial x} G = \frac{-jk}{4} H_1^{(2)}(k\rho) \frac{x' - x}{\rho} \quad (4.49)$$

$$\frac{\partial}{\partial x} G_s = \frac{1}{2\pi} \frac{x' - x}{\rho^2} \quad (4.50)$$

$$\frac{\partial}{\partial y} G = \frac{-jk}{4} H_1^{(2)}(k\rho) \frac{y' - y}{\rho} \quad (4.51)$$

$$\frac{\partial}{\partial y} G_s = \frac{1}{2\pi} \frac{y' - y}{\rho^2} \quad (4.52)$$

$$\begin{aligned} \frac{\partial^2}{\partial x^2} G = \frac{jk^2}{4\rho^2} & \left(\frac{(x - x')^2}{2} (H_0^{(2)}(k\rho) - H_2^{(2)}(k\rho)) + \right. \\ & \left. \frac{(y - y')^2}{\rho} H_1^2(k\rho) \right) \end{aligned} \quad (4.53)$$

$$\frac{\partial^2}{\partial x^2} G_s = \frac{-k^2}{8\pi} \left(1 + \frac{4}{k^2 \rho^2} + 2 \log(k\rho) \right) \quad (4.54)$$

$$\frac{\partial^2}{\partial y^2} G = \frac{jk^2}{4\rho^2} \left(\frac{(y-y')^2}{2} \left(H_0^{(2)}(k\rho) - H_2^{(2)}(k\rho) \right) + \frac{(x-x')^2}{\rho} H_1^2(k\rho) \right) \quad (4.55)$$

$$\frac{\partial^2}{\partial y^2} G_s = \frac{\partial^2}{\partial x^2} g_s \quad (4.56)$$

$$\frac{\partial^2}{\partial x \partial y} G = \frac{jk}{4\rho^2} \left(\frac{k(x-x')(y-y')}{2} \left(H_0^{(2)}(k\rho) - H_2^{(2)}(k\rho) \right) + \frac{(x-x')(y-y')}{\rho} H_1^2(k\rho) \right) \quad (4.57)$$

$$\frac{\partial^2}{\partial x \partial y} G_s = \frac{4 + k^2 \rho^2}{4\pi \rho^4} (x-x')(y-y') \quad (4.58)$$

All results involving G_s can be integrated analytically.

5

Impact of the human body at millimeter waves

5.1 Introduction

THE number of devices connected to mobile networks have increased significantly over the last decade. It is estimated to be 22 billion at the end of 2018 with an expected growth up to 40 billion by the end of 2025 [140]. Besides smartphones, tablets and laptops, connected devices now include wearables, home assistants, thermostats, security sensors, connected speakers and screens, IoT devices, etc. For those applications, wireless connectivity is mandatory as wired connections cannot be afforded. On top of that, most of the services such as video calls, high-resolution video streaming or virtual reality handsets require a high throughput and a low latency for a better user's experience. High data rates and devices mobility are to be reached with the fifth-generation (5G) cellular networks standards. Those are defined over a large frequency spectrum, ranging from the sub-6 GHz and the beyond 6 GHz bands. Millimeter-wave frequencies are a good candidate for which a large unlicensed bandwidth is available, thus enabling data rates of the order of ten times those of today's networks. The small electrical size of antennas

at those frequencies reduces the size and weight of equipment which is an advantage for small connected devices such as wearables, for example.

The main drawback of mmWaves is the increased attenuation for a fixed transmitter and receiver position, compared to sub-6 GHz frequency bands. It is well-known that the free-space path loss (FSPL) increases with the square of the wavelength, as expressed by the Friis transmission formula:

$$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2 \quad (5.1)$$

where $P_{r(t)}$ is the received(transmitter) power, $G_{r(t)}$ in the receiving(transmitting) antenna gain and d is the distance between the transmitter and the receiver. Millimeter-wave spectrum also suffers from a higher penetration loss, for example when the signal travels through walls, windows or other neighboring scatterers. In non-line-of-sight (NLOS) conditions, significantly higher power loss is expected when compared to lower frequency bands. The FSPL is compensated by increasing the gain of the transmitter and the receiver [141]. This can be achieved by using directional elements (for example, a horn antenna [104]) or by using antenna arrays where gain increases with the effective aperture of the array, thus forming a narrower beam. Phased arrays as analyzed in Chapter 3 are good candidates for 5G applications; besides their increased gain, control of the relative phase between each active element enables beamforming capabilities. By constructive interference, it is possible to steer the beam in a chosen direction. This is especially useful in order to compensate for the losses in NLOS conditions, where the transmitter makes use of the environment in order to find an alternative path through reflections on walls, ceilings, and other neighboring scatterers.

The 5G applications examples mentioned above have the common characteristic to be operated in an indoor environment. This entails offices, houses, factories, stations or airport halls, among others. These environments are of particular interest, as most of the mobile availability is required by users located inside buildings. Traditional macro-cells with base stations located outside do not meet the 5G requirements in those environments. Besides the problem of increased path loss, capacity required by a larger number of devices is not easily achieved and penetration through building reduces significantly the signal power. The use of mmWave frequency band will require the development of indoor radio networks, providing connectivity to users and devices in smaller cells delimited by office

rooms or stories, for example.

In such indoor environments, the user itself is a scatterer of interest, which plays an important role in the propagation mechanism of the electromagnetic waves. This has been investigated by the blockage effect of the hand on a cellphone, for example [142]. At those frequencies, the electrical size of a human body is very large (hundreds of wavelengths), thus producing a larger shadow in the forward direction due to the signal blockage. Besides static considerations, a walking person has a dynamic impact on the channel, inducing variation of tens of dB during tens of milliseconds [143]. On top of that, many people are likely to be present at the same in a specific environment, increasing the complexity to the time-varying characteristics of the channel.

In order to evaluate *a priori* system performances in such complex scenarios, channel modeling is crucial for the design and implementation of communication systems, as stated in Chapter 1. Channel models enable the estimation of signal strength in space, as well as other important channel characteristics (power delay profile, coverage, etc.). The goal of this chapter is to provide models and validate models of the signal attenuation in the presence of a walking human. The main challenge is to develop a simplified model which mimics with enough accuracy the empirically observed electromagnetic effects induced by a human body over a communication link, while it should be simple enough to be computed with a fairly low complexity and easily embedded in classical propagation software.

This chapter of the dissertation aims at evaluating quantitatively the effect of the presence of a human body over an indoor wireless communication link. This is achieved by the development of models capturing the electromagnetic characteristics of a human and of an indoor environment, as well as the evaluation of numerical techniques used for the computation of the scattering mechanisms occurring in those scenarios. Those models and methods proposing different levels of accuracy for various computational complexities are assessed with experimental data, kindly shared and provided by Christopher Slezak from New-York University. The first part of this chapter focusses on the LOS blockage due to a perpendicular walk by a person. Different numerical models and methods presented in the previous chapters are compared in terms of accuracy and numerical complexity for this specific application. Those theoretical results are compared with the available experimental data. Once those models are validated, they are embedded in a more complex and realistic scenario, where

the human shadowing is evaluated in a multipath indoor scenario. It is shown how the received power can be maximized by taking advantage of the reflections on the environment, thus creating other propagation paths when the LOS is obstructed. The shadowing under multipath propagation is modeled by considering a ray-tracing approach combined with the human models developed in the first part of this chapter. Those simulation results are also compared with the experimental data acquired during the measurement campaign. The validity and the applicability of the models are finally discussed as a conclusion of this part of the thesis.

5.2 Models for the LOS human blockage

This chapter tackles the numerical modeling of human blockage at mmWaves. The problem consists of evaluating the power available at a receiver when a person's walk intersects the LOS, this corresponds to another interesting forward scattering scenario. This is a likely situation for indoor communications, as access points are usually located close to the walls, with devices and people distributed randomly in an office, for example. This problem is addressed deterministically by developing a good electromagnetic model of a human. The parameters defining the model (shape, dimensions, material, etc.) are carefully selected in order to provide a realistic representation of human scattering at mmWaves. The efficiency of the model also depends on the computational complexity of the numerical method employed for solving the numerical problem. A good balance between the accuracy of the numerical method and the computation time is therefore investigated.

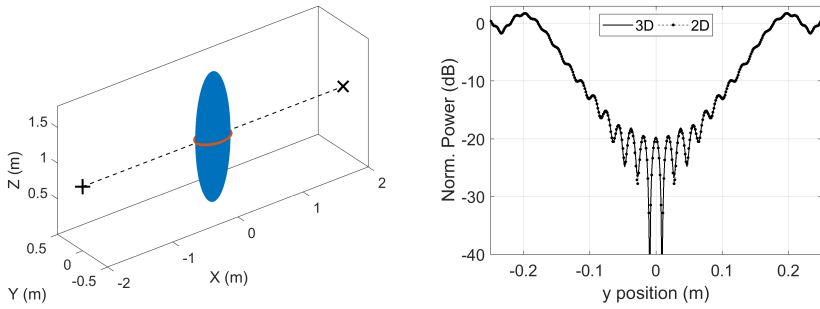
Two main approaches are commonly employed when modeling the diffraction by humans: on one hand, realistic phantoms made of (inhomogeneous) materials mimicking the electrical properties are measured and compared to real human observations [144, 145, 146]; on the other hand, geometrical meshes are designed with the help of CAD tools and are used as inputs of electromagnetic simulators predicting the interactions of the body with electromagnetic waves [147, 148, 149, 150]. For both approaches, the development of an accurate human model for microwave propagation is not a trivial task, by reason of the large number of parameters describing it.

Phantoms provide a model close to a realistic human body and are widely used in medical science or electromagnetic compatibility. Besides

the phantom size and shape, other parameters such as the dielectric constant of the body parts are not easily determined [151]. It is nevertheless well accepted that at mmWaves, the penetration depth is small, so that phantoms should be covered with a dielectric having the same properties [145] as the skin. More simplistic phantoms, such as a metallic cylinder are also used to model the whole body, or some parts of it (legs, arms, torso, etc.) [152, 153]. Different materials also affect the diffraction, whereas the high conductivity of the skin at mmWaves promotes highly conductive materials, such as water or metal. It is shown in [145] that both metallic and water-filled cylinder provides a good estimate of the shadowing by a leg. In backscattering, the water-filled phantom model is more accurate than the metallic cylinder which overestimates the amount of reflected power. Clothing also has a significant impact on the transmitted power, as indicated in [154], where adding a cotton cloth around a metallic cylinder strongly reduces the attenuation for the VV polarization (i.e. vertical polarization for both the transmitter and the receiver).

Another way to study the human diffraction relies on mesh-based electromagnetic simulations. This approach provides more flexibility for the tuning of the model parameters (dimensions, material, etc.) and enables a high repeatability. Nevertheless, the generation of such meshes is painstaking due to the highly detailed geometry of the body. As for the phantom based approach, some simplifications of the geometry are commonly used. Some of those adaptations allow analytical solutions providing more physical insights, or simplify the generation of meshes employed by numerical tools. In that sense, the human body is commonly modeled by cylinders [155, 156], ellipsoid or sheets. In the continuity of this thesis which studies the scattering by large scatterers, the mesh-based approach is considered in this work.

As the number of discretizing basis functions grows with the square of the size of a meshed surface, the generation of human meshes at mmWaves becomes a real challenge. In order to simplify the mesh generation, to reduce the memory requirements and accelerate the computation of the scattered fields, the human is assumed to be infinitely long in the main body axis (i.e. the vertical dimension, aligned with the human spine). Under this assumption, the initial 3D problem is reduced to a simplified 2D problem, thus reducing significantly the number of discretizing functions representing the body shape. This assumption is verified through a numerical example in Figure 5.1.



(a) The 3D(2D) model is represented by the blue(orange) ellipsoid(ellipse). The transmitter(receiver) is represented by the black plus(cross) sign, located at $[-2, 0, 0]$ m ($[2, 0, 0]$ m).

(b) Comparison of the received power for the 3D and 2D models illustrated in Figure 5.1a.

Fig. 5.1 Validation of the approximation of infinite cylinders.

The 3D human model corresponds to a PEC tri-axial ellipsoid with $[\hat{x}, \hat{y}, \hat{z}]$ principal axes of length $[0.47, 0.28, 1.8]$ m represented by $\sim 6 \cdot 10^6$ flat triangles, with a surface of $\sim \lambda^2/36$ m². The 2D model is an ellipse defined as the intersection between the 3D model ellipsoid and the xy -plane. The human is moving perpendicularly to the LOS in the $+\hat{y}$ direction. The transmitter and the receiver are respectively represented by a plus sign and a cross marker and are positioned at $[-2, 0, 0]$ m and $[2, 0, 0]$ m, as illustrated on Figure 5.1a. Both are modeled by vertical electric dipoles radiating in free space, with a central frequency of 60 GHz.

The comparison of the received power using the 2D and 3D models is obtained using the classical physical optics and is displayed on Figure 5.1b. For the 2D case, an infinite line source is considered. The comparison is very good, with an absolute error between the 3D model and the 2D approximation is of the order of a tenth of a dB. Moving from 3D to 2D reduces significantly the complexity of the study; in the example in Figure 5.1a, the ellipsoid is meshed using $5.8 \cdot 10^6$ triangular facets, whereas the 2D ellipse is discretized by $2.4 \cdot 10^3$ linear segments. As the problem is solved for each human position, the two-dimensional simplification drastically reduces the computation time. Based on the results of Figure 5.1b, the assumption of infinite cylinders for the modeling of the human body at mmWaves is employed throughout this chapter. This assumption will

be further validated with the experimental data of the measurement campaign.

The shadowing loss estimated with the aforementioned numerical techniques relies on parametric models. Those parameters are selected so that the models provide an accurate representation of the physical scattering mechanisms. The impact of the most important parameters is evaluated below. This analysis is helpful for a more comprehensive comparison with the models and the realized experiments. Besides the geometrical shape of the human model, its dimensions also have an impact on the scattered fields. In this work, a flat plate, a circular cylinder and an ellipsoid are considered as candidates for modeling the body shape. The electrical properties of the material are also investigated by considering a PEC model as well as lossy dielectric materials. As mmWaves span a wide frequencies spectrum, the impact of the signal frequency is also studied. In addition to the model itself, the characteristics of the communication system (i.e. the transmitter and the receiver arrays radiation patterns) also impact the variation of the received power with respect to the person's position. Finally, the results obtained by different electromagnetic solvers are compared, in regards with their associated computational complexity.

