



ENGINEERING RECIPROCAL STRUCTURES

Applications to Floors and Domes

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Preamble

Reciprocal Structures (RS) resemble "pick-up sticks", achieving high architectural aesthetics through their lack of apparent hierarchy and enabling spans over longer distances with shorter beams. The latter is particularly relevant when building with timber, which is naturally limited in length. In addition, recent developments allow us to consider the irregular shape of logs from design through to construction, using 3D scanning and modelling tools, as well as robot arms for high-precision cutting.

Due to the reciprocity between beams, challenges in geometric design, mechanical behaviour, and construction phasing increase with the number of modules, particularly in spatial RS. These challenges converge in the connection design, which must allow easy beam assembly, tolerate fabrication or assembly inaccuracies, ensure sufficient strength and be correctly modelled. As a result, there is significant apprehension and a general lack of confidence among designers towards reciprocal structures, as built examples are generally limited to single modules or planar structures. Finally, this structural type is not considered optimal and therefore chosen mainly for its aesthetics.

This thesis first examines an RS log floor comprising a four-beam module. It starts with optimising the floor geometry with respect to the thickness and volume of the material used, under different design criteria. Afterwards, this pattern is compared with the "traditional" pattern formed by joists spanning in the shortest direction, as well as with other conventional patterns that use transversal primary beams to reduce joist span. For optimisation and comparison, to limit the number of floor parameters included in the design criteria (floor geometry, loading, material properties, and in-service performance parameters), this work uses the morphological indicator method. This method reduces the number of parameters to be considered by combining them into dimensionless indicators. This allows for the generation of universal guidelines.

For a square floor, an optimal RS pattern always minimizes the volume compared to the traditional pattern and is as good as the patterns composed of transverse beams. Furthermore, the RS pattern can be the most suitable solution for minimising the floor thickness, depending on how the beams are connected. These results show that the reciprocal floor pattern combines an

optimal solution with a solution for using short-length logs.

In the first part, the structural analysis was based on conventional RS assumptions and did not account for a specific connection type. However, the behaviour of an RS is highly dependent on the node modelling. To better understand the link between the physical connection and its model, and the impact on RS behaviour, the second part of this work broadens the discussion to include any material.

The second part is divided into two sections. The first section reviews the proposed connections and supports used in built RS and research prototypes, with a focus on their numerical modelling. The propositions generally fix rotational movements, as in welded connections, compromising the essence of reciprocal behaviour and aligning them more closely with the principles of gridshells. Indeed, a reciprocal structure typically consists of stacked elements, with rotationally free ends.

One aim of using rotational fixed connections is to prevent local and global geometric instabilities, including rigid-body rotations about the longitudinal axis. To address this need, the second section presents four methods from the literature for assessing geometric stability and compares their effectiveness. Only one applies to all connections and support types, and to all RS geometries.

This general method showed that hinged connections and supports can generate geometrically stable space RS. Indeed, the rigid-body rotation about the longitudinal axis is prevented by the distance between the beams' axes at the connection, which creates a lever arm that blocks this rotation. However, geometries with specific central symmetry exhibit global geometric instability, necessitating the application of this method at the early design stage.

The literature review of the second part also showed that connection modelling was difficult to validate experimentally. This discrepancy is mainly due to the complex mechanical behaviour of RS, which is difficult to capture, and to the connections, whose complex designs generally exhibit nonlinear behaviour, including friction, locking effects, and semi-rigid responses.

The last part of the thesis demonstrates that hinged connections enable a reliable numerical model. It develops a spherical hinge connection comprising rods attached to the ends of the bars. It proposes a methodology for integrating this connection into a numerical model to obtain reliable computational results.

To achieve this, large-scale steel structural prototypes equipped with such a connection were constructed and tested. The experimental measurements

were performed using a redundant measurement system, notably a motion capture system, which is highly effective for measuring three-dimensional displacements and had never previously been used for reciprocal structures.

The results show a strong similarity between the numerical and experimental values of displacements and stresses.

These three parts correspond to journal papers (in order: 3, 2, and 1 from the list below), with additional context and links to integrate them.

All Journal Publications

1. STEINMETZ Martin, LATTEUR Pierre, and SGAMBI Luca. Predictability of numerical models for reciprocal structures. *Engineering Structures*, 2026, vol. 365.
2. STEINMETZ Martin, SGAMBI Luca, and LATTEUR Pierre. Geometric Stability of Reciprocal Structures. *Journal of Structural Engineering*, 2026, vol. 152, no 3.
3. STEINMETZ Martin, LATTEUR Pierre, and SGAMBI Luca. Optimal log floor patterns. *Engineering Structures*, 2023, vol. 297.
4. GOOSSE Justin, STEINMETZ Martin, and LATTEUR Pierre. A non-destructive protocol for the industrial proof-loading of tree trunks used for beams and floors. *Wood Material Science & Engineering*, 2025.
5. ALMEIDA João, STEINMETZ Martin, RIGOT Fabrice, and DE COCK Sébastien. Shape-memory NiTi alloy rebars in flexural-controlled large-scale reinforced concrete walls: Experimental investigation on self-centring and damage limitation. *Engineering Structures*, 2020, vol. 220.
6. LEZAACK Matthieu B., SIMAR Aude, MARCHAL Yannick, STEINMETZ Martin, FAES Koen, and ALMEIDA João. Dissimilar friction welding of NiTi shape memory alloy and steel reinforcing bars for seismic performance. *Science and Technology of Welding and Joining*, 2022, vol. 27, no 6.

All Conference Publications

1. STEINMETZ Martin, SGAMBI Luca, and LATTEUR Pierre. Optimal reciprocal log floor patterns. In : *Proceedings of IASS Annual Symposia. International Association for Shell and Spatial Structures (IASS)*, 2026.
2. STEINMETZ Martin, SGAMBI Luca, and LATTEUR Pierre. Reliable experimental testing methodology for reciprocal structures. In : *Proceedings of IASS Annual Symposia. International Association for Shell and Spatial Structures (IASS)*, 2024.

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Chapter 1

General Introduction

This research begins by discussing the use of logs for reciprocal structures. Later, it took a different direction, focusing on modelling connections by testing reciprocal steel structures to limit variability in the geometrical and mechanical properties during the experimental campaign. In the final chapter, the author outlines further research outlooks on applying the experience gained and the results obtained to the analysis and design of log reciprocal structures. These outlooks may serve as a starting point for further research.

1.1 Context of the research

Timber use in buildings has notably increased in recent years. This trend can be attributed to several factors, primarily driven by efforts to mitigate anthropogenic climate change [68]. It is worth noting that concrete structures have, on average, 42% higher embodied greenhouse gas emissions than their timber alternatives [19]. Among the various products employed [Figure 1.1], adhesive-bonded Engineered Wood Products (EWP) have gained prominence, including Cross-Laminated Timber (CLT), glued-laminated timber (Glulam), Oriented Strand Board (OSB), and Laminated Veneer Lumber (LVL). Indeed, EWP offers greater size availability and increased strength by averaging out defects, making it a preferred choice among engineers.

However, EWP have a higher embodied energy due to their manufacturing processes, particularly compared to kiln-dried sawn timber [Figure 1.2]. Moreover, within the processing chain, approximately 50% of the original log volume is lost as waste [61], and the cut wood grain limits fibre continuity and exposes the knots and other defects [91]. Furthermore, the extensive processing stages significantly escalate the cost of timber construction.

In the late 90s, researchers in Europe and North America [31, 42, 62, 78] followed, two decades later, by Bukauskas et al. [10] conducted comprehensive reviews of the current state-of-the-art "whole timber" construction of small to large diameter, including natural form and mechanically rounded timber structures. They highlight the potential of using logs as structural elements, but note challenges with connections, mechanical grading, and processing.

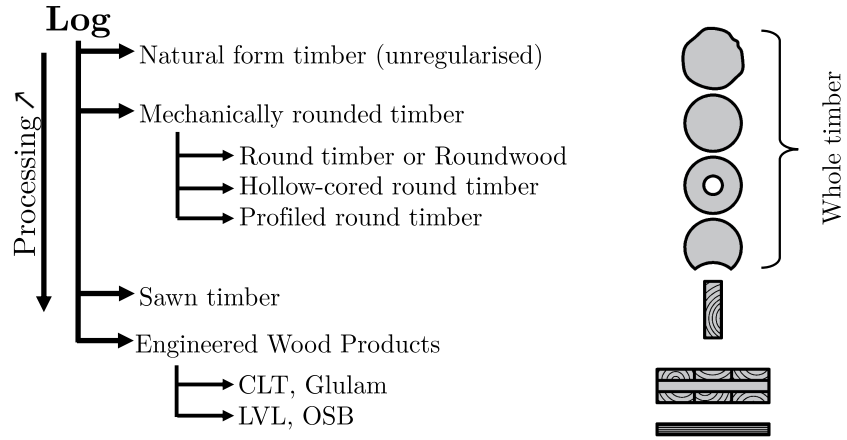


Figure 1.1: Timber products from least to most log processing with the typical final section.

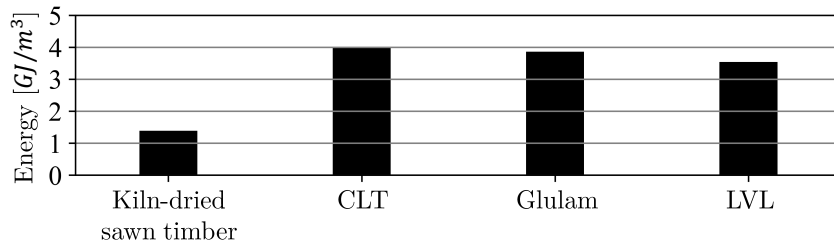


Figure 1.2: Total non-renewable primary energy consumption for production (Scope A1-A3), associated with timber products manufactured in the UK. (Values based on [92])



Figure 1.3: a) Log reciprocal structure constructed at UCLouvain (Belgium) presented in Geno et al. [26]. b) Final wood-wood connection cut using a robotic arm.

In recent years, researchers at UCLouvain [26] constructed a log structure, shown in Figure 1.3, using an innovative wood-wood connection. To minimally process the logs, they used a combination of recent developments: advanced 3D scanning and modelling tools to capture and interact with each log's unique geometry, and robotic arms to precisely cut the connection.

As logs are naturally limited in length, the beams are disposed so that each supports the others, spanning longer distances and leveraging structural reciprocity. However, these researchers faced significant challenges in predicting the structure's mechanical behaviour. Consequently, this research delves into the mechanical behaviour of reciprocal structures.

1.2 Reciprocal structure: definitions and notations

Geometrically, a Reciprocal Structure (RS) is a structure composed of beams that mutually support each other along their span, without any apparent hierarchy. It consists of one or several simple modules, each comprising at least three beams [Figure 1.4a]. Assembling such simple modules allows the generation of geometries of virtually unlimited variety [Figure 1.4]. An RS offers a dual advantage: on the one hand, it presents an unusual and striking architectural appearance, allowing, for example, central roof lights; on the other hand, it enables spanning large distances by superimposing small elements.

An RS follow three fundamental design rules [40]:

- Each beam supports at least one other beam, along its span.
- Each beam end is supported either on another beam or a support.
- Each support carries only a single beam.

RS behaves structurally as a series of stacked beams, with joints connecting only two beams at a time and never at their ends. On the contrary, it should not be confused with gridshells, which are space structures composed of crossed grids of beams, whose overall rigidity comes from the membrane effect and from rigid or partially rigid nodes.

In the literature, RS are also referred to as "reciprocal frames" [4, 9, 47, 71, 95], "multi-reciprocal grids" [67], "Reciprocally (or mutually) supported elements space structure" [63, 64, 65], "Self-Supporting Interlocking Grids for Multiple Applications" [23] or "nexorades (or nexor structures)" [6, 18, 24, 70, 87]. For the latter, each beam is called a "nexor" (from the Latin word for "link"). Some researchers prefer the term "fan" for a module, as it conveys the rotational motion of the assembled beams. To clarify, Figures 1.5, 1.6, and 1.7 present the notation used in this manuscript.

Firstly, beam types can be defined according to the number of supports and connections. The most common types are illustrated in Figure 1.5.

Secondly, this work distinguishes the following types of reciprocal structures composed of straight beams:

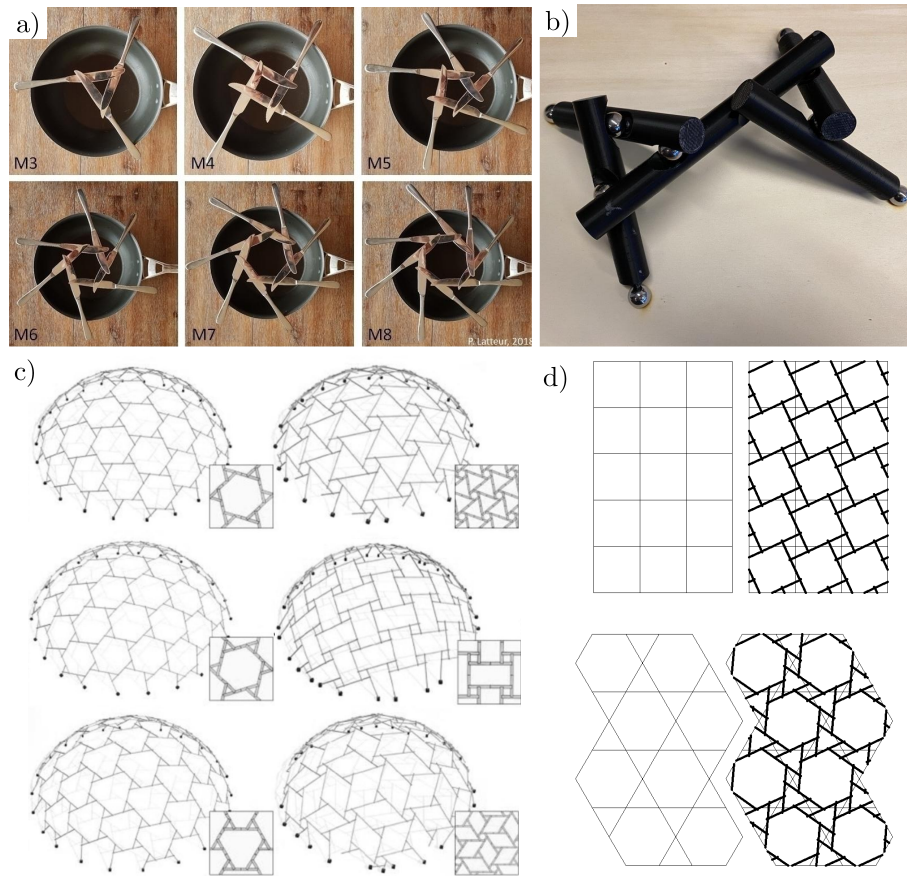


Figure 1.4: a) Simple modules with 3 to 8 beams [40]. b) 3D printing reciprocal structure. c) Different dome patterns [71]. d) Initial and reciprocal tilings.

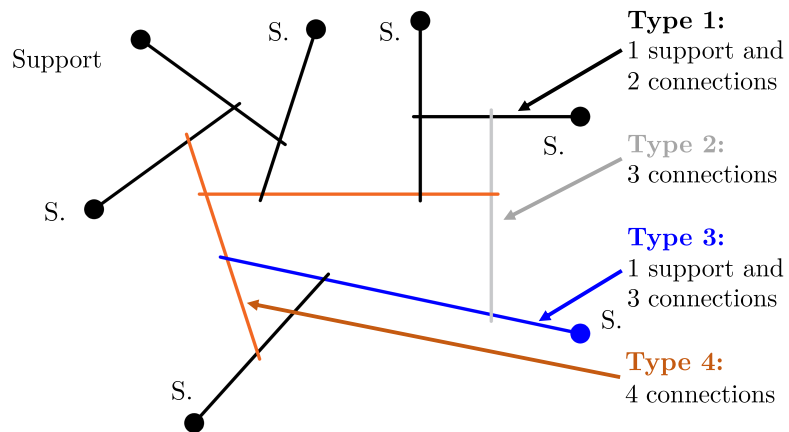


Figure 1.5: Beam types in a schematically represented RS. Beam ends with circles are associated with supports (S.).

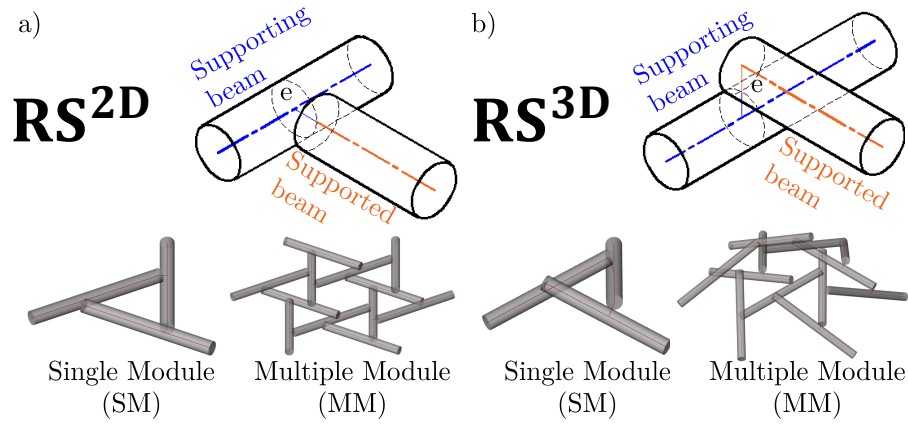


Figure 1.6: a) Plane reciprocal structures. b) Spatial reciprocal structures.

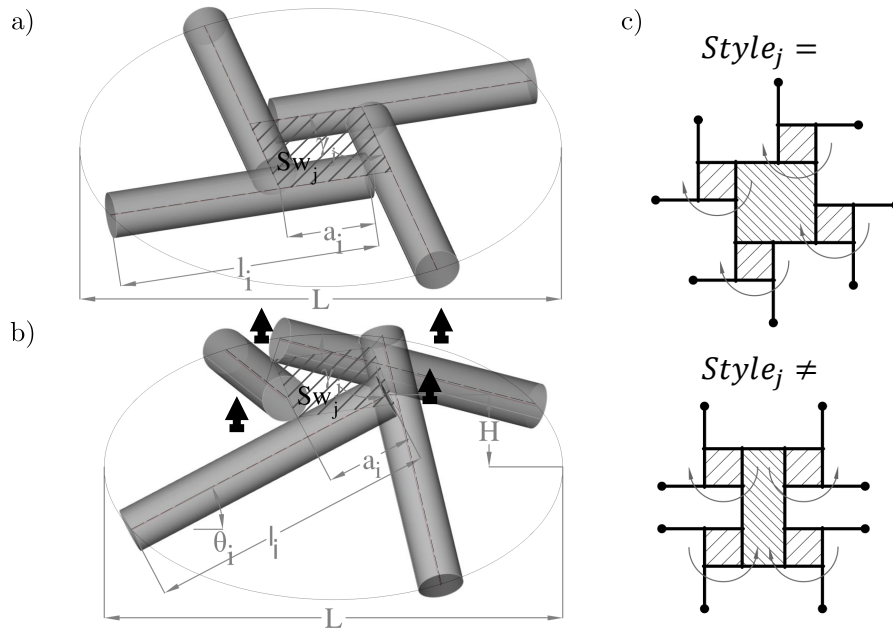


Figure 1.7: a) Parameters of an **RS^{2D}**. b) Parameters of an **RS^{3D}**, arrangements are represented by the vertical arrows. c) Style of an RS.

- Plane reciprocal structures subjected to out-of-plane loading, referred to as **RS^{2D}**, in which the beam axes lie within the same plane [Figure 1.6a].
- Spatial reciprocal structures, referred to as **RS^{3D}** [Figure 1.6b]. In **RS^{3D}**, the supported beam may be suspended from the supporting beam.

In both cases, the connection is characterised by an eccentricity e as indicated in Figure 1.6. Whether dealing with RS^{2D} or RS^{3D} , a distinction is made between [Figure 1.6]:

- A reciprocal structure composed of a single module (SM RS^{2D} or SM RS^{3D}), and composed only of type 1 beams.
- A reciprocal structure composed of multiple modules (MM RS^{2D} or MM RS^{3D}).

Finally, besides the eccentricity e between beams at a connection, multiple parameters describe the geometry of an RS [Figure 1.7]:

- Span of a structure L as the shortest distance between two supports.
- Density d_j of beams forming module j . d_{tot} indicates the total number of beams in the RS.
- Orientation angle γ_j between two beams within the same module j .
- Length l_i between the two ends of beam i .
- Engagement length a_i ($< l_i$) between one end of beam i and the eccentricity of the supported beam. If beam i supports a second beam, the second engagement length is denoted b_i ($< l_i$).
- Engagement ratio α_i ($\in]0; 1[$) between the engagement length a_i and the total length of beam i ; β_i ($\in]0; 1[$) in the case for a second engagement length.
- Engagement window Sw_j defined as the area within module j .
- Style St_j representing the direction of rotation of module j (clockwise or counter-clockwise), relative to the normal of the engagement window of module j .

And specifically for an RS^{3D} :

- Height H as the distance between the lowest support and the highest connection point.
- Inclination angle θ_i of the axis of beam i relative to the horizontal plane.
- Bottom/top arrangement at a connection.

1.3 Built reciprocal structures

1.3.1 Historical use

Throughout history, RS were used for their aesthetic or spiritual appeal, their ease of assembly and disassembly, or their technical capability to span longer distances. Large historical overviews of RS are proposed in the work of Popovic Larsen [47] and Thonissen [81]. Famous reciprocal principles emerged :

- In the Middle Ages, V. de Honnecourt wrote down the layout of a framework composed of a four-beam SM RS^{2D} for spanning rectangular surfaces [Figure 1.8a]. He proposed using this method for floors and roofs when the beams are too short to span. The Serlio's floor (sixteenth century) and the Wallis's grillage (seventeenth century) are extensions of

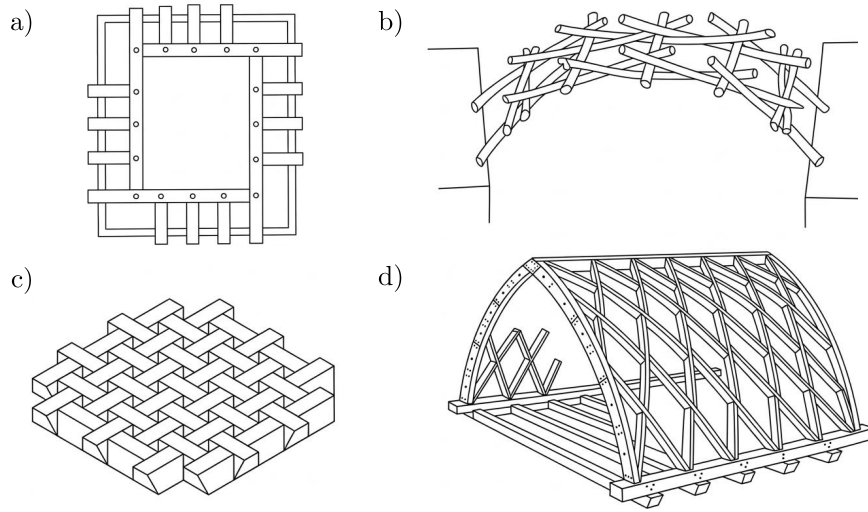


Figure 1.8: a) Honnecourt floor. b) Da Vinci bridge. c) Abeille vault. d) Zollinger roof.

this floor into a regular MM RS^{2D}.

- During the Renaissance, L. da Vinci designed the now-known-as the "Da Vinci bridge", depicted in Figure 1.8b. However, a similar principle was used for the "Rainbow Bridge" in China before L. da Vinci's drawings, without any proof of communication between these two parts of the world.
- The Abeille vault is made of stone voussoirs arranged to form a flat vault. Each stone is geometrically constrained by its neighbours, ensuring that the assembly remains in equilibrium through compression and friction without the need for mortar [Figure 1.8c].
- The Zollinger roof (twentieth century) is an MM RS^{3D} based on timber lamellas arranged in four-beam modules with an engagement ratio of almost 0.5, generating a rhombus-shaped engagement window [Figure 1.8d]. Due to the small eccentricity, it produces a slightly curved vault. This technique could save up to 60% of timber and 40% of construction labour time compared to a conventional roof framework.

1.3.2 Present constructions

Xu et al. [95] list 94 RS worldwide (before 2024), mainly in Europe and Asia, and principally constructed in timber. This list comprises research prototypes and has a very broad definition of RS. The following is a shortened list of RS built in the last decades, presented for SM RS in Figure 1.9 and MM RS in Figures 1.10 and 1.11, that will be discussed in more detail in the thesis:

- Figure 1.9a: The Roundhouses (Great Britain), proposed by Wrench [93], provide central roof lighting. Even with relatively short logs, the SM RS^{3D} enables a significant amount of available interior space.

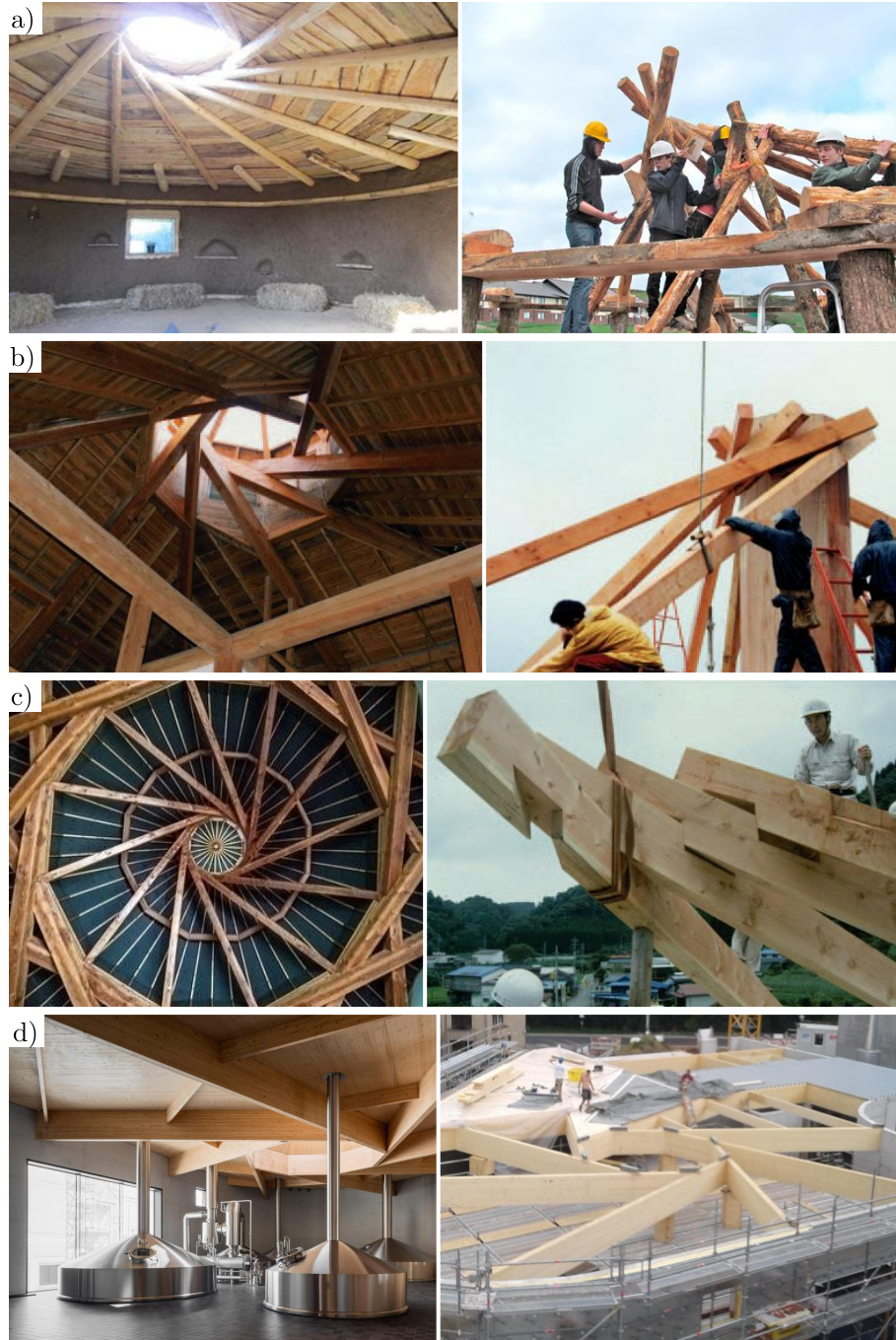


Figure 1.9: Built SM RS: a) Roundhouse (Great Britain) [93]. b) New Farmhouse (Japan) designed by Yoichi Kan [47]. c) roof of exhibition hall in the Seiwa Bunraku Puppet Theatre complex (Japan) [47]. d) Omer Brewery (Belgium) [<https://wowengineering.com/>].

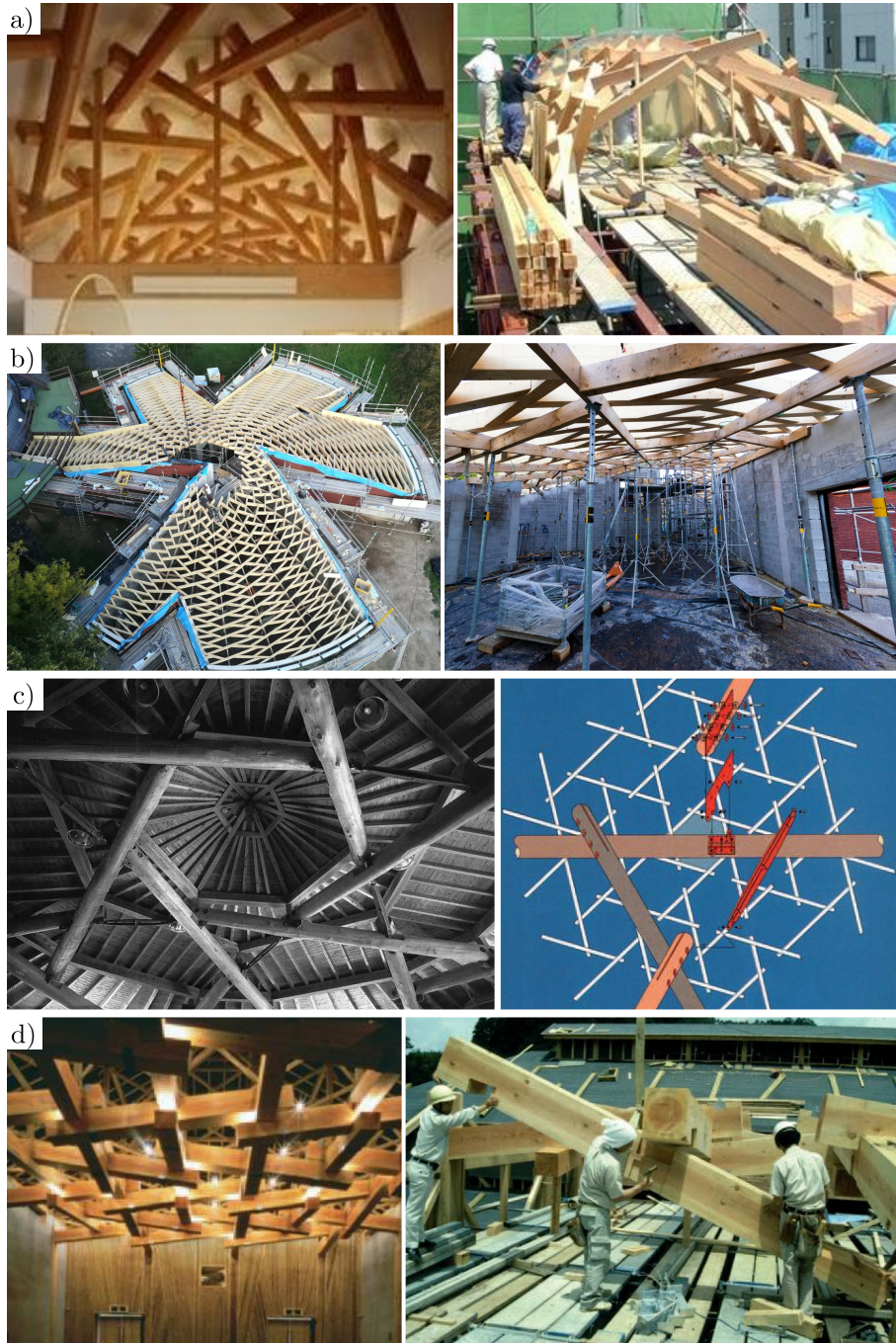


Figure 1.10: Built MM RS: a) Iseya town house (Japan) [81]. b) Frans Masereel Pavilion (Belgium) [<https://www.timberframing.be/>]. c) Stonemason Museum (Japan) [47]. d) Auditorium of Seiwa Bunraku Puppet Theatre Complex (Japan) [47].

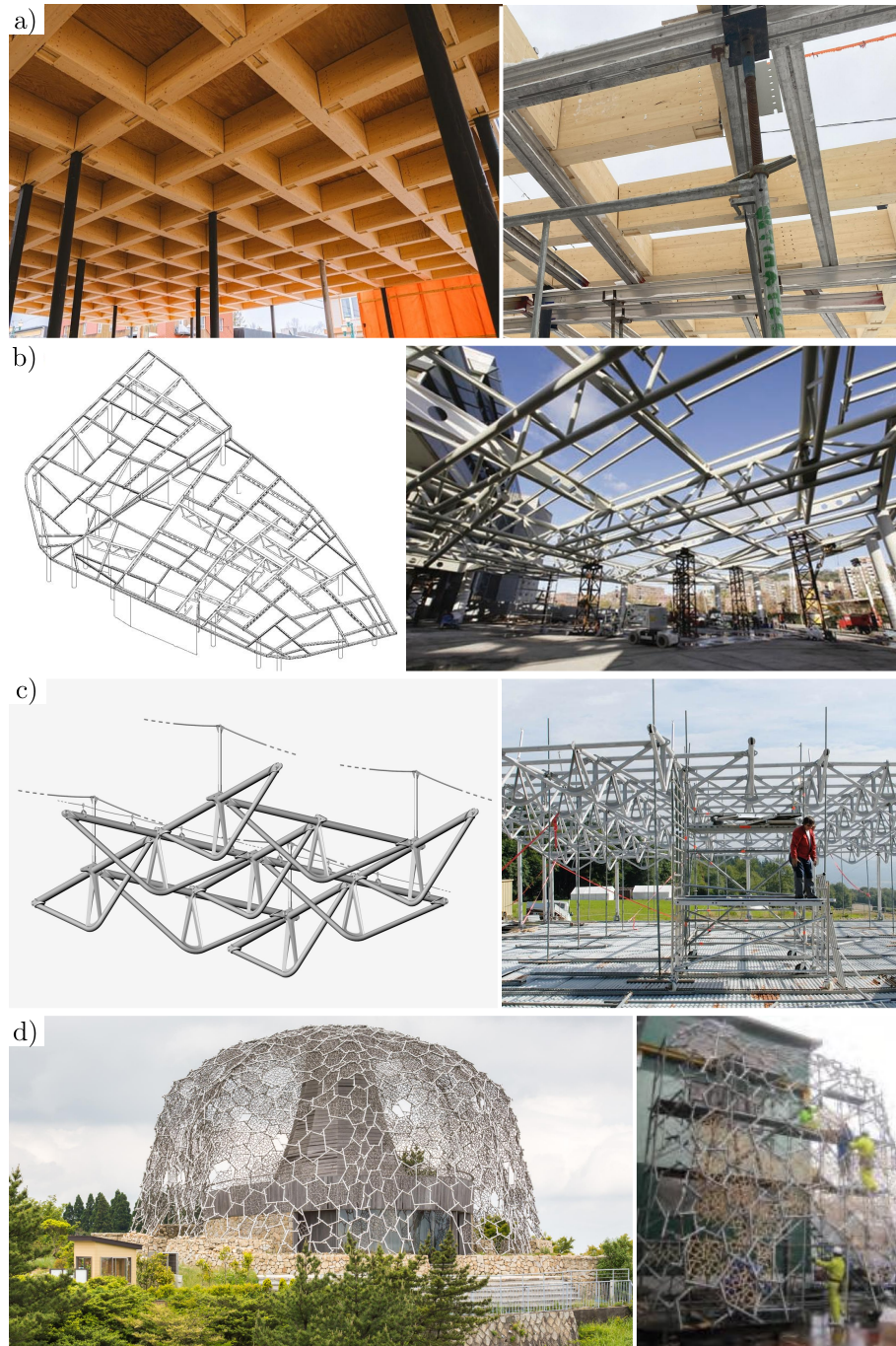


Figure 1.11: Built MM RS: a) Mont Laurier Library (Canada) [<https://www.ville-montlaurier.qc.ca/>]. b) Extension of Euskalduna conference centre (Spain) [37]. c) Cover of Bibracte's archaeological site (France) [25]. d) Mount Rokko-Shidare observatory (Japan) [28].

- Figure 1.9b: New Farmhouse (Japan) designed by Yoichi Kan. The eight-beam module is supported on a square plan, which complicates the orientation angle between beams.
- Figure 1.9c: The roof of the exhibition hall in the Seiwa Bunraku Puppet Theatre complex (Japan), designed by Kazuhiro Ishii and completed in 1992, consists of SM RS^{3D} with twelve beams supported by two SM RS^{2D} in opposite styles. The timber beams are joined using traditional carpentry joints called "Vatariago joints", without any steel elements.
- Figure 1.9d: The main hall of the extension of the Omer Brewery (Belgium) was finished in 2016. It is covered with SM RS for additional zenithal lightning and a particular architectural design.
- Figure 1.10a: Iseya town house (Japan), built in 2004 by Noboru Moriyama, has a barrel vault-shaped MM RS^{3D} made of 111 beams. To ease assembly and pre-fabrication, there are only 6 different types of timber elements.
- Figure 1.10b: The roof of the Frans Masereel Pavilion (Belgium), completed in 2019, constructed by Timber Framing, consists of a timber MM RS^{3D} made up of beams of identical cross-section. The large number of beams requires high-precision cutting, which is achieved using CNC machines [8].
- Figure 1.10c: The Stonemason Museum (Japan), designed in 1993, comprises two dome RS made of roundwood. Additional steel bars (in red) are added between connections, "which helped make the structure stable" [47]. When adding these elements to link the beam ends, this building does not comply with the definition of RS geometry.
- Figure 1.10d: The auditorium of the Puppet Theatre complex (Japan) is covered by an MM RS made of large timber beams supporting the inclined roof [47]. With an engagement ratio of exactly 0.5, each supporting beam supports both supported beams at the same point.
- Figure 1.11a: The Mont-Laurier Library (Canada) is covered by a plan MM RS as a square grid with sides measuring 1.2 m. This structure will be finished in 2026 and is designed to allow easy disassembly, adaptable column position and reuse.
- Figure 1.11b: The extension of the Euskalduna conference centre (Spain) was completed in 2012 and is made of an irregular pattern with steel beams of variable cross-section. This allows for a large open surface without columns [37].
- Figure 1.11c: To cover Bibracte's archaeological site (France), Gelez et al. [25] proposed, in 2008, an MM steel RS^{2D} supported on peripheral columns carrying a stretched fabric. The reciprocity of beams allows for easy adaptation of the covered area.
- Figure 1.11d: The dome-like structure on top of Mount Rokko-Shidare

(Japan) was built in 2010 to provide shading and air movement on the observatory platform [28]. It is composed of steel tubes of 1 to 2 *m* in length, welded together in an irregular pattern. The latter is filled in with timber sticks, also in a reciprocal pattern.

1.4 Main challenges of reciprocal structures

The built examples presented above had to face the three main challenges of RS:

- Iterative geometry design.
- Limited reliability in predicting structural behaviour.
- Assembly/disassembly complexity.

These challenges, outlined below, are being addressed through recent findings and technological advancements.

1.4.1 Iterative geometry design

As shown in the Section 1.2, many parameters describe the geometry of a module. These parameters are interdependent, particularly for RS^{3D} , in a way that modifying the position of one beam impacts the whole geometry. This interdependency is illustrated in Figure 1.12 by three SM with different eccentricities (sum of radii of beams): H , θ_i and l_i increase for constant L and α_i .

Fist, the geometry of SM RS^{2D} or RS^{3D} can be determined analytically. By choosing a limited set of control parameters, one can easily generate the corresponding geometry [70, 7, 3].

Complexity increases for multiple module geometries. For MM RS^{2D} , the literature of methods can be divided into two types:

- Additive methods: Simple modules are used as building blocks progressively assembled. The number of control parameters for the last added modules is reduced to comply with RS's definition.
- Adaptive methods: Modules are generated from adapting the edges of a grillage or mesh. Anastas et al. [1] review and propose different ways to "reciprocalize" a mesh. Song et al. [71] propose different tilings suitable for RS [Figure 1.4].

