



Constraining Binary Mergers in Active Galactic Nuclei Disks Using the Nonobservation of Lensed Gravitational Waves

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Abstract

The dense and dynamic environments within active galactic nuclei (AGN) accretion disks may serve as prolific birthplaces for binary black holes (BBHs) and one possible origin for some of the BBHs detected by gravitational-wave (GW) observatories. We show that a considerable fraction of the BBH in AGN disks will be strongly lensed by the central supermassive black hole (SMBH). Thus, the nonobservation of lensed GW signals can be used to constrain the fraction of BBH binaries residing in AGN disks. The nondetection of lensing with current $\mathcal{O}(100)$ detections will be sufficient to start placing constraints on the fraction of BBHs living within accretion disks near the SMBH. In the next-generation detectors era, with $\mathcal{O}(10^5)$ BBH observations and no lensed events, we will be able to rule out most migration traps as dominant birthplaces of BBH mergers; moreover, we will be able to constrain the minimum size of the accretion disk. On the other hand, should AGNs constitute a major formation channel, lensed events from AGNs will become prominent in the future.

Unified Astronomy Thesaurus concepts: Active galactic nuclei (16); Gravitational wave astronomy (675); Strong gravitational lensing (1643); Gravitational waves (678)

1. Introduction

Active galactic nuclei (AGN) are promising sites for stellar-origin black hole (BH) mergers (K. E. S. Ford et al. 2019; K. E. S. Ford & B. McKernan 2022). These binary black holes (BBHs) reside in the accretion disk of the AGN, orbiting the center supermassive black hole (SMBH), and merge within a rapid timescale (A. Secunda et al. 2019). The resulting gravitational waves (GWs; W. Ishibashi & M. Gröbner 2020) could be detected by current and future ground-based detectors (J. Aasi et al. 2015; F. Acernese et al. 2015; D. Reitze et al. 2019; K. Ackley et al. 2020; T. Akutsu et al. 2021; A. Grado 2023). These mergers are expected to be particularly abundant near the migration traps—regions in the disk where inward and outward torques exerted by the gas disk cancel (T. A. Thompson et al. 2005; B. McKernan et al. 2012; Y. Yang et al. 2019; P. Peng & X. Chen 2021). The detection of such BBHs may provide information about their AGN environment (N. C. Stone et al. 2016; B. McKernan et al. 2020b; Y.-Z. Wang et al. 2021), such as the mass of the SMBH and the age of the AGN (A. Vajpeyi et al. 2022). Remnant BHs from such mergers, rapidly moving through the gas disk due to the GW recoil, could produce a transient electromagnetic (EM) flare (A. Loeb 2007; B. McKernan et al. 2019; M. J. Graham et al. 2020, 2023; J.-M. Wang et al. 2021). This could be observable for an observer perpendicular to the disk (face-on). When viewed from the plane of the disk (edge-on), the flare would be blocked by the

gaseous torus surrounding the disk. However, if the BBH is located behind the SMBH, then the GW signal can be strongly lensed by the SMBH, splitting it into two “images” of the same GW (M. Millon et al. 2023; R. Takahashi & T. Nakamura 2003). These images would appear as repeated waves with similar properties, except for differing amplitudes, arrival times, and phases. These lensed signals can be detected using standard lensing search methods (K. Haris et al. 2018; A. Liu et al. 2023). In this Letter, we show that a fraction of such mergers produce detectable lensed GWs, with a lensing probability that depends on their radial distribution on the accretion disk. Consequently, the nondetection of lensed GW events will constrain the number of such systems and exclude regions of the disk as the dominant BBH merger sites.

2. The AGN-BBH System

We consider a BBH residing within the accretion disk of an AGN, and orbiting around a SMBH of mass M_{SMBH} at a fixed radius R_{orb} . We refer to this kind of binary as AGN-BBH. We define the x -axis by the projection of the observer onto the orbital plane, and ϕ_S is the azimuthal angle of the BBH with respect to that x -axis (Figure 1). For an observer at an angle ι , the probability of observing a binary that is located at a radius⁸ $r = R_{\text{orb}}/R_{\text{Sch}}$ as strongly lensed (i.e., the binary falls within the volume defined by the Einstein radius θ_E , Equation (A6)), is given by

$$p(\text{lensing} | r, \iota) = \frac{1}{\pi} \arccos\left(\frac{x}{\sin \iota}\right), \quad (1)$$

⁸ The Schwarzschild radius is $R_{\text{Sch}} = 2GM_{\text{SMBH}}/c^2$.



