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Metamaterial devoted to Magnetic Resonance Imaging: Numerical Analysis and Experimental Validation

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Abstract

Recent years have seen a growing interest in a new type of structures, often periodic, called "metamaterials". These new artificial materials exhibit many new appealing properties not found in nature, and open many new possibilities in the design of microwave and optical components. The most spectacular property of metamaterials is their ability to manipulate the near-field of an object and to image it with sub-wavelength resolution.

Besides this, among medical imaging techniques, Magnetic Resonance Imaging (MRI) has become increasingly competitive, its main advantage being its non-ionizing property. This radio-frequency technique appears to be a very interesting field of investigation for metamaterials due to its narrow bandwidth. This imaging technique requires constant improvements of the imaging quality in order to better understand the human body mechanisms and also provide better and faster diagnosis. The imaging process in MRI does not involve any optical means such as focusing or collimation.

The objective of the present thesis is to demonstrate numerically and experimentally that the ability of a metamaterial to manipulate the near-field can be used to realize new devices devoted to deep internal imaging or to concentrate MR signal in order to improve the signal-to-noise ratio of MR images. Both the numerical simulation and the experimental aspects are approached in this work.

In a first instance, the numerical tools developed to study metamaterials are presented. The basics of the Method of Moments is recalled. The MoM enables to study finite or infinite periodic structures made of metal and/or dielectrics. Since it requires to mesh only the surface of the structures, this method is particularly suited for the simulation in open-space of periodic structures like metamaterials. Several numerical techniques that

can be combined with the MoM are presented, namely: the Array Scanning Method which is used to compute the response of a periodic structure to a non-periodic excitation and the Macro Basis Function approach which enables to accelerate the analysis of large periodic structures of finite extent.

In a second time, these numerical methods are applied to the analysis and design of two types of metamaterials that can be used in 3Tesla MRI. The most promising structure is the wire medium which has the ability to transfer or concentrate the magnetic field onto significant distance with low losses. Straight wire media are investigated numerically from the point of view of their coefficient of transmission and eigenmodes. Special attention is given to the recuperation of the imaged fields, through the insertion of the receiving coil inside the metamaterial. Convergent geometries are numerically analyzed in order to determine their capacity to improve the RF-homogeneity of MR images by concentrating the magnetic field. Finally, curved wire media are investigated, the obtained results indicate that flexible endoscopic devices based on wire medium metamaterials may be envisaged.

Finally, we present the medical images obtained in clinical condition with the different types of wire media. These results confirm the ability of wire media to transfer MR images with low losses and open the possibility to build new MRI devices devoted to internal imaging.

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Introduction

1

1.1 Objective of the Thesis

Recent years have seen a growing interest in a new type of structures, often periodic, called "metamaterials". These new artificial materials exhibit many new appealing properties not found in nature, and open many new possibilities in the design of microwave and optical components. The most spectacular property of metamaterials is their ability to manipulate the near-field of an object and to image it with sub-wavelength resolution.

Besides this, among medical imaging techniques, Magnetic Resonance Imaging (MRI) has become increasingly competitive, its main advantage being its non-ionizing property. This radio-frequency technique is now used routinely for medical diagnosis. In this technique, the generation of an image is based on the detection of spatial variations of the radio-frequency energy absorbed and re-emitted by the protons of the imaged object. An MR image is built from the signals measured by a receiving coil which has no information about the relative location of these protons. Therefore, the imaging process in conventional MRI is mainly based on signal processing and does not involve any optical means such as focusing or collimation.

The objective of the present thesis is to demonstrate numerically and experimentally that the ability of a metamaterial to manipulate the near-field can be used to collimate or concentrate the MR signal. These properties open either the possibility to enlarge the field of view of a given surface coil by concentrating the RF magnetic field generated by a large area onto a high SNR surface coil, or the possibility to create more reliable MRI endoscopic sensors.

In this thesis, both the numerical simulation and the experimental aspects are approached. New numerical techniques devoted to the study of metamaterials are presented. These methods are applied to the analysis and design of two types of metamaterials that can be used in MRI. Then, it is demonstrated in MRI conditions that a wire medium metamaterial can be employed without creating artifacts to manipulate the fields generated by the protons, and consequently it can be used to realize new devices for MRI providing MR images with a better quality.

1.2 Principle of Metamaterial

In terms of optical physics, every material can be defined by its index of refraction n and its impedance Z . The index of refraction describes the velocity of light in a given material assuming that the medium is isotropic and unbounded. The impedance is defined as the ratio between electric and magnetic fields in such a medium. Two properties of transparent materials are directly related to refractive index and impedance: first, light rays change direction when they cross the interface from vacuum to the material, an effect that is used in lenses, and second light reflects partially from surfaces that have different refractive index and impedance.

From an electromagnetic point of view, the index of refraction can be expressed in terms of the permittivity ϵ which is the response of a medium to an electric field, and the permeability μ which is the response to a magnetic field, through the following relation:

$$n = \sqrt{\epsilon\mu} \quad \text{with } \epsilon > 0 \text{ and } \mu > 0 \quad (1.1)$$

In nature, common indexes are always greater than one and include 1 for vacuum, 1.0002 for air, 1.3 for water and 2.4 for diamond.

In 1968 V.G. Veselago examined the feasibility of media characterized by simultaneously a negative permittivity and a negative permeability [Ves68]. He concluded that such media are allowed by Maxwell's equations and he showed that, for a monochromatic uniform plane wave traveling in such a medium, the direction of the Poynting vector is anti-parallel to the direction of the phase velocity, contrary to the case of plane wave propagation in conventional simple media (see Fig. 1.1). Hence, these media exhibit a negative group velocity and the wavevector $\vec{k}$ forms a left-handed set with the vectors

$\vec{E}$ and $\vec{H}$. This is the reason why they are often called Left Handed Material (LHM).

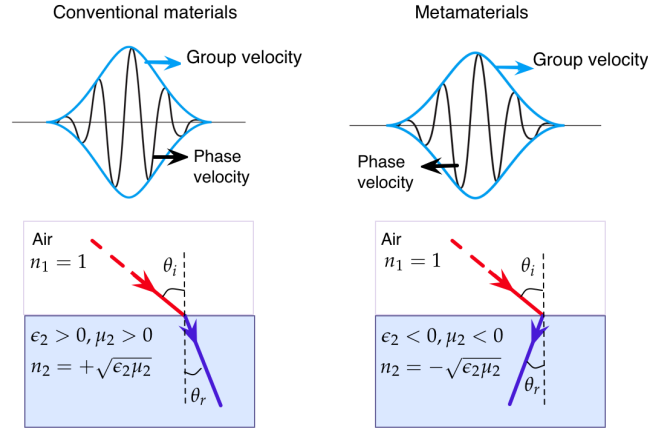


Figure 1.1 Comparison of the behavior of a ray incident on the interface of a conventional material (glass, water,...) or a negative index metamaterial. In the metamaterial case, since both refractive indices have opposite signs, the refracted wave is on the same side as the incident wave.

As a consequence of these characteristics, Veselago showed that in left-handed media, the Doppler effect is reversed, i.e. the red shift occurs in approaching conditions and the blue shift occurs when the source and receiver move away. Another interesting property of such structures is that the index of refraction is negative, therefore, the Snell's Law is reversed, and the refracted angle is opposite to the one obtained for a conventional medium. Indeed, according to Snell's law, a wave coming from a medium with a refractive index n_1 , on the interface of a medium with refractive index n_2 and an incidence angle θ_i with respect to the normal at the interface will be refracted in the second medium with an angle θ_r . Snell's law can be written as follows:

$$\frac{n_1}{n_2} = \frac{\sin(\theta_r)}{\sin(\theta_i)}$$

If n_1 and n_2 have opposite signs, which is the case at the interface of a metamaterial, the refracted wave will be on the same side as the incident wave, as illustrated in Fig. 1.1.

Veselago suggested to use this phenomenon to create a rectangular slab of LHM could focus electromagnetic radiation. More recently, Pendry predicted an intriguing property of such a metamaterial lens [Pen00]. Unlike conventional optical components, it will focus both the propagative and evanescent waves, and will thus be capable of achieving diffraction-free imaging. As one can imagine, this unique property has received considerable interest in view of the new applications it offers. Such media were called *metamaterials* where the prefix meta (meaning “beyond” in Greek) suggests that it possesses properties that surpass those available in nature. The most interesting application is the perfect flat lens, a lens which theoretically enable sub-wavelength imaging.

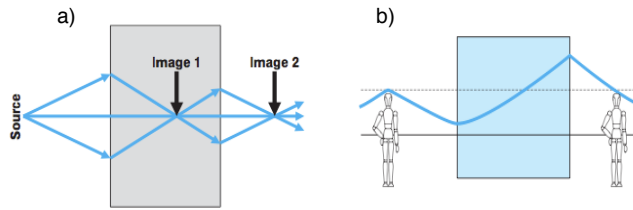


Figure 1.2 Properties of the superlens: (i) it has no focal axis and the rays are focused two times, inside and outside the lens, (ii) it can enhance the evanescent waves generated by the source.

The principle of the superlens is illustrated in Fig. 1.2. Consider a source in the air placed in front of a slab of metamaterial with a refractive index $n_2 = -1$ and suppose that the thickness of the slab is sufficient and the source is not too far from the slab. In this case, the electromagnetic field emitted by the source will create a first image of the source inside the lens and another one outside (side opposed to the source). Since the planar metamaterial lens has no optical axis, the object can be translated vertically, while keeping the focusing capability. In addition to this optical property, the superlens can amplify the evanescent waves. To illustrate the concept of evanescent waves, let us consider a monochromatic planar source (in the xy plane), the electromagnetic

field in free space can be expressed as a Fourier sum over all wave-vectors:

$$E(x, y, z, t) = \sum_{k_x, k_y} E(k_x, k_y, k_z) e^{i(k_x x + k_y y + k_z z - \omega t)}$$

where $k_x^2 + k_y^2 + k_z^2 = \omega^2 / c^2$. This electromagnetic field can be decomposed in two sets of waves depending on the value of the spatial wavevectors k_x and k_y . Indeed, if $k_x^2 + k_y^2 < \omega^2 / c^2$, the wavevector k_z is real and the waves are propagative. On the other hand, if $k_x^2 + k_y^2 > \omega^2 / c^2$ the wavevector k_z is imaginary and the associated waves are evanescent and are only observable in the close vicinity of the source (called the near-field zone). Since they carry the fine details of the object, this latter set of waves is very important to achieve high-resolution. In conventional imaging the near-field components are never detected, since conventional optical lenses deal only with the propagative waves and dephase them in order that they focus in the image plane. This lack of information is the origin of their limited resolution. A superlens does not have such a limitation since it can amplify the near-field components generated by the source and reconstitute it with sub-wavelength resolution on the other side of the lens. The superlens could be used to improve lithography techniques employed for the fabrication of electronic components. It could also be used to increase the capacity storage of devices such as DVDs,... Finally, there would be a wide range of applications in the telecommunications field for materials having backward properties such as novel filters, antennas and waveguides.

1.3 Realization of Unusual Material Characteristics

From the point of view of the realization, given that media having a negative index do not naturally exist in nature, metamaterials are built with the help of substances with common indexes, and arranging them in such a way that the structure of the object at a scale much smaller than the wavelength (but possibly very much larger than the atomic scale) determines its electromagnetic properties and thus a negative index metamaterial. The most popular metamaterial structure consists of periodically arranged thin metallic wires and split-ring resonators [PHRS98, PHRS99].

1.3.1 Negative Permittivity

It is in 1998 that J.B. Pendry published a paper [PHRS98] about an artificial metallic structure which exhibits a negative permittivity. The structure pro-

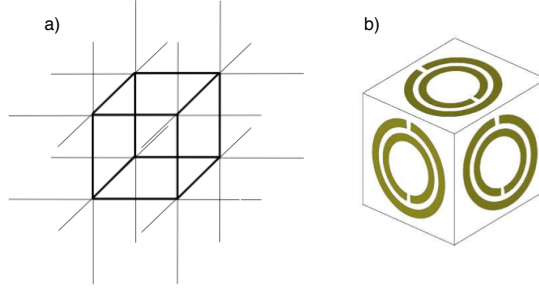


Figure 1.3 (a) Periodic array of thin wires employed to obtain a negative permittivity; (b) unit cell of a three-dimensional array used to obtain a negative permeability.

posed is made of a three-dimensional cubic lattice of thin metallic wires with a periodicity smaller than the wavelength. This structure is shown in Fig. 1.3.a and can model the response of a diluted plasma whose effective permittivity is given by the following formula:

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} \quad (1.2)$$

where ω_p is the so-called plasma frequency. As in the case of a plasma, the metallic lattice exhibits a negative permittivity for frequencies below the plasma frequency.

1.3.2 Negative Permeability

The structure employed to obtain a negative permeability consists of a three-dimensional array of split-ring resonators (SRR's) whose periodicity is smaller than the wavelength (traditionally about one tenth of a wavelength) of the exciting electromagnetic field [PHRS99]. Each SRR is made of two concentric circles of metal, each of them being split. So the induced current cannot flow within one ring but rather in the other ring which is capacitively coupled. The unit cell of the periodic structure is illustrated in Fig. 1.3.b. At frequencies slightly beyond the resonance frequency of the structure (which can be regarded as a LC resonator in free-space), the average permeability μ has a negative real part. Usually, the phenomenon of negative permittivity appears within a narrow bandwidth, since the SRR's are small in comparison to the

wavelength. The permeability can be expressed as a function of frequency as follows [PHRS99]:

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + j\nu\omega} \quad (1.3)$$

where ω_0 is the resonance frequency of the SRR's, ν is a loss term and F depends on geometrical parameters of the lattice.

1.3.3 Combination of both Structures

Until 1999, it was not physically possible to realize a material with simultaneously negative permittivity and permeability. It was D.R. Smith and his team who suggested to combine the two structures proposed by J.B. Pendry to realize a left-handed material [SWV⁺00]. The principle is straightforward and is explained in Fig. 1.4 for a one-dimensional case; it consists of combining an array of rods with an array of SRR's. The material thus realized exhibits over a narrow bandwidth a negative index of refraction. The phenomenon has been verified experimentally for different frequencies [SWV⁺00, SSS01].

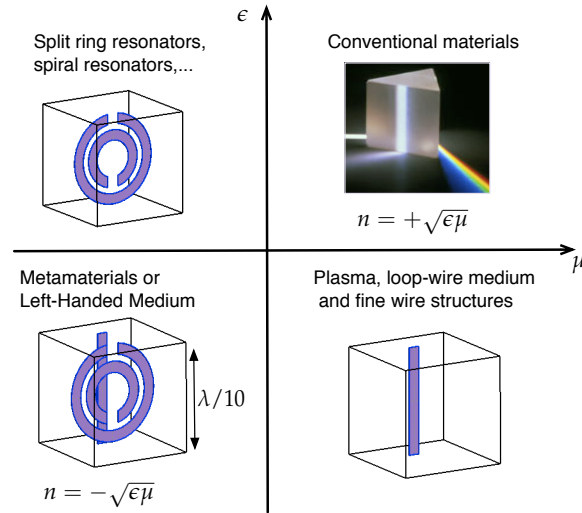


Figure 1.4 The different structures developed in the metamaterial field to create different negative values of permittivity ϵ and permeability μ .

Regarding the performances in transmission of metamaterials based on SRR's and rods, currently they are limited. This is due to two factors: (i) the SRR's are very sensitive to the losses in the dielectric board [PN05]. Indeed, given that SRR's are small in comparison to the wavelength, the electromagnetic field flowing through them is very concentrated and even high-quality dielectric material leads to important absorption of energy; (ii) the negative-index phenomenon is narrow-band, hence it is very challenging to obtain both $\mu=-1$ and $\epsilon=-1$ at the same time (and on a wide bandwidth). Consequently, the imaging capability is often limited by the mismatch between the lens and its surrounding medium [SSR⁺03]. Moreover, the realization of such "perfect lenses" requires to repeat the SRR pattern in the three dimensions of space to obtain an isotropic behavior. From this point of view, it remains a real challenge, especially at optical frequencies [Sha07].

Given that the constraint on both the permittivity and the permeability is difficult to meet, J.B. Pendry has shown that, in the electrostatic (magnetostatic) approximation, i.e. for $k_x^2 + k_y^2 \rightarrow \infty$ (with k_x and k_y the spatial wavenumber of a two-dimensional source in the xy plane), the dependence of the transmission coefficient for the p-/s-polarized field on the permeability/permittivity can be eliminated [PHRS98]. Therefore under this hypothesis, such a system can be realized by using noble metals [Pen00, RPS⁺02] which act as plasma in a given range of optical frequencies and can exhibit large negative permittivity. An example is a thin slab of silver which act as a near-perfect lens for near-field imaging was first observed for silver films in 2003 [LFYZ03].

In 2005, it has been suggested to use another principle to transport the sub-wavelength details of an image at a significant distance. The principle of operation is based on the fact that, with certain strongly anisotropic materials, the evanescent waves can be canalized, by being transformed into propagating waves in the medium [Ma07, WPT06, BSI05]. In comparison to metamaterials based on SRR's associated with rods, these media also exhibit less losses. Practically, for frequencies up to the infrared band, the anisotropic medium can be created from an array of parallel conducting wires, also called wire medium [SBS07]. In the visible frequency range, such kind of media could be created from a layered-metal dielectric structure [BSI05]. The wire medium is further described in the next section and is analyzed in depth in chapter 4 and chapter 5.

1.4 Applications of Metamaterials

Initially the term metamaterial was used to describe artificial media possessing a left-handed behavior. This term is now accepted in a more general sense as engineered media whose electromagnetic responses are different from those of their constituent components and whose behavior cannot be found in nature. Some of these new artificial materials and applications are described in this section.

1.4.1 High-gain Antennas

We have seen in section 1.3.1 that an array of three-dimensional wires behaves as a diluted plasma and that its permittivity below the plasma frequency is negative. Looking at Eq. (1.2) we can see that for frequencies just above the plasma frequency, the value the permittivity becomes close to zero, hence by Eq. (1.1) the refractive index is also close to zero.

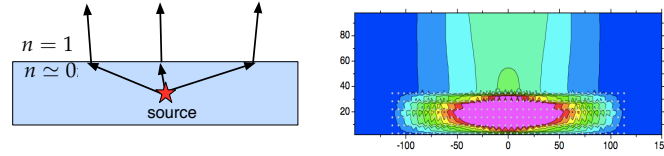


Figure 1.5 (a) The field radiated by a source placed inside a structure with a zero index of refraction is refracted almost perpendicularly to the interface; (b) the amplitude of the electric field in a structure made of parallel conducting wires and excited with a small embedded monopole [TSP⁺02].

If we place a source inside such an array, Snell's law tells us that the plane waves radiated by the source will be refracted close to the normal to the interface of the medium. This property can be exploited to design high-gain antennas. The concept is illustrated in Fig. 1.5 showing the electric field amplitude in a structure made of parallel conducting wires and excited by an embedded monopole [TSP⁺02]. We can clearly see that the electric field outside the structure is concentrated near the normal at the interface.

1.4.2 Frequency Selective Surfaces

Frequency-selective surfaces are surfaces tailored to exhibit specific filtering characteristics [Var03], those surfaces are sometimes -by extension- considered as a type of metamaterials. They can be used to provide coatings with specific frequency filtering characteristics. This type of surface is used in the field of electromagnetic compatibility of civil devices like cell phones. They are also used to design Fabry-Perot resonators designed to increase the gain of antennas [ROS⁺07].

1.4.3 High Impedance Surfaces

Artificial magnetic conductors (AMCs) or high impedances surfaces metamaterials can be used to improve the radiation efficiency of patch antennas and of dipoles located close and parallel to ground planes. When designing patch antennas, compact circuit design is best achieved on high dielectric constant substrates. However, using these high permittivity dielectrics tends to increase the amount of surface waves and hence increases the chances of unwanted side lobes. A solution to this problem consists of using PBG surfaces which enable to stop the propagation of those surface waves. An example of realization is presented in Fig. 1.6.b, for the case of a patch antenna.

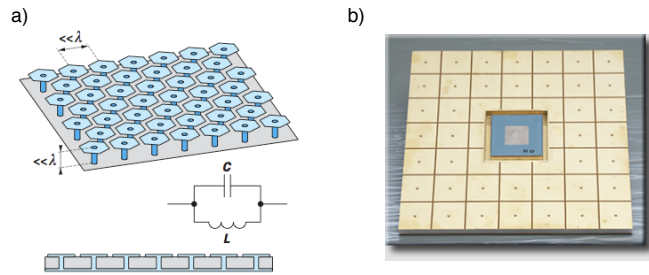


Figure 1.6 (a) A well-known AMC structure, the Sievenpiper class of mushroom high impedance surfaces, copied from [SZB⁺07], (b) Patch antenna placed on a mushroom-type PBG ground plane to suppress surface waves and increase radiation efficiency, copied from [BMVLH06].

High impedances surfaces are also very interesting to enhance the directivity of antennas and to reduce their size. Let us explain the advantage brought

by AMC's for the design of compact antennas. Traditionally, to enhance the directivity of antennas and to prevent back radiation a perfect electric conductor (PEC) is placed behind the antennas, which acts as a reflector. In the case of a PEC reflector, the antenna must be placed at a minimum distance $d = \frac{2n+1}{2}\lambda/2$ with $n=0,1,2,\dots$ from the reflector, otherwise at broadside, the antenna would be short-circuited by its image. This constraint is problematic when we want to design compact antennas. The solution is to replace the PEC reflector with a Perfect Magnetic Conductor (PMC) which has the advantage that it can be placed very close to the antenna. Currently, no PMC material has been discovered, but periodic structures show that the same characteristics in a limited frequency band can be used as artificial PMCs. An example of such a structure is the mushroom-like EBG structure depicted in Fig. 1.6.a., the structure consists of an array of hexagonal metallic patches connected to a PEC ground plane with wires [SZB⁺07].

1.4.4 Cloaking

The possibility of cloaking an object from detection by electromagnetic waves has recently become a topic of considerable interest. By creating a specific distribution of ϵ and μ , and thus a distribution of refractive index n , one can curve the space for light in a nearly arbitrary way, making it possible to propagate light not only in the backward direction (when n is negative) but also along nearly any curved line [PSS06]. As a result, a myriad of fascinating devices are achievable using transformation optics and metamaterials. One of the most exciting applications is an electromagnetic cloak that can bend light around an object, similar to the flow of water around a stone, making invisible both the cloak and an object hidden inside, as illustrated in Fig.1.7.

Such complicated gradient-index media are difficult to create with conventional materials but are much easier to build with artificially structured metamaterials, in which spatial variations of the material parameters can be achieved by modifying the geometry and placement of the constituent element. Recently, experimental realization of a cloak has been demonstrated, that conceals a perturbation on a flat conducting plane, under which an object can be hidden. The design has a broad operational bandwidth and exhibiting extremely low loss [LJM⁺09].

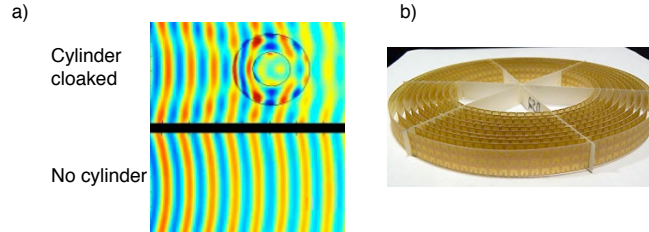


Figure 1.7 (a) A metamaterial shield placed around a cylinder object and having the correct index of refraction indices can bend the light around the cylinder. Consequently, the cylindrical object becomes invisible since the incident wave is not perturbed. (b) Practical realization of a cloaking device, copied from [PSS06]

1.4.5 Strongly Anisotropic Material

As we have seen in Section 1.3.3, an alternative solution to transfer images with sub-wavelength resolution imaging is the use of strongly anisotropic materials. This kind of materials has specific electromagnetic properties which enable the transfer of an image from one side of the medium to other side with sub-wavelength resolution. The characteristics of such materials is that they have a diagonal permittivity tensor with one very large component, involving spatial dispersion, i.e. leading to a dispersion relationship that depends on the wavenumber itself [Ma07, WPT06, BSI05]. Inside such a material the most part of the spatial spectrum of the source is transformed into propagating eigenmodes which all have the same longitudinal component of the wave vector. Consequently, this transformation enable to transfer the high details of a source on a significant distance (of the order of the wavelength). These materials does not involve negative refraction as metamaterials based on SRR's and rods.

A typical realization of such materials is the wire medium, which consists of an array of parallel metallic wires operating at frequencies smaller than the plasma frequency of the array. In order to be matched to the host medium, the length of the wires is set to an integer number of half wavelengths, so that the medium operates as a Fabry-Perot resonator. The structure is depicted in Fig. 1.8 with an example of a magnetic field transfer. The wire medium offers new perspectives, since it operates over a wide frequency band and exhibits low losses. Moreover, its resolution is limited only by the spacing

between wires, so it is theoretically possible to reach any resolution, as far as the structure can be mechanically realized [BS06].

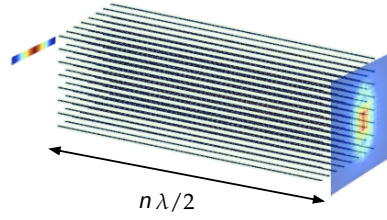


Figure 1.8 Illustration of the ability of a straight wire medium to transfer with sub-wavelength resolution the magnetic field generated by a small dipole. In this simulation, only the component of the magnetic field perpendicular to the dipole is represented on the image plane.

In this thesis, we propose to employ a wire medium in Magnetic Resonance Imaging. It will be used mainly for its waveguiding property, which opens new prospects to realizing more reliable devices devoted to 3-D non-intrusive imaging. As will be explained in sections 3.4 and 4.2.1, in a first instance the ability of wire media to transfer sub-wavelength details will not be exploited, since the resolution in MRI depends only on the strength of the gradient fields (see Section 3.2.4). Wire media are analytically and numerically analyzed in Chapter 3 and their capacity to transfer images in MRI conditions will be demonstrated in clinical conditions in Chapter 4.

1.5 Outline of the Thesis

In Chapter 2 we describe the numerical tools employed in this thesis. We briefly recall the basics of the Method of Moments, which is a method particularly efficient for the simulation in open-space of periodic structures like metamaterials. We also introduce the methods that can be employed to simulate dielectric structures and infinite arrays. In the latter case, the periodicity is accounted for through the use of the periodic Green's function. We also describe how eigenmodes of an infinite structure can be computed from the impedance matrix of the MoM. When dealing with metamaterials, one often

needs to compute the response of a periodic structure to a non-periodic excitation. A way to achieve this is to exploit the so-called "Array Scanning Method" (ASM) [MB79,CJW05,CPD04]. The method is explained and an example of application is given for the computation of the spatial response of a wire medium. This spatial response can be very useful to compensate for image deformation introduced during the field transfer. Finally, we present a solution that can be combined with the MoM to accelerate the analysis of periodic structures of finite extent, namely the macro basis functions (MBF) approach. A key point to obtain accurate results with the MBF approach lies the choice of the size of the basis and the method used to construct it. Here a new method to create rapidly an MBF basis is presented and its accuracy is demonstrated by comparison with results obtained in full resolution.

In Chapter 3, we briefly describe the principle of Magnetic Resonance Imaging (MRI), as well as its current limitations. We demonstrate the advantages that a wire medium collimator could bring in MRI, from the point of view of the image quality. We present the different MRI coils employed during the experiments led at the hospital of Liège to evaluate the performance of wire media. We describe the steps used to fabricate a MRI surface coil which was especially dedicated for the MRI measurement inside wire media. Finally, the impact of the presence of a wire medium on the performance of a surface coil is presented.

In Chapter 4, we first present the analytical models developed in the literature to analyze straight wire media. In particular, we detail the modes supported by the structure, we identify the eigenmodes and explain the origin of the sub-wavelength imaging. Then we demonstrate numerically the use of wire media as energy collimators to transmit or concentrate the magnetic field to a certain distance from the MRI tube center, in a region with negligible static field. Straight wire media are investigated numerically with the Method of Moments from the point of view of their coefficient of transmission and eigenmodes. Special attention is given to the recuperation of the imaged fields, through the insertion of the receiving coil inside the metamaterial. Convergent geometries are also numerically analyzed in order to determine their capacity to improve the homogeneity of MR images. Finally, curved wire media are investigated numerically, the obtained results indicate that flexible endoscopic devices based on wire medium metamaterials may be envisaged.

In Chapter 5, we present the experimental results obtained during the experiments carried out at the hospital of Liège. We first present a systematic method to compare the quality of the MR images obtained during experiments. The results obtained with the different geometries investigated numerically are tested. It is confirmed that the optimal way to recover the MR signal (higher signal-to-nois ratio) is obtained by inserting the MR surface coil inside the metamaterial. These experimental results confirm the ability of wire media to transfer MR images and open the possibility to build new MRI devices devoted to deep internal imaging.

Numerical Methods

2

2.1 Introduction

In electromagnetics, the starting point of all treatments are the four Maxwell's equations [Har61], which have to be solved for taking into account the geometry of the problem. Given a homogeneous medium with constituent parameters ϵ , μ and σ , the Maxwell's equations in the frequency domain reads:

$$\begin{aligned}\nabla \times \vec{E} &= -\vec{M} - j\omega\mu\vec{H} \\ \nabla \times \vec{H} &= \vec{J} + j\omega\epsilon\vec{E} \\ \nabla \cdot \vec{D} &= q_e \\ \nabla \cdot \vec{B} &= q_m\end{aligned}$$

where $\vec{D} = \epsilon\vec{E}$, $\vec{B} = \mu\vec{H}$, $\vec{J} = \sigma\vec{E}$, and the time dependence $e^{j\omega t}$ is assumed. Although the magnetic current M and charge q_m are not physical realizable quantities, they are often employed as mathematical tools to solve radiation problems [Gib08]. Unfortunately, most of the structures studied today are very complex and cannot be solved analytically. Hence, accurate computer aided analysis and design become essential. The requirements put on these numerical methods are very severe since they have to be accurate in terms of final results and at the same time, not too computationally expensive.

In this chapter, our aim is to develop numerical tools to efficiently simulate metamaterials and allow us to characterize them (index of refraction, eigenmodes, etc). Among different numerical methods presented in the literature, we use the Method of Moments (MoM). As we will see later on, the MoM

is particularly well suited to the analysis of open periodic structures such as metamaterials and antenna arrays. Moreover, additional approaches can be combined with the MoM such as the Macro Basis Functions approach and the Array Scanning Method in order to study more complex materials such as large finite structures.

The outline of this chapter is as follows: Section 1 presents and compares most common numerical methods encountered nowadays; Section 2 describes the principle of the Method of Moments, the equivalence principle which is employed to simulate dielectric media and the periodic Green's functions which are used to simulate infinite periodic structures. Section 3 presents the Array Scanning Method (ASM) which is used to compute the Point Spread Function (PSF) of a metamaterial unit cell. The PSF is an interesting quantity when it comes to compensate imaging deformation introduced by a metamaterial slab. Section 5 presents the method used to extract the constitutive parameters of a metamaterial slab from its S-parameters. An example of application is provided with the design of a metamaterial devoted to MRI imaging. Finally, Section 6 presents the Macro Basis Functions (MBF) approach which allows us to efficiently simulate transmission through a large finite metamaterial.

2.2 Overview of Some Numerical Methods

When carrying out numerical simulations it is important to have a basic knowledge of the different methods available in order to choose the best numerical approach suited to a particular problem [Sad01]. In this section, we present the principle and characteristics of the three most used numerical methods.

- The Finite Difference method (FD)
- The Finite Element Method (FEM)
- The Method of Moments (MOM)

The principles of these methods are briefly described hereafter.

The Finite Difference method

One of the first techniques developed is the so-called Finite Difference method (FD). The FDM method [Yee66] is often referred to as the least analytical method, since the mathematical processing it requires is minimal. How-

ever, this feature renders the FD method applicable to a wide range of complex structures, as well as to frequency or time domain problems (FDTD) [Taf98]. The finite difference method involves three steps: first the discretization of the solution region into a grid of nodes, second the approximation of the differential Maxwell's equations by finite differences, the space or time dependent variable at a point in the solution region being related to its values at the neighboring points, and third solution of the sparse system of equations with the help of the boundary conditions.

Primarily, the method was very inefficient for open structures since it required to mesh the free-space region which lead to unreasonable computation times. Fortunately, a lot of work has been carried out to develop proper absorbing layer conditions [MBK81], allowing the meshes to be limited only to the immediate vicinity of the structures. Nevertheless, many designs such as open structures remain very challenging to simulate with FD methods mainly because of the very large mesh required and are better addressed by other methods such as the Method of Moments.

The Finite Element Method

Compared to the Finite Difference method and the Method of Moments, the Finite Element Method (FEM) is more difficult to implement. Nevertheless, it is also a more powerful and flexible numerical technique for handling problems involving complex geometries and inhomogeneous media. The systematic aspect of the method makes it possible to construct general-purpose computer programs for solving a wide range of problems.

The basic concept of the FEM [SP96] is the subdivision of the mathematical model into disjoint (non-overlapping) components of simple geometry called finite elements. The response of each element is expressed in terms of a finite number of degrees of freedom characterized as the value of an unknown function, or functions, at a set of nodal points. The response of the mathematical model is then considered to be approximated by that of the discrete model obtained by connecting or assembling the collection of all elements. The difference between the FEM and the FDM is that the former can handle more complicated geometries (not limited to a rectangular mesh), and the quality of approximation between grid points is better.

As for the FD method, the disadvantage of the FEM appears when it is applied to open structures since in this case, the region to analyze has to be trun-

cated. This is not easy since, in some situations, the fields may be slowly decaying and a bad truncation can lead to important errors. As for FD methods, the investigation of proper truncation is being extensively studied in the literature and one solution is also to define proper absorbing conditions [SB94].

The Method of Moments

The Method of Moments (MoM) [Har93, Gib08] has been successfully applied to a wide variety of electromagnetic problems, among others scattering problems, microstrip circuits, radiating structures,... The main advantage of the MoM is that only surfaces need to be meshed (interfaces between different homogeneous media) instead of a whole volume. Hence, it leads to less unknowns in comparison with FD or FEM methods, which both require heavy computation when dealing with open boundaries. Both structures made of metal and/or dielectrics can be easily studied. The MoM is particularly well suited to the simulation of periodic materials such as metamaterials. Concerning the analysis of *infinite* periodic structures, periodicity can be taken into account with the help of periodic Green's functions while concerning the analysis of *finite* periodic structures, simulations can be improved with the help of the macro basis functions approach. The comparison of the results obtained with both methods enables to study edge effects. The MoM usually requires three steps : first, the derivation of an appropriate integral equation, second the discretization of the integral equation into a matrix equation using basis and testing functions and its evaluation to form a matrix, and finally the solution of the matrix equations.

In this thesis, our main goal is to deal with open environments and simulate periodic structures such as metamaterials, e.g. wire media. For this reason, we prefer to use the Method of Moments which is a technique better suited for the analysis of metamaterials and based on more physical considerations.

2.3 Principle of the Method of Moments

In this section, we not only describe the Method of Moments implemented in an in-house code developed at UCL but we also explain the strategies used to simulate finite and infinite structures made of metal and/or dielectric material.

2.3.1 Treatment of Metallic structures

Let us consider a metallic surface S on which we want to determine the current generated by an incident electromagnetic wave E^{inc} . In the Method of Moments, the first step consists in discretizing the surface into a set of triangles and rectangles as illustrated in Fig. 2.2.a, for a double-split-ring resonator. In our case, the discretization is carried out with the help of the Gmsh program [GR09]. Triangular sub-domains are very popular, nevertheless in order to reduce the number of unknowns and sometimes to account for the symmetry of the structure, we will also use rectangular sub-domains. Once the mesh is obtained, a basis function $\bar{f}_n$ is associated to each internal vertex n of the discretization (i.e. not the boundary edges).

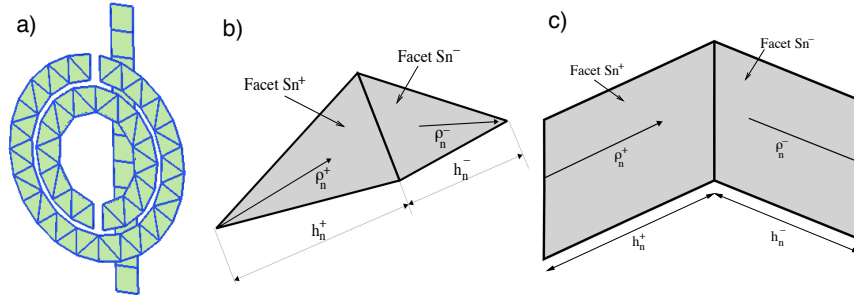


Figure 2.1 (a) A double split-ring resonator and a strip meshed with triangles and rectangles; (b) geometry of a Rao-Wilton-Glisson basis function; (c) geometry of a rooftop basis function.

In the MoM, the surface current $\bar{J}$ is approximated, with the help of these basis functions $\bar{f}_n$ as a vectorial sum as follows:

$$\bar{J} = \sum_{n=1}^N I_n \bar{f}_n \quad (2.1)$$

where N is the total number of internal vertices in the mesh of the surface S , and I_n is a weighting coefficient associated with the basis function f_n . The associated basis functions have particular properties defined in [RWG82] and reminded hereafter for convenience.

$$\bar{f}_n(\bar{r}) = \begin{cases} \bar{\rho}_n^+ / h_n^+ & \text{if } \bar{r} \text{ is in } S_n^+ \\ \bar{\rho}_n^- / h_n^- & \text{if } \bar{r} \text{ is in } S_n^- \\ 0 & \text{elsewhere} \end{cases}$$

Consequently, since the normal component of $\bar{f}_n$ at the edge n is unity; each coefficient I_n may be interpreted as the normal component of current density flowing through the edge n . In the case of triangular sub-domains, we use the Rao-Wilton-Glisson basis functions [RWG82], and for rectangular sub-domains we use rooftop basis functions. Both types of basis functions are depicted in Figs. 2.2.b and 2.2.c.