As MM RS^{3D} generate a spatial "envelope" geometry, the two previous methods need to be adapted. The second type of methods for MM RS^{2D} is subdivided in two, and their names are inspired by Anastas et al. [1]:

- Additive methods: Simple modules "building blocks" are now SM RS^{3D} . As the number of parameters increases for RS^{3D} , the number of control parameters is further reduced.
- "Modules Follow Form" methods: The RS^{3D} can be generated whether by projecting an MM RS^{2D} on a target surface [71] or meshing the target surface with a particular pattern and "reciprocalizing" through

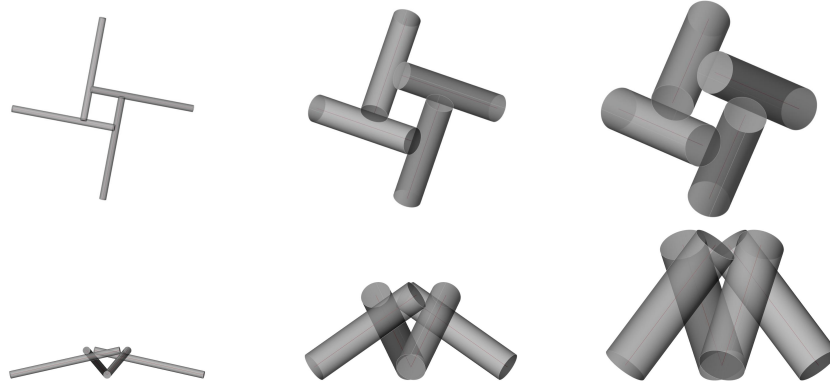


Figure 1.12: Three $SM RS^{3D}$ with increasing e for identical L and α_i , but with varying H , θ_i , and l_i .

(adapted) methods presented for $MM RS^{2D}$ [1, 43, 82]. In both cases, an optimisation procedure is necessary to (at least) normalise the eccentricities across the structure [1, 43, 63, 71, 82]. Other parameters could be included in the procedure, such as orientation angles [71] or engagement length [43].

- "Form Follows Modules" methods: The form of an $MM RS^{3D}$ can also be the result of an iterative algorithm starting from an imposed initial mesh and initial parameters. Different methods are proposed: hybrid optimisation method [48], self-adaptive form generation method [80], form finding and shape optimisation [79], dynamic relaxation [18], Form-Finding Tool [81], and genetic algorithms [6].

As the designer generally wants to control the final geometry, literature mostly focuses on the second type. However, it is important to note that the final geometry resulting from the last two types does not necessarily comply with all the imposed parameters. In other words, the iterative algorithm may not converge to a satisfactory solution. Finally, iteration may also break symmetry or regularity, leading to difficulties during fabrication and assembly.

1.4.2 Limited reliability in predicting structural behaviour

The load path in RS is counterintuitive, as each beam supports another, unlike conventional systems with clear primary and secondary bearing members. The left part of Figure 1.13 shows the clear lead path distribution that interacts with closed-loop diffusion (right part). Therefore, it has the consequence that:

- Manual analytical computation of the equilibrium equations is exponentially labour-intensive as the beam density increases, requiring the use of numerical calculation tools [59].
- Beams are principally subjected to bending (and associated shear forces),

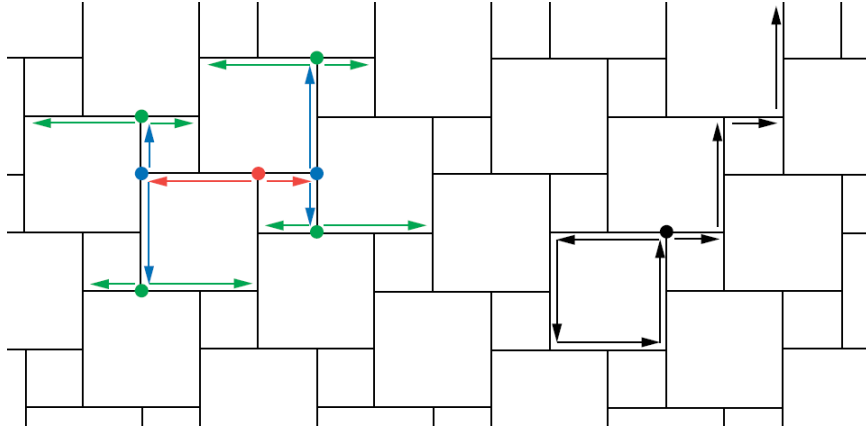


Figure 1.13: Conventional load path distribution with applied force on at the red dot (left) interacts with closed-loop diffusion of the load (right) [35].

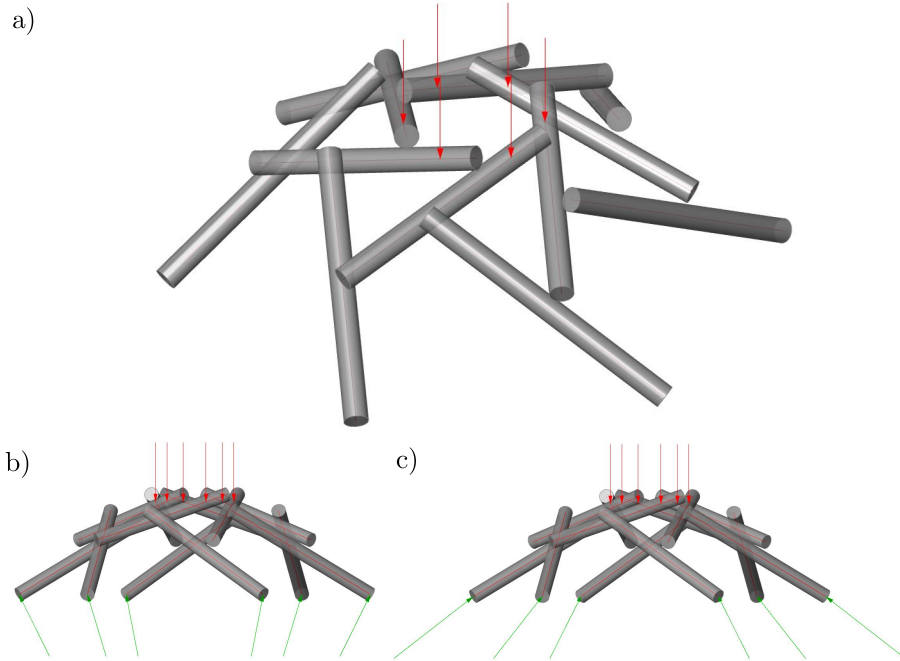


Figure 1.14: a) Isometric view of the geometry of an MM RS^{3D} with six point loads with pinned supports. b) Reaction forces for hinged connections. c) Reaction forces for completely fixed connections.

but RS^{3D} are subjected to all combinations of internal forces due to eccentricity between beam axes. In addition, RS^{3D} undergoes three-dimensional displacements, even under vertical loading. For less stiff structures, geometric nonlinearity becomes evident, requiring advanced structural analysis.

- The nature of the connections strongly influences the structure’s global response. The latter is illustrated in Figure 1.14, which presents the reaction forces for two RS with the same geometry, loading and pinned supports, but with completely fixed connections [Figure 1.14b] or hinged (pinned) connections [Figure 1.14c]. It is therefore important to model the connection correctly, as conservative assumptions may lead to unpredictable load paths and local failure.

1.4.3 Assembly/disassembly complexity

The assembly sequence must be carefully planned from the first design stage of an RS, and is largely influenced by the connection and support design. Since each beam supports the others, issues can occur during the installation of the final beam(s) or when geometric divergence exceeds tolerance limits. For instance, the more a connection restricts movement, the less the structure can adapt to geometric deviations during construction. In the extreme case where beams are welded together, such as in the structure of the Rokko Mountain Observatory [Figure 1.11d], any error during assembly makes it impossible to install subsequent beams [28].

Different strategies are documented and depend on the number of modules, the material used, and the connections/supports. They may employ temporary supports or scaffolds until all the beams are in place. To illustrate, the strategies used for erecting the RS of Figures 1.9, 1.10, and 1.11 are presented:

- The most common approach for SM RS is using one or more temporary supports, as presented in Figures 1.9a, 1.9c, and 1.9d. This method is especially delicate for SM RS^{3D}; as shown clearly in Figure 1.9c, where the connection must tolerate movement until the last piece is placed and the support is removed. For this particular building, the constructors built a 1:5 scale model to ensure buildability.
- Another solution for SM RS, used for erecting the New Farmhouse Figure 1.9b, is to employ a central core that supports all the beams and ensures they are correctly aligned with each other.
- Piece-by-piece erection is also possible for MM RS [Figures 1.10a, 1.10d, 1.11a and 1.11c]. This gradual process requires temporary supports only at the edges. For Bibracte’s archaeological site [Figure 1.11c], it also allows the structure’s geometry to be adapted to the specific zone to be covered.
- It may be preferred to assemble the entire structure on scaffolding until the last beam, to prevent the effects of structural deformations caused by self-weight [Figure 1.10b, 1.10d and 1.11a]. For the Frans Masereel Pavilion [Figure 1.10b], this solution was selected to allow tolerance in the placement of supports on the peripheral concrete walls.

1.4.4 Research objectives

Among the three challenges, geometry generation has been extensively investigated in the literature, with numerous methods already proposed. In contrast, assembly and disassembly processes are highly dependent on the RS geometry and the type of connections used, and must therefore be considered on a case-by-case basis. Therefore, this manuscript focuses primarily on predicting and assessing the structural behaviour of RS.

The RS behaviour is particularly complex because there is no structural hierarchy among beams, and it is closely linked to connection design and modelling. As a result, there is significant apprehension and a general lack of confidence among designers towards reciprocal structures, as built examples are generally limited to single modules or RS^{2D}. The few MM RS^{3D} presented above are the roofs of Stonemasson Museum [Figure 1.10c] and the Mount Rokko-Shidare observatory [Figure 1.11d]. However, both examples deviate from reciprocal structures. In the first example, the reciprocal geometry was redesigned by adding steel bars to link the connections between timber beams. For the second, the tubes were welded, blocking all movement at the connection and generating a mechanical gridshell-like behaviour. Due to these challenges, this structural type is mainly chosen for its aesthetics or when limited beam lengths are necessary, but it is not considered an optimal solution.

This thesis aims to establish a robust design framework for RS to encourage broader adoption. It addresses all types of building materials, since the challenges extend beyond logs, and discusses all typologies of RS, with particular attention to MM RS^{3D} typologies, which are currently less used.

The objectives of this thesis are to:

- Assess the structural efficiency of reciprocal structures in comparison with conventional structural typologies.
- Establish the conditions under which RS^{3D} are geometrically stable to exhibit their intended mechanical behaviour.
- Develop and experimentally validate a connection concept for RS^{3D} that enables reliable numerical modelling and predictable structural behaviour.

1.5 Thesis layout

1.5.1 Chapter 2: Optimal reciprocal log floor patterns

Before engaging in more complex RS, this chapter focuses on a four-beam module RS floor system to demonstrate RS's structural efficiency. This type of RS has already been constructed as one of its earliest applications. However, the selection of the engagement ratio is currently mainly driven by aesthetic preferences or practical considerations. Therefore, this chapter investigates

the optimal geometry of an SM RS^{2D} made of logs. The floor spans a rectangular area and needs to minimise both the overall structural thickness and the volume of material used. After that, the optimal RS is compared with more conventional floor patterns, composed only of joists or including transverse primary beams.

The morphological indicators method is employed to reduce the number of variables involved in the optimisation process and enable the derivation of universal design rules. The only parameters considered are the floor width-to-length ratio, the engagement ratio, and the number of joists.

Compared with conventional floor systems, there is little interest in using RS patterns for floors with a small width-to-length ratio (i.e. rectangular-shaped floors). However, for squared floors, they perform as well as the optimal conventional floor in terms of material savings and even better in terms of thickness for specific situations. This shows that RS patterns could be used more widely, not just to reduce the length of primary beams and joists.

1.5.2 Chapter 3: Geometric stability of reciprocal structures

As mentioned, the design of connections particularly affects the mechanical behaviour of reciprocal structures, which can exhibit geometric instabilities, such as rigid-body rotations of beams about their longitudinal axes. As a result, significant uncertainty remains, leading to a general lack of confidence among designers, who are prone to excessively blocking connections. This issue is further compounded by the absence of well-defined design guidelines for connections and a universal method for assessing geometric stability.

This chapter reviews the literature on connections and supports for reciprocal structures across various materials, as well as existing methods for evaluating geometric stability. It aims to clarify appropriate design strategies for RS with respect to global geometry, connections, and supports by comparing the effectiveness of several methods for assessing geometric stability.

The first part of the chapter highlights that connections are generally rotationally fixed, bringing their structural behaviour closer to that of gridshells, even though they are geometrically reciprocal. In addition, semi-rigid connections are designed for ease of construction or aesthetic reasons, but are difficult to model accurately and reliably.

The second part compares four methods for evaluating the geometric stability of RS, assuming infinitely rigid beams and ideal connections. This approach enables stability assessment at the preliminary design stage without requiring predefined cross-sections. Geometrically unstable configurations cannot meet structural requirements and must be discarded early in the design process. While most existing methods are limited to specific geometries or connection types, one general method—previously unused for RS—is identified and used as a basis for proposing design guidelines.

1.5.3 Chapter 4: Predictability of reciprocal structures

The review of Chapter 3 also indicates that the actual behaviour of reciprocal structures can differ significantly from numerical predictions, particularly for RS^{3D} . These discrepancies mainly arise from the complex behaviour of connections, which often exhibit nonlinear characteristics such as friction, locking, and semi-rigid behaviour.

This chapter aims to demonstrate that hinged connections enable the development of reliable numerical models by proposing a spherical hinge connection. This connection is proposed for an RS^{3D} made of steel tubes, to avoid uncertainties linked to geometrical and mechanical properties of logs.

The spherical hinge connection comprises rods attached to the ends of the tubes, and its numerical modelling is based on local testing of the connection. To validate its model, large-scale steel structural prototypes equipped with the proposed hinged connection were constructed and tested. The instrumentation includes a motion capture system to measure three-dimensional displacements. This system has not previously been used for reciprocal structures. The results show a strong similarity between the numerical and experimental values of displacements and stresses.

1.5.4 Chapter 5: General conclusion

This chapter reviews the main findings of this thesis. After that, it presents two outlines based on these findings, with a focus on RS^{3D} and its structural behaviour. The first focuses on the connections in RS made of logs and their numerical modelling. One solution is to adapt the connection proposed in Chapter 4. The second discusses the method for evaluating the optimal geometries of RS^{3D} and assesses the feasibility of using morphological indicators.

1.5.5 Appendix

To facilitate access, the lists of tables and figures, the bibliography, and the list of symbols are compiled at the end of the manuscript for all chapters.

Chapter 2

Optimal reciprocal log floor pattern

This chapter combines the results presented in the following publications:

- STEINMETZ Martin, LATTEUR Pierre, and SGAMBI Luca. *Optimal log floor patterns*. Engineering Structures, 2023, vol. 297, p. 116970.
- STEINMETZ Martin, SGAMBI Luca, and LATTEUR Pierre. *Optimal reciprocal log floor patterns*. In: Proceedings of the IASS Annual Symposium. International Association for Shell and Spatial Structures (IASS), 2026.

This chapter aims to demonstrate the structural efficiency of RS compared to other conventional structures. It focuses on RS log floors composed of a four-beam module, as among the earliest applications for reducing the required lengths of joists and primary beams. However, their engagement ratios are still largely defined by architectural preferences or construction practices rather than structural optimisation. This chapter addresses this gap by investigating the structural efficiency of RS log floors as an alternative to "traditional" flooring, which consists of single-span parallel joists aligned along the shortest span.

The primary objective is to identify floor configurations that minimise both structural thickness and material volume. Given the large number of parameters governing floor behaviour (floor geometry, loading, material properties, and in-service performance parameters), this work employs the morphological indicator method. This approach combines the project parameter into a few dimensionless indicators, reducing its complexity while enabling general design guidelines. The second objective is to demonstrate the potential of an RS floor pattern compared to ten other "conventional" solutions, including the "traditional" pattern, as well as configurations incorporating transverse primary beams to reduce joist span.

The results demonstrate that, for square floor configurations, RS patterns systematically reduce material volume compared to the "traditional" pattern. Their performance is comparable to that of the conventional pattern using transverse beams, but, depending on the beam connection configuration, they can also provide the most efficient solution for minimising structural thickness. These findings emphasize the potential of reciprocal floor patterns as

structurally efficient solutions that minimize material use and the lengths of necessary beams.

2.1 Context on log floors

To use logs in floor systems, they must be connected longitudinally and transversally. First, longitudinal connections are primarily needed to couple joists, ensuring load redistribution and providing sufficient rigidity to efficiently transmit horizontal forces to the bracing system. As depicted in Figure 2.1, various practical solutions exist to address this matter:

- Pouring a concrete screed to cover the floor provides substantial in-plane and transversal rigidity [11], while the added mass of concrete enhances dynamic behaviour [36]. However, the stiffness and strength of the timber-concrete composite structure highly depend on the connection system [96]. Additionally, the concrete compromises this solution's sustainability and recyclability, and the requirement for metal connectors further increases the cost.
- An alternative approach involves combining logs with prestressed transverse steel rebars or embedded rebar within a non-shrink grout core [44]. However, both methods require direct contact between logs to ensure consistent prestressing and proper grout sealing. Consequently, the logs are positioned side by side with flat surfaces cut on both lateral sides, resulting in increased cutting waste that offsets the advantages of using natural-form timber.
- Another option is to employ wood-based panels, such as OSB, for connecting log joists. Nonetheless, achieving sufficient connection stiffness is challenging. A noteworthy example of log flooring can be observed at the Lyss school in Switzerland, where 160 mm-diameter logs were arranged in parallel with a span of 3.3 m, positioned beneath plywood panels [13]. Similarly, Chapman and Dodd [14] employed 200 mm-diameter roundwood, spaced 0.6 m apart, for floors with a 6 m span (with additional centre support). For fire resistance, their solution included a lower layer of plasterboard and an upper panel composed of two external plywood panels with an inner layer of a sand-sawdust mix.

Second, many options are available for transversal connections [Figures 2.1d-f]. It should be noted that the connection between floor logs significantly affects the overall floor thickness. On the one hand, numerous examples of historic and modern steel-free connection options exist [84]. These steel-free, machine-milled solutions are considered more cost-effective because they require only local cutting of the logs. Among them, the simply notched connection [Figure 2.1d] and the JJP connection [26] [Figure 2.1e] have the benefit of conserving a large portion of the supporting beam in addition to allowing dimensional movements due to moisture variation. On the other hand,

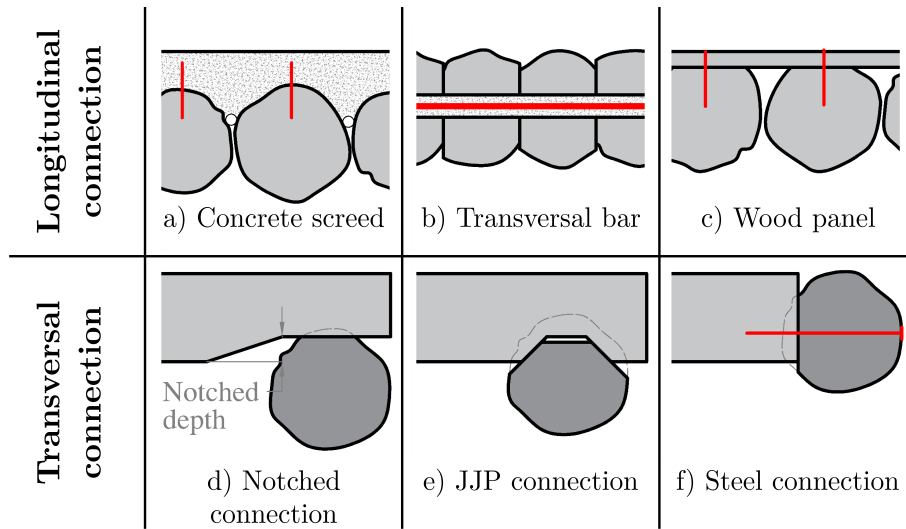


Figure 2.1: Solutions to achieve longitudinal and transversal connections between joists.



Figure 2.2: Mock-up of a reciprocal log floor 4 m × 4 m (birch wood).

steel components can facilitate in-plane connections between logs, thereby reducing the total floor thickness.

To illustrate, Figure 2.2 showcases a mock-up log floor with a reciprocal pattern made by the author. The joists, reciprocal-framed primary beams, peripheral beams, and supports are minimally processed and connected at the same level with self-tapping screws, as shown in Figure 2.1f and discussed further in section 2.5. This design demonstrates the feasibility of employing logs in floor construction while preserving their inherent qualities and reducing the need for extensive processing. Moreover, this floor pattern allows for reducing both the primary and joist lengths, particularly interesting given that logs are naturally limited in length.

2.2 Research objectives

This chapter analyses a rectangular reciprocal floor composed of a module with four logs, similar to the mock-up shown in Figure 2.2. It focuses on determining the optimal engagement ratio [Figure 2.3]. Indeed, the choice of the engagement ratio is usually guided by aesthetic or construction considerations [40]. The performance of the floor is evaluated for both total volume and thickness: minimising the volume of materials used is crucial to reduce flooring costs, and limiting the floor thickness is essential for achieving buildings with fewer façade surfaces or the potential for additional stories.

To evaluate the potential of reciprocal floors, this chapter follows with a comprehensive comparative analysis of alternative patterns. Traditionally, architects and engineers design rectangular floors with single-span parallel joists along the shortest span, presuming this is the most suitable choice. However, the effectiveness of this "traditional" solution is not always guaranteed. For instance, Jelušič and Kranaja [32] showed that adding transversal primary beams is more cost-effective for the scope of their study.

Determining the overall thickness and volume of a floor requires designing each element following local building codes [2, 5, 12, 72], considering five essential design criteria for timber structures: 1) bending resistance, 2) shear resistance, 3) short-term deflection limitation, 4) long-term deflection limitation, and 5) vibration comfort. These criteria depend on various parameters, such as the dimensions of the rectangular area to be covered, the applied loading, the joist spacing, the mechanical properties of timber, and the in-service design parameters for deflection and vibration limitation. In parametric analysis, these continuous and discrete variables significantly increase computational demands and require precise parameter definitions [45].

To address this challenge, this chapter is based on the Morphological Indicator (MI) method [41], which groups project parameters (dimensions, mechanical properties, loads, etc.) into a few dimensionless numbers, called Morphological Indicators. At the early design stage, with limited knowledge

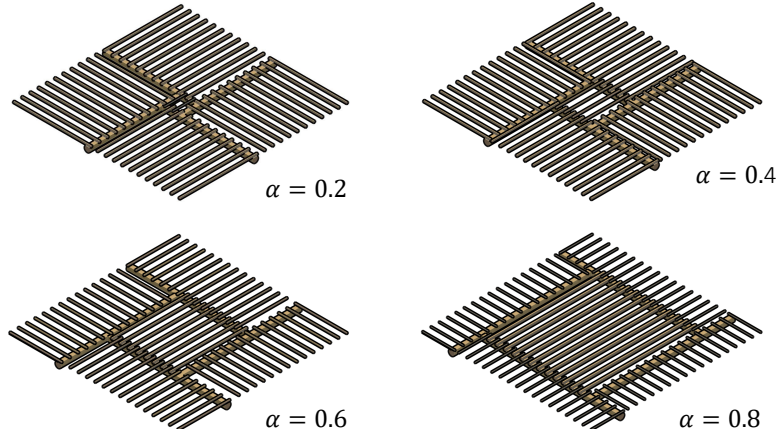


Figure 2.3: Four reciprocal modules with four beams, with different engagement ratios from 0.2 to 0.8.

of these parameters, the MI can be computed quickly for each option and then compared to determine the optimal solution. This approach has been initially developed for fully-stressed trusses, cables and beams [41, 69]. Afterward, it was extended to other structural typologies, such as tensegrity structures [20, 39], or to account for planar stability or vibration [86]. This method was once used for floors by Latteur et al. [40] but covered a limited number of patterns and considered only the volume and a few design criteria. In this chapter, this method is used for expressing a pattern's minimum thickness or volume solely in terms of three dimensionless numbers: the floor's width-to-length ratio, the number of joists, and the engagement ratio. Knowing these factors at an early design stage enables comparison of solutions and, therefore, the selection of the most suitable engagement ratio or pattern among the proposed alternatives, regardless of the project parameters.

This chapter first establishes the MI of thickness and volume of a floor, illustrated by the "traditional" solution, which uses single-span logs to cover a rectangular area. Secondly, the MI are computed for a reciprocal floor pattern, with the focus on determining the optimal engagement ratio for a rectangular reciprocal floor. Finally, the optimal reciprocal floors are compared with the "traditional" solution and patterns comprising primary beams transversal to the joists for a comprehensive comparison and to propose general design guidelines.

2.3 Method

Before introducing the different MI, this section begins with the general assumptions of the case study. Subsequently, the minimum diameter and corresponding volume required for a single beam are presented, addressing the five design criteria mentioned above. The expressions are extended to the

floors and analytically illustrated for the "traditional" pattern. Based on these expressions, the MI of total thickness and volume of the reciprocal floor are computed. However, since the analytical computation of the structural response is complex, the method is adapted to enable the use of a finite element software.

2.3.1 Assumptions and scope of the study

The notation and design assumptions for the studied floors are as follows:

- Two situations are studied: the floor is supported either by four peripheral walls (Case 1) or by four columns with peripheral beams (Case 2) [Figure 2.4].
- The floor covers a rectangular area of length L_1 [m] and width L_2 [m]. The geometric ratio of the floor $\lambda = L_2/L_1 \in]0; 1]$ [Figure 2.4]. In practice, values of λ smaller than 0.2 are not relevant. For Case 2, Peripheral Long Span Beams (PLSB) span along L_1 , and Peripheral Short Span Beams (PSSB) span along L_2 . The floor studied is part of a grid of walls or columns with uniform spacing.
- Joists (J), Primary Beams (PB), and Peripheral long and short Span Beams (PSB), made of logs, are considered cylindrical with diameter ϕ_i [m] and length l_i [m], generating an area $A_i = \phi_i^2 \pi / 4$, an effective shear area $A_{v,i} = 3A_i / 4$, a volume $V_i = A_i l_i$, a moment of inertia $I_i = \phi_i^4 \pi / 64$, and a section modulus $W_{el,i} = \phi_i^3 \pi / 32$.
- The spacing between joists is denoted d [m] (typically $d \in [0.25; 0.6]$, as imposed by the decking). Since logs have a limited available length ($L_1 \in [4 \text{ m}; 9 \text{ m}]$), $n = L_1/d \in [5; 35]$ generally equals 20 (for $d = 0.4 \text{ m}$ and $L_1 = 8 \text{ m}$, for instance).
- Ten traditional floor patterns are considered and shown in Figure 2.5. Each pattern differs in joist span direction (X or Y patterns), and the number of bays ($K \in [1; 5]$). Pattern X1 is considered the "traditional" solution.
- Particularly for reciprocal floors [Figure 2.6], the lengths of the primary reciprocal beams are denoted l_a and l_b and may have different diameters. The engagement ratios α_a and $\alpha_b \in]0; 1]$, defined as the ratio between the engagement length of a primary beam and its total length, respectively l_a or l_b [Figure 2.6b]. For simplicity, it is assumed that $\alpha = \alpha_a = \alpha_b$.
- The mechanical properties of the logs are the Young's modulus E [N/m²], the bending strength f_b [N/m²], and the shear strength f_v [N/m²]. These strengths already consider safety coefficients and wood defects. These properties are identical to all the logs composing the same floor layer (joists, primary beams and peripheral beams).
- The decking is subjected to a uniformly distributed load q [N/m²], in-

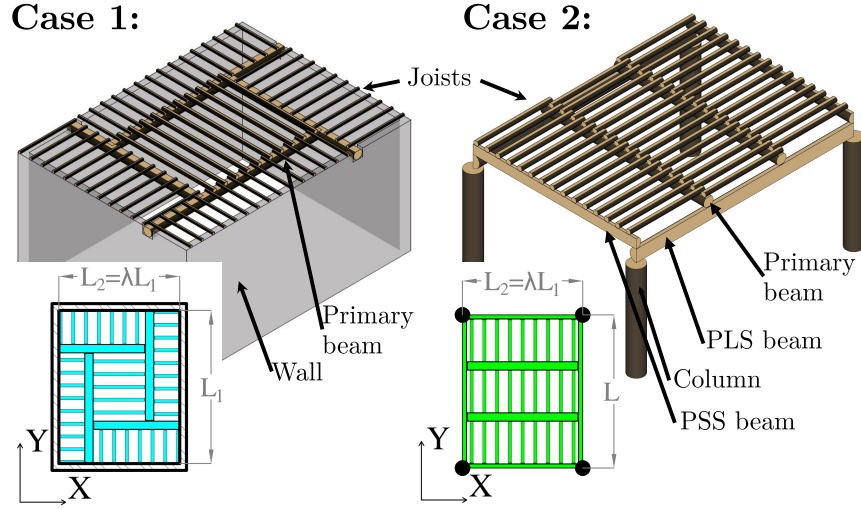


Figure 2.4: General notation presented for Case 1 and Case 2 on a reciprocal pattern and pattern Y3, with joist, primary beams, and peripheral beams.

cluding the self-weight of the floor and permanent and variable vertical loads. This load is factored at the Ultimate Limit State (ULS) q_{ULS} , at the Serviceability Limit State (SLS) short-term q_{SLS} , and SLS long-term $q_{SLS,Lt}$. The total load on the floor is $F [N] = q L_1 L_2$, denoted F_{ULS} , F_{SLS} and $F_{SLS,Lt}$ at the respective limit states. According to load distribution, the joists, reciprocal primary beams, and peripheral beams are loaded as shown in Figure 2.6. For vibration design, this load is converted into a modal mass $\eta_{cov} F_{SLS}/g [kg]$, where $g [m/s^2]$ is the gravitational acceleration and $\eta_{cov} [-] \in [0; 1]$ is the participating mass ratio. η_{cov} depends on the support conditions, the load distribution and the type of loading. For example, the modal mass is expected to be half the permanent and self-weight load for simply supported beams under uniformly distributed loading. Interaction between decking, joists, primary beams, and peripheral beams is neglected.

- The building project specifications impose three in-service performance levels: the maximal relative deflection factor for short- and long-term behaviour (μ_{SLS} and $\mu_{SLS,Lt}$, respectively) and the lower bound fundamental vibration frequency ($f_{1,lim} [Hz]$). The first two represent the maximal allowable deflection as a fraction of the element's span, knowing that $\mu_{SLS} \leq \mu_{SLS,Lt}$. The lower-bound frequency prevents floor resonance, guarantees occupant comfort, and, for example, in EUROCODE 5 [12], is set at least to 8 Hz for beams on two supports.
- Connections between logs are considered perfectly hinged, neglecting any rotational and transversal stiffness.
- All the floor elements are considered protected from fire.

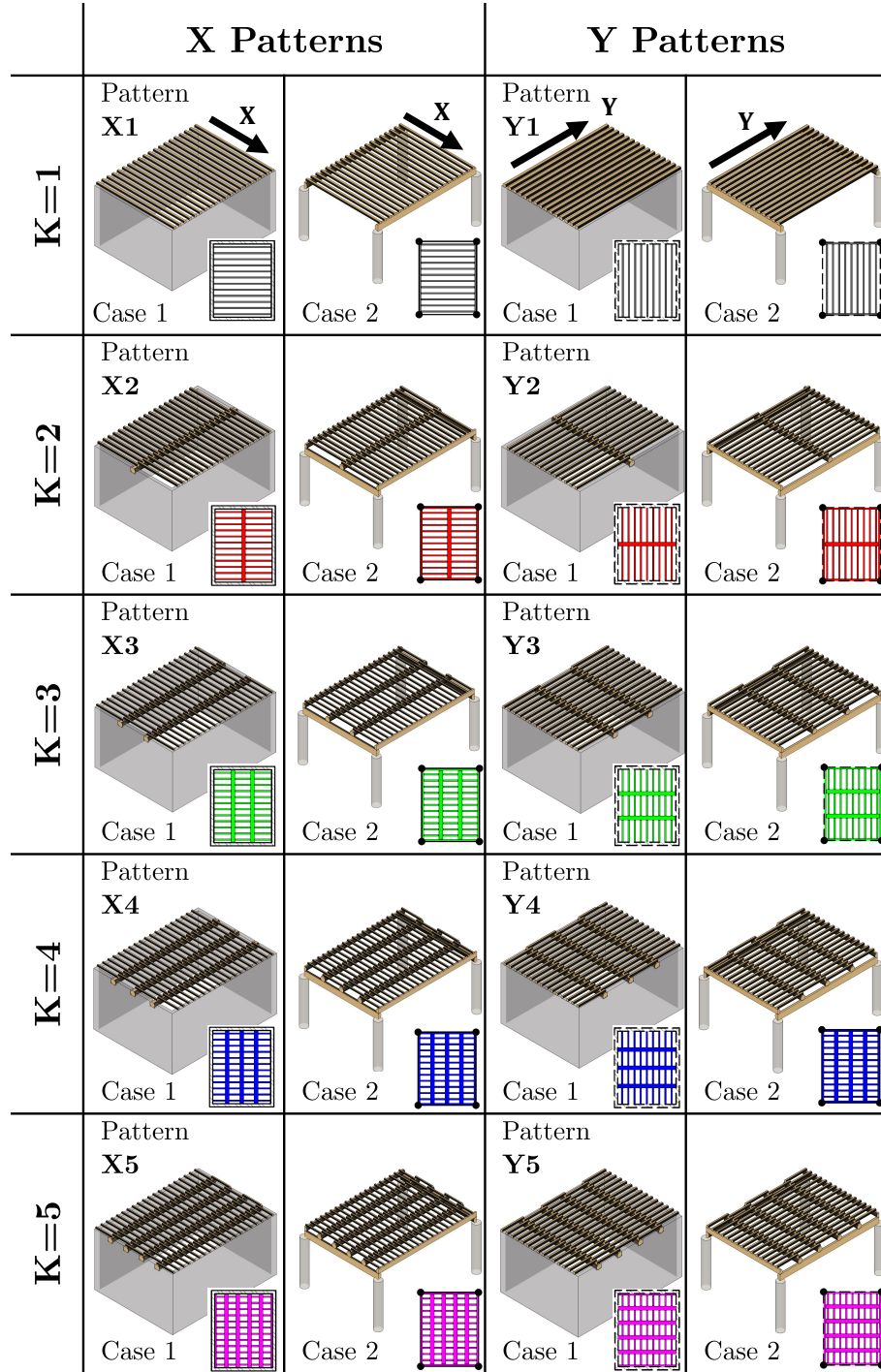


Figure 2.5: Ten different floor patterns for Cases 1 and 2, here presented with $\lambda = 0.75$.

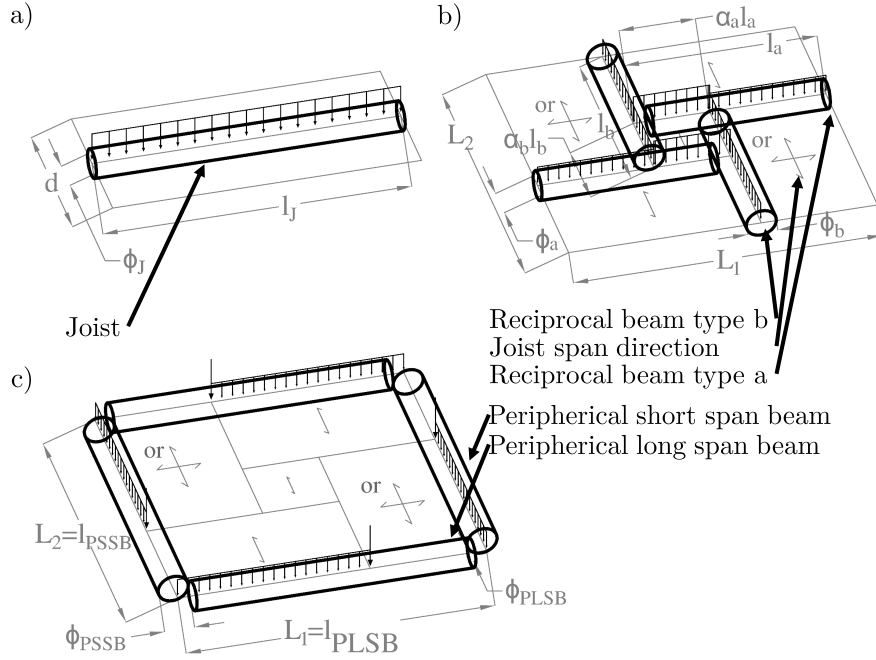


Figure 2.6: Particular notation for reciprocal floors.

2.3.2 Thickness and volume of a single beam

Considering a single beam i presented in Figure 2.7 and the coefficient $\varepsilon_{M,i}$, $\varepsilon_{V,i}$, $\varepsilon_{D,i}$ and $\varepsilon_{F,i}$ linked to the distribution of the total external load F_i , one has:

- A maximum ultimate bending moment:

$$M_{ULS,i} \text{ [Nm]} = \varepsilon_{M,i} F_{ULS,i} l_i$$

- A maximum ultimate shear force:

$$V_{ULS,i} \text{ [N]} = \varepsilon_{V,i} F_{ULS,i}$$

- A maximum service short-term deflection:

$$\delta_{max,i} \text{ [m]} = \varepsilon_{D,i} F_{SLS,i} l_i^3 / (E I_i)$$

- A maximum service long-term deflection:

$$\delta_{max,Lt,i} \text{ [m]} = \varepsilon_{D,i} F_{SLS,Lt,i} l_i^3 / (E I_i)$$

- A first service fundamental frequency:

$$f_{1,i} \text{ [Hz]} = \varepsilon_{F,i} \sqrt{E I_i g / (\eta_{cov,i} F_{SLS,i} l_i^3)}$$

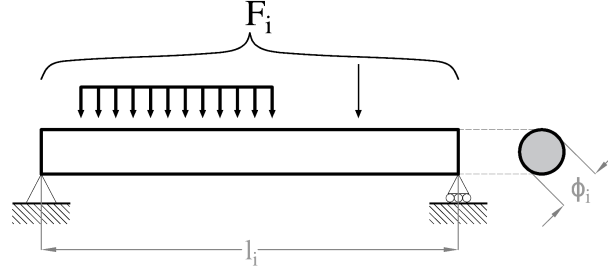


Figure 2.7: Simply supported log approximated by a cylindrical beam of diameter ϕ_i and length l_i , and subjected to total equivalent force F_i .

For a uniformly distributed load and self-weight, one has: $\varepsilon_{M,i} = 1/8$, $\varepsilon_{V,i} = 1/2$, $\varepsilon_{D,i} = 5/384$ and $\varepsilon_{F,i} = \pi/2$.

This beam must meet the five design criteria: 1) bending resistance, 2) shear resistance, 3) short-term deflection limitation, 4) long-term deflection limitation, and 5) vibration comfort.

The bending resistance criterion is given by Equation 2.1 and leads to the minimum value of the beam diameter $\phi_{Rm,i}$ in Equation 2.2 and volume $v_{Rm,i}$ in Equation 2.3:

$$\frac{M_{ULS}}{W_{el,i}} \leq f_b \quad (2.1)$$

$$\phi_{Rm,i} = \left(\frac{32\varepsilon_{M,i}}{\pi} \right)^{1/3} \left(\frac{F_{ULS,i} l_i}{f_b} \right)^{1/3} \quad (2.2)$$

$$v_{Rm,i} = l_i \left(\frac{\pi \phi_{Rm,i}^2}{4} \right) = 3.691 \varepsilon_{M,i}^{2/3} \left(\frac{F_{ULS,i} l_i^{5/2}}{f_b} \right)^{2/3} \quad (2.3)$$

The shear resistance criterion is given by Equation 2.4 and leads to the minimum value of the beam diameter $\phi_{Rv,i}$ in Equation 2.5 and volume $v_{Rv,i}$ in Equation 2.6. The minimum diameter $\phi_{Rv,i}$ is expressed in the variables of $\phi_{Rm,i}$ thanks to the linking parameter $Z_R [-] = (F_{ULS,i}^{1/6} f_b^{1/3}) / (l_i^{1/3} f_v^{1/2})$. This parameter depends only on the timber strength class and the loading and is discussed in Section 2.3.3.1.

$$\frac{V_{ULS}}{A_{v,i}} \leq f_v \quad (2.4)$$

$$\phi_{Rv,i} = \left(\frac{16\varepsilon_{V,i}}{3\pi} \right)^{1/2} Z_R \left(\frac{F_{ULS,i} l_i}{f_b} \right)^{1/3} \quad (2.5)$$

$$v_{Rv,i} = l_i \left(\frac{\pi \phi_{Rv,i}^2}{4} \right) = \frac{4\varepsilon_{V,i}}{3} Z_R^2 \left(\frac{F_{ULS,i} l_i^{5/2}}{f_b} \right)^{2/3} \quad (2.6)$$

The short-term deflection limitation criterion is given by Equation 2.7 and leads to the minimum value of the beam diameter $\phi_{S,i}$ in Equation 2.8 and volume $v_{S,i}$ in Equation 2.9:

$$\delta_{max,i} = \frac{\varepsilon_{D,i} F_{SLS,i} l_i^3}{E I_i} \leq \mu_{SLS} l_i \quad (2.7)$$

$$\phi_{S,i} = \left(\frac{64 \varepsilon_{D,i}}{\pi} \right)^{1/4} \left(\frac{F_{SLS,i} l_i^2}{\mu_{SLS} E} \right)^{1/4} \quad (2.8)$$

$$v_{S,i} = l_i \left(\frac{\pi \phi_{S,i}^2}{4} \right) = \sqrt{4 \varepsilon_{D,i}} \pi \left(\frac{F_{SLS,i} l_i^4}{\mu_{SLS} E} \right)^{1/2} \quad (2.9)$$

The long-term deflection limitation criterion is adapted from Equation 2.7 with force $F_{SLS,Lt}$ and long-term maximal relative displacement factor $\mu_{SLS,Lt}$. Based on the linking parameter $Z_{Lt}[-] = ((F_{SLS,Lt,i} \mu_{SLS}) / (F_{SLS,i} \mu_{SLS,Lt}))^{1/4}$, this criterion leads to the minimum value of the beam diameter $\phi_{S,Lt,i}$ in Equation 2.10 and volume $v_{S,Lt,i}$ in Equation 2.11:

$$\phi_{S,Lt,i} = Z_{Lt} \phi_{S,i} \quad (2.10)$$

$$v_{S,Lt,i} = l \left(\frac{\pi \phi_{S,Lt,i}^2}{4} \right) = Z_{Lt}^2 v_{S,i} \quad (2.11)$$

When $Z_{Lt} \leq 1$, the short-term deflection limitation criterion is more restrictive than the long-term criterion.

