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Weyl-invariant Einstein-Cartan gravity: unifying the strong CP and hierarchy puzzles

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Abstract: We show that the minimal Weyl-invariant Einstein-Cartan gravity in combination with the Standard Model of particle physics contains just one extra scalar degree of freedom (in addition to the graviton and the Standard Model fields) with the properties of an axion-like particle which can solve the strong CP-problem. The smallness of this particle's mass as well as of the cosmological constant is ensured by tiny values of the gauge coupling constants of the local Lorentz group. The tree value of the Higgs boson mass and that of Majorana leptons (if added to the Standard Model to solve the neutrino mass, baryogenesis and dark matter problems) are very small or vanishing, opening the possibility of their computability in terms of the fundamental parameters of the theory due to nonperturbative effects.

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Contents

1 Introduction

Gauge symmetries are pivotal for our understanding of Nature. In the Standard Model (SM) of particle physics, the strong and electroweak interactions arise from the gauging of the internal $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ groups. Gravity can also be derived as gauge theory of the Poincaré group [1–3], which singles out the Einstein-Cartan (EC) formulation of General Relativity (GR) [4–7] with the tetrad and connection the associated gauge fields; confer for instance the classic review [8]. Yet another important gauge symmetry is that of Weyl [9–12], which changes locally the length of rulers.[1]

This paper aims to show that the minimal theory which embraces the SM and Weyl-invariant quadratic gravity in the EC formulation has several attractive features. It contains one extra (pseudo)scalar degree of freedom in addition to the graviton and the particles of the SM. This new degree of freedom has all the properties of an axion-like particle (ALP). The constructed theory provides insights into several puzzles of the SM. One of the gauge couplings associated with the Lorentz group determines the classical value of the cosmological constant, and so we know from phenomenology that this parameter has to be extremely small. It is then logical to expect that the other gauge coupling(s) be equally small. Amazingly, those determine the Higgs and ALP masses (the former being generated dynamically in

[1] A small side remark on terminology is now in order. Since the flat limit of theories invariant under Weyl transformations are automatically conformal theories [13], the notions of Weyl/conformal are sometimes mixed or even used interchangeably. Nevertheless, they are distinct from each other, see e.g. [13–15], thus Weyl transformations should not be simply thought of as the gravitational analog of conformal transformations. Rather, the notion of conformal symmetry only makes sense on specific backgrounds.

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our framework). Moreover, since our theory automatically accommodates a coupling of the ALP to the topological charge density of QCD, it can relax the strong CP-angle $\bar{\theta}$ close to zero, i.e., leads to a purely gravitational solution to the strong CP-problem. Evidently, all of this fully resonates with the expectation that gravity is the weakest force [16]. *In short, we provide a common gravitational origin for the seemingly independent tunings of the cosmological constant, Higgs mass, and θ-angle.*

Taken at face value, the classical theory constructed along these lines contradicts to observations — the Higgs boson mass is too small. However, this should not be considered as the weakness of the proposal, *this is actually its strength.* Indeed, the classical action is not the whole story. The smallness or absence of the Higgs and ALP masses persists in scale-invariant quantum perturbation theory, as the zero values of these parameters correspond to fixed points of their renormalization group evolution [17, 18]. This makes them in principle computable in terms of the parameters of the underlying theory, once nonperturbative effects are taken into account [19–24]. As fas as the ALP is concerned, these effects are well understood and are associated with the non-trivial topological structure of the QCD vacuum [25, 26], leading to nonperturbative mass generation [27, 28]. If the QCD contribution is larger than the gravitationally induced tree-level mass, the strong CP problem is solved in exact similarity with the conventional axion [29–31]. Concerning the Higgs boson mass, its nonperturbative origin is more speculative and requires further investigations, which we leave for future work. A possibility certainly worth exploring is that the hierarchy between the Fermi and Planck scales be attributed to the semiclassical suppression of gravitational-Higgs instanton amplitudes, as proposed in [22, 24] (see also [23, 32]).

Exact or approximate scale and Weyl invariance have both been evoked previously in attempts to solve the hierarchy problem, see [18, 23, 32–80] for a non-exhaustive list of references. An important ingredient of previous studies of *exact* symmetries is an extra dilaton field χ, implementing the idea of spontaneously broken scale invariance. The main drawback of these theories is that both the Weyl and scale symmetries allow for a dilaton-Higgs interaction of the type $\alpha\chi^2 H^\dagger H$, where H is the Higgs field. In the presence of an ALP a, the allowed terms in the action are $c_a\chi^2 a^2$. With the dilaton vacuum expectation value of the order of the Planck mass, phenomenology dictates that the coupling constants α, c_a have to be very small without any obvious reason why this should be the case. In the theory we present in this paper, there is no any additional scalar field, all degrees of freedom are contained in the gauge fields of the gauged Poincaré group. The "dangerous" dilaton type couplings are either absent or automatically tiny.

Similar considerations comprise [81, 82] based on the idea of "restricted Weyl invariance" introduced in [83],[2] the works [85–87] based on the Weyl conformal geometry and [88] using global scale invariance. All have a gravitational sector which contains the associated scalar curvature squared, as well as couplings to the SM via a gravitational-Higgs portal. The usual Einstein-Hilbert action with a nonvanishing cosmological constant is recovered in the spontaneously broken phase. The Higgs sector also gets nontrivially modified in that a tachyonic mass is induced, so electroweak symmetry breaking may be traced to gravity. However, no ALP of gravitational origin was considered in [81, 82, 85–88].

[2]See however [84].

It is important to point out that our approach differs significantly from the conventional axion solution to the strong CP-problem, which is based on postulating a Peccei-Quinn (PQ) symmetry [29–31]. In the most popular models [89–92], at least six new degrees of freedom are required in addition to the SM fields and graviton.[3] Apart from the question about the physical origin of the assumed PQ symmetry, one "unnaturally" small number — the CP-violating angle $\bar{\theta} \lesssim 10^{-10}$ — is replaced by two others: the ratio of the electroweak v and the PQ F scales, $(v/F)^2 \lesssim 10^{-14}$, and the parameter measuring the "quality" of the symmetry, $(M/F)^2 \lesssim 10^{-50}$, where M is the scale around which the Peccei-Quinn symmetry is explicitly broken. In contrast, our model only brings into a single new degree of freedom, namely the axion itself. Moreover, no additional tuning is introduced apart from the smallness of the cosmological constant, which exists anyway. In summary, one may say that we get three tunings for the price of one.

This observation is particularly interesting since "quantum breaking" has provided indications that eternal de Sitter states must not exist in a consistent theory of quantum gravity [93–99] (see also [100–103] for similar conjectures in string theory). However, the presence of $\bar{\theta}$-vacua in QCD would lead to de Sitter states, and so the only way out is to make the $\bar{\theta}$-angle unphysical by the addition of an axion — its existence becomes a mandatory consistency requirement, independently of any naturalness considerations [104–106]. Our proposal completes the picture as gravity also provides the necessary ALP.

Finally, we mention that our approach is fully compatible with the neutrino Minimal Standard Model (νMSM) [107, 108], which is a minimal extension of the SM in the neutrino sector capable of addressing simultaneously the experimental problems of the latter: neutrino masses and oscillations, dark matter, baryon asymmetry of the Universe. Weyl symmetry forces the tree-level Majorana masses of the heavy neutral leptons (HNLs) of the νMSM to be zero, which would be incompatible with phenomenology since successful baryogenesis cannot take place. In full analogy to the situation with the Higgs, we can speculate that non-perturbative effects generate masses for the HNLs. This is in line with the common lore that gravity breaks all global symmetries (see e.g. [109]). In this case, the classical action would have a global $B - L$ symmetry in the absence of HNL masses (B and L being the baryon and lepton numbers, respectively), and the breaking of $B - L$ à la Nambu-Jona-Lasinio [110, 111] can potentially lead to Majorana masses for the HNLs, even though the order of magnitude of this effect remains obscure and has never been computed. As an additional bonus, the EC formulation of GR provides a mechanism for generating the HNLs in the early Universe so that the lightest of them can provide the observed abundance of dark matter in a wide range of masses [112].

The paper is organized as follows. In section 2, we define the action of the theory, guided by the principle of having only Weyl-invariant terms at most quadratic in curvatures and with a consistent particle spectrum. The principle of minimality relates the Weyl gauge field to the vector part of torsion. In section 3, we integrate out torsion to obtain the action in the metric formulation of GR such that the presence of the dynamical pseudoscalar mode

[3] Two degrees of freedom are contained in the PQ-field. In KSVZ [89, 90], one complex scalar field and a new massive quark are required, while DFSZ [91, 92] relies on two complex scalar fields, where one is a doublet with respect to the SU(2) weak isospin and the other one is a singlet.