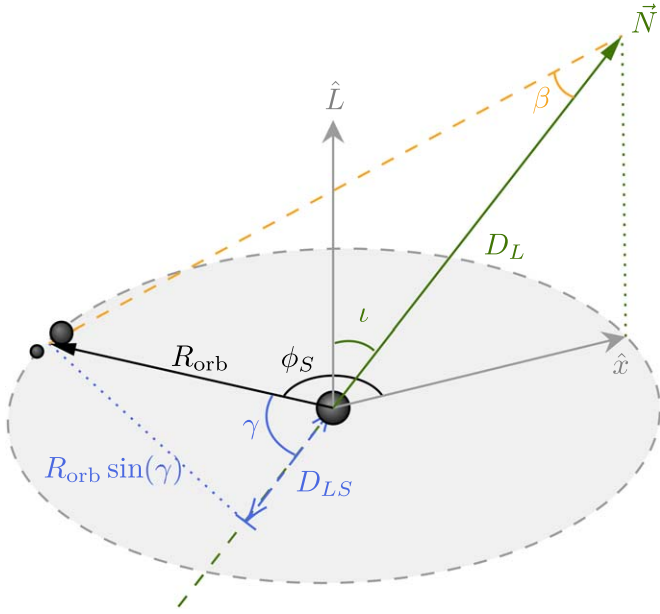


Figure 1. Schematic diagram of the AGN-BBH system. Diagram showing a BBH (left) orbiting an SMBH (center). The gray ellipse is the orbital plane of the BBH around the SMBH, defined by the orbital angular momentum $\hat{L}$. The accretion disk is modeled as a thin disk, coplanar with the orbital plane. The observer is located at N and represented by the green vector; its continuation below the plane is the green dashed line, and they form the optical axis. The inclination angle is i , and the projection of N onto the orbital plane defines the x -axis, from which the azimuthal angle of the BBH, ϕ_S , is measured. The angles subtended by the source and the SMBH, and the source and the observer are denoted by γ and β , respectively.

where x is defined as

$$x = \frac{\sqrt{1 + r^2} - 1}{r}. \quad (2)$$

Assuming that observers are distributed isotropically around the SMBH (i.e., no preferred orientation for AGNs), we can marginalize over i to compute the lensing probability for BBH located at a radius r in the AGN disk

$$f_{\text{lensing|AGN}} = p(\text{lensing} | r) \quad (3)$$

$$= \int_{\arcsin x}^{\pi/2} \frac{1}{\pi} \arccos\left(\frac{x}{\sin i}\right) \sin(i) di \quad (4)$$

$$\approx \frac{1}{2r}. \quad (5)$$

The final approximation is only valid for large r (for $r \gtrsim 10$, see Figure 2) and the full derivation can be found in Appendix A.

3. Constraining the Fraction of BBHs in AGN Disks

Knowing the probability of observing a lensed BBH in an AGN disk, we show that the nondetection of lensed GW events can then be used to constrain the fraction of BBHs that reside in AGN disks (f_{AGN}) among all BBH detections. First, we define f_{AGN} as the fraction of GW events that originated from AGN among all N_{obs} observed GW events:

$$f_{\text{AGN}} \equiv \frac{N_{\text{obs. BBH from an AGN}}}{N_{\text{obs.}}}, \quad (6)$$

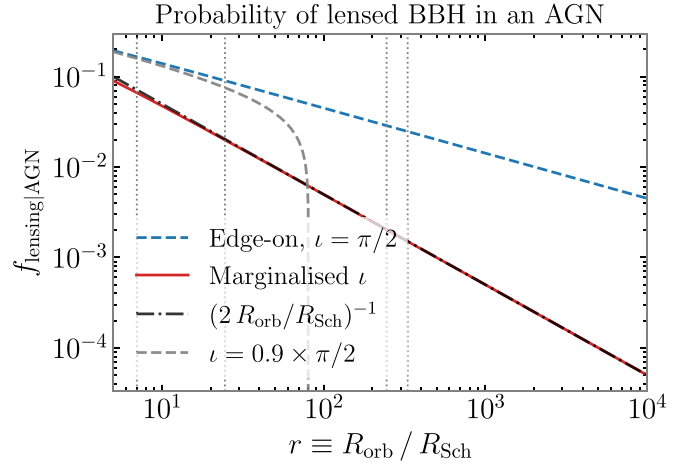


Figure 2. BBH lensing probability as a function of R_{orb} . The dashed blue and gray lines represent the lensing probability (Equation (1)) at fixed inclinations $i = \pi/2$ and $9\pi/20$, respectively. The former follows the $1/\sqrt{r}$ trend, and the latter has a finite radius cut-off, at around $80 R_{\text{Sch}}$. Shown in red is the marginalized probability over inclination (Equation (4), or (A15)), which matches very closely $1/(2r)$ (black dashed-dotted line, Equation (5)), except for at small radius (for $r \lesssim 10$). Vertical dotted lines indicate the locations of some migration traps mentioned in the literature (e.g., E. Sirko & J. Goodman 2003; T. A. Thompson et al. 2005; J. M. Bellovary et al. 2016; P. Peng & X. Chen 2021), where a majority of binary black holes are expected to migrate to and merge.