We have seen that the incident electric field induced a current on the surface S . This current in its turn produces a scattered field $\bar{E}^{sc}$ which can be related to the current $\bar{J}$ through the following equation:

$$\bar{E}^{sc} = -j\omega\bar{A} + \frac{1}{j\omega\epsilon\mu} \nabla (\nabla \cdot \bar{A}) \quad (2.2)$$

with ϵ the permittivity and μ the permeability of the host medium, ω the angular frequency, and $\bar{A}$ the vector potential defined as :

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi} \int_S \bar{J}(\bar{r}') \frac{e^{-jk|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|} dS$$

The factor

$$G(r, \bar{r}') = \frac{e^{-jk|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|} \quad (2.3)$$

is called the Green's function of free space. The vectors $\bar{r}'$ and $\bar{r}$ are vectors defined in Fig. 2.2.

The electric-field integral equation (EFIE) is obtained by imposing that the total electric field on the surface S be equal to zero, i.e:

$$\bar{E}_{tan}^{sc} - \bar{E}_{tan}^{inc} = 0 \quad (2.4)$$

Using expressions (2.1) and (2.2), we can expand the expression of the reflected electric field $\bar{E}_{tan}^{sc}$ in equation (2.4) as:

$$\sum_{n=1}^N \left(-j\omega\mu + \frac{1}{j\omega\epsilon} \nabla \nabla \right) \cdot \int_S I_n \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|} \bar{f}_n dS = \bar{E}_{tan}^{inc} \quad (2.5)$$

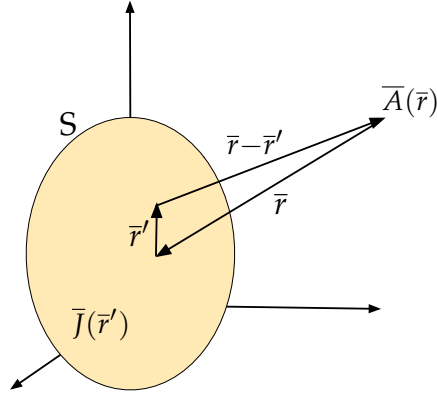


Figure 2.2 Vector potential $\bar{A}$ created by a surface current $\bar{J}$ flowing on a metallic surface S .

In order to solve the integral equation and find the N coefficients I_n , the boundary condition represented by Eq. (2.5) must be verified or tested N times on the surface S . In the MoM, the testing is realized by taking the inner product of Eq. (2.5) with N weighting functions w_m . There are two popular ways of choosing these testing functions:

- Point matching: $w_m = \delta(\bar{r} - \bar{r}_m)$; in this case, Eq. (2.5) is exactly enforced at N points $\bar{r}_m$, located on the surface S . It leads to a relatively simple evaluation of the system of equations, but generally leads to less accurate solutions that are very sensitive to the choice of the N points, and the convergence of the MoM may not be optimal.
- Galerking testing: $w_m = \bar{f}_m$. The set of testing functions is the same as the set of basis functions used to approximate the surface current. With this choice, Eq. (2.5) is enforced in an average sense over the subdomains on which the N basis functions are defined.

In our case, we use the Galerking testing. Consequently the testing of Eq. (2.5) leads to the following system of N equations with N unknowns that can be written in matrix form as follows:

$$\begin{bmatrix} \vdots \\ \dots & Z_{mn} & \dots \\ \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ I_n \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ V_m \\ \vdots \end{bmatrix} \quad (2.6)$$

$$m, n = [1, 2, \dots, N] \quad (2.7)$$

where

$$\begin{aligned} Z_{mn} &= \int_S \bar{f}_m(\bar{r}) \left(-j\omega\mu + \frac{1}{j\omega\epsilon} \nabla \nabla \right) \cdot \int_{S'} \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|} \bar{f}_n(\bar{r}') dS' dS \\ V_m &= \int_S \bar{f}_m(\bar{r}) \cdot \bar{E}^{inc}(\bar{r}) dS \end{aligned}$$

The solution of this system of equations leads to the determination of the coefficient I_n and also to the surface current $\bar{J}$ with the help of Eq (2.1).

2.3.2 Treatment of Periodic Structures

Solving large periodic structures such as metamaterials or large antenna arrays can be excessively time-consuming because of the large amount of unknowns included in such structures. A convenient approach to simulate infinite periodic structures is to use the infinite-array approximation. In infinite arrays, the periodicity is accounted for by replacing the Green's function in free space given in Eq. (2.3) by the Green's function related to an infinite array of electric sources with constant amplitudes and a linear phase approximation.

Let us consider a two-dimensional infinite array of sources located in the plane xy and separated by a distance a and b along the x - and the y -axis, respectively. The periodic Green's function for this array is the solution of the Helmholtz equation:

$$(\nabla^2 + k^2)G = - \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x - ma) \delta(y - nb) e^{-jm\psi_x} e^{-jn\psi_y} \quad (2.8)$$

where $\psi_x = k_x a$ and $\psi_y = k_y b$ are the phase shifts along x and y . By wave superposition, the solution of Eq.(2.8) leads to the following spatial formulation of the Green's function:

$$G(x, y, z) = \sum_{m=-\infty}^{\infty} e^{-jm\psi_x} \sum_{n=-\infty}^{\infty} e^{-jn\psi_y} \frac{e^{jkr_{mn}}}{4\pi r_{mn}} \quad (2.9)$$

where $r_{mn} = \sqrt{(x - ma)^2 + (y - nb)^2 + z^2}$. Unfortunately, this spatial summation converges very slowly for most observation positions (x, y, z) . Its use in a numerical method would be very time-consuming. The solution is to use other formulations [JM90] and to express the Green's function either as a sum of plane waves or as a sum of cylindrical waves. Depending on where the periodic Green's functions needs to be evaluated, the adequate formulation is chosen to ensure the fastest convergence.

- *Sum of plane waves*: this spectral formulation converges rapidly as long as z is not too close to the plane xy , i.e. $z > \epsilon$, otherwise the convergence is slower and it is better to express the Green's function as a sum of cylindrical waves.

$$G(x, y, z) = \frac{1}{2jab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\gamma_{mn}|z|} e^{j(\beta_m x + \beta_n y)}}{\gamma_{mn}} \quad (2.10)$$

where

$$\gamma_{mn} = \begin{cases} \sqrt{k^2 - \beta_m^2 - \beta_n^2} & \text{when } k^2 > \beta_m^2 + \beta_n^2 \\ -j\sqrt{\beta_m^2 + \beta_n^2 - k^2} & \text{when } k^2 < \beta_m^2 + \beta_n^2 \end{cases}$$

$$\text{with } \beta_m = \frac{2\pi m}{a} - k_x \quad \beta_n = \frac{2\pi n}{b} - k_y$$

- *Sum of cylindrical waves*: The doubly periodic array of sources can be seen as the superposition of an infinite number of infinite linear arrays along the x -axis [CSST03]. The associated Green's function can then also be seen as the superposition of Green's functions associated to infinite lines along x , with a phase shift ψ_x . Each linear array is represented by a spectral sum of cylindrical waves via the zero-order Hankel function of the second kind.

$$G(x, y, z) = \frac{1}{4ja} \sum_{p=-\infty}^{\infty} e^{-jp\psi_y} \sum_{n=-\infty}^{\infty} e^{-jk_n x} H_0^{(2)}(\gamma_n R_p) \quad (2.11)$$

where

$$\gamma_n = \begin{cases} \sqrt{k^2 - k_n^2} & \text{when } k^2 > k_n^2 \\ -j\sqrt{k_n^2 - k^2} & \text{when } k^2 < k_n^2 \end{cases}$$

$$k_n = \frac{\psi_x}{a} + n \frac{2\pi}{a}$$

Here, $R_p = \sqrt{(y - pb)^2 + z^2}$ corresponds to the distance to successive lines of dipoles, denoted by p . This formulation is employed when $z \leq \epsilon$ and for observation points located far from the x-axis. For observation points located close to the x-axis many evanescent waves need to be taken into account and the summation converges slowly, in this case the direct summation in Eq. (2.9) is used.

In the MoM code used for this thesis, extrapolation techniques such as Shank's [SS91] or Levin's [CC06, SS93] are utilized in order to accelerate the convergence of the previous formulations. The Levin's transform is more elaborated than the Shank's transform, it involves coefficients whose calculation may take a significant portion of the overall computation time but it proved to be powerful for the acceleration of periodic Green's functions. On the other hand, to further speed up the process, the Green's function is tabulated over a 3D regular grid of observation points, after extraction of the singularity related to the nearest point. When the Green's function needs to be evaluated, a second order interpolation technique is used on the values stored in the table and the singular part is re-introduced where necessary.

2.3.3 Treatment of Dielectric Material

Given that the MoM is based on the calculation of currents, it is ideally suited for the treatment of metallic objects. When the studied structure contains also dielectric material the procedure is slightly modified and the surface equivalence principle is exploited [Har61].

To illustrate the concept, let us consider the example in Fig. 2.3.a where an external electromagnetic field is incident on a dielectric body with permittivity ϵ^{in} and permeability μ^{in} . Fields outside the body can be represented with the help of equivalent sources, distributed either in the volume V or over the surface S of the dielectric. This principle is exploited in the surface equivalence principle, where the dielectric body is removed and replaced by fictitious equivalent currents on the interface, namely an equivalent surface current density $\bar{J}$ and an equivalent magnetic surface current density $\bar{M}$, defined as follows,

$$\begin{aligned}\bar{J} &= \hat{n} \times \bar{H} \\ \bar{M} &= \bar{E} \times \hat{n}\end{aligned}$$

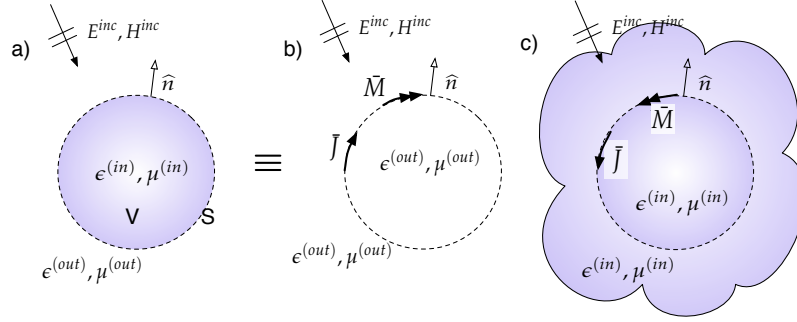


Figure 2.3 (a) Geometry of a dielectric object with permittivity ϵ^{in} and permeability μ^{in} , (b) Equivalent surface electric $\vec{J}$ and magnetic $\vec{M}$ currents are shown on its boundary S. They radiate in a homogeneous medium, produce the original field outside and a vanishing field inside, when the incident field is added. (c) The equivalent currents produce the original field inside the boundary S.

where $\hat{n}$ is the external normal pointing outward the object and $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields. The field generated by $\vec{J}$ and $\vec{M}$ is such that its superposition with the incident field produces the original field outside and zero field inside the volume V. Therefore, one can virtually consider the presence of any material inside because of the absence of field. In particular, one could assume a perfect conductor inside, or a homogeneous material as the one outside, so as to use the free-space Green's function to determine the field radiated outside by the equivalent currents $\vec{J}$ and $\vec{M}$ (Fig. 2.3.b). Analogously, the equivalent currents can be used to evaluate the field inside the surface S when they radiate in the environment as shown in Fig. 2.3.c.

The unknown electric and magnetic currents can be determined by ensuring the continuity of tangential electromagnetic fields along the surface of the dielectric object (PMCHWT approach). There we must find first the expression of the fields outside and inside the object. The fields outside the dielectric body generated by the equivalent currents can be written in the form

$$E^{out}(\vec{r}) = E^{inc}(\vec{r}) + \int_S \left(\frac{k_{out}^2 + \nabla \nabla \cdot}{j\omega\epsilon^{out}} (\vec{J}G^{out} - \vec{M} \times \nabla' G^{out}) \right) dS \quad (2.12)$$

$$H^{out}(\vec{r}) = H^{inc}(\vec{r}) + \int_S \left(\frac{k_{out}^2 + \nabla \nabla \cdot}{j\omega\mu^{out}} (\vec{M}G^{out} + \vec{J} \times \nabla' G^{out}) \right) dS \quad (2.13)$$

where E^{inc} and H^{inc} are the incident fields, G^{out} is the free space Green's function outside the medium and $\nabla'G$ denotes the gradient with respect to the coordinates of the source point r' . Inside the dielectric object, fields are expressed as:

$$E^{in}(\bar{r}) = - \int_S \left(\frac{k_{in}^2 + \nabla \nabla \cdot}{j\omega\epsilon_{in}} (\bar{J}G^{in} - \bar{M} \times \nabla'G^{in}) \right) dS \quad (2.14)$$

$$H^{in}(\bar{r}) = - \int_S \left(\frac{k_{in}^2 + \nabla \nabla \cdot}{j\omega\mu_{in}} (\bar{M}G^{in} - \bar{M} \times \nabla'G^{in}) \right) dS \quad (2.15)$$

The negative sign in Eqs. (2.14) and (2.15) is due to the fact that the equivalent currents seen from inside the dielectric body have an opposite sign to those seen from outside. G^{in} is the Green's function for the medium inside the dielectric. One should recall that when fields are evaluated on the surface of the dielectric object, the integrals involved in Eqs. (2.12),(2.13),(2.14) and (2.15) are singular. The singularity must be extracted and integrated analytically. The two sets of equations are completed with the boundary conditions enforcing the continuity of tangential fields over the surface of the dielectric object.

$$\begin{aligned} \hat{n} \times \bar{E}^{in} &= \hat{n} \times \bar{E}^{out} \\ \hat{n} \times \bar{H}^{in} &= \hat{n} \times \bar{H}^{out} \end{aligned}$$

Introducing Eqs. (2.12),(2.13),(2.14) and (2.15) in these expressions yields to the integral equations for the electric and magnetic fields (continuity or PMCHWT formulation):

$$\begin{aligned} \hat{n} \times \left(E_{inc} + \frac{k_{in}^2 + \nabla \nabla \cdot}{j\omega\epsilon_{in}} \int_S G^{in} \bar{J} dS + \frac{k_{out}^2 + \nabla \nabla \cdot}{j\omega\epsilon_{out}} \int_S G^{out} \bar{J} dS \right. \\ \left. - \oint_S \bar{M} \times \nabla' (G^{in} + G^{out} dS) \right) = 0 \\ \hat{n} \times \left(H_{inc} + \frac{k_{in}^2 + \nabla \nabla \cdot}{j\omega\mu_{in}} \int_S G^{in} \bar{M} dS + \frac{k_{out}^2 + \nabla \nabla \cdot}{j\omega\mu_{out}} \int_S G^{out} \bar{J} dS \right. \\ \left. + \oint_S \bar{J} \times \nabla' (G^{in} + G^{out} dS) \right) = 0 \end{aligned}$$

where $\oint$ denotes a principal value integral. It is interesting to notice that since two equivalent currents are associated with each triangles of the mesh of the

dielectric body, the computation effort is significantly higher when the problem consists of simulating/analyzing dielectric structure. A discussion of the equivalence principle can be found in [Har61], and its use in the MoM is detailed in [Har93, PS02, CRSC09].

2.3.4 Eigenmode Analysis

Iso-frequency surfaces and dispersion curves of a periodic structure can be computed numerically with the MoM code. The knowledge of dispersion curves of a metamaterial is very helpful. They provide information about frequency ranges and directions for which eigenmodes may exist and determine the ability of a medium to transmit an electromagnetic wave [SGG03]. Let us explain how we can compute them with the help of the MoM.

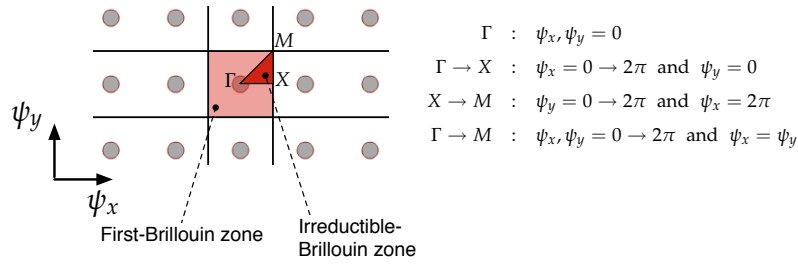


Figure 2.4 Illustration of the First-brillouin zone and the irreducible Brillouin zone for a crystal with a squared lattice

We have seen in Eq. (2.6) that the MoM system of equations of a structure meshed with N basis functions can be written as follows:

$$[Z] [I] = [V] \quad (2.16)$$

with Z the MoM impedance matrix (N -by- N), I the column vector (N -by-1) containing the N unknowns describing the currents and V the excitation vector (N -by-1). By definition, the eigenmodes of a structure are modes that can propagate; hence the current in the structure is non zero, although there is no excitation. Therefore, to find eigenmodes of a structure we have to solve the following MoM system:

$$[Z] [I] = [0] \quad \text{with} \quad [I] \neq [0] \quad (2.17)$$

Therefore, the dispersion relation $f = f(k)$ and the iso-frequency surfaces can be computed by searching for the values of phase shifts $\psi_x = k_x a$ and $\psi_y = k_y b$ which cancel out the determinant of the matrix Z ; i.e. the values of ψ_x and ψ_y that produce at least one zero eigenvalue. Thanks to the periodicity of the structure the search of eigenmodes only needs to be carried out in a unit cell of the reciprocal lattice of the periodic structure called the first-Brillouin zone (ψ_x and ψ_y between 0 and 2π) [C.K96]. Moreover, if the periodic structure possesses additional symmetries, such as mirror planes, the first Brillouin zone may be reduced by eliminating the redundant region. In this case, we obtain the irreducible Brillouin zone, as illustrated in Fig. 2.4

In this thesis, the search of eigenmodes has been mainly used to compute iso-frequency surfaces of a straight wire medium. The knowledge of the iso-frequency surfaces is very useful and allows us to determine when eigenmodes may exist in the structure. An example of computation of an iso-frequency surface is shown in Fig. 2.5. As we will see in Section 4.2.3 for frequencies f lower than the frequency of resonance of the wire medium $f_{\lambda/2}$, eigenmodes may propagate in the xy plane. The existence of these eigenmodes corresponds to the black curves in the iso-frequency plot.

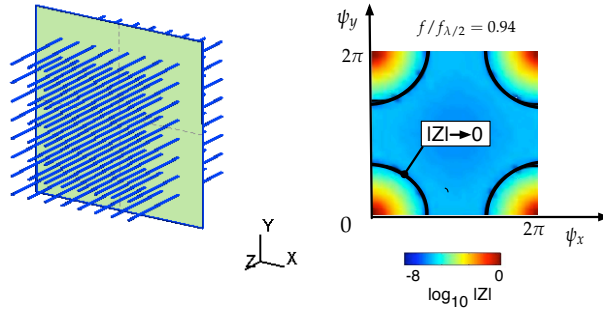


Figure 2.5 Illustration of the existence of eigenmodes in the plane xy when the operational frequency is lower than the resonance frequencies.

2.4 Computing the Point Spread Function of a Wire Medium

The Array Scanning Method [MB79] allows for the determination of fields radiated by an isolated source in an infinite periodic structure. The response to a point source is computed as a continuous superposition of periodic excitations with different phase progressions, also involving the evanescent part of the spectrum. In its discretized form, and in the two-dimensional array case, the ASM reads [MB79,CJW05,CPD04]:

$$I_{n,m} = \frac{1}{N^2} \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} I^\infty(\psi_p, \psi_q) e^{-jn\psi_p} e^{-jm\psi_q} \quad (2.18)$$

where $\psi_p = p \frac{2\pi}{N}$ and $\psi_q = q \frac{2\pi}{N}$

where $I_{n,m}$ is the current in cell (n, m) when one cell every N is excited, starting with cell $(0,0)$, and $I^\infty(\psi_p, \psi_q)$ is the current in the unit cell with all cells excited and inter-element phase shifts ψ_p, ψ_q along x and y , respectively. Since this formulation amounts to reproducing the source every N cells in both principal directions, N needs to be large enough in order to avoid interference between fields due to all sources [DC05]. Fortunately, for collimating structures, like the wire medium, the convergence is very fast, since images from different sources tend not to mix with each other.

The principle of the ASM is illustrated with the help of an ASM 2×2 performed with a straight wire medium. The purpose of the simulation is to determine the image of a dipole collimated through an infinite-by infinite wire medium. The unit cell is constituted of 2×2 wires along x - and y -axis, as shown in Fig. 2.6.a. The simulation is carried out at a frequency for which the magnetic field is well collimated ($f/f_{\lambda/2} = 1.015$). The component along x of the magnetic fields is observed on the other side of the medium with the help of additional testing functions placed in an image plane of size $2 \text{ cm} \times 2 \text{ cm}$. Since $N=2$, four simulations are necessary, each performed with different phase shifts (ψ_p, ψ_q) , namely: $(0, 0)$, $(\pi/2, 0)$, $(0, \pi/2)$ and $(\pi/2, \pi/2)$. The magnetic field related to the different $I^\infty(\psi_p, \psi_q)$ are depicted in the first line in Fig. 2.6. When they are superimposed following the Array scanning method, we obtain the magnetic field illustrated in Fig. 2.6.b, which corresponds to the image of a dipole repeated every 2 unit cell ($N=2$) along x and y through an infinite-by-infinite wire medium.

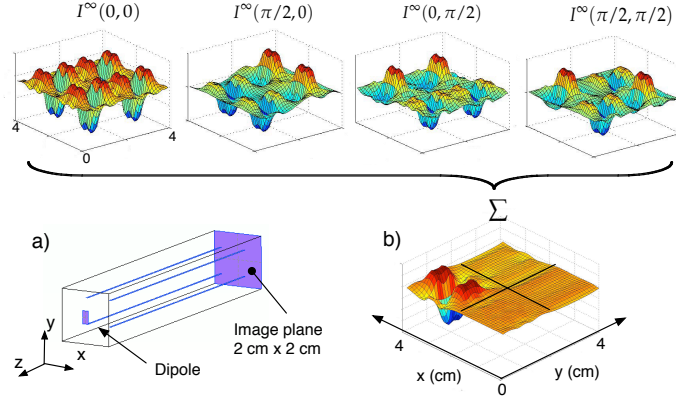


Figure 2.6 (a) Unit cell used for ASM 2×2 , (b) the magnetic fields obtained for the four different phase shifts interfere with each other and provide the image of the dipole (repeated every 2 unit cells) through the wire medium.

It is interesting to compare transmission results obtained with the ASM, and with a full-wave simulation for a straight wire medium. The image of a very small dipole, of the order of the lattice constant, is computed through a medium constituted of 12×12 wires, and is presented in the first column in Fig. 2.7. The magnetic field along two polarizations is computed along the x and y axes. The second column in Fig. 2.7 shows the image of the dipole obtained by using the ASM with $N = 3$; this configuration is equivalent to having the dipole repeated every 3 unit cells in both array directions. The unit cell is built with an array of 4×4 wires instead of one single wire. Hence, the source is implicitly repeated every 12 periods or 12 wires. Both simulations are carried out slightly above $f_{\lambda/2}$, frequency for which the length of the collimator is $\lambda/2$, as suggested in [BMM⁺03] and [NDCT06], in order to achieve a good focusing of the source image. More precisely, at frequency, $f/f_{\lambda/2} = 1.015$. The difference between magnetic fields for the two methods is shown in the third column of Fig. 2.7, for both magnetic field components. Since the collimation is quite effective, edge effects in the finite array and repeated ("aliased") sources in the infinite array are not strongly visible, such that both solutions are quite close to each other.

The point spread function or the spatial impulse response can be obtained by computing with the ASM, the image for different positions of the small

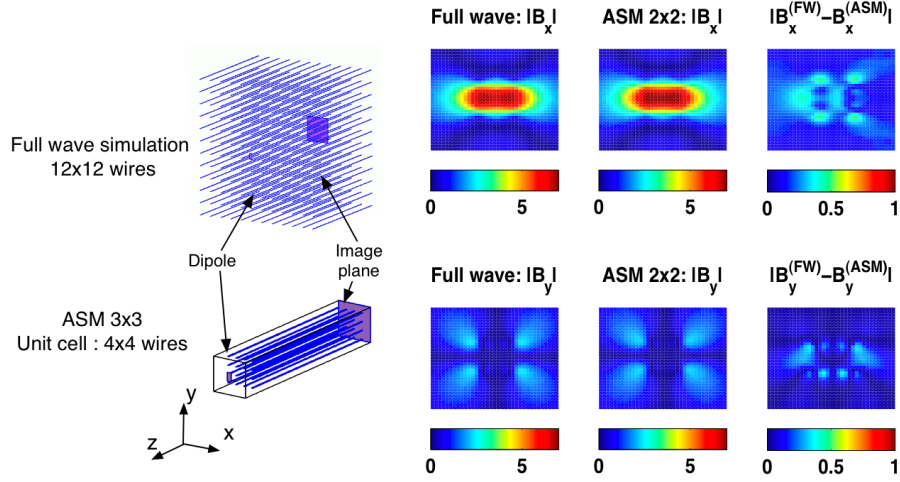


Figure 2.7 Comparison of the magnetic-field image of a dipole obtained, at $f/f_{\lambda/2} = 1.01$, with full-wave simulation inside a 12×12 array (first column), and with an ASM 3×3 (second column). In the latter case, the unit cell contains 4×4 wires. For both methods, two polarizations are shown, either along the dipole direction (first line), or perpendicular to it (second line). In both cases, the field of view corresponds to 4×4 cells. The difference between the fields obtained for both type of arrays is presented in the third column, for both polarizations.

dipole inside a period of the structure. In view of the non-homogeneous structure of the medium, the linear system is not spatially “invariant” and the PSF is dependent on the position of the source within a period of the structure. When computed the PSF can then be used to generate rapidly, by convolution, the image of more complicated sources. It can also be used as a Green’s function exploited in an integral-equation approach to simulate the behavior of structures containing objects on either side of the wire medium without discretizing the latter.

2.5 Extraction of the Constitutive Parameters of a Metamaterial

In this section, we present the method used to extract the constitutive parameters of a metamaterial from its computed S-parameters. The extraction

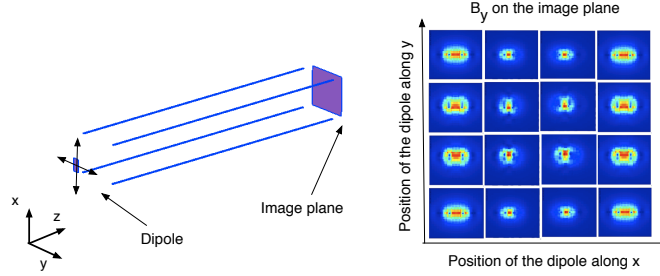


Figure 2.8 Magnetic field obtained on the image plane for different positions of the dipole source within the unit cell. Simulations are carried out with an ASM 5×5 .

method is employed to analyze a metamaterial made of loop-wires and spirals which may be used as a lens with very short focal length in order to improve the resolution of Magnetic Resonance Imaging [Pen00,ZH01,WPY⁺01]. The RF-field, detected by the antennas in last-generation MRI systems (3Tesla static field), has a frequency close to 123 MHz and a bandwidth of 500 kHz. The main challenge in the design of the metamaterial is the reduction of the unit cell size to one hundredth of a wavelength, while having an index of refraction close to minus one.

Hereafter, in a first step we show the permittivity and permeability extracted from MoM simulation results with the help of the method presented in [CGB⁺04]. In order to validate our simulation tools the results obtained are compared with results described in the literature [AGK⁺04,CGB⁺04]. Then in a second step, we present the basis structures (loop-wires and spirals), developed in the framework of this thesis [RSD⁺06,HXS⁺07], to achieve the required unit cell size reduction and obtain a metamaterial operating at the MRI frequency.

2.5.1 Infinite Array Analysis

In this subsection, we study a metamaterial filled periodically with double split ring resonators (SRR) and their associated strips. In order to enable comparisons, the dimensions of the structure are the same as in [AGK⁺04]. The geometric parameters of the SRR and the rod are the following: $d = t = 0.2$ mm, $w = 0.9$ mm, and $r = 1.6$ mm, the significance of the symbols is explained in Fig. 2.9.a. The whole structure is made of thin perfect electric conductor. The

unit cells used for the infinite-by-infinite array simulation are made either of 3 or 5 layers of SRR's and rods as presented in Figs.2.9.b and c.

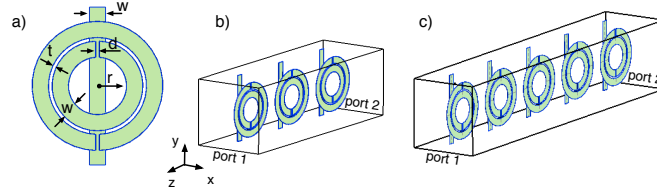


Figure 2.9 (a) Schematic of a single SRR combined with a rod, (b) and (c) unit cells used for the infinite-by-infinite array simulations. Each unit cell is periodized in the xy plane.

S-parameters calculated with a wave incident (along the z axis) on each slab of metamaterial are presented in Fig. 2.10.c. We can see that there is a peak in reflection data at 1.85 GHz and a peak in transmission data at 2.1 GHz, current distributions related to these two frequencies are respectively depicted in Fig. 2.10.a and Fig. 2.10.b, for the 3-layers case. As expected, for zero transmission, only the first layer of SRR's is excited. The number of layers has an impact on the band-gap depths as we can see the transmission band-gap around 2.2 GHz, it reaches -10dB with 3 layers while with 5 layers it goes down to -22dB.

In comparison, with results presented in Fig. 2.11.b and obtained with a structure constituted of 5 layers, the S_{21} curves have approximatively the same behavior. Nevertheless, the depth of the second band-gap around 2.2GHz in Fig. 2.10.c is narrower and less deep with the MoM. Moreover, in our case the transmission and reflections curves are shifted down by about 1.8 GHz due to the absence of FR4 dielectric substrate (FR4 dielectric constant $\epsilon = 4.4$). Indeed, our first peak in transmission is positioned at 2.1GHz while the corresponding peak in Fig. 2.11.b is at 3.9GHz. A common rule of thumb to take into account the presence of a dielectric substrate, consists of using the square root of the dielectric constant to approximate the relative frequency shift. The application of this rule leads to a relative frequency shift of 2.01 GHz for the first peak in transmission which is approximatively what we have obtained. Actually, the frequency shift measured is lower since the pre-

vious rule supposes that the structure is entirely surrounded by the dielectric medium which is not the case here.

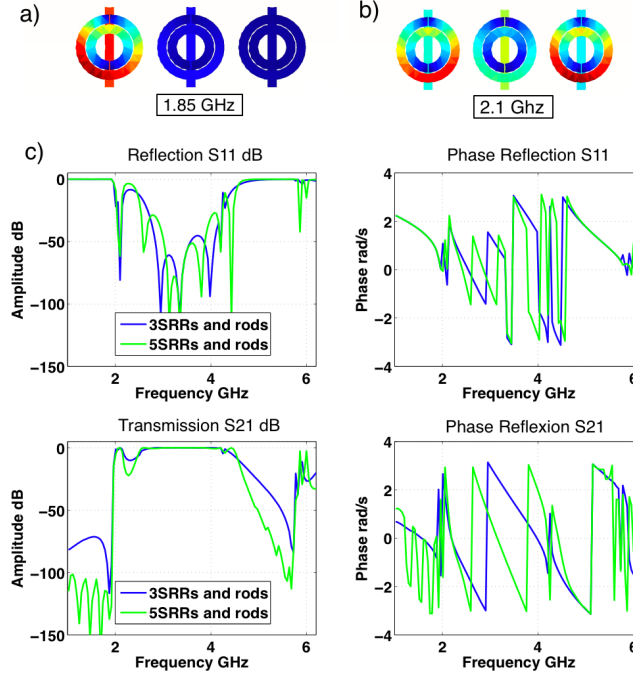


Figure 2.10 Current distribution computed at (a) 1.85GHz and at (b) 2GHz with a structure composed of 3 layers of SRR's and rods and excited at port 1; (c) S-parameters obtained with the unit cells presented in Fig. 2.9.b and c. and containing either 3 layers or 5 layers of SRR's and rods. Compared with Fig. 2.11 the left-handed peak obtained with the MoM (around 2GHz) is at a lower frequency due to the absence of dielectric substrate.

2.5.2 Constitutive Parameters Extraction

Several methods exist to extract the constitutive parameters (permeability and permittivity) from S-parameters: [VT06], [Sai04], [CGB⁺04], [SSMS02], [NG70], [HSD01]. These methods suppose that only one mode is propagating through the structure. The mode is observed through its ability to reflect and transmit the single propagating Floquet mode for normal incidence only, this

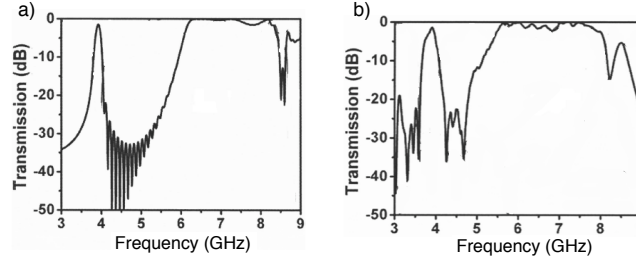


Figure 2.11 (a) Curve S_{21} obtained numerically with FDTD in [AGK⁺04], the unit cell used for the simulation is constituted of 5 SRR's and rods in depth, the left handed peak corresponds to the peak at 4.5GHz (b) S_{21} -parameter obtained experimentally with the help of two horns. In this case, the metamaterial has finite dimensions and it is constituted of $18 \times 15 \times 5$ unit cells along the x, y and z axis, respectively. In both cases, the SRR's and rods are placed on a 1.6-mm thickness FR4 board ($\epsilon = 4.4$).

may be viewed as a limited projection of that mode. In practice, the retrieval process may fail in some instances, more specifically when the thickness of the effective slab is not properly estimated or when reflection and transmission data are small in magnitude or noisy [Zio03, SBH03]. In this thesis we use the method described in [CGB⁺04], since it proposes a solution to these issues.

Constitutive parameters extracted from S-parameters are presented in Fig. 2.12. These parameters have the same behavior as those obtained in [CGB⁺04] for a similar metamaterial constituted of two adjacent square SRR's and their associated strip. We have to note that there is a frequency region for which no constitutive parameters can be found. This region corresponds to the resonance region, its properties are still being debated in the literature [ST07, KME⁺03] and for this reason the retrieved results are interrupted in frequency by the resonance region.

2.5.3 Analysis of a Compact Metamaterial

In this subsection, we present the two basis structures developed to design a metamaterial that would operate at MRI frequencies. The constraints on the metamaterial are important: it must have an index of refraction close to minus one and have unit cells of the order of one hundredth of a wavelength. This

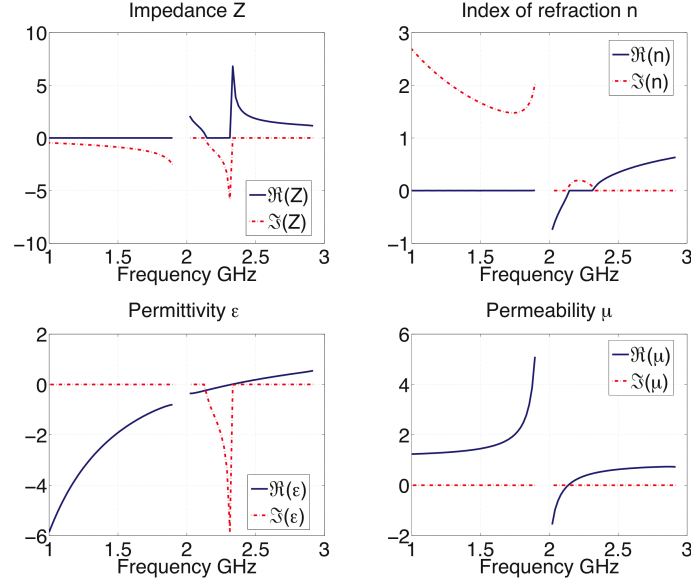


Figure 2.12 Constitutive parameters of the 3-layers metamaterial shown in Fig.2.9.b. These parameters are extracted with the method described in [CGB⁺04] from S-parameters obtained with the MoM and presented in Fig. 2.10.c.

constraints ensure to obtain a lens with a sufficient resolution (of the order of the unit cell) and which is matched to the air. The structures employed to obtain these properties are an array of loop-wires combined with an array of spirals.

2.5.3.1 Loop-wire Medium Analysis

Two structures have been proposed in the literature to obtain negative permittivity: an array of closely spaced thin wires [PHRS98] and an array of loop wires [SVW⁺99]. The former is widely used but its scaling at radio frequencies, leads to huge unit cells, difficult to fabricate and to introduce in MRI systems. The latter structure allows us to reduce the plasma frequency and to achieve the desired negative permittivity at low frequencies. In this section, we show how the permittivity increases, when the loop-wires inductance is increased [RSD⁺06]. To demonstrate this idea, a parametric study has been carried out, showing the influence of the loop wires dimensions on the plasma frequency and, hence, on the permittivity.

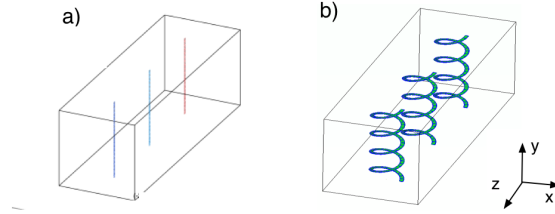


Figure 2.13 (a) Array of thin wires, (b) array of loop wires.

