PRODUCTION AND FINANCIAL POLICIES UNDER ASYMMETRIC INFORMATION

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Abstract. We propose an extension of the standard general equilibrium model with production and incomplete markets to situations in which (i) private investors have limited information on the returns of specific assets, (ii) managers of firms have limited information on the preferences of individual shareholders. The extension is obtained by the assumption that firms are not traded directly but grouped into 'sectorial' funds. In our model the financial policy of the firm is not irrelevant; we define a decision criterion for the firm that takes into account both its production and financial decisions. With this criterion, we prove the existence of equilibria. Then we discuss the nature of the inefficiencies introduced by the presence asymmetric information. In an appendix we illustrate the properties of the model in the CAPM framework.

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1. Introduction

We introduce information asymmetries in the standard general equilibrium model with production and incomplete markets (see e.g. Geanakoplos at al. (1990)). In that model, shares of stock of individual firms are traded on a competitive stock exchange; whereas the firms choose production plans that are Pareto-efficient from the viewpoint of owners-shareholders.

Two types of information failures naturally come to mind in this framework: (i) private investors may have limited information about the returns from specific assets; they cannot differentiate finely individual firms belonging to a broad class, like an industrial sector; (ii) managers of firms may have limited information about the identity and preferences of individual investors-shareholders; they do not know whether and how the preferences of anonymous shareholders may differ from those of identified shareholders.

Market responses to these information failures include financial arrangements whereby individual assets are pooled into "funds" consisting of several assets. In some cases, like "mortgage funds" or "venture capital funds", the underlying assets are not marketable. In other cases, like "index funds", the underlying assets are marketable, but uninformed investors treat them as substitutable.

In this paper, we introduce the simplest possible framework encompassing these features. Some assets (firms) are not traded, but are grouped into funds traded on a stock exchange. The allocation of individual assets to the funds is an a priori given partition. Elements of the partition are labeled "sectors". Each firm is initially owned by a set of individuals. The firm can raise outside funding by selling equity to the fund in which it belongs. In order to model limited information on the part of investors, we assume that the equity of all firms in a given sector is bought by the fund at a single price. That price emerges from the market value of the fund on the stock exchange. Given that price, the initial owners of a firm choose the share of their equity supplied to the fund. As for the funds, they act passively, applying to their equity purchases the price emerging from their market value, and refraining from interfering with the decisions of individual firms. Finally, the individuals trade in shares of the funds and a safe bond.

In interpreting this simple model, one may think about the funds as veils reflecting the inability of investors to differentiate firms belonging to the same sector on the one hand, the inability of firms to identify anonymous shareholders on the other hand. The resulting inefficiencies are then due to these information asymmetries. Alternatively, one may think about the funds as vehicles for financing non-marketable assets. The resulting inefficiencies are then due as well to the restrictive asset structure.

This simple structure is based on the work of Dubey et al. (1990), then Bisin and Gottardi (1999) and Bisin et al. (2001), on equilibrium in pure exchange economies with asymmetric information[1] . As will be seen, the extension to production economies enriches the analysis. Also, the financial decision (how much equity is supplied to the fund) is of genuine interest.

Building on the standard general equilibrium model with production and incomplete markets, we first adapt the model to the specification just outlined (section 1). An equilibrium still requires that asset markets clear, and firm decisions are optimal.

Next (section 2), we address the question of the decision criterion of the firms. Because firms are effectively run by initial owners, we adopt the criterion introduced by Grossman and Hart (1979), namely Pareto efficiency for initial owners.

[1]See also Citanna and Villanacci (2004) for a model of sectorial pools in which the price paid by the fund to each borrower depends on the individual's promise to repay.

According to that criterion, a firm maximizes profits at shadow prices obtained as weighted averages of the shadow prices of initial owners, with weights corresponding to ownership fractions. Initial owners of a given firm may hold shares of the fund to which the firm belongs, thereby changing their stakes in the firm's profits. We show that the indirect stakes are irrelevant, both when shareholders ignore the influence of firm-level decisions on the fund's value and returns, or when these shareholders assess that influence on the basis of their own shadow prices ("competitive price perceptions", in the Grossman-Hart terminology).

With well-defined decision criteria for the firms, we turn to existence and efficiency of equilibria (section 3). Under standard assumptions, equilibria exist. (The proof is given in Appendix A.) Our definition of "constrained Pareto efficiency" is standard. In order to bring out the specific inefficiencies resulting from information asymmetries, we turn to two special cases where equilibria of production and asset exchange are constrained-Pareto efficient, namely: (i) the model of multiplicative uncertainty (a special case of "partial spanning") in Diamond (1967); (ii) the model of mean - variance investor preferences and non-stochastic endowments known as CAPM. For both cases, we show by means of simple examples that the efficiency result does not carry over to our model. In case (i), this is due to inefficient investment levels; in case (ii), it is due to inefficient supply of equity.

We then return to the general model, and compare the first-order necessary conditions for constrained Pareto optimality with the corresponding conditions for equilibrium of firms and investors. Not surprisingly, they differ: the former take into account the implications of firm-level decisions for the anonymous shareholders holding fund shares, the latter do not. Taking into account the preferences of all final shareholders (direct or indirect) corresponds to the decision criterion for firms introduced by Drèze (1974). Pareto efficiency for all final shareholders, as postulated under that criterion, would require the funds to elicit the preferences (shadow prices) of fund holders, then to intervene as active shareholders of the firms on the basis of these preferences (duly aggregated). We establish that such intervention would eliminate the additional inefficiencies due to information asymmetries but not those already present in the standard model. This closes the comparison between efficiency under our specification and under the standard specification.

In section 4, we take up informally further implications and/or extensions of the preceding analysis. First, we discuss implications for the so-called Modigliani-Miller theorem. Second, we consider the possibility that a fund structure might emerge because no separating equilibria of the economy with one firm per fund satisfy incentive compatibility. Third, we outline an extension of our specification to the case where sectors are defined by specific requirements, subject to which individual firms choose the sector to which they belong. The informal discussion of the latter two topics is based on illustrations of our results in the CAPM framework, illustrations developed in Appendix B.

2. The economy and equilibrium

The economy has two time periods. Uncertainty is over S possible states of nature, $\mathcal{S} = \{1, .., s, .., S\}$, and is resolved in the second period. There are J firms $\mathcal{J} = \{1, .., j, .., J\}$, and H consumers, $\mathcal{H} = \{1, .., h, .., H\}$. There is one commodity per state.

A firm $j \in \mathcal{J}$ is characterized by a production set $Y^j \in \mathbb{R}^{S+1}$. A feasible production plan is $y = (-y_0, y_{\mathbf{1}})$, where $y_0 \in \mathbb{R}_+$ denotes input and $y_{\mathbf{1}} \in \mathbb{R}^S_+$ a state contingent output vector. We assume that production possibilities can equivalently be represented by a non decreasing, quasi-convex, differentiable transformation function, $F^j : \mathbb{R}^{S+1} \to \mathbb{R}$ with $F^j(0) = 0$. Each consumer $h \in \mathcal{H}$ is fully characterized by $\left(u^h, \omega^h, s^h\right)$. Where $u^h : \mathbb{R}_+^{S+1} \to \mathbb{R}$ is a monotonic, quasi-concave and differentiable utility function. Commodities endowment is $\omega^h \in \mathbb{R}_+^{S+1}$. Initial firm ownership share is $s^h = (.., s_j^h, ..) \in [0,1]^J$, and $\sum_h s_j^h = 1$, for all j.

There is asymmetric information between initial owners and potential investors. Initial owners have inside information on their firm production plans which they may not be able to credibly communicate to the market. We model this by assuming that firms are partitioned in M types, or sectors, $\mathcal{M} = \{1, .., m, .., M\}$. Because of asymmetric information, investors are only willing to pay a (sector) price q^m for a promise to deliver $y_{\mathbf{1}}^j$, for all $j \in m$. Thus, a firm in sector m can only obtain outside financing by issuing equities at the sector price q^m. In each sector there is an exogenous bound $K^m \in [0,1]$ on the share of equity which can be issued to obtain financing. This allows for a refinement of our notion of a sector in which firms are grouped depending on the maximum share of outside financing, which is reasonable if the amount of outside financing can be taken as a signal on the quality of production plans. Here we take K^m as an exogenous characteristic of a sector. One could instead follow Dubey,Geanakoplos and Shubik (1990) and, for any given technological sector, take as exogenous a list of contracts, each one indexed by a different bound on outside financing, among which firms in the sector would choose freely. We will illustrate this possibility in the context of a specific example in section 5. An economy is defined by

$$\mathcal{E} = \left\{\mathcal{H}, \mathcal{S}, \mathcal{M}, \mathcal{J}, \left(u^h, \omega^h, s^h\right), \left(Y^j\right), \left(K^m\right)\right\}$$

Markets. In the first period agents trade in assets. There are M security markets where financial resources are intermediated between consumers and firms. Each security is traded competitively at price q^m and yields a state dependent payoff $r^m \in \mathbb{R}^{S+1}$, which reflects the average return from investment in sector m. We shall later specify r^m as the outcome of a financial intermediation activity.

There is also a market for a riskless bond, $m = 0$, with returns $r_0^0 = 0$, $r_s^0 = 1$ for all $s \in \mathcal{S}$.

The asset structure is described by the $(S+1) \times (M+1)$ matrix of securities $R = [r^0, \ldots r^m \ldots]$. Markets are incomplete, $M + 1 < S$.

Firms. A firm j, given a technology Y^j, and an evaluation criterion π^j, optimally chooses production and financing. Investment costs, y_0^j, are financed by direct contributions of initial shareholders and by outside financing. The only form of outside financing for $j \in m$ is represented by the possibility of the firm to sell equities, claims on future return from production. A share ζ^j of equities issued today, at price q^m, entitles the firm to receive $\left(q^m + y_0^j\right)\zeta^j$. Investors receive a share of future returns $\zeta^j y_{\mathbf{1}}^j$.[2] We assume that control rights fully remain in the hands of initial shareholders.

[2] An alternative formulation is to let $\widehat{q}^m$ to represent the price of a unit of capital invested in sector m: $\widehat{q}^m$ is the price that a firm j in sector m is entitled to receive to let an outside investor participate with a unit of capital in a firm's investment that promises a future stream of returns $(1/y_0^j)y_{\mathbf{1}}^j$. In our formulation, we assume that outside investors regard alternative investment projects y (and equities) in each sector as ex-ante identical. For example we could think that sectors are defined by taking into account also the dimensional class of the firms.

For given present value vector criterion[3], $\pi^j = (1, \pi^j_{\mathbf{1}}) \in \mathbb{R}^{S+1}$ and asset prices q, $j \in m$ solves,

$$\begin{array}{l} \max_{y^j, \zeta^j} - \left(1 - \zeta^j\right) y^j_0 + q^m \zeta^j + (1 - \zeta^j)\pi^j_{\mathbf{1}} y^j_{\mathbf{1}} \text{ s.t.} \\ y^j \in Y^j,\ \zeta^j \in [0, K^m] \end{array} \tag{1}$$

Financial intermediaries. Intermediaries are passive funds. An intermediary m buys claims on future returns from production activities in sector m, at an initial outlay $\sum_{j \in m} \zeta^j (q^m + y^j_0)$. The intermediary raises funds through the asset market. In a notation consistent with the one used for firms, we model the intermediary as issuing $\overline{\zeta}^m \equiv \sum_{j \in m} \zeta^j$ "shares" at a unit price $\tilde{q}^m$, with unitary returns[4]:

$$r^m = \sum_{j \in m} \frac{\zeta^j}{\overline{\zeta}^m} y^j.$$

If consumer h buys θ^h_m shares, market clearing requires $\overline{\zeta}^m = \overline{\theta}^m \equiv \sum_h \theta^h_m$. The price $\tilde{q}^m$ should see to that.

Note that market clearing for shares implies zero profit (i.e. no dividends) in the second period:

$$\overline{\theta}^m r^m_1 = \sum_{j \in m} \zeta^j y^j. \tag{2}$$

In the first period, the accounting profit of fund m is:

$$\tilde{q}^m \overline{\theta}^m - r^m_0 \overline{\theta}^m - q^m \overline{\zeta}^m - \sum_{j \in m} \zeta^j y^j_0 = \Pi^m_0.$$

Here market clearing implies $\tilde{q}^m - \frac{\Pi^m_0}{\overline{\theta}^m} = q^m$, that is the market price of a share of the fund net of dividend is always equal to the buying price of firms' shares in the sector. It simplify notation to use throughout this price net of dividends, q^m, and omit any reference to the first period profits of the fund.

