Preamble-Based Channel Estimation for Filterbank Multicarrier Wireless Systems

Stijn Van Caekenberghe[1,2,3] Sofie Pollin[3] André Bourdoux[3]
Liesbet Van der Perre[2,3] Jérôme Louveaux[1]

[1] Université catholique de Louvain, Place du Levant 2, Louvain-la-Neuve, Belgium

[2] Katholieke Universiteit Leuven, Kasteelpark Arenberg 10, Heverlee, Belgium

[3] IMEC, Kapeldreef 75, Heverlee, Belgium

stijn.vancaekenberghe@student.uclouvain.be pollins@imec.be bourdoux@imec.be

vdperre@imec.be jerome.louveaux@uclouvain.be

Abstract

Three preamble-based channel estimation techniques are discussed that can be used in filterbank multicarrier systems. The first one was described by Lélé in [1] for FBMC with IOTA filters. The second one is based on the auxiliary pilot method described by Stitz in [2]. The last one is suggested in this paper. The three techniques are compared with BER curves as well as by looking at the origin of their errors.

1 Introduction

Filterbank multicarrier (FBMC) is a multicarrier modulation technique, just like OFDM. It provides better spectral containment than OFDM and illiminates the need for a cyclic prefix. The price of this all is an increased complexity. There are many flavors of FBMC, depending on the filters used in the filterbank. Recently, FBMC using wavelet filters has been standardized in IEEE 1901 for powerline communications. And back in 2003, FBMC using IOTA filters (Isotropic Orthogonal Transform Algorithm) was on the table of the 3GPP for LTE. In this paper however, we'll investigate channel estimation for FBMC using root raised cosine filters. This kind of FBMC was used in the European FP7 Phydyas project.

2 Filterbank Multicarrier Systems

In FBMC, the filterbank is put in a transmultiplexer configuration. In this configuration, the synthesis filterbank (SFB) is located in the transmitter and the analysis filterbank (AFB) is located in the receiver. They are shown on figures 1 and 2. At the transmitter, the input QAM symbols (with a rate of $1/T$, T being the inverse of the subcarrier spacing) are first converted to OQAM. After this conversion the symbol rate is $2/T$. Their sample index will be denoted by n. In this case the symbols are denoted by $d^R_{k,n}$ before, and by $d_{k,n}$ after the multiplication with $\theta_{k,n}$ (k being the subcarrier number). This multiplication makes sure that the symbols $d_{k,n}$ are alternately purely real or purely imaginary (since $d^R_{k,n}$ is purely real). In the receiver we have a similar case. The received OQAM symbols are denoted by $x_{k,n}$, and by $x^R_{k,n}$ after multiplication with $\theta^*_{k,n}$ and taking the real part. At the output of the transmitter the symbol rate is M/T, where M is the number of subchannels. In this case, the sample index will be denoted by m. The output will be denoted by $s[m]$