Principle

Metals have a characteristic response to electromagnetic radiation due to the plasma resonance of the electron gas [Rot62, PHS⁺96]. The effective dielectric constant of a metallic structure is determined by the plasma frequency ω_p according to

$$\epsilon_{eff} = 1 - \frac{\omega_p^2}{\omega^2} \quad (2.19)$$

The plasma frequency is inversely proportional to the inductance of the unit cell. Increasing the inductance can be accomplished by making the wire radius very thin or by using a wire-loop medium, which leads to structures easier to fabricate, especially for magnetic resonance imaging. The link between the plasma frequency and the inductance of the unit cell is presented hereafter.

An analytical model for the inductance of an array of wires of radius r and spaced with distance d is given in [QMW⁺05],

$$L_{wires} = L' d^2 = \mu_0 \frac{\ln(d/r_{wire})}{2\pi} d^2 \quad (2.20)$$

where L' is the inductance density in the unit cell. The plasma frequency can be expressed as a function of L_{wires} , by using the line theory for metamaterial developed in [CCI01]:

$$\epsilon_{eff} = 1 - \frac{1}{\omega^2 L_{wires} \epsilon_0} = 1 - \frac{\omega_p^2}{\omega^2} \quad (2.21)$$

$$\omega_p = \frac{1}{L_{wires} \epsilon_0} = \frac{1}{d^2 L' \epsilon_0} \quad (2.22)$$

To evaluate the plasma frequency for a loop-wire medium, we must adapt the expression of the inductance density, given in (2.20), by using a new expression which takes into account the loops. In [SVW⁺99], it is proposed to add to

the classical inductance density, the contribution of the inductance of a loop given in [Vor94]; the new inductance L' is then given as:

$$L' = \frac{\mu_0 R N^2 (\ln(8R/r_{wire}) - 1.75)}{2\pi R N} + \mu_0 \frac{\ln(d/r_{wire})}{2\pi} \quad (2.23)$$

The first term is the inductance density of N loops of radius R , the second is the contribution of an array of wires of radius r_{wire} separated by a distance d . This expression consider that there is no interaction between adjacent loops. Replacing this expression in (2.22) gives the corresponding plasma frequency. Hereafter, simulations are carried out with strips. A rule of thumb to adapt the expression (2.23) to the strip case is to consider the following geometric conversion: $w_{strip} = 4r_{wire}$ [GRS05]. A parametric study of loop-wire media is now presented.

Parametric study of loop wires

In this section, we evaluate the influence of geometric parameters of loop-wire medium on the real part of the permittivity and we compare with the permittivity of a classic wire-medium. For the parametric study, the loop wire medium dimensions vary: for the strips width from 0.05 cm to 0.25 cm, for the number of loops from 4 to 16, and for the radius from 0.3 cm to 1.3 cm. Dimensions of one unit cell is 4 cm $\times$ 4 cm $\times$ 4 cm.

- *Influence of the number of turns:* Fig. 2.14.a. depicts the permittivity analytically obtained for various radii. The reference comparison for is the solid curve which is the real permittivity computed for three strips of width 0.1 cm. We can see that, as long as the number of turns remains small, there is no real gain to use a loop-wire medium. To have a significant permittivity reduction, the number of turns and the radius need to be increased.
- *Influence of the radius:* The influence of the radius is presented in Fig. 2.14.b. As seen before, the value of the radius is quite large. The greater the radius the larger the permittivity for a given frequency. It stems from the fact that an increase of the radius increases the amount of magnetic flux in the loop, and consequently the inductance.
- *Influence of width of the strips:* The influence of the width of the strips is presented in the Fig. 2.14.c. It can be seen that the larger the strips, the

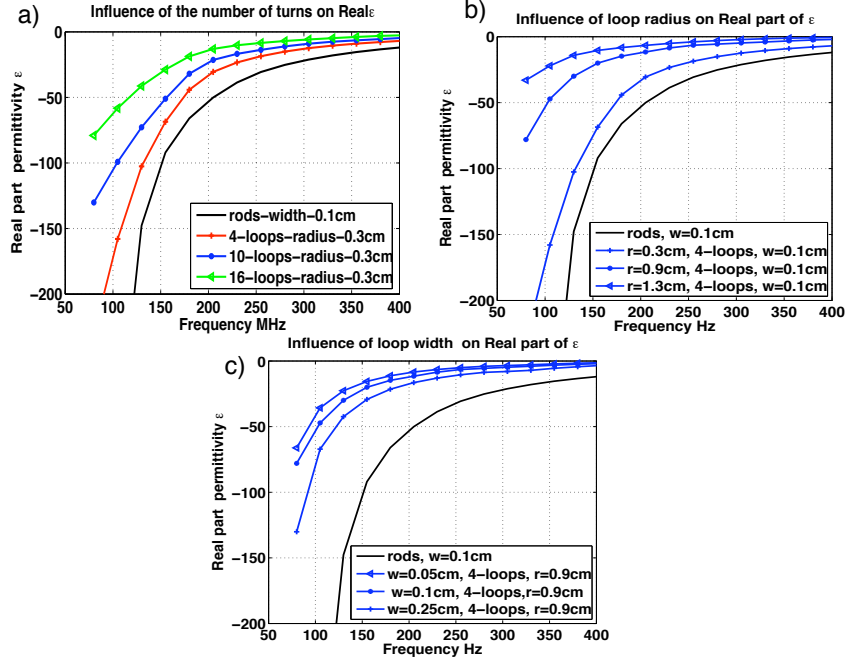


Figure 2.14 Analytic results, influence of on the real part of the permittivity of: (a) the number of loops, (b) the radius of the loops, and (c) the width of the loops.

larger is the absolute value of the permittivity. It stems from two main effects: the first one is a decrease of the magnetic flux which flows in the helix when the strips width increases, and the second one is an increase of the wire inductance, like in a classical array of rods when the rod is narrower.

In order to validate the model for the inductance given in Eq.2.23, we have compared the permittivity obtained with the analytical model and with simulations results. In order to have a significant phenomenon and stable results, simulations were made with three unit cells in depth (along z) illustrated in Fig. 2.13. The plasma frequency of reference is evaluated with rods of width 0.1 cm. The comparison is shown for various geometrical parameters of the loops, in Fig. 2.15. Comparisons are made at the frequency of interest for MRI, i.e. 123 MHz. Results are in agreement, except for large radii, where it seems

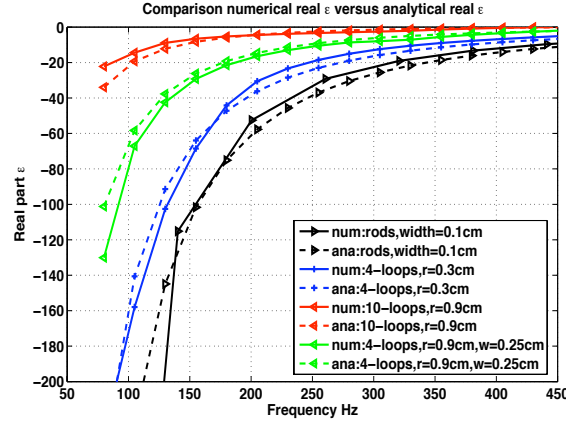


Figure 2.15 Comparison between the permittivity obtained with the analytical model and extracted from simulations results.

that the analytical method underestimates the value of the permittivity. For large radii, the coupling must be taken into account into the model, to have more accurate results.

2.5.3.2 Analysis of Spiral Media

Infinite array analysis: In parallel to loop-wire media, a medium with high negative permeability has been designed. This metamaterial is based on spiral particles, which allow a compact structure in comparison to the wavelength [BMM04] and does not contain magnetic elements which could create artifacts in MRI. The most challenging problem is to design the spiral structure so that negative permeability occurs at the operational frequency of 3-Tesla MRI systems (123 MHz). The difficulty is due to the limited bandwidth of the resonant effect and to its sensitivity to several factors that have been analyzed and modeled with the help of an equivalent circuit approach [HXS⁺07].

In this section, we present simulation results obtained in the infinite-by-infinite array case and computed with Method of Moments. In order to evaluate the mutual inductance between spirals, two unit cells are considered, as presented in Figs. 2.16.a and 2.16.b. When isolated, the spirals have a resonance frequency of 123 MHz (frequency of the MRI system for a 3 Tesla static

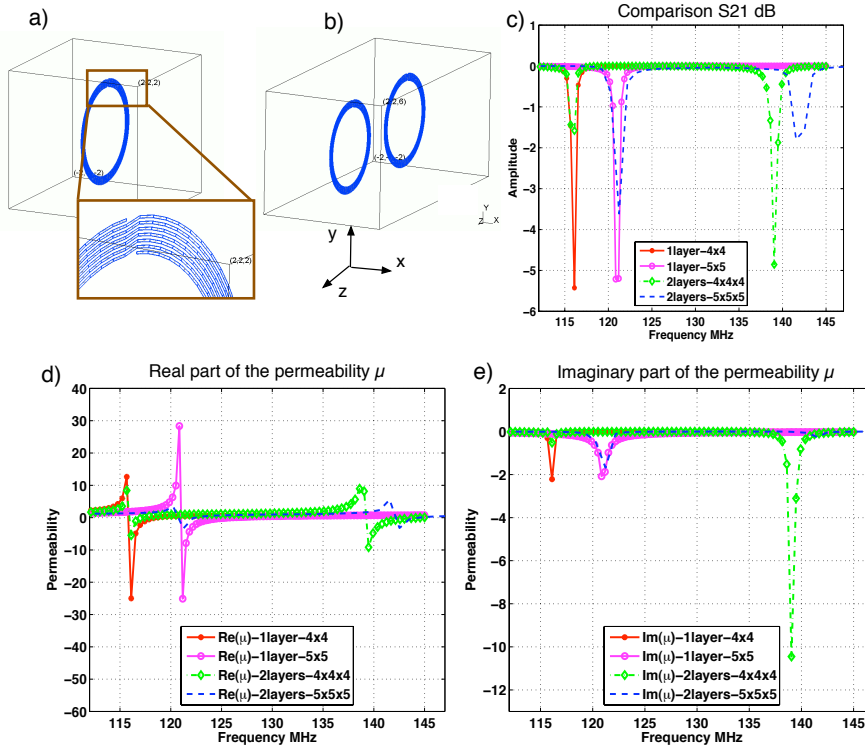


Figure 2.16 Infinite array approach geometry of unit cell with (a) one layer or (b) two-layer; (c) S-parameters computed using home-made MoM code, (d) and (e) real and imaginary part of μ extracted from simulated S-parameters [CGB⁺04].

field). The width and the interstice between turns of the spirals are respectively $w = 0.160$ mm and $s = 0.2$ mm. The external spiral diameter is $D = 3$ cm and the unit cell size varies from $4 \times 4 \times 4$ cm to $5 \times 5 \times 5$ cm. Each spiral is meshed with 1700 basis functions, since their size is around $\lambda/1500$, special care must be taken during the simulation to avoid instabilities [LLB03].

For one layer of spirals, there is a dip in transmission (S21), as shown in Fig. 2.16.c, which corresponds to a resonance of the corresponding permeability associated to artificial magnetism of spirals (Fig. 2.16.d): the resonant frequency is 117 MHz, and there is only one dip, induced by the extremum of the imaginary part of permeability (solid red line). Mutual coupling within the layer has lowered the resonant frequency. When the unit cell size for one

layer is increased, the resonant frequency tends to go up (bullet-solid line). Above this frequency, the real part of the permeability is negative over less than 5 MHz. When a second layer of spirals is added, a second peak at a higher frequency appears (bullet-dotted, dotted lines); this peak is due to the in-plane mutual inductance between spirals [HXS⁺07]. The depth of the second peak depends on the unit cell size. When the unit cell size is increased (dotted curve), the peak due to the mutual inductance tends to disappear.

Physical explanation of multiple resonance frequencies: The emergence of multiple resonance frequencies can be explained with an equivalent circuit approach [HXS⁺07]. Fig. 2.17.a depicts the equivalent circuit per unit length of propagation: it models the interaction between the incident electromagnetic (EM) wave and one spiral resonator. In this circuit, the spiral is modeled by the resistance R_s , inductance L_s , and capacitance C_s of the resonant circuit. The spiral inductance L_s is calculated using formulas developed by Rosa [Ros06], the capacitance C_s is estimated from the capacitance between concentric cylinders and fringing capacitances, following the method reported in [DMH08], while R_s is simply obtained as the total resistance of the conducting strip forming the spiral. Elements L_o and C_o model the propagation of the EM wave in vacuum. The interaction between the EM wave and the spiral is rendered by the mutual inductance M_o , calculated analytically by taking into account the induction flux created by the magnetic field of the EM wave into the spiral [DMH08]. This equivalent circuit is implemented in the ADS software from Agilent for the calculation of parameter S_{21} rendering the propagation of the EM wave through the spiral. Fig. 2.17.b shows the result for S_{21} : in accordance with results presented at Fig. 2.16.c, a sharp absorption peak is observed around 117 MHz, characteristic of the resonant behavior of the spiral.

Fig. 2.17.c shows the equivalent circuit for two layers of spiral resonators. Two equivalent circuits of Fig. 2.17.a are cascaded and their resonant parts are coupled through mutual inductance M modeling the in-plane coupling between inductances from each layer. Figs. 2.17.d and.e show the transmission coefficient S_{21} calculated with ADS for two values of the mutual inductance M between spirals. Results show that the number and position of resonances are strongly dependent on the coupling between spirals: for low coupling (Fig. 2.17.e), there is only one resonance, corresponding to that of a single res-

onator (i.e. 126 MHz), while for increased coupling (Fig. 2.17.d), a secondary resonance appears, while the main one is shifted toward lower frequencies (119 MHz, in this example). The same frequency shift is observed in the first part, when the mutual coupling is lowered by increasing the unit cell size (Fig. 2.16).

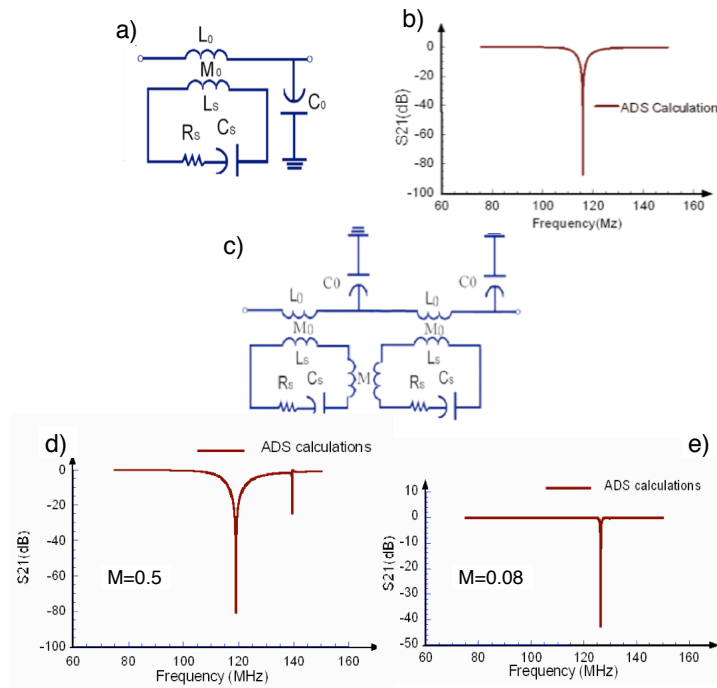


Figure 2.17 (a) Equivalent circuit for the interaction between EM wave and one spiral resonator, (b) transmission parameter S_{21} calculated with ADS, (c) equivalent circuit for two layers of spiral resonators, (d) transmission coefficient S_{21} calculated with ADS for $M = 0.5 \mu\text{H}$, (e) transmission coefficient S_{21} calculated with ADS for $M = 0.08 \mu\text{H}$

2.5.3.3 Metamaterials based on loop-wires and spirals

In this subsection, we simulate metamaterials whose unit cell is composed of a spiral and a loop-wire (or a rod). The purpose is to evaluate the impact of loop-wire medium on the value of the index of refraction. The spiral used

is the same as in section 2.5.3.2 and has a resonant frequency of 123 MHz when simulated alone. Three types of negative permittivity structures are simulated, namely: an array of strips, an array of loop-wires with 4-loops per unit cell and an array of loop-wires with 10-loops per unit cell. Dimensions of one unit cell are $4\text{ cm} \times 4\text{ cm} \times 4\text{ cm}$. The three types of unit-cell analyzed are illustrated in Fig. 2.18. The distance between the axis of the loop wire and the center is set to 2 cm.

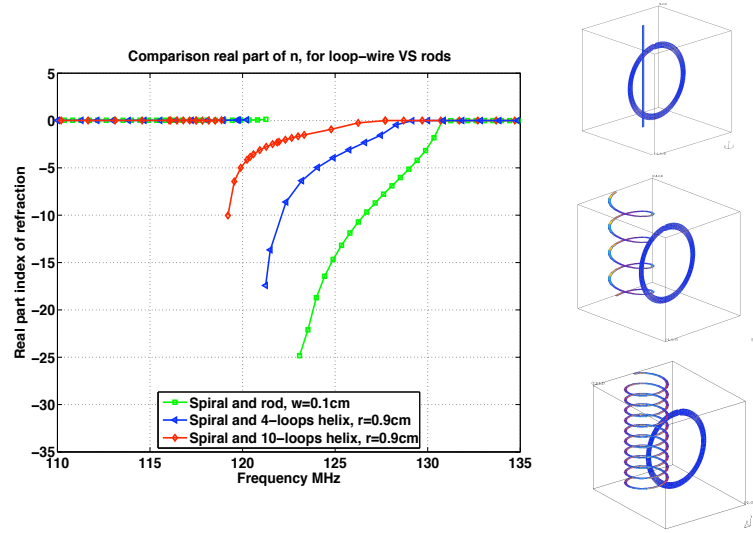


Figure 2.18 Real part of the index of refraction obtained with three different geometries of metamaterial. By varying the number of turns of the loop-wire inside an unit-cell, the value of the negative index of refraction can be tuned to obtain the desired value.

One layer of each unit-cell is simulated in infinite-by-infinite array with a MoM code. The index of refraction is extracted from S-parameters using the method suggested by [CGB⁺04]. The real parts of the extracted index of refraction are shown in Fig. 2.18, we see that a negative index of refraction is present, and that the use of loop-wire medium allow to decrease its value, as predicted. As for classical rods, the presence of the loop-wire medium causes a frequency shift of the resonance peak of the spiral.

2.6 Simulation of a Large-Scale Finite Array

When we consider complex unit cells of an artificial material, made of a periodic structure of finite extent, the method of moments (MoM) may lead to a systems of equations with a very large number of unknowns, and may require a very large computation time. A solution to accelerate the analysis of such structures is to use a macro basis functions (MBF) approach combined with the MoM. Indeed, this approach dramatically reduces the system of equations, is not iterative and provides some physical insight.

The MBF approach is based on the principle that fields on sub-domains of a large structure can be approximated by the superposition of solutions obtained when the sub-domains are isolated or embedded in small structures.

Moreover, in passive periodic structures, fields (or equivalent currents) that can be excited on a given cell have relatively few physical degrees of freedom [MVDO01] for most types of excitations, at least for excitation from outside the sub-domain (assuming a minimum distance of a fraction of the wavelength from the structure). In other words, the physically possible discretized solutions belong to a relatively small subspace of the space representing all possible mathematical solutions (with dimension equal to the number of unknowns). An alternative basis set for this physical subspace can be formed by the solution for a given unit cell under a few illuminations [SM00,MVDO01,PM03,MLV07]. This is the case of metamaterials since the elements to be analyzed are smaller than the wavelength, the fields radiated by a given structure can be described with very few parameters [CCI07], [BF07].

A key point to obtain accurate results with the MBF approach is the choice of the size of the basis and the method used to construct it. A relatively open problem then consists of finding a systematic way to construct such a basis. Several methods, developed for antenna array applications and for metamaterials, have been proposed in the literature. A first one consists of exciting one cell from a number of directions [PM03] or from point sources located on a surface enclosing the structure [MLV07]. A singular value decomposition (SVD) procedure then enables the reduction of the number of generated Macro Basis Functions (MBFs), based on their linear dependence below a certain threshold. Another method for MBF generation is based on the illumination of an isolated element (primary), which in turn, illuminates other elements (secondaries) of the array [YPM03]. Alternative methods are based on the solu-

tions obtained with small arrays (e.g. 3×3 arrays) [LCZ07]. Finally, another approach starts from the superposition of infinite-array solutions [CS08] for a given set of phasings along the periodic directions, $k_{x,0}a$ and $k_{y,0}b$, which finds its justification in the Array Scanning Method. Reference [PM03] shows how to obtain the reduced system of equations, while an accelerated technique, which avoids computation of the original system of equations, based on multipole decompositions, is described in [Cra06].

In this section, a new method to create rapidly an MBF basis is presented. It creates a basis which provides more accurate simulation results, as compared to classical approaches, in particular when the metamaterial contains connected unit cells like rods. We first start by presenting the principle of the MBF approach [SM00], [YPM03].

2.6.1 Principle of the macro basis functions (MBF)

Let us consider a general problem shown in Fig. 2.19.a to introduce the MBF approach. The structure represented is a metamaterial filled periodically with double split ring resonators (SRR) and their associated strips. Further details about the dimensions of the structure are given in the next section. We want to determine the value of the currents flowing on the entire structure in order to determine the coefficient of transmission of the metamaterial. The structure contains in total, U unit cells, each meshed exactly in the same manner with N basis functions $\tilde{f}(n)$, as shown in Fig. 2.19.b. The general large-scale problem can be expressed under a conventional MoM system of equations as written in Eq.(2.6):

$$[Z] [I] = [V] \quad (2.24)$$

with Z the MoM impedance matrix ($N.U$ -by- $N.U$), I the column vector ($N.U$ -by-1) containing the $N.U$ weighting coefficients describing the currents flowing on the structure and V is the excitation vector ($N.U$ -by-1).

The current $\tilde{J}$ flowing on the entire structure can be expressed in terms of the basis functions defined with the mesh of a unit cell, as follows,

$$\tilde{J} = \sum_{u=1}^U \sum_{n=1}^N x_{u,i} \tilde{f}_i^{(u)} \quad (2.25)$$

where $x_{u,i}$ is the weighting coefficients associated with the basis functions $\tilde{f}_i^{(u)}$, i.e. the basis function i on the unit cell u . The total number of unknowns is

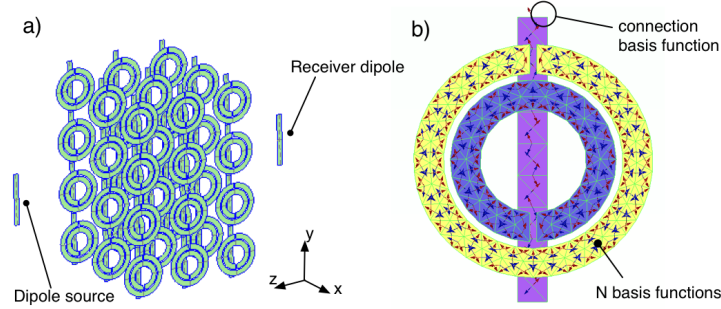


Figure 2.19 (a) The general problem constituted of U unit cells, (b) each unit cell is meshed with N basis functions and is made of the combination of two split-ring resonators and a strip. The whole structure is in perfect electric conductor.

equal to NU and the system rapidly becomes difficult to solve directly, when the number of unit cells increases.

Let us suppose that we have created a basis containing M typical current distributions $\bar{J}_m$ which can flow on a unit cell with $M \ll N$. The basis can be written with a N -by- M matrix: $[C] = [\bar{J}_1, \bar{J}_2, \dots, \bar{J}_M]$. The simplification of the MoM system of equations is realized by expressing the current distributions flowing on every unit cell of the structure as a linear combination of the M current distributions in the basis. The current distribution on the global structure can then be written as follows:

$$\bar{J} = \sum_{u=1}^U \sum_{m=1}^M \alpha_{u,m} \bar{J}_m \quad \text{with} \quad \bar{J}_j \in [C] \quad (2.26)$$

In this reduced system of equations, the coefficients $\alpha_{m,u}$ are the new unknowns. The solution time is strongly reduced, since the number of unknowns of the large-scale problem passes from $N.U$ to $M.U$ (with $M \ll N$). Moreover, the MBF approach has also a stabilization effect (better conditioning), which is interesting for fine meshes with respect to the wavelength, as it is generally the case for metamaterials. The key point for an accurate MBF approach is the basis calculations. Several methods will be compared below.

The implementation of the MBF method in a MoM is very straightforward, if the sub-impedance matrix $Z_{sub}^{(p,p')}$ between a unit cell p and a unit cell p' is

known (with p and p' comprised between 1 and NU .) With this notation the general system in Eq. (2.24) can be written as:

$$\begin{bmatrix} Z_{sub}^{(1,1)} & \dots & Z_{sub}^{(1,U)} \\ \vdots & \ddots & \vdots \\ Z_{sub}^{(U,1)} & \dots & Z_{sub}^{(U,U)} \end{bmatrix} \begin{bmatrix} I_{sub}^{(1)} \\ \vdots \\ I_{sub}^{(U)} \end{bmatrix} = \begin{bmatrix} V_{sub}^{(1)} \\ \vdots \\ V_{sub}^{(U)} \end{bmatrix}$$

The reduction of the system size is realized by multiplying the previous system on the left with a matrix C_{diag} containing U times the matrix C on its diagonal, and on the right with C_{diag}^T the matrix C_{diag} transposed. The following reduced system of equations is obtained:

$$\begin{bmatrix} CZ_{sub}^{(1,1)}C^T & \dots & CZ_{sub}^{(1,U)}C^T \\ \vdots & \ddots & \vdots \\ CZ_{sub}^{(U,1)}C^T & \dots & CZ_{sub}^{(U,U)}C^T \end{bmatrix} \begin{bmatrix} \alpha_{sub}^{(1)} \\ \vdots \\ \alpha_{sub}^{(U)} \end{bmatrix} = \begin{bmatrix} CV_{sub}^{(1)} \\ \vdots \\ CV_{sub}^{(U)} \end{bmatrix}$$

Once the system is solved and the coefficients $\alpha_{sub}^{(m)}$ are determined, the current $\bar{J}$ is calculated using Eq.(2.26)

2.6.1.1 Example: Transmission through a Large-Scale Metamaterial

An example for the use of Macro Basis Functions in the analysis of finite metamaterials is given below [RC07]. The structure analyzed is composed of $3 \times 4 \times 3$ unit cells along the x , y , and z axis, respectively and presented in Fig. 2.19.a. The strips of neighbouring unit cells are connected between each other along the y -axis. Our aim is to determine the coefficient of transmission of the structure by employing two dipoles placed on both sides of the metamaterial.

More precisely, the array is illuminated, on one side, by a dipole and observed by a receiving dipole on the other side. Both dipoles are short in comparison to the wavelength and have a height of 2 cm. Every unit cell of the array is made of the combination of a metallic split-ring resonator and a strip, in order to realize a negative-index material [PS04] (see Fig. 2.19 and Fig. 2.22).

The dimensions of SRR's and strips are the same as described in Section 2.5.1 and are chosen in order to provide a negative index of refraction around 2 GHz when they are simulated in an infinite-by-infinite array. Each unit cell occupies a cube with sides of 2.6 cm, and is made of a thin perfect electric conductor meshed with 94 RWG [RWG82] basis functions.

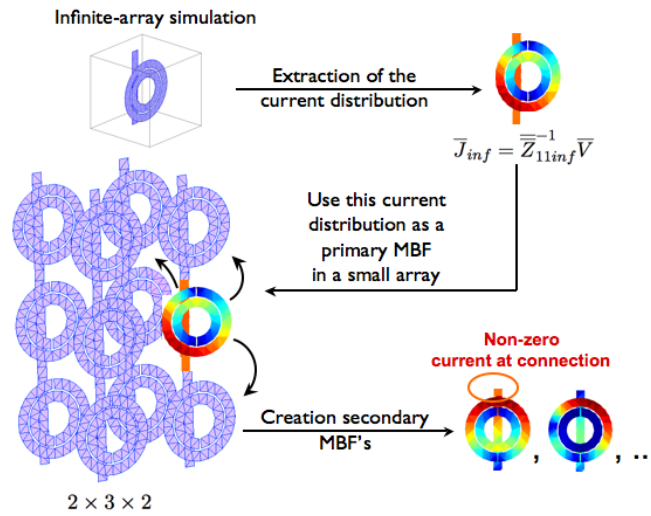


Figure 2.20 Scheme for creating a set of MBFs from an infinite-array solution.

The transmission between both dipoles is observed versus frequency, and is defined as the ratio between the current flowing at the center of each dipole with the presence of the metamaterial, normalized with respect to the same ratio in absence of the metamaterial. The transmission is computed by simulating the structure in three different manners: by using MBF's from the infinite-array solution, by using MBF's from a $2 \times 3 \times 2$ -array solution, and by full-wave simulation (gaussian elimination). The latter solution is used as a reference solution to calculate the absolute error of the transmission obtained with the MBF methods and requires solving a system containing 3384 unknowns.

For the first method based on the MBF's from the infinite-array solution, the primary MBF corresponds to the infinite-array solution obtained with a plane-wave excitation. The secondary MBF's [YPM03] are obtained by placing the primary MBF in a small-scale interconnected structure, constituted of

$2 \times 3 \times 2$ unit cells, and by computing on the neighboring elements, taken individually, the currents induced by the fields radiated by the primary MBF (see Fig. 2.20). The advantages of this approach are that (i) only two MoM impedance matrices need to be inverted, corresponding to infinite-array and single-element cases, respectively, and (ii) unit cells connecting to the primary MBF solution inherently exhibit continuous currents across the connection.

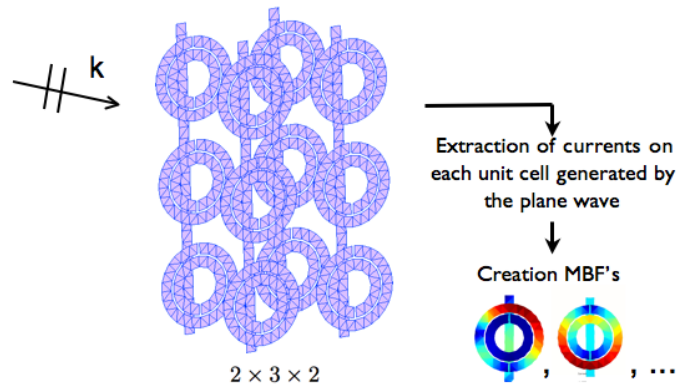


Figure 2.21 Scheme for creating a set of MBFs from a finite-array solution.

Regarding the second method based on the MBF's from the finite-array solution, the approach is a more straightforward -but also computationally more expensive- method for obtaining the MBFs [RCH06], which consists of analyzing by brute force a small-scale interconnected structure, made of $2 \times 3 \times 2$ cells, excited by a plane-wave (see Fig. 2.21). The MBF's are extracted from the currents induced on each unit cell of the structure.

Both MBF approaches allow to reduce dramatically the number of unknowns of the complete system of equations. Indeed, since in both methods, 12 MBF's are obtained, the number of unknowns is reduced from 3384 unknowns to 432 unknowns.

Fig. 2.22 compares the different approaches. The top line corresponds to the gaussian elimination transmission, as defined above, for the $3 \times 4 \times 3$ array. Overall, the presence of the metamaterial leads to a certain blockage (transmission coefficient below zero). Since the structure is not very large compared to the wavelength, that blockage is limited. The peak near 1.9 GHz is believed

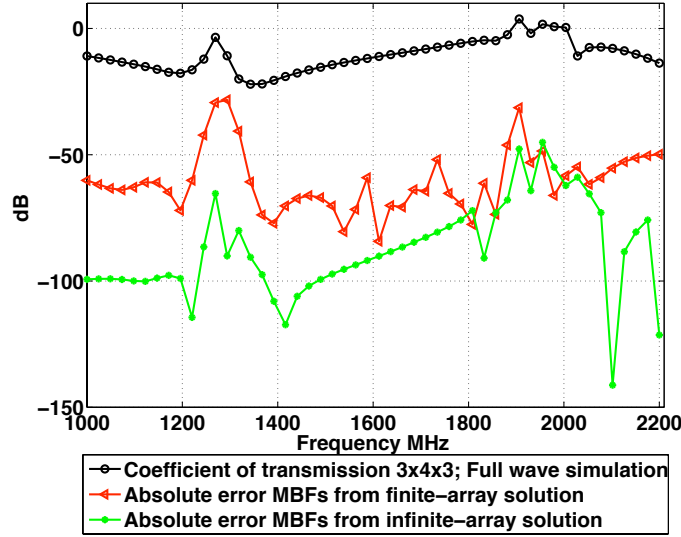


Figure 2.22 Transmission coefficient, as defined in Section 1.8.3, for a periodic meta-material made of $3 \times 4 \times 3$ unit cells.

to correspond to the negative-index region, where a pass-band is expected. Indeed, the dimensions of the SRR's and the rods are exactly the same as those presented in Section 2.5.1 where an infinite-by-infinite array simulation has demonstrated the existence of a negative index of refraction around 2GHz. We have verified that the peak around 1.3 GHz is related to a resonance of the finite wires by simulating larger structure (see hereafter). The two other lines represent, on the same dB scale, the errors obtained with the two MBF-based approaches; the error is computed as the magnitude of the difference between the MBF-based solution and the brute-force solution. The line with bullets shows the errors when the MBF's are obtained from the small finite-array analysis. The curve with triangles shows the errors when the MBF's are obtained from the infinite-array solution. It can clearly be seen that the error from the latter method is much smaller, i.e. -60 dB, as compared to the reference solution. This method is also much faster, since it requires the solution of unit-cell problems only.

Fig. 2.23 further analyses the physical behavior of the coefficient of transmission obtained previously with the $3 \times 4 \times 3$ array. For the convenience, this

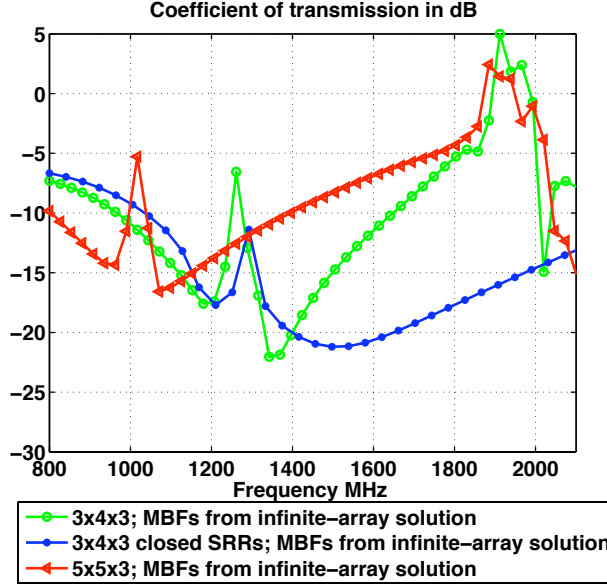


Figure 2.23 Transmission coefficients for metamaterials made of $3 \times 4 \times 3$ and $4 \times 5 \times 4$ unit cells. The left handed peak around 1.9GHz disappears when the SRR's are closed.

curve is repeated in Fig. 2.23 with the circled solid line. In order to verify that the peak near 1.9 GHz corresponds to a negative-index region, a $3 \times 4 \times 3$ array constituted of *closed* SRR's and rods has been simulated. As expected, in this case the LHM peak disappears as illustrated with the solid line with dots, since when the SRR's are closed the magnetic behavior disappears. On the other hand, we have seen that the coefficient of transmission with a $3 \times 4 \times 3$ array shows a peak in transmission around 1.3GHz which is caused by a resonance of the rods of the structure. Indeed, the height of the rods in $3 \times 4 \times 3$ array corresponds approximatively to half a wavelength at 1.3GHz. To further analyze the phenomenon, the coefficient of transmission of a $5 \times 5 \times 3$ array has been computed and represented with the solid lines with triangles. We can see that since the structure is larger the peak shift down at around 1.05 GHz.

Introduction to Magnetic Resonance Imaging

3

3.1 Introduction

Magnetic resonance imaging (MRI) is an imaging technique used primarily in radiology to produce high quality images of the inside of the human body. This technique is mainly used to distinguish pathologic tissue (such as a brain tumor) from normal tissue. Compared to computed tomography scans and traditional X-rays, the main advantage of MRI is that it is harmless to the patient since it uses strong magnetic fields and non-ionizing radiation in the radio frequency range.

We have seen in the introduction that left-handed materials are narrow band; from this point of view MRI appears to be a very interesting field of investigation for such materials since it operates on a narrow bandwidth. Another advantage being that the operational frequency of MRI is low and therefore the accurate realization of left-handed structure is relatively easy. Among former experiments involving metamaterials, one should note [WPY⁺01] where so-called Swiss-Roll materials were used to guide fields toward receiving loops and [FMJ08] where a metamaterial lens with a relative permittivity close to -1 has been used to improve the sensitivity of a surface coil. In this thesis, we will show that wire-media can be employed to improve the quality of MR images. Indeed, they can be used as an energy collimator to transmit the magnetic field to a certain distance from the MRI tube, to a region with negligible static field. This provides several advantages: first, the possibility to increase the radio-frequency homogeneity of the MR images by using a

convergent wire medium; second, the opportunity to create new flexible endoscopic devices.

In this Chapter we present the principle of MRI as well as its current limitations. We demonstrate how a wire medium can be used to collimate the MR images. We present the material used to evaluate the performance of wire media in MRI. In particular, we describe the steps to design a MRI surface coil for a 3 Tesla MRI scanner.

The Chapter is outlined as follows: Section 1 presents the MRI principle and the process which leads to MR images. Section 2 describes common artifacts often encountered in MRI. Section 3 explains the advantages brought by wire media in MRI to the RF-inhomogeneity artifact. Finally, Section 4 presents the coil used for the experimental measurements and describes the important steps to design a MR coil with a high sensitivity. The impact of a straight wire medium on the resonance frequency of a surface coil is demonstrated with the help of measurements.