then, the probability of observing a lensed BBH from AGN is

$$f_{\text{lensed} \wedge \text{AGN}} = f_{\text{lensing|AGN}} \times f_{\text{AGN}}, \quad (7)$$

where the conditional probability characterizes the probability of detecting a lensed binary from an AGN, which is subject to the AGN geometry and potential detection loss due to demagnified images. We will address the latter issue in later sections. For the remainder of this Letter, we will assume each lensed AGN-BBH will be correctly detected and identified. For a given number of GW observations and f_{AGN} , this probability can be used to estimate the expected number of lensed events via the binomial distribution.

A nondetection of lensed events can, therefore, be used to constrain the fraction of AGN events among all observed binaries. Following S. Basak et al. (2022), we define the upper bound of this AGN fraction to be the 90th percentile of the conditional distribution of f_{AGN} , i.e., the $f_{\text{AGN}}^{\text{upper}}$ satisfies

$$90\% = \int_0^{f_{\text{AGN}}^{\text{upper}}} df_{\text{AGN}} p(f_{\text{AGN}} | N_{\text{obs}}, N_{\text{lensed}} = 0). \quad (8)$$

Figure 3 shows the constraints on f_{AGN} if we do not detect any lensed events, assuming different numbers of observed events, and across a range of orbital radii around the SMBH. With the $\mathcal{O}(100)$ events from the LIGO–Virgo–KAGRA (LVK) third observation run (O3) (R. Abbott et al. 2024), from the left panel, we find that if all AGN-BBHs merge near the “last migration trap” ($\sim 10 R_{\text{Sch}}$) (P. Peng & X. Chen 2021), then the fraction of AGN-BBHs events cannot be over 47% of all LVK binaries. This will be limited to within 5% when we have $\mathcal{O}(1000)$ (unlensed) observations. Most other migration traps reported are located between 100 and $1000 R_{\text{Sch}}$ (J. M. Bellovary et al. 2016; E. Grishin et al. 2024), as an example, the light gray (hatched from the top-left) curve represents the constraint for binaries at $245 R_{\text{Sch}}$. In this case,

