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Group 1 : Genus 3 Trisections

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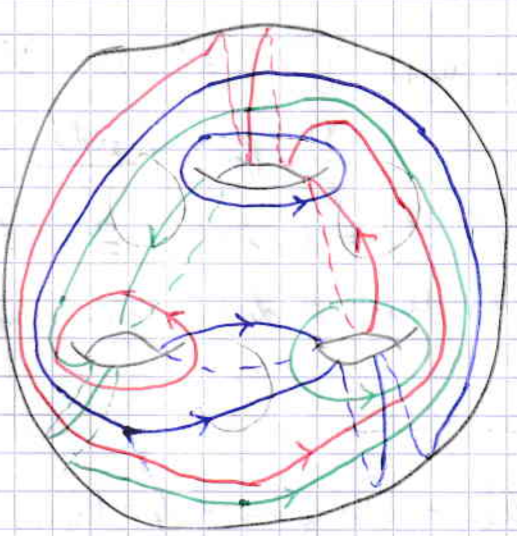
Réf : [Meier, Trisections and spun 4-manifolds]

(Remark 2.5)

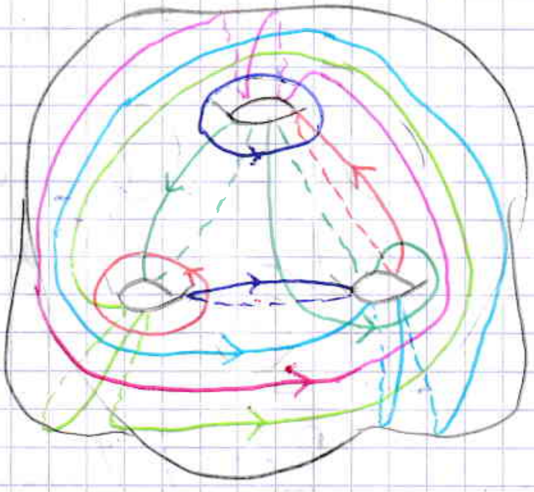
Twisted Spun lens Space $\xleftarrow{\text{Gluck Twist of first type}}$ Spun lens Space $\xrightarrow{\text{Gluck Twist of second type}}$ Spun lens Space with a specific Gluck Twist

$S^*(L(p,q)) \cong S(L(p,q))$, both $\cong S_p$

S'_p



$\begin{pmatrix} p=2 \\ q=1 \end{pmatrix}$



Regular 6-chain + curves $(p/q, p/q, p/q)$

Twisted 6-chain + curves $(p/q, p/q, p/q)$

Conj : Let X^4 be a irreducible 4-manifold with a genus 3 trisection $(3,1)$ more precisely), then X is one of the previous S_p or S'_p for $p \in \mathbb{N}^*$.

Th : (Pao, Prop 2.3)

p odd $\Rightarrow$ $S'_p \cong S_p$

p even $\Rightarrow$ $S'_p \not\simeq S_p$ (not homotopy equivalent)

- $\chi(S_p) = 2$

Facts:
- $\pi_1(S_p) = \mathbb{Z}/p\mathbb{Z}$
- $H_2(S_p) = \mathbb{Z}/p\mathbb{Z}$
  ($\Rightarrow$ no intersection form)
- $\widetilde{S_p} = \#^{p-1}(S^2 \times S^2)$ (universal cover and p-fold)

Question:
- $\chi(S'_p) = ?$

For p even,
- $\pi_1(S'_p) = ?$
- $H_2(S'_p) = ?$
- $\widetilde{S'_p} = ?$

Questions:
- Do we have $\widetilde{S'_p} = \#^{p-1}(S^2 \tilde{\times} S^2)$?
- Do we have $S^2 \tilde{\times} S^2 = \mathbb{C}P^2 \# \overline{\mathbb{C}P^2}$?
- Does this check out with $\widetilde{S_p} = \#^{p-1}(S^2 \times S^2)$
  $\cong$ $\widetilde{S'_p}$ for p odd ?
  - Do we have $(S^2 \tilde{\times} S^2) \# (S^2 \tilde{\times} S^2) \cong (S^2 \times S^2) \# (S^2 \times S^2)$?
    - Does this imply that $\#$ decomposition is not unique ?

Ref: [Aranda, Moeller - Diagrams of $*$-trisections, Section 7]

A diagram with a regular 6-chain and three slopes $\frac{a}{b}, \frac{c}{d}, \frac{p}{q}$ (at Farey distance ± 1) trisects a manifold X:
- $X \simeq S_p$ if $\left(\frac{p}{q}, \frac{p}{q}, \frac{p}{q}\right)$
- X reducible (i.e. $\#$ of $S^1 \times S^3, S^2 \times S^2, \mathbb{C}P^2, \overline{\mathbb{C}P^2}$) if at least two slopes are distinct.

Project: Generalise this to twisted 6-chains with
- $\left(\frac{p}{q}, \frac{p}{q}, \frac{p}{q}\right)$ and p even (and $q=1$), i.e. describe S'_p better
- other cases $\left(\frac{a}{b}, \frac{c}{d}, \frac{p}{q}\right)$: Is X still reducible ?
  (If not, it may disprove the conjecture)

<u>Idea</u>: From a <u>given</u> candidate X^4 with a $(3,1)$-trisection T

- Compute $\pi_1(X)$ (known to be cyclic) from T
  - If $\pi_1(X) = \mathbb{Z}$, then X is not S_p nor S'_p and disproves the conjecture. ✓
  - If $\pi_1(X) = \mathbb{Z}/p\mathbb{Z}$, search if $X = S_p$ or $X = S'_p$
- Compute a trisection $\tilde{T}$ of $\tilde{X}$ (p-fold and universal cover of X)

  (How ??)
- From $\tilde{T}$, compute invariants of $\tilde{X}$ such ~~as~~ $H_2(\tilde{X})$, Intersection Form ... to <u>recognise</u> $\tilde{X}$
  - If $\tilde{X} \neq \#^{p-1}(S^2 \times S^2) = \tilde{S_p}$ and $\tilde{X} \neq \tilde{S'_p}$ $(= ??)$, then X disproves the conjecture
  - If $\tilde{X} = \tilde{S_p}$ or $\tilde{X} = \tilde{S'_p}$, can you conclude somehow by re-quotienting that $X = S_p$ or $X = S'_p$?

    (No idea)

<u>Related Goal</u>: Find <u>promising examples</u>

- Twisted 6-chain with Farey triplet $\neq S'_p$
- $(3,1)$ trisections obtained by double branched covers on spin of 2-bridge knots.
- .... ?