5.2.1 Impact of the shape

Three different shapes are considered: a flat plate, a circular cylinder and an elliptical cylinder. The flat plate is the coarsest representation of the human shape. For a human walk perpendicular to the LOS, the flat plate is aligned with the torso axis of the person. The circular cylinder represents the human body with more accuracy than a simple flat surface, whereas the elliptical cylinder accounts for the different widths of the torso and the shoulders. Those different models are compared assuming a person whose main dimensions are fixed to 0.47 m and 0.28 m for the shoulders and torso axis, respectively. Those correspond to the dimensions of the two main axes of the elliptical cylinder. The flat plate model has a width of 0.28 m and the same value applies for the radius of the circular cylinder.

The power at the receiver is obtained with the physical optics method at 60 GHz, where the transmitter and the receiver are vertical elementary dipoles (i.e. only the TE component is considered here). The distance between the transmitter and the receiver is 4 m and the person walks along

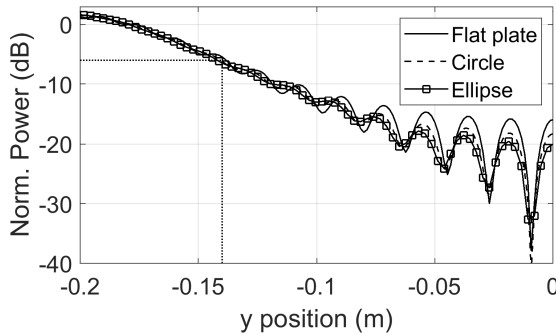


Fig. 5.2 Normalized received power for different model shapes, with respect to the perpendicular walk position depicted in Figure 5.1a.

the $\hat{y}$ direction at a halfway distance between the transmitter and the receiver, as represented in Figure 5.1a.

The comparison of the normalized received power with respect to the human position is illustrated in Figure 5.2. By symmetry, only the first half of the person's walk is represented. The circular and elliptical solutions are very similar in the shadow transition region. In the deep shadow (i.e. for small values of y), the elliptical model creates an attenuation 1 – 2 dB stronger than the circular model. The flat plate solution estimates a smaller attenuation than the ellipse (4 dB) in the deep shadow region, and exhibits higher oscillations in the transition region. The dotted line represents the -6 dB received power which corresponds to the y position where the human intersects the LOS. Those results are coherent with the comparison of the different models in [154].

5.2.2 Influence of the scatterer dimensions

Considering the same walk scenario of Figure 5.1a and the circular cylinder model, the impact of the cylinder radius is illustrated in Figure 5.3. By symmetry, only the first half of the walk is represented. As expected, the -6 dB limit is reached for larger y positions when the radius increases, as mentioned in the previous paragraph. The rate at which the received power decays in the transition region is independent of the radius of the cylinder. Nevertheless, the received power in the deep shadow region (i.e.

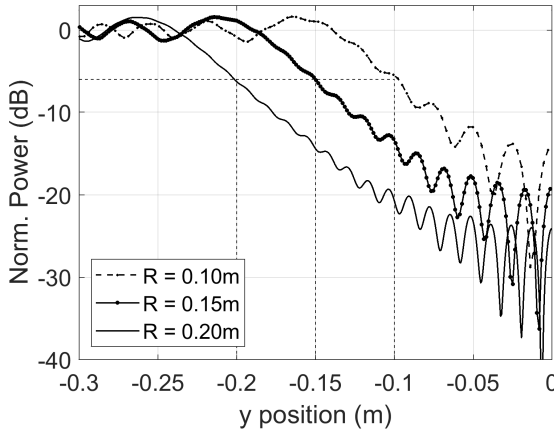


Fig. 5.3 Attenuation versus the perpendicular distance, for cylinders of different radii, computed with the PO. Dotted lines correspond to the -6 dB level crossing.

when the person center is close to the LOS) becomes smaller when the scatterer size increases. It is up to 10 dB smaller when the radius grows from 10 cm to 20 cm. For a fixed distance between the blocker and the receiver, the total field strength reduces when the size of the blocker increases, thus creating a larger shadow in the forward region. This effect has been illustrated exhaustively in Chapter 4. It is also observed that the interference between the incident and the scattered fields creates more oscillations of the received power when the size of the scatterer increases.

5.2.3 Influence of the material

The constitutive material of the human model has an impact on the attenuation. When considering PEC material, all the incident power is reflected at the interface between the air and the metal. In the case of dielectric material, some part of the incident energy is transmitted through the body. The human body is made of dielectric material (e.g. skin, fat, water, muscles, etc.); using the equivalent electrical properties of those material seems more realistic than assuming PEC models. Since penetration depth is quite small at mmWaves, the skin ($\epsilon_r = 7.98 - j10.91$ [151]) is considered since it may play a significant role on the human diffraction. As the body is made

up of roughly 60 % of water, the dielectric constant of pure water at 20° is also considered ($\epsilon_r = 11.9 - j19.5$ [157]). The received power during a blockage event is also compared to the PEC solution, which has an infinite conductivity.

The comparison of the normalized received power is presented in Figure 5.4 for the three mentioned materials. This result is obtained using the MECA method [158], which is an extended version of the physical optics for dielectric objects. The scatterer used for this simulation is the same as the 2D ellipse in Figure 5.1. For more visibility, only a small part of the human walk is shown, which corresponds to the deep shadow locations. One can observe that at the considered frequency, the attenuation is not significantly impacted by the choice of the material constituting the body. This is because the chosen material has a quite high permittivity and conductivity. The PEC model has a slightly higher attenuation (less than 0.5 dB) in the deep shadow region than the considered dielectrics. For the latter ones, some part of the incident power is transmitted through the body towards the receiver. Still, the fraction of transmitted power is small, as the human skin has a high absorption at mmWaves [159]. Based on this result, it is reasonable to consider the human as a PEC material. Results in [145] also indicate that this assumption is valid in the forward scattering region at similar frequencies.

5.2.4 Influence of the frequency

Besides the unlicensed 60 GHz band, the mmWave bands span a much broader frequency spectrum. The impact of the frequency on the human shadowing is illustrated in Figure 5.5 for 24 GHz, 40 GHz and 60 GHz, using the PEC circular cylinder model and the physical optics. The decay rate of the received power when the human moves closer to the LOS increases with the frequency. This is due to the quasi-optical behavior of the blockage at higher frequencies, also observed in the power behind a wind farm in Figure 2.8. The same observation also holds for the attenuation in the deep shadow region, with a power reduction of 5 dB between 24 GHz and 60 GHz. Note that the position for which the received power is reduced by a half (i.e. -6 dB) is kept constant when the frequency varies. Similar results have been observed for 60 GHz and 300 GHz in [160]. Asymptotically, the normalized received power would be maximal in LOS and null in NLOS when the frequency tends to infinity.

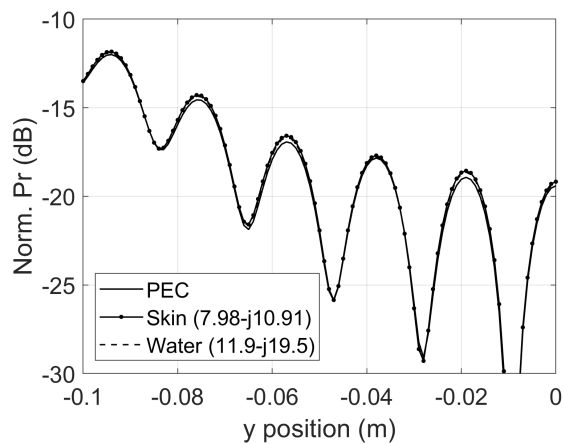


Fig. 5.4 Attenuation versus the perpendicular distance, for different materials, at 60 GHz.

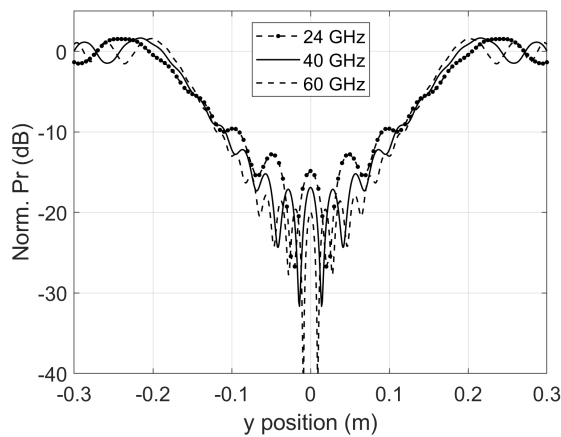


Fig. 5.5 Attenuation versus the perpendicular distance for different frequencies.

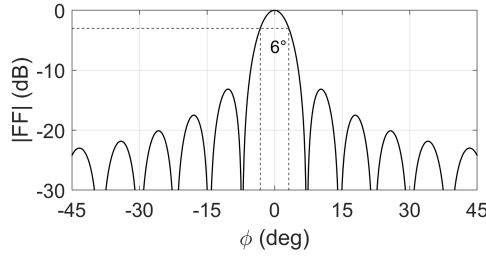


Fig. 5.6 Far-field radiation pattern of the transmitter and the receiver antenna arrays, with a -3 dB beamwidth of 6° .

5.2.5 Influence of the radiation pattern

The examples provided in the previous sections assumed that the scatterer is illuminated by an omnidirectional antenna. This implies that the incident power on the scatterer is constant when the distance between the human body and the transmitter/receiver is larger than the far-field distance d_{FF} , commonly expressed as

$$d_{\text{FF}} = \frac{2D^2}{\lambda} \quad (5.2)$$

where D is the largest dimension of the human. At mmWaves, this condition is not always met, as d_{FF} may exceed tens of meters. A more precise modeling would include the phase and the amplitude of the impinging and transmitted wave by including the radiation pattern of both the transmitter and the receiver, as suggested in [104, 18].

The impact of the radiation pattern is illustrated in Figure 5.7, for a fixed receiver position and different transmitter locations. For each example, the received power is represented for three different radiation patterns, identical for the transmitter and the receiver. Besides the plane wave assumption and the omnidirectional radiation pattern, the third antenna model is a directional antenna with a -3 dB beamwidth of 6° , as represented in Figure 5.6. It is observed that the attenuation by the human is stronger when the source is located close to it (up to 10 dB for a distance of 1 m, Figure 5.7a). For large increasing distances, the received power tends to the far-field illumination, as observed when comparing solid and dashed lines in Figure 5.7d. Those simulations also highlight the effect of using

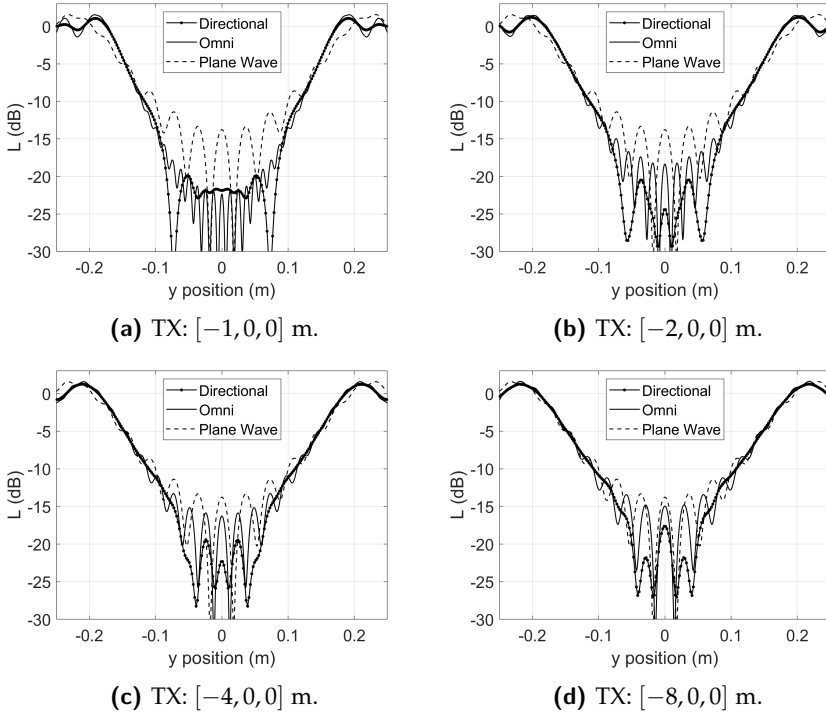


Fig. 5.7 Comparison of the attenuation for different positions of the transmitter and the human position. The receiver is located at $[2, 0, 0]$ m. The solid lines corresponds to a transmitter and receiver radiation pattern presented in Figure 5.6; the dotted line assumes an omnidirectional radiation pattern at both TX/RX; the dashed line corresponds to an incident plane wave and an omnidirectional receiving array.

directive arrays on the diffraction mechanisms. For short distances, the received power is significantly impacted by the radiation pattern; this is clearly observable in Figures 5.7a and 5.7b. An accurate modeling of the human shadowing would thus require a good knowledge of the characteristics of the antennas.

5.2.6 Influence of the numerical method

Once the physical model has been chosen, different numerical techniques are available for the computation of the scattered fields. Besides the currents methods presented and developed in this thesis (MoM, PO and MPO), the knife-edge diffraction (KED) model is also considered in this part of the thesis, as it is commonly employed as a diffraction model in a propagation context. Those different numerical approaches are detailed here below.