Finally, the vibration comfort criterion, Equation 2.12, yields the minimum values of the element's diameter $\phi_{V,i}$ and volume $v_{V,i}$, as expressed in Equations 2.13 and 2.14, respectively.

$$\varepsilon_{F,i} \sqrt{\frac{E I_i g}{\eta_{cov,i} F_{SLS,i} l_i^3}} \geq f_{1,lim} \quad (2.12)$$

$$\phi_{V,i} = \left(\frac{64}{\pi \varepsilon_{F,i}^2} \right)^{1/4} \left(\frac{f_{1,lim}^2 \eta_{cov,i} F_{SLS,i} l_i^3}{E g} \right)^{1/4} \quad (2.13)$$

$$v_{V,i} = l_i \left(\frac{\pi \phi_{V,i}^2}{4} \right) = \left(\frac{4\pi}{\varepsilon_{F,i}^2} \right)^{1/2} \left(\frac{f_{1,lim}^2 \eta_{cov,i} F_{SLS,i} l_i^5}{E g} \right)^{1/2} \quad (2.14)$$

The minimal beam diameter should be the maximum value of all criteria of Equations 2.2, 2.5, 2.8, 2.10 and 2.13, while the minimal beam volume should be the maximum value of all criteria of Equations 2.3, 2.6, 2.9, 2.11 and 2.14. However, the long-term design criteria of Equations 2.10 and 2.11

are neglected for the rest of this work, assuming that $Z_{Lt} \leq 1$. The opposite condition ($Z_{Lt} > 1$) occurs when the beam is exposed to high moisture variation (e.g. outside use), and/or when a somewhat restrictive long-term deflection criterion is combined with a rather weakly restrictive short-term deflection criterion, which is senseless. If this happens, however, the short-term thickness and volume determined in Equation 2.8 and Equation 2.9 must be multiplied by Z_{Lt} and Z_{Lt}^2 , respectively.

2.3.3 Thickness and volume of a floor

The above Equations 2.2, 2.3, 2.5, 2.6, 2.8, 2.9, 2.13, and 2.14 must be adapted to the joists, primary beams, and PSS and PLS beams (for Case 2) before combining them for expressing the total thickness or volume a floor. The general procedure is exposed and illustrated for the "traditional" pattern X1 supported on corner columns (Case 2).

First, each expression is adapted for each beam, which:

- Have a length l_i expressed in terms of L_2 as $\omega_i L_2$. For pattern X1, the joists span in the smallest direction, such as $\omega_{X1,J} = 1$, and the peripheral beams span in the other direction, $\omega_{X1,PLSB} = 1/\lambda$.
- Are loaded by a load F_i , which is a fraction of the total load F . Mathematically, the way to adapt the above equations is to replace the parameters $\varepsilon_{M,i}$, $\varepsilon_{V,i}$, $\varepsilon_{D,i}$ and $\varepsilon_{F,i}$. For pattern X1, these values equal, respectively, by $1/(8n)$, $1/(2n)$, $5/(384n)$ and $\pi/(2n)$ for the joists, and $1/8$, $1/2$, $5/384$ and $\pi/2$ for the PLSB.

Secondly, these adapted diameters and volumes can then be regrouped to compute the following, where i represents joists (J), primary beams (PB) and peripheral beams (PSB):

- The total volume of the floor is taken as the sum of all log volumes: $V_{Tot} = \sum(\psi_i v_i)$, where ψ_i represents the particular number of beams of each floor layer. For Case 1, the total volume regroupes the volume of joists and primary beams. For Case 2, the volume of PSSB and PLSB must be considered in addition to the Case 1 volume. One has:

$$V_{Tot}^{Case\ 1} = \psi_J v_J + \psi_{PB} v_{PB} \quad (2.15a)$$

$$V_{Tot}^{Case\ 2} = \psi_J v_J + \psi_{PB} v_{PB} + \psi_{PSSB} v_{PSSB} + \psi_{PLSB} v_{PLSB} \quad (2.15b)$$

For pattern X1 in Case 2: $\psi_{X1,J} = n$, $\psi_{X1,PLSB} = 1$, and all other $\psi_{X1,i} = 0$.

- The total floor thickness is computed as the sum of the diameter of the log multiplied by a reduction factor $\xi_i \in [0; 1]$: $T_{Tot} = \sum(\xi_i \phi_i)$, with i representing joists, primary and peripheral beams, if any. This factor accounts for possible cutting at the connection zone (e.g., notch depth) as a fraction of the diameter, as shown in Figure 2.8 for different options. For Case 1, the total thickness is the sum of the joist thickness and the primary beam thickness. For Case 2, the thickness of PSSB or

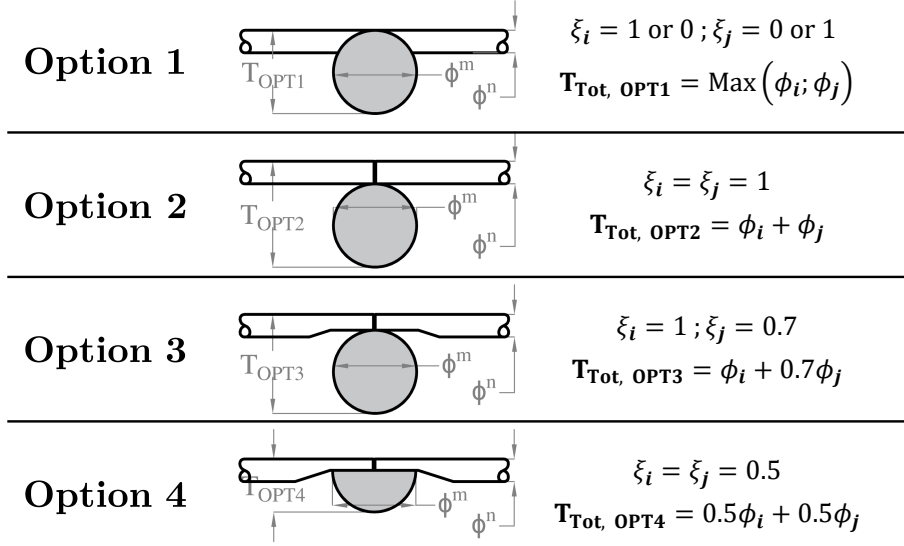


Figure 2.8: Different options of floor thickness definition, with "i" representing the peripheral beams or primary beams and "j" representing joists or primary beams.

PLSB must be considered in addition to the thickness in Case 1. One has:

$$T_{\text{Tot}}^{\text{Case 1}} = \xi_J \phi_J + \xi_{PB} \phi_{PB} \quad (2.16a)$$

$$T_{\text{Tot}}^{\text{Case 2}} = \xi_J \phi_J + \xi_{PB} \phi_{PB} + \xi_{PLSB} \phi_{PLSB} + \xi_{PSSB} \phi_{PSSB} \quad (2.16b)$$

The results presented in the following sections will focus only on the first two thickness options in Figure 2.8: Option 1 supposes that the different beams are in the same plane, and Option 2 that the uncut beams are simply superimposed. Other options will be discussed in section 2.5. For pattern X1 in Case 2 with Option 1: $\xi_{X1,J} = 0$ & $\xi_{X1,PLSB} = 1$ if $\phi_{X1,J} < \phi_{X1,PLSB}$, else $\xi_{X1,J} = 1$ & $\xi_{X1,PLSB} = 0$.

2.3.3.1 Thickness and volume of a floor considering the resistance criterion

Satisfying the bending resistance criterion, the adaptation of Equations 2.2 and 2.3 for any type of floor beam i leads to Equations 2.17 and 2.18, where $S_{Rm,i}$ and $W_{Rm,i}$ represent the "Thickness indicator under the bending criterion" and the "Volume indicator under the bending criterion", generically called "Morphological Indicators" (MI) [41]. These indicators regroup all the non-project parameters and are therefore dimensionless, only depending on λ , n and α .

$$\phi_{Rm,i}(\lambda, n, \alpha) = \left(\frac{32\varepsilon_{M,i} \omega_i}{\pi} \right)^{1/3} \left(\frac{F_{ULS} L_2}{f_b} \right)^{1/3} = S_{Rm,i} \left(\frac{F_{ULS} L_2}{f_b} \right)^{1/3} \quad (2.17)$$

$$\begin{aligned}
v_{Rm,i}(\lambda, n, \alpha) &= 3.691 (\varepsilon_{M,i} \omega_i^2)^{2/3} \left(\frac{F_{ULS} L_2^{5/2}}{f_b} \right)^{2/3} \\
&= W_{Rm,i} \left(\frac{F_{ULS} L_2^{5/2}}{f_b} \right)^{2/3} \quad (2.18)
\end{aligned}$$

The previous equations lead to the total thickness or volume of the whole floor using Equations 2.15 and 2.16:

$$T_{Rm,Tot} = S_{Rm,Tot}(\lambda, n, \alpha) \left(\frac{F_{ULS} L_2}{f_b} \right)^{1/3} \quad (2.19)$$

$$V_{Rm,Tot} = W_{Rm,Tot}(\lambda, n, \alpha) \left(\frac{F_{ULS} L_2^{5/2}}{f_b} \right)^{2/3} \quad (2.20)$$

Following the same steps for the shear resistance criterion, it is possible to adapt the Equations 2.5 and 2.6 for any type of floor beam i, leading to Equations 2.21 and 2.22, where $S_{Rv,i}$ and $W_{Rv,i}$ represent the "MI of thickness under the shear criterion" and the "MI of Volume under the shear criterion".

$$\phi_{Rv,i}(\lambda, n, \alpha) = \left(\frac{16\varepsilon_{V,i} \omega_i}{3\pi} \right)^{1/2} Z_R \left(\frac{F_{ULS} L_2}{f_b} \right)^{1/3} = S_{Rv,i} \left(\frac{F_{ULS} L_2}{f_b} \right)^{1/3} \quad (2.21)$$

$$v_{Rv,i}(\lambda, n, \alpha) = \frac{4\varepsilon_{V,i} \omega_i^2}{3} Z_R^2 \left(\frac{F_{ULS} L_2^{5/2}}{f_b} \right)^{2/3} = W_{Rv,i} \left(\frac{F_{ULS} L_2^{5/2}}{f_b} \right)^{2/3} \quad (2.22)$$

The linking parameter Z_R [–] is now defined as $(F_{ULS}^{1/6} f_b^{1/3}) / (L_2^{1/3} f_v^{1/2})$, which can be written as $(q_{ULS}^{1/6} f_b^{1/3}) / (\lambda^{1/6} f_v^{1/2})$. Each term can be estimated: $q_{ULS} \in [0.002; 0.012]$ [N/mm^2] and $f_b^{1/3} / f_v^{1/2} \in [1.88; 2.44]$ [$mm^{1/3}/N^{1/6}$] when taking EUROCODE 5 safety factors. Therefore, $Z_R \in [0.668/\lambda^{1/6}; 1.169/\lambda^{1/6}]$ and is taken equal to $0.918/\lambda^{1/6}$ in this work.

The previous equations lead to the total thickness or volume of the whole floor using Equations 2.15 and 2.16:

$$T_{Rv,Tot}(\lambda, n, \alpha) = S_{Rv,Tot}(\lambda, n, \alpha) \left(\frac{F_{ULS} L_2}{f_b} \right)^{1/3} \quad (2.23)$$

$$V_{Rv,Tot}(\lambda, n, \alpha) = W_{Rv,Tot}(\lambda, n, \alpha) \left(\frac{F_{ULS} L_2^{5/2}}{f_b} \right)^{2/3} \quad (2.24)$$

Mathematically, Equations 2.19 and 2.23, as well as 2.20 and 2.24, can be merged to express, respectively, the minimum thickness T_R and volume V_R respecting both criteria :

$$T_{R,Tot}(\lambda, n, \alpha) = \text{Max}[S_{Rm,Tot}, S_{Rv,Tot}] \left(\frac{F_{ULS} L_2}{f_b} \right)^{1/3} \quad (2.25)$$

$$V_{R,Tot}(\lambda, n, \alpha) = \text{Max}[W_{Rm,Tot}, W_{Rv,Tot}] \left(\frac{F_{ULS} L_2^{5/2}}{f_b} \right)^{2/3} \quad (2.26)$$

Equations 2.25 and 2.26 can be respectively rewritten into Equations 2.27 and 2.28, representing the "MI of thickness under the resistance constraint" and the "MI of volume under the resistance constraint" :

$$\frac{f_b^{1/3} T_{R,Tot}}{F_{ULS}^{1/3} L_2^{1/3}} = S_{R,Tot}(\lambda, n, \alpha) \quad (2.27)$$

$$\frac{f_b^{2/3} V_{R,Tot}}{F_{ULS}^{2/3} L_2^{5/3}} = W_{R,Tot}(\lambda, n, \alpha) \quad (2.28)$$

The previous equations have a physical meaning: regardless of the material characteristics f_b , the total load F_{ULS} and the span L_2 , and considering both bending and shear resistance criteria:

- The minimum floor thickness is obtained for the pattern that minimises the function $S_{R,Tot}$ for a given λ , n , and α .
- The minimum volume or mass is obtained for the pattern that minimises the function $W_{R,Tot}$ for a given λ , n , and α .

As an example, considering the pattern X1 of Case 2, as presented in Figure 2.5, one has:

$$\begin{aligned} - S_{R,Tot}^{X1,Case\ 2}(\lambda, n) [-] &= \text{Max}[S_{R,J}^{X1}, S_{R,PLSB}^{X1}] \\ &= \text{Max}[S_{Rm,J}^{X1}, S_{Rv,J}^{X1}, S_{Rm,PLSB}^{X1}, S_{Rv,PLSB}^{X1}] \\ &= \text{Max}[1.128/n^{1/3}, 0.921Z_R/n^{1/2}, 1.128/\lambda^{1/3}, 0.921Z_R/\lambda^{1/2}] \\ - W_{R,Tot}^{X1,Case\ 2}(\lambda, n) [-] &= W_{R,J}^{X1} \psi_J^{X1} + W_{R,PLSB}^{X1} \psi_{PLSB}^{X1} \\ &= \text{Max}[0.923/n^{2/3}, 0.667Z_R^2/n] * n + \text{Max}[0.923/\lambda^{4/3}, 0.667Z_R^2/\lambda^2] \end{aligned}$$

2.3.3.2 Thickness and volume of a floor considering the stiffness criterion

Satisfying the short-term deflection limitation criterion, the adaptation of Equations 2.8 and 2.9 for any floor beam type leads to Equations 2.29 and 2.30, where $S_{S,i}$ and $W_{S,i}$ represent the "MI of thickness under the stiffness constraint" and the "MI of Volume under the stiffness constraint".

$$\phi_{S,i}(\lambda, n, \alpha) = \left(\frac{64\epsilon_{D,i} \omega_i^2}{\pi} \right)^{1/4} \left(\frac{L_2^2 F_{SLS}}{\mu_{SLS} E} \right)^{1/4} = S_{S,i} \left(\frac{L_2^2 F_{SLS}}{\mu_{SLS} E} \right)^{1/4} \quad (2.29)$$

$$v_{S,i}(\lambda, n, \alpha) = (4\pi \varepsilon_{D,i} \omega_i^2)^{1/2} \left(\frac{L_2^4 F_{SLS}}{\mu_{SLS} E} \right)^{1/2} = W_{S,i} \left(\frac{L_2^4 F_{SLS}}{\mu_{SLS} E} \right)^{1/2} \quad (2.30)$$

The previous equations, written dimensionless for a whole floor using Equations 2.15 and 2.16, lead respectively to the total MI of thickness $S_{S,Tot}$ and the total MI of volume $W_{S,Tot}$:

$$\frac{\mu_{SLS}^{1/4} E^{1/4} T_S}{F_{SLS}^{1/4} L_2^{1/2}} = S_{S,Tot}(\lambda, n, \alpha) \quad (2.31)$$

$$\frac{\mu_{SLS}^{1/2} E^{1/2} V_S}{F_{SLS}^{1/2} L_2^2} = W_{S,Tot}(\lambda, n, \alpha) \quad (2.32)$$

Equations 2.31 and 2.32 have a physical meaning: regardless of the material characteristic E , the total load F_{SLS} , in-service parameter μ_{SLS} and the span L_2 , and considering only the short-term deflection limitation criterion:

- The minimum floor thickness is obtained for the pattern that minimises the function $S_{S,Tot}$ for a given λ , n , and α .
- The minimum volume or mass is obtained for the pattern that minimises the function $W_{S,Tot}$ for a given λ , n , and α .

As an example, considering the pattern X1 of Case 2, as presented in Figure 2.5, one has:

- $S_{S,Tot}^{X1,Case\ 2}(\lambda, n) [-] = \text{Max}[S_{S,J}^{X1}, S_{S,PLSB}^{X1}]$
 $= \text{Max}[0.718/n^{1/4}, 0.718/\lambda^{1/2}]$
- $W_{S,Tot}^{X1,Case\ 2}(\lambda, n) [-] = W_{S,J}^{X1} \psi_J^{X1} + W_{S,PLSB}^{X1} \psi_{PLSB}^{X1}$
 $= 0.405n^{1/2} + 0.405/\lambda$

2.3.3.3 Thickness and volume of a floor considering the vibration criterion

Satisfying the vibration limitation criterion, the adaptation of Equations 2.13 and 2.14 for any type of floor beam leads to:

$$\begin{aligned} \phi_{V,i}(\lambda, n, \alpha) &= \left(\frac{64\omega_i^3}{\pi \varepsilon_{F,i}^2} \right)^{1/4} \left(\frac{f_{1,lim}^2 \eta_{cov} L_2^3 F_{SLS}}{E g} \right)^{1/4} \\ &= S_{V,i} \left(\frac{f_{1,lim}^2 \eta_{cov} L_2^3 F_{SLS}}{E g} \right)^{1/4} \end{aligned} \quad (2.33)$$

$$\begin{aligned}
v_{V,i}(\lambda, n, \alpha) &= \left(\frac{4\pi\omega_i^5}{\varepsilon_{F,i}^2} \right)^{1/2} \left(\frac{f_{1,lim}^2 \eta_{cov} L_2^5 F_{SLS}}{E g} \right)^{1/2} \\
&= W_{V,i} \left(\frac{f_{1,lim}^2 \eta_{cov} L_2^5 F_{SLS}}{E g} \right)^{1/2} \quad (2.34)
\end{aligned}$$

Again, the previous equations, dimensionlessly written for a whole floor using equations 2.15 and 2.16, lead respectively to the "MI of thickness under the vibration constraint" $S_{V,Tot}$ and the "MI of volume under the vibration constraint" $W_{V,Tot}$:

$$\frac{E^{1/4} g^{1/4} T_{V,Tot}}{f_{1,lim}^{1/2} \eta_{cov}^{1/4} L_2^{3/4} F_{SLS}^{1/4}} = S_{V,Tot}(\lambda, n, \alpha) \quad (2.35)$$

$$\frac{E^{1/2} g^{1/2} V_{V,Tot}}{f_{1,lim}^{1/2} \eta_{cov}^{1/2} L_2^{5/2} F_{SLS}^{1/2}} = W_{V,Tot}(\lambda, n, \alpha) \quad (2.36)$$

Equations 2.35 and 2.36 have a physical meaning: regardless of the material characteristic E , the total co-vibration mass $F_{SLS} \eta_{cov}/g$, in-service parameter $f_{1,lim}$ and the span L_2 , and considering only the vibration comfort criterion:

- The minimum floor thickness is obtained for the pattern that minimises the function $S_{V,Tot}$ for a given λ , n , and α .
- The minimum volume or mass is obtained for the pattern that minimises the function $W_{V,Tot}$ for a given λ , n , and α .

As an example, considering the pattern X1 of Case 2, as presented in Figure 2.5, one has:

- $S_{V,Tot}^{X1,Case\ 2}(\lambda, n) [-] = \text{Max}[S_{V,J}^{X1}, S_{V,PLSB}^{X1}]$
 $= \text{Max}[1.695n^{1/2}, 1.695/\lambda^{3/4}]$
- $W_{V,Tot}^{X1,Case\ 2}(\lambda, n) [-] = W_{V,J}^{X1} \psi_J^{X1} + W_{V,PLSB}^{X1} \psi_{PLSB}^{X1}$
 $= 2.257n^2 + 2.257/\lambda^{5/2}$

2.3.3.4 Designing tool in the early design stage

This part explains how MI can be used for fast decision-making at the early design stage. Supposing, for example, an $6\text{ m} \times 6\text{ m}$ square floor ($\lambda = 1$) supported by columns (Case 2) with a required joist spacing of 300 mm ($n = 20$). Given the previous equations computed for pattern X1 for Case 2, the indicator of thickness and volume under a specific constraint (for example, stiffness in Section 2.3.3.2) can be calculated as follows: $S_{S,Tot}^{X1,Case\ 2}(\lambda, n) = \text{Max}[0.718/n^{1/4}, 0.718/\lambda^{1/2}] = 0.718$ and $W_{S,Tot}^{X1,Case\ 2}(\lambda, n) = 0.405n^{1/2} + 0.405/\lambda = 2.216$.

Considering a uniformly distributed service loading $q_{SLS} = 5\text{ kN/m}^2$ (including variable, permanent and self-weight load), a Young modulus $E =$

11000 MPa for all the logs, and a maximal relative short-term displacement factor $\mu_{SLS} = 1/350$ of each log's span, the total thickness (given Option 1) can be calculated as:

$$T_{S,Opt1} = S_{S,Tot}^{X1,Case\ 2,Opt1} \frac{F_{SLS}^{1/4} L_2^{1/2}}{\mu_{SLS}^{1/4} E^{1/4}} = 0.718 \frac{(5 * 36 * 10^3)^{1/4} 6000^{1/2}}{(1/350)^{1/4} 11000^{1/4}} = 484 \text{ mm}$$

The total volume can be calculated as:

$$V_S = W_{S,Tot}^{X1,Case\ 2} \frac{F_{SLS}^{1/2} L_2^2}{\mu_{SLS}^{1/2} E^{1/2}} = 2.214 \frac{(5 * 36 * 10^3)^{1/2} 6000^2}{(1/350)^{1/2} 11000^{1/2}} = 6.03 * 10^9 \text{ mm}^3$$

It is important to note that these results can be easily recalculated for different loading conditions and mechanical properties by adjusting the relevant parameters.

2.3.4 Thickness and volume of a reciprocal floor

The same procedure can be applied to compute the developed indicators for reciprocal floors. The above Equations 2.17, 2.18, 2.29, 2.30, 2.33, and 2.34 must be adapted to the joists, reciprocal primary beams and peripheral beams with the following parameters:

- The value of ω_i depends of λ and α . The joists always span in the smallest direction such as $\omega_J \in [(1-\alpha)/(2-\alpha), \min[1/(2-\alpha), (1-\alpha)/(\lambda(2-\alpha))], \alpha/(2-\alpha)]$, and the other beam types equals $\omega_{PB,a} = 1/(\lambda(2-\alpha))$, $\omega_{PB,b} = 1/(2-\alpha)$, $\omega_{PSSB} = 1$, and $\omega_{PLSB} = 1/\lambda$.
- The coefficients $\varepsilon_{M,i}$, $\varepsilon_{V,i}$, $\varepsilon_{D,i}$ and $\varepsilon_{F,i}$ are a function of λ , n , α , and the loading distribution on the beams. In particular, the maximal displacement and vibration frequency are considered for the four beams of the reciprocal module together.

However, the values of these four coefficients ε_i are particularly difficult to determine analytically for the primary and peripheral reciprocal beams. Therefore, this work proposes determining the MI of thickness and volume through numerical computation. This procedure inverts the equations of the designing tool presented in the previous subsection, following the steps presented in Figure 2.9:

- Step A: For any values of λ , n , and α , impose arbitrarily all the other necessary project parameters to construct the numerical model: L_1 (with $L_2 = \lambda L_1$), ϕ_J , $\phi_{PB,a} = \phi_{PB,b}$, ϕ_{PLSB} , ϕ_{PSSB} , q (with $F = q L_1 L_2$), η_{cov} , and E . This work uses Karamba3D [58], finite element software within the Rhinoceros/Grasshopper interface, which facilitates parametric design.
- Step B: For each beam layer i (Joist, Primary reciprocal Beams, and Peripheral long and short Span Beams), the following values are extracted from the numerical response: the maximum bending moment $M_{ULS,i}$,

For $\forall (\lambda, n, \alpha)$:

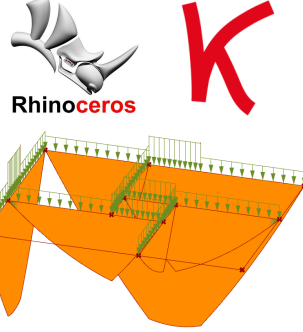
A. Construct the numerical model by imposing (arbitrary):

L_1 (with $L_2 = \lambda L_1$), q (with $F = q L_1 L_2$),
 ϕ_i , Z_R , η_{cov} , and E

B. Extract numerical response of each beam i (= J, PB, PSB):

$M_{ULS,i}$, $V_{ULS,i}$, $\delta_{Max,i}$, and $f_{1,i}$

C. Combine MI of each beam i :



Constraint		Individual indicator
Thickness	Resistance	$Max \left[\left(\frac{32 M_{ULS,i}}{\pi L_2 F_{ULS}} \right)^{1/3}, \left(\frac{16 V_{ULS,i}}{3\pi F_{ULS}} \right)^{1/2} Z_R \right] = S_{R,i}(\lambda, n, \alpha)$
	Stiffness	$\left(\frac{\phi_i^4 (\delta_{Max,i}/l_i) E}{L_2^2 F_{SLS}} \right)^{1/4} = S_{S,i}(\lambda, n, \alpha)$
	Vibration	$\left(\frac{\phi_i^4 E g}{f_{1,i}^2 L_2^3 \eta_{cov} F_{SLS}} \right)^{1/4} = S_{V,i}(\lambda, n, \alpha)$
Volume	Resistance	$Max \left[\left(\frac{8\pi^{3/2} M_{ULS,i} l_i^{3/2}}{L_2^{5/2} F_{ULS}} \right)^{2/3}, \left(\frac{4 V_{ULS,i} l_i}{3 F_{ULS} L_2} \right) Z_R^2 \right] = W_{R,i}(\lambda, n, \alpha)$
	Stiffness	$\left(\frac{\pi^2 \phi_i^4 l_i^2 E (\delta_{Max,i}/l_i)}{16 L_2^4 F_{SLS}} \right)^{1/2} = W_{S,i}(\lambda, n, \alpha)$
	Vibration	$\left(\frac{\pi^2 \phi_i^4 l_i^2 E g}{16 f_{1,i}^2 L_2^5 \eta_{cov} F_{SLS}} \right)^{1/2} = W_{V,i}(\lambda, n, \alpha)$

Figure 2.9: Procedure based on numerical analysis to determine thickness and volume MI according to the different constraints.

the maximum shear force $V_{ULS,i}$, the maximum vertical displacement $\delta_{max,i}$, and the fundamental frequency $f_{1,i}$.

- Step C: Equations 2.17, 2.18, 2.21, 2.22, 2.29, 2.30, 2.33, and 2.34, expressing the thickness or volume of floor beam under each constraint, are rearranged to compute the corresponding MI.

Finally, these computed indicators for each beam type are combined following Equations 2.15 and 2.16, as presented for pattern X1. This last step does not differ from the procedure presented in the previous subsection. The particular number of elements for computing the total volume are: depending on the joist direction, $\psi_J \in [2n/(2-\alpha), \text{Max}[2\lambda n/(2-\alpha), 2(1-\alpha)n/(2-\alpha)], \alpha L_2/(2-\alpha)]$, and, for the other beam types, $\psi_{PB,a} = 2$, $\psi_{PB,b} = 2$, $\psi_{PSSB} = 1$, and $\psi_{PLSB} = 1$.

When the logs are supposed to be in the same plane (Option 1), the total thickness is computed using the reduction factors $\xi_i = 1$ for the biggest dimensionless thickness, and $\xi_i = 0$ for the others. For Option 2, all reduction factors equal 1 for Case 1. However, in Case 2, when peripheral beams are required, joists must be supported on the same reference plane. Therefore, $\xi_J = 1$ but the reduction factors $\xi_i = 1$ for the biggest beam diameter between the primary beam (if any) and the peripheral beam, and $\xi_i = 0$ for the other.

2.4 Results

The mathematical expressions of the morphological indicators under resistance, stiffness, and vibration constraints cannot be directly compared. However, they can be used individually as design tools in the early design stage, as shown in Section 2.3.3.4 or for conducting comparative analyses, considering one criterion at a time and independently of the other two.

This section is divided into three parts. First, the reciprocal floor pattern is studied. After developing the MI to the reciprocal pattern, it is possible to determine the optimal engagement ratio α that minimises thickness and volume, for a given case study, λ and n . After that, the indicators of thickness and volume for the ten "conventional" floor patterns are presented and compared with those of the reciprocal ones. The MI of the conventional patterns are generated using also the numerical procedure presented in Figure 2.9. The computed indicators for each beam type are combined following Equations 2.15 and 2.16: $\psi_J = n \lambda K$ for X patterns or $\psi_J = n K$ for Y patterns, $\psi_{PB} = K - 1$ for Case 1 or $\psi_{PB} = K$ for Case 2, and $\psi_{PSB} = 1$ or 0 depending of the presence of peripheral beams. As for the reciprocal patterns, the Option 1 total thickness is computed using the reduction factors $\xi_i = 1$ for the biggest beam thickness, and $\xi_i = 0$ for the others. For Option 2, ξ_i equals 1. Finally, the results of the comparison are illustrated in a project scenario for expressing the gain or loss in terms of absolute values.

2.4.1 Optimal reciprocal floor

Following the procedure presented in Section 2.3.4, Figures 2.10, 2.11 and 2.12 display the thickness and volume MI, respectively, both before and after their combination with Equations 2.15 and 2.16. The following observations, based on these graphs, are universal and do not depend on the subsequently chosen project parameters. Note that curves with different λ values cannot be compared, as they correspond to floors with different geometries and total loads. Particularly for RS floors, $\lambda < 0.5$ is not relevant and is therefore not presented in this subsection.

The graphs in Figure 2.10 and 2.11 show that:

- Reducing n (i.e., increasing the spacing between joists) increases the thickness (diameter) of the joists but reduces the total material volume.

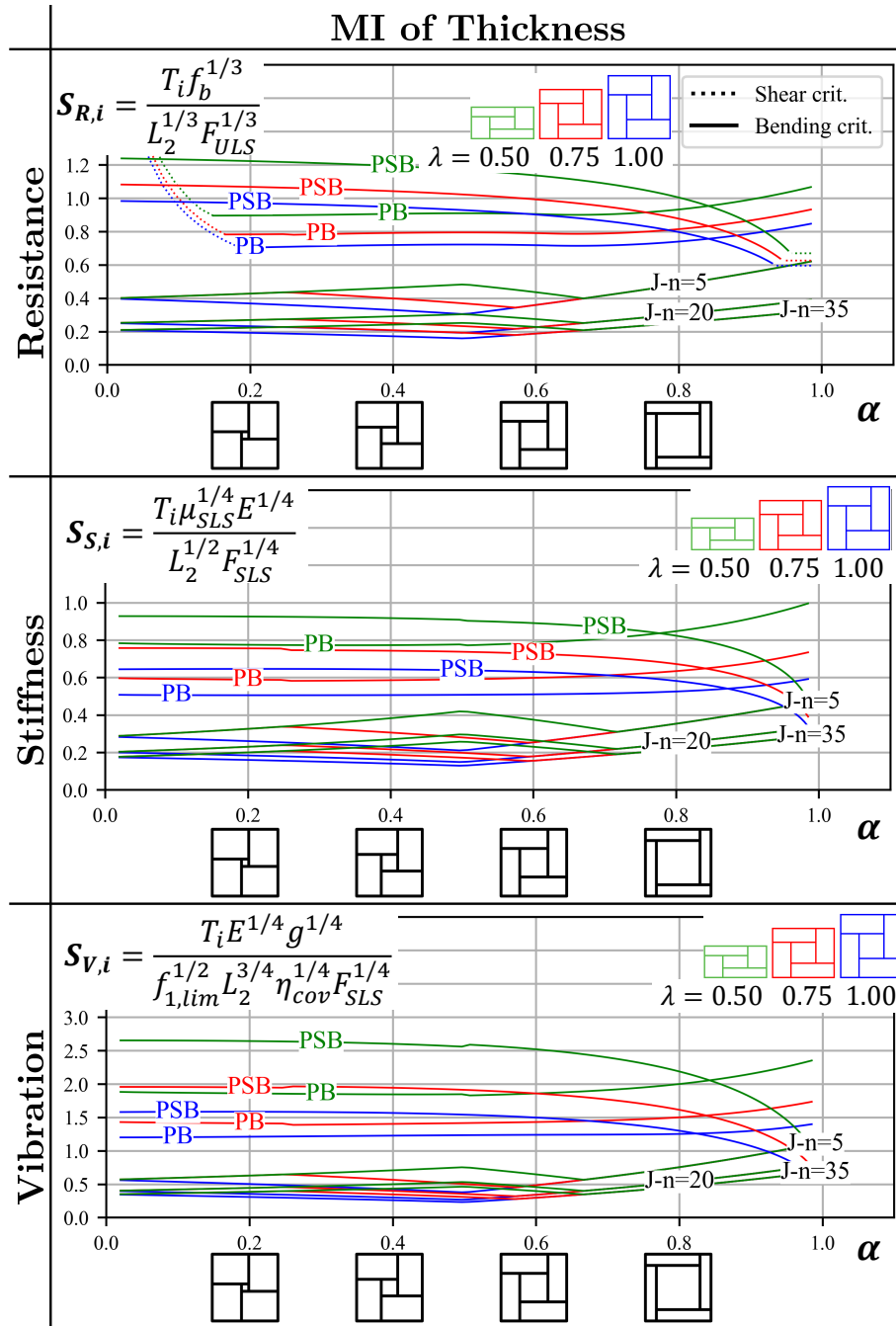


Figure 2.10: Thickness MI for each layer of a reciprocal floor as a function of engagement ratio α and number of joists n . Each colour corresponds to a specific λ .

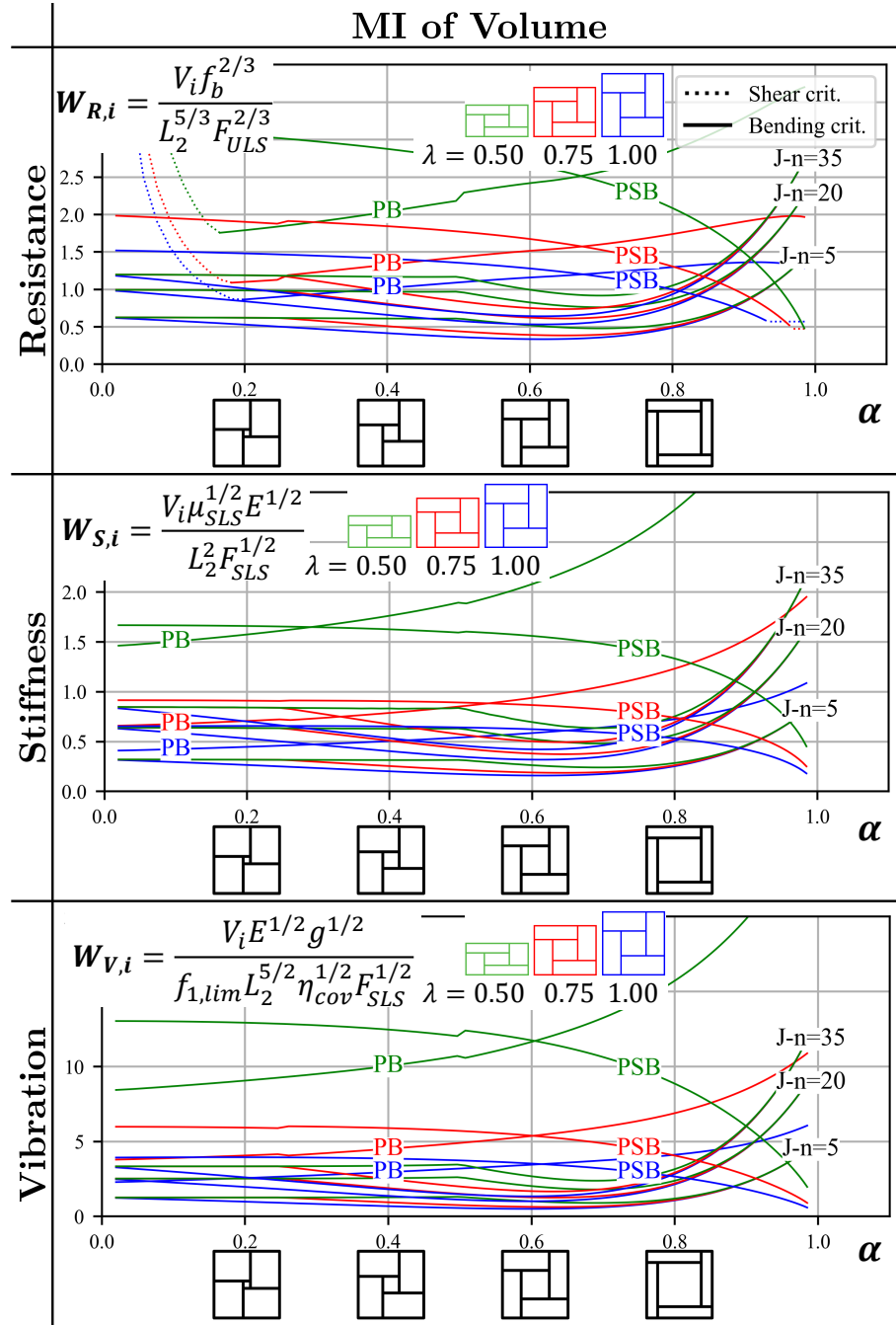


Figure 2.11: Volume MI for each layer of a reciprocal floor as a function of engagement ratio α and number of joists n . Each colour corresponds to a specific λ .

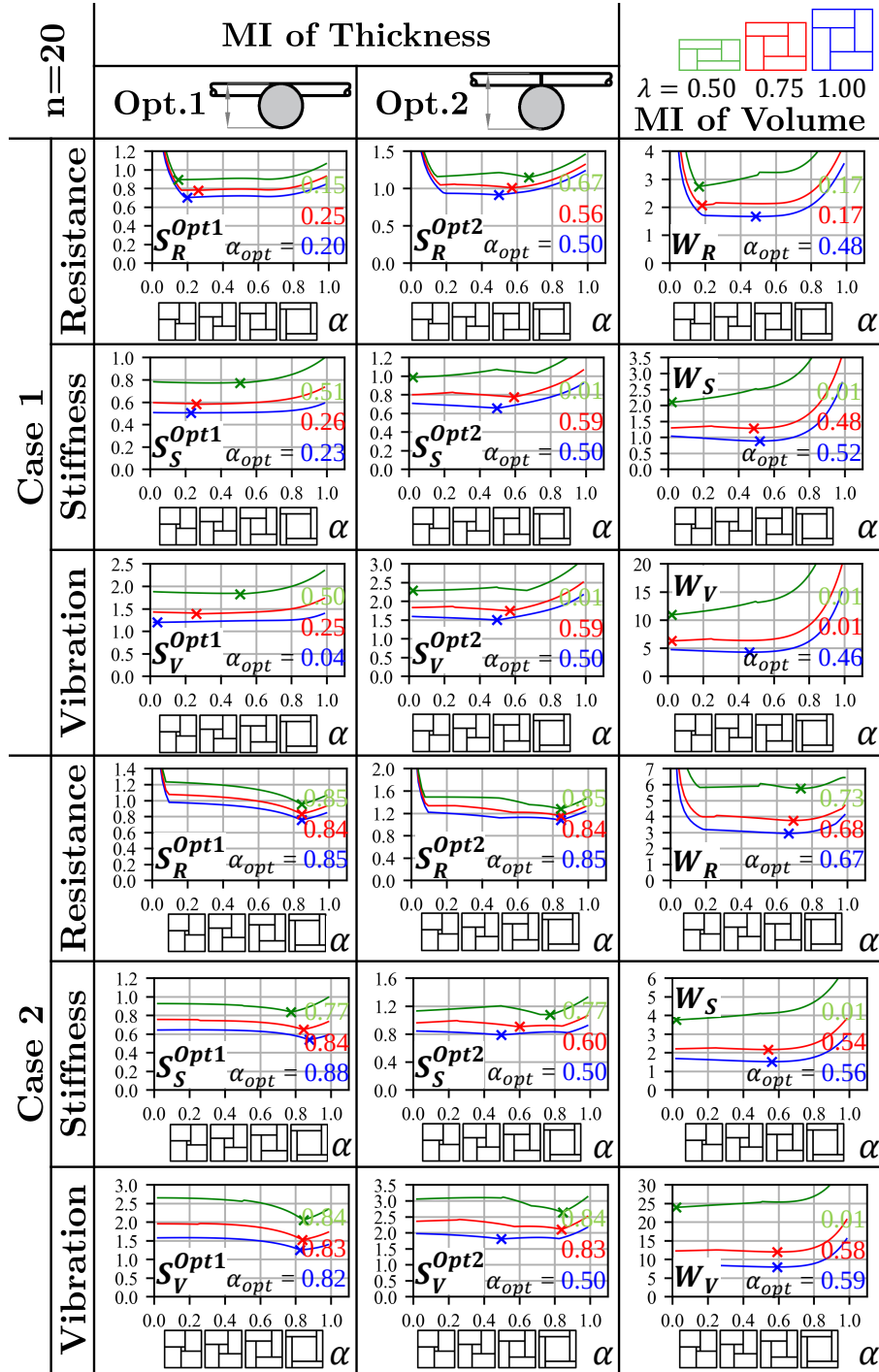


Figure 2.12: Total thickness and volume MI for reciprocal floors with $n = 20$. α_{opt} is marked with a cross. Each colour corresponds to a specific λ .

