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Urban Morphodynamics

Reconciling Location Theory and Complex Systems

Doctoral Dissertation presented by

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“Mais, moi, j’ai sans doute un peu plus voyagé, dit sèchement Humboldt. Et il lui assurait qu’il existait des rues plus pouilleuses encore. C’était une grande erreur de s’éloigner comme il l’avait fait lorsque tant de gens se réunissaient, des gens avec lesquels on pouvait mettre en place des projets.

Des projets, aboya Gauss. Des bavardages, des plans, des intrigues. Des palabres avec dix princes et cent académies jusqu’à ce qu’on obtienne l’autorisation de planter son baromètre quelque part. Ce n’était pas de la science, ça.

Tiens donc, s’écria Humboldt, et c’était quoi, la science, dans ce cas?

Gauss tira sur sa pipe. Un homme seul à son bureau. Une feuille de papier devant lui, à la rigueur une lunette astronomique et, devant la fenêtre, un ciel dégagé. Un homme qui n’abandonnait pas avant d’avoir compris. Ça, c’était peut-être de la science.

Et si cet homme faisait des voyages?

Gauss haussa les épaules. Ce qui se cachait au loin, dans des grottes, des volcans ou des mines, était aléatoire et insignifiant. Le monde n’en devenait pas plus clair pour autant.

[...]

Mais tandis que les premiers faubourgs de Berlin défilaient devant eux et que Humboldt imaginait Gauss en train d’observer au téléscope, à ce moment précis, les corps célestes dont il pourrait résumer la trajectoire grâce à des formules simples, il fut soudain incapable de dire lequel des deux était allé très loin et lequel était toujours resté chez lui.”

Daniel Kehlmann, *Les Arpenteurs du Monde*, Traduit de l’allemand par Juliette Aubert. Actes Sud (Babel), 2007, Paris, France.

I

Introduction and background literature

1

Introduction

This chapter introduces the context and motivation of the present thesis. Section 1.1 starts by describing what is scientific modelling, and what is its place in the process of policy-making. It emphasizes the distinction between predictive and explicative models, as well as the specific role of explicative (theoretical) models in the social sciences. More specifically, section 1.2 introduces the role of urban planning in addressing the global challenge of sustainable urban development, and the support provided by urban models in this purpose. It argues that microsimulation models, which have a growing success in applied research, currently present some operational drawbacks that can be fixed by an appropriate theoretical framework. Contributing to the development of such a theory is the purpose of this thesis. To this end, three mathematical models, addressing different issues of Urban Morphodynamics using different tools, are developed. Their research questions and their structure in the outline of this thesis are presented in section 1.3.

1.1 Context: models and policies

Although there is no standard agreement, a model can be defined as a purposeful reduction of reality's complexity in order to answer a set of questions (Minsky, 1965; Frigg and Hartmann, 2018). In that sense, modelling is a cognitive exercise that is commonly carried out by any one. What distinguishes scientific modelling is its formalism, which explicitly poses the assumptions and mechanics of its models. From the above definition, it is clear that modelling is an essential step in any decision-making process, and scientific modelling in particular seems relevant for the design of public policies (see Fig. 1.1). In that perspective, the main goal of models is to predict the consequences of a public policy in order to assess its success. Yet models have another fundamental role that is complementary to prediction: understanding.

Prediction is only one of the many ways models can support decision making (Epstein, 2008). Moreover, it has been known at least since the nineteenth century that "*knowledge insufficient for prediction may be most valuable for guidance*" (Mill, 1858, p.564). Nevertheless, *predictive* models remain often opposed to *explicative* models. This traditional opposition was particularly exposed during the fifties, in a popular debate opposing the two Nobel Prize-winning economists Paul Samuelson and Milton Friedman. At that time, Friedman published a book to argue in favour of predictive models (Friedman, 1953), whilst Samuelson defended the explicative models that had been used in Economics for decades. This opposition is better understood from a philosophical perspective. On the one hand, the predictive ability of a model is often increased by considering the specificities of the system under study. On the other hand, understanding the mechanics of a real system often require to abstract from the details in order to compare with similar empirical objects. Thus, from the perspective of philosophy of science, the predictive models often anchor in a idiographic approach of scientific knowledge, whilst explicative models are closer to a nomothetic approach. According to the terminology introduced by the Kantian philosopher Wilhelm Windelband in the late nineteenth century, an *idiographic* approach of science studies the specificities of its study object, whilst a *nomothetic* approach tends to generalize it. An illustration of these approaches is provided by the imaginary conversation quoted at the beginning of this thesis, which involves the mathematician Carl Friedrich Gauss and the geographer Alexander von Humboldt. Because of their nomothetic motivation, many explicative models are theoretical models since they provide the building blocks of scientific theories.

