TRANSACTIONS DEMAND FOR MONEY AND CONSUMER BEHAVIOR

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I. INTRODUCTION

This paper is the result of efforts to improve upon the construction and estimation of dynamic demand systems by introducing the transactions demand for money into these. It is hoped that it will also provide a better framework for the estimation of the latter and the testing of related hypotheses such as the real-balance effect, the absence of money illusion and the interest-elasticy of the transactions demand for cash.

Patinkin has worked out the first satisfactory integration of monetary and demand theory. Using an intertemporal framework, he allowed money to enter into the utility function, arguing that the security which money balances provide against the inconveniences resulting from running out of cash invest them with utility. Simultaneously an alternative inventory theoretic approach, based on the work of Baumol and Tobin (1956), has developed. Here, the transactions demand for money is rationalized as if money were a producer's good. This theory seems to have gained a wide acceptance, although it presents the obvious deficiency of treating consumption expenditures as exogenously given and therefore unexplained. Recently, C. Lluch and M. Morishima, have tried to remedy this deficiency, without abandoning the idea that the demand of money has to be explained by the objective cost of transforming "bonds" into money rather than by the subjective bother of so doing. Although these efforts are promising, they do not seem to be too rewarding from the point of view of the construction and estimation of demand systems.

Given that the two approaches are operationally equivalent, in the sense that they cannot be identified from the econometric point of view (Patinkin (1965, p. 160)), I feel entitled to revert to the utility theory approach, the more so as it permits to make use of the tool box developed by demand analysts. In doing so, this paper will also try to introduce recent techniques of intertemporal analysis based on optimal control theory. These techniques provide a handsome way not only to formalize Patinkin's ideas, as emphasized by Douglas, but also to derive a system of dynamic demand equations which lend themselves easily to econometric implementation.

Although money is endowed with utility, so that the demand for it can be treated formally as resulting from utility maximization, it should be clear that specific substitution and complementarity relationships between money and commodities have to be ruled out, in general. The utility function for money will therefore be treated as a branch utility function which is strongly separable from the branch utility function of the commodities. This is, of course, in direct contradiction with the usual interpretation of the Patinkin approach, such as in Lloyd and Dusansky and Kalman.

Strong separability is also a desirable property to the extent that, as a matter of research strategy,one wants to arrive at results that are operationally equivalent with those of alternative approaches in which money does not enter the utility function, and not too far away from those of standard demand theory. Lloyd and Berglas and Razin have shown that with weak separability virtually all of the empirical implications of traditional demand theory may be reinstated for the commodity demand equations. With strong separability, all "general" restrictions of demand theory, which we call "Slutsky conditions' after Barten (1967), remain valid without modifications for all demand equations (for commodities <u>and</u> for cash balances).

Strong separability further makes it possible to answer the question whether the presence of cash balances <u>as such</u> implies "money illusion", independently of possible interactions between money and commodities. Given that the homogeneity restriction is now defined over all commodities <u>and</u> money, the answer is : yes.

Observed cash balances are nominal balances. These are the ones to be predicted. Nevertheless, one wants to reason in terms of real balances, to take effects of price changes on the demand for money, and effects of changes in real balances on the demand for commodities, into account. In the Patinkin approach, these effects are the result of the introduction of real balances (in fact nominal balances and all commodity prices) in the utility function (see e.g. Patinkin, Lloyd, Dusansky and Kalman, Berglas and Razin). In the approach developed below, only nominal balances appear in the utility function, to avoid possible confusion between the present issue (and the related problem of money illusion) and Veblen effects. It will be shown that the system resulting from utility maximization takes care of real balance effects, so that there is no need to include prices in the utility function if one prefers not to do so. In econometric applications, in which fairly aggregated data are used, this seems to be a desirable feature.

The present approach implies that the scope of portfolio theory is restricted to nonmonetary wealth items. In the general equilibrium approach to monetary theory worked out by Tobin (1969) , expenditure decisions are supposed to be separable from portfolio choices, where the latter include both money and non-liquid assets. Here, cash balances held for transactions purposes are taken out of the domain of portfolio choices and linked together with consumption expenditures in what one may call transactions choices. That this is not too unrealistic may be illustrated by the fact that many households have a checking account in one bank and a savings account in a savings bank across the street.

In the analysis that follows, prices and the rate of interest are supposed to be known with certainty, to exclude the precautionary and speculative motives for holding money. Wealth effects based on changes in the prices of durables will also be ignored to simplify the analysis.