3.2 Principle of Magnetic Resonance Imaging

To understand MRI, we have to start from the physical principles which describe the behavior of nuclei placed in a strong magnetic field. A short synopsis will be given here. For further details, we address the readers to the extensive literature on this topic, see e.g. [KVP03, Jin99, CH89, Hor07, Pud02]. Three steps are necessary to create a MR image, they are presented, hereafter.

3.2.1 Step 1: Alignment and Precession:

MRI is based on the fact that the human body is predominantly constituted of water molecules which contain atoms of hydrogen also called protons. Each proton spins about its axis and produces a small magnetic moment m , as illustrated in Fig. 3.1.a. We now consider a set of protons, as shown in Fig. 3.1.b, in the absence of an applied magnetic field, the magnetic moments m of the different protons are randomly oriented. When placed in a strong static magnetic field B_0 , the magnetic moments of the protons align with the magnetic field according to two directions either parallel or anti-parallel, as shown in Fig. 3.1.c. A certain energy level is associated with each direction. Both states are stable, although the energy associated with the parallel direction is clearly lower than the latter. Moreover, it is well known [Gri04] that the application

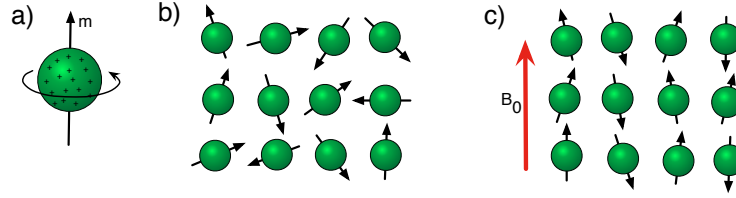


Figure 3.1 (a) Magnetic moment m generated by a spinning distributed charge; (b) in the absence of a magnetic field, all the magnetic moments m are randomly oriented; (c) in the presence of a magnetic field, all the moments m align with the magnetic field either in a parallel or anti-parallel way.

of an aligning force to a spinning object causes a circular movement called *precession*, hence, here the application of the magnetic field B_0 causes the protons to precess, as illustrated in Fig. 3.2.a. The number of time per second that the axis of the magnetic moment m revolves around the magnetic field direction is called the Larmor frequency. This frequency is directly related to the magnitude of the local magnetic field B_0 , according to the relation:

$$f_{Larmor} = \gamma B_0 / 2\pi \quad (3.1)$$

where γ is the gyro-magnetic ratio. For the hydrogen nuclei its value is 42,57MHz/T [Jin99]. The magnetic moments m of a collection of protons can be represented as a series of vectors as shown in Fig. 3.2.b.

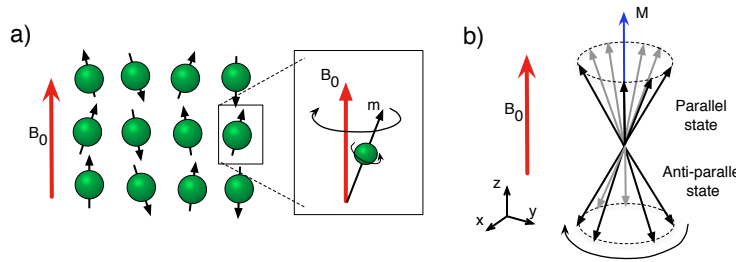


Figure 3.2 (a) The presence of a static magnetic field causes the protons to align and precess at the Larmor frequency; (b) the net-magnetization vector M resulting from the sum of all m is parallel to B_0 .

Each vector can be decomposed with its perpendicular and parallel component relative to B_0 . For a large number of spins, the individual components of the protons perpendicular to B_0 tend to cancel, leaving only the components in the direction parallel to B_0 . Moreover, given that the energy associated with the parallel state (in the same direction as B_0) is lower, most protons adopt this state and the net-magnetization vector M , resulting from all the magnetic moments m , is in the same direction as B_0 .

3.2.2 Step 2: Resonance of the Protons

The resonance is a phenomenon that occurs when a nucleus is exposed to an oscillating perturbation that has a frequency close to its own natural frequency of oscillation. In MRI, the resonance of the protons occurs when a radio-frequency (RF) pulse matching exactly the Larmor frequency f_{Larmor} is applied. In the case of such excitation the protons absorb the energy from the pulse. This phenomenon has two consequences on the net-magnetization vector M :

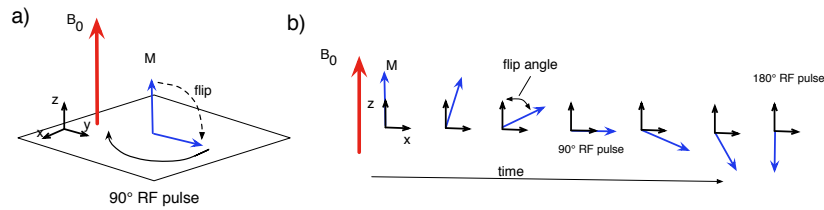


Figure 3.3 (a) A 90° RF pulse causes the net-magnetization M to flip by 90° ; (b) depending on the duration of the RF pulse, M is tilted by 90° , 180° or comes back to its initial position.

First, the net-magnetization vector M moves away from the direction of B_0 . Indeed, the energy of the RF-pulse is absorbed by the protons, the population of protons in the high energy state, i.e. in the anti-parallel state, increases. The angle to which the net-magnetization is tilted relative to the main magnetic field depends on the amplitude and the duration of the RF-pulse. For example, after a 90° RF pulse the number of protons in both states is the same, and consequently, M_z , the component of M along the z axis completely disappears. On the other hand, after a 180° RF pulse, the vector M rotates into

a position opposite to B_0 since more spins are in the anti-parallel state, rather than in the parallel one. If the RF-pulse is applied indefinitely, M comes back and returns to its initial position.

Second, the magnetic moments m of the protons, move to the same position on the precessional path, and consequently they become in phase with each other as illustrated in Fig. 3.4, and a net-magnetization vector M_{xy} emerges in the xy plane.

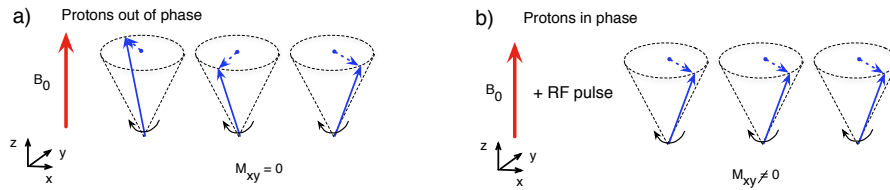


Figure 3.4 The RF-pulse causes the protons to come in phase with each other, a vector M_{xy} emerges in the plane XY.

In brief, after an excitation with a 90° RF pulse -which is a common excitation in MRI-, the longitudinal component of M , M_z disappears (nb of parallel state = nb of anti-parallel state) and a rotating component M_{xy} emerges (due to the phasing of the protons).

3.2.3 Step 3: Relaxation

It is thanks to the relaxation of the protons that the MRI phenomenon is observable. As we have seen after the excitation, the net magnetization vector M is tilted by 90° . This state is unstable and when the RF-pulse is switched off, the vector M starts to realign with B_0 as illustrated in Fig. 3.5. This realignment is due to a phenomena inverse to those present for the excitation.

The construction of an image in MRI is based on the measure of the time of increase of M_z , called T1 and the time of decrease of M_{xy} , called T2. It is important to notice that both these times $T1$ and $T2$ are independent since they are related to different phenomena based on different mechanisms. Indeed T2 depends on the coherence of rotations, while it is not the case for T1. Since the relaxation times T1 and T2 differ from one type of tissue to another, an MR

image of an organ can be created by mapping the different relaxation times according to their positions. The relaxation times are defined as follows

- T1 is linked with the longitudinal growth of the M_z component. It is defined as the time for the component M_z to recover 63% of its initial longitudinal magnetization $M_z^{t_0}$ before the excitation. The underlying phenomenon is caused by nuclei giving up their energy to the surrounding environment. T1 varies with the molecular structure and state of the matter, it is longer for the liquid than for the solid and is very short for adipose tissue [KVPG03]. Typically, for biological tissues T1 has a value comprised between 500 and 1000 milliseconds.
- T2 is linked with the decay of the component M_{xy} and is defined as the time for M_{xy} to lose 63% of its maximal transversal magnetization obtained at the end of the excitation. Here the relaxation phenomenon is caused only by protons exchanging energy with their neighboring protons, i.e. spin-spin interactions. Indeed, the protons are locally in different molecular environments, and they experience slightly different magnetic fields. Consequently, they precess at different frequencies, and lose their phasing rapidly when the RF-pulse is stopped. T2 is shorter in the liquid compared to the solid or tissues constituted of large molecules. T2 is usually 10 times shorter than T1, therefore its value is usually comprised between 50 and 100 milliseconds for biological tissues.

For practical reasons, an additional relaxation time $T2^*$ is always defined. Indeed, the dephasing of the magnetic moments m has two origins: the inhomogeneities in the magnetic field, and the spin-spin interactions.

- $T2^*$ is related to the decay of M_{xy} , like T2, but it takes into account both the dephasing due to the spin-spin interactions and due to the inhomogeneity of the static magnetic field. The latter, at the level of the protons, is not perfectly homogeneous and accelerates the dephasing of the protons. Consequently $T2^*$ is always shorter than T2.

3.2.4 Measurement of the MR signal

The construction of MR images requires the measurement of the relaxation times previously defined. In practice, the strong magnetic field B_0 prevents the measurement of the variation of M_z and *the variation of M_{xy} is the only com-*

ponent that can be measured. The growth of the component M_z is a continuous process which does not produce an RF signal [KVPG03]. Consequently, T_1 cannot be measured, nevertheless it is indirectly used to isolate some types of organs by playing on the repetition time of the MRI sequences. T_2 and T_2^* are evaluated respectively by using MRI sequences such as 90° RF pulse and more complex MRI sequences, like the "spin-echo", which compensate the dephasing of the protons due to the inhomogeneity of B_0 .

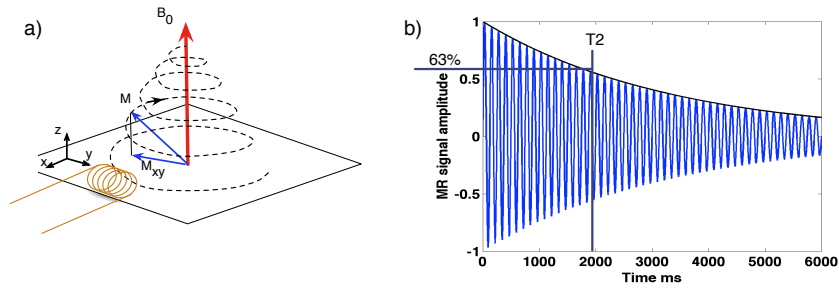


Figure 3.5 (a) After a 90° RF pulse, the net-magnetization vector M starts to realign with B_0 , only its component M_{xy} can be measured in MRI; (b) voltage induced in the coil by M_{xy} .

Relaxation times T_2 and T_2^* are measured with a radio-frequency coil. The coil is positioned close to the studied sample and such that its axis lies in the plane xy , as illustrated in Fig. 3.5.a. During the relaxation the rotating component M_{xy} induces a voltage in the receiver coil. The resulting signal at the output of the coil is a damped sinus, as presented in Fig. 3.5.b. The relaxation times of the sample are deduced from the damping factor.

If we want to create an MR image, an additional mechanism which allows us to determine the relaxation times of a small cube of matter instead of a whole body is necessary. In order to map the relaxation times according to their positions and construct an MR image, the sample is virtually sliced in 3D-pixels called "voxels", as illustrated in Fig. 3.6.b. The image is constructed by imaging each voxel one after each other. The actual location within the sample from which the relaxation RF-signal is emitted is determined by superimposing to the magnetic field B_0 three magnetic field gradients, placed orthogonally to one another. By applying these magnetic gradients, we cause the

hydrogen atoms to experience a different magnetic field strength dependent on their position along that gradient. As stated above, the Larmor frequency is dependent on the local external magnetic field. Therefore, if a monochromatic RF-pulse is applied to the whole sample, only the atoms experiencing a static magnetic field sufficiently close to the appropriate field will absorb the RF energy. This principle is illustrated for a one-dimensional case in Fig. 3.6.a.

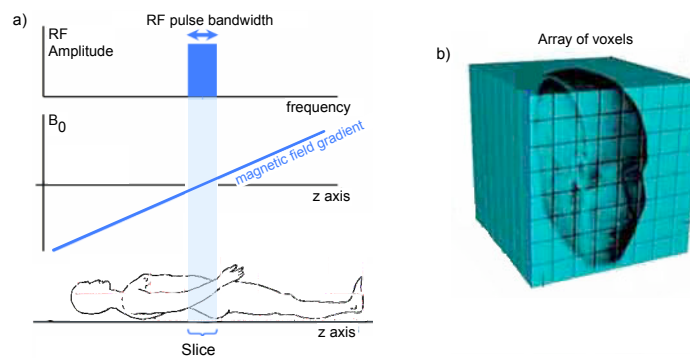


Figure 3.6 (a) The superposition to the magnetic field B_0 of a 1D-gradient field causes the hydrogen atoms to experience a different magnetic field strength depending on their position along that gradient; (b) the use of three gradient fields enables to decompose a sample in 3D pixels.

The resolution in MRI is related to the strength of the magnetic gradients and can vary depending on the studied organ. For a 3 Tesla MRI scanner the smallest voxel size is of the order of 0.35mm^3 . Usually, the resolution is sufficient in MRI to conduct medical diagnostics, although a constant trade-off has to be made between voxel size and signal to noise ratio (SNR) [SRVE00, EGHR86]. Indeed, the excitation of small voxels provides more details but decreases the SNR, and conversely; since the intensity of the signal at each frequency is proportional to the number of protons present in the voxel.

3.3 Artifacts in MRI

An image artifact is a structure which appears in an MR image but which is not present in the original imaged object. It results from a limitation or malfunction in the hardware or software of the MRI device, or is a consequence of

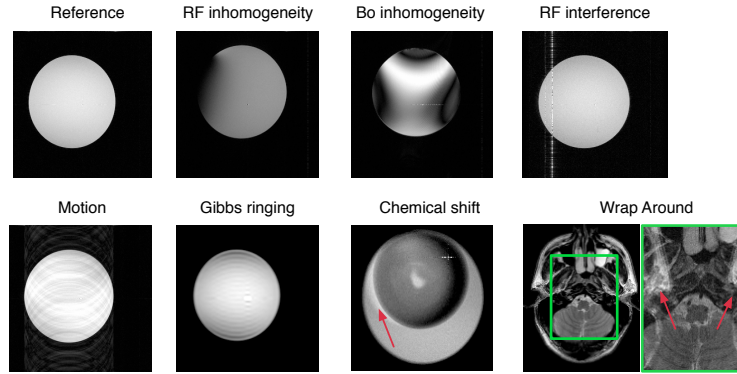


Figure 3.7 Artifacts usually encountered in magnetic resonance imaging [Roo97].

environmental/body influences (blood flow, implants,...). Artifacts may be very noticeable or just a few pixels can be out of balance but they can lead to confusion and to misdiagnosed pathology. Hereafter, we present a brief description of artifacts often encountered in MRI [HR02, Roo97] and illustrated in Fig. 3.7.

- *RF inhomogeneity*: It is a variation in intensity across an image. The cause is either a nonuniform excitation field or a nonuniform sensitivity in a receive only coil. Some RF coils, such as surface coils, naturally have variations in sensitivity and will always display this artifact. Indeed, when small organs are observed with a surface coil, it usually requires the use of a small loop antenna to capture local magnetic fields with high signal-to-noise ratio. However, the main consequence of the use of such a small coil is that it captures only a very localized field and creates in-homogenous images for surrounding organs. An example of RF inhomogeneity is illustrated in Fig. 3.8, for the case of a nail tumor and in Fig. 3.7 for the case of a bottle. The presence of this artifact in other coils represents the failure of an element in the RF coil or the presence of non-ferromagnetic material in the imaged object. An RF inhomogeneity problem does not have to be confused with the artifact caused by the inhomogeneity of B_0 , which produces distorted images.
- *B_0 inhomogeneity*: All magnetic resonance imaging assumes a homogeneous B_0 magnetic field, an such even when a body is present in the

tube. In the latter case the homogeneity is corrected with the help of shim coils, carrying a relatively small current that are used to provide auxiliary magnetic fields. An in-homogeneous B_0 magnetic field will cause distorted images. The distortions can be either spatial, intensity, or both.

- *RF interference*: The presence of external sources of RF energy, external to the MRI system can interfere with MR signals. When the image is constructed artifacts may occur because the amplitude of the MR signal is increased at the frequencies of the interfering signals.
- *Gradient*: Artifacts caused by the gradient system are sometimes very similar to those described as B_0 inhomogeneities. A gradient which is not constant with respect to the gradient direction will distort the image.
- *Motion*: The motion of a small portion of the imaged body results in a blurring of that small portion in the final MR image. The motion is usually caused by the heart beating or the patient breathing. The solution to a motion artifact is to immobilize the imaged object or to synchronize the imaging process with the periodicity of the motion (heart beating). Systems are now under development to carry out such corrections.
- *Chemical shift*: A chemical shift artifact is caused by the difference in chemical shift (Larmor frequency) of fat and water. The artifact manifests itself as a misregistration between the fat and water pixels in an image. The effect is that fat and water spins in the same voxel are encoded as being located in different voxels.
- *Wrap Around*: A wrap around artifact is the occurrence of a part of the imaged anatomy, which is located outside of the field of view (FOV), inside of the image. This artifact is caused by the selected field of view being smaller than the size of the imaged object. It is caused by a corruption in the spatial encoding of objects outside the FOV which cannot be distinguished from objects inside the FOV
- *Gibbs Ringing* : Gibbs ringing is a series of lines parallel to a sharp intensity edge in an image. The ringing is caused by the reconstruction of the image by a Fourier transform from a finite sampled signal (truncation in the k-space).

3.4 Wire medium in MRI

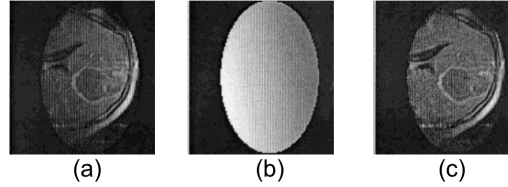


Figure 3.8 (a) MR image corrupted by RF inhomogeneity. The in-homogeneous part of (a) is compensated by the gradient scale in (b). (c) MR image free of RF-inhomogeneity.

A priori, we could think that the capability of a metamaterial lens to magnify the images with sub-wavelength resolution could be used to improve the resolution of MR images. In practice, this is not the case given that the resolution in MRI depends primarily on the strength of the gradient field (which determines the voxel size). Moreover, the current resolution is the order of 0.3mm^3 (for a 3 Tesla MRI scanner) and is sufficient to conduct most of medical diagnostics.

A major issue in MRI is the low signal-to-noise ratio (SNR) caused by the important dielectric loss in the organs. For the moment the solution to improve the SNR is to use small surface coils having a high SNR, but the price to pay is a reduction of the field of view or sensitivity. The difficulty with the surface coils become apparent if the volume of interest lies deep within the body of the patient because in this case the received magnetic field is then very weak. Moreover, most of the surface coils have an in-homogeneous sensitivity also called RF-inhomogeneity (Fig. 3.8.a) which must be compensated in order to provide reliable diagnostic.

In this thesis we study the use of wire media in order to collimate or concentrate the MR signal. Indeed, wire media can be used as an energy collimator to capture the signal deep within the body and transmit the RF magnetic -carrying the information about the MR images - to a certain distance from the MRI tube center. Wire media can also be used to concentrate the RF magnetic field emitted by a large area onto a small surface coil (having a high SNR). These two properties provide several potential applications in MRI:

- The perspective of using a straight/curved wire medium to design more reliable endoscopic sensors. As said before small surface coils have high SNR but a limited sensitivity, when the volume of interest lies deep within the body of the patient it is necessary to employ small surface coils inserted within the body in order to be closer from the energy source. Following discussions with medical doctors, it appears that traditional endoscopes (insertable surface coils) are not sufficiently reliable. Their shape can sometimes change (typically changing ellipticity of the loop) when introduced in a body, which in turn shifts the resonance frequency. A solution is to use a flexible wire medium with very small cross-section. The wire medium has as weakly resonant nature -the resonating elements are not very small in comparison to the wavelength- and imaging is realizable with strongly curved wire media. When it operates in the canalization regime [BSI05], the collimation phenomenon remains very effective for structures having a cross-section corresponding to about a hundredth of the wavelength [RGC09]. Therefore, curved wire media could offer an alternative solution to the coil used for intra-vascular imaging [QLZP⁺99] since their cross-section remain constant .
- The perspective of using a wire medium as an energy collimator toward a region with very low static fields, and hence the possibility of using simpler receivers with higher SNR. Indeed, we will see in chapter 4 that to work properly, the length of a wire medium should be close to a multiple of half a wavelength ($\lambda_{3TMR I} = 243$ cm), hence collimation at very large distances can be obtained. Consequently, the radio frequency magnetic field can be transported away from the strong static magnetic field and preamplifiers containing magnetic materials can be used in the preamplifier placed after the MRI coil, without creating artifacts. In view of the absence of constraint on materials used, better noise figures may be expected from the amplifiers.
- The possibility to increase the field of view of a small surface coil by combining it with a convergent wire medium, [STPS07, IST⁺07, RDC07]. Indeed a convergent wire medium can be used in order to concentrate the field emitted by a large area onto a small surface coil with a high SNR. As we will see in chapter 4 the weakly resonant na-

ture of the wire medium does not make fabrication accuracy a critical issue and allows us to imagine a convergent geometry.

Regarding the use of a wire medium in MRI, we would like to notice that the use of a single wire instead of a coil to detect the MR signal has already been explored in [OA97], although in this case the wire is used as a detector, not as a collimator. Moreover, the use of short probes as in [OA97] may pose important matching difficulties.

In order to understand the principle of operation of a wire medium in MRI, we have to remember that the net-magnetization vector M can be decomposed along two components M_{xy} and M_z , as explained in Section 3.2.4. Only the component M_{xy} induces a RF signal which is captured with the help of a receiving surface coil whose axis is perpendicular to the direction of the static magnetic field [KVP03]. Consequently, if we place a wire medium - which has the property to transfer the component of a magnetic field perpendicular to the wires (TM) [BZS⁺06] - with the wires along the z -direction, then the component M_{xy} will be transferred from one side to the other side of the wire medium.

The next Section presents the coils used at the University hospital of Liège (with a 3 Tesla MRI scanner) to evaluate the performance in transmission of different wire media. In particular, a surface coil specifically designed for the measurement of wire media is presented, as well as the main steps of its construction (matching, balancing,...).

3.5 Coils in MRI

3.5.1 Coils Employed for the Measurements

As we have seen in Section 3.2, an MR image is created by exciting a sample with a powerful pulse, and then by measuring during the relaxation step the outgoing signal through a surface coil. The MRI scanner used for the measurement is a 3 Tesla MRI scanner having an operational frequency (Larmor frequency) of 123.23 MHz and a bandwidth of 500 kHz. The power of the excitation pulses depends on the specific absorption rate (SAR) of the patient, which is the amount of energy absorbed by the patient. During a MRI scan the SAR is constantly monitored in order to avoid excessive heating of the patient, the maximum SAR is usually limited to 4W/kg in European countries [MRI].

The surface coil [ATHI76, AGW⁺80] is the basic tool of MR signal reception and is one of the principal determinants of a high SNR. In order to provide such a high SNR, they have a sensitivity profiles tailored to particular target regions. A trade-off in image quality exists between depth of penetration of a surface coil and peak SNR as a function of coil dimensions. Smaller coils offer high sensitivity but they also show a rapid falloff of sensitivity with distance from the coil center, resulting in limited penetration and thus restricted useful field of view, whereas larger coils are better suited for imaging of deeper regions of interest. In chapter 4, the performance in transmission of the different wire media have been evaluated thanks to three types of coils, namely : a body coil, a shoulder coil and a small surface coil. They have the following characteristics:

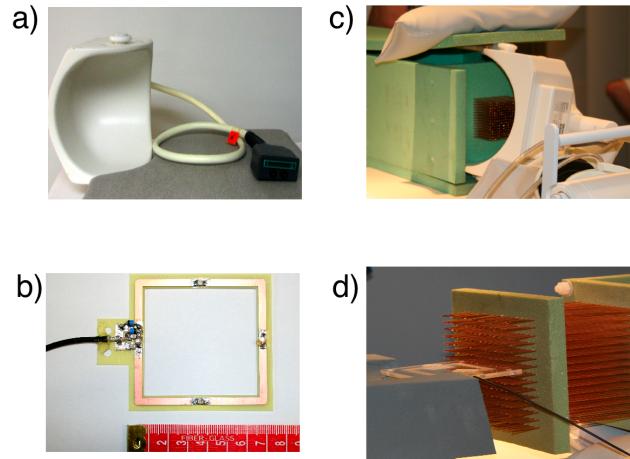


Figure 3.9 MRI coils for 3 Tesla MRI scanner: at the beginning of the project the shoulder coil (a) was the only available coil, it offers very limited possibilities in terms of placement near the wire medium (c). In a later stage of the project we have use a home-made surface coil (b) that can be inserted inside the wire medium (d) to obtain a stronger received signal.

- The body coil: it is located in the tube itself and is essentially used to excite the sample and provide a homogeneous excitation of the whole sample. The body coil can be used either as a transmitter or as a receiver, although due to its low SNR, it is often associated with surface coils for a more sensitive reception.
- The shoulder coil: it is used as a receive only surface coil dedicated to the analysis of the shoulder. For the first experiments, it was the only MRI coil available to measure the magnetic field outputs. The coil contains three loops perpendicular to each other and can capture all polarizations (Fig. 3.9.a). The exact position of the antennas within this coil is unknown. However, the possibilities in terms of placement near the wire medium were very limited as illustrated in Fig. 3.9.c.
- The surface coil: this coil was realized in a later stage of the research program in collaboration with the KULeuven. The coil was designed so that it can be inserted into the wire medium (Fig. 3.9.d). Indeed, it was demonstrated numerically in [RLC08] that the insertion of the receiving loop at the back side of the wire medium improves the signal strength. Since the proximity of a wire medium tends to shift the resonant frequency of the coil towards higher frequency, variable capacitors were used to enable the tuning of the coil in order to exhibit a resonance frequency close enough to the MRI operational frequency. The coil is presented in Fig. 3.9.b, its sensitivity is of the order of its dimensions which are 6×6 cm. The design rules employed to realize this surface coil are described in next section.

3.5.2 Design Steps of a Surface Coil

Basically, the coil must convert the rotating magnetization into a current that is amplified and processed by a receiver. The coil deals with very weak signals and implies a number of conditions that define the RF design.

3.5.2.1 Matching

In order to optimize the transfer of energy, the first step consists in matching the coil to the MRI coaxial cable ($Z_0 = 50 \Omega$ and is purely resistive). The dimensions of the coil are imposed by the analyzed object. A common rule of thumb to determine the size of the coil while having a good SNR is to use a coil whose radius is approximatively the size of the analyzed organ (hand,

wrist,...). Once the dimensions of the coil are determined, its inductance L can be calculated with the following formula [Vor94]:

$$L = \mu_0 a \left(\ln \left(\frac{8a}{w/4} \right) - 2 \right) \quad (3.2)$$

with w the width of the strip and a the radius of the coil. The resistance of the coil is noted R and is usually small. The matching is realized by adding two variable capacitors C_M and C_R . The complete matching circuit is presented in Fig. 3.10. The matching procedure is the same as for antennas and consists of

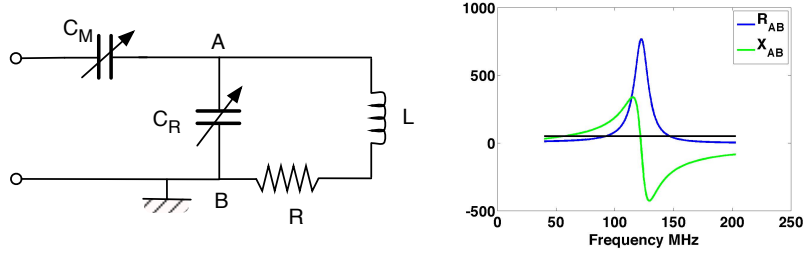


Figure 3.10 (a) The matching circuit, (b) real and imaginary part of the circuit at the port AB

two steps: first, adjust the resonance frequency of the coil, through the capacitor C_R in order to obtain a real part for the impedance at port AB equal to Z_0 , and second, compensate the reactive component of the impedance at port AB by connecting in series an appropriate reactance C_M .

Let us start with the determination of the capacitor C_R . If we add in parallel to the coil inductance L and the parasitic resistor r_c , a capacitor C_R we obtain at port AB a RLC circuit. The circuit exhibits a large variation of its resistive impedance component near the resonance ω_R , and usually exhibits a resistance equal to Z_0 (50Ω) for two frequencies ω_{inf} , ω_{sup} . Usually the lower frequency is preferred due to the ease to compensate an inductive component with a variable, non magnetic, high quality capacitor. Hereafter, we suppose that we have choose to work at ω_{inf} . The impedance of the RLC circuit at the port AB can be written as:

$$Z_{AB} = R_{AB} + jX_{AB} \quad (3.3)$$

with

$$R_{AB} = \frac{r_c / (\omega C_R)^2}{r_c^2 + (\omega L - 1/(\omega C_R))^2} \quad (3.4)$$

$$X_{AB} = \frac{-r_c^2 / (\omega C_R) - L / C_R (\omega L - 1/(\omega C_R))}{(r_c^2 + (\omega L - 1/(\omega C_R))^2)} \quad (3.5)$$

The capacitor C_R is determined by imposing that at ω_{inf} , the real part of the impedance of the RLC circuit is equal to Z_0 ,

$$Z_0(Z_0 R^2 + L^2 \omega)^2 C_R^2 \omega^2 - 2Z_0 L \omega C_R \omega + Z_0 - R = 0 \quad (3.6)$$

A real solution exists if the following inequality is verified,

$$r + (L\omega)^2 / R > Z_0 \quad (3.7)$$

If we define the quality factor Q of the inductance as the ratio of the reactance to the resistance $L\omega/r_c$, we can rewrite the condition

$$Q + 1/Q > Z_0 / L\omega \quad (3.8)$$

The left member is always greater or equal to 2. Given that $Z_0 = 50 \Omega$ in MRI, the inequality is always fulfilled when $L\omega > 25$ and consequently a matching solution exist. On the contrary, when $L\omega < 25$, a matching solution does not exist for the quality factors located between the following two roots:

$$\frac{\frac{Z_0}{L\omega} - \sqrt{(\frac{Z_0}{L\omega})^2 - 4}}{2} < Q < \frac{\frac{Z_0}{L\omega} + \sqrt{(\frac{Z_0}{L\omega})^2 - 4}}{2} \quad (3.9)$$

For example for $L\omega = 10$, the excluded Q is from 0.1 to 10. Q is rarely lower than 20 and consequently a solution almost always exists.

Regarding the capacitor C_M , since we have decided to work at $\omega = \omega_{inf}$, the reactive impedance of the RLC circuit X_{AB} is inductive and is given by Eq. (3.5). The capacitor C_M is chosen in order to cancel this reactive impedance and therefore has the following value:

$$C_M = 1/(X_{AB}\omega_{inf}) \quad (3.10)$$

Obviously, if we decide to work at the higher frequency $\omega = \omega_{sup}$, the reactance X_{AB} must be compensated with an inductor.

3.5.2.2 Balancing

One of the major drawbacks in MRI is its lack of sensitivity. Much of the loss in sensitivity lies in the MR coil (surface coil). In this section, we present the sources of noise, and how their effects can be reduced by employing a balanced circuit.

The losses in the probe circuit can originate in numerous places. The most elemental is the resistance R associated with the inductance L of the coil itself, already mentioned in Section 3.5.2.1. This resistance is proportional to the quality factor Q of the circuit. Other losses occur in the sample under observation itself. Usually when a sample of biological tissue is introduced into a MR coil, rather large losses appear and the quality factor Q falls dramatically. Much of these losses can be attributed directly to the conductivity of the ions in tissue and strongly affect the signal to noise ratio of the MRI. The sample losses can be classified following their origin which is either inductive or dielectric.

Inductive losses are a consequence of the conductivity of the sample and originate from eddy currents induced in the sample by the alternating RF magnetic field. Little can be done to reduce these losses, except making the sample less conductive [AG79].

Dielectric losses can also be regarded as a consequence of the conductivity of the sample. The response of a sample to alternating fields is characterized by a complex permittivity $\epsilon_s = \epsilon' + j\epsilon''$ where ϵ' is related to the stored energy and ϵ'' represents the loss of energy within the sample. These losses are related to the distributed capacitance that is present between the MR coil and the lossy materials. If we look at the electric field generated by a surface coil placed on a sample of biological tissue, we can see, as illustrated in Fig. 3.11, that some of the electric field lines are coil-to-coil and some of them are coil-to-ground. The latter comes from the fact that for the purpose of shielding the sample is generally contained in a metallic enclosure attached to the ground.

The losses associated with the coil-to-coil electric fields are represented, in the equivalent circuit in Fig. 3.11, with the resistor R_{cc} and the capacitor C_{cc} [HL79,GR79]. The losses associated with the coil-to-ground electric field lines may be modeled by a distributed coil-to-ground lossy capacitor [MBK83]. However, there is generally some distance between the ground to which the sample is coupled and the one to which the MRI is coupled. This results in

a ground loop. The equivalent circuit is shown in Fig. 3.11. The lossy capacitors $R1_{CG}$ and $R2_{CG}$ produce dielectric losses and a shift of the resonance frequency upon introduction of the sample. The ground loop may induce an additional frequency shift (due to the reactive part of its impedance), as well as resistive and radiative losses when the physical dimensions of the sample are comparable to a wavelength [Ter43]. As a result of the high dielectric constant and low conductivity of tissues [SKSS82], the current in the ground loop can be relatively high.

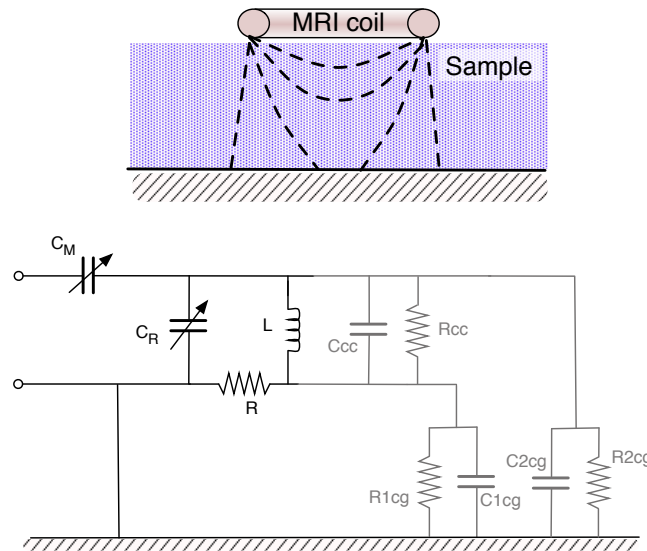


Figure 3.11 Top: electric field lines generated by a MR loop coil; bottom: equivalent circuit of the dielectric losses.

In [MBK83] it is demonstrated that a balanced coil increases the sensitivity of the coil. The improvement comes from the reduction of the dielectric losses due to the coil-to-ground distributed capacitor. The losses are approximately halved by the use of a balanced circuit instead of an unbalanced one. To create a balanced matching network, a very simple solution is to reuse the matching network presented previously and to split the matching capacitor C_M as shown in Fig. 3.12. Since the same current flows in opposite directions and because they have the same impedance, the voltage with respect to the

ground at the two coils ends is almost equal and of opposed phase, realizing the electrical balancing. Actually, the voltages are not completely opposed due to the the resistance of the coil.

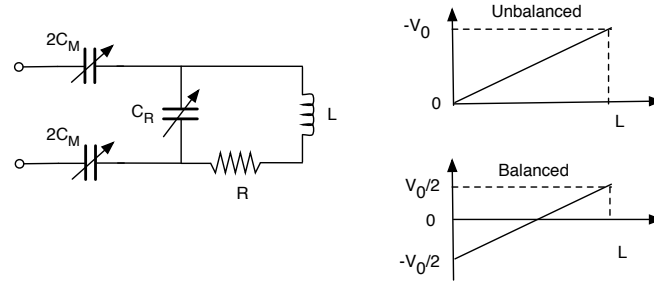


Figure 3.12 Balanced circuit

There are several advantages of using a balanced circuit. The voltage along the coil is supposed to vary linearly along the coil wire. If l is the position along the coil wire, and L is the total coil length, the potential at the point l is given by $V_0 l/L$, for an unbalanced circuit and by $V_0(l/L - 1/2)$ for a balanced circuit. If the losses are modeled by the distributed resistance R_{cg} , then the power dissipated in this distributed resistor is equal to:

$$P_{R_{cg}} = \frac{1}{2} \int_0^L \frac{V(l)^2}{R_{cg}} dl \quad (3.11)$$

shows that the balanced circuit gives coil-to-ground electric losses equal to a quarter of that unbalanced one. By analogy, it is also possible to show that the energy stored in the coil-to-ground distributed capacitor C_{cg} is equal to one quarter of the energy stored in the ground capacitors of the unbalanced circuit. Thus, if R_{cg} and C_{cg} are the parallel resistor and capacitor equivalent to the coil-to-ground lossy capacitor of the unbalanced circuit, $4R_{cg}$ and $C_{cg}/4$ are the corresponding elements for the balanced circuit.