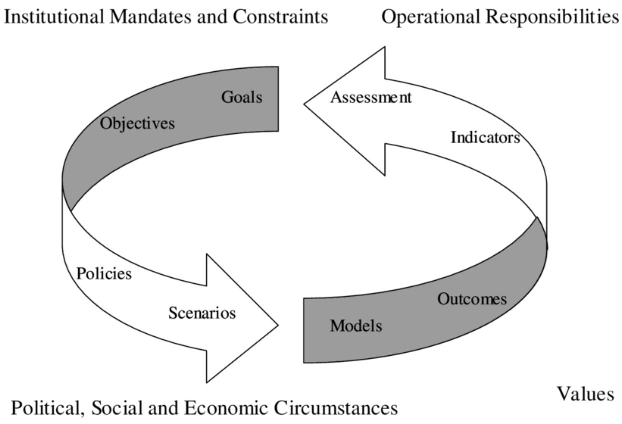

Figure 1.1. Models in the policy process, from Waddell and Ulfarsson (2004). Modelling takes part in an iterative process of policy making. On the one hand, political institutions set up long-term goals and particular objectives for which they formulate policies whose effects are embedded into anticipated scenarios. On the other hand, models explore the effects of theses scenarios on important outcomes, and produce indicators that are used to assess the achievement (or not) of initial goals. Most of the times, initial goals have to be adapted and and the policy making process starts again.

But what is a theory? In Mathematics, a theory is a set of theorems, and theorems are "*statements deduced in a purely logical way from previously chosen and fixed statements called axioms*" (Plisko, 2002). By analogy with this definition, theoretical models in general can be defined as statements deduced in a purely logical way from previously chosen and fixed statements called assumptions. In that perspective, a set of interrelated theoretical models is a theory. For example, some of the theoretical models presented in the next chapter constitute the monocentric city theory. Nevertheless, this analogy should not be regarded as an equivalence. As purely abstract statements, mathematical theorems cannot (and does not need to) be supported by empirical evidence. This is not the case of theories in other sciences, whose essential feature is, according to the philosopher Karl Popper, to be falsifiable by empirical experiment. In that sense, theories in social sciences are even more specific since, contrary to most of the natural sciences, their study topic cannot be isolated and directly experimented. Moreover, reliable natural experiments are rare. Thus, theoretical models have a unique role in social sciences. They are used as "virtual laboratories", which can take the form of a set of equations or a computer program, and where the reality is simplified in order to conduct unbiased experiments (Simon, 1969). That role is essential in supporting decision making because if predictive models provide a (hopefully) reliable description of possible futures, explicative models explain why a particular future is expected in some particular conditions. This understanding of causal chains is fundamental to identify the levers of action that are within the reach of political action. Note that the impossibility to isolate the experimental object from the experimenter also explains why in social science, qualitative and quantitative theories coexist. Fundamentally, quantitative and qualitative models are only differentiated by the language they use to formalise scientific modelling (Papageorgiou, 1982). Quantitative models are based on mathematical, statistical or computational techniques (Donmoyer, 2008), whilst qualitative models often use text, narrative or visual supports (Given, 2008; Glesne, 2010).

The importance of models in policy making applies to almost all regulated structures of modern civilizations, including cities. In the following section, the societal challenge posed by the management of cities around the world is presented, as well as the role played by urban planning and urban modelling in tackling the related issues. In particular, it is argued that current limitations of predictive urban models can be overcome by specific explicative models.