Knife-edge diffraction (KED)

This model assumes the human body as an absorbing opaque screen perpendicular to the shoulders' axis. The model used in this work is the one presented in [18], simplified here in the case of a walk perpendicular to LOS. The total diffraction is computed as the coherent sum of the fields diffracted by both edges of the KED. The diffraction gain $L_A(L_B)$ induced by edge $A(B)$ is written as

$$L_A = \frac{1+j}{2} \left[\left(\frac{1}{2} - C(\nu_A) \right) - j \left(\frac{1}{2} - S(\nu_A) \right) \right] \quad (5.3)$$

$$L_B = \frac{1+j}{2} \left[\left(\frac{1}{2} - C(\nu_B) \right) - j \left(\frac{1}{2} - S(\nu_B) \right) \right] \quad (5.4)$$

where $C(\nu)$ and $S(\nu)$ are the Fresnel integrals, defined as

$$C(\nu) = \int_0^\nu \cos(t^2) dt \quad (5.5)$$

$$S(\nu) = \int_0^\nu \sin(t^2) dt \quad (5.6)$$

and $\nu_{A,B}$ is dimensionless quantity defined as

$$\nu_{A,B} = \pm h_{A,B} \sqrt{\frac{2}{\lambda} \frac{dT + dR}{dT dR}} \quad (5.7)$$

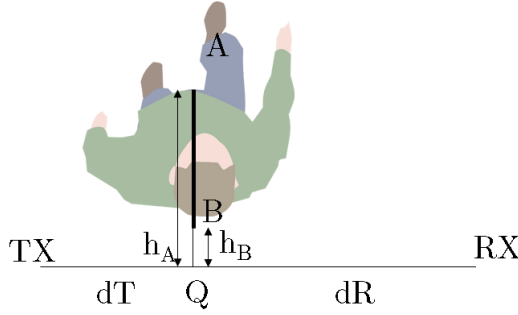


Fig. 5.8 Variables useful for the derivation of the shadowing attenuation of a KED (eq. (5.7)).

The variables involved in eq. (5.7) are represented on Figure 5.8. The $\pm$ sign in eq. (5.7) depends on the position of the knife-edge with respect to the LOS.

The total diffraction gain L_{KED} is obtained as

$$L_{KED} = L_A + L_B \quad (5.8)$$

and is valid for omnidirectional antennas. The model of the diffraction in eqs. (5.3) and (5.4) can be modified in order to include the radiation pattern of the transmitting and receiving antenna arrays, and is given by

$$L'_{(A,B)} = L_{(A,B)} \sqrt{D_{(A,B)-Tx}} \sqrt{D_{(A,B)-Rx}} \quad (5.9)$$

where $D_{(A,B)-(Tx,Rx)}$ is the directivity of the considered array in the direction of the two edges A and B of the knife-edge. This modification has a considerable impact on the received power, as illustrated in Figure 5.7. This model is based on very simplified representation of the human considered as a single flat plate, whereas its computational complexity in $\mathcal{O}(1)$ is very attractive.

An extension of this model has been proposed in [154], where the person is represented by two perpendicular knife-edges (one aligned with the torso, the second aligned with the shoulders) and where the diffraction over the head is also taken into account using an additional knife-edge.

Physical Optics (PO) model

The second model considered is the physical optics (PO) approximation presented in Chapter 4. PO approximates the surface currents on the illuminated part of the scatterer as

$$\mathbf{J}_{\text{PO}}^{\text{lit}} = 2\hat{\mathbf{n}} \times \mathbf{H}_i \quad (5.10)$$

with $\mathbf{H}_i$, the incident magnetic field. By contrast with the KED model presented before, more realistic representations of the human body can be analyzed, such as cylinders, ellipsoids, cuboids, etc. As the PO has a computation time and memory requirements which grow linearly with the number of discretizing facets, this method enables the study of very large scatterers. This method has been extended for multiple blockers in [161]. The physical optics approach enables the study of more complex scenarios if they are available (e.g. the mesh of a realistic phantom), as it is proposed in [147], where a realistic human representation based on NURBS is analyzed with accelerated physical optics.

Modified Physical Optics (MPO)

The modified physical optics models presented extensively in Chapter 4 is considered as a candidate for the modeling of human shadowing. It has been shown that this method improves the classical physical optics approximation in the forward region. The attenuation of the body when it intersects the LOS is thus a nice application to the forward scattering phenomenon, although its complexity is higher than the KED and the PO.

Method of moments (MoM)

The method of moments (MoM) is compared to the previously mentioned methods. As 2D scenarios are considered, MoM is still suitable for moderate size problems.

Figure 5.9 reveals that the PO estimates a received power larger than the MoM solution. This is not surprising in regards with the results of Chapter 4, where it was observed that the PO creates a smaller shadow in the forward region. In the deep shadow region (i.e. for human positions close to the LOS), the modified physical optics converges towards the MoM solution, besides some deviation in the transition region (around $y = \pm 0.8$ m). The knife-edge diffraction model estimates a significantly smaller attenuation than the other methods, as already highlighted in [162]. Among the

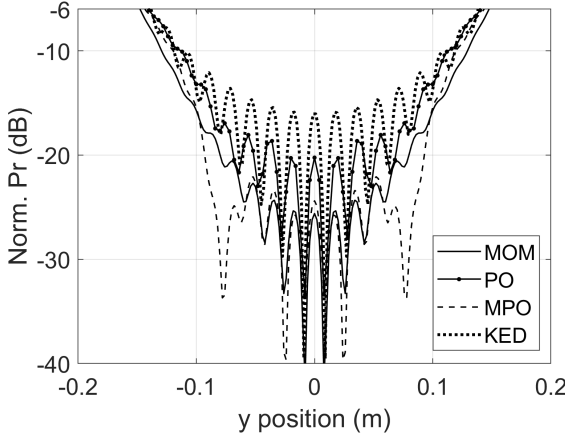


Fig. 5.9 Received power during the shadowing event, computed with different numerical methods.

four different numerical techniques, the maximal attenuation varies within a range of 10 dB.

A comparison of the total computation time for the four methods illustrated is provided in Table 5.1. For the MoM, this includes the computation and inversion of the impedance matrix; for the MPO, this corresponds to the precomputation of the FMM quantities and the time required for three MPO iterations. For the four methods, the computation time encompasses the computation of the received power for a human walking along the $+\hat{y}$ direction in the interval $y \in [-0.2, 0.2]$ m, with a step of $\lambda/4$ (i.e. 320 positions). One can observe that the cost of the KED approach is significantly lower than the current based methods. Moreover, its computational complexity does not vary with the frequency or the size of the scatterer.

5.2.7 Comparison between models and experimental data

The different shadowing models are validated in this section by comparison with the experimental values acquired and kindly shared by Christopher Slezak from New-York University (NYU). For all the considered models, it is assumed that the body is a perfect electric conductor, as results of Section 5.2.3 have shown that this assumption is valid in transmission at mmWaves. The layout of the experiment is illustrated in Figure 5.10a,

Table 5.1 Computation time of the received power for the example in Figure 5.9.

Method	Comp. time (s)
MoM	1920
PO	203
MPO	960
KED	0.11

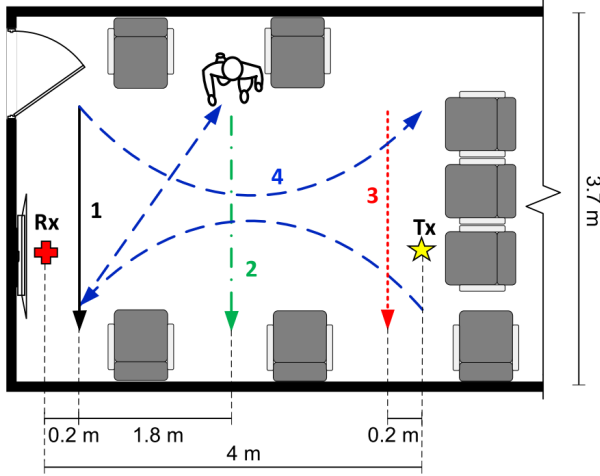
where only the experiments involving a single blocker moving perpendicularly with respect to LOS (paths 1, 2 and 3) are considered. More details on the measurement campaign are found in [163, 143, 164]. The real environment is depicted in Figure 5.10b for one of the realization of the path 2 experiment.

The number of independent realizations of the same experiment is listed in Table 5.2. In order to filter out the experimental noise and to smoothen the data, the measurements corresponding to the same experience are averaged in the time domain. This averaging requires pre-processing steps of the raw data including:

1. **Normalization of the power:** for each realization of the experiment, the received power is normalized by the time average of the received signal in LOS conditions. It is assumed that the human does not have any impact on the transmitted signal at the beginning of each realization.
2. **Temporal inversion:** for a given path on Figure 5.10a, data are acquired with the human moving from left to right, and from right to left. Some data are reversed in the time domain, so that each realization corresponds to a walk in the same direction. This is important

Table 5.2 Number of independent realizations for the experiments "Path 1", "Path 2" and "Path 3" represented in Figure 5.10a.

Name of the experiment	Number of realizations
Path 1	4
Path 2	6
Path 3	3



(a) Schematic top view of the different scenarios of the measurements campaign [163].



(b) Real environment, viewed from the receiver side. The snapshot is taken from a measurement corresponding to the "path 2" in Figure 5.10a.

Fig. 5.10 Schematic and realistic view of the measurement environment.

as the environment and the antennas radiation patterns are not symmetrical.

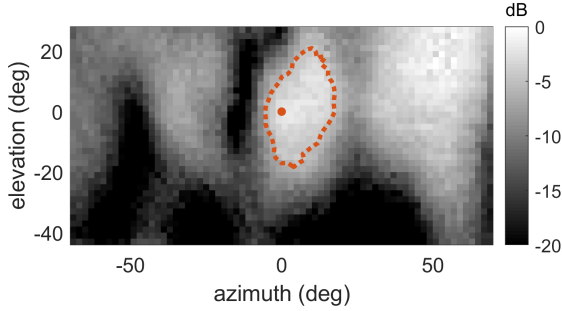
3. **Selection of the LOS multipath component:** in the delay domain, only the index corresponding to the main multipath component is kept. By selecting this particular multipath component, the effect of the reflections on the surrounding objects is discarded.
4. **Data cropping:** in order to illustrate the LOS attenuation by the human blockage, the data is cropped to the samples where the shadowing loss is larger than 6 dB. From Figure 5.3, this corresponds to the position for which the human intersects the LOS. The velocity of the walking person is assumed constant during this interval, which seems fair by visual inspection of the videos synchronized with the acquired samples.

In order to include the radiation pattern effect, the (normalized) radiation pattern is extracted for the elevation angle $\theta = \pi/2$ and all the available azimuth angle ϕ (ranging from $\phi \in [-70, 70]^\circ$), as the transmitter and the receiver are located at the same height.

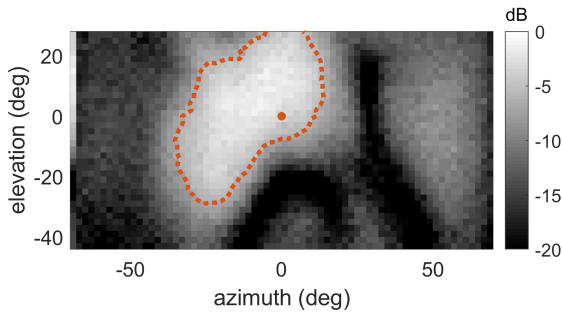
Human located halfway between TX and RX

The first comparison is the simplest case where the human is following the path 2, with a walk perpendicular to the LOS at roughly equal distance between the transmitter and the receiver, and both array main beams facing each other. The pair of beams fulfilling this condition is obtained by inspection of the empty room measurements and by selecting the (TX,RX) pair among the 12×12 candidates that maximizes the received power. This corresponds to the 11th and the 7th beam for the transmitter and the receiver, respectively. The associated normalized radiation patterns are illustrated in Figures 5.11a and 5.11b.

The normalized time-variant power delay profile for this specific steering angles combination is represented in Figure 5.12. The human shadowing occurs around 3.3 and 3.8 s for the first realization of the "path 2" experiment. In order to analyze the attenuation of the LOS component, the corresponding line is extracted from the 2D measurement matrix. This is performed for each of the six realizations of the experiment "path 2". After power normalization and temporal inversion, the data are cropped in order to keep only the attenuation values higher than 6 dB. The signals are



(a) Normalized radiation pattern of the transmitting array for the 11th precoder.



(b) Normalized radiation pattern of the receiving array for the 7th precoder.

Fig. 5.11 Pair of steering vectors for the transmitting and receiving arrays maximizing the received power. The red dots represent to the (0,0) azimuth/elevation angle corresponding to the LOS direction, with associated -3 dB region around this direction.

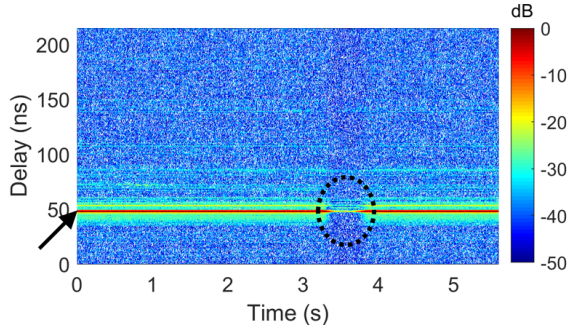


Fig. 5.12 Time dependent power-delay profile for the first realization of a "path 2" scenario. The LOS is indicated by the black arrow. The shadowing event is clearly observable around 3.5 s (dashed circle).

finally interpolated under the assumption that the human walking speed is constant over time and is the same for all the realizations. The total duration of the shadowing event is the averaged value of the durations of the six shadowing events. The resulting signal of the "path 2" experiment is then obtained as the average of the six pre-processed signals. The averaged signal for this experiment is illustrated on Figure 5.13, as well as the standard deviation associated with each snapshot of the six realizations of the experiment. From this figure, one can observe a large variation of the attenuation in the deep shadow region, up to 9.7 dB. This is explained by the small number of realizations (only six for this example) and the assumption of constant speed during each realization, as well as the movement of the arms which is different for each realization. The shadowing events last between 317 ms and 442 ms with a mean value of 364 ms. This value is coherent with the results from other works and reveals the necessity to compensate for the attenuation through beamforming.