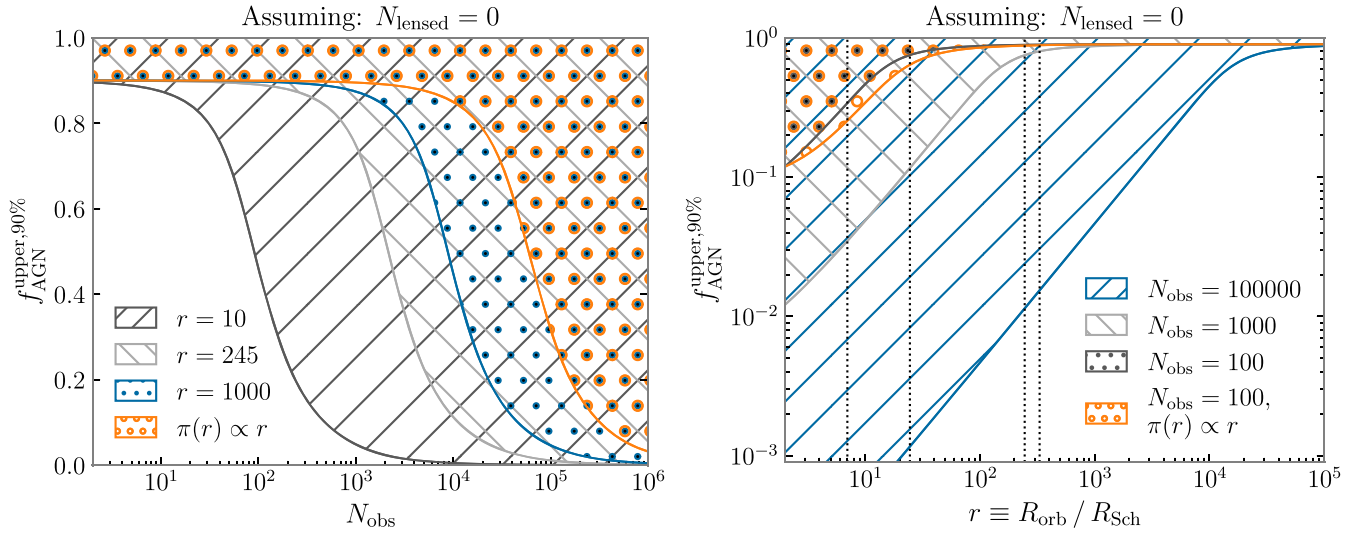


Figure 3. Constraints on the fraction of AGN-BBH events based on the nondetection of lensed events. The left and right panels show the 90th percentile upper bound of f_{AGN} as a function of the number observed BBH events N_{obs} and the radial distance r from the SMBH, respectively. The hatched area represents the f_{AGN} that is ruled out by the nondetection of lensed events. In the left panel, each curve corresponds to a different fixed radial distribution of BBH around the SMBH; in particular, two gray and blue curves (hatched from the top-right and the top-left, and small circles) represent BBHs from fixed radii $r = 10$, 245, and 1000, respectively. The orange (circles) curve accounts for the whole disk case by assuming the number of BBHs grows linearly with the radius (i.e., BBHs distributed uniformly on the disk, with $2 \leq r \leq 10^4$). The right panel shows the disk inner region that will be excluded as the dominant BBHs production site, for three different numbers of observed events, as well as for the number of mergers growing linearly at $N_{\text{obs}} = 100$ (orange, circles). Note that in the last case, the x-axis represents the maximum disk radius, $2 \leq r \leq r_{\text{max}}$, with the same meaning as in Figure 4. It is worth noting that, following from the definition Equation (8), $f_{\text{AGN}}^{\text{upper},90\%}$ is 0.9 for regions with no constraint. Finally, the vertical dotted lines denote the trap locations reported in J. M. Bellovary et al. (2016) and P. Peng & X. Chen (2021), at $r \in \{7, 24.5, 245, 331\}$, respectively.

constraining the AGN-BBH fraction to less than 50% will require ~ 2200 events.

We also show, in orange (circles), the constraint when the BBHs are distributed uniformly over the entire disk (number density growing linearly with the radius, with $2 \leq r \leq 10^4$).⁹ In the right panel of Figure 3, we show the upper limit of f_{AGN} as a function of orbital radius instead. This is useful for understanding the constraints on different AGN disk models that predict AGN-BBHs located at a narrow range of radii. The dark gray curve (with small circles) is the constraint achieved with 100 events, which can only place meaning bounds for AGN-BBHs located within $100 R_{\text{Sch}}$ from the center, and at most down to $\sim 12\%$. In the era of third-generation detectors when the number of BBH events are expected to reach $\mathcal{O}(10^5)$ per year (M. Evans et al. 2023), a nondetection of lensed events can limit the fraction of GW events from AGNs down to 0.5%–5% if the AGN-BBHs are located in the range $100 \leq r \leq 1000$, where the migration traps are expected to form, denoted by vertical gray dotted lines. This will enable us to rule out these trap locations (J. M. Bellovary et al. 2016; E. Grishin et al. 2024) as dominant AGN-BBH production sites. We note that, however, there are possible trap locations at $r > 10^4$, as suggested in H. Tagawa et al. (2020), which will require much longer observation time to be constrained.

Finally, Figure 4 shows the f_{AGN} constraint taking the contribution from the whole disk into account as a function of their maximum disk radii, R_{max} , while assuming four different disk models with 10^5 BBH observations. We find that other than small fluctuations at particular radii due to the different density profiles, the overall lensing probability drops with r_{max}

regardless of models, and hence results in the largely similar constraint on f_{AGN} . This result also suggests that the nondetection of lensing is most effective at informing the minimum size of the disks, while the effects from different disk models only become significant for $r_{\text{max}} \gtrsim 2000$.

4. Discussion

M. J. Graham et al. (2020) reported a plausible electromagnetic counterpart to the BBH merger event GW190521, which is consistent with a flare produced by the motion of the remnant BH through an AGN disk. Following this, M. J. Graham et al. (2023) performed a search for electromagnetic counterparts to candidate BBH triggers from O3 and found, at most, seven pairs of GW-EM signals,¹⁰ with an expectation of three pairs when $f_{\text{AGN}} = 0.5$. If those were real, they imply the lower bound of $f_{\text{AGN}} \approx 1$ (see Equation (14) thereof). If we further assume all these binaries are GW190521-like, located in the migration trap at $r \approx 331$ (T. A. Thompson et al. 2005; M. J. Graham et al. 2020), then we find that with ~ 100 (1000) observations, the probability of observing at least one lensed event is 14% (78%).

Some limitations of this work are worth mentioning. For one, we used a thin disk model for the accretion disk. This assumption could be refined, but even in a full disk model, the typical aspect ratio is less than 10%. Preliminary investigations do not show significant changes to our conclusions.