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originating from the gravitational Holst term, the ALP, is made explicit. We also fix here the free parameters of the theory, associated with the cosmological constant and the ALP mass. In section 4, we show how the ALP can indeed solve the strong CP problem. We proceed in section 5, by adding the Higgs sector. We demonstrate that the gravitationally-generated Higgs mass is tiny and that the gravitational solution to the strong CP-puzzle persists. In section 6, we discuss how the theory can be analytically continued to Euclidean spacetimes and point out that it may be possible to obtain a positive-definite action. In section 7, we conclude. In appendix A, we show that a nontrivial redefinition of field variables casts in diagonal form the kinetic sector of the scalars (in its full generality) and with the Higgs canonical. In appendix B, we present the Einstein-frame equations of motion for the gravitational and scalar sectors of the theory.

Conventions. We use the conventions of [113, 114]. Lowercase Greek and capital Latin letters denote spacetime and Lorentz indices, respectively. Both the spacetime $g_{\mu\nu}$ and Minkowski η_{AB} metrics have signature $(-1,+1,+1,+1)$. For the gamma matrices we use

$$\{\gamma_A,\gamma_B\}=-2\eta_{AB}\,,\quad \gamma_5=-i\gamma^0\gamma^1\gamma^2\gamma^3=i\gamma_0\gamma_1\gamma_2\gamma_3\,, \tag{1.1}$$

and for the Levi-Civita tensor we take

$$\epsilon_{0123}=1=-\epsilon^{0123}\,, \tag{1.2}$$

for spacetime and Lorentz indices.

Making the Poincaré group local necessitates the introduction of the gauge fields associated with translations and Lorentz transformations. In the standard jargon these are called tetrad e^A_μ and spin connection ω^{AB}_μ. Their respective field-strength tensors are torsion

$$T^A_{\mu\nu}=\partial_\mu e^A_\nu-\partial_\nu e^A_\mu+\omega^A_{\mu B}e^B_\nu-\omega^A_{\nu B}e^B_\nu\,, \tag{1.3}$$

and curvature

$$F^{AB}_{\mu\nu}=\partial_\mu\omega^{AB}_\nu-\partial_\nu\omega^{AB}_\mu+\omega^A_{\mu C}\omega^{CB}_\nu-\omega^A_{\nu C}\omega^{CB}_\mu\,, \tag{1.4}$$

respectively.

The covariant derivative of a spacetime vector A^μ is defined as

$$\nabla_\nu A^\mu=\partial_\nu A^\mu+\Gamma^\mu{}_{\nu\rho}A^\rho\,, \tag{1.5}$$

where $\Gamma^\mu{}_{\nu\rho}=e^\mu_A\left(\partial_\nu e^A_\rho+\omega^A_{\nu B}e^B_\rho\right)$ is the affine connection. The associated "affine" curvature tensor R reads

$$R^\rho{}_{\sigma\mu\nu}=\partial_\mu\Gamma^\rho{}_{\nu\sigma}-\partial_\nu\Gamma^\rho{}_{\mu\sigma}+\Gamma^\rho{}_{\mu\lambda}\Gamma^\lambda{}_{\nu\sigma}-\Gamma^\rho{}_{\nu\lambda}\Gamma^\lambda{}_{\mu\sigma}\,, \tag{1.6}$$

while "affine" torsion is simply given by the antisymmetric part of the connection

$$T^\mu_{\nu\rho}\equiv\Gamma^\mu{}_{\nu\rho}-\Gamma^\mu{}_{\rho\nu}\,. \tag{1.7}$$

For later convenience in the computations we decompose this into its three irreducible pieces: the vector

$$v^\mu = g_{\nu\rho} T^{\nu\mu\rho} \,, \tag{1.8}$$

the pseudovector

$$a^\mu = E^{\mu\nu\rho\sigma} T_{\nu\rho\sigma} \,, \tag{1.9}$$

and the sixteen-component reduced torsion tensor

$$\tau_{\mu\nu\rho} = T_{\mu\nu\rho} + \frac{1}{3}\left(g_{\mu\nu} v_\rho - g_{\mu\rho} v_\nu\right) - \frac{1}{6} E_{\mu\nu\rho\sigma} a^\sigma \,, \tag{1.10}$$

that satisfies

$$g^{\mu\rho} \tau_{\mu\nu\rho} = E^{\mu\nu\rho\sigma} \tau_{\nu\rho\sigma} = 0 \,. \tag{1.11}$$

To declutter the notation we also introduced the densitized ϵ

$$E^{\mu\nu\rho\sigma} = \frac{\epsilon^{\mu\nu\rho\sigma}}{\det(e)} \,, \qquad E_{\mu\nu\rho\sigma} = \det(e) \epsilon_{\mu\nu\rho\sigma} \,, \tag{1.12}$$

with $\det(e)$ the determinant of the tetrad.

2 The simplest scenario: induced cosmological constant and axion-like particle of gravitational origin

In the Einstein-Cartan formulation of GR [4–7], we are interested in the most general theory that fulfills the following criteria (see [80, 114–116] for similar approaches):[4]

1. The gravitational action is composed of terms that are at most quadratic in the field strength tensors $F^{AB}_{\mu\nu}$ and $T^A_{\mu\nu}$ (equivalently $R_{\mu\nu\rho\sigma}$ and $T^\mu{}_{\nu\rho}$).

2. The theory is Weyl invariant at the classical level.[5]

3. The particle spectrum is healthy (in particular, does not contain ghosts).

Criterion 1. follows the standard approach of Poincaré gauge theory [1–3] (see [125]), which relies on curvature-squared terms as minimal ingredient for obtaining a larger particle spectrum than in GR. Since curvature and torsion appear as field strength tensors on equal footing, also torsion-terms are restricted to quadratic power. We note, however, that Weyl invariance does not allow for terms quadratic in $T^A_{\mu\nu}$ since they would have to come with a dimensional coefficient, and the same goes for contributions linear in curvature $F^{AB}_{\mu\nu}$. With

[4]Among the various formulations of GR (see [113, 117, 118] for overviews), metric-affine gravity [119–122] stands out a priori since it does not require any assumptions about the vanishing of curvature, torsion or non-metricity (cf. [113, 114, 123, 124]). What makes the EC formulation special, however, is that it can be derived as gauge theory of the Poincaré group [1–3]. Therefore, it is a natural starting point for our goal of unification with Weyl symmetry.

[5]The quantum aspects of this theory, including the Weyl anomaly will be considered in a separate publication.

(possibly Weyl-invariant) terms formed from quartic powers of $T^A_{\mu\nu}$ excluded, we are left with curvature-squared terms as sole ingredient for our theory.[6]

As is well known, new propagating degrees of freedom that result from curvature-squared terms may be ghosts (and/or tachyons of spin-1 and higher) and are thus plagued by inconsistencies [126–132], even in the absence of higher derivatives (as is the case here). This is because the Poincaré group is not compact, meaning that not all kinetic and mass terms appear with the correct signs in the action. While unwanted particles can be removed from the linearized spectrum by specific parameter choices [132–178], problems almost inevitably reappear in the full theory because of strongly-coupled modes [138, 174, 179–203].

We shall therefore focus on the only known curvature-squared terms that lead to a healthy particle spectrum for arbitrary parameter choices.[7] These are composed of the scalar curvature F and its parity-odd counterpart $\tilde{F}$ — the so-called Holst curvature [214–217]—defined as

$$F = \frac{1}{4}\varepsilon_{ABCD}E^{\mu\nu\rho\sigma}e^A_\mu e^B_\nu F^{CD}_{\rho\sigma}\,, \quad \tilde{F} = E^{\mu\nu\rho\sigma}F^{AB}_{\mu\nu}e_{\rho A}e_{\sigma B}\,, \tag{2.1}$$

where the curvature tensor $F^{AB}_{\mu\nu}$ was defined previously in (1.4). First, we shall consider the most vanilla, stripped-to-its-bare-essentials, toy-model that nevertheless still captures the main idea — this is described by the following action

$$S = S_{\mathrm{gr}} + S_{\mathrm{f}}\,. \tag{2.2}$$

Here, S_{gr} is the purely gravitational piece, the *quadratic EC gravitational theory*, which we take to be[8]

$$S_{\mathrm{gr}} = \int \mathrm{d}^4x\,\det(e)\left[\frac{1}{f^2}F^2 + \frac{1}{\tilde{f}^2}\tilde{F}^2\right]\,, \tag{2.3}$$

with f and $\tilde{f}$ dimensionless constants, the couplings of the gauged Lorentz group. Both gauge couplings will eventually have to be required to be very small: the former (f) controls the cosmological constant and the latter ($\tilde{f}$) the mass of a pseudoscalar mode of gravitational origin. The theory (2.3) misses the parity-odd term $F\tilde{F}$, which would be admissible according to our criteria. Including it does not change our conclusions, as long as the coefficient in front of it is sufficiently small, i.e., does not exceed $1/f^2$ or $1/\tilde{f}^2$.