The comparison between the measurements and the numerical models from Section 5.2 is presented in Figure 5.14. The model employed for this comparison is an infinite cylinder with an elliptical section for the MOM and the PO models defined by a large and small radius $R = 0.325$ m and $r = 0.165$ m, respectively. The KED model is represented by two knife-edges separated by a flat plate of width $2R$. All the models are made up of PEC material, as lossy dielectric do not have a significant impact on the received power. The measured radiation patterns for both the transmitter

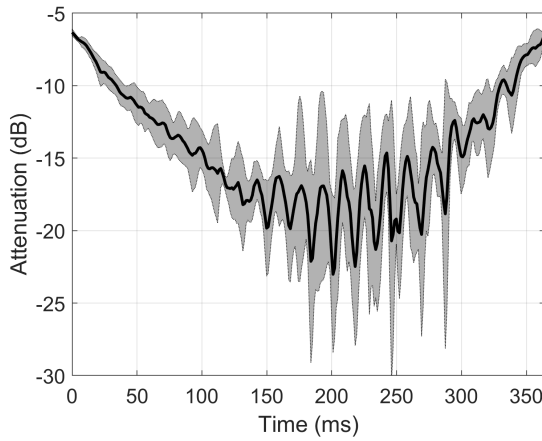


Fig. 5.13 Measured attenuation for the scenario "path 2" and the beam pair $(TX, RX) = (11, 7)$, with respect to the time, assuming a constant speed of 0.9 ms. The black line is the averaged value of the six realizations of the same experiment and the grey area corresponds to interval defined by one standard deviation around the mean value.

and the receiver arrays (Figure 5.11) are included in the computation of the scattered fields as it has a significant impact on the attenuation. From Figure 5.14a, it is observed that the KED model estimates the smaller attenuation, followed by the PO model and with the MoM model predicting the largest attenuation. Surprisingly, the KED model shows the best performances whereas it has a coarser representation of the body shape compared to the elliptical model. A quantitative comparison of the different models accuracy is obtained with the help of Figure 5.14b through the comparison of the cumulative distribution function (CDF) of the attenuation. It is quite clear that the flat plate model is the most accurate representation of the human blockage. Moreover, the attenuation computed with the model is performed at very low complexity using the KED model. The cylinder representation overestimates the attenuation when computing the attenuation with the physical optics approximation. The attenuation is even larger when using the method of moments; besides its high computational complexity, it is thus not an appropriate method for this application.

As presented in Section 5.2.5, the radiation pattern has a significant impact on the attenuation. This observation is also confirmed by the mea-

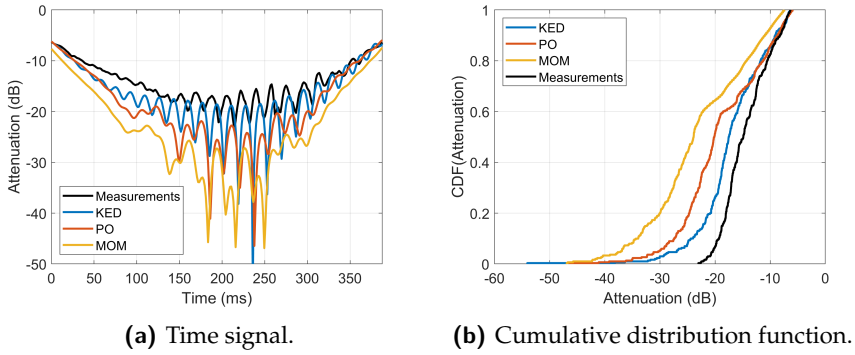
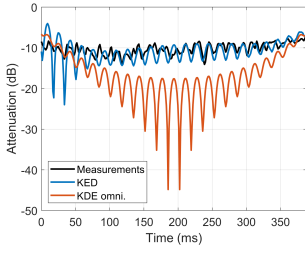


Fig. 5.14 Comparison between measurements and simulation for the "path 2" scenario.

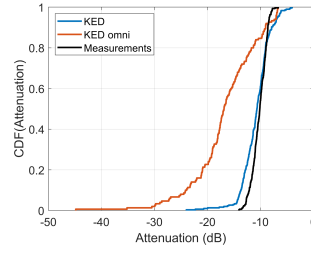
surements. In Figure 5.15, the attenuation is plotted for the KED model, using an omnidirectional radiation pattern and the measured radiation pattern, for different combinations of transmitter and receiver beams. Attenuation in Figure 5.15a is almost constant over the obstruction time. This is explained by inspection of the radiation pattern of the beam 1 of the transmitter which exhibits a null in the region around the LOS direction. Some asymmetry is visible in Figure 5.15c, which is also explained by a pronounced asymmetry in the corresponding radiation pattern around the LOS direction. In all of these cases, the KED model including the radiation pattern shows a good accuracy. Under the assumption of omnidirectional antennas, the estimation of the attenuation may become very inaccurate (see for example Figure 5.15b).

Human located close to the receiving array

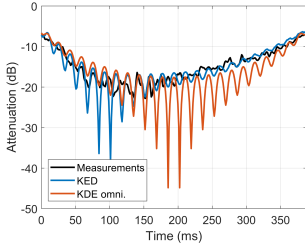
Another interesting scenario realized during the measurements campaign corresponds to a human blocker walking closely to one of the two antenna arrays. The scenario "path 3" is considered here, where the person walks perpendicularly to the LOS and close to the receiving array. The received power for the beam pair which maximizes the received power (i.e. (TX,RX) = (11,7)) is represented in Figure 5.16. When the human walks closer to the receiving array, the received power when the LOS is obstructed is significantly smaller (approximately 10 dB) than when the person was walking at a position halfway between the transmitter and the receiver (see Figure 5.13). This behavior is not surprising as the shadow in the forward



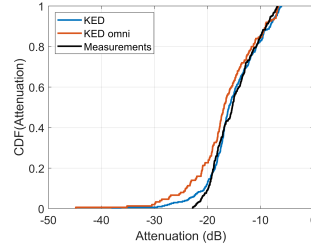
(a) (TX,RX) = (1,6): time signal.



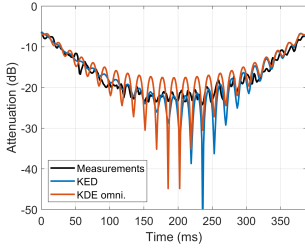
(b) (TX,RX) = (1,6): CDF.



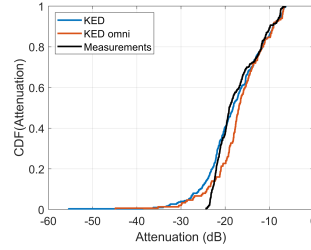
(c) (TX,RX) = (4,10): time signal.



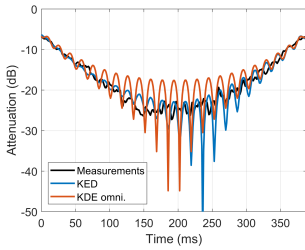
(d) (TX,RX) = (4,10): CDF.



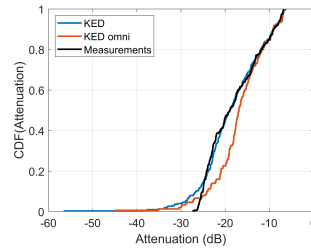
(e) (TX,RX) = (9,1): time signal.



(f) (TX,RX) = (9,1): CDF.



(g) (TX,RX) = (9,5): time signal.



(h) (TX,RX) = (9,5): CDF.

Fig. 5.15 Attenuation for the "path 2" scenario and different transmitter and receiver beam pairs using the KED model. The orange line corresponds to the omnidirectional TX and RX array and is thus identical for each beam pair.

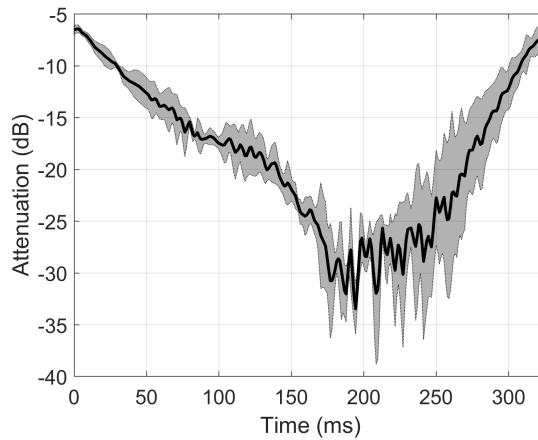


Fig. 5.16 Measured attenuation for the scenario "path 3" and the beam pair (TX,RX) = (11,7), with respect to the time, assuming a constant speed of 0.9 m/s. The black line is the averaged value of the six realizations of the same experiment and the grey area corresponds to interval defined by one standard deviation around the mean value.

direction is stronger when the receiver is close to the scatterer, thus blocking the transmitted signal.

The received power profile in Figure 5.16 presents a certain asymmetry which is much more pronounced than the "path 2" case (see Figure 5.13). This is because, for the same perpendicular distance, the range of angles of arrival for which the human body blocks the LOS is much broader when the person is moving close to the receiver. The receiver gain variation over this larger azimuthal range is larger (see Figure 5.11b), thus inducing this asymmetrical attenuation pattern. The measured received power is compared with the numerical models in Figure 5.17. One can observe that both the PO and MoM methods induce a shadow behind the obstacle much larger than what is observed experimentally. The KED model models quite accurately the decaying and rising part of the attenuation, whereas it underestimates the attenuation between 150 ms and 275 ms (see Figure 5.17a). This lower accuracy (as compared to the case where the human walks at a larger distance from both arrays) is explained by the smaller number of measurements and the lack of precise knowledge of the human position. Small deviations between the modeled and the actual positions

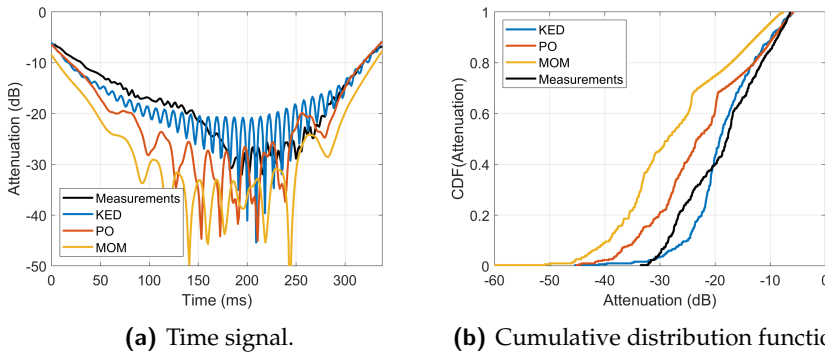


Fig. 5.17 Comparison between measurements and simulation for the "path 3" scenario.

can lead to large differences. The CDFs in Figure 5.17b clearly show the overestimated attenuation using the PO and the MoM models, while the KED is still reasonably close to the experimental results. Note that when the blocker is close to the receiving antennas, the probability of experiencing an attenuation lower than -20 dB is around 40 %, while it is less than 7 % when the person is at a larger distance (Figure 5.14b).

5.2.8 Conclusion

The first part of this chapter investigated the shadowing attenuation on the LOS link for a perpendicular human walk. This phenomenon is modeled by computing numerically the fields scattered by simplified electromagnetic problems. The elongated geometry of the human body is represented by infinite cylinders of different cross sections (i.e. circular and elliptical) and by plates with an infinite length in the vertical dimension. The comparison with experimental results has shown that the coarsest knife-edge diffraction model provides the most accurate representation of the received power, which also has a computation time and implementation complexity significantly smaller than other investigated approaches.

Attenuation of the signal can reach up to 20 dB for a perpendicular human walk at 60 GHz when the person is roughly 2 m away from both the transmitter and the receiver. This value is increased when the person moves closer to the receiver, with an attenuation up to 30 dB. This preliminary analysis was focusing only on the LOS component. The next section

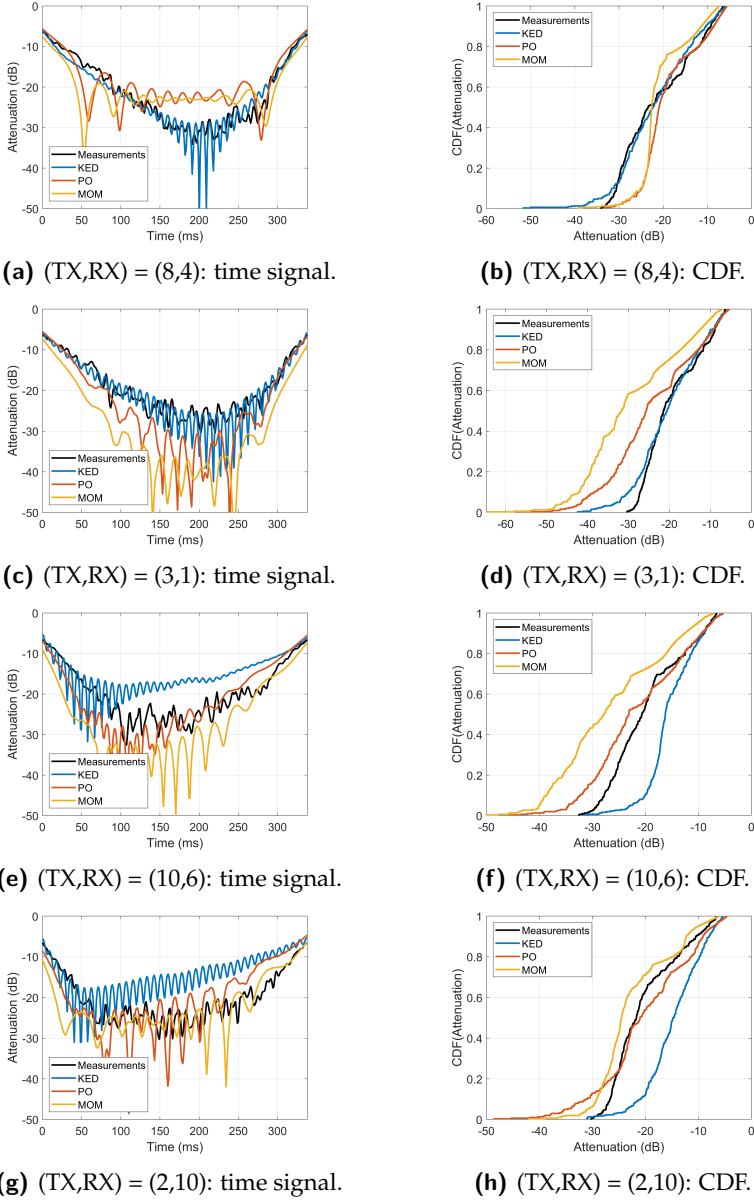


Fig. 5.18 Attenuation for the "path 3" scenario and different transmitter and receiver beam pairs using the KED model.

investigates the effect of the multipath propagation on the human blockage and how beamforming techniques at the transmitter and the receiver can mitigate this strong attenuation.