Consumers. A consumer h solves the following problem

$$\begin{array}{ll} & \max_{x^h, \theta^h} u^h(x^h) \text{ s.t.} \\ (\lambda^h_0) & x^h_0 - \omega^h_0 - \sum_{m \in \mathcal{M}} \sum_{j \in m} s^h_j (- \left(1 - \zeta^j\right) y^j_0 + \\ & +q^m \zeta^j) + \sum_{m \in \mathcal{M}} \left(q^m - r^m_0\right) \theta^h_m + q_0 \theta^h_0 = 0 \\ (\lambda^h_s) & x^h_s - \omega^h_s - \sum_j s^h_j (1 - \zeta^j) y^j_s - \sum_{m \in \mathcal{M}} r^m_s \theta^h_m - \theta^h_0 = 0,\ s \in \mathcal{S} \\ & x^h \geq 0,\ \theta^h_m \geq 0\ m \in \mathcal{M}. \end{array}$$

In our model shareholders are assumed to have perfect knowledge of the firm technology and choices. However, because shareholders' information cannot be credibly conveyed to other potential investors, initial ownership is not directly tradable: as investor in the asset market, every agent faces the same set of sectorial pooled assets. Notice that, although firm shares are not traded directly, firm ownership provides an extra degree of risk sharing with respect to the one attainable on financial markets. This is limited by our assumption of no short sales of assets: trading in asset m, an initial owner of firm j in sector m can only increase his final stake in firm j profits.

[3]See section 3

[4]Remember that $y^j = (-y^j_0, y^j_{\mathbf{1}})$, $y^j_0 \in \mathbb{R}_+$, so that $r^m_0 \leq 0$.

Equilibrium. We now give a definition of competitive equilibrium for our economy.

Definition 1. *A competitive equilibrium for an economy $\mathcal{E}$, and for given firm evaluation criteria $\{\pi^j : j \in \mathcal{J}\}$, is a profile of allocations, security prices, and returns $((x, \theta, y, \zeta), q, R)$ such that*
1) (x^h, θ^h) solves the consumer problem at (q, R), $h \in \mathcal{H}$,
2) (y^j, ζ^j) solves the firm problem at (π^j, q), $j \in \mathcal{J}$,
3) funds' returns are $r^m = \sum_{j \in m} \frac{\zeta^j}{\overline{\zeta}^m} y^j$ if $\overline{\zeta}^m > 0$, $m \in \mathcal{M}$,
4) asset markets clear, $\overline{\zeta}^m = \overline{\theta}^m$, $m \in \mathcal{M}$.

It is easy to verify that individual budget balance and asset market clearing imply commodity market clearing in every state. We postpone the proof of existence of an equilibrium until section 4, after a decision criterion of the firm, π^j, will be specified.

3. The decision criterion of the firm

When firms are held by a single individual (single ownership), the decision problem of a firm is embedded in the consumer problem. The owner of firm j simultaneously chooses his level of consumption, production, and financing. When there are multiple initial owners, there is a potential problem of how to specify a decision criterion of the firm. In our economy there are three sources of conflict of interests that complicate this task. The first is common to any model with an incomplete market structure, and is related to the fact that, even at equilibrium, individuals do not typically agree on the ex ante evaluation of production plans and investments. The second potential conflict of interest arises when some of the initial owners are also investors of the sectorial fund. Should their additional holdings (as investors in the fund) affect their weight in the aggregation of shareholders preferences ? The third potential conflict may emerge between initial owners and owners of the sectorial fund who are not initial shareholders of the firm. In facing these potential conflicts, the firm may, or again may not, take into account the implications of its decisions for the return and market price of its sectorial fund.

In dealing with the problem of defining a decision criterion of the firm, we follow a common approach in the literature and assume that, for a firm j, π^j, is "efficient" in the perspective of shareholders, along the lines of Drèze (1974) and Grossman and Hart (1979). Because control rights remain fully in the hands of the initial owners, the Grossman - Hart criterion is the relevant one. We denote by $\mathcal{H}^j = \{h \in \mathcal{H} : s_j^h > 0\}$ the set of consumers who are initial owners of firm j.

Definition 2. *At a given equilibrium $((x, \theta, y, \zeta), q, R)$, the decision of a firm j, (y^j, ζ^j), is initial owners constrained efficient if there does not exist a feasible marginal change $(dy^j, d\zeta^j)$ and transfers $(\tau^h)_{h \in \mathcal{H}^j}$ such that*

$$\begin{array}{l} \sum_{h \in \mathcal{H}^j} \tau^h = 0 \\ \text{and, for all } h \in \mathcal{H}^j \\ dx_0^h = s_j^h(-(1-\zeta^j)dy_0^j + (q^m + y_0^j)d\zeta^j) + \tau^h \\ dx_s^h = s_j^h(1-\zeta^j)dy_s^j - y_s^j d\zeta^j,\ s \in \mathcal{S} \\ du^h > 0. \end{array}$$

Letting λ^h be a vector of multipliers at the solution of consumer h's problem we have

$$\begin{array}{rcl} du^h & = & \lambda_0^h dx_0^h + \sum_s \lambda_s^h dx_s^h \\ & = & \lambda_0^h(s_j^h(-(1-\zeta^j)dy_0^j + (q^m + y_0^j)d\zeta^j)) + \lambda_0^h \tau^h + \\ & & + \sum_s \lambda_s^h (s_j^h(1-\zeta^j)dy_s^j - y_s^j d\zeta^j). \end{array}$$

Thus, at the given equilibrium, the decision of firm j is initial owners constrained efficient if and only if the following inequality holds for all feasible $(dy^j, d\zeta^j)$

$$0 \geq q^m d\zeta^j + (1-\zeta^j)\sum_h s_j^h \xi^h \cdot dy^j - \sum_h s_j^h \xi^h y^j d\zeta^j \tag{3}$$

where summations are across h in $\mathcal{H}^j$, and $\xi^h = \left(1, .., \lambda_s^h/\lambda_0^h, ..\right) \in \mathbb{R}^{S+1}$.

For given present value vector criterion, $\pi^j \in \mathbb{R}^{S+1}$, at an optimum for firm j, first order conditions imply,

$$0 \geq q^m d\zeta^j + \left(1-\zeta^j\right)\pi^j \cdot dy^j - \pi^j y^j d\zeta^j \tag{4}$$

where dy^j satisfies $\nabla F^j \cdot dy^j = 0$.

Comparing equations (3) and (4), it is immediate to see the following.

Fact 3. *At any equilibrium allocation, the decision of firm j is initial owners constrained efficient if*

$$\pi^j = \sum_h s_j^h \xi^h \tag{5}$$

The firm criterion in Fact 3 coincides with the Grossman - Hart criterion: the firm maximizes the expected value of initial owners' cash flow rights, by using a weighted average of its initial owners' state prices.

Price perceptions. Up until now we have assumed that the initial owners, when evaluating a marginal change in the firm's policy, take the payoff of the sectorial fund as given. Formally, this could only be justified if the share of each firm in the sectorial mutual fund is negligible. Even if we assume that there are many firms in each sector, there might exist equilibria in which few of them participate to the mutual fund. If we let the owners take into account the effect of their decisions on the payoff of the sectorial mutual fund, is natural to assume that they also consider the effect on the fund price. The definition of initial owners constrained efficiency must then be modified to take into account the impact of the firm's decisions on r^m and q^m:

$$dx_0^h = s_j^h(-(1-\zeta^j)dy_0^j + (q^m + y_0^j)d\zeta^j) + \zeta^j dq^m - (dq^m - dr_0^m)\theta_m^h + \tau^h$$
$$dx_s^h = s_j^h(1-\zeta^j)dy_s^j - y_s^j d\zeta^j + \theta_m^h dr_s^m,\ s \in \mathcal{S}.$$

Equation (3) then becomes,

$$\begin{aligned} 0 \geq\ & q^m d\zeta^j + \sum_h (\zeta^j s_j^h - \theta_m^h) dq^{m,h} + \sum_h (1-\zeta^j) s_j^h \xi^h \cdot dy^j + \\ & + \sum_h \left(\xi^h \cdot dr^m\right)\theta_m^h - \sum_h s_j^h \xi^h y^j d\zeta^j. \end{aligned} \tag{6}$$

Following Grossman - Hart, we assume that in evaluating this effect the owners have *competitive price perceptions* : given his equilibrium state prices ξ^h, owner h expects the price change to match the discounted change in the fund payoff, $dq^{m,h} = \xi^h \cdot dr^m$, where dr^m is evaluated using the expression characterizing the intermediary portfolio:

$$dr^m = \frac{\zeta^j}{\bar{\zeta}^m} dy^j + \frac{1}{\bar{\zeta}^m}\left(y^j - r^m\right) d\zeta^j$$

Equation (6) then becomes,

$$\begin{aligned} 0 \geq\ & q^m d\zeta^j + \zeta^j \sum_h s_j^h \xi^h \cdot dr^m + (1-\zeta^j)\sum_h s_j^h \xi^h \cdot dy^j - \\ & - \sum_h s_j^h \xi^h y^j d\zeta^j. \end{aligned} \tag{7}$$

We should compare this equation with the firm's first order conditions. But we face a difficulty. Contrary to the standard model, here the financial policy of the firm is relevant and q^m enters the firms decision problems. If we allow the owners to contemplate the effects of a marginal change of policy on the price q^m, this should also be reflected in the definition of the firm's problem. Assuming competitive price perceptions for the firm would mean that, at a given present value vector criterion, $\pi^j \in \mathbb{R}^{S+1}$, the firm calculates $dq^m = \pi^j dr^m$. Its first order conditions would then imply,

$$0 \geq q^m d\zeta^j + \zeta^j \pi^j \cdot dr^m + \left(1 - \zeta^j\right) \sum_s \pi^j \cdot dy^j - \pi^j y^j d\zeta^j \tag{8}$$

where dy^j satisfies $\nabla F^j \cdot dy^j = 0$.

Comparing equation (8) with (7) we see that Fact 3 continues to hold.

This may sound surprising in an economy with asymmetric information. Since initial owners have private information on the firm production plans, potential conflicts of interest arise if some of them are also investors in the sectorial mutual fund. Thus, one may think that an evaluation criterion that only takes initial holdings into account leads to adverse selection, and thus to inefficient decisions at the level of the firm (Myers and Majluf (1984)). This does not happen if agents have competitive price perceptions, because individuals think that any change in the return from holding the mutual fund will be reflected in a corresponding price variation. To see this, observe that in going from equation (6) to equation (7), the terms containing θ_m^h vanish, while if $dq^{m,h} \neq \xi^h \cdot dr^m$ this is not true any more. For example, if production choices are made "after" the security market meets, there will not be any effect on the fund's price but the impact on its returns might still be relevant for initial owners who also invested in the fund. From equation (6) one sees that in this case the production decision of the firm is initial owners constrained efficient if taken according to the criterion

$$\pi_s^j = \sum_h [(1 - \zeta^j) s_j^h + \frac{\zeta^j}{\overline{\zeta}^m} \theta_m^h] \xi_s^h$$

The weight of initial owner h corresponds to his direct and indirect participation (as investor in the fund) in the firm. In this sense, the latter criterion would be in the spirit of Drèze's (1974).

4. Equilibrium existence and efficiency

Existence. We are now going to address the issue of existence of equilibria when firms' decision criterion is the Grossman-Hart criterion, (GH criterion).

Assumption 4.
1) $\forall h, u^h : \mathbb{R}_+^{S+1} \to \mathbb{R}$ *is continuous, quasi-concave, strictly monotone;*
2) $\forall h, \omega^h \gg 0$*;*
3) $\forall j, Y^j$ *is a closed, strictly convex subset of* $\mathbb{R}^{S+1}$ *and* $0 \in Y^j$*;*
4) $\left(\sum_h \omega^h + \sum_j Y^j\right) \cap \mathbb{R}_+^{S+1}$ *is compact.*

Theorem 5. *Under assumption* 4*, for every economy in* $\mathcal{E}$ *a competitive Grossman-Hart equilibrium exists.*

The proof follows the proof of Theorem 1 in Dubey et al. (1990). As in that paper, an equilibrium without fund asset trading can always be sustained by setting $q^m = 0$, $r^m = 0$ for all $m \in \mathcal{M}$, $\zeta^j = 0$ for all $j \in \mathcal{J}$. To prove existence of non-trivial equilibria, one has to deal with the discontinuity of the asset payoff when the denominator is zero. Two additional complications arise here because the firm's objective function needs not be jointly concave in its arguments (production and financial policy) and from the fact that the funds' payoff enters the first period budget constraint, which complicates slightly the argument for market clearing. Details are provided in Appendix A.

Efficiency. The following definition of constrained feasibility extends that of Diamond (1967) to our model.

