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Constrained Optimized Resource Allocation for Multiuser Multicarrier Wireless Systems

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Abstract

Multicarrier transmission schemes have been widely adopted in wireless communication systems. Multiple users can share a multicarrier wireless channel using multiple access mechanisms such as multi-carrier code division multiple access (MC-CDMA) or orthogonal frequency division multiple access (OFDMA). In multiuser systems, users are allowed to have different quality of service (QoS) requirements. On another hand, users have channels of different quality. Therefore the achievement of fairness among the users while satisfying users' QoS requirements is an important issue. Due to the limited resources available at the base station, e.g. bandwidth and power, optimized allocation of these resources to users is crucial for delivering the best possible QoS to the user with least cost. This thesis proposes a new class of algorithms which achieves the users' fairness and QoS requirements under total power constraint.

In most of previous works on multicarrier resource allocation, it has been assumed that perfect channel state information (CSI) is available at the transmitter. However, perfect CSI at the transmitter is rarely possible due to channel estimation errors and channel feedback delay. This thesis develops analytical framework and derives practical algorithms for downlink resource allocation assuming that only statistical channel information is available at the base station.

The most intuitive and up to now, the most widely used strategy to increase coverage in wireless networks, is to shrink the size of the cell, effectively increasing the number of base stations (BS) over a given area. Although this strategy likely will increase coverage because users are much closer to their serving BSs, its benefit is limited because of the exceeding cost of BSs. For each BS, the provider must pay for antenna space and the wired backhaul to the network, not to mention the digital and radio frequency equipment itself. Instead, an increasingly attractive strategy is to insert relaying stations into the cell whose sole purpose is to aid communication from BS to mobile station (MS) and vice versa. Such communications are called cooperative communications. This thesis extends the analytical framework and algorithms proposed for resource allocation in non-cooperative communication systems to resource allocation strategies in cooperative communication systems.

Notations and Acronyms

Mathematical notations

j	imaginary unit: $j = \sqrt{-1}$
x	a scalar
x^*	the complex conjugate of x
$\text{Re}(x)$	the real part of x
$\text{Im}(x)$	the imaginary part of x
$ x $	the absolute value or norm of x
$\mathbf{A}^T$	the transpose of matrix $\mathbf{A}$
$\mathbf{A}^H$	the conjugate transpose of matrix $\mathbf{A}$
$\mathbf{A}^{-1}$	the inverse of matrix $\mathbf{A}$
$\text{E}[x]$	the expected value (or mathematical expectation) of x
$\cong$	is approximately equal to
$[x]^+$	the positive part of x , i.e. $[x]^+ \triangleq \max(0, x)$

Acronyms

3GPP	Third Generation Partnership Project
ACK	ACKnowledgment
ARQ	Automatic Repeat Request
AWGN	Additive White Gaussian Noise
BBBA	Branch-and-Bound Bit Allocation
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CDMA	Code Division Multiple Access
cdf	cumulative distribution function
CFO	Carrier Frequency Offset
CIR	Channel Impulse Response
CP	Cyclic Prefix
CPSC	Cyclic Prefixed Single Carrier
CSI	Channel State Information
CSIT	Channel State Information at the Transmitter side
DFE	Decision Feedback Equalization
DFT	Discrete Fourier Transform
DMMT	Discrete Matrix Multitone
DMT	Discrete Multi-Tone
DVB-H	Digital Video Broadcasting-H
EBCMCS	Enhanced Broad-Cast Multi-Cast Services
EBPA	Equal BER Power Allocation
EDGE	Enhanced Data Rate for GSM Evolution
EIA	Electronics Industry Association
ESBA	Exhaustive Search Bit Allocation
EV-DO	Evolved Data Optimized
FEC	Forward Error Correction
FDMA	Frequency Division Multiple Access
FER	Frame Error Rate
FFT	Fast Fourier Transform
FLASH OFDM	Fast Low-Latency Access with Seamless Handoff OFDM
FLO	Forward Link Only
FSR	Frame Success Rate
GABA	Greedy Algorithm Bit Allocation

GBN	Go-Back-N
GPRS	General Packet Radio Service
GSM	Global System for Mobile communications
GP	Geometric Programming
HDR	High Data Rate
HSPA	High Speed Packet Access
IDFT	Inverse Discrete Fourier Transform
IEEE	Institute of Electrical and Electronics Engineers
IFFT	Inverse Fast Fourier Transform
i.i.d.	independent and identically distributed
ISI	Inter-symbol Interference
LTE	Long Term Evolution
MAC	Media Access Control
MC	Multi-Carrier
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MMDS	Multichannel Multipoint Distribution Services
MMSE	Minimum Mean Square Error
MRC	Maximum Ratio Combining
MSE	Mean Square Error
MTSO	Mobile Telephone Switching Office
MUD	Multi User Detection
NAK	Negative Acknowledgment
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
OPA	Optimal Power Allocation
PAPR	Peak to Average Power Ratio
PEP	Pairwise Error Probability
pdf	probability density function
PSK	Phase Shift Keying
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
RRBA	Round Relaxed Bit Allocation
SAW	Stop-And-Wait
SC	Single Carrier
SISO	Single Input Single Output

SINR	Signal to Interference plus Noise Ratio
SNR	Signal-to-Noise Ratio
SR	Selective-Repeat
SUD	Single User Detection
SVD	Singular Value Decomposition
TDMA	Time Division Multiple Access
TIA	Telecommunications Industry Association
UMB	Ultra-Mobile Broadband
UMTS	Universal Mobile Telecommunications System
WCDMA	Wideband Code Division Multiple Access
WiMAX	Worldwide Interoperability for Microwave Access
ZF	Zero Forcing
ZFBF	Zero-forcing beamforming

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Chapter 1

Introduction

Wireless communications have made it possible for people to communicate with anyone, from anywhere, at any time. In recent years, wireless communications have grown to contribute significantly to the safety, comfort, and convenience in almost every aspect of life. Today, developing countries in the Asia-Pacific and Africa are fastest growing adopters of cellular wireless technology. According to ICT indicators database from the International Telecommunication Union, the Nigeria mobile cellular subscriber base increased from 1.56 million in 2002 to 40.39 million in 2007 and reached 62.98 million at the end 2008 [1].

We begin this introductory chapter by briefly outlining the evolution of wireless communications. The motivations of the thesis are provided. We then present the overall organization and end this chapter with the summary of the contributions.

1.1 Evolution of Wireless Communications

As of Dec. 2008, the worldwide cellular telephony subscriber base has reached 3.77 billion customers [2], roughly 55% of the world population of 6.77 billion. This is projected to increase to 5.5 billion by 2013, roughly 77.35% of the projected world population at that time of 7.11 billion [3]. On the other hand, the popularity of the Internet is also growing tremendously, with 1.6 billion worldwide users as of Dec. 2008, approximately a 342.2% growth since 2000 [4]. Given these trends in our ever more connected world, it is conceivable why wireless communications is moving from providing simple voice service to delivering heterogeneous data applications.

1.1.1 Evolution of Cellular Systems

The history of cellular wireless communications dates back to more than a century. In the late 1960s, Bell laboratories first developed the concept of cellular wireless communications [5], wherein spectrum within a geographical region can be reused by breaking the region into a tessellation of hexagonal small areas known as cells. Each cell is assigned a set of frequencies, and, due to the physical phenomenon of radio strength attenuation with increasing distance, these frequencies can be reused either by adjacent cells, or in the second tier of cells, and so on. Fig. 1.1 shows an example 7-cell cluster cellular network, where F_i denotes the frequency set used in a particular cell [6].

A basic wireless communication system consists of a base station (BS), a mobile telephone switching office (MTSO) and multiple mobile units. Operation within a cell is controlled by a BS. All base stations in a given geographical area are connected via a high-speed communication link to a MTSO. The MTSO acts as a central controller for the network allocating channels within each cell, coordinating handoffs between cells when a mobile traverses a cell boundary, and routing calls to and from mobile units. The two communication links between a BS and multiple mobile units are known as the downlink and uplink. A downlink (or forward link) is a communication link from a BS to mobile unit whereas an uplink (or reverse link) is a communication link from a mobile unit to a BS.

Initial cellular system designs were mainly driven by the high cost of base stations. For this reason early cellular systems used a relatively small number of cells to cover an entire city or region. The cell base stations were placed on tall buildings or mountains and transmitted at very high power with cell coverage areas of several square kilometers. These large cells are called macrocells. Signal power was radiated uniformly in all directions, so a mobile moving in a circle around the base station would have approximately constant received power if the signal was not blocked by an attenuating object. This circular contour of constant power yields a hexagonal cell shape for the system, since a hexagon is the closest shape to a circle that can cover a given area with multiple nonoverlapping cells. Cellular systems in urban areas now mostly use smaller cells with the base stations close to street level transmitting at much lower power. These smaller cells are called microcells or picocells, depending on their size. This evolution to smaller cells occurred for two reasons: the need for higher capacity in areas with high user density and the reduced size and cost of base station electronics. A

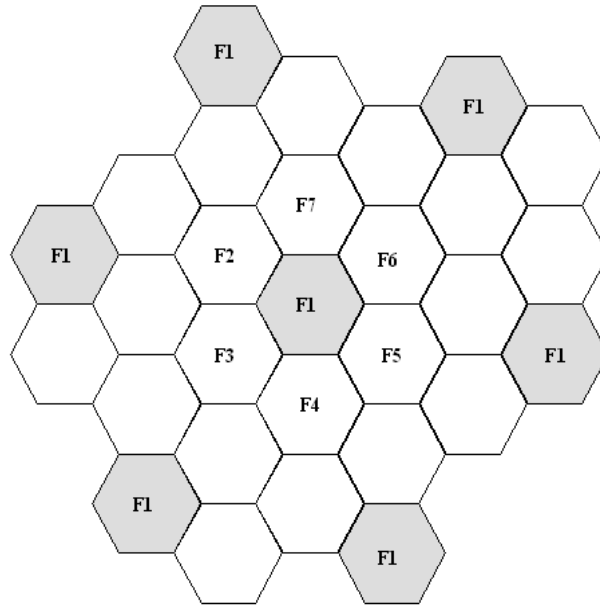


Figure 1.1: Cellular wireless system with hexagonal cells

cell of any size can support roughly the same number of users if the system is scaled accordingly.

First generation (1G)

The first generation of cellular systems used analog communications, since they were primarily designed in the 1960s, before digital communications became prevalent. These 1G systems used analog frequency modulation (FM), where each subscriber making a call was assigned a separate uplink and downlink FM channel. This method of spectrum sharing, wherein the users and the transmission direction are assigned disjoint partitions in frequency, is called frequency division multiple access (FDMA) with frequency-division duplexing (FDD). In FDMA and FDD, it is important to separate the channels sufficiently such that inter-channel interference can be mitigated using practical filters. Fig. 1.2 shows a typical FDMA with FDD setup, where users are assigned an uplink and downlink frequency channel for the entire duration of the connection.

Many of the first generation cellular systems in Europe were incompatible, and the Europeans quickly converged on a uniform standard for second generation digital systems.

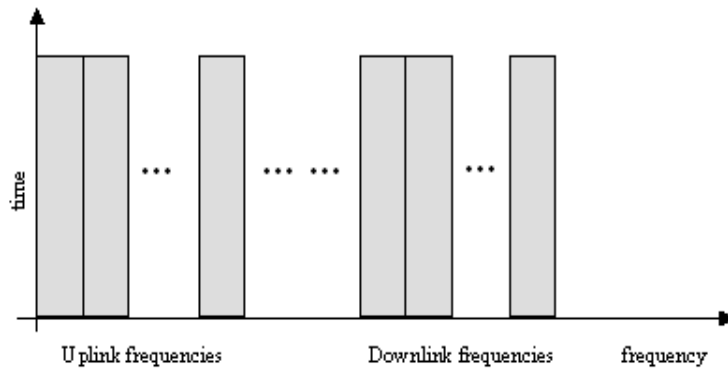


Figure 1.2: Frequency division multiple access (FDMA) with frequency division duplexing (FDD)

Second generation (2G)

Second generation cellular systems moved from analog to digital due to its many advantages. The components are cheaper, faster, smaller, and require less power. Voice quality is improved due to error correction coding. Digital systems also have higher capacity than analog systems since they can use more spectrally-efficient digital modulation and more efficient techniques to share the cellular spectrum. They can also take advantage of advanced compression techniques and voice activity factors. In addition, encryption techniques can be used to secure digital signals against eavesdropping. Digital systems can also offer data services in addition to voice, including short messaging, email, Internet access, and imaging capabilities. Thus, in the early 1990s, second generation (2G) cellular networks that use digital modulation were developed. The most widely used 2G standard in the world today, with approximately 3 billion subscribers, is the global system for mobile communications (GSM). GSM similarly uses separate sets of uplink and downlink frequencies (FDD), but users share the spectrum using separate time slots. This method of spectrum sharing is called time-division multiple access (TDMA), and is shown in Fig. 1.3 with FDD. GSM was designed to support low rate data services of up to 9.6 kbps. GSM systems also provide data rates of up to 100 kbps by aggregating all timeslots together for a single user. This enhancement is called general packet radio Service (GPRS). A more fundamental enhancement, enhanced data rate for GSM evolution (EDGE), further increases data rates using a high-level modulation format combined with forward error correction (FEC) coding. The IS-95 systems

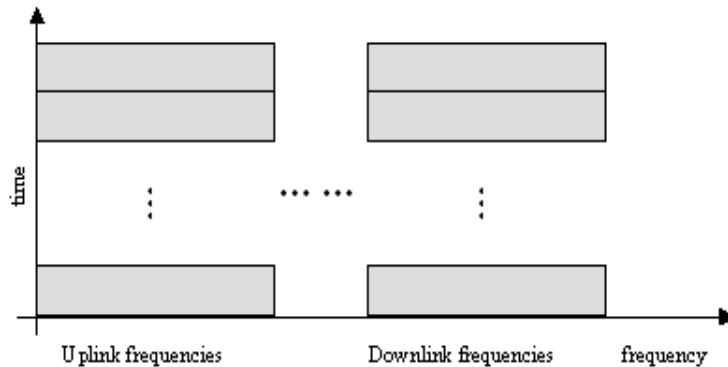


Figure 1.3: Time division multiple access (TDMA) with frequency division duplexing (FDD)

support higher data rates using a time-division technique called high data rate (HDR) [7].

Third generation (3G)

The two most popular 3G standards, i.e. universal mobile telecommunications system (UMTS) which evolved from GSM, and cdma2000 which evolved from the IS-95 2G standard, are both based on FDD with code division multiple access (CDMA) technology. CDMA allows different users to transmit at the same time and frequency, but using different codes. Fig. 1.4 shows a diagram of CDMA where there are separate uplink and downlink frequencies (FDD), and users are separately assigned different codes, but use the same time and frequency blocks. These codes, when designed to be orthogonal, i.e. non-interfering, can then effectively separate the users from each other, allowing simultaneous links to be maintained with minimal interference. 3G cellular standards were designed to support different data rates depending on mobility and location: 384 kbps for pedestrian use, 144 kbps for vehicular use, 2 Mbps for indoor office use and 9.6 kbps for satellite applications.

Fourth generation and beyond (3GPP-LTE)

Third generation partnership project - long term evolution (3GPP-LTE) is the fourth generation cellular standard, and is aligned with existing 3G deployments, e.g. UMTS. The 3GPP-LTE standard reached the functional freeze milestone in December 2008. 3GPP-LTE uses orthogonal frequency

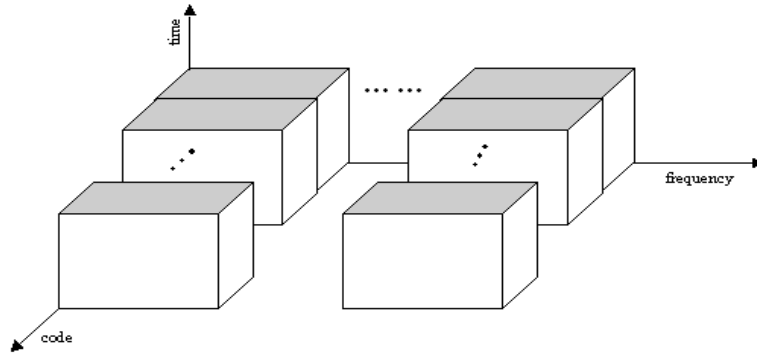


Figure 1.4: Code division multiple access (CDMA) with frequency division duplexing (FDD)

division multiple access (OFDMA) for the downlink, and single-carrier frequency division multiple access (SC-FDMA) on the uplink. OFDMA and SC-FDMA are the state-of-the-art in multiple access technologies, wherein users are assigned separate subchannels that effectively divide up the wideband spectrum into a multitude of narrowband spectrum chunks. OFDMA is based on the modulation method called orthogonal frequency division multiplexing (OFDM), which similarly uses a multitude of narrowband subcarriers that are orthogonal with each other and carry lower data rate streams, which sum up to a high data rate transmission. We discuss OFDM and OFDMA in more detail in chapter 2.

Fig. 1.5 shows OFDMA using either Time Division Duplex (TDD) or FDD, where a wideband channel is divided up into narrowband subchannels that are orthogonal to each other, such that users can be assigned a mutually exclusive subset of these subchannels without interfering with each other. Both OFDMA and SC-FDMA allow for intelligent scheduling and resource allocation so as to most efficiently use the existing wireless spectrum. 3GPP-LTE supports scalable bandwidths of 1.25, 2.5, 5, 10, 15 and 20 MHz and is expected to provide peak downlink data rate up to 326 Mbps and peak uplink data rate up to 86 Mbps for 4×4 antennas and 20 MHz bandwidth. The standard assumes a full Internet protocol (IP) network architecture, where the standard voice service is delivered via voice-over-IP (VOIP).

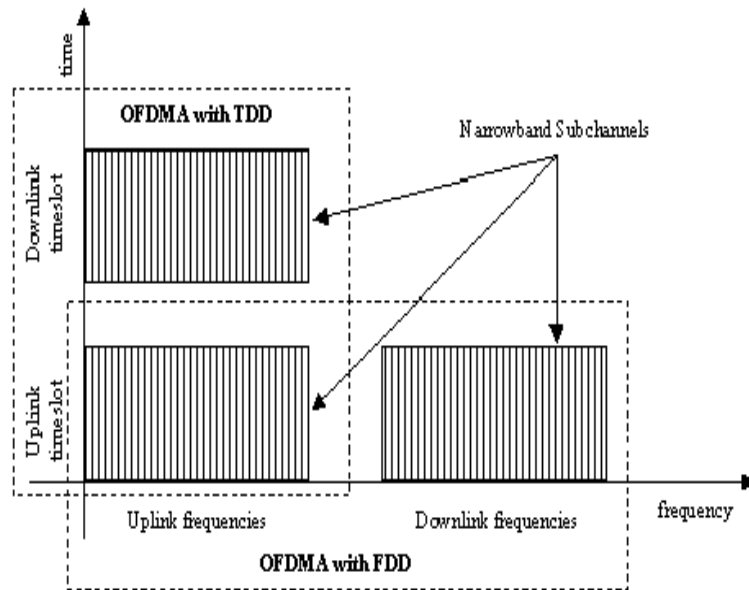


Figure 1.5: Orthogonal frequency division multiple access (OFDMA) with either TDD or FDD

1.1.2 Evolution of Broadband Wireless Access (BWA)

Before we delve into BWA, let us review the state of broadband access today. Digital subscriber line (DSL) technology, which delivers broadband over twisted-pair telephone wires, and cable modem technology, which delivers over coaxial cable TV plant, are the predominant mass-market broadband access technologies today. Both of these technologies typically provide up to a few megabits per second of data to each user, and continuing advances are making several tens of megabits per second possible. Since their initial deployment in the late 1990s, these services have enjoyed considerable growth. The availability of a wireless solution for broadband could potentially accelerate this growth. Broadband access not only provides faster Web surfing and quicker file downloads but also enables several multimedia applications, such as real-time audio and video streaming, multimedia conferencing, and interactive gaming. Broadband connections are also being used for voice telephony using voice-over-Internet Protocol (VoIP) technology. More advanced broadband access systems, such as fiber-to-the-home (FTTH) and very high data rate digital subscriber loop (VDSL), enable such applications as entertainment-quality video, including high-definition TV (HDTV)

and video on demand (VoD). As the broadband market continues to grow, several new applications are likely to emerge, and it is difficult to predict which ones will succeed in the future.

So what is broadband wireless access? Broadband wireless access is about bringing the broadband access experience to a wireless context, which offers users certain unique benefits and convenience [8]. There are two fundamentally different types of broadband wireless services. The first type attempts to provide a set of services similar to that of the traditional fixed-line broadband but using wireless as the medium of transmission. This type, called fixed broadband wireless access, can be thought of as a competitive alternative to DSL or cable modem. The second type of wireless broadband access, called mobile broadband wireless access, offers the additional functionality of portability, nomadicity, and mobility. Nomadicity implies the ability to connect to the network from different locations via different base stations; mobility implies the ability to keep ongoing connections active while moving at vehicular speeds.

First generation BWA

In the late 1990s, one of the more important deployments of broadband wireless access happened in the so-called multichannel multipoint distribution services (MMDS) band at 2.5GHz. The MMDS band was historically used to provide wireless cable broadcast video services, especially in rural areas where cable TV services were not available. The advent of satellite TV ruined the wireless cable business, and operators were looking for alternative ways to use this spectrum. First generation MMDS systems required that subscribers install at their premises outdoor antennas high enough and pointed toward the tower for a clear line-of-sight (LOS) transmission path.

Second generation BWA

Second generation broadband wireless access networks were able to overcome the LOS issue and to provide more capacity. This was done through the use of a cellular architecture and implementation of advanced-signal processing techniques to improve the link and system performance under multipath conditions. Several start-up companies developed advanced proprietary solutions that provided significant performance gains over first generation networks. Most of these systems could perform well under non-line-of-sight (NLOS) conditions, with customer-premise antennas typically mounted under the eaves or lower. Many solved the NLOS problem by using such techniques as OFDM, CDMA, and multi-antenna processing.

Emergence of Standards Based BWA Technology

In 1998, the Institute of Electrical and Electronics Engineers (IEEE) formed a group called 802.16 to develop a standard for what was called a wireless metropolitan area network, or wireless MAN. “Metropolitan” in this sense indicates not the target geography but instead the target scale. The standard supports networks that are about the size of a city. It is by no means limited to urban applications. Some of the most likely applications are in rural areas in which high-quality broadband access is not readily available. IEEE Standard 802.16 was designed to evolve as a set of air interfaces based on a common medium access control (MAC) protocol but with physical layer specifications dependent on the spectrum of use and the associated regulations [9].

IEEE 802.16-2001

In 2002, the IEEE 802.16-2001 [10] standard was published to provide a common air interface for fixed broadband wireless access systems between 10-66 GHz. The physical layer of IEEE 802.16-2001 uses single-carrier 4, 16, and 64 quadrature amplitude modulation (QAM) and TDMA, and a choice between FDD and TDD. Although spectrum is abundant in the 10-66 GHz range of carrier frequencies, the short wavelengths introduce significant deployment challenges, which include strict antenna alignment specifications due to required line-of-sight propagation, and significant attenuations brought about by atmospheric disturbances like rain and snow.

IEEE 802.16-2004

In 2004, the IEEE 802.16-2004 [11] standard was published to provide a common air interface for NLOS operation of fixed broadband wireless access systems between 2-11 GHz. This standard is intended for NLOS residential applications, where LOS operation is typically impractical due to natural (e.g. trees and hills) and man-made (e.g. buildings and bridges) obstructions between the base station to lower rooftop antennas. Three physical layer mechanisms are proposed in the standard: single-carrier, 256-subcarrier OFDM using TDMA, and 2048-subcarrier OFDMA options. Both TDD and FDD options are available in this standard, and peak data rates of up to 23 Mbps in the downlink are possible.

IEEE 802.16e-2005

In 2005, the IEEE 802.16e-2005 [12] standard was published, which extends the IEEE 802.16-2004 standard for combined fixed and mobile broadband wireless access. Focusing primarily on mobility enhancements to IEEE

System Bandwidth	Sampling Frequency	FFT Size
1.25 MHz	1.429 MHz	128
2.5 MHz	2.857 MHz	256
5 MHz	5.714 MHz	512
10 MHz	11.429 MHz	1024
20 MHz	22.857 MHz	2048

Table 1.1: IEEE 802.16e OFDMA Scalability Parameters [13]

802.16-2004, this standard similarly supports the three physical layer mechanisms as above. The primary difference is that the OFDMA physical layer in IEEE 802.16e supports varying numbers of subcarriers that scale with the various supported bandwidths, thereby keeping the subcarrier spacing fixed [13] (see Table 1.1). Both TDD and FDD modes are also available in IEEE 802.16e, and peak data rates of 46 Mbps in the downlink and 23 Mbps for the uplink with the 2048-subcarrier, 20 MHz OFDMA physical layer option.

As it turns out, the IEEE 802.16 specifications are a collection of standards with a very broad scope. In order to accommodate the diverse needs of the industry, the standard incorporated a wide variety of options. In order to develop interoperable solutions using the 802.16 family of standards, the scope of the standard had to be reduced by establishing consensus on what options of the standard to implement and test for interoperability. The IEEE developed the specifications but left to the industry the task of converting them into an interoperable standard that can be certified. The worldwide interoperability for microwave access (WiMAX) Forum was formed to solve this problem and to promote solutions based on the IEEE 802.16 standards [8]. WiMAX technology is designed to accommodate both fixed and mobile broadband applications. Korea's Electronics and Telecommunications Research Institute (ETRI) developed a mobile broadband solution known as wireless broadband (WiBro). The WiBro technology is compatible with IEEE 802.16e-2005 and mobile WiMAX.

Three different network topologies are currently supported in IEEE 802.16: Point-to-Multipoint (PMP) mesh and relay. In PMP, a Base Station (BS) distributes traffic directly to roof-mounted Subscriber Stations (SSs). The basic idea behind mesh and relay networks is to perform data transmission via intermediate relay stations and thus to provide extended coverage range

and an improved overall link quality. Rather than connecting directly to the BS through a low quality link, a SS can choose to connect through another SS where both links are of a higher quality.

1.2 Motivation

In a downlink wireless system, a centralized basestation needs to communicate to multiple users, with limited resources, e.g. total transmit power and available frequency bandwidth. Given the freedom of separating users in the time, frequency, code, or spatial domain, how the basestation allocates the resources among users is critical to system performance. Earlier generations of wireless systems adopted static resource allocations such as time or frequency division multiple access, where the basestation takes turns to serve one user in a designated time slot or frequency band, irrespective of the user channels. The wireless channel is, however, time-varying and frequency selective. The channels experienced by different users are largely independent because of users' different locations. The basestation should allocate the limited resources among users adaptively by taking the user channel conditions into consideration and enhance the system performance. In the multiuser multicarrier wireless systems, adaptive resource allocation is more challenging because of the additional degree of freedom for resources. Resources to be allocated are time slots, subcarriers, power, etc.

In the multiuser multicarrier wireless systems, adaptive transmission schemes such as subcarrier and power allocation often involve greedy algorithms that maximize the system throughput [14]. These algorithms usually allocate maximal resources to those users whose channel conditions are above a certain threshold and restrain those whose channel conditions are below that threshold. When the path loss differences among users are large, the users with higher average channel gains will be allocated most of the resources for a significant portion of time. The users with lower average channel gains may be unable to receive any data. Therefore users with better channel quality often enjoy the privilege of good quality of service (QoS) while other users suffer with poor QoS. To counter this unfair scenario a *max-min* resource allocation scheme, where the worst user's rate is maximized, has been proposed [15]. However, the *max-min* scheme can only provide maximum fairness among users. In most wireless systems of interest, different users require different data rates, which may be accommodated by allowing users to subscribe to different levels of service. This motivates us to develop resource allocation methods and algorithms to satisfy user's specific

requirements and constraints.

The perfect feedback of channel state information (CSI) has been an essential assumption for most resource allocation schemes in multicarrier wireless systems. In reality, perfect channel information is not always available since wireless communication channel is much more volatile than its wired counterpart. Perfect CSI at the transmitter (CSIT) is rarely possible due e.g. to channel estimation errors and channel feedback delay. When perfect CSIT is not available, resource allocation strategies can be based on statistical channel information.

An idea referred to as subchannelization has been introduced in OFDMA based systems such as WIMAX [8]. Instead of considering each separate subcarrier assignment, a subchannel made of a subset of adjacent or distributed subcarriers is considered. Then the need for knowledge of radio channel quality is reduced from per-subcarrier to per-subchannel resolution and resources are allocated on per-subchannel basis. This motivates us to propose resource allocation schemes with reduced feedback.

The most intuitive and up to now, the most widely used strategy to increase coverage in wireless networks, is to shrink the size of the cell, effectively increasing the number of base stations (BS) over a given area. Although this strategy likely will increase coverage because users are much closer to their serving BSs, its benefit is limited because of the exceeding cost of BSs. For each BS, the provider must pay for antenna space and the wired backhaul to the network, not to mention the digital and radio frequency equipment itself. Instead, an increasingly attractive strategy is to insert fixed relays into the cell whose sole purpose is to aid communication from BS to mobile station (MS) and vice versa. Such communications are called cooperative communications. This motivates us to extend our framework to resource allocation in cooperative communication systems.

Since we have modelled the resource allocation problem for various users' requirements and systems constraints, this motivates us to provide a trade-off analysis between achieved performances and the constraints. We also analyze a trade-off between performance degradation and reduced feedback/complexity of our schemes in comparison with the existing schemes.

1.3 Organization of the Thesis and Summary of Contributions

The main focus of this dissertation is to develop adaptive resource allocation methods and low computational algorithms for multiuser multicarrier wireless systems under certain constraints. The basic idea behind adaptive resource allocation is to exploit the characteristics of the fading channel to determine which users to schedule, how to allocate subcarriers (or subchannels) to them and how to determine the appropriate power levels for each user on each subcarrier (or subchannel) in order to achieve optimal transmission rates. We provide answers to the issues involving computational complexity, outdated channel information, power consumption and fairness constraints in downlink resource allocation for various multiuser multicarrier wireless systems. Both PMP and cooperative networks topologies are considered. A brief description of each chapter in this thesis and its contribution follows:

Chapter 2: Background

In Chapter 2, we outline the characteristics of the basic channel and system models of downlink multiuser multicarrier systems and provide sufficient and concise mathematical background on the Newton's method, geometric programming and Lagrange dual approach for constrained optimization.

Chapter 3: Resource Allocation in Multicarrier Systems

Chapter 3 contextualizes our research problem by surveying the existing resource allocation strategies for multicarrier systems in the literature. The chapter begins with surveying various adaptive resource allocation techniques for multicarrier systems. The problem of downlink resource allocation in multiuser multicarrier systems where the base station only has statistical channel knowledge is identified. Users and systems requirements and constraints are described and optimization goals are outlined. Then our constrained resource allocation problem is formulated.

Chapter 4: Constrained Resource Allocation for OFDM-CDM Systems with perfect CSIT

In Chapter 4, we develop multiuser power allocation algorithms in OFDM system with code division multiplexing (OFDM-CDM). A zero-forcing beamformer is used at the mobile unit and the basestation optimizes its power

allocation to maximize the sum-rate subject to transmit power and fairness constraints. For the purpose of simplicity, we assume the availability of perfect channel state information at the transmitter (perfect CSIT) because the main goal is to compare the performances of proposed power allocation algorithms in terms of fairness and to analyze the trade-off between achievable sum-rate and the fairness requirement. Due to the perfect CSIT assumption, this work may be perceived to be idealistic but shall form the basis of our research and help us to model more practical problems in later chapters. We summarize the original contribution of this chapter as follows:

- **Optimal and sub-optimal power allocation**

We first solve the power allocation by the duality approach and propose two algorithms: one with higher computational complexity based on *Newton Raphson* method and the other with lower computational complexity based on *Bisection* method. We also obtain a global optimal solution of the problem using a geometric programming framework.

- **Fairness constraints and minimum rate requirements**

We formulate the power allocation problem that maximize the sum-rate of the system while maintaining fairness among the users rates and satisfying minimum rate requirements. We analyze the trade-off between achieved sum-rate and the constraints.

Original results have been presented in [16] and [17].

Chapter 5: Constrained Resource Allocation for OFDMA Systems with statistical channel information

The previous chapter assumed a perfect instantaneous channel knowledge at the transmitter. However, perfect CSIT is rarely possible due to channel estimation. Moreover, in some fading environments, there may not be a feedback link sufficiently fast to convey the full channel state information (CSI) to the transmitter. In this chapter, we consider practical resource allocation strategies for downlink multiuser OFDMA systems, where the base station has only statistical channel information in terms of statistics of the partial CSIT or in terms of channel distribution information (CDIT). We summarize the original contribution of this chapter as follows:

- **partial CSIT based Optimal subchannel and power allocation**
We use the Lagrange dual decomposition framework to derive an algorithm that efficiently finds the optimal allocation that maximize the expected weighted sum rate while satisfying users' minimum data rate and system fairness requirements under total power constraint.
- **Impact of the channel estimation error**
The impact of the channel estimation error on the performance of the system is illustrated by the means of simulations.
- **Trade-off analysis**
We also analyze the trade-off between achieved expected weighted sum rate and the constraints.
- **CDIT based Optimal subchannel and power allocation**
We derive the optimal power allocation in the form of *multi-level water-filling* power allocation. The important difference is that, in contrast to the CSIT-based *multi-level water-filling* power allocation where the optimal power allocation depends on the instantaneous channels realizations, the optimal allocation based on CDIT is dependent on the mean of the channel distribution, and thus needs to be computed only when the statistic of the channel has changed.
- **Feedback Reduction and Complexity Analysis**
We show that with CDIT, the amount of feedback required for the resource allocation reduces significantly. Using a dual decomposition framework, the computational complexity has been significantly decreased. The proposed resource allocation method has a complexity of $\mathcal{O}(KM)$ for K users and M subchannels.

Original results have been presented in [18], [19], [20], [21] and [22].

Chapter 6: Constrained Resource Allocation in Cooperative Communication

In the previous chapters, PMP communications have been considered. In PMP communications, a base station distributes traffic directly to roof-mounted subscriber stations. In contrast, in cooperative communications, data are transmitted via intermediate relay stations in order to provide extended coverage range and an improved overall link quality. Chapter 6 is dedicated to constrained resource allocation in cooperative communication.

We summarize the original contribution of this chapter as follows:

- **constrained resource allocation for OFDMA mesh networks**

We use the Lambert-W function to solve the constrained resource allocation for OFDMA mesh networks. By using the Lambert-W function, resource allocation can not only be expressed in closed form but it can also quickly be determined without resorting to complex algorithms since a number of popular mathematical softwares, including Matlab, contain the Lambert-W function as a function component.

The associated results have been partly submitted to [23].