5.3 Overcoming the human blockage in a multipath environment

This section analyzes the human blockage in a realistic office environment where multipath propagation is induced by the reflections on the walls, floor, ceiling and other neighboring scatterers. We are focusing on the total power at the receiver, computed as the sum of the power associated to each multipath component. Those contributions reach the receiver with a given delay depending on the distance traveled by each of those possible paths. During a shadowing event created by a human walk (as considered in Section 5.2), the goal is to evaluate how much power the system can spare by dynamically choosing the "best" propagation path through beamforming techniques.

First, different beamforming strategies are compared numerically for a given static environment scenario. The proposed numerical framework is then extended to time-varying scenarios, in order to assess the advantage of dynamic beam steering during a shadowing event. In order to validate the theoretical beamforming gain, a ray-tracing approach is proposed, thus enabling a more realistic representation of the environment and an efficient integration of the human blockage using the knife-edge diffraction model from the previous section. Numerical results are finally compared with the measurements.

5.3.1 Beamforming gain in static scenarios

The increased path loss at mmWaves is compensated by increasing the gain at the transmitter and/or the receiver, thus creating narrow beams. Phased arrays are considered for this purpose; despite their implementation complexity, they enable the steering of the transmitter and the receiver beam through beamforming. This procedure involves varying the amplitude and/or phases at each active element, thus creating a constructive interference in a specific direction. This is particularly useful in indoor environments, where wave interactions with furniture, walls, ceilings, etc.

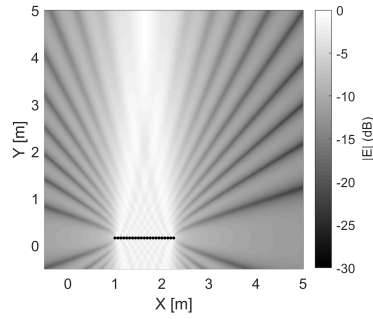
create unwanted interference. This is all the more true at mmWaves for which small objects become electrically large.

These interferences can be observed by comparing the field distribution in Figures 5.19a and 5.19b, where the transmitting antenna array in Figure 5.19a radiates in free space, whereas Figure 5.19b illustrates the field distribution in presence of PEC obstacles (orange lines). The scattered fields are computed with the MoM accelerated with the FMM in 2D at 2.4 GHz and 10 GHz and considering a uniform linear array (ULA) of 21 omnidirectional elements with a $\lambda/2$ spacing, represented by black dots. The elements are fed in phase with equal amplitudes, so that the main beam points towards the broadside direction. The diffraction around the cylinder and the shadows behind the scatterers are clearly visible in Figures 5.19b and 5.19c. Note that the shadow behind the scatterers is more pronounced when the frequency increases.

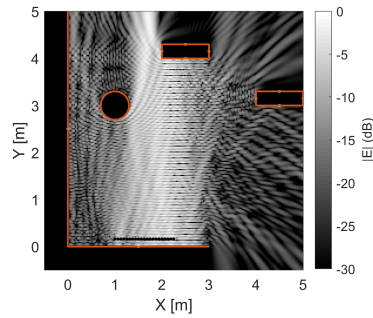
In the example of Figure 5.19b, the transmitter “blindly” sends a signal in a fixed direction, which creates zones where the signal is strongly attenuated. For devices located in NLOS (e.g. behind a scatterer), the received power can be too small for a reliable communication. This situation is overcome by sending the signal in specific directions, so that the sum of the different paths increases the power received by a device at a given location. Three different beamforming approaches are considered here. For theoretical considerations, digital beamforming architecture is assumed, as it provides a maximal number of degrees of freedom (i.e. phase and amplitude for each radiating element). The hardware complexity of this architecture can be mitigated by analog or hybrid beamformers [141], even if those system implementation considerations are not treated here.

Maximum ratio transmission (MRT) : This precoder uses complex weights (amplitude and phase) for each active element such that the SNR of their sum is maximized [165]. With no *a priori* knowledge of the channel, the weights are determined based on the correlation matrix of the signals received at each antenna. Considering a system with N_d different devices (assumed isotropic sources) and an access point antenna array of N_a elements, the vector of the received signals at each antenna $\mathbf{y}$ is of size $(N_a \times 1)$, with $N_d < N_a$. The correlation matrix $\mathbf{C}$ of size $(N_a \times N_d)$ is defined as :

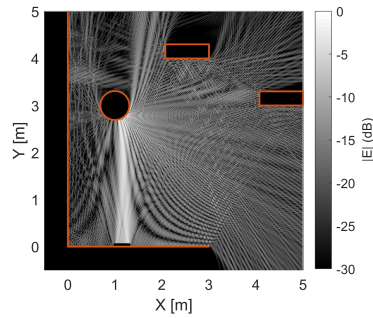
$$\mathbf{C} = \mathbb{E} \left(\mathbf{y} \mathbf{y}^H \right) \quad (5.11)$$



(a) Field distribution of the transmitter array in free space, with the main beam pointing towards the broadside direction for a central frequency of 2.4 GHz.



(b) Field distribution in presence of PEC scatterers at 2.4 GHz.



(c) Field distribution in presence of PEC scatterers at 10 GHz. The inter-element spacing is $\lambda/2$, thus reducing the physical size of the array at higher frequencies.

Fig. 5.19 Impact of the environment on the field distribution at for different frequencies.

where uncorrelated Additive White Gaussian Noise (AWGN) is present at each receiver. By ergodicity, the expectation is computed as the average over the sampled signals at different time instants. The matrix $\mathbf{C}$ is then factorized using the eigenvalue decomposition (EVD) as follows :

$$\mathbf{C} = \mathbf{U}_C \mathbf{\Lambda}_C \mathbf{U}_C^H \quad (5.12)$$

where the subscript C refers to the correlation matrix and superscript H is the hermitian operator. The weights of the beamformer corresponding to the N_d devices are selected as the complex conjugate of the N_d vectors of $\mathbf{U}_C$ associated to the N_d non-zero eigenvalues of $\mathbf{\Lambda}_C$. Working with the correlation matrix, as already done for localization [166], rather than instantaneous snapshots avoids potential problems with residual Doppler cancellation and enables efficient noise suppression using time averaging.

Note that under the assumption of uncorrelated sources, eigenvectors of the correlation matrix are exactly equal to the left singular vectors of the channel matrix $\mathbf{H}$ of size $(N_a \times N_d)$, defined as

$$\mathbf{y} = \mathbf{H} \mathbf{x} + \mathbf{v} \quad (5.13)$$

where $\mathbf{x}$ of size $(N_d \times 1)$ represents the uncorrelated signals transmitted by each device and $\mathbf{v}$ is the noise vector for each antenna with $v_i = \mathcal{N}(0, \sigma) + j\mathcal{N}(0, \sigma)$. Inserting eq. (5.13) in eq. (5.11), under the assumption that $\mathbf{H}$ is constant, one can derive

$$\mathbf{C} = \mathbb{E} \left(\mathbf{H} \mathbf{x} \mathbf{x}^H \mathbf{H}^H \right) = \mathbf{H} \mathbb{E} \left(\mathbf{x} \mathbf{x}^H \right) \mathbf{H}^H \quad (5.14)$$

If vectors $\mathbf{x}$ are uncorrelated over time and the various devices, $\mathbb{E} \left(\mathbf{x} \mathbf{x}^H \right)$ is the identity matrix. Under those assumptions, one can write

$$\mathbb{E} \left(\mathbf{y} \mathbf{y}^H \right) = \mathbf{H} \mathbf{H}^H \quad (5.15)$$

Using the singular value decomposition (SVD) of $\mathbf{H} = \mathbf{U}_H \mathbf{\Sigma}_H \mathbf{V}_H^H$, we find that $\mathbf{C} = \mathbf{U}_H \mathbf{\Sigma}_H^2 \mathbf{U}_H^H$. Using eq. (5.12), we find that

$$\mathbf{U}_C = \mathbf{U}_H \quad (5.16)$$

$$\mathbf{\Lambda}_C = \mathbf{\Sigma}_H^2 \quad (5.17)$$

It has been shown that the weights are equally obtained using EVD

on the correlation matrix $\mathbf{C}$ or SVD on the channel matrix $\mathbf{H}$. Nevertheless, knowledge of $\mathbf{H}$ is not always available in practice and requires that pilots are sent which increase the system complexity.

Equal gain transmission (EGT) : Equal gain transmission can be derived from MRT, where the amplitude of the active elements is the same. This reduces the number of degrees of freedom, even though the phase of the transmitted signal has more impact on the beam steering than the amplitude. This approach enables a lower complexity hardware implementation and relaxes the constraint on the power amplifiers used for MRT, where the ratio between the peak and average power is significantly higher than for EGT [167].

Codebook (CB) : The previous approaches (MRT and EGT) require the channel state information (CSI), or compute correlation at the access point, which increases the complexity of the system. Although those techniques allow continuous values of the beamformer's weights, some practical implementations rely on predefined steering vectors (forming a so-called "codebook"). The best weights combination is selected among a set of discrete values thanks to the feedback between the access point and the user. A good compromise between optimal SNR and complexity has to be reached by a careful selection of the number and directions of the different array beams.

Two different codebooks are considered here: they are obtained from a 21-elements ULA as used in Figure 5.19 with a range of steering angles between $[-\pi/4, \pi/4]$. The first and the second codebook contain 6 and 12 discrete angles linearly spaced in this interval, (CB_6 and CB_{12} , respectively).

The beamforming gain G_{BF} is compared for the different beamforming techniques presented above. This index of performance is defined as

$$G_{\text{BF}} = \frac{P_{\text{BF}}}{P_{\text{noBF}}} \quad (5.18)$$

where P_{noBF} is the power at a given position when an unitary precoder (i.e. no beam steering is applied) is employed, and P_{BF} is the power at a given position using one of the considered beamforming approaches (MRT, EGT, CB_6 and CB_{12}). The scenario is the same as the one represented in Figures 5.19b and 5.19c, where the optimal precoder is computed for every

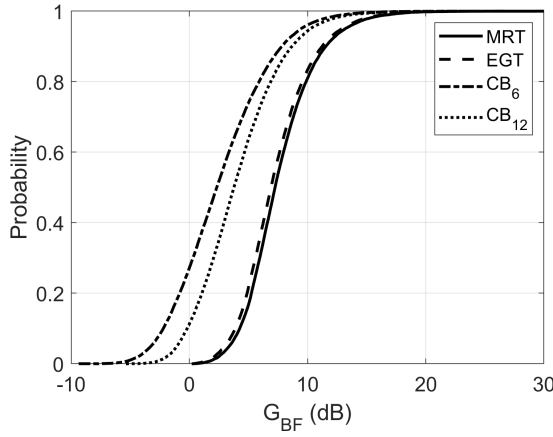


Fig. 5.20 Comparison of the cumulative distribution function (CDF) of the beamforming gain at 10 GHz for the same environment as in Figure 5.19c for different beamforming strategies.

position in the 2D space, which spans $X \in [0, 5]$ m and $Y \in [0, 5]$ m with a step of λ in both dimensions. For the codebook approach, the beam that maximizes the power among the 6 or 12 beamformer's weights is selected.

The CDF of the beamforming gain is represented at 10 GHz in Figure 5.20. As expected, the MRT precoder outperforms the codebook approach. One can observe that the EGT approach presents similar performances than the MRT, with a lower implementation complexity. For those two techniques, the beamforming gain G_{BF} does not reach negative values: steering the beam using MRT and EGT increases the received power of a receiver located anywhere in the considered 2D space. This is not the case for the codebook based approach. For some positions of the receiver, the power is lower as compared to the broadside transmission case. This is because the codebook set does not include the unitary precoder. As expected, the beamforming gain increases when the number of predefined steering vectors comprising the codebook grows. Note that the probability to achieve a positive beamforming gain for both codebook approaches is still high (75% and 90% for CB₆ and CB₁₂, respectively).

This example in a simplified static environment has demonstrated how phased arrays combined with beamforming strategies can increase the

power at a desired position in space. For the sake of implementation complexity, the beamforming gain has been illustrated through 2D examples. In practice, it is possible to steer the beam both in the azimuth and elevation using two-dimensional arrays [168, 169]. However, simulations with the MoM in 3D become too tedious.

5.3.2 Dynamic beamforming in a time-varying environment

In the previous section, the beamforming gain has been compared for different beamforming approaches in a static environment. In this section, a walking person considered as a moving scatterer is introduced, thus creating a dynamic scattering environment. It has been shown in Section 5.2 that the attenuation induced by a person intersecting the LOS can reach up to 30 dB. As beamforming techniques from Section 5.3.1 are able to increase the power at a specific location by taking advantage of multipath effects, the goal of this section is to evaluate how dynamic beamforming is able to overcome the significant LOS attenuation.

As for the static case, the dynamic beamforming capabilities are modeled by a MoM solver in two-dimensions. However, as the environment is varying over time, the MoM impedance matrix changes for every human position, thus leading to a huge computation time. This solver is accelerated by an effective ordering of the impedance block-matrix, where only a few blocks are updated for each position. Timing and memory efficiency are further improved with the help of FMM, thus enabling the fast computation of the scattered fields for this time-varying problem.

In order to present and illustrate the framework developed for the analysis of the beamforming gain in a time-varying scenario, let us consider the test case in Figure 5.21. It is assumed that the scatterers are infinitely long in the z dimension, thus reducing the problem to a two-dimensional scenario, as it is done in Section 5.3.1 and validated in Section 5.2. The transmitting ULA referred as "TX" is the same as the one used in Section 5.3.1 and is located at (0,0) m and the receiver (omnidirectional antenna) is located at (3,4.5) m. The static scatterers are represented by the dark gray polygons, numbered from 1 to 4. The human body is represented by the light gray cylinder with a circular cross section with a radius of 0.25 m labelled with the letter "H". The validity of this model have been discussed in Section 5.2. The black arrow represents the direction of the human motion and the dashed line is the LOS.