Definition 6. *(Constrained feasibility) An allocation* (x, y, θ, ζ) *is constrained feasible if it satisfies the following constraints:*

$$\begin{array}{ll}
(\lambda_0) & \sum_h (x_0^h - \omega_0^h - r_0\theta^h) + \sum_j (1-\zeta^j) y_0^j = 0 \\
(q^m) & \sum_h \theta_m^h - \sum_{j\in m} \zeta^j = 0,\ m \in \mathcal{M} \\
(q^0) & \sum_h \theta_0^h = 0 \\
(\lambda_s^h) & x_s^h - \omega_s^h - r_s\theta^h - \theta_0^h - \sum_j s_j^h \left(1-\zeta^j\right) y_s^j = 0,\ s\in\mathcal{S}, h\in\mathcal{H} \\
(v^j) & F^j\left(y^j\right) \le 0,\ j \in \mathcal{J} \\
 & r^m = (1/\sum_{j\in m}\zeta^j)\sum_{j\in m}\zeta^j y^j,\ y^j \in Y^j,\ m\in\mathcal{M}.
\end{array}$$

Definition 7. *(Constrained Pareto Optimality) A constrained feasible allocation* (x, y, θ, ζ) *is constrained Pareto optimal (CPO) if there does not exist a constrained feasible allocation* $(x', y', \theta', \zeta')$ *that Pareto dominates it.*

We will not pursue a full analysis of constrained suboptimality. By comparing the first order conditions at a constrained optimal solution and at an equilibrium of our model we identify a new potential source of suboptimality, specific to the asymmetric information setting. To prove that suboptimality of equilibria indeed is a generic property would require a more detailed analysis[5]. Given our limited objective, we will for simplicity consider only interior solutions. A *CPO* allocation satisfies the first order conditions of the problem of maximizing $\sum \alpha^j u^j$ over the set of constrained feasible allocations, for some vector α of welfare weights. At an interior solution of this problem, first order conditions are

$$\begin{array}{ll}
\theta^h: & q = \xi^h R,\ h\in\mathcal{H} \\
x^h: & \nabla u^h(x^h) = \xi^h,\ h\in\mathcal{H} \\
y_s^j: & v^j \nabla F_s^j = \left(1-\zeta^j\right)\sum_h s_j^h \xi_s^h + \sum_h \theta_m^h \xi_s^h \frac{\partial r_s^m}{\partial y_s^j},\ s\in\mathcal{S}\cup\{0\} \\
\zeta^j: & q^m - \sum_h s_j^h \sum_s \xi_s^h y_s^j + \sum_h \sum_s \theta_m^h \xi_s^h \frac{\partial r_s^m}{\partial \zeta^j} = 0,\ j\in\mathcal{J}, m\in\mathcal{M}
\end{array} \tag{9}$$

where $\xi^h = \left(1, .., \lambda_s^h/\alpha^h\lambda_0^h, ..\right) \in \mathbb{R}^{S+1}$, $\nabla u = (1, .., \partial_s u\,/\,\partial_0 u, ..) \in \mathbb{R}^{S+1}$ is the normalized gradient of $u(x)$, and y^j satisfies technological feasibility, $F^j\left(y^j\right) \le 0$.

At an interior equilibrium, the following first order conditions for consumer h and firm j are satisfied:

$$\begin{array}{ll}
\theta_m^h: & q^m = \xi^h r^m, m\in\mathcal{M}\cup\{0\} \\
x^h: & \nabla u^h(x^h) = \xi^h \\
y_s^j: & v^j\nabla F_s^j = \left(1-\zeta^j\right)\pi_s^j,\ s\in\mathcal{S}\cup\{0\} \\
\zeta^j: & q^m - \sum_s \pi_s^j y_s^j = 0
\end{array} \tag{10}$$

where $\nabla F^j = \left(1, .., \partial_s F^j/\partial_0 F^j, ..\right) \in \mathbb{R}^{S+1}$, and π^j is the Grossman-Hart criterion (GH criterion).

Comparing the first order conditions for constrained optimality obtained for y^j and ζ^j in (9) with the corresponding equilibrium first order conditions in (10), one sees that the former have additional terms. These terms are a consequence of asymmetric information, and represent a further source of inefficiency with respect to the well known ones which arise in standard economies with incomplete markets (see Drèze (1974), and Geanakoplos et al. (1990)).

[5]See Bisin et al (2001) for such an analysis in a model of an exchange economy with asymmetric information.

Looking more closely, we see that the additional inefficiency results because the firm fails to take into account the effect of its investment and of its financial policy on the return of the fund. Again, this may be interpreted either by saying that the firm simply acts competitively on the markets, taking the return and price from investing in the fund as given; or by considering that the firm lacks specific information on actual fund holders (namely the terms $\theta's$ in the first order conditions with respect to ζ and y in (9)).

To distinguish the specific sources of inefficiencies emerging in our model from those which are typical of an incomplete markets economy, we consider two benchmark models in which, without information asymmetries, equilibria deliver constrained efficient allocations. Precisely, for productive inefficiencies we appeal to an example of an economy in which two firms have technologies characterized by 'multiplicative uncertainty', as in Diamond (1967). While for financial inefficiencies we present an example based on the CAPM.

Example 1: productive inefficiency. Suppose that in our economy two firms, that we label $j = L, H$, have the following very simple technologies, characterized by 'multiplicative uncertainty' (Diamond (1967)):

$$Y^L = \{y \in \mathbb{R}^{S+1} \mid y = (-y_0, \ldots, \phi_s f^L(y_0), \ldots)\}$$

$$Y^H = \{y \in \mathbb{R}^{S+1} \mid y = (-y_0, \ldots, \phi_s f^H(y_0), \ldots)\}.$$

Suppose further that the first technology is less efficient than the second, $f^L(\cdot) = \alpha f^H(\cdot) = \alpha f(\cdot)$, with $0 < \alpha < 1$ and $f : \mathbb{R} \to \mathbb{R}$ strictly increasing and strictly concave. According to definition 7, first order conditions imply that productive efficiency (equality of marginal products) obtains when input levels satisfy,

$$\frac{\partial f(y_0^H)}{\partial y_0^H} = \alpha \frac{\partial f(y_0^L)}{\partial y_0^L}$$

which requires $y_0^H > y_0^L$. To understand this condition, observe that operating both technologies, rather than one, does not provide any advantage in terms of risk sharing: once the initial level of investments have been chosen, the second period returns, $\left(y_1^L, y_1^H\right)$, are collinear.

We now argue that productive efficiency cannot be achieved at equilibrium. Indeed, suppose that investors cannot distinguish between the two firms, and that firms obtains outside financing through the fund market. This implies that firms H and L belong to the same sector m with price q^m and return

$$r_s^m = \frac{\zeta^H \phi_s f(y_0^H) + \zeta^L \phi_s \alpha f(y_0^L)}{\zeta^H + \zeta^L}$$

Assume each of the two firms $j = L, H$ belongs to a single owner, $h = j$. The single owner's choices of ζ^j and y_0^j satisfy the first order conditions:

$$\begin{array}{ll} \zeta^j : & \partial u_0^j (y_0^j + q^m) - \sum_s \partial u_s^j \phi_s f^j(y_0^j) = 0 \\ y_0^j : & \left(1 - \zeta^j\right) \left[\partial u_0^j - \sum_s \partial u_s^j \phi_s \frac{\partial f^j(y_0^j)}{\partial y_0^j}\right] = 0 \end{array}$$

leading to

$$\sum_s \frac{\partial u_s^j}{\partial u_0^j} \phi_s = \frac{y_0^j + q^m}{f^j(y_0^j)} = \frac{1}{\partial f^j(y_0^j)/\partial y_0^j}$$

Using the properties of the technologies described above, the last equality leads to $y_0^H = y_0^L$: with respect to the efficient allocation of inputs, incomplete information leads to overinvestment in the 'lemon' project $j = L$, relative to the investment in the good project $j = H$[6].

Example 2: financial inefficiency. Next, we would like to present an example showing that in our model efficiency may be impaired by financial decisions. To do so we consider a $CAPM$ economy. Initial endowments, ω, are deterministic, and preferences are mean-variance, defined over date 1 consumption by $u^h(E(x_{\mathbf{1}}), Var(x_{\mathbf{1}}))$, with u^h monotonically increasing in the mean term and decreasing in the variance; where, hereafter, all statistical moments are defined as weighted averages of the variables across s with weights equal to the common prior probabilities of the events s in $\mathcal{S}$. Finally, in line with the general model, we assume that the only risky asset traded in the economy is a single mutual fund with price q and return r.

Since endowments are deterministic, individuals face two sources of uncertainty: one due to portfolio holdings and the other due to firm ownership. Efficient risk sharing is obtained when the latter risk is completely hedged, i.e. when $\zeta^j = 1$ for all j. Indeed, when $\zeta^j = 1$, the individual optimal consumption satisfies the two - fund - separation property. Since this is true for all j, $r_s^m = \frac{1}{J}\sum_{j\in m} y_s^j$, i.e. the fund coincides with 'the market'. In other words, when endowments are deterministic, the first best allocation is constrained feasible. Moreover, if consumers/entrepreneurs have identical preferences, and have identical variance σ, then an efficient (first best) allocation is attained when $\theta^j = 1$, and $\zeta^j = 1$, for all j. At this allocation the variance of holding the fund, σ_r, and the agents' portfolio variances, are equal to the minimum.[7]

To show financial inefficiency, we argue that the first best allocation of this economy is not sustainable as a competitive equilibrium.

For expositional purpose, suppose that each consumer j is the single owner of firm j, with $(\omega_0^j, \omega_{\mathbf{1}}^j) = (1, 0)$, and that the economy has two firms, $j = L, H$ and one sector. To focus on financial decisions, let the technology j consist of a unique production plan with a fixed level of initial investment, $y_0^j = 1$, and a stochastic component $y_{\mathbf{1}}^j$. A project j has mean μ^j and variance σ, with $\mu^H = \mu$, $\mu^L = \alpha\mu$, $\alpha \in (0,1)$, and covariance c. Thus, the date zero budget constraint of j is $\theta_0^j = (q+1)\left(\zeta^j - \theta^j\right)$. Using this in the second period budget constraint yields $x_{\mathbf{1}}^j = \left(1-\zeta^j\right) y_{\mathbf{1}}^j + r_{\mathbf{1}}\theta^j + (q+1)\left(\zeta^j - \theta^j\right)$. From this it is straightforward to compute the mean and the variance of $x_{\mathbf{1}}^H$. Second, consider the following marginal, financial, reallocation for H: $d\zeta^H = d\theta^H$. This is feasible and implies $d\theta_0^H = 0$. Evaluating the effects of this change on $H's$ expected utility yields:

$$\begin{aligned}\left(\frac{du^H}{d\zeta^H}\right)_{\zeta=\theta=1} &= (u_1[-\mu + E(r_{\mathbf{1}})] + \\ &+ u_2[-2\left(1-\zeta^H\right)\sigma + 2\theta^H\sigma_r + 2Cov(y^H, r)((1-\zeta^H) - \theta^H)])_{\zeta=\theta=1} \\ &= u_1[-\mu + \frac{\mu(1+\alpha)}{2}] = -u_1[\frac{(1-\alpha)}{2}\mu] < 0\end{aligned}$$

where $u_1 > 0$, $u_2 < 0$ are the partial derivatives of u with respect to the mean and the variance of H's consumption; and where at $\zeta = \theta = 1$, $Cov\left(y^H, r\right) = \sigma_r = (\sigma + c)/2$. Thus, by lowering ζ^H below 1, H increases her expected utility.

[6] For simplicity we wrote the first order conditions only for the case of interior solutions. Notice though that, if $\zeta^L = K$, the result is actually reinforced, because in this case the owners' first order conditions imply $y_0^H < y_0^L$.

[7] When there are only two projects, $j = H, L$, with identical variance, σ, then $\sigma_r = \left(\delta^2 + (1-\delta)^2\right)\sigma + 2\delta(1-\delta)c$, where, $\delta = \zeta^H / \left(\zeta^H + \zeta^L\right)$. σ_r attains a unique minimum at $\delta^* = 1/2$ when $c \neq \sigma$. At $\delta = 1/2$, $\sigma_r = (\sigma + c)/2$.

Intuitively, the resulting inefficiency is due to the fact that H does not take into account the implications of $d\zeta^H$ on the mean and variance of the funds return r. At equilibrium, $\zeta^H/\bar{\zeta} < 1/2$, denoting an overrepresentation in the fund portfolio of the 'lemon' project, $j = L$, relative to the good project, $j = H$.