Chapter 7: Conclusion

Chapter 7 summarizes the contributions of the thesis, and outlines interesting avenues for future investigations, which include constrained resource allocation for MIMO-OFDMA, multi-cell constrained resource allocation considering inter-cell interferences, and constrained resource allocation for OFDMA relaying networks.

Chapter 2

Background

2.1 Introduction

In this chapter, we discuss the technical background of this research by outlining the characteristics of the wireless channel and system models of multicarrier systems. We provide sufficient and concise mathematical background on the constrained optimization theory and describe the three main mathematical tools for constrained optimization which are used in this research. These are Newton's method, geometric programming and Lagrange dual approach.

2.2 Wireless Propagation

The radio signals are electromagnetic waves. The propagation of electromagnetic waves from a base station transceiver to a mobile receiver through a transmission medium is illustrated in Fig. 2.1. This propagation is generally governed by three main natural phenomena: reflection, diffraction and scattering [5]. They cause variations in the strength of the waves as the waves propagate. Depending on the time scale on which these variations occur, they can be grouped into two categories: large scale fading (slowly varying in time and space) and small scale fading.

2.2.1 Large Scale Fading

The large scale fading refers to the degradation of the signal strength due to the path loss as a function of the distance between the transmitter and the receiver and shadowing effects caused by the surrounding environmental

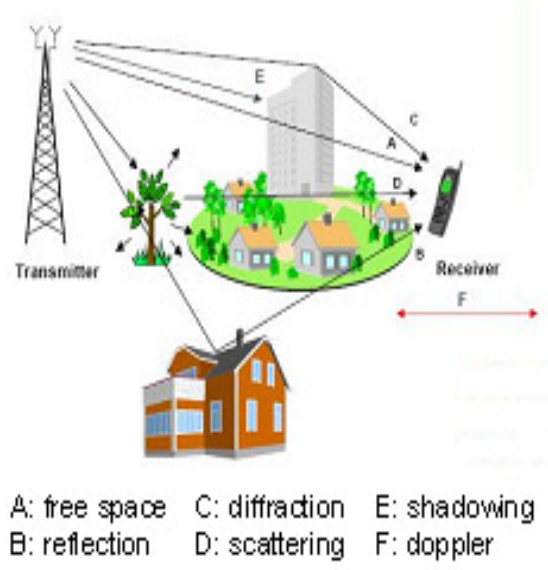


Figure 2.1: Illustration of wireless propagation.

clutter. The path loss is the gradual loss of received signal power with the distance from the transmitter. Shadowing describes the random effects which occur at different locations which have the same transmitter receiver separation, but different surroundings on the propagation path.

Path Loss

The first obvious difference between wired and wireless channels is the amount of transmitted power that reaches the receiver.

When the transmitter and the receiver have a clear unobstructed LOS path between them, the free space propagation model can be used to predict the received signal strength. The free space power received by a receiver antenna which is separated from a radiating transmitter antenna by a distance d , is given by the Friis free space equation [5],

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2}, \quad (2.1)$$

where P_t is the transmitted power, $P_r(d)$ is the received power which is a function of the transmitter-receiver separation distance d , G_t is the trans-

mitter antenna gain, G_r is the receiver antenna gain, and λ is the wavelength.

The *path loss*, which represents signal attenuation, is defined as the difference (in dB) between the effective transmitted power and receiver power. The path loss for the free space model when the antennas are assumed to have unity gain is given by

$$PL(dB) = 10 \log \frac{P_t}{P_r} = -10 \log \left[\frac{\lambda^2}{(4\pi)^2 d^2} \right]. \quad (2.2)$$

The free space propagation model is only valid for distances d which are in the *far-field* of the transmit antenna. The far-field of the transmit antenna is defined as a distance beyond a far-distance d_f which is a function of the largest linear dimension of the transmit antenna aperture and the carrier wavelength. For this reason, a known reference distance $d_0 \geq d_f$ is defined and the received power at any distance $d \geq d_0$ can be computed as:

$$P_r(d) = P_r(d_0) \left(\frac{d_0}{d} \right)^2, \quad d \geq d_0 \geq d_f, \quad (2.3)$$

where $P_r(d_0)$ is the received power at the reference distance d_0 .

When there is no unobstructed LOS path between the transmitter and the receiver, the prediction of the free space propagation model is far too optimistic. Both analytical and empirical models indicate that the average received signal power decreases according to [5]:

$$P_r(d) = P_r(d_0) \left(\frac{d_0}{d} \right)^\alpha, \quad (2.4)$$

where α is the path loss exponent which indicates the rate at which path loss increases with distance d . The value of α depends on the specific propagation environment and typically ranges from 2 (i.e., free space attenuation) to 6 (strong attenuation, e.g., because of city building).

Shadowing

Equation (2.4) only gives a distance-dependent relationship of the received power. However, many factors other than distance can have a large effect on the total received power. For example, as illustrated in Fig. 2.1, trees and buildings may be located between the transmitter and the receiver and cause temporary degradation in received signal strength. Since modeling the locations of all objects in every possible communication environment is

generally impossible, the standard method of accounting for these variations in signal strength is to introduce a random effect called *shadowing*. With shadowing, the empirical received power formula becomes

$$P_r(d) = P_r(d_0) \left(\frac{d_0}{d} \right)^\alpha X_s, \quad (2.5)$$

where X_s is a sample of the shadowing random process. Hence, the received power is now also modeled as a random process. In effect, the distance trend in the path loss can be thought of as the mean, or expected, received power, whereas the shadowing value causes a perturbation from that expected value. The shadowing value X_s is typically modeled as a log-normal random variable, that is,

$$X_s = 10^{\frac{x_s}{10}},$$

where x_s is a Gaussian (normal) distribution with mean 0 and variance σ_s^2 . With this formulation, the standard deviation σ_s is expressed in dB. The standard deviation σ_s is calculated based on measurements that are taken over a wide range of locations that have the same transmitter receiver separation but with different environmental clutter. Typical values for σ_s are in the 6-12 dB range.

Apart from this simple model of 2.5, there exist various empirical models that take into account the terrain profile of a particular area in the propagation predictions (such as Okumura model [24] and Hata model [25]).

2.2.2 Small Scale Fading

The small scale fading refers to the rapid fluctuations in the envelope of the received signal over very small distances or time periods. These are caused by multipath delay effects and mobility in the wireless environment.

Multipath Effect

As shown in Fig. 2.1, the transmitted signal in a wireless environment may follow multiple paths to reach the receiver. As different replicas of the signal travel in different paths, they may arrive at the receiver at different times with widely varying amplitude and phase. This causes rapid fluctuations in the received signal strength due to the way these replicas of the signal add up. There are two related parameters that are used to describe the multipath delay effects. The multipath delay spread represents the difference in time between the earliest and the latest replica of the signal to arrive at the receiver. The coherence bandwidth which is inversely proportional to the

delay spread, provides a measure of the range of frequencies over which the channel can be considered flat.

When the bandwidth of the transmitted signal is smaller than the coherence bandwidth of the wireless channel, then the signal will experience flat fading. Flat fading channels are also known as narrow band channels, since the signal bandwidth is narrow compared to the coherence bandwidth of the channel. In the time domain, this corresponds to the symbol period of the signal being larger than the delay spread of the signal.

On the other hand, frequency selective fading occurs when the bandwidth of the transmitted signal is larger than the coherence bandwidth of the wireless channel. In this case, the signal will have frequency components that are spaced apart by more than the coherence bandwidth and these components will experience different attenuations. Frequency selective fading channels are also known as wide band channels. In the time domain, this corresponds to the symbol period of the signal being smaller than the delay spread. As a result, the signal components of different symbols overlap introducing what is known as inter-symbol interference (ISI).

There are two approaches to deal with ISI. The first is to avoid ISI by dividing the channel bandwidth into several narrow-band subchannels and adding a cyclic prefix (CP) to every symbol. The second is to manage ISI at the receiver by equalization. Corresponding air-interfaces for these two approaches are orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA).

Doppler Effect

The Doppler effect captures the frequency dispersion caused by the relative motion between the transmitter and the receiver or by the movements of the objects surrounding them. This frequency dispersion is an apparent shift in frequency of the received signal, and is referred to as Doppler Shift. The Doppler Shift is directly proportional to the velocity and the direction of the relative motion of the receiver with respect to the transmitter [5]. There are two related parameters that are used to describe the Doppler effect: Doppler spread and the coherence time. The Doppler spread characterizes the spectral broadening of the signal, and is defined as the range of frequencies over which the received spectrum of the carrier signal is non-zero. The coherence time is inversely proportional to the maximum Doppler shift, and is a measure of the time invariance of the channel. The coherence time represents the period over which two received signals have a strong amplitude correlation.

When the Doppler spread is high relative to the signal bandwidth, the signal experiences fast fading. Fast fading causes the channel to fluctuate faster than the variations in the signal. In the time domain, this corresponds to the symbol period of the signal being larger than the coherence time of the channel, which causes significant distortion of the symbol. The spectrum broadening due to the Doppler shift cannot be ignored at the receiver.

Conversely, if the signal bandwidth is much greater than the Doppler spread, then the effects of the Doppler shift is negligible at the receiver. This corresponds to a slow fading channel. The channel variation is much slower than the symbol period of the signal, and the signal is not distorted to a significant degree. In the time domain, this corresponds to the coherence time of the channel being much larger than the symbol period of the signal.

2.2.3 Impulse Response Model of a Multipath Channel

Let $x(t)$ represent the transmitted bandpass waveform, the received waveform $y(t)$ can be expressed as

$$y(t) = x(t) \otimes h(t, \tau), \quad (2.6)$$

where $h(t, \tau)$ is the impulse response of the time varying multipath radio channel. The impulse response $h(t, \tau)$ completely characterizes the channel and is a function of both the time variation due to the motion t and the channel impulse delay τ .

If the multipath channel is assumed to be bandlimited bandpass channel, then $h(t, \tau)$ may be equivalently described by a complex baseband impulse response $h_b(t, \tau)$ with the input and output being the complex envelope representations of the transmitted and received signals, respectively. That is,

$$\frac{1}{2}r(t) = \frac{1}{2}c(t) \otimes \frac{1}{2}h_b(t, \tau),^1 \quad (2.7)$$

where $c(t)$ and $r(t)$ are related to $x(t)$ and $y(t)$, respectively, through [5]

$$x(t) = \Re \{c(t) \exp(j2\pi f_c t)\}, \quad y(t) = \Re \{r(t) \exp(j2\pi f_c t)\}, \quad (2.8)$$

where f_c is the carrier frequency.

¹The factors 1/2 are due to the properties of the complex envelope, in order to represent the passband radio system at baseband.

Since the received signal in a multipath channel consists of a series of attenuated, time-delayed, phase shifted replicas of the transmitted signal, the baseband impulse response of a multipath channel can be expressed as

$$h_b(t, \tau) = \sum_{l=0}^{L-1} a_l(t, \tau) \exp(-j2\pi f_c \tau_l(t)) \delta(\tau - \tau_l(t)), \quad (2.9)$$

where L is the total number of possible multipath component. In equation (2.9), $a_l(t, \tau)$, $\tau_l(t)$ and $\theta_l(t, \tau) = 2\pi f_c \tau_l(t)$ represent the real amplitude, excess delay and phase shift, respectively, of l th multipath component at time t .

If the channel impulse response is assumed to be time invariant, or is at least wide sense stationary over a small time interval, then the channel impulse response may be simplified as

$$h_b(\tau) = \sum_{l=0}^{L-1} a_l \exp(-j\theta_l) \delta(\tau - \tau_l). \quad (2.10)$$

2.2.4 A Statistical description of the Transmission Channel

Signal statistics is another way of developing a mathematical understanding of the propagation channel.

Gaussian Distribution

The distribution resulting from the superposition of an infinite number of statistically independent random variables is, based on the central limit theorem, a Gaussian distribution with probability density function (pdf)

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x - \mu_x}{2\sigma_x^2}\right). \quad (2.11)$$

A complete description of the Gaussian distribution x is provided through its mean value μ_x and the variance σ_x^2 .

When the additive noise is included in the model described in subsection 2.2.3, the standard assumption is made that the noise is zero-mean additive white Gaussian with a given variance σ_n^2 .

Rayleigh Distribution

On the assumption that all component waves are approximately incident at a plane and approximately have the same amplitude, a Rayleigh distribution

can be used to describe the statistical time varying nature of the received envelope of the signal. This assumption applies in particular when the receiver has no LOS connection with the transmitter because of lack of dominance of any particular signal component.

The pdf of a Rayleigh distribution $r(t)$ is given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) & \text{for } (0 \leq r \leq \infty) \\ 0 & \text{for } r < 0 \end{cases} \quad (2.12)$$

where σ^2 is the time-average power of the received signal before *envelope detection*.

The mean value r_{mean} and the variance σ_r^2 of the Rayleigh distribution are given by

$$r_{mean} = E[r] = \sigma \sqrt{\frac{\pi}{2}}, \quad \sigma_r^2 = E[r^2] - E^2[r] = \sigma^2 \left(2 - \frac{\pi}{2}\right). \quad (2.13)$$

The Rice Distribution

There are many cases in which the assumption of component waves having the same amplitude does not apply, especially when a line-of-sight connection dominates. The envelope of the signal is then described on the basis of a Rice distribution.

The pdf of a Rice distribution $r(t)$ is given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{(r^2 + A^2)}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right) & \text{for } (A \geq 0, r \geq 0) \\ 0 & \text{for } r < 0 \end{cases} \quad (2.14)$$

where the parameter A denotes the peak amplitude of the dominant signal and $I_0(\bullet)$ is the modified Bessel function of the first kind and zero-order.

The Rice factor $K_r = A^2/(2\sigma^2)$ is used to completely specified the Rice distribution. There is no closed-form solution for the mean value and variance of the Rice distribution. These parameters can only be determined using approximation formula and tables.

The Rayleigh distribution is a special case of the Rice distribution for $A = 0$.

2.3 Multicarrier Modulation

Multicarrier modulation provides an elegant solution to the problem of intersymbol interference induced by multipath channels.

2.3.1 The principle of Multicarrier Modulation

The principle of multicarrier (MC) modulation is to split the transmission bandwidth into several narrowband subchannels, so-called subcarriers, which are used for parallel data transmission. Each subcarrier experiences flat fading. Equalization is performed on a subcarrier basis and thus is much simpler than the single carrier transmission systems. Fading across different subcarriers provides frequency diversity. A single fade only affects a limited number of the subcarriers. By coding across the symbols in different subcarriers, the information conveyed by those affected subcarriers can be recovered at the receiver [26].

Orthogonal Frequency Division Multiplexing (OFDM) is an efficient realization of MC transmission. OFDM offers high spectral efficiency by allowing the overlapping of the spectrum of the subcarriers. In order for this to work, the interference between subcarriers must be mitigated. This is achieved by making the subcarriers mutually orthogonal.

The orthogonality between subcarriers corresponds to a precise mathematical relationship between the subcarrier signals over a symbol period T_s . Suppose that the set $\{\Psi_n(t)\}$ represent the subcarriers signals, with $n = 1, 2, \dots, N_c$. Then, orthogonality between subcarriers corresponds to the condition that over a symbol period T_s , the signal contribution to a subcarrier n from all other subcarriers is zero:

$$\int_0^{T_s} \Psi_n^*(t) \Psi_m(t) dt = A \times \delta[n - m] = \begin{cases} A, & \text{if } n = l \\ 0, & \text{otherwise,} \end{cases} \quad (2.15)$$

where $*$ is the complex conjugate operator, A is a constant, and $\delta[\bullet]$ is the Kronecker delta function.

Orthogonal sets of subcarrier signals can be obtained by using the sinusoidal wave family and also letting the spacing between subcarriers, Δf , equal to the reciprocal of the symbol period: $\Delta f = 1/T_s$. The orthogonal signals $\{\Psi_n(t)\}$ are of the form:

$$\Psi_n(t) = \begin{cases} e^{j2\pi f_n t}, & \text{if } t \in [0, T_s] \\ 0, & \text{otherwise,} \end{cases} \quad (2.16)$$

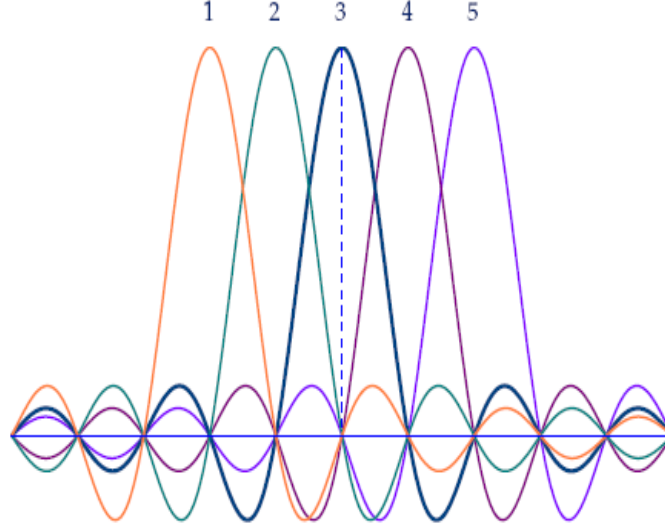


Figure 2.2: Example of an OFDM signal spectra.

with f_n being the center frequency of subcarrier $n = 1, 2, \dots, N_c$.

The orthogonality ensures that the spectrum of each subcarrier has a null at the center frequency of each of the other subcarriers in the system as illustrated in Fig. 2.2.

Fig. 2.3 shows the savings of bandwidth with OFDM compared to a conventional system with the same subcarrier bandwidth. A saving of almost 50% of bandwidth can be achieved with OFDM due to the overlapping of subcarriers.

2.3.2 OFDM Transmission

A schematic diagram of an OFDM transmission is depicted in Fig. 2.4. Let us concentrate on the transmission of N_c serially transmitted symbols. The N_c serial symbols are converted into N_c parallel substream elements denoted as s_n , $n = 1, 2, \dots, N_c$. The N_c parallel source symbols s_n , $n = 1, 2, \dots, N_c$ are modulated simultaneously on N_c orthogonal subcarriers. The N_c modulated symbols are referred to as an OFDM symbol further on. The complex envelope of the OFDM symbol with rectangular pulse shaping is given by

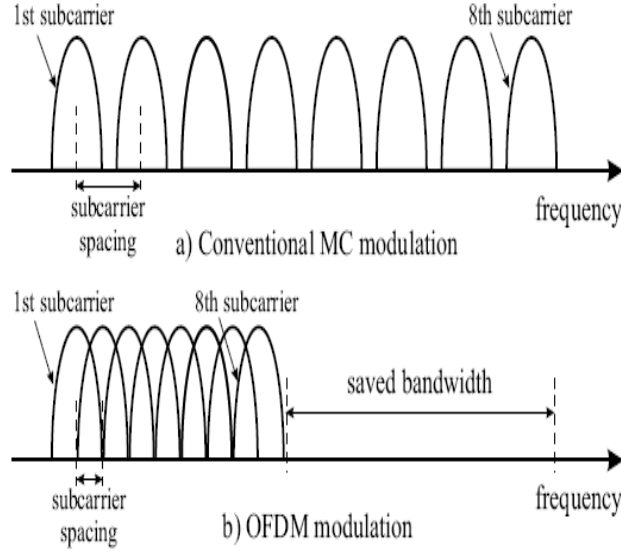


Figure 2.3: Spectral efficiency of OFDM.

$$x(t) = \frac{1}{\sqrt{N_c}} \sum_{n=1}^{N_c} s_n \text{rect} \left(\frac{t}{T_s} \right) e^{j2\pi f_n t}, \quad (2.17)$$

where the subcarrier frequencies f_n equal

$$f_n = (n-1) \cdot \frac{1}{T_s}, \quad n = 1, 2, \dots, N_c, \quad (2.18)$$

and the center of the frequency spectrum is located at $(N_c - 1)/(2T_s)$. The factor $1/\sqrt{N_c}$ is used for energy normalization.

Sampling the signal $x(t)$ at time instances $t = (m-1)T_s/N_c$, $m = 1, 2, \dots, N_c$ results in

$$x_n = \frac{1}{\sqrt{N_c}} \sum_{n=1}^{N_c} s_k e^{j2\pi(n-1)(m-1)/N_c}, \quad n = 1, 2, \dots, N_c, \quad (2.19)$$

which represents the inverse discrete Fourier transform (IDFT) of the signal s_k . This is a very important property of OFDM due to which MC modulation can be implemented in the discrete domain by using an IDFT. As assumed in the following, if $N_c = 2^i$, $i = 1, 2, \dots$, the computationally more

effective inverse fast Fourier transform (IFFT) can be applied to modulate the transmission signal instead of IDFT.

Due to the time-dispersive behavior of the mobile radio channel, transmission of several subsequent OFDM symbols can result in mutual inter-symbol interference (ISI). Compared to the serial transmission of data, parallel OFDM transmission increases the symbol duration N_c times and, thus, the symbol duration becomes large compared to the duration τ_{max} of the channel impulse response (CIR), reducing the ISI. Moreover, ISI can be completely eliminated by introducing a guard interval. Usually a guard interval in the form of a cyclic prefix is added in front of an OFDM symbol. With a cyclic prefix inter-carrier interference (ICI) does not occur. This is in contrast to the insertion of an empty guard interval where the problem of ICI appears. By setting the duration of the cyclic prefix larger than τ_{max} ISI is completely avoided. However, the OFDM symbol duration is increased.

Finally, after cyclic prefix insertion the resulting signal is transmitted through a digital-to-analog converter (DAC) and a low-pass filter (LPF) whose output, in an ideal situation and excluding the cyclic prefix, equals $x(t)$ as defined in equation (2.17).

The signal $x(t)$ is sent through a frequency-selective wireless channel with CIR $h_{cir}(\tau, t)$ which represents the response of the channel at time t due to an impulse applied at time $t - \tau$. In addition, the transmission signal is affected by an additive noise signal $n(t)$ which is modelled as additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2 . Then the signal at the receive antenna is given by

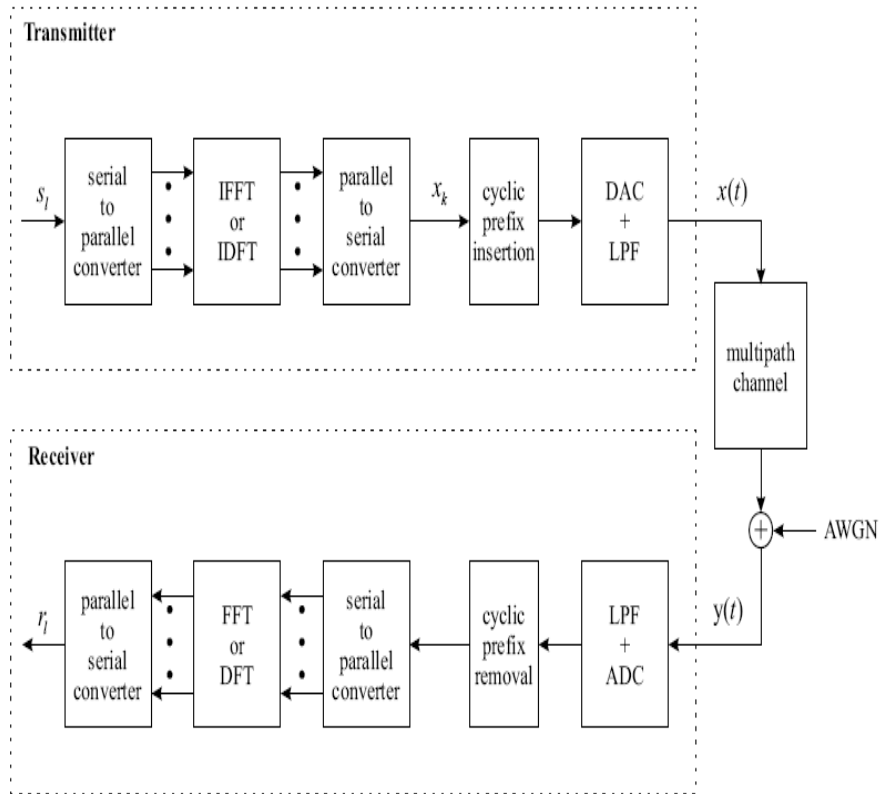
$$y(t) = \int_0^{\tau_{max}} x(t - \tau) h_{cir}(\tau, t) d\tau + n(t). \quad (2.20)$$

At the receiver, the signal is passed through a LPF and an analog-to-digital converter (ADC). Next, the cyclic prefix is removed and the remaining ISI-free part is demodulated by a discrete Fourier transform (DFT) or a more computationally efficient fast Fourier transform (FFT). The output of the FFT block is represented by a sequence r_n , $n = 1, 2, \dots, N_c$ equal to

$$r_n = H_n s_n + n_n, \quad n = 1, 2, \dots, N_c, \quad (2.21)$$

where H_n is the n th fading coefficient over the n th subcarrier and n_n represents the AWGN component on the n th subcarrier.

Several subsequent OFDM symbols are grouped together to form a so-called OFDM frame.

**Figure 2.4:** Block diagram of an OFDM transmission

2.4 Multiuser Multicarrier Systems

Modern multicarrier systems present a very challenging multiuser communication problem. Many users in the same geographic area require high on-demand data rates in a finite bandwidth. Multiple access techniques allow different users to share the available bandwidth by allotting each user some fraction of the total system resources.

2.4.1 Multiple Access Strategies for OFDM

OFDM by itself is not a multiple access strategy, but a modulation technique. Existing multiple access schemes can be combined with OFDM.

Multiple access strategies typically attempt to provide orthogonal, i.e. non-interfering, communication channels for each active link. The most common way to divide the available dimensions among the multiple users is through the use of frequency, time, or code division multiplexing. In frequency division multiple access (FDMA), each user receives a unique carrier frequency and bandwidth. In Time Division Multiple Access (TDMA), each user is given a unique time slot, either on demand or in a fixed rotation. Orthogonal Code Division Multiple Access (CDMA) systems allow each user to share the bandwidth and time slots with many other users, and rely on orthogonal binary codes to separate out the users.

Frequency Division Multiple Access

Frequency division multiple access (FDMA) can be readily implemented in OFDM systems by assigning different users their own sets of subcarriers. There are a number of ways this allocation can be performed. The simplest method is a static allocation of subcarriers to each user. For example, in a 64 subcarrier OFDM system, user 1 could take subcarriers 1-16, with users 2, 3, and 4 using subcarriers 17-32, 33-48, and 49-64, respectively. The allocations are enforced with a multiplexer for the various users before the IFFT operation. Naturally, there could also be uneven allocations with high data rate users being allocated more subcarriers than lower rate users.

An improvement upon static allocation is dynamic subcarrier allocation based upon channel state conditions. For example, due to frequency selective fading user 1 may have relatively good channels on subcarriers 33-48, while user 3 might have good channels on subcarriers 1-16. Obviously, it would be mutually beneficial for these users to swap the static allocations

given above. There are well-developed theories for how the dynamic allocation of subcarriers should be performed, and we discuss these in detail in Chapter 3.

Time Division Multiple Access

In addition to or instead of FDMA, multiple users can also be accommodated with TDMA. In reality, OFDM based systems such as WiMAX, employ both FDMA and TDMA since there will generally be more users in the system than can be simultaneously carried on a single OFDM symbol. Furthermore, users often will not have data to send, so it is crucial for efficiency's sake that subcarriers be dynamically allocated in order to avoid waste.

Static TDMA methodology is appropriate for constant data-rate applications like voice and streaming video, but in general, a packet-based system like WiMAX can employ more sophisticated scheduling algorithms based on queue lengths, channel conditions, and delay constraints to achieve much better performance than static TDMA.

Code Division Multiple Access

As already explained, multicarrier modulation provides an elegant solution to the problem of ISI induced by multipath channels. Large symbol lengths ensure that the signals arriving from different paths will combine coherently without additional processing. Code division multiple access (CDMA) has been proposed to exploit this effect of frequency diversity for each user, while minimizing multiple access interference (MAI). CDMA has been chosen as an air interface for several cellular systems, e.g., IS-95, CDMA2000, and UMTS.

The success of MC systems as well as of CDMA systems motivated several researchers to investigate the potentials of techniques which combine the benefits of MC and CDMA [27, 28, 29]. Such systems are called multicarrier code division multiple access (MC-CDMA) systems. In MC-CDMA, each user is assigned a CDMA code, which is used to differentiate between signals belonging to different users at the receiver. The users' symbols are spread prior to multicarrier modulation and each chip is transmitted on a different subcarrier.

OFDM code division multiplexing (OFDM-CDM) is one of the MC-CDMA versions proposed by S. Kaiser [30] for the Downlink transmission. Resource allocation strategies for OFDM-CDM will be studied in Chapter 4.

Orthogonal Frequency Division Multiple Access

Orthogonal Frequency Division Multiple Access (OFDMA) is essentially a hybrid of FDMA and TDMA: users are dynamically assigned subcarriers (FDMA) in different time slots (TDMA). OFDMA allows multiple users to transmit simultaneously on different subcarriers per OFDM symbol. The advantages of OFDMA start with the advantages of single user OFDM in terms of robust multipath suppression and frequency diversity. In addition, OFDMA is a flexible multiple access technique that can accommodate many users with widely varying applications, data rates, and QoS requirements. Because the multiple access is performed in the digital domain (before the IFFT operation), dynamic and efficient bandwidth allocation is possible. This allows sophisticated power allocation and time and frequency domain scheduling algorithms to be integrated in order to best serve the user population. Some of these algorithms are discussed in Chapters 4, 5 and 6.

Expression (2.21) represents the received signal for the single user system. In the context of multiuser, a general Downlink received signal for an arbitrary k th user at its n th subcarrier is modeled as

$$r_{k,n} = H_{k,n}s_{k,n} + n_{k,n}, \quad (2.22)$$

where $s_{k,n}$ and $r_{k,n}$ are the transmitted and the received signals, respectively, $n_{k,n}$, is the AWGN noise and $H_{k,n}$ is the fading coefficient calculated from the Fourier transform of the discrete CIR.

The corresponding channel signal-to-noise ratio (SNR) for k th user's n th subcarrier is expressed as

$$\gamma_{k,n} = \frac{|H_{k,n}|^2}{\sigma_{k,n}^2}, \quad (2.23)$$

where $\sigma_{k,n}^2$ is the noise variance of AWGN.

Assuming unit power for each subcarrier, the Shannon's capacity [31] for the multiuser OFDMA is defined as

$$C = \sum_{k=1}^K \sum_{n=1}^{N_c} \frac{B}{N_c} \log_2 (1 + \gamma_{k,n}), \quad (2.24)$$

where B is the overall bandwidth.

Mode	FFT Size (Points)			
	128	512	1024	2048
Downlink AMC	2	8	16	32
Downlink FUSC	2	8	16	32
Downlink PUSC	3	15	30	60

Table 2.1: Number of subchannel for each subchannelization mode

OFDMA Downlink Subchannelization

An OFDMA symbol consists of a number of subcarriers equal to the size of the Fourier transform. The OFDMA symbols are constructed from data subcarriers (for data transmission) and pilot subcarriers (for channel estimation). To support multiple access, the data subcarriers are divided into groups called subchannels. As illustrated in Fig. 2.5, subchannels can be formed using adjacent subcarriers or distributed subcarriers. The subchannelization and desubchannelization modules map and demap the raw constellation data to particular subcarriers within subchannels. There are three types of subchannelizations, namely, band adaptive modulation and coding (AMC), partially used subchannelization (PUSC), and fully used subchannelization (FUSC). Band AMC performs adaptive modulation and coding across bands of adjacent subcarriers. With both PUSC and FUSC, the subchannels are composed of distributed subcarriers. The data and pilot subcarrier indexes are generated differently for the FUSC and PUSC. For downlink PUSC, the set of used subcarriers is partitioned into subchannels and pilot subcarriers are allocated from within each subchannel. For downlink FUSC, fixed and variable pilot tones are added for each OFDMA symbol independently and remaining subcarriers are divided into subchannel that are used exclusively for data. The FUSC method uses all the subchannels and employs full-channel diversity. Thanks to the diversity provided by the FUSC, the performance degradation due to fast fading in mobile environments is minimized. Table 2.1 shows the number of subchannels for each mode.

In general, AMC is well suited for stationary, portable, and low mobility applications, whereas PUSC and FUSC are the best alternatives for mobile applications. In Chapter 5, we employ AMC for OFDMA resource allocation based on partial channel state information and FUSC for the CDIT-based resource allocation.

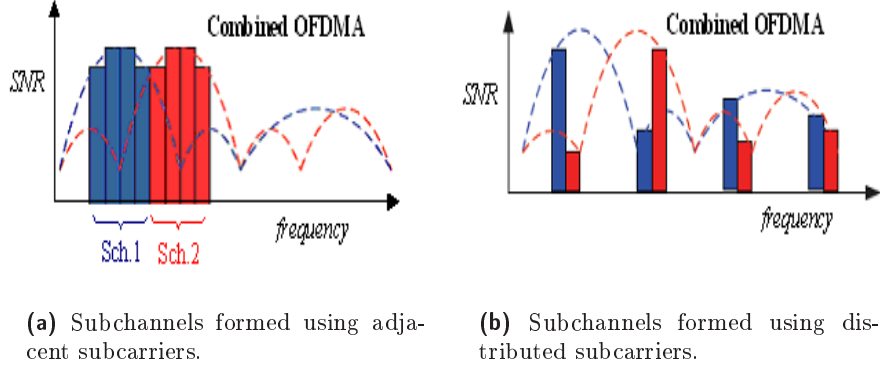


Figure 2.5: OFDMA Subchannelization

2.4.2 Multiuser Diversity

The main motivation for adaptive resource allocation in multiuser multicarrier systems like OFDMA is to exploit multiuser diversity. Multiuser diversity describes the gains available by selecting a user from a subset of users that have “good” conditions.

Although multiuser multicarrier systems have a number of subcarriers, we will focus temporarily on the allocation for a single subcarrier amongst multiple users for illustrative purposes.

Consider a K -user system, where the subcarrier of interest experiences i.i.d. Rayleigh fading, that is, each user’s channel gain is independent of the others, and is denoted by h_k . The probability density function (pdf) of user k ’s channel gain $p(h_k)$ is given by

$$p(h_k) = \begin{cases} 2h_k e^{-h_k^2}, & \text{if } h_k \geq 0 \\ 0, & \text{otherwise.} \end{cases} \quad (2.25)$$

Now suppose the base station only transmit to the user with the highest channel gain, denoted as $h_{max} = \max\{h_1, h_2, \dots, h_K\}$. It is easy to verify that the pdf of h_{max} is

$$p(h_{max}) = 2Kh_{max} \left(1 - e^{-h_{max}^2}\right)^{K-1} e^{-h_{max}^2}. \quad (2.26)$$

Fig. 2.6 shows the pdf of h_{max} for different values of K . As the number of users increases, the pdf of h_{max} shifts to the right, which means the probability of getting a large channel gain improves.