- The shear criterion only affects very small and very large values of α , for PB and PSB, respectively, and does not affect joist design.
- Neglecting the shear criterion, the diameters follow the logical order: $S_J \leq S_{PB} \leq S_{PSB}$, except for $\alpha > 0.82$. This does not extend to volume MI, since length and number of elements also play a role.
- Curves with $\lambda = 0.5$ and 0.75 show discontinuities at $\lambda = 1 - \alpha$, due to a change in joist span direction. Joists always span in the shortest direction. The discontinuities in PB and PSB occur in opposite directions.

After combining the MI of all layers ($n = 20$), Figure 2.12 shows the total MI for the two cases: 1) supported on peripheral walls, and 2) supported on corner columns. The optimal engagement ratio (α_{opt}) is indicated. Considering each constraint independently, the results show:

- No single value of α is optimal in all cases, and the weighted average of all α_{opt} , with double weight for volume, equals 0.47, denoted α_{mean} . However, choosing $\alpha \geq 0.7$ is generally not relevant, except for Case 2, where minimising thickness is the objective.
- For Case 1, thickness and volume MI vary little within $\alpha \in [0.2; 0.6]$. The lower bound depends only on the shear criterion and thus on Z_R . For nearly square floors ($\lambda = 0.75$ or $\lambda = 1$), it is generally advantageous to reduce α .
- For Case 2, the thickness of MI are mainly governed by the peripheral beam thickness. When $\alpha \geq 0.85$, the reciprocal module thickness becomes dominant. Conversely, volume MI are only weakly influenced by additional volume of the peripheral beams.

2.4.2 Comprehensive comparison

Each graph in Figures 2.13 and 2.14 represents the thickness (with two options) and the volume indicators, depending on the shape factor $\lambda \in [0.2; 1]$ (and a fixed value $n = 20$). The ten conventional X and Y patterns are presented, as well as two reciprocal patterns denoted RM and RO. As no value of α is optimal in all cases, the RM pattern is the reciprocal floor with the average optimal ratio ($\alpha_{mean} = 0.47$). Secondly, the reciprocal pattern RO is a reciprocal floor with an optimal engagement ratio specific to the MI and the floor geometry (α_{opt}). Since α_{opt} varies, these results are purely theoretical but illustrate the maximum potential gain (or minimum loss) achievable with reciprocal floors.

To highlight the loss or gain compared to the "traditional" pattern X1, the values of the indicators in Figures 2.13 and 2.14 are divided by the corresponding value associated with pattern X1.

2.4.2.1 Case 1: Floor supported by four walls

Regarding **both option's floor thicknesses**, the first two columns of Figure 2.13 show that:

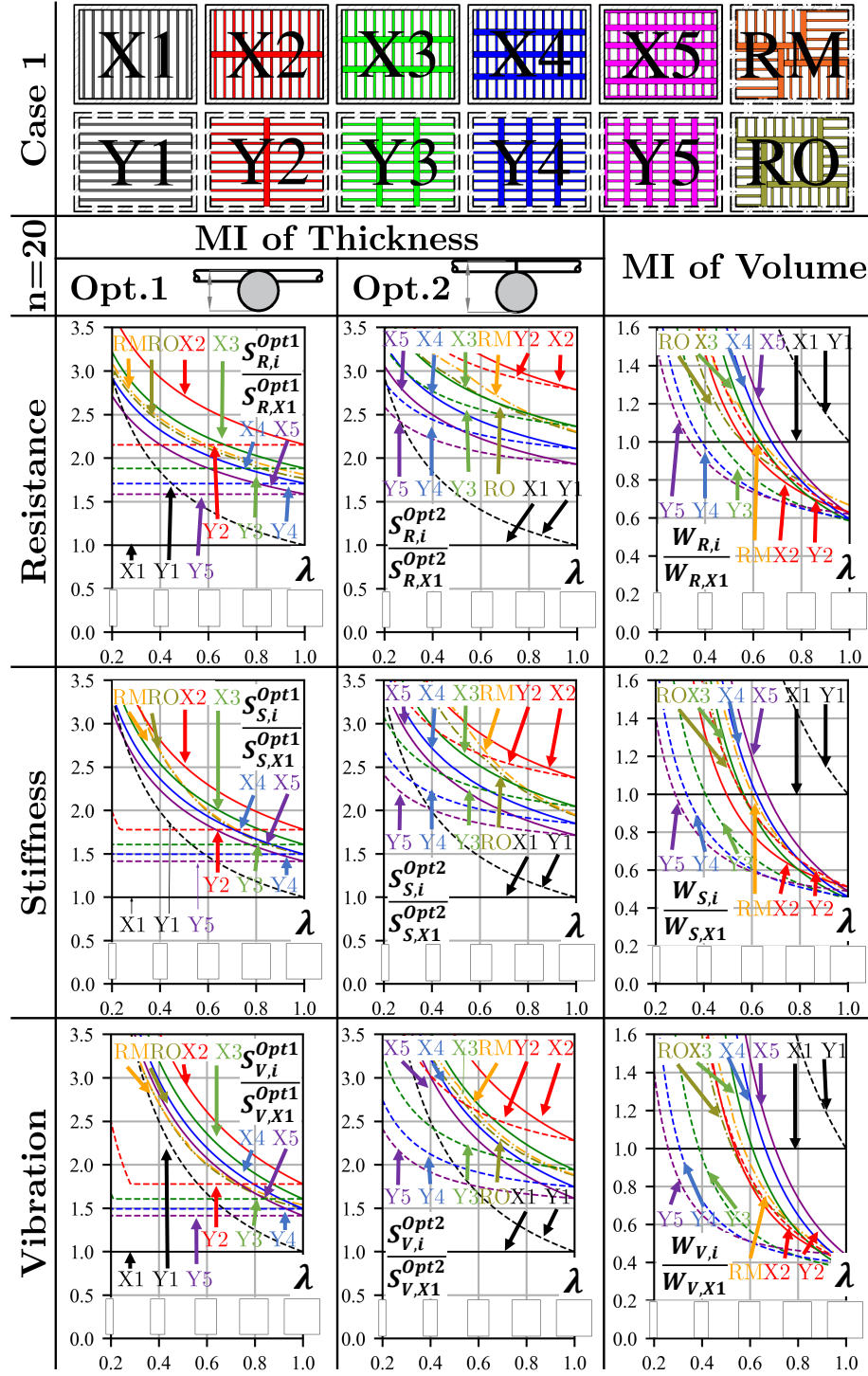
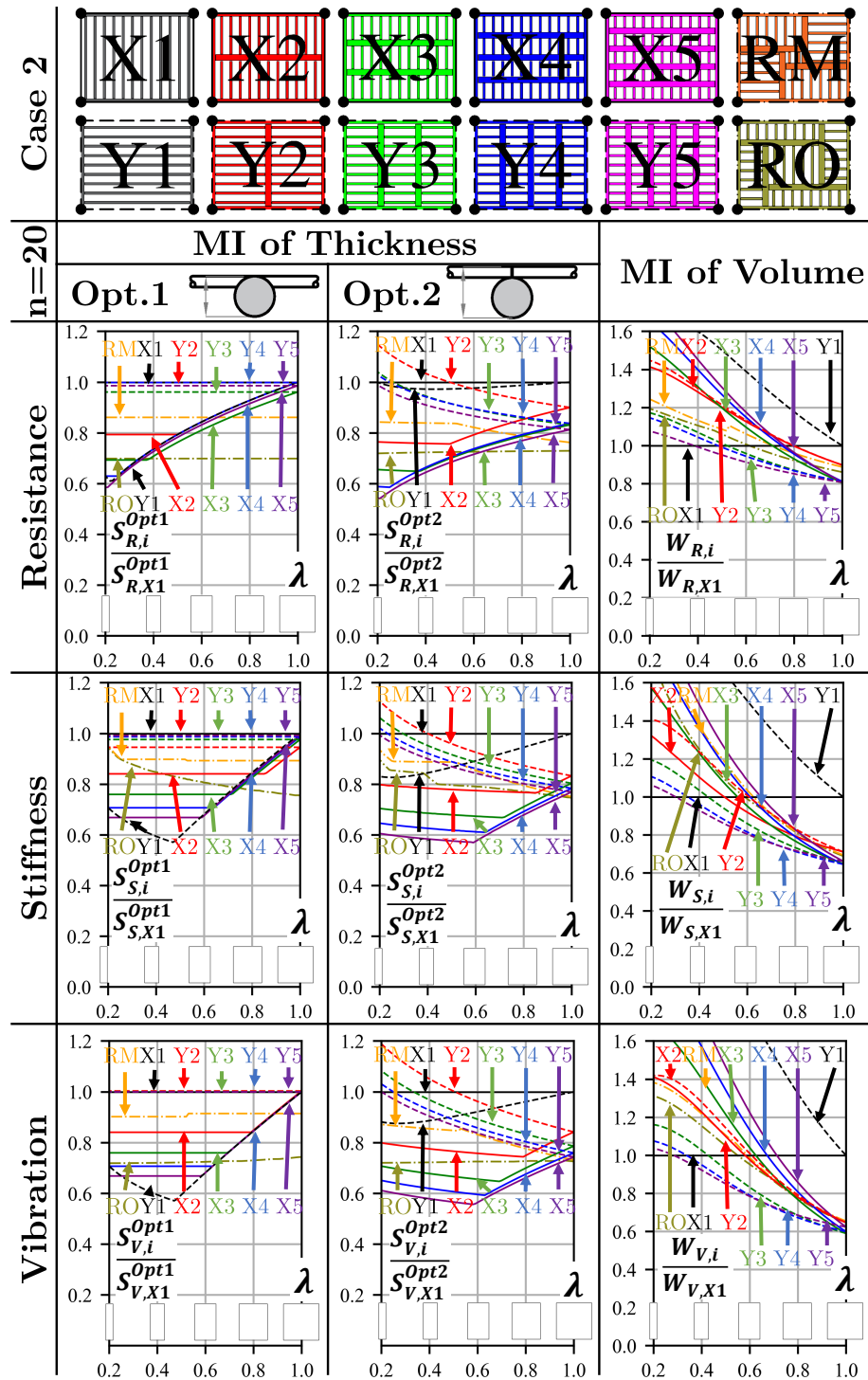


Figure 2.13: Indicators of thickness and volume (relative to pattern X1) for Case 1.



- Pattern X1 consistently performs the best among all the options.
- Pattern Y5 is a viable alternative to pattern X1 when aiming to maximize smaller timber elements. For square floors, pattern Y5 still exhibits 59%, 41% and 41% more thickness for Option 1, but this difference lowers to 93%, 71% and 61% for Option 2, considering the resistance, stiffness and vibration constraints, respectively.
- Patterns X2 and Y2 are generally the least suitable solution compared to other X and Y patterns.
- Patterns Y2, Y3, Y4 and Y5 consistently have less thickness than their corresponding X patterns with the same number of primary beams.
- Reciprocal patterns RO and RM do not offer better results than Y5 and Y4 patterns.

Regarding the **floor's volume**, the third column of Figure 2.13 shows that:

- Pattern X1 is more efficient for smaller shape factors but can become more than twice as heavy as other patterns for more "square" floors ($\lambda \rightarrow 1$).
- Patterns Y3, Y4 and Y5 provide excellent alternatives to pattern X1 and are almost equivalent when $\lambda = 1$, with a gain in volume between 40% to 60%, depending on the constraint.
- Reciprocal Patterns are less optimal than patterns Y5, Y4, and Y3 but, for $\lambda = 1$, they have all similar gain, compared to pattern X1.

2.4.2.2 Case 2: Floor supported by corner columns

In Case 2, where peripheral beams are required as shown in Figure 2.4, the floor's thickness for Option 1 and Option 2 exhibits distinct behaviour and will be discussed separately.

Regarding the **floor's thickness for Option 1**, the first column of Figure 2.14 shows that:

- The optimal solution for thickness is no longer the "classical" pattern X1, but rather the pattern Y1. Indeed, the total thickness is mainly generated by the PSSB or PLSB, which makes the patterns X1, Y2, Y3, Y4 and Y5 highly inefficient.
- Among the conventional pattern, patterns Y1, X2, X3, X4, and X5 are optimal, offering a thickness gain of up to 40% compared to pattern X1.
- The reciprocal patterns are the best solutions for square floors. Indeed, the reciprocal module distributes (almost) evenly on the peripheral beams, which limits their thickness (diameter), compared to other patterns.

Regarding the **floor's thickness for Option 2**, the second column of Figure 2.14 shows that:

- As for Option 1, patterns X5, X4 and X3 are always the most efficient between conventional patterns. Pattern Y1 only shows a maximum thick-

ness gain of 16% compared to pattern X1.

- Reciprocal patterns are still the best solutions for square floors. However, their gain with the X5 pattern is lower than with Option 1.

Regarding the **floor's volume**, the third column of Figure 2.14 shows that for Case 2:

- Pattern X1 is more efficient for smaller shape factors but can become significantly heavier than other patterns for more "square" floors ($\lambda \rightarrow 1$).
- Patterns Y3, Y4, and Y5, as well as reciprocal patterns, provide the best alternatives to pattern X1 and are equivalent when $\lambda = 1$, with a volume gain up to 40% compared to pattern X1.
- Pattern Y1 is consistently the least suitable solution.

2.4.2.3 Summary of the results

Table 2.1 summarises the results and presents the order of magnitude of the gain (negative value, e.g. -55%) or loss (positive value, e.g. +75%) compared to pattern X1. This order of magnitude is based on the mean of the results across the different constraints with $n = 20$. The results of both considered reciprocal patterns are shown in all situations.

Regarding small shape factors ($\lambda \rightarrow 0.2$), the first column of this table shows that:

- **For Case 1**, pattern X1 is consistently better for both thickness options and volume (light grey region). Patterns with primary beams consume more volume and have at least 47% more thickness.
- **For Case 2**, pattern X1 provides a smaller volume. However, patterns Y1, X5, and X4 result in a smaller thickness for both options (dark grey area). However, pattern X1 still optimises volume.

Regarding large shape factors ($\lambda \rightarrow 1$), the second column of this table shows that:

- **For Case 1**, pattern X1 is always better for thickness but not for volume (grey region: Y3, Y5, and Y4).
- **For Case 2**, reciprocal patterns exhibit a smaller thickness. In terms of volume optimisation, patterns with more than 2 primary beams ($K \geq 3$) are the best solutions, followed by reciprocal patterns. For square floors, the volume is minimised when $K=3$, so no additional primary beams are needed.

In addition, general remarks can be made:

- Increasing the number of primary beams limits the thickness and volume in all situations. Therefore, dividing the joists' span only by a factor of 2 ($K = 2$) is not optimal compared to other patterns with more primary beams. Nevertheless, each supplementary primary beam leads to additional connections and on-site installation costs.

		$\lambda \rightarrow 0.2$ □						$\lambda \rightarrow 1$ □					
Case 1	Thick. Opt1	X1	Y5	Y4	Y3	RO	RM	X1	Y5	Y4	Y3	RO	RM
		+47%	+57%	+72%	+301%	+304%		+47%	+57%	+70%	+58%	+62%	
	Thick. Opt2	X1	Y5	Y4	Y3	RO	RM	X1	Y5	Y4	Y3	RO	RM
Case 2		+147%	+176%	+221%	+364%	+380%		+75%	+90%	+112%	+104%	+104%	
	Volume	X1	Y5	Y4	Y3	RO	RM	Y3	Y4	Y5	Y2	RO	RM
		+43%	+81%	+159%	+443%	+583%		-53%	-52%	-48%	-48%	-47%	-47%
Case 2	Thick. Opt1	X5	Y1	X4	X3	RO	RM	X3	X2	X5	X4	RO	RM
		-36%	-32%	-32%	-26%	-21%	-10%	-2%	-2%	-1%	-1%	-27%	-11%
	Thick. Opt2	X5	X4	X3	X2	RO	RM	X5	X4	X3	X2	RO	RM
Case 2		-41%	-37%	-31%	-31%	-22%	-12%	-22%	-21%	-19%	-14%	-27%	-25%
	Volume	X1	Y5	Y4	Y3	RO	RM	Y3	Y4	Y5	Y2	RO	RM
		+6%	+11%	+18%	+42%	+53%		-32%	-31%	-30%	-25%	-27%	-26%

Table 2.1: Review of the observation. Under each pattern, the gain (-) or loss (+) of thickness or volume compared to pattern X1. The highlighted cells show that the pattern X1 (light grey), patterns Y3, Y4 & Y5 (grey), patterns Y1, X3, X4 & X5 (dark grey), and reciprocal patterns (white) are minimising thickness or volume.

- Except for square floor support on columns, reciprocal patterns do not outperform patterns with $K \geq 4$, although they require smaller lengths of logs to cover the same area.

2.4.3 Application

This section illustrated the previous results to show the gain or loss of each pattern in absolute terms. The comparison is presented in Table 2.2 based on the project situation introduced in Section 2.3.3.4. In this table, the thickness and volume of patterns X1, Y3 and RM are computed based on their respective MI and the assumed project parameters. The final total thickness and volume of a pattern corresponds to the maximum of the values computed. For this situation, the vibration constraint is clearly the designing one and show a thickness gain of 47 [mm] for the reciprocal pattern compared to the two others. The gain raises to 218 [mm] for pattern X1 for thickness Option 2. In terms of volume, patterns Y3 and RM respectively have 3.23 and 2.72 [m^3] compare to pattern X1.

2.5 Discussion

This discussion mainly focuses on the connection between floor joists and beams, since they play a significant role in determining the overall thickness of a floor. This section explores the design implications of alternative connection definitions and assesses their feasibility based on construction feedback obtained from the mock-up shown in Figure 2.2.

Figure 2.1 illustrated that the notched connection restricts local cutting to

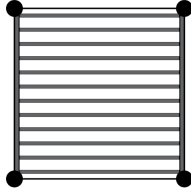
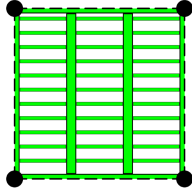
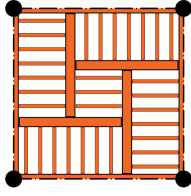
Assumptions :		a) Pattern X1	b) Pattern Y3	c) Pattern RM
- $L_1 = L_2 = 6 [m]$, - $d = 0.4 [m]$ - $E = 11000 [MPa]$, - $f_b = 15 [MPa]$ - $F_{ULS} = 252 [kN]$, - $F_{SLS} = 180 [kN]$, - $\eta_{cov} F_{SLS}/g = 9 [t]$ - $\mu_{SLS} = 1/350$, - $f_{1,lim} = 8 [Hz]$				
Thickness		S_{Tot}^{X1} $T_{Tot}^{X1} [mm]$	S_{Tot}^{Y3} $T_{Tot}^{Y3} [mm]$	S_{Tot}^{RM} $T_{Tot}^{RM} [mm]$
Opt. 1	Resistance	1.08 504	1.04 485	0.93 435
	Stiffness	0.72 484	0.70 472	0.64 432
	Vibration	1.70 553	1.70 553	1.55 601
	Floor thickness	553 [mm]	553 [mm]	601 [mm]
Opt. 2	Resistance	1.48 690	1.23 574	1.13 526
	Stiffness	1.06 712	0.85 573	0.79 535
	Vibration	2.50 814	1.96 640	1.83 709
	Floor thickness	814 [mm]	640 [mm]	709 [mm]
Volume		W_{Tot}^{X1} $V_{Tot}^{X1} [m^3]$	W_{Tot}^{Y3} $V_{Tot}^{Y3} [m^3]$	W_{Tot}^{RM} $V_{Tot}^{RM} [m^3]$
Resistance		3.43 6.27	2.76 5.05	3.04 5.57
Stiffness		2.21 6.03	1.43 3.91	1.54 4.19
Vibration		12.35 7.88	7.29 4.65	8.09 5.16
Floor volume		7.88 [m³]	4.65 [m³]	5.16 [m³]

Table 2.2: Comparison of patterns X1, Y3 and RM for a particular project situation.

the supporting log, unlike the JJP connection. This characteristic helps preserve the highly stressed zone of the supporting beam while reducing the potential for thickness reduction. However, the notched connection still affects the shear resistance at the end of the supported element. As a result, general recommendations limit the notch depth to 15% of the supported beam's diameter, which can be increased to 30% when anti-splitter bolts are added [90]. Other codes accept notches up to $\frac{1}{4}$ of the diameter without requiring strengthening measures [17].

This least-restrictive recommendation corresponds to thickness Option 3, as defined in Figure 2.8. In this situation, the reduction $\xi_j = 0.7$ needs to be applied to the joists' thickness when $K \geq 2$ for Case 1, and for all patterns for Case 2. When using this thickness option, it is observed that pattern Y5 still has at least 55% more thickness than pattern X1 for Case 1 (vibration constraint). For Case 2, notching reduces the thickness gain achieved with the Y1, X5, X4, X5, and reciprocal patterns. For a square floor, the mean maximal gain (obtained by RO) equals 14%. However, the shear resistance criterion must be adjusted to account for the reduced cross-section due to notching.

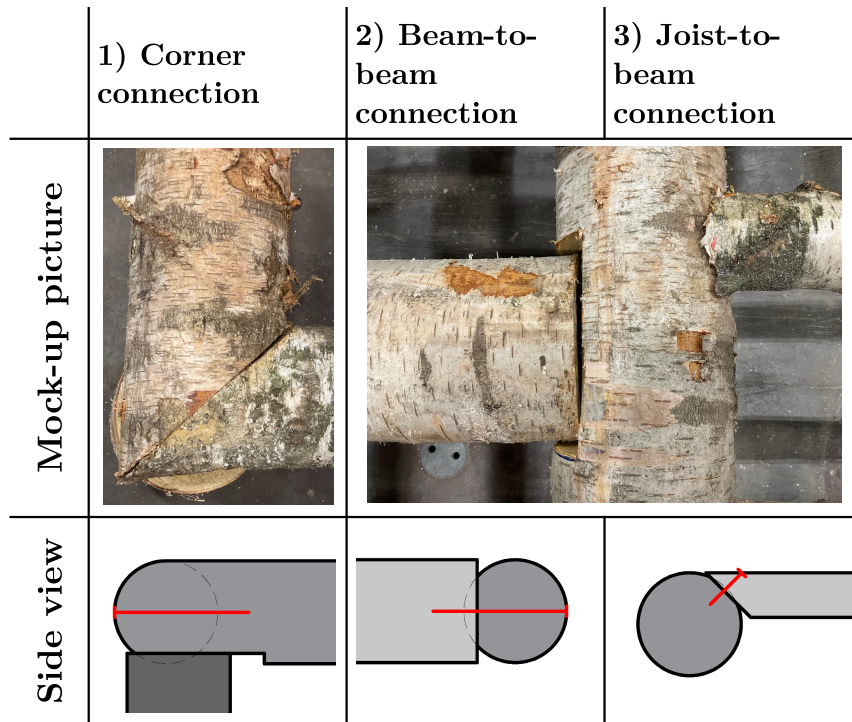


Figure 2.15: Connections types for the mock-up floor of Figure 2.2.

In the mock-up floor shown in Figure 2.2, the beams and joists are in the same plane, and three connection types are used to achieve this. These connection types are illustrated in Figure 2.15:

- Type 1: This connection type involves connecting peripheral beams at the supports. The beams are assembled at the corners with a contact plane cut at 45° , and a notch is cut to ensure good contact with the supports. Two self-tapping screws are placed in the logs' axes to secure the connection.
- Type 2: This connection type involves a 50 mm cut-out in the supporting log to create a good contact surface with the end of the supported log. Self-tapping screws are then placed through the supporting beam along the longitudinal axis of the supported beam.
- Type 3: This connection type is used to connect the joists with the primary beams. The ends of the joists are cut at an appropriate angle to match the curvature of the primary beam, ensuring an identical upper floor level throughout. Screws are then placed at both ends of the joists to secure the connection.

While the first connection type (Type 1) was chosen for its ease of fabrication, it has limitations for buildings because it cannot achieve vertical column-column connections. However, the second and third connection types (Type 2

and Type 3) can be used in buildings and allow for in-plane connections with minimal processing. Furthermore, these connection types limit cuts to the side of the supporting log, preserving the areas under higher bending stress.

One potential drawback of the second connection type (Type 2) is the presence of gaps between logs, as visible in Figure 2.15. These gaps can result from cutting and assembly tolerances as well as hygrometric variations. These gaps can be mitigated by ensuring that the log moisture content is consistent with the ambient indoor humidity and by having adequate knowledge of log geometry. Reducing the gaps prevents flexure in the screws and mitigates the risk of increased fire damage when exposed to steel parts.

2.6 Conclusion

Due to recent advances in parametric design and robotic fabrication, the use of minimally processed logs is now possible in cost-effective, sustainable structural systems. In this context, this chapter focused on the design of log floor systems. The pattern X1, consisting of parallel joists spanning the shortest direction, is traditionally regarded as the ideal solution. This work challenges that assumption by exploring alternative configurations, including reciprocal floor systems, that reduce the necessary beam lengths.

To support this investigation, the Morphological Indicator (MI) method is introduced as a unified framework for comparing floor patterns. Based solely on geometric parameters (i.e. floor dimensions ($L_1 \geq L_2$) and joist spacing), floor patterns can be rapidly compared, selected, or discarded. Once a configuration is chosen, its actual thickness or material volume can be readily derived from its MI value under specific design criteria.

The MI method, however, has limitations as each design criterion is observed independently: the assumption of a single governing criterion for all structural elements may not hold in layered systems. As such, the method does not constitute a fully comprehensive optimisation framework.

For reciprocal patterns, no unique optimal value of engagement ratio α exists, but clear trends emerge:

- For nearly square floors, values within the range $\alpha \in [0.2; 0.7]$ minimises material use. When the ratio L_2/L_1 reduces, α needs to be minimal.
- Thickness is largely impacted by how the beams are supported and connected. When the floor is supported by columns, α is best set to 0.85. In other cases, no general role can be expressed.

Compared to more conventional floor patterns, the reciprocal floor can be advantageous for (nearly) square floors. In terms of material efficiency, they are more interesting than pattern X1 and perform similarly to the other studied floors. However, they are more optimal in terms of floor thickness in specific conditions.

Chapter 3

Geometric stability of reciprocal Structures

The previous chapter focused on a relatively simple RS and showed the interest in using RS compared with conventional solutions. Before extending the study to more complex geometries, it is important to discuss the connection detailing of an RS as it impacts the mechanical behaviour of the structure. Therefore, this chapter more broadly addresses modelling connections in all materials and their impact on RS stability, drawing from a journal paper rearranged for clarity:

- STEINMETZ Martin, SGAMBI Luca, and LATTEUR Pierre. Geometric Stability of Reciprocal Structures. *Journal of Structural Engineering*, 2026, vol. 152, no 3, p. 04025288.

This chapter is divided into two sections. The first reviews the proposed connections and supports used in built RS and research prototypes, with a focus on their numerical modelling. The propositions generally fix rotational movements, as in welded connections, compromising the essence of reciprocal behaviour and aligning them more closely with the principles of grid-shells. As a reminder, a reciprocal structure typically behaves like simply stacked elements, with rotationally free ends. This chapter also shows that semi-rigidities are sometimes present in proposed connections. They are due to friction, tolerances, and non-linearities in the joint. However, neglecting them can lead to numerical predictions that are far removed from reality. On the other hand, assessing them numerically or experimentally is also complicated.

One aim of using rotational fixed connections is to prevent local and global geometric instabilities, including rigid-body rotations about the longitudinal axis. To address this need, the second section presents five methods from the literature for assessing geometric stability and compares their effectiveness. Only one is applicable to all connections and support types, independently of the cross-section properties, and to all RS geometries.

This method showed that hinged connections and supports can generate geometrically stable space RS. Indeed, the rigid-body rotation about the longitudinal axis is prevented by the distance between the beams' axes at the connection, which creates a lever arm that blocks this rotation. However, ge-

ometries with specific central symmetry exhibit global geometric instability, necessitating the application of this method at the early design stage.

3.1 Research objectives

During the design of an RS, the choice of supports and connections, particularly the types of constraints they introduce, must be carefully considered for several reasons:

- They can be the source of local instabilities, such as rotating rigid beams around their longitudinal axis.
- The actual behaviour of supports and connections is often challenging to model numerically with sufficient accuracy [63], which may result in the imprecise calculation of internal forces within the beams [21], especially in RS^{3D}.
- The connections determine the eccentricity between beams, which influences the overall shape and curvature of the structure [6]. Modifying one or more connections can therefore affect the global geometry, which must then be generated iteratively [49].
- Supports and connections should allow straightforward construction and facilitate the mutual assembly of the beams while maintaining the designed geometry [1].
- Translational constraints at the supports may generate horizontal reaction forces that are sometimes incompatible with the project requirements, even though such constraints may enhance structural stability.

Therefore, this chapter aims to critically review various types of supports and connections proposed in the literature and to compare five methods for verifying the geometric stability of RS. This research does not consider instabilities due to local or global buckling.

The following section reviews different types of supports and connections, both in built structures and laboratory prototypes. Then, five methods are compared to evaluate the geometric stability of RS^{2D} and RS^{3D}, including methods explicitly developed for RS and those that can be adapted to RS. The first method is based on the structure's stiffness matrix and thus depends on the cross-section and material properties. The remaining four methods assume infinitely rigid beams and require fewer computational resources. Given that the geometric design of RS is often iterative [18], these methods are particularly valuable for accelerating the design process.

The support and connection models considered in this chapter are illustrated in Figure 3.1. Each may have six Degrees of Freedom (DoFs), which can be free, semi-rigid, or fixed. The fixed DoFs are indicated with arrows (T_x , T_y , T_z , R_x , R_y , and R_z). In particular, when only the three translational DoFs are fixed (free R_x , R_y , and R_z), the support or connection is called "hinged". An RS_{HINGED} has only hinged supports and connections. Furthermore, the beam

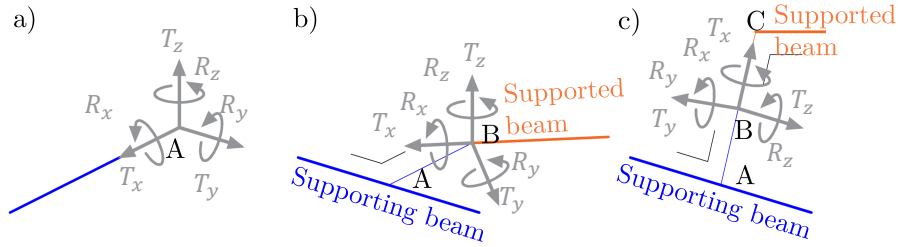


Figure 3.1: Six free, semi-rigid or fixed DoF, presented by arrows when fixed: a) Support, b) RS^{2D} connection placed at point B, and c) RS^{3D} connection placed at point B.

thickness's eccentricity is modelled using a fictitious element AB for RS^{2D} [Figure 3.1b] or two fictitious elements AB and BC for RS^{3D} [Figure 3.1c], with clamped connections at A and C. If any presented supports or connections deviate from these assumptions, their modelling approach will be explicitly specified.

3.2 State of the art regarding supports and connections of RS

3.2.1 Supports

Figure 3.2 presents examples of supports used in laboratory-tested RS or described in the scientific literature:

- Figure 3.2a: Two support solutions for an RS^{3D} built with bamboo canes are proposed by Godina Rodriguez [66]. In the first case, the bamboo beam is placed on the ground, and the support is modelled as restraining only vertical translation. The second solution incorporates cables connecting the three supports, thereby limiting lateral displacements [zoom of Figure 3.2a]. These cables are not considered infinitely rigid and are thus modelled as semi-rigid horizontal translational DoF, with stiffness values derived from their mechanical properties.
- Figure 3.2b: This refers to an experimental metallic structure, in which the support tubes were cut obliquely and rested directly on the ground [63]. These supports are modelled by fixing only the vertical translation, or by adding semi-rigid horizontal translational DoFs to account for friction. The stiffness of these semi-rigidities is determined based on measured displacements during testing.
- Figure 3.2c: For this RS^{2D} built with steel tubes, a previous research of author, not exposed in this thesis, proposes a support that fixes rotation about the tube's axis (R_x) using a sufficiently wide plate welded under the tube and resting on a rolling metal cylinder. Vertical translation (T_z) is fixed. Lateral translation perpendicular to the tube's axis (T_y) is not mechanically restrained, but since the loading is purely vertical, it is modelled as fixed.
- Figure 3.2d: For this RS^{3D} , also made of steel tubes, another previous

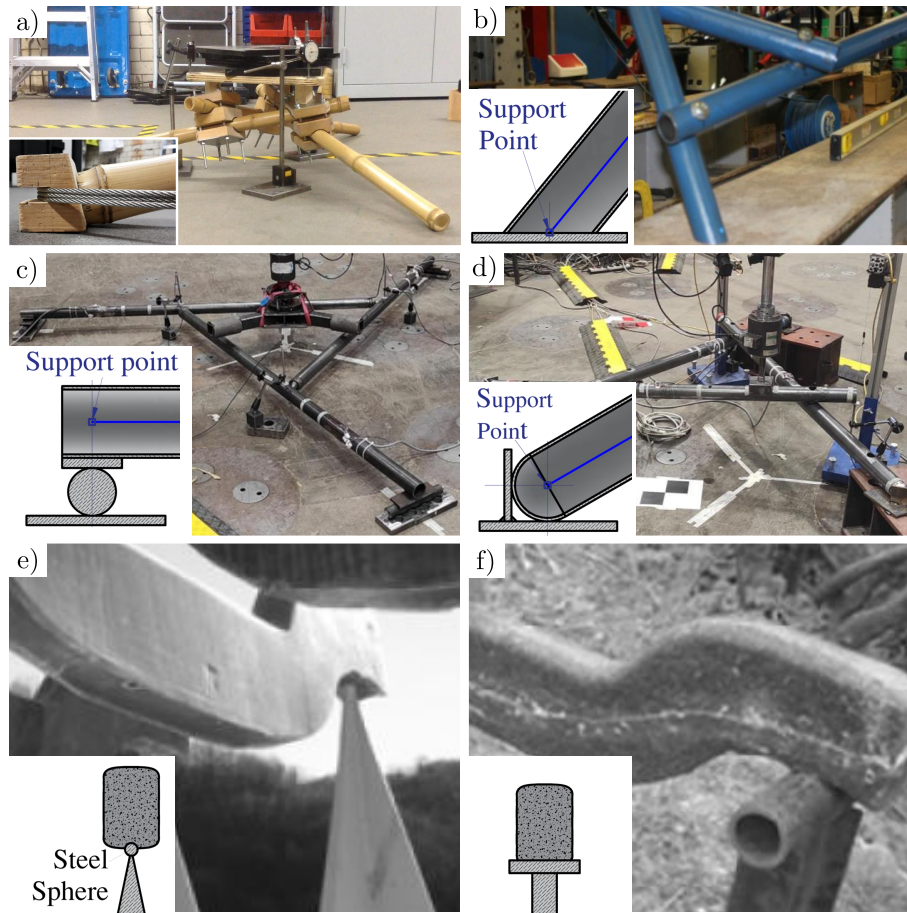


Figure 3.2: Various supports for experimental RS: a) Support for bamboo RS [66], b) Circular tube laid on a horizontal surface [63], c) Tube with welded plate, laid on support roll, d) Circular tube with a hemisphere welded at the end, e) Concrete beam on fixed sphere [55], and f) Concrete beam on tube [55].

research of the author proposes to weld a hemisphere at the tube's end, which fits into a corner bracket. This allows the support to be modelled as a hinge located along the tube axis, at the centre of the hemisphere. This solution is valid as long as there is no tension in the tube and horizontal thrusts are either permitted at the supports or absent.

- Figures 3.2e and 3.2f: For an RS^{2D} , built with fibre-reinforced concrete, Pizzigoni [55] propose two types of supports: the beams are placed on fixed metal spheres [Figure 3.2e] or metal tubes [Figure 3.2f]. In the first case, the support is modelled as a hinge. Only the rotational DoF about the tube axis is left free in the second case.

Figure 3.3 pertains to the supports of several existing timber structures:

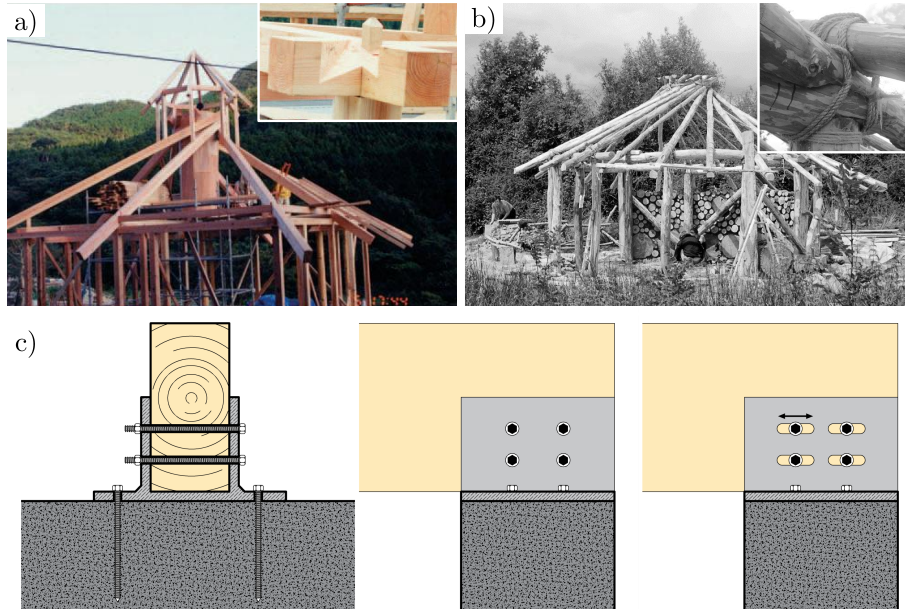


Figure 3.3: Different supports of constructed RS: a) wood-wood support for the roof of the "New Farmhouse" (Japan) [Figure 1.9b] [56], b) Log support with ropes [93], and c) Frans Masereel Pavilion support [Figure 1.10b].

- Figure 3.3a: The squared timber beam is inserted into a tenon and rests on a flat surface [56]. All three translational DoFs are fixed in this configuration. The rotation about the beam's axis is prevented by secondary beams resting atop the reciprocal structure.
- Figure 3.3b: Logs are secured at the supports using ropes, effectively fixing all three translational DoFs [93].
- Figure 3.3c: For the supports of the pavilion shown in Figure 1.10b, the timber beams are laterally restrained by four through-rod and a steel beam shoe anchored into the substructure. In this case, all three translational DoFs are fixed. However, for specific supports where horizontal reaction forces were deemed unacceptable for the substructure, the holes in the brackets were made oblong to release translation along the beam axis (T_x) [M. Guillaume, personal communication, April 2025].

Figure 3.4 illustrates other possible solutions proposed by the author, either for timber RS or for steel RS:

- Figure 3.4a: A variant of the solution in Figure 3.3c, applied to RS^{3D} structures in timber.
- Figure 3.4b: The solutions shown in Figures 3.4c and 3.4a also apply to round timber, provided the beam is locally machined into a rectangular section at the support area.