1.2 Motivation: urban models in support of urban planning

In 1987, the World Commission on Environment and Development of the United Nations published the Brundtland report, entitled "*Our Common Future*" (WCED, 1987). This report resulted from three years of work to better understand the interactions between environmental changes and human activities. It called for a major shift in human development practices in order to address environmental, social and economic issues on an equal basis, a new paradigm it named "sustainable development". Although the identification of the threat posed by unbalanced human development was already a huge step, in the last thirty years it has appeared that countering it may be an even greater challenge. In order to tackle this challenge, in 2015, the United Nations set up the 2030 Agenda for Sustainable Development, which contains 17 Sustainable Development Goals (United Nations, 2015). Implementing these goals is an incredible task, which necessitates a coordinated worldwide effort in which cities have a central role to play.

Indeed, around the years 2015 and 2016, 54% of the world population was urban, and this population was responsible for 80% of the global gross domestic product (UN-Habitat, 2016). Yet at the same time, urban citizens around the world produced 70% of the world's carbon dioxide emissions, and 90% of them breathed air that did not meet the World Health Organization's air quality guidelines value for particulate matter (UN-Habitat, 2016; United

Nations, 2017). This overwhelming weight in human activity explains why the 11th Sustainable Development Goal of the United Nations is to make cities more sustainable (United Nations, 2017). Sustainable cities ought to be "*an environmentally, socially, and economically healthy and resilient habitat for existing populations, without compromising the ability of future generations to experience the same*" (ICLEI, 2017). There are many ways to achieve sustainable urban development, and the last thirty years have seen the emergence of a particularly large number of decentralised initiatives in this direction (UN-Habitat, 2016; Worldwatch Institute, 2016). Yet older institutional tools also reinvent themselves in a fruitful way, which is the case of urban planning.

Urban planning was institutionalized at the end of the nineteenth century, in reaction to the rapid and chaotic growth of industrial cities (Batty, 2008). Initially, most urban planners were seeking for a rational, regular and geometric planning that split up urban spaces into distinct functional zones, like the "Radiant City" of Le Corbusier (1933). Thus, up to the sixties, urban planning was essentially about imposing a rational urban plan using zoning land uses and transport policies (Batty, 2008; Marshall, 2012a). From the sixties, critics arose against this top-down traditional conception of urban planning. The segregation of land uses and transport modes started to be regarded as alienating the urban life, limiting its intrinsic richness (Jacobs, 1961; Alexander, 1965a,b). A new approach of urban planning arose whose task "*became less one of producing the simple order of 'rational' urban plans, but one of how best to generate and maintain the functional complexity – or complex functionality – traditionally possessed by cities*" (Marshall, 2012a, p.192). This view of a more integrated and participative urban planning, which accommodates of mixed land uses, has gained popularity in the last thirty years, generating movements like the New Urbanism (Duany and Plater-Zyberk, 1991; Aldous, 1992; Katz, 1993; Calthorpe, 1995; Hebbert, 2003; Marshall, 2008; Moroni, 2015). In particular it is now seen as a tool for achieving sustainable urban development (Frey, 1999; Williams et al., 2000; Jenks et al., 2003; Mander et al., 2006; Moroni, 2007; Banister, 2008; Jenks and Jones, 2009; Jha et al., 2013; Bierlaire et al., 2015; Bertolini, 2017). Yet, in that perspective, additional efforts are needed. In particular, some authors have recently argued that a novel understanding of urban dynamic processes, based on rigorous scientific research, has to support those new movements of urban planning (Batty and Marshall, 2012; Marshall, 2012b). This is where urban modelling comes in.

Just as urban planning, urban modelling also underwent a conceptual shift in the last fifty years, which can be summarized as a shift from "*macro-statics to micro-dynamics*" (Batty, 2008). From the fifties, urban models designed to support policy making were essentially restricted to transport models, which were extended to land use and transport interaction (LUTI) models during the sixties, after the seminal work of Lowry (1964). Those models were mainly predictive models. However, the paradigm shift occurring in urban planning after the sixties went along with a shift in modelling practices (it is actually hard to establish which one occurred first, and how those movements mutually influenced each other). From a methodological perspective, the increasing computing power and the new interest for the self-organizing dynamic of cities have supported the development of a new modelling tool: the microsimulations.