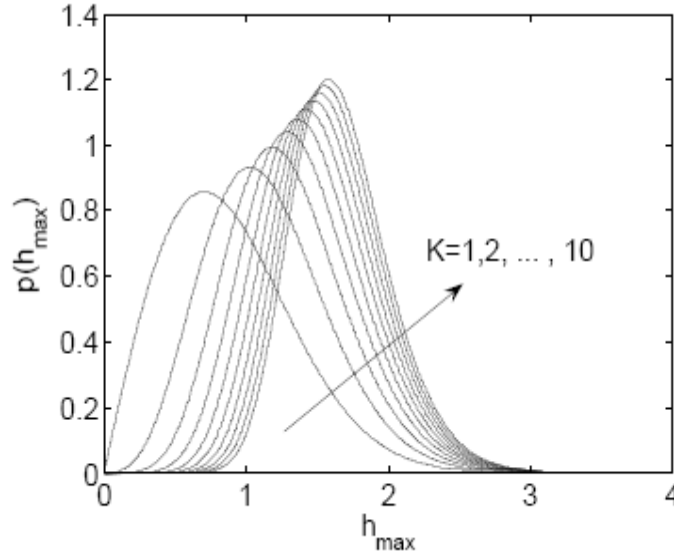


Figure 2.6: Probability density function of $h_{\max}$.

This increased channel gain improves the capacity as illustrated in Fig. 2.7. This figure shows that the multiuser diversity gain improves as the number of users in the system increases, but the majority of the gain is achieved from just the first few users. Specifically, it has been proven using extreme value theory that in a K -user system the average capacity scales as $\log \log K$ [32], assuming just Rayleigh fading. If i.i.d. log-normal shadowing is present for each of the users, which is a reasonable assumption, the scaling improves to $\sqrt{\log K}$ [33].

Although we will focus on the gains in terms of throughput (capacity) in this thesis, it should be noted that in some cases the largest impact from multiuser diversity is on the link reliability and overall coverage area.

2.5 Mathematical Background

2.5.1 Constrained Optimization Theory

Constrained optimization problems are made up of three basic ingredients:

1. an objective function which we want to minimize or maximize,

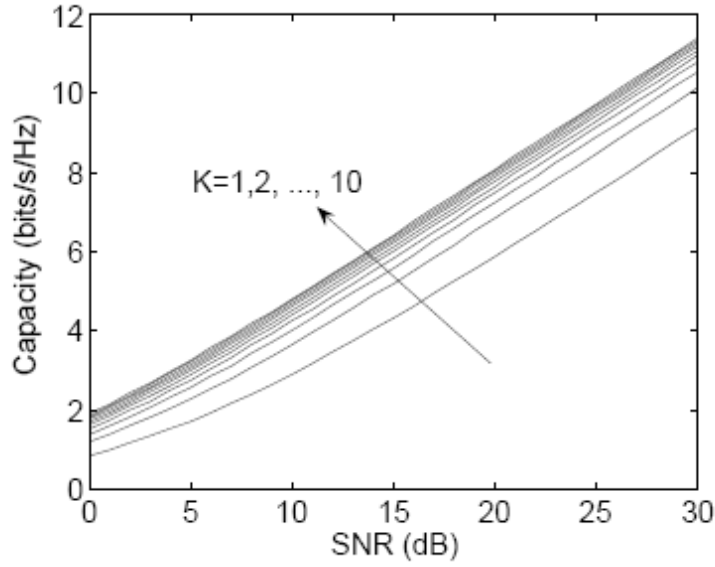


Figure 2.7: Capacity increase thanks to multiuser diversity gain.

2. a set of unknowns or variables which affect the value of the objective function,
3. a set of constraints that allow the unknowns to take on certain values but exclude others.

The constrained optimization problem is then:

Find values of the variables that minimize or maximize the objective function while satisfying the constraints.

Let $f_o : \mathbb{R}^n \rightarrow \mathbb{R}$, $x \in \mathbb{R}^n$ and $\Phi(x)$ be the objective function, the set of variables and the set of constraints respectively. The following notation will be used in this thesis.

$$\begin{aligned} \min_x \quad & f_o(x) \\ \text{subject to} \quad & \Phi(x) \end{aligned} \tag{2.27}$$

stands for *minimize* $f_o(x)$ subject to constraints $\Phi(x)$
and

$$\begin{array}{ll} \max_x & f_o(x) \\ \text{subject to} & \Phi(x) \end{array} \quad (2.28)$$

stands for *maximize* $f_o(x)$ subject to constraints $\Phi(x)$.

The constraints $\Phi(x)$ can be an arbitrary combination of equations $f_i(x) = b_i$, weak inequalities $f_i(x) \leq b_i$, strict inequalities $f_i(x) < b_i$, $i = 1, \dots, K$. The functions $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$, $i = 1, \dots, K$, are the constraint functions and the constants b_i , $i = 1, \dots, K$, are the limits or bounds for the constraints.

A point $\tilde{x}$ is said to satisfy the constraints Φ if $\Phi(\tilde{x})$ is true.

The following describes constrained optimization problems more precisely, restricting to maximization problems that are discussed in this thesis.

Global and local optimization

A point $x^* \in \mathbb{R}^n$ is said to be a global maximum of f_o subject to constraints Φ if x^* satisfies the constraints and for any point $\tilde{x}$ that satisfies the constraints, $f_o(x^*) \geq f_o(\tilde{x})$.

A value y_o is said to be the global maximum value of f_o subject to constraints Φ if for any point $\tilde{x}$ that satisfies the constraints, $y_o \geq f_o(\tilde{x})$.

The global maximum value y_o is attained if there exists a point x^* such that $\Phi(x^*)$ is true and $f_o(x^*) = y_o$. Such a point x^* is necessarily a global maximum.

If f_o is a continuous function and the set of points satisfying the constraints Φ is compact (closed and bounded) and nonempty, then a global maximum exists. Otherwise a global maximum may or may not exist.

A point $x^* \in \mathbb{R}^n$ is said to be a local maximum of f_o subject to constraints Φ if x^* satisfies the constraints and, for some $\epsilon > 0$, if $\tilde{x}$ satisfies $|\tilde{x} - x^*| < \epsilon$, then $f_o(x^*) \geq f_o(\tilde{x})$.

The value of the objective function f_o at the point x^* is called local maximum.

A local maximum may not be a global maximum. A global maximum is always a local maximum.

Throughout this thesis, maximum will mean global maximum.

Convex and non-convex optimization

A convex optimization problem is one in which the objective and constraint functions are convex, which means they satisfy the inequality

$$f_i(\alpha x_1 + \beta x_2) \leq \alpha f_i(x_1) + \beta f_i(x_2) \quad (2.29)$$

for all $x_1, x_2 \in \mathbb{R}^n$ and all $\alpha, \beta \in \mathbb{R}$ with $\alpha + \beta = 1$, $\alpha \geq 0$; $\beta \geq 0$ [34].

A fundamental property of convex optimization problems is that any locally optimal point is also (globally) optimal. Maximum of convex functions is convex. Sum of convex functions is also convex.

A non-convex optimization problem is one in which the objective or the constraint functions are not convex. Traditional techniques for general non-convex optimization problem involve compromises between local optimization and global optimization methods. Local optimization methods are fast and can handle large problems, but they require initial guess and they don't provide any information about the distance to the global optimum. Global optimization methods find the global optimum but worst-case complexity grows exponentially with problem size.

Feasibility and trade-off analysis

The optimization problem is said to be feasible if there exists at least one feasible point and infeasible otherwise. The feasibility problem is to determine whether the constraints are consistent, and if so, find a point that satisfies them.

In the trade-off analysis, we vary the constraints, and see the effect on the optimized objective. The bound values b_i of the original problem (2.28), with inequality constraint, are replaced by $\tilde{b}_i$ to form a perturbed problem. When $\tilde{b}_i = b_i$, the perturbed problem coincides with the original problem. When $\tilde{b}_i > b_i$, it means that we have relaxed the i th constraint; when $\tilde{b}_i < b_i$, it means that we have tightened the constraint. The trade-off curves are found by solving the perturbed problem for many values of the bound $\tilde{b}_i$.

2.5.2 Optimization Tools

The methods used to solve constrained optimization problems depend on specific problem types. Constrained optimization problems can be categorized according to several criteria. Depending on the type of functions involved there are linear and nonlinear (polynomial, transcendental, ...) constrained optimization problems. Additionally, constrained optimization

problems can be divided into integer and mixed integer optimization problems depending on the type of variables involved.

The ability to solve a constrained optimization problem varies considerably, and depends on factors such as the particular forms of the objective and constraint functions, how many variables and constraints there are, and special structure, such as sparsity (a problem is sparse if each constraint function depends on only a small number of the variables). In the following we give an overview of the optimization methods and tools used in this thesis.

Newton's method for convex optimization

Newton's method is a basic numerical tool in solving convex optimization problems with equality constraints,

$$\begin{aligned} \min_x & f_o(x) \\ \text{s.t. } & \mathbf{A}x = \mathbf{b} \end{aligned} \quad (2.30)$$

where the objective function f_o is convex and twice continuously differentiable, and $A \in \mathbb{R}^{p \times n}$ with $\text{rank} A = p < n$. The assumptions on A mean that there are fewer equality constraints than variables, and that the equality constraints are independent.

The main parameter of Newton's method is the Newton step Δx_{nt} . To derive the Newton step Δx_{nt} for the equality constrained problem (2.30) at the feasible point x , the objective function f_o is replaced with its second-order Taylor approximation near x , to form the problem

$$\begin{aligned} \min_v & \hat{f}_o(x+v) = f_o(x) + \nabla f_o(x)^T v + (1/2)v^T \nabla^2 f_o(x)v \\ \text{s.t. } & A(x+v) = b. \end{aligned} \quad (2.31)$$

Problem (2.31) is a (convex) quadratic minimization problem with equality constraints, and can be solved analytically. The Newton step, Δx_{nt} , at x is defined as the solution of the convex quadratic problem (2.31). In other words, the Newton step Δx_{nt} is what must be added to x to solve the problem when the quadratic approximation is used in place of f_o .

The Newton step Δx_{nt} is characterized by

$$\begin{bmatrix} \nabla^2 f_o(x) & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} \Delta x_{nt} \\ w \end{bmatrix} = \begin{bmatrix} -\nabla f_o(x) \\ 0 \end{bmatrix} \quad (2.32)$$

where w is the associated optimal dual variable for the quadratic problem (2.31).

It can be observed that when the objective function f_o is exactly quadratic, the Newton update $x + \Delta x_{nt}$ exactly solves the equality constrained minimization problem, and in this case w is the optimal dual variable for the original problem 2.30.

The Newton's method is very fast when it converges (superlinear convergence) and its convergence is quadratic near the optimal point x^* . But the Newton's method may not converge when started far from a nonsingular local optimum.

We refer the reader to sources such as Boyd [34], Luenbeger [35], Lin [36] and Goodman [37] for more details on Newton's method for constrained optimization.

Geometric programming

Geometric programming (GP) is a special form of convex optimization for which very efficient algorithms have been developed [34]. There are two equivalent forms of GP: the standard form and the convex form. The first is a constrained optimization of a type of function called posynomial, and the second form is obtained from the first through a logarithmic change of variable. GP uses monomial and posynomial functions. A monomial is defined as a function: $\mathbb{R}_{++}^n \rightarrow \mathbb{R}$

$$f_m(x) = dx_1^{a^{(1)}} x_2^{a^{(2)}} \dots x_j^{a^{(j)}} \dots x_n^{a^{(n)}}, \quad (2.33)$$

where $d \geq 0$, $a^{(j)} \in \mathbb{R}$ and x stands for a vector of variables x_j . A posynomial is a sum of monomials:

$$f_p(x) = \sum_{k=1}^K d_k x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \dots x_j^{a_k^{(j)}} \dots x_n^{a_k^{(n)}}, \quad (2.34)$$

where $d_k \geq 0$; $k = 1, 2, \dots, K$; $a_k^{(j)} \in \mathbb{R}$ and $j = 1, 2, \dots, n$.

Geometric programming in standard form is an optimization problem in the following form:

$$\begin{aligned} & \min_x f_0(x) \\ & \text{s.t.} \quad \begin{aligned} h_i(x) &= 1, \quad i = 1, \dots, m, \\ g_j(x) &\leq 1, \quad j = 1, \dots, r. \end{aligned} \end{aligned} \quad (2.35)$$

where f_0 and g_j are posynomials and h_i are monomials in variable x .

Geometric programming in the above standard form is not a convex optimization problem. However, with a logarithmic change of the variables: $y_i = \log x_i$, and constants $b_{ik} = \log d_{ik}$, $b_j = \log d_j$, we can turn it into the following convex form:

$$\begin{aligned} \min_y P_0(y) &= \log \sum_k \exp(a_{0k}^T y + b_{0k}) \\ Q_i(y) &= a_i^T y + b_i = 0, \\ \text{s.t. } P_j(y) &= \log \sum_k \exp(a_{jk}^T y + b_{jk}) \leq 0, \end{aligned} \quad (2.36)$$

where $a_{i,k}$ is given by $a_{i,k} = (a_{ik}^{(1)}, a_{ik}^{(2)}, \dots, a_{ik}^{(n)})$ and the optimization variables are y .

The logarithm of a sum of exponentials is a convex function [34], thus (2.36) is a convex optimization problem.

When some of the variables x_j in problem (2.35) are boolean variables, the problem is called mixed boolean geometric programming (MBGP). By fixing the boolean variable, the MBGP can be transformed into a number of GPs.

A variety of GP applications can be found in communication systems (see e.g. [38], [39]). Solving a GP is very easy since efficient algorithms have been developed and are available for free download. However, GP modeling can be much trickier than modeling via classical NLP since GP are constrained in the form the objective and the constraints functions can take. Some problems cannot be represented, or even approximated, as GPs.

Lagrange Dual Method

Consider the constrained minimization problem

$$\begin{aligned} f_o^* &= \min_x f_o(x) \\ \text{s.t. } h_i(x) &= 0, \quad i = 1, \dots, m, \\ g_j(x) &\leq 0, \quad j = 1, \dots, r. \end{aligned} \quad (2.37)$$

where $x \in \mathbb{R}^n$ is a vector of optimization variables, $f_o : \mathbb{R}^n \rightarrow \mathbb{R}$, $h_i : \mathbb{R}^n \rightarrow \mathbb{R}$, $i = 1, \dots, m$ and $g_j : \mathbb{R}^n \rightarrow \mathbb{R}$, $j = 1, \dots, r$ are continuously differentiable functions. When we say that a vector is less than zero, such that $g < 0$, we mean that each component of g is less than zero, and similiary for other

comparisons. The idea of Lagrange dual approach [40, 41] is to solve (2.37) by forming its Lagrangian dual

$$L_p(x, \lambda, \mu) = f_o(x) + \sum_{i=1}^m \lambda_i h_i(x) + \sum_{j=1}^r \mu_j g_j(x), \quad (2.38)$$

where $\lambda = (\lambda_1, \dots, \lambda_m)$ and $\mu = (\mu_1, \dots, \mu_r)$ are the vectors of Lagrangian multipliers, of which each λ_i corresponds to the equality constraint of $h_i(x) = 0$ and each μ_j corresponds to the inequality constraint of $g_j(x) \leq 0$.

The dual objective can be defined as an unconstrained minimization of the Lagrangian as follow

$$q(\lambda, \mu) = \min_x L_p(x, \lambda, \mu). \quad (2.39)$$

The dual optimization problem is then

$$q^* = \max_{\lambda, \mu \geq 0} q(\lambda, \mu). \quad (2.40)$$

The dual problem (2.40) is concave.

Let x^* be the local minimum of the primal problem (2.37). Then there exist unique Lagrange multipliers vectors $\lambda^* = (\lambda_1^*, \dots, \lambda_m^*)$ and $\mu^* = (\mu_1^*, \dots, \mu_r^*)$ such that

$$\begin{aligned} h_i(x^*) &= 0; \quad i = 1, \dots, m, \\ g_j(x^*) &\leq 0; \quad j = 1, \dots, r, \\ \mu_j^* &\geq 0; \quad j = 1, \dots, r, \\ \frac{\partial}{\partial x} L_p(x^*, \lambda^*, \mu^*) &= 0; \\ \mu_j^* g_j(x^*) &= 0. \end{aligned} \quad (2.41)$$

The system of equations (2.41) forms the Karush-Kuhn-Tucker (KKT) conditions, which are necessary and sufficient for optimality. The last equation in (2.41) is the complementary slackness constraint. This constraint states that the inequality constraint for the original primal problem is satisfied with equality if and only if the dual variable associated with the inequality constraint is strictly greater than zero.

We always have $q^* \leq f^*$ where the difference between these two optimal values is defined as the optimal duality gap

$$\Gamma_d = f_o^* - q^*. \quad (2.42)$$

When the objective function and the constraint functions are convex, standard convex optimization results [34] guarantee that there is no duality gap, i.e., the primal problem (2.37) and the dual problem (2.40) have the same optimal value.

Even if there is a duality gap, the optimal dual value gives a lower bound to the optimal primal value.

For further details on Lagrangian dual methods for constrained optimization, we recommend [34], [42], [40], [43] and [44].

In this thesis, we consider constrained maximization problems in the form

$$\begin{aligned} f^* &= \max_x f(x) \\ \text{s.t.} \quad h_i(x) &= 0, \quad i = 1, \dots, m, \\ g_j(x) &\leq 0, \quad j = 1, \dots, r. \end{aligned} \tag{2.43}$$

The maximization problem (2.43) can be solved using the above Lagrange dual approach since maximizing the function f is equivalent to minimizing its negative.

Chapter 3

Resource Allocation in Multicarrier Systems

3.1 Introduction

There are a number of different ways to take advantage of various form of diversity such as frequency diversity and multiuser diversity provided by the multiuser multicarrier systems. Frequency diversity refers to different subcarriers within a wireless link having different channel gains due to the frequency selective nature of the channel, while multiuser diversity refers to different users experiencing different channel conditions due to their different locations in the network. These diversities imply that a subcarrier that is in a deep fade for one user may not be in deep fade for the other users. The idea behind adaptive resource allocation is to exploit these diversities by determining which users to schedule, how to allocate subcarriers to them, and how to determine the appropriate power and rate levels for each user on each subcarrier based on the channel conditions the users see on the subcarriers.

The idea of using channel state information (CSI) at the transmitter to improve the performance of communication systems has been around since at least 1968 [45]. In multicarrier systems, the channel state information for each subcarrier can be estimated with the use of pilot/training symbols [46]. In a system with time division duplex, the transmitter can estimate the instantaneous channel conditions due to the reciprocity of the uplink and the downlink channels. On the other hand, in a system with frequency division duplex, there is no channel reciprocity since the uplink and the downlink fade independently. The channel state information needs to be conveyed

by the receiver to the transmitter. Perfect CSI based resource allocation schemes perform the allocations based on instantaneous channel conditions and work with the assumption of perfect CSI at the transmitter. As such, these schemes need to adapt to the instantaneous channel variations.

The resource to be allocated can be:

- Subcarrier (or subchannel),
- Power,
- Rate,
- Spatial antennas,
- Code resource,
- Transmission time, ...

There are various optimization goals such as to:

- Maximize the average, expected or ergodic (weighted) sum rate,
- Find the capacity region,
- Minimize the outage probability,
- Maximize the goodput, ...

The optimization goals have to be met while satisfying user's specific requirement and system constraints such as:

- Total power constraint,
- Minimum data rate,
- Minimum bit error rate (BER) or packet error rate (PER),
- Fairness constraint,
- Limited feedback, ...

Adaptive resource allocation has been shown to provide substantial performance gain in multicarrier wireless systems, when full CSI is available at the transmitter [14]. In this chapter, we contextualize our research problem by surveying various adaptive resource allocation schemes in the literature.

3.2 Single User Resource Allocation

In a single user multicarrier system, the user has full access to all subcarriers. In this context, the resource allocation problem is to determine an appropriate transmit power allocation across the subcarriers. When the perfect channel state information (CSI) is available at the transmitter, the power allocation for the user across orthogonal subcarriers with additive white Gaussian noise can be obtained with the water filling method [47]. Water filling finds the optimal power allocation across the subcarriers that maximizes the capacity for a given total power constraint.

When the power allocation is determined by the water filling technique, power p_n allocated to subcarrier n satisfies $p_n = (L_0 - \sigma_n^2)^+$ where $x^+ = \max(x, 0)$ and σ_n^2 is the effective noise spectral density on subcarrier n (i.e., noise spectral density divided by the fading coefficient of subcarrier n) and the water level L_0 is a constant that is the same for all subcarriers. The more the effective noise spectral density on a subcarrier is, the less the power allocated to that subcarrier will be. Since p_n is a non-negative quantity, if $\sigma_n^2 \geq L_0$, then $p_n = 0$, i.e., subcarrier n will not be used for transmission at all. The value of L_0 is determined by solving the equation $\sum_{n=1}^{N_c} (L_0 - \sigma_n^2)^+ = P_{tot}$, where N_c is the number of subcarriers in the system and P_{tot} is the power budget.

3.3 Downlink Multiuser Resource Allocation

Referring to the downlink multiuser system, users estimate and feedback channel state information to a centralized base station, where subcarrier and power allocation is determined. Due to the individual user's QoS requirement and to the limited availability of resources at the base station, e.g. bandwidth and power, intelligent allocation of these resources to the users is crucial for delivering the best possible QoS to the users with the least cost. This is especially important in the multi-service wireless standards such as WiMAX or 3GPP-LTE.

The problem of multiuser resource allocation in multicarrier systems has been an area of active research over the past several years. The resource allocation is usually formulated as a constrained optimization problem. The research in this area can be broadly classified into two categories, based on the optimization objectives. The objective in the first category of work is to minimize overall transmit power given individual user quality of service (QoS) requirements (e.g., data rate requirements, bit error rates, delays).

The body of work in the second category aims to maximize the sum of user data rates (or throughput) under a power constraint with some fairness criteria (e.g., proportional fairness among the users, minimum rate constraints). The first objective is appropriate for fixed-rate applications (e.g. voice) while the second is more appropriate for bursty applications like data and other IP applications. Both categories of problems are combinatorial in nature due to the discrete nature of the subcarrier allocation.

3.3.1 Power Minimization Problem

In [14], power minimization problem was investigated. The authors applied a constraint relaxation technique, which allowed the binary integer parameter of subcarrier assignment to take on real values, which in turn implies a time-sharing of each subcarrier among users. This converted the problem into a convex minimization problem with a convex feasible region, and allowed the use of iterative convex optimization algorithms to find the global minimum transmit power. They proposed a heuristic algorithm, the so-called Hungarian algorithm, based on constructive assignment and iterative improvement. Although an iterative process is required in this algorithm, it is guaranteed to converge to an optimal solution. Unfortunately, the algorithm requires a large number of iterations to converge, and is too complex for cost-effective real-time implementation.

Following the Hungarian approach, [48] proposed an iterative algorithm for power minimization and bit loading. The algorithm is considered as suboptimal for adaptive modulation. To reduce computational complexity, [49] proposed a two step algorithm to solve the problem of minimizing the total power consumption under bit error rate and transmission rate constraints. The first step of the algorithm is the bandwidth allocation, which determines the number of subcarriers to be allocated to each user based on the average SNR of the users. The second step then allocates the required number of subcarriers (determined by the first step) to users, by applying a greedy approach considering the channel quality of the individual subcarriers. In this step, the algorithm examines the subcarriers, assigning one subcarrier at a time to the user who has the highest SNR on it and has not yet received the required number of subcarriers. The subcarrier allocation produced by this algorithm is not unique, and depends on the order in which the subcarriers are considered.

In [50], the performance of bandwidth-constrained power minimization and power minimization schemes in terms of outage probability and packet error

rate under user data rate satisfaction are compared. It was shown that, in severe shadowing environment with both frequency selective and flat fading, the bandwidth-constrained power allocation scheme outperforms the unconstrained ones.

3.3.2 Rate Maximization Problem

The problem of rate maximization was also widely studied in the literature. In [51], the rate maximization problem was investigated, wherein the objective was to maximize the total sum continuous rate over all users subject to power and bit error rate constraints. It was shown in [51] that in order to maximize the sum rate, each subcarrier should be allocated to the user with the best gain on it, and the power should be allocated using the water filling algorithm across the subcarriers. However, no fairness among the users was considered in [51]. Thus, the users that have the best channel conditions will be assigned all the resources, which leaves many users without a chance to use the spectrum at all. The same authors extended the problem formulation to consider ergodic rates in [52], i.e. the expected value of the sum rate is maximized, which utilizes the temporal dimension when ergodicity of the channel gains is assumed to improve the data rate performance. However, [52] likewise suffers from the unfairness problem.

The fairness issue was partially addressed in [15]. The authors studied the *max-min* problem, where the minimum user's data rate is maximized using subcarrier time-sharing methods as in [14]. However, the *max-min* optimization problem can only provide maximum fairness among the users. In most wireless systems of interest, different users require different data rates, which may be accommodated by allowing users to subscribe to different levels of service. A very useful method for service differentiation was proposed in [53] by maximizing the sum data rate under a *proportional rate constraint* which is the method that balances the trade-off between sum rate and fairness.

Fairness issues in a wireline multiaccess channel were taken into account in [54] and [55]. The authors introduce the concept of balanced capacity to characterize the multiuser channel performance with total power constraints in [54] and they extend the concept to individual power constraints in [55]. This concept of balanced capacity is closely related to the proportional fair allocation presented in [53].

Utility functions, used in economics to quantify the benefit of usage of certain resources, have been also used in wireline and wireless networks

to evaluate the degree to which a network satisfies service requirements of users' applications, rather than in terms of system-centric quantities like sum-rate, outage probability, power, etc. [56]. The basic idea of utility-pricing structures is to map the resource use (bandwidth, power, etc.) or performance criteria (data rate, delay, etc.) into the corresponding utility or price values and optimize the established utility-pricing system. In wireline networks, utility and pricing mechanisms have been used for flow control [57]. In wireless networks, utility theory has been investigated in [58] for a cross-layer approach of resource allocation. It was shown that trade-offs between efficiency and fairness can be realized by maximizing a concave utility function of the user's data rate, instead of maximizing the data rates themselves.

Several other methods that use various heuristic algorithms for fair resource allocation have also been proposed, e.g. a multiuser bargaining algorithm for generalized proportional fairness based on Nash bargaining solutions and coalitions was developed in [59].

We also note that considerable related work on resource allocation has been done for multicarrier DSL systems, e.g. [60], [61], [62] and [63].

3.4 Availability of Channel State Information

The resource allocation schemes that we have seen in Sections 3.2 and 3.3 perform the allocation based on instantaneous channel conditions and work with the assumption of perfect CSI at the transmitter.

In many practical systems, the assumption of perfect CSI at the transmitter is not realistic. First, it may be very costly in terms of resources to perform an error-free channel estimation. Second, the signaling overhead due to the feedback of CSI from the receiver to the transmitter will be significant. In addition, the requirement of perfect CSI is only suitable for static or very slow time-varying channels such as in wireline systems. Since mobile radio channels vary quickly in time, it may not be possible for the transmitter to keep the channel state information up to date.

When the perfect CSI is not available at the transmitter, the use of partial CSI will still lead to better performance compared to fixed allocation (that does not take into account channel state information at all) [64]. The effect of imperfect channel knowledge on the performance of OFDM systems was studied in [65].

When the channel state can be accurately tracked at the receiver, the statistical channel model at the transmitter can be based on channel distribution information feedback from the receiver. The knowledge of the channel distribution at the transmitter is referred to as CDIT. An *opportunistic cumulative distribution function (CDF)*-scheduling based subcarrier and power allocation scheme was proposed in [66]

Wireless systems such as WiMAX or 3GPP-LTE use OFDMA for improved performances in multipath environments. One of the future aspects of OFDMA is the subchannelization which allows to group a total number of subcarriers into subsets of subcarriers called subchannels [67]. Standards such as IEEE 802.16e mobile WiMAX enable downlink subchannelization. Thanks to the subchannelization, the need for knowledge of radio channel quality is reduced from per-subcarrier to per-subchannel resolution and resources are allocated on per-subchannel basis. Recently, [68] studied resource allocation strategies for IEEE 802.16e mobile WiMAX which the goals to maximize goodput and minimize power consumption while ensuring feasible algorithm complexity and system scalability.

3.5 Resource Allocation in Cooperative Communication

All of the aforementioned works mainly dealt with a single hop transmission where each node attempts to communicate its message directly to the destination node. While such schemes can be implemented in a straightforward manner, alternative schemes in which nodes are allowed to cooperate have the potential to improve the QoS that is offered to the transmitting nodes by enlarging the achievable rate region and by reducing the probability of outage; e.g., [69] and [70]. The basic principle of cooperative communication is for the nodes to mutually relay their messages to the destination node, and hence power and other communication resources, such as time-frequency dimensions, must be allocated to the direct transmission and to the relay link. The realization of the potential improvement in quality of service provided by cooperation is contingent on this allocation (among other things).

In [71], centralized resource allocation algorithm for power control, bandwidth allocation, relay selection and relay strategy choice in an OFDMA based relay network has been designed. In [72], auction-based resource allocation mechanisms that determine relay selection and relay power allocation in a distributed fashion for multiple relays networks have been proposed.

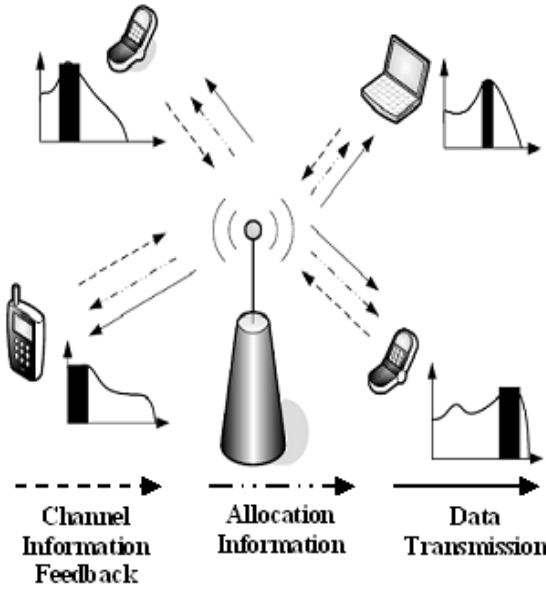


Figure 3.1: Downlink multiuser resource allocation

In [73], dynamic resource allocation in wireless relay channel where communication is constrained by delay and average power limitations has been considered. The authors analyze the system performance in terms of minimum outage probability under partial CSIT assumption. In [74], power allocation for ergodic capacity maximization in relay networks based on CDIT under high SNR regime has been studied.

3.6 The Problem Addressed in this thesis

This thesis investigate the constrained resource allocation problem in multiuser multicarrier systems. We focus on the downlink in which the centralized base is transmitting to mobile users with different channel conditions as illustrated in Fig. 3.1.

Downlink multiuser wireless networks are constrained by:

- the limited resource (bandwidth, power, etc.) available at the base station,
- individual users' quality of service requirements (minimum rate, fair-

ness, etc.)

- the availability of the channel information at the base station (partial CSIT, only CDIT, only subchannel average SNR, etc.)

We observe that in most of aforementioned works in resource allocation for multiuser multicarrier systems, the formulation and algorithms only consider one or two of the three abovementioned categories of constraints. Thus all main practical requirements are not being taken in to account when the resource allocation is performed. We formulate optimization problems considering all three categories of constraints in order to provide more realistic resource allocation schemes.

If the base station had perfect CSI of all the users, this knowledge could be used in solving the resource allocation problem. Independent to the work in [53], we first address the proportional fair resource allocation problem for OFDM-CDM systems under perfect CSIT with the objective to maximize the instantaneous sum rate while maintaining short-term proportional fairness (PF) among the users' data rates. By PF resource allocation, we mean the resource allocation algorithm should avoid starvation of any particular user by satisfying user rate proportionality constraints. Under short-term PF constraints, the user data rate requests have to be satisfied at every time-slot (or channel realization). The main goals is to compare the performances of proposed power allocation algorithms in terms of short-term PF and to analyze the trade-off between achievable sum-rate and the PF requirements.

Since PF is a soft QoS metric and its strict enforcement over each time-slot is often not necessary, the short-term PF resource allocation may be too stringent for some applications. Moreover, in some fading environment, perfect CSI is not available and the resource allocation needs to be performed based on knowledge of the statistics of the channel conditions. Previous research efforts have assumed that algorithms to find optimal or near-optimal solution to the constrained resource allocation with statistical channel information are too computationally complex for real-time implementation and uniform allocation was proposed. However, individual users' quality of service requirements may not be satisfied under uniform allocation. To fill this gap, we address the problem of constrained resource allocation based on statistical channel information such as combined partial CSIT or CDIT. Our approach is based on a Lagrangian dual decomposition. Using this dual optimization framework and some special mathematical functions, we solve the problem of maximizing the ergodic weighted sum rate of

multiuser ODFMA systems under power, minimum rate and long-term PF constraints. The optimal solution is obtained with a linear complexity.

We also extend the dual optimization approach to multiuser resource allocation in power/rate constrained cooperative networks with limited feedback and we provide analytical methods to find the optimal allocation.

Chapter 4

Constrained Resource Allocation for OFDM-CDM Systems with Perfect CSIT

4.1 Introduction

Orthogonal frequency division multiplexing with code division multiplexing (OFDM-CDM) is one of the MC-CDMA versions proposed by S. Kaiser [30] for the Downlink transmission. OFDM-CDM is a multiplexing scheme which can better exploit diversity than conventional OFDM systems. In an OFDM-CDM system, each data symbol is spread over several sub-carriers and/or several OFDM symbols, exploiting additional frequency- and/or time diversity.

By using orthogonal spreading codes, self-interference between data symbols can be minimized. Nevertheless, self-interference occurs in fading channels due to a loss of orthogonality between the spreading codes. To reduce this degradation, an efficient data detection and decoding technique is required. Data detection techniques can be classified as either single-user detection (SUD) or multi-user detection (MUD). The principle of SUD is to detect the user signal of interest by not taking into account any information about the multiple access interference. In the MUD, the a priori knowledge about the spreading codes of the interfering users is exploited in the detection process. The methods of MUD can be divided into interference cancellation (IC) and joint detection [75]. The optimal detector applies joint detection with maximum likelihood criterion. Since the complexity of maximum likelihood

detection grows exponentially with the number of users, its use is limited in practice to applications with a small number of users. Simpler joint detection techniques can be realized by using block linear equalizers based on zero-forcing (ZF) or minimum mean squared error (MMSE) criterion.