Another advantage of the balanced circuit is that, if capacitors from coil-to-ground are regularly distributed along the coil, a balanced circuit causes the currents in the ground loop to be equal to zero [Ter43]. This is the reason why the capacitor C_R previously introduced is often segmented in several parts on

a MRI coil. For unbalanced coils, the coil-to-ground capacitive path causes ground currents to flow, which create resistive and radiative losses [DBR⁺85].

3.5.2.3 Common Mode

We have seen that the balanced coil circuit should be driven in a differential mode, in this case the two currents on the inner and on the outer conductors are equal and opposite in order to have a net current in the transmission cable (coaxial) ideally at zero.

In practice, this is seldom the case, a common mode is very likely to occur. The common mode corresponds to currents circulating towards and from the coil, and is frequently evidenced by hand effect. These currents perturb the differential mode, by modifying the tuning and the matching of the probe coil, picking up noise from the surrounding and radiating energy. Moreover, when dealing with animals or humans it may cause serious burns, therefore it is important to cancel it.

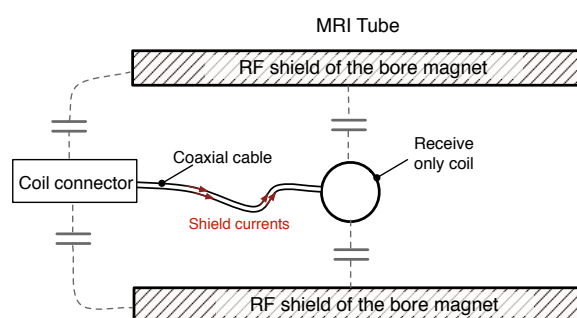


Figure 3.13 Shield currents are induced in unintentional loops formed by the capacitance between the coil and coaxial cable and the RF shield of the MRI magnet

The common mode is caused by the electromotive force generated in the *outer* part of the coaxial cable of the receiving coil, by the body coil transmitter. Three conductors can be identified: the central conductor of the coaxial, the internal surface and the external surface of the coaxial shield. Indeed unintentional loop circuits are created given that the coil and the cable are capacitively coupled to the shield of the bore magnet, and are also connected through the system connector. The voltage induced on these loops when the body coil is

transmitting creates the shield currents. To trap these currents, the classical solution consists in using a balun [Ter43,PDF97]. Recently, an ingenious device has been proposed [SJM04], it is a variation of the bazooka balun [Pau06]. The main advantage of this new trap is that it does not have to be connected or soldered to the cable and it can fit over any cables assemblies without affecting the coil parameters.

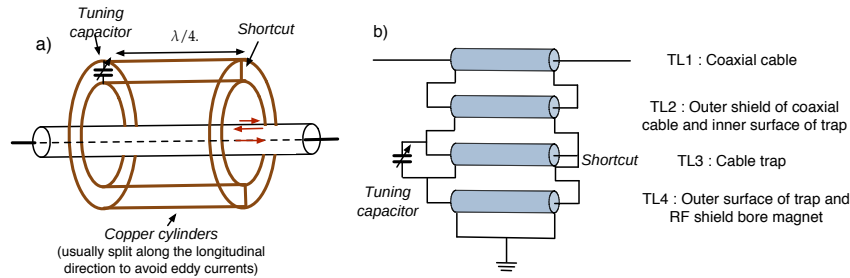


Figure 3.14 (a) Realisation of trap, it is formed by two concentric cylinders in copper separated by a dielectric; (b) Identification of the different transmission lines.

Practically, the construction of the trap is very simple. It is formed by two concentric cylinders in copper separated by a dielectric. The trap is presented in Fig.3.14.a. We can see that the line is short circuited at one end and is tuned with a capacitor on the other end in order to obtain a length equal to $\lambda/4$. This essentially creates a tuned-LC circuit. The equivalent circuit of the trap is illustrated in Fig.3.14.b. The transmission line, T1, is the coaxial cable, with the shield current to be attenuated. The current flowing on the shield of the T1 transmission line forms the center conductor on the T2 transmission line, whose outer conductor is the inner surface of the cable trap. The inner conductor of the cable trap and the outer conductor of the cable trap form the T3 transmission line. The T3 transmission line is tuned to resonate at the frequency of the shield current, creating high impedance to the shield currents. The outer conductor of the floating cable trap forms the fourth transmission line, T4, with the ground of the shield in the MRI system. Given that the shield currents flowing on the out surface of the coaxial, becomes the center conductor of the third transmission T3, which exhibits an high impedance; the shield

currents are significantly reduced. This trap was employed when building our MRI surface coil, the complete reception chain is presented in Fig. 3.15.

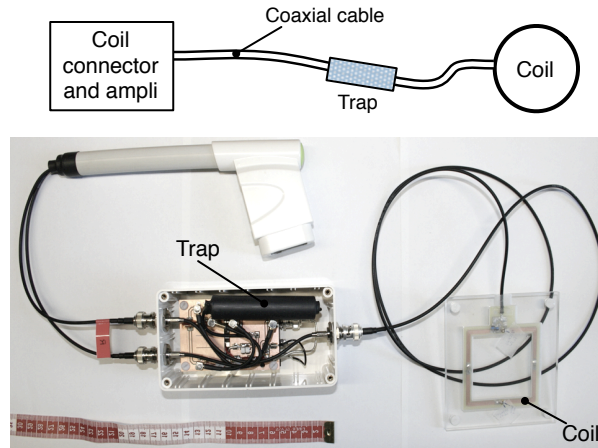


Figure 3.15 MRI surface coil and its preamplifiers used to acquire MR images transferred through the wire medium.

3.5.2.4 Detuning

We have seen that during the excitation process a powerful signal is sent in order to excite the protons contained in a sample. During this excitation step, the receiving coil has to be protected in order to avoid to burn its constitutive elements (preamplifier, capacitors,...). A common technique used in MRI consists of detuning the coil, it means that the resonant frequency of the coil is shifted toward lower frequencies in order to prevent the absorption of energy generated during the excitation process. In practice, the coil is detuned electronically thanks to a bias circuit, and PIN diodes. PIN diodes are employed as switches because they can be activated within a few microseconds.

The bias circuit is presented in Fig. 3.16. The coaxial cable R carries the RF signal from the receiving coil to the probe interface and the DC bias voltages required to switch on/off the PIN diodes. The bias voltage during the excitation phase is set to +10V (I-mode), and during the reception of the MRI signal is set to -30V (U-mode).

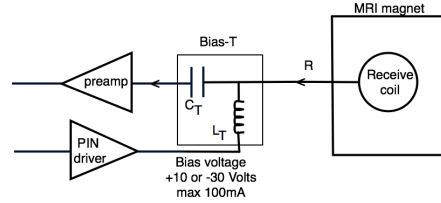


Figure 3.16 Bias circuit used to detune the coil, the current is limited to 100mA.

The circuit employed to detune the coil is presented in Fig. 3.17.a, we can see the two diodes D1 and D2 which enable to shortcut the coil capacitor and to shift down the resonant frequency of the coil. In practice, the detuning circuit is always placed just after the coil output, as shown in Fig. 3.17.b.

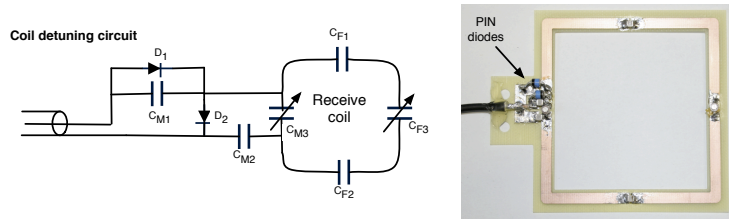


Figure 3.17 (a) Detuning circuit, (b) practical realization of the detuning circuit.

The two states of the detuning circuit are illustrated in Fig. 3.18. The response time to pass from one state to the other is typically around 5 microseconds. When the bias voltage is positive the PIN diodes are activated. The capacitors C_{M2} and C_{M3} are in parallel shifting down the resonant frequency of the coil by approximatively 5MHz, hence the current induced onto the coil is very low. Moreover, the coaxial cable is short-circuited, preventing parasitic currents to enter into the preamplifier. The bias current is short circuited, nevertheless there is no need to add a resistor since the current of the bias circuit is limited to 100 mA. When the bias voltage is negative, the PIN diodes are inactive and the coil can receive a MR signal, since its resonant frequency matches the MRI operational frequency.

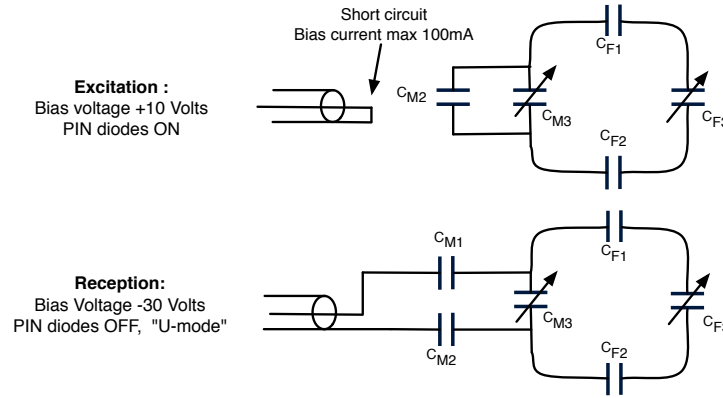


Figure 3.18 The two states of the detuning circuit.

3.5.3 Impact of a Wire Medium on a Surface Coil

In this section, we analyze the impact of the coupling between the receiving surface coil and the straight wire medium. The reflection coefficients S_{11} of the surface coil measured with a vector network analyzer are depicted in Fig. 3.19 for three situations, when the coil is: in free space, against a water bottle (mimicking the human body), and inside a wire medium. The coil is tuned in free space thanks to two variable capacitors (C_M and C_R) in order to resonate at 123.2 MHz and present an input impedance equal to 50Ω . Due to its small size ($6 \text{ cm} \times 6 \text{ cm}$) the coil has a high quality factor and is very sensitive to the capacitor values. When placed in free-space the S_{11} parameter of the coil reached -30 dB at 123.2 MHz. When the coil is placed against the water bottle the resonant frequency is shifted toward lower frequencies due to capacitive loading, while when the coil is inserted into a wire medium the resonant frequency is shifted toward higher frequencies by approximately 3 MHz due to inductive coupling. For this reason during experiments made at the hospital with a wire medium, the coil had to be tuned when placed inside the different wire media in order to have a resonant frequency close enough the MRI operational frequency. The tuning has been realized with the help of waves generator and a voltage standing wave ratio meter initially devoted for radio communications; therefore the quality of the tuning was approximate.

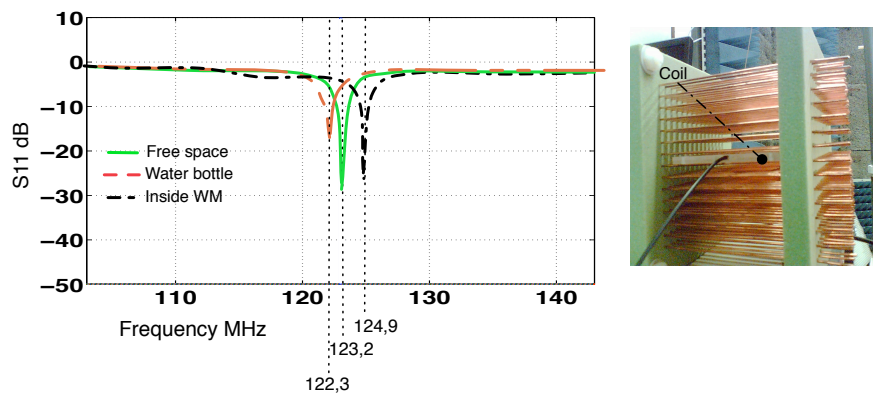


Figure 3.19 S11 parameters of a surface coil placed: in free-space, against a water bottle or inside a wire medium

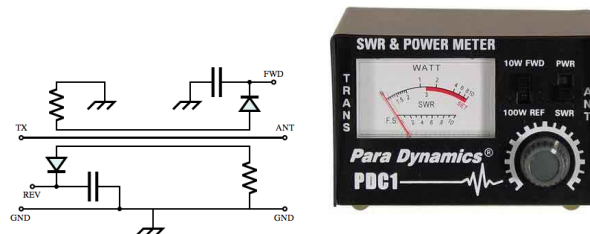


Figure 3.20 Schematic of voltage standing wave ratio meter. The transmitter (TX) and the antenna (ANT) are connected via an internal transmission line. This main line is coupled to two smaller sense lines which are connected to resistors at one end, and diode rectifiers at the other. One sense line senses the forward wave, and the other the reflected wave. The diodes convert the alternative voltages induced on the sense lines to DC voltages. The ratio of the two voltages obtained at port RF and FWD are employed to determine the reflection coefficient Γ and hence the SWR.

Wire medium: Numerical Analysis

4

4.1 Introduction

The resolution of common imaging systems is restricted by the diffraction limit. The limitation comes from the fact that the evanescent waves emitted or scattered by an object and which carry the sub-wavelength details are never detected. As we have seen in the Introduction, a solution to obtain sub-wavelength resolution is to use metamaterials, which are capable of manipulating the near fields and allow to operate with objects located at significant distances. Several types of metamaterials can be distinguished, depending on their underlying principle.

A first kind of metamaterial proposed in 2000, used to regenerate the evanescent waves, is based on the design of a periodic structure which exhibits for certain frequency range, a negative permeability μ and a negative permittivity ϵ at the same time. The most popular unit cell structure used to realize such properties, is the split-ring resonator (SRR) associated with a rod [Pen00]. Unfortunately, in practice, structures based on SRR's and rods are very sensitive to losses [PN05] and their imaging capability is often limited by the mismatch between the metamaterial lens and the host medium [SSR⁺03]. Also it is difficult to believe that the resolution may be better than the size of unit cells of the periodic structures, which is about a tenth of the wavelength.

In 2005, another type of metamaterial has been proposed to transport the sub-wavelength details, without involving negative refraction [BSI05]. The principle of operation is based on the use of a material with strong anisotropy to canalize the evanescent waves [Ma07, WPT06, BSI05]. Inside such a material the most part of the spatial spectrum of the source is transformed into prop-

agating eigenmodes (of the medium) which all have the same longitudinal component of the wave vector. In other words, when a source is placed close from an anisotropic medium, the propagating and the evanescent waves produced by the source excite the material eigenmodes at the front interface. The eigenmodes propagate from the front interface to the back interface without disturbances and deliver the distribution of the electric near fields generated by the source at the entry of the metamaterial. Finally, at the back interface of the anisotropic medium, the waves refracts into the host medium to form the image of the source.

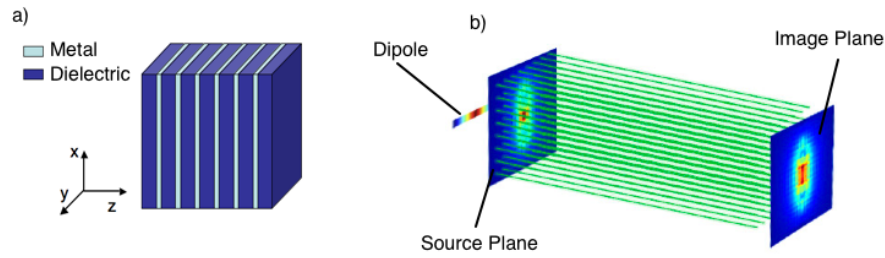


Figure 4.1 (a) Layered dielectric metal, (b) Canalization of the magnetic field generated by a small dipole through a straight wire medium.

Practically, in the visible frequency range, such an anisotropic medium can be created from a layered-metal dielectric structure [BSI05] as shown in Fig. 4.1.a, while for frequencies up to infrared band anisotropic medium can be realized from an array of parallel conducting wires called “wire medium”, as illustrated in Fig. 4.1.b [SBS07]. Indeed, the wire medium cannot be directly scaled to optical frequencies because metals lose their conducting properties, hence the imaging properties are altered [OiKK05]. In comparison to metamaterials based on SRR’s combined with rods, wire media exhibit practically no loss and a wide bandwidth [BS06, SBS07]. Moreover, the resolution of the collimator is limited by the spacing between the wires [LJJP03], so it is theoretically possible to reach any resolution, as far as the structure can be mechan-

ically realized. Regarding the transmitted fields, the collimator is capable of transmitting the TM component of the source spectrum only; consequently, on the other side of the medium, only the normal components (along the wires) of the electric field produced by the source are completely restored [BHS06].

In this chapter, we demonstrate numerically the use of a wire medium as an energy collimator to transmit or concentrate the magnetic field to a certain distance from the MRI tube center, to a region with negligible static field. The use of a wire medium for MRI opens several prospects : the improvement of the RF-homogeneity of MR images, the enhancement of the SNR thanks to the use of receivers (which can contain magnetic material) having better noise figures and the design of a more reliable endoscopic sensor. Straight wire media are investigated numerically with the Method of Moments from the point of view of their coefficient of transmission and eigenmodes. Special attention is given to the recuperation of the imaged fields, through the insertion of the receiving coil inside the metamaterial. Simulations results are compared with analytical results presented in the literature [BZAH07, BHS06, ZBH09, SBS08, SBS07]. Wire media with convergent geometries are also numerically analyzed in order to determine their ability to improve the homogeneity of MR images [STPS07, IST⁺07, RDC07]. Finally, extreme configurations, in terms of curvature and in terms of reduction of the cross-section are investigated numerically; the obtained results indicate that flexible endoscopic devices based on wire medium metamaterials may be envisaged.

The remainder of this chapter is organized as follows. Section 1 describes analytically the modes supported by straight wire media and analyzes eigenmodes which may exist for certain frequencies and their impact on the transmission. Section 2 presents the numerical results obtained with the Method of Moments, we focus on peculiarities of MRI operational conditions, i.e. (i) the quantity of interest is the magnetic field and, (ii) the transported image needs to be observed: loop antennas with finite dimensions are used to this end. These aspects are essential, indeed MRI deals with very low SNR and the absorption of protons energy by the coils must be optimal. Straight and non-straight wire media are analyzed; their eigenmodes are identified with the help of infinite- array approaches and compared with analytical predic-

tions. Section 3 presents images obtained with convergent/divergent lenses, as well as through a 63 degrees curved wire medium.

4.1.1 Infinite Wire Medium Analysis

The wire medium is a unique artificial material which possesses strong spatial dispersion effects. For a long time, this medium has been known as an artificial dielectric with plasma-like properties at low frequencies [Bro60, Rot62, PHS⁺96]. It was usually described as a uniaxial crystal, whose permittivity component along the wires is expressed by the following plasma formula.

$$\epsilon = \epsilon_0 \left(1 - \frac{k_0^2}{k^2}\right) \quad (4.1)$$

with $k_0 = \omega_0 \sqrt{\epsilon_0 \mu_0}$ which plays the role of an equivalent plasma frequency. Recently in [BMM⁺03], it was shown that if a wave, traveling in a wire medium has a nonzero component of its wave vector along the wires ($k_z \neq 0$), the previous plasma model should be corrected. Actually, the wire medium can be approximated as a homogeneous dielectric slab with both *frequency* and *spatial* dispersions. Consequently, the permittivity tensor of the wire medium cannot be described by using a model which depends on only the frequency (also called “local model”) but rather by a model which depends on frequency and wave vector (also called “non-local model”).

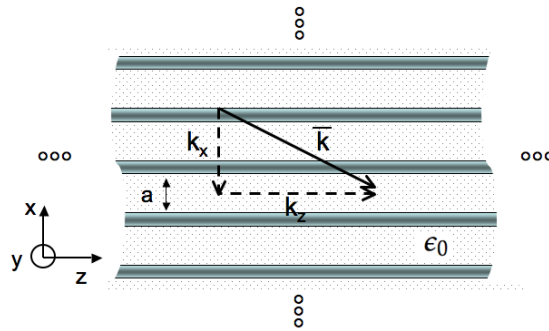


Figure 4.2 Infinite wire medium (along the three dimensions) supporting an electromagnetic wave with wave vector $\vec{k}$.

Let us consider an array of conducting rods oriented along the z -direction, as depicted in Fig. 4.2. The rods have an infinite length and are embedded in the air, with permittivity ϵ_0 . It was proven that the non-local model of the permittivity tensor of a wire medium is of the form [BMM⁺03]:

$$\bar{\epsilon}(\omega, k_z) = \begin{pmatrix} \epsilon_0 & 0 & 0 \\ 0 & \epsilon_0 & 0 \\ 0 & 0 & \epsilon_{zz}(\omega, k_z) \end{pmatrix}$$

with

$$\epsilon_{zz}(\omega, k_z) = \epsilon_0 \left(1 - \frac{k_p^2}{k^2 - k_z^2} \right) \quad (4.2)$$

where $k = \omega/c$ is the wave vector in free space, k_z its component along z and k_p is the wavevector corresponding to the plasma frequency of the wire medium. This latter depends on the lattice periods a and b , and on the radius of the wires r , as follows [BTA02]:

$$k_p^2 = \frac{2\pi/(ab)}{\ln(\frac{\sqrt{ab}}{2\pi r}) + F(a/b)} \quad (4.3)$$

$$F(\psi) = \frac{1}{2}\ln(\psi) + \frac{\pi}{6}\psi + \sum_{n=1}^{\infty} \frac{\coth(\pi n\psi) - 1}{n} \quad (4.4)$$

For the case of square grid ($a=b$), $F(1)=0.5275$ and the expression reduces to

$$k_p^2 = \frac{2\pi/a^2}{\ln \frac{a}{2\pi r} + 0.5275} \quad (4.5)$$

It is interesting to notice that the plasma frequency is inversely proportional to the wire spacing. The previous model is valid only in the quasistatic case, i.e. when $a, b \ll \pi/k$, the spacing between the wires is small in comparison to the wavelength [BMM⁺03]. Eq. (4.5) is not the only expression present in the literature to compute the plasma frequency. Other formulas are presented in [PHS⁺96], [SS01] and [MTB02]. These models have been compared in [MTB02]: a little difference is present between models except for Pendry's model [PHS⁺96] which does not take into account the interaction between wires.

Given the expression of the permittivity tensor, the modes which can propagate inside the wire medium can be analyzed. A modal analysis has been

performed in [BTA02, BMM⁺03]; these articles reveal that in a wire medium with the wires oriented along z , and for a wavevector with a component along the z -direction which is not equal to the wavevector in free space, $k_z \neq k_0$, two sets of propagating modes can be distinguished: the *ordinary waves* whose electric field component along the wires is equal to zero i.e. $E_z = 0$, and the *extraordinary waves* with $E_z \neq 0$. Hereafter, we briefly summarize, the steps to obtain the dispersion equations of these two sets of waves.

Let us assume a wave which propagates inside an infinite wire medium with a field dependence of the type $e^{j(\omega t - k_x x - k_z z)}$, as illustrated in Fig. 4.2. In the medium, the Maxwell equations in the Fourier domain are as follows,

$$\begin{aligned}\nabla \times \mathbf{E} &= -j\omega\mu_0\mathbf{H} \\ \nabla \times \mathbf{H} &= j\omega\epsilon_0\tilde{\epsilon} \cdot \mathbf{E}\end{aligned}$$

If we introduce the above dependence of the wave fields and expand the expression of the curl, we obtain the following system, with six equations and six unknowns,

$$-j(k_x\hat{x} + k_z\hat{z}) \times (E_x\hat{x} + E_y\hat{y} + E_z\hat{z}) = -j\omega\mu_0(H_x\hat{x} + H_y\hat{y} + H_z\hat{z}) \quad (4.6)$$

$$-j(k_x\hat{x} + k_z\hat{z}) \times (H_x\hat{x} + H_y\hat{y} + H_z\hat{z}) = -j\omega\epsilon_0(E_x\hat{x} + E_y\hat{y} + \frac{\epsilon_{zz}}{\epsilon_0}E_z\hat{z}) \quad (4.7)$$

The dispersion equations of the two sets of propagative modes can be deduced from this system of equations.

Ordinary waves: The dispersion equation of the ordinary waves, is obtained by solving the system and imposing that $E_z = 0$. From (4.6) and (4.7) we obtain the following dispersion equation,

$$(k_x^2 + k_z^2 - k^2)E_y = 0 \quad (4.8)$$

the same equation can be written for H_x and H_z ; the components E_x , E_z and H_y are equal to zero. From Eq. (4.8), we can see that the ordinary waves, or equivalently the transverse electric (TE) modes, does not interact with the wires and propagate as if they were in air.

Extraordinary waves: For this set of modes, the dispersion equation is obtained by imposing that the electric field along the wires is non zero, $E_z \neq 0$. Moreover, since we are in a waveguide and have already analyzed the TE

modes, we impose the magnetic field along the wires to be equal to zero, i.e. $H_z = 0$. These conditions lead to the following dispersion equation,

$$\left(\frac{k_x^2}{\epsilon_{zz}(\omega, k_z)} + \frac{k_z^2 - k^2}{\epsilon_0} \right) E_x = 0 \quad (4.9)$$

the relation also holds for the fields E_z and H_y , the fields H_z , E_y and H_x are equal to zero. The equation (4.9) cannot be solved explicitly as a function of k_z because the permittivity is itself a function of k_z . To solve it, we have to distinguish two situations depending on the value of the longitudinal wavevector k_z , and consequently, the longitudinal permittivity ϵ_{zz} :

- $k_z \neq k$, in this case ϵ_{zz} has a finite value, the dispersion equation (4.9) is solved by introducing the expression of the permittivity (4.2); it gives the following dispersion equation [BTA02]:

$$k_x^2 + k_z^2 = k^2 - k_p^2 \Rightarrow k_z = \sqrt{k^2 - k_p^2 - k_x^2}$$

It corresponds to transverse magnetic (TM) waves, we can see that there is propagation for $k^2 > k_p^2$, the cut-off frequency is at $k^2 = k_p^2$. It is the expected behavior for a plasma like structure.

- $k_z = k$, in this case, if we replace this relation in Eq. (4.2), we can see that the component of the permittivity along the wires has an infinite value $\epsilon_{zz} \rightarrow \infty$ and the dispersion equation simply reduces to,

$$k_z = k \quad ; \quad \forall k_x$$

Actually, it corresponds to transverse electromagnetic modes (TEM), since the field $D_z = \epsilon_{zz} E_z$ must keep a finite value, the field E_z is equal to zero. It is interesting to note that in this case there is no restriction on the spatial wave vector k_x , in other words if we consider a wave incident at the interface of a wire medium, for any angle of incidence, including complex ones, the longitudinal wave vector remains the same. This phenomenon is known as the “canalization” regime and is the underlying principle which enables the wire medium to transport images with sub-wavelength resolution and without distortion [BHS06, WPT06, Ma07]. Hereafter, we will always operate below the plasma frequency, so that TM modes are blocked and TEM modes can propagate inside the wire medium and ensure the propagation of the evanescent waves.

4.1.2 Finite Wire Medium Analysis

The transmission of a slab of finite length is analyzed in details in [BS06], in this Section, we briefly recall the important results. The computation of the transmission requires the solution of a boundary conditions problem. In the case of a medium with strong spatial dispersion, we have to note that usual boundary conditions i.e. the continuity of tangential components of the electromagnetic fields are insufficient [AG66, APW74, BS72]. For the wire medium case, it has been proven that the additional boundary condition is that the normal component of the electric field must be continuous at the the interface between the wire medium and the host medium [Sil06].

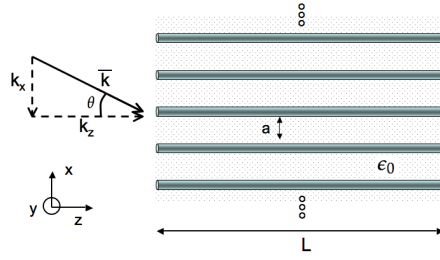


Figure 4.3 Wave incident on a finite length wire medium.

In [BS06], transmission through a wire medium is analytically calculated by considering an incident wave on a wire medium and by matching the six boundary conditions (tangential and normal) at the front and back interface. If we considered a wire medium constituted of wires of length L , oriented along the z -direction, and unbounded along x and y direction, the transmission and the reflection coefficients can be written as follows (Eqs.(8) and (9) in [BS06]),

$$T = \frac{H(z)e^{jk_z(z-L)}}{H_{inc}} \quad \text{with } z > L \quad (4.10)$$

$$= \left[1 + A \tanh \left(\frac{L\sqrt{k_p^2 + k_x^2 - k^2}}{2} \right) - B \tan \left(\frac{kL}{2} \right) \right]^{-1} \\ - \left[1 + A \coth \left(\frac{L\sqrt{k_p^2 + k_x^2 - k^2}}{2} \right) + B \cotan \left(\frac{kL}{2} \right) \right]^{-1} \quad (4.11)$$

$$R = \frac{H(z)e^{jk_z z}}{H_{inc}} - e^{-2jk_z z} \quad \text{with } z < 0 \quad (4.12)$$

$$= 1 - \left[1 + A \tanh \left(\frac{L \sqrt{k_p^2 + k_x^2 - k^2}}{2} \right) - B \tan \left(\frac{kL}{2} \right) \right]^{-1} \\ - \left[1 + A \operatorname{ctanh} \left(\frac{L \sqrt{k_p^2 + k_x^2 - k^2}}{2} \right) + B \operatorname{ctan} \left(\frac{kL}{2} \right) \right]^{-1} \quad (4.13)$$

where for the coefficients A and B can be expressed as,

$$A = \frac{k_x^2 \sqrt{k_p^2 + k_x^2 - k^2}}{(k_x^2 + k_p^2) \sqrt{k_x^2 - k^2}} \quad (4.14)$$

$$B = \frac{k k_p^2}{(k_x^2 + k_p^2) \sqrt{k_x^2 - k^2}} \quad (4.15)$$

k_p is the plasma frequency calculated with Eq. (4.5), and $k_x = k \sin(\theta)$ is the spatial wave vector which depends on the incident wave angle.

Let us illustrate with an example the impact of the frequency of operation f , on the coefficient of transmission of a given wire medium of length L . Consider a wire medium of length $L = 122$ cm, constituted of wires of 1 mm radius and spaced by 1 cm. These values correspond to the dimensions given in Chapter 4 for the straight wire medium used in MRI. We define the frequency $f_{\lambda/2}$ as the frequency for which the length of the wire medium is equal to half a wavelength, i.e.: $L = \lambda/2 = c/(2f_{\lambda/2})$. In our case $f_{\lambda/2}$ is equal to 123.23 MHz, which corresponds to the 3 Tesla MRI frequency. The transmission and reflection coefficients T and R are computed with the help of Eq.(4.11) and Eq.(4.13) for three ratios of frequency $f/f_{\lambda/2}$ equal to: 1.025, 1 and 0.98. The absolute value of these coefficients is shown in Fig. 4.4 versus the ratio k_x/k . The black vertical line in Fig. 4.4 corresponds to the limit of visibility $k_x = k$ (assuming that $k_y = 0$) and defines two areas depending on the value of k_x . The left area, with $k_x \leq k$ is related to waves propagative along z , while the right part, with $k_x > k$ is related to evanescent waves along z . We can see that the coefficient of transmission of propagative waves is always close to one, whatever the operational frequency f (and the ratio $f/f_{\lambda/2}$). When the

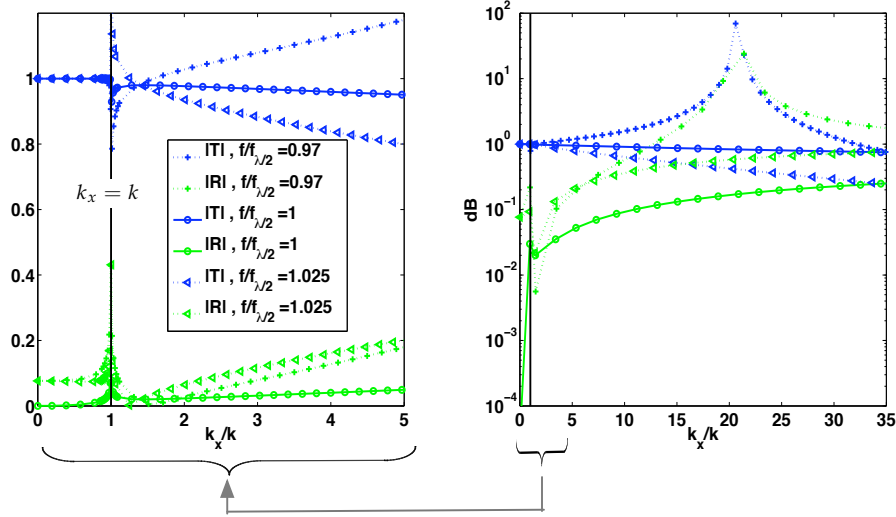


Figure 4.4 Left: transmission and reflection coefficients calculated for three frequency ratio $f/f_{\lambda/2}$. Right: for ratio $f/f_{\lambda/2}$ larger than one the coefficient of transmission tends to reach very high values for certain spatial harmonics k_x , due to the excitation of eigenmodes in the wire medium as explained [BS06].

angle of incidence θ is such that $k_x = k$, the transmission falls at zero. Hopefully, the stop-band effect appears only on a very limited bandwidth. On the other hand, regarding evanescent waves we can notice that for frequencies of operation lower than $f_{\lambda/2}$ (i.e. $f/f_{\lambda/2} = 0.97$), the coefficient of transmission tends to reach very high values for certain k_x . The phenomenon is further illustrated in Fig. 4.4 with the figure on the right where a large scale for k_x was used. Actually, the peak in transmission is an anomaly caused by eigenmodes traveling across the wires in the xy plane. The phenomenon is further analyzed in Section 4.2.3 with the help of numerical simulations [BS06, RLC08]. Eigenmodes disappear for frequencies of operation larger than $f_{\lambda/2}$. Moreover, it is demonstrated in [BS06] that when the plasma frequency k_p goes to infinity the transmission tends to 1 for any k_x . To conclude, we can see that the coefficient of transmission is optimal when the ratio $f/f_{\lambda/2}$ is close to one, i.e. when the Fabry-Perot condition is fulfilled: the length of the structure is equal to an integer number of half-wavelength, $kL = n\pi$.

Actually, it is demonstrated in [DNC06, NDCT06] that the length of the wires must be slightly longer than $\lambda/2$, since end-effects occur in a finite wire medium. To explain the origin of this extra-length, we introduce the transmission line model developed in [DNC06] to model the transmission properties of a plane wave through a straight wire medium. Suppose a wire medium of length L , placed in the air and excited by a plane wave incident with an angle θ , as illustrated in Fig. 4.5.a. The E-field polarization is parallel to the plane of the strips (TM waves). The wire medium can be represented by a very simple transmission line model depicted in Fig. 4.5.b. Indeed, only TEM modes can propagate between the wires below the plasma frequency and at the interface of the structure, the longitudinal component of the electric field of TM waves will vanish and only the component tangential to the interface propagates into the wire medium.

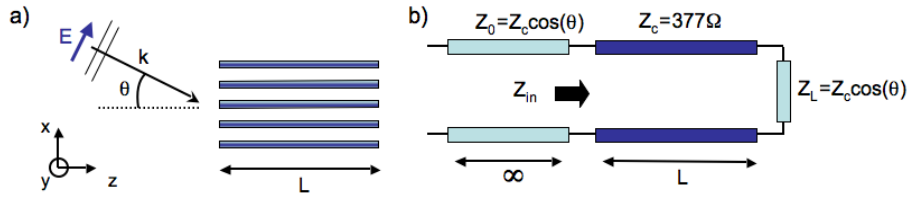


Figure 4.5 (a) Wire medium excited by a plane wave incident with an angle θ (b) Transmission line model for a plane wave (TM-waves) incident onto a wire medium

With this model, the input impedance and the reflection coefficient of the equivalent transmission line can be written as follows:

$$Z_{in} = Z_c \frac{Z_0 + jZ_c \tan(kL)}{Z_c + jZ_0 \tan(kL)} \quad \text{and} \quad \Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

According to this model, the transmission of the wire medium is fully transmitted each time the length of the wires is a multiple of half-wavelength. But in practice, an extra length is necessary. Indeed, at resonance according to the transmission line model, the currents at the ends of the wire medium should be non-zero to keep a finite input impedance for the wire medium, which is physically not possible for finite wires. Those non-zero currents are annihilated by the currents associated with the evanescent waves appearing at the

transition between free space and wire medium. Consequently, extra length is necessary for this annihilation to occur [BS06]. To illustrate this principle, in Fig. 4.6, the current flowing on the transmission line model (Fig. 4.5.b) is compared with the current obtained numerically with MoM by simulating an infinite-by-infinite wire medium. In all cases the excitation is a plane wave having an incident angle of 50° , the length of the wire medium is set to 122 cm and the frequency of operation is set to 123 MHz. Three different wire spacings are considered in the numerical simulations namely: 1cm, 2cm and 3cm. We can see that in the transmission line model the current at the extremity of the wire medium is non zero to keep a finite input impedance. In the numerical simulations, an extra length, which depends on the wire spacing, is necessary for the recombination to occur. Consequently the effective length of the wires is different from their physical length.

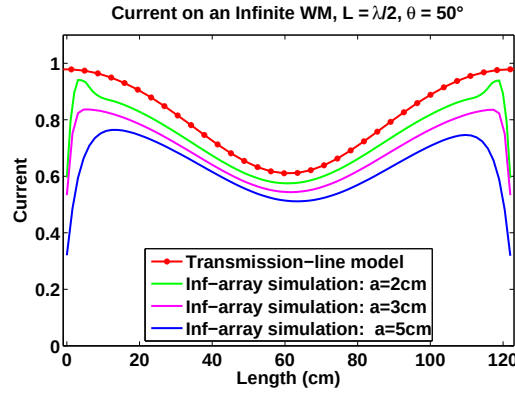


Figure 4.6 Comparison of the absolute value of the current flowing on the transmission line model in Fig. 4.5.b or obtained numerically with MoM by simulating an infinite-by-infinite wire medium. We can clearly see that the current obtained numerically tends to zero at the ends of the wires which is not the case in the transmission line model.