II. THE MODEL

Let x be the n vector of flows of commodities purchased, m the nominal stock of money held for transactions purposes, p the n vector of prices of the commodities and r the rate of interest. To each quantity x_i corresponds a state variable s_i, measuring "stocks of habits" and/or physical stocks. To m corresponds a state variable w representing the consumer's (non human) wealth, i.e. his "net worth" including cash holdings.

The consumer's instantaneous utility function is defined as

$$(1) \qquad U = u\,(x,s) + z\,(m,w)$$

with the interpretation that only the elements of x and m contribute directly to utility, while the state variables s and w "dynamize" the utility function by pushing the marginal utilities of x_i and m upwards or downwards. When commidity i is habit forming, an increase in s_i leads to an increase in its marginal utility. When it is a durable, the effect is negative . As for m, an increase in wealth is likely to affect its marginal utility (z_m) in a way that has to be determined empirically. u and z are strongly separable. Both u and z are twice differentiable, u is quasi-concave and $z_{mm} = \partial^2 u/\partial m^2 < 0$.

The consumer is supposed to maximize, at each point in time, the integral of discounted utilities

$$(2) \qquad \int_0^\infty e^{-\gamma t} \left[\, u(x,s) + z\,(m,w) \,\right] dt$$

with respect to x and m, subject to the following n+1 constraints :

$$(3) \qquad \dot{s}_i = x_i - \delta_i\, s_i \qquad\qquad (i = 1,\ldots,n)$$

and

$$(4) \qquad \dot{w} = r\,(w - m) + Y - p'x$$
$$= (Y-y) + rw$$

where

(5) $y = p'x + rm.$

Equation (3) assumes s_i to depreciate at the constant exponential rate δ_i. Equation (4) is the wealth constraint utilized e.g. by Douglas. It takes all non-human wealth items into account and is more general than the traditional constraint saying that $p'x$ is equal to an initial endowment of cash balances plus a given "cash receipt" or "income". Y represents labor income, while y may be interpreted as "transactions income", i.e. that part of income which is set aside for transaction purposes. Both $\dot{s}_i$ and $\dot{w}$ may become negative, e.g. if $x_i < \delta_i s_i$ or $(Y - y) < 0$.

The model is based on the assumption that spending decisions (including the decision on how much cash to hold for spending purposes) are separable from portfolio decisions (on how to allocate $(w - m)$ among non-liquid assets). Its solution determines the quantities bought (x) and cash holdings (m) and therefore total consumption $(p'x)$. The state variables are determined by (3) and (4), while p, r and Y are exogenously given together with initial values for s and w. The rate of interest r reflects the composition of $(w - m)$ and is net of transaction costs.

From the point of view of optimal control theory, $n+1$ controls (x and m) are to be determined subject to the $n+1$ differential equations (3) and (4). The current value Hamiltonian is

(6) $H = u(x,s) + z\,(m,w) + \Psi'\dot{s} + \lambda\dot{w}$

where Ψ is an n vector of auxiliary variables measuring the current implicit value of an additional unit of s_i, while λ is the current marginal utility of savings $(\dot{w})$.

The Maximum Principle gives the necessary conditions

$$\text{(7a)} \quad \frac{\partial u}{\partial x_i} + \Psi_i = \lambda p_i$$

$$\text{(7b)} \quad \frac{\partial z}{\partial m} = \lambda r$$

$$\text{(8a)} \quad \dot{\Psi}_i = \delta_i \Psi_i - \frac{\partial H}{\partial s_i} = (\gamma + \delta_i) \Psi_i - \frac{\partial u}{\partial s_i}$$

$$\text{(8b)} \quad \dot{\lambda} = (\gamma - r)\lambda$$

together with (3), (4) and transversality conditions on Ψ and λ.

Conditions (7a) refer to the optimal quantities and are the same as those derived in Philips (1974, Chap. X). Condition (7b) formalizes the Fisherian idea that the marginal utility of holding money (normalized by the marginal utility of savings) should be equal to the rate of interest. Conditions (8a) imply that the change in the implicit price of a stock must be equal to minus its net contribution to utility $\left[\frac{\partial u}{\partial s_i} - (\delta_i + \gamma) \Psi_i\right]$. (8b) leads to the familiar Fisherian result that the rate of time preference (γ) must be equal to the rate of interest in long-run equilibrium (when $\dot{\lambda} = 0$).

The marginal utility of holding money ($\partial z/\partial m$) is assumed to be a positive function of the probable lack of syncronization between payments and receipts, and the consumer's subjective evaluation of the inconvenience of running short of cash. It is simultaneously affected by the importance of accumulated wealth, as explained earlier.