Recent studies on OFDM-CDM systems have focused on detection techniques [30], [75], and [76]. However, in OFDM-CDM systems, users can have different data rates varying from low-rate speech to high-rate video transmission. Besides that, users have channels of different quality. Therefore the achievement of fairness among the users is an important issue. On the other hand, as users of wireless communication networks become less satisfied with best effort transmission, Quality of Service (QoS) has also become an important research and commercial issue. Therefore, efficient radio resource allocation combined with joint detection techniques are crucial to improve the performance of OFDM-CDM systems in terms of achievable throughput, QoS and/or fairness. In section 4.2, we make use of ZF detector in order to formulate a more tractable constrained resource allocation problem for OFDM-CDM systems. We apply Newton method and bisection method to derive an optimal allocation. We evaluate the performance of presented methods in terms of fairness.

In OFDM-CDM systems, a multiple input multiple output (MIMO) architecture can be used to further improve performance of the systems. The use of MIMO introduces additional degrees of freedom which can be exploited to increase the bit rate or the robustness of the system, or a combination thereof [77], [78]. However, in multipath channels, when several users are active, MIMO OFDM-CDM systems may suffer from self-interference caused by the loss of orthogonality between the spreading codes, as well as from interference arising from the transmission of multiple signals. Therefore the challenges for MIMO-OFDM-CDM systems come from both the interference management techniques and the resource allocation methods. Zero-forcing beamforming (ZF-BF) has been presented as a practical scheme to deal with the interference in MIMO-OFDM [79]. In section 4.3, transmit ZF-BF is deployed at the base station, allowing a more tractable downlink constrained resource allocation formulation for MIMO-OFDM-CDM systems and geometric programming is applied to find the optimal allocation. We also analyze the trade-off between achieved performance and the constraints.

For the purpose of simplicity, we assume the availability of perfect channel state information at the transmitter (perfect CSIT) because the main goals are to compare the performances of proposed resource allocation algorithms

in terms of fairness and to analyze the trade-off between achieved sum-rate and the fairness and QoS requirements.

4.2 Constrained Resource Allocation for OFDM-CDM Using ZF detection

4.2.1 System Model

We consider a Downlink OFDM-CDM system with the block diagram depicted in Fig. 4.1. A transmit data $\mathbf{d}$ is demultiplexed into K users data symbols. Every user data symbol $d^{(k)}$ is spread over its specific spreading code $\mathbf{c}^{(k)} = [c_1^{(k)}, c_2^{(k)}, \dots, c_L^{(k)}]^T$ of length L . The spreading codes are used to multiplex multiple users symbols. This spreading procedure provides additional frequency diversity that can be exploited to improve the performance of the system.

However, as mentioned above, the drawback of OFDM-CDM is that self-interference (SI) might occur among the multiplexed transmitted symbols either due to non-orthogonal spreading codes or due to a frequency selective fading channel which destroys the orthogonality of the spreading codes. To mitigate the effect of self-interference between spreading codes, orthogonal Walsh-Hadamard (WH) spreading codes are used. Then, the overall transmit signal of the K users can be expressed as

$$\mathbf{s} = \mathbf{c}^{(k)} \sum_{k=1}^K d^{(k)} = \mathbf{c}\mathbf{d}, \quad (4.1)$$

where $\mathbf{c} = [\mathbf{c}^{(1)}, \mathbf{c}^{(2)}, \dots, \mathbf{c}^{(k)}, \dots, \mathbf{c}^{(K)}]$ is an $(L \times K)$ -dimensional matrix representing the spreading code of K users and $\mathbf{d} = [d^{(1)}, d^{(2)}, \dots, d^{(k)}, \dots, d^{(K)}]^T$ denotes the transmit data symbol vector.

The sequence $\mathbf{s}$ is transmitted by an OFDM module that comprises the inverse discrete Fourier transform (IDFT) and insertion of a cyclic prefix (CP). This cyclic prefix, longer than the maximum channel delay spread, is inserted between OFDM symbols in order to eliminate inter-symbol interference (ISI). The resulting OFDM-CDM signal propagates through a frequency selective fading channel. At the receiver, the received signal is processed by the inverse OFDM module that is composed of serial to parallel conversion, CP removal and discrete Fourier transform (DFT). The received signal is given by

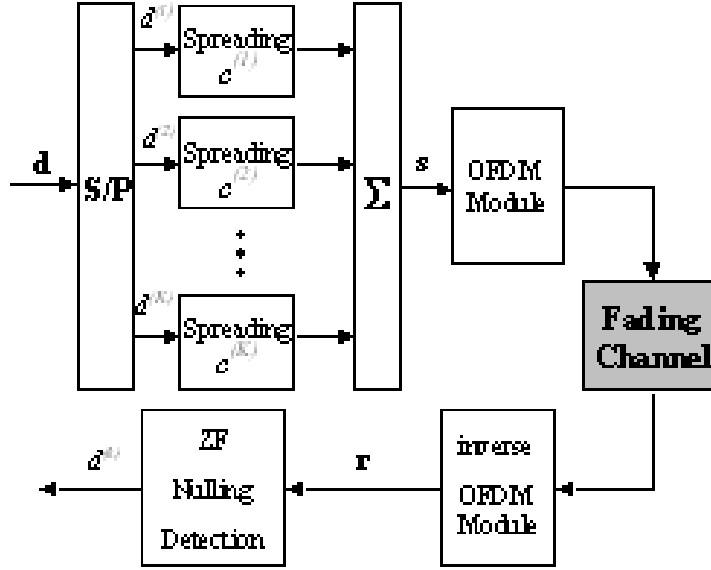


Figure 4.1: Block diagram of a downlink OFDM-CDM system

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{H}\mathbf{c}\mathbf{d} + \mathbf{n}, \quad (4.2)$$

where $\mathbf{H}$ is an $(L \times L)$ -dimensional diagonal matrix representing the complex fading channel and $\mathbf{n} = [n_1, n_2, \dots, n_L]^T$ is an L -dimensional vector representing the additive white Gaussian noise (AWGN).

Let $\mathbf{H}_{sp} = \mathbf{H}\mathbf{c}$. The received signal in (4.2) can be rewritten as

$$\mathbf{r} = \mathbf{H}_{sp}^{(k)} d^{(k)} + \sum_{j=1, j \neq k}^K \mathbf{H}_{sp}^{(j)} d^{(j)} + \mathbf{n}, \quad (4.3)$$

where $\mathbf{H}_{sp}^{(k)}$ is the k th column of $\mathbf{H}_{sp}$.

Equation (4.3) shows that the received signal $\mathbf{r}$ is composed of the desired signal for a given user k , the interferences from other users and the noise. To detect the symbol $d^{(k)}$ of user k , we use a low-complex ZF detector in order to null out all the interferences. The nulling vector $\mathbf{w}^{(k)}$ has to satisfy the following ZF condition

$$\begin{cases} \mathbf{w}^{(k)} \mathbf{H}_{sp}^{(k)} = 1 \\ \mathbf{w}^{(k)} \mathbf{H}_{sp}^{(j)} = 0, \forall j \in [1, K] \ \& \ j \neq k. \end{cases} \quad (4.4)$$

The vector $\mathbf{w}^{(k)}$ satisfying the ZF condition in (4.4) can be expressed as

$$\mathbf{w}^{(k)} = D^T (\mathbf{H}_{sp}^H \mathbf{H}_{sp})^{-1} \mathbf{H}_{sp}^H, \quad (4.5)$$

where $D = [0, 0, \dots, 1, \dots, 0]^T$ (all zeros excepted at the k th position).

The signal at the output of the ZF detector is deduced as

$$\hat{d}^{(k)} = \mathbf{w}^{(k)} \mathbf{r} = D^T (\mathbf{H}_{sp}^H \mathbf{H}_{sp})^{-1} \mathbf{H}_{sp}^H \mathbf{H}_{sp} \mathbf{d} + \mathbf{w}^{(k)} \mathbf{n}, \quad (4.6)$$

what is equivalent to

$$\hat{d}^{(k)} = d^{(k)} + \mathbf{w}^{(k)} \mathbf{n}. \quad (4.7)$$

We can observe from (4.7) that the data symbol $d^{(k)}$ of user k can be detected without interferences. We will see in the next section that the use of ZF detector leads to a more tractable formulation of resource allocation for OFDM-CDM.

The resulting noise power is given by

$$\tilde{N}_k = \mathbf{w}^{(k)} E [\mathbf{n} \mathbf{n}^H] (\mathbf{w}^{(k)})^H = N_0 \|\mathbf{w}^{(k)}\|^2, \quad (4.8)$$

where N_0 is the noise power of the AWGN.

The effective overall channel gain to noise ratio of user k , defined as the signal-to-noise power ratio, SNR of user k for unit transmit power, is deduced as

$$\gamma_k = \frac{1}{\Gamma N_0 \|\mathbf{w}^{(k)}\|^2}, \quad (4.9)$$

where Γ is the SNR gap related to the required bit error rate (BER).

Assuming that the data symbols of user k is allocated a transmit power p_k , the user k 's data rate normalized by the total bandwidth is given by

$$R_k = \log_2 (1 + p_k \gamma_k). \quad (4.10)$$

4.2.2 Problem Formulation

Considering a downlink transmission, the issue is how to allocate the available total power P_{tot} to the K users in order to maximize the sum of user data rates while satisfying proportional fairness among user data rates under total power constraint. Mathematically, this constrained maximization problem can be formulated as

$$\begin{aligned}
& \max_{p_k} \quad \sum_{k=1}^K \log_2 (1 + p_k \gamma_k) \\
& \text{subject to:} \\
& R_1 : R_k = \alpha_1 : \alpha_k; \forall k \in [2, \dots, K] \\
& \sum_{k=1}^K p_k \leq P_{tot},
\end{aligned} \tag{4.11}$$

where α_k are the set of predetermined values, which are used to ensure proportional fairness among the users.

4.2.3 Proposed Solution

The maximization problem (4.11) can be transformed into the following minimization problem

$$\begin{aligned}
& \min_{p_k} \quad - \sum_{k=1}^K \log_2 (1 + p_k \gamma_k) \\
& \text{subject to:} \\
& R_1 : R_k = \alpha_1 : \alpha_k; \forall k \in [2, \dots, K] \\
& \sum_{k=1}^K p_k \leq P_{tot}.
\end{aligned} \tag{4.12}$$

The minimization problem (4.12) belongs to the class of constrained optimization problems, where the objective function is to be minimized subject to a linear inequality constraint (total transmit power constraint) and non-linear equalities constraints (proportional fairness constraints). As sum of convex functions, the objective function is convex [34]. Thus we can solve the problem using Lagrange dual approach. The Lagrangian of the primal problem (4.12) is given by

$$L(p_k, \lambda_k) = \sum_{k=1}^K \log_2(1 + p_k \gamma_k) + \lambda_1 \left(\sum_{k=1}^K p_k - P_{tot} \right) + \sum_{k=2}^K \lambda_k \left(\log_2(1 + p_1 \gamma_1) - \frac{\alpha_1}{\alpha_k} \log_2(1 + p_k \gamma_k) \right), \quad (4.13)$$

where λ_1 is the Lagrangian multiplier associated with the power constraint and $\lambda_k, k = 2, \dots, K$ are the Lagrangian multipliers associated with the proportional fairness constraints.

By applying the gradient-based optimality conditions [34], we derive (4.13) over λ_1 and λ_k and set the derivatives to zero to obtain

$$\begin{cases} \sum_{k=1}^K p_k - P_{tot} = 0 \\ \log_2(1 + p_1 \gamma_1) - \frac{\alpha_1}{\alpha_k} \log_2(1 + p_k \gamma_k) = 0. \end{cases} \quad (4.14)$$

We also derive (4.13) over p_1 and p_k and set the derivative to zero to obtain the following KKT conditions

$$\begin{cases} \frac{\gamma_1}{1 + p_1 \gamma_1} + \lambda_1 \ln 2 + \left(\sum_{k=2}^K \lambda_k \right) \cdot \frac{\gamma_1}{1 + p_1 \gamma_1} = 0 \\ \frac{\gamma_k}{1 + p_k \gamma_k} + \lambda_1 \ln 2 + \lambda_k \frac{\alpha_1}{\alpha_k} \cdot \frac{\gamma_k}{1 + p_k \gamma_k} = 0, \forall k \in [2, \dots, K]. \end{cases} \quad (4.15)$$

We can interpret the underlying constrained optimization problem as finding a solution of the above reduced KKT equations composed of primal feasibility equations (4.14) and dual feasibility equations (4.15).

In equations (4.15), λ_1 acts as slack variable and can be eliminated. Then, after some mathematical manipulations, λ_2 and $\lambda_k, k = 3, \dots, K$ are deduced as

$$\lambda_2 = \frac{\frac{1}{p_2} - \frac{1}{p_1}}{\frac{\alpha_1}{\alpha_2} \cdot \frac{1}{p_2} + \frac{1}{p_1}}; \quad \lambda_k = \frac{\frac{1}{p_k} - \left(1 + \sum_{i=2}^{k-1} \lambda_i\right) \frac{1}{p_1}}{\frac{\alpha_1}{\alpha_k} \cdot \frac{1}{p_k} + \frac{1}{p_1}}. \quad (4.16)$$

The power constraint is such that the Slater's condition [34] is satisfied. Therefore, equations (4.14)-(4.16) show that there is a set $\{\widetilde{\lambda}_k\}_{k=1}^K$ that together with $\{\widetilde{p}_k\}_{k=1}^K$ satisfy the reduced KKT conditions, where $\{\widetilde{p}_k\}_{k=1}^K$

is the set of powers to allocated to users in order to maximize the sum rate while maintaining proportional fairness among the users data rates. We propose two methods to find the $\{\tilde{p}_k\}_{k=1}^K$.

4.2.3.1 Newton-Raphson Method

Considering equations (4.14)-(4.16), the *Newton-Raphson* method [35], which is a variant of the Newton method, can be deployed to find the optimal powers $\{\tilde{p}_k\}_{k=1}^K$ together with the Lagrangian multipliers $\{\tilde{\lambda}_k\}_{k=1}^K$.

We define a square system of equations

$$g(\mathbf{p}) = \mathbf{0}, \quad (4.17)$$

where

$$\begin{aligned} g_1(\mathbf{p}) &= \sum_{k=1}^K p_k - P_{tot} = 0 \\ g_k(\mathbf{p}) &= \log_2(1 + p_1\gamma_1) - \frac{\alpha_1}{\alpha_k} \log_2(1 + p_k\gamma_k) = 0, \quad \forall k \in [2, K], \end{aligned} \quad (4.18)$$

where $\mathbf{p} = [p_1, p_2, \dots, p_K]^T$.

We also define the Hessian matrix of $g(\mathbf{p})$ as

$$\mathcal{H}(\mathbf{p}, \lambda) = \begin{bmatrix} \mathcal{W}(\mathbf{p}, \lambda) & (\mathcal{J}(\mathbf{p}))^T \\ \mathcal{J}(\mathbf{p}) & \mathbf{0} \end{bmatrix} \quad (4.19)$$

where $\mathcal{J}(\mathbf{p})$ is the Jacobian matrix of $g(\mathbf{p})$, expressed as

$$\mathcal{J}(\mathbf{p}) = \begin{bmatrix} 1 & 1 & \cdot & \cdot & 1 \\ \frac{\gamma_1}{(1+p_1\gamma_1)\ln 2} & \frac{-\alpha_1\gamma_2}{\alpha_2(1+p_2\gamma_2)\ln 2} & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 \\ \frac{\gamma_1}{(1+p_1\gamma_1)\ln 2} & 0 & \cdot & 0 & \frac{-\alpha_1\gamma_K}{\alpha_K(1+p_K\gamma_K)\ln 2} \end{bmatrix} \quad (4.20)$$

and $\mathcal{W}(\mathbf{p}, \lambda) = \frac{\partial^2 L}{\partial \mathbf{p}^2}$ is an $(K \times K)$ -diagonal matrix with the diagonal elements given by

$$\mathcal{W}_{kk} = \frac{1}{\ln 2} \cdot \left(1 - \sum_{j=2}^K \lambda_j \right) \frac{\gamma_k^2}{(1 + p_k \gamma_k)^2}. \quad (4.21)$$

It is interesting to note that the Hessian matrix $\mathcal{H}$ and the Jacobian matrix $\mathcal{J}$ are sparse. This reduced the computational complexity.

The update directions $\Delta \mathbf{p}$ and $\Delta \lambda$ are given by

$$\begin{aligned} \Delta \mathbf{p} &= \mathcal{H}^{-1}(\mathbf{p}, \lambda) \left(-\frac{\partial L}{\partial \mathbf{p}} \right) \\ \Delta \lambda &= \mathcal{H}^{-1}(\mathbf{p}, \lambda) \left(-\frac{\partial L}{\partial \lambda} \right). \end{aligned} \quad (4.22)$$

The powers $\mathbf{p}$ and the multipliers λ are updated as

$$\begin{aligned} \mathbf{p}_{i+1} &= \mathbf{p}_i + \Delta \mathbf{p} \\ \lambda_{i+1} &= \lambda_i + \Delta \lambda. \end{aligned} \quad (4.23)$$

4.2.3.2 Bisection Method

The bisection method is a root-finding algorithm which repeatedly bisects an interval then selects a subinterval in which a root must lie for further processing. It is a very simple and robust method [80]. We can use this method to find the optimal power allocation.

The second expression in (4.14) can be transformed into

$$\log_2(1 + p_k \gamma_k) = \frac{\alpha_k}{\alpha_1} \log_2(1 + p_1 \gamma_1), \quad (4.24)$$

which is equivalent to

$$1 + p_k \gamma_k = (1 + p_1 \gamma_1)^{\frac{\alpha_k}{\alpha_1}} \quad (4.25)$$

From equation (4.25), we deduce that the power p_k is related to p_1 as

$$p_k = \sigma_k \left(\frac{1}{\gamma_1} + p_1 \right)^{\delta_k} - \frac{1}{\gamma_k}, \quad (4.26)$$

where $\sigma_k = \frac{\gamma_1^{\alpha_k/\alpha_1}}{\gamma_k}$ and $\delta_k = \frac{\alpha_k}{\alpha_1}$.

Replacing (4.26) in the power constraint, we obtain

$$\sum_{k=1}^K \left(\sigma_k \left(\frac{1}{\gamma_1} + p_1 \right)^{\delta_k} - \frac{1}{\gamma_k} \right) - P_{tot} = 0. \quad (4.27)$$

We use the bisection method to solve equation (4.27) for p_1 and we deduce the powers p_k ($k = 2, \dots, K$) from equation (4.26).

4.2.4 Fairness Analysis

As mentioned in Chapter 3, the fairness for all the users awaiting transmission is an important design criterion. The proportional fairness strikes a balance between maximizing the total system throughput and maintaining fairness among users [57].

To evaluate the performance of the proposed methods in term of fairness and overall system satisfaction, we define the user's fairness index f_k and the overall system degree of satisfaction DoS as

$$f_k = 1 - \frac{\left| R_1 - \frac{\alpha_1}{\alpha_k} R_k \right|}{R_1}, \quad (4.28)$$

$$DoS = 1 - \sqrt{\frac{\sum_{k=1}^K (1 - f_k)^2}{K}}. \quad (4.29)$$

The strictly proportional fairness is achieved for $f_k = 1$. This leads to the maximum system degree of satisfaction ($DoS = 1$).

4.2.5 Simulation Results and Discussion

We consider a single cell OFDM-CDM systems consisting of one base station and K user terminals. The maximum number of users considered is 16. The channel bandwidth is $B = 1$ MHz and the number of subcarriers for OFDM modulation is $N = 256$. The wireless channel is modeled as a frequency selective fading channel consisting of six independent Rayleigh fading path, with an exponentially decaying profile. Perfect frequency interleaving is assumed. Moreover, perfect channel state information is assumed to be available at the base station. Walsh Hadamard codes of length $L = 16$ are used for spreading. The PSD of AWGN is $N_0 = -82$ dBW/Hz. The total transmit power is $P = 1$ W. For every number of users K power of 2, 100 simulations have been run in order to have a better statistical evaluation. For the Newton's method, all the starting points are set to P_{tot}/K .

In Fig. 4.2, the overall system performances of the proposed Newton-Raphson method (marked QN) and Bisection method (marked BI) are compared with the one of equal power allocation (marked EP), which simply distributes the transmission power equally among the users. This figure depicts the overall degree of satisfaction in term of fairness DoS versus the number of users K for high SNRs. The figure shows that the EP method achieves a very poor overall degree of satisfaction. The system degrees of satisfaction obtained with the QN method and the BI method is equal to 100% for all values of number of users.

In Fig. 4.3, the performances of the three methods are compared in term of user's fairness. The fairness index for every user is depicted for the predetermined proportional fairness constants set

$$\{\alpha_k\}_{k=1}^K = \{1, 2, 1, 4, 2, 1, 4, 2, 2, 2, 4, 1, 1, 1, 2, 4\}.$$

The fairness index achieved by the QN method and the BI method is 1 for every user. This shows that the QN method and the BI method achieve the optimal proportional fairness among the users. In contrast the fairness index obtained with the equal power allocation fails down to 0.24 for users with the high rate request.

The optimal power allocation, i.e., the power allocation at which the PF constraint and the total power constraint (with equality) are satisfied, is obtained at the 18er iteration with the QN algorithm and at the 10er iteration with the BI algorithm respectively. This shows that for the power allocation formulation (4.11), the bisection method is computational less complex than the Newton's method. Another drawback of the Newton's method is that it may be more slower when started far from the optimum.

4.3 Constrained Resource Allocation for MIMO-OFDM-CDM Using ZFBF

4.3.1 System Model

Fig. 4.4 shows the proposed downlink MIMO-OFDM-CDM transmitter with M transmit antennas and the receiver of the k th user. For complexity reasons, we consider mobile terminal (MT) equipped with a single antenna. At the base station (BS), each data symbol multiplies a beamforming vector $\mathbf{w}_k$ of size M . Each one of the M elements is further spread with an orthogonal Walsh-Hadamard code of length L . We denote the code vector

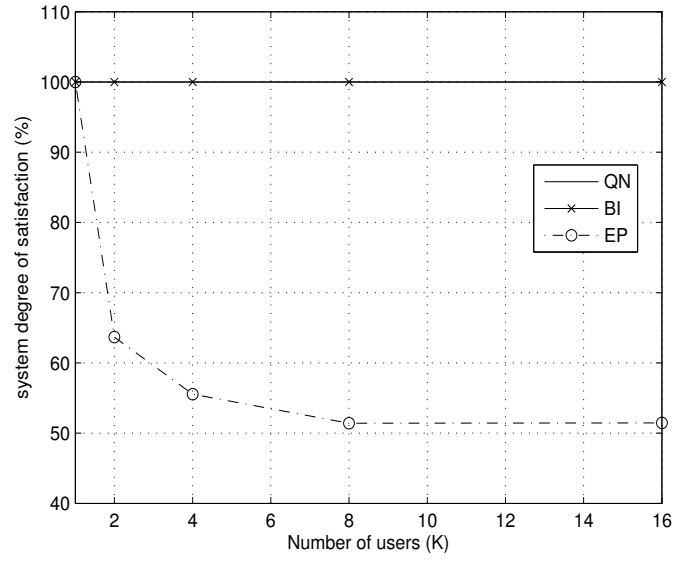


Figure 4.2: Overall system degree of satisfaction versus number of user K .

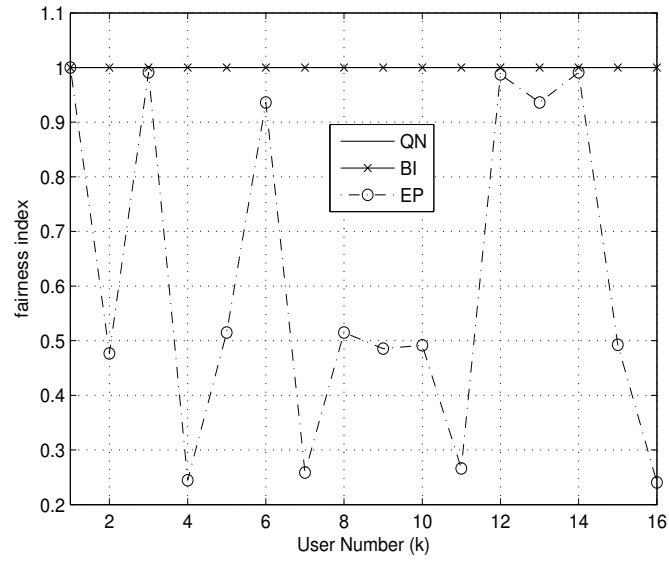


Figure 4.3: User's fairness index.

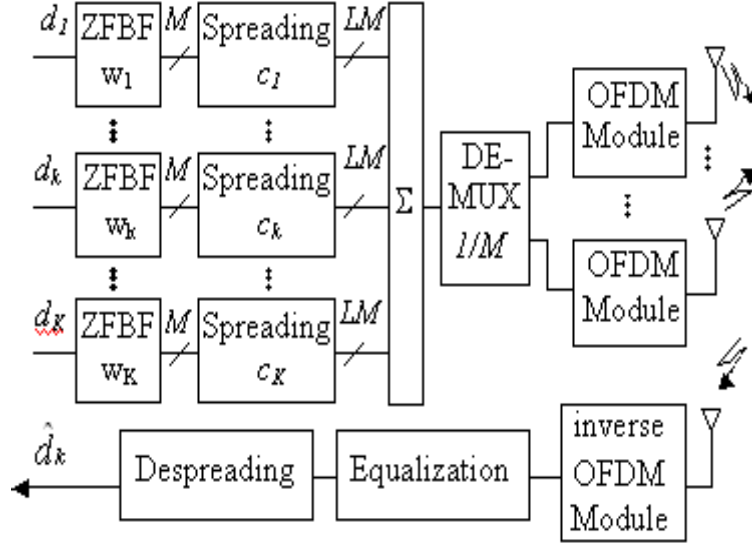


Figure 4.4: Block diagram of a downlink OFDM-CDM system

of user k by $\mathbf{c}_k = [c_{k,1}, \dots, c_{k,l}, \dots, c_{k,L}]^T$, where $c_{k,l}$ is the l^{th} chip. The LM chips of all MTs are summed and demultiplexed, in order to be transmitted on the appropriate antenna. On each antenna branch, OFDM modulation is implemented and a cyclic prefix (CP) is inserted to absorb the multipath spread of the channel.

Thanks to the CP and assuming that the coherence time of the channel is much longer than the duration of the OFDM symbol, the propagation channel for MT k can be represented by a flat fading coefficient on each sub-channel. Then LM -dimensional vector $\mathbf{h}_k$ gathers the fading channel gains for the k th user.

From the signal received by a single antenna at MT k , the CP is removed and OFDM demodulation is carried out. Since the channel is frequency selective, the orthogonality among users signals may be corrupted. We assume that the equalization is performed by a single user detector [81]. The L -dimensional equalization vector is denoted $\mathbf{g}_k$. The power allocated by the BS to the MT k is denoted p_k . After despreading with the appropriate sequence $\mathbf{c}_k$, the estimate of transmitted symbol $\hat{d}_k$ is given by:

$$\begin{aligned}
\hat{d}_k &= \tilde{\mathbf{c}}_k^T (\tilde{\mathbf{g}}_k \circ \mathbf{h}_k \circ \tilde{\mathbf{w}}_k \circ \tilde{\mathbf{c}}_k) \sqrt{p_k} \cdot d_k \\
&+ \tilde{\mathbf{c}}_k^T \left(\tilde{\mathbf{g}}_k \circ \mathbf{h}_k \circ \sum_{j=1, j \neq k}^K (\tilde{\mathbf{w}}_j \circ \tilde{\mathbf{c}}_j) \sqrt{p_j} \cdot d_j \right) \\
&+ \mathbf{c}_k^T (\mathbf{g}_k \circ \mathbf{n}_k),
\end{aligned} \tag{4.30}$$

where $\circ$ defines the vector multiplication element by element.

Expression (4.30) is composed of the desired signal for user k , the multiple access interference (MAI) and the noise ($\mathbf{n}_k$ is an L -dimensional AWGN vector). To represent the recombination of signals at the single receive antenna, an extended vector notation indicated by $\tilde{\cdot}$ is defined. For each MT k , $\tilde{\mathbf{c}}_k$ is an LM -dimensional spreading vector representing M times vector $\mathbf{c}_k$ (the same spreading is applied on all transmit antenna branches). Similarly $\tilde{\mathbf{g}}_k$ is an LM -dimensional equalization vector representing M times vector $\mathbf{g}_k$ (equalization vector applied to the signal coming from one transmit antenna) and $\tilde{\mathbf{w}}_k$ is also an LM -dimensional vector repeating L -times vector $\mathbf{w}_k$.

The interference that the signal of a user k produces at another MT j is given by

$$I_{k,j} = \tilde{\mathbf{c}}_j^T (\tilde{\mathbf{g}}_j \circ \mathbf{h}_j \circ \tilde{\mathbf{w}}_k \circ \tilde{\mathbf{c}}_k) \sqrt{p_k} = \mathbf{v}_{j,k}^T \tilde{\mathbf{w}}_k \sqrt{p_k}, \tag{4.31}$$

where $\mathbf{v}_{j,k} = \tilde{\mathbf{c}}_j \circ \tilde{\mathbf{g}}_j \circ \mathbf{h}_j \circ \tilde{\mathbf{c}}_k$.

Transmit zero-forcing beamforming (ZFBB) is used at the base station. The use of transmit ZFBB has two main advantages: reducing the MAI at MTs by pre-formatting the signal so that the received signal at the decision point is free from interferences and allowing moving most of the computational burden from MT to BS, keeping the MT at a low complexity level. Using ZFBB, the weight vector for user k is obtained by constraining the desired signal part of its own decision variable to a constant while canceling its MAI contribution at all other MTs at the same time. This leads to the following set of ZF conditions:

$$\begin{cases} \mathbf{v}_{k,k}^T \tilde{\mathbf{w}}_k \sqrt{p_k} = \sigma_k \\ \mathbf{v}_{j,k}^T \tilde{\mathbf{w}}_k \sqrt{p_k} = 0, \forall j \neq k. \end{cases} \tag{4.32}$$

Assuming that the pre-filtering weight vector always has unit norm, the effective signal-to-noise ratio of user k for unit transmit power can be ex-

pressed as

$$\gamma_k = \frac{\left| \mathbf{v}_{k,k}^T \right|^2}{\Gamma L \cdot E \left[(\mathbf{g}_k \circ \mathbf{n}_k)^H (\mathbf{g}_k \circ \mathbf{n}_k) \right]}, \quad (4.33)$$

where Γ is the SNR gap related to the required BER.

The user k 's data rate normalized by the total bandwidth is given by

$$R_k = \log_2 (1 + p_k \gamma_k). \quad (4.34)$$

4.3.2 Problem Formulation

The main goal here is to analyze the trade-off between the achieved performances and the constraints. Starting from the basic problem (4.11), we define a relaxation coefficient α_f ($\alpha_f \leq 1$) for the fairness constraint and we form a *perturbed* problem

$$\begin{aligned} & \max_{p_k} \sum_{k=1}^K \log_2 (1 + p_k \gamma_k) \\ & \text{subject to:} \\ & (1 - \alpha_f) \omega_k R_1 \leq R_k \leq (1 + \alpha_f) \omega_k R_1, \\ & \sum_{k=1}^K p_k \leq P_{tot}, \end{aligned} \quad (4.35)$$

where $\omega_k = \frac{\alpha_k}{\alpha_1}$ are the proportionality weights.

We also formulate a resource allocation problem that takes into account the individual QoS requirements instead of proportional fairness criterion. The minimum transmission rates are used as QoS metric. We define a QoS relaxation coefficient α_q ($\alpha_q \leq 1$) and we form a *perturbed* problem

$$\begin{aligned} & \max_{p_k} \sum_{k=1}^K \log_2 (1 + p_k \gamma_k) \\ & \text{subject to:} \\ & R_k \geq (1 - \alpha_q) R_{k,min} \\ & \sum_{k=1}^K p_k \leq P_{tot}, \end{aligned} \quad (4.36)$$

where $R_{k,min}$ is user k 's minimum rate requirement.

4.3.3 Proposed Solution

Since the main concern here is not the absolute achieved throughput, but the analysis of the trade-off between the maximized throughput and the constraints, we will focus on high SNR regime and we will use a geometric programming (GP) approach, presented in 2.5.2, to solve the perturbed problems (4.35) and (4.36). GP based algorithms efficiently and globally find the optimal solution if the problem is feasible. If the constraints are so that the problem is infeasible, GP based algorithms clearly return the non-feasibility of the problem.

For high SNR regime, problem (4.35) is equivalent to:

$$\begin{aligned}
 & \min_{p_k} \log_2 \prod_{k=1}^K (1/p_k \gamma_k) \\
 & \text{subject to:} \\
 & \log_2 (1/p_k \gamma_k) \leq -(1 - \alpha_f) \omega_k \log_2 (1/p_1 \gamma_1), \\
 & \log_2 (p_k \gamma_k) \leq (1 + \alpha_f) \omega_k R_1, \\
 & \sum_{k=1}^K p_k \leq P_{tot},
 \end{aligned} \tag{4.37}$$

which in turn is equivalent to:

$$\begin{aligned}
 & \min_{p_k} \frac{1}{\prod_{k=1}^K \gamma_k} \prod_{k=1}^K (p_k^{-1}) \\
 & \text{subject to:} \\
 & \left(\frac{\gamma_1^{-(1-\alpha_f)\omega_k}}{\gamma_k} \right) p_k^{-1} p_1^{-(1-\alpha_f)\omega_k} \leq 1, \\
 & \left(\gamma_k \gamma_1^{-(1+\alpha_f)\omega_k} \right) p_k p_1^{-(1+\alpha_f)\omega_k} \leq 1, \\
 & \frac{1}{P_{tot}} \sum_{k=1}^K p_k \leq 1.
 \end{aligned} \tag{4.38}$$

In the minimization problem (4.38), the objective function and the first two constraints are monomials in variables p_k while the third constraint is a posynomial in variables p_k . Thus, problem (4.38) is a geometric programming problem in its standard form and GP algorithm can be used to efficiently and globally find the optimal variables p_k .

For high SNR regime, problem (4.36) is equivalent to:

$$\begin{aligned}
 & \min_{p_k} \log_2 \prod_{k=1}^K (1/p_k \gamma_k) \\
 & \text{subject to:} \\
 & \log_2 (1/p_k \gamma_k) \leq -(1 - \alpha_q) R_{k,min}, \\
 & \sum_{k=1}^K p_k \leq P_{tot},
 \end{aligned} \tag{4.39}$$

which in turn is equivalent to:

$$\begin{aligned}
 & \min_{p_k} \frac{1}{\prod_{k=1}^K \gamma_k} \prod_{k=1}^K (p_k^{-1}) \\
 & \text{subject to:} \\
 & \frac{1}{\gamma_k e^{-(1-\alpha_q) R_{k,min}}} (p_k^{-1}) \leq 1, \\
 & \frac{1}{P_{tot}} \sum_{k=1}^K p_k \leq 1.
 \end{aligned} \tag{4.40}$$

In the minimization problem (4.40), the objective function and the first constraint are monomials in variables p_k while the second constraint is a posynomial in variables p_k . Thus, problem (4.40) is also a geometric programming problem in its standard form and GP solver [38] can be used to efficiently and globally find the optimal variables p_k .

