A programmable Ising machine using Vortex Spin-Torque Nano-Oscillators

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ABSTRACT

Spin torque nano-oscillators are promising spintronic devices that proved to be suitable for various computing architectures using amplitude or frequency encoding schemes. The nonlinear phase dynamics offers further potential and remains largely unexplored in experiments. Here, we demonstrate how the stochastic phase dynamics of injection-locked spin torque nano-oscillators can be leveraged to generate perfectly unbiased random numbers, and based on this how to emulate Ising spins to be used for a proof-of-concept Ising machine capable of addressing combinatorial optimization problems.

Keywords: Ising machine, Spin torque nano-oscillators, Second-harmonic injection-locking, phase binarization

1. INTRODUCTION

Solving computational problems of ever-increasing complexity and size goes in hand with an increase in the energy consumption related to the computing architecture that imposes a constant data transfer between memory and processing units. Therefore, novel hardware approaches are needed to remedy the issue of energy consumption, while improving at the same time crucial parameters such as computation speed, compactness and integrability. One of these novel approaches are Ising machines (IM), which are physics-inspired hardware platforms aiming to efficiently solve combinatorial optimization problems (COPs). IMs emulate the Ising model $H_{\mathrm{Ising}}(\{\mathbf{S}\}) = \sum_{<i,j>} J_{ij} S_i \cdot S_j - \mu \sum_i h_i S_i$, that describes a 2D lattice of binary-valued spins S_i, S_j. Such an IM can solve combinatorial optimization problems since it inherently seeks to minimize its total energy, where spin-spin coupling described by the coupling matrix J_{ij} is used to map arbitrary COPs to the IM. The convergence of the system's energy towards its global minimum, given by the spin configuration $\mathbf{s}$, while escaping local minima, corresponds to the algorithm to find the solution to the mapped COP. A practical implementation of an IM requires four ingredients: a physical building block emulating an Ising spin, tuneable interconnections between the spins with individual bias to (re-)define the optimization problem, a process to switch between the binary spin states and an annealing scheme to explore the energy landscape of the coupled system in search for the global energy minimum.

A promising approach to build an Ising machine is to use the phase of a non-linear oscillator as an Ising spin.[1] While a free-running oscillator has an arbitrary phase, synchronizing it to an external signal provides a stable phase reference. A particular variant is the second harmonic injection-locking (SHIL), where the frequency of the external signal has twice the oscillator's natural frequency $f_{\mathrm{SHIL}} \approx 2 f_{\mathrm{Osc}}$. In this case, the oscillator's phase

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can only take binary values $\Delta\phi_1$ and $\Delta\phi_2$ that are π shifted, $\Delta\phi_2 = \Delta\phi_1 + \pi$, with respect to the SHIL signal. The phase can therefore act as an Ising spin.

In this context, spin torque driven oscillators are promising candidates for building large arrays of coupled oscillator networks envisioned for broadband microwave communication and unconventional computing applications.[2–5] In this study we will employ magnetic tunnel junction based devices that are non-linear, low power and tuneable microwave oscillators with magnetic structures similar to magnetic non-volatile memories. They can be fabricated in large numbers in microelectronic chips, making them excellent competitors for integration with CMOS technology. A spin-polarized direct current applied to such a device can generate self-sustained oscillations of the free-layer magnetization, which produces microwave voltage oscillations through magneto-resistive effects. Due to the nonlinear properties of the magnetization dynamics, varying the applied direct current allows for tuning of the spin-torque nano-oscillator (STNO) frequency f_{STNO} as well as for efficient synchronization of STNOs to external microwave signals within a frequency span called the injection-locking range. An example is shown in Fig. 1 'a) and (b) for the devices studied here, which are magnetic tunnel junction based STNOs with a vortex state free layer (called here vortex-STNO).

2. TOWARDS A SPINTRONIC ISING MACHINE BASED ON VORTEX SPIN-TORQUE NANO-OSCILLATORS

2.1 Phase binarization through second harmonic injection locking (SHIL)

As shown in Fig. 1 (c), the phase of the vortex-STNOs under SHIL ($f_{\mathrm{SHIL}} \approx 2f_{\mathrm{STNO}}$),[6,7] can take binary values. Furthermore, thermal fluctuations induce stochastic phase jumps between these two discrete phase states that differ by π and that are completely equivalent with respect to the driving signal .[8] In addition, control of the phase jump rate is achieved via the frequency mismatch between the driving signal frequency f_{SHIL} and the STNO frequency f_{STNO}. As the phase jump process is completely thermally driven, it leads to the generation of a perfectly unbiased and random bitstream that can be exploited for true random number generation of cryptographic quality.[8] Hence, the vortex-STNO phase, under SHIL and thermal fluctuations, emulates a stochastic Ising spin. Coupling several vortex-STNOs will then represent an Ising spin system, where the mutual coupling will bias the phase state configuration of the full STNO network. Thermal fluctuations will allow for an efficient means to search for the global energy minimum by exploring stochastically the different phase state configurations. The global energy minimum is then inferred from the phase state configuration with the highest probability.