For what follows it is most convenient to work in terms of the metric $g_{\mu\nu}$ and the affine connection $\Gamma^\mu{}_{\nu\rho}$. The action (2.3) in these variables is straightforwardly seen to read as

$$S_{\mathrm{gr}} = \int \mathrm{d}^4x\sqrt{g}\left[\frac{1}{f^2}R^2 + \frac{1}{\tilde{f}^2}\tilde{R}^2\right]\,, \tag{2.4}$$

[6]It would be interesting to investigate the consequences of abandoning criterion 1. Because of Weyl invariance, this would lead to terms quartic in torsion as sole additional ingredient, which a priori are not necessarily inconsistent and likely do not qualitatively alter our conclusions.

[7]In general, there are 10 quadratic invariants which can be constructed from the curvature. They are listed in [204]. In fact, our proposal holds even if we include in the action all possible Weyl-invariant terms (quadratic in curvatures, quartic in torsion, as well as their mixings). We expect that this can give a renormalizable theory that will nevertheless propagate extra degrees of freedom [205–207]. Some of those are ghosts; whether this jeopardizes the stability of the theory is under debate, see e.g. refs. [208–213].

[8]It should be noted that there is freedom in choosing the overall sign of the gravitational action. We shall discuss that in section 6.

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with $g = -\det(g_{\mu\nu})$, and

$$R = g^{\sigma\nu} R^{\rho}{}_{\sigma\rho\nu}\,, \quad \tilde{R} = E^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}\,, \tag{2.5}$$

where $R^{\rho}{}_{\sigma\mu\nu}$ is the (affine) curvature tensor given in (1.6).

Before plunging into computations, it is instructive to do a simple counting exercise to get a grasp on the particle spectrum of (2.4). In four spacetime dimensions, the connection and tetrad carry twenty-four and sixteen degrees of freedom, respectively — not all of them are dynamical. First of all, it is obvious that the equations of motion are at most second order in the derivatives of the fields. Combined with the fact that (2.4) is invariant under local translations (four parameters), Lorentz transformations (six parameters) and Weyl rescalings (one parameter), leaves nineteen potentially propagating degrees of freedom, at most; keep in mind that the Poincaré group "hits twice" [218]. Taking into account that *in this specific theory* sixteen of the remaining fields are not dynamical,[9] leaves in total three propagating degrees of freedom: the two polarizations of the massless spin-2 graviton associated with R^2 and one massive pseudoscalar associated with $\tilde{R}^2$.[10] The presence of the latter has first been pointed out in [219](see also the recent papers [116, 175, 220–222]). It is important to note that adding more curvature-squared invariants in (2.4), in principle increases the number of propagating modes, one exception being a term proportional to $R\tilde{R}$. For instance, had we included the (Weyl tensor)^2 then besides the massless graviton and pseudoscalar, the theory would now propagate five extra degrees of freedom corresponding to a massive spin-2 field,[11] which moreover is known to be a ghost.

For analyzing the effect of curvature-squared terms, we shall follow the well-known approach of introducing auxiliary fields (see [85, 116, 220–222]). In our case,[12] we need a dilaton χ carrying dimensions of mass and a dimensionless "axion" ϕ to recast (2.4) in the following equivalent form

$$S_{\rm gr} = \int \mathrm{d}^4x\sqrt{g}\left[\chi^2 R + M_P^2\phi\tilde{R} - \frac{f^2\chi^4}{4} - \frac{\tilde{f}^2 M_P^4\phi^2}{4}\right]. \tag{2.6}$$

Note that since the assignment of mass dimensions to the fields χ and ϕ is arbitrary, we made this particular choice only to simplify the computations.

Moving on, the Weyl-invariant fermionic action $S_{\rm f}$ appearing in (2.2) is[13] [80, 114, 115, 225, 226]

$$S_{\rm f} = \int \mathrm{d}^4x\sqrt{g}\left[\frac{i}{2}\overline{\Psi}\gamma^\mu D_\mu\Psi + \text{h.c.} + (\zeta_V^a V_\mu + z_A^a A_\mu)\, a^\mu\right], \tag{2.7}$$

[9]These are the components of the reduced torsion tensor.

[10]The pseudoscalar actually descends from the temporal component of the pseudovector torsion, exactly like a pseudoscalar descends from the temporal component of a pseudovector dual to the (massive) three-form.

[11]This is an aftermath of the corresponding components of the reduced torsion tensor acquiring kinetic term and therefore becoming dynamical.

[12]The auxiliary-field method may also be employed even when other curvature invariants are present. A well-known and studied in details example is the Einstein-Hilber action supplemented by the square of the Weyl tensor. The dynamics of the latter is equivalently captured [223, 224] by a rank-2 symmetric field coupled to the Einstein tensor and with mass term of the Pauli-Fierz type. It can be shown that the kinetic term for the tensor field appears with wrong sign, signaling the presence of a spin-2 ghost.

[13]For simplicity we confine ourselves to one generation. When discussing more species, however, one may or may not assume that gravity is generation-blind. In the latter situation, the torsion-fermion couplings will carry a generation index.

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where ζ_V^a, z_A^a are real constants, and

$$V_\mu = \bar{\Psi}\gamma_\mu\Psi\,, \quad A_\mu = \bar{\Psi}\gamma_5\gamma_\mu\Psi\,, \tag{2.8}$$

are the vector and axial fermionic currents, respectively. To preserve Weyl invariance, fermions here have a minimal kinetic term and do not have a direct coupling of the torsion vector v^μ to V_μ and A_μ. We note that this corrects [80], where the coupling $v^\mu V_\mu$ had mistakenly been included. The fermionic covariant derivative D_μ is defined as

$$D_\mu = \mathcal{D}_\mu + \frac{1}{8}\omega_\mu{}^{AB}\left(\gamma_A\gamma_B - \gamma_B\gamma_A\right)\,, \tag{2.9}$$

with $\mathcal{D}_\mu$ the appropriate(flat spacetime) SM covariant derivative, the exact form of which depends on the specific nature of the fermion (left/right handed, lepton/quark).

3 Equivalent metric theory

The first step in elucidating the dynamics of the action S consists of using its Weyl invariance to eliminate the dilaton by taking

$$\chi = \frac{M_P}{\sqrt{2}}\,, \tag{3.1}$$

such that

$$S_{\rm gr} = M_P^2 \int \mathrm{d}^4x\sqrt{g}\left[\frac{R}{2} + \phi\tilde{R} - \frac{\tilde{f}^2 M_P^2\phi^2}{4} - \frac{f^2 M_P^2}{16}\right]. \tag{3.2}$$

In the classical theory, we get a cosmological constant which is small (by the smallness of the gauge coupling f) but cannot be zero.[14] Needless to say, other perturbative and non-perturbative contributions to vacuum energy are expected to arise due to quantum effects. The Lorentz-group gravitational gauge coupling should be chosen in a way that the total value of the vacuum energy accounting for quantum contributions coming from the Standard Model physics coincides with the observed value. As the SM contribution is expected to be of the order M_W^4 (M_W is the W-boson mass), an estimate of f would be $f \sim (M_W/M_P)^2 \sim 10^{-32} \ll 1$, though no convincing computation can be presented.