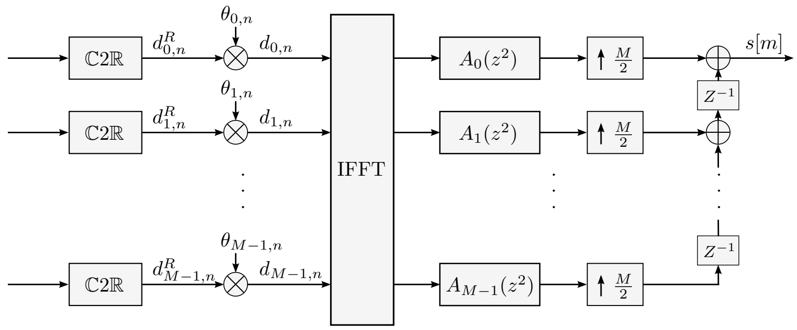

Figure 1: Synthesis filterbank in the transmitter

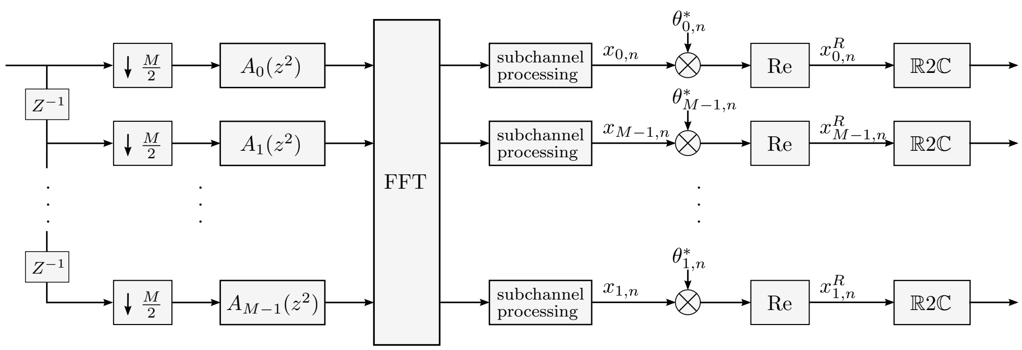

Figure 2: Analysis filterbank in the receiver

At the output of the SFB we have:

$$
\begin{aligned}
s[m] &= \sum_{k=0}^{M-1} \sum_{n=-\infty}^{\infty} d^R_{k,n} \theta_{k,n} a \left[m - \frac{nM}{2} \right] e^{\frac{j2\pi}{M} km} \\
&= \sum_{k=0}^{M-1} \sum_{n=-\infty}^{\infty} d_{k,n} a \left[m - \frac{nM}{2} \right] e^{\frac{j2\pi}{M} km}
\end{aligned}
\tag{1}
$$

where

$$\theta_{k,n} = j^{k+n \mod 2} \tag{2}$$

and $a[m]$ being the prototype filter taps. In this paper, the prototype filter $A(z)$ is a root raised cosine filter as defined in [3]. The length of this filter is equal to KM, with K being called the overlapping factor. The polyphase decomposition of this filter is defined by:

$$A(z) = \sum_{k=0}^{M-1} z^{-k} A_k(z^M) \tag{3}$$

To be closer to the implementation, this formulation of the FBMC system is slightly different than what is common. Because of this formulation however, the symbols are reordered at the output of the receiver (as can be seen on figure 2). In case of an ideal

	$n-3$	$n-2$	$n-1$	n	$n+1$	$n+2$	$n-3$
$k-1$	$-0.0429j$	-0.1250	$0.2058j$	0.2393	$-0.2058j$	-0.1250	$0.0429j$
k	-0.0668	0.0002	0.5644	1.0000	0.5644	0.0002	-0.0668
$k+1$	$0.0429j$	-0.1250	$-0.2058j$	0.2393	$0.2058j$	-0.1250	$-0.0429j$

Table 1: Transmultiplexer response for $K = 4$ and even k

channel, the output symbols $x_{k,n}$ of the FFT for the nth symbol and the kth subcarrier are

$$x_{k,n} = \sum_{k'=0}^{M-1} \sum_{n'=-\infty}^{\infty} d_{k',n'} t_{k-k',n-n'} \tag{4}$$

In this equation, $t_{k,n}$ is the transmultiplexer response. This response is determined by the prototype filter $A(z)$ in use. An example transmultiplexer response is given in table 1. Most of the interference coming from the transmultiplexer response of one data symbol on another one is removed by only selecting the real (imaginary) part, as the $d_{k,n}$ that were originally sent were alternately purely real or purely imaginary. Note however, that there is actually a little intersymbol interference (ISI) happening at $t_{k,n-2}$ and $t_{k,n+2}$ due to the fact that this is a Near Perfect Reconstruction (NPR) filterbank. This ISI will increase on frequency selective channels.

3 The Pairs of Pilots Preamble

Before starting with the analysis of the preamble-based channel estimation techniques, some new symbols need to be defined. The preamble at the transmitter side will be denoted by $p_{k,n}$ (located before the IFFT in figure 1). The received preamble will be denoted by $y_{k,n}$ (located after the FFT in figure 2). The $p_{k,n}$ of the preambles under discussion are shown on figure 3. The preamble-based channel estimation techniques will be compared by using a 1-tap Zero Forcing (ZF) equalizer.