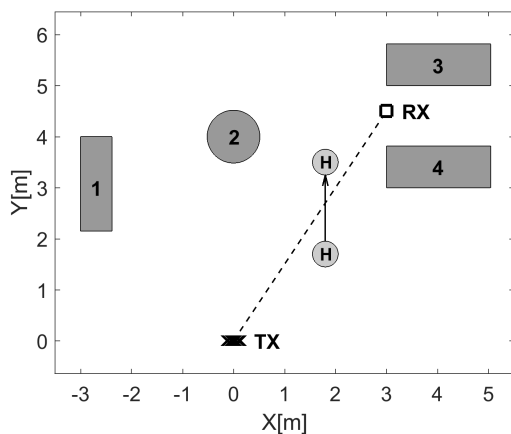


Fig. 5.21 Scenario of the time-varying environment. The dark gray shapes numbered from 1 to 4 represent the static scatterers and the light gray object represents the human person moving in the $+\hat{y}$ direction (black arrow). The dashed line corresponds to the LOS. The transmitting array is an ULA of 21 omnidirectional elements with a $\lambda/2$ spacing and is located at $(0,0)$. The receiver is a single omnidirectional element.

The problem is solved using MoM and benefits from the FMM acceleration and compression of the impedance matrix $\mathbf{Z}$. The system of equations to be solved for computing the MoM currents $\mathbf{i}$ with an excitation $\mathbf{v}$ reads

$$\mathbf{Z} \mathbf{i} = \mathbf{v} \quad (5.19)$$

where $\mathbf{Z}$ can be written, for the example of Figure 5.21:

$$\mathbf{Z} = \begin{pmatrix} \mathbf{Z}_{HH} & \mathbf{Z}_{H1} & \mathbf{Z}_{H2} & \mathbf{Z}_{H3} & \mathbf{Z}_{H4} \\ \mathbf{Z}_{1H} & \mathbf{Z}_{11} & \mathbf{Z}_{12} & \mathbf{Z}_{13} & \mathbf{Z}_{14} \\ \mathbf{Z}_{2H} & \mathbf{Z}_{21} & \mathbf{Z}_{22} & \mathbf{Z}_{23} & \mathbf{Z}_{24} \\ \mathbf{Z}_{3H} & \mathbf{Z}_{31} & \mathbf{Z}_{32} & \mathbf{Z}_{33} & \mathbf{Z}_{34} \\ \mathbf{Z}_{4H} & \mathbf{Z}_{41} & \mathbf{Z}_{42} & \mathbf{Z}_{43} & \mathbf{Z}_{44} \end{pmatrix} \quad (5.20)$$

In eq. (5.20), the **red blocks** correspond to the self-impedance terms of each scatterer. Note that the computation of those self-terms is accelerated and compressed thanks to the FMM presented in Chapter 2. Those are computed once for all and do not vary with the position of the human. The **green blocks** correspond to the mutual impedance terms of the static environment (i.e. the dark gray polygons in Figure 5.21). Those blocks are also computed once for all and will not vary when the person moves. As FMM groups of a pair of static scatterers are in the far region, the computation of those groups does not require brute-force MoM and is fully accelerated with the multipole expansion. Finally, the **orange blocks** represent the mutual impedance terms between the human and the environment. Due to the human motion and the variation of relative distances between the human and the static scatterers, those blocks have to be recomputed at each human position. The human walk is taken into account by using a quasi-static approach, which is valid when the simulation steps are small enough with respect to the wavelength. This can lead to a large number of different positions even for displacements of the order of the meter. By assuming that the human does not get too close to the scatterers (by a distance smaller than the size of the FMM groups), only the translation operators of the orange blocks have to be recomputed at each position.

Once the matrix $\mathbf{Z}$ has been updated for each new position, the new linear system of equations (eq. (5.19)) has to be solved for $\mathbf{i}$. This is written as

$$\mathbf{i} = \mathbf{Z}^{-1} \mathbf{v} \quad (5.21)$$

By inspection of eq. (5.20), only the first line and column vary for each position. It is thus convenient to use the Schur complement [170], which expresses eq. (5.20) as

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{pmatrix} \quad (5.22)$$

where

$$\mathbf{A} = \mathbf{Z}_{HH} \quad \mathbf{B} = (\mathbf{Z}_{H1} \quad \mathbf{Z}_{H2} \quad \mathbf{Z}_{H3} \quad \mathbf{Z}_{H4})$$

$$\mathbf{C} = \mathbf{B}^T \quad \mathbf{D} = \begin{pmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \mathbf{Z}_{13} & \mathbf{Z}_{14} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \mathbf{Z}_{23} & \mathbf{Z}_{24} \\ \mathbf{Z}_{31} & \mathbf{Z}_{32} & \mathbf{Z}_{33} & \mathbf{Z}_{34} \\ \mathbf{Z}_{41} & \mathbf{Z}_{42} & \mathbf{Z}_{43} & \mathbf{Z}_{44} \end{pmatrix}$$

The Schur complement solves the system eq. (5.22) by solving for $\mathbf{i}_1$

$$(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})\mathbf{i}_1 = \mathbf{v}_1 - \mathbf{B}\mathbf{D}^{-1}\mathbf{v}_2 \quad (5.23)$$

Once $\mathbf{i}_1$ is known, $\mathbf{i}_2$ is obtained by:

$$\mathbf{i}_2 = \mathbf{D}^{-1}(\mathbf{v}_2 - \mathbf{C}\mathbf{i}_1) \quad (5.24)$$

As $\mathbf{A}$ and $\mathbf{D}$ do not vary with the human position, $\mathbf{D}^{-1}$ is computed only once. If the human is the only moving part of the scenario and is represented by P basis functions and if the static scatterers are discretized with a total of M basis functions, then for N different positions of the human, using the Schur complement by solving eq. (5.22) instead of solving eq. (5.19) reduces the number of operations by a factor

$$\frac{\text{\#operations (eq. 5.19)}}{\text{\#operations (eq. 5.22)}} = \frac{N(P+M)^3}{NP^3 + M^3} \quad (5.25)$$

where $P \ll M$ and $N \gg 1$. Note that $\mathbf{D}$ may become large. In this case, $\mathbf{i}_2$ is rather computed iteratively using the FMM compressed representation of $\mathbf{D}$ instead of performed a direct inversion as suggested in eq. (5.24).

For each position of the human, the precoder weights assuming the MRT strategy are computed. This is performed numerically by putting a fictive source at the receiver position which radiates fields inducing equivalent currents on the scatterers, by solving eq. (5.19). The weights of the

beamformer are taken as the complex conjugate of the total field (i.e. the fields scattered by the MoM currents added to the incident field radiated by the fictive source), which correspond to the reversed time signal. Dynamic beamforming updates the precoder for each human position, so that the receiver power is always maximized whatever the human location is. For a comparison with the static scenario, the static beamformer's weights are computed in the absence of the human body and are kept constant when the human is walking.

The impact of the dynamic beamforming operation is illustrated in Figure 5.22 for the scenario of Figure 5.21, where the total attenuation with respect to the human position is compared with the static beamforming approach. The walk is modeled using a quasi-static approach, where the walking is step $6 \text{ mm} \simeq \lambda/5$. The precoder for the dynamic case is updated for each of those quasi-static positions. Practically, that update rate could be reduced, as recomputing the beamformer weights increases the complexity of the system and may impact significantly the data rate, although this consideration is out of the scope of this work.

Thanks to the continuous update of the optimal weights, one can observe that the total attenuation is significantly reduced, with up to 20 dB when the LOS is deeply obstructed (red surface), where the optimal MRT limits the attenuation to 10 dB in NLOS conditions. This is an ideal case hardly met in practice for two reasons: firstly, the precoder cannot be update continuously; secondly, the reflectors are assumed as perfect electric conductors, whereas in reality, dielectric materials do not reflect 100 % of the incident energy. It is also observed that beam switching is not required in LOS conditions, as results in Figure 5.22 show a difference of attenuation below 0.2 dB. Under the assumption of PEC reflectors and 2D propagation, a person does not significantly impact the received power when it does not obstruct the LOS. This observation is based on simulation results at 10 GHz, whereas the same behavior is expected to be valid for mmWaves, as the electromagnetic waves tend to behave as optical rays for higher frequencies [171].

5.3.3 Comparison between models and experimental data

The advantage of using dynamic beamforming techniques in a multipath scenario is now compared with the experimental results. The acquired

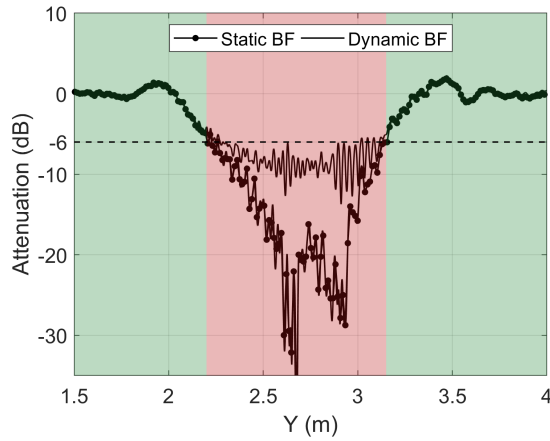
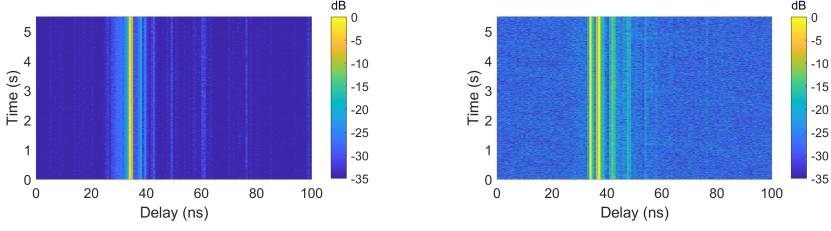


Fig. 5.22 Comparison of the attenuation using static and dynamic MRT precoder versus time, assuming a walk aligned with the $\hat{y}$ axis and a frequency of 10 GHz. The green(red) parts corresponds to the LOS(NLOS) conditions, defined by the -6 dB dashed line.

data are available in the form of time-varying power delay profiles for 144 TX/RX beams pairs. The measured PDP for two different beams pairs is represented in Figure 5.23 when no person is present in this room. Since the room is empty, the PDP variation in the time domain is not significant. However, the different multipath components are clearly visible in the delay domain. The LOS is clearly visible in Figure 5.23a around 33 ns, for the (TX,RX) = (11,7), i.e. when the TX and RX beams are pointed towards each other. This delay corresponds to the travel time between the transmitter and the receiver, as well as the propagation delay through the cables connecting the receiver to the acquisition unit (located close to the transmitter, see Figure 5.10b). The other multipath components reach the receiver with a larger delay after reflections on the environment (walls, floor, ceiling and other scatterers).

The effect of beamforming is illustrated in Figure 5.23b. In this example, the beam pair is selected so that the TX and RX beams point towards the right wall, as seen from the transmitted side. One can clearly observe that the MPC around 37 ns has a higher power than when the beams were facing each other. In this case, the MPC associated to the 1st order reflection on the right wall has a higher amplitude than the LOS component. This is



(a) Beams pointing towards each other ((TX,RX) = (11,7)).

(b) Beam pointing towards the lower wall ((TX,RX) = (12,10)).

Fig. 5.23 Power delay profile of the empty room (no person present) for different beam pairs.

a nice example illustrating how it is possible to allocate more power to different MPCs using beamforming techniques when the LOS is obstructed.

In this work, we are interested in the total power available at the receiver. The total power includes the contribution of the LOS and all the others delayed MPCs. This value P_{tot} without loss of generality is a time-dependent variable obtained from the measurements as

$$P_{\text{tot}}(t) = \int_0^{\tau_{\text{max}}} |\text{PDP}(t, \tau)|^2 d\tau \quad (5.26)$$

In the previous sections, this power was estimated using a 2D MoM solver with PEC scatterers. This method was very useful for a theoretical analysis of beamforming capabilities in a multipath environment. However, due to its huge complexity, it is not suited to the analysis of realistic 3D scenarios. Moreover, it is implemented only for PEC material (authors in [172] provide an implementation for dielectrics, but is restricted to 2D scenarios in the sub-6 GHz band). Even if results from Section 5.2.3 have proven that the PEC assumption for the human body is reasonable for the human body, this hypothesis does not hold for the modeling of office dry walls.

For these reasons, a ray-tracing simulator has been implemented in order to simulate the indoor propagation effects. This tool based on the geometrical optics enables the simulation of 3D scenarios and the modeling of 1st and 2nd orders reflections on dielectric flat surfaces at a very low computational cost. Moreover, it is easily combined with the KED model for the human body, which represents the human shadowing with a good accuracy. On top of that, by tracking the distance traveled by each ray, esti-

imating the power delay profile such as Figure 5.23 is an easy task. This will be particularly helpful for the validation of the ray-tracer and the identification of the measured multipath components.

Ray-tracing simulator including the diffraction by a walking person

Ray-tracing simulators are particularly helpful when modeling propagation mechanisms, especially for mmWaves, where the received signal is assumed as a combination of rays leaving the transmitter and reaching the receiver after interaction with the environment. In this work, only LOS propagation and reflections (up to the second order) are taken into account, even if diffraction by edges and corners can also be integrated [173]. In addition to the features of a classical ray-tracing simulator, the attenuation due to the walking person is also included. For each position of the human, it consists in the computation of the attenuation of the KED model for each ray.

The ray-tracing simulator workflow is summarized as follows:

1. Define the environment (dimensions and material),
2. Compute the existing rays (LOS, 1st and 2nd order reflections) between the transmitter and the receiver,
3. Compute the reflection coefficients at each reflection point,
4. Compute the attenuation of each ray for each position of the person,
5. Apply the beamforming gain factor to each ray depending on the angle of arrival/departure,
6. Compute the power delay profile by applying the proper delay associated with each ray and for each time instant.

The measurement environment is represented in Figure 5.24. This representation is a simplified model of the realistic scenario which only includes the walls, the floor and the ceiling which are assumed as flat surfaces. Other objects such as sofas, TV screen, door, etc. are not represented because their positions are not known precisely and they are quite complex to represent. Walls are plaster panels and the floor and ceiling are in concrete, with complex relative permittivity $\hat{\epsilon}_r = \epsilon_r - j\frac{\sigma}{\omega\epsilon_0}$ provided in Table 5.3.