Active funds. It is interesting to observe that the first order conditions for CPO would be fulfilled at equilibrium if we had used a different notion of competitive equilibrium, with a more active role for the funds. Indeed, suppose that the funds actively represent the interest of their owners at the shareholders' meetings of the firms which they finance: they vote on the choice of the production plan, and at the same time negotiate a firm specific price, $q^j = (q^m + \tau^j)$ for the financing of the firm. If we let π^m be the vector of fund's state prices, the relevant first order conditions for firm $j \in m$ in (10) become:

$$\begin{array}{ll} y_s^j: & v^j \nabla F_s^j = \left(1 - \zeta^j\right) \pi_s^j + \zeta^j \pi_s^m, \ s \in \mathcal{S} \cup \{0\} \\ \zeta^j: & (q^m + \tau^j) - \sum_s \pi_s^j y_s^j = 0 \end{array} \tag{11}$$

Let the fund's state price be a weighted average of the individual state prices of its owners :

$$\pi_s^m = \sum\nolimits_h \frac{\theta_m^h}{\bar{\theta}^m} \xi_s^h$$

and the transfer τ^j be such as to cover the discrepancy between the market valuation of the production plan and its valuation in the eyes of the fund: $\tau^j = \sum_s \pi_s^m y_s^j - q^m$. Then, using the expressions for the derivatives of the fund's return with respect to y^j and ζ^j, one sees that the first order conditions in (11) coincide with the corresponding ones in (9): the active role of the fund leads firm j to internalize the externality. Moreover, the transfers add up to zero for the fund:

$$\sum\nolimits_{j \in m} \tau^j \zeta^j = \sum\nolimits_s \pi_s^m \sum\nolimits_{j \in m} \zeta^j y_s^j - q^m \bar{\zeta}^m = [\sum\nolimits_s \pi_s^m r_s^m - q^m] \bar{\zeta}^m = 0$$

This construction assumes that the funds a) take an active role in the production decision of the firms they finance, b) know the preferences of their shareholders and act on their behalf, c) are allowed to share the information of the initial owners when contracting with them on the transfer τ. Our assumption of totally passive funds is probably too extreme, but it seems to us a comparatively better approximation of reality.

5. Concluding remarks

Modigliani-Miller. The M-M theorem establishes the irrelevance of firm financial policies at two different levels: the first one, a partial equilibrium level, in which the irrelevance operates with respect to the firm value as perceived by the firm itself; a general equilibrium level, in which the irrelevance proposition refers to the overall patterns of the returns from investments and on the equilibrium allocations and prices of the economy as a whole.[8] As De Marzo (1988) shows, the first level of the M-M theorem extends to economies with incomplete financial markets (GEI).[9] On the other hand, the second, general equilibrium, level of the theorem does (generically) fail whenever a modification of the financial policy changes the span of the available assets.

[8] The first extension of M-M theorem to general equilibrium is due to Stiglitz (1969, 1974).

[9] Here, one should keep in mind that an additional assumption of M-M theorem is that agents can take arbitrary positions in every asset (i.e. short sales are allowed). Alternatively, if no-short sales of some assets are assumed, one should restrict attention to equilibria for which such constraints do not bind (Geanakoplos et al. (1990) show that there is a generic set of GEI economies in which this property holds).

In our model, individual equities are not traded on the markets (except in the limiting case in which only one firm is financed by a sectorial fund). This, in turn, implies that no-arbitrage may not hold at the level of the single firm, although it is applicable to the securities issued by the funds. Thus, different financial policies (i.e. different choices of ζ) result in a different expected value of the firm in the eyes of their initial shareholders (i.e. the first level of M-M theorem fails). This clearly emerges from the discussion on the objective of the firm, regardless on whether or not competitive price perceptions are considered: in firm j's first order conditions, eqs. (4) and (8) the terms involving $d\zeta^j$ would cancel out only if the asset y^j were traded (i.e. firm j was the only one in its sectorial fund) so that, at equilibrium, $q^{m,j} = \xi^h y^j$ for all h. Also, firms decisions on equity issues have external effects on the return from the sectorial fund, as can be seen by a direct comparison of the planner's first order conditions with those of the firms. The latter type of externalities does, typically, result in a change of the asset span (i.e. the second level of the M-M theorem fails too).

Rational Expectations. The standard equilibrium notion in models of general equilibrium with asymmetric information is the concept of rational expectations equilibrium (REE)[10]. In this last section we make a few remarks on the differences between our notion of equilibrium and REE.

Information revelation. We model asymmetric information by imposing that firms which investors cannot distinguish are exogenously lumped together in a sector, and their equities cannot be traded separately in the market. In the RE model, each firm is traded on the market, and uninformed investors refine their information on the characteristics of the firm by observing its equilibrium price. By assuming that firms are traded only through aggregate sectorial funds we are in effect imposing a restriction on prices, forcing them to reflect only the average productivity of the firms represented in the fund.

Incentives. The concept of REE has been criticized because, while it assumes a high degree of sophistication on the part of individuals when they extract information from observed market price, it pays no attention at all to the incentives that they may have in preventing their private information to be revealed[11]. By forcing prices to depend only on the sectorial returns, which is known to the market, our model avoids this type of critique. Of course, a non revealing REE would also be immune to the critique. Most of the time though such an equilibrium does not exist: if firms with different productivities can be traded separately, the prices will in general be fully revealing. In Appendix B we calculate the fully revealing REE for the example presented in section 4. The L type firm sells for a lower price than the H type firm. But this is hard to interpret: if the quality of the production plan is known only to the entrepreneur, it is hard to see what could prevent the L type entrepreneur from trying to sell his equity at the higher price. The emergence of sectorial funds, which allow for the existence of pooling equilibria, can be interpreted as a market response to the lack of incentive compatibility of fully revealing REE.

Signaling. The two preceding remarks focused on the fact that, thanks to the presence of sectorial funds, our model allows for the existence of pooling equilibria in which all the firms belonging to a given sector trade for the same price. As pointed out by Dubey et al. (1990), signaling can be introduced in this type of models if we allow firms in the same sector to choose among different funds, each characterized

[10]See for example the survey by Radner (1982).

[11]See for example Blume and Easley (1989)

by a specific observable constraint. To illustrate, we could introduce, for each sector m a list of funds with different bounds on outside financing $K^{m,n}$, $n = 1, 2, \ldots N$, among which firms in sector m could choose (we assume exclusivity: each firm can obtain financing from only one fund). At equilibrium the different funds may trade for a different price, and some of them may be inactive. We refer again to the appendix for an illustration: in our two firms example, if we introduce a sectorial fund which finances firms only for a small share of their investment, this allows the H type entrepreneur to credibly signal his quality to the market; at equilibrium the quality of each firm is revealed, and incentives are properly taken into account; indeed, the L type entrepreneur prefers to sell a higher share of his project, even at the cost of obtaining a lower price.

Knowledge of fund returns. Our model specifies that market investors: (i) lack information on the production and financial decisions of individual firms, (ii) know exactly the return to sectorial funds. This may be justified by arguing that it is much easier to forecast the average return of a sector than each individual firm's choices, but here we would like to point out that the assumption that consumers have correct expectations on the funds' returns could be easily dispensed with. One could extend the model by rewriting the individual problem as the choice of a portfolio of funds given a price vector and an *expectation* of returns tomorrow. The simplest case would be to assume that each individual has point expectations on the matrix of returns R. This expectations might turn out to be totally wrong, and individuals might be disappointed in the second period. Nevertheless, with one good per state and the assumed structure of funds' returns, market clearing on the funds market in the first period would still be enough to guarantee a feasible redistribution of the total resources in every state in the second period. More generally, one could follow the temporary equilibrium approach[12], and endow individuals with subjective distributions on future returns, possibly depending (in a continuous way) on observed prices. Existence would then require additional assumptions, of the same nature of those needed in standard temporary equilibrium models[13]. The analysis on the criterion of the firm could also be extended in a straightforward manner, taking into account that the relevant individual preferences would be over subjective distributions on second period consumption. What would be more problematic is the discussion of efficiency; if individuals are allowed to have wrong expectations, it is not clear how to extend the notion of constrained optimality.

[12]See Grandmont (1988).

[13]A model of pooled assets was actually studied along these lines in a very innovative paper on default by Jerry Green already thirty years ago (see Green (1974)).

6. Appendix A

We consider the model in section 2 of the paper, assuming -for simplicity- $K^m = K < 1$ all m.

Actions in the economy are $(x, \theta, y, \zeta) \in \mathbb{R}^{(S+1)H} \times \mathbb{R}^{(M+1)H} \times \mathbb{R}^{(S+1)J} \times [0, K]^J$. We restrict actions to a cube $C = [-c, c]^k$, where $c > 0$ and $k = (S+M+2)H+(S+2)J$. Using assumption 4, we will later show that if c is big enough equilibria will be in the interior of this cube. For all h, j, define $X_C^h = X^h \cap C$, and $Y_C^j = Y^j \cap C$, and let $Y_C = \times_j Y_C^j$, $X_C = \times_h X_C^h$. Similarly, let the set of truncated asset portfolios be, Θ_C^0, Θ_C^m; $\Theta_C = \Theta_C^0 \times_{m\in\mathcal{M}} \Theta_C^m$. We label $\mathcal{E}_C$ the corresponding (truncated) economy.

Equilibrating variables are $(q, R, \pi, \lambda) \in \mathbb{R}^{M+1} \times \mathbb{R}^{(S+1)(M+1)} \times \mathbb{R}^{(S+1)J} \times \mathbb{R}^{(S+1)H}$. We normalize consumers' state prices $\lambda^h \in \Delta^S$, and restrict firms' state prices to the unit cube $\pi^j \in [0, 1]^{S+1}$. Let $Y^m = \sum_{j\in m} Y^j$ be the aggregate production set of sector m, and $\mathcal{Y} = \times_m Y^m$. We shall take $\mathcal{R} = \{R \in \mathbb{R}^{(S+1)(M+1)} \mid r^0 = (0, 1^S), (r^m)_{m\in\mathcal{M}} \in \mathcal{Y}\}$ as the domain of possible asset matrices. This implies that, for assets that are not traded at equilibrium, the expected payoffs are convex combinations of feasible production plans in the sector.

For given $\varepsilon > 0$, we restrict market prices q in a compact set:

$$\mathcal{P}_\varepsilon = \left\{ q \in \mathbb{R}^{M+1} \mid 0 \le q^m \le \frac{1}{\varepsilon}, m \in \mathcal{M} \cup \{0\} \right\}$$

Firm j choice correspondence is
$S_{C,\varepsilon}^j : \mathcal{P}_\varepsilon \times \mathcal{R} \times ([0, 1]^{S+1})^J \to Y_C^j \times [0, K]$,
$S_{C,\varepsilon}^j (q, R, \pi) = \arg\max_{y,\zeta} \pi_0^j [-\left(1 - \zeta^j\right) y_0^j + q^m \zeta^j] + (1 - \zeta^j)\pi_{\mathbf{1}}^j y_{\mathbf{1}}^j$,
$S_{C,\varepsilon} = \times_{j\in\mathcal{J}} S_C^j$.

If $(y, \zeta) \in S_{C,\varepsilon}^j (q, R, \pi)$, let
$\Pi_0^j (q, R, \pi) = [-\left(1 - \zeta^j\right) y_0^j + q^m \zeta^j]$

$\Pi_{\mathbf{1}}^j (q, R, \pi) = (1 - \zeta^j) y_{\mathbf{1}}^j$
denote firm j's state contingent dividends.

Consumer h demand correspondence is,
$D_{C,\varepsilon}^h : \mathcal{P}_\varepsilon \times \mathcal{R} \times ([0, 1]^{S+1})^J \to X_C^h \times \Theta_C$,
$D_{C,\varepsilon}^h (q, R, \pi) = \arg\max_{x,\theta} \left\{u^h (x) : (x, \theta) \in B_{C,\varepsilon}^h (q, R, \pi)\right\}$, where

$$B_{C,\varepsilon}^h (q, R, \pi) = \left\{ (x, \theta) \in X_C^h \times \Theta_C : \begin{array}{l} A^h (x, \theta, q, R, \pi) \le 0 \\ \theta_m \ge 0, m \in \mathcal{M} \end{array} \right\}$$

and

$$A^h (x, \theta, q, R, \pi) = \left(\begin{array}{l} (x_0 - \omega_0^h) + (q - r_0)\theta^h - \sum_j s_j^h \Pi_0^j (q, R, \pi) \\ x_{\mathbf{1}} - \omega_{\mathbf{1}}^h - R_{\mathbf{1}}\theta - \sum_j s_j^h \Pi_{\mathbf{1}}^j (q, R, \pi) . \end{array} \right);$$

$D_{C,\varepsilon} = \times_h D_{C,\varepsilon}^h$.