The first preamble to be discussed is the Pairs of Pilots preamble on figure 3(a). This preamble is described in [1]. On a noiseless channel the received preamble would be:

$$y_{k,0} = \begin{cases} H_k \left(p_{k,0}^R + j u_{k,0}\right) & \text{if } k \text{ is even} \\ H_k \left(u_{k,0} + j p_{k,0}^R\right) & \text{if } k \text{ is odd} \end{cases} \tag{5}$$

$$y_{k,1} = \begin{cases} H_k \left(u_{k,1} + j p_{k,1}^R\right) & \text{if } k \text{ is even} \\ H_k \left(p_{k,1}^R + j u_{k,1}\right) & \text{if } k \text{ is odd} \end{cases} \tag{6}$$

In these equations $u_{k,n}$ originates from the interference caused by the transmultiplexer response of symbols nearby. They do not create any error since they are alternately purely real or purely imaginary, while data symbols are originally purely imaginary or purely real. This only holds when $t_{k,n-2}$ and $t_{k,n+2}$ are negligible. When they are not, there will be real (imaginary) interference on a real (imaginary) data symbol, which causes ISI. In the case of a frequency flat (sub)channel, they are indeed negligible (as can be seen in table 1).

$$\begin{array}{ll} 1 & 0 \\ -1j & 0 \\ 1 & 0 \\ -1j & 0 \\ & \vdots \\ 1 & 0 \\ -1j & 0 \end{array}$$

(a)

$$\begin{array}{llll} 1 & 0 & 0 & 0 \\ -1j & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -1j & 0 & 0 & 0 \\ & & \vdots & \\ 1 & 0 & 0 & 0 \\ -1j & 0 & 0 & 0 \end{array}$$

(b)

$$\begin{array}{ll} 1 & p_0^{aux} \\ 0 & 0 \\ 1 & p_2^{aux} \\ 0 & 0 \\ & \vdots \\ 1 & p_{M-2}^{aux} \\ 0 & 0 \end{array}$$

(c)

$$\begin{array}{llll} 1 & 0 & 0 & 0 \\ -1j & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1j & 0 & 0 & 0 \\ & & \vdots & \\ 1 & 0 & 0 & 0 \\ 1j & 0 & 0 & 0 \end{array}$$

(d)

Figure 3: $p_{k,n}$ of the different preambles: (a) Pairs of Pilots Preamble; (b) Pairs of Pilots Preamble with extra zeros; (c) Preamble with the Auxiliary Pilot; (d) Enlarged Preamble

The ZF equalizer taps can now be calculated. For even k we have ($p^R_{k,0} = 1$, $p^R_{k,1} = 0$):

$$\frac{1}{H_k} \approx \frac{p^R_{k,0} y^*_{k,1} + j p^R_{k,1} y^*_{k,0}}{\mathrm{Re}\left(y_{k,0} y^*_{k,1}\right)} = \frac{y^*_{k,1}}{\mathrm{Re}\left(y_{k,0} y^*_{k,1}\right)} \tag{7}$$

For odd k this is ($p^R_{k,0} = -1$, $p^R_{k,1} = 0$):

$$\frac{1}{H_k} \approx \frac{p^R_{k,1} y^*_{k,0} + j p^R_{k,0} y^*_{k,1}}{\mathrm{Re}\left(y_{k,0} y^*_{k,1}\right)} = \frac{-j y^*_{k,1}}{\mathrm{Re}\left(y_{k,0} y^*_{k,1}\right)} \tag{8}$$