As a side remark, we shall point out that we can equivalently interpret the theory (3.2) in the generic metric-affine formulation of GR, in which none of the three geometric properties curvature, torsion and non-metricity is excluded a priori. In this case, both R and $\tilde{R}$ have additional contribution from non-metricity (see [113, 114] for details), but the behavior of the model (3.2) will still be very similar because the Holst term $\tilde{R}$ exhibits an extended-projective (EP) symmetry that is generated by a pair of vectors [116]. This invariance can be used

[14] As shown in details in [227, 228], at the perturbative level, admissible backgrounds for this type of theories are spacetimes with non-vanishing curvature (such as cosmological ones). The reason is that around Minkowski spacetime, the action at quadratic order in excitations exhibits *accidental gauged invariances*, i.e. gauge invariances not inherited from the full non-linear theory. This results into degrees of freedom being introduced at higher orders via interactions, corresponding to (infinitely) strongly coupled dynamics. In other words, the self-consistency of the theory dictates that f (as well as $\tilde{f}$) not vanish.

to set non-metricity to zero, which results in an effective equivalence of metric-affine and Einstein-Cartan gravity.[15]

To proceed with computations, the next step is to split the connection into a torsion-free and a torsionful part; schematically

$$\Gamma \sim \mathring{\Gamma} + v + a + \tau\,, \tag{3.4}$$

where $\mathring{\Gamma}$ is the usual Levi-Civita connection that depends on the (derivatives of the) metric, while v_μ, a_μ and $\tau_{\mu\nu\rho}$ are the torsion vector (1.8), pseudovector (1.9) and reduced tensor (1.10), respectively. In turn, the curvature tensor (1.6) is also decomposed into its Riemannian and post-Riemannian pieces, which translates into the following expressions for the scalar and Holst curvatures (2.5)

$$R = \mathring{R} + 2\mathring{\nabla}_\mu v^\mu - \frac{2}{3}v_\mu v^\mu + \frac{1}{24}a_\mu a^\mu + \frac{1}{2}\tau_{\mu\nu\rho}\tau^{\mu\nu\rho}\,, \tag{3.5}$$

$$\tilde{R} = -\mathring{\nabla}_\mu a^\mu + \frac{2}{3}a_\mu v^\mu + \frac{1}{2}E^{\mu\nu\rho\sigma}\tau_{\lambda\mu\nu}\tau^{\lambda}{}_{\rho\sigma}\,, \tag{3.6}$$

where $\mathring{R}$ is the usual, metrical, Ricci scalar.

The third step is to plug ((3.4), (3.5),3.6) into (3.2) and (2.7), to obtain (after some integrations by parts)

$$S = \int d^4x\sqrt{g}\Bigg[M_P^2\Bigg(\frac{\mathring{R}}{2} - \frac{v_\mu v^\mu}{3} + \frac{a_\mu a^\mu}{48} + \frac{2\phi a_\mu v^\mu}{3} + \frac{\tau_{\mu\nu\rho}}{2}\left(\frac{\tau^{\mu\nu\rho}}{2} + \phi E^{\nu\rho\kappa\lambda}\tau^{\mu}{}_{\kappa\lambda}\right) - \frac{\tilde{f}^2 M_P^2\phi^2}{4} - \frac{f^2 M_P^2}{16}\Bigg) + a^\mu J_\mu^a + \frac{i}{2}\left(\overline{\Psi}\gamma^\mu \mathring{D}_\mu\Psi + \text{h.c.}\right)\Bigg]\,, \tag{3.7}$$

where

$$J_\mu^a = M_P^2\partial_\mu\phi + \zeta_V^a V_\mu + \zeta_A^a A_\mu\,, \tag{3.8}$$

and we introduced the shifted constant [114, 115]

$$\zeta_A^a = z_A^a - \frac{1}{8}\,. \tag{3.9}$$

[15] As it stands, the theory (3.2) fails to be EP-invariant, but this can be easily remedied with a small modification:

$$S_{\rm EP} = M_P^2\int d^4x\sqrt{g}\Bigg[\frac{\mathring{R}}{2} + b_1(2v_\mu + Q_\mu + \hat{Q}_\mu)(2v^\mu + Q^\mu + \hat{Q}^\mu) + b_2 a_\mu a^\mu + b_3 a_\mu(2v^\mu + Q^\mu + \hat{Q}^\mu) + \phi\tilde{R} - \frac{\tilde{f}^2 M_P^2\phi^2}{4} - \frac{f^2 M_P^2}{16}\Bigg]\,, \tag{3.3}$$

where b_1, b_2, b_3 are real constants and $Q_\mu = Q_{\mu\alpha}{}^{\alpha}$ and $\hat{Q}_\mu = Q^{\alpha}{}_{\mu\alpha}$ with non-metricity $Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu}$. The metric-affine model (3.3) is EP-invariant and hence fully equivalent to the EC-theory (3.2) for $b_1 = -1/12$, $b_2 = 1/48$ and $b_3 = 0$ (cf. eq. (3.5)). What is more, eq. (3.3) is the most general gravitational theory that is symmetric under EP-transformations. Hence, one could replace Weyl- by EP-invariance as criterion for selecting consistent models with an ALP of gravitational origin. (Then the term $R\tilde{R}$ would indeed be excluded).

Inspection of (3.7) reveals that the connection appears algebraically in the action and can thus be integrated out. A straightforward computation leads to the following equations of motion for v, a and τ

$$v_\mu = -\frac{24\phi J^a_\mu}{M_P^2\,(1+16\phi^2)}\,, \quad a_\mu = -\frac{24 J^a_\mu}{M_P^2\,(1+16\phi^2)}\,, \quad \tau_{\mu\nu\rho} = 0\,. \tag{3.10}$$

The last step amounts to substituting (3.10) into the action (3.7) to end up with

$$\begin{aligned} S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{g}\bigg(& M_P^2 \mathring{R} - \frac{24M_P^2}{1+16\phi^2}(\partial_\mu\phi)^2 - \frac{\tilde{f}^2 M_P^4\phi^2}{2} - \frac{f^2 M_P^4}{8} \\ & + i\left(\overline{\Psi}\gamma^\mu \mathring{D}_\mu\Psi + \text{h.c.}\right) - \mathscr{L}_{\phi V} - \mathscr{L}_{\phi A} - \frac{3}{2M_P^2}\left(\mathscr{L}_{VV} + \mathscr{L}_{AA} + \mathscr{L}_{VA}\right)\bigg)\,, \end{aligned} \tag{3.11}$$

where

$$\mathscr{L}_{\phi V} = \frac{48\zeta_V^a}{1+16\phi^2} V^\mu \partial_\mu\phi\,, \quad \mathscr{L}_{\phi A} = \frac{48\zeta_A^a}{1+16\phi^2} A^\mu \partial_\mu\phi\,, \tag{3.12}$$

are contact interactions between the derivative of the ALP and the axial and vector fermionic currents, while

$$\mathscr{L}_{VV} = \frac{16{\zeta_V^a}^2}{1+16\phi^2} V_\mu V^\mu\,, \tag{3.13}$$

$$\mathscr{L}_{VA} = \frac{32\zeta_A^a\zeta_V^a}{1+16\phi^2} V_\mu A^\mu\,, \tag{3.14}$$

$$\mathscr{L}_{AA} = \frac{16{\zeta_A^a}^2}{1+16\phi^2} A_\mu A^\mu\,, \tag{3.15}$$

are the usual four-fermi interactions generically present in the EC framework.

It is clear that the theory under consideration is the usual Einsteinian gravity with a (nonvanishing) cosmological constant — whose value depends on the first Lorentz gauge coupling, meaning that $f \lll 1$ — coupled to fermions and an axion-like particle with mass controlled by the second Lorentz gauge coupling $\tilde{f}$. In addition, the fermionic sector comprises nontrivial four-fermi interactions as well as mixings between ϕ and the vector and axial fermionic currents. It is the latter coupling that is the crux of it all as we shall show now.

Let us mention that the ALP nature of ϕ descending from $\tilde{R}^2$ was also pointed out in [172]. The authors discussed its dynamics, including the couplings to fermionic currents and the interactions of ϕ with electromagnetism and gravity due to the chiral anomaly. However, no coupling to QCD was included and hence the strong CP problem was not considered.

4 A solution to the strong CP problem

We focus on low energies, which amounts to taking $\phi \ll 1$. From (3.11) we obtain

$$S \approx -\frac{1}{2}\int \mathrm{d}^4x\sqrt{g}\left[(\partial_\mu a)^2 + \frac{\tilde{f}^2 M_P^2}{48}a^2 - 4\sqrt{6}\zeta_A^a \frac{a}{M_P}\mathring{\nabla}_\mu A^\mu\right]\,, \tag{4.1}$$

where we defined $a = 2\sqrt{6}M_P\phi$ to have a canonically normalized field[16] and retained only the terms which are relevant for what follows.