Market auctioneer correspondence $\Phi_{C,\varepsilon} : X_C \times (\Theta_C)^H \times Y_C \times [0, K]^J \times \mathcal{R} \to \mathcal{P}_\varepsilon$,

$$\Phi_{C,\varepsilon} (x, \theta, y, \zeta, R) = \arg\max_{q\in\mathcal{P}_\varepsilon} \left\{ (\overline{x}_0 - \overline{\omega}_0 + \overline{y}_0) + \sum_m q^m (\overline{\theta}_m - \overline{\zeta}_m) + q^0 \overline{\theta}_0 - (\sum_j \zeta^j y_0^j + \sum_m r_m^0 \overline{\theta}_m) \right\},$$

where we use the notation, $\overline{x}_0 = \sum_h x_0^h$, $\overline{\omega}_0 = \sum_h \omega_0^h$, $\overline{y}_0 = \sum_j y_0^j$, $\overline{\zeta}_m = \sum_{j\in m} \zeta^j$, $\overline{\theta}_m = \sum_h \theta_m^h$, $\overline{\theta}_0 = \sum_h \theta_0^h$.

Consumer h state-price correspondence $\Lambda^h_{C,\varepsilon} : X_C \times \Theta_C \times \mathcal{P}_\varepsilon \times \mathcal{R} \times ([0,1]^{S+1})^J \to \Delta^S$,

$$\Lambda^h_{C,\varepsilon}(x,\theta,q,R,\pi) = \arg\max_{\lambda^h} \left\{ \ \lambda^h A^h(x,\theta,q,R,\pi) \ \right\},$$

$\Lambda_{C,\varepsilon} = \times_h \Lambda^h_{C,\varepsilon}$.

Firm j state-price correspondence, $\pi^j_C : (\Delta^S)^H \to [0,1]^{S+1}$
$\pi^j_C(\lambda) = ((\Pi_h \lambda^h_0), \ldots (\sum_h s^h_j \lambda^h_s \Pi_{h' \neq h} \lambda^{h'}_0))$, $\pi_C = \times_{j \in \mathcal{J}} \pi^j_C$.

Perturbed *asset payoff* correspondence $\mu_{C,\varepsilon} : Y_C \times [0,K]^J \to \mathcal{R}$,
$\mu_{C,\varepsilon}(y,\zeta) = R_\varepsilon$.
where $R_\varepsilon \in \mathcal{R}$ has typical element,

$$R^m_{\varepsilon,s} = \sum\nolimits_{j \in m} \frac{\zeta^j + \varepsilon}{\sum_{j \in m}(\zeta^j + \varepsilon)} y^j_s$$

Let $\sigma = (x,\theta,y,\zeta,q,R,\pi,\lambda) \in \Sigma_{C,\varepsilon}$ with
$\Sigma_{C,\varepsilon} = X_C \times (\Theta_C)^H \times Y_C \times [0,K]^J \times \mathcal{P}_\varepsilon \times \mathcal{R} \times (\Delta^S)^H \times ([0,1]^{S+1})^J$.
Equilibrium correspondence, $G_{C,\varepsilon} : \Sigma_{C,\varepsilon} \to \Sigma_{C,\varepsilon}$, where
$G_{C,\varepsilon} = D_{C,\varepsilon} \times S_{C,\varepsilon} \times \mu_{C,\varepsilon} \times \Phi_{C,\varepsilon} \times \Lambda_{C,\varepsilon} \times \pi_C$.

Lemma 8. *Let 1 - 3 in assumption* 4 *hold.* $G_{C,\varepsilon}$ *has a fixed point.*

Proof:

Firms: We omit the index j. $S_{C,\varepsilon}$ is non empty, compact valued at each $(q,R,\pi) \in \mathcal{P}_\varepsilon \times \mathcal{R} \times ([0,1]^{S+1})^J$. To see that it is also convex valued and u.h.c, notice that, whatever its choice of $\zeta \in [0,K]$, the firm chooses $y \in Y_C$ to maximize $\sum_s \pi_s y_s - \pi_0 y_0$. The solution, $y(q,R,\pi)$ is, under our assumptions, a continuous function. The optimal ζ is then determined as follows:

$$\zeta(q,R,\pi) = \begin{cases} K & \text{if} \quad \pi_0 q^m > \sum_s \pi_s y_s(q,R,\pi) - \pi_0 y_0(q,R,\pi) \\ [0,K] & \text{if} \quad \pi_0 q^m = \sum_s \pi_s y_s(q,R,\pi) - \pi_0 y_0(q,R,\pi) \\ 0 & \text{if} \quad \pi_0 q^m < \sum_s \pi_s y_s(q,R,\pi) - \pi_0 y_0(q,R,\pi) \end{cases}$$

which is convex valued and upper hemi-continuous (u.h.c.).
S_C is then the product of two convex valued, u.h.c. correspondences.

Consumers: We omit the index h. Since $B_{C,\varepsilon}$ is compact and u is continuous, $D_{C,\varepsilon}$ is nonempty. Since $B_{C,\varepsilon}$ is convex and u is quasi-concave, $D_{C,\varepsilon}$ is convex valued. By the continuity of u and Berge's maximum theorem, $D_{C,\varepsilon}$ is u.h.c if the budget correspondence is continuous.

$\Pi^j(q,R,\pi)$ is a u.h.c. correspondence, and (x,θ) in $B_{C,\varepsilon}(q,R,\pi)$ is a linear function of Π. Thus, continuity of the budget correspondence with respect to (q,R,Π) implies continuity with respect to (q,R,π).

$B_{C,\varepsilon}(q,R,\Pi)$ is l.h.c. Consider a point $(x,\theta) \in B_{C,\varepsilon}(q,R,\Pi)$, and a sequence $(q,R,\Pi)_n \to (q,R,\Pi)$. Let $\alpha_n = 1 - \frac{1}{n}$. Since $\omega \gg 0$, $(\alpha_n x, \alpha_n \theta) \in B_{C,\varepsilon}(q,R,\Pi)$ for all n, and, choosing subsequences, $(\alpha_n x, \alpha_n \theta) \in B_{C,\varepsilon}(q,R,\Pi)_n$ for n big enough.

The graph of the budget correspondence $B_{C,\varepsilon}$ is closed in $\mathcal{P}_\varepsilon \times \mathcal{R} \times ([0,1]^{S+1})^J \times X^h_C \times \Theta_C$ and $X^h_C \times \Theta_C$ is compact, so $B_{C,\varepsilon}$ is u.h.c.

Auctioneer: $\Phi_{C,\varepsilon}$, $\Lambda^h_{C\varepsilon}$ are nonempty, compact and convex valued u.h.c. correspondences. π^j_C and $\mu_{C,\varepsilon}$ are continuous functions.

We can apply Kakutani's theorem to $G_{C,\varepsilon} = D_{C,\varepsilon} \times S_{C,\varepsilon} \times \mu_{C,\varepsilon} \times \Phi_{C,\varepsilon} \times \Lambda_{C,\varepsilon} \times \pi_C$.
■

Lemma 9. *Let 1 - 3 in assumption* 4. *A competitive equilibrium for the truncated economy* $\mathcal{E}_C$ *exists.*

Proof: At a fixed point σ_ε consumers and firms are maximizing on their truncated choice sets. By definition of $\Lambda^h_{C,\varepsilon}$ consumers' state prices are Lagrange multipliers of the utility maximization problem, and by definition of π_C firms' state prices are obtained by aggregation of initial shareholders' state prices according to the GH criterion. We now use the definition of $\Phi_{C,\varepsilon}$ to show that at the fixed point σ_ε, for small ε we obtain approximate market clearing of assets and goods at time $t = 0$, and of good at time 1. To cut on notation we define $\overline{\zeta}_0 = 0$.

Monotonicity of utility implies that the consumers' first period budget equations are satisfied as equalities at σ_ε. Summing across consumers we obtain

$$(\overline{x}_0 - \overline{\omega}_0 + \overline{y}_0) + \sum\nolimits_m q^m(\overline{\theta}^m - \overline{\zeta}^m) - (\sum\nolimits_j \zeta^j y^j_0 + \sum\nolimits_m r^0_m \overline{\theta}_m) = 0$$

where all variables are indexed by ε.

Inserting the value of $r^m_0 = \sum_{j\in m} \frac{\zeta^j+\varepsilon}{\sum_{j\in m}(\zeta^j+\varepsilon)} y^j_0$, and letting J^m denote the number of firms in sector m, we can rewrite this equation as

$$(\overline{x}_0 - \overline{\omega}_0 + \overline{y}_0) + \sum\nolimits_m (q^m - r^m_0)(\overline{\theta}_m - \overline{\zeta}_m) - \varepsilon \sum\nolimits_m (J^m r^m_0 + \sum\nolimits_{j\in m} y^j_0) = 0$$

Define $V_{0,\varepsilon} = \varepsilon \sum_m (J^m r^m_0 + \sum_{j\in m} y^j_0)$ and $V_{s,\varepsilon} = \varepsilon \sum_m (J^m r^m_s - \sum_{j\in m} y^j_s)$, for $s \in \mathcal{S}$. Then

$$(\overline{x}_0 - \overline{\omega}_0 + \overline{y}_0) - \sum\nolimits_m r^m_0(\overline{\theta}^m - \overline{\zeta}^m) - V_{0,\varepsilon} + \sum\nolimits_m q^m(\overline{\theta}^m - \overline{\zeta}^m) = 0 \tag{12}$$

If $(\overline{x}_0 - \overline{\omega}_0 + \overline{y}_0 - \sum_m r^m_0(\overline{\theta}_m - \overline{\zeta}_m)) > V_{0,\varepsilon}$, the optimality of the auctioneer's choice would be contradicted by setting $q = 0$. Thus

$$(\overline{x}_0 - \overline{\omega}_0 + \overline{y}_0 -) \sum\nolimits_m r^m_0(\overline{\theta}^m - \overline{\zeta}^m)) \leq V_{0,\varepsilon} \tag{13}$$

If $(\overline{\theta}^m - \overline{\zeta}^m) > 0$ for some asset with $q^m < \frac{1}{\varepsilon}$ then the auctioneer could profitably rise q^m. Similarly, if $(\overline{\theta}^m - \overline{\zeta}^m) \leq 0$ for some asset with $q^m > 0$ the auctioneer could profitably lower q^m. Therefore

$$(\overline{\theta}^m - \overline{\zeta}^m) = 0 \quad \text{if } 0 < q^m < \tfrac{1}{\varepsilon} \tag{14}$$

If for some $m \in \mathcal{M} \cup \{0\}$ $q^m = \frac{1}{\varepsilon}$ then it must be that $(\overline{\theta}^m - \overline{\zeta}^m) \geq 0$, otherwise the auctioneer could gain by lowering q^m. Then from (12) we have

$$\frac{1}{\varepsilon}(\overline{\theta}^m - \overline{\zeta}^m) = V_{0,\varepsilon} - (\overline{x}_0 - \overline{\omega}_0 + \overline{y}_0) + \sum\nolimits_m r^m_0(\overline{\theta}^m - \overline{\zeta}^m) - \sum\nolimits_{m'\neq m} q^{m'}(\overline{\theta}^{m'} - \overline{\zeta}^{m'})$$

Using (14), the last term is non-negative and we can write

$$\frac{1}{\varepsilon}(\overline{\theta}^m - \overline{\zeta}^m) \leq V_{0,\varepsilon} + \overline{\omega}_0 - \sum\nolimits_{m'\neq m} r^{m'}_0 K$$

Therefore

$$0 \leq (\overline{\theta}^m - \overline{\zeta}^m) \leq \varepsilon[V_{0,\varepsilon} + \overline{\omega}_0 - \sum\nolimits_{m'\neq m} r^{m'}_0 K] \quad \text{if } q^m = \tfrac{1}{\varepsilon} \tag{15}$$

If for some $m \in \mathcal{M} \cup \{0\}$ $q^m = 0$ then it must be that $(\overline{\theta}^m - \overline{\zeta}^m) \leq 0$, otherwise the auctioneer could gain by rising q^m. If the inequality is strict, we can always turn it into an equality. For $m \in \mathcal{M}$, at $q^m = 0$ all firms are willing to set $\zeta^j = 0$, implying $\overline{\theta}_m = \overline{\zeta}_m = 0$.