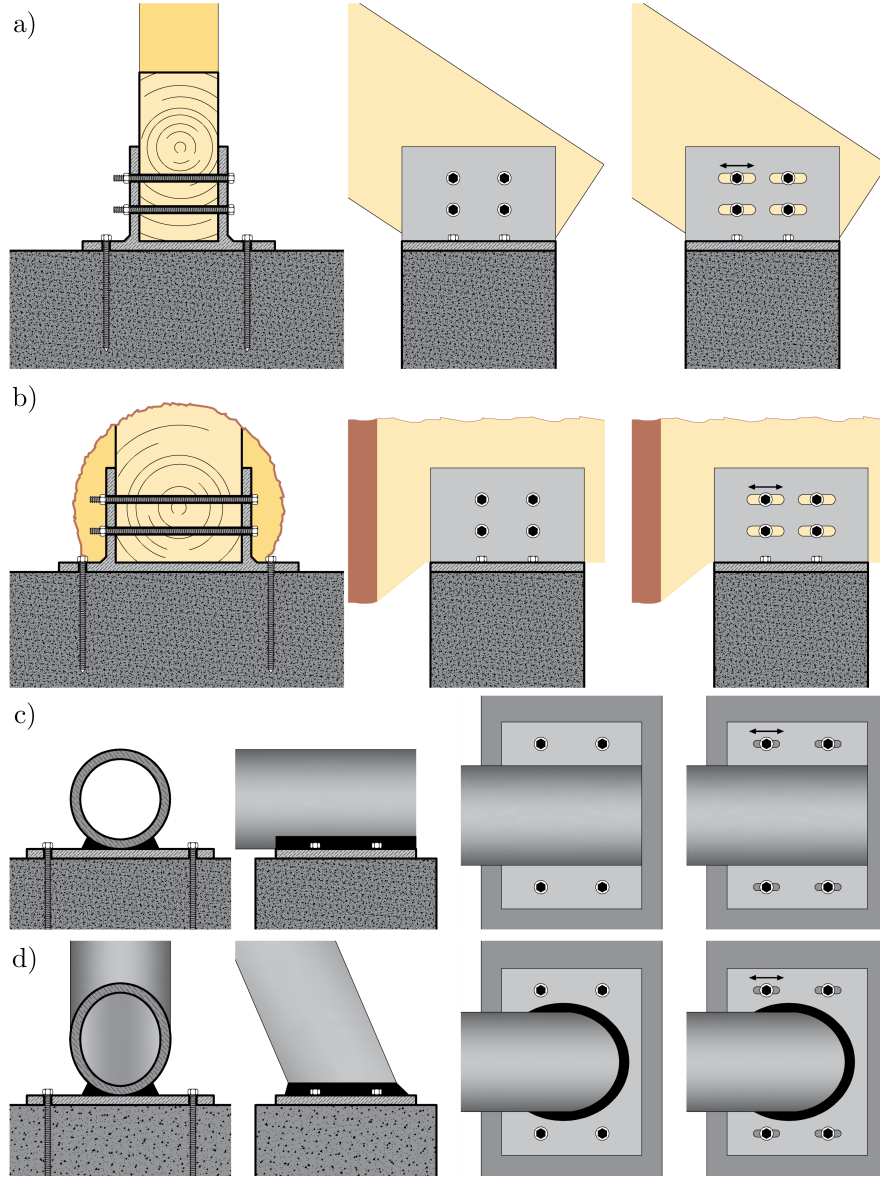


Figure 3.4: Various RS supports proposed by the author: a) Squared timber beams support for RS^{3D} with or without oblong holes, b) Round timber support, c) Support for RS^{2D} with welded tube to fixed plate, and d) Support for RS^{3D} with welded tube to fixed plate.

- Figures 3.4c and 3.4d: Supports for RS^{2D} and RS^{3D} using metal tubes. A metal plate is welded locally beneath the tube, fixing rotation (R_x) about the tube axis. The fixed DoFs are T_x , T_y , T_z , and R_x . Using oblong holes releases the translational DoF along the beam axis, thereby avoiding horizontal reaction forces.

3.2.2 Connections

3.2.2.1 Connections for timber RS

The connections must account for the variable mechanical properties of timber that depend on fibre orientation, dimensional changes due to moisture, and, in the case of logs, potentially irregular geometry. A distinction is made between wood-wood connections [Figure 3.5] and connections incorporating fastening elements [Figure 3.6].

A wood-wood connection is achieved by cutting the supporting and/or supported beams locally to create contact surfaces. These cuts may be carried out manually [Figure 3.5a] or with various tools, such as a robotic arm equipped with a cutting tool [Figure 3.5b] [26, 38, 87].

Gustafsson [29] modelled and experimentally tested a $5\text{ m} \times 5\text{ m}$ RS^{2D} composed of squared timber with notched half-lap connections to ensure that the beam axes remain in the same plane [Figure 3.5c]. At the boundaries, the beam ends rest on trestles and are modelled as hinged supports. The connection is modelled with six semi-rigid DoFs, with stiffness values determined using the method proposed by Descamps and Guerlement [16]: for each movement (translation or rotation) of the supported beam, the pressure zones generated on the contact surfaces are converted into equivalent stiffness values based on the mechanical properties of the timber, neglecting friction. For small loads, the measured displacement was approximately 3 times the calculated value, and the failure load was nearly 30% lower than the estimated value, indicating a brittle failure mode. These results highlight the difficulty of accurately modelling such connections based solely on contact surfaces. Despite this modelling difficulty, the connection enables rapid assembly and disassembly, which facilitates structural reuse. This type of connection can also generate RS^{3D} structures if the notches do not precisely halve the beam height, thus creating the necessary eccentricity, as proposed by Anastas et al. [1].

The strength of wood-wood connections improves when multiple faces are in contact; however, this increases uncertainty in stress transfer and modelling. In addition, it is difficult to predict which faces will be in contact due to machining tolerances, hygrometric variations, applied loads, and global structural deformation, all of which may vary [26]. To limit these uncertainties, the connection geometry, shown in Figure 3.5a and described by Geno et al. [26], ensures that the contact planes are maintained despite dimensional variations, as the central area is not a contact surface [Figure 3.5a]. The MM RS^{3D} structure, made of logs, presented in Figure 1.6b, was constructed based on this type of connection [26]. Its design was based on an idealised connection model, in which the fixed DoFs are the three translational movements (locked only in compression) and rotation around the eccentricity axis. This rotation

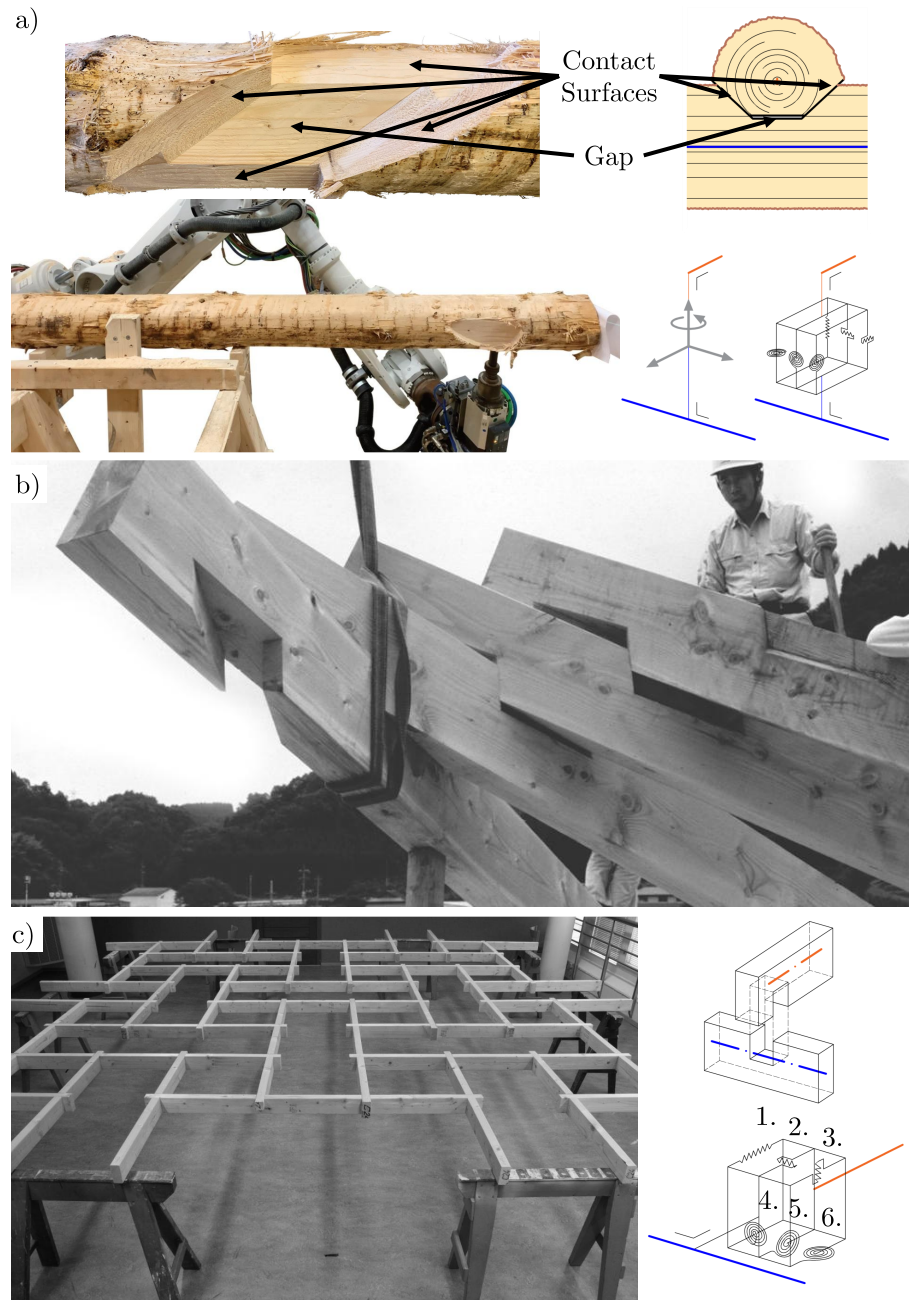


Figure 3.5: Wood-wood connection: a) Log-log connection [26], assembly diagram and models, b) Bunraka Theatre (Japan) connection [Figure 1.9c] [56], and c) RS^{2D} half-lap connection tested by [29], assembly diagram and modelling with an element of 6 semi-rigid DoF with uncoupled linear stiffness: 1. Longitudinal, 2. Transversal, 3. Lateral, 4. Torsional, 5. In transverse bending and 6. In lateral bending.

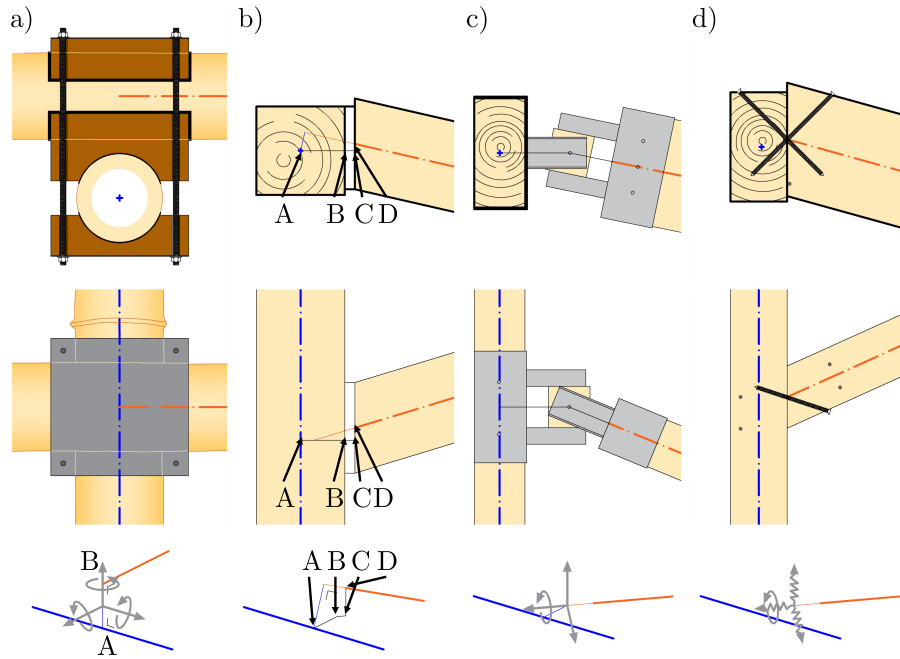


Figure 3.6: Timber Connection examples with section, top views, and modelling: a) Connection between two bamboo canes, b) Lateral glued connection, c) Lateral U-joint, and d) Masureel Centre screwed connection [Figure 1.10b].

mobilises the four oblique contact surfaces in compression. This modelling choice aimed to ensure safety rather than accurately represent the structural behaviour.

Vestartas et al. [87] present an RS^{3D} composed of logs assembled with a cut similar to that shown in Figure 3.5a, modelled by an element with six semi-rigid DoFs connecting the beam axes. The stiffness values are determined experimentally, as proposed by these authors for a wood-wood connection between engineered timber boards [46]. The numerical results are promising but have yet to be experimentally validated.

Many other types of joints between timber beams exist, using screws, rods, connecting metal pieces, rope, adhesive, and so on [8, 15, 29, 34, 51, 66]. Among these solutions, four are schematically represented in Figure 3.6:

- Figure 3.6a: Godina Rodríguez [66] developed a connection between two bamboo canes using three timber blocks, CNC (Computer Numerical Control) cut to match the curvature of the canes and ensure their respective orientations. These blocks are rigidly fixed using four steel rods. Steel plates are added on either side of the assembly to optimise the transfer of forces between the rods and the timber blocks. A neoprene membrane is placed at the bamboo-timber interface to distribute compressive stresses. Despite the neoprene, this connector is

modelled as a rigid arm AB. The displacements provided by the numerical model are consistent with experimental measurements on an SM RS^{3D} with three canes for the two types of supports presented in Figure 3.2a. However, for an MM RS^{3D} structure composed of two single modules of three beams, geometrically similar to the RS shown in Figure 1.4b, the numerical model yields stiffer results than observed experimentally. These authors also highlight that the challenges of these tests are causing imprecision: unknown precise mechanical properties of the bamboo and inaccuracies between the tested and numerical geometries.

- Figure 3.6b: Kohlhammer et al. [34] propose a model for a connection between two squared beams glued laterally. To represent the local semi-rigid behaviour of the bonded area, the glued assembly is modelled with three small beams, AB, BC, and CD, rigidly connected, but accounting for the glued areas and the timber properties perpendicular to the grain for beams AB and CD, and for the adhesive properties in beam BC (whose length is exaggerated in the diagram). This modelling captures the semi-rigid behaviour of the connection but has not been experimentally validated.
- Figure 3.6c: Corio et al. [15] present a connection between two squared timber beams with a U-joint for RS^{3D} structures, although it has not been experimentally verified. This type of joint allows two rotational DoFs at the centre of the timber block. Only the rotation around the eccentricity axis is fixed. However, this connection would fail under excessive torsion from the supported beam, generated, for example, by eccentric loads.
- Figure 3.6d: For the Masereel Centre in Belgium [Figure 1.10b], the chosen connection consists of five screws and is modelled with the three translational DoFs considered semi-rigid. As the beams are subsequently covered with timber panels, the rotational DoF around the axis of the carried beam is assumed to be fixed. Some highly loaded connections are reinforced with steel plates or glued dowels to increase their strength (M. Guillaume, personal communication, April 2025).

3.2.2.2 Connection for concrete RS

To the author's knowledge, there is only one reference concerning reciprocal structures made of concrete: an RS^{2D} prototype [Figure 3.7] [55]. These fibre-reinforced concrete prototypes were developed to assess the feasibility of demountable, reusable reciprocal structures. The beams are connected using a steel sphere at the interface. The sphere maintains the same contact point even after beam rotation due to bending. Numerical results from a solid finite element model show good agreement with experimental measurements

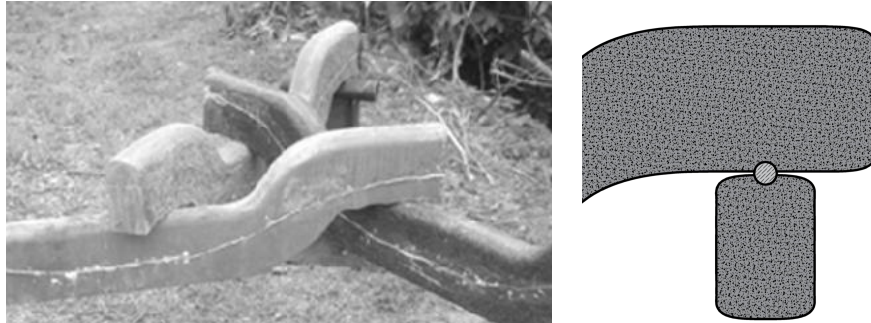


Figure 3.7: *SM RS^{2D} with three fibre concrete beams with connection detailing [22].*

under static loading conditions [22]. The supports are shown in Figures 3.2e and 3.2f. The connection can be modelled as a hinge in a beam-based model, provided the loading does not cause separation between the beams.

3.2.2.3 Connections for metal RS

Three examples of built metal structures with a reciprocal geometry are shown in Figure 1.11b, 1.11c and 1.11d, although such structures remain extremely rare. Due to its ease of welding, steel is generally preferred for gridshells. A few prototypes of RS^{3D} in metal have been tested in laboratories and are typically made of circular tubes, as their constant eccentricity, regardless of orientation, simplifies connections. Figure 3.8 presents examples of connection solutions discussed in the literature or proposed by the author:

- Figure 3.8a: Welded connection [57]: This solution is used to join the tubes of the main structure of the Mount Rokko-Shidare Observatory, shown in Figure 1.11d. It can be modelled with all six DoFs fixed by the welds. However, this connection complicates the construction process: the welds on curved surfaces are challenging to execute, and each tube must be accurately positioned to maintain the final geometry [28]. Moreover, the welds create rigid connections between all members, raising questions about whether the structure remains truly reciprocal or belongs to the category of "gridshell" structures.
- Figure 3.8b: Rigid bolted connection [60]: This connection is designed to ensure a rigid joint between tubes, irrespective of their relative orientation. However, its bulk impacts both aesthetics and architectural expression. For modelling purposes, a rigid arm AB is fixed at point A, with only five DoFs fixed at point B. The remaining DoF is the rotation about the axis perpendicular to the plane containing the axis of the carried tube and the line AB, which is semi-rigid. The connection has been verified through testing on an SM RS^{3D} comprising six beams with support ends welded to a rigid frame, showing a discrepancy of less than

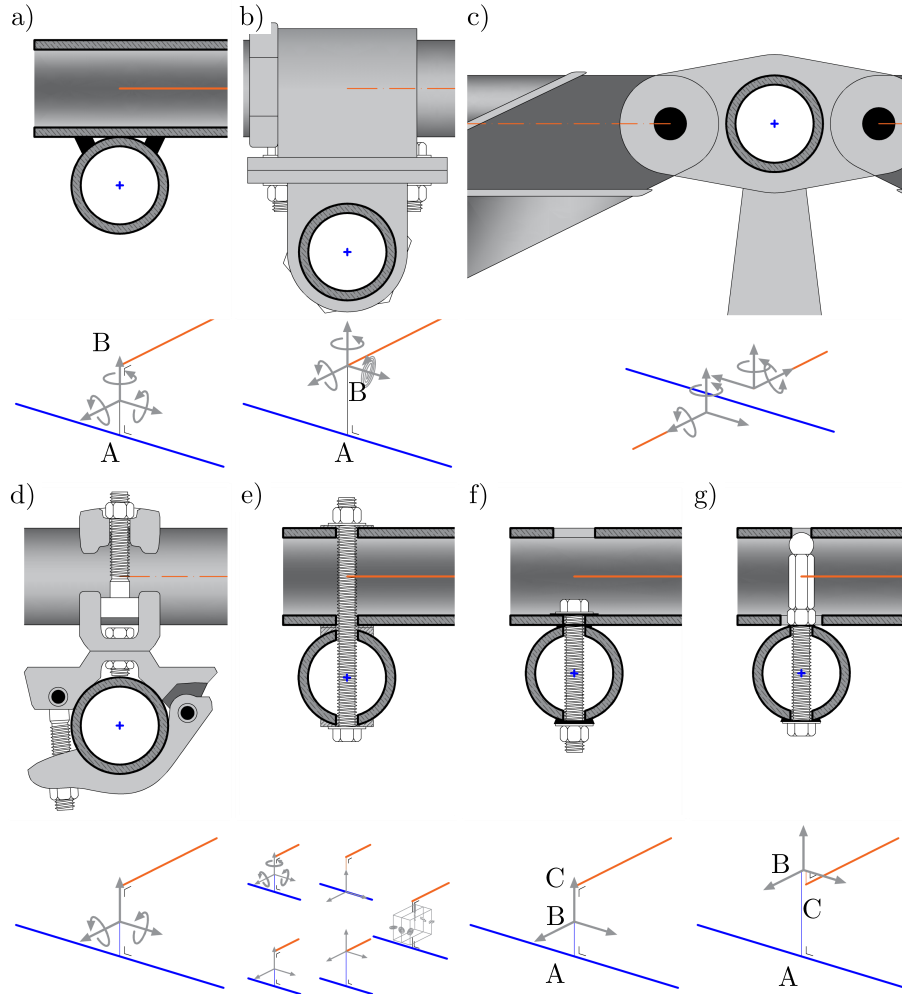


Figure 3.8: a) Welded connection. b) Rigid bolted connection. c) Bolted pinned connection. d) Scaffolding swivel connector. e) Fully through-bolt connection. f) Partially through-bolt connection. g) Ball-based connection.

10% between numerical simulations and experimental measurements.

- Figure 3.8c: Bolted pinned connection [25]: The connection used for the RS^{2D} presented in Figure 1.11c is realised with a single bolt that allows rotation about its axis, while the five other DoFs are considered fixed. These authors point out that this type of connection provides a structural behaviour equivalent to a hinge, under vertical loading and at first order.
- Figure 3.8d: Scaffolding swivel connector [70]: This simple solution allows rapid and economical construction of an RS^{3D} . In terms of modelling, the eccentricity can be represented by a rigid arm that allows

free rotation about its pivot. Nevertheless, drawbacks include poor aesthetics and the fact that rotation of the members about their axes is prevented only by friction during tightening, making it challenging to model accurately.

- Figure 3.8e: Fully through-bolt connection [65]: A bolt passes entirely through both tubes, with saddle washers at the contact interfaces to better distribute stresses between the tubes. Rizzuto and Larsen [65] model this connection in four ways: in the first, all six DoFs are restrained; in the others, a hinge is introduced at one end of the eccentricity or at its midpoint. These authors observe that the first modelling approach correlates better with experimental results than the others. Subsequently, Rizzuto [63] used the same connection for a larger RS^{3D}, modelling the connection with six semi-rigid DoFs, as shown in Figure 3.5a. The stiffnesses were determined from a local model using 2D shell elements to model the tubes and the bolt. However, experimental validation was not possible due to horizontal movement of the supports [Figure 3.2b], which altered the tested geometry and disrupted displacement measurements.
- Figure 3.8f: Partially through-bolt connection: This alternative to the previous solution aims to simplify the connection behaviour and reduce the number of fixed DoFs. The connection can be modelled by a hinge positioned at the tube interface (point B), linked to the tube axes by rigid arms (AB and BC). The modelling results and experimental displacement measurements (for a simple RS^{3D} with three beams, as shown in Figure 3.2d) differ by an average of 5.5%. However, stress measurements show a mean discrepancy of 15.3%, as the rotational DoFs between the tubes are slightly semi-rigid due to contact between the bolt head and the supported tube at point B.
- Figure 3.8g: Ball-based connection: An alternative solution to release rotational DoFs completely is by placing a ball inside the supported tube, allowing free rotation relative to the ball's centre, with an opening made in the carried tube at the midpoint of the eccentricity. The model then consists of a perfect hinge at the ball's centre, connected to the tube axes via a short beam AB, which accounts for the fastener properties, and a rigid arm BC. Modelling results show displacements and stress magnitudes within 5% of experimental measurements on a simple RS^{3D} geometrically identical to that shown in Figure 3.2d. However, this assembly only functions if the ball remains in contact with the supported tube, thus limiting its applicability.

3.2.3 Discussion on semi-rigid DoF in connections and supports

As explained in this section, the supports and connections often exhibit specific semi-rigid degrees of freedom (DoFs) due to friction, clearances, machining tolerances, lever arms between bolts, and other factors. Such semi-rigidities are often unavoidable; however, the author recommends minimising them as far as possible for three main reasons:

- They affect the transmission of forces between beams and may cause localised plastic deformations or failures that are difficult to predict.
- In principle, they should be modelled as semi-rigid DoFs, but their behaviour is often non-linear and extremely difficult to evaluate. Nonetheless, neglecting them in numerical models leads to overestimating deflections and internal forces within the beams, which is safe at the structural level, though not necessarily at the local level.
- Finally, the greater the number of restrained DoFs in the connections, the more the structure departs from an actual reciprocal structure and approaches the behaviour of a gridshell. Although gridshells have advantages, this shift results in the loss of the distinctive architectural character of reciprocal structures and of the potential for modular construction, in which elements can be stacked to span large distances using small beams.

Nevertheless, the fewer the fixed DoFs, the less assured the local or global stability is. Consequently, the following sections of this chapter are dedicated to methods for verifying local and global stability.

3.3 Existing approaches for RS geometric stability assessment

This section presents and compares the effectiveness of five methods for verifying RS stability. Geometric instability must be distinguished from elastic instability:

- Geometric instability is a mechanism with one or more "rigid movements". Rigid movement refers to a beam's motion that is independent of any elastic deformation caused by internal forces. Geometrically unstable structures are called "kinematically indeterminate".
- Elastic instability occurs when excessive force creates the buckling of either a single beam or the whole structure. It depends on the structure's geometry, the mechanical and cross-sectional properties of the beams, and the loading.

This chapter focuses solely on geometric instability and compares four methods, namely those developed by Gelez et al. [25], Brocato [9], Xia et al. [94], and Wackerfuß [89]. In this chapter, they will be referred to as follows:

- Method 1: "Local Stability Method" (LSM).
- Method 2: "Kinematic Analysis Method" (KAM).

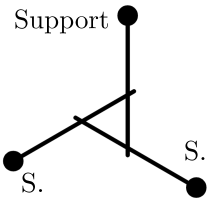
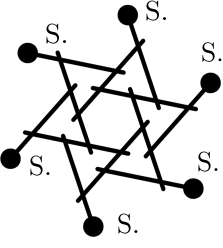
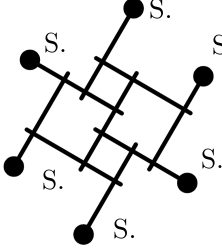
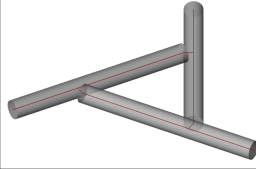
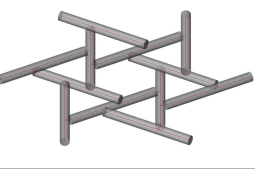
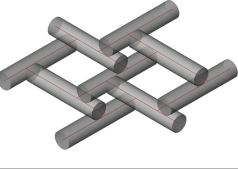
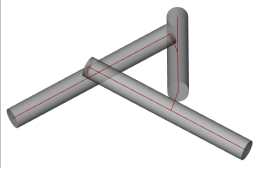
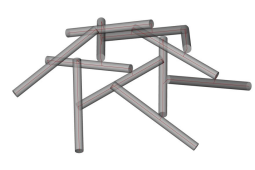
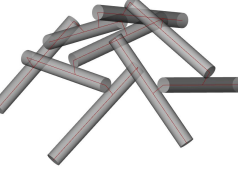
	Triangular (SM)	Hexa- Triangular (MM)	Quad- quadrangular (MM)
Scheme			
RS^{2D}			
RS^{3D}			

Figure 3.9: Varying RS^{2D} and RS^{3D} with their pattern and diagram. Supports (S.) are indicated with circles.

- Method 3: "Kinematic Compatibility Matrix Analysis Method" (KC-MAM).
- Method 4: "Constraint Matrix Analysis Method" (CMAM).

In addition, a "Method 0", the well-known finite element method, will serve as a benchmark to validate the results of the four previous methods, as it accounts for the mechanical and cross-sectional properties of the beams and therefore distinguishes itself from the other four methods.

These methods are compared based on the patterns illustrated in Figure 3.9. Unless stated otherwise, the modelling assumptions for the supports and connections are based on those presented in Figure 3.1. Note that the presented methods suppose that the connections and supports can withstand tension and compression. However, when RS are modelled as simply stacked elements, the connections are generally assumed to remain in compression. The validity of this assumption must be verified during the structural design check.

3.3.1 Method 0: Stiffness Matrix Analysis Method (SMAM)

"Method 0" determines the stability of a structure by analysing the eigenvalues λ_i of its stiffness matrix $\mathbf{K}$. These eigenvalues are computed by solving $\mathbf{K}\mathbf{v} = \lambda_i\mathbf{v}$, where $\mathbf{v}$ is an eigenvector of the stiffness matrix. A null value of at least one eigenvalue implies a mechanism, which is described by its corresponding eigenvector [33]. This analysis, referred to as the "Stiffness Matrix Analysis Method" (SMAM), applies to both RS^{2D} and RS^{3D}, regardless of the fixed Degrees of Freedom (DoFs) at the supports and connections. However, this analysis is computationally expensive and requires defining the beam's material and cross-sectional properties.

3.3.2 Method 1: Local Stability Method (LSM)

The "Local Stability Method" consists of analysing the global stability of an RS^{2D} or RS^{3D} based on the local stability of each beam. The method is founded on the principle that if the six possible movements of each beam are restrained, considering only the fixed DoFs at its ends (at connections and/or supports), then the structure is deemed stable [25]. This method, for example, confirms the stability of the RS^{2D} presented in Figure 3.10a [25]. Indeed, each beam is locally stable when considering only the fixed DoFs at its end: on one side, three translational DoFs (T_x , T_y , and T_z) are fixed, and on the other side, two translational DoFs (T_y and T_z) and one rotational DoF (R_x) are fixed. Applying the same principle to the RSs shown in Figures 3.10b and 3.10c would lead to the conclusion that both structures are unstable, which is incorrect in the case of Figure 3.10c. Although each beam has three fixed translational DoFs at each end, they can still rotate around their longitudinal axis (R_x). While the structure in Figure 3.10b is indeed unstable, the RS^{3D} of Figure 3.10c is, in fact, stable, as can be demonstrated using other methods discussed in this chapter. This discrepancy is explained by the fact that the LSM neglects the stabilisation provided by the supported beams.

The drawback of this method is that it may classify specific structures as unstable when, in fact, they are stable. In other words, this method can guarantee that an RS is stable but not that it is unstable. It has the advantage of being simple, quick to implement, and conservative, but it may unnecessarily limit the designer's choice of structural solutions.

3.3.3 Method 2: Kinematic Analysis Method (KAM)

Using a vector analysis, this method enables the determination of rigid-body movements between beams in an RS^{3D} model, considering only the fixed DoFs at the connections. Based on the number of these rigid movements, it is possible to determine the minimum number of DoFs that must be fixed at the supports to stabilize the RS. The connection between beams is assumed to be

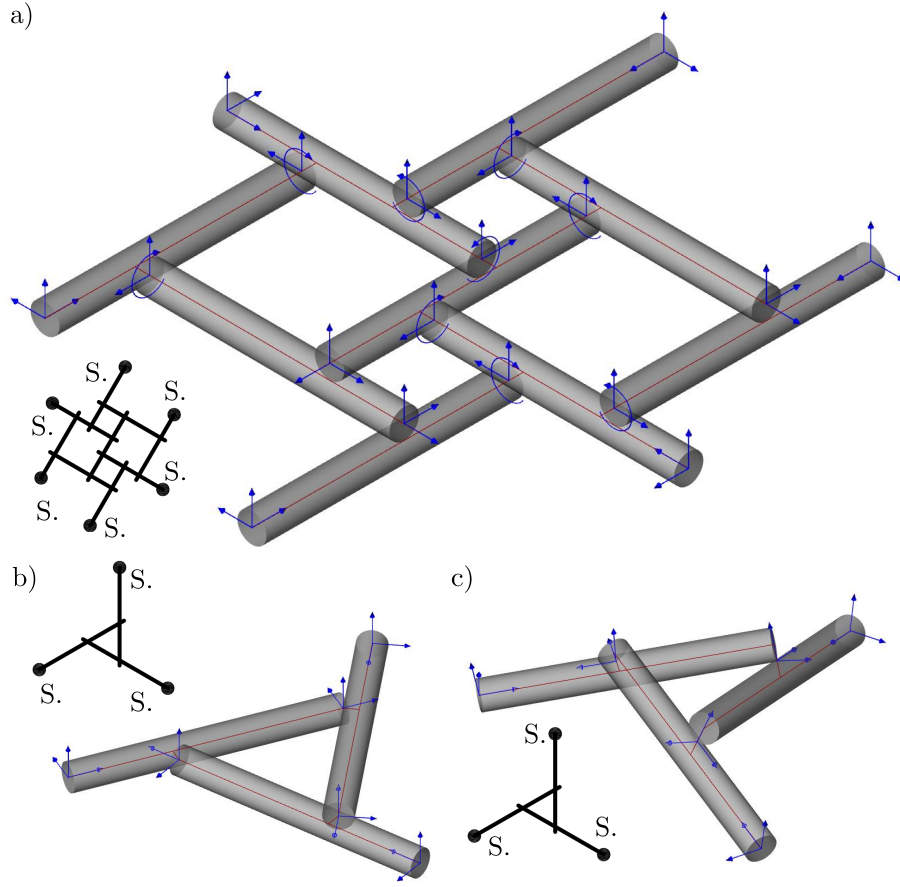


Figure 3.10: RS with their diagram and a perspective view with arrows indicating the fixed DoFs at connections and supports: a) stable RS^{2D} , b) unstable RS^{2D} , and c) stable RS^{3D} .

identical throughout and is based on the following assumptions [Figure 3.11a] [9]:

- It is a variant of Figure 3.1c, in which the supporting beam is assumed to have no thickness (the eccentricity AB is fully embedded in the supported beam).
- In addition to fixing the three translational DoFs, the connection also prevents the rotation about the y-axis R_y , passing through the connection point A [Figure 3.11a]. Thus, each connection allows two relative rotations between two beams: one about the eccentricity axis AB and the other about the axis of the supporting beam.

Based on these assumptions and without considering the fixed DoFs at the supports, Brocato [9] states that an MM RS^{3D} composed of n_{int} connections and f single modules exhibits $2n_{int} - 5f$ rigid body movements of beams. To

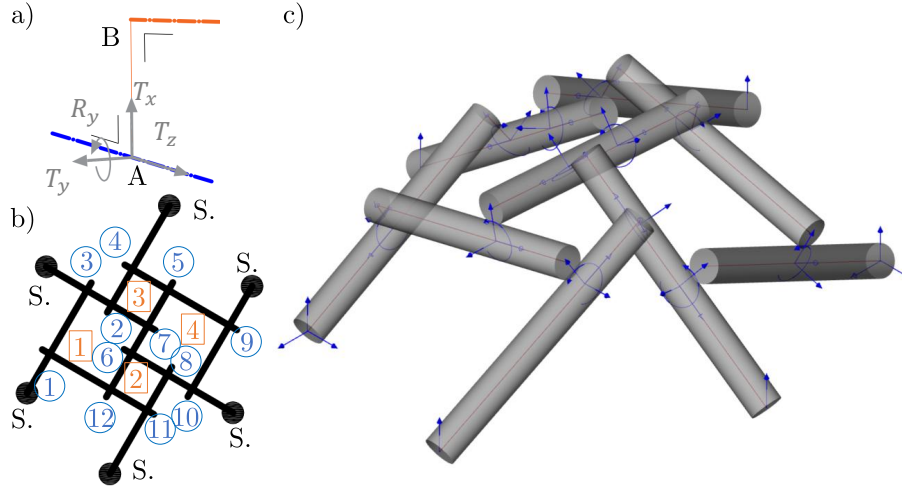


Figure 3.11: a) RS^{3D} connection type considered by this method. b) RS^{3D} with $n_{int} = 12$ connections (circled numbers) and $f = 4$ single modules (framed numbers). c) Isometric view of the RS with displayed fixed DoF at connections and supports.

guarantee the global stability of the RS, this author concludes that the number of fixed DoFs that must be provided at the supports should be greater than or equal to $2n_{int} - 5f + 6$.

Brocato [9] illustrates this formula with the MM RS^{3D} shown in Figure 3.11b ($n_{int} = 12$, $f = 4$), for which it is necessary to provide $2n_{int} - 5f + 6 = 2 * 12 - 5 * 4 + 6 = 10$ support reactions distributed among the six support points. This author suggests, for example, using two hinged supports and four roller supports, blocking only the vertical direction.

This method allows for a rapid and straightforward determination of the number of DoFs that must be fixed at the supports to ensure the stability of an RS. However, it is limited to a unique type of connection, assuming a null thickness for the supporting beam. This author has not extended the method to RS^{2D} , for which different connection assumptions would be required.

3.3.4 Method 3: Kinematic Compatibility Matrix Analysis Method (KCMAM)

The Kinematic Compatibility Matrix Analysis Method was developed by Parigi et al. [52] for RS^{1D} structures with hinged connections. An RS^{1D} is a special case of RS^{2D} where both the external loading and the structure are defined within the same plane. Subsequently, Xia et al. [94] extended it to RS^{2D} and RS^{3D} structures.

Parigi et al.'s method is illustrated for an RS^{1D} composed of three beams in Figure 3.12a, each possessing three rigid-body displacements (U , V , and ϕ) indicated at its centre of gravity. Figure 3.12b shows the fixed movements at each beam's support and connection nodes. These fixed movements at

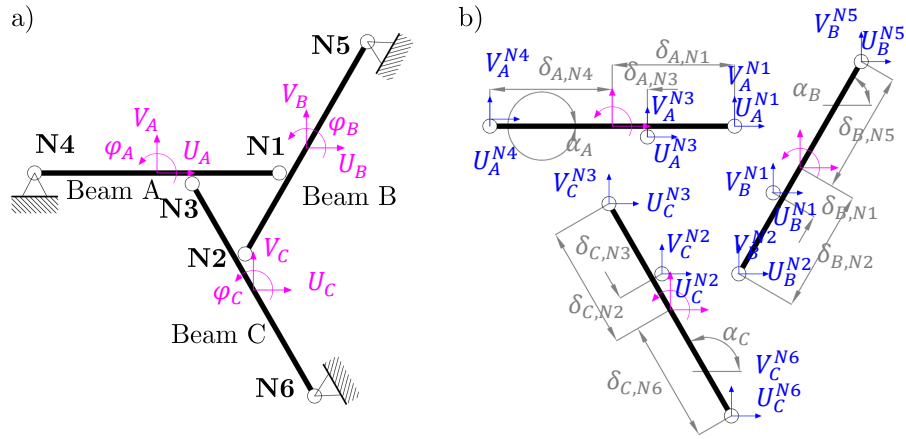


Figure 3.12: a) RS^{1D} with three beams (A, B and C), each having three rigid body movements (U , V , and ϕ). b) RS decomposed to express the bonded movements fixed at the supports and between the beams.

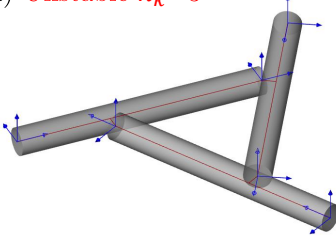
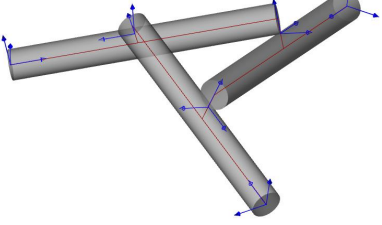
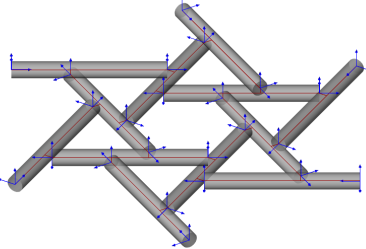
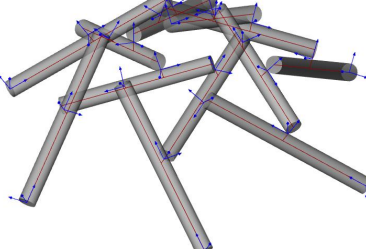
	RS^{2D}_{Hinged}	RS^{3D}_{Hinged}
 Triangular	a) Unstable $n_k=3$ 	b) Stable 
 Hexa— Triangular	c) Unstable $n_k=12$ 	d) Stable 

Figure 3.13: RS with triangular and hexa-triangular patterns, hinged supports, and connections.

the supports and connections can be expressed as kinematic compatibility equations between the restrained movements:

- At the supports: $U_A^{N4} = V_A^{N4} = U_B^{N5} = V_B^{N5} = U_C^{N6} = V_C^{N5} = 0$
- At the connections: $U_A^{N1} = U_B^{N1}, V_A^{N1} = V_B^{N1}, U_B^{N2} = U_C^{N2}, V_B^{N2} = V_C^{N2}, U_A^{N3} = U_C^{N3}$ and $V_A^{N3} = V_C^{N3}$.

These node movements are then expressed in terms of the rigid-body motions of the beams. For example, at connection node N1, $U_A^{N1} = U_B^{N1}$ becomes $U_A - \delta_B^{N1} \sin(\alpha_A) \varphi_A = U_B - \delta_B^{N1} \sin(\alpha_B) \varphi_B$. Finally, the system can be written in the following matrix form: $\mathbf{B}\mathbf{d} = \mathbf{0}$ where $\mathbf{B}$ is the kinematic compatibility matrix and $\mathbf{d}$ is the vector of rigid body movements of the beams.

For a structure with d_{tot} beams, if r is the rank of matrix $\mathbf{B}$, the number of mechanisms is given by $n_k = 3d_{tot} - r$, and the structure is thus unstable if $n_k > 0$. The RS shown in Figure 3.12 presents no mechanisms ($n_k \leq 0$).

Xia et al. [94] extended the method to three-dimensional space, applying it to RS^{2D} and RS^{3D} structures with fully hinged supports and connections (RS_{HINGED}), respectively located at the end of the centroidal axis and at the half eccentricity, as shown in Figure 3.1. In three dimensions, the number of mechanisms is $n_k = 6d_{tot} - r$, accounting for the three additional rigid-body displacements.