III. GENERALIZED SLUTSKY CONDITIONS

A continuous replanning in the sense of R. Strotz being permitted, the consumer is, at any point in time, at the beginning of an optimal plan. At a given point in time, stocks and wealth are given so that equations (3) and (4) - which determine net investments in stocks and savings - can be ignored. On the other hand, the value taken by λ, which is the initial value in equation (8b) , is determined by the system (7a), (7b) and (5). Indeed, equation (5) is an instantaneous constraint governing the allocation between total expenditures $p'x$ and cash holdings m. The instantaneous values of Ψ are determined by the given values of s. To derive the Slutsky conditions at one moment in time, we therefore have to use only equations (7a), (7b) and (5), written in self-explanatory matrix notation as

$$
\begin{aligned}
&u_x + \Psi = \lambda p \\
&z_m = \lambda r \\
&y = p'x + rm,
\end{aligned}
\tag{9}
$$

in which y, p, r and Ψ are given. As $y = Y + rw - \dot{w}$, Y, w (and the remainder $\dot{w}$) are also supposed to be given.

On solving (9) we get the system of n+2 equations

$$
\begin{aligned}
&x = x(y,p,r,s,w,\Psi) \\
&m = m(y,p,r,s,w,\Psi) \\
&\lambda = \lambda(y,p,r,s,w,\Psi)
\end{aligned}
\tag{10}
$$

The total differential of the vector of marginal utilities u_x is $du_x = Udx + Vds$, where U is the matrix of second-order derivatives of u and x

$$
V = \left[\frac{\partial^2 u}{\partial x_i \ \partial s_j}\right] . \qquad (i,j = 1,\dots,n)
$$

Total differentiation of (9) gives, after rearrangement,

$$(11)\quad \left[\begin{array}{cc|c} U & o & p \\ 0' & z_{mm} & r \\ \hline p' & r & o \end{array}\right] \begin{bmatrix} dx \\ dm \\ -d\lambda \end{bmatrix} = \begin{bmatrix} \theta & \lambda I & o & -V & o & -I \\ o & 0' & \lambda & 0' & -z_{mw} & 0' \\ 1 & -x' & -m & 0' & o & 0' \end{bmatrix} \begin{bmatrix} dy \\ dp \\ dr \\ ds \\ dw \\ d\Psi \end{bmatrix}$$

where 0 is an n vector with all elements equal to zero. Total differentiation of (10) leads to

$$(12)\quad \begin{bmatrix} dx \\ dm \\ -d\lambda \end{bmatrix} = \begin{bmatrix} x_y & X_p & x_r & X_s & x_w & X_\Psi \\ m_y & m_p' & m_r & m_s' & m_w & m_\Psi' \\ -\lambda_y & -\lambda_p' & -\lambda_r & -\lambda_s' & -\lambda_w & -\lambda_\Psi' \end{bmatrix} \begin{bmatrix} dy \\ dp \\ dr \\ ds \\ dw \\ d\Psi \end{bmatrix}$$

Substitution from (12) into (11) and solving gives

$$(13)\quad \lambda_y = (p'U^{-1}p)^{-1} \left[1 - \frac{r^2}{v} (p'U^{-1}p)^{-1}\right]$$

$$\text{with}\quad v^{-1} = \left[z_{mm} + r^2 (p'U^{-1}p)^{-1}\right]^{-1} .$$

In standard demand analysis, the derivative of the Lagrangian multiplier with respect to total expenditures is $(p'U^{-1}p)^{-1}$. Here we have the derivative of the marginal utility of savings with respect to transactions income.

Using (13) it is an easy matter to rediscover the well-known results from which Barten derived the "Slutsky conditions", i.e. the general restrictions all demand equations must obey. We have

$$(14)\quad \lambda_p = -\lambda_y (\lambda U^{-1} p + x)$$

$$= - \lambda_y x - \lambda x_y$$

$$x_y = \lambda_y U^{-1} p \tag{15}$$

$$\begin{aligned} X_p &= \lambda U^{-1} - \lambda\lambda_y^{-1} \quad x_y \, x_y' - x_y \, x' \\ &= K - x_y \, x' \end{aligned} \tag{16}$$

where K is the usual substitution matrix. However, y represents transactions income. An increase in y therefore reflects an increase in wealth due for example to an increase in cash as a result of increased labor income. The derivatives x_y thus measure what is called the "real-balance effect" in the literature (see e.g. Patinkin (1965, p. 18-21)) : they measure the effect of a change in the real quantity of money on the demand for commodities.