Putting all this together, we obtain that at σ_ε,

$$\overline{x}_0 - \overline{\omega}_0 - \overline{y}_0 \leq V_{0,\varepsilon} + \sum\nolimits_m r^m_0(\overline{\theta}^m - \overline{\zeta}^m) \leq V_{0,\varepsilon} \tag{16}$$

and, for all $s \in \mathcal{S}$,

$$\begin{array}{rcl} \overline{x}_s - \overline{\omega}_s - \overline{y}_s & = & \sum_m r_s^m \overline{\theta}_m - \sum_j \zeta^j y_s^j \\ & = & \sum_m (\overline{\theta}^m - \overline{\zeta}^m) r_s^m - \varepsilon \sum_m (J^m r_s^m - \sum_{j \in m} y_s^j) \\ & \leq & \varepsilon [V_{0,\varepsilon} + \overline{\omega}_0 - \sum_{m' \neq m} r_0^{m'} K] \mid (M r_s^m + 1) + \mid V_{s,\varepsilon} \mid \end{array} \tag{17}$$

When $\varepsilon \to 0$, all elements of the sequence σ_ε lie in compact sets, except possibly q. We now show that if along the sequence of fixed point for some m the price q^m was going to infinity we would contradict consumer optimality. Indeed, if q^m was becoming unboundedly large some firm $j \in m$ will choose $\zeta^j > 0$. therefore at the fixed point some consumer must be buying fund m. But then he could reduce its holding of fund m a bit and buy large quantities of x_0. If C is big enough to contain the maximal feasible amount of x_0, this contradicts consumer maximization. Thus all elements in the sequence remain bounded and the sequence converge to some σ which, using (16) and (17), is an equilibrium for the truncated economy $\mathcal{E}_C$. ■

The last step is to remove the artificial bound C. Let $C_n = [-c_n, c_n]^k$ be a sequence of larger and larger cubes. For each n there is an equilibrium $\sigma_n = (x, \theta, y, \zeta, q, R, \pi, \lambda)_n$. By *4* in assumption 4 and feasibility we know that x_n, y_n and R_n converge. But then, using the period 1 budget equations we see that θ also converge. C can therefore be chosen big enough so that all choices are in the interior of the cube. Using 1, 2, an interior solution of the consumer's C-constrained problem must be a solution to the unconstrained problem as well. Using *3*, the same is true for firms because, at fixed ζ, the objective of the firm is convex in y.

7. Appendix B

In this section, we further analyze our equilibrium notion and its welfare properties in a mean-variance model with a single sector ($M = 1$).

Let us assume that consumers' initial endowments, ω^j, is stochastic, and preferences are mean-variance, defined over date 1 consumption, $u^j(x_{\mathbf{1}}) = a^j E(x_{\mathbf{1}}) - \frac{a^{j^2}}{2} Var(x_{\mathbf{1}})$; where, hereafter, all statistical moments are defined as weighted averages of the variables across s with weights equal to the common prior probabilities of the events s in $\mathcal{S}$.

We also assume that each consumer j is the single owner of firm j. To focus on financial decisions, we assume that the technology j consists of a unique production plan with a level of initial, fixed, investment, y_0^j, and a stochastic component $y_{\mathbf{1}}^j$ of mean μ^j, and variance σ^j. Monotonicity[14] of u^j implies that we can normalize the price of the riskless asset to one.

The budget constraint of j is defined over the domain of the decision variables $(x_{\mathbf{1}}^j, \zeta^j, \theta^j)$ such that,

$$\theta_0^j - \left(\omega_0^j + q\zeta^j - (q + r_0)\,\theta^j - \left(1 - \zeta^j\right) y_0^j\right) = 0$$

$$x_{\mathbf{1}}^j - \left(\omega_{\mathbf{1}}^j + \theta_0^j + \left(1 - \zeta^j\right) y_s^j + r_s \theta^j\right) = 0, \;\; x_{\mathbf{1}}^j > 0, \;\; 0 \leq \zeta^j \leq K, \;\; \theta^j \geq 0.$$

The decision problem of entrepreneur j can then be written as follows.

$$\begin{array}{l} Max_{x_{\mathbf{1}}, \zeta, \theta} \; u^j(x_{\mathbf{1}}) = a^j E(x_{\mathbf{1}}) - \frac{a^{j^2}}{2} Var(x_{\mathbf{1}}), \text{ s.t.} \\ x_s = \omega_s + \omega_0 + (1 - \zeta)\left(y_s^j - y_0^j\right) + q\zeta + (r_s - r_0 - q)\,\theta, \; s \in \mathcal{S} \\ x_{\mathbf{1}} > 0, \; 0 \leq \zeta \leq K, \; \theta \geq 0 \end{array} \tag{18}$$

[14] The utility function u^j is monotonic on the set of feasible allocations if the risk aversion coefficient a^j is sufficiently small.

In this extended CAPM - economy, an interior optimum for individual j yields a demand of the pooled asset,

$$\theta^j = \frac{E(r) - q}{a^j \sigma_r} - \beta^j_{\omega,r} - \left(1 - \zeta^j\right)\beta^j_r \tag{19}$$

where $E(r) = E(r_{\mathbf{1}}) - r_0$, $\beta^j_r = Cov(y^j_{\mathbf{1}}, r_{\mathbf{1}})/\sigma_r$, $\beta^j_{\omega,r} = Cov(\omega^j_{\mathbf{1}}, r_{\mathbf{1}})/\sigma_r$, $\sigma_r = Var(r_{\mathbf{1}}) \neq 0$ (see e.g. Sharpe (1964)). The first term is the net expected return by j. The remaining two, reflect the extent to which the pooled asset allows individual j to hedge against his income risk. This risk has two components, one originated by individual endowments, and the other by the individual participation to his production activity. At equilibrium, the return of the pooled asset, r, is a linear function of the returns of the firms participating to the fund: more generally, for a fund m, $r^m_s = \sum_{j\in m} \delta^j y^j_s$, where $\delta^j = \zeta^j/\overline{\zeta}^m$ is the share of project j in the composition of fund m. If agents conjecture that this is the structure of r_s, then their typical demand will reduce to,
$\theta^j = \frac{\sum_k \delta^k E\left(y^k\right) - q}{a^j \sigma_r} - \beta^j_{\omega,r} - \left(1 - \zeta^j\right)\beta^j_r$
where $E\left(y^k\right) = E\left(y^k_{\mathbf{1}}\right) - y^k_0$.

Similarly, supply of equities by entrepreneur j to the fund is

$$\zeta^j = \frac{q - E(y^j) + a^j \sigma^j}{a^j \sigma^j} - \beta^j_{\omega,y} + \theta^j \beta^j_y \tag{20}$$

where $\sigma^j = \sigma^j(y^j_0)$, $\beta^j_{\omega,y} = Cov(\omega^j_{\mathbf{1}}, y^j_{\mathbf{1}})/\sigma^j$, and $\beta^j_y = Cov(y^j_{\mathbf{1}}, r_{\mathbf{1}})/\sigma^j$.

Next, let us define aggregate income as $I_{\mathbf{1}} = \sum_j \left(\left(1 - \zeta^j\right) y^j_{\mathbf{1}} + r_{\mathbf{1}}\theta^j + \theta^j_0 + \omega^j_{\mathbf{1}}\right)$. Summing the individual demand θ^j over individuals, we obtain the security market line for the fund pooled asset, $q = E(r^m) - \overline{a}Cov(r_{\mathbf{1}}, I_{\mathbf{1}})$, where $\frac{1}{\overline{a}} = \sum_j \frac{1}{a^j}$ is an index of aggregate risk aversion. At an equilibrium in which agents trade in the fund market ($\theta^j > 0$, for all j),[15]

$$q = E(r) - \overline{a}\sigma_r \sum_j \left(\beta^j_r + \beta^j_{\omega,r}\right) \tag{21}$$

Reinterpreting q as the multiplier for the planner's problem (see definition 7), we can express the planner's decision as one in which the only control variables are in the J dimensional vector ζ. Indeed, by direct substitutions

$$\theta^j(\zeta) = \frac{\overline{a}}{a^j} \sum_{j'} \left(\beta^{j'}_r + \beta^{j'}_{\omega,r}\right) - \left(1 - \zeta^j\right)\beta^j_r - \beta^j_{\omega,r} \tag{22}$$

for all j, where the terms β are functions of r and thus of ζ.

Using equation (22), and the conditions for constrained feasibility in definition (7), the planner's problem reduces to,

$$W(\zeta) = \max_{\zeta\in[0,1]^J} \sum_h u^h\left(x^h(\zeta)\right) \tag{23}$$

where u^h are our mean-variance utility, and consumption allocations do only depend on ζ. We will illustrate this construction in some of our examples below.

7.1. **Examples.** Consider a class of mean-variance economies, presented above, with the following features. There are two entrepreneurs, identified with theirs firm types, $j \in \{L, H\}$, who are given a deterministic, initial, endowment vector ω, with $\omega_0 = 1$, and $\omega_s = 0$, for all $s \in \mathcal{S}$. Each entrepreneur j is also endowed with a production plan that requires one unit of input to be activated, and that produces an uncertain future return $y^j_{\mathbf{1}}$. $y^j_{\mathbf{1}}$ is privately observed by j at $t = 0$:

---

[15]Market clearing implies that $I_{\mathbf{1}} = \sum_j \left(y^j_{\mathbf{1}} + \omega^j_{\mathbf{1}}\right)$, and
$q^m = \sum_{j\in m} \frac{\zeta^j}{\overline{\zeta}^m}\left(\mu^j - y^j_0 - \overline{a}\left(Cov(y^j, \sum_i \omega^i_{\mathbf{1}}) + \sum_k \sigma^{jk}\right)\right)$ where $\sigma^{jk} = Cov\left(y^j, y^k\right)$.

the competing entrepreneur, cannot distinguish between a production plan of type H and one of type L. Production plans, or technologies, are $y_{\mathbf{1}}^j = \mu^j + u_{\mathbf{1}}^j$, where $\mu^j > 0$ is a scalar and $u_{\mathbf{1}}^j$ is a stochastic term. For the two projects, H, L, the mean of the returns are $E\left(y_{\mathbf{1}}^H\right) = \mu$, $E\left(y_{\mathbf{1}}^L\right) = \alpha\mu$, with $0 < \alpha < 1$, the variance is $\sigma = Var\left(u_{\mathbf{1}}^j\right)$ (uniform across j), and the covariance is $c = Cov\left(u_{\mathbf{1}}^H, u_{\mathbf{1}}^L\right)$. As for the general model, we assume that the only risky asset traded in the economy is a sectorial mutual fund with price q and return r. Below we will show that, for this class of economies, this assumption is fully justified: separating equilibria are never incentive compatible (see section 7.1.4 below).

Our examples are presented in the following three subsections and cover, in the order, the cases of positively correlated production plans ($c > 0$), uncorrelated plans ($c = 0$), and negatively correlated plans ($c < 0$). Finally, in the context of the latter example we illustrate the main points raised in section 5, when discussing information revelation, incentives, and signaling.

Our three examples illustrate different characteristics and properties of equilibria. When production projects are positively correlated, investing in the fund does not provide any risk sharing to the entrepreneurs. Yet, an entrepreneur may still want to use equity financing to reduce the variance associated to firm ownership. This can be true even for a financially unconstrained entrepreneur, holding a relatively "high quality" project. The reason why the equilibrium may fail to be constrained efficient is that, in doing so, agents do not correctly perceive the effects of their decisions on the fund asset price and return. Thus, at equilibrium the proportions of high quality projects in the fund can be inefficiently low. In this context, we also compare our equilibria with those discussed in Leland and Pyle (1977).

On the other hand, when project are either uncorrelated or negatively correlated, a non-trivial pooling equilibrium has the property that holding a share of the fund provides risk sharing to the entrepreneurs. Again, as argued in section 4, because agents fail to internalize the effect of their decisions on the fund price and return, equilibria display adverse selection and financial inefficiencies.

7.1.1. *Positively correlated production plans.* Assume that production plans have a common stochastic component, $u_{\mathbf{1}}$; then, $c = \sigma_r = \sigma$, $\beta_r^H = \beta_y^H = \beta_r^L = \beta_y^L = 1$. To avoid trivial equilibria, of the type $\zeta^L = 1 = 1 - \zeta^H$, $\theta^L = \zeta^L$, $\theta^H = 0$, assume that there is a third agent, a "pure buyer", b, who -for simplicity- owns only a consumption endowment, ω^b. ω^b is also assumed to be perfectly, negatively, correlated with the two production plans, $Cov\left(\omega_{\mathbf{1}}^b, y^j\right) = -\sigma$, and to satisfy $\omega_0^b = 0$, $E\left(\omega_{\mathbf{1}}^b\right) = 1$.