From the above we notice that the ALP has a perturbative "gravitationally-induced" mass

$$m_{a,\text{grav}} = \frac{\tilde{f} M_P}{4\sqrt{3}}\,, \tag{4.4}$$

whose value is determined solely by $\tilde{f}$; in addition and most importantly, the field couples linearly to the (anomalous) divergence of the axial current. In curved backgrounds this reads [229]

$$\begin{aligned}\mathring{\nabla}_\mu A^\mu = &-\frac{g_1^2}{16\pi^2}E^{\mu\nu\rho\sigma}B_{\mu\nu}B_{\rho\sigma} - \frac{g_2^2}{16\pi^2}E^{\mu\nu\rho\sigma}\mathrm{Tr}\left(\mathcal{G}_{\mu\nu}\mathcal{G}_{\rho\sigma}\right)\\ &-\frac{g_3^2}{16\pi^2}E^{\mu\nu\rho\sigma}\mathrm{Tr}\left(G_{\mu\nu}G_{\rho\sigma}\right) - \frac{1}{384\pi^2}E^{\mu\nu\rho\sigma}\mathring{R}_{\mu\nu\kappa\lambda}\mathring{R}_{\rho\sigma}{}^{\kappa\lambda}\,,\end{aligned} \tag{4.5}$$

with g_1, g_2, g_3 the electromagnetic, weak and strong couplings, whereas $B_{\mu\nu}, \mathcal{G}_{\mu\nu}, G_{\mu\nu}$ the corresponding field-strength tensors.

Let us now focus on the ALP & QCD contributions in the effective theory[17]

$$S \approx -\frac{1}{2}\int \mathrm{d}^4x\sqrt{g}\left[(\partial_\mu a)^2 + m_{a,\text{grav}}^2 a^2 - \frac{g_3^2}{8\pi^2}\left(\bar{\theta} - \frac{a}{f_a}\right)E^{\mu\nu\rho\sigma}\mathrm{Tr}\left(G_{\mu\nu}G_{\rho\sigma}\right)\right]\,, \tag{4.6}$$

where we introduced the ALP "decay constant"[18]

$$f_a = \frac{M_P}{2\sqrt{6}\zeta_A^a}\,, \tag{4.7}$$

and for completeness we included the physical $\bar{\theta}$ angle defined as

$$\bar{\theta} = \theta + \arg\det(M_u M_d)\,, \tag{4.8}$$

where θ is the QCD topological angle, and M_u and M_d are the up- and down- type quark mass matrices.

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[16] There is no difficulty in canonicalizing the ALP for all field values. This is achieved by introducing

$$a = \sqrt{\frac{3}{2}}M_P\,\mathrm{arcsinh}\,(4\phi)\,, \tag{4.2}$$

to obtain

$$S \supset -\frac{1}{2}\int \mathrm{d}^4x\sqrt{g}\left[(\partial_\mu a)^2 + \frac{\tilde{f}^2M_P^4}{32}\sinh^2\left(\sqrt{\frac{2}{3}}\frac{a}{M_P}\right) - 12\zeta_A^a\,\mathrm{arctan}\sinh\left(\sqrt{\frac{2}{3}}\frac{a}{M_P}\right)\mathring{\nabla}_\mu A^\mu\right]. \tag{4.3}$$

In deriving this expression we used (4.2) to write $\mathscr{L}_{\phi V}$ and $\mathscr{L}_{\phi A}$ in terms of the canonical field, then absorbed the coefficient functions into the four-derivative of a and finally integrated by parts; note that we dropped the term proportional to the divergence of the vector current since it vanishes, $\mathring{\nabla}_\mu V^\mu = 0$. It can easily be shown that for $a \ll M_P$ (corresponding of course to $\phi \ll 1$), the action (4.3) boils down to (4.1), as it should.

[17] As is evident from eq. (4.5), gravity can pose a threat to the solution to the strong CP-problem if it introduces a gravitational theta term [105, 230, 231]. However, the solution can be provided by the SM itself [232–234] in the form of a composite ALP. Then the physical QCD axion is an admixture of these pseudoscalars, so gravity is equipped with a mechanism protecting it against itself as advocated in [235]. In addition, the non-perturbative gravity contribution to the ALP mass, similar to that of [22, 24], must be smaller than the QCD one.

[18] Interestingly, f_a is non-zero even when fermions are coupled minimally, since in this case $\zeta_A^a = -\frac{1}{8}$.

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Non-perturbative QCD effects generate the standard potential for the ALP and thus the effective action at small energies becomes (see e.g., [236])

$$S \approx -\frac{1}{2}\int \mathrm{d}^4x\sqrt{g}\left[(\partial_\mu a)^2 + m_{a,\mathrm{grav}}^2 a^2 + 2m_\pi^2 f_\pi^2\sqrt{1-\frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{\bar{\theta}}{2}-\frac{a}{f_a}\right)}\,\right], \tag{4.9}$$

with m_π, f_π the pion mass and decay constant, and m_u, m_d the up and down quark masses. As long as the perturbative mass is smaller than the nonperturbative one induced by QCD, then the gravi-ALP solves the strong-CP problem.

In this particular toy-model, this translates into

$$\frac{\tilde{f}}{\zeta_A^a} \lesssim 10^{-5}\frac{m_\pi f_\pi}{M_P^2}\frac{\sqrt{m_u m_d}}{m_u+m_d} \sim \mathcal{O}\left(10^{-43}\right), \tag{4.10}$$

where we took into account the observational bound on the $\bar{\theta}$-angle around 10^{-10} (see [237]). Contrary to the PQ treatment, our proposal does not require the introduction of extra fermions or modifications to the SM; the axion is already present in the gravitational sector of the theory. What is more, the gravitational origin of $\tilde{f}$ (as well as f) justifies the expectation that the coupling be vanishingly small — remember, gravity after all is the weakest force.

Finally, we shall mention previous attempts [238–240] offering a gravitational solution to the strong CP puzzle via an axion-like particle associated with torsion. Crudely speaking, these models correspond to starting from the gravitational action of our toy-model written in terms of auxiliary fields, fixing the Weyl gauge and then setting $\tilde{f} = 0$ — strictly speaking, this limit cannot and should not be taken. It does not come as a surprise that the effective low-energy dynamics and conclusions of our construction bears similarity to the ones of the aforementioned works. However, it must be stressed that the starting points differ radically. Contrary to our approach, where the pseudoscalar particle is in the spectrum of the theory from the onset, in these articles it is introduced in a somewhat mysterious manner. Their gravitational sector comprises only the Einstein-Hilbert term, whereas the field is a Lagrange multiplier that either enforces the conservation of the axial torsion current (inspired by the considerations of [241]) or the vanishing of the Nieh-Yan topological term (inspired by the results of [242] concerning the axial anomaly in torsionful spacetimes); interestingly, it was later understood that this is one and the same requirement [240]. The bottom-line is that the pseudoscalar becomes dynamical only after torsion is integrated out, leading to a mismatch in the dynamical degrees of freedom.

5 Adding the Higgs field

For the purposes of illustration, it suffices to consider the SM Higgs field h in the unitary gauge and couple it to the quadratic EC gravity in a Weyl-invariant manner. In other words, we now take

$$S = S_{\mathrm{gr}} + S_{\mathrm{f}} + S_{\mathrm{Higgs}} \tag{5.1}$$

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with the purely gravitational $S_{\rm gr}$ given in (2.4), the fermionic one given in (2.7), and

$$S_{\rm Higgs} = \frac{1}{2}\int {\rm d}^4x\sqrt{g}\Big[\xi_h h^2 R + \zeta_h h^2 \tilde{R} + c_{aa}h^2 a_\mu a^\mu + c_{\tau\tau}h^2\tau^2_{\mu\nu\rho} + \tilde{c}_{\tau\tau}h^2 E^{\mu\nu\rho\sigma}\tau_{\lambda\mu\nu}\tau^\lambda{}_{\rho\sigma} - \left(D^W_\mu h\right)^2 - \frac{\lambda h^4}{2}\Big], \tag{5.2}$$

is the most general action built on the basis of Weyl invariance and the requirement of retaining terms with at most two derivatives of the fields. Here, $\xi_h, \zeta_h, c_{aa}, c_{\tau\tau}$ and $\tilde{c}_{\tau\tau}$ are (real but otherwise arbitrary) nonminimal gravi-scalar couplings. To ensure the Weyl invariance of the Higgs's kinetic term, the torsion vector must appear with fixed coefficient(s) in the action and can be conveniently packed with the derivative of h to define the "Weyl-covariant derivative" D^W_μ see e.g. [80]. Explicitly,

$$D^W_\mu h = \partial_\mu h + \frac{v_\mu}{3}h\,. \tag{5.3}$$

In principle, we could include a Yukawa interaction between the Higgs and fermions, however this is irrelevant for what follows. Also irrelevant are the full expressions involving the fermionic sector, which are rather long and do not add to the discussion; we need only worry about the interaction of the ALP with the axial fermionic current, and we shall come back to that in the next section. For the time being we will only consider the graviscalar sector of (5.1).