In brief, microsimulations are the modern forms of the virtual laboratories mentioned above. Starting from functional units such as spatial cells (in cellular automata) or "individuals" (in agent-based models), microsimulations simulate their dynamic interactions, between them and with their environment, in order to study emerging collective dynamics. This method is intuitive and adaptive, which makes it suitable for the development of both predictive or explicative models. Thus, because of the applied problems raised by urban planning, microsimulations are more and more used in predictive models of urban development (Waddell and Ulfarsson, 2004; Wegener and Fuerst, 2004; Wegener, 2004; Batty, 2008; Ortùzar and Willumsen, 2011; Acheampong and Silva, 2015). Yet this approach has some limitations.

A first problem, inherent to many predictive models, is that their predictive capacity often results from the multiplicity of the involved processes. These processes add to the models large numbers of variables and parameters, which raise two difficulties. First, there is a risk of over-calibration in the sense that large numbers of free parameters yield numerous degrees of freedom from which almost any empirical result can be calibrated. This problem is nicely summarized by the mathematician John von Neumann, which has been quoted by the physicist Enrico Fermi, in saying "*with four parameters I can fit an elephant, and with five I can make him wiggle his trunk*" (see Dyson, 2004).[1] Second, large numbers of parameters prevent from exploring the model behaviour with sensitivity analysis. Indeed, the number of points to be explored in the parameters space is an exponential of the parameters sampling size, whose power is the total number of parameters. Thus, sensitivity analysis quickly became intractable.

A second problem, which is exposed by White et al. (2015c) on the basis of their experience in using cellular automata to support land use policy making in Belgium, is the validation procedure. In order to formulate realistic predictions, microsimulations are stochastic systems that can provide several

[1]Note that this well-known quote has now a strict sense since Mayer et al. (2010) have fitted an elephantine shape using a Fourier coordinate expansion with four complex parameters.

predictions for a single parametrization. This multiplicity may correspond to a real diversity of possible futures, such that an *ex-post* validation procedure should not penalize it (as long as the actual system behaviour was in the set of predicted trajectories). Thus dedicated validation procedures are required. Although this technical discussion is not to be developed here, one may note that Marks (2007) calls "complete but inaccurate" those models that reproduce all the observed behaviours of a complex system, along with some unobserved ones.

Altogether, large numbers of parameters and the multiplicity of predicted outcomes strive against further applications of microsimulation models in supporting policy making, because they make them suspicious to policy makers. However, those problems could be addressed by improving the explicative feature of these urban models, that is, to support them with dedicated theoretical models that derive the models' behaviours from explicit micro-assumptions.

Suppose that such theoretical model exists. Then its parameters have a clear interpretation, which is derived from behavioural assumptions, and they are related to each other by formal relationships (described, for example, by mathematical expressions). First, this tackles the problem of overcalibration, not by reducing the number of parameters, but by limiting their freedom. Indeed, the clear interpretation of parameters enable to confront them to empirical data and to quickly distinguish irrational values. Second, it reduces the dimensionality of the parameters space to be explored in sensitivity analysis, because formal relationships between parameters delineate sub-spaces out of which the model's behaviour is irrelevant. Third, it also opens new possibilities of model validation. Indeed, the validation can now be made on the basis of explicit behavioural assumptions, for which data are often easier to get. Finally, a clear exposition of the mechanics of the model not only reduces uncertainty on its validity domain, but it also enables to identify new levers of policy actions. For all those reasons, an adequate theoretical framework would substantially reinforce the use of predictive models in operational research.

The possibility of such theoretical framework to be developed is explored in this thesis. Its object of study is the evolution of the spatial structure of an urban system from elementary interactions between its components. By analogy with physical geography (Wright and Short, 1984; Murray et al., 2009), this research topic is called Urban Morphodynamics. In the following chapters, three mathematical models of cities are proposed that provide new insights on Urban Morphodynamics by using different representations of urban morphology and different dynamic assumptions. The structure of these chapters, as well as their specific research questions, are presented in the next section.

1.3 Objective and outline of the thesis

The general objective of this thesis is to contribute to a new theoretical approach of Urban Morphodynamics that would enable urban modelling to address the challenges raised by new urban planning practices. To this end, it presents three theoretical models of Urban Morphodynamics which, according to the definition, have three distinct features: their explicit consideration of space, their explicit consideration of time and their explicit consideration of individuals, either isolated or grouped into subsystems. The works presented in this thesis are all anchored in explicit behavioural assumptions at the micro-level. They will, however, address different research questions, using different representations of space (hence urban morphology) and time. Note that this thesis anchors in quantitative theories of urban geography and so, unless otherwise specified, in the following the word "models" implicitly refers to quantitative models. The contributions of this thesis are organized in seven chapters, including this introduction, which are organized in three parts.

