

Abstract

Unbounded domains often appear in engineering applications, such as acoustic or elastic wave radiation from a body immersed in an infinite medium. To simulate the unboundedness of the domain special boundary conditions have to be imposed: *the Sommerfeld radiation condition*.

In the present work we focused on steady-state wave propagation. The objective of this research is to obtain accurate prediction of phenomena occurring in exterior acoustics and elastodynamics and ensure the quality of the solutions even for high wavenumbers.

To achieve this aim, we develop higher-order domain-based schemes: Spectral Element Method (*SEM*) coupled to Dirichlet-to-Neumann (*DtN*), Perfectly Matched Layer (*PML*) and Infinite Element (*IEM*) methods. Spectral elements combine the rapid convergence rates of spectral methods with the geometric flexibility of the classical finite element methods. The interpolation is based on *Chebyshev* and *Legendre* polynomials.

This work presents an implementation of these techniques and their validation exploiting some benchmark problems. A detailed comparison between the DtN, PML and IEM is made in terms of accuracy and convergence, conditioning and computational cost.

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