4.3.4 Trade-off Analysis

In the trade-off analysis, we vary the constraints and see the effect on the optimal value. By increasing the coefficients α_f the fairness constraints are relaxed, so do the QoS constraints when α_q increases. The relaxation of the constraints leads in general to an improvement of the optimal objective. The trade-off curves are found by solving the perturbed problems (4.35) and (4.36) for many values of the relaxation coefficients α_f and α_q to be varied.

4.3.5 Simulation Results and Discussion

We consider a wireless network with one BS and eight MTs. The total channel bandwidth of the system is 1 MHz and the number of subcarriers

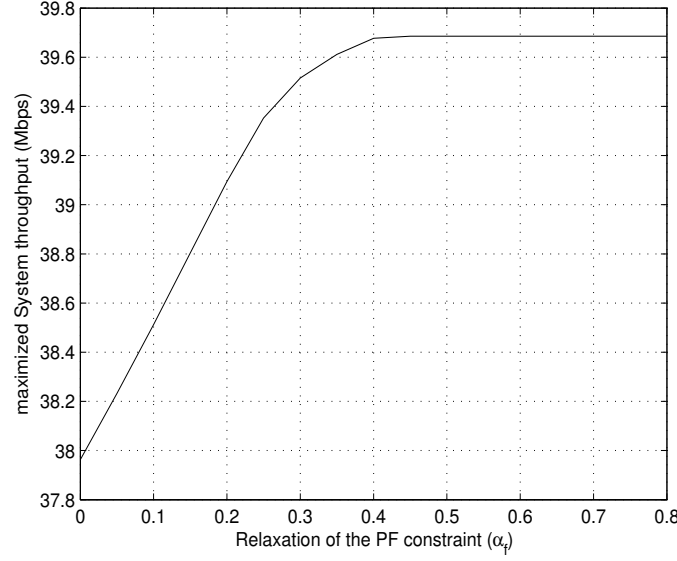


Figure 4.5: Throughput-fairness trade-off.

for OFDM modulation is 256. Walsh Hadamard codes of length 16 are used for spreading. The wireless channel from a transmit antenna to a user is modeled as a frequency selective fading channel consisting of six taps generated as i.i.d zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables. The noise PSD is equal to -80 dBW/Hz. The total power is 1W. The minimum data rate requirements are 2.2, 4.4, 8.8, 4.4, 2.2, 4.4, 8.8 and 4.4 Mbps for users 1 to 8 respectively. The QoS and fairness relaxation coefficients vary from 0 (no relaxation) to 0.8. The channel is assumed to be perfectly known at the transmitter.

Fig. 4.5 and Fig. 4.6 show the maximized system throughput for different relaxation coefficients of the fairness constraints and the QoS constraints respectively for BS equipped with only one transmit antenna. For one transmit antenna, as fairness/QoS constraints are relaxed, the maximized system throughput increases sharply first, then more slowly until the fairness/QoS constraints are no longer active.

Fig. 4.7 and Fig. 4.8 illustrate how the relaxation of the constraints is beneficial for the global system throughput in the case of multiple transmit antennas. These figures also show that for every fairness constraint and

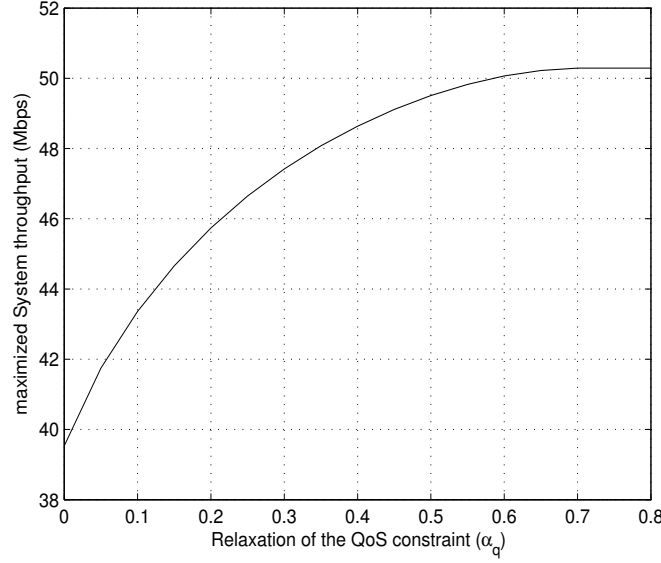


Figure 4.6: Troughput-QoS trade-off.

every QoS demand, the system throughput increases with the number of transmit antenna thanks to SNR increase provided by multiuser diversity, which translates into a higher number of bits in the constellations. With multiple transmit antennas, high throughputs can be achieved without sacrificing fairness nor QoS.

Looking at the solution obtained for different values of α_f and α_q , the system designer may then make a choice about the configuration he considers to be the most appropriate.

4.4 Conclusion

In this Chapter, constrained resource allocation schemes for OFDM-CDM and MIMO-OFDM-CDM systems under perfect CSIT assumption have been presented.

The optimal performance in terms of proportional fairness is achieved using the Newton-Raphson based algorithm. Newton-Raphson method can often converge remarkably quickly, especially if the iteration begins sufficiently

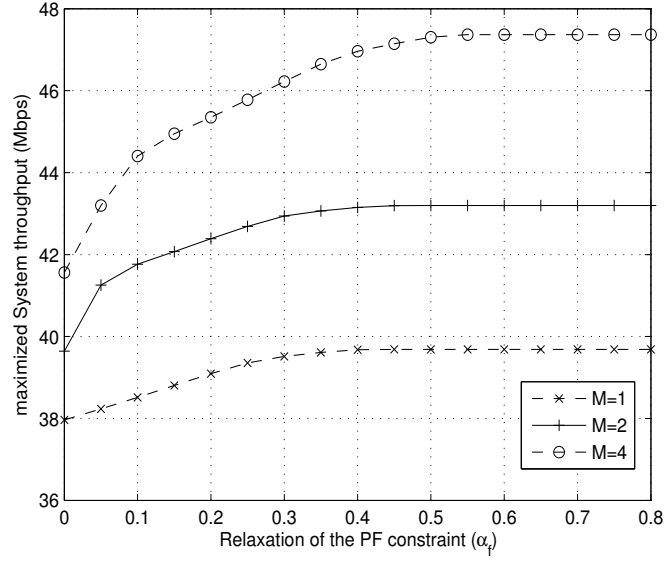


Figure 4.7: Throughput-fairness trade-off for multiple antennas.

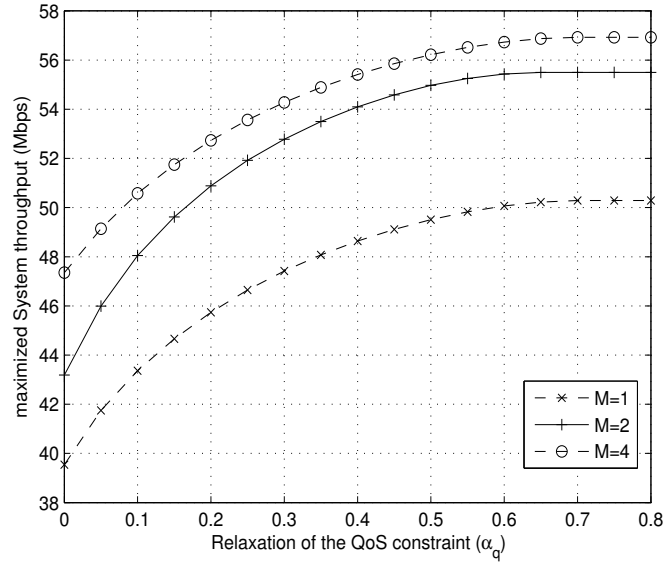


Figure 4.8: Throughput-QoS trade-off for multiple antennas.

near the desired root. Unfortunately, if the initial root is too far from the optimal root, Newton's method may fail to converge.

We have also developed a low complexity power allocation algorithm, based on the bisection method, which achieves the same performance as the Newton's method. The bisection method is simple and robust.

We have proposed the use of geometric programming framework to solve the constrained resource allocation problem in multiuser MIMO-OFDM-CDM systems. Geometric Programming based algorithms efficiently and globally find the optimal solution. We also analyze the trade-off between achieved throughput and the fairness and QoS constraints through GP formulation. Geometric programming turns out to be an efficient tool for solving the optimization problem with relaxed constraints.

Due to the perfect CSIT assumption, this work may be perceived to be idealistic but shall form the basis of our research and help us to model more practical problems in later chapters.

Chapter 5

Constrained Resource Allocation for OFDMA Systems with Statistical Channel Information

5.1 Introduction

The previous chapter studied constrained resource allocation strategies for OFDM-CDM systems with perfect CSIT. In this chapter, multiuser constrained resource allocation is explored in the context of OFDM based frequency division multiple access (OFDMA) systems with statistical channel information. In contrast to OFDM-CDM systems where multi-user interferences arise, an OFDMA system does not suffer from any multiuser interference when there are no synchronization errors.

An OFDMA system can be defined as one in which each user is assigned a subset of subcarriers (or subchannel) for use, and each subcarrier (or subchannel) is assigned exclusively to one user [82]. Because of its structure, OFDMA enables efficient and easy adaptation over time, frequency and power across users.

Recall that one of the future aspects of OFDMA is the subchannelization which allows to group a total number of subcarriers into subsets of subcarriers called subchannels [82]. The major advantage of subchannelization is the provision of frequency diversity. A byproduct of the subchannelization is that the need for knowledge of radio channel quality is reduced from

per-subcarrier to per-subchannel resolution and resources are allocated on per-subchannel basis.

Like in OFDM-CDM systems presented in Chapter 4, in OFDMA-based systems, users will be allowed to have different requirements simultaneously, for instance, real-time data, voice, or background downloading of data. On another hand, users have channels of different quality. With classical best effort transmission, unfair resource allocation leads to starvation of some users in bad channel conditions. Therefore the achievement of fairness among the users while satisfying users' minimum data rate requirements is an important issue.

Most of the previous works on OFDMA constrained resource allocation have considered only the case where instantaneous channel state (CSI) is available at the transmitter and various algorithms based on instantaneous CSI have been developed. Some of these algorithms have been reviewed in Chapter 3. However, as mentioned in section 3.4, in many practical systems, the assumption of perfect CSI at the transmitter is quite unrealistic due channel estimation errors and channel feedback delay. In this chapter, we focus on the case where only the statistical channel information is available at the transmitter.

Resource allocation strategies for single user OFDM under imperfect CSI has been quite well studied. It has been shown in [64], that the use of partial CSI will still lead to better performance for OFDM system compared to fixed allocation (that does not take into account channel state information at all). The effect of channel estimation error on the performance of adaptive OFDM has been analyzed in [65].

In the OFDMA system, the difficulty arises from the fact that the exclusive subcarrier assignment restriction makes the problem combinatorial in nature. Recently, a resource allocation algorithm using Lagrange dual decomposition has been developed for OFDMA systems assuming imperfect CSIT [83]. But the formulation in [83] considered only total power constraint; fairness and QoS requirements are not taken into account. Our formulation of partial CSIT based resource allocation considers power, fairness and QoS constraints, which is a more realistic formulation for multimedia applications.

First, we define an expected weighted sum rate of a multiuser OFDMA system that includes the expected data rate and the fairness requirement. We develop resource allocation strategies that use the statistics of the imperfect CSI to adaptively assign subchannels and distribute the total power

budget to users. The objective is to maximize the *expected* weighted sum rate under minimum data rate requirements and total power constraint. This constrained optimization problem is formulated as a non-convex finite dimensional stochastic problem. We use the Lagrange dual decomposition framework to efficiently find an optimal solution.

Instantaneous resource allocation strategies are suitable for quasi-static or slow fading environments. However, when the channel variations are fast, the transmitter may not be able to adapt to the instantaneous channel realization. Hence, CSI-aware resource allocation is not suitable for environments with high mobility. When the channel state can be accurately tracked at the receiver, the statistical channel model at the transmitter can be based on channel distribution information feedback from the receiver. We refer to knowledge of the channel distribution at the transmitter as CDIT.

We develop constrained resource allocations strategies for mobile OFDMA with only CDIT. We use CDIT to adaptively assign subchannels and distribute the total power to users with the objective to maximize the *ergodic* weighted-sum rate under *long-term* fairness, minimum data rate requirements, and a total power constraint. This constrained optimization problem is formulated as an infinite dimensional stochastic problem. We propose an analytical solution based on the Lagrange dual decomposition framework.

The proposed method has a complexity of $\mathcal{O}(KM)$ for K users and M subchannels. Simulation results are provided to compare the performance of this method with other allocation schemes and to illustrate the trade-off between maximized weighted-sum rate and the constraints.

This chapter is organized as follows. Section 5.2 describes the system model that is used. Section 5.3 discuss the constrained resource allocation strategies for the partial CSIT case. Section 5.4 discuss the constrained resource allocation strategies for the CDIT case. The conclusion of this chapter is presented in Section 5.5.

5.2 System Model

We consider a single cell downlink OFDMA communication from a base station (BS) to K mobile user terminals, over a realistic frequency-selective fading channel with total bandwidth B . The block diagram of the physical layer of the system is shown in Fig. 5.1. Functional blocks are according

to [12, 84] with the exception of subchannel and resource allocation block, which is our contribution to improve system performance.

OFDMA enables downlink subchannelization. In an OFDMA system with subchannelization options, the BS splits up the downlink bandwidth into M different subchannels. Then the data to be transmitted to K different mobile user terminals are modulated into a given constellation and amalgamated using Downlink subchannelization. After the Downlink subchannelization, the resulting frequency domain OFDMA symbols are converted into time domain OFDMA symbols using inverse FFT. Then a cyclic prefix is added to each symbol to provide immunity against multipath propagation. Finally the signal is filtered by a pulse shaping filter before it is transmitted from the base station to the user terminals.

Each user k is adaptively assigned a number of different subchannels to send and receive data. An indicator $\rho_{k,m}$ is used to represent whether the m th subchannel is assigned to user k . Note that in a single cell OFDMA system, each subchannel can be assigned to at most one user at any time, i.e., $\sum_{k=1}^K \rho_{k,m} \in \{0, 1\}, \forall m$. Due to the consideration for the reduction of the signaling overhead in OFDMA based systems such as WIMAX, the power is equally distributed across subcarriers within each subchannel.

5.3 Constrained Resource Allocation for OFDMA Systems with Partial CSIT

5.3.1 Partial CSIT Channel Model

Let $h_{k,m}(n)$ denote the actual channel gain for the n th subcarrier of the m th subchannel of user k , which is the product of the distance attenuation and the fading gain. Classically, the resource allocation policy for OFDMA systems can be obtained as a function of $h_{k,m}(n)$. In contrast, our resource allocation policy is based on the average estimated channel gain sum over all subcarriers constituting each subchannel. To simplify the analysis, we characterize the channel estimation error as additive Gaussian noise, independent of the channel itself, so that the imperfect CSI of user k on subcarrier n of subchannel m can be modeled as

$$h_{k,m}(n) = \hat{h}_{k,m}(n) + e_{k,m}(n), \quad (5.1)$$

where $\hat{h}_{k,m}(n)$ is the estimate of $h_{k,m}(n)$, $e_{k,m}(n)$ is the estimation error on the given subcarrier which is a zero-mean circulant symmetric complex Gaussian (ZMCSCG) random variable with variance $\sigma_{e,n}^2$.

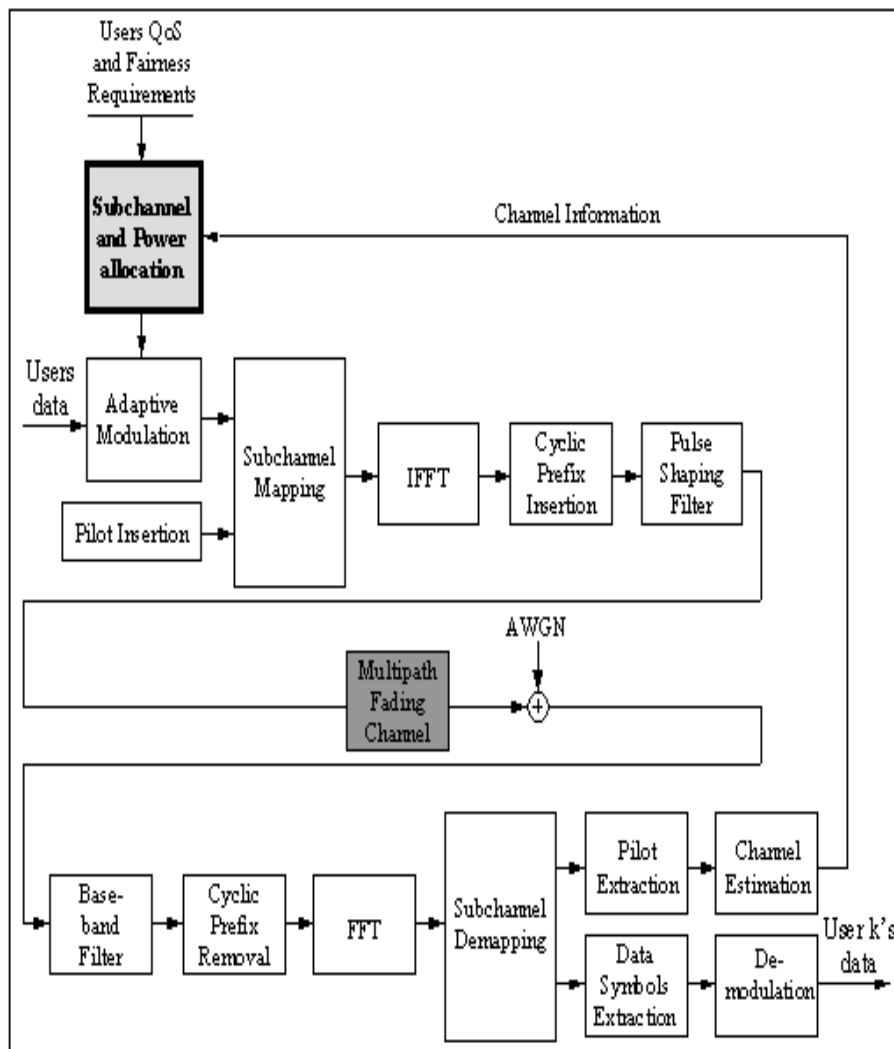


Figure 5.1: Block diagram of a downlink OFDMA system with constrained resource allocation

The link quality of an OFDMA system can be described by an effective SNR level. This SNR level can be determined in different manners. In this work, the effective SNR is the mean SNR averaged over all N subcarriers constituting a subchannel. The actual and the estimated effective SNR per unit power for user k within the m th subchannel, $\gamma_{k,m}$ and $\hat{\gamma}_{k,m}$, are given by

$$\begin{aligned}\gamma_{k,m} &= \frac{1}{N} \sum_{n=1}^N \frac{|h_{k,m}(n)|^2}{\Gamma_k \sigma_n^2} \\ \hat{\gamma}_{k,m} &= \frac{1}{N} \sum_{n=1}^N \frac{|\hat{h}_{k,m}(n)|^2}{\Gamma_k \sigma_n^2}\end{aligned}\tag{5.2}$$

σ_n^2 is the background noise variance and Γ_k is referred to as SNR gap related to the required BER of user k .

We consider that each user has i.i.d. Rayleigh fading on each subcarrier, i.e. $\hat{h}_{k,m}(n)$ are Gaussian distributed with zero mean. Hence the conditional distribution of the actual channel gain $h_{k,m}(n)$ given the estimate $\hat{h}_{k,m}(n)$ are also a Gaussian distribution with mean $\hat{h}_{k,m}(n)$:

$$h_{k,m}(n) \mid \hat{h}_{k,m}(n) \sim \mathcal{CN}(\hat{h}_{k,m}(n), \sigma_e^2).\tag{5.3}$$

From equation (5.3), we deduce that $\gamma_{k,m}$ conditioned on $\hat{\gamma}_{k,m}$ is a *non-central Chi-squared* distributed random variable with $2N$ degrees of freedom with probability density function (pdf)

$$T_{\gamma_{k,m}}(\gamma_{k,m} \mid \hat{\gamma}_{k,m}) = \frac{1}{\gamma_{e/n}} e^{-\frac{\hat{\gamma}_{k,m} + \gamma_{k,m}}{\gamma_{e/n}}} \left(\frac{\gamma_{k,m}}{\hat{\gamma}_{k,m}} \right)^{\frac{N-1}{2}} I_{(N-1)} \left(\frac{2}{\gamma_{e/n}} \sqrt{\hat{\gamma}_{k,m} \gamma_{k,m}} \right)\tag{5.4}$$

where $I_v(x)$ is the v th-order modified Bessel function of the first kind and $\gamma_{e/n} = \sigma_e^2 / \sigma_n^2$ is the ratio of the subchannel estimation error variance to the background noise variance.

For lower channel estimation error ($\sigma_e^2 \ll \sigma_n^2$), we can use a *Gamma* distribution to approximate (5.4) as

$$T_{\gamma_{k,m}}(\gamma_{k,m} \mid \hat{\gamma}_{k,m}) \approx \frac{\beta^\alpha}{\Gamma(\alpha)} \gamma_{k,m}^{(\alpha-1)} e^{-\beta \gamma_{k,m}},\tag{5.5}$$

where $\alpha = (\hat{\gamma}_{k,m} \gamma_{e/n}^{-1} + 1)^2 / (2\hat{\gamma}_{k,m} \gamma_{e/n}^{-1} + 1)$ and $\beta = \alpha / (\hat{\gamma}_{k,m} + \gamma_{e/n})$ are the shape parameter and the rate parameter of the *Gamma* pdf respectively.

5.3.2 Problem Formulation

In downlink OFDMA systems where the base station has only partial CSI, the estimated effective SNR $\hat{\gamma}_{k,m}$ can be fed back to the base station and the base station can determine the resource allocation based on $\hat{\gamma}_{k,m}$ and the user's specific QoS requirement. In such system with partial CSIT, the expected performance based on this partial CSI is a useful metric. We define the expected weighted sum rate of the multiuser system as

$$U_{\gamma}(\hat{\gamma}) = E_{\gamma} \left\{ \sum_{k=1}^K \sum_{m=1}^M \frac{\rho_{k,m}}{\bar{R}_k^{\alpha_f}} \log_2 (1 + p_{k,m} \gamma_{k,m}) \middle| \hat{\gamma}_{k,m} \right\} \quad (5.6)$$

where $\gamma = [\gamma_1^T, \dots, \gamma_K^T]^T$ with $\gamma_k = [\gamma_{k,1}, \dots, \gamma_{k,M}]$, $E_{\gamma} \{ \cdot \}$ represents the statistical expectation with respect to γ , $p_{k,m}$ denotes the power allocated to user k on subchannel m , α_f is the fairness index, and $\bar{R}_k$ is the average past data rate of user k which are updated according to

$$\bar{R}_k(t+1) = \left(1 - \frac{1}{\tau_c}\right) \bar{R}_k(t) + \frac{1}{\tau_c} r_k(t), \quad (5.7)$$

where $r_k(t)$ is the rate allocated to user k at time t and τ_c is the parameter that controls the latency of the system. This way, we consider both current rate as well as rates given to the users in the past, what is suitable for long-term fairness evaluation.

Setting α_f to 1 results in the *long-term proportional fair* allocation. For $\alpha_f = 0$, this results in *maximum ergodic sum rate* allocation. This is an explicit formulation of the expected weighted sum rate with imperfect CSI [85], where the fairness parameter can be adapted by the system operator according to the system and users requirements.

The resource allocation problem investigated in this section is how to adaptively assign subchannels to the users and distribute the total power P_{tot} in order to maximize the expected weighted sum rate (5.6) while satisfying user's specific minimum data rate and fairness requirements under a total power constraint. Mathematically, this constrained optimization problem

is formulated as

$$\begin{aligned}
 & \max_{\gamma} E_{\gamma} \left\{ \sum_{k=1}^K \sum_{m=1}^M \frac{\rho_{k,m}}{\bar{R}_k^{\alpha_f}} \log_2 (1 + p_{k,m} \gamma_{k,m}) \middle| \hat{\gamma}_{k,m} \right\} \\
 & \text{subject to:} \\
 & E_{\gamma} \left\{ \sum_{m=1}^M \rho_{k,m} \log_2 (1 + p_{k,m} \gamma_{k,m}) \middle| \hat{\gamma}_{k,m} \right\} \geq R_k, \\
 & \sum_{k=1}^K \sum_{m=1}^M p_{k,m} \leq P_{tot}, \\
 & \sum_{k=1}^K \rho_{k,m} = 1, \forall m.
 \end{aligned} \tag{5.8}$$

The maximization is performed on the subchannel assignment $\rho_{k,m}$ and on the powers $p_{k,m}$. The first constraint in (5.8) corresponds to the data rate requirement R_k for each user given the user's target BER requirement. This constraint is used as QoS metric. We assume that appropriate admission control is performed such that the minimum data rates R_k are feasible. The second constraint is the total power constraint. The third constraint indicates that one and only one user is assigned a given subchannel at a time. Note that the optimization problem (5.8) involves both continuous variables $p_{k,m}$ and boolean variables $\rho_{k,m}$. Such an optimization problem is neither convex nor concave with respect to $(\rho_{k,m}, p_{k,m})$.

In the next subsection, we propose a method to efficiently solve the problem.

5.3.3 Proposed Solution

We can solve problem (5.8) using Lagrange duality methods. Following the approach in [14], we relax $\rho_{k,m}$ to be a real value $\rho_{k,m} \in [0, 1]$ instead of a boolean. Then $\rho_{k,m}$ can be regarded as time-sharing factor that defines the proportion of time that the k th user is assigned to the subchannel m . Doing so, the power constraint needs to be rewritten as

$$\sum_{k=1}^K \sum_{m=1}^M \rho_{k,m} p_{k,m} \leq P_{tot} \tag{5.9}$$

taking into account the time sharing. The issue with this formulation is that the powers on unused subchannels ($\rho_{k,m} = 0$) are undefined. We will show

later how this issue can be solved. In practice, the corresponding power should of course be zero.

We can write the Lagrangian of the primal problem (5.8) as

$$\begin{aligned}
 L = & \sum_{k=1}^K \sum_{m=1}^M E_{\gamma_{k,m}} \left\{ \frac{\rho_{k,m}}{R_k^{\alpha_f}} \log_2 (1 + p_{k,m} \gamma_{k,m}) | \hat{\gamma}_{k,m} \right\} \\
 & + \sum_{k=1}^K \lambda_k \sum_{m=1}^M E_{\gamma_{k,m}} \{ \rho_{k,m} \log_2 (1 + p_{k,m} \gamma_{k,m}) | \hat{\gamma}_{k,m} \} \\
 & - \mu \sum_{k=1}^K \sum_{m=1}^M \rho_{k,m} p_{k,m} - \sum_{m=1}^M \nu_m \sum_{k=1}^K \rho_{k,m} \\
 & - \sum_{k=1}^K \lambda_k R_k + \mu P_{tot} + \sum_{m=1}^M \nu_m
 \end{aligned} \tag{5.10}$$

where λ_k, μ , and ν_m are Lagrangian multipliers. Let $\rho_{k,m}^*$ and $p_{k,m}^*$ denote the optimal solution.

We first investigate the problem for fixed values of μ and λ_k . If these are fixed, problem (5.10) can be solved independently in each subchannel. By Karush-Kuhn-Tucker (KKT) first optimality condition [34], $\rho_{k,m}^*$ and $p_{k,m}^*$ should satisfy the following

$$\frac{\partial L}{\partial p_{k,m}^*} \begin{cases} < 0, & p_{k,m}^* = 0 \\ = 0, & p_{k,m}^* > 0 \end{cases} \tag{5.11}$$

$$\frac{\partial L}{\partial \rho_{k,m}^*} \begin{cases} < 0, & \rho_{k,m}^* = 0 \\ = 0, & 0 < \rho_{k,m}^* < 1 \\ > 0, & \rho_{k,m}^* = 1 \end{cases} \tag{5.12}$$

We deduce from equation (5.11) that $p_{k,m}^*$ has to satisfy the following condition

$$\begin{cases} p_{k,m}^* = 0, & \text{if } \rho_{k,m}^* E_{\gamma_{k,m}} \{ \gamma_{k,m} | \hat{\gamma}_{k,m} \} < \rho_{k,m}^* \frac{\mu \ln 2}{A_k} \\ \rho_{k,m}^* E_{\gamma_{k,m}} \left\{ \frac{\gamma_{k,m}}{1 + p_{k,m}^* \gamma_{k,m}} | \hat{\gamma}_{k,m} \right\} = \rho_{k,m}^* \frac{\mu \ln 2}{A_k}, & \\ \text{if } \rho_{k,m}^* E_{\gamma_{k,m}} \{ \gamma_{k,m} | \hat{\gamma}_{k,m} \} \geq \rho_{k,m}^* \frac{\mu \ln 2}{A_k} \end{cases} \tag{5.13}$$

where $A_k = \overline{R}_k^{(-\alpha_f)} + \lambda_k$.

As already mentioned, when $\rho_{k,m}^* = 0$, the value of $p_{k,m}$ is undefined, and any value can be taken without any influence on the objective function or on the constraints. On the other hand, for any other value (it must always be positive), $\rho_{k,m}$ vanishes out of the expression and we get

$$\begin{cases} p_{k,m}^* = 0, & \text{if } E_{\gamma_{k,m}} \{ \gamma_{k,m} | \hat{\gamma}_{k,m} \} < \frac{\mu \ln 2}{A_k} \\ E_{\gamma_{k,m}} \left\{ \frac{\gamma_{k,m}}{1 + p_{k,m}^* \gamma_{k,m}} | \hat{\gamma}_{k,m} \right\} = \frac{\mu \ln 2}{A_k}, & \\ \text{if } E_{\gamma_{k,m}} \{ \gamma_{k,m} | \hat{\gamma}_{k,m} \} \geq \frac{\mu \ln 2}{A_k}. \end{cases} \quad (5.14)$$

The second equation provides, given μ and A_k , the way to compute the optimal value of the power $p_{k,m}^*$. As it is independent of $\rho_{k,m}$, we can arbitrarily decide to keep the same value for $p_{k,m}^*$ when $\rho_{k,m} = 0$.

Using the approximated pdf (5.5) and assuming non-zero power allocation, the optimal power is the solution of

$$\frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty \frac{\gamma_{k,m}^\alpha}{1 + p_{k,m} \gamma_{k,m}} e^{-\beta \gamma_{k,m}} d\gamma_{k,m} - \frac{\mu \ln 2}{A_k} = 0. \quad (5.15)$$

By the means of variable change, equation (5.15) produces

$$\frac{\alpha}{p_{k,m}} \left(\frac{\beta}{p_{k,m}} \right)^\alpha e^{\frac{\beta}{p_{k,m}}} \Gamma \left(\alpha, \frac{\beta}{p_{k,m}} \right) - \frac{\mu \ln 2}{A_k} = 0, \quad (5.16)$$

where $\Gamma(a, x)$ is the *upper incomplete Gamma* function [86].

To the best of our knowledge, there is no closed-form solution to (5.16), but it can be solved numerically for $p_{k,m}$ using a zero-finding procedure like bisection method.

Equation (5.12) is equivalent to

$$A_k G_k(p_{k,m}^*) - \nu_m \begin{cases} < 0, & \rho_{k,m}^* = 0 \\ = 0, & 0 < \rho_{k,m}^* < 1, \\ > 0, & \rho_{k,m}^* = 1 \end{cases} \quad (5.17)$$

where

$$G_k(p_{k,m}) = E_{\gamma_{k,m}} \{ \log_2(1 + p_{k,m} \gamma_{k,m}) | \hat{\gamma}_{k,m} \} - \mu p_{k,m} \quad (5.18)$$

The Lagrangian parameter ν_m needs to be chosen such that $\sum_{k=1}^K \rho_{k,m} = 1$. Since the derivatives do not depend on $\rho_{k,m}$ and are all affected in the same way by ν_m , ν_m can be chosen in such a way that only one of the derivatives (the one with the largest value of $A_k G_k(p_{k,m}^*)$) is positive and the others are negative. In the end, only one $\rho_{k,m}^* = 1$, and the others are equal to zero. So the solution of the relaxed problem is itself of the boolean type, which means that subchannel m is only assigned to one user. Practically, the subchannel assignment for a given subchannel m can be found by solving (5.15) for all possible users, and then choosing the one maximizing $A_k G_k(p_{k,m}^*)$. We can conclude that a given subchannel m is exclusively assigned to the user k_m^* satisfying

$$k_m^* = \arg \max_k A_k G_k(p_{k,m}^*). \quad (5.19)$$

Finally, given a set of Lagrangian multipliers $\{\mu, \lambda_1, \dots, \lambda_K\}$ and a fairness parameter α_f , we are able to obtain the (approximate) optimal power allocation using (5.15) and then choose the subchannel assignment using (5.19).

The optimal values of μ and $\lambda_1, \dots, \lambda_K$ still need to be found. They correspond to the ones that satisfy the total power constraint (with equality) and the individual rate constraints. If some of the individual rate constraints are exceeded, the corresponding λ_k is equal to zero. We use subgradient search [40] as line search method to find for the optimal multipliers μ^* and $\lambda_1^*, \dots, \lambda_K^*$. From initial values $\mu^{(0)}$ and $\lambda_1^{(0)}, \dots, \lambda_K^{(0)}$, the subgradient method updates the dual variables according to the following iterations:

$$\begin{aligned} \mu^{(i+1)} &= [\mu^{(i)} - s g_\mu^{(i)}]^+ \\ \lambda_1^{(i+1)} &= [\lambda_1^{(i)} - s g_{\lambda_1}^{(i)}]^+ \\ &\vdots \\ \lambda_K^{(i+1)} &= [\lambda_K^{(i)} - s g_{\lambda_K}^{(i)}]^+ \end{aligned} \quad (5.20)$$

where s is a sufficiently small positive step-size and $[x]^+ = \max(x, 0)$.

In (5.20), $g_\mu^{(i)}$ and $g_{\lambda_k}^{(i)}$, $k = 1, \dots, K$, denote the subgradient of the Lagrangian function (5.10) with respect to μ and λ_k respectively and can be expressed as

$$\begin{aligned}
 g_{\mu}^{(i)} &= P_{tot} - \sum_{k=1}^K \sum_{m=1}^M p_{k,m}^* \left(\mu^{(i)}, \lambda_k^{(i)} \right) \\
 g_{\lambda_k}^{(i)} &= \sum_{m=1}^M E_{\gamma_{k,m}} \left\{ \log_2 \left(1 + p_{k,m}^* \left(\mu^{(i)}, \lambda_k^{(i)} \right) \gamma_{k,m} \right) | \hat{\gamma}_{k,m} \right\} - R_k,
 \end{aligned} \tag{5.21}$$

where $p_{k,m}^*(\mu, \lambda_k)$ is solution of (5.15) for given multipliers μ and λ_k .