These authors analysed the stability of RS_{HINGED}^{2D} and RS_{HINGED}^{3D} structures, with triangular and hexa-triangular patterns [Figure 3.13]. The two RS^{2D} structures were declared unstable with, respectively, 3 and 12 instability modes. In contrast, the RS^{3D} structures were found to be stable. Despite these interesting results, the method developed by Xia et al. [94] applies only to RS with fully hinged connections and supports. However, accounting for fixed rotational DoF appears possible, but was not developed by the authors.

3.3.5 Method 4: Constraint Matrix Analysis Method (CMAM)

The Constraint Matrix Analysis Method was developed by Wackerfuß [88, 89] in a general context, without being explicitly applied to RS. This section summarises the method and examines how it can be advantageously applied to RS. The method is illustrated in Figure 3.14 using an unstable plane structure composed of two bars connected by a hinge and supported on two pinned supports. The bars connect nodes N1 to N2 and N3 to N4. A fictitious element of zero length represents the hinge, linking nodes N2 and N3, which are in fact coincident. Each node has two translational DoFs expressed in the (x, y) coordinate system.

Three types of constraints are considered, which can be expressed in matrix form in Equation 3.1:

- Invariability of bar length : $\Delta L^{N1,N2} = U_1 - U_3 = 0$ et $\Delta L^{N3,N4} = U_5 - U_7 = 0$ (rows 1 and 2).

- Fixed displacements at the supports: $U_1 = U_2 = U_7 = U_8 = 0$ (rows 3 to 6). Connection conditions: $U_3 = U_5$ and $U_4 = U_6$ (rows 7 and 8).

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.1)$$

The matrix is then reduced to triangular form by Gaussian Elimination to determine the number of non-zero rows, corresponding to the matrix rank r . It should be noted that this operation may rearrange the matrix rows and change the order of DoFs in the vector in Equation 3.2:

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_5 \\ U_3 \\ U_7 \\ U_2 \\ U_8 \\ U_4 \\ U_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.2)$$

Based on the matrix rank r (number of non-zero rows, here $r = 7$) and the total number p of DoFs (number of columns, here $p = 8$), the number of mechanisms can be calculated as $n_k = p - r$, which is 1 in the case of the structure in Figure 3.14. The corresponding instability mode, shown in orange in Figure 3.14, can be obtained as follows:

1. Complete the matrix by inserting a 1 in the diagonal element of the zero row (in this case, imposing $U_6 = 1$). If $n_k > 1$, a 1 is successively placed in each zero row's diagonal elements to compute each instability mode.
2. Solve the system to compute the other displacements, which provides a displacement vector representing the instability mode.

The general matrix form can be written as: $\mathbf{C}_{m \times p} \mathbf{u}_p = \mathbf{0}_m$ where the constraint matrix $\mathbf{C}$ contains the m constraints (rows), and $\mathbf{u}$ is the vector of p DoFs (columns). The matrix $\mathbf{C}$ is thus not necessarily square, and it is a necessary (but not sufficient) condition for stability that $m \geq n$.

To generalise this method to RS^{2D} and RS^{3D}, certain aspects illustrated in Figure 3.15 must be specified:

- The method defines two types of rigid elements: Wire and Meshed. A Wire Element is modelled as a bar connecting two nodes, as in the example of Figure 3.14. A Meshed Element is modelled as a rigid polygon

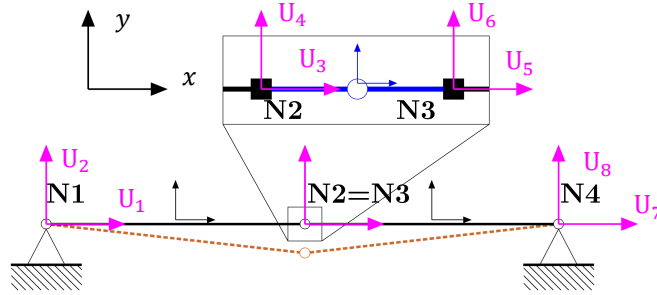


Figure 3.14: Illustration of the Constraint Matrix Analysis Method with an example of a two-bar unstable structure.

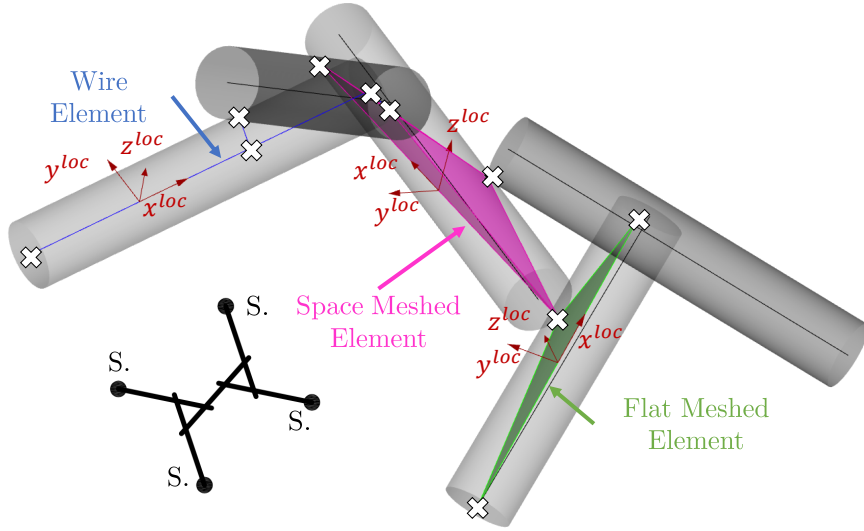


Figure 3.15: Modelling of RS beams using Wire, Flat Meshed, and Space Meshed Elements, with their respective local coordinate system.

decomposed into triangles (Flat Meshed Element) for planar cases, or as a rigid polyhedron decomposed into tetrahedra (Space Meshed Element) for spatial cases.

- Structural elements, supports, and joints each have their local reference frame. It is therefore necessary to transform constraints from the local to the global coordinate system, in which DoFs are expressed. Figure 3.15 shows the local reference systems for different structural elements. The local frame of a Wire Element is defined by its axis (x^{loc}) and another chosen direction. The local frame of a Meshed Element is defined by three nodes forming a triangle.
- The spatial movement of Meshed Elements can be fully defined by the three translational DoFs of the nodes forming the element. In con-

trast, for Wire Elements, an additional rotational DoF about x^{loc} must be added to account for the possibility of axial rotation.

Specifically, for RS, their beams can be modelled as Wire Elements representing their median fibres, but this also implies modelling many Wire Elements to represent the eccentricities between the beam axes [Figure 3.15]. This drawback, combined with the need to introduce a rotational DoF per Wire Element, suggests that using Flat Meshed Elements for RS^{2D} and Flat or Space Meshed Elements for RS^{3D}, depending on the beam type [Figure 1.5], is a more suitable approach.

3.3.5.1 Comparison of methods

The four methods previously presented, along with the "Method 0: Stiffness Matrix Analysis Method", were tested on three RS^{2D} and three RS^{3D} structures, labelled a, b, c, d, e, and f, as illustrated in Figure 3.16. As a reminder, the "hinged" notation refers to structures where all supports and joints are hinged (i.e. R_x , R_y , and R_z are free).

Table 3.1 summarises the results obtained and provides the size of the analysed matrices:

- The SMAM applies to all RS configurations but requires a significantly larger matrix analysis than the other matrix-based methods. It also requires defining the material and cross-section properties of the beams. To ensure that the stiffness matrix is minimally affected by these modelling choices, it is necessary to impose artificially high stiffness properties to isolate instability modes related solely to the geometry and configuration of blocked or free DoFs.
- The LSM is a universal method that guarantees an RS's stability. However, it classifies some structures as unstable, which are, in fact, stable; thereby, it considerably restricts the designer's freedom. Although all three RS^{3D} structures analysed are labelled as unstable by the LSM, they are stable.
- The KAM and KCMAM methods provide a correct assessment of RS stability but are not universally applicable in their current developments: they are limited to specific configurations of fixed DoFs at supports or joints and/or to certain RS types.
- For RS_{HINGED}, the KCMAM and the CMAM (with Meshed Elements) produce matrices of identical dimensions. The CMAM correctly determines the stability of RS structures and is the only method, based on the assumption of infinitely rigid beams, that can be applied unconditionally to all RS^{2D} and RS^{3D} types, regardless of the assumptions made regarding supports and connections.

This comparison demonstrates that the CMAM, although not initially developed for RS applications, is the only method, under the assumption of rigid

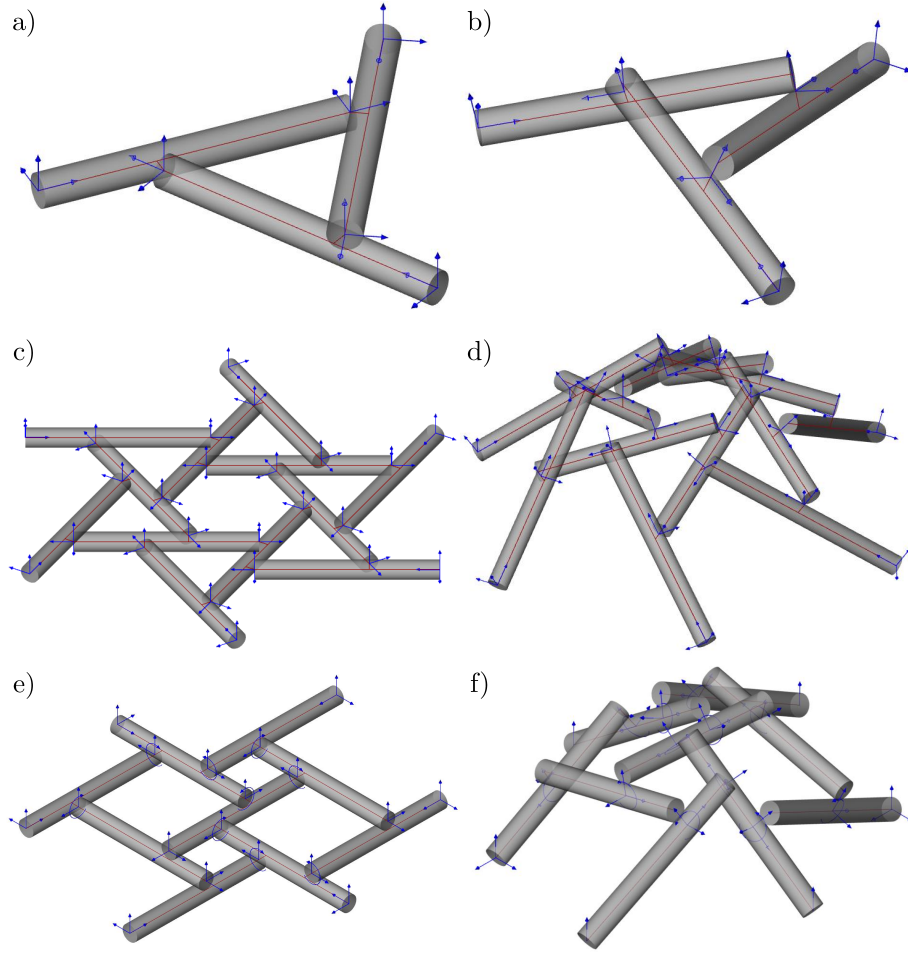


Figure 3.16: Geometry of the six RS studied in Table 3.1.

beams, that allows for stability verification across all RS types, independently of the hypotheses made on supports and connections. It also enables the identification of potential instability modes and their corresponding geometric configurations. However, by adding the possibility of fixing rotational DoF in the KCMAM, the latter would be as applicable and valuable as the CMAM.

3.4 Discussion

3.4.1 Special case of RS_{HINGED}

Previously, the semi-rigid DoFs that can occur in particular supports and connections were discussed, and the author recommends limiting them as much as possible through sound, thoughtful design. If all DoFs are rigidly fixed (e.g., by welding the steel beams together), the structure no longer strictly qualifies as an RS but becomes a gridshell. While gridshells are of interest,

Struct.		RS ^{2D}			RS ^{3D}		
N°		a	b	c	d	e	f
Patt.							
Stable?		No	No	Yes	Yes	Yes	Yes
Hinged?		Yes	Yes	No	Yes	Yes	No
Methods	0) SMAM	$n_k = 3$ [54×54]	$n_k = 12$ [252×252]	Stable [180×180]	Stable [72×72]	Stable [360×360]	Stable [240×240]
	1) LSM	Unstable	Unstable	Stable*	Unstable	Unstable	Unstable
	2) KAM	ND	ND	ND	ND	ND	Stable*
	3) KCMAM	$n_k = 3^*$ [18×18]	$n_k = 12^*$ [72×72]	ND	Stable* [18×18]	Stable* [72×72]	ND
	4) CMAM	$n_k = 3$ [18×18]	$n_k = 12$ [72×72]	Stable [90×90]	Stable [18×18]	Stable [72×72]	Stable [94×90]

Table 3.1: Summary of the results obtained using the five methods on six different structures. Matrix sizes are indicated in each case. Asterisks (*) denote results obtained from the cited references; the author produced the others. ND = Not Developed by the author or not applicable.

they depart from the definition of RS, which relies instead on the "stacking" of hinged-connected beams. The RS_{HINGED} represents a special case of RS in which all supports and connections allow complete rotational freedom about all three axes. These structures are always statically determinate: for a configuration with d_{tot} beams and $n_{tot} = 2d_{tot}$ nodes (support or connection), the total number of fixed DoFs at the nodes is $3n_{tot}$, which equals the total number of rigid body motions ($6d_{tot} = 3n_{tot}$).

The author recommend favouring the use of RS_{HINGED} whenever possible, as they offer two significant advantages:

- They are statically determinate, making them easier to assemble. In statically indeterminate RSs, improper placement of one beam can make it impossible to place the subsequent one(s) [28].
- Supports and connections are identical at every node, simplifying their fabrication.

The RS_{HINGED}^{2D} structures A and B from Figure 3.16, whose instability modes are shown in Figures 3.17a and 3.17b, are unstable because their beams can freely rotate about their longitudinal axis. To ensure stability, it is necessary

to prevent axial rotation at each beam, whether at connections or supports. In contrast, the RS_{HINGED}^{3D} shown in Figures 3.16d and 3.16e are stable, as the presence of supported beams prevents axial rotation of the supporting beams. Unlike RS^{2D} configurations, the beam axes in RS^{3D} are not coplanar; they are offset by an eccentricity that acts as a lever arm, resisting rotation.

However, certain RS_{HINGED}^{3D} structures can still be unstable, as illustrated by the RS in Figures 3.17c and 3.17d. In these cases, the instability arises from a rotational movement of the central beam around its longitudinal axis, which induces motion in the other beams. This instability mode tends to appear in RS_{HINGED}^{3D} with symmetrical patterns featuring a central symmetry axis through the beam's centre, such as the bi-triangular and quad-quadrangular patterns.

It is important to note that all four RS illustrated in Figure 3.17 are stable in their displaced configurations (i.e. under second-order effects and after large displacements), as is the case for the double-beam structure of Figure 3.14. Xia et al. [94] demonstrated this for the two RS_{HINGED}^{2D} of Figures 3.17a and 3.17b. For the two RS_{HINGED}^{3D} cases [Figures 3.17c and 3.17d], the displacement of the beams inevitably breaks the central symmetry, thereby rendering them second-order stable in their deformed configuration.

In conclusion:

- RS_{HINGED}^{2D} structures are always unstable, regardless of the pattern. It is necessary to design connections and/or supports that prevent axial rotation of the beams, at least at one end.
- RS_{HINGED}^{3D} structures are often stable, except for particular geometries that exhibit a central axis of symmetry passing through the centre of a beam. If necessary in some cases, fixing the supports can be a simple and effective solution for transforming an unstable RS_{HINGED}^{3D} into a stable RS_{HINGED}^{3D} .

3.4.2 Accuracy and efficiency of KCMAM and CMAM

Stability verification for the KCMAM and CMAM methods relies on the rank of the kinematic compatibility matrix $\mathbf{B}$ and the constraint matrix $\mathbf{C}$. The literature [54, 89, 94] recommends computing the rank using Singular Value Decomposition (SVD), a more robust algebraic method than Gaussian Elimination. Compared to the latter, an SVD of a matrix is more numerically stable (less impacted by rounding effects or inaccuracies in the positioning of nodes) and provides more insight than just its rank.

An SVD decomposes a matrix $\mathbf{A}_{k \times l}$ into two orthogonal matrices ($\mathbf{P}_{k \times k}$ and $\mathbf{D}_{l \times l}$), and a diagonal matrix $\mathbf{S}_{k \times l}$ containing the "singular values" s_i (all non-negative) and all non-diagonal entries being zero. This results in the factorisation: $\mathbf{A} = \mathbf{P} \mathbf{S} \mathbf{D}^T$. The singular values are arranged in descending

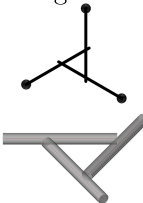
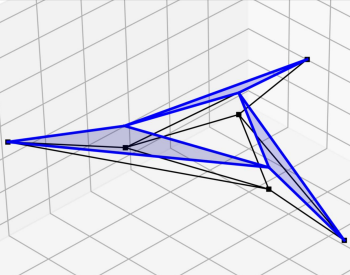
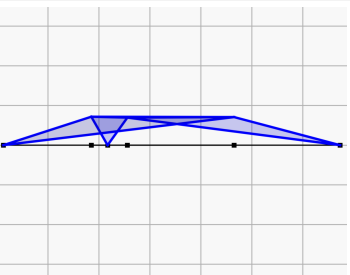
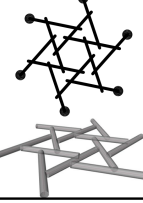
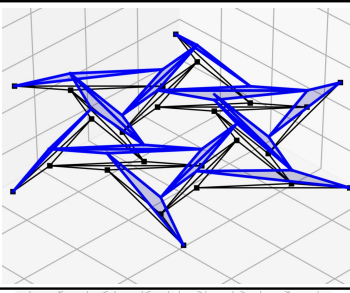
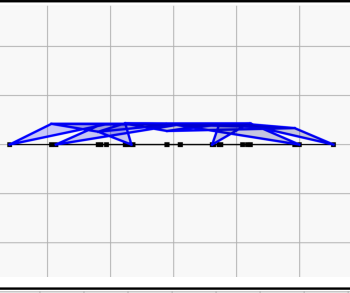
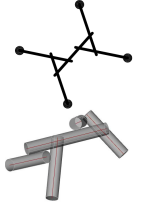
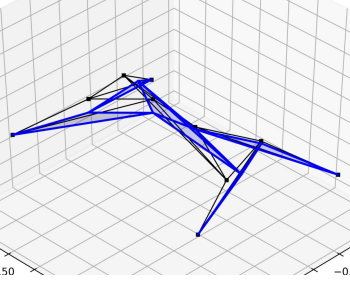
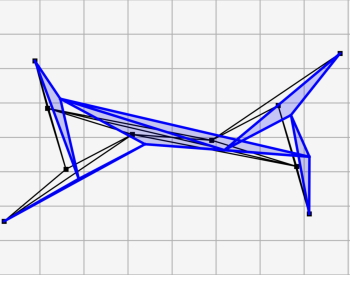
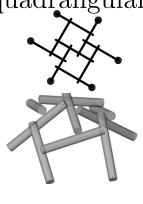
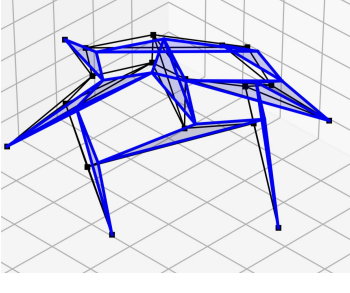
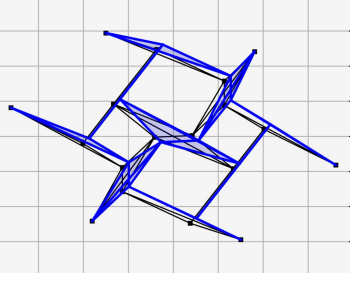
Patterns	Isometric view of instability mode	Lateral or top view of instability mode
a) RS_{Hinged}^{2D} Triangular 		
b) RS_{Hinged}^{2D} Hexa-triangular 		
c) RS_{Hinged}^{3D} Bi-triangular 		
d) RS_{Hinged}^{3D} Quad- quadrangular 		

Figure 3.17: *Instability modes of various RS_{HINGED}^{2D} et RS_{HINGED}^{3D} .*

order: $s_{\max} \geq \dots \geq s_i \geq \dots \geq s_{\min} \geq 0$. It should be noted that, when the matrix A is square and symmetric, its singular values coincide with its eigenvalues. If A is singular, at least its smallest singular value ($s_{\min}$) will be exactly or numerically zero.

RS^{3D}_{HINGED}		Wire Elements			Meshed Elements		
		s_{min} [-]	s_{max} [-]	κ [-]	s_{min} [-]	s_{max} [-]	κ [-]
Triangular: STABLE		0.063	6.70	106	0.078	2.11	27
Hexa- triangular: STABLE		0.028	5.26	188	0.026	3.06	119
Bi- triangular: UNSTABLE		$8.21 \cdot 10^{-16}$	9.56	$116 \cdot 10^{14}$	$1.93 \cdot 10^{-16}$	3.02	$157 \cdot 10^{14}$
Quad- quadrangular: UNSTABLE		$8.48 \cdot 10^{-16}$	8.56	$101 \cdot 10^{14}$	$9.07 \cdot 10^{-16}$	3.03	$33 \cdot 10^{14}$

Table 3.2: Values of s_{min} , s_{max} , and κ for 4 RS^{3D}_{HINGED} modelled with Wire Elements or Meshed Elements.

Based on the singular values of A , a "Condition Number" $\kappa = s_{max}/s_{min} \in [1; \infty]$ can be defined [83]. According to Trefethen and Bau [83], the matrix A is considered "well-conditioned" if κ is "small" ($\kappa \leq 10^3$) and "ill-conditioned" if κ is "large" ($\kappa \geq 10^6$).

These algebraic tools allow a qualitative assessment of the stability of an RS based on the condition number κ of the constraint or kinematic compatibility matrix. A "small" κ value indicates that the RS is likely to be stable, whereas a "large" value suggests instability.

Table 3.2 provides the values of s_{min} , s_{max} , and κ for the constraint matrix (CMAM) associated with four RS^{3D}_{HINGED} structures modelled using either Wire Elements or Meshed Elements, whose stability was previously discussed. The values of s_{min} , s_{max} , and κ vary little between the two modelling approaches. Moreover, the κ values reveal an order-of-magnitude difference of approximately 10^{14} between stable and unstable RSs. The two stable structures have a condition number below 10^3 , supporting the validity of the criterion above.

Figure 3.18 illustrates the variation in the condition number κ for the unstable structures of Table 3.2 and the beam of Figure 3.14, when their geometries are slightly modified.

For the beam of Figure 3.18a, the condition number κ decreases rapidly

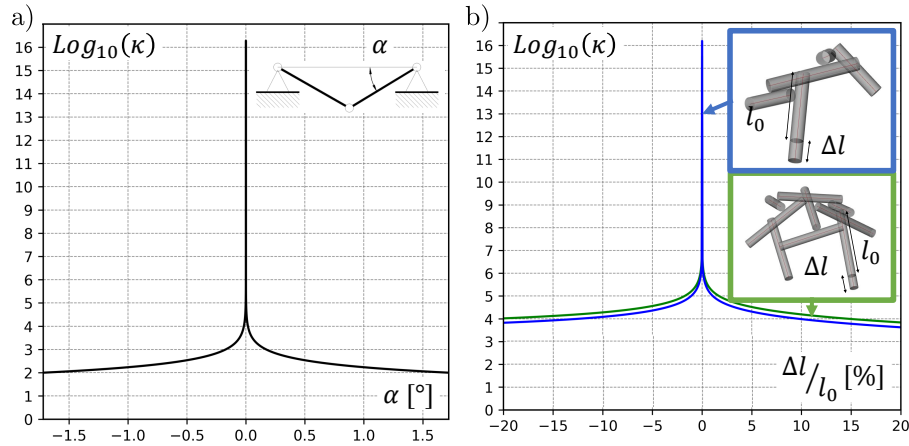


Figure 3.18: κ values (logarithmic scale) for structures with geometric variation: a) central hinged beam, and b) Bi-triangular and Quad-quadrangular patterns.

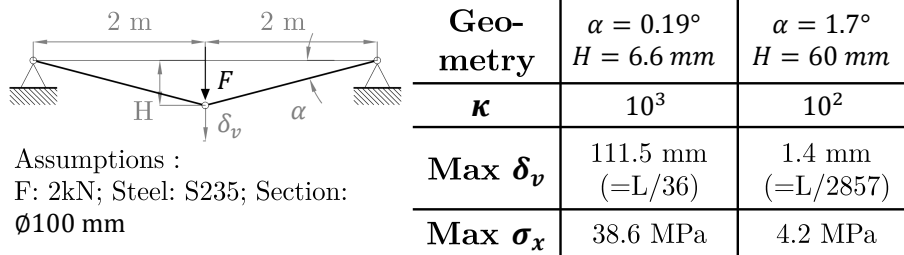


Figure 3.19: Structural analysis of central hinged beam using SCIA Engineering Software.

with even a slight change of the angle α . suggesting that the structure is stable ($\kappa = 10^3$) for $\alpha = \pm 0.19^\circ$. However, depending on the cross-sectional area of the beam, the displacement of the central node can be high and must be checked by "Method 0" since $\alpha = 0.19^\circ$ is a very small angle.

As for Figure 3.18b, the geometry of the Bi-triangular or Quad-quadrangular $\text{RS}_{\text{HINGED}}^{3\text{D}}$ is slightly non-symmetrically modified by increasing the length of one support beam. After such a modification, these structures are no longer symmetric and are therefore considered stable when computing the rank after Gaussian Elimination. However, the condition number falls within a "moderate" range (between 10^3 and 10^6), which requires a complete stability calculation using method 0.

Figure 3.19, related to the simple unstable double beam, shows the middle node displacement for a specific case with several values of α . Figure 3.20a, related to the stable Hexa-Triangular RS in Table 3.2, provides the value of the maximum nodal displacement for a specific case, which is limited to 2.4 mm, confirming the good stability of the structure and its low displace-

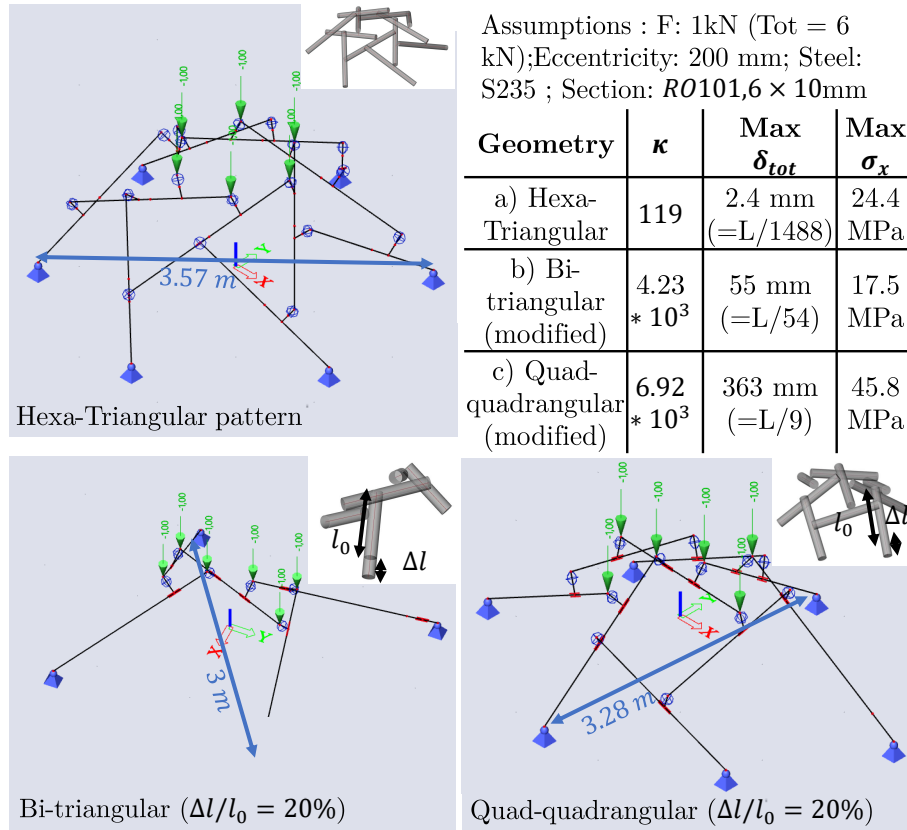


Figure 3.20: Structural analysis of structures using SCIA Engineering Software: a) Stable Hexa-triangular pattern, b) Unstable Bi-triangular pattern (modified), and c) Unstable Quad-quadrangular pattern (modified).

ments. Figures 3.20b and 3.20c, related to the unstable Bi-Triangular and Quad-quadrangular RS of Table 3.2, modified with a relative elongation of 20% of the support beam, provide the values of the maximum nodal displacement, respectively 55 and 363 mm, which can be considered excessive. The structures of Figure 3.20 have identical cross-sections, materials, total loads, eccentricities and similar spans.

These examples show that the qualitative discussion on the structure's stability, based on κ , needs to be confirmed by a comprehensive structural analysis. Additionally, the proposed threshold at 10^3 does not ensure limited displacements for reasonable cross-sections.

3.5 Conclusion

Five methods for verifying the stability of Reciprocal Structures (RS) have been studied and compared. Except for "Method 0", these methods assume that the beams are infinitely rigid, enabling the determination of RS geomet-

Method		Considered RS		Advantages	Dis-advantages
N°	Name	Blocked DoF	RS type		
0	SMAM	Arbitrary	RS ^{2D} & RS ^{3D}	Applicable to all RS.	Computationally demanding; requires complete numerical modelling and definition of all material and cross-section properties.
1	LSM	Arbitrary	RS ^{2D} & RS ^{3D}	Intuitive and fast; no matrix calculations.	Identifies some stable structures as unstable.
2	KAM	Imposed	RS ^{3D}	Simple formula, no matrix computations	Not yet developed for all RS.
3	KCMA M	Imposed	RS ^{2D} & RS ^{3D}	Reduces the matrix size to be analysed.	Not yet developed for all RS.
4	CMAM	Arbitrary	RS ^{2D} & RS ^{3D}	Universally applicable to all RS.	

Table 3.3: Advantages and disadvantages of RS's five stability verification methods.

ric stability at a preliminary design stage without assigning cross-sections. Indeed, unstable RS need to be rejected as they would never satisfy the building codes, or require an excessive amount of materials. Table 3.3 summarises the advantages and disadvantages of each, demonstrating that the CMAM method is the most effective and universal, as it applies to all types of RS regardless of the DoF fixed at supports and connections. In addition, this method enables a qualitative discussion of the RS stability based on the condition number κ , thereby expanding the scope of rejected unstable RS.

These methods have shown that certain RS_{HINGED} can be geometrically stable under specific conditions. For RS_{HINGED}^{2D}, rotational DoFs about the longitudinal axis of each beam must be blocked either at the connections or at the supports. In contrast, most RS_{HINGED}^{3D} are geometrically stable, as this axial rotation is inherently prevented due to the eccentricity between the supporting and supported beams, which creates a stabilising lever arm. Finally, RS_{HINGED}^{3D} are geometrically unstable if they exhibit central symmetry through the centre of a beam. In some cases, fixing the supports can be a simple and effective solution for transforming an unstable RS_{HINGED}^{3D} into a stable RS_{HINGED}^{3D}.

Furthermore, an examination of the supports and connections used in existing RS, whether in laboratory specimens, scaled prototypes, or full-scale structures, reveals that:

- Designers often opt to fix an excessive number of DoFs, leading to increased stiffness but at the expense of ease of assembly and a behaviour closer to that of gridshells;
- Supports and connections often exhibit semi-rigid DoFs, which should be avoided in practice. These arise from friction, machining tolerances, or assembly gaps, and are challenging to model accurately, despite their significant influence on overall structural behaviour.
- The design and modelling of supports remains an under-discussed topic, although they play a critical role in ensuring the structural stability of RS.

The author therefore recommends favouring RS_{HINGED} using hinged supports and connections, and where necessary, blocking axial rotation at least at one end of each beam or fixing the supports. However, no connection in the literature exhibits truly hinge-like behaviour, as they invariably exhibit some degree of semi-rigidity.

Chapter 4

Predictability of reciprocal structures

The previous chapter showed that hinged RS^{3D} are stable for most geometries and comply with the theoretical mechanical behaviour of RS. However, its literature review presents no hinge connections. Moreover, results from experimental campaigns on proposed connections showed discrepancies with their numerical models, because the connections exhibited nonlinear or semi-rigid behaviour, including friction, locking effects, and semi-rigid responses, and because the complex behaviour of the RS was difficult to monitor.

This chapter develops a spherical hinge connection and demonstrates that it enables a reliable numerical model. This connection is proposed for an RS^{3D} made of steel tubes, to avoid uncertainties linked to geometrical and mechanical properties for logs. The development was carried out through iterations that simultaneously evolved the experimental method to yield robust instrumentation and procedures. Notably, a motion capture system was used effectively to measure three-dimensional displacements and had never been used for reciprocal structures.

This chapter describes only the last connection solution and regroups the results of a journal paper with the method presented in a conference paper:

- STEINMETZ Martin, LATTEUR Pierre, and SGAMBI Luca. Predictability of numerical models for reciprocal structures. *Engineering Structures*, 2026, vol. 365.
- STEINMETZ Martin, SGAMBI Luca, and LATTEUR Pierre. Reliable experimental testing methodology for reciprocal structures. In: *Proceedings of IASS Annual Symposia*. International Association for Shell and Spatial Structures (IASS), 2024. p. 1-10.

First, it develops the spherical hinge connection comprising rods attached to the ends of the bars and proposes a methodology for integrating it into a numerical model. To validate this method, large-scale steel structural prototypes equipped with the proposed hinged connection were then constructed and tested. The results show a strong similarity between the numerical and experimental values of displacements and stresses.

4.1 Introduction and objectives

The previous chapter highlights the wide variety of possible RS connections, while emphasizing that designers of RS^{3D} generally select connections that restrain one or more rotational degrees of freedom between beams, in addition to the three translational degrees of freedom. These rotational constraints promote the stability of RS^{3D}, but they also complicate construction and, above all, contradict the definition of reciprocal structures, which are supposed to be composed of beams simply resting on one another [9]. Selecting completely hinged (pinned) connections and supports that restrain only the three translational degrees of freedom is therefore recommended.

In addition, the previous chapter presented different experimental campaigns aimed at validating the behaviour of RS connections :

- The squared timber connection using a half-lap joint [29] [Figure 3.5c].
- The clamped connection for bamboo poles, by Godina Rodríguez [66] [Figure 3.6a].
- The rigid connection between steel tubes [60] [Figure 3.8b].
- The through-bolt and saddle washers for connecting circular steel tubes [64, 63] [Figure 3.8e].
- The metallic sphere between fibre-reinforced concrete double S beams [55] [Figure 3.7].

Across these studies, in addition to the predictability of numerical analyses of connection behaviour, measurement accuracy remains a major challenge, as the 3D displacement of RS is difficult to capture.

The objective of this chapter is to demonstrate that hinged connections, whose behaviour has been experimentally characterised, allow a reliable numerical model to be obtained for RS. To this end, this chapter:

- Develops a connection comprising a spherical hinge linked by rods to the ends of the members, which makes it possible to construct reciprocal structures more easily and inspire real-world projects.
- Proposes a methodology to reliably integrate this type of connection into a numerical model.
- Assess the assembly difficulties of a large-scale MM RS^{3D} prototype and of the modules composing it. This structure is shown in Figure 4.1 and presented in Section 4.3.
- Validates this hinged connection by comparing numerical and experimental results for nodal displacements and member stresses.

4.2 Development of a reliable hinged connection for RS^{3D}

The developed connection was the subject of several successive variants tested on an SM RS^{3D} composed of three beams (S235). Numerical results were compared with experimental measurements in terms of nodal displacements and stresses in the tubes :



Figure 4.1: General view of the RS-24B prototype, built outdoors.

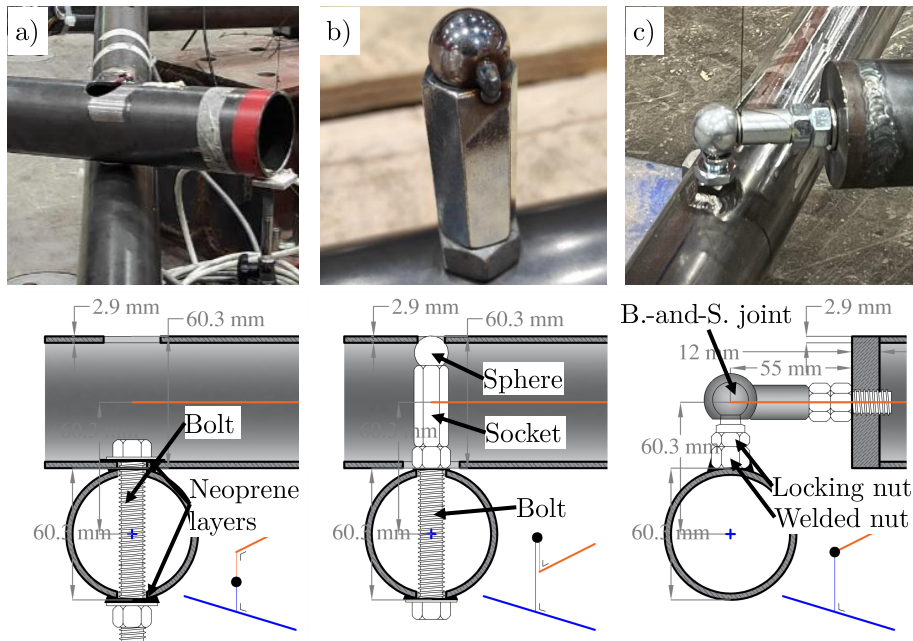


Figure 4.2: Three proposed steel tube connections and their numerical modelling.

- The first tested connection is inspired by the one proposed by Rizzuto [63], presented in Figure 3.8e. To limit rotational constraints, the M12 threaded rod (8.8 class) passes only through the supporting tube [Figure 4.2a]. A neoprene layer is added between the tubes (1 mm) and at the interface between the bolt head and the inside of the supported tube (2 mm). This connection is then modelled by a hinge at the contact point between the two tubes, connected to the tube axes by two infinitely rigid elements whose lengths equal the tubes' radii. The tests

showed that this type of connection does not behave linearly, with significant discrepancies between numerical results and measurements: about 10% for nodal displacements and 20% for bending stresses. This behavioural difference is due to the supported tube being restrained in rotation about its axis by the bolt head, despite the spacing provided by the neoprene, with no bolt preload.

- In the second proposed solution [Figure 4.2b], the top of the supported tube is perforated and placed on a sphere so that it can rotate freely on it in the three directions, thanks to a sufficiently large opening on the opposite lower face. The sphere is welded to a hexagonal socket nut, which is connected to the supporting tube by an M12 steel bolt passing through the nut. An additional nut between the socket nut and the supporting tube is necessary to ensure that the eccentricity between the tubes equals the sum of the radii of both tubes. On average, the numerical results at the measurement points correspond well with the experimental measurements, with deviations of only a few per cent. Although this test confirms that perfect hinges ensure reliable numerical results, this connection is not very relevant because it does not prevent uplift or disengagement.
- The third connection is illustrated in Figure 4.2c. The supported member ends with a flat plate that has a threaded hole. During assembly, the threaded rod is inserted into this hole, a nut is tightened, and the female part of a commercial ball-and-socket M12 joint is connected. At the level of the supporting tube, the male part is screwed into a M12 nut welded to the tube and secured with a locking nut. The successive metallic parts of the connection are modelled by short solid cylindrical elements whose stiffness is determined experimentally (this procedure is detailed later in this section). The numerical results show good agreement between experimental tests and numerical calculations, with differences of about 1% for nodal displacements and 4% for bending stresses. However, this connection was not adopted because inserting the male part of the joint into the supporting tube nut is difficult, as perfect alignment with the welded nut is required for screwing. In addition, the welded nut engages only the upper part of the tube, which may cause local ovalisation.
- The fourth solution, an improvement of the previous one, is presented in Figure 4.3 and was selected for the construction of the prototypes. It ensures ease of assembly while preserving the hinge between the two beams and improving force transmission. At the supporting tube, the male part of the ball-and-socket M16 joint is screwed into a sleeve ensuring continuity with the threaded rod, which is itself inserted into a tubular reinforcement welded inside the supporting tube. A locking

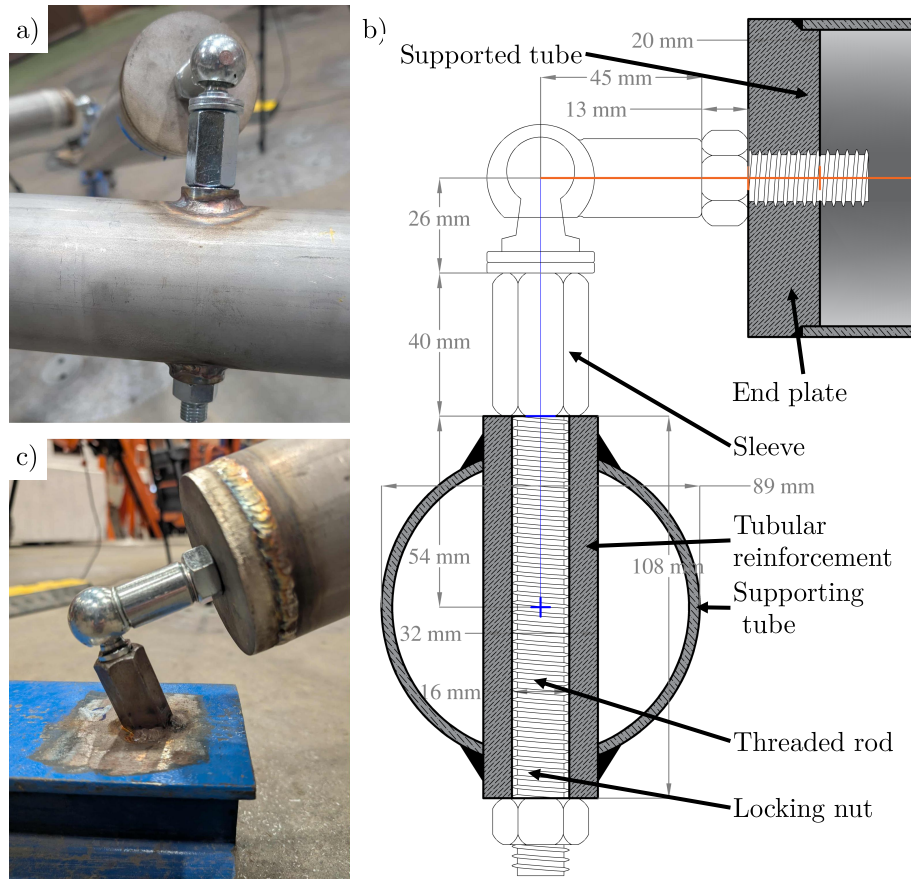


Figure 4.3: Final connection proposal: a) connection between members, b) descriptive diagram, and c) support connection.

nut on the opposite side of the reinforcing tube secures the connection against uplift. The thickness of this tubular reinforcement is relatively large due to welding constraints. This connection can also be used to create hinged supports [Figure 4.3c].