The reflections points and the reflection coefficients are computed with MATLAB, inspired by [175]. Based on this information, the power P_i and

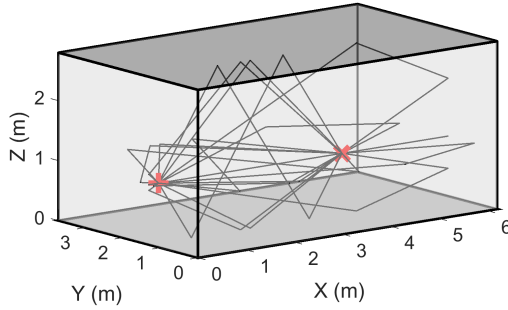


Fig. 5.24 Representation of the room used for the ray-tracing simulations. The transmitter is located at $[0.2, 1.3, 1]$ and is represented by the red "+" sign; the receiver is located at $[4, 1.3, 1]$ and is represented by the red "x" sign. The black lines represent the ray leaving the transmitter and reaching the receiver through multipath propagation (LOS, 1st and 2nd order reflections).

propagation time τ_i associated to ray i is then computed. The reflections are limited to the second order, as more reflections do not have a significant impact on the total received power. The total number of rays is composed of a single LOS ray, M 1st order rays and N 2nd order rays (see Figure 5.25). The m^{th} ray of the first order is decomposed in two rays

- from TX to R_m^1
- from R_m^1 to RX.

The n^{th} ray of the second order is decomposed in three rays

Table 5.3 Complex relative permittivity used for the ray-tracing simulator [174].

Material	$\hat{\epsilon}_r$ (60 GHz)
Plasterboard	$2.94 - j0.063$
Concrete	$5.31 - j0.269$

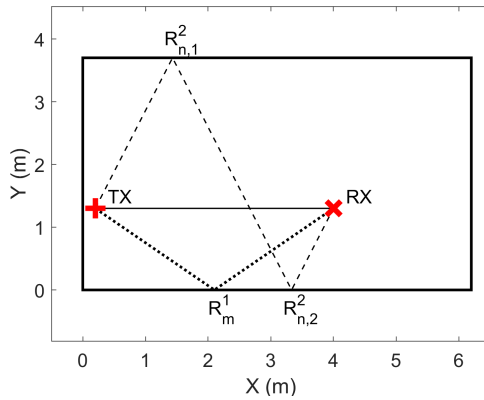
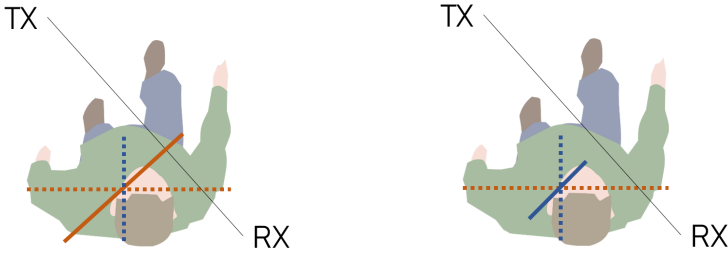


Fig. 5.25 Top view of Figure 5.24. Decomposition of the m^{th} ray of the first order (dotted line) and the n^{th} ray of the second order (dashed line). The LOS ray remains unchanged (solid line).

- from TX to $R_{n,1}^2$
- from $R_{n,1}^2$ to $R_{n,2}^2$
- from $R_{n,2}^2$ to RX.

The unique LOS ray is not reflected on any surface by definition and is thus not modified. The number of rays grows from $(1 + M + N)$ to $(1 + 2M + 3N)$. The decomposition in sub-rays is used because first and second order rays can experience an attenuation due to the knife-edge at different time instants.

For each position of the human, the attenuation of each of the $(1 + 2M + 3N)$ rays is computed with the help of the knife-edge diffraction model of eq. (5.8). In a scenario of multipath propagation, all the rays are not perpendicular to the human walk anymore. In addition to the knife-edge representing the torso of a person in Section 5.2.7, a second knife-edge is added along the shoulders of the human, as proposed in [154]. Thanks to this second knife-edge, the model takes into account the diffraction by the body for a walk parallel to the LOS, where a single knife-edge model would fail. The effective knife-edge width corresponds to the knife-edge



(a) Knife-edge representing the shoulders. (b) Knife-edge representing the torso.

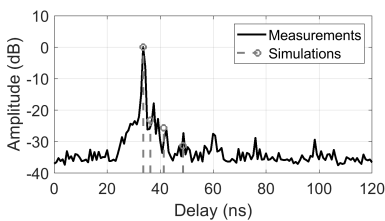
Fig. 5.26 Knife-edge diffraction models for a non-perpendicular walk.

aligned with the shoulders(torso) projected perpendicularly to the LOS, as illustrated on Figure 5.26.

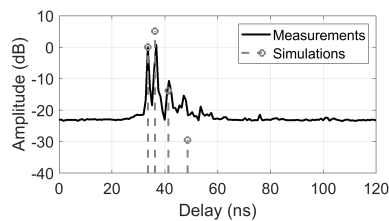
Empty room measurements

The ray-tracing simulator is first validated based on the empty room measurements and for the two beam pairs corresponding to Figure 5.23. As the radiation patterns of the transmitting and receiving arrays have not been measured over a full sphere, rays for which the gain corresponding to their angle of arrival/departure is not available are simply discarded. This condition removes the rays which are reflected from the floor and the ceiling, for the specific measurement scenario illustrated in Figure 5.24.

Simulations and measurements are compared in Figure 5.27 for $(TX,RX) = (11,7)$ and $(TX,RX) = (12,10)$. For the measurements, the power-delay profile is averaged in the time domain, since the scenario is static. The PDP



(a) $(TX,RX) = (11,7)$.



(b) $(TX,RX) = (12,10)$.

Fig. 5.27 Comparison of the PDP of the empty room, for two beams pairs.

are normalized to the LOS power and synchronized with the LOS delay. One can observe from Figure 5.27a that the delay of the MPCs is similar to the measurements. Slight differences come from the approximate knowledge of the room dimensions and the transmitter and receiver positions. The power associated with the four modeled MPCs is also quite accurate, within a few dB. Some measured MPCs at 60 ns, 76 ns and 98 ns are not captured by the ray-tracer and corresponds to discarded rays reflected on the floor and the ceiling, to higher order reflections or to reflections on scatterers (e.g. the TV screen) which are not modeled in the simplified scenario of Figure 5.24. When the beamforming pair steers the TX and RX beams towards the right wall (Figure 5.27b), the LOS component has a smaller power than the first MPC. This effect is also captured by the ray-tracing simulations, which overestimates the associated power by 4 dB, while the MPCs corresponding to a delay of 48 ns is underestimated. Note that the measured values of (TX,RX) = (12,10) are more sensitive to noise, as the dynamic range for this experiment is smaller than for the case where the beams are facing each other (20 dB versus 35 dB). This reduced dynamic range is also visible in Figure 5.23.

Higher accuracy is not expected because of the lack of a precise ray-tracer calibration. Besides the uncertainties on the room geometry and arrays positions, the dielectric properties of the room material are not known and the rays reflected on the floor and ceiling are not taken into account. Nevertheless, results in Figure 5.27 indicate that the ray-tracer is able to model the main propagation effects, including the effect of beamforming.

Multipath with a walking person

The impact of the multipath propagation on the received power is illustrated in Figure 5.28. For the measurements, the power associated to the LOS path is compared to the integrated power of the PDP in the delay domain. Both powers are normalized to the power of the LOS component when no person is present in the room. This comparison is performed for "path 2" and "path 3", in Figures 5.28a and 5.28b respectively. Before the main blocking event, the measured total received power is nearly 2 dB higher than the LOS component. For each time snapshot, the pair of transmitter and receiver beams that maximizes the total received power is selected among the 144 possibilities; this corresponds to the "Opt. BF" curves in Figure 5.28. The attenuation of the received signal is limited to 10 dB when the person is blocking the LOS, by steering the transmitter and the re-

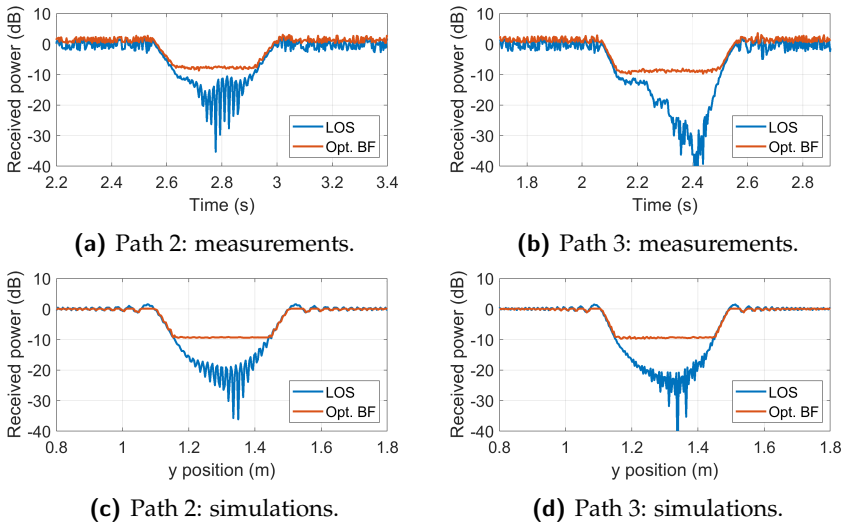


Fig. 5.28 Impact of the multipath and beam switching on the total received power, for different human position: comparison measurements and simulations.

ceiver towards the wall with the (TX,RX) pair (12,10). Sending more power towards the wall when the person blocks the LOS also reduces the fluctuation of the signal strength, which is kept constant during the shadowing event. Interestingly, the total received power is roughly the same when the human walks closely to the receiver. Because of the blocker proximity, the LOS component is strongly attenuated (up to 40 dB), whereas this is not the case when the system selects the optimal beam pair. On top of that, the array beams are not very narrow, power can thus be received from a broad set of angles of arrival without being strongly attenuated.

The results obtained with the ray-tracing tool and the multiple knife-edge diffraction model for the human body are represented in Figures 5.28c and 5.28d for "path 2" and "path 3", respectively. One can observe that the proposed numerical framework also enables an accurate modeling of the multipath propagation mechanisms and the beamforming capabilities, as the estimated received power is estimated to approximately -10 dB for both the simulated and measured results. Obtaining a roughly constant received power during NLOS events through beamforming was also obtained from the results of Figure 5.22.

Despite the coarse representation of the scenario, the ray-tracing simulator combined with the knife-edge diffraction model provides good insights when modeling the time varying power delay profile. Figure 5.29 compares the measured and the modeled PDP for the "path 2" and "path 3" scenarios for $(TX, RX) = (11, 3)$. The delay reference for the simulated PDPs is calibrated in order to match the delay of the measured LOS component. This accounts for the delay induced by the distance between the transmitter and the receiver, as well as the delay of the cables, etc. The simulated taps for which the power is lower than the dynamic range of the experimental receiver are set to -40 dB [163]. It is observed that the simulated PDP capture the main taps of the power delay profile. On the other hand, measured taps around 76 ns and 98 ns are not visible in the simulated PDP; they correspond to reflections on scatterers which are not included in the simplified model of Figure 5.24 or to higher order reflections. The blockage of the LOS (number 1 in red in Figures 5.29a to 5.29d) is clearly visible in both the measurements and the simulations. Depending on the distance between the blocker and the receiver, the same multipath component is blocked at different time instants. This is visible by comparing the number 2 in Figures 5.29a and 5.29b and the number 3 in Figures 5.29c and 5.29d. Those events are also clearly captured by the ray-tracing simulator. Note that in the time domain, there is no absolute synchronization between the measurements and the simulations. This explains why the shadowing events in different tap occur at different times. Moreover, the dimensions of the room, the material used and the positions of the transmitter and the receiver are not known precisely. It is thus not expected that the simulated PDP fit perfectly the measurements, even if the accuracy reached in Figure 5.29 allows the modeling of the main propagation mechanisms.

5.4 Discussion and conclusion

This chapter sums up all the numerical methods employed and developed in this thesis for the analysis of the scattering by large objects, through the example of the human scattering at mmWaves. Besides the electromagnetic solvers, a particular attention was given to the choice of a proper model. The first idea was to consider cylinders as a more accurate representation of the human shape, compared to the simpler knife-edge diffraction model. The cylinder model delivered good performance for the mod-

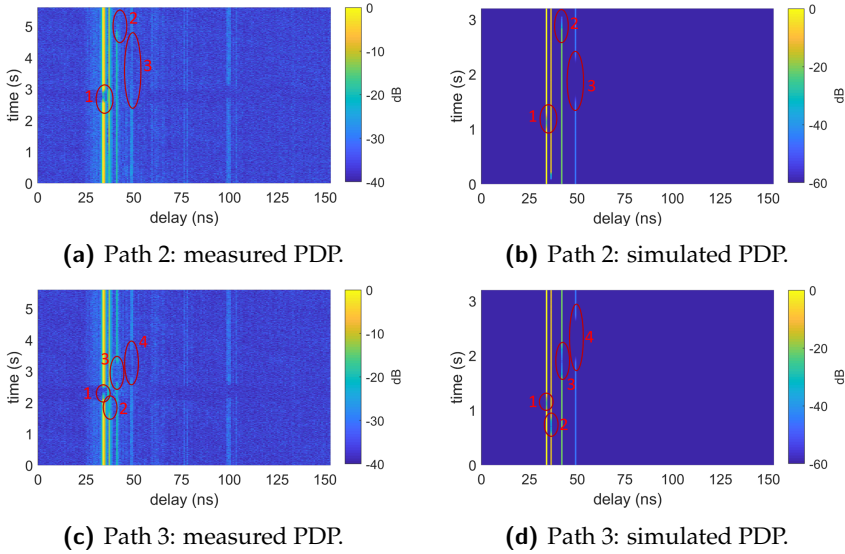


Fig. 5.29 Measured and modeled time-varying PDP for "path 2" and "path 3" scenarios, for the beam combination (11,3).

eling of the diffraction around a human leg in [145]. Results from Section 5.2 indicate that this is not the case anymore when the whole body is considered. Even if the elliptical cylinder seems to fit the actual body shape, this is not a sufficient condition to be considered as an accurate model from an electromagnetic point of view. For example, the MoM solver models creeping waves that flow all around the scatterer surface which creates a large shadow area in the forward direction. Those creeping waves likely have a smaller intensity (if they exist) on a real person, due to the irregularities of the human body or the clothes which generate a surface roughness not negligible at mmWaves [176, 152]. Results showed that the knife-edge diffraction model by assuming the body as a blocking screen is a quite accurate model. A tedious meshing of the human body combined with the physical optics as performed in [147] does not provide a significant improvement of accuracy. Besides its performances and its ease of implementation, the computational complexity of the KED with respect to other currents based methods is smaller by two to three orders of magnitude. It is thus the best candidate for the evaluation of the human shadowing in a multipath environment carried out in Section 5.3.