The proposed resource allocation algorithm is summarized below.

Algorithm 5.1

- Initialize the dual variables $\mu, \lambda_k (k = 1, \dots, K)$, the step-size s and ϵ
- Iterate until

$$\begin{aligned}
 P_{tot} - \sum_{k=1}^K \sum_{m=1}^M p_{k,m}^* \left(\mu^{(i)}, \lambda_k^{(i)} \right) &= \epsilon \\
 \sum_{m=1}^M E_{\gamma_{k,m}} \left\{ \log_2 \left(1 + p_{k,m}^* \left(\mu^{(i)}, \lambda_k^{(i)} \right) \gamma_{k,m} \right) | \hat{\gamma}_{k,m} \right\} - R_k &= \epsilon.
 \end{aligned}$$

1. Get the new power allocation $p_{k,m}^* \left(\mu^{(i)}, \lambda_k^{(i)} \right)$ by solving (5.15)
2. Update the dual variables $\mu, \lambda_k (k = 1, \dots, K)$ using (5.20).

Complexity Analysis

For each set of multipliers μ and $\lambda_1, \dots, \lambda_K$, we need to compute KM power allocation from equation (5.16). Thus line search must be done on each multiplier, resulting in equation (5.16) being solved $(K + 1)$ times. Each power allocation value calculation requires a zero-finding routine which we assume requires I_p function evaluations to converge. Let I_{λ} and I_e be the number of iterations required for each line search and the number of function evaluations required to compute the expected user's rate respectively. The overall complexity is $\mathcal{O}((K + 1)I_{\lambda}(I_p + I_e)KM)$. Typically, a zero-finding using bisection search and a subgradient search converge within a few iterations. Taking this into account, the constant terms can be ignored, and the overall complexity can be reduced to $\mathcal{O}((K + 1)KM)$.

The naïve approach

In order to assert the relevance of our approach, it has to be compared to other approaches. One of them is the so called naïve approach. In the naïve approach, it is assumed that the resource allocation is based only on the estimated SNR $\hat{\gamma}_{k,m}$. Under this assumption, the optimization goal is to maximize the weighted sum rate instead of the expected weighted sum rate.

Mathematically, resource allocation based on the naïve approach is formulated as follows

$$\begin{aligned}
 & \max \sum_{k=1}^K \sum_{m=1}^M \frac{\rho_{k,m}}{\bar{R}_k^{\alpha_f}} \log_2 (1 + p_{k,m} \hat{\gamma}_{k,m}) \\
 & \text{subject to:} \\
 & \sum_{m=1}^M \rho_{k,m} \log_2 (1 + p_{k,m} \hat{\gamma}_{k,m}) \geq R_k, \\
 & \sum_{k=1}^K \sum_{m=1}^M p_{k,m} \leq P_{tot}, \\
 & \sum_{k=1}^K \rho_{k,m} = 1, \forall m.
 \end{aligned} \tag{5.22}$$

From the KKT optimality conditions, the optimal power allocation, solution of (5.22) is given by

$$\tilde{p}_{k,m}^* = \left[\frac{A_k}{\mu \ln 2} - \frac{1}{\hat{\gamma}_{k,m}} \right]^+. \tag{5.23}$$

Expression (5.23) is in the form of *multi-level water-filling* power allocation with cut-off subchannel SNR $(\mu \ln 2)/A_k$ below which we do not transmit any power, and above which we transmit more power when the SNR $\gamma_{k,m}$ is higher. This is different from the conventional *water-filling* in that the water level is not only determined by the dual variables λ_k, μ , but also by the averaged data rate $\bar{R}_k$ and the fairness index α_f .

We also deduce from KKT optimality condition that, a given subchannel m is exclusively assigned to the user $\tilde{k}_m^*$ ($\rho_{\tilde{k}_m^*,m} = 1, \rho_{k,m} = 0$ for $k \neq \tilde{k}_m^*$) satisfying

$$\tilde{k}_m^* = \max_k A_k (\log_2 (1 + p_{k,m}^* \hat{\gamma}_{k,m}) - \mu p_{k,m}^*). \tag{5.24}$$

Then $\tilde{p}_{k,m}^*$ and $\rho_{\tilde{k}_m^*,m}$ are used to compute the maximized weighted sum rate as

$$U_{max} = \sum_{k=1}^K \frac{1}{\bar{R}_k^{\alpha_f}} \sum_{m=1}^M \rho_{\tilde{k}_m^*,m} \log_2 (1 + \tilde{p}_{k,m}^* \gamma_{k,m}). \quad (5.25)$$

5.3.4 Simulation Results and Discussion

To illustrate the performance of the proposed resource allocation method, we perform simulations for an OFDMA system with 4 users and 512 subcarriers divided into 8 subchannels using AMC subchannelization. The performance is evaluated in multipath channel environments specified in the IEEE 802.16a channel modeling subcommittee report [87]. We employ the SUI-3 omni antenna channel model. To simulate imperfect CSI, we generate i.i.d. realizations of the estimated channel and its prediction error. The relative path loss difference between user 1 and users 2, 3 and 4 are 0dB, 3dB and 5dB respectively while the minimum data rate requirement is the same for all users. The target bit error rate is set to 10^{-4} for all users.

In Fig. 5.2, the performance of the proposed adaptive resource allocation (ARA) is compared to that of the optimal resource allocation based on perfect CSI, the uniform power allocation under static subchannel assignment (SRA) and the naïve allocation. The result shows that the proposed adaptive resource allocation brings significant gain over static resource allocation thanks to the successfully exploitation of the multiuser diversity. We also observe that the proposed method outperforms the naive approach, especially for low SNRs.

Fig. 5.3 shows the maximized expected weighted sum rate as a function of the estimated SNR $\hat{\gamma}_{k,m}$ for various values of the estimation error ratio $\gamma_{e/n}$. We can observe that when $\gamma_{e/n}$ is small (the effect of the estimation error is less dominant than the one of the background noise), the optimization under imperfect CSI still provides significant performances compared to the perfect CSI case. The expected weighted sum rate decreases as $\gamma_{e/n}$ grows, especially for high SNRs. This was expected, since the expected weighted sum rate depends on the pdf of the channel distribution which is sensitive to the channel estimation error.

Fig. 5.4 and Fig. 5.5 illustrate the trade-off between maximized expected weighted sum rate and fairness and QoS constraint respectively. To analyze the trade-off, we replace the minimum rate R_k by $(1 - \alpha_q)R_k$. We vary the coefficients α_f and α_q and see the effect on the optimal value. By decreasing the coefficients α_f , the fairness constraints are relaxed. The relaxation

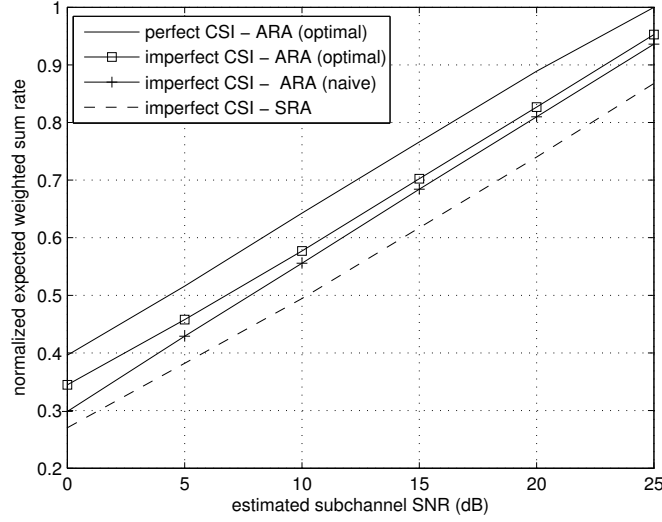


Figure 5.2: Maximized expected weighted sum rate versus estimated SNR $\hat{\gamma}_{k,m}$ for different resource allocation schemes and an estimation error ratio $\gamma_{e/n} = -5dB$.

of the constraints leads in general to an improvement of the optimal objective. The trade-off curves are found by solving the constrained optimization problem for many values of fairness and minimum rate requirements. As already observed in Fig. 4.5 and Fig. 4.6 of the previous chapter, we can see here that, as the constraints are relaxed, the expected weighted sum rate increases. Looking at the solution obtained for different fairness and minimum rate requirements, the system designer may then make a choice about the configuration he considers to be the most appropriate.

5.4 CDIT-Based Constrained Resource Allocation for OFDMA Systems

5.4.1 CDIT Channel Model

We assume that the actual channel gains, $h_{k,m}(n)$, are perfectly known at user terminals so that user terminals can compute the actual effective SNRs $\gamma_{k,m}$, but there is no fast feedback link to perfect these instantaneous channel informations to the base station. Hence, the base station has only

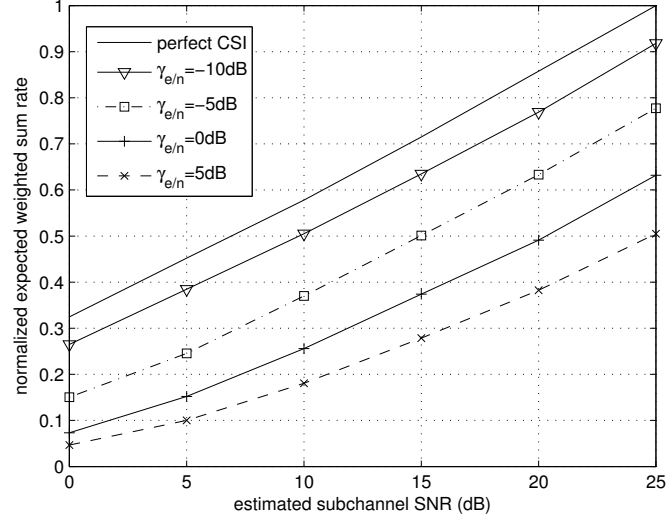


Figure 5.3: Maximized expected weighted sum rate versus estimated SNR $\hat{\gamma}_{k,m}$ for various estimation error ratios $\gamma_{e/n}$.

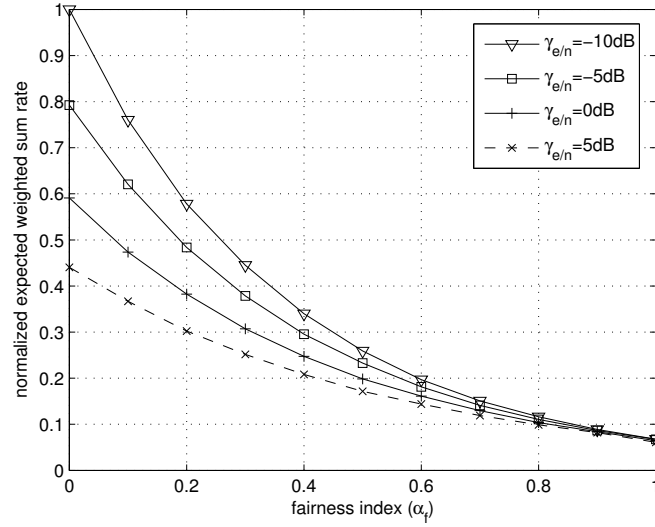


Figure 5.4: Trade-off between maximized expected weighted sum rate and fairness requirement for various estimation error ratios $\gamma_{e/n}$.

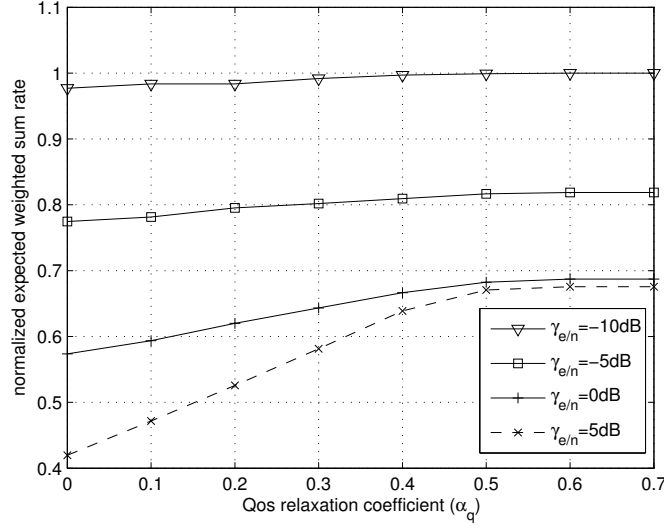


Figure 5.5: Trade-off between maximized expected weighted sum rate and minimum rate requirement for various estimation error ratios $\gamma_{e/n}$.

channel distribution information (CDI) in terms of the distribution of the effective SNR, but no knowledge of the instantaneous channel realizations.

As for the partial CSIT case, we consider that each user has i.i.d. Rayleigh fading on each subcarrier, i.e. $h_{k,m}(n)$ are Gaussian distributed with zero mean. Hence, the effective SNR of user k on subchannel m , $\gamma_{k,m}$, is a *central Chi-squared* (χ^2) distributed random variable with $2N$ degrees of freedom and with probability density function (pdf)

$$\tilde{T}_{\gamma_{k,m}}(\gamma_{k,m}) = \frac{1}{\Gamma(N)\bar{\gamma}_{k,m}} \left(\frac{\gamma_{k,m}}{\bar{\gamma}_{k,m}} \right)^{(N-1)} e^{-\frac{\gamma_{k,m}}{\bar{\gamma}_{k,m}}}, \quad (5.26)$$

where $\bar{\gamma}_{k,m}$ is the mean of the $\gamma_{k,m}$ distribution and Γ denotes the *Gamma* function [86].

Assuming fast channel variations but lower fluctuation of the variations around the mean, the pdf (5.26) can be approximated to

$$\tilde{T}_{\gamma_{k,m}}(\gamma_{k,m}) \approx \frac{1}{\Gamma(N)\bar{\gamma}_{k,m}} e^{-\frac{\gamma_{k,m}}{\bar{\gamma}_{k,m}}}. \quad (5.27)$$

We assume the duration of the transmission codewords is long enough to

undergo all channel realizations. We further assume perfect CDIT, thereby allowing to take the expectation over the distribution.

5.4.2 Problem Formulation

The ergodic weighted sum rate of the multiuser system is defined as

$$\tilde{U}_\gamma = E_\gamma \left\{ \sum_{k=1}^K \frac{1}{\bar{R}_k^{\alpha_f}} \sum_{m=1}^M \rho_{k,m} \log_2 (1 + p_{k,m} \gamma_{k,m}) \right\} \quad (5.28)$$

where γ , $p_{k,m}$, $\bar{R}_k$ and α_f are defined as in the previous section.

The issue is how to adaptively assign the M subchannels to the K users and distribute the total power budget P_{tot} in order to maximize the ergodic weighted sum rate (5.28) while satisfying user's minimum rate and system fairness requirements under a total power constraint.

Mathematically, this constrained optimization problem is formulated as

$$\begin{aligned} f^* = \max_{\rho_{k,m}, p_{k,m}} \quad & E_\gamma \left\{ \sum_{k=1}^K \frac{1}{\bar{R}_k^{\alpha_f}} \sum_{m=1}^M \rho_{k,m} \log_2 (1 + p_{k,m} \gamma_{k,m}) \right\} \\ \text{subject to:} \quad & \\ E_\gamma \left\{ \sum_{m=1}^M \rho_{k,m} \log_2 (1 + p_{k,m} \gamma_{k,m}) \right\} \geq R_k, & \quad (5.29) \\ E_\gamma \left\{ \sum_{k=1}^K \sum_{m=1}^M \rho_{k,m} p_{k,m} \right\} \leq P_{tot}. & \end{aligned}$$

The first constraint in equation (5.29) is for the user's specific minimum data rate demand. We assume that appropriate admission control is performed such that the minimum data rates R_k are feasible. The second constraint is system limitation on transmits powers.

Note that the optimization problem (5.29) involves both continuous variables $p_{k,m}$ and boolean variable $\rho_{k,m}$. Such an optimization problem is neither convex nor concave with respect to $(\rho_{k,m}, p_{k,m})$.

5.4.3 Proposed Solution

We can solve problem (5.29) using the Lagrange dual decomposition framework. Following the same approach as in the previous section, we relax

$\rho_{k,m}$ to be $\rho_{k,m} \in [0, 1]$. Then $\rho_{k,m}$ can be regarded as time-sharing factor. Thanks to the linearity property of the expectation, the Lagrangian function of the primal problem (5.29) can be expressed as

$$\begin{aligned} L_{\gamma}(p_{k,m}, \rho_{k,m}, \lambda_k, \mu) = & \sum_{k=1}^K \sum_{m=1}^M E_{\gamma_{k,m}} \left\{ \frac{\rho_{k,m}}{\overline{R}_k^{\alpha_f}} \log_2 (1 + p_{k,m} \gamma_{k,m}) \right\} \\ & + \sum_{k=1}^K \lambda_k \sum_{m=1}^M E_{\gamma_{k,m}} \{ \rho_{k,m} \log_2 (1 + p_{k,m} \gamma_{k,m}) \} \\ & - \mu \sum_{k=1}^K \sum_{m=1}^M \rho_{k,m} p_{k,m} - \sum_{k=1}^K \lambda_k R_k + \mu P_{tot} \end{aligned} \quad (5.30)$$

where λ_k and μ are Lagrangian multipliers. Let $p_{k,m}^*$ and $\rho_{k,m}^*$ denote the optimal solution of (5.30). We first investigate the problem for fixed values of λ_k and μ . By KKT first optimality condition [34], $\rho_{k,m}^*$ and $p_{k,m}^*$ should satisfy the equations (5.11) and (5.12).

By applying KKT equations (5.11) and (5.12) to (5.30) and considering non-zero power allocation and $\rho_{k,m}^* \in (0, 1)$, we obtain

$$\rho_{k,m}^* E_{\gamma_{k,m}} \left\{ \frac{\overline{R}_k^{(-\alpha_f)} + \lambda_k}{\ln 2} \cdot \frac{\gamma_{k,m}}{1 + p_{k,m}^* \gamma_{k,m}} - \mu \right\} = 0, \quad (5.31)$$

and

$$E_{\gamma_{k,m}} \left\{ \frac{\overline{R}_k^{(-\alpha_f)} + \lambda_k}{\ln 2} \cdot \ln (1 + p_{k,m}^* \gamma_{k,m}) - \mu p_{k,m}^* \right\} = 0. \quad (5.32)$$

We deduce from equation (5.31) that $p_{k,m}^*$ has to satisfy the following condition

$$\rho_{k,m}^* E_{\gamma_{k,m}} \left\{ \frac{\gamma_{k,m}}{1 + p_{k,m}^* \gamma_{k,m}} - \frac{\mu \ln 2}{A_k} \right\} = 0, \quad (5.33)$$

where $A_k = \overline{R}_k^{(-\alpha_f)} + \lambda_k$.

When $\rho_{k,m}^* = 0$, the value of $p_{k,m}$ is undefined, and any value can be taken without any influence on the objective function or on the constraints. On the other hand, for any other positive value, $\rho_{k,m}$ vanishes out of the expression and we get

$$E_{\gamma_{k,m}} \left\{ \frac{\gamma_{k,m}}{1 + p_{k,m}^* \gamma_{k,m}} - \frac{\mu \ln 2}{A_k} \right\} = 0. \quad (5.34)$$

We can use the approximated pdf of the effective SNR distribution (5.27) to transform equation (5.34) into

$$\int_0^\infty \left(\frac{\gamma_{k,m}}{1 + p_{k,m}^* \gamma_{k,m}} - \frac{\mu \ln 2}{A_k} \right) \cdot \frac{1}{\Gamma(N) \bar{\gamma}_{k,m}} e^{-\frac{\gamma_{k,m}}{\bar{\gamma}_{k,m}}} d\gamma_{k,m} = 0, \quad (5.35)$$

which is equivalent to

$$\frac{\mu \ln 2}{A_k} p_{k,m}^{*2} \bar{\gamma}_{k,m} - p_{k,m}^* \bar{\gamma}_{k,m} + \exp \left(\frac{1}{p_{k,m}^* \bar{\gamma}_{k,m}} \right) E_1 \left(\frac{1}{p_{k,m}^* \bar{\gamma}_{k,m}} \right) = 0, \quad (5.36)$$

where

$$E_1(x) = \int_1^\infty \frac{e^{-tx}}{t} dt$$

is the exponential integral function of x [88].

Equation (5.32) is equivalent to

$$E_{\gamma_{k,m}} \left\{ \frac{A_k}{\mu \ln 2} \ln (1 + p_{k,m}^* \gamma_{k,m}) - p_{k,m}^* \right\} = 0. \quad (5.37)$$

Using the pdf (5.27), equation (5.37) can be transformed into

$$\int_0^\infty \left(\frac{A_k}{\mu \ln 2} \ln (1 + p_{k,m}^* \gamma_{k,m}) - p_{k,m}^* \right) \cdot \frac{1}{\Gamma(N) \bar{\gamma}_{k,m}} e^{-\frac{\gamma_{k,m}}{\bar{\gamma}_{k,m}}} d\gamma_{k,m} = 0, \quad (5.38)$$

which is finally equivalent to

$$\frac{A_k}{\mu \ln 2} \exp \left(\frac{1}{p_{k,m}^* \bar{\gamma}_{k,m}} \right) E_1 \left(\frac{1}{p_{k,m}^* \bar{\gamma}_{k,m}} \right) - p_{k,m}^* = 0. \quad (5.39)$$

From equations (5.36) and (5.39), we derive (see appendix 5.6.1)

$$p_{k,m}^* = \left[\frac{A_k}{\mu \ln 2} - \frac{1}{\bar{\gamma}_{k,m}} \right]^+, \quad (5.40)$$

where $[x]^+ = \max(0, x)$. The expression (5.40) is in the form of *multi-level water-filling* power allocation with cut-off subchannel SNR $(\mu \ln 2)/A_k$ below which we do not transmit any power, and above which we transmit more power when $\bar{\gamma}_{k,m}$ is higher. The important difference is that, in contrast to the CSIT-based allocation where the $p_{k,m}^*$ depend on the instantaneous channels realizations, the optimal allocation here is dependent on the mean

of the channel distribution, and thus needs to be computed only when the statistics of the channel has changed.

We have from, (5.32) and (5.39), that

$$\rho_{k,m}^* = \begin{cases} 1 & \text{if } G_{k,m}(p_{k,m}^*) > 0 \\ \in (0, 1) & \text{if } G_{k,m}(p_{k,m}^*) = 0, \\ 0 & \text{if } G_{k,m}(p_{k,m}^*) < 0 \end{cases} \quad (5.41)$$

where

$$G_{k,m}(p_{k,m}^*) = \frac{A_k}{\mu \ln 2} \exp\left(\frac{1}{p_{k,m}^* \bar{\gamma}_{k,m}}\right) E_1\left(\frac{1}{p_{k,m}^* \bar{\gamma}_{k,m}}\right) - p_{k,m}^*. \quad (5.42)$$

Due to the exclusive subchannel assignment constraint in OFDMA, we can conclude that for each subchannel m , if $G_{k,m}$ are all different, then only the user with the largest $G_{k,m}$ can use that subchannel. In other words,

$$\rho_{k_m^*,m}^* = 1, \quad \rho_{k,m}^* = 0, \quad \forall k \neq k_m^*, \quad (5.43)$$

where

$$k_m^* = \arg \max_k G_{k,m}(p_{k,m}^*). \quad (5.44)$$

Substituting (5.40) into the Lagrange function (5.30) and thanks to the exclusive subchannel assignment constraint, we obtain the following per-subchannel dual problem

$$g^* = \min_{\lambda_k, \mu} L_\gamma^*(\lambda_k, \mu), \quad (5.45)$$

where $L_\gamma^*(\lambda_k, \mu)$ is the dual function given by

$$\begin{aligned} L_\gamma^*(\lambda_k, \mu) = & E_{\gamma_{k,m}} \left\{ \bar{R}_k^{(-\alpha_f)} \log_2 \left(\frac{\bar{R}_k^{(-\alpha_f)} + \lambda_k}{\mu \ln 2} \bar{\gamma}_{k,m} \right) \right\} \\ & + \lambda_k E_{\gamma_{k,m}} \left\{ \log_2 \left(\frac{\bar{R}_k^{(-\alpha_f)} + \lambda_k}{\mu \ln 2} \bar{\gamma}_{k,m} \right) \right\} \\ & - \lambda_k R_k - \mu \left(\frac{\bar{R}_k^{(-\alpha_f)} + \lambda_k}{\mu \ln 2} - \frac{1}{\bar{\gamma}_{k,m}} \right) + \mu P_{tot}. \end{aligned} \quad (5.46)$$

Next we turn to the optimization of the dual function (5.46) over μ and λ_k . First we consider the optimization over λ_k for μ fixed to find. We

differentiate (5.46) with respect to λ_k and set the derivative to 0 to obtain

$$\left. \frac{\partial L_\gamma^*}{\partial \lambda_k} \right|_{\lambda_k = \lambda_k^*} = \log_2 \left(\frac{\overline{R}_k^{(-\alpha_f)} + \lambda_k}{\mu \ln 2} \overline{\gamma}_{k,m} \right) - R_k = 0. \quad (5.47)$$

The optimum λ_k^* is derived from (5.47) as

$$\lambda_k^*(\mu) = 2^{R_k} \frac{\mu \ln 2}{\overline{\gamma}_{k,m}} - \frac{1}{\overline{R}_k^{(\alpha_f)}}. \quad (5.48)$$

If some of the individual rate constraints are exceeded, the corresponding λ_k is equal to zero.

Substituting (5.48) into (5.46) we obtain

$$L_\gamma^*(\mu) = \max_{\lambda_k} L_\gamma^*(\lambda_k, \mu) = \frac{R_k}{\overline{R}_k^{(\alpha_f)}} - \frac{2^{R_k} \mu}{\overline{\gamma}_{k,m}} + \frac{\mu}{\overline{\gamma}_{k,m}} + \mu P_{tot} \quad (5.49)$$

We next consider the optimization of $L_\gamma^*(\mu)$ over μ . The function $L_\gamma^*(\mu)$ can be shown to be a convex function of μ , which can then be minimized via a one-dimensional search with geometric convergence. The optimal values μ^* correspond to the ones that satisfy the total power constraint (with equality).

We can conclude that, if $G_{k,m}$ are all different, then a given subchannel m is exclusively assigned to the user $\tilde{k}_m^*$ satisfying

$$\tilde{k}_m^* = \arg \max_k G_{k,m}(\tilde{p}_{k,m}^*), \quad (5.50)$$

where $\tilde{p}_{k,m}^*$ is the optimal power allocation given by

$$\begin{aligned} \tilde{p}_{k,m}^* &= \left[\frac{\overline{R}_k^{(-\alpha_f)} + \lambda_k^*}{\mu^* \ln 2} - \frac{1}{\overline{\gamma}_{k,m}} \right]^+, \quad \text{if } k = \tilde{k}_m^*, \\ &= 0, \quad \text{if } k \neq \tilde{k}_m^*. \end{aligned} \quad (5.51)$$

5.4.4 Relative Duality Gap

The relative duality (optimality) gap which indicates how far we are from the optimal value can be expressed as:

$$d^* = \frac{g^* - f^*}{f^*} \geq 0, \quad (5.52)$$

where $f^* > 0$ and $g^* > 0$ given in (5.29) and (5.45) are the optimal values of the primal and dual problems. The inequality follows from the positivity of f^* and the weak duality theorem [40].

Without the minimum rate constraints in (5.29), problem (5.29) becomes a standard convex optimization problem, and then the duality gap is zero. Due to the nonlinearity of the minimum rate constraints, the convexity of problem (5.29) does not hold. However, the nonconvex optimization problem (5.29) for the investigated OFDMA based system fulfills the *time-sharing* condition as defined in [41]. Then when the power constraint is met tightly, i.e. with equality, the duality gap is zero, and thus solving the dual problem (5.45) also solves the primal problem (5.29).

5.4.5 Feedback Reduction and Complexity Analysis

First, thanks to the subchannelization, the need for knowledge of radio channel quality in OFDMA systems is reduced from per-subcarrier to per-subchannel resolution and resources are allocated on per-subchannel basis. Second, under CDIT-based allocation, instead of feeding back the instantaneous channel coefficients to the transmitter the users simply feed back the mean of the subchannel SNR distribution. Putting these two facts together, the amount of feedback required for the resource allocation reduces significantly.

Using a dual decomposition framework, the optimization problem has been reduced to a per-subchannel optimization, and the computational complexity has been significantly decreased.

Since the optimal λ_k^* and μ^* depend on the mean of the subchannel SNR distribution and not on the instantaneous values, they need to be computed only when the statistic of the channel has changed. We need to run the line search to compute for μ^* . This is followed by the computation of KM values of multipliers λ_k^* using (5.48) and power allocation values $\tilde{p}_{k,m}^*$ using (5.51). We assume the line search to require I_μ iterations. The computation of λ_k^* and $\tilde{p}_{k,m}^*$ requires $\mathcal{O}(KM)$ operations. The overall complexity order for the CDIT-based resource allocation is thus $\mathcal{O}(KMI_\mu)$. Since I_μ is just constant independent of K and M , the complexity is $\mathcal{O}(KM)$. Once μ^* , λ_k^* and $\tilde{p}_{k,m}^*$ have been determined, we do not need to update them as long as the statistics of the fading channel remain the same.

The optimal power allocation for perfect CSIT-based resource allocation can be obtained from (5.23) as

$$\tilde{p}_{k,m}^* = \left[\frac{A_k}{\mu \ln 2} - \frac{1}{\gamma_{k,m}} \right]^+. \quad (5.53)$$

This is a *multi-level water-filling* power allocation with cut-off subchannel SNR $(\mu \ln 2)/A_k$. The difference between (5.53) and (5.40) is that the power allocation in (5.53) depends on the instantaneous subchannel SNR $\gamma_{k,m}$ while the one in (5.40) depends on the mean of the SNR distribution $\bar{\gamma}_{k,m}$. Both expressions (5.40) and (5.53) are in the form of *multi-level water-filling* power allocation with cut-off subchannel SNR $(\mu \ln 2)/A_k$. Thus, the complexity in term of water filling is the same. The main difference between (5.40) and (5.53) is the amount of feedback required to performed resource allocation. Recall that, for the CSIT-based scheme, the allocation is perform after each OFDMA symbol period. Let N_s be the number of OFDMA symbol periods after which the CDIT-based resource allocation is performed. Then a rough estimation tells us that the complexity of the CDIT-based allocation is reduced by $\frac{1}{N_s}$ compared to the perfect CSIT scheme.

5.4.6 Simulation Results and Discussion

To illustrate the performance of the proposed resource allocation method, we perform simulations for a three-users ODFMA-based mobile WiMAX system with bandwidth divided into $M = 8$ subchannels. The subchannels are formed using the FUSC which is suitable for mobile applications. The FFT size of the OFDMA is 512 points. The performance is evaluated in multipath channel environments modeled as a tapped-delay line with six taps as specified in the ITU M.1225 Vehicular A channel model [89]. We consider a scenario where user 2 is every time closer to the base station than users 1 and 3 and the relative mean SNR difference between user 2 and users 1 and 3 are -5dB and -3dB respectively while the minimum data rate demand of user 3 is higher than the one of users 2 and 1. The target bit error rate is set to 10^{-3} (without channel coding). The performances are evaluated using simulations over 10,000 instances of independent channel realizations. For all the simulations, the total power is set to $P_{tot} = KM$.

In Fig.5.6, the performance of the proposed CDIT-based resource allocation is compared to those of optimal resource allocation based on perfect CSIT, resource allocation based on partial CSIT with various channel estimation error variance, and a uniform allocation (which uses static FDMA and simply distributes the power equally among the users). The result shows that

the proposed adaptive resource allocation brings significant gain over resource allocation based on partial CSIT with higher estimation error. We can observe that when $\gamma_{e/n}$ is small, that means the effect of the estimation error is less dominant than the one of the background noise, the optimization under partial CSI is closed to the one under perfect CDIT. For very low estimation errors, the partial CSIT based scheme outperforms the perfect CDIT scheme. The weighted sum rate degrades quickly as the estimation error grows, especially for high SNRs. The highest weighted sum rate is obtained with perfect CSIT but the difference in term of performance is not so significant compared to the difference of complexity between CDIT-based and CSIT-based allocation schemes. The proposed method outperforms the uniform power allocation.

Fig.5.7 shows the user's rate for different allocation schemes when the target bit error rate is the same for all users and is set to 10^{-3} , users minimum data rate demands are constrained to $R_1 = 0.75, R_2 = 1.50, R_3 = 2.25$ in bps/Hz and the fairness parameter α_f is set to 0. We observe that under optimal allocation based on perfect CDIT, the need of all users in term of data rate is satisfied. This is not the case under allocation based on partial CSIT with high estimation error nor under uniform allocation where the high data rate demand of user 3 is not satisfied. The uniform allocation is strictly unfair.

Fig.5.8 illustrates the trade-off between the maximized weighted sum rate and the fairness constraint when the minimum rate demand is relaxed to make the optimization problem feasible. The average user rates are updated according to (5.7) with $\tau_c = 20$. The maximum weighted sum rate is achieved when $\alpha_f = 0$, which is very unfair. We can see that, as the fairness constraint is enforced, the weighted sum rate decreases. For $\alpha_f = 1$, the allocation is strictly fair but inefficient in term of sum rate.

The trade-off between reduced complexity and performance degradation of the proposed CDIT-based resource allocation in comparison with the perfect CSIT allocation is shown in Fig.5.9. Adapting the transmission strategy to the short-term channel statistics, i.e. reducing N_s , increases the performance but also increases the complexity. However, if the transmission is adapted to the long term channel statistics, i.e. for larger N_s , the complexity decreases significantly but with a penalty on the performance. For a CDIT-based allocation with a distribution taken over 16 OFDMA symbol periods, the complexity is reduced by 93.75% while the performance degradation in terms of weighted sum rate is less than 15%.

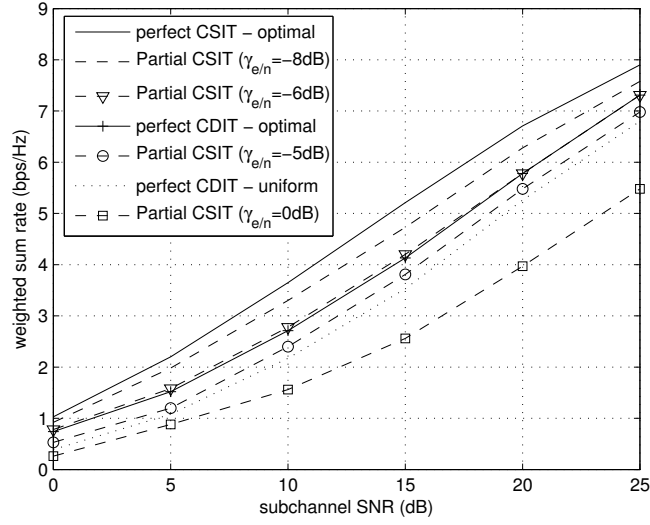


Figure 5.6: Maximized weighted sum rate vs. mean SNR for different resource allocation schemes and fairness parameter $\alpha_f = 0$.