Following more or less verbatim the steps presented in the toy-model of the previous section, we can easily massage (5.1) into its metric-equivalent form and work out its dynamics. After fixing the Weyl gauge, see (3.1), and integrating out torsion we end up with the following Jordan-frame action

$$S_{\rm gr} + S_{\rm Higgs} = \frac{1}{2}\int {\rm d}^4x\sqrt{g}\left[\left(M_P^2+\xi_h h^2\right)\mathring{R} - \gamma_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b - \frac{\lambda h^4}{2} - \frac{\tilde{f}^2M_P^4\phi^2}{2} - \frac{f^2M_P^4}{8}\right], \tag{5.4}$$

where summation over all repeated indexes is understood. Here $\varphi^a = (h,\phi)$ and $\gamma_{ab} = \gamma_{ab}(h,\phi)$ is the metric of the internal two-dimensional field-derivative space with components

$$\gamma_{hh} = \frac{N}{D}\,, \quad \gamma_{h\phi} = \frac{48\left(3\zeta_h - (1+6\xi_h)\phi\right)M_P^4 h}{D}\,, \quad \gamma_{\phi\phi} = \frac{24\left(6M_P^2 + (1+6\xi_h)h^2\right)M_P^4}{D}\,, \tag{5.5}$$

where

$$N = 6(1+16\phi^2)M_P^4 + 36\left(4(\zeta_h^2 + c_{aa}) - \xi_h^2 - 16\xi_h\zeta_h\phi\right)M_P^2h^2 - 6\xi_h\left(24\zeta_h^2 + (\xi_h + 24c_{aa})(1+6\xi_h)\right)h^4\,, \tag{5.6}$$

$$D = 6(1+16\phi^2)M_P^4 + (1+12\xi_h+144c_{aa}+96\zeta_h\phi)\,M_P^2h^2 + \left(24\zeta_h^2 + (\xi_h+24c_{aa})(1+6\xi_h)\right)h^4\,. \tag{5.7}$$

We now transform the action in the Einstein frame to eliminate the mixing between h and gravitons. This is achieved with the usual Weyl rescaling of the metric

$$g_{\mu\nu} \;\mapsto\; \Omega^{-2}g_{\mu\nu}\,, \quad \Omega^2 = \frac{M_P^2+\xi_h h^2}{M_P^2}\,. \tag{5.8}$$

Once this is effectuated, we find

$$S_{\rm gr}+S_{\rm Higgs}=\frac{1}{2}\int {\rm d}^4x\sqrt{g}\left[M_P^2\mathring{R}-\tilde{\gamma}_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b-\frac{\lambda h^4}{2\Omega^4}-\frac{\tilde{f}^2M_P^4\phi^2}{2\Omega^4}-\frac{f^2M_P^4}{8\Omega^4}\right], \tag{5.9}$$

where the components of the transformed field space metric $\tilde{\gamma}_{ab}$ read

$$\tilde{\gamma}_{hh}=\frac{1}{\Omega^2}\left(\gamma_{hh}+\frac{6\xi_h^2h^2}{M_P^2\Omega^2}\right), \quad \tilde{\gamma}_{h\phi}=\frac{\gamma_{h\phi}}{\Omega^2}, \quad \tilde{\gamma}_{\phi\phi}=\frac{\gamma_{\phi\phi}}{\Omega^2}. \tag{5.10}$$

For completeness, we show the resulting equations of motion in a cosmological background in appendix B.

To make our point, it suffices to consider the low-energy limit of the theory, so we can take $h\ll M_P$ and $\phi\ll 1$ to obtain

$$\begin{aligned} S_{\rm gr}+S_{\rm Higgs}\approx\frac{1}{2}\int {\rm d}^4x\sqrt{g}\bigg[&M_P^2\mathring{R}-24M_P^2(\partial_\mu\phi)^2-\frac{\tilde{f}^2M_P^4\phi^2}{2}-\frac{f^2M_P^4}{8}\\ &-(\partial_\mu h)^2+\frac{f^2\xi_hM_P^2}{4}h^2-\frac{\lambda}{2}\left(1+\frac{3f^2\xi_h^2}{4\lambda}\right)h^4\bigg]+\dots, \end{aligned} \tag{5.11}$$

where we kept only the leading terms and the ellipses stand for higher orders in h and ϕ. The first line in the above is exactly the gravi-pseudoscalar sector of the previous section's toy-model, see (3.11), and thus the dynamics are identical. From the second line of (5.11) we observe that a tachyonic mass for the Higgs equal to

$$m^2_{\rm h,grav}=-\frac{f^2\xi_hM_P^2}{4}, \tag{5.12}$$

has been gravitationally induced. Since the gauge coupling f sets the cosmological constant (see the discussion after eq. (3.2)), the tree value of the Higgs boson mass for all practical purposes can be taken to be vanishingly small for reasonable values of the nonminimal coupling ξ_h. In addition, we notice that the Higgs's quartic self-coupling gets shifted by a gravitational contribution also proportional to f^2, which can be safely neglected.

We find it remarkable that although the inclusion of the Higgs sector is rather non-trivial, the gravitational and ALP dynamics are practically left unaltered as compared to the toy-model. For completeness and as mentioned earlier, we now turn to the modifications to the relevant part of the fermionic sector and show that the gravi-axion solution to the strong CP problem persists.

To this end, we focus on the contact term capturing the interaction of the ALP with the axial current. After a straightforward but cumbersome computation we find that in the Einstein frame this reads

$$\mathscr{L}_{\phi A}=48M_P^2\zeta_A^a\frac{6M_P^2+(1+6\xi_h)h^2}{D}A^\mu\partial_\mu\phi, \tag{5.13}$$

with D defined previously in eq. (5.7). As usual when Weyl-transforming in the presence of fermions, in order to make the fermionic kinetic term canonical, we dressed Ψ with the appropriate power of the conformal factor Ω (given in (5.8)), i.e. we redefined

$$\Psi\mapsto\Omega^{\frac{3}{2}}\Psi. \tag{5.14}$$

It is easy to show that in the low-energy limit, the corresponding part of the action becomes exactly what we found in section 4

$$S_{\phi A} = \int \mathrm{d}^4x \sqrt{g} \mathscr{L}_{\phi A} \approx \int \mathrm{d}^4x \sqrt{g} \frac{a}{f_a} \mathring{\nabla}_\mu A^\mu \,, \tag{5.15}$$

where we introduced the canonical ALP a and integrated by parts; f_a is the ALP decay constant (4.7).

6 Euclidean continuation and sign of the gravitational action

The Euclidean path integral formulation of the *standard metric gravity* is ill-defined since the Euclidean action for gravity is not positive definite unlike those of matter fields [243]. At the same time, Euclidean formulation of *field theory* is an indispensable tool for addressing the non-perturbative tunnelling processes in the semiclassical approximation and accessing the strong coupling regime by doing lattice simulations. So, several ideas were proposed to make sense of the path integral in gravity.[19] One of them is associated with a specific prescription for the analytic continuation of the conformal part of the metric into the Euclidean domain [243, 245]. Yet another argues for abandoning the Euclidean formulation altogether and using the non-perturbative causal dynamical triangulations (for a review, see [246]). A similar in-spirit approach uses the Minkowskian path integral with deformed trajectories going through the complex "thimbles" for non-perturbative definition [247]. We think it is fair to say that the question has not been settled yet. It is, however, essential for the understanding of several fundamental questions such as the wave function of the Universe and the initial conditions for inflation [248], the cosmological constant problem [249, 250] the role of wormholes and connected to them "baby universes" (for a recent review see [251]).

Perhaps the failure to provide the unique acceptable recipe for going into Euclidean space-time is a drawback of the considered theory (Einstein metric gravity) rather than the analytic continuation itself. We will argue below that the problem may be solved in the Weyl-invariant EC gravity considered in this paper.