The connection shown in Figure 4.3 consists of a succession of elements with specific mechanical properties. In the numerical model, these elements are represented by a hinge connected by two cylinders, whose fictitious diameters are determined from experimental tests on the connection. Figure 4.4 illustrates how these tests were performed. The actuator force is applied progressively [Figure 4.4a]: three loading–unloading cycles up to 2 kN, followed by three cycles up to 3.5 kN, three cycles up to 5 kN, and finally a monotonic loading up to failure.

This loading protocol is applied to the connection placed in two configurations [Figures 4.4b and 4.4c]. For the part located in the extension of the

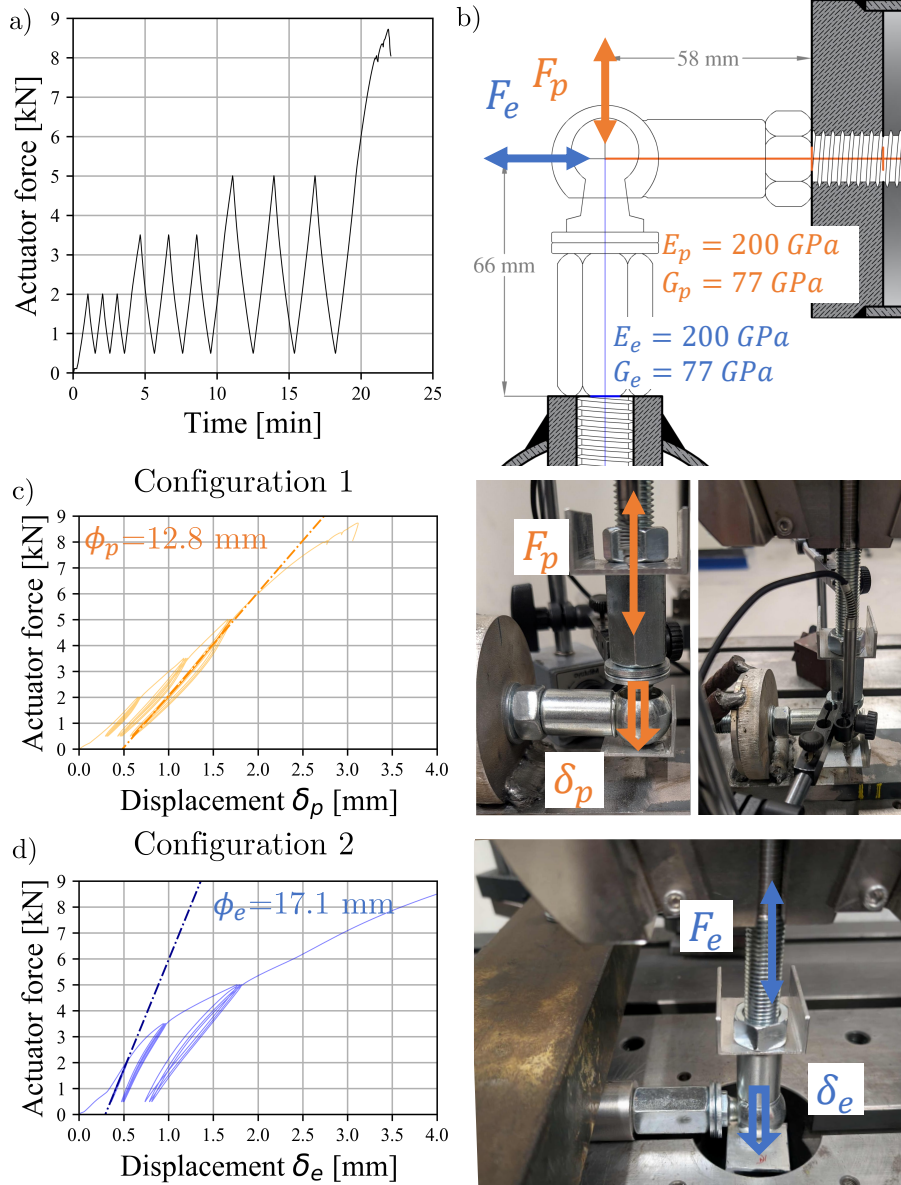


Figure 4.4: Connection characterisation: a) loading protocol, b) Configuration 1 with the load is applied to the supporting part, c) Configuration 2 with the load is applied to the eccentricity part, and d) Scheme of the connection.

supported beam (denoted p, Configuration 1), elastic behaviour is observed up to at least 5 kN, followed by a slightly plastic phase before failure of the rod at the flat plate. For the eccentricity part (denoted e, Configuration 2), the initial stiffness is higher. Still, a clear elastic limit appears between the cyclic loads of 2 kN and 3.5 kN, before reaching a ductile failure by thread stripping

near the interface between the reinforcement and the sleeve. The loading protocol applied to the prototypes tested in the laboratory [Section 4.3.3] must therefore ensure that these forces (Configuration 1: 5 kN and Configuration 2: 2 kN) are not exceeded at any connection.

The equivalent diameters of the two cylinders (ϕ_p and ϕ_e) are determined assuming fixed-end conditions. It is then possible to relate the stiffness obtained from the test (δ/F) to the geometric parameters (κ_s , l , A , and I , defined in Table 4.1) and material properties (E and G) of each cylinder, considering only bending effects (bending moment and associated shear force) as presented in Equation 4.1.

$$\frac{\delta}{F} = \frac{l^3}{3 E I} + \frac{l}{G \kappa_s A} \quad (4.1)$$

Equation 4.2 makes it possible to determine the diameter of the cylinders.

$$\phi = \sqrt{\frac{2F l}{\pi \kappa_s G \delta} + \sqrt{\left(\frac{2F l}{\pi \kappa_s G \delta}\right)^2 + \frac{64F l^3}{3\pi E \delta}}} \quad (4.2)$$

The stiffness is determined during the last cycle, yielding values of 5 kN and 2 kN for configurations 1 and 2, respectively, using linear regression over the entire loading phase. Assuming an isotropic material ($E_e = E_p = 200000 \text{ MPa}$ and $G_e = G_p = 77000 \text{ MPa}$), and knowing the equivalent beam lengths ($l_p = 58 \text{ mm}$ and $l_e = 66 \text{ mm}$) indicated in Figure 4.4d, the equivalent diameters are computed using Equation 4.2: $\phi_p = 12.8 \text{ mm}$ and $\phi_e = 17.1 \text{ mm}$. It is important to emphasise that, in the context of a real project involving connections of the same type but different sizes, laboratory tests will also be required to introduce the correct connection data (the two-cylinder diameters) into the numerical model.

4.3 Description of the tests performed on the prototypes

The proposed connection is used for connecting the 24 stainless steel 304L tubes (88.9×3.05) of a large-scale MM RS^{3D} prototype that covers an area of 9.4 m in diameter and reaches a maximum height of 3.5 m [Figure 4.5a]. This structure, denoted RS-24B, is composed of four rings (denoted A, B, C and D), each consisting of six beams [Figure 4.5c]. The six beams of rings A, B, and C are identical and measure, respectively, 161 mm, 221 mm, and 251 mm, which simplifies fabrication. The lengths of the supporting beams in ring D vary from 330 to 571 mm to ensure the desired inclination [Figure 4.5c]. This structure could only be tested outdoors at its final site due to its large size [Section 4.5].

4.3.1 Tested prototypes

Before testing the RS-24B, prototypes with fewer bars are tested in the laboratory to validate the numerical modelling and to understand the effect of

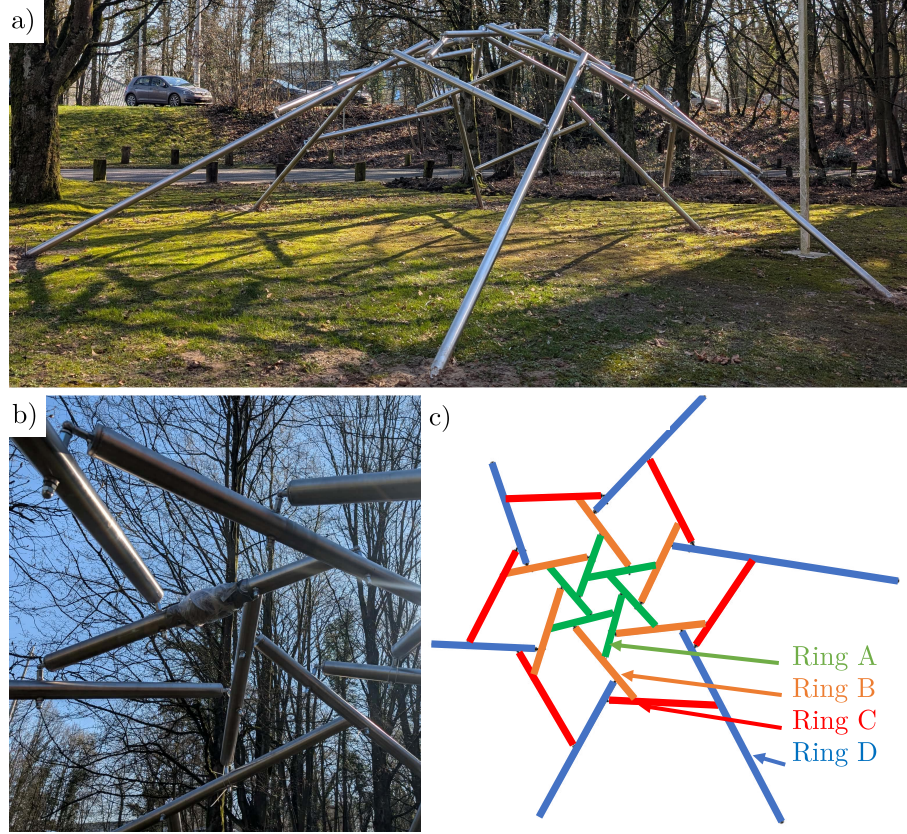


Figure 4.5: RS-24B prototype, built outdoors: a) General view, b) Central rings, and c) Plan view with ring notation.

modelling uncertainties related to connections on the structure's response. Specifically, the following structures were tested in the laboratory [Figure 4.6]:

- A single three-beam module RS-3B, composed of two beams (denoted A1 and A2, belonging to ring A) and one beam (denoted B1, belonging to ring B).
- Ring A, denoted RS-6B (six beams).
- Rings A and B together, denoted RS-12B (12 beams).

The stainless-steel tubes (grade 304L) have a cross-section of $88.9\text{ mm} \times 3.05\text{ mm}$, and their material and geometric properties are presented in Table 4.1.

As explained in the introduction, erecting a reciprocal structure is complex because the structure remains unstable until all beams are assembled. For the RS prototypes tested in the laboratory, the supports were first fixed to the test floor with a positioning accuracy ensured by the Motion Capture System (MCS) described in Section 4.3.2. A marker is placed on top of the

	Property	Symbol	Value	Unit
Material 304L	Young's Modulus	E	193000	MPa
	Yield strength at 0.2%	f_y	215	MPa
	Poisson's Ratio	ν	0.3	-
Geometric 88.9x3.05	Outer Radius	r_{ext}	44.45	mm
	Inner Radius	r_{int}	41.40	mm
	Area	$A = \pi(r_{ext}^2 - r_{int}^2)$	822.60	mm^2
	Moment of Inertia	$I = \frac{\pi}{4}(r_{ext}^4 - r_{int}^4)$	758802	mm^4
	Polar Moment of Inertia	$I_p = \frac{\pi}{2}(r_{ext}^4 - r_{int}^4)$	1517604	mm^4
	Radius Ratio	$m = r_{int}/r_{ext}$	0.931	—
	Shear Coefficient	$\kappa_s = \frac{6(1+\nu)(1+m^2)^2}{m^4 + (34+24\nu)m^2 + (7+6\nu)}$	0.532	—

Table 4.1: Material and geometric properties of the circular stainless steel 304L tubes

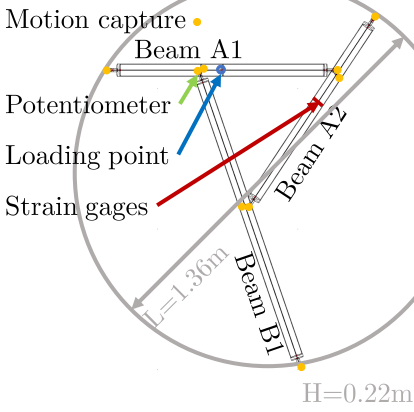
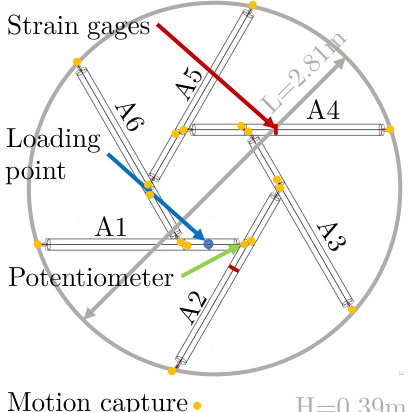
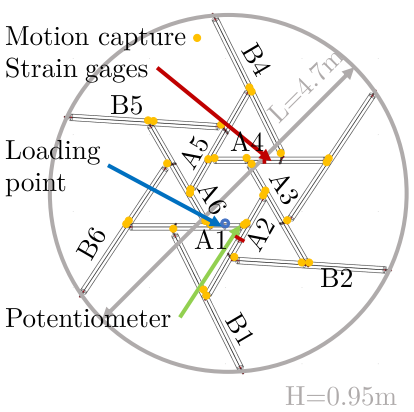
	Lab view	Scheme
RS – 3B		 Motion capture • Beam A1 Potentiometer Loading point Strain gages Beam A2 Beam B1 $L=1.36\text{m}$ $H=0.22\text{m}$
RS – 6B		 Strain gages A6 A5 A4 A3 A2 A1 Loading point Potentiometer $L=2.81\text{m}$ Motion capture • $H=0.39\text{m}$
RS – 12B		 Motion capture • Strain gages B4 B5 A5 A4 A3 A2 A1 B6 B2 B1 Loading point Potentiometer $L=4.7\text{m}$ $H=0.95\text{m}$

Figure 4.6: Structures RS-3B, RS-6B, and RS-12B. L is the diameter of the circle passing through the support points, and H is the total height.

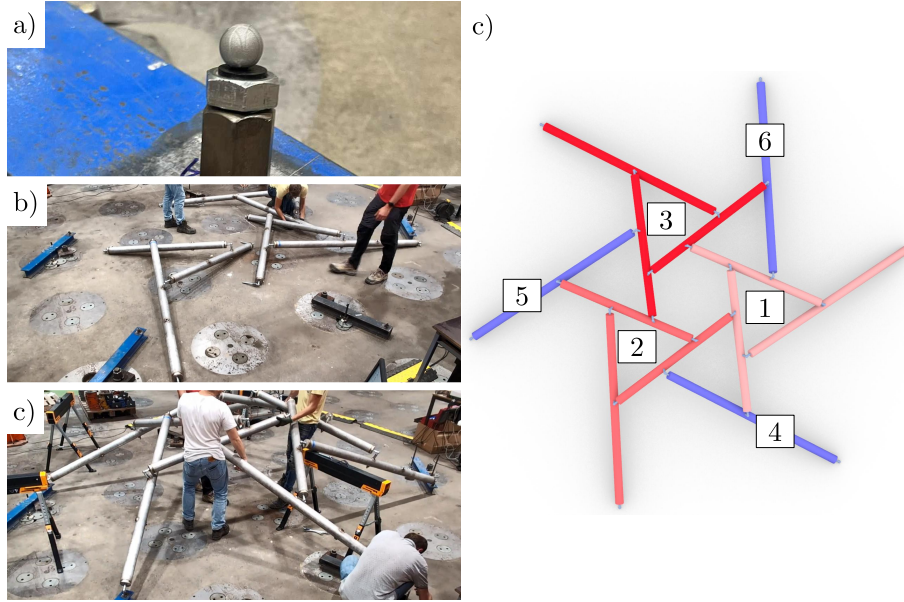


Figure 4.7: a) Positioning of support using the MCS. b) Six assembly steps (shown above) of RS-12B in the laboratory, with photographs of the pre-assembly and assembly step 5.

sleeve, allowing the support to be positioned with millimetre accuracy using live tracking [Figure 4.7a].

The assembly of RS-3B and RS-6B is relatively simple because they consist of a single module and require only one temporary support to hold the first beam. The assembly of RS-12B and RS-24B is more complex. The erection of RS-12B follows the six assembly steps shown in Figure 4.7b, which require pre-assembling the three-beam modules and using three temporary supports. The assembly of RS-24B is described in Section 4.5.1.

4.3.2 Measurement instrumentation

The instrumentation must be redundant and account for structural points moving in all directions and for most members being subjected to six internal forces. The measurement locations are shown in Figure 4.6.

Measurements were performed using the following instruments [Figure 4.8]:

- The cable potentiometer [Figure 4.8b] measures the vertical displacement of the connection point expected to experience the largest displacement. Its advantage is that it is only slightly affected by horizontal displacement components if placed sufficiently far from the cable attachment point.
- The Motion Capture System (MCS) measures three-dimensional dis-

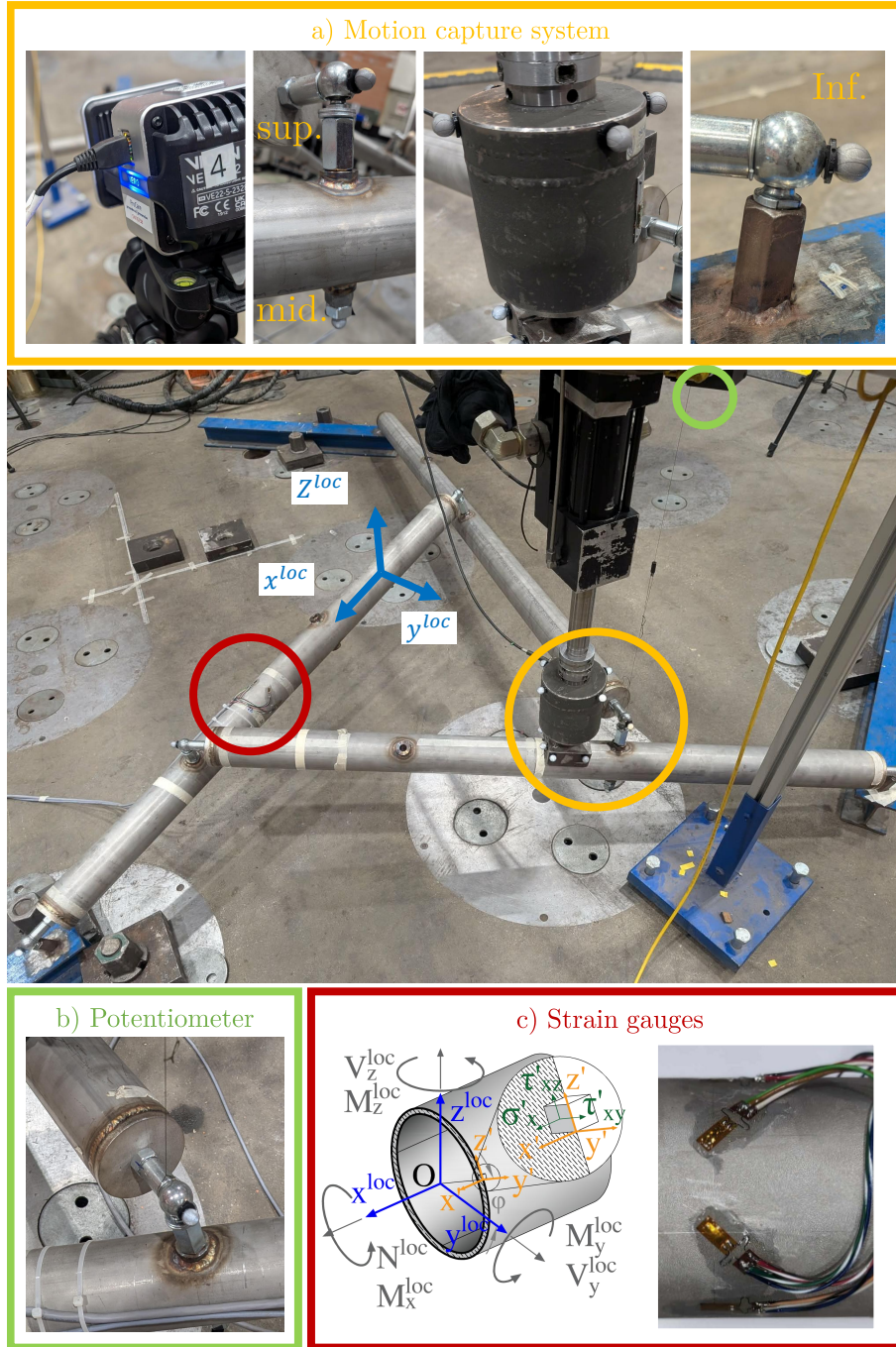


Figure 4.8: Instrumentation details on RS-3B (similar for the other structures tested): a) motion capture system, b) cable potentiometer, and c) strain gauges.

placements of reflective markers (small white plastic spheres) using eight optical cameras [Figure 4.8a]. This method allows a large number of measurement points, provided each marker is observed by at least two cameras [53]. The measurement accuracy is approximately ± 0.05 mm. Markers are placed at the beam ends, beneath the connection rods, and on the load cell.

- Six strain gauges (SG) are glued around the perimeter of the same cross-section to determine the internal forces in the beam's local coordinate system (defined in Figure 4.8c according to their positive direction) and the maximum von Mises stress $\sigma_{vM,Max}$. This cross-section is located midway between the two central connections of Beam A2 and, for RS-6B and RS-12B, also between those of Beam A4. This location corresponds to a highly stressed region. Still, it is located 239 mm ($\geq 4 r_{ext}$) from each connection, ensuring the validity of Saint-Venant's principle.

4.3.2.1 Post-processing of strain gauge measurements

At a particular cross-section of a steel tube, six strain gauges glued along the perimeter enable the determination of internal forces (N^{loc} , V_y^{loc} , V_z^{loc} , M_x^{loc} , M_y^{loc} and M_z^{loc}) according to the local coordinate system, as defined in Figure 4.9a. Based on the internal forces, the maximum von Mises stress and its position can be calculated.

First, the strain of a single strain gauge glued to the outside of the tube is analysed. Initially, the position angle φ_i and orientation angle ω_i of the gauge, defined in Figure 4.9b, and the material and geometric properties of the steel tube, defined in Table 4.1, were determined. To adjust both the local and the strain gauge coordinate system, angle μ_i must be integrated in ω_i . For the following explanations, both coordinate systems are aligned ($\mu_i = 0$).

During the test, the strain ε_i'' measured by this gauge is theoretically a function of the six local internal forces existing in the considered cross-section, whose properties are defined in Table 4.1. Iriarte et al. [30] give it for circular steel tubes in Equation 4.3.

$$\begin{aligned} \varepsilon_i''(\varphi_i, \omega_i) = & \left(\frac{1 - (1 + \nu) \sin^2 \omega_i}{E A} \right) N^{loc} + \left(- \frac{(1 + \nu) \sin(2\omega_i) \sin \varphi_i}{\kappa_s E A} \right) V_y^{loc} \\ & + \left(- \frac{(1 + \nu) \sin(2\omega_i) \cos \varphi_i}{\kappa_s E A} \right) V_z^{loc} + \left(\frac{(1 + \nu) \sin(2\omega_i)}{E I_p / r_{ext}} \right) M_x^{loc} \\ & + \left(\frac{(1 - (1 + \nu) \sin^2 \omega_i) \sin \varphi_i}{E I / r_{ext}} \right) M_y^{loc} + \left(- \frac{(1 - (1 + \nu) \sin^2 \omega_i) \cos \varphi_i}{E I / r_{ext}} \right) M_z^{loc} \quad (4.3) \end{aligned}$$

Equation 4.3 can be vectorially represented as $\varepsilon_i'' = [w_i][F]$, where the row vector $[w_i]$ regroups the terms E , A , κ_s , I , I_p , r_{ext} , ν , φ_i and ω_i , and the column vector $[F]$ contains the six internal forces. This relation can be generalized in

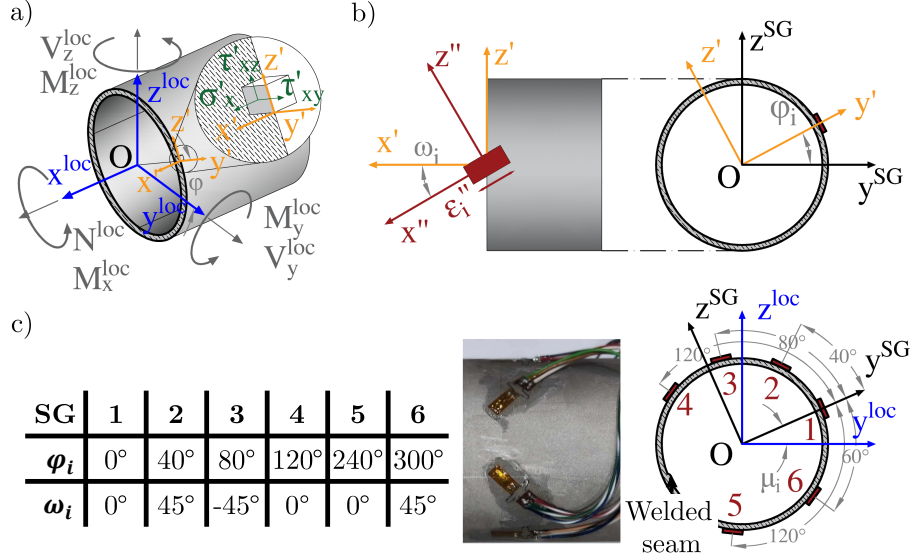


Figure 4.9: a) Internal forces in the local coordinate system, presented in their positive direction: the normal force N^{loc} , the shear forces V_y^{loc} and V_z^{loc} , the torsional moment M_x^{loc} , and the bending moments M_y^{loc} and M_z^{loc} . The normal stresses σ'_x , and the shear stresses τ'_{xz} , and τ'_{xy} , in the $x'y'z'$ coordinate system are indicated for any point along the perimeter, depending of the position angle φ . b) Convention for the position angles φ_i and orientation angles ω_i for strain gauges. c) Table and representation of the angles of the six strain gauges (SG from 1 to 6) along the same cross-section.

matrix form, as presented in Equation 4.4, in the case where the six SGs are placed along the same perimeter.

$$\begin{bmatrix} \varepsilon_1'' \\ \varepsilon_2'' \\ \varepsilon_3'' \\ \varepsilon_4'' \\ \varepsilon_5'' \\ \varepsilon_6'' \end{bmatrix} = \begin{bmatrix} [w_1] \\ [w_2] \\ [w_3] \\ [w_4] \\ [w_5] \\ [w_6] \end{bmatrix} \begin{bmatrix} N^{loc} \\ V_y^{loc} \\ V_z^{loc} \\ M_x^{loc} \\ M_y^{loc} \\ M_z^{loc} \end{bmatrix} \quad (4.4)$$

The six internal forces can then be determined by inverting the matrix composed of the vectors $[w_i]$. To ensure that this square matrix is invertible, the position and orientation angles of the six SG selected in this study are indicated in Figure 4.9c. Other angle combinations are proposed by Iriarte et al. [30]. However, care must be taken not to position an SG on the longitudinal welded seam of the tube to avoid measurement disturbance.

Finally, the internal forces allow determining the intensity and position of von Mises stresses on the perimeter of the tube, denoted as σ_{vM} , which are computed using Equation 4.5 and expressed in (the only non-null) terms of

normal stresses σ'_x [Equation 4.6] and shear stresses τ'_{xz} [Equation 4.7] defined in Figure 4.9a.

$$\sigma_{vM} = \sqrt{(\sigma'_x)^2 + 3(\tau'_{xz})^2} \quad (4.5)$$

$$\sigma'_x = \frac{+N^{loc}}{A} + \frac{-M_y^{loc} \sin\varphi + M_z^{loc} \cos\varphi}{I/r_{ext}} \quad (4.6)$$

$$\tau'_{xz} = \frac{+M_x^{loc}}{I_p/r_{ext}} + \frac{-V_y^{loc} \sin\varphi + V_z^{loc} \cos\varphi}{\kappa_s A} \quad (4.7)$$

These two stresses depend on the position of the point considered along the perimeter defined by the angle φ . The maximum intensity $\sigma_{vM,Max}$ and its position angle φ_{Max} along the perimeter can then be deduced.

4.3.3 Loading protocol

The loading protocol is identical for the three prototypes [Figure 4.10]:

- The actuator force $F_{Actu.}$ is applied to Beam A1 through a steel plate to avoid local stress concentrations. The loading rate is slow (0.025 mm/s) to obtain a sufficient number of measurement points.
- Eight loading–unloading cycles are first applied progressively between $F_{min} = 0.25 \text{ kN}$ and F_{Max} equal respectively to 6.25 kN, 4.75 kN and 3.25 kN for RS-3B, RS-6B and RS-12B.
- Five additional loading cycles (cycles 9–13) are then applied up to F_{Max} . Only these cycles are discussed in the results, which are presented relative to the beginning of cycle 9.

4.4 Comparison between experimental measurements and numerical results

4.4.1 Numerical model assumptions

The structures were analysed using the commercial software SCIA, with the following assumptions:

- Each tube is modelled by its centroidal axis. The local coordinate system used to determine internal forces is shown in Figure 4.8.
- Each connection and support [Figure 4.3] is modelled as a hinge connected to the tubes by two cylindrical elements [Section 4.2] whose mechanical characteristics are presented in Figure 4.4.
- The actuator force is applied on the centroidal axis of Beam A1 at the intersection between the vertical actuator axis and the tube axis [Figure 4.10b].
- Nodes corresponding to the positions of the MCS markers are connected to the model by massless beams.

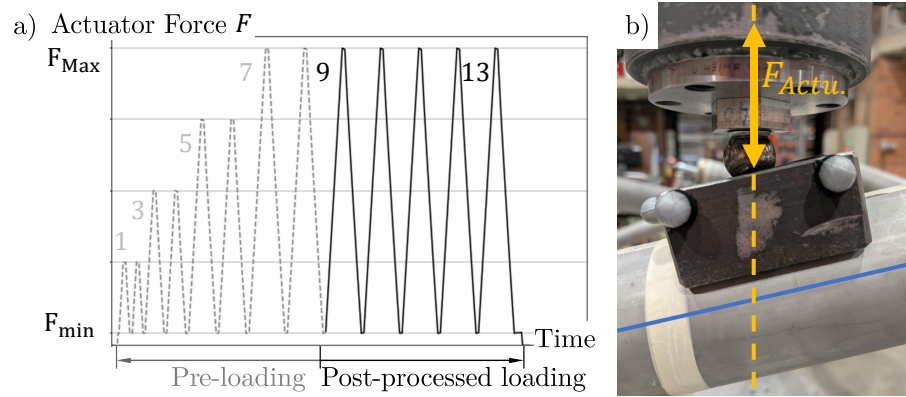


Figure 4.10: a) Loading protocol for RS-3B, RS-6B, and RS-12B tested in the laboratory. b) Position of the actuator force on Beam A1.

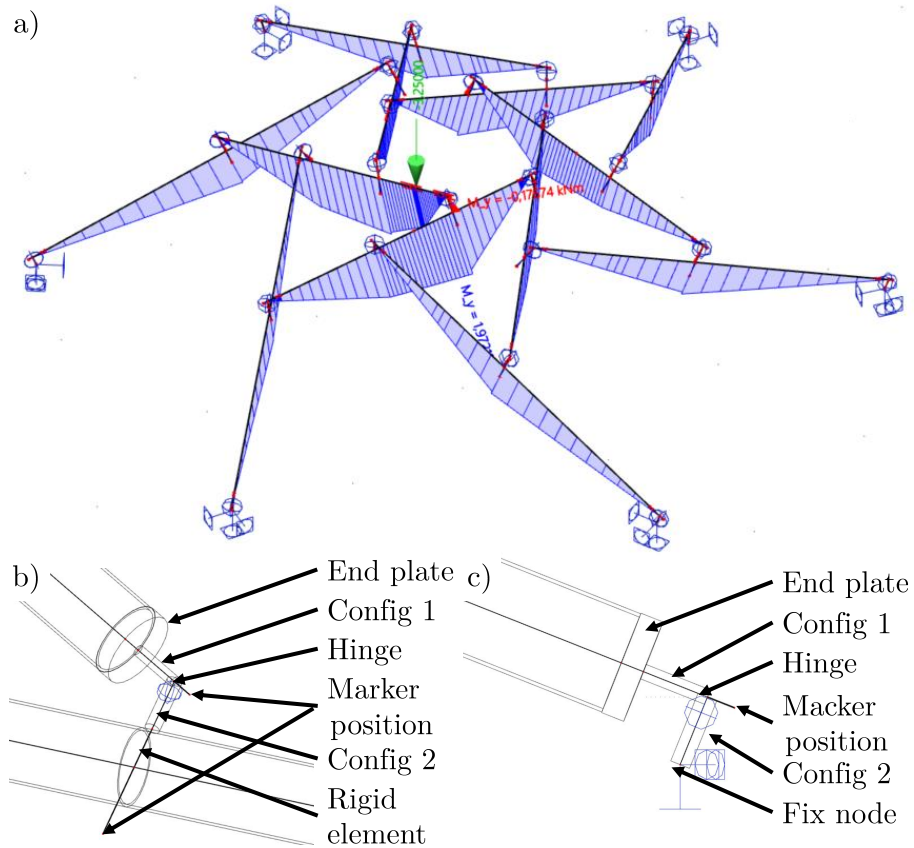


Figure 4.11: a) Numerical model of RS-12B, with bending moment for maximal actuator force. b) Detailed modelling of the connection. c) Detailed modelling of the support.

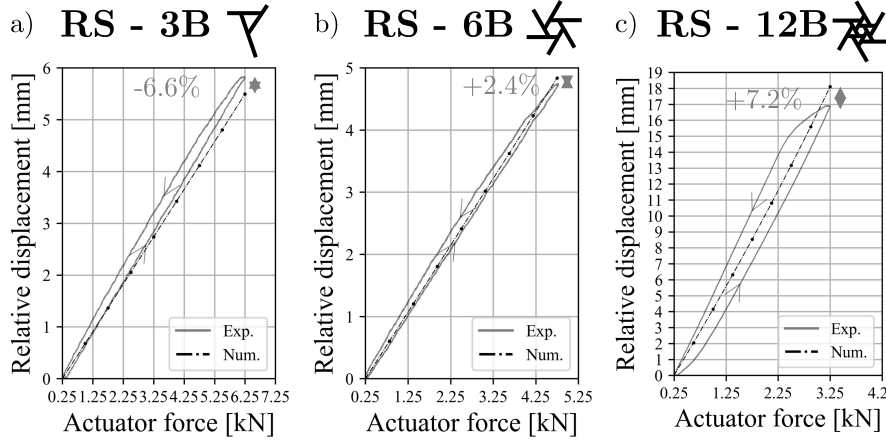


Figure 4.12: Comparison between the displacements (relative to the beginning of cycle 9) measured by the potentiometer and those predicted by the numerical model for: a) RS-3B, b) RS-6B, and c) RS-12B.

- A second-order analysis is performed to account for geometric nonlinearities, since some prototypes, such as RS-12B or RS-24B, are relatively flexible. Material nonlinearity is not considered because the loading is intentionally limited to remain within the elastic range of both tubes and connections.

The numerical modelling and stability checks were performed before testing the four structures. Figure 4.11a shows the RS-12B model with bending moment response, as well as the modelling details of the connection [Figure 4.11b] and support [Figure 4.11c].

In the following, the difference between a numerical result (denoted Num.) and an experimental measurement during the loading phase of the last cycle (cycle 13, denoted Exp.) is expressed as an “error” defined by: $100 * (Num. - Exp.) / Exp.$ [%].

4.4.2 Test results

4.4.2.1 Measurements using the potentiometer

Figure 4.12 compares the cable potentiometer measurements for the last five cycles (superimposed) with the numerical displacements at the same point. The errors are respectively -6.6% for RS-3B, +2.4% for RS-6B, and +7.2% for RS-12B. Although the potentiometer measures only a single point of the structure, the agreement between experimental and numerical results is very good for the three prototypes.

4.4.2.2 Measurements using the Motion Capture System

Figure 4.8a defines the notation: for a given beam A_1 , the marker located at its highest end is denoted A_1^{sup} , at its lowest end A_1^{inf} , and the intermediate

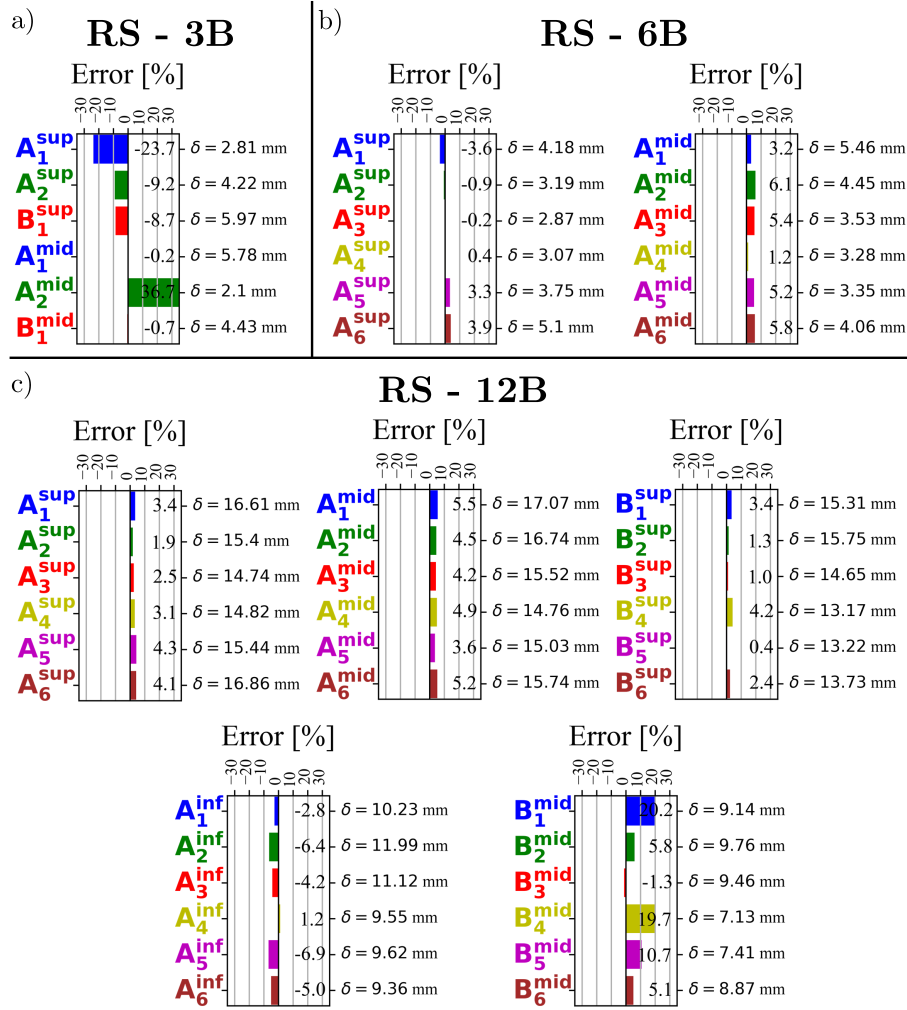


Figure 4.13: For prototypes RS-3B, RS-6B, and RS-12B, error value on vertical displacement at the markers.

one A_1^{mid} . For example, the connection between A_1 and A_2 has two markers: above, the marker of the supported beam (A_1^{sup}) and below, the marker of the supporting beam (A_2^{mid}).