It is expected that the extra length is of the order of twice the decay distance of the evanescent waves. The attenuation coefficient, α , of the evanescent waves in the wire medium can be deduced from the dispersion equation (4.9),

$$\alpha = 1/j\sqrt{k^2\cos^2(\theta) - k_p^2} = 1/jk_z \quad (4.16)$$

Assuming that the evanescent waves have a significant effect along a distance $1/\alpha$ at each end, the difference between real and effective length of wires should be of the order of $2/\alpha$. The coefficient $2/\alpha$ is plotted in Fig. 4.7.b and 4.7.c for wire spacings a comprised between 1 cm and 4 cm and for two frequencies of operation: 123 MHz and 1600 MHz [DNC06]. We can see that when the plasma frequency is large in comparison to the operational frequency, the coefficient $2/\alpha$ becomes independent of the incident angle θ . Moreover, the relation between wire spacings and the factor $2/\alpha$ is almost linear, hence the extra length is approximatively of the order of the wire spacing. Consequently when designing a wire medium, it is recommended to use the smallest wire spacing in order to obtain a linear relation.

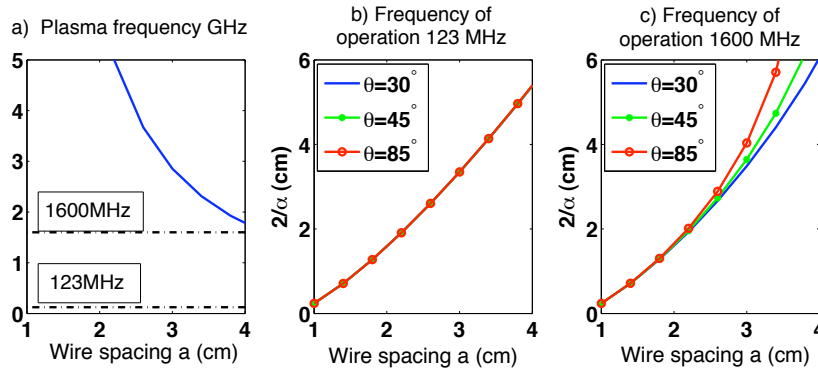


Figure 4.7 (a) Plasma frequency computed for different wire spacings, (b) and (c) extra-length computed at 123 MHz and 1600 MHz, respectively.

The impact of an extra length can also be evaluated through the analysis of the coefficient of reflection given in Eq.(4.13). It is plotted in Fig. 4.8 for two different wire spacings respectively equal to 1 cm and 2 cm and for three different angles of incidence, namely: 30° , 45° and 60° . The plasma frequency which depends on the wire spacing is large in comparison to the operational frequency set to 123 MHz ($= f_{\lambda/2}$). We can see that the coefficients of reflection reach zero for ratio $f/f_{\lambda/2}$ equal to 1.01 and 1.02 for wires respectively spaced

by 1 cm and 2 cm. It confirms the necessity to use a wire medium slightly longer than $\lambda/2$ to reach the optimal coefficient of transmission.

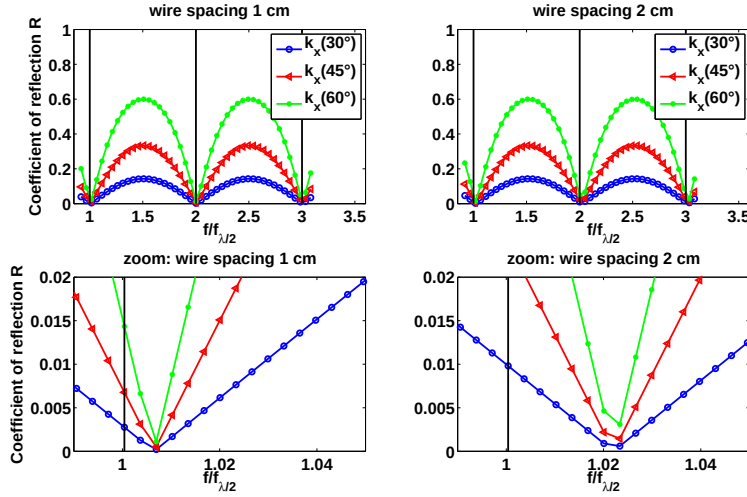


Figure 4.8 Coefficients of reflection computed with Eq. (4.13) for two different wire spacings.

4.1.3 Eigenmodes: Analytical Model

We have seen in Fig. 4.4 that for certain $k_x > k$, the coefficient of transmission tends to reach very large values. This phenomenon can be explained by eigenmodes traveling across the wires. The dispersion equation of these eigenmodes can be found by using the boundary conditions in Eq. (4.11) and searching for non-trivial solutions while the external excitation is set to zero. The following dispersion equation is found [BS06]:

$$\left(1 + A \tanh \left(\frac{d \sqrt{k_p^2 + k_y^2 - k^2}}{2} \right) - B \tan \left(\frac{kd}{2} \right) \right) \cdot \left(1 + A \coth \left(\frac{d \sqrt{k_p^2 + k_y^2 - k^2}}{2} \right) + B \cotan \left(\frac{kd}{2} \right) \right) = 0 \quad (4.17)$$

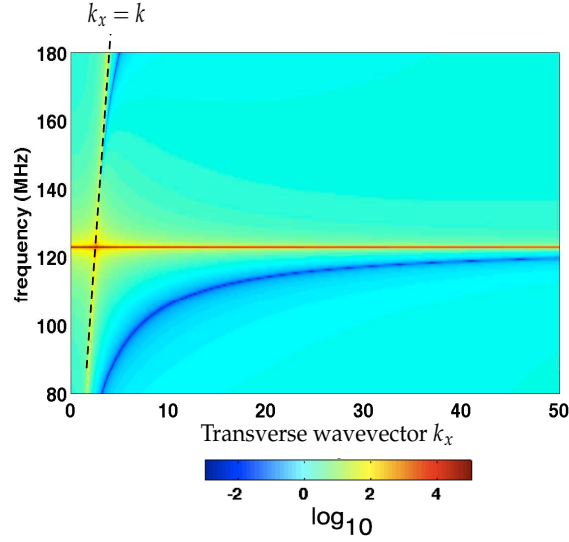


Figure 4.9 Identification of eigenmodes which can be excited in a straight wire medium.

This equation can be illustrated through the computation of eigenmodes in a wire medium devoted to MRI. The wire medium has a length $L = 122$ cm and is made of wires spaced by 1 cm. Eigenmodes are found by iteratively searching the wavenumbers (k, k_x) which satisfy Eq. (4.17), the spatial wave vector k_y is assumed to be equal to zero. They are represented by the dark blue lines in Fig. 4.9. We can see that for frequencies comprised between 123 MHz and 145 MHz eigenmodes cannot propagate across the wires. Therefore, the maximum bandwidth that we can expect for the studied wire medium is of the order of 18%. The next section presents numerical results obtained with the method of moments for different types of wire media and compares them with previously described analytical predictions.

4.2 Numerical Analysis of Straight Wire Medium

The main purpose of this section is to confirm the predicted analytical results (eigenmodes, bandwidth,...) and overall to determine the optimal man-

ner to catch the MR images with good signal-to-noise ratio in order to ultimately transform them into a voltage.

4.2.1 Calculation of the Open-Loop Voltage

The goal of this section is to evaluate the field transmission through a finite wire medium. Hereafter, the transmission between two small loops, with a size of the order of the lattice constant, and placed on both sides of the collimator, is evaluated. The setup considered to determine the transmission is sketched in Fig. 4.10.a. The first loop is excited with a delta-gap source and resembles the radiation of the protons in MRI, the second loop acts as an MRI receiving antenna. In order to evaluate the transmission through the wire medium, the open-loop voltage, V_{open} , is calculated in the open gap of the receiving loop. We verified that the inductance of the second loop is not significantly dependent on the presence of the wire medium; which means that there is a straightforward link between the open-loop voltage and the voltage across the input impedance of the front-end amplifier. An in-house MoM simulation software [RWG82] has been used. This code allows us to calculate the currents on the wires, as well as the electric and magnetic fields in any portion of space. From the currents flowing along the wires, two possibili-

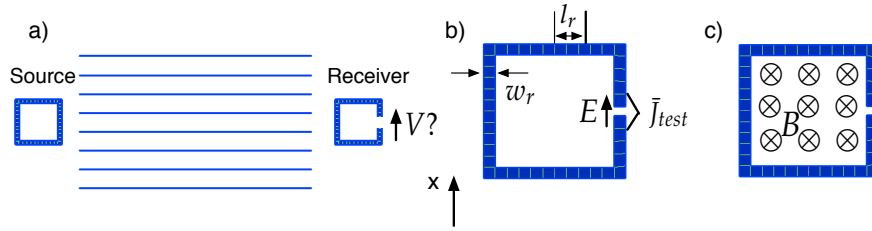


Figure 4.10 a) Simulation setup: loops are placed on both sides of the wire medium. b) method 1: the open-circuit voltage is calculated from the electric field, measured by using a virtual testing function in the open gap of the loop, c) method 2: the voltage is determined from the measure of the magnetic field flux through the loop.

ties exist to determine the open-loop voltage. The first one is a direct method based on the computation of the electric field in the air gap, and the second one computes the open-loop voltage from the magnetic flux flowing within

the receiving loop. Both methods use as input datas; the currents induced on the wire medium. Some details for the calculation procedure and results obtained with both methods are presented hereafter.

First method

In this method, the open-loop voltage is determined from the incident and scattered electric field, $\vec{E}_{tot}$. The electric field present across the air-gap of the receiving loop is computed with the help of a rooftop testing function, $\vec{J}_{test}$, bridging the gap. We suppose that the width of the testing function is defined on a rectangle of width w_r and length l_r , as shown in Fig. 4.10.b, and that the entire structure is discretized with N basis functions. The expression that needs to be evaluated is:

$$V_{open} = - \int_{Open-gap} \vec{E}_{tot} \cdot d\vec{l} = - \frac{\int_S \vec{E}_{tot} \cdot \vec{J}_{test} dS}{w_r} \quad (4.18)$$

where S is the domain of the normalized rooftop testing function, $\vec{J}_{test}$. The currents flowing on the whole structure are obtained in discretized form (in terms of coefficients x_i) with the help of the MoM [RWG82] applied to the electric-field integral equation.

$$\bar{\bar{Z}}_{EJ} \bar{x} = -\bar{V} \quad (4.19)$$

with $\bar{V}$ the excitation vector and, $\bar{\bar{Z}}_{EJ}$ the matrix of interactions between the basis functions and the testing functions belonging to the source and to the wire medium. From those currents, the electric field can be computed by processing an extra line of the impedance matrix, $\bar{\bar{Z}}_{(N+1)}$. This extra line represents the interaction between the testing function $N+1$, that bridges the gap on the receive side and the other basis functions of the structure. The electric field in the open gap tested by function $\vec{J}_{test}$ satisfies

$$\int_S \vec{E}_{tot} \cdot \vec{J}_{test} dS = \sum_{i=1}^N x_i Z_{(N+1,i)}$$

Consequently, from Eq. 4.19, the open-loop voltage can be written as:

$$V_{open} = - \frac{\sum_{i=1}^N x_i Z_{(N+1,i)}}{w_r} = \frac{-\bar{Z}_{N+1} \cdot \bar{x}}{w_r} \quad (4.20)$$

Second method

In the second method, the open-loop voltage, V_{open} , is determined from the magnetic field, $\vec{H}$, flowing through the loop:

$$V_{open} = -j\omega\mu_0 \int_{S_{loop}} \vec{H} \cdot \hat{n} dS_{loop} \quad (4.21)$$

with $\omega = 2\pi f$ and f the frequency, μ_0 the permittivity of air and $\hat{n}$ the unit vector normal to the loop. The magnetic field is computed numerically: an observation plane filled with identical rooftop testing functions, $\vec{J}_j$, oriented along the axis of the loop, is placed in the loop, as illustrated in Fig. 4.10.c. The sampling should be sufficiently fine and the testing function sufficiently small in order to catch the logarithmic singularity of the fields on the edges of integration domain S_{loop} . The magnetic field tested in the middle of the testing function, $\vec{J}_j$, can be approximated as:

$$H^j \simeq -\frac{\int_S \vec{H} \cdot \vec{J}_j dS}{S/2} \quad (4.22)$$

where S is the surface of the domain on which the rooftop testing function (with average magnitude 1/2) is defined. The integral in the latter expression is evaluated, with the MoM [YOT05], with the help of the following system matrix product:

$$\int_S \vec{H} \cdot \vec{J}_j dS \simeq \bar{\bar{Z}}_{HJ} \bar{x} \quad (4.23)$$

where $\bar{\bar{Z}}_{HJ}$ is the MoM impedance matrix for basis functions supporting electric currents on the structure and the testing functions multiplying magnetic fields in the observation plane. $\bar{x}$ is the vector of current coefficients obtained from the solution of the MoM system (4.19). From the tested magnetic fields, H^j in (4.22), the flux integral (4.21) is approximated numerically.

In Fig. 4.11, the transmission obtained with both methods is compared for a 12×12 wire medium excited with a loop of size 0.5×0.5 cm. The lattice constant between wires is set to $a = 0.5$ cm, the length of wires is $L = 16$ cm, wires are replaced by strips of width 0.05 cm. The curves have approximately the same shape; in terms of absolute values, a difference of approximately 5% is noticeable. This is probably due to errors appearing in the numerical integration of the logarithmic singularity of fields along the edges of the loop.

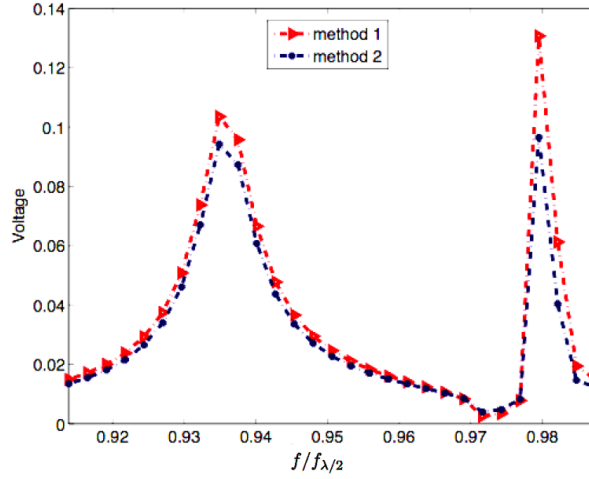


Figure 4.11 Comparison between open-circuit voltages obtained with methods 1 and 2.

4.2.2 Frequency Response Analysis

In this section, we will show transmission results obtained with the methods previously described for various positions of the loop receiver inside the collimator. The analyzed structure is the same as presented in Fig. 4.10.a; the dimensions of the straight wire medium are scaled in order to have the same geometrical parameters as for the MRI experiment described in Section 5.3.2. The spacing between wires is set to $a = \lambda/243$. The copper wires are modeled by strips whose width is twice the wire diameter, i.e. $w_{strip} = \lambda/1215$, in order to preserve their inductance per unit of length, and their length is close to $L = \lambda/2$, at the working frequency. Actually, the length of the wires must be slightly longer than $\lambda/2$, since end effects occur for a finite wire medium [NDCT06] as we have seen in the Section 4.1.2. The width of the wire medium (orthogonal to the wires) is $W = \lambda/16$ at $f_{\lambda/2}$, i.e. it is significantly narrower than for other collimation experiments shown in the literature [BZAH07]. This means that truncation effects may be very important and may become dramatic if the structure supports eigenmodes propagating transversally [NDCT06].

Transmission results are presented in Fig. 4.12.b for various positions of the receiving loop along the axis of the collimator. The frequency axis is nor-

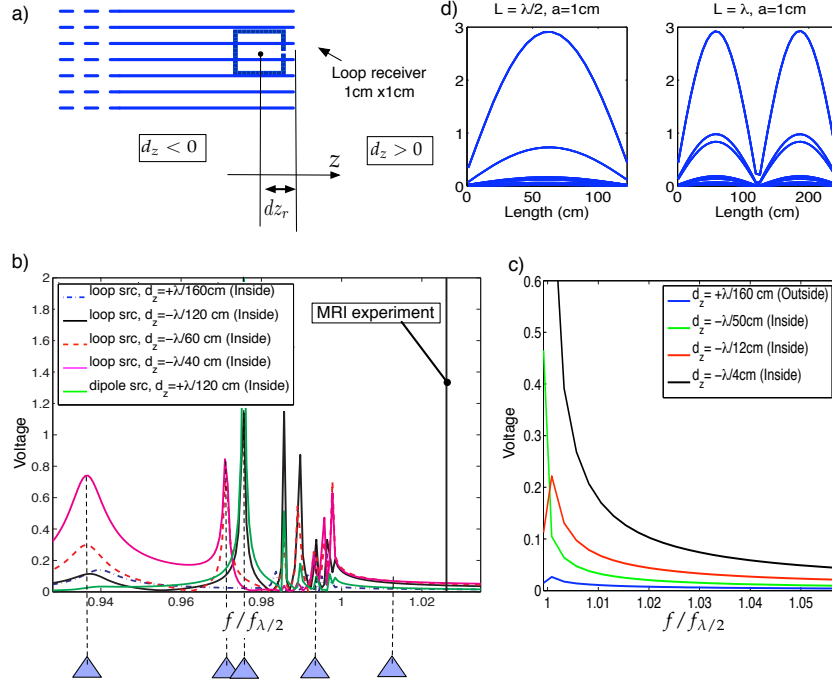


Figure 4.12 (a) The receiver is translated along the axis of the collimator, (b) MoM simulations results for a 14×14 -array collimator, obtained either with a dipole source or with a loop source, and for various positions, d_z , for the loop receiver. Arrows are at frequencies where the magnetic field has been mapped in Fig. 4.13, (c) zoom on the frequency region where the transmission is stable, we can see that the insertion of the receiver loop provides a higher voltage, (d) profile of the currents flowing on the wires of a 10×10 straight wire medium, working in the canalization regime and having a length equal to $\lambda/2$ or to λ . For the $\lambda/2$ wire medium, we can see that the highest currents are present at the center.

malized with respect to $f_{\lambda/2}$, the frequency for which the length of the wire medium is $\lambda/2$. The source employed is either a small square loop whose width, at $f = f_{\lambda/2}$, is $\lambda/970$, or a small dipole with a $\lambda/970$ height, placed in front of the wire medium. For five key frequencies, represented with blue arrows in Fig. 4.12.b, the magnetic fields along the axis of the loop receiver are depicted in Fig. 4.13. The testing plane is placed inside the wire medium and depicts the field flowing through the receiving loop placed at $d_z = -\lambda/128$

at $f = f_{\lambda/2}$, with d_z the position of the center of the receiving loop, along the axis of the collimator and measured from the end of the medium. The first column in Fig. 4.13 shows the position of the testing plane across the structure. The position and the cross section of the sources are shown with the line at the center of each magnetic-field plot. The analysis of the transmission curves allows us to distinguish four different regions:

- from $f/f_{\lambda/2} = 0.93$ to 0.95 , a smooth peak is present; this peak tends to disappear if the loop source is replaced by a dipole (dotted line). The analysis of the magnetic field in Fig. 4.13 shows that the magnetic field is widely spread. When the loop receiver is inserted deeper, this peak tends to increase (solid line and dashed lines).
- from $f/f_{\lambda/2} = 0.96$ to 0.98 , higher peaks are present. Usually, they tend to shift towards lower frequencies when the receiver is inserted deeper in the wire medium (this peak usually is the place where the voltage on the receiver is the highest). The analysis of the corresponding magnetic field, in Fig. 4.13, shows that the field is no longer concentrated near the center of the collimator.
- from $f/f_{\lambda/2} = 0.98$ to 1 is a region which gives rise to a quite unstable field transmission. The origin of this instability is explained by eigenmodes [BS06, RLC08] propagating transversally (perpendicular to the wires) and undergoing multiple reflections at the edges of the periodic structure.
- Above $f/f_{\lambda/2} = 1$, the field from the source is stable and concentrated, as shown in the last column of Fig. 4.13. On the other hand, in that figure, the maximum value of the field is weaker in comparison to the two preceeding frequencies, which explains the lower voltage measured in Fig. 4.12.b. This is the region where the MRI experiments were conducted. In Fig. 4.12.c this region is enlarged, we can see that the voltage slowly goes up when the receiver is inserted deeper inside the wire medium. It can be explained by analyzing the profile of the currents flowing on the wires of a finite wire medium in Fig. 4.12.d.

As we will see in Section 5.3.2, the MRI experiment was conducted in the fourth region i.e. slightly above $f/f_{\lambda/2} = 1$, more precisely $f_{IRM}/f_{\lambda/2} = 1.027$, which corresponds to the position of the black vertical line in Fig. 4.12.b.

Nevertheless, in view of the large bandwidth of the collimation phenomenon, such a shift is easily affordable.

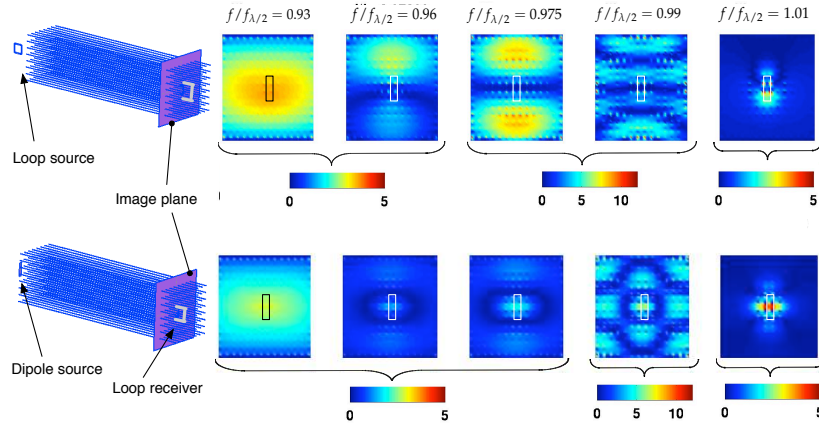


Figure 4.13 Magnetic field through the middle of the receiving loop, the testing plane is at $dz = -\lambda/128$, the loop source is represented with the small white box in the center and is also illustrated in the first column. Top line source: loop; bottom line source: dipole.

4.2.3 Numerical Analysis of Eigenmodes

We have seen that the coefficient of transmission is unstable for a ratio $f/f_{\lambda/2}$ slightly smaller than one. To further understand this chaotic behavior phenomenon and to validate the analytical predictions given in Section 4.1.3, the component along x of the magnetic field flowing through the receiving loop is computed in an infinite-array context with the help of the Array Scanning Method [MB79, CJW05, CPD04]. The geometrical configuration is presented in Fig. 4.14, the unit cell is constituted of 2×2 wires, the source employed is a small dipole with a $\lambda/20$ height, on the other side the magnetic field is observed with the help of testing functions oriented along x and placed against the back side of the wire medium. Fig. 4.14 shows the images of the dipole obtained for different ratios $f/f_{\lambda/2}$ by using the 5×5 ASM; i.e. the spectral integral is replaced by a 5-points integral with constant weights. In view of the discretized approach to the ASM, this method is equivalent to

having the source dipole repeated every 10 cells in both directions (a factor of 5 in view of the order of the ASM and a factor of 2 because the unit cell is made of 2×2 wires).

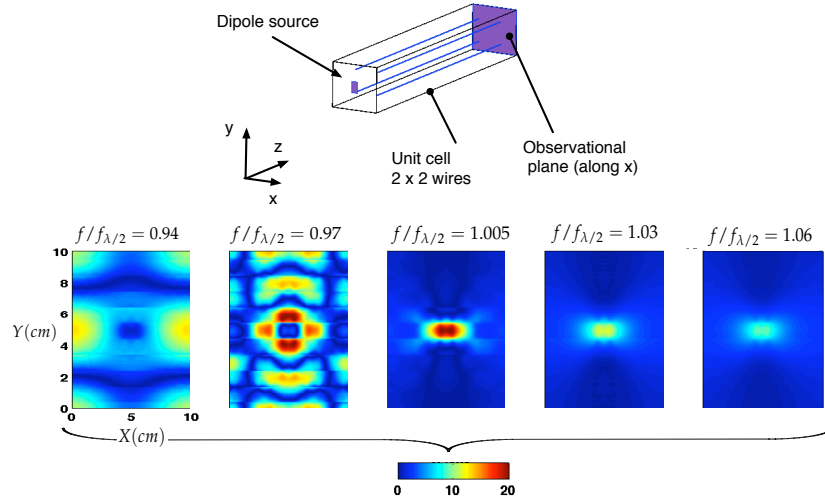


Figure 4.14 Component of the B-field along x , in the XY plane positioned at the back side of the wire medium, obtained with a 5×5 ASM and within a unit cell comprising 2×2 strips. Arbitrary units.

It can be seen that, in the problematic frequency band, the magnetic field is uncollimated since it follows patterns with increasingly fast spatial variations, while for the other frequencies the field is neatly collimated. The explanation given in [BS06, BZS⁺06, RLC08] is that eigenmodes may appear in that frequency band. This is now confirmed with the help of the ASM. The spatial variations are very similar to those found in finite arrays [RLC08] at identical frequencies (see Section 4.2.2). In both cases, a standing-wave pattern appears, with a similar periodicity. In the finite array, that pattern results from multiple reflections of eigenmodes on the edges of the array. In the ASM result, it is due to interference of the bound waves generated by all the replicas of the source, located every N cells in both directions, in view of the $N \times N$ -points discrete ASM (aliasing phenomenon).

More precisely, the eigenmodes can be identified by determining for which inter-element phase shifts (ψ_x, ψ_y) and for which frequencies, the determinant

of the MoM impedance matrix Z is zero (or below a given threshold). Iso-frequency results are given in Fig. 4.15 for different ratios $f/f_{\lambda/2}$. We can see that for a ratio $f/f_{\lambda/2}$ less than one, the determinant of the impedance matrix tends to zero along certain (Ψ_x, Ψ_y) curves, illustrated by the dark lines in the first two iso-frequency plots. That means that an eigenwave for these phase shifts may propagate transversally in the wire medium (XY plane) and perturb the transmission along the z direction. Moreover, in view of the wide spatial spectrum of the incident field -which has a non-zero projection on the eigenmode fields- and since the wire medium is used in the near field, the possibility of exciting the eigenmodes is important. On the other hand, for a ratio $f/f_{\lambda/2}$ slightly larger than one, the transversal eigenmodes disappear and the magnetic field is well collimated over a wide bandwidth, as illustrated on the three pictures on the right in Fig. 4.14. For this reason, this is the frequency range where the MRI experiments with wire media were conducted.

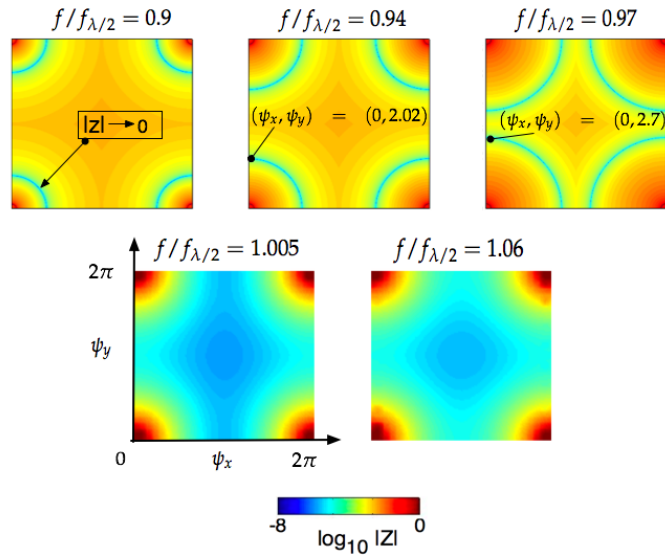


Figure 4.15 Iso-frequency curves for various frequencies (normalized) computed with the MoM. The plotted quantity is the determinant of the MoM impedance matrix. Dark lines correspond to the lowest values and indicate eigenmodes. We can see that for frequency ratio larger than one, no eigenmodes are present

In order to validate results obtained with the MoM, we have computed analytically iso-frequency curves with the help of Eq. (4.17). Iso-frequency results are given in Fig. 4.16. In order to allow comparison between both methods, the phase shifts (ψ_x, ψ_y) corresponding to the eigenmode propagating along the y-axis is given for the ratios $f/f_{\lambda/2} = 0.94$ and $f/f_{\lambda/2} = 0.97$. We can see that the obtained phase shifts are approximatively identical with both approaches, the MoM seems to slightly overestimate the eigenmode spatial frequency.

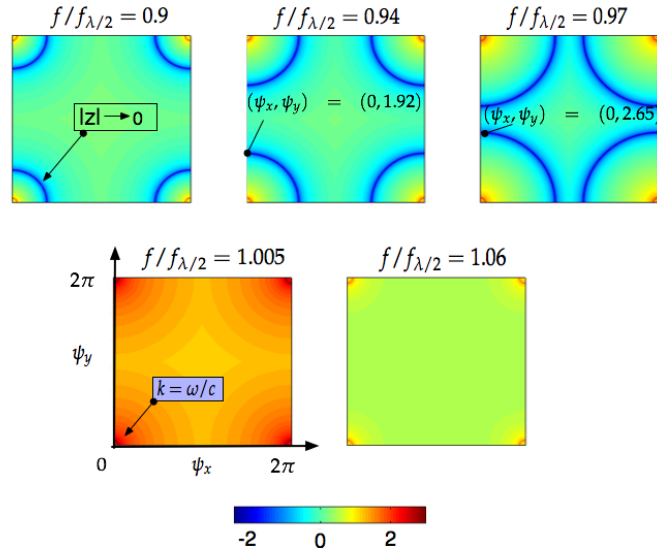


Figure 4.16 Iso-frequency curves obtained analytically with the help of Eq. 4.17. Compared with iso-frequencies computed analytically in Fig. 4.15, Ψ_x and Ψ_y associated with eigenmodes curves take approximatively the same values.

4.2.4 Influence of the Density of Wires

As can be seen from Fig. 4.17, for a constant cross-section of the wire medium ($10 \times 10 \text{ cm}^2$), the amplitude of the signal has a weak dependence on the density of wires; with a better quality, however, for the densest lense. This result is in agreement with the analytical expression of the coefficient of transmission given in Section 4.1.2. Indeed it can be demonstrated that when the density of wires increases, the plasma frequency k_p goes to large values and

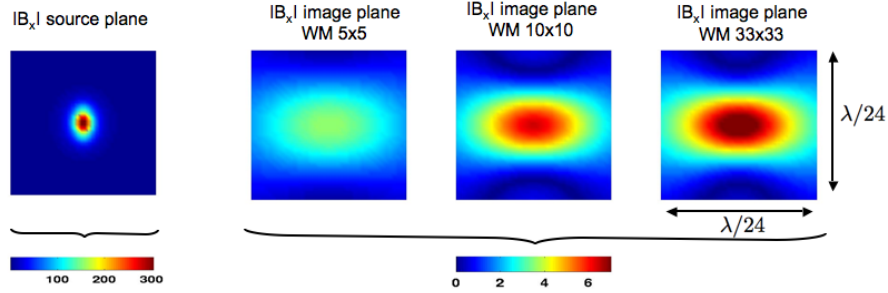


Figure 4.17 Magnitude of the x-component of the magnetic field for different wire densities (the dipole source is oriented along y)

the coefficient of transmission tends to 1 for any spatial wave vectors [BS06]. The density of wires also has an effect on the iso-frequency plots. The phenomenon is illustrated in Fig. 4.18, where we have plotted the eigenmodes curves for two different density of wires. From this figure, we can see that the curves tend to go down when the wire density increases and that the frequency range free of eigenmodes is wider, this effect could be interesting to improve the bandwidth of wire medium near the resonant frequency $f_{\lambda/2}$.

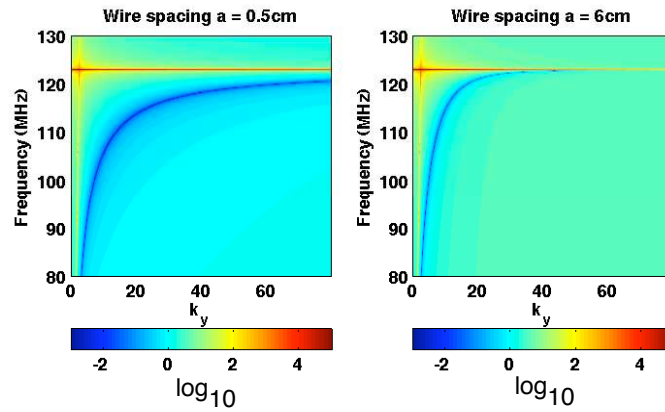


Figure 4.18 Curves of eigenmodes calculated analytically for two different wire densities

4.2.5 Numerical Analysis of the Impact of an Extra Length

In Fig. 4.19, we analyze numerically the impact of an extra length on the coefficient of reflection of straight wire medium. An infinite-by-infinite wire medium made with strips of width $w = 0.3$ cm and of length $L = 122$ cm is simulated with the help of the Method of Moments. The wire is finely meshed in order to be sensitive to the different length (height of the basis function $= \lambda/100$). The structure has the geometrical parameters as in Section 4.1.2. the reflection coefficient S_{11} is computed numerically for two different wire spacings, namely $a = 1$ cm and $a = 2$ cm. As expected, compared to Fig. 4.8 the amplitudes of reflection coefficients are identical, they are null for ratio $f/f_{\lambda/2}$ slightly larger than one. It means that the optimal frequency of operation is higher than the resonance frequency $f_{\lambda/2}$ of the wire medium (frequency for which the length of the wire-medium corresponds to half a wavelength). Consequently for a given wire medium an extra length is necessary to lower its resonance frequency and reach the optimal transmission. Regarding the extra-length needed, contrary to the analytical part, it depends on the incident angle and tends to decrease when the wire spacing increases.

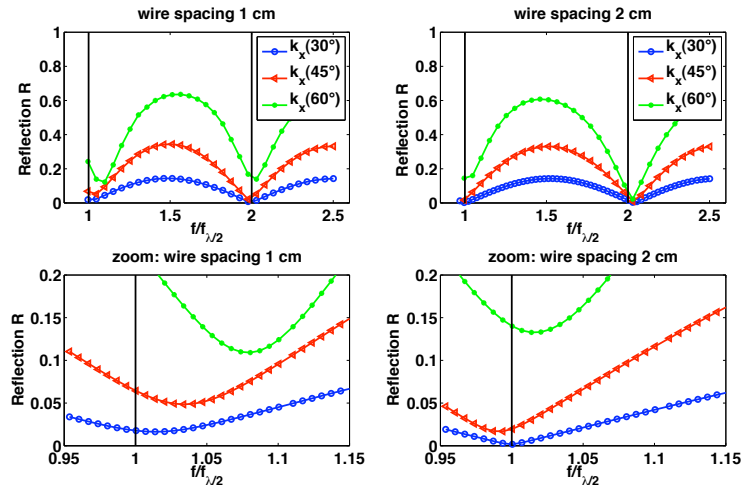


Figure 4.19 Coefficients of reflection computed for an infinite-by-infinite wire medium. Two different wire spacings are considered.

4.2.6 Antenna Coupling

Since we are interested in using a straight wire-medium in MRI, special attention is given to the recuperation of the MR signal. In this sense, the use of multiple receiving antennas can significantly improve the signal reception and improve the field of view of the wire medium. In this section, we demonstrate this principle. The structure employed for the analysis is a 10×10 wire medium, with wires spaced by 1 cm and of length 122 cm; therefore the cross section is equal to 9 cm^2 . The source employed is a dipole small compared to the lattice constant and placed at 2.5 cm from the front side of the wire medium. Nine square receiving antennas are used, each having a width and a height equal to 3 cm.

In Fig. 4.20.a, the blue points represent the wires of the wire medium. The green lines correspond to the different antenna positions inserted in the back side of the wire medium. Nine antenna positions are envisaged. The goal of this section, is to evaluate how their response can be combined to obtain a homogeneous reception of the signal.

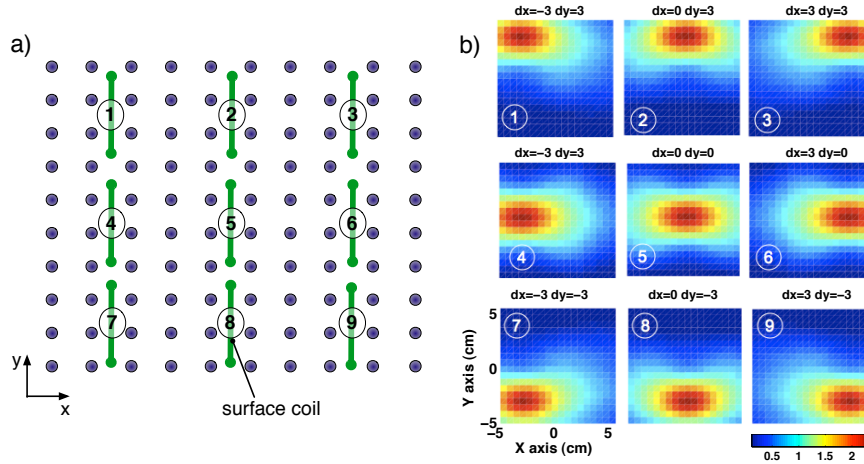


Figure 4.20 (a) View of the back side of the wire medium and of the different antenna positions considered, (b) sensibility of the antennas for the nine considered positions.

To study the sensitivity of the set of (all) antennas, the sensitivity of each antenna must be determined separately. In this sense, the open-loop voltage is analyzed for different positions of the dipole between -4.5 and 4.5 cm along the x- and y-axis (the coordinates of the center of the medium are (0,0)). The sensitivity results for the different positions are given in Fig. 4.20.b.

Various combinations of antenna sensitivities are envisaged. The global sensitivities are presented in Fig. 4.21 when 4, 5 and 6 antenna sensitivities are combined. The weighting coefficients employed are given in Tab. 4.1, the first line refers to the antenna position.