When the BBH is located at a large distance from the optical axis, the second lensed image will be demagnified. This will reduce the horizon distance to lensed AGN-BBH systems from which both images are detectable. We have estimated the loss

⁹ In this simple model, with $\pi(r) \propto r$, the marginalized lensing probability is $\approx 1/r_{\text{max}}$. Thus, the lensing probability for this distribution is the same as that of the migration traps model, where the binaries are located at $r = r_{\text{max}}/2$ (see Equation (5)).

¹⁰ There are 12 event pairs listed in Table 3 of M. J. Graham et al. (2023); but we find, at most, seven of those can be simultaneously true, for some of the 12 pairs share the same GW or EM signal.

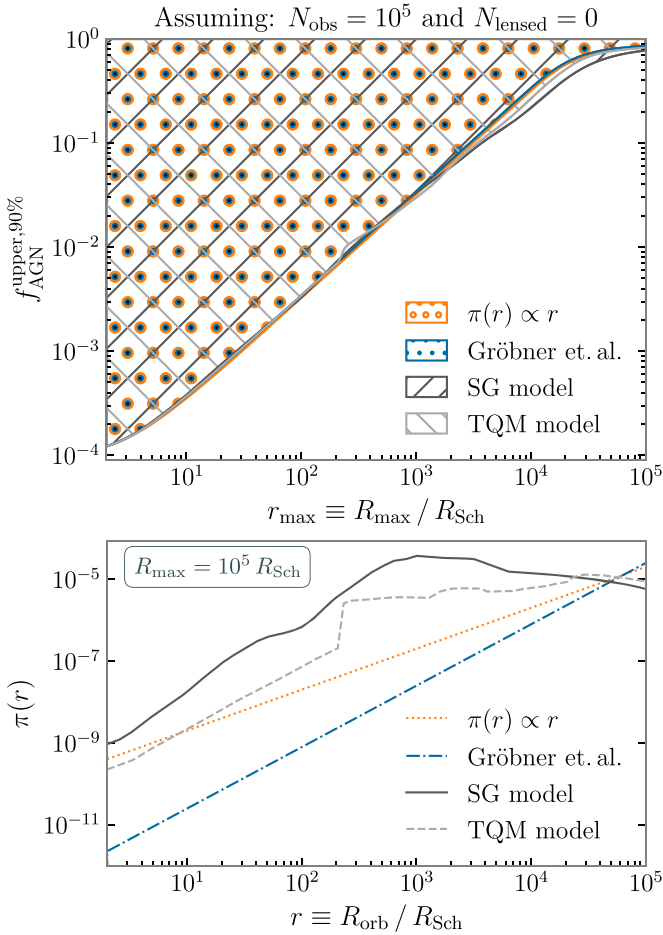


Figure 4. Constraints on the fraction of AGN-BBH events when they are distributed over the entire disk. The 90th percentile upper bound of f_{AGN} is shown as a function of the maximum AGN disk radius, assuming four disk models and with 10^5 observations, in the upper panel. The hatched area represents the f_{AGN} that is ruled out by the nondetection of lensed events. The orange curve denotes the same simple disk model as in Figure 3. The blue curve assumes the BBHs follow the radial distribution given by Equation (19) of M. Gröbner et al. (2020). The last two, dark and light gray, have their radial distributions constructed from the density and height profiles from E. Sirko & J. Goodman (2003; SG) and T. A. Thompson et al. (2005; TQM), respectively. In the bottom panel, we show the several radial distributions we employed, in log-log scale, normalized to $r_{\text{max}} = 10^5$, following the same color codes as above.

of binaries due to this is 60% in the worst-case scenario. Furthermore, when the sensitivity of the detectors is higher (e.g., next-generation detectors), and the horizon distance extends far beyond where these binaries are expected to form, first, we find that effects due to demagnification will be negligible. Moreover, when that is the case, then there are no selection effects induced by the redshift distribution. In this case, the f_{AGN} that we will report is not limited to the *observed* population of AGN-BBH, but the *actual* fraction of binaries in AGN disks. Note that even though there could be additional effects on the gravitational lensing of binaries in the vicinity of the SMBH, such as wave-optics effects at low GW frequencies, which will be relevant for future space-based GW detectors (R. Takahashi & T. Nakamura 2003; D. J. D’Orazio & A. Loeb 2020; M. Pijnenburg et al. 2024), or the potentially highly magnified overlapping signals (R. K. L. Lo et al. 2024), they shall be addressed by future infrastructures for detection of lensed GWs, which shall not hamper the merit of this work as