As a result, there is no reason why the traditional adding-up condition $p'x_y = 1$ should remain valid. In fact,

$$p'x_y = \lambda_y \, p'U^{-1}p = 1 - \frac{r^2}{v} \quad (p'U^{-1} \, p)^{-1} \quad . \tag{17}$$

The adding-up condition has to be redefined in terms of x and m. Similarly, the traditional homogeneity condition will have to be reconsidered in view of the results that follow. But the symmetry condition $k_{ij} = k_{ji}$ and the negativity condition $k_{ii} < 0$ remain valid, as they are based on the properties of $u(x,s)$.

The demand for money being treated as an "ordinary" demand for a commodity, it should have the same properties, except for the specific cross substitution effects which are eliminated by the assumption that $u(x,s)$ and $z\,(m,w)$ are strongly separable. It is not surprising, then, to find

$$\lambda_r = - \lambda_y m - \lambda m_y \tag{18}$$

$$x_r = - \lambda\lambda_y^{-1} \, x_y \, m_y - x_y m, \tag{19}$$

in which $(- \lambda\lambda_y^{-1} x_y m_y)$ is the "general" cross substitution effect of a change in the rate of interest, and $(- x_y m)$ is its income effect. We also find

$$(20) \qquad m_y = v^{-1} (p'U^{-1} p) r,$$

so that

$$(21) \qquad v^{-1} = r^{-1} \lambda_y^{-1} m_y x_y' p$$

and

$$(22) \qquad z_{mm}^{-1} = \lambda_y^{-1} r^{-1} m_y ,$$

and furthermore

$$(23) \qquad m_p = - \lambda\lambda_y^{-1} x_y m_y - m_y x$$

$$\begin{aligned}(24) \qquad m_r &= \lambda v^{-1} - m_y m \\ &= \lambda r^{-1} \lambda_y^{-1} m_y x_y' p - m_y m \\ &= \lambda\lambda_y^{-1} r^{-1} m_y (1-rm_y) - m_y m \\ &= \lambda z_{mm}^{-1} - \lambda\lambda_y^{-1} m_y^2 - m_y m\end{aligned}$$

using (20), (21), (22) and the new adding-up condition

$$\begin{aligned}(25) \qquad p'x_y + rm_y &= \lambda_y p'U^{-1}p + r^2 v^{-1} (p'U^{-1} p)^{-1} \\ &= 1\end{aligned}$$

The "Cournot aggregation" conditions can now be written as

$$\begin{aligned}(26) \qquad X_p p + x_r r &= \lambda\lambda_v^{-1} x_y - \lambda\lambda_y^{-1} x_y x_y' p - x_y x'p - \lambda\lambda_p^{-1} x_y m_y r - x_y rm \\ &= \lambda\lambda_y^{-1} x_y - \lambda\lambda_y^{-1} x_y - x_y y \\ &= - x_y y\end{aligned}$$

and

$$(27) \qquad m_p'p + m_r r = -\lambda\lambda_y^{-1} m_y x_y' p - m_y x'p - m_y rm + \lambda\lambda_y^{-1} m_y - \lambda\lambda_y^{-1} m_y^2 r$$

$$= -\lambda\lambda_y^{-1} m_y x_y' p - m_y (x'p + rm) + \lambda\lambda_y^{-1} m_y x_y' p$$

$$= - m_y y.$$

Equation (26) and (27) are then used to redefine the homogeneity conditions as

$$(28) \qquad Kp + k_{xr} r = X_p p + x_y x'p + x_r r + x_y rm$$

$$= - x_y y + x_y y$$

$$= 0$$

$$(29) \qquad k_{mp}' p + k_{mr} r = m_p' p + m_y x'p + m_r r + m_y rm$$

$$= - m_y y + m_y y$$

$$= 0,$$

using the notation

$$k_{xr} = - \lambda\lambda_y^{-1} x_y m_y = k_{mp}$$

$$k_{mr} = \lambda z_{mm}^{-1} - \lambda\lambda_y^{-1} m_y^2$$

to designate the substitution effects not included in K. It is of some interest to remark that a compensated change in the rate of interest has the same effect on the demand for commodity i as a compensated change in its price on the demand for money. This is of course nothing but the symmetry condition applied to a commodity on the one hand and money on the other hand. We also have $k_{mr} < 0$.