After a validation of the models for the LOS blockage, those have been embedded in a realistic indoor scenario. Apart from being attenuated by a human body, the transmitted signal is reflected on the environment before reaching the receiver. The reflections and diffractions on the neighboring scatterers are modeled when using numerical solvers as the 2D MoM tool developed for the demonstration of the beamforming capabilities in Section 5.3.1. However, this method is not well suited to propagation at mmWaves due to the very large meshes required for the modeling of a realistic 3D environment. A ray-tracing simulator has been developed, which provides (for a computational cost independent of the frequency) a good estimation of the multipath components, as demonstrated in Section 5.3.3. For those simulations, only the 1st and 2nd order reflections are taken into account. More accuracy between the simulations and the measurements is expected with a finer geometrical representation of the environment and by taking the diffraction effects into account.

Once the human model and the numerical tools have been developed, they are used for the estimation of the received power when the human body is walking perpendicularly to the LOS. When considering only the LOS component, the attenuation reaches up to 20 dB when the person is walking halfway between the transmitter and the receiver at a distance of 2 m. The non-symmetrical radiation patterns of both the transmitter and the receiver have been included in the numerical simulation as they have a significant impact on the attenuation. This effect is even more important when the array beams become narrower, as presented in [104] for horn antennas at 73 GHz. The LOS attenuation grows up to 35 dB when the human moves closer to the receiver, as also mentioned in [104, 177]. It was shown in Section 5.3.3 that the multipath propagation mitigates the human shadowing attenuation. When the LOS is blocked, other propagation paths are still reaching the receiver, thus limiting the total attenuation. This is possible because the TX and RX array beams are not too narrow, as observed in Figure 5.11. When combined to beamforming and by selecting the pair of beams that maximizes the received power, the attenuation during the shadowing event can be kept constant around 10 dB. This conclusion also holds when the blocker is close to the receiver, as observed in Figure 5.28b.

6

Conclusion

The goal of this thesis was to study numerical techniques for the estimation of the fields scattered by electrically very large objects. For such scatterers, the physical optics approximation is a good candidate, as it provides an accurate estimation of the scattered fields in the specular region with an attractive complexity, growing linearly with the number of discretizing facets. However, it was observed that a poor accuracy was achieved for observation angles located away from the specular region. By comparing the currents on the surface with those obtained with a full-wave method, our intuition was that it was due to the null currents assumed by the PO on the surfaces that are not visible by the source, as well as the poor accuracy of the currents in the transition region. This is particularly remarkable for the TE polarization (see Figure 4.9) where the magnitude of the currents abruptly drops to zero in the shadow region. In order to provide a better estimate of the currents in this region, our idea was to redistribute the initial source on a fictitious equivalent surface surrounding the object. The initial source can be expressed with a continuous distribution of electric and magnetic elementary currents flowing on the equivalence surface and obtained with the help of the equivalence theorem. With the initial source distributed all around the scatterer, every elementary portion of the surface is illuminated by at least one elementary electric or magnetic equivalent

current: the classical physical optics approximation is then used in order to compute the currents in the initial shadow region.

This method enables a significant accuracy improvement of the surface currents in the transition region as well as in the shadow region where the currents are not null anymore. The corrected currents provide a smaller error on the scattered fields than the classical PO with respect to a MoM reference, in particular in the forward region, thus meeting the above-mentioned needs. The improvement is particularly noticeable for elongated objects comprising surfaces nearly aligned with the direction of propagation, as shown in Section 4.3.2. Although the complexity of the modified physical optics is still of the order of $\mathcal{O}(N^{1.5})$ with the help of the decomposition of the Green's function in multipoles, the same complexity is preserved when considering multiple-scattering scenarios, where a naive implementation has a complexity in $\mathcal{O}(N^2)$. This is not as good as the $\mathcal{O}(N \log N)$ achieved in [17], although our method enables a significant improvement of accuracy than the classical iterative physical optics, as outlined in Section 4.4.2. The accuracy provided by the latter is quite poor when considering elongated scatterers (see Figure 4.28) or when successive multiple complete obstructions occur (see Figure 4.30). Future work would consider other propagation operators for the rapid evaluation of the currents on the equivalence surface. In particular, the decomposition of the Green's function into homogeneous plane waves requires fewer terms than the multipole decomposition for distant sub-domains as presented in Section 2.4.

The validity of the PO approximation depends on the geometry characteristics of the considered scatterers and is well suited for large and smooth objects. In an antenna context, the physical optics is generally employed for the analysis of reflectors [178] or lenses [14]. However, it generally cannot be applied to the analysis of the feeding element itself because of its complex geometry. In order to analyze another type of large electromagnetic problems (antenna arrays), we have developed an appropriate numerical method based on an accelerated full-wave approach. Even though the considered element itself is small in terms of unknowns, it is not possible to compute the array characteristics using a brute-force method of moments for an array containing hundreds of elements. The high computational complexity of this problem is tackled by representing the currents on the antennas as a combination of macro-basis functions (MBFs) which are assumed identical for all the elements in the array. A common method

consists in computing the MBFs using a multiple interaction process (thus obtaining *primary* and *secondary* MBFs) [73]. In our approach, the MBFs are computed at each iteration of the iterative solver, where the matrix-vector products are accelerated with the FMM. The formulation of the FMM for identical and disconnected elements is particularly convenient in terms of memory usage and number of operations. The combination of the MBFs accelerated with the FMM enables the computation of all the embedded element patterns in arrays with hundreds of elements, within minutes. Based on this fast solution, the effect of the finite ground plane is included by computing rapidly the currents on the ground plane with the physical optics approximation. For large ground planes and many antennas, this operation is accelerated by tabulating the magnetic field radiated by the MBFs prior to a phase extraction. This allows a non-regular sampling which increases with the radial distance with respect to the antenna. The proposed hybrid approach improves the accuracy on the radiated fields (with an error below -30 dB in a $\pm 30^\circ$ around the main beam for an array of 8 elements) with a much smaller complexity than using a brute-force approach, which is in general impractical. However, the effect of the finite ground plane becomes less visible in the main beam when the number of antennas and thus the area of the ground plane increases.

Using the fast computation tools developed for the analysis of large antenna arrays, a similar formalism is employed for the computation of the radar response of wind turbines. Due to their large electrical size that may reach thousands of wavelengths, they reflect a significant amount of power back to the radar, they create a large shadow as far as a few kilometers in the forward region, and false echoes behind the wind farm, partly due to the multipath propagation inside the wind farm. This application is a good illustration for the study of the forward scattering mechanisms by large objects. Due to their time-varying geometry (rotation and pitch of the blades, orientation of the mast) and given their huge electrical size, a full-wave analysis with the precise geometry is not an option, especially when multiple wind turbines are considered. Due to their vertically elongated geometry, and based on the results in [9], the assumption of infinite cylinders was adopted. The problem is now reduced to a two-dimensional scenario where the wind turbines are considered as 2D circular scatterers, reducing drastically the complexity of the problem. Under this assumption, it is possible to carry out the computation using a full-wave approach and the formalism developed for the analysis of antenna arrays with identical

elements (Chapter 3). However, the wind turbines are much more distant than the antennas in an array: in this case, it was observed that the reactions integrals between sub-domains are computed more efficiently using a smaller number of inhomogeneous plane waves than the number of plane waves required with the FMM. This decomposition into waves that are at the same time propagating and evanescent also enables the rapid computation of the scattered fields in a large area behind a wind farm, where a significant attenuation is observed up to a few kilometers behind the wind farm (see Section 2.5.2). In Section 2.5.3, the same formalism has been used for the modeling of the extended zone of echoes observed behind a wind farm. As suggested by the authors in [47] and [48], these observations may be caused by the multipath propagation within the wind farm, where the width of the echo in the radial direction is related to the finite impulsion of the radar, while the width in the azimuth direction is related to the beamwidth of the radar. By taking into account up to three reflections, false echoes are detected up to a few kilometers behind the wind farm, depending on the wind farm geometry. Some further analysis of measured data is necessary for discriminating the effect of the multiple scattering between masts, the effects of the (rotating) blades and the additional clutter introduced by reflections on the ground.

The last considered application of the diffraction by large objects in the forward region focused on the attenuation caused by a human body over a 60 GHz communication link in an indoor environment. In Chapter 5, the different numerical techniques used in the rest of the thesis are compared in terms of accuracy and computation time using different models and are validated using measured data. Surprisingly, the simplest investigated model which represents the diffraction around the human body as the diffraction by a pair of PEC flat plates provides a very accurate comparison with the measured attenuation in various scenarios. This consideration is the key message of Chapter 5: in some cases, when the scenario or object to model is very complex, one should consider which enhancement is expected with the development of a very precise model, which may be difficult (if not impossible) to develop and would increase the computation time. For our application and the considered frequency, it was pointless to use a model which includes the precise anatomy of the human, which may be even impossible to obtain and would have lead to hours of computation for a single excitation. Indeed, based on the observation of the measured data in Section 5.2.7, a significant variability is observed between different

realizations of the same experiment. Hence, a simple model was looked for, which combines a good accuracy with respect to the measurement and a low implementation and computational complexity. A double knife-edge diffraction model was selected based on those criteria. This approach models quite accurately the attenuation of a human walking perpendicularly to the LOS at a very low computational cost. The received signal is attenuated by roughly 20 dB when the human moves a few meters away from both access points. This value may reach 40 dB when the human is located closer to one of the access points (this is a likely situation as the access points can be mounted on walls, for example). As this blockage lasts for a few hundreds of millisecond, the signal is transmitted through alternative propagation paths using mainly reflections with the environment and beamforming techniques enabled by phased arrays (studied in Chapter 3). By performing beam switching at both the transmitter and the receiver, the attenuation is capped to 10 dB, which increases the SNR by 30 dB in some cases (see Section 5.3). The beamforming capabilities are well estimated by combining the knife-edge diffraction model, the radiation pattern of the arrays and the main characteristics of the environment with a simple ray tracing simulator, at a fairly low computational cost. Again, a simple representation of the scenario is preferred to a very complex and detailed description. It is expected that a higher accuracy can be achieved by a proper calibration of the ray-tracer (position of the walls and human, characterization of the materials in the considered frequency band, etc.). Such tools are very helpful for a better understanding of the channel characteristics in a specific environment, including walking humans. The same approach can be adapted to other indoor scenarios and further validations would include the effect of multiple humans.

As illustrated throughout this manuscript, many modern applications require the modeling of the interactions of electromagnetic waves with very large structures. Even though the computational power is still increasing, it is important to understand the physics behind the problem under analysis, so that appropriate approximations and proper numerical tools can be selected. This choice is critical in order to provide a pertinent model with a relevant accuracy within a reasonable computation time, depending on the targeted application.

List of publications

Journal papers

- **T. Pairon**, C. Oestges and C. Craeye, "Improved physical optics computation near the forward scattering region: application to 2D scenarios," *IEEE Transactions on Antennas and Propagation*, in early access.
- V. Semkin, J. Haarla, **T. Pairon**, C. Slezak, S. Rangan, V. Viikari and C. Oestges, "Analyzing Radar Cross Section signatures of diverse drone models at mmWave frequencies," *IEEE Access*, vol. 8, pp. 48958-48969, Mar. 2020.
- T. Feuillen, **T. Pairon**, C. Craeye and L. Vandendorpe, "Localization of rotating targets using a monochromatic continuous-wave radar," *IEEE Antennas and Wireless Propagation Letters*, vol. 16, pp. 2598-2601, Aug. 2017.

Conference papers

- O. Mhadhbi, J. Cavillot, **T. Pairon** and C. Craeye, "Acceleration of physical optics interactions using inhomogeneous plane waves," *European Conference on Antennas and Propagation (EuCAP)*, Copenhagen, Denmark, 2020.
- **T. Pairon**, J. Cavillot, S. Karki and C. Craeye, "Effect of finite ground plane on irregular arrays with metallic elements", *International Conference on Electromagnetics in Advances Applications (ICEAA)*, Granada, Spain, 2019.

- **T. Païron**, J. Cavillot and C. Craeye, "Fast numerical techniques for irregular arrays devoted to satellite SAR on finite ground plane," *40th ESA Antenna Workshop*, Noordwijk, The Netherlands, 2019.
- **T. Païron**, C. Oestges, M. Drouguet and C. Craeye, "Efficient evaluation of energy focusing based on eigen-beamforming", *European Conference on Antennas and Propagation (EuCAP)*, Krakow, Poland, 2019.
- **T. Païron**, S. Karki, S. Monni and C. Craeye, "Efficient analysis of arrays of compact metallic elements devoted to satellite SAR", *European Conference on Antennas and Propagation (EuCAP)*, Krakow, Poland, 2019.
- H. Bui-Van, **T. Païron** and C. Craeye, "Fast iterative techniques for the simulations of large antenna arrays," *International Conference on Electromagnetics in Advances Applications (ICEAA)*, Cartagena de Indias, Colombia, 2018.
- S. Karki, J. I. Echeveste, **T. Païron** and C. Craeye, "Sequentially shifted beamforming from a conformed conical array," *47th European Microwave Conference (EuMC)*, Nuremberg, Germany, 2017.
- J. Léger, **T. Païron** and C. Craeye, "Analytical formulation of the micro-Doppler spectrum for rotating targets in the near-field," *European Conference on Antennas and Propagation (EuCAP)*, Paris, France, 2017.
- **T. Païron**, C. Craeye and C. Oestges, "Doppler spectrum of a rotating smooth cylinder," *European Conference on Antennas and Propagation (EuCAP)*, Davis, Switzerland, 2016.

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