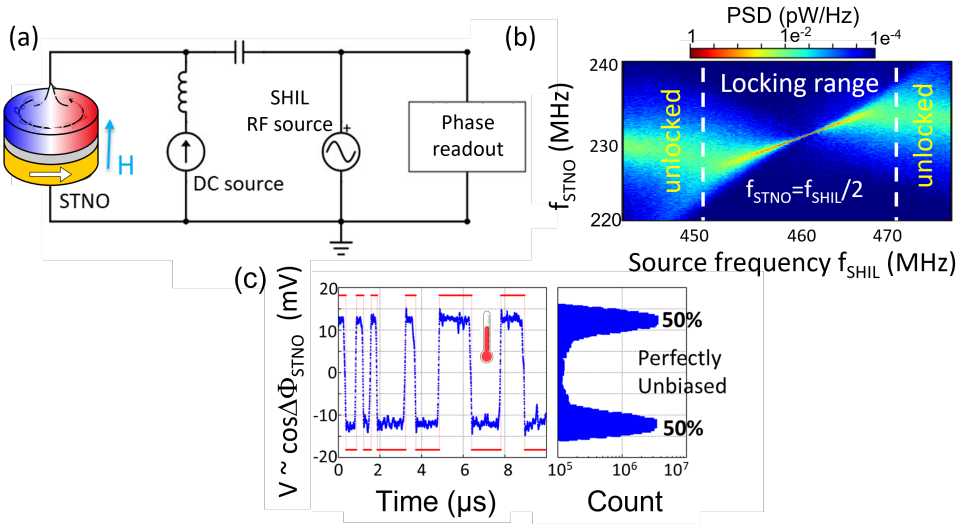


Figure 1. (a) Schematic of the experimental setup for the second harmonic injection locking of a single vortex-STNO. (b) STNO frequency f_{STNO} vs. SHIL RF source frequency f_{SHIL} demonstrating injection locking. (c) Example of a time trace of the STNO phase as a function of time.

2.2 Phase state probabilities of two coupled vortex-STNOs

To demonstrate the effect of the coupling, we investigated first the phase state probabilities of two coupled vortex-STNOs. The coupling is achieved via a field-line antenna nanofabricated on top of the vortex-STNOs. The signal of STNO1 is then fed into the field-line of STNO2, and vice versa, producing locally a microwave field at frequency $f_{\mathrm{STNO}} = f_{\mathrm{SHIL}}/2$, see Fig. 2 (a), that influences the phase dynamics of the injection-locked vortex-STNO. In order to control the coupling strength, a variable resistance is included in the coupling path. The phase state of each vortex-STNO can then be read out via a specifically designed phase-readout circuit. An example is shown in Fig 2 (b), where the phases $\Delta\phi_{\mathrm{STNO1}}$ and $\Delta\phi_{\mathrm{STNO2}}$ are shown to fluctuate as a function of time between two possible binarized phase states 0 and π. This leads to permanent transitions from one of the four phase state configurations of the two coupled vortex-STNOs to another with $\{\Delta\phi_{\mathrm{STNO1}}, \Delta\phi_{\mathrm{STNO2}}\} = \{(0,0),(\pi,\pi),(0,\pi),(\pi,0)\}$. These states are highlighted in Fig 2 (b). It is noted that for this example of two coupled Ising spins the configurations $\{(0,0),(\pi,\pi)\}$ are degenerate and are called parallel and similarily the configurations $\{(0,\pi),(\pi,0)\}$ are degenerate and are called antiparallel.

