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Weights under DEA Control**

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Summary

Non-parametric efficiency analysis, such as Data Envelopment Analysis (DEA) relies so far on endogenous local or exogenous general weights, based on revealed preferences or market prices. However, as DEA is gaining popularity in regulation and normative budgeting, the strategic interest of the evaluated industry calls for attention. We offer endogenous general prices based on a reformulation of DEA where the units collectively propose the set of weights that maximize their efficiency. Thus, the sector-wide efficiency is then a result of compromising the scores of more specialized smaller units, which also gives a more stable set of weights. The potential application could be to precipitate collective bargaining on cost efficiency for non-marketed resources and products. The models are applied to panel data from 285 Danish district heating plants, where the open evaluation of multiple non-priced outputs is relevant. The results show that sector wide weighting schemes favor input/output combinations that are less variable than would individual units.

Keywords : DEA, efficiency, weights,

JEL Classification: D24.

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The scientific responsibility is assumed by the authors.

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ENDOGENOUS GENERALIZED WEIGHTS UNDER DEA CONTROL

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Abstract

Non-parametric efficiency analysis, such as Data Envelopment Analysis (DEA) relies so far on endogenous local or exogenous general weights, based on revealed preferences or market prices. However, as DEA is gaining popularity in regulation and normative budgeting, the strategic interest of the evaluated industry calls for attention. We offer endogenous general prices based on a reformulation of DEA where the units collectively propose the set of weights that maximize their efficiency. Thus, the sector-wide efficiency is then a result of compromising the scores of more specialized smaller units, which also gives a more stable set of weights. The potential application could be to precipitate collective bargaining on cost efficiency for non-marketed resources and products. The models are applied to paneldata from 285 Danish district heating plants, where the open evaluation of multiple non-priced outputs is relevant. The results show that sector wide weighting schemes favor input/output combinations that are less variable than would individual units.

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1 Introduction

Weighting of resources consumed and outputs rendered is inherent in any performance evaluation technique that results in a set of measures that is of lower dimensionality than the original production space. The methodology for determining the relative prices is one of the pivotal challenges in performance evaluation. Whereas market prices may be observed or elicited in certain circumstances, they may not necessarily reflect the social welfare effects due to externalities and horizon problems. Technical valuations or specifications may postulate prices for a given technology, but this may be doubtful in regulative contexts. Non-parametric frontier approaches such as the Data Envelopment Analysis (DEA) by Charnes, Cooper and Rhodes (1978,79) addresses this issue by allocating sets of *individual endogenous weights* that put the individual unit in the best possible light. In this manner DEA provides the evaluator with a conservative performance estimate that is valid for a range of preference functions. Under a convex frontier specification, the analysis explicitly provides the evaluator with dual information that later may be used to refine the preference model of the evaluator by inserting additional constraints. In an open retrospective evaluation, where the modelling rests entirely at the discretion of the analyst or collectively of the units, such an approach may support organizational learning and development. Unrestricted weights are relevant in the determination of technical efficiency, i.e. the general ability to produce many outputs using few inputs.

Recently, however, DEA has gained a widespread use also in more normative contexts, such as industry and sector evaluations aiming at informing forwardlooking decisions in regulation (cf. Agrell et al., 2005), budgeting or incentive management (cf. Agrell et al., 2002). For surveys of applications in the domains of utilities' regulation, see Jamasb and Pollitt (2000), for a full survey cf. Seiford and Thrall (1990). This change of perspective implies higher demands to analyze the strategic behavior of the units, as well as the methodological consistency of the evaluation. A performance measure, albeit conservative, that counterintuitively discourages relevant economic actions will inevitably lead to dysfunctional behavior. On the other hand, an overly cautious approach using individual dual prices comes at a social cost in terms of the discriminatory capacity of the method.

The use of individual weights is also troublesome from an allocative point of view. If, for example, two units - to put their performance in its best possible light - stipulate the value of labor to capital as being (1:10) and (10:1) respectively, there is clearly a social loss from the allocation of capital and labor. In the DEA literature, allocative efficiency is studied with given

market prices. In such cases, allocative efficiency can be evaluated along with technical efficiency to give, for example, the cost efficiency of units. The latter is then an example of *common exogenous weights*. In the absence of unanimous market information that can provide these weights, the DEA approach has usually been to restrain the analysis to individual endogenous weights, alternatively supplemented with partial price information.