In economies with perfectly, positively, correlated production projects, investing in the fund does not provide any risk sharing to entrepreneurs, $\sigma_r = \sigma$. Thus, a (financially unconstrained) entrepreneur like H would only sell a share of her projects to reduce the variance associated to her production plan. If this is true, one expects that H would rather be willing to short-sale a share of the sectorial fund than to hold it in positive amount. To make this intuitive argument precise, notice that, since $\beta_y^j = 1$, $\beta_{\omega,y}^j = 0$, equation (21) yields,

$\zeta^j - \theta^j = \frac{q - \mu^j + 1 + a^j\sigma}{a^j\sigma} + \frac{\lambda^j}{a^j}$

where λ^j is the Lagrange multiplier (LM) associated to the constraint $\theta^j \geq 0$. If agents have identical preferences ($a^i = a$, $i = H, L, b$), equilibrium price is $q = \mu\left(\delta + \alpha\left(1 - \delta\right)\right) - 1 - a\sigma + \frac{1}{a}\sum_i \lambda^i$, where $\delta = \zeta^H / \overline{\zeta}$.[16] This implies that,

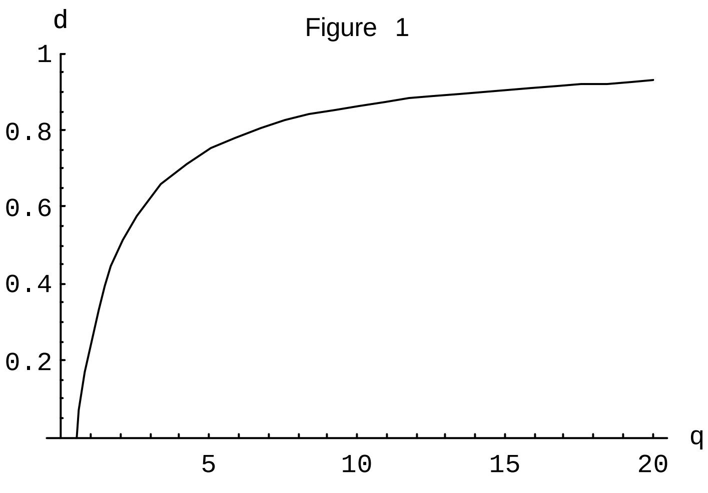
[16] Accounting for corner solutions on fund portfolio decisions, the equilibrium price is $q = E\left(r\right) - \overline{a}\sigma_r \sum_j \left(\beta_r^j + \beta_{\omega,r}^j\right) + \sum_j \frac{\lambda^j}{a^j}$.

$\zeta^H - \theta^H = \frac{\mu(\delta+\alpha(1-\delta)-1)}{a\sigma} + \frac{\lambda^H}{a}\left(\frac{1+a\sigma}{a\sigma}\right) + \frac{\lambda^L}{a}\left(\frac{1}{a\sigma}\right).$
Then, there exist economies in which, at equilibrium, $\zeta^H > 0$, $\zeta^L > 0$, and $\theta^H = 0$, $\theta^L > 0$. Precisely, when $\theta^H = 0$,
$\zeta^H = \frac{\mu(\delta+\alpha(1-\delta)-1)}{a\sigma} + \frac{\lambda^H}{a}\left(\frac{1+a\sigma}{a\sigma}\right) > 0,$
holds for parameter values such that the negative mean-effect, $\mu\left(\delta + \alpha\left(1-\delta\right) - 1\right) \leq 0$, is counterbalanced by a positive variance-effect, $\left(1 + a\sigma\right)/a\sigma$, at $\lambda^H > 0$.

An example is provided for the following parameterization.
$\mu = 3, \sigma = .5, \alpha = .8, a = 3, K = 1$.
$\zeta^H = .487, \zeta^L = 1,$
$\theta^H = 0\,(\lambda^H = .403), \theta^L = .243, \theta^b = 1.244,$
$\sigma_r = .5, E\left(r_{\mathbf{1}}\right) = 2.597,$
$q = 1.231, \delta = .328,$
$\theta_0^H = 1.087, \theta_0^L = 1.687, \theta_0^b = -2.774,$
$E(x_{\mathbf{1}}^H) = 2.625, E(x_{\mathbf{1}}^L) = 2.320, E(x_{\mathbf{1}}^b) = 1.455,$
$Var(x_{\mathbf{1}}^H) = .1314, Var(x_{\mathbf{1}}^L) = Var(x_{\mathbf{1}}^b) = 0.0297,$
$u^H = 7.285, u^L = 6.827, u^b = 4.230, \sum_j u^j = 18.342$

At equilibrium, the conjectures on r, which uninformed consumers form at the market price q, are validated. In fact, since δ is monotonic in q, $\delta = 1 - 3/(2(q+1))$, uninformed consumers can correctly predict δ, the share of type H project in the fund, by observing the asset market price (see Figure 1). Thus, knowing the linear structure of r, different values of q signal to the uninformed different returns of the pooled asset, r. This equilibrium property is analogous to Theorem 1 in Leland and Pyle (1977). There uninformed buyers learn about the fundamental value of an equity by inverting the supply of equity by the informed entrepreneurs. Here the uninformed, observing q, can recover δ, but still cannot recover the fundamental value of the two projects. This does also explain why a pooling equilibrium exists in our example.

Figure 1

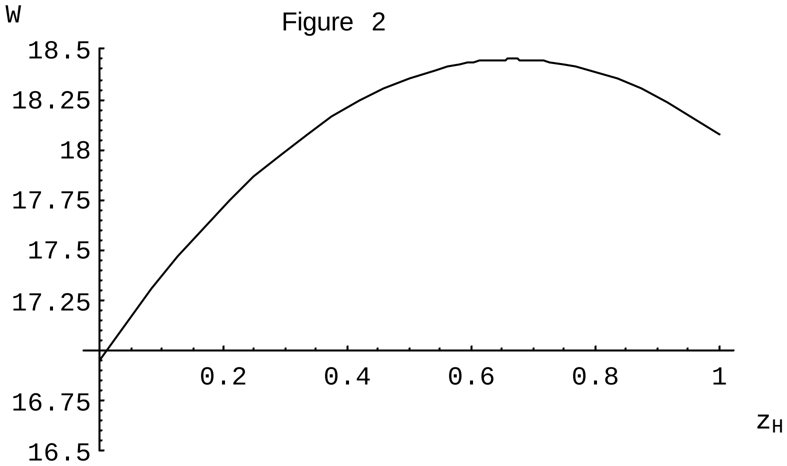


Welfare properties of equilibria. In this context, equilibria need not be a CPO. Unlike private agents, the planner internalizes the effect that a change in ζ^H, and in δ, has on the expected value from holding the fund. An interior welfare maximum, $W = 18.45$, is achieved at $\zeta^H = \zeta^L = 1$, $\theta^L = \theta^H = 1/3$, $\theta = 4/3$. At the CPO, risk sharing yields, $Var(x_{\mathbf{1}}^H) = Var(x_{\mathbf{1}}^L) = Var(x_{\mathbf{1}}^b) = 0.0056$.

This is not the unique CPO. There is a corner solution to the planner's problem that shares some properties with the equilibrium: $\zeta^H > 0$, $\zeta^L > 0$, and $\theta^H = 0$, $\theta^L > 0$. The welfare function is

$$W\left(\zeta^H\right) = 16.95 + 4.5\zeta^H - 3.375\left(\zeta^H\right)^2$$

A welfare maximum, $W = 18.45$, is achieved at $\zeta^H = 2/3$ (see Figure 2).



The CPO allocation is,
$\zeta^H = 2/3, \zeta^L = 1, \delta = 2/5$
$\theta^H = 0, \theta^L = 1/3, \theta^b = 4/3$
$\sigma_r = \sigma = .5, E\left(r_{\mathbf{1}}\right) = 2.64, q = 1.14$
$E(x_{\mathbf{1}}^H) = .9999, E(x_{\mathbf{1}}^L) = .88, E(x_{\mathbf{1}}^b) = 4.52$
$Var(x_{\mathbf{1}}^H) = Var(x_{\mathbf{1}}^L) = Var(x_{\mathbf{1}}^b) = 0.0056$
$u^H = 2.75, u^L = 2.39, u^b = 13.31$
Comparing the CPO allocation with the equilibrium allocation, one can observe that the share of project H in the fund, δ, is higher in the first. Differently from private investors, the planner when choosing δ internalizes the effects of its decision on both $E\left(r\right)$ and q. Here, in particular, augmenting δ, over the equilibrium value, increases $E\left(r\right)$ more than proportionally with respect to its shadow-price, q. The latter effect is special to corners in which $\theta^j \geq 0$ is binding for some j, and operates through a weakening of the constraint, and a reduction of its multiplier, λ^j, toward zero. Going back to our parametric example, at the CPO, we have an higher $E\left(r\right)$, a lower q, and a lower LM ($\lambda^H = 0$), with respect to equilibrium values. Precisely, in going from the equilibrium to the CPO, and using the equations that determine q as an equilibrium price and a shadow-price respectively, one finds that $\Delta E\left(r\right) - \Delta q = \lambda_{eq}^H / a > 0$, with $\lambda_{eq}^H = .403$ denoting the equilibrium value of the LM.

7.1.2. *Uncorrelated production plans.* In the same class of economies, we now consider the case of two entrepreneurs who are endowed with production plans with uncorrelated stochastic component, $c = 0$. Since plans have the same variance, σ, one can check that the return from holding the fund has variance, $\sigma_r = D\sigma$, where $D = \left(\delta^2 + (1-\delta)^2\right)$ is an index of the homogeneity of the fund composition: D attains a maximum value of 1 when δ assumes an extreme value of 0 or 1, and has a minimum, $D = 1/2$, at $\delta = 1/2$. Hence, for $0 < \delta < 1$, $\sigma/2 \leq \sigma_r < \sigma$, and the entrepreneurs can lower their income variance also by selling a share of their firm to the fund and buying a portfolio of the fund. Finally, $\beta_r^H = \delta/D$, $\beta_r^L = (1-\delta)/D$, $\beta_y^H = \delta$, $\beta_y^L = (1-\delta)$.

Dropping the "pure buyer" and retaining the above parameterization (see section 7.1.1), we have the following equilibrium.

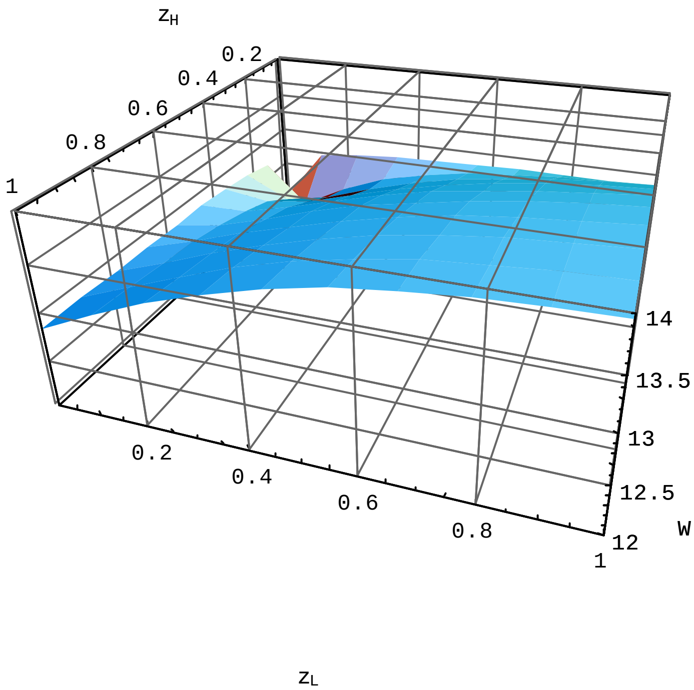
$\zeta^H = 1/3, \zeta^L = 1,$
$\theta^H = 8/15, \theta^L = 4/5,$
$\sigma_r = .312, E(r_{\mathbf{1}}) = 2.55,$
$q = 4/5, \delta = 1/4,$
$\theta_0^H = -.36, \theta_0^L = .36,$
$E(x_{\mathbf{1}}^H) = 3.72, E(x_{\mathbf{1}}^L) = 1.68,$
$Var(x_{\mathbf{1}}^H) = 2/5, Var(x_{\mathbf{1}}^L) = 1/5,$
$u^H = 9.36, u^L = 4.14, \sum_j u^j = 13.5$

Welfare properties of equilibria. Equilibria are not CPO. The central planner internalizing the effect of the choice of $\zeta' s$ on the fund mean and variance, can improve upon the competitive allocation of risk. In the context of our last example, the welfare function is

$$W\left(\zeta^H, \zeta^L\right) = .225 \frac{20{\zeta^L}^2 \zeta^H + 57{\zeta^L}^2 - 20{\zeta^L}^2{\zeta^H}^2 + 20{\zeta^H}^2\zeta^L - 10\zeta^L\zeta^H + 57{\zeta^H}^2}{{\zeta^L}^2 + {\zeta^H}^2}$$

and achieves a unique, global, maximum, $W = 13.95$, at $\zeta^H = \zeta^L = 1$, that is at $\delta = 1/2$.