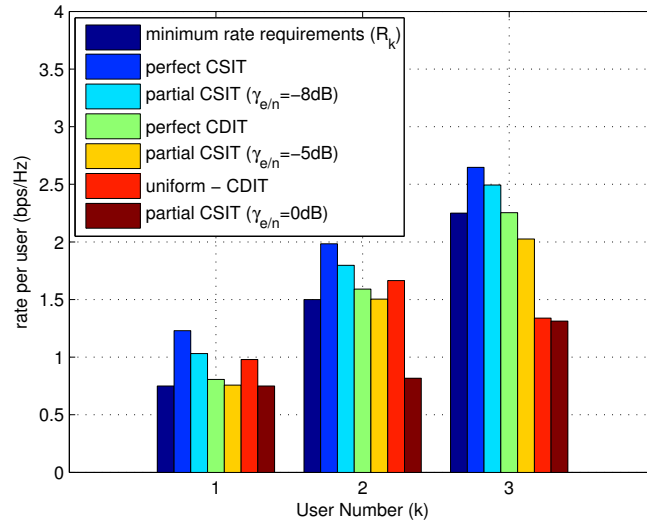


Figure 5.7: Users rates for different resource allocation schemes, mean SNR of 15 dB and fairness parameter $\alpha_f = 0$.

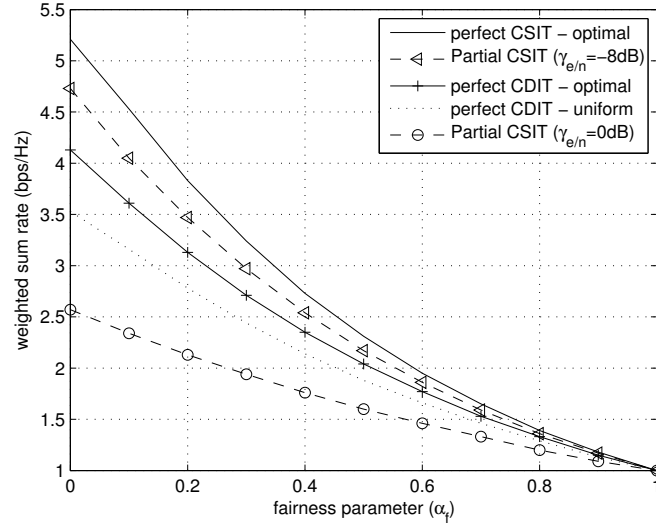


Figure 5.8: Trade-off between maximized weighted sum rate and fairness for different resource allocation schemes and mean SNR of 15 dB.

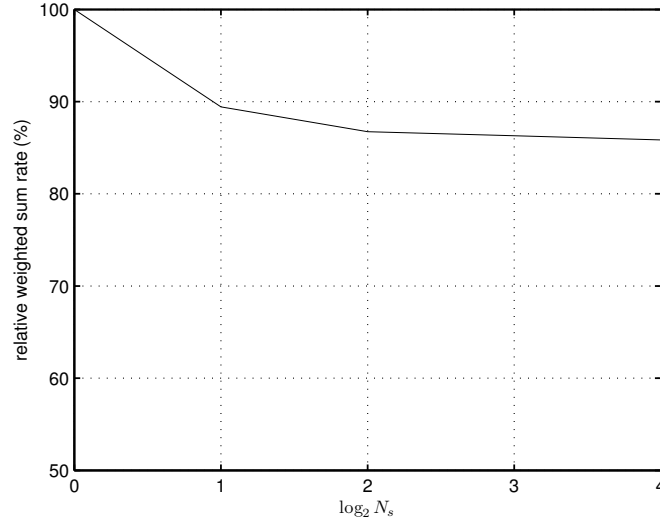


Figure 5.9: Trade-off between reduced complexity and performance degradation of the CDIT-based resource allocation compared with the perfect CSIT resource allocation.

5.5 Conclusion

In this chapter, we have presented constrained resource allocation strategies for OFDMA systems with statistical channel information in terms of statistics of the partial CSIT and CDIT. For the allocation based on the statistics of the partial CSIT, we have formulated a resource allocation problem to maximize the expected weighted sum rate under user's specific minimum rate, system fairness and total power constraints. For the CDIT-based case, we have formulated the problem with the goal to maximize the ergodic weighted sum rate under the same constraints. Though the two problems are originally nonlinear optimization problems, we have shown that the problems can be reformulated as Lagrangian dual problems. From these, methods have been proposed to efficiently solve the problems.

We have illustrated the impact of the channel estimation error on the performances of the system with partial CSIT. The performance degradation due to CSI errors, caused by noisy channel estimation, is moderate for the level of error which can be achieved by a practical channel estimator.

Closed form expression for CDIT-based power allocation has been derived. Through complexity analysis, we have shown that the proposed CDIT-based method can find the optimal solution with significant lower computational complexity than the optimal CSIT-based allocation schemes. The use of the proposed CDIT-based method leads to significant feedback reduction while the performance degradation is not so significant.

For both schemes, we have analyzed the trade-off between the maximized weighted sum rate and the constraints and we have illustrated the analysis through trade-off curves. Looking at these trade-off curves, the system designer may then make a choice about the configuration he considers to be the most appropriate.

In fading environments, even with statistical channel information only, adaptive resource allocation strategies provide performance gain for OFDMA systems in comparison with the static resource allocation which uses static FDMA and simply distributes the power equally among the users. Since user mobility is the principal driving force for OFDMA-based systems such as mobile WiMAX, CDIT-based resource allocation strategies are of particular interest. The methods proposed in this chapter can be applied to other OFDMA-based wireless systems such as Long Term Evolution (LTE).

5.6 Appendix

5.6.1 Appendix: Derivation of (5.40)

From equation (5.39), we obtain

$$\exp\left(\frac{1}{p_{k,m}^* \bar{\gamma}_{k,m}}\right) E_1\left(\frac{1}{p_{k,m}^* \bar{\gamma}_{k,m}}\right) = p_{k,m}^* \cdot \frac{\mu \ln 2}{A_k}. \quad (5.54)$$

Replacing (5.54) in (5.36), we have

$$\frac{\mu \ln 2}{A_k} \bar{\gamma}_{k,m} p_{k,m}^{*2} - \bar{\gamma}_{k,m} p_{k,m}^* + \frac{\mu \ln 2}{A_k} \cdot p_{k,m}^* = 0, \quad (5.55)$$

which when solved for $p_{k,m}^*$ gives us (5.40).

Chapter 6

Constrained Resource Allocation in Cooperative Communications

6.1 Introduction

In the previous chapters, PMP communications have been considered. In PMP communications, a base station distributes traffic directly to roof-mounted subscriber stations. In such a situation, the most intuitive and up to now, the most widely used strategy to increase coverage, is to shrink the size of the cell, effectively increasing the number of BSs over a given area. Although this strategy likely will increase coverage because users are much closer to their serving BSs, its benefit is limited because of the exceeding cost of BSs. For each BS, the provider must pay for antenna space and the wired backhaul to the network, not to mention the digital and radio frequency equipment itself.

Instead, an increasingly attractive strategy is to insert fixed relays into the cell whose sole purpose is to aid communication from BS to mobile station (MS) and vice versa. Such communications are called cooperative communications. In cooperative communications, data are transmitted via intermediate relay stations in order to provide extended coverage range and an improved overall link quality. Cooperative communications take advantage of the broadcast nature of wireless channels, use relay nodes as virtual antennas, and thus realizes the benefits of MIMO communications in situations where physical multiple antennas are difficult to deploy.

Wireless Mesh networks (WMN) and relay networks are the two main networks using cooperative communication. The main distinguishing characteristic of mesh and relay networks is the possibility of multi-hop communication. In the Mesh networks, traffic can be routed through other mobile stations (MS) and can also occur directly between them. Nodes are comprised of mesh routers and mesh clients and thus routing process is controlled not only by base station (BS) but also by mobile station MS [90]. Each node can forward packets on behalf of other nodes that may not be within direct wireless transmission range of their destination. In case of relay networks, the network infrastructure consists of relay stations (RSs) that are mostly installed, owned and controlled by service provider. An RS is not directly connected to wire infrastructure and has the minimum functionality necessary to support multi-hop communication. The important aspect is that traffic always leads from or to BS. The realization of the performance improvement provided by cooperation in wireless mesh and relay networks is contingent on resource allocation (among other things).

This chapter is dedicated to constrained resource allocation in cooperative communications.

Wireless mesh networks (WMN) have emerged as a key technology for next generation wireless networking [90]. To compete with existing broadband technologies (such as cable modem or DSL) and become a viable solution, WMNs have to meet two main performance criteria: to provide high throughput in order to meet the ever-growing demand of network applications and to guarantee fairness among different clients. With the growing demand for multimedia applications, WMNs are expected to support heterogeneous traffic types (e.g. voice, video, data, etc.) with various QoS requirements. Efficient resource allocation is one of the keys to achieve these performances. By assigning different subcarriers to different nodes, simultaneous transmission are fostered in a cooperative manner so as to increase system throughput. Since different subcarriers experience different channel fading characteristics, power allocation with respect to channel conditions and users requirements is proved to be crucial. In [91], a fair subcarrier and power allocation scheme to maximize the Nash bargaining fairness has been proposed. Instead of solving a centralized global optimization problem, the authors proposed a distributed hierarchical approach where the mesh router allocates groups of subcarriers to the mesh clients, and each mesh client allocates transmit power among its subcarriers to each of its outgoing links. In [92], an efficient intra-cluster packet-level resource allocation approach taking into account power allocation, subcarrier assignment, packet

scheduling, and QoS support has been studied. The authors employ the utility maximization framework to find the joint power-frequency-time resource allocation that maximizes the sum rate of a WMN while satisfying users minimum rate requirements.

The benefits of optimal resource allocation in cooperative relay networks with perfect and or imperfect CSI has also been investigated by several authors. In [93], capacity bounds and power allocation for wireless relay channels have been studied. The authors explored power allocation strategies that take into account practical constraints on the transmission/reception duplexing at the relay node and on the synchronization between the source node and the relay node. In [94], resource allocation strategies to optimize the energy efficiency of relay networks under power supply constraint have been proposed. In [95], a dynamic resource allocation algorithm to maximize the delay-limited capacity of cooperative relay networks with statistical channel information has been developed. In [96], the capacity improvement provided by optimal power allocation and cooperation strategies in relay networks has been studied.

The above mentioned strategies and most of the previous proposed constrained resource allocation schemes for cooperative networks require rigorous optimization processes using numerical methods since closed-form solutions are rarely found. In this thesis, we present a new result on how the Lambert-W function can be used to efficiently find closed-form solution of constrained resource allocation problems for cooperative networks.

The Lambert-W function has several uses in physical and engineering applications [97]. Let us give some examples of applications. In [98], the Lambert-W function is used for the purpose of diode parameters determination in diode I-U curve fitting. In [99], the Lambert-W function is integrated to a tolerance optimization problem in the context of manufacturing operations. Recent work in [100] shows how the Lambert-W function also has uses in astronomy to calculate the position of an orbiting body in a central gravity field.

The remainder of this chapter is organized as follows. In Section 6.2, we provide a concise introduction to the Lambert-W function. In Section 6.3, we demonstrate how the Lambert-W function is applied to a subcarrier allocation problem in wireless mesh networks. The use of the Lambert-W function to a power allocation problem in relay networks with statistical channel information is discussed in Section 6.4. Finally, conclusions are drawn in Section 6.5.

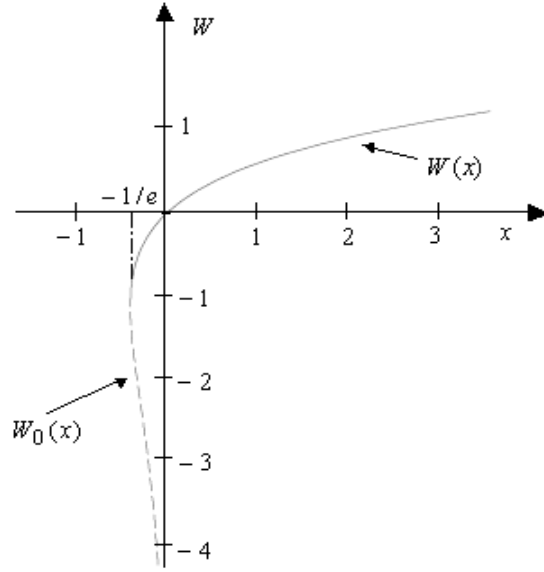


Figure 6.1: *The two real branches of the Lambert-W function*

6.2 The Lambert-W Function

The Lambert function $W(x)$ is defined to be the multivalued inverse of the function $f(x) = xe^x$ [97]. That is, Lambert $W(x)$ can be any function solution of the transcendental equation

$$W(x)e^{W(x)} = x. \quad (6.1)$$

Actually, for some values of x , equation (6.1) has more than one root in W , in which case the different solutions are called branches of W . Since the values of interest in our work are real, we will concentrate on real-value branches of W . If x is real, then for $-1/e \leq x < 0$, there are two possible real values of $W(x)$ (see Fig. 6.1). The branch satisfying $W(x) \leq -1$ is denoted by $W_0(x)$. The branch satisfying $W(x) \geq -1$ is denoted by $W(x)$ and it is referred to as the principal branch of the Lambert-W function.

The n th derivative of the Lambert-W function is given by [97].

$$\frac{d^n W(e^x)}{dx^n} = \frac{q_n(W(e^x))}{(1 + W(e^x))^{2n-1}} \quad \text{for } n \geq 1, \quad (6.2)$$

in which the polynomials q_n , given by

$$q_n(w) = \sum_{i=0}^{n-1} \left\langle \left\langle \begin{matrix} n-1 \\ i \end{matrix} \right\rangle \right\rangle (-1)^i w^{i+1}, \quad (6.3)$$

contain coefficients expressed in terms of second-order Eulerian numbers and $q_1(w) = w$.

In (6.3), the second-order Eulerian number $\left\langle \left\langle \begin{matrix} n \\ m \end{matrix} \right\rangle \right\rangle$ corresponds to the number of permutations of the *multiset* $\{1, 1, 2, 2, \dots, n, n\}$ with m ascents which have the property that for each i , all the numbers appearing between the two occurrences of i in the permutation are greater than i [101].

For more mathematical details on the Lambert-W function, we refer the reader to [97] and references therein.

There are two significant benefits from using the Lambert-W function in the context of resource allocation for cooperative networks. Most resource allocation schemes for cooperative networks require rigorous optimization processes using numerical methods since closed-form solutions are rarely found. By using the Lambert-W function, resource allocations can not only be expressed in closed-form but they can also quickly be determined without resorting to complex algorithms since a number of popular mathematical softwares, including Maple and Matlab, contain the Lambert-W function as component.

The application of the Lambert-W function to obtain a closed-form solution for resource allocation problems in wireless mesh and relay networks constitutes the principal contribution of this chapter.

6.3 Constrained Resource Allocation for Wireless Mesh Networks

6.3.1 System Model

We consider a single cluster OFDMA WMN that consists of one mesh router (MR) and K mesh clients (MC) as illustrated in Fig. 6.2. The MR serves as a gateway for the MCs to the external network (e.g. Internet). The MCs can communicate with the MR and with each other through multi-hop routes via other MCs. We label the MR as node 0 and the MC nodes as $k = 1, \dots, K$. A link (k, j) exists between node k and node j when they are within transmission range of each other, i.e., they are neighboring nodes.

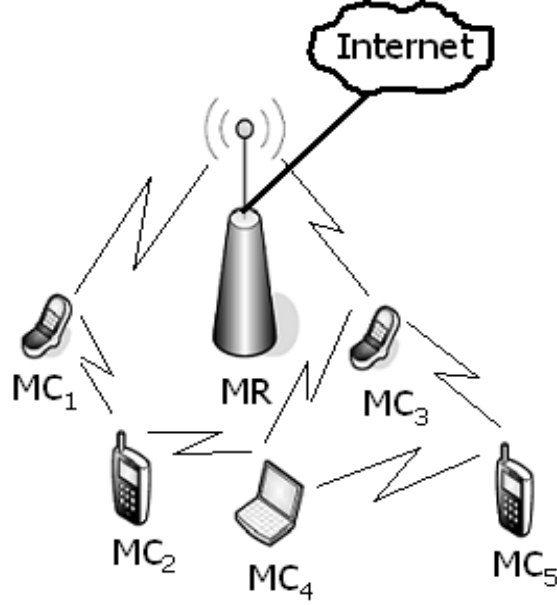


Figure 6.2: Illustration of a wireless mesh network

There are a total of N subcarriers in the system. Each subcarrier has a bandwidth B . The channel gain of subcarrier n on link (k, j) , which connects MC k to MC j , is denoted by G_{kj}^n and the transmit power of MC k on subcarrier n is denoted by p_k^n . Each MC k has a transmit power limit of $\bar{p}_k$ and a minimal rate requirement of R_k . We assume a slow-fading channel such that the channel gains are constant within each OFDMA frame. Let n_k be the number of subcarriers to allocate to MC k , using only information available at the MR, i.e. the average channel gain of all outgoing links at MC k , $\bar{G}_k$. The rate realized at MC k from allocation n_k , is given by

$$r_k(n_k) = n_k B \log_2 \left(1 + \frac{\bar{G}_k \bar{p}_k}{\Gamma \sigma_n^2 n_k} \right), \quad (6.4)$$

where σ_n^2 is the thermal noise power, and Γ is the SNR gap related to the required BER.

The data rate of link (k, j) on subcarrier n is given by

$$r_{kj}^n = B \log_2 \left(1 + \frac{G_{kj}^n p_k^n}{\Gamma \sigma_n^2} \right) = B \log_2 (1 + p_k^n \gamma_{kj}^n). \quad (6.5)$$

There are various constraints associated with resource allocation in OFDMA-based WMNs. At each node k , the sum of the transmit power on the allocated subcarriers is bounded by a maximum power level $\bar{p}_k$. We assume that each subcarrier can only be allocated to one transmission link in a cluster. Different traffic types require different packet transmission rates. For example, voice packets require a constant rate; video traffic has minimum, mean, and maximum rate requirements; while data traffic is usually treated as background traffic whose source rate is dynamic. In our problem formulation, we only take the minimum rate requirement of these three traffic types, if any, into account.

The resources to schedule are defined as a set of subcarriers, and the total transmit power available at each node. We consider a distributed hierarchical resource allocation, where the MR performs a rough resource allocation with limited information (the average channel gain of all outgoing links at MC or the statistical channel information) and the MCs perform more refined resource allocation with full information that is available locally.

6.3.2 Resource Allocation at Mesh Router

6.3.2.1 Problem Formulation

The MR determines the number of subcarriers to be assigned to each MC k based on $\bar{G}_k$ and R_k . However, the MR does not allocate power levels on the respective subcarriers because the MR does not know G_{kj}^n . Instead, power allocation is performed by the MC. Mathematically, the subcarriers allocation problem at the mesh router can be formulated as

$$\begin{aligned} & \max_{n_k} \sum_{k=1}^K n_k B \log_2 \left(1 + \frac{\alpha_k}{n_k} \right) \\ & \text{subject to:} \\ & n_k B \log_2 \left(1 + \frac{\alpha_k}{n_k} \right) \geq R_k \\ & \sum_{k=1}^K n_k \leq N \end{aligned} \tag{6.6}$$

where $\alpha_k = \frac{\bar{G}_k \bar{p}_k}{\Gamma \sigma_n^2}$.

6.3.2.2 Solution method

We propose a solution method based on KKT solving and the Lambert-W function.

First we express the Lagrangian of the primal problem (6.6) as

$$L(n_k, \lambda_k, \mu) = \sum_{k=1}^K n_k B \log_2 \left(1 + \frac{\alpha_k}{n_k} \right) - \mu \left(\sum_{k=1}^K n_k - N \right) + \sum_{k=1}^K \lambda_k \left(n_k B \log_2 \left(1 + \frac{\alpha_k}{n_k} \right) - R_k \right) \quad (6.7)$$

where λ_k and μ are the Lagrangian multipliers associated with the different constraints.

By KKT first optimality condition, we derive (6.7) over n_k for fixed (λ_k, μ) and set the derivative to zero to obtain

$$\ln \left(1 + \frac{\alpha_k}{n_k} \right) - \frac{\frac{\alpha_k}{n_k}}{1 + \frac{\alpha_k}{n_k}} - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k} = 0. \quad (6.8)$$

By solving equation (6.8) for n_k , the optimal allocation can be obtained as (see appendix 6.6.1)

$$n_k^* = \frac{-\alpha_k W \left(-\exp \left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k} \right) \right)}{1 + W \left(-\exp \left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k} \right) \right)}. \quad (6.9)$$

The optimal values of μ and λ_k still need to be found. They correspond to the ones that satisfy the total subcarrier constraint (with equality) and the individual rate constraints. We replace n_k in equation (6.7) by n_k^* to form the dual problem

$$\min_{\lambda_k, \mu} L(n_k^*, \lambda_k, \mu). \quad (6.10)$$

The optimal λ_k^* , for fixed μ , are found using KKT conditions. To derive $L(n_k^*, \lambda_k, \mu)$ over λ_k , we make use of the formula of the n th derivative of the Lambert-W function given in (6.2). Applying (6.2) for $n = 1$, we obtain that the optimal λ_k^* has to satisfy (see appendix 6.6.2 for details of the derivation)

$$f_m(\lambda_k^*) - \frac{R_k}{\alpha_k} = 0, \quad (6.11)$$

where

$$f_m(\lambda_k^*) = \frac{w_k^*}{1 + w_k^*} \cdot \left[\left(\frac{B}{\ln 2} + \frac{\mu}{(1 + \lambda_k^*)(1 + w_k^*)^2} \right) \ln(-w_k^*) + \frac{\mu}{(1 + \lambda_k^*)(1 + w_k^*)} + \frac{\mu^2}{(1 + \lambda_k^*)^2 (1 + w_k^*)^2} \right]$$

where $w_k^* = W \left(-\exp \left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k^*} \right) \right)$.

It can be shown that f_m is a strictly increasing function of λ_k^* and $f_m(\lambda_k^*) > 0, \forall \lambda_k^* \geq 0$, thus the inverse function of f_m , f_m^{-1} , exists. The optimal λ_k^* can then be deduced as

$$\lambda_k^* = f_m^{-1} \left(\frac{R_k}{\alpha_k} \right). \quad (6.12)$$

Now we turn to find the optimal μ^* . Replacing λ_k in (6.9) by the optimal value obtained in (6.12) and replacing (6.9) in the constraint $\sum_{k=1}^K n_k = N$, we obtain

$$\sum_{k=1}^K \frac{-\alpha_k W \left(-\exp \left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k^*} \right) \right)}{1 + W \left(-\exp \left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k^*} \right) \right)} = N. \quad (6.13)$$

Let

$$g_m(\mu) = \sum_{k=1}^K \frac{-\alpha_k W \left(-\exp \left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k^*} \right) \right)}{1 + W \left(-\exp \left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k^*} \right) \right)}. \quad (6.14)$$

It is shown in appendix 6.6.3 that an inverse function for g_m , g_m^{-1} , exists. Thus

$$\mu^* = g_m^{-1}(N). \quad (6.15)$$

6.3.2.3 Extension of the Solution Method to Mesh Router with Statistical Channel Information

As wireless mesh nodes typically have no mobility, it may be sufficient to perform resource allocation only when the statistics of the channel as changed instead of instantaneously. When only channel distribution information (CDI) is available at the MR, we can use these CDI to perform resource allocation. We formulate an ergodic rate maximization problem at the MR with only CDI as

$$\begin{aligned} \max_{n_k} \quad & E_{\alpha} \left\{ \sum_{k=1}^K n_k B \log_2 \left(1 + \frac{\alpha_k}{n_k} \right) \right\} \\ \text{subject to:} \quad & E_{\alpha_k} \left\{ n_k B \log_2 \left(1 + \frac{\alpha_k}{n_k} \right) \right\} \geq R_k \\ & \sum_{k=1}^K n_k \leq N \end{aligned} \quad (6.16)$$

where $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_k, \dots, \alpha_K]$, and $E_{\alpha} \{.\}$ represents the statistical expectation with respect to α .

Using the solution method proposed in 6.3.2.2, the optimal subcarrier allocation solution of (6.16) is obtained by solving the following equation for $\tilde{n}_k$

$$E_{\alpha_k} \left\{ \ln \left(1 + \frac{\alpha_k}{\tilde{n}_k} \right) - \frac{\frac{\alpha_k}{\tilde{n}_k}}{1 + \frac{\alpha_k}{\tilde{n}_k}} - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k} \right\} = 0. \quad (6.17)$$

To express the left hand side of (6.17), we need to find the probability density function (pdf) of the random variable

$$\tilde{f}_m(\alpha_k) = \ln \left(1 + \frac{\alpha_k}{\tilde{n}_k} \right) - \frac{\frac{\alpha_k}{\tilde{n}_k}}{1 + \frac{\alpha_k}{\tilde{n}_k}} - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k}. \quad (6.18)$$

It can be observed that $\tilde{f}_m$ is strictly increasing and non-negative with respect to α_k . Thus there exists a unique inverse function, $\tilde{f}_m^{-1}$, of $\tilde{f}_m$.

Let $\tilde{F}_{\alpha_k}(\alpha_k)$ and $\tilde{T}_{\alpha_k}(\alpha_k)$ denote the cumulative distribution function (cdf) and the pdf of α_k . We assume that $\tilde{F}_{\alpha_k}(\alpha_k)$ and $\tilde{T}_{\alpha_k}(\alpha_k)$ are known at the MR.

First, using the same derivation as in appendix 6.6.1 where the right hand side of equation (6.53) is $\tilde{f}_m$ instead of 0, we can express the inverse function of $\tilde{f}_m$ as

$$\tilde{\alpha}_k(\tilde{f}_m) = \frac{-\tilde{n}_k^* \left(1 + W \left(-\exp \left(-1 - \frac{\mu + \tilde{f}_m}{\frac{B}{\ln 2} + \lambda_k} \right) \right) \right)}{W \left(-\exp \left(-1 - \frac{\mu + \tilde{f}_m}{\frac{B}{\ln 2} + \lambda_k} \right) \right)}. \quad (6.19)$$

Using expression (6.19) for the root, we derive the cdf of $\tilde{f}_m$ as [102]

$$\tilde{F}_{\tilde{f}_m}(\tilde{f}_m) = \tilde{F}_{\alpha_k}(\tilde{\alpha}_k(\tilde{f}_m)). \quad (6.20)$$

The pdf of $\tilde{f}_m$ is then given as the derivative of (6.20) with respect to $\tilde{f}_m$ as

$$\tilde{T}_{\tilde{f}_m}(\tilde{f}_m) = \tilde{T}_{\alpha_k}(\tilde{\alpha}_k(\tilde{f}_m)) \frac{\left(1 + \tilde{\alpha}_k(\tilde{f}_m) \right)^2}{\frac{\frac{B}{\ln 2} + \lambda_k}{\tilde{n}_k^{*2}}}. \quad (6.21)$$

Finally, using (6.21), the optimal subcarrier assignment $\tilde{n}_k^*$ is obtained by solving the following equation for $\tilde{n}_k^*$

$$\int_0^\infty \tilde{f}_m \tilde{T}_{\tilde{f}_m}(\tilde{f}_m) d\tilde{f}_m = 0. \quad (6.22)$$

For given multipliers λ_k and μ , equation (6.22) can be solved numerically.

The optimal values of λ_k ($k \in [1, K]$) and μ still need to be found. They correspond to the ones that satisfy the individual rate constraints and the total subcarriers constraint (with equality). If some of the individual rate constraints are exceeded, the corresponding λ_k is equal to zero. We use an iterative search algorithm, similar to algorithm (5.1), to find the optimal set of λ_k and μ . The algorithm is summarized below.

Algorithm 6.1

- Initialize the dual variables $\mu, \lambda_k (k = 1, \dots, K)$, the step-size s and ϵ
- Iterate until

$$g_{\mu}^{(i)} = N - \sum_{k=1}^K \tilde{n}_k^* \left(\mu^{(i)}, \lambda_k^{(i)} \right) = \epsilon$$

$$g_{\lambda_k}^{(i)} = E_{\alpha_k} \left\{ \tilde{n}_k^* \left(\mu^{(i)}, \lambda_k^{(i)} \right) B \log_2 \left(1 + \frac{\alpha_k}{\tilde{n}_k^* \left(\mu^{(i)}, \lambda_k^{(i)} \right)} \right) \right\} - R_k = \epsilon.$$

1. Get the new subcarrier allocation $\tilde{n}_k^* \left(\mu^{(i)}, \lambda_k^{(i)} \right)$ by solving (6.22)
2. Update the dual variables $\mu, \lambda_k (k = 1, \dots, K)$ using (5.20).

6.3.2.4 Trade-off between Feedback Reduction and Performance Degradation

The amount of feedback required to perform resource allocation at MR with instantaneous channel information is different from the one required for CDI-based resource allocation. Recall that, for the instantaneous allocation scheme, the allocation is performed after each OFDMA symbol period. Let N_s be the number of OFDMA symbol periods after which the resource allocation with statistical channel information is performed. Then a rough estimation tells us that the feedback required to perform CDI-based resource allocation is reduced by $\frac{1}{N_s}$ compared to the instantaneous allocation.

6.3.3 Subcarrier and Power Allocation for Mesh Clients

6.3.3.1 Problem Formulation for each Mesh Client k

Each MC k allocates its available subcarriers that have been assigned to it by the MR and its available transmit power among these subcarriers based on the channel gain G_{kj}^n , and the minimum rate requirement for each outgoing link (k, j) , R_{kj} . Mathematically, the subcarrier and power allocation problem at each MC k can be formulated as

$$\max_{\rho_{kj}^n, p_k^n} \sum_{j \in A_k} \sum_{n \in \Omega_k} \rho_{kj}^n B \log_2 (1 + p_k^n \gamma_{kj}^n)$$

subject to:

$$\begin{aligned} \sum_{n \in \Omega_k} \rho_{kj}^n B \log_2 (1 + p_k^n \gamma_{kj}^n) &\geq R_{kj} \\ \sum_{n \in \Omega_k} \sum_{j \in A_k} \rho_{kj}^n &\leq n_k^* \\ \sum_{j \in A_k} \rho_{kj}^n &\leq 1, \forall n \in \Omega_k \\ \sum_{n \in \Omega_k} p_k^n &\leq \bar{p}_k. \end{aligned} \tag{6.23}$$

In equation (6.23), ρ_{kj}^n is the subcarrier allocation indicator ($\rho_{kj}^n = 1$ if link (k, j) is allocated subcarrier n , $\rho_{kj}^n = 0$, otherwise), n_k^* is the optimal allocation solution of (6.6), Ω_k is the set of subcarriers allocated to MC k ($n_k^* = |\Omega_k|$), and A_k is the set of active links communicating with MC k .

6.3.3.2 Proposed Solution

Problem (6.23) is solved at each MC k using the values of Ω_k ($n_k^* = |\Omega_k|$) obtained from the MR, and R_{kj} 's and G_{kj}^n 's obtained locally. This problem can be solved using a Lagrangian dual approach. The Lagrangian of the primal problem (6.23) can be expressed as

$$\begin{aligned} L(p_k^n, \rho_{kj}^n, \lambda_j, \mu_k, \nu, \eta_k) &= \sum_{j \in A_k} \sum_{n \in \Omega_k} \rho_{kj}^n B \log_2 (1 + p_k^n \gamma_{kj}^n) \\ &+ \sum_{j \in A_k} \lambda_j \left(\sum_{n \in \Omega_k} \rho_{kj}^n B \log_2 (1 + p_k^n \gamma_{kj}^n) - R_{kj} \right) \\ &- \nu \left(\sum_{j \in A_k} \sum_{n \in \Omega_k} \rho_{kj}^n - n_k^* \right) - \sum_{n \in \Omega_k} \eta_k \left(\sum_{j \in A_k} \rho_{kj}^n - 1 \right) \\ &- \mu_k \left(\sum_{n \in \Omega_k} p_k^n - \bar{p}_k \right), \end{aligned} \tag{6.24}$$

where λ_j, μ_k, ν , and η_k are Lagrangian multipliers associated with the different constraints.

By interpreting KKT conditions for fixed multipliers and a given subcarrier assignment, the optimal power allocation p_k^{n*} has to satisfy

$$f_k(p_k^{n*}) - \mu_k = 0, \quad (6.25)$$

where

$$f_k(p_k^{n*}) = \sum_{j \in A_k} \left(\frac{B}{\ln 2} + \lambda_j \right) \cdot \frac{\rho_{kj}^n \gamma_{kj}^n}{1 + p_k^{n*} \gamma_{kj}^n}. \quad (6.26)$$

As a sum of strictly decreasing functions, f_k is a strictly decreasing function of p_k^{n*} . Thus, the inverse function of f_k , f_k^{-1} , exists. The optimal power allocation can then be deduced as

$$p_k^{n*} = f_k^{-1}(\mu_k). \quad (6.27)$$

Replacing (6.27) in the power constraint

$$\sum_{n \in \Omega_k} p_k^n = \bar{p}_k,$$

we have

$$\sum_{n \in \Omega_k} f_k^{-1}(\mu_k) = \bar{p}_k. \quad (6.28)$$

Let

$$g_k(\mu_k) = \sum_{n \in \Omega_k} f_k^{-1}(\mu_k). \quad (6.29)$$

It can be shown that $g_k(\mu_k)$ is a strictly decreasing function. Thus g_k^{-1} exists and μ_k can be expressed as

$$\mu_k = g_k^{-1}(\bar{p}_k). \quad (6.30)$$

Replacing (6.30) in (6.27), we obtain

$$p_k^{n*} = f_k^{-1}(g_k^{-1}(\bar{p}_k)). \quad (6.31)$$

Also by KKT optimality conditions, the criterion of optimal subcarrier allocation can be deduced as follows: for each subcarrier n , choose the link j^* such that

$$j^* = \arg \max_j B \log_2 (1 + p_k^{n*} \gamma_{kj}^n) \quad (6.32)$$

and set $\rho_{kj^*}^n = 1$.

It can be observed from (6.25)-(6.32) that the criteria of optimal subcarrier and power allocation are coupled.