Part I consists in two chapters. This introductory chapters aims at presenting the societal motivation of this thesis, its general objective and its structure. Chapter 2 is a literature review of the quantitative theories of Urban Geography from the perspective of Urban Morphodynamics. Using a chronological structure, it argues that the two main strands of this literature, which are Location Theory and Complex systems, share a common conception of how urban spatial structures emerge from agglomeration and dispersion forces that are, ultimately, related to spatial frictions and spatial interactions. Although both approaches show some conceptual and methodological divergences, this common foundation is an opportunity to conciliate the domain knowledge of Location Theory with the modelling framework of Complex Systems in an integrated theory. Consequently, the four next chapters will address specific research questions that arise when comparing those two disciplines, and propose theoretical models that gather from both of them. They are organised following spatial and temporal axes (Fig. 1.2).

In Part II, two chapters present two different models where urban dynamics is static (or quasi-static) and deterministic. First, Chapter 3 addresses the question of whether scaling regularities in urban monocentric structures can be explained by the classic monocentric city model of Urban Economics? To answer this question, it introduces power laws in the monocentric city model of Alonso (1964). Results show that this augmented model is compatible with observed scalings of urban land and population density profiles, and that it satisfactorily represents European cities. Second, Chapter 4 addresses the question of which non-circular and non-monocentric urban configurations can emerge in the non-monocentric city model of Urban Economics in a dynamic context with heterogeneous adjustment speeds. To do so, a dynamic urban model is proposed, based on an agent-based implementation of the non-monocentric city model of Fujita and Ogawa (1982). Results show that under strong agglomeration effects, urban development is monotonic and ends up with circular, monocentric long-term configurations whilst for low agglomeration effects, elongated and multicentric urban configurations may emerge.

In Part III, new mathematical tools are used to study non-monocentric configurations in a dynamic and stochastic setting (Fig. 1.2). In particular, a new modelling framework is introduced that enables to catch the self-organizing and chaotic features of Complex Systems, whilst including advanced features of Urban Economics and, in particular, being completely valid from a decision-theoretic perspective. The general framework, which can be regarded as a dynamic model of collective discrete choices, is presented in Chapter 5. It gathers from Discrete Choice Theory, Synergetics and Stochastic Calculus in order to reduce the collective dynamics of agents to a set of Itô stochastic differential equations whose diffusion and drift coefficients are explicitly related to individual variables. The perspectives of empirical application of this framework are also discussed. Chapter 6 addresses the question of what is the influence of heterogeneous preferences on the adjustment dynamic of New Economic Geography models. In order to answer this question, the developed stochastic modelling framework is applied to the footloose entrepreneur model of (Forslid and Ottaviano, 2003). Results highlight the existence of noise-induced transitions in the interregional distribution dynamic of high-skilled workers. Using stochastic notions of equilibrium selection and stability, it proposes a distinction between different dynamic scenarios that refines the predictions of the deterministic model.

Finally, in Part IV, Chapter 7 comes back on the global objective of this thesis, which is to contribute to a new theoretical approach of Urban Morphodynamics. It brings a new regard on the results of chapters 3 to 6, in light of the convergent formalism of Location Theory and Complex systems that has been underlined in chapter 2. Especially, it argues that these chapters sketch a progression towards an common formalism of Urban Morphodynamic theory. To conclude, this chapter highlights the research difficulties that remain to be overcome before using this modelling framework in the establishment of an operational urban morphodynamic theory.