A first remark is that the denominator of these fractions can become quite small. This is because for some combinations of $p_{k,n}$ and data symbols $d_{k,n}$ the real part of $y_{k,0} y^*_{k,1}$ can be very small, even when the amplitudes of $y_{k,0}$ and $y_{k,1}$ are big. A considerable noise enhancement will occur due to this. A second remark is that in real life the (sub)channel will be frequency selective. Because of this selectivity, the ISI cannot be ignored. Taking this ISI $u^{ISI}_{k,n}$ into account, the error ϵ on the estimated ZF equalizer taps for even k can be expressed as:

$$\epsilon = \frac{u^{ISI}_{k,0} y^*_{k,1} + j u^{ISI}_{k,1} y^*_{k,0}}{\mathrm{Re}\left(y_{k,0} y^*_{k,1}\right)} \tag{9}$$

It's important to note that this error can have a large absolute value due to the often very small denominator. That's why in the plot of the equalization taps on figure 4 (upper right corner) large peaks occur for the Pairs of Pilots estimation on a subchannel by subchannel basis. A first measure to cope with these large peaks could be a low pass filter (LPF). By using such a low pass filter however, subchannels in the neighbourhood will get influenced by this large peak that was originally concentrated on one sole subchannel. This is illustrated on figure 4 (lower right corner) for a second order Butterworth LPF. A better measure is the removal of the peaking estimate and replacing it with an interpolated value of the two neighbouring subchannels. For doing that, a peak detection algorithm is needed. An easy algorithm looks whether the absolute value of the estimate is more than a certain threshold ψ larger than the absolute value of the estimates of the neighbouring subchannels. This is illustrated on figure 4 (lower left corner) for $\psi = 0.2$. This peak detection algorithm (and the ψ-value) can be optimized, but this is not the focus of this paper. The only goal here is to understand the source of the errors in the Pairs of Pilots estimation.

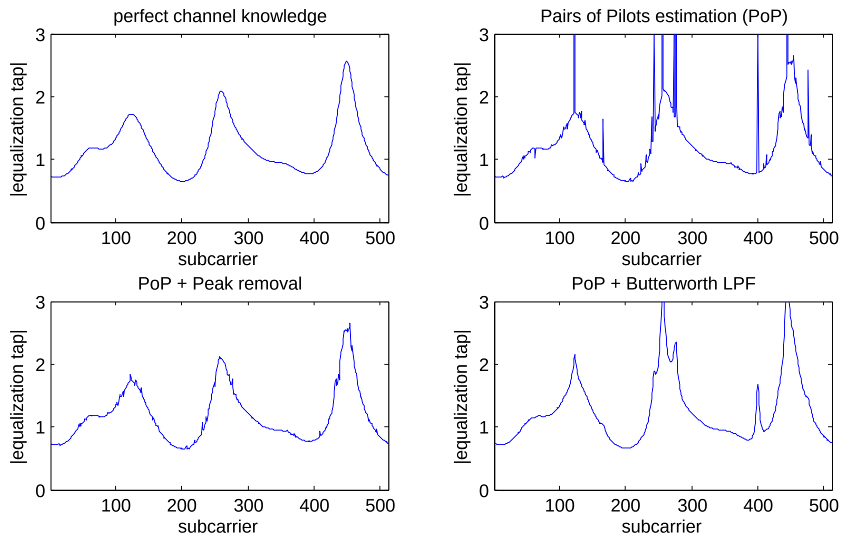

Figure 4: Equalizer taps for Pairs of Pilots estimation

The bit error rate (BER) curve on figure 5 shows the bad performance of the Pairs of Pilots estimation. The BER curve was also plotted for when the preamble is enlarged with zeros, as on figure 3(b). This case is very interesting, because the ISI error of (9) is negligible in this case. It only suffers from noise enhancement. By comparing this with the original one, it can been seen that up to an E_b/N_0 of 10 dB, the Pairs of Pilots estimation is mainly effected by noise enhancement. From $E_b/N_0 = 10$ dB on, the ISI error is dominant. The peak removal only shifts this value to 15 dB, it's unable to avoid the ISI error.

Because of the many drawbacks of the Pairs of Pilots estimation (ISI and noise enhancement), it's clear that better solutions are needed. The next two preamble estimation techniques will prove to perform better.