Table 4.1 Weight coefficients employed to create the sensitivity maps in Fig. 4.21

N° coil	1	2	3	4	5	6	7	8	9
4 MRI coils	0	1	0	0.5	0	0.5	0	1	0
5 MRI coils	1	0	1	0	0.3	0	1	0	1
6 MRI coils	1	0	1	0.5	0	0.5	1	0	1

From Fig.4.21, we can see that when six coils are combined a very homogeneous sensitivity can be obtained. The amount of antennas necessary is relatively low compared to certain antennas arrays used nowadays.

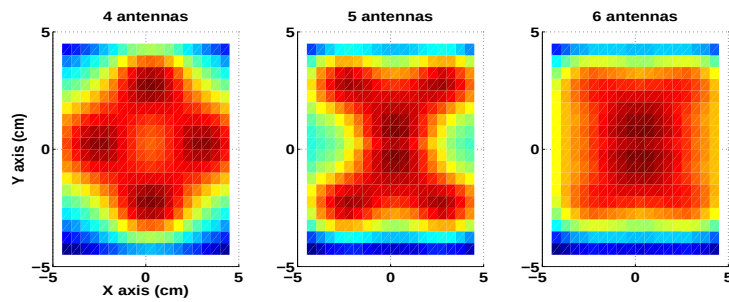


Figure 4.21 Global sensitivity obtained by combining various antenna sensitivities

Note that complex fields (Fig.4.20.b) and complex weighting coefficients may be used. Moreover, weights may be depth dependent (distance between

source and front side) and depend also on whether or not the source is inside a dielectric object.

4.3 Analysis of a Convergent Wire Medium

We have seen that a straight wire medium has a wide bandwidth and is constituted of elements which are not very small in comparison with the wavelength. Consequently, its weakly resonant nature does not make fabrication accuracy a critical issue and allow us to imagine variations of the structure, such as convergent, divergent geometries. The magnification effect using a divergent wire medium has been studied in [RDC07,STPS07,IST⁺07], where sub-wavelength imaging has been obtained with success.

In this section, we numerically analyzed the magnetic field distribution transmitted through a convergent wire medium. Our aim is to evaluate the possibility to use a convergent wire medium to improve the RF homogeneity of MR images. We have seen that when small organs are studied, it usually requires the use of a small loop antenna to capture local magnetic fields with a high SNR. However, the main consequence of the use of a small surface coil is that it captures only a very localized field and creates in-homogenous images. From this point of view, convergent wire media could bring a solution by concentrating the surrounding fields on a small coil with a high SNR.

4.3.1 Impact of the convergence on transmission

First we have to determine the influence of the geometry on the canalization of images. Three forms of geometries are studied, namely: a convergent, a divergent and a straight wire medium. They are illustrated in Fig. 4.22. Each one is made of an array of 15×15 strips of width 0.3 cm. The strips are considered as perfect electric conductors. In the straight geometry, they are spaced by a distance of 1 cm. In the convergent geometry, depending on the side observed, the wires are spaced by 1.6 cm on one side and by 0.6 cm on the other side. For the three structures, the central wire has a length equal to 123 cm, therefore we work at a frequency ratio $f/f_{\lambda/2} = 1.01$, i.e. in the frequency range where the magnetic field is collimated.

The source of excitation employed is a 1 cm-height dipole, positioned close to the wire medium at 0.5 cm so that the wire medium can capture evanescent waves. In order to extract magnetic fields on both sides of the lens two virtual

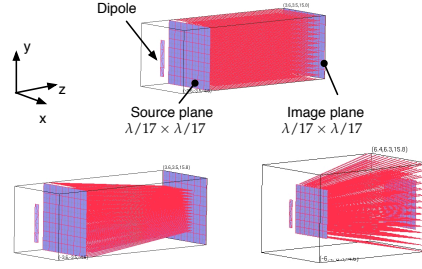


Figure 4.22 The three geometries of wire medium considered.

planes are placed at 1 mm from the front side of the lens (source plane) and from the back side of the lens (image plane). The observation is obtained by adding fictitious testing functions in the MoM problem, to compute the corresponding interaction matrix block, and by multiplying the current induced on the structure by that block. The field transmission is shown in relative scale for the three geometries in Fig. 4.23.

We can see that the canalization effect is still present for all different geometries. For the convergent structure, a concentration of fields is observed. This is the sought effect in order to improve the homogeneity of medical images. However the magnitude of the field is lower, this caused by a modification of the input impedance, as explained in the next section.

4.3.2 Impact of the Convergence on Matching

In view of the amplitude of the transferred magnetic field obtained in the previous section, the influence of the convergence/divergence is expected to be important. In order to see the impact of different geometries on the matching of the material, the difference between the absolute value B_x obtained in presence of wire media with different geometries is computed in Fig. 4.24. We can see that in comparison with the straight wire medium the input impedance of the convergent/divergent wire medium is different. This modification could be exploited, in medical scenarios, to observe without mismatch the organs which usually have a high dielectric constant (ϵ' between 30 and 70).

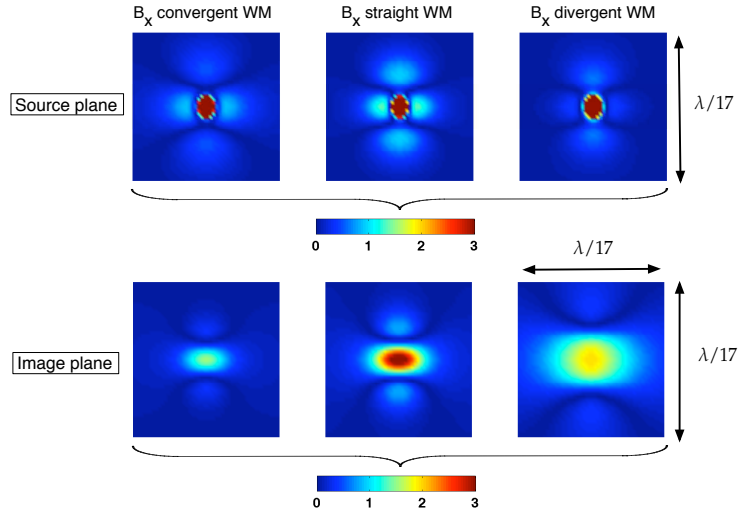


Figure 4.23 Component along x of the magnetic field computed on the source plane (top line) and on the image plane (bottom line). We can see that the B-field on the image plane is concentrated in the convergent case, although its intensities are lower than in the straight case. The dimensions of the observational planes are $\lambda/17 \times \lambda/17$, they have the same cross-section as the straight wire medium.

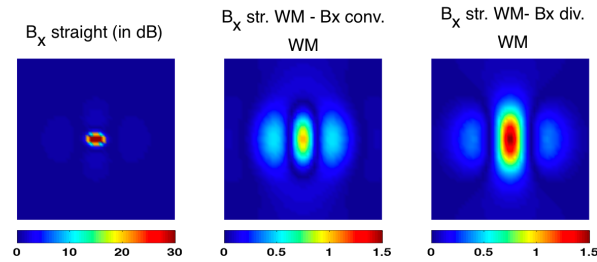


Figure 4.24 Left: B_x on the source plane with the straight wire medium. Right: difference field for different geometries.

In order to see if the modification of impedance can be exploited to match the wire medium to biological bodies, we have carried out a simulation involving sources embedded in a dielectric object. The simulation is performed

with the MoM and with the help of the equivalence principle (Section 2.3.3). The setup simulated is presented on top in Fig. 4.25, the source is a small dipole included in a dielectric bottle. The bottle has dimensions approximately identical to the cross section of the straight wire medium. Two dielectric values ϵ' are tested namely: $\epsilon' = 1$ and $\epsilon' = 30$. We consider that there is no loss, so the imaginary part of the dielectric is set to zero. The results are presented in Fig. 4.25. We can see that when the bottle is in air we obtain the same behavior as in the previous section, i.e. the straight wire medium gives the highest magnetic field on the image plane. On the other hand, when a source embedded in a dielectric object is employed, it is the convergent/divergent wire medium which outputs the highest field. Further simulations including losses need to be carried out to confirm this result.

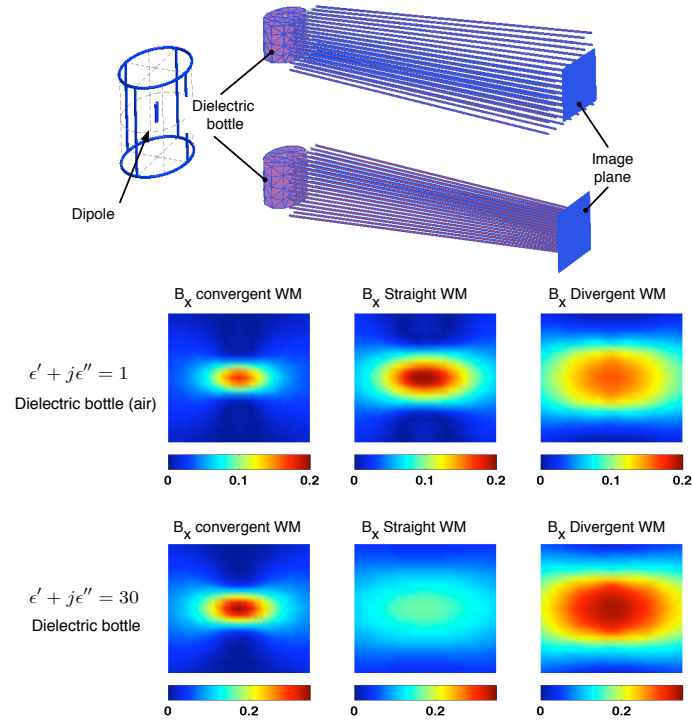


Figure 4.25 (Top) Setup employed to test the influence of a source embedded in a dielectric object. (Bottom) B_x on the image plane obtained with a source embedded in a dielectric with $\epsilon' = 1$ on the first line and $\epsilon' = 30$ on the second line

4.3.3 Antenna Coupling

We can perform the same exercise as in section 4.2.6 with the straight wire medium and evaluate the impact of the use of multiple antennas (or coils) on the signal reception. The structure employed for the analysis is a 10×10 convergent wire medium, with a reduction factor of 3. The wires are spaced by 1 cm at the front side and by 0.33 cm at the back side. Hence, the cross-section of the wire medium passes from 9 cm^2 to 3.3 cm^2 . The central wire has a length of 122 cm in order to operate around 123 MHz. The source employed is a dipole small compared to the lattice constant and placed at 2.5 cm from the front side of the wire medium. Three square receiving open loop (coils) are used whose dimensions are $3 \text{ cm} \times 3 \text{ cm}$. Given that the coils employed have the same height as the cross-section of the back side of the wire medium, only three antenna positions are considered, they are depicted in Fig. 4.26.a with the green lines.

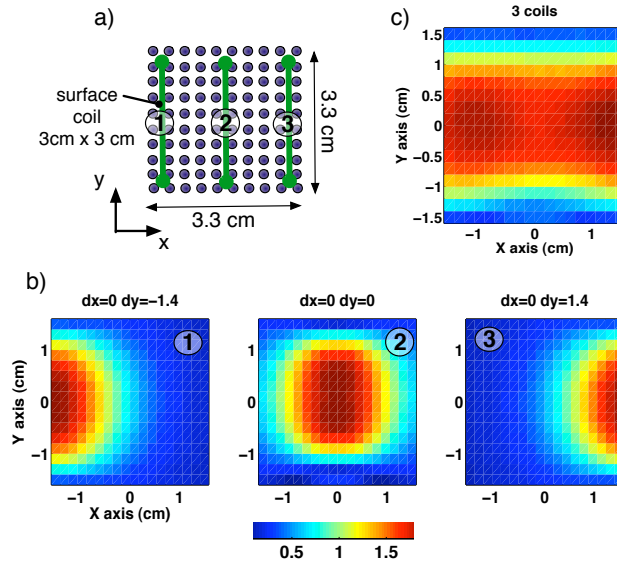


Figure 4.26 (a) View of the back side of the convergent wire medium (3.3 cm^2) and of the three different antenna positions considered, the antennas used are $3 \text{ cm} \times 3 \text{ cm}$ square loop whose axis is perpendicular to wires, (b) sensibility obtained for one antenna for the three considered positions, (c) global sensibility obtain through the linear combination of the different sensibility maps with the weighting coefficients in Tab. 4.2.

For each antenna position, the sensitivity for different source positions is computed and presented in Fig. 4.26.b. The sensitivity of each antenna is determined by computing the open-loop voltage at the reception for different positions of the dipole at the front side of the wire medium. The positions of the dipoles are comprised between -4.5cm and 4.5 cm along the x- and y- axis. Compared to the straight wire medium case (section 4.2.6), we can see that the sensitivity is higher for a given position of the coil given that the convergent geometry concentrates the magnetic field.

The three antenna signals can be combined in order to obtain a uniform global sensitivity. An example is given in Fig. 4.26.c where we have combined the three sensitivity maps with the weighting coefficients given in Tab. 4.2. We can clearly see that a very uniform sensitivity can be obtained by using only three coils.

Table 4.2 Weight coefficients employed to create the sensitivity maps in Fig. 4.26.c

N° coil	1	2	3
3 MRI coils	1	0.5	1

4.4 Analysis of Curved Wire Medium

4.4.1 Impact of Curvature on Transmission

We saw that a straight wire medium exhibits a wide bandwidth and that collimation still occurs for cross-sections relatively small compared to the wavelength. In other words, the collimation phenomenon is well posed since, to take place, it does not require very strict conditions. Consequently, one may expect that a smooth bending for the lens will not significantly hamper the collimation. In this section, we present the simulation results obtained with a wire medium having different curvatures, namely with circular arc angles of 0° , 60° and 90° . Our aim is to determine the influence of the curvature on the quality of the transmitted image and on the operational bandwidth. The simulated structure is made of an array of 11×11 perfect electric conducting strips. The length of the central wire is set to $\lambda/2$. The strips have a width of $\lambda/1200$ and are spaced by $\lambda/243$. In order to evaluate the bandwidth of

the different wire media, we have placed a small electric dipole with a $\lambda/243$ height close to the front side of the medium, and at the backside of the collimator, we have observed the component of the magnetic field perpendicular to both the wires and to the source dipole (this is the component collected by receiving loops of standard MRI equipment). The simulated configuration for a 90° wire medium is illustrated in Fig. 4.27.a. The different structures are tested for a frequency ratio $f/f_{\lambda/2}$ greater than one, for which collimation is expected for a straight wire medium.

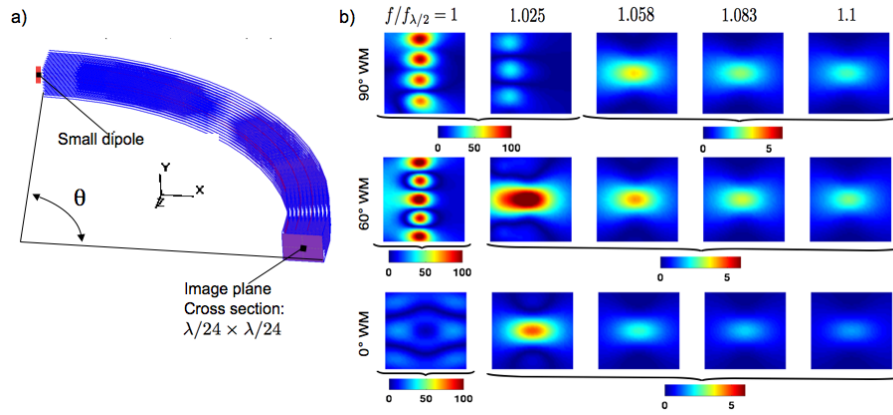


Figure 4.27 a) Simulation setup for a 90° wire-medium, b) component (along the x-axis) of the transferred magnetic field coming from the dipole.

The magnitude of the component along x of the magnetic fields (B_x) transmitted through the different curved media is shown in Fig. 4.27.b. These fields are observed thanks to testing functions placed against the back side of the medium and forming an image plane with exactly the same cross section as the medium. All images are given with the same color scale, except for those presenting an eigenmode. We can see that sub-wavelength imaging is realizable over a bandwidth of at least 7% for all the envisaged curvatures. The curvature tends to shift up the frequency band of operation. In all cases, the amplitude of the magnetic field decreases with frequency. For frequency ratios $f/f_{\lambda/2}$ very close to one, we can see that an eigenmode is excited, with propagation along the y axis, judging from the standing-wave pattern in that direction. Indeed, this direction is the only one for which the length and shape

of the wires is constant, and hence also the only one that permits propagation of eigenmodes similar to those obtained in the non-curved case.

4.4.2 Impact of Curvature on Matching

A question which arises is the influence of the curvature on the reflection at the entrance of the wire medium. In view of the amplitude of the transferred field obtained in the previous section, which is almost independent from curvature, the influence of the curvature on the input impedance is expected to be low. In order to see the impact of different geometries on the matching of the material and to see whether reflections occur, the difference between the absolute value of B_x on the source field, obtained in free space and in presence of wire media with different curvatures, is presented in Fig. 4.28. One can see that reflection increases with curvature; however it remains very low as compared to the incident fields.

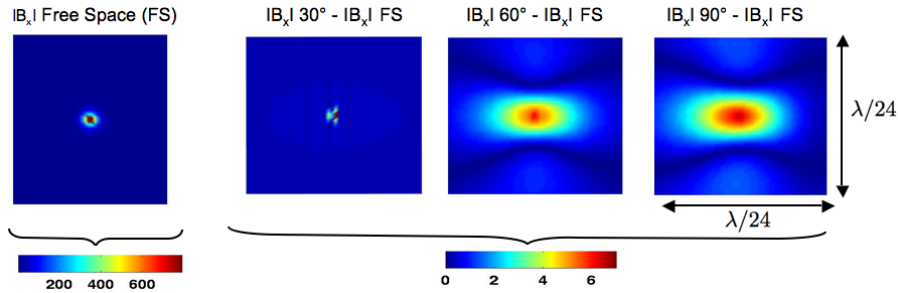


Figure 4.28 Left: free-space field near the source without wire medium. Right: difference field for different curvatures.

4.5 Toward MRI Endoscopy

In view of the somehow surprising image quality predicted (see next Section for experimental confirmation) for curved structures and for structures very narrow compared to the wavelength, an additional simulation has been carried out for the case of a medium with an extremely small cross-section, and a curvature of the order of 360° . The simulated example corresponds to a looping structure shown in Fig. 4.29. Only 5 wires are found across the width

of the structure, which has a cross-section of 2.4 cm, while its length is one wavelength at 123.5 MHz.

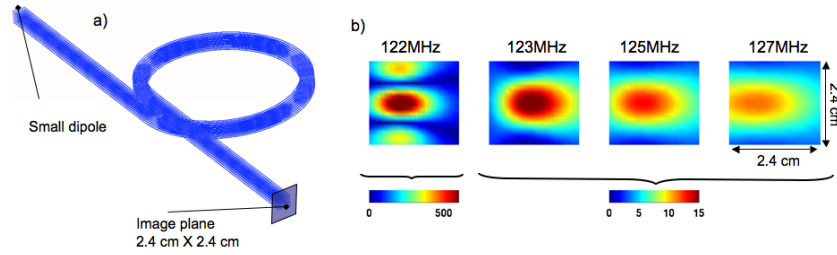


Figure 4.29 (a) 5×5 wire medium; the total length of the wire medium is equal to 244 cm, which is almost equal to one wavelength at 123 MHz; spacing between wires: 0.6 cm, (b) collimated magnetic field at different frequencies.

A chaotic behavior, similar to the eigenmode conditions referred to above, is found at 122 MHz, while a good collimation (with a spot size of the order of $\lambda/200$) is obtained from 123 MHz to 125 MHz. This surprisingly stable result opens the way to even more narrow structures that may be embedded in flexible materials and may be used as endoscopes, for instance by introducing them through the oesophagus. Curved wire media could offer an alternative solution to the coil used for internal imaging [QLZP⁺99]. Following discussions with medical doctors, it appears that traditional endoscopes are not sufficiently reliable. Their shape can sometimes change (typically changing ellipticity of the loop) when introduced in a body, which in turn shifts the resonance frequency. This would not be the case of the wire medium, in view of its robustness versus curvature.

Wire medium: experimental results

5

5.1 Introduction

We have numerically demonstrated in chapter 3 that a wire medium can be used as an energy waveguide in order to transmit or concentrate the magnetic field. In this chapter we confirm the predicted numerical results by presenting experiments carried out at the hospital of Liège with three different types of wire medium. The obtained results indicate that wire media can transmit MR signals onto significant distance and with low losses, moreover their performances remain very good even for important curvatures, consequently flexible endoscopic devices based on wire-medium metamaterials may be envisaged in near future.

In this chapter, we first start by presenting former experiments involving metamaterials in magnetic resonance imaging. Then, we present the medical images obtained in clinical conditions with the different wire media as well as images obtained with an array of spiral resonators.

5.2 Former Experiments Involving Metamaterials

In this Section, we briefly remind two important experimental results obtained in MRI and involving metamaterials, namely: [WPY⁺01] where so-called Swiss-Roll materials were used to guide fields toward a receiving loop and [FMJ08] where a metamaterial lens with a relative permittivity close to -1 has been used to improve the sensitivity of a surface coil.

5.2.1 Array of Swiss Rolls

The individual Swiss Rolls consist of a central cylindrical mandrel upon which is wound a spiral of conductor with an insulating backing, so that there is no electrical contact between the layers. The rolls are stacked into an array to provide the bulk material as illustrated in Fig. 5.1.a. When an alternating magnetic field, with a frequency in the vicinity of the resonance, is applied along the axis of the cylinders, the Swiss rolls start to behave as magnetic wires. The real part of their effective permeability start to increase as shown in Fig. 5.1.b. Consequently it means that in MRI, Swiss Rolls must be oriented perpendicularly to the MRI tube in order to be excited by the component M_{xy} generated by the protons.

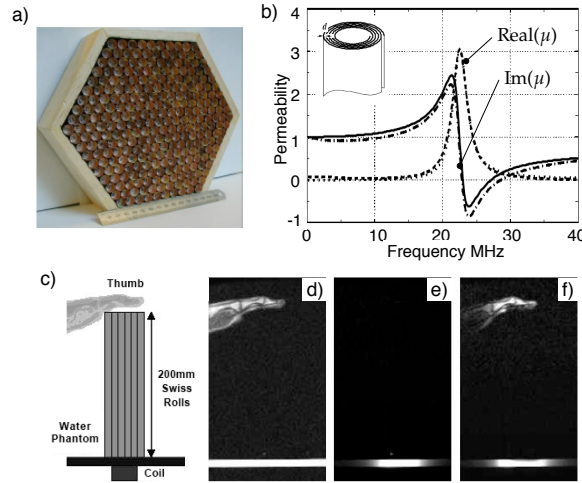


Figure 5.1 Figures reproduced from [WPY⁺01]: (a) Bulk material formed by an array of Swiss Rolls; (b) measured and calculated permeability the bulk material designed to give a peak in the $\text{Real}(\mu)$ at 21.3 MHz; (c) setup employed for the experiment; (d) reference image obtained with the body coils in the magnet; (e) MR image obtained with the small surface coil, when the thumb is supported on an inert plastic block, (d) The image from the same coil when 19 Swiss Rolls are inserted. Now the image of the thumb can be clearly seen.

In the experiment driven by M.C. Wiltshire et al., the Swiss rolls were tuned in order to have a resonant permeability slightly above the 21.3 MHz

operating frequency of a 0.5 Tesla magnetic resonance imaging (MRI). The maximum permeability realizable was $\mu = 2.23$ and occurs at 21.23 MHz. The Swiss Rolls were used as flux guides in an imaging experiment without disturbing the DC field or the field gradients. The object under test was a thumb held 20 cm above a water phantom which provides a reference plane (Fig. 5.1.c). The MRI receiver employed was a 3.7 cm diameter surface coil. The 20 cm space between the phantom and the thumb was filled either with an inert plastic block (not shown) or with Swiss Rolls. The image obtained with the plastic block and with the Swiss Rolls are respectively depicted in Figs. 5.1.e and f. In the latter image we can see that the Swiss Rolls enable to transfer the MR signal generated by the thumb to the surface coil without distortion.

5.2.2 Slab with a Permeability close to -1

In the MRI field, it is well known that surface coils provide a signal-to-noise (SNR) ratio much larger than that obtained with body coils. However, whereas the sensitivity of body coils is uniform, the sensitivity of surface coils, as well as the SNR, decreases rapidly with the distance from the coil. In this experiment driven by M.J. Freire et al. [FMJ08], the authors demonstrate that they manage to improve the sensitivity of surface coil by using a metamaterial with $\mu=-1$ at the MRI operating frequency. Indeed an ideal metamaterial lens has the ability of focusing the electromagnetic field, translating the field distribution from the source plane to the image plane, and vice versa. Therefore, the use of such a lens combine with a surface would increase the coil sensitivity. The sensitivity of surface coil is depicted in Fig. 5.2.a and b, with and without the presence of metamaterial slab.

To practically demonstrate this effect the authors have fabricated a slab consisting of a three-dimensional array of copper metallic rings loaded with nonmagnetic capacitors. The dimensions of the lens are 27 cm $\times$ 27 cm $\times$ 3 cm. The slab exhibits a $\mu=-1$ at 63.85 MHz, i.e. the frequency of operation of a 1.5 T MRI scanner. The setup used for the experiments is shown in Fig. 5.2.c and d, the objects analyzed are the knees of one of the authors. In Fig. 5.2.c the image obtained with the surface coil only. In Fig. 5.2.c the image obtained with the surface coil and the metamaterial slab between both knees. We can see that the sensitivity is higher. As a conclusion the authors feel that these results introduce a new concept in MRI, showing that $\mu = -1$ metamaterial lens can be

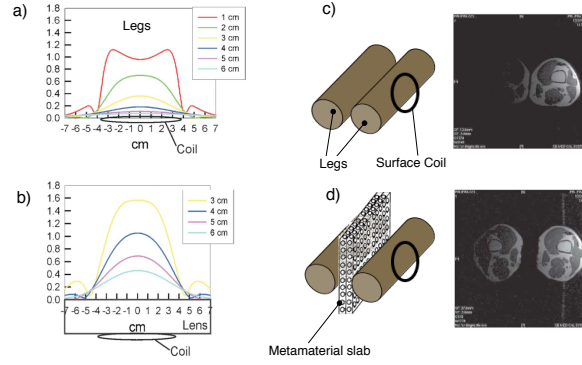


Figure 5.2 Figures reproduced from [FMJ08] : Calculation of the normalized magnetic field intensity for different distances in centimeter of (a) standard surface coil of 7.6 cm in diameter and (b) of the same coil placed on the lens. (a) sensitivity MR image of the knees of one of the authors (c) without the lens and (d) with the lens between the knees.

applied in the frame of conventional MRI technology in order to improve the sensitivity of surface coils.

5.3 Experiments Realized in MRI

In this section, experimental results obtained with different wire media are presented. All the experiments are realized with a 3 Tesla Siemens[®] MRI scanner which has a working frequency of 123.26 MHz and is installed at the University Hospital of Liège (CHU). They are all fabricated by SIRRIS Belgium and they are built with copper rods maintained together with slabs of Divinicell[®]. The Divinicell material foam provides a rigid structure while having low losses, it has a relative dielectric constant of $\epsilon_r = 1.07$ (see Appendix A for its characterization). We start this section with a brief presentation of the method used to normalize the different MR images and so enable their comparison in order to determine the quality of the transmission trough the different wire media; then we present the experimental results .

5.3.1 Equalization of the MR Images

Each time an MR image is acquired, window and level parameters must be adjusted to maximize the contrast and the organ visibility. In this section, we explain a simple method to normalize the images obtained during the different experiments, regarding their levels of contrast. This method called *histogram equalization* gives the possibility to compare them, on a specific basis, such as texture, in order to conclude on the level of transmission achieved by the different wire media.

Let us start with the definition of the histogram of an image. This is a graph showing the number of pixels of an image at each different intensity value, as shown in Fig. 5.3.a. For example for an 8-bit grayscale image, there are 256 different possible intensities, and consequently the histogram will graphically display 256 numbers showing the distribution of pixels amongst those grayscale values.

The concept of histogram equalization [Pra78] is to rescale the grey levels of the original image and to force the histogram of the modified image to follow a desired distribution (uniform distribution, Rayleigh distribution, the distribution of a given image,...). In this manner, the high intensities that lie close together will be stretched out so that hidden features of the image will become more visible. Usually histogram equalization produces unrealistic effects in images; nevertheless it is very useful for medical images: to reveal problems of inhomogeneity, to improve the performance of segmentation realized in a post-processing way and to provide the opportunity to compare the MR images obtained under different circumstances [YGvV00].

The equalization method [Pra78, Qur05] is briefly summarized hereafter. The gray levels of the input image f and of the output image g can be modeled as two random variables f_c and g_d , respectively. The method seeks to find a gray level transformation of the form $g_d = T(f_c)$ for which the input amplitude variable $f_c \in [f_1 \dots f_C]$ is mapped into an output variable $g_d \in [g_1 \dots g_C]$ so that the output probability of distribution p_g follows a uniform distribution (in our case), as illustrated in Fig. 5.3.

In order to find the transformation T , one imposes that the area underneath the two probability density functions p_f and p_g is the same. It gives the following relation:

$$\int_{g_{min}}^g p_g(g) dg = \int_{f_{min}}^f p_f(f) df \quad (5.1)$$

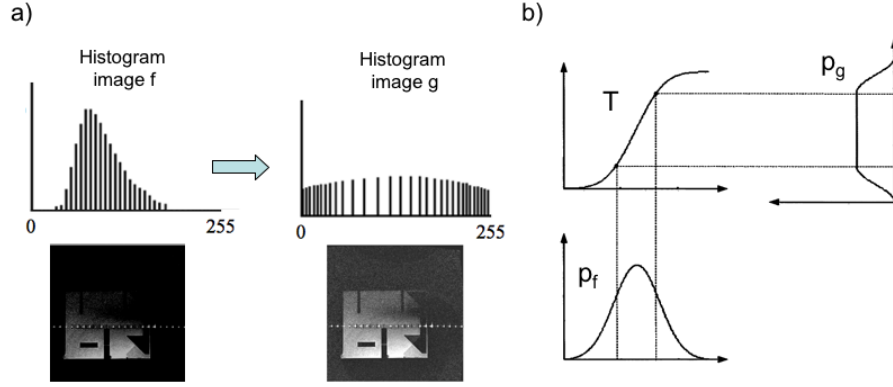


Figure 5.3 a) Equalization of the histogram of an image f in order to obtain an image g whose histogram is stretched out (details are revealed in the equalized MR image g), b) principle of the histogram equalization through the use of the transform function T , [Qur05].

The integral on the right is the cumulative distribution $P_f(f)$ of the input image f , i.e. the probability that the random variable f_c exhibits a value lower than or equal to f ; consequently:

$$\int_{g_{min}}^g p_g(g) dg = P_f(f) \quad (5.2)$$

For histogram equalization, we impose that the output density function be uniform $p_g(g) = \text{Constant}$, this means that

$$p_g(g) = \frac{1}{g_{max} - g_{min}} \quad (5.3)$$

with g_{min} and g_{max} the extreme gray levels of image g . The histogram equalization transfer function becomes, after integration of (5.2),

$$g = (g_{max} - g_{min})P_f(f) + g_{min} \quad (5.4)$$

Thus, an image which is transformed using its cumulative histogram yields an output histogram which is flat.

In this work, we have used a slightly more complicated method, called *contrast limiting adaptive histogram equalization* (CLAHE). This method was originally developed for medical imaging (X-ray) and has proven to be successful

for enhancement of low-contrast images [Pra78]. Indeed, ordinary histogram equalization simply uses a single histogram for an entire image, it enhances the details but can produce significant noise amplification especially for low contrast images. The CLAHE algorithm gives a solution to this problem, by partitioning the images into smaller regions and applying by the histogram equalization to each one of them. Hereafter, all the MR images obtained during the experiments are equalized using the CLAHE method, in order to obtain after equalization a uniform distribution for the gray levels on the background. The experimental results are presented by types of wire-medium.

5.3.2 Results Obtained with the Straight Wire Medium

This subsection presents the results obtained with the straight wire-medium. Two types of coils have been used to realize the field measurement; namely, the shoulder coil and the loop coil already presented in Section 3.5.1. The first coil was used because at the beginning we had to use the available MRI equipment available at CHU, i.e. a coil dedicated for the analysis of shoulder.

Receiving device : shoulder coil

The dimensions of the collimator are presented in Fig. 5.4.a. The straight wire medium is made of 15×15 rods with 1 mm radius and spaced by 1 cm. The collimator length is equal to 124.8 cm, i.e. 3.1 cm (2.7%) longer than $\lambda/2$. As seen in Section 4.1.2, this extra length should be of the order of the spacing between wires. In our case, the analytical difference between real and effective lengths of wires should be of the order of $2/\alpha = 0.96$ cm [DNC06]. Shorter length than 124.8 cm have been avoided because in case of malfunction lengthening the (expensive) structure was not possible.

For this first MRI tests, two types of bodies have been observed: a bottle with a diameter of 14 cm filled with phantom liquid, and a hand. The second source is essentially used to test the ability of the medium to transmit a weak signal. Indeed, a hand contains many different media and is thinner; consequently, its analysis requires the use of small voxels in order to capture the fine details. As explained in Section 3.2.4, the use of small voxels implies a weaker signal-to-noise ratio (SNR). The experimental process is presented in Fig. 5.4.b. The sources are successively placed at the center of the MRI tube, with the wire medium placed in front and against them (face marked with "WM" in the bot-

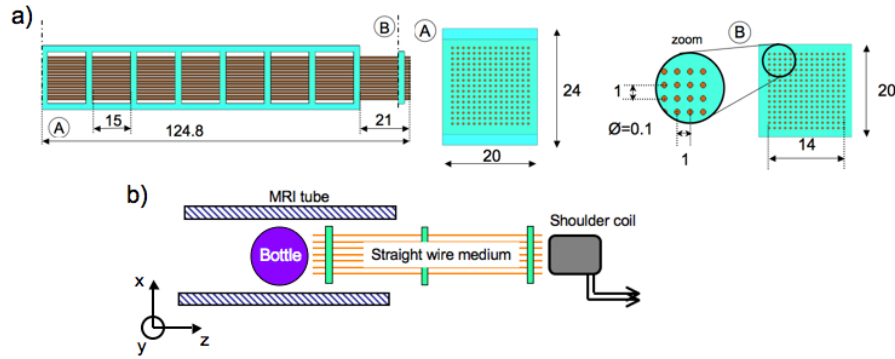


Figure 5.4 a) Dimensions (in cm) of the 15×15 -array straight wire medium, b) top view of the setup used in the MRI scanner to test the collimator, the bottle axis was along the y axis.

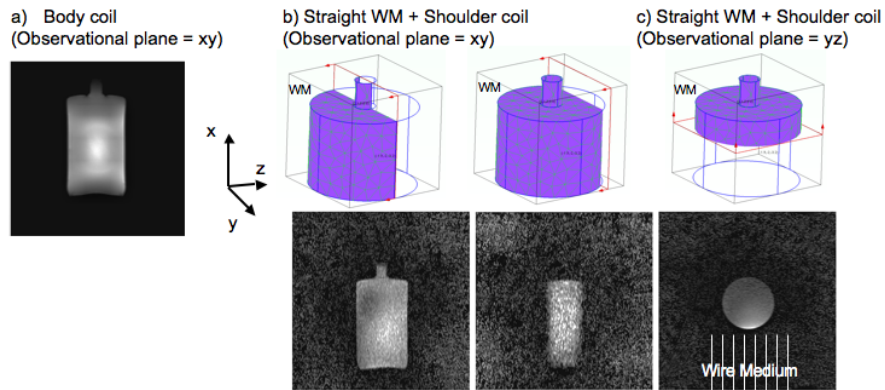


Figure 5.5 (a) MR image for a bottle obtained with the body coil and without the wire medium; MR images of a bottle transferred through the straight collimator, several observational planes are considered vertically (b) and horizontally (c). The wire medium is placed against the face marked with "WM".

the case). Several observational planes are considered horizontally (in the yz plane) and vertically (in the xy plane) in the bottle and in the hand, at several distances from the front side of the collimator. The observational planes are illustrated for the bottle case at the top of Fig. 5.5, and for the hand case at the top of Fig. 5.6. The signal acquisition is carried out with an MRI coil originally

devoted to shoulder analysis, presented in Fig. 3.9.a. Regarding the orientation of the coil, the medium was placed against the shoulder coil, since the cross section of the collimator was wider than that of the antenna. This kind of antenna was chosen because it contains three loops perpendicular to each other and can capture all polarizations. However, the possibilities in terms of placement of the antenna were very limited (Fig. 3.9.c). More specifically dedicated antennas, e.g. small resonant loops, have been realized in a later stage of the research program, the related results are presented in the next section. For comparison, a conventional MRI capture (without collimator) has been carried out with the help of the body coil for the bottle and for the hand, corresponding MRI images are shown, respectively, in Figs. 5.5.a and 5.6.a. All other images in Figs. 5.5.b and 5.6.b have been acquired through the wire medium for different observational planes with the shoulder coil.