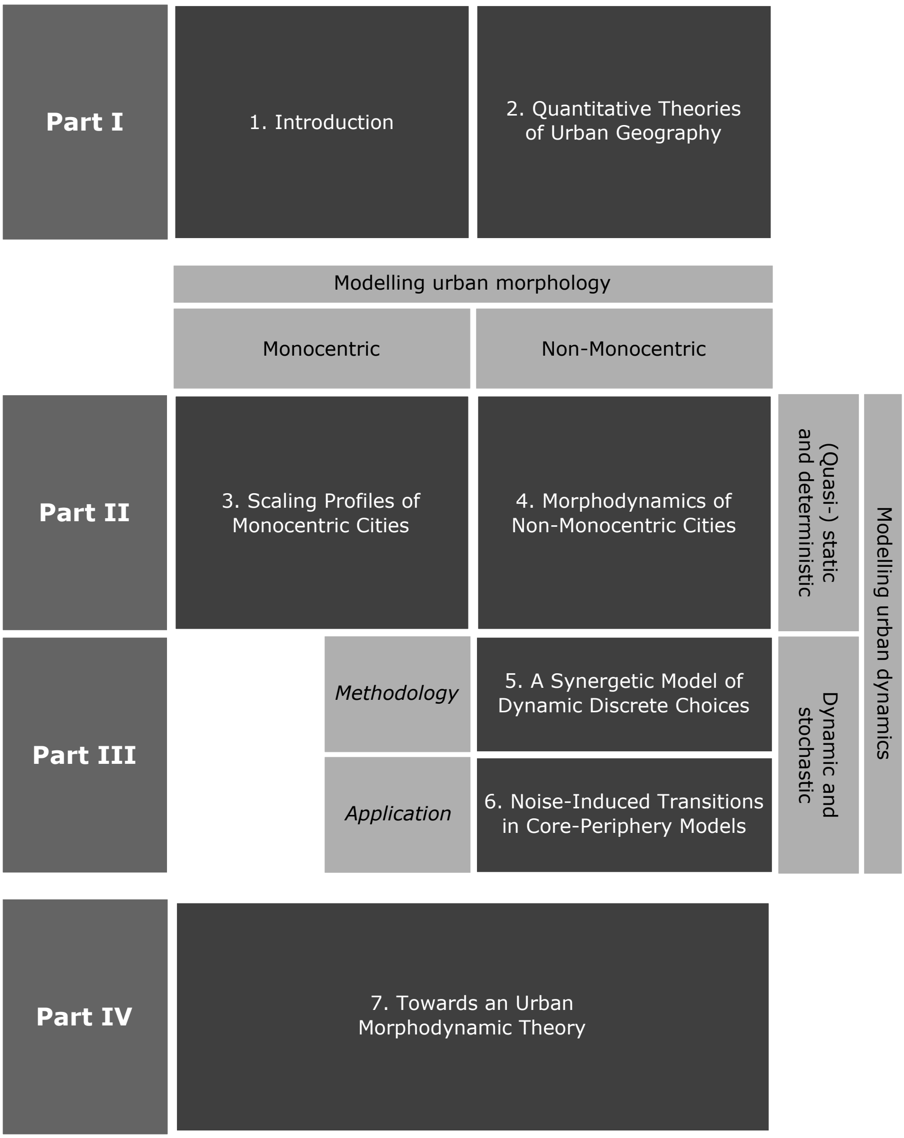

Figure 1.2. **Schematic outline of the thesis.**

2

Quantitative Theories of Urban Geography

This chapter reviews the literature of theoretical models in quantitative Urban Geography from the perspective of Urban Morphodynamics. Thus, the models presented below have been selected for their influence on how space, time or microbehaviours have been treated in urban modelling. The chapter is organized chronologically in order to follow the evolution of modelling practices over more than a century and to highlight biases due to the historical context. More precisely, section 2.1 presents the early developments of Location Theory up to the fifties. Afterwards, section 2.2 develops its modern disciplines, Urban Economics and New Economic Geography, whilst section 2.3 exposes the literature of Complex Systems, starting from Social Physics and Spatial Interaction Theory. Finally, section 2.4 compares Location Theory and Complex Systems from the perspective of Urban Morphodynamics. In turns out that although they share a common conception of how urban spatial structures emerge from agglomeration and dispersion forces, both fields evolved differently after the fifties because of disciplinary and motivational differences. Recently, urban microsimulation models have opened new perspective of reconciling them, but they lack of coordination in this purpose. The present thesis aims at addressing this issue by developing integrated mathematical models of urban morphodynamics.