Figures 4.13 and 4.14 presents, for the three structures:

- Histograms of the vertical displacement error during the last loading cycle (except for support markers, whose vertical displacement is nearly zero).
- A plan view showing the measured horizontal displacement vectors of the markers (coloured) and the numerically calculated displacement (darkened).

For RS-3B, the marker closest to the potentiometer measurement (B_1^{sup})

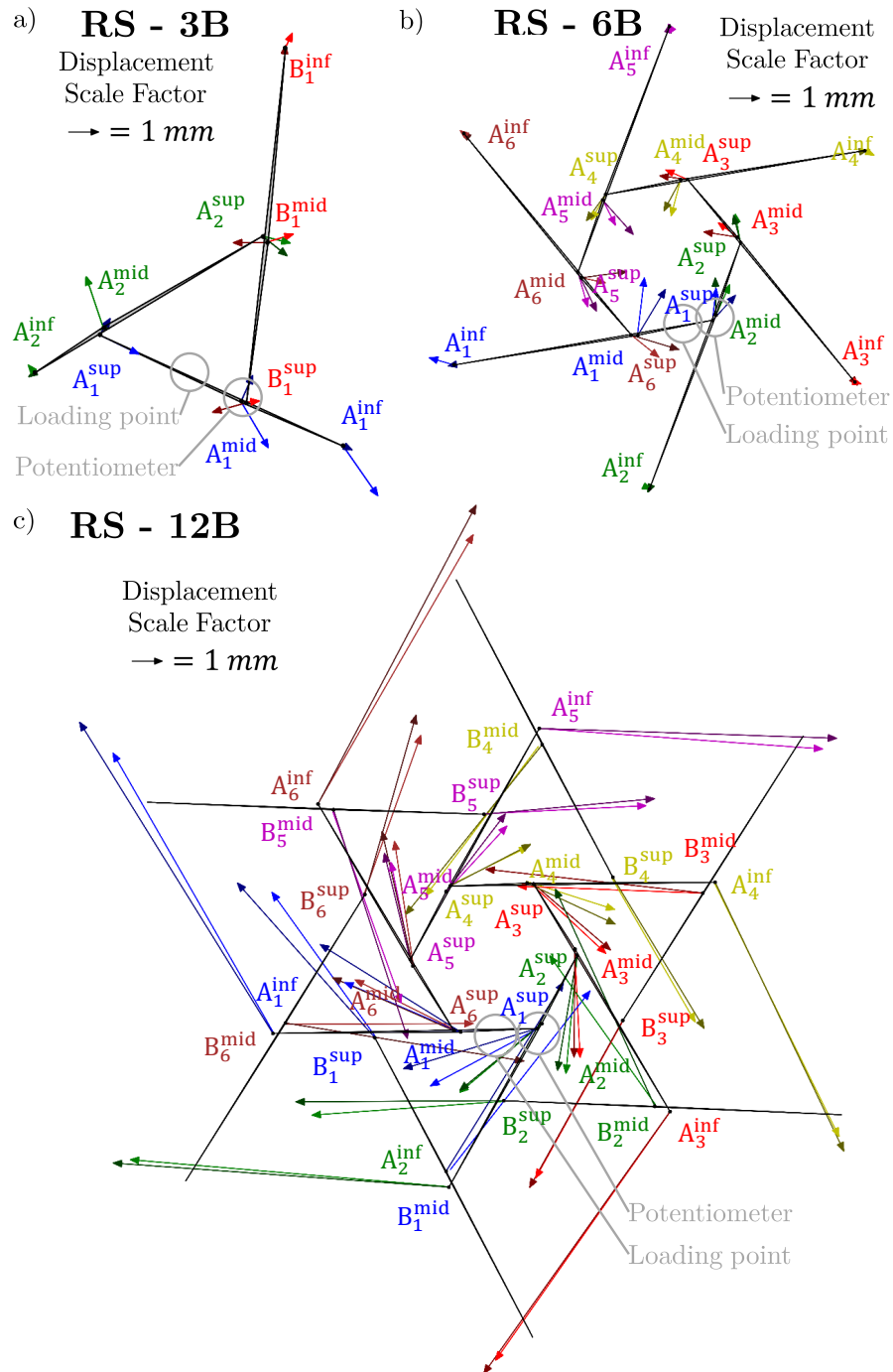


Figure 4.14: For prototypes RS-3B, RS-6B, and RS-12B, plan view of the structure's behaviour with measured displacement vectors (in colour) and calculated displacement vectors (in dark).

shows an error of -8.7%, similar to the potentiometer's measurement (-6.6%). However, larger errors occur at some measurement points, reaching up to 36.7% [Figure 4.13a]. These discrepancies are attributed to the horizontal components of the support reactions, which induce unexpected elastic bending of the support beams [Figure 4.6]. In particular, the marker A_1^{inf} presents a total horizontal displacement of 1.87 mm, compared to the computed 0.14 mm [Figure 4.14a].

For RS-6B, the marker closest to the potentiometer measurement (A_1^{sup}) shows an error of -3.6%, close to the potentiometer error (+2.4%) [Figure 4.13b]. The model predicts vertical displacements very well, with errors ranging from -3.6% to +6.1%, even though some supports moved horizontally due to the flexure of the support beams (A_1^{inf} and A_4^{inf}) [Figure 4.14b].

For RS-12B, the marker closest to the potentiometer measurement (A_1^{sup}) shows an error of +3.4%, similar to the potentiometer measurement (+7.2%). The model predicts the vertical displacements very well overall [Figure 4.13c], with errors for the 18 central markers (A^{sup} , A^{mid} and B^{sup}) ranging from +0.4% to +5.5%. For five of the twelve peripheral markers (B^{mid} and A^{inf}), the errors are larger, reaching -6.9% and +20.2%. Regarding the horizontal behaviour shown in Figure 4.14c, the model correctly predicts the “spinning-top” effect, with a slight overestimation for all markers. Unlike the other tests, the support nodes were not instrumented to prevent the cameras from being placed too far from the central markers. It is therefore not possible to determine whether some support points moved.

4.4.2.3 Measurements using strain gauges

Beams A2 and A4 are equipped with strain gauges enabling the determination of internal forces and maximum stresses. Figure 4.15 presents only the values of the total bending moment $M_{yz,tot} = \sqrt{(M_y^{loc})^2 + (M_z^{loc})^2}$ and the von Mises stress σ_{vM} during the last five loading cycles, which are perfectly superimposed. The other internal forces were also measured but are negligible, generating less than 1.5% of the von Mises stress. For RS-3B, only Beam A2 is instrumented. Figure 4.15a shows that the maximum bending moment is very close to the maximum numerical value, with an error of -2.5%. The maximum von Mises stress is also very close to the numerical value, differing by only 0.1%. For RS-6B [Figure 4.15b], the errors are also very small. Beams A2 and A4 exhibit maximum deviations of -1.5% and -2.3%, respectively, for the maximum bending moment, and -0.8% and -0.9% for the maximum von Mises stresses.

For RS-12B [Figure 4.15c], Beam A4 shows an error of -3.4% for the bending moments and -4.2% for the von Mises stresses. However, these errors are larger for Beam A2, reaching +10.9% and +10.6%, respectively.

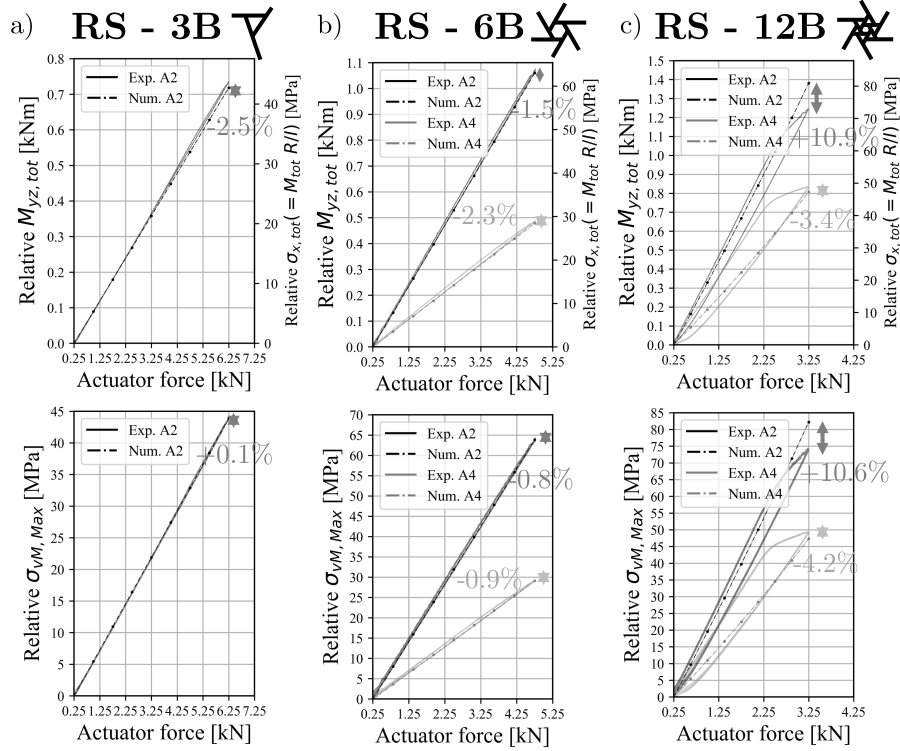


Figure 4.15: Comparison between the total bending moment and von Mises stresses (relative to the beginning of cycle 9) and those predicted by the numerical model for: a) RS-3B, b) RS-6B, and c) RS-12B.

4.4.3 Discussion

Validation of the numerical model and relevance of the hinged connection
With a few exceptions, which can be explained by more flexible support than expected, the numerical model's predictions are very good for the three structures. The redundancy of the experimental measurements, obtained with three different instruments, confirms that:

- The choice of a hinged connection is appropriate for reciprocal structures.
- The connection modelling approach described in Section 4.2, including experimental characterisation according to the defined protocol, is relevant and reproducible for other types of reciprocal structures and for real projects.

It should be noted that a clear hysteresis is visible for RS-12B in both displacements and stresses. This behaviour can be explained by friction at the ball-and-socket joints, as observed during local tests on the connection [Figure 4.4]. Since these joints were reused across different tests and assemblies, it would have been advisable to apply additional lubrication in addition to

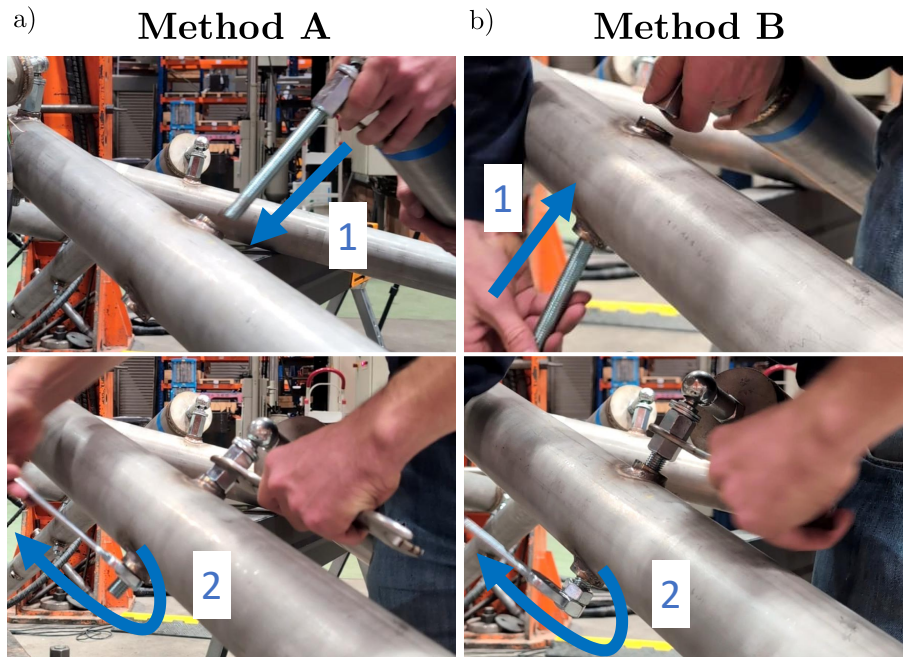


Figure 4.16: Assembly phases for Methods A and B.

the factory-applied grease. Other sources of hysteresis can be excluded as the structures are tested in their elastic range, and the loading rate is very slow.

4.4.3.1 Ease of assembly

The proposed connection [Figure 4.3] enables rapid assembly and disassembly of the structures: RS-12B can be assembled in less than 30 minutes by four people (excluding the time required to anchor the supports to the test floor). The proposed connection allows two assembly methods:

- **Method A** [Figure 4.16a]: the threaded rod is first fixed to the sleeve, then inserted into the tubular reinforcement (Phase 1). The connection is then secured by the locking nut (Phase 2). This method is very fast but requires sufficient separation between the two tubes during assembly. If this is not possible—for example, when the supported tube is already connected to other tubes—Method B must be used.
- **Method B** [Figure 4.16b]: the sleeve attached to the joint is first aligned with the tubular reinforcement, after which the threaded rod is inserted into the tubular reinforcement (Phase 1). In Phase 2, the threaded rod is screwed into the sleeve.

4.4.3.2 Choice of instrumentation

The Motion Capture System (MCS) is highly effective because it enables measurement of displacements at many points and facilitates the assembly and

positioning of the supports. In addition, it enables observation of the spinning-top effect associated with the nodes' horizontal displacements. However, the markers must be positioned as accurately as possible at locations that are easily identifiable in the numerical model, to facilitate their correspondence with the model nodes. Strain gauges are also advantageous because they are less sensitive to parasitic effects caused by support displacements, since the structures are isostatic. Finally, the cable potentiometer provides a more traditional measurement, enabling, at least, local validation of the results obtained with the MCS.

4.5 Assembly and testing of the full structure RS-24B

The RS-24B structure was constructed directly on an outdoor site because of its large size (9.4 *m* in diameter with a height of 3.5 *m*, as shown in Figure 4.1).

4.5.1 Assembly

The assembly was carried out in four stages, illustrated in Figure 4.17:

- **Preparation** [Figure 4.17a]: concrete anchoring slabs are placed in holes excavated to a depth of at least 25 *cm*. A temporary platform is built at the centre, with temporary supports equipped with threaded pads allowing vertical adjustment of a few centimetres.
- **Pre-assembly, transport and placement of RS-12B** [Figure 4.17b]: the RS-12B, weighing at least 180 *kg*, is first assembled in the laboratory and braced with cables to preserve its geometry during transportation. It is then placed on the temporary platform by lifting equipment. The tubes of ring B rest on pads of different heights to ensure the desired inclination.
- **Assembly of the peripheral tubes** [Figures 4.17c, 4.17d, and 4.17e]: the tubes of the two lower rings are installed one after another. Once the structure is complete, the supports are fixed to the slabs using chemical anchors.
- **Finishing**: the foundations are covered with soil, and the temporary support system is removed.

This assembly sequence is inspired by that used at the Centre Masureel [Figure 1.10b], where the inner modules were first assembled and temporarily propped at the desired height, before the supporting beams were installed. Nevertheless, other assembly strategies were also considered:

- Installing and fixing the lower rings before bringing the pre-assembled RS-12B. This option avoids constructing the temporary platform but carries a higher risk. In addition, the required geometric tolerance is larger because the supports' positions relative to the foundations cannot be adjusted.



Figure 4.17: a) Site preparation. b) The lifting equipment is placing the RS-12B on the temporary supports. c) Tubes of rings C and D are being installed sequentially. d) View of the central area. e) Tube supports are anchored in the concrete slabs.

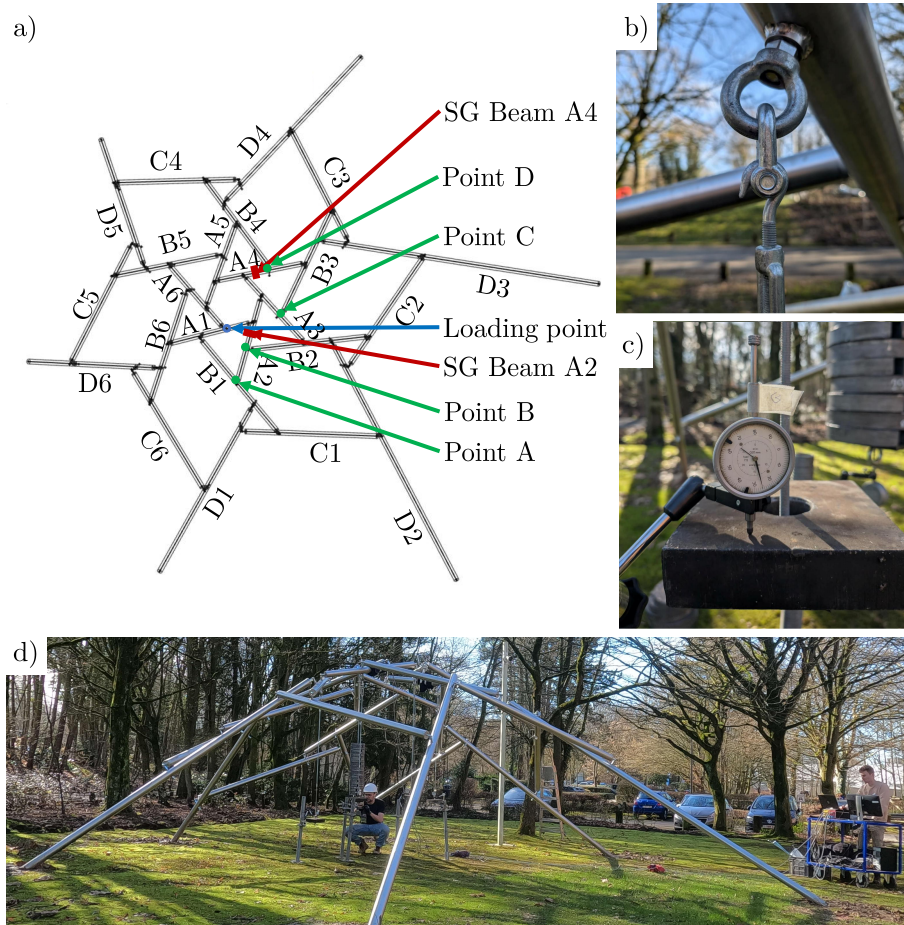


Figure 4.18: a) Plan view of RS-24B with the locations of measurement and loading points. b) Loading attachment and displacement measurement device. c) Displacement measurement device. d) Overall view of the test site.

- Assembling the structure tube by tube, starting from the lower rings. Despite the large number and size of temporary supports required, this solution would have been adopted if the lifting equipment had been unable to access the site.

4.5.2 Experimental setup

Figure 4.18a shows a plan view of RS-24B, depicting the entire experimental setup. Because the test was carried out in situ, this setup differs from that used for the three other RS prototypes:

- The load is applied by suspending masses from a lifting ring attached beneath the connection linking Beams A6 and A1 [Figure 4.18b].
- The loading protocol consists of two cycles from 0 to 2354 N, with 196 N increments applied with masses of 20 kg.

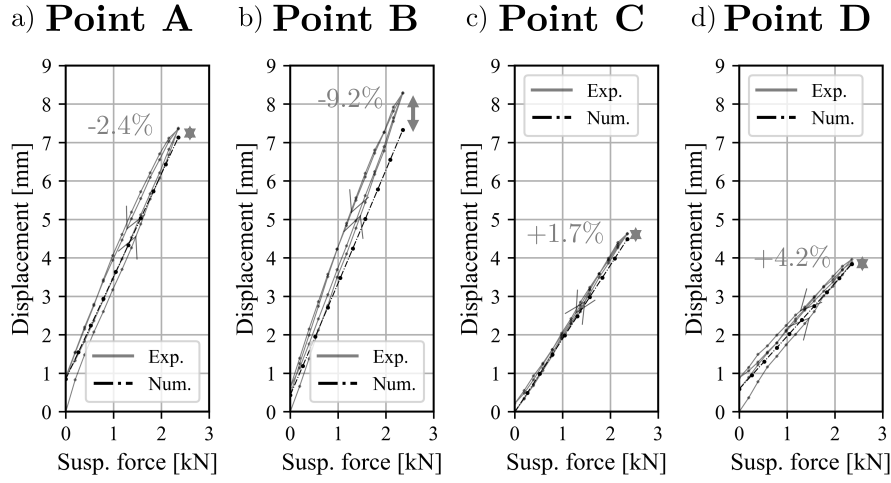


Figure 4.19: Vertical displacement at four points of RS-24B under suspend: a) Point A, b) Point B, c) Point C, and d) Point D.

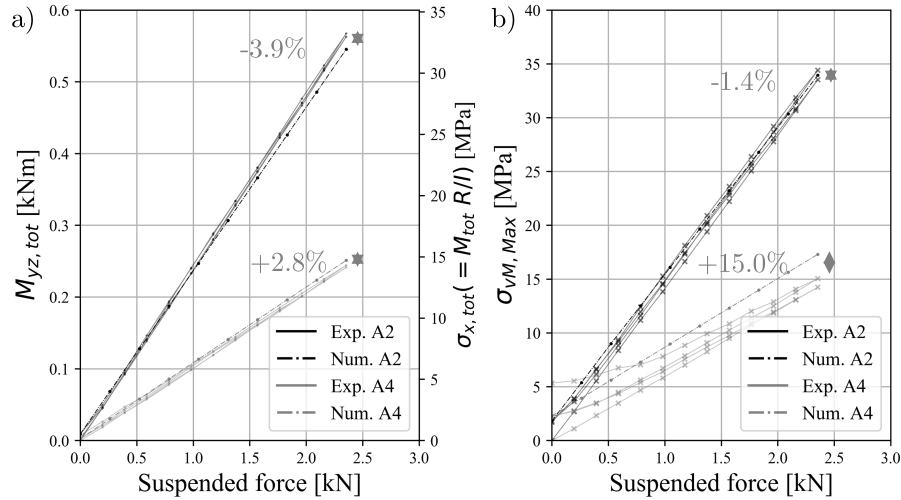


Figure 4.20: Bending moment and von Mises stress for Beams A2 and A4 of RS-24B.

- The vertical displacement is measured using mechanical dial comparators placed at four points. To limit the influence of the spinning-top effect, each dial comparator is mounted on a ballasted plate and attached to the structure, as with the loading device [Figure 4.18c].
- Strain gauge measurements could also be performed because the gauges had been protected before assembly.

4.5.3 Test results

As for the three other RS prototypes, the experimental results are compared with numerical predictions. The geometry of the numerical model was obtained from a LiDAR survey of the constructed structure. The resulting point cloud enables the determination of the tubes' centroidal axes and hinge positions. The supports, modelled as perfectly fixed, are located at the tube ends at the interface with the concrete slab. Their depth was measured before being covered.

The vertical displacements measured at four points of the structure are denoted A to D [Figure 4.18a]. The first cycle shows a significant settlement effect, whereas the second does not. Consequently, the numerical results are compared with the measurements during the loading phase of the second cycle. The displacement measurements show good agreement with the numerical model [Figure 4.19]. The errors are -2.4% at Point A, -9.2% at Point B, +1.7% at Point C, and +4.2% at Point D. Figure 4.20 presents the bending moments and stresses calculated from the strain-gauge measurements on Beams A2 and A4. The bending moments are very similar to the numerical results, with an error of -3.9% for Beam A2 and +2.8% for Beam A4. Regarding the von Mises stresses, the error is very small for Beam A2 (-1.4%), but larger for Beam A4 (+15.0%). This difference can be explained by a greater discrepancy between the numerical and experimental shear force results.

4.6 Conclusion

This chapter demonstrates that real metallic reciprocal structures can be designed using reliable numerical modelling, thereby increasing structural safety. This is achieved by implementing hinged connections, which must first be tested in the laboratory according to a precise protocol.

To achieve this objective, the chapter proposes a hinged connection and a characterisation protocol. This protocol is validated through tests on several large-scale prototypes: RS-3B, RS-6B and RS-12B (tested in the laboratory), and RS-24B (tested in situ). The results show excellent agreement between numerical and experimental results, with discrepancies generally below a few per cent. The occasional deviations observed can mainly be explained by secondary effects, notably the supports' unexpected flexibility or geometric imperfections.

The redundancy of the measurement systems (potentiometer, Motion Capture System and strain gauges) strengthens the reliability of the conclusions and enables a detailed analysis of the three-dimensional structural behaviour, including the horizontal "spinning-top" effects characteristic of RS^{3D}.

In addition to the mechanical validation, this work highlights the constructive advantages of the proposed connection. As the number of beams increases, the assembly sequence becomes more complex and the required

geometric tolerances become more stringent. The developed connection facilitates these operations thanks to its simplicity of implementation and the ability to adopt different assembly strategies.

Chapter 5

General Conclusion

5.1 Summary of Findings

Reciprocal structures (RS) are space structures that resemble “pick-up sticks”, in which each element supports and is supported by others, with no clear hierarchy. They enable striking architectural forms and large spans using relatively small beams. This characteristic motivates the use of log beams in RS, as logs are limited in length and may offer a more sustainable construction solution. As highlighted throughout this research, reciprocity results in iterative geometric design processes, non-intuitive mechanical behaviour, and sequential construction phasing. These features explain both the appeal of RS and designers’ hesitation to adopt it in standard practice.

This thesis primarily aimed to improve understanding of and predictability for the structural behaviour of RS and to establish a more robust design framework to support their broader adoption. It addressed a wide range of materials, since the identified challenges are not material-specific, and considered multiple RS typologies, with particular attention to MM RS^{3D} systems.

The structural efficiency of reciprocal structures was assessed by comparing rectangular RS log floor systems with conventional floor configurations, either composed solely of joists or incorporating transverse primary beams. The results show that square RS floors can be structurally competitive with conventional floor systems under appropriate design conditions. These conclusions are enabled by the morphological indicator method, which reduces a complex design space to a limited set of governing parameters, allowing general design trends to be identified.

The second key outcome concerns the geometric stability of RS, addressing concerns about mechanical behaviour, particularly instabilities arising from rigid-body rotation of beams about their longitudinal axes. Present RS connections are often rotationally fixed to prevent these instabilities, which contradicts the true RS behaviour as a series of stacked elements with rotationally free ends. This thesis demonstrates that hinged connections can indeed produce stable spatial RS. In these cases, the longitudinal rigid-body

rotation is maintained by the lever arm created by the distance between the beam axes at the connection point. However, this role does not apply to planar RS, where all beam axes are aligned, or to spatial RS geometries with central symmetry. In these cases, fixing the rotations of certain connections or supports is mandatory to stabilise the RS. These conclusions are independent of the beams' loading and cross-sectional properties, as determined using a stability assessment method from the literature that was adapted for RS.

In addition to stability considerations, this thesis addressed the reliability of numerical modelling for reciprocal systems. Significant discrepancies between numerical predictions and experimental observations are often attributed to nonlinear and semi-rigid connection behaviour. To address this, large-scale steel prototypes were constructed and tested, leading to the development and validation of a spherical hinge connection for RS^{3D}. An experimental methodology was also established to characterise this behaviour, enabling more reliable numerical modelling. The experimental campaign relied on a redundant measurement system, including motion-capture technology, which enabled accurate measurement of the three-dimensional spinning-top effect and the structure's geometric nonlinearity.

More generally, this work highlights that connection design cannot be treated as a secondary detailing step. It fundamentally governs the geometry, structural behaviour, and erection sequence of reciprocal structures and must therefore be integrated from the earliest stages of design. In this respect, successful RS design requires a fully integrated approach in which form generation, connection behaviour, and construction strategy are developed concurrently.

5.2 Applicability of RS

RS offer a combination of modularity, adaptability, and architectural expressiveness that makes them suitable for roofs, floors, and pavilions, as illustrated by the built projects [Figures 1.9, 1.10, and 1.11]. The successful application of RS depends on decisions made at the earliest design stages, since the reciprocal principle influences not only the structural form but also the connection detailing and the erection strategy. Nevertheless, the use of reciprocity is particularly relevant in three situations:

- When the designer needs to construct with small structural elements, light enough for one or two people to handle, RS allows the necessary length of the element to be reduced, making it more transportable and easier to manipulate on intricate construction sites or in remote locations, as demonstrated by the Bribracte project [25]. In the same vein, reciprocal structures are also attractive for temporary structures, where rapid and repeated assembly and disassembly are often required.

- When the designer needs to construct using reused structural members recovered from dismantled buildings, the RS's shape can be adapted to incorporate members of different lengths and sections. This allows for irregularity, as in the extension of the Euskalduna conference centre [Figure 1.11b].
- When the designer integrates the building's end-of-life and evolution, symmetry and repetition in the shape allow for simplified fabrication, maintenance, replacement, and eventual dismantling, while facilitating future reuse of the components or extensions of the building. For example, the Iseya townhouse [Figure 1.10a] was constructed using only six beam types, and the regular floors of the Mont-Laurier library [Figure 1.11a] used 3 types of beams.

5.3 Outlooks

The findings of this work enable more reliable RS design and demonstrate the promising use of floor RS in specific geometries. These results can serve as a basis for exploring other potential benefits and expanding the use of log for RS^{3D}.

5.3.1 Log RS^{3D}

Chapter 4 focused on the experimental validation of the modelling of a hinged connection. The beams were therefore made of steel to limit any discrepancies during testing arising from their geometric and mechanical properties.

As mentioned, the use of logs in RS is particularly relevant because they have finite lengths. However, several challenges must be addressed to enable broader application: selecting and preparing logs, integrating with the rest of the building, and detailing connections.

A primary issue concerns selecting and preparing logs. Wood species directly influence the available dimensions and mechanical properties of the elements. Depending on their structural role, logs may require characterization of both geometric and mechanical properties. Geometric data can be obtained through manual measurements or advanced scanning techniques. Logs exhibiting excessive curvature, conicity, or spiral grain are generally unsuitable for structural applications. Strength grading may be performed using non-destructive methods, such as elastic loading, dynamic excitation, X-ray scanning, or visual inspection, all of which are based on the statistical relationship between elastic properties and strength. Recently, Goosse et al. [27] proposed a proof-loading protocol to determine both the modulus and the load-bearing capacity with minimal damage to the log.

Another key challenge lies in integrating the building envelope with log RS^{3D}. Different strategies for covering RS are presented in Chapter 1 [Figures 1.9, 1.10 and 1.11], but they need to be adapted to the irregular shape of logs:

- Use spacers combined with a substructure supporting the roof [Figures 1.9b, 1.9c, 1.10a, and 1.10d]. The logs are then punctually connected to this secondary structure, which needs to be properly detailed. Alternatively, roofs can be suspended, with the RS placed outside, or made of stretched fabric.
- Add timber panels in the engagement windows [Figures 1.10d, 1.10b and 1.11a]. The panel is then placed in a continuous groove to ensure plane support. The panel can additionally serve as bracing [43]. Though, this solution is feasible only for RS^{2D} or RS^{3D} with small eccentricity.

The design of connections between logs constitutes another major challenge. This work began by highlighting the difficulties of reliably modelling the JJP connection [Figure 3.5a], and Chapter 3 extended to the Wood-wood connections proposed in the literature. For RS, both stiffness and strength need to be correctly predicted. In addition, these properties need to be evaluated for the 6 DoF of the connections, as the connection can not reasonably be simplified to a hinged connection, and no symmetry can be used to limit the characterization. Given the variability in timber properties, the number of experimental tests would be increased proportionally. One procedure could start by validating a 3D finite-element model of the connection to ensure the correct deformation distribution. This validation is performed through local testing using, in addition to standard instrumentation, digital image correlation for visible exposed surfaces and a fibre-optic system for contact surfaces. This instrumentation was recently used for a beam-to-column dovetail connection under tension [85]. Based on the numerical model, the stiffness of each DoF can be extracted, along with the impact of creep and moisture variations on the geometry.

Alternatively, steel-based connections provide a more controlled solution. In this context, adapting the joint developed in Chapter 4 [Figure 4.3] to logs represents a promising direction. The threaded rods are then replaced with double-threaded stud screws, screwed directly at the log end or radially. However, the capacity of the proposed connection for larger structures. Therefore, a hinged connection with higher capacity is needed.

5.3.2 Optimal RS^{3D}

The results obtained in Chapter 2 call for extending RS^{3D} and comparing it with gridshells.

The first part is to quantify the influence of each geometric parameter

on the structural response. In particular, it remains unclear whether performance is improved by minimizing or maximizing:

- Member density for a particular tessellation pattern [Figure 1.4].
- The geometric ratio H/L (i.e., structural curvature).
- the engagement ratio α .
- the ratio L_b/L (i.e. the relative size of individual elements).
- the proportion of a given module style (i.e. alternative or identical style for all modules).

Based on these results, designers can wisely select the input variables for the second type of geometry-generation "Modules Follow Form" method [Section 1.4.1].

It should be emphasized that existing studies [18, 50, 8, 34, 79] rely on distinct optimization objectives, RS patterns, global geometry, and structural assumptions, which limit the direct comparability of their results. In some cases, these differences may even lead to contradictory conclusions. To address this question systematically, a general methodological framework needs to be defined, structured around two key steps.

First, the geometry generation in RS^{3D} remains a major challenge due to the interdependence of geometric parameters. For covering a large number of geometries, the procedure should be as automated and robust as possible. In this context, the third type of methods appears to be the most suitable, as the final geometry does not need to be an input parameter. However, it is necessary to distinguish driving and dependent parameters to determine the convergence and acceptability of the generated geometry. A practical approach is to select a set of tessellated patterns and the covered plane surface, thereby fixing the bar density and support position. For each initial geometry, key parameters such as module style, engagement ratio, and eccentricity must be prescribed. The generated geometries that converge toward the target parameter values (within a defined tolerance) can then be subjected to structural analysis.

Second, the evaluation of structural performance may be conducted following the methods presented in this thesis:

- The morphological indicators method, presented in Chapter 2.
- The discussion around the condition number κ from the CMAM, presented in Chapter 3.

The first option can be limited to minimizing the MI of volume, as the thickness is of secondary importance in spatial structures. To adapt the proposed method to RS^{3D} , there are two main concerns: the presence of normal forces and torsional moments, and the loading pattern with a horizontal component. For the first one, integrating these internal forces would necessitate additional assumptions. However, as the MI method is mainly used at the early design stage, one could neglect the impact of these forces on stresses

and displacements. In addition, Chapter 4 showed that stresses are principally generated by bending moments. Next, the choice of loading pattern is an intricate question that must be tailored to the final generated geometry. The ratio between the total vertical load and the total horizontal load may be added as an additional variable influencing the MI.

The alternative option is based on the comparison of the condition number κ , derived from the CMAM, which has the advantage of being independent of the cross-section and the loading pattern. However, as a global indicator, it does not capture localized effects and stress concentrations.

The second research question concerns whether optimized RS configurations are more efficient than alternative structural systems. The existing literature on this topic remains limited.

Some comparative studies can nevertheless be identified. Douthe and Baverel [18] report that RS exhibit strength comparable to gridshells, although they are generally less stiff. Gelez et al. [24] highlight RS^{2D}'s superior ability to redistribute concentrated loads. Similarly, Garavaglia et al. [22] observe comparable collapse loads between RS^{2D} and gridshell systems. More generally, Brocato [9] suggests that the structural efficiency of RS may be improved by minimizing axial forces and maximizing bending moments within the elements, thereby reducing forces transmitted through the joints. This assumption is based on the idea that RS connections primarily transfer forces through friction, implying that beams behave as if they were simply superimposed. Under this hypothesis, RS may be particularly advantageous for structures with low curvature.

Given a prescribed envelope surface, a corresponding gridshell geometry can be generated, enabling direct comparison between the two systems. Such comparisons should also consider asymmetric loading cases, as structural performance is expected to be highly sensitive to load patterns, particularly the relative contributions of permanent and variable loads.

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List of symbols

Latin symbols		
Symbol	Units	Property
a	$[m]$	Engagement length of a beam
A	$[m^2]$	Area of a beam
A_v	$[m^2]$	Effective shear area of a beam
b_i	$[m]$	Secondary engagement length of beam i
CLT		Cross-Laminated Timber
B		Kinematic Compatibility Matrix
C		Constraints Matrix
d	$[m]$	Centre-to-centre distance between two joists
d_j, d_{tot}	$[-]$	Number of beams of module j or an RS
DoF		Degree of freedom in a connection or support
E	$[N/mm^2]$	Elastic modulus
EWP		Engineered Wood Products
$f_1, f_{1,lim}$	$[Hz]$	Fundamental and limited fundamental frequency of a beam
f	$[-]$	Number of single modules in an RS
f_b	$[N/mn^2]$	Design bending strength of a beam
f_v	$[N/mn^2]$	Design shear strength of a beam
f_y	$[N/mn^2]$	Yield strength of a beam
$F, F_{SLS}, F_{SLS,LT}, F_{ULS}$	$[N]$	Total equivalent force on a floor
$F_{Actu}, F_{Actu,max}, F_{Actu,min}$	$[N]$	Force applied by the actuator : maximum and minimum
g	$[m/s^2]$	Gravity acceleration
G	$[N/mm^2]$	Shear modulus
Glulam		Glued-laminated timber
H	$[m]$	Height of the structure
I	$[mm^4]$	Second moment of inertia of a beam
I_p	$[mm^3]$	Polar moment of inertia of a beam
J		Joist
K		Stiffness matrix
K	$[-]$	Number of bays of a pattern

Symbol	Units	Property
l	[m]	Length of a beam
L	[m]	Longest distance between two supports
L_1, L_2	[m]	Longest and shortest dimensions of a floor
L_b	[m]	Reciprocal framed beam length
LVL		Laminated Veneer Lumber
M_{ULS}	[Nmm]	Maximal bending moment at ULS somewhere
M_x^{loc}		Local torsional moment
M_y^{loc}, M_z^{loc}	[Nmm]	Local bending moments around y and z axis
$M_{yz, tot}$		Total bending moment combining M_y^{loc} and M_z^{loc}
MCS		Motion Capture System
MI		Morphological Indicator
MM RS		Multiple Module Reciprocal Structure
n	[−]	Number of joists
n_{int}, n_{tot}	[−]	Number of connections or supports
n_k	[−]	Number of mechanisms
N^{loc}	[N]	Local normal force
OSB		Oriented Strand Board
p	[−]	Total number of DoF
PB		Primary Beam
PLSB		Peripheral Long Span Beam
PSB		Peripheral Beam
PSSB		Peripheral Short Span Beam
$q, q_{SLS}, q_{SLS,LT}$	[N/m ²]	Uniformly distributed surface load
q_{ULS}		
r	[−]	Rank of matrix
r_{ext}	[mm]	Outer radius of tube
r_{int}	[mm]	Inner radius of tube
R_x, R_y, R_z		Rotational degrees of freedom
RS, RS ^{2D} , RS ^{3D}		Reciprocal structures
s_i, s_{min}, s_{max}	[−]	Singular values
S	[−]	Morphological indicator of thickness
SDR		Small-Diameter Roundwood
SG		Strain gauge
SLS		Short-term service limit state
SLS,LT		Long-term service limit state
SM RS		Single module reciprocal structure
St		Style of a module
Sw	[m ²]	Engagement window of a module
T	[−]	Thickness of whole floor

Symbol	Units	Property
T_x, T_y, T_z		Translational degrees of freedom
ULS		Ultimate service limit state
$U, V, \varphi, \mathbf{d}$		Rigid body motions
$U_i, \mathbf{u}$		Global translational DoF
V	$[m^3]$	Volume of a whole floor
V_y^{loc}, V_z^{loc}	$[N]$	Local shear forces
V_{ULS}	$[N]$	Maximum shear force at ULS somewhere
$\mathbf{v}$		Eigenvector
W_{el}	$[mm^3]$	Section modulus of a beam
W	$[-]$	Morphological indicator of volume
X, Y		Floor directions
$x^{loc}, y^{loc}, z^{loc}$		Local coordinate system
Z_{Lt}	$[-]$	Linking parameter between SLS and long-term SLS
Z_R	$[-]$	Linking parameter between bending and shear resistance criteria

Greek symbols		
Symbol	Units	Property
α, β	[-]	Engagement ratio for engagement length a and b
γ	[°]	Orientation angle between two beams in a module
δ_e, δ_p	[mm]	Experimental displacement of the hinge connection
$\delta_{max}, \delta_{max,Lt}$	[mm]	Maximal displacement of a beam
ε_i''	[-]	Deformation of a strain gauge i
$\varepsilon_M, \varepsilon_V, \varepsilon_D, \varepsilon_F$	[-]	Coefficients linked to the load distribution
η_{cov}	[-]	Participation mass ratio
θ_i	[°]	Inclination angle of beam i
κ	[-]	Condition number
κ_s	[-]	Shear coefficient
λ	[-]	Shape factor (L_2/L_1)
λ_i	[-]	Eigenvalue of stiffness matrix $\mathbf{K}$
μ	[°]	Angle between the local and the strain gauges coordinate systems
$\mu_{SLS}, \mu_{SLS,Lt}$	[-]	Maximum relative deflection factor
ν	[-]	Poisson ratio
ν_i	[m ³]	Volume of log i
ξ^i	[-]	Thickness reduction factor
σ_{vM}	[N/mm ²]	von Mises stress
σ_x	[N/mm ²]	Normal stress
τ_{xz}	[N/mm ²]	Shear stress
ϕ_i	[mm]	Diameter/thickness of log i
φ	[°]	Position angle of a Strain Gauge
ψ_K	[-]	Occurrence of an element in a pattern
ω	[°]	Orientation angle of a Strain Gauge
ω_K	[-]	Length adaptation factor