The aim of this paper is to develop a set of *common endogenous weights* that puts the evaluated industry in the best possible light. By using only *ex post* production data, we may derive collective evaluations that are applied across the sample. This will enable us to make cost effectiveness analysis even in cases where relevant market prices do not exist and no other preference information is available. In a normative context, this corresponds to a conservative, yet intra-industry consistent estimate of performance that preempts collective and individual complaints on its validity. Whenever our endogenous common weights are not assessed, the evaluator runs the risk that the evaluated units collectively assert these relative prices and then internally redistributes the allocated incentives among the units. Since the common weights maximize the collective incentive, it also opens for strategic behavior on behalf of the units.

The contribution of the paper is twofold.

First, it extends the methodological discussion on preference modelling in DEA with a treatment of a class of endogenous *and* collective evaluations. A particular application of our approach is in the evaluation of non-balanced activities, where individual weights would have been zerovalued. The common weights here express a comprehensive assessment on these activities, a weighted average of the social benefits.

Second, it addressed a relevant issue in the normative application of DEA in e.g. bargaining or regulation. The suggested approach may be directly used in negotiations with associations that represent the collective of evaluated units.

The outline of this paper is as follows. Section 2 presents the traditional approach and derives the individual endogenous weights. Our model is presented in Section 3, along with some properties and interpretations. An extensive illustration using regulatory panel data from the energy sector is given in Section 4. The paper is closed with some conclusions in Section 5.

2 The Traditional Approach

In the following we address a traditional setting of evaluated DMUs $j = 1, \dots, n$, transforming an inputvector $x^j \in \mathbb{R}_+^r$ $j = 1, \dots, n$ to a vector of outputs $y^j \in \mathbb{R}_+^s$ $j = 1, \dots, n$. Let (X, Y) be the set of observed input-output combinations. The task is to determine a set of input prices $u \in \mathbb{R}_+^r$ and output prices $v \in \mathbb{R}_+^s$ such as to maximize the productivity measure for the unit under evaluation. To avoid degenerated cases, assume that x^j and y^j each contain at least one positive element for all $j = 1, \dots, n$.

Assuming constant returns to scale as in Charnes et al. (1978), we arrive at the classical "dual" CRS model [P1]:

$$\begin{array}{ll} \text{Max}_{u^i, v^i} & \frac{v^i y^i}{u^i x^i} \\ \text{st} & \frac{v^i y^j}{u^i x^j} \leq 1 \quad \forall j = 1, \dots, n \\ & u^i \in \mathbb{R}_+^r \quad v^i \in \mathbb{R}_+^s \end{array} \quad (\text{P1})$$

The optimal solution P1 gives a set of individual endogenous weights (u^i, v^i) , these are the weights putting DMU i in the best possible light.

In an economic context, the program is equivalent to the maximization of a net profit $v^i y^i - u^i x^i$, given a set of normalized input prices ($u^i x^i = 1$) and subject to the condition that all observed units run with nonpositive profits $v^i y^j - u^i x^j \leq 0$.

$$\begin{array}{ll} \text{Max}_{u^i, v^i} & v^i y^i \\ \text{st} & u^i x^i = 1 \\ & v^i y^j - u^i x^j \leq 0 \quad \forall j = 1, \dots, n \\ & u^i \in \mathbb{R}_+^r \quad v^i \in \mathbb{R}_+^s \end{array} \quad (\text{P1}^*)$$

Technically, therefore, the weights define a hyperplane that dominate all observed input-output combinations and minimizes the potential improvement in profit by DMU i by doing as well as the best DMUs. Hence, a unit that exhibits a net profit lower than 0 is dominated by some more productive units.

The dual program P1* above is equivalent to the primal program P2 below for the decision variables (θ^i, λ) , where θ^i is the radial distance measure for DMU i and λ the convex weights on (X, Y) that dominate (x^i, y^i) .

$$\begin{array}{ll} \text{Min}_{\theta^i, \lambda} & \theta^i \\ \text{st} & \theta^i x^i \geq \sum_{j=1}^n \lambda_j x^j \\ & y^i \leq \sum_{j=1}^n \lambda_j y^j \\ & \lambda \in \mathbb{R}_+^n \end{array} \quad (\text{P2})$$

The primal description of the production possibility set gives a minimal convex hull that contains all observed units under constant returns to scale. Analogous formulations may also be made under various scale assumptions and distance measures, cf. e.g. Charnes et al. (1989).