In order to verify whether the coupling influences the phase state configuration, and which configuration is more probable, the Pearson correlation ρ is calculated from the phase time traces, with

$$\rho = \frac{\mathrm{cov}(\Delta\phi_{\mathrm{STNO1}}, \Delta\phi_{\mathrm{STNO2}})}{\sigma_{\mathrm{STNO1}} \cdot \sigma_{\mathrm{STNO2}}} \tag{1}$$

Here cov is the covariance and σ_i is the standard deviation. As can be seen in Fig. 2 (c), the correlation is positive for small values of resistance, hence strong coupling, favoring a parallel configuration, while for resistances in the range of $\mathrm{R} = 400 - 2000\ \Omega$, the correlation is negative, favoring an antiparallel state. For larger resistances, the coupling becomes too weak, the correlation is zero, indicating that the phases of the two vortex-STNOs are independent. This result demonstrates that (i) the coupling signal generated by each vortex-STNO is strong enough to influence the phase state of the other vortex-STNO, (ii) the coupling can be tuned via the resistances and (iii) the coupling sign can change.

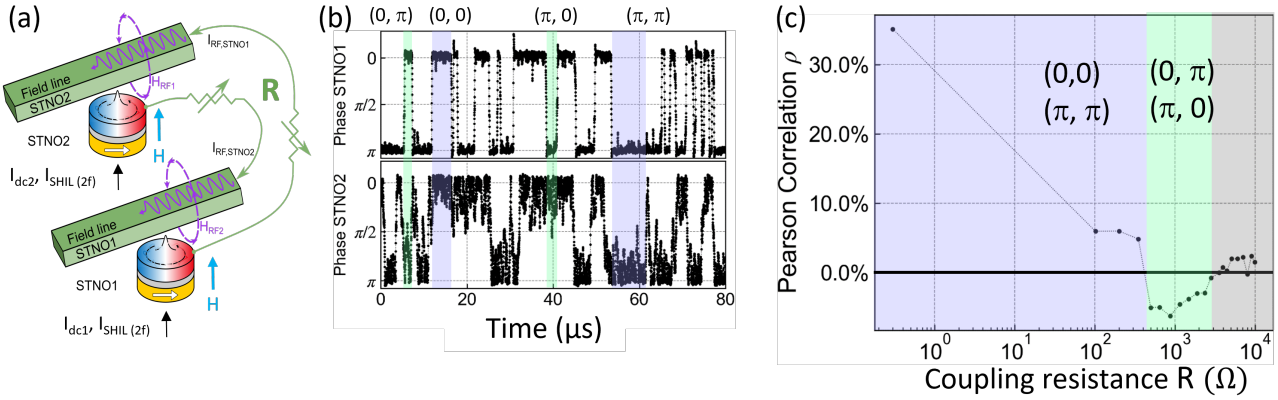


Figure 2. (a) Schematic illustration of the coupling of two vortex-STNOs via a field-line and a coupling resistance R. (b) Time traces of the phases of both vortex-STNOs and (c) Pearson correlation ρ calculated from (b).

2.3 N=3 Ising spin system

Having demonstrated the basic brick of a two-spin Ising system based on vortex-STNOs, we developed a fully interconnected platform that enables tuneable coupling via field-lines between up to N=5 vortex-STNOs. Fig. 3 (a) depicts the coupling of a network of N=3 SHIL vortex-STNOs and Fig. 3 (b) shows the time traces of the phase states for the three STNOs, where some of the eight possible phase state configurations (PSC) are highlighted. From these time traces, the pairwise Pearson correlation ρ_{ij} of the three pairs of STNO phases is extracted and used to estimate the coupling constant $J_{ij} \propto \rho_{ij}$. This is then inserted into the Ising Hamiltonian H_{Ising} to calculate the corresponding Hamiltonian energy $H_{\mathrm{Ising}} \propto \sum \rho_{ij} S_i S_j$, with S_i the experimental phase states of

the STNOs obtained from Fig. 3 (b). As can be seen in Fig. 3 (c), the Hamiltonian values H_{Ising} are in perfect agreement with the probability distribution for the eight phase state configurations PSC_i, calculated from their respective mean dwell times τ_{PSCi}. In particular, the states of highest probability correspond to those of lowest energy, thus confirming that the system of the three coupled vortex-STNOs represents an Ising Hamiltonian.

In conclusion, this work demonstrates that all the necessary requirements are met to build a spatial and re-configurable Ising machine based on vortex-STNOs.