long as these lensed signals are correctly identified. Nevertheless, if there are other stellar objects in the vicinity of the SMBH with much lower masses, then they could induce additional microlensed images, similar to the scenario considered by A. Mishra et al. (2021). Moreover, for sources at distances as close as $\mathcal{O}(10) R_{\text{Sch}}$, the spin nature of GW can induce geodesics that are frequency- and polarization-dependent (A. I. Harte & M. A. Oancea 2022; M. A. Oancea et al. 2024a, 2024b), effectively leading to birefringent effects to the lensed images. Also, the thin-lens approximation might not hold strongly at this distance. To this end, we have checked that changes in the deflection angle are sufficiently small at $r = 10$ using the exact lens equation (V. Bozza 2008). Nevertheless, this part of the disk will only contribute to a small proportion of the whole disk, and such deviation is negligible. Finally, while we have here excluded event counts from binary neutron stars (BNSs), the accretion disk could also host BNSs, the merger rate of which is correlated with that of BBHs (B. McKernan et al. 2020a). For the next-generation detectors, where the expected BNS detections are higher than that of BBH, this shall render the prospective constraints even stronger.

5. Conclusion

In this work, we showed that a considerable fraction of the BBH in AGN disks will be strongly lensed by the central SMBH. Thus, the nonobservation of lensed GW signals can be used to constrain the fraction of BBH binaries residing in AGN disks. While the current constraints are modest, as the number of observed GW signals increases, this will allow us to set tight constraints on the fraction of AGN-BBH systems among all detections. In particular, we will be able to rule out some locations of migration traps as the dominant BBH merger sites, as well as to put a lower bound on the size of the accretion disk. Future GW observatories that are expected to detect $\mathcal{O}(10^5)$ events per year should tighten these bounds further to the subpercent level within their first year of operation. On the other hand, if AGN-BBH is a major formation channel for BBHs, then we can expect to observe a sizable number of lensed events in the future. Previous studies based on the existence of AGN-BBH systems (K. E. S. Ford & B. McKernan 2022; M. J. Graham et al. 2023) and spatial correlation (I. Bartos et al. 2017; N. Veronesi et al. 2022) placed some lower bounds on the fraction of AGN-BBH events. Here, we present the upper bound of f_{AGN} based on the nondetection of lensed events, which will further allow us to sandwich the possible f_{AGN} range and constrain different AGN models. Besides evaluating the spatial correlation of the electromagnetic counterpart of AGN-BBH GW signals, confirmation of such AGN-BBH events can also rely on measuring the recoil direction (J. Calderón Bustillo et al. 2022; S. H. W. Leong et al. 2024, in preparation), and the deviations in waveform morphology due to the orbital motion around the SMBH (A. Torres-Orjuela et al. 2021; A. Vijaykumar et al. 2022, 2023; S. L. Morton et al. 2023), the dense gaseous environment, which particularly affects the ringdown signal (S. H. W. Leong et al. 2023). As a final remark, we note that confirming and characterizing such AGN-BBH events is also possible via the characteristic lensing signatures from such systems (L. Gondán & B. Kocsis 2022), or potentially the large opening angle between the two lensed GW images and Doppler lensing, which we shall defer to future work to explore.

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Software: NumPy (C. R. Harris et al. 2020), SciPy (P. Virtanen et al. 2020), Matplotlib (J. D. Hunter 2007), Astropy (Astropy Collaboration et al. 2013, 2018; Astropy Collaboration et al. 2022), Mathematica (Wolfram Research, Inc 2022), Cython (S. Behnel et al. 2011).

Appendix A Thin-disk Geometry

Throughout the text, we assume the source is confined to an infinitely thin disk, and a BBH on the disk can be represented by the following unit vector:

$$\hat{\mathbf{S}} = (\cos \phi_S \quad \sin \phi_S \quad 0), \quad (\text{A1})$$

and the optical axis is

$$\hat{\mathbf{O}} = -\hat{\mathbf{N}} = (-\sin \iota \quad 0 \quad -\cos \iota). \quad (\text{A2})$$

They form an angle γ :

$$\cos \gamma = \langle \hat{\mathbf{S}}, \hat{\mathbf{O}} \rangle = -\cos \phi_S \sin \iota, \quad (\text{A3})$$

and from the geometry in Figure 1, the following relation holds:

$$D_S \tan \beta = R_{\text{orb}} \sin \gamma, \quad (\text{A4})$$

where β is the source angle. In this appendix, we shall derive the expressions shown in Section 2 with a relaxed condition, namely, we assume lensing will occur as long as $\beta/\theta_E = y \leq y_{\text{max}}$. In the main text, all results had y_{max} set to 1, for a more conservative constraint.