4 Preamble with Auxiliary Pilot

A second preamble technique can be derived from the auxiliary pilot method explained in [2], and is illustrated on figure 3(c). The second symbol of this preamble, $p_{k,1}$, is used to remove the (predicted) interference on the first symbol $p_{k,0}$ coming from the transmultiplexer response of the data symbols following the preamble. This second symbol is called the auxiliary pilot, denoted by p_k^{aux}, and is calculated by looking at the first 2 data symbols on the same subchannel and on the neighbouring subchannels (this is $d_{k,0}$, $d_{k,1}$, $d_{k-1,0}$, $d_{k-1,1}$, $d_{k+1,0}$, $d_{k+1,1}$). Note that this is only done for even carriers. Otherwise the complexity would increase dramatically as for the calculation of one auxiliary pilot, the neighbouring auxiliary pilots should be taken into account and vice versa.

On a noiseless channel the received preamble would be:

$$\begin{aligned} y_{k,0} &= H_k p_{k,0}^R \\ y_{k,1} &= H_k \left(u_{k,1} + j p_{k,1}^R \right) \\ &= H_k \left(u_{k,1} + p_k^{aux} \right) \end{aligned} \tag{10}$$

This is of course again under the assumption that the channel is frequency flat.

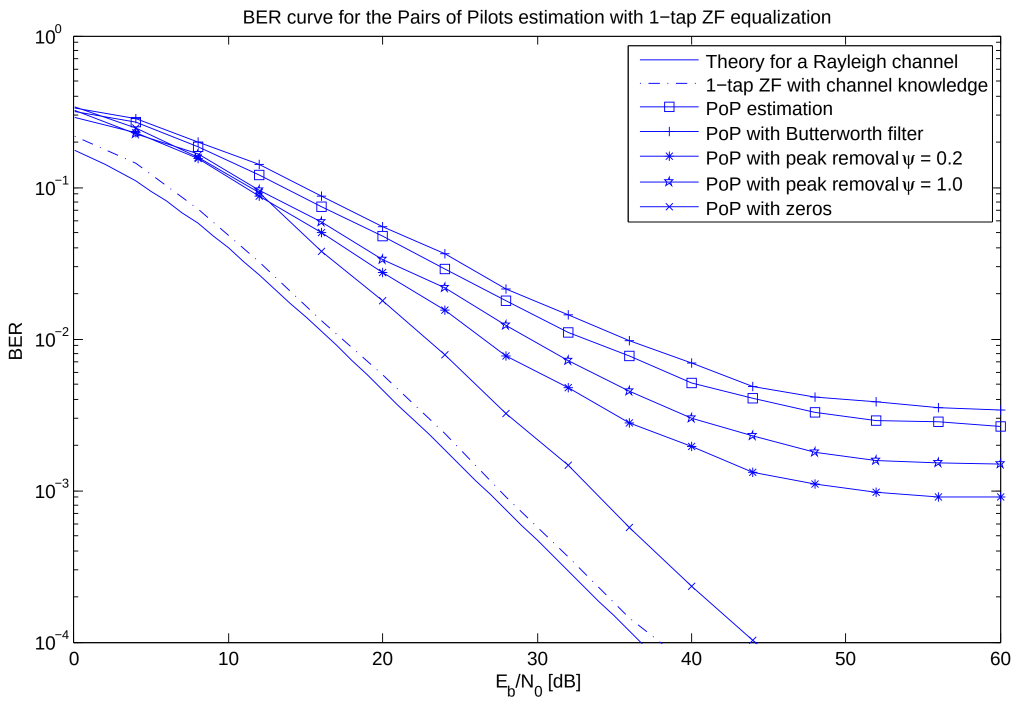

Figure 5: BER curve for the Pairs of Pilots estimation

With $p_{k,0}^R = 1$, the ZF equalizer taps for even k can be calculated as:

$$\frac{1}{H_k} \approx \frac{1}{y_{k,0}} \tag{11}$$

The ZF equalizer taps for odd k are calculated by interpolating the ZF equalizer taps from the neighbouring even k subcarriers. The error ϵ caused by ISI is in this case is:

$$\begin{aligned} \epsilon &= \frac{1}{H_k} - \frac{1}{H_k\left(1+u_{k,0}^{ISI}\right)} \\ &= \frac{u_{k,0}^{ISI}}{H_k\left(1+u_{k,0}^{ISI}\right)} \end{aligned} \tag{12}$$