2.1 Early developments of Location Theory

The birth and early developments of Location Theory (Fig. 2.1), which took place before 1950, can be summarized to three main contributions (see Ponsard, 1983; Fujita, 2010; Capello, 2014; White et al., 2015a): the seminal agricultural land use model of von Thünen (1826), the industrial location models of Weber (1909) and Hotelling (1929), which gave birth to Spatial Competition Theory, and finally the Central Place Theory of Christaller (1933) and Lösch (1940).

In the early nineteenth century, the Prussian Johann Heinrich von Thünen proposed a formal theory of the spatial structure of agricultural land uses. His work has spread far beyond Location Theory and his influence on the development of Geography (Grotewold, 1959; Sinclair, 1967; Portugali, 1984; Huriot, 1994; O'Kelly and Bryan, 1996) and Economics (Samuelson, 1983; Dorfman, 1986; Fujita, 1989; Fujita et al., 1999b; Fujita, 2010) cannot be overestimated. For example, regarding Economics, Paul Samuelson wrote that von Thünen "*not only created* marginalism *and* managerial economics*, but also elaborated one of the first models of* general equilibrium *and did so in terms of realistic* econometric *parameters*" (Samuelson, 1983, p.1468, original emphasis. See also Dorfman, 1986). A complete overview of his influence is thus beyond the scope of this chapter, which simply highlights the contributions that turned out to be fundamental in Location Theory.

In his book (von Thünen, 1826, see also Huriot, 1994, for a commented french translation of the 1842 edition), von Thünen proposed a set of simplifying assumptions known as the assumption of "Isolated State": a featureless homogeneous agricultural plane with a radial transport network organized around a single punctiform market. One can easily see the link with the flat lands of Prussia which were, at that time, essentially used to agricultural production. Yet this abstract formalism was incredibly anticipative. von Thünen further assumed that, due to different conservation properties of the agricultural products, travelling to the market centre costs more to some producers than others. Those heterogeneous costs limit their ability to pay for the land in a competitive market. As a result, "*land uses determine land values, through competitive bidding among farmers,* [and] *land values distribute land uses, according to their ability to pay*" (Alonso, 1960, p.153). The model outcome is that the land market structures agricultural land uses as concentric rings around the market centre. Thus, von Thünen (1826) proposed the first model of monocentric spatial structure of land uses.

Three aspects of this model were particularly relevant for the development of Location Theory. First, regarding the modelling assumptions, the abstract formalism of the Isolated Space enabled von Thünen to provide a first discussion of the complex notion of *accessibility* (Ben-Akiva and Lerman, 1979; Morris et al., 1979; Litman, 2003; Horner, 2004; Ferreira and Batey, 2007; Vandenbulcke et al., 2009; Geurs et al., 2010; Tannier et al., 2012), here reduced to the accessibility to a market centre. Hence this was a seminal example of the effect of transport cost, and more generally of heterogeneous accessibility levels, on land use spatial structure. Second, the modelling of the land market using the *bid rents* of competitive farmers will be used in an urban context more than a century later by Alonso (1964), in what constitutes the first model of Urban Economics (see Section 2.2.1).

In the second half of the nineteenth century, industrial activity became prominent in shaping the economic landscape and it naturally motivated a new step in the development of Location Theory. Thus, at the beginning of the twentieth century, the economist and geographer Alfred Weber (Weber, 1909, see also Weber, 1929) proposed a model of geographical location of industrial activities based on the minimization of total transport costs (to input locations, e.g. labour or natural resources, to final markets or even between production sites, see Fujita, 2010; Capello, 2014). His model inherited from von Thünen (1826) the formalization of geographical space as a featureless plane except for punctiform input and market locations. However, Weber took a major step towards endogenous co-location problems since he considered that the location that minimizes total transport costs has to be compared to alternative locations, the increase in transport cost being balanced by the existence of agglomeration economies (Capello, 2014). Although he did not expand on the nature of agglomeration economies, his intuition prefigured the interpretation of agglomeration economies and transport costs as competing centripetal and centrifugal forces, an interpretation that is at the heart of the New Economic Geography (see Section 2.2.2 and Fig. 2.1).