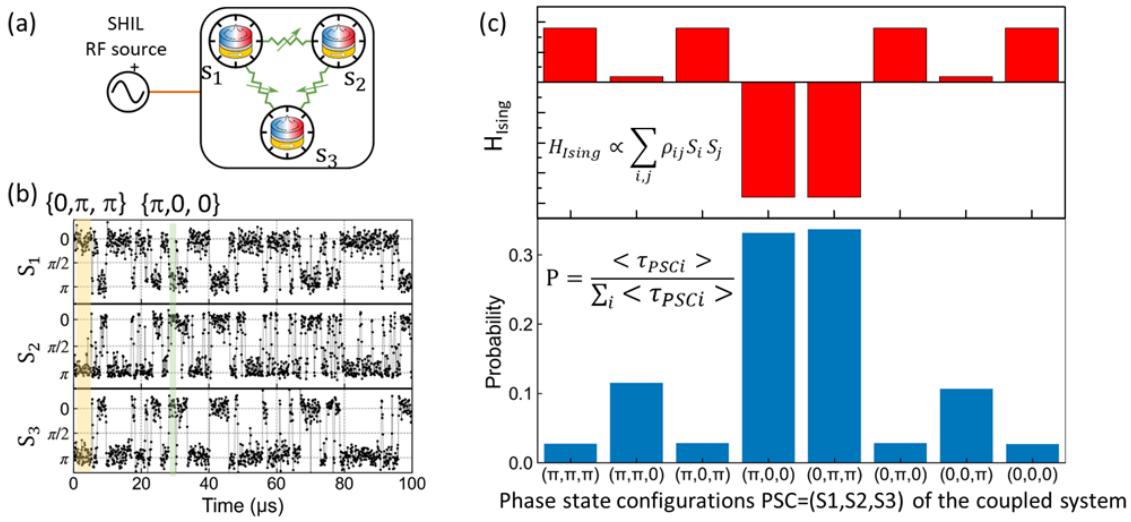


Figure 3. (a) Schematic of 3-node Ising-spin system based on second harmonic injection locked vortex-STNOs. (b) Time traces of the phase states S_i of all three vortex-STNOs. (c) Probability distribution for the eight possible phase state configurations PSC of the coupled system, revealing that the configurations $(\pi,0,0)$ and $(0,\pi,\pi)$ of highest probability correspond to those of lowest Hamiltonian value of H_{Ising}.For calculating H_{Ising}, $J_{ij} \propto \rho_{ij}$ and the values of S_i were defined as: "0" phase state $S_i = 1$ and "π" phase state $S_i = -1$.

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REFERENCES

[1] Wang, T., Wu, L., Nobel, P., and Roychowdhury, J., "Solving combinatorial optimisation problems using oscillator based ising machines," *Natural Computing* **20**, 287–306 (May 2021).

[2] Grollier, J., Querlioz, D., Camsari, K. Y., Everschor-Sitte, K., Fukami, S., and Stiles, M. D., "Neuromorphic spintronics," *Nature Electronics* **3**, 360–370 (Mar. 2020).

[3] Chen, T., Dumas, R. K., Eklund, A., Muduli, P. K., Houshang, A., Awad, A. A., Durrenfeld, P., Malm, B. G., Rusu, A., and Akerman, J., "Spin-torque and spin-hall nano-oscillators," *Proceedings of the IEEE* **104**, 1919–1945 (Oct. 2016).

[4] Houshang, A., Zahedinejad, M., Muralidhar, S., Checiński, J., Khymyn, R., Rajabali, M., Fulara, H., Awad, A. A., Dvornik, M., and Åkerman, J., "Phase-binarized spin hall nano-oscillator arrays: Towards spin hall ising machines," *Physical Review Applied* **17** (Jan. 2022).

[5] Albertsson, D. I., Zahedinejad, M., Houshang, A., Khymyn, R., Åkerman, J., and Rusu, A., "Ultrafast ising machines using spin torque nano-oscillators," *Applied Physics Letters* **118** (Mar. 2021).

[6] Lebrun, R., Jenkins, A., Dussaux, A., Locatelli, N., Tsunegi, S., Grimaldi, E., Kubota, H., Bortolotti, P., Yakushiji, K., Grollier, J., Fukushima, A., Yuasa, S., and Cros, V., "Understanding of phase noise squeezing under fractional synchronization of a nonlinear spin transfer vortex oscillator," *Physical Review Letters* **115** (June 2015).

[7] Litvinenko, A., Sethi, P., Murapaka, C., Jenkins, A., Cros, V., Bortolotti, P., Ferreira, R., Dieny, B., and Ebels, U., "Analog and digital phase modulation and signal transmission with spin-torque nano-oscillators," *Physical Review Applied* **16** (Aug. 2021).

[8] Phan, N.-T., Prasad, N., Hakam, A., Valli, A. S. E., Anghel, L., Benetti, L., Madhavan, A., Jenkins, A. S., Ferreira, R., Stiles, M. D., Ebels, U., and Talatchian, P., "Unbiased random bitstream generation using injection-locked spin-torque nano-oscillators," *Physical Review Applied* **21** (Mar. 2024).