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Modelling and analysis of two-wheelers equipped with crowned wheels via a constrained MBS approach

Q. Docquier, P. Fisette
iMMC, Institute of Mechanics, Materials and Civil Engineering
Université Catholique de Louvain
Place du Levant 2, 1348 Louvain-la-Neuve, Belgium
e-mail: [quentin.docquier,paul.fisette]@uclouvain.be

State of the art

Over the past two decades, bicycle dynamics has been the subject of many scientific investigations, from the modelling of the rider behavior and the bicycle geometrical optimization to the prototyping of steer-by-wire bicycle. To this extent, the Whipple bicycle model consisting of four rigid ideally-hinged parts (i.e., the two wheels, a frame and a front assembly) has been extensively used. The popularity of this model is mainly due to its simplicity of implementation and utilization which has been faciliated by the work of Meijaard and al. in 2007 [1]. In their article, they derived the linearized equations of motions for a fully parametrized bicycle. Through the use of non-holonomic rolling constraints for the wheel-ground contact and considering a given rotational speed for the rear wheel, a set of 2 linearized equations are obtained in terms of the bicycle roll angle, ϕ and steer angle, δ (see [1] for the details).

$$M\ddot{q} + vC_1\dot{q} + [gK_0 + v^2K_2]q = f \tag{1}$$

Besides the assumptions of rigid bodies, the axisymmetric wheels make ideal *knife-edge* rolling point-contact with the level ground. This assumption neglects the crown radius of the tires and limits the model applicability to bicycle equipped with narrow tires. Extension of the model for this particular aspect has been realized and verified with a multibody system (MBS) modelling program [2]. However, very little is known on the tire crown radius and its influence on the bicycle dynamics and stability. The modelling and analysis of the pure rolling motion for such profiled wheels could be of great help for the understanding of both bicycle and motorcycle dynamics (e.g., the influence of fat tires on the stability).

Pure rolling motion through a relative MBS formalism

To model the bicycle dynamics, a multibody system approach is considered as it performs an automatic derivation of the equations of motion and allows for an easy modification of the system morphology and parameters. The derivation and treatment of non-holonomic rolling constraints within a multibody system formalism is far from obvious. In addition, to comply with the whipple model assumptions, rigid contact constraints have to be considered between the wheels and the ground. The equations of motion for a constrained MBS can be written in terms of q, the *relative* generalized coordinates (see [3] for details):

$$M(q)\,\ddot{q} + c(q, \dot{q}, frc, trq, g) = Q(q, \dot{q}) + J\lambda \tag{2}$$

$$h(q) = 0 \tag{3}$$

$$\dot{h}(q, \dot{q}) = J(q)\dot{q} = 0 \tag{4}$$

$$\ddot{h}(q, \dot{q}, \ddot{q}) = J(q)\ddot{q} + \dot{J}\dot{q}(q, \dot{q}) = 0 \tag{5}$$

The pure rolling constraints denoted as $g_{pr}(q, \dot{q}) = 0$ and the rigid contact constraint $h_{rc}(q) = 0$ are simply added to the non-kinematic constraints $h_l(q) = 0$ (i.e. loop closure). Gathering these m constraints together, one can write at position and velocity level:

$$h(q) = \begin{pmatrix} h_l(q) \\ / \\ h_{rc}(q) \end{pmatrix} = 0 \qquad \dot{h}(q, \dot{q}) = \begin{pmatrix} \dot{h}_l(q, \dot{q}) \\ g_{pr}(q, \dot{q}) \\ \dot{h}_{rc}(q, \dot{q}) \end{pmatrix} = 0 \qquad [m * 1]$$

The challenge resides in deriving the corresponding lines of the terms $J(q)$ and $\dot{J}\dot{q}(\dot{q}, q)$ for both the pure rolling and rigid contact constraints. Once completed, the set of DAE equations (2-5) can be reduced to an ODE system by using the so-called coordinate partitioning technique [3]. The reduced set of 2 equations can then be linearized and used to perform modal analysis in a similar way than in [1] on the basis of equation (equation (1).

Preliminary results

The implementation of the wheel ground constraints is done within the MBS software, Robotran [4]. Results for the bicycle benchmark developped in [1], on the form of a stability analysis, have first been obtained (Fig. 1). For the benchmark bicycle equipped with a front disc wheel and a rear wheel with finite crown radius $R = 0.3$ m, the weave mode is less stable but the capsize mode is more stable.

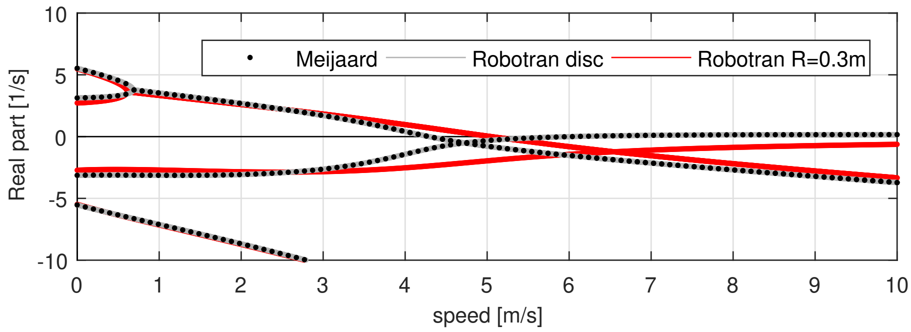

Figure 1. Real part of the eigenvalues for the bicycle benchmark and different rear tire width.

Based on the proposed tire/ground contact model, this research focuses on the influence of the tire transverse radius on the bicycle stability in the low-speed range. The goal is to determine how the dynamic behaviour is affected by analysing the evolution of the modes while increasing or decreasing the tire transversal radius on both the front and the rear wheel. Finally, the developed model is used to analysis the dynamic stability of adapted child bikes whose rear wheel has been replaced by a crowned roller [5]. To do so, the roller profile influence has to be analysed alongside other important dynamical parameters such as the crowned roller inertia or the height of the rotation axis compared with that of the bicycle center of mass.

References

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