It is well known from empirical studies that units under evaluation will claim very diverse prices, since each unit is emphasizing its comparative advantages. This has two implications.

Firstly, in some cases, it is therefore useful (or economically asked for) to introduce an exogenous set of price restrictions. The imposition of restrictions on the DMU specific prices in DEA models has been proposed by many authors to reflect partial price information, preference information or other subjective information about the relative importance of the inputs and outputs, cf. e.g. Ali, Cook and Seiford(1991), Golany(1988) and Halme, Joro, Korhonen, Salo, and Wallenius (1999). This is easily done by adding constraints on u^i and v^i in the formulation P1 *. Introducing e.g. $v_h^i \geq v_k^i$ could reflect that output h is at least as important or valuable as output k . More generally, it is useful and straightforward to introduce such information by requiring $u^i \in U^i$ and $v^i \in V^i$, where U^i and V^i are convex polyhedral sets in the strictly positive orthants, $U^i \subset \mathbb{R}_{++}^r$ and $V^i \subset \mathbb{R}_{++}^s$. The resulting version of P1 * in this case remains a simple linear programming problem.

Secondly, it motivates the search for industry-wide prices, which can be expected to be less extreme than the individual prices. However, as we shall see in the numerical illustration, this depends on the underlying technology.

3 The New Approach

We now revisit the classical CRS model to determine a common set of weights (u, v) for all units, so that the overall efficiency of the set of units is maximized. Consider the following program P3:

$$\begin{array}{ll}
 \text{Max}_{u,v} & \frac{v \sum_{i=1}^n y^i}{u \sum_{i=1}^n x^i} \\
 \text{st} & \frac{vy^j}{ux^j} \leq 1 \quad \forall j = 1.., n \\
 & \frac{v \sum_{i=1}^n y^i}{u \sum_{i=1}^n x^i} \leq 1 \\
 & u \in \mathbb{R}_+^r \quad v \in \mathbb{R}_+^s
 \end{array} \tag{P3}$$

where the objective function expresses the aggregate productivity, the first constraint the individual productivity normalization as in CRS and the second constraint the collective normalization. The interpretation is

straightforward. We seek the prices that make the joint production plan look as attractive as possible, subject to the usual normalization constraints that no observed production, joint or individual, can have a benefit-cost ratio exceeding 1. Some immediate remarks to the formulation are appropriate.

Remark 1 *The second constraint $\frac{v \sum_{i=1}^n y^i}{u \sum_{i=1}^n x^i} \leq 1$ is redundant.*

Proof. $vy^j/ux^j \leq 1$ implies $vy^j - ux^j \leq 0$ for all j , which implies $\sum_{j=1}^n (vy^j - ux^j) \leq 0$.

In turn this implies $\sum_{j=1}^n vy^j / \sum_{j=1}^n ux^j \leq 1$. ■

Remark 2 *One of the n first constraints will always be binding.*

Proof. Assume that (u^*, v^*) is an optimal solution to the program and that none of the n first constraints are binding, i.e. $v^*y^j/u^*x^j < 1$ for all j . This implies $v^*y^j - u^*x^j < 0$ for all j and we may therefore increase all elements of v^* marginally without violating the constraints. This would increase $\sum_{j=1}^n v^*y^j / \sum_{j=1}^n u^*x^j$ and it thus contradicts the optimality of (u^*, v^*) . ■

An alternative interpretation of the objective function is stated without its (trivial) proof.

Remark 3 *The objective function can be rewritten as a weighted average of the usual benefit cost ratios: $\sum_{i=1}^n (\frac{ux^i}{\sum_{j=1}^n ux^j} \frac{vy^i}{ux^i})$*

The solution to program $P3$ above is not unique. The normalization is (as well as in the original CRS model) arbitrary and any inflation or deflation of all prices with the same factor would not affect the solution.