Indeed, working in the EC framework has yet another advantage, in that it incorporates a straightforward and unambiguous way to analytically continue from Lorentzian to Euclidean spacetimes: the Lorentz-temporal components of the tetrad and connection get multiplied by the imaginary unit while their purely (Lorentz-)spatial ones remain intact (see also [252, 253])

$$e^0_\mu \to -ie^0_\mu \,, \quad e^j_\mu \to e^j_\mu \,, \quad \omega^{0j}_\mu \to -i\omega^{0j}_\mu \,, \quad \omega^{jk}_\mu \to \omega^{jk}_\mu \,, \quad j,k = 1,2,3 \,, \tag{6.1}$$

somewhat resembling the Gibbons-Hawking-Perry prescription [243]. Then the curvature and torsion tensors are Euclideanized as follows

$$F^{0j}_{\mu\nu} \to -iF^{0j}_{\mu\nu} \,, \quad F^{jk}_{\mu\nu} \to F^{jk}_{\mu\nu} \,, \quad T^0_{\mu\nu} \to -iT^0_{\mu\nu} \,, \quad T^j_{\mu\nu} \to T^j_{\mu\nu} \,. \tag{6.2}$$

Applying this prescription to S_{gr} given in (2.4), we find that

$$S_{\mathrm{gr}} \to -iS_{\mathrm{gr}} \tag{6.3}$$

meaning that the Euclidean action is negative

$$S^{\mathrm{E}}_{\mathrm{gr}} = -\int \mathrm{d}^4x \, \det(e) \left[\frac{1}{f^2} F^2 + \frac{1}{\tilde{f}^2} \tilde{F}^2 \right] . \tag{6.4}$$

[19] At the one-loop level and above the Ricci-flat backgrounds the conformal factor problem is an artefact of the naive analytic continuation of the Einstein-Hilbert action [244].

Interestingly, however, there is an "ambiguity" in choosing the overall sign of the EC quadratic action (2.3), see a related discussion in [254]. We could have equally well taken as our starting point

$$S'_{\rm gr} = -S_{\rm gr} = -\int {\rm d}^4x \det(e) \left[\frac{1}{f^2}F^2 + \frac{1}{\tilde{f}^2}\tilde{F}^2\right] . \tag{6.5}$$

The only difference as far as the classical dynamics is concerned would be that the classical background would correspond to an anti-de-Sitter spacetime, while the (tiny) gravitationally-induced masses for the ALP and Higgs would change their signs. Since there can be other positive contributions to the cosmological constant, and in any case the tree-level masses of the scalars are negligibly small, this is a perfectly acceptable choice too.

The Euclidean action of the sign-reverted theory is now positive-definite, opening the possibility of its consistent path-integral formulation. This could provide an even more solid starting point for exploring the idea of non-perturbative Higgs mass generation [22], as it is based on a positive-definite Euclidean gravitational action. It would also be interesting to reanalise the whole set of questions mentioned at the beginning of this section within the framework advocated in our work. We leave this for the future.

7 Conclusions and discussions

In our paper we demonstrated that the combination of the Standard Model with the Weyl-invariant Einstein-Cartan gravity leads to a remarkably economical description of all known interactions. In addition to the fields of the Standard Model (or νMSM) and the graviton, it automatically contains just one extra pseudoscalar degree of freedom, or in other words an axion-like particle (ALP), with all requisite properties to solve the strong CP problem. The smallness of the cosmological constant and the ALP mass arise from tiny values of the dimensionless gauge couplings of Einstein-Cartan gravity. Moreover, the tree-level values of the Higgs (and heavy neutral lepton masses in the νMSM) are very small or vanishing, opening up the possibility of the potential computability of these parameters from non-perturbative effects. Finally, yet another attractive feature of our theory is that a particular sign choice can make its Euclidean action bounded from below, allowing a more consistent path-integral formulation.

Of course, our study is far from being complete. From the theoretical side, the quantum theory of the Weyl-invariant Einstein-Cartan gravity remains to be developed, with understanding of its high-energy limit and non-perturbative effects, which may lead to the computation of the Higgs boson and heavy neutral lepton masses. This theory is not pertubatively renormalizable. We may think about its ultraviolet completion along the lines of asymptotic safety [255–257], classicalization [258–260], or non-renormalizable resummation of amplitudes proposed in [261]. From the phenomenological side, the most interesting questions are related to cosmology. It would be important to understand whether inflation can take place in this theory and what would be the predictions of observables. The presence of a new light scalar field poses the question whether this can be a suitable dark matter candidate. We plan to return to these problems in the future.

A few comments are now in order. The first one concerns the role of Weyl symmetry for our findings. A step back would be replacing Weyl invariance by a smaller symmetry —

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global scale invariance. In this case, the action of the theory contains several extra terms. In addition to the axion-like and Standard Model particles there is an extra propagating scalar mode — an exactly massless dilaton, being the Goldstone boson of the spontaneously broken scale invariance. This theory enjoys all the attractive features of the Weyl-invariant setup, including the dynamical generation of the Planck scale, solution of the strong CP problem, etc. In spite of the presence of a dynamical dilaton, no long-range fifth force shows up [45, 61]. One can go even further and remove the requirement of the global scale invariance. The resulting action now allows many extra terms, including those with an explicit Planck mass. The particle spectrum contains, as in the previous case, two extra degrees of freedom of gravitational origin. One of them is the ALP we encountered here, which can still be invoked to solve the strong CP problem, and the other is the so-called scalaron, well-known from the Starobinsky inflationary model [262]. The computability of the Higgs and heavy neutral lepton masses is however lost, as these terms are now allowed at tree level.

As a second comment, we believe that our proposal to solve the strong CP problem has three advantages as compared to existing ones. First of all, our extension of the SM and GR can be viewed as minimal since no new particles are required apart from a single real scalar degree of freedom — the ALP itself that is in any case required for solving the strong CP-puzzle. Second, we do not add any additional global symmetries (or equivalently tunings of parameters), but instead "recycle" the observed smallness of the cosmological constant for explaining why the perturbative Higgs and ALP are also tiny. Finally, gravity has provided hints that an axion must exist as a consistency requirement, so it is intriguing that we have found a self-consistent and purely gravitational origin for such a particle.

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A Diagonalization of scalar sector

In this appendix we show that not only it is possible to fully diagonalize the scalar kinetic sector of (5.9) by getting rid of the derivative mixing between the Higgs and ALP, but also make canonical the kinetic term of the former. Introduce[20]

$$\Phi = \frac{M_P}{1+6\xi_h}\log\left(\frac{6+(1+6\xi_h)\frac{h^2}{M_P^2}}{|3\zeta_h-(1+6\xi_h)\phi|}\right)\,, \qquad H=\sqrt{6}M_P\,\mathrm{arctanh}\left(\frac{\frac{h}{M_P}}{\sqrt{6+(1+6\xi_h)\frac{h^2}{M_P^2}}}\right)\,, \tag{A.1}$$

[20]The field redefinitions can be easily inverted to yield

$$\phi=\frac{3}{1+6\xi_h}\left(\zeta_h-\frac{2e^{-(1+6\xi_h)\frac{\Phi}{M_P}}}{1-(1+6\xi_h)\tanh^2\left(\frac{H}{\sqrt{6}M_P}\right)}\right)\,, \quad h=\frac{\sqrt{6}M_P\tanh\left(\frac{H}{\sqrt{6}M_P}\right)}{\sqrt{1-(1+6\xi_h)\tanh^2\left(\frac{H}{\sqrt{6}M_P}\right)}}\,.$$

Notice also that some expressions may blow up at certain values of the fields. Here, these are merely "coordinate singularities:" the scalar curvature of the kinetic manifold remains regular at these points.

to obtain

$$S_{\rm gr}+S_{\rm Higgs}=\frac{1}{2}\int {\rm d}^4x\sqrt{g}\bigg[M_P^2\mathring{R}-(\partial_\mu H)^2-\tilde{\gamma}_{\Phi\Phi}(\partial_\mu\Phi)^2-V(H,\Phi)\bigg]\,, \tag{A.2}$$

with

$$\tilde{\gamma}_{\Phi\Phi}=\frac{3}{2}\frac{\cosh^4\left(\frac{H}{\sqrt{6}M_P}\right)}{\cosh^2\left(\frac{H}{\sqrt{6}M_P}\right)\left(\frac{1-\frac{\zeta_h}{2}e^{(1+6\xi_h)\frac{\Phi}{M_P}}}{1+6\xi_h}\right)^2+\left(\frac{e^{(1+6\xi_h)\frac{\Phi}{M_P}}}{24}\right)^2\left(1+144c_{aa}\sinh^2\left(\frac{H}{\sqrt{6}M_P}\right)\right)}\,, \tag{A.3}$$

and

$$\begin{aligned}V(H,\Phi)=&\;18\lambda M_P^4\sinh^4\left(\frac{H}{\sqrt{6}M_P}\right)\\&+\frac{f^2M_P^4}{8}\left(1-6\xi_h\sinh^2\left(\frac{H}{\sqrt{6}M_P}\right)\right)^2\\&+\frac{18\tilde{f}^2M_P^4e^{-2(1+6\xi_h)\frac{\Phi}{M_P}}}{(1+6\xi_h)^2}\bigg[\cosh^2\left(\frac{H}{\sqrt{6}M_P}\right)\\&-\frac{\zeta_h}{2}e^{(1+6\xi_h)\frac{\Phi}{M_P}}\left(1-6\xi_h\sinh^2\left(\frac{H}{\sqrt{6}M_P}\right)\right)\bigg]^2\,.\end{aligned} \tag{A.4}$$

Interestingly, the first two terms in the potential (A.4) are identical to the ones of ref. [87], where the Weyl-geometric cousin of our theory was constructed.