By comparing at the previous formula and (9), it can be concluded that this channel estimation method suffers from much less error caused by ISI than the PoP method does. This is also confirmed on figure 7, where the BER curve doesn't have that saturation that is caused by the ISI error. There is also no noise enhancement. There is, however, an error that is caused by $u_{k,1}$, which is not entirely removed by the auxiliary pilot on a frequency selective channel. This error was not present in the case of the Pairs of Pilots estimation. But its effect is rather small compared to the ISI error and noise enhancement error of the latter.

5 Enlarged Preamble

Instead of trying to cope with the ISI error, it can also be avoided by enlarging the preamble as on figure 3(d). This decreases the complexity at the transmitter side,

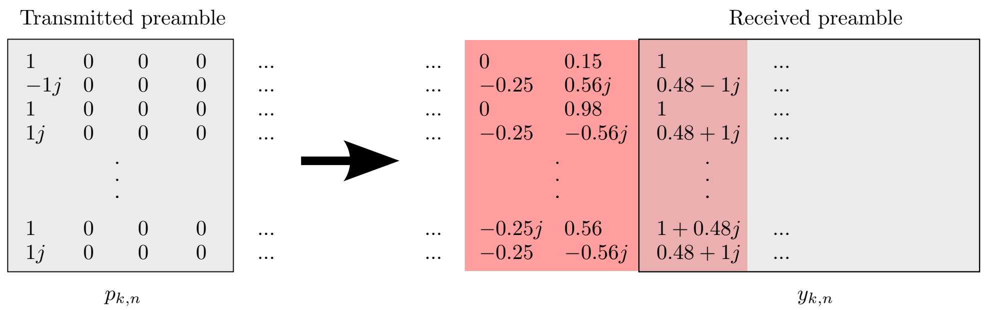

Figure 6: The symbols used for MRC when the Enlarged Preamble is used.

since no auxiliary pilot has to be calculated. The disadvantage is of course a longer preamble. Let $y_{k,0}^{ideal}$ be the received preamble on an ideal channel, then the channel can easily be estimated by:

$$H_k \approx \frac{y_{k,0}}{y_{k,0}^{ideal}} \tag{13}$$

Note that there is always a delay in the demodulation in the transmitter when using FBMC. The longer the filters are, the longer this delay is. For $K = 4$ for example, the demodulation can start at $n = 7$. The symbols at $n < 7$ can be used for channel estimation when this enlarged preamble is used. They don't suffer from ISI originating from the data symbols. In the case of $K = 4$, the most useful symbols are located at $n = 6$ and $n = 5$ (let's call them $y_{k,-1}$ and $y_{k,-2}$ respectively). Symbols at lower n are too small to make a difference.

On figure 6, an example of the $y_{k,n}^{ideal}$ values is given. The channel estimate can be improved by doing a maximum ratio combining (MRC) over the marked area in this figure:

$$H_k \approx \frac{\displaystyle\sum_{n=-2}^{1} y_{k,n} y_{k,n}^{ideal*}}{\displaystyle\sum_{n=-2}^{1} |y_{k,n}^{ideal}|^2} \tag{14}$$

On the BER plot of figure 7, the enlarged preamble doesn't seem to perform as well as the preamble with auxiliary pilot. This is caused by the energy normalization of the preambles. As the latter only uses its even subcarriers k, it is allowed to put more energy on them. When the enlarged preamble is modified to only use the even subcarriers, the performance is similar to the preamble with auxiliary pilot, as can be seen on figure 7.