The problem $P3$ can also be reformulated as a game theoretic problem $P4$, where the industry picks prices to maximize the value added and the regulator selects the benchmark ratio, as the most promising from the set of individual processes:

$$\begin{aligned} \text{Max}_{u,v} \quad & \frac{v \sum_{i=1}^n y^i}{u \sum_{i=1}^n x^i} \text{Max} \{ \frac{vy^j}{ux^j} | j = 1 \dots n \} \\ \text{st} \quad & u \in \mathbb{R}_+^r, v \in \mathbb{R}_+^s \end{aligned} \tag{P4}$$

However, a more conventional primal reformulation of $P3$ is given below as $P5$, which is equivalent to the superefficiency (Andersen and Petersen, 1993) evaluation of an aggregate unit $(\sum_{i=1}^n x^i, \sum_{i=1}^n y^i)$. The equivalence between $P3$ and $P5$ is proved in Proposition 1 below.

$$\begin{array}{ll}
Min_{\theta, \lambda} & \theta \\
st & \theta(\sum_{i=1}^n x^i) \geq \sum_{j=1}^n \lambda^j x^j \\
& \sum_{i=1}^n y^i \leq \sum_{j=1}^n \lambda^j y^j \\
& \lambda \in \mathbb{R}_+^n
\end{array} \quad [P5] \tag{P5}$$

Here, the interpretation falls out immediately from the formulation as the amount of the pooled inputs that could have been saved in the production of the pooled outputs by allocating the production in the best possible way among the available production processes. Assuming proportional weights, it is also the optimal production plan in a centralized planning setting with given subprocesses.

The primal formulation in $P5$ is also related to the measure of overall potential gains from mergers developed in Bogetoft and Wang (1997) and Bogetoft, Strange and Thorsen (2002), and to the measures of structural efficiency suggested by Farrell (1957) and Försund and Hjalmarsson (1979).

Proposition 1 *The dual variables associated with the two sets of constraints in $P5$ will be the optimal weights or prices u and v in $P3$.*

Proof. Usual dualization of $P5$ gives the program

$$\begin{array}{ll}
Max_{u,v} & v(\sum_{i=1}^n y^i) \\
st & u(\sum_{i=1}^n x^i) \leq 1 \\
& vy^j - ux^j \leq 0 \quad \forall j = 1..,n \\
& u \in \mathbb{R}_+^r, v \in \mathbb{R}_+^s
\end{array} \tag{P6}$$

Without loss of generality, we may require that $u(\sum_{i=1}^n x^i) = 1$ and program $P6$ is thus equivalent to

$$\begin{array}{ll}
Max_{u,v} & \frac{v \sum_{i=1}^n y^i}{u \sum_{i=1}^n x^i} \\
st & \frac{vy^j}{ux^j} \leq 1 \quad \forall j = 1..,n \\
& u \sum_{i=1}^n x^i = 1 \\
& u \in \mathbb{R}_+^r \quad v \in \mathbb{R}_+^s
\end{array}$$

Again, this is equivalent to $P3$ due to redundancy of the second constraint in $P3$ (Remark 4) and the possibility to scale all prices. ■

As can be easily seen from the program $P6$, the common weight problem essentially maximizes the total payment the industry can claim, given the knowledge that the regulator has about the best production practices. Hereby, any Pareto efficient solution for the industry is supported and could potentially be implemented using appropriate sidepayments. This suggests

that if the industry is bargaining for incentive payments based on the prices u, v the units should collectively agree on the common weights. Note also from the game-theoretical formulation above that the bargaining power of the regulator is given by his information about the efficiency of individual processes. This limits the rents the collective of units can claim using extreme weights.

So far we have not made use of, nor assumed the existence of, market prices of the inputs or outputs. The revenue and cost terms calculated are merely used to define the reimbursement scheme. An interesting possibility is that the attempt to optimize incentives under DEA control may be costly when considering the true market prices. This is the case if the reduced incentive cost forces the DMU away from the locally allocatively efficient productions.

4 Numerical Illustration

To illustrate the proposed model, we use the panel data in Agrell and Bogetoft (2004) of district heating plants in Denmark. Define x_1 as the operating expenditure in MDKK, x_2 as the primary fuel input in GJ, y_1 as the heat energy delivered in GJ, y_2 as the electrical energy delivered in GWh, y_3 as the heat capacity utilized in MW, y_4 as the total length of pipelines. The input-oriented model uses the non-discretionary pipeline length as a proxy for customer density. The dataset contains 285 DMUs for 1998/99 and 234 DMUs for 1999/00, each representing the performance of a district heat plant. Real input prices w_1, w_2 are known for each DMU. Some descriptive statistics about the data are presented in Tables 1 and 2.