$$\frac{\beta}{y_{\text{max}}} \leq \theta_E = \sqrt{\frac{2R_{\text{Sch}}D_{LS}}{D_S D_L}} \quad (\text{A5})$$

$$\approx \frac{R_{\text{Sch}}}{D_L} \sqrt{2 \frac{D_{LS}}{R_{\text{Sch}}}}. \quad (\text{A6})$$

We made use of the fact that $D_S = D_L + D_{LS} \approx D_L$ on the second line. Applying this constraint to Equation (A4), we conclude

$$2 \cos \gamma \geq \frac{r}{y_{\text{max}}} \sin^2 \gamma, \quad (\text{A7})$$

where, as before, we defined $r = R_{\text{orb}}/R_{\text{Sch}}$, and assumed $D_S \approx D_L$ and the small angle approximation $\tan \beta \approx \beta$. Then,

we express γ in terms of ϕ_S and ι via Equation (A3) as

$$\cos(\pi - \phi_S) \geq \frac{y_{\text{max}}^2}{r \sin \iota} \left(\sqrt{1 + \left(\frac{r}{y_{\text{max}}^2} \right)^2} - 1 \right), \quad (\text{A8})$$

or equivalently, if we write $\bar{r} = r/y_{\text{max}}^2$,

$$\phi_{S,\text{min}} = \arccos \left(\frac{1 - \sqrt{1 + \bar{r}^2}}{\bar{r} \sin \iota} \right). \quad (\text{A9})$$

This means that at a given r and ι , only BBHs that are within $[\phi_{S,\text{min}}, 2\pi - \phi_{S,\text{min}}]$ can be observed as lensed BBHs. It translates into a probability:

$$\begin{aligned} p(\text{lensing} | r, \iota) &= \frac{1}{2\pi} [(2\pi - \phi_{S,\text{min}}) - \phi_{S,\text{min}}] \\ &= \frac{1}{\pi} \arccos \left(\frac{x}{\sin \iota} \right) \end{aligned} \quad (\text{A10})$$

where x was defined as

$$x = \frac{\sqrt{1 + \bar{r}^2} - 1}{\bar{r}}. \quad (\text{A11})$$

Note that when $\iota = \pi/2$ (edge-on) and in the large r limit, this probability drops as $\sqrt{y_{\text{max}}^2/r}$. However, due to the disk geometry, any deviation from the edge-on angle will result in a drastic drop in the probability of being lensed for binaries that are slightly farther away in the disk, as shown by the gray dashed line in Figure 2. We note that this gray line is the most conservative bound, for if the disk has finite height, this line becomes a gray band and extends the maximum radius more to the right. At each given radius, Equation (A8) also defines a minimum inclination:

$$\iota_{\text{min}} = \arcsin(x), \quad (\text{A12})$$

we find that for $5 R_{\text{Sch}}$ (and beyond), no lensed binaries will be observed for inclination smaller than 0.96 rad. From this we can define the probability of a lensed BBH in an AGN at a radius r by marginalizing over the observer angle:

$$\begin{aligned} f_{\text{lensing|AGN}} &= p(\text{lensing} | r) \\ &= \int_{\iota_{\text{min}}}^{\pi/2} \frac{1}{\pi} \arccos \left(\frac{x}{\sin \iota} \right) \sin(\iota) d\iota \end{aligned} \quad (\text{A13})$$

$$= \int_1^x \frac{1}{\pi} \arccos(y) \frac{1}{y^2} \frac{-x^2}{\sqrt{y^2 - x^2}} dy \quad (\text{A14})$$

$$= \frac{1 - x}{2} \quad (\text{A15})$$

the extra factor of $\sin(\iota)$ accounts for the isotropic probability distribution of the observer around the SMBH. In the large r limit, $x \rightarrow 1 - y_{\text{max}}^2/r$, which gives us the approximation in Equation (5) when $y_{\text{max}} = 1$; otherwise, we see that $f_{\text{lensing|AGN}} \propto y_{\text{max}}^2$.

Appendix B The AGN Constraint

Assuming that we have N_{obs} BBHs, and a fraction $f_{\text{lensed}\wedge\text{AGN}}$ of them is both lensed and from an AGN, then the expected number of such lensed GW coming from an AGN is the mean of its probability mass function, which is given by the binomial distribution:

$$p(N_{\text{lensed}} | N_{\text{obs}}, k) = \binom{N_{\text{obs}}}{N_{\text{lensed}}} k^{N_{\text{lensed}}} (1 - k)^{N_{\text{obs}} - N_{\text{lensed}}}, \quad (\text{B1})$$

where $k = f_{\text{lensed}\wedge\text{AGN}}$ in this case. In order to find the upper bound of f_{AGN} , we made the assumption that all lensed events originate from an AGN, hence we replace the $N_{\text{lensed}\wedge\text{AGN}}$ with simply N_{lensed} . One can always relax this assumption with the knowledge of the proportions of lensed systems in our Universe.