The homogeneity restriction now implies that all demand equations, including the demand for money, are homogeneous of degree zero in the prices, transaction income and the rate of interest.

Finally, the impact of the state variables and Ψ can be described by the following results :

(30) $$\lambda_s = V x_y$$

(31) $$\lambda_w = v^{-1} (p'U^{-1}p) r z_{mw}$$
$$= z_{mw} m_y$$

(32) $$X_s = -\frac{1}{\lambda} KV$$

(33) $$x_w = -\frac{1}{\lambda} k_{mp} z_{mw} = -\frac{1}{\lambda} z_{mw} k_{xr}$$

(34) $$m_s = -\frac{1}{\lambda} V k_{mp}$$

(35) $$m_w = -\frac{1}{\lambda} k_{mr} z_{mw}$$

(36) $$X_\psi = -\frac{1}{\lambda} K$$

(37) $$m_\psi = -\frac{1}{\lambda} k_{mp}.$$

Equations (32) to (35) show the impact on demand of a change in the state variables to be equal to K times $-1/\lambda$ times the matrix of second order partial derivatives of the utility function with respect to the state variables. According to (36) and (37), the impact of a change in the implicit value of a stock is equal to the substitution effect except for sign and the positive constant $1/\lambda$.

IV. MONEY ILLUSION AND REAL BALANCE EFFECTS

Money illusion is defined in the context of our model by the fact that the demand for a commodity is affected by an equi-proportionate change in transactions income (y) and all commodity prices. The absence of money illusion, i.e. the homogeneity condition on ordinary demand equations, is one of the "Slutsky conditions " in standard demand theory. It is also the restriction that is the most difficult to reconcile with observed behavior. While the other conditions (symmetry and negativity) could be consistent with empirical evidence, the homogeneity restriction never passed any test based on a system of demand equations (see Deaton). Furthermore, there is increasing evidence to the same effect on the macro-economic level, as exemplified by work by Branson and Klevorick, Cukierman and Craig.

It is worthwhile, therefore, to try to generalize demand theory in such a way that the homogeneity restriction is relaxed. Dusansky and Kalman have done so by elaborating a demand system, including the demand for money, in which money illusion generally does exist. However, their "generalization" of demand theory is rather inappropriate, as it amounts to a generalization to all commodities of features which cannot be but particular to very specific commodities. Following Lloyd, Dusansky and Kalman introduce cash balances in the utility function in such a way that the specific substitution effect between cash and any commodity could be non-zero. All commodity prices also appear as [redacted] arguments in the utility function, with no other justification than the necessity of reasoning in terms of real balances. One can understand that a change in the price of commodity i may alter its marginal utility because it has snob appeal, or scarcity value, or simply because people judge quality by price, as in earlier work by Kalman.

These phenomena are obviously sufficient to destroy the homogeneity condition, but could hardly be interpreted as a general cause of money illusion. Finally, Dusansky and Kalman's approach relates the absence of money illusion to the existence of a particular class of utility functions (of which the Cobb-Douglas function is a member).

It is preferable to try to explain money illusion simply and more generally by the presence of money, not by the assumption of Veblen effects (which may provide an additional justification for some commodities or some consumers) or specific relationships between some commodities and cash balances (which may also exist). This is done in the model described a above. The resulting demand equations for commodities are not homogeneous in transactions income and prices, in general, according to equation (28). For the quantities demanded to remain unchanged, a doubling of all prices and of transactions income has to be accompanied by a doubling of the rate of interest. The same is true, of course, for cash balances according to equation (29).

Notice also that our model includes real balance effects, although nominal balances do appear and prices do not appear in the utility function. Price changes affect nominal balances through their effect on the marginal utility of savings (λ) in equation (7b), not through a direct impact on the marginal utility of holding money ($\partial z/\partial m$). And a change in real balances (i.e. a change in y with constant prices) affects all commodity demands according to equation (15), without supposing any non-zero specific substitution effect between cash and commodities. But the decomposition of the Slutsky equation into a substitution effect and an income effect is maintained : there is no third term representing "derived" real balance effects, so that the results of standard demand theory are formally salvaged. In interpreting the Slutsky equation, it should be remembered, though, that the "income" derivatives x_y now have a different interpretation.

The general and particular restrictions reported in Section III are all directly measurable with the help of a (dynamic) system of demand equations, to which one equation is added for nominal transactions balances (the interest foregone r being the "price" of one unit of cash). To work out a suitable empirical implementation , one simply has to use any of the available specifications of the instantaneous utility functions u and z.

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