At the CPO the allocation is symmetric, perfectly egalitarian, and attains an efficient risk sharing: the fund variance and the consumption variance are minimized, $\sigma_r = Var(x_{\mathbf{1}}^H) = Var(x_{\mathbf{1}}^L) = 1/4$.



7.1.3. *Negatively correlated production plans.* Consider the case in which production plans are negatively correlated and the economy has no aggregate uncertainty. We take no-aggregate uncertainty to mean, $Var(I_{\mathbf{1}}) = Var(y_L + y_H) = 0$. This, in turn, is easily seen to imply that $c = -\sigma$, i.e. production projects are perfectly negatively correlated. Finally, $\beta_r^H = -1 = \beta_r^L$, $Cov\left(r, I_{\mathbf{1}}\right) = 0$, $\sigma_r = \sigma\left(2\delta - 1\right)^2$, where the fund variance is a function the composition of the fund with a minimum of zero at $\delta = 1/2$ (i.e. $0 \leq \sigma_r \leq \sigma$).

The reader can check that, for every economy with $\alpha\mu > 1$, and $\sigma > 0$, there exists an equilibrium in which entrepreneur L sells all his firm, $\zeta^L = 1$, and does not trade in the fund, $\theta^L = 0$. While entrepreneur H retains full control of her firm, $\zeta^H = 0$, and chooses $\theta^H = 1$. Since H does not sell her shares to the fund, the latter is possible only because H issues the bond. Finally, at equilibrium, $r = y^L$, $E\left(r_{\mathbf{1}}\right) = \alpha\mu$, $\sigma_r = \sigma$, and, assuming identical preferences, equation (21) yields, $q = \alpha\mu - 1 + a\sigma + \frac{\lambda^L}{a}$, where λ^L is the LM associated to $\theta^L \geq 0$.

Since in our example the return from investing in the fund coincides with the return from investing in firm L, the equilibrium displays adverse selection of the type first demonstrated by Akerlof: at equilibrium, only the *lemon* project is financed on the market, and the asset price reflects its true value.

Yet, here the result is somewhat an implication of our assumption that only two firm types exist in the economy. In the next example we show that with three firm types, non-degenerate pooling equilibria exist.

Let us modify the setup of our previous example as follows. There are three entrepreneurs, $j \in \{L, M, H\}$. L and H are as above, while entrepreneur M has a technology with $E\left(y_{\mathbf{1}}^M\right) = \alpha'\mu$, where $\alpha < \alpha' < 1$, and variance σ. Firms' projects have identical covariance, c. No-aggregate uncertainty, $Var\left(I_{\mathbf{1}}\right) = 0$, implies $c = -\sigma/2$. Since at an interior equilibrium (with $\theta^j > 0$ for all j), the fund price is $q = E\left(r_{\mathbf{1}}\right) - 1 + a\sigma_r$, we have $\theta^j = -\left(1 - \zeta^j\right)\beta_r^j$, where $\beta_r^j = (\delta^j - 2\sum_{k \neq j}\delta^k)/\sum_k(\delta^{k^2} - 2\delta^j\delta^k)$, for all j.

We check that a non-degenerate pooling equilibrium, with $\zeta^L = 1 = 1 - \zeta^H$, $0 < \zeta^M < 1$, exists in the context of a parametric example.

Again, letting $\alpha = .8$, $\mu = 3$, $\sigma = .5$, $a^j = 3$ for all j, and assuming $\alpha' = .9$, we find the following equilibrium:

$\zeta^H = 0, \zeta^M = .9141, \zeta^L = 1,$
$\theta^H = 1.9141, \theta^M = \theta^L = 0,$
$\sigma_r = .126, E\left(r_{\mathbf{1}}\right) = 2.543,$
$q = 1.571, \delta^H = 0, \delta^M = .478, \delta^L = .522,$
$\theta_0^H = -4.921, \theta_0^M = 2.350, \theta_0^L = 2.571,$
$E(x_{\mathbf{1}}^H) = 2.947, E(x_{\mathbf{1}}^M) = 2.582, E(x_{\mathbf{1}}^L) = 2.571,$
$Var(x_{\mathbf{1}}^H) = .960\,71, Var(x_{\mathbf{1}}^M) = .004, Var(x_{\mathbf{1}}^L) = 0$
$u^H = 8.823, u^M = 7.73, u^L = 7.7134, \sum_j u^j = 24.266$

The equilibrium is a non-degenerate pooling, with entrepreneurs L and M who sell shares of their firms to the fund, and entrepreneur H who is the only one buying the fund.

Welfare properties of equilibria. The equilibrium in the two-entrepreneurs example is Pareto efficient. With both entrepreneurs achieving full insurance: $Var\left(x_{\mathbf{1}}^H\right) = Var\left(x_{\mathbf{1}}^L\right) = 0$, and utility levels $u^L = \alpha u^H = a\alpha\mu$. Yet, not all the efficient allocations may be decentralized as competitive equilibria. For example, let us consider the egalitarian first best allocation, $\zeta^H = \zeta^L = \theta^H = \theta^L = 1$, and $\theta_0^L = \theta_0^H = 0$. At this allocation, efficient risk sharing for both individuals is achieved, with $\sigma_r = Var\left(x_{\mathbf{1}}^H\right) = Var\left(x_{\mathbf{1}}^L\right) = 0$, and utility levels are $u^L = u^H = a\frac{\mu}{2}\left(1 + \alpha\right)$. In order to decentralize this allocation the natural candidate equilibrium price is $q = E\left(r_{\mathbf{1}}\right) - 1 = \frac{\mu}{2}(1 + \alpha) - 1$. Yet, if asset re-trade were allowed, at this price q, entrepreneur H would lower the supply of shares in her firm below 1. This is because, unlike the planner, H does not internalize the effects of her financial decisions on the composition of the fund.

Equilibrium efficiency is a specific property of our two-agent economy with no aggregate risk, and perfectly negatively correlated projects. We are now going to show that if a third agent is introduced in this economy, the first welfare theorem need not hold.

Consider the three-entrepreneurs economy, with $j = L, M, H$. Under the same parameterization, the egalitarian Pareto efficient allocation yields an utility of $u^j = 8.1$ for all j. This is higher than the equilibrium utility for $j = L, M$, but lower than the equilibrium utility of H. Yet, since at the efficient allocation the aggregate utility is 24.3 (higher than the equilibrium aggregate utility, $24, 266$), a strict Pareto improvement can be achieved using lump sum transfers or, equivalently, through a centralized allocation of portfolios of the riskless bond. In other words, our example illustrates that equilibria may be Pareto inefficient exactly because they fail to provide full insurance.

7.1.4. *REE, information revelation, signaling.* We compare the equilibrium predicted by our model to the one obtained if we use the standard notion of rational expectations (see, for example, Grossman (1976)). At an (interior) fully revealing rational expectations equilibrium (REE) prices are

$q^j = E\left(y^j\right) - \overline{a} \sum_{k \in \mathcal{J}} Cov\left(y^j, y^k\right),$ for all $j \in \mathcal{J}$

If, we put ourselves in the context of section 7.1, the supply of equity by $j \in \{L, H\}$ is

$$\zeta^j = \frac{q^j - \mu^j + 1}{a^j \sigma^j} - \theta^j \beta^j$$

$\beta^j = Cov\left(y^j, y^{-j}\right) / \sigma^j$, and θ^j that denotes the demand of the equity issued from the other firm by agent j,

$$\theta^j = \frac{\mu^j - 1 - q^j}{a^j \sigma^j} - \left(1 - \zeta^j\right) \beta^j$$

Since, because of the assumption of no-aggregate uncertainty, $q^j = \mu^j - 1$, substituting q^j into the expression for ζ^j, yields $\zeta^j = \theta^j = 1/2$, $j \in \{L, H\}$, whenever entrepreneurs have identical coefficients of risk aversion ($a = a^L = a^H$). Equity issues do not convey any information about the quality of the investment projects. Moreover, $q^H > q^L$, so that the equilibrium is not incentive compatible (IC): if the market can only distinguish entrepreneurs from their supply of equities, L would obtain financing $q^H \zeta$, thereby destroying the separating equilibrium.

For example, under the parameters values used in (7.1.3), $c = -\sigma = .5$, $K = 1$, the REE in which two separate equity contracts $\{L, H\}$ are traded competitively has prices $q_H = 2$, $q_L = 1.4$, and payoffs $u_L = 7.2$, $u_H = 9$. As expected this equilibrium is a first best, and achieves a fully efficient diversification of risk, with $Var\left(x_{\mathbf{1}}^H\right) = Var\left(x_{\mathbf{1}}^L\right) = 0$. Yet, the equilibrium is not IC: by pretending to be of type H, entrepreneur L achieves a utility level of 8.1, higher than 7.2.

For the same parameter values, the equilibrium with the sectorial mutual fund has the following figures,

$\zeta^H = 0, \zeta^L = 1, \theta^H = 1, \theta^L = 0, \theta_0^H = -\theta_0^L = -2.4$

$\sigma_r = \sigma = .5, E\left(r_{\mathbf{1}}\right) = \alpha\mu = 2.4, q = 1.4$

$E(x_{\mathbf{1}}^H) = 3, E(x_{\mathbf{1}}^L) = E\left(r_{\mathbf{1}}\right) = 2.4,$

$Var(x_{\mathbf{1}}^H) = Var(x_{\mathbf{1}}^L) = 0$

$u^H = 9, u^L = 7.2,$

Our model can be easily extended to capture separating equilibria. Following Dubey et al. (1990) this can be achieved by indexing equity contracts with respect

to their bounds K^m, and allowing firm to sort among sectors. In our first example we could introduce two contracts, one with $K^1 = 1$ and the other with $K^2 = .1$. Then a separating equilibrium exists in which firm L sells the first contract and firm H sells the second, more constrained one. Of course at this equilibrium the prices of the two contracts will differ. Keeping the other parameters as above we obtain:
$\zeta^L = .9, \zeta^H = .1, \theta_0^H = -\theta_0^L = -1.86,$
$q^L = 1.4, q^H = 2,$
$E(x_{\mathbf{1}}^H) = 3, E(x_{\mathbf{1}}^L) = 2.4,$
$Var(x_{\mathbf{1}}^H) = 1, Var(x_{\mathbf{1}}^L) = 0$
$u^H = 9, u^L = 7.2$

To check that this is an equilibrium of our model, we must verify that the L firm does not prefer to go on the market for asset 2, and sell a share $\zeta^L = .1$ of its equity at the going price $q^H = 2$. Indeed the payoff of this choice is $u^L = 5.94$, which is less than its equilibrium payoff, $u^L = 7.2$. A similar argument shows that firm H has no incentive to go on market 1. The notion of incentive compatibility embodied in this definition of equilibrium is based on two ideas: i) contracts are *exclusive*, i.e. each firm can sell only one contract, and ii) individuals are competitive, i.e. when considering a deviation from the proposed equilibrium they take prices as given.

This analysis can be replicated in the context of the second example presented above. With three firms, there exists a REE with $\zeta^j = 2\theta_k^j = 1,\ j \neq k,\ j,k \in \{H, L, M\}$, and prices $q^H > q^M > q^L$, where θ_k^j denotes the demand of equity k by j. Using the above parameterization,
$\zeta^j = 2\theta_k^j = 1, k \neq j,$ and $j,k \in \{H, L, M\}$
$\theta_0^L = -.45, \theta_0^M = 0, \theta_0^H = .45,$
$q^L = .65, q^M = .95, q^H = 1.25$
$u^L = 6.075, u^M = 6.975, u^H = 7.875$
This equilibrium fails to be IC: by pretending to be of type H, entrepreneur L achieves a utility level of 7.875, higher than 6.075.

In this economy, an IC-REE exists with $K^1 = 1$, $K^2 = .1$, $K^3 = .05$:
$\zeta^L = .408, \theta_L^M = .217, \theta_L^H = .191$
$\zeta^M = \theta_M^L = .1, \theta_M^H = 0$
$\zeta^H = \theta_H^L = .05, \theta_H^M = 0$
$q^L = .4, q^M = 1.106, q^H = 1.481$
$\theta_0^L = .237, \theta_0^M = -.093, \theta_0^H = -.144,$
$u^L = 5.215, u^M = 6.205, u^H = 6.974$
This equilibrium is IC: by pretending to be of type H (M), entrepreneur L achieves a utility level of 5.089 (5.213), lower than 5.215; by pretending to be of type H, entrepreneur M achieves a utility level of 6.118, lower than 6.205; the remaining IC-constrains are also satisfied. Thus, at this equilibrium, firm H sells contracts of type 3, M of type 2, and L of type 1, and securities issuing are fully revealing.

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