B Equations of motion of the gravi-scalar sector

In this appendix we write down the equations of motion of the gravitational & scalar sectors of the theory expressed in the Einstein frame, see (5.9), as these can be useful for studying the implications of the theory in cosmological settings. We follow closely [263].

For the reader's convenience, we remind that the action can be conveniently expressed in the following compact form

$$S_{\rm gr}+S_{\rm Higgs}=\int {\rm d}^4x\sqrt{g}\left(\frac{M_P^2}{2}\mathring{R}-\frac{1}{2}\tilde{\gamma}_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b-V\right)\,, \tag{B.1}$$

where $\varphi^a=(h,\phi)$, and the components of the nontrivial field-space metric $\tilde{\gamma}_{ab}$ can be read-off from eqs. (5.10), (5.5)–(5.7) in the main text; we also introduced

$$V\equiv V(h,\phi)=\frac{1}{2\Omega^4}\left(\frac{\lambda h^4}{2}+\frac{\tilde{f}^2M_P^4\phi^2}{2}+\frac{f^2M_P^4}{8}\right)\,, \tag{B.2}$$

with Ω the conformal factor defined in (5.8).

Variation of (B.1) w.r.t. the metric results into the Einstein equations

$$\mathring{G}_{\mu\nu}=\tilde{\gamma}_{ab}\left(\partial_\mu\varphi^a\partial_\nu\varphi^b-\frac{1}{2}g_{\mu\nu}\partial_\rho\varphi^a\partial^\rho\varphi^b\right)+g_{\mu\nu}V\,, \tag{B.3}$$

where $\mathring{G}_{\mu\nu}$ is the (torsion-free) Einstein tensor.

On the other hand, varying the action w.r.t. φ^a gives

$$\frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}g^{\mu\nu}\varphi^a\right)+\widetilde{\mathscr{T}}^a_{bc}\partial_\mu\varphi^b\partial^\mu\varphi^c-\tilde{\gamma}^{ab}\partial_b V=0\,, \tag{B.4}$$

where the inverse metric reads

$$\tilde{\gamma}^{ab}=\frac{1}{\tilde{\gamma}_{hh}\tilde{\gamma}_{\phi\phi}-\tilde{\gamma}^2_{h\phi}}\begin{pmatrix}\tilde{\gamma}_{\phi\phi} & \tilde{\gamma}_{h\phi}\\ \tilde{\gamma}_{h\phi} & \tilde{\gamma}_{hh}\end{pmatrix}\,, \tag{B.5}$$

and

$$\widetilde{\mathscr{T}}^a_{bc}=\frac{1}{2}\tilde{\gamma}^{ad}\left(\partial_b\tilde{\gamma}_{cd}+\partial_c\tilde{\gamma}_{bd}-\partial_d\tilde{\gamma}_{bc}\right)\,, \tag{B.6}$$

are the field-space metric's Christoffel symbols:

$$\widetilde{\mathscr{T}}^h_{hh}=\frac{\tilde{\gamma}_{\phi\phi}\partial_h\tilde{\gamma}_{hh}+\tilde{\gamma}_{h\phi}\left(\partial_\phi\tilde{\gamma}_{hh}-2\partial_h\tilde{\gamma}_{h\phi}\right)}{2\left(\tilde{\gamma}_{hh}\tilde{\gamma}_{\phi\phi}-\tilde{\gamma}^2_{h\phi}\right)}\,, \tag{B.7}$$

$$\widetilde{\mathscr{T}}^h_{h\phi}=\widetilde{\mathscr{T}}^h_{\phi h}=\frac{\tilde{\gamma}_{\phi\phi}\partial_\phi\tilde{\gamma}_{hh}-\tilde{\gamma}_{h\phi}\partial_h\tilde{\gamma}_{\phi\phi}}{2\left(\tilde{\gamma}_{hh}\tilde{\gamma}_{\phi\phi}-\tilde{\gamma}^2_{h\phi}\right)}\,, \tag{B.8}$$

$$\widetilde{\mathscr{T}}^h_{\phi\phi}=-\frac{\tilde{\gamma}_{h\phi}\partial_\phi\tilde{\gamma}_{\phi\phi}+\tilde{\gamma}_{\phi\phi}\left(\partial_h\tilde{\gamma}_{\phi\phi}-2\partial_\phi\tilde{\gamma}_{h\phi}\right)}{2\left(\tilde{\gamma}_{hh}\tilde{\gamma}_{\phi\phi}-\tilde{\gamma}^2_{h\phi}\right)}\,, \tag{B.9}$$

$$\widetilde{\mathscr{T}}^\phi_{hh}=-\frac{\tilde{\gamma}_{h\phi}\partial_h\tilde{\gamma}_{hh}+\tilde{\gamma}_{hh}\left(\partial_\phi\tilde{\gamma}_{hh}-2\partial_h\tilde{\gamma}_{h\phi}\right)}{2\left(\tilde{\gamma}_{hh}\tilde{\gamma}_{\phi\phi}-\tilde{\gamma}^2_{h\phi}\right)}\,, \tag{B.10}$$

$$\widetilde{\mathscr{T}}^\phi_{h\phi}=\widetilde{\mathscr{T}}^\phi_{\phi h}=-\frac{\tilde{\gamma}_{h\phi}\partial_\phi\tilde{\gamma}_{hh}-\tilde{\gamma}_{hh}\partial_h\tilde{\gamma}_{\phi\phi}}{2\left(\tilde{\gamma}_{hh}\tilde{\gamma}_{\phi\phi}-\tilde{\gamma}^2_{h\phi}\right)}\,, \tag{B.11}$$

$$\widetilde{\mathscr{T}}^\phi_{\phi\phi}=\frac{\tilde{\gamma}_{hh}\partial_\phi\tilde{\gamma}_{\phi\phi}+\tilde{\gamma}_{h\phi}\left(\partial_h\tilde{\gamma}_{\phi\phi}-2\partial_\phi\tilde{\gamma}_{h\phi}\right)}{2\left(\tilde{\gamma}_{hh}\tilde{\gamma}_{\phi\phi}-\tilde{\gamma}^2_{h\phi}\right)}\,. \tag{B.12}$$

Let us now particularize to a flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime with line element

$$ds^2=-dt^2+a^2(t)d\vec{x}^2\,, \tag{B.13}$$

where $a(t)$ is the scale factor. Considering homogeneous fields $\varphi^a=\varphi^a(t)$, the equations of motion ((B.3), B.4) boil down to

$$3M_P^2H^2=\frac{1}{2}\tilde{\gamma}_{ab}\dot{\varphi}^a\dot{\varphi}^b+V\,, \tag{B.14}$$

$$M_P^2(2\dot{H}+3H^2)=-\frac{1}{2}\tilde{\gamma}_{ab}\dot{\varphi}^a\dot{\varphi}^b+V\,, \tag{B.15}$$

$$\ddot{\varphi}^a+\widetilde{\mathscr{T}}^a_{bc}\dot{\varphi}^b\dot{\varphi}^c+3H\dot{\varphi}^a=-\tilde{\gamma}^{ab}\partial_b V\,, \tag{B.16}$$

where, as usual, "$\cdot$" denotes differentiation w.r.t. the coordinate time t, and $H=\dot{a}/a$ is the Hubble parameter.

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For future reference, let us mention that during the "slow-roll" inflationary phase ($|\dot{H}| \ll H^2$), the above simplify considerably and become

$$3M_P^2 H^2 \approx V\,, \tag{B.17}$$

$$3H\dot{\varphi}^a \approx -\tilde{\gamma}^{ab}\partial_b V\,. \tag{B.18}$$

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