The biggest error is again caused by $u_{k,n}$, which is not entirely taken into account by $y_{k,n}^{ideal}$ on a frequency selective (non-ideal) channel. But there is no significant ISI error or noise enhancement.

6 Conclusion

The auxiliary pilot and enlarged preamble techniques from sections 4 and 5 do not suffer from noise enhancement, because they do not a have real part in denominator

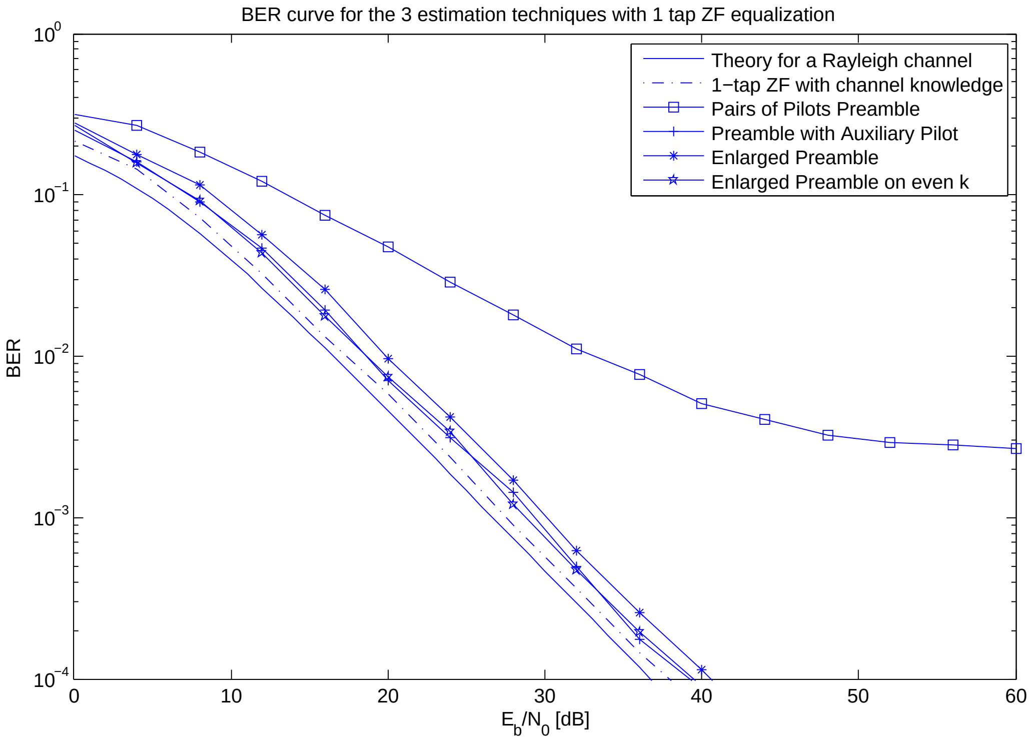

Figure 7: BER curve for the three preamble-based estimation techniques

(as is the case in the Pairs of Pilots technique). They also don't suffer as much from ISI as the first one. But in counterpart they suffer from $u_{k,n}$ interference. The complexity of the enlarged preamble technique is low, especially in the transmitter, at the cost of a longer preamble compared to the auxiliary pilot technique. Note that the PAPR of all the preambles can be improved by randomizing the sign of $p_{k,0}$.

References

[1] C. Lélé, J.-P. Javaudin, R. Legouable, A. Skrzypczak, P. Siohan, "Channel estimation methods for preamble-based OFDM/OQAM modulations", IEEE International Symposium on Power Line Communications and Its Applications, p 59 - 64 , March 2007.

[2] T. H. Stitz, T. Ihalainen, A. Viholainen, M. Renfors, "Pilot-Based Synchronization and Equalization in Filter Bank Multicarrier Communications", EURASIP Journal on Advances in Signal Processing, 2010.

[3] M. Bellanger, "Specification and design of a prototype filter for filter bank based multicarrier transmission", IEEE International Conference on Acoustics, Speech, and Signal Processing, p 2417 - 2420, May 2001.