Notice that $k = f_{\text{lensed}\wedge\text{AGN}}$ is an implicit function of R_{orb} and f_{AGN} , and therefore we could write

$$p(N_{\text{lensed}} | N_{\text{obs}}, k) = p(N_{\text{lensed}} | N_{\text{obs}}, f_{\text{AGN}}, R_{\text{orb}}), \quad (\text{B2})$$

and from Bayes' theorem, we obtain

$$p(f_{\text{AGN}} | N_{\text{obs}}, N_{\text{lensed}}) \propto \int p(N_{\text{lensed}} | N_{\text{obs}}, f_{\text{AGN}}, R_{\text{orb}}) \pi(R_{\text{orb}}) dR_{\text{orb}}, \quad (\text{B3})$$

and $\pi(R_{\text{orb}})$ is the probability distribution of BBHs across the AGN disk, which we show in the lower panel of Figure 4. For the Gröber et al. model, we adopted the radial part of Equation (14) in M. Gröbner et al. (2020) as our radial distribution. For the SG (E. Sirko & J. Goodman 2003) and TQM (T. A. Thompson et al. 2005) models, we constructed the radial distribution from the density ($\rho(r)$) and aspect ratio ($h(r)/r$) profiles, as follow: $\pi(r) \propto r \times h(r) \times \rho(r)$. Ideally, $\pi(r)$ should be proportional to the local merger rate of the disk, but if we assume all BBHs merge at the same rate, then the net rate is only proportional to the local density. Thus, it is sufficient to know the radial distribution of the binaries.

Appendix C Effects from the Demagnified Images

For each pair of images that are lensed by the SMBH, one of them is always demagnified, and the amplitude of the signal will be reduced. Effectively, this means that the horizon distance of a given binary will be shortened. In this section, we will estimate the maximum loss in merger rate due to the demagnified images.

First, for the unlensed signals, we model their merger rate as a function of the horizon redshift (z_{H}) via

$$\mathcal{R}_{\text{U}}(z_{\text{H}}) = \int_0^{z_{\text{H}}} \frac{R_{\text{Oguri}}(z) dV_{\text{c}}}{1 + z} dz, \quad (\text{C1})$$

here, dV_{c}/dz is the differential comoving volume, and we assume the BBH distribution follows R_{Oguri} , from M. Oguri (2018).

Then, for each binary, at a dimensionless position y with respect to the SMBH lens, will receive a (de)magnification $\mu_{\pm}$, which depends on y assuming the SMBH is a point mass lens. Therefore, for each position y , the effective horizon distance

is related to the unlensed horizon distance as follows (B. Allen et al. 2012):

$$\frac{D_{\text{H}}^{\text{eff}}}{\sqrt{\mu_{\pm}(y)} D_{\text{H}}} = \left(\frac{1 + z(D_{\text{H}}^{\text{eff}})}{1 + z(D_{\text{H}})} \right)^{5/6}, \quad (\text{C2})$$

note that the horizon redshift and distance are defined naturally as $z_{\text{H}} = z(D_{\text{H}})$, assuming the Planck-2018 cosmology. Therefore, the magnified and demagnified merger rates can be computed by integrating Equation (C1) up to the effective horizon redshift for each y , which we further marginalized over by assuming $p(y) \propto y$:

$$\mathcal{R}_{\pm}(D_{\text{H}}) = \int_0^{y_{\text{max}}} p(y) \int_0^{z_{\text{H}}^{\text{eff}}(y)} \frac{R_{\text{Oguri}}(z) dV_{\text{c}}}{1 + z} dz dy. \quad (\text{C3})$$

We found that for the horizon distance ranges from (100 to 10^6) Mpc, the ratio between the demagnified merger rate and the unlensed one, $\mathcal{R}_{-}(D_{\text{H}})/\mathcal{R}_{\text{U}}(D_{\text{H}})$, is about 90% for $D_{\text{H}} < 10^3$ Mpc and is minimal at around 3×10^4 Mpc at ~40%. This means that for the LIGO binaries, it amounts to multiplying the lensing probability of Equation (7) by 0.9, which would not significantly affect our results above, as the dominant source of uncertainty originates from the uncertainty in the orbital radii. On the other hand, with third-generation detectors, the horizon distance is far beyond 3×10^4 Mpc, in which case the ratio is close to unity, and there will be no bias resulting from demagnification.

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