The following efficiencies under constant returns to scale are assessed for each yearly dataset: input-oriented technical efficiency $TE(year)$ given by a $P2$ formulation, cost efficiency given local prices $CE(w^{year})$, cost efficiency given average input prices $CE(\bar{w}^{year})$, aggregate cost efficiency $ACE(w^{year})$ and cost efficiency under the new approach $CEC(u^{*,year}, v^{*,year})$ using the $P6$ formulation. We also introduce the notation $C(w) = wx$ for the total realized cost at valuation w . For clarity, we restate the programs $CE(\cdot)$ and $ACE(\cdot)$ below.

$$CE^i(w^i) = \min \left\{ w^i \sum_{j=1}^n \lambda^j x^j \mid \sum_{j=1}^n \lambda^j x^j \leq x^i, \sum_{j=1}^n \lambda^j y^j \geq y^i, \lambda \in \mathbb{R}_+^n \right\}$$

Table 1: Descriptive statistics, district heating plants in Denmark 1998/99, Agrell and Bogetoft (2004).

		Mean	Median	Min	Max	Standard.dev.
x_{opex}	kkkr	6,319	2,301	138	226,039	18,580
$w_{fuel}x_{fuel}$	kkkr	17,998	4,633	194	854,646	65,698
x_{fuel}	GJ	314	84	10	11,919	1,037
c	kkkr	24,318	7,247	625	961,049	78,604
z_{pipes}	km	54	25	0	1597	127
y_{heat}	GJ	259	64	7	10,308	907
y_{elec}	GWh	9	0	0	280	24
y_{cap}	MW	78	21	2	2,405	241

Table 2: Descriptive statistics, district heating plants in Denmark 1999/00, Agrell and Bogetoft (2004).

		Mean	Median	Min	Max	Standard.dev.
x_{opex}	kkkr	7,550	2,535	60	268,941	23,724
$w_{fuel}x_{fuel}$	kkkr	21,490	5,577	238	932,759	76,421
x_{fuel}	GJ	352	86	10	12,018	1,145
c	kkkr	29,040	8,119	693	1,068,944	93,260
z_{pipes}	km	63	26	0	1,629	147
y_{heat}	GJ	296	64	1	10,658	1,050
y_{elec}	GWh	10	0	0	267	25
y_{cap}	MW	87	21	2	2,499	273

$$ACE^i(w^i) = \min \left\{ \sum_{j=1}^n (w^j x^j) \lambda^j \mid \sum_{j=1}^n \lambda^j (w^j x^j) \leq w^i x^i, \sum_{j=1}^n \lambda^j y^j \geq y^i, \lambda \in \mathbb{R}_+^n \right\}$$

The aggregate units $x^{98} = \sum_{j=1}^{285} x^{j,98}$ and $x^{99} = \sum_{j=1}^{234} x^{j,99}$ and corresponding output aggregations are included in the calculations, but are inefficient under any assumptions.

The results of the assessments are presented in Table 3 below. As expected, the technical efficiency estimates $TE(.)$ are the highest, well above 0.80 for even this fairly aggregated two-input model. The proposed common weights model in Table 3 denoted $CEC(u^*, v^*)$ yields in both cases the degenerated dual prices $u^* = \{0, 1\}$ and $v^* = \{1, 0, 0, 0\}$, effectively transforming the efficiency problem into a two-dimensional issue of heat losses. As the heat losses are limited by the thermophysical configuration of the network, the overall efficiencies are high. Inputs such as operating expenditure have higher variability towards output, which favors selected DMUs, but lower the bulk of the scores. Irrespective of whether an extremely favorable observation is by skill or luck, the regulator-evaluator is using this variability to gauge all units in DEA. The industry collectively can hedge itself against this bargaining power by de-emphasizing inputs (e.g. operating expenditures) with higher variability towards outputs in favor of the inputs that are naturally bounded by proportionality. This explains the seemingly counterintuitive result. The lowest individual score for 1999, 0.290, is likely due to reporting errors rather than actual heatlosses. The $CE(\bar{w})$ model assesses the cost efficiency under the premises that all units are subject to average fuel prices. As the majority of the units are single sourced heat plants, this assumes change of technology. The resulting scores are lower, around 0.74, as the trade-off between operating expenditure and fuel cost becomes more realistic. The next model $CE(w)$ changes the competition by exposing DMUs to evaluation by local prices, which of course may be lower or higher than the market average. The efficiency level is roughly as with average prices, albeit with some extreme dips for certain technologies. Finally, the $ACE(w)$ model assumes that the markets behind the DMUs may purchase power and heat to average prices from any other units. Here, the integrated efficiency of operating expenditure, fuel purchases and fuel efficiency is estimated. In this case, the resulting efficiencies are low, around 0.60, highlighting the large discrepancies in overall cost efficiency on the district heat market.