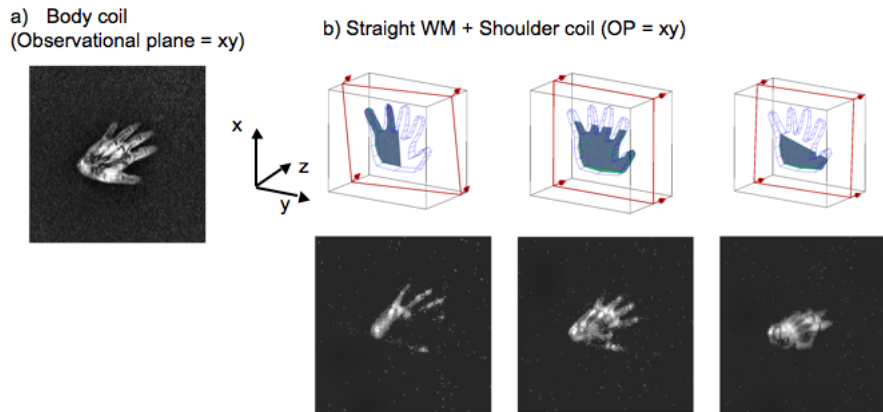


Figure 5.6 a) MR image for a hand obtained with the body coil and without the wire medium, b) transmission of MR images obtained with a straight collimator for a hand, for three observational planes oriented in the xy plane. The coordinates refer to Fig. 5.4.b

We can see that, due to the non-magnetic behavior of the collimating medium, and since the MR coils measured only the TM component of the signal (Section 3.2.4) -which is exactly the component transmitted by the wire medium- a MR image acquisition is possible through the wire medium. For

the first body, the bottle is visible, even for a section taken at $\lambda_{IRM}/24 = 10$ cm from the front side of the wire medium. It is in accordance with what Belov and al. [ZBH07] predicted for a source located at $\lambda/20$ from the front side of the collimator: although the SNR becomes weak in the latter case, the shape of the bottle is still visible. For the second body, the transmission of the MR image of a hand is successful, despite the small voxel size considered for the analysis of the hand. In the absence of the wire medium, and with everything else at the same position, nothing was visible.

Regarding the SNR, in comparison with a conventional MRI capture, the SNR was lower when using the wire medium. Two parameters can explain this difference: first, the length of the wire medium is not optimal for a maximum transmission at 123 MHz, second, the shoulder antenna used for measurements could not be placed directly on the back side of the collimator but at approximatively 20 cm from it (curved structure and thick plastic casing).

Receiving device : loop coil

In this section, we present the results obtained with the straight wire medium and measured with the loop coil, shown in Fig. 3.9.c. In comparison to the shoulder coil, the loop coil can be inserted between the wires, and as predicted by the numerical simulations, this insertion should improve the SNR. Regarding the source, we have used an ACR phantom (American College of Radiology) instead of a bottle. This phantom, presented in Fig. 5.7.b, has the advantage of containing geometrical structures, and allows us to detect whether the wire medium introduced an image deformation.

In a first instance, we have reproduced the previous experiment (Section 5.3.2), in order to observe the influence of the insertion of the antenna on the signal strength. Fig. 5.7.d presents the results acquired through the straight wire medium for two vertical (yz-plane) observational planes realized in the ACR phantom, which is positioned on its side (along the z axis). The straight wire medium used here has the same dimensions as those one given in Fig 5.4.a. Given that the wire medium shifts in frequency the resonance of the coil (loading effect), the latter is fine-tuned in order to resonate at the MRI frequency when inserted in the medium. The coil tuning was accomplished with the help of a SWR meter and initial devoted to radio field (see Section 3.5.3). Due to the very narrow-band of the coil and its high sensitivity, it was a real challenge to obtain the optimal parameters in terms of matching

and resonance frequency. The images transferred through the wire medium are presented in Fig. 5.4.d. For comparison, a conventional MRI capture of the ACR phantom (without collimator) has been carried out with the surface coil and is presented in Fig. 5.7.c. The setup employed to obtain the reference image is shown in Fig. 5.7.a, the ACR phantom axis was oriented along the y axis and the surface coil was placed above it.

We can see that in comparison to the image acquired with the shoulder coil, presented in Fig. 5.5.b, the signal is stronger and the image has a better quality. We have considered that the bottle and the ACR phantom are equivalent sources (amount of protons), given that they have approximatively the same volume and contain the same liquid. The background of the transferred image is homogeneous only in a small area, located close to the front side of the wire medium. This phenomenon comes from the small dimensions of the receiving surface coil, which limit the area with a homogeneous sensitivity.

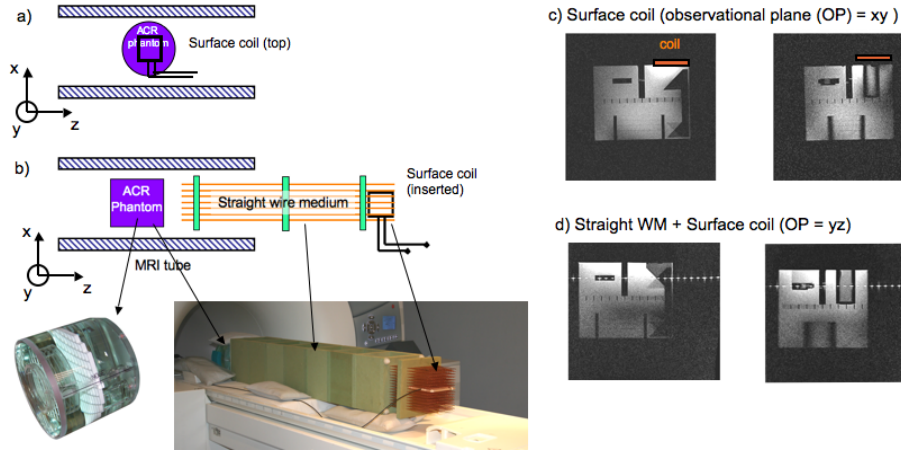


Figure 5.7 Top view of the setup employed in MRI : (a) to obtain the reference image, (b) to transfer MR images through the collimator. (c) MR images of the ACR phantom acquired with the surface coil with the setup in (a); (d) MR images transferred through the straight wire medium with the setup in (b).

A second experiment consists of studying the influence of the length of the wire medium on the coefficient of transmission. For this purpose, we have plugged a copper extension at the end of the straight wire medium. The exten-

sion connected to the previous studied wire medium is illustrated in Fig. 5.8.a, and is constituted of an array of small copper tubes, each tube containing in its turn a copper wire, which can be slid inside or outside its tube, in order to exhibit a variable length from 0 cm to 6 cm. With this setup, several lengths of straight wire medium are realized and tested, namely: 122 cm, 123 cm and 126 cm. Regarding the position of the surface coil, it is always inserted between the wires and located at the end of the medium. The images transferred through the medium for these different lengths are presented in Fig. 5.8.b.

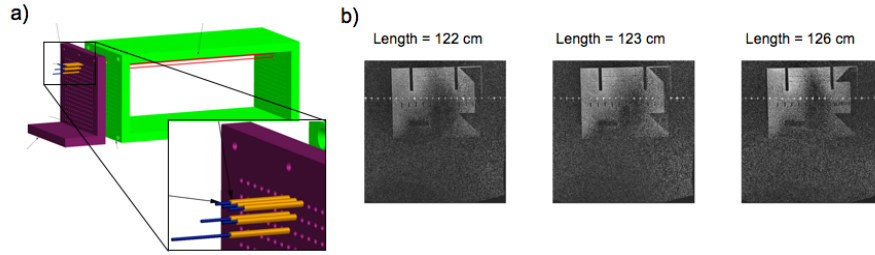


Figure 5.8 (a) Extension plugged to the wire medium in order to obtain different lengths, (b) MR images obtained through the wire medium for three different lengths.

As we can see, an image transmission through the different collimator is always possible. The different length does not lead to a significant difference in term of image quality. It is in accordance with the numerical simulations, shown in the chapter 4, where we have seen that when we use a wire medium at a frequency such that the ratio f/f_λ is larger than one, the magnetic field is well collimated. The ratios associated to the different lengths are presented in Tab. 5.3.2

Lenght	Resonant frequency	$f/f_{\lambda/2}$
122 cm	122.95 MHz	1.002
123 cm	121.95 MHz	1.01
126 cm	119.04 MHz	1.035

Since we always use these wire media at the MRI frequency, 123.23 MHz, i.e. higher than the resonant frequency of the different medium, they exhibit a stable coefficient of transmission (see Fig. 4.12.c in Section 4.2.2).

In comparison to the image transmission obtained in Fig 5.7.d, we can see that the image noise is more important when the extension is used. It may come from the losses from the electric connections of the connection tubes which were probably not good enough at RF frequencies.

5.3.3 Results Obtained with the Convergent Wire Medium

In this section, we present the results obtained with the convergent wire medium shown in Fig. 5.9.a. The wire medium is made of 15×15 rods with 1 mm radius. Depending on the side observed, the wires are spaced either by 1 cm or by 0.5 cm. The central wire has a length equal to 126 cm. The convergent wire medium is tested in two opposite directions as presented in Fig. 5.9.b and in Fig. 5.9.c, in order to test the effectiveness of the convergent or divergent geometry. For the measurement, a small MRI surface coil has been employed and is inserted between the wires, at the back side of the collimator. Given that numerical simulations show that a convergent wire medium has the ability to concentrate the electromagnetic field, here we would like to determine whether this phenomenon can be exploited to improve the homogeneity of the MR images by concentrating the fields on a small loop coil with high SNR.

For both geometries, MR images can be successfully transferred. The resulting images for the convergent and divergent geometries are shown in Fig. 5.10.c and in Fig. 5.10.d, respectively. For all cases the receiving surface coil is inserted between the wires, positioned at the end of the wires and is tuned in order to have a standing wave ratio equal to unity. Moreover, the ACR phantom axis is oriented along the z axis. Previous results obtained with a straight wire medium are reproduced for comparison purposes in Fig. 5.10.b. Conventional MRI captures obtained with the setup in Fig.5.7.a with a small surface coil are reproduced in Fig. 5.10.a. The position of the coil is represented by the red box; we can clearly see the stronger signal-to-noise ratio in the close vicinity of the loop coil.

Regarding the divergent geometry, the MR images transferred through the medium exhibit approximatively the same quality as those obtained with the straight wire medium in Fig. 5.10.b. The intensity level of the transferred im-

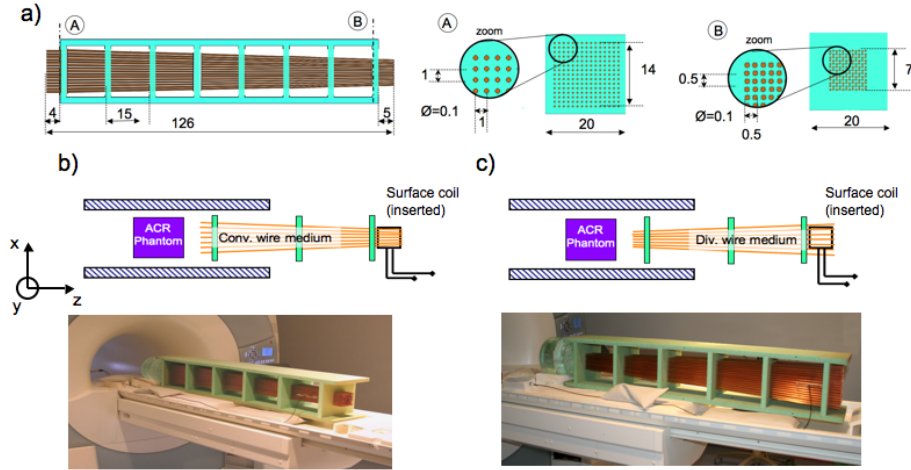


Figure 5.9 (a) Dimensions of the convergent wire medium (in cm); Top view of the setup used in MRI to test (b) the convergent wire medium, (c) the divergent wire medium.

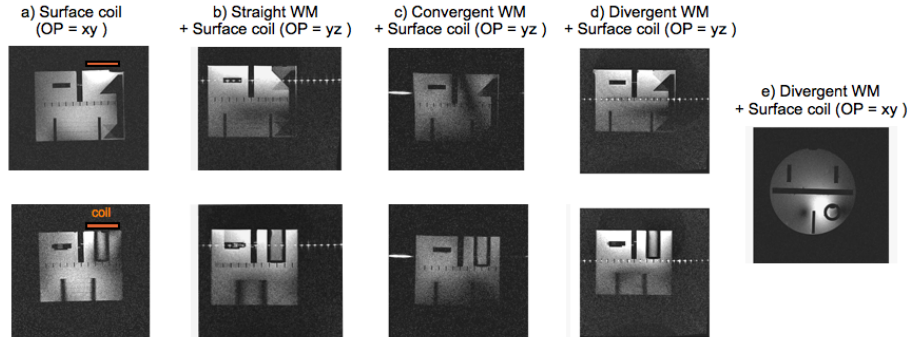


Figure 5.10 (a) MR images of the ACR phantom acquired with the surface coil in the configuration shown in Fig. 5.7.a. MR images acquired with the surface coil and transferred through different wire media: (b) the straight wire medium, (c) the convergent wire medium, (d) and (e) the divergent wire medium. Different observational planes are considered: vertically (in the yz plane) or horizontally (in the xy plane). The ACR phantom is oriented along the z axis when it is tested with WM. In (e) we can clearly see a bright spot which corresponds to the front side of the divergent WM.

ages, as for the straight wire medium case, is homogeneous only on a small area, close to the front side of the wire medium (bottom part of the image). That is caused by the small dimensions of the loop coil, which limits its sensitivity. Several receiving coils may need to be combined to obtain a better result (section 4.2.6). Regarding the MR images obtained with the convergent wire medium, the bright areas are more homogeneous, but compared to the divergent case, the signal strength is weaker. The losses may be caused by a modification of the input impedance of the wire medium. Indeed, when a wire medium is straight and has a length equal to an integer number of half-wavelength, its input impedance is equal to the impedance of its host medium (which the air in our case); this property disappears when the geometry of the wire medium is modified [RDC07]. Consequently, the mismatch can create reflections at the entrance of the wire medium and decrease the signal strength. The modification of the impedance appears for the convergent and divergent geometries, but maybe not in the same way, which may explain the difference of quality. The input impedance of the divergent wire medium may be closer to the impedance of the ACR phantom and lead to better transmission of MR images. Nevertheless, the impedance modification could be exploited, in medical scenarios, to observe without mismatch the organs which usually have a high dielectric constant, and, hence, an impedance higher than that of air. These hypotheses need to be assessed through further simulations and experiments.

5.3.4 Results Obtained with a 63° Curved Wire Medium

A 63° wire medium has been fabricated by SIRRIS Belgium and tested in MRI at CHU. The wire-medium, presented in Fig. 5.11.a, is made of 15×15 rods with 1 mm radius and spaced by 1 cm, maintained together with Divinicell[®] foam (relative dielectric constant of 1.07). Contrary to the other geometries where the wires are maintained together thanks to Divinicell slabs, here the wires are completely immersed in a continuous Divinicell in order to maintain the desired curvature. Consequently, the presence of the Divinicell reduces the length of the wire-medium by approximatively 3%. The end of the medium is cut in diagonal in order to obtain wires with almost the same length over the whole medium. The inner, central, and outer wires have radii equal to 108 cm, 115 cm and 122 cm, respectively. The choice of the curve is imposed by the diameter of the MRI tube, which limits the maximum realiz-

able curvature. The setup used at the hospital is illustrated in Fig. 5.11.b. In order to catch the maximum perpendicular component of the net-magnetization coming from the excited protons, the front side of the wire-medium is oriented perpendicular to the static field. The sources employed are the ACR phantom, and a hand. Both sources are placed against the front side of the wire-medium. Regarding the receiver, it is a small surface coil specifically designed for that measurement. This thin coil is placed and tuned inside the wire-medium, in order to exhibit the same resonance frequency as the MRI.

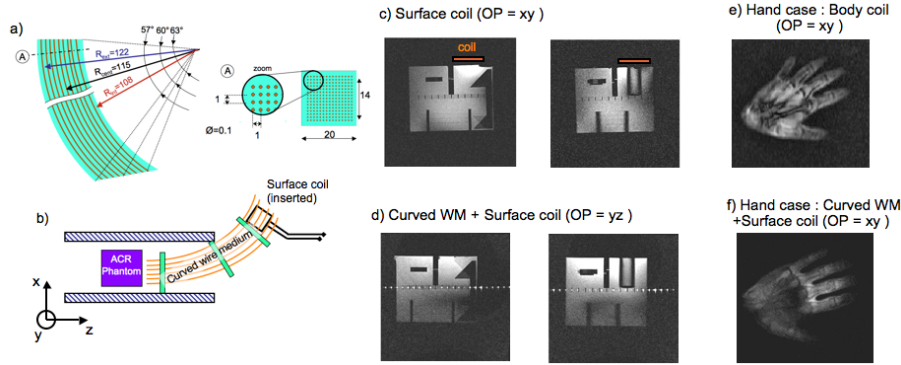


Figure 5.11 (a) Dimensions of the curved wire medium, (b) top view of the setup employed in MRI to transfer the MR image through the 63° curved WM, (c) conventional MRI of an ACR phantom (setup in Fig. 5.7.a), (d) MR images in the yz plane of the ACR phantom, transferred through the curved wire medium, (e) conventional MRI of a hand performed with the body coil and (f) MR image of a hand transferred through the curved WM, the details of the hand are still visible.

Several images of the phantom and the hand have been acquired through the 63° curved wire medium. They are presented in Figs. 5.11.d and 5.11.f. For comparison purpose, conventional MRI captures of the hand and of the ACR phantom are shown in Fig. 5.10.c and e. They were respectively acquired with the body coil and with the surface coil (setup in Fig. 5.7.a). We can see that the transmission of the images is successful through the curved wire-medium and that the use of a MRI coil inserted within the structure enables to obtain an image quality similar to the quality obtained with the surface coil directly

placed on the source. Even for a weak source like a hand (which contains few protons) the transfer is effective.

5.3.5 Performance of the Different Wire Media

The quality of the MR images is essential to provide reliable diagnosis and depends on several parameters dependent or not on the operator. Some factors such as the signal-to-noise ratio and the contrast-to-noise ratio enable to evaluate the quality. In the MRI field, these factors are defined as follows:

Signal-to-Noise Ratio: The signal to noise ratio is used in MRI to describe the relative contributions to a detected signal of the true signal and of the background noise. It can be evaluated from an MR image with the following formula:

$$SNR = S_f / \sigma \quad (5.5)$$

where S_f is the average signal in an homogeneous and bright area, and σ is the standard deviation of the gray levels of the background. One common method to increase the SNR is to average several measurements of the signal. The SNR can also be improved by increasing the field of view and slice thickness (voxels size) with a corresponding loss of spatial resolution.

Contrast-to-Noise Ratio: The contrast to noise ratio represents the variation of signal intensity between two adjacent structures. Improving CNR increases the perception of the distinct differences between two areas of interest. Assuming an MR image with two adjacent areas A and B, the contrast to noise ratio can be evaluated by subtracting the SNR related to the two areas as follows,

$$CNR = |SNR_A - SNR_B| \quad (5.6)$$

In order to evaluate the performance of the different wire media, we have computed in Tab. 5.1 the SNR (of the bright area) and the CNR of the different MR images presented in the previous sections. More precisely, we have used the equalized MR images obtained directly with the surface coil (Fig. 5.4.c) or obtained with the different geometries of wire media (straight Fig. 5.4.d, convergent Fig. 5.10.c, divergent Fig. 5.10.d and curved Fig. 5.11.d). These different SNR's have been computed with the help of masks which enable to isolate the bright homogeneous areas or the background of the different figures.

Table 5.1 SNR (of the bright area) and CNR of the MR images obtained with the surface coil or with the different geometries of wire medium.

	Surface Coil (Ref.)	Straight WM	Convergent WM	Divergent WM	Curved WM
SNR	7.53	7.14	6.16	6.90	7.59
CNR	4.42	4.28	1.64	4.23	4.71

From Tab. 5.1, we can see that the different geometries lead to very similar SNR's except for the convergent and divergent wire media. It is important to notice that the calculations have been performed on equalized MR images and only on the homogeneous bright areas. We have hidden the inhomogeneous area close to the surface coil, otherwise the RF-homogeneity would be interpreted according to the SNR definition as noise. Regarding the CNR, all wire media except the convergent wire medium lead to a CNR close to the reference CNR.

5.3.6 Results Obtained with a 3×3 Array of Spirals

In this section, we present the results obtained in MRI with a μ negative structure. The structure is composed of an array of 3×3 copper spirals, deposited over 250 μ -thickness Teflon board. The spirals have the same dimensions as employed in Section 2.5.3.2 for the numerical simulations. Therefore, they exhibit a negative μ at 3T-MRI operating frequency, i.e. 123.3 MHz. The aim of the experiment is to study the effect of the amplification of evanescent waves on the MR resolution and to evaluate the impact of the presence of a resonant structure inside the tube on the imaging process. Fig. 5.12.a presents the setup employed, the array of spirals is placed either against a bottle filled with phantom liquid or against chicken meat. All the structure is encapsulated in the head coil, for an optimal signal reception. Two conventional MR images of the reference objects have been acquired with the head coil and without the array (Figs. 5.12.b and.c). Images obtained in the presence of the metamaterial slab are presented for different observational planes in Figs. 5.12.d, .e, and .f. We can see that for voxels close to the metamaterial slab, the RF-magnetic field is perturbed and an artifact appears. Nevertheless, the stronger field generated by the array could be exploited to obtain a hyper signal, and hence,

locally a higher signal-to-noise ratio to improve the sensitivity of a surface coil. An hyper signal is visible on the right in Fig. 5.12.f. Finally, it is interesting to notice that although wire media are also resonating structures, they does not produce such kind of effects due to their weak resonance nature.

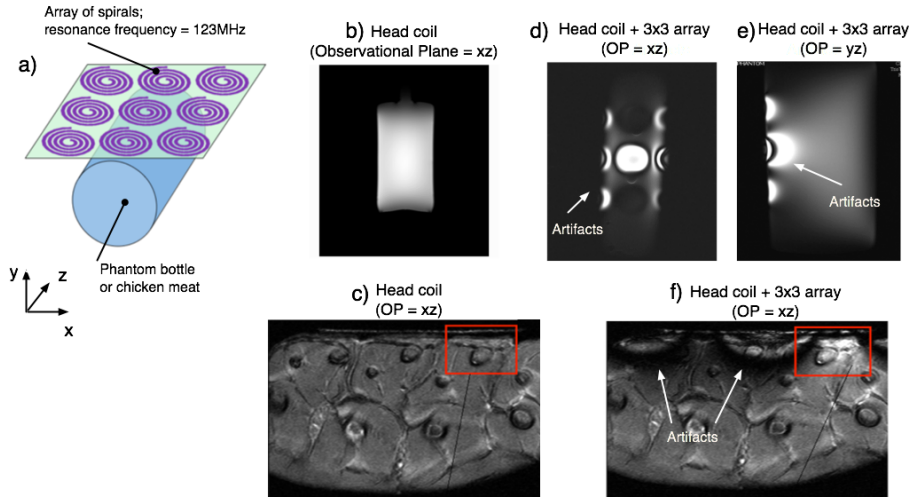


Figure 5.12 (a) Setup used in MRI to test the array of spirals; (b),(c) Reference image acquired without the array; (d),(e),(f) MR images obtained in the presence of the array of spirals, strong artifacts and hyper signal are present. The latter could be use to improve the SNR locally.

5.3.7 Conclusion

Due to its non-magnetic behavior and to its wide bandwidth, we showed that collimation can be achieved with strongly perturbed wire media, with convergent/divergent shapes, with strongly reduced cross-section and with very strong curvature. In view of the already good resolution of MRI imaging, the effect looked for here is the collimation toward a region in which the static magnetic field would be reduced, and where amplifiers containing magnetic material -with better noise factors- could be used. In this context, despite the excellent resolution of the wire-medium structure, the effect looked for so far is more the power transfer than the imaging capabilities. This supposes also that great care be brought to the recuperation of the collimated fields with

sufficient SNR through a standard MRI receiving chain. Hence, attention has also been given to the matching and the positioning of the receiving loop. In this respect, the use of a flat loop that could be inserted inside the material has proven very beneficial. While the convergent/divergent prototype seemed to exhibit some lower SNR, probably due to matching difficulties, excellent SNR has been obtained in clinical conditions with a curved prototype.

Compared to Swiss rolls, straight and curved wire media have the same objective: transfer the MR signal onto a certain distance. Nevertheless, wire media present several advantages: first, they are easier to fabricate and can be bent; second, their cross-section can be significantly reduced in order to create new devices for endoscopy. Finally, as wire media must be excited with TM waves, the array of a wires must be oriented along the axis of the MRI tube, which is very convenient if we want to create devices for endoscopy or reach a region where the static magnetic field is reduced. Given that Swiss rolls must be excited with a magnetic field along their axis, Swiss rolls must be oriented perpendicularly to the MRI tube axis which leads to limitation in terms of applications.

Compared to the slab with $\mu = -1$ which leads to a higher sensitivity, the convergent wire medium has the same goal. Indeed, the convergent wire medium has the ability to concentrate the magnetic field. This property could be used to concentrate the MR signal onto a high SNR surface coil and hence it would be possible to observe deeper tissues. The experiments previously presented did not manage to prove this phenomenon. Matching problem can be the origin of the lower SNR obtained. These hypotheses need to be assessed through further simulations and experiments.

Conclusion

6

We saw that metamaterials exhibit many new appealing properties not found in nature, the most spectacular property being their ability to manipulate for certain frequency range the near-field of an object, and to image it with sub-wavelength resolution. Besides this, among medical imaging techniques, Magnetic Resonance Imaging (MRI) appears to be a very interesting field of investigation for metamaterials due to its narrow bandwidth. This imaging technique requires constant improvements of the imaging quality in order to better understand the human body mechanisms and also provide better and faster diagnosis.

The main goal of this work was to demonstrate that the ability of a metamaterial to manipulate the near-field can be used to collimate or focus MR images. The use of a metamaterial in the field of MRI offers the possibility to improve the quality of the images and to realize new endoscopic sensors. Both the numerical simulation and the experimental aspects were approached.

6.1 Design Methods and Tools

New numerical techniques devoted to the study of metamaterials were presented and applied to the analysis and design of two types of metamaterials that can be used in 3-Tesla MRI, namely: a metamaterial based on loop-wires and spirals, and a metamaterial made from an array of wires.

In this work, we have employed the Method of Moments, which appears to be an efficient numerical method for the simulation in open-space of periodic structures like metamaterials. The Method of Moments is implemented in an in-house code which is capable of simulating metallic object (EFEI for-

mulation), homogeneous dielectric objects (PCHWT formulation) or mixed structures. In comparison with other methods like FEM, FDTD,... which require to mesh the entire volume comprising the structure, the MoM has the advantage to require only the discretization of surfaces (metallic surfaces or interfaces between homogeneous media); hence the MoM deals with less unknowns and is faster.

We have seen that metamaterials are often large periodic structures. Because of the large number of unknowns involved in these structures, brute-force simulation can be excessively time-consuming. Efficient numerical methods taking into the periodicity must be employed. A solution to overcome this problem is to use the infinite-array approximation, since it reduces the unknowns of the problem to a single unit cell and dramatically reduces the computation time. The periodicity is accounted for by using the periodic Green's function. To ensure the fastest possible convergence, three formulations for the computation of this function were recalled, depending on the position of the observation point. The infinite-array approximation also has the advantage to enable the identification of the eigenmodes of a structure. These eigenmodes corresponds to the phase shifts which cancel the determinant of the MoM impedance matrix.

Another method that can be combined with the Method of Moments to accelerate the analysis of periodic structures of finite extent is the Macro Basis Functions (MBF) approach. The MBF approach is based on the principle that fields on sub-domains of a large structure can be approximated by the superposition of solutions obtained when the sub-domains are isolated or embedded in small structures. A key point to obtain accurate results with the MBF approach lies in the choice of the size of the basis and the method used to construct it. In this work, a new method to create rapidly an MBF basis is presented and its accuracy is demonstrated thanks to the simulation of a metamaterial based on SRR's and the comparison of the obtained results with a full resolution (by Gaussian elimination) approach. This method gives more accurate results than traditional methods based on MBF and is particularly powerful for the simulation of structures involving connections between unit cells.

Additional methods for the design of metamaterials were presented. A first one is the array scanning method (ASM) which enables to determine the

fields radiated by an isolated source in an infinite periodic structure. In the field of metamaterials, this method is interesting for the computation of the spatial response of a structure. The spatial response can be used to compensate the image deformation introduced during the imaging process through a metamaterial. In its discretized form, the principle of the ASM consists in combining N^2 infinite-array simulations with every cell excited. As a consequence, the excitation period in both directions is multiplied by N . If N is large enough, the portions of the structure surrounding an excited cell behave effectively as if there were only one cell excited. An example of the computation of the spatial response of a wire medium is given to illustrate the capability of the method.

We have also recalled a method from the literature which is used to extract the constitutive parameters of a metamaterial slab from its S-parameters. The extraction method is validated by comparing the obtained simulation results with those from the literature. An example of application is provided with the design of a metamaterial made from loop-wires and spirals devoted to MRI imaging. The main challenge of this design was to obtain a compact unit cell. It is demonstrated that in this sense the miniaturization using a loop-wire medium combined with spirals is an interesting approach.

6.2 Design of Wire media Devoted to MRI

We have seen that metamaterials based on SRR's and rods are narrow-band and very sensitive to losses in the supporting dielectric material, an interesting alternative is to use wire media recently described in the literature. Such structures have the ability to canalize the evanescent waves on a significant distance by transforming them into propagating waves in the medium. They have interesting properties: practically no loss, a wide bandwidth and their resolution depends only on the spacing between the wires, so it is theoretically possible to reach any resolution, the only limitation being fabrication accuracy. Regarding the transmitted fields, wire media are capable of transmitting the transverse magnetic component of the source spectrum only; consequently, on the other side of the medium, only the normal components (along the wires) of the electric field produced by the source are completely restored.

In the numerical part, we have demonstrated with the help of the MoM the ability of wire media to collimate or focus magnetic fields. Since our final aim

is to use wire media in MRI, special attention is given to the observation of collimated or image fields in order to analyze matching of the structure with the host medium and determine the optimal position of the receiving loop. In this respect, we have demonstrated that the use of a flat loop that could be inserted inside the material is very beneficial in terms of signal strength. We have also shown that the combination of several loops inside a straight wire medium enables to obtain a uniform sensitivity with respect to the object at the opposite end of the medium.

We have presented simulation results obtained in infinite and in finite array analysis. Both methods show the ability of the wire medium to collimate the magnetic fields from a source in a wide frequency range. The comparison of the simulation results obtained with both methods allows us to see that, in the canalization regime, truncation effects are very limited. The coefficient of transmission of the straight-wire medium has been evaluated thanks to two small loops placed on both sides of the structures, feeding one loop and computing the open-loop voltage on the other one. The analysis of transmission curves shows that an extra-length is necessary to reach the optimal coefficient of transmission, and that an anomalous behavior is present in some frequency ranges. That chaotic behavior can be explained with the presence of eigenmodes propagating across the wires. This phenomenon, already announced in the literature, is demonstrated with the help of an infinite-array simulation and of the array-scanning method. The obtained results were validated by comparing the iso-frequency surfaces obtained numerically and analytically. Regarding the position of the receiving antenna, our numerical analysis showed that a shift towards inside the medium does not perturb the operation of the wire medium while providing a better transmission coefficient.

Besides this, the weakly resonant nature of such collimators does not make fabrication a critical issue and allows us to imagine variations of the structure such as convergent, divergent and curved geometries. Convergent and divergent geometries have been simulated. The results obtained showed that such types of geometry have the capability to concentrate or magnify the transferred magnetic field. However, such kind of geometry has an important impact on the input impedance. Hence, in free space, the convergent and divergent wire medium are not matched anymore and, compared to the straight-wire medium case, the amplitude of the transferred magnetic field is lower.

Nevertheless, this modification of impedance could be used in MRI to match wire media to the human body, which has a high dielectric constant ϵ . Therefore, additional simulations involving sources embedded in dielectric objects have been carried out; as expected the convergent and divergent geometries have an input impedance that is matched to high dielectric media. On the other hand, simulations have also been carried out with wire media having a very strong curvature. We have shown that the collimation of the magnetic fields is possible for curvatures up to 90° . Moreover, it has been demonstrated that the curvature does not significantly affect the input impedance of the wire medium.

Finally, simulations combining straight and curved wire media have been realized. These simulations predicted a good collimation quality with prototypes with a cross-section as small as $\lambda/100$, with a length of one wavelength and a shape involving a 360° loop. Assuming a further reduction of the cross-section by about a factor of two and the introduction of the wire medium in a flexible material, endoscopic applications seem to be in sight.

6.3 Experimental Testing of Wire media

In this last part, we have experimentally demonstrated that, due to the non-magnetic behavior of the collimating medium, and since the energy radiated by the protons is not polarized, MRI image acquisition is possible through a wire medium. In view of the already good resolution of MR imaging, our goal is to collimate the magnetic field toward a region in which the static magnetic field would be reduced, and where amplifiers containing magnetic materials -with better noise figures- could be used.

The experiments carried out in MRI conditions confirmed the numerical results, compared to a conventional imaging capture excellent signal to noise ratio has been obtained with straight and curved prototypes. While the convergent/divergent prototype seemed to exhibit some lower SNR, probably due to matching difficulties. Different types of objects have been observed namely, an ACR phantom and a hand in order to test the ability to transfer a weak signal. Indeed, the hand contains a lot of bones and hence behaves as a weak source.

6.4 Further projects

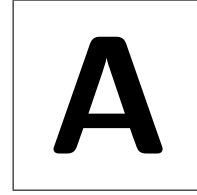
We demonstrated numerically and experimentally the good performance of wire media in term of collimation in clinical conditions. This opens new perspectives to realize innovative MRI devices. Structures based on wire media could be used either to pick up the energy at very local points with a high signal-to noise ratio, and inversely they could be used to excite precisely and with a certain excitation pattern small areas of the body.

The aspect which still requires more investigations is the optimal reception of the MR signal, in particular with the help of an array of coils. We have demonstrated that the use of several coils can produce an uniform sensitivity over a given plane. But the model used to combine the signals was very straightforward and does not take into account coils coupling and complex weighting coefficients. Therefore, complex fields and complex weighting coefficients may be used. Moreover, weights may be made depth dependent (distance between source and front side) and dependent on whether the source is inside a dielectric object or not.

Further analysis may be carried out regarding the capability of the convergent and divergent wire media to change their input impedance. Since MRI usually deals with body having a high dielectric constant, this property could be very interesting to further improve the SNR. It would be interesting to model the input impedance in terms of the rate of convergence. Moreover, more complex structures such as an array of helixes or even interconnected wires (e.g. with capacitors) may also be investigated to improve the matching.

From the experimental point of view, it would be very interesting to confirm the behavior of an endoscope made from wire medium, having strong curvatures and a very small cross-section and to compare the image quality with the image quality obtained with traditional endoscopes.

Divinicell characterization



This appendix presents the characterization results obtained for a slab of Divinicell foam. We have used a line-line dielectrometric method to characterize the Divinicell [HSD01].

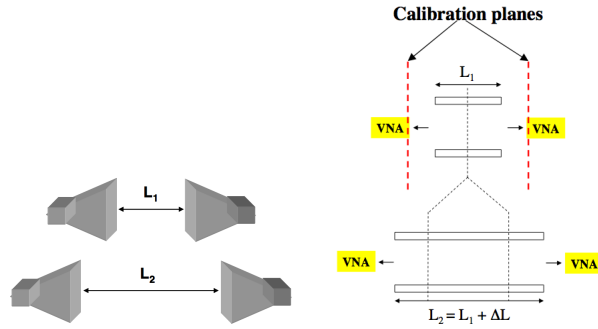


Figure A.1 Two samples with a different length are measured.

With the vector analyzer, we have measured the magnitude and phase of the scattering parameters, for two samples with a different length. Then the complex permittivity and loss tangent can be deduced as follows: [HSD01]:

$$S_{12}^{(L2)} = S_{12}^{(L1)} e^{\gamma \Delta L} \Rightarrow e^{\gamma \Delta L} \quad (\text{A.1})$$

$$\epsilon_r = \epsilon' - j\epsilon'' = \frac{-c^2 \gamma^2}{\omega^2} \quad (\text{A.2})$$

$$\text{tg} \delta = \frac{\epsilon''}{\epsilon'} \quad (\text{A.3})$$

The constitutive parameters obtained for the Divinicell foam are presented in Fig. A.2. We can see that the Divinicell has on average a real permittivity ϵ' around 1.07. Regarding the losses in the medium they are very low and the $\tan(\delta)$ is expected to have a value around 0.0015. Due to measurement noise and its low value the extracted loss tangent takes negative values and was very difficult to evaluate.

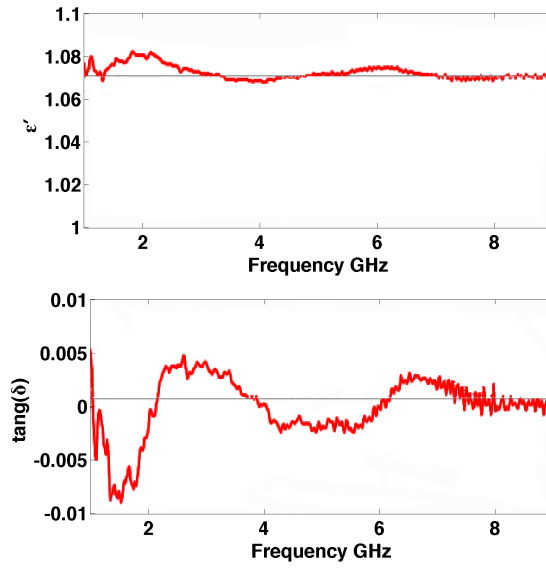


Figure A.2 Permittivity and loss tangent of Divinicell foam, the loss tangent takes negative value due to measurement noise.

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- X. Radu, D. Garraay and C. Craeye, "Toward a wire medium endoscope for MRI imaging," *Metamaterials*, August 2009.
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- X. Radu, A. Lapeyronnie and C. Craeye, "Numerical and Experimental Analysis of a Wire Medium Collimator for MRI," *Electromagnetics*, Vol. 28:7, pp. 431-543, May 2008.

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