One way to jointly find the optimal power and subcarrier allocation using (6.31) and (6.32) is by exhaustive search. That is, we find $2^{n_k^* |A_k|}$ values of p_k^{n*} using (6.31), one for each possible value of the (Boolean) variables ρ_{kj}^n , and then choose the one of these optimal values that satisfy (6.32). For a small number of subcarriers per user, this can be of practical interest since, in practical WMNs, there is only a few active links per user. For a large number of subcarriers per user, exhaustive search may be computational more complex.

To reduce the computational complexity at the MC, we propose a sub-optimal resource allocation that first fixes the power allocation, invokes the subcarrier allocation based on the criterion (6.32), reallocates the subcarriers until all the constraints are met, and performs power allocation using (6.31).

For the reallocation procedure, we use the same approach as in [103]. Let A_k^u be the set of unsatisfied links communicating with node k . Consider the following cost function of reallocating a subcarrier to the i th link instead of the originally assigned j th link:

$$J_{i,n} = \frac{1}{r_{kj} - r_{ki}} \cdot \frac{\rho_{ki}^n B \log_2 (1 + p_k^n \gamma_{ki}^n)}{R_{ki} - r_{ki}}, \quad i \in A_k^u \quad (6.33)$$

where

$$r_{kl} = \sum_{n \in \Omega_k} \rho_{kl}^n B \log_2 (1 + p_k^n \gamma_{kl}^n).$$

The cost in (6.33) represents the relevance of reallocating the n th subcarrier to the i th link instead of the originally assigned j th link. The first term measures the decrease in throughput for subcarrier swapping, while the second term indicates the tendency of satisfying the minimum rate requirement of the i th link if n th subcarrier is reallocated to it. The larger the second term, the more likely the minimum rate requirement of i th link can be met. Therefore, we should choose the subcarrier with the maximum cost, i.e.

$$i^* = \arg \max_{i \in A_k^u} J_{i,n}. \quad (6.34)$$

If the minimum rate constraint for the j th link are not violated, perform subcarrier swapping by setting $\rho_{ki^*}^n = 1$ and $\rho_{kj}^n = 0$.

The resource allocation scheme at each MC k with reallocation procedure can be summarized as

Algorithm 6.2

1. *Equally distribute the transmit power over all subcarriers allocated to MC k and over all the active links communicating with MC k*
2. *For each subcarrier n allocated to MC k , choose the link according to (6.32)*
3. *For each link j communicating with MC k , compute the data rate $r_{k,j}$ of link (k, j)*
4. *If the minimum rate constraint for link (k, j) is violated and subcarrier n is not assigned to any other unsatisfied link (k, i)*
 41. *Compute the cost function using equation (6.33)*
 42. *Perform subcarrier swapping using equation (6.34)*
5. *Perform power allocation on each link (k, j) and each subcarrier n using equation (6.31)*

6.3.4 Simulation Results

In this section, we evaluate the performance of the proposed constrained resource allocation scheme through simulations. We consider a cluster with 4 wireless nodes with the scheduling tree topology shown in Fig. 6.3 and a total number of subcarriers $N = 128$ over a 1-MHz band. The relative effective SNR difference between MC 1 (the closest MC to the MR) and MC 2, 3, and 4 are 3dB, 6dB, and 10 dB respectively. The minimum rate requirements are chosen to be the same for all MCs, the maximal power at each MC k is $\bar{p}_k = 50mW$, the thermal noise power is $\sigma_n^2 = 10^{-11}W$. For the CDI-based allocation at MR, we assume an exponential distribution of α_k .

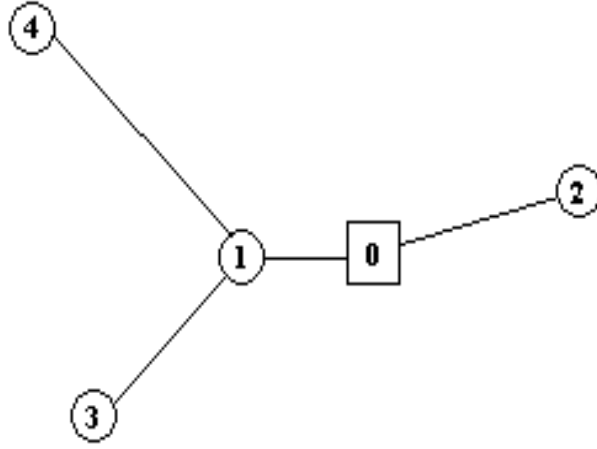


Figure 6.3: Network topology used in the simulations

We name the proposed scheme with optimal allocation at MR and MCs as *full optimal resource allocation (FORA)* and the scheme using optimal allocation at MR and sub-optimal reallocation procedure (6.33) at MCs as *MC-based sub-optimal resource allocation (SORA)*. For comparison, we also implement the following resource allocation schemes:

1. *MR-based uniform resource allocation (MURA)* where each MC is assigned the same number of subcarriers and the proposed optimal allocation is performed at each MC.
2. *Full uniform resource allocation (FURA)* where each MC is assigned the same number of subcarriers and transmit power at each MC is uniformly distributed over the assigned subcarriers and the active links.

We evaluate system performance in terms of sum rate, and satisfaction of minimum rate requirements.

In Fig.6.4, the performance of the proposed optimal and sub-optimal resource allocation (FORA and SORA) is compared to that of the uniform resource allocation at MR under optimal power allocation at MCs (MURA) and the full uniform allocation (FURA). The result shows that the proposed optimal and sub-optimal resource allocations bring significant gain over uniform resource allocation, especially for low SNRs. We also observe that the achievable sum rate obtained with the sub-optimal algorithm based on reallocation at MCs is closer to the optimal one.

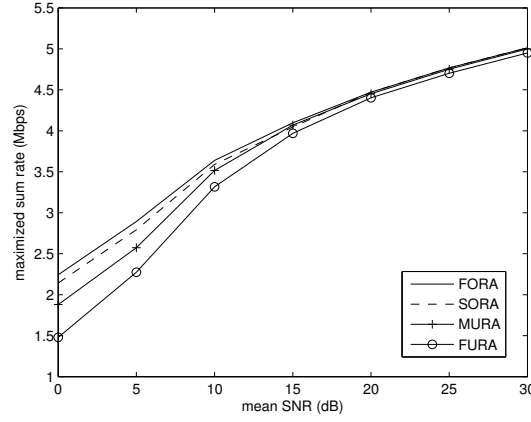


Figure 6.4: Maximized sum rate versus mean SNR for various resource allocation schemes

Fig. 6.5 shows the user's rate for different allocation schemes when the users minimum data rate demands are constrained to $R_k = 1Mbps$ for all MCs. We observe that under optimal and sub-optimal allocation, the need of all users in terms of data rate is satisfied. This is not the case under uniform allocation. With uniform allocation, there is an over-allocation for closer MCs to the MR (MCs 1 and 2) while the rate demand of farer users with bad channel conditions (MCs 3 and 4) are not satisfied.

6.4 CDIT-based Power Allocation for Relay Networks

6.4.1 Problem Formulation

Consider the relay network operating in receiver cooperation mode as illustrated in Fig. 6.6. The transmitter at the source node sends a signal x . Let y_1 denote the received signal at the relay node. We assume that the relay node operates in the full duplex mode, i.e. the relay can receive and transmit simultaneously on the same frequency channel [93]. Thus, the received signals at the relay node and the destination node are given by

$$\begin{aligned} y_1 &= h_2x + z_1 \\ y &= h_1x + \sqrt{g}h_3x_1 + z, \end{aligned} \tag{6.35}$$

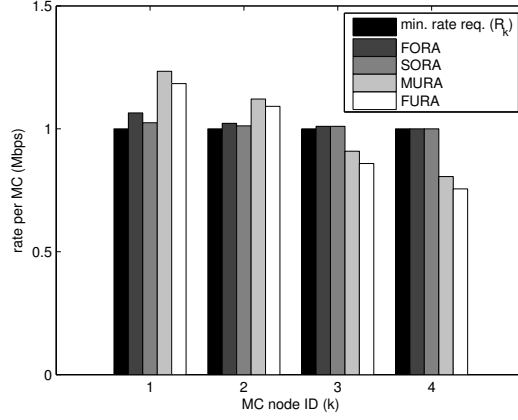


Figure 6.5: Per mesh client rate for $R_k=1\text{Mbps}$

where z_1 and z are independent identically distributed (i.i.d) zero mean circularly symmetric complex Gaussian (ZMCSCG) additive noise with unit variance.

According to [93], the capacity cut set bound of the relay network of Fig. 6.6 operating in a full duplex mode with perfect CSI can be expressed as

$$C_{inst} = \max_{0 \leq \rho, \beta \leq 1} \min \left\{ \frac{1}{2} \log_2 (1 + \beta P (1 - \rho^2) (\gamma_1 + \gamma_2)) , \right. \\ \left. \frac{1}{2} \log_2 \left(1 + \beta P \gamma_1 + \left((1 - \beta) g + 2\rho \sqrt{\beta (1 - \beta) g} \right) P \gamma_3 \right) \right\}, \quad (6.36)$$

where ρ represents the correlation between the transmit signals of the transmitter and the relay, and $\gamma_i = |h_i|^2$.

We assume Rayleigh fading where the channel gain h_i , $i = 1, 2, 3$, is i.i.d. ZMCSCG with unit variance; hence the corresponding channel power gain is i.i.d. exponential with unit mean. The average channel power gain between the relay and the receiver is then g . We assume that g characterizes only path-loss attenuation, hence $g = 1/d^\alpha$, where d is the distance between the relay node and the receiver node and α is the path-loss power attenuation exponent. As in receiver cooperation mode the relay is assumed to be close to the receiver, the scenario of interest is when d is small.

We consider a fast fading environment, where the receiver has CSI to perform coherent detection, but there is no fast feedback link to convey the CSI to the transmitter. Hence the transmitter has only CDI, but no knowledge

of the instantaneous channel realizations. We assume the duration of the transmission codewords is long enough to undergo all channel realizations, and thus ergodic capacity is used to characterize the transmission rate of the channel.

We assume the channel has unit bandwidth. We further assume an average network power constraint on the system:

$$E \left[|x|^2 + |x_1|^2 \right] \leq P, \quad (6.37)$$

where the expectation is taken over repeated channel uses.

The total power P is optimally allocated between the transmitter and the relay, i.e.:

$$E \left[|x|^2 \right] \leq \beta P, \quad E \left[|x_1|^2 \right] \leq (1 - \beta)P, \quad (6.38)$$

where $\beta \in [0, 1]$ is parameter to be optimized based on CDI and node geometry g .

It has been shown in [93] that the capacity upper bound in the asynchronous channel model, i.e. the channel model where the nodes don't have complete CSI, can be found by setting the correlation ρ to zero. Since the CDI channel model falls into this case, the ergodic capacity upper bound can be found by taking the expectation of (6.36) over the channel distribution and setting $\rho = 0$. Making use of the high SNR ($P \gg 1$) approximation $\log(1 + xP) \approx \log(xP)$, the ergodic capacity upper bound is then given by

$$C_{erg} = \max_{0 \leq \beta \leq 1} \min \left\{ E \left[\log_2 (\beta P (\gamma_1 + \gamma_2)) \right], \right. \\ \left. E \left[\log_2 (\beta P \gamma_1 + (1 - \beta) P g \gamma_3) \right] \right\}. \quad (6.39)$$

The problem is to find the optimal power allocation, i.e. the optimal value of β , that gives the capacity upper bound $C_{erg}(\beta)$ of (6.39). Mathematically the power allocation problem can be formulated as

$$\max_{0 \leq \beta \leq 1} \min \left\{ E \left[\log_2 (\beta P (\gamma_1 + \gamma_2)) \right], \right. \\ \left. E \left[\log_2 (\beta P \gamma_1 + (1 - \beta) P g \gamma_3) \right] \right\}. \quad (6.40)$$

6.4.2 Solution Method

To find the optimal power allocation, we propose an approach that uses the Lambert-W function.

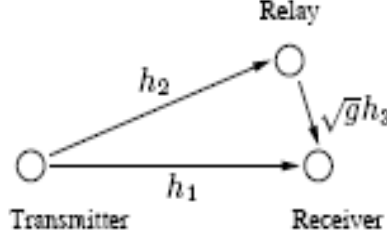


Figure 6.6: Illustration of a relay network operating in receiver cooperation mode.

First we need to evaluate the expected value of the capacity expression over the channel fading distribution. For this end, we make use of the following formula for i.i.d. exponential random variables X_1, X_2 with unit mean:

$$E[\log(a_1 X_1 + a_2 X_2)] = \begin{cases} \frac{a_1 \log a_1 - a_2 \log a_2}{a_1 - a_2} - \log e^\gamma & \text{if } a_1 \neq a_2 \\ \log a_1 + \log e^{1-\gamma} & \text{if } a_1 = a_2, \end{cases} \quad (6.41)$$

where a_1, a_2 are positive scalar constants and γ is Euler's constant.

Applying formula (6.41), the first term inside the $\min\{\cdot\}$ in expression (6.39) is given by

$$E[\log_2(\beta P(\gamma_1 + \gamma_2))] = \log_2 P + \log_2 \beta + \log_2 e^{1-\gamma}, \quad (6.42)$$

while the second term evaluates to

$$E[\log_2(\beta P \gamma_1 + (1 - \beta) P g \gamma_3)] = \log_2 P + \frac{g(1 - \beta) \log_2(g(1 - \beta)) - \beta \log_2 \beta}{g(1 - \beta) - \beta} - \log_2 e^\gamma. \quad (6.43)$$

Expression (6.42) is an increasing function of β . It is easy to show that expression (6.43) is a decreasing function of β (for g of interest, i.e. $g > 1$). Thus the optimal value β^* solution of the maximization problem (6.40) can be found by equating expressions (6.42) and (6.43) as

$$\log_2 P + \log_2 \beta^* + \log_2 e^{1-\gamma} = \log_2 P + \frac{g(1 - \beta^*) \log_2(g(1 - \beta^*)) - \beta^* \log_2 \beta^*}{g(1 - \beta^*) - \beta^*} - \log_2 e^\gamma. \quad (6.44)$$

Equation (6.44) is equivalent to

$$\ln(\beta^*) + 1 = \frac{g(1 - \beta^*) \ln(g(1 - \beta^*)) - \beta^* \ln(\beta^*)}{g(1 - \beta^*) - \beta^*}, \quad (6.45)$$

where $\ln(x)$ is the natural logarithm of x .

After some algebraic manipulations, (6.45) can be transformed into

$$\frac{-\beta^*}{g(1 - \beta^*)} \exp\left(\frac{-\beta^*}{g(1 - \beta^*)}\right) = -\frac{1}{e}. \quad (6.46)$$

It can be recognized that equation (6.46) is in the form of a transcendental equation (6.1). Thus we have

$$W\left(-\frac{1}{e}\right) = \frac{-\beta^*}{g(1 - \beta^*)}. \quad (6.47)$$

The optimal value β^* is deduced from (6.47) as

$$\beta^* = \frac{Kg}{Kg + 1}, \quad (6.48)$$

where $K = -W\left(-\frac{1}{e}\right) = 1$.

It is easy to verify that $0 \leq \beta^* \leq 1$.

Interesting is to observe that the optimal power allocation is obtained in closed-form and depends only on g , i.e. on the distance d between the relay node and the destination node and the path-loss power attenuation exponent α .

6.4.3 Comparison with CSIT-based Power Allocation

In order to assert the relevance of the CDIT-based approach, it has to be compared to the allocation scheme based on perfect CSIT. Perfect CSIT is unrealistic, but for the purpose of comparison, let assume perfect CSIT. Then the power allocation can be formulated to maximize the instantaneous capacity instead of the ergodic capacity. Mathematically, the CSIT-based power allocation can be formulated as

$$\max_{0 \leq \rho, \beta \leq 1} \min \left\{ \frac{1}{2} \log_2 (1 + \beta P (1 - \rho^2) (\gamma_1 + \gamma_2)), \right. \\ \left. \frac{1}{2} \log_2 \left(1 + \beta P \gamma_1 + \left((1 - \beta) g + 2\rho \sqrt{\beta(1 - \beta)g} \right) P \gamma_3 \right) \right\}. \quad (6.49)$$

The optimal values of ρ and β solution of (6.49) have been found in [96] as

$$\rho^* = \frac{1}{\sqrt{g^2 + 2g + 2}}, \quad (6.50)$$

$$\beta^* = \frac{g^2 + 2g + 2}{g^2 + 3g + 2}. \quad (6.51)$$

The instantaneous capacity upper bound for high SNR regime is deduced as

$$C_{inst} = \log_2 \left(\frac{2(g+1)}{g+2} P \right). \quad (6.52)$$

Both CDIT-based and CSIT-based optimal power allocation expressions (6.48) and (6.51) are in closed-form and very fast to compute. Thus, the complexity is almost the same. The main difference between the two allocation schemes is the amount of feedback required to perform power allocation. Recall that, for the CSIT-based scheme, the allocation is performed after each symbol period. Let N_s be the number of symbol periods after which the CDIT-based resource allocation is performed. Then a rough estimation tells us that the feedback needed to perform CDIT-based power allocation is reduced by $\frac{1}{N_s}$ compared to the perfect CSIT scheme.

6.4.4 Numerical Results

Numerical results are provided to evaluate the performance of the proposed CDIT-based power allocation scheme in terms of achieved capacity. In all the simulations, we assume a path-loss power attenuation exponent of 2, and hence $g = 1/d^2$. The distance d between the relay node and the receiver node varies from 0.1 to 1.

In Fig. 6.7, the ergodic capacity achieved using the proposed power allocation scheme is compared to the one obtained with uniform power allocation ($\beta = 0.5, \forall d \in [0.1, 1]$). We consider system average network power constraints of $P = 10$ and $P = 100$. It can be observed that the achieved capacity using the proposed optimal power allocation method outperforms the capacity obtained with uniform allocation.

Fig. 6.8 illustrates the achieved capacity using the proposed CDIT-based optimal power allocation (6.48) in comparison with the capacity of the CSIT-based optimal power allocation (6.51). The CSIT-based capacity is averaged over the same number of channel realizations N_s over which the distribution is taken to evaluate the ergodic capacity. The result shows that the gap between the average capacity and the ergodic capacity is small.

Thus, even with CDIT only, optimal power allocation improves performance of relay networks.

The trade-off between reduced feedback and performance degradation of the proposed CDIT-based optimal power allocation in comparison with the perfect CSIT-based optimal power allocation is shown in Fig.6.9. We observe that adapting the transmission strategy to the short-term channel statistics, increases the performance but also increases the amount of feedback. However, if the transmission is adapted to the long term channel statistics, the amount of feedback decreases significantly but with a penalty on the performance. For a CDIT-based allocation with a distribution taken over 16 symbol periods, the amount of feedback is reduced by 93.75% while the performance degradation in terms of capacity is less than 12%.

6.5 Conclusion

In this chapter, we have addressed constrained resource allocation problems for wireless mesh and relay networks. For mesh networks, we have formulated a distributed subcarrier allocation problem to maximize the sum rate while satisfying minimum rate demand. For relay networks, we have formulate power allocation problem to maximize the ergodic capacity under total power constraint. Both cases of perfect and statistical channel knowledge at the transmitter have been considered. We have shown how the Lambert-W function can be use to efficiently find the optimal resource allocation in closed-form. By using the Lambert-W function, resource allocation can quickly be determined since a number of popular mathematical softwares, including Maple and Matlab, contain the Lambert-W function as a component. The Lambert-W function can be combined with the Lagrange dual approach to solve variety of wired and wireless resource allocation problems without resorting to complex numerical algorithms.

Simulation results have been provided to compare the performance of the proposed scheme with other allocation schemes.

Looking at the proposed solution methods based on the Lambert-W function, we can predict that the Lambert-W function can be very useful for solving a variety of constrained resource allocation problems in wireless communications.

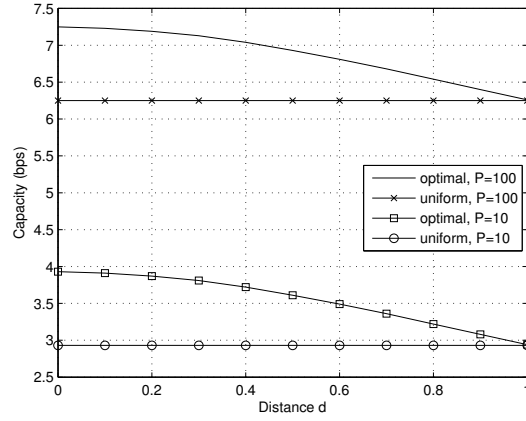


Figure 6.7: *Maximized capacity vs. distance d for CDIT-based optimal power allocation and uniform power allocation.*

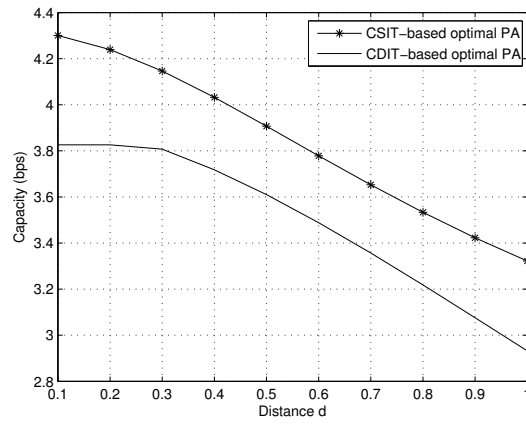


Figure 6.8: *Maximized capacity vs. distance d for CDIT-based optimal power allocation and CSIT-based optimal power allocation*

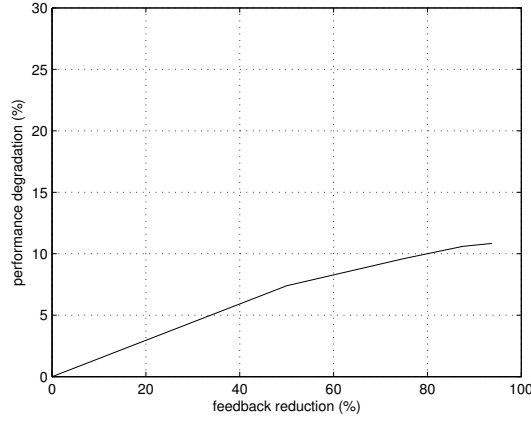


Figure 6.9: Trade-off between feedback reduction and performance degradation of the CDI-based allocation compared with the perfect CSIT-based allocation.

6.6 Appendix

6.6.1 Appendix: Derivation of (6.9)

Equation (6.8) can be rewritten as

$$\ln \left(\frac{n_k}{n_k + \alpha_k} \right) - \frac{n_k}{n_k + \alpha_k} + 1 + \frac{\mu}{\frac{B}{\ln 2} + \lambda_k} = 0. \quad (6.53)$$

Equation (6.53) can be transformed into

$$-\frac{n_k}{n_k + \alpha_k} e^{\left(-\frac{n_k}{n_k + \alpha_k}\right)} = -\exp \left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k} \right) \quad (6.54)$$

Expression (6.54) is in the form of the *Lambert-W* function $W(x)$, which is the solution to $W(x)e^{W(x)} = x$. Thus, from (6.54) we can deduce

$$W \left(-\exp \left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k} \right) \right) = -\frac{n_k}{n_k + \alpha_k} \quad (6.55)$$

which when solved for n_k gives (6.9).

6.6.2 Appendix: Derivation of (6.11)

Let $w_k(\lambda_k, \mu) = W\left(-\exp\left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k}\right)\right)$.

Replacing n_k in (6.7) by its optimal obtained in (6.9), we deduce the dual function as

$$\begin{aligned} L(\lambda_k, \mu) &= \sum_{k=1}^K \frac{-\alpha_k w_k(\lambda_k, \mu)}{1 + w_k(\lambda_k, \mu)} B \log_2 \left(-\frac{1}{w_k(\lambda_k, \mu)} \right) \\ &\quad + \sum_{k=1}^K \lambda_k \left(\frac{-\alpha_k w_k(\lambda_k, \mu)}{1 + w_k(\lambda_k, \mu)} B \log_2 \left(-\frac{1}{w_k(\lambda_k, \mu)} \right) - R_k \right) \\ &\quad - \mu \left(\sum_{k=1}^K \frac{-\alpha_k w_k(\lambda_k, \mu)}{1 + w_k(\lambda_k, \mu)} - N \right). \end{aligned} \quad (6.56)$$

Expression (6.56) can be rewritten as

$$\begin{aligned} L(\lambda_k, \mu) &= \sum_{k=1}^K \left((\lambda_k + 1) \frac{\alpha_k w_k(\lambda_k, \mu)}{1 + w_k(\lambda_k, \mu)} B \log_2(-w_k(\lambda_k, \mu)) \right) \\ &\quad - \mu \sum_{k=1}^K \frac{-\alpha_k w_k(\lambda_k, \mu)}{1 + w_k(\lambda_k, \mu)} - \sum_{k=1}^K \lambda_k R_k + \mu N \end{aligned} \quad (6.57)$$

Applying (6.2) for $n = 1$ and using the property of the derivative of a composite function, we obtain the derivative of w_k with respect to λ_k for fixed μ as

$$\frac{dw_k(\lambda_k, \mu)}{d\lambda_k} = \frac{w_k(\lambda_k, \mu)}{1 + w_k(\lambda_k, \mu)} \cdot \frac{\mu \ln 2}{B} \cdot \frac{1}{(1 + \lambda_k)^2}. \quad (6.58)$$

Using (6.58), we calculate the derivative of $L(\lambda_k, \mu)$ with respect to λ_k for fixed μ as

$$\begin{aligned}
 \frac{dL(\lambda_k, \mu)}{d\lambda_k} = & -R_k + \frac{B}{\ln 2} \cdot \frac{\alpha_k w_k(\lambda_k, \mu)}{1 + w_k(\lambda_k, \mu)} \cdot \ln(-w_k(\lambda_k, \mu)) \\
 & + \frac{\mu}{1 + \lambda_k} \cdot \frac{\alpha_k w_k(\lambda_k, \mu)}{1 + w_k(\lambda_k, \mu)} \cdot \ln(-w_k(\lambda_k, \mu)) \\
 & + \frac{\mu}{1 + \lambda_k} \cdot \frac{\alpha_k w_k(\lambda_k, \mu)}{(1 + w_k(\lambda_k, \mu))^2} \\
 & + \frac{\mu^2 \ln 2}{B(1 + \lambda_k)^2} \cdot \frac{\alpha_k w_k(\lambda_k, \mu)}{(1 + w_k(\lambda_k, \mu))^3}.
 \end{aligned} \tag{6.59}$$

Applying KKT optimality conditions, we set the derivative (6.59) to zero to obtain

$$\begin{aligned}
 -\frac{R_k}{\alpha_k} + \frac{B}{\ln 2} \cdot \frac{w_k(\lambda_k^*, \mu)}{1 + w_k(\lambda_k^*, \mu)} \cdot \ln(-w_k(\lambda_k^*, \mu)) \\
 + \frac{\mu}{1 + \lambda_k^*} \cdot \frac{w_k(\lambda_k^*, \mu)}{1 + w_k(\lambda_k^*, \mu)} \cdot \ln(-w_k(\lambda_k^*, \mu)) \\
 + \frac{\mu}{1 + \lambda_k^*} \cdot \frac{w_k(\lambda_k^*, \mu)}{(1 + w_k(\lambda_k^*, \mu))^2} \\
 + \frac{\mu^2 \ln 2}{B(1 + \lambda_k^*)^2} \cdot \frac{w_k(\lambda_k^*, \mu)}{(1 + w_k(\lambda_k^*, \mu))^3} = 0,
 \end{aligned} \tag{6.60}$$

where λ_k^* is the optimal value of λ_k .

We see that equation (6.60) can be rewritten in the form of equation (6.11).

6.6.3 Appendix: Proof of (6.15)

Let

$$g_{k,m}(\mu) = \frac{-\alpha_k \tilde{w}_k(\mu)}{1 + \tilde{w}_k(\mu)}$$

where

$$\tilde{w}_k(\mu) = W \left(-\exp \left(-1 - \frac{\mu}{\frac{B}{\ln 2} + \lambda_k^*} \right) \right).$$

The derivative of $\tilde{w}_k$ with respect to μ is given by

$$\frac{d\tilde{w}_k(\mu)}{d\mu} = \frac{w_k(\mu)}{1 + w_k(\mu)} \cdot \left(-\frac{1}{\frac{B}{\ln 2} + \lambda_k^*} \right). \tag{6.61}$$

Using (6.61), we can calculate the derivative of $g_{k,m}(\mu)$ with respect to μ as

$$\frac{dg_{k,m}(\mu)}{d\mu} = \frac{\alpha_k \cdot \frac{w_k(\mu)}{1+w_k(\mu)} \cdot \left(-\frac{1}{\frac{B}{\ln 2} + \lambda_k^*}\right)}{(1+w_k(\mu))^2} < 0. \quad (6.62)$$

Thus

$$\frac{dg_m(\mu)}{d\mu} = \sum_{k=1}^K \frac{dg_{k,m}(\mu)}{d\mu} < 0. \quad (6.63)$$

namely, $g_m(\mu)$ is a strictly decreasing function of μ . Then $g_m(\mu) = N$ gives $\mu = g_m^{-1}(N)$. This completes the proof.

Chapter 7

Conclusions

7.1 Summary

The goal of this thesis was to investigate resource allocation strategies for wireless multiuser multicarrier systems with various constraints and users requirements. We have addressed several problems associated with the optimization of the achievable sum-rate for downlink transmission under minimum rate requirement, fairness and power constraints. The main contributions of this thesis are summarized as follow:

We have developed multiuser power allocation algorithms in downlink OFDM-CDM system with the objective to maximize the sum-rate of the system subject to total transmit power and fairness constraints. We have extended the study to MIMO-OFDM-CDM with transmit beamforming. For the purpose of simplicity, we have assumed the availability of perfect CSIT at the transmitter. We have evaluated the performances of the proposed algorithms in terms of fairness and analyzed the trade-off between achievable sum-rate and the fairness requirements. This work formed the basis of our research and helped us to model more practical problems.

We have formulated more practical resource allocation problems for downlink OFDMA systems under minimum rate requirement, fairness and power constraints. Instead of perfect CSIT assumption, a realistic assumption of the availability of statistical channel information has been made. To solve the underlying stochastic constrained optimization problem, we have proposed various efficient algorithms based on the Lagrange dual framework. In OFDMA resource allocation problems studied in this thesis, the objective function is separable across the subcarriers, and the number of constraints are in the order of the number of users. This makes dual optimization

techniques an ideal approach to solve them, since the duality gap is zero in these problems as shown in 5.4.4. Furthermore, we saw that the solution to the dual problem involves very simple closed-form power and subcarrier allocation and its complexity is of $\mathcal{O}(KM)$ for K users and M subchannels. We have also shown that with statistical channel information in terms of channel distribution information, the amount of feedback required for the resource allocation reduces significantly. We have analyzed the trade-off between reduced complexity and performance degradation of the proposed resource allocation schemes compared with the perfect CSIT based scheme.

We have also formulated a constrained resource allocation problem for OFDMA-based WMNs. To efficiently solve the problem, we have proposed an approach that combines the merits of the Lagrange dual approach and the Lambert-W function. We saw that, by using the Lambert-W function, resource allocation can not only be expressed in closed form but it can also quickly be determined without resorting to complex algorithms since a number of popular mathematical softwares, including Matlab, contain the Lambert-W function as a component.

We can conclude that, in fading environments, even with statistical channel information only, optimized resource allocation strategies provide performance gain for wireless networks in terms of achieved sum-rate, fairness, QoS and computational complexity.

7.2 Future Work

In the prolongation of this thesis, several interesting research directions deserve future works.

7.2.1 Other Formulation

Goodput maximization

This thesis focused on throughput maximization problems. This criterion is appropriate for applications like speech or video transmission. However, some applications like data file transfer require that all bits be transmitted correctly. For such applications, we would like to maximize the goodput instead. The goodput can be defined to as the number of information bits delivered without error to the user by unit of time. Recently, the problem of bits and power allocation for goodput maximization in coded OFDM

systems with automatic repeat request (ARQ) mechanism has been addressed in [104]. The authors of [104] assumed perfect CSIT. It is interesting to study if the optimization approaches developed in our thesis can still be used in multiuser goodput maximization problem for OFDMA systems when only statistical channel information is available at the transmitter.

Power or BER minimization

Although this thesis focused on throughput maximization problems, the developed methods can be applied to power/BER minimization. Since the average power and average BER for OFDMA systems are similarly separable objectives across the subcarriers, a similar dual optimization approach can be used to find the optimal resource allocation.

7.2.2 MIMO-OFDMA

In Chapter 4 of this thesis, we addressed the problem of power allocation for MIMO-OFDM-CDM systems with transmit beamforming. For the OFDMA case, we have focused on systems with single transmit and receive antennas. However, MIMO techniques have been implemented in OFDMA-based standards such as IEEE 802.16e. Extending the optimization strategies proposed in this thesis to MIMO-OFDMA-based systems is certainly an interesting problem. The algorithms and methods proposed in this thesis for OFDMA can be extended in straightforward manner to MIMO-OFDMA with transmit beamforming.

7.2.3 Multi-cell OFDMA

In this thesis, we focused on the single-cell scenario, and ignored the effect of inter-cell interferences. In practical networks, inter-cell interferences significantly affect the performance, and thus should be properly controlled, avoided, and/or canceled. Although centralized control may not be feasible from a practical perspective, a theoretical study of the sum-rate assuming that all resource allocation across cells can be performed centrally is interesting. In this case, the dual optimization framework is still applicable, however, the power allocation procedure would involve a non-convex per-user and per-subcarrier non-linear program. Some recent works addressed this issue, assuming perfect CSIT (see e.g. [105] and [106]).

A more interesting research study would be in the semi-coordinated case, wherein a small amount of information, e.g. the loading of the cell or the total interference power experienced by a neighboring cell, is exchanged across

base stations. In this case, we can use the dual optimization approach, with the addition of a *penalty* term that is a function of the information that is exchanged, such that the marginal dual is decreased for problematic users, e.g. users that are in the cell-edge.

7.2.4 Multi-relay networks

As mentioned in Chapter 6, an interesting method to increase the coverage of wireless networks is the concept of cooperative communications. In this thesis, we focused on wireless mesh networks and one-relay networks. However, another attractive relay network topology is the multi-relay networks where multiple relay nodes are used to forward the transmission from the source node to the destination node. Constrained resource allocation for multi-relay networks is more challenging due to the complex form of the capacity or the ergodic capacity. The extension of the optimization tools used in this thesis to multi-relay networks remains a topic of interest.

Curriculum vitae

Education

1988-1993 : Master of Engineering in Electronics and Communications (first class honor), ENSET, University of Douala, Cameroon.

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List of publications

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- F. Brah, J. Louveaux, and L. Vandendorpe, "CDIT-based constrained resource allocation for mobile WiMAX systems," *Eurasip Journal on Wireless Communications and Networking*, Vol. 2009, pp. 1-8, Jan. 2009.

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