Table 3: Cost efficiency estimates for various model specifications.

	n	Mean	r	Min	Standard.dev.
$TE(98)$	285	0.827	24	0.610	0.093
$TE(99)$	234	0.814	19	0.330	0.107
$CEC_{98}(u^{98}, v^{98})/C(u^{98})$	285	0.805	11	0.600	0.088
$CEC_{99}(u^{99}, v^{99})/C(u^{99})$	234	0.792	11	0.290	0.105
$CE_{98}(\bar{w}^{98})/C(\bar{w}^{98})$	285	0.745	9	0.440	0.122
$CE_{99}(\bar{w}^{99})/C(\bar{w}^{99})$	234	0.737	11	0.310	0.136
$CE_{98}(w^{98})/C(w^{98})$	285	0.730	9	0.120	0.139
$CE_{99}(w^{99})/C(w^{99})$	234	0.725	12	0.160	0.160
$ACE_{98}(w^{98})/C(w^{98})$	285	0.600	7	0.300	0.137
$ACE_{99}(w^{99})/C(w^{99})$	234	0.614	10	0.260	0.144

An interesting break-down of the results for the five models is made in Table 4. The two first columns give the minimal cost estimates for each model, in applicable prices. The four columns to the right tabulate the optimal production profile for each model, and the current production is given in the top row. As expected from the model formulation, the proposed model minimizes the overall inefficiency, as $C(u) = x_2$. However, to assess the budget value $C(w)$, the average prices $\bar{w}$ are used. Note that this model implies a hefty substitution rate between opex and fuel, reducing the fuel input to an absolute minimum. The technical efficiency model TE implies a proportional reduction of both inputs, disregarding the actual substitution rate. The overall budget under local prices w is lower than for the common weights model, the results are marginally different than for average prices $\bar{w}$. The case illustrates thus the difference between the objective to maximize average efficiency (as for TE) and to maximize aggregate efficiency. The 64 DMUs that have lower score in the new aggregated model than in the technical efficiency model are primarily units that are comparatively stronger in partial opex-efficiency. In one outlier case, a small technically efficient unit drops to 0.51 in aggregate efficiency. The following two models $CE(\bar{w})$ and $CE(w)$ suggest substantial reductions of opex at comparatively higher levels of fuel than the aggregate model. The detailed outcome of these models give raise to far more revolutionary changes of the organization and technology in the market than the aggregate and technical efficiency models. The overall cost efficiency model $ACE(w)$ evaluated at local budgets, indicates a further 1000 MDkr reduction of controllable costs, which of course presumes complete flexibility in scale and scope of operations.

Table 4: Minimal cost estimates and input requirements.

	$C(w)$		OPEX		Fuel	
	1998	1999	1998	1999	1998	1999
x	6 856	6 804	1795	1770	89	82
$x^u(u^*, v^*)$	6 047	5 637	1846	1489	74	68
$x^{TE}(w)$	5 878	5 711	1520	1478	76	69
$x^{CE}(\bar{w})$	5 425	5 247	1096	895	76	71
$x^{CE}(w)$	5 390	5 217	1016	927	77	71
x^{ACE}	4 228	4 276	-	-	-	-

It is interesting to note that the costs using local prices may well increase when using the production plan generated using common weights incentives, $x^u(u^*, v^*)$. This reflects the potential conflict of reducing incentive costs and achieving allocative efficiency.

5 Conclusions

In this paper we derive endogenous sector-wide prices for DEA evaluations. This is useful when there are no exogenous general weights available, nor relevant to use local endogenous prices. The resulting model can be interpreted as a game theoretic model, where the industry suggests prices to collectively maximize net revenue or compensation and a principal selects a benchmarking unit to constrain the set of acceptable prices. This interpretation is specifically valid in the frequent scenarios evoked in applied work where the regulated or evaluated units are 'consulted' in order to derive or validate the specification of the activity model(s) used in the assessment. The common weight approach can then be seen as a focal point for the firms in a cooperative game against the evaluator, providing a basis for the side payments necessary to implement the solution.

We illustrate the model using panel data from Danish district heating plants. The outcome has several intriguing characteristics, among those the risk reducing strategy of emphasizing input-output dimensions with low variability across the sample. The empirical study also illustrates the potential direct distortion of total allocative efficiency when reacting strategically to collective incentives. Further work intends to explore the cooperative game properties of interactive specification of models for performance assessment.

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