Twenty years latter, Hotelling (1929) studied the competition that occurs between firms in search of their optimal location. He considered that if consumer have to supply themselves from competing firms, then they will buy from the firm with the lowest total price (including transport cost). As a result, firms are local monopoles and they compete strategically when choosing their location and their prices (Thisse, 2011). Although Hotelling's results turned to be inaccurate (see d'Aspremont et al., 1979), his influence on Location Theory is undeniable. Hotelling (1929) revived the pioneer work of Launhardt (1882) on optimal location and price competitions. His work his often considered as a the first model of Spatial Competition Theory (Eaton and Lipsey, 1977; Gabszewicz and Thisse, 1986) and also as a pioneering discussion of the principle of differentiation in industrial organization (Rosen, 2002; Thisse, 2011).

During the thirties, just after the work of Hotelling (1929), the German geographer Walter Christaller proposed a model of hierarchical urban system, which had no precedent in Location Theory: the Central Place Theory (Christaller, 1933, see Fig. 2.1). Building upon the isolated space of von Thünen (1826), and in a close relationship to Spatial Competition Theory, he assumed a population of consumer evenly spread in a featureless region. Residents consume a hierarchy of goods, produced in “central places”, and have to incur a costly transport to those market places. Naturally, they go to the closest market place where they can find the good they want, hence delineating market areas. Christaller assumed that different market goods require different minimal market sizes for their production to be profitable, because of different degrees of scale economies in production. Then, under the assumption of an homogeneous population distribution, he showed how a hierarchy in market size thresholds yields a hierarchical spatial structure of market places into an hexagonal latices (Fujita, 2010).

The main drawback of the Central Place Theory exposed by Christaller (1933) is the lack of economic and behavioural micro-foundations (Fujita, 2010). Yet at the same time, following the model of Hotelling (1929), a serial of economic studies on imperfect competition (including the monopolistic competition of Chamberlin, 1933, and Dixit and Stiglitz, 1977, that will be used in Chapter 5) were in development. This spurred the economist August Lösch to propose a spatial equilibrium model of oligopolistic industry, where the market areas of firms of the same industry form an hexagonal lattice (Lösch, 1940). His work reinforced the importance of Central Place Theory in the development of Location Theory, whose micro-foundations are still under study (Fujita et al., 1999a; Tabuchi, 2009; Faden, 2010; Ikeda and Murota, 2014).

The seminal works presented above were carried out during the first half of the twentieth century, and were influenced by the industrial revolution that has occurred in the preceding century. Hence the focus of Location Theory was on industrial locations, and the residential function of cities was not much discussed. Yet, the rapid urbanization that has followed the industrial revolution in developed countries caught the attention of another research group: the sociological geographers. In Chicago, sociological geographers proposed to study cities following principles of evolutionary dynamics such as invasion, domination and succession between social groups. The first model of Burgess and Park (1925) assumed an exogenous city centre in a two-dimensional homogeneous plane and resulted in a pattern of concentric rings of different social classes. Although his model was qualitative and followed a totally different approach than those of geographers working in Location Theory, the similarity with the agricultural model of von Thünen

(1826) is striking and shows the importance of the monocentric urban structure at that time.

Fifteen years later, Hoyt (1939) disturbed the concentric model by assuming the heterogeneity of the space. The principle is that the city can be divided into sectors whose different properties are not evenly quoted by the different social classes so that the city social pattern is still segregated, but not perfectly concentric any more (Hoyt, 1939). Introducing a dynamic consideration, Harris and Ullman (1945) went one step further and stated that cities grow through the integration of subcentres. In that sense, they also made the first multicentric city model of urban sociology. Merging all the previous works, White (1987) proposed a general model where the concentric social segregation is locally disturbed by subcentres. Since all these models where applied and developed in Chicago, this storyline is known as the *Chicago's school of social geography.* Since these models were qualitative, they belong to another strand of the literature. However, they initiated a specific approach of urban morphologies that is still active today (Kwan, 2013) and that cannot be omitted in the present review.

To sum up, early works of Location Theory regarded cities as market areas or (agricultural) production areas organized around a production and/or market centre. This view is not surprising given the historical influence of the industrial revolution. Thus, morphologically, cities were not clearly defined. On the one hand, in von Thünen (1826) and Weber (1909), cities are reduced to punctiform demand centres. On the other hand, in Hotelling (1929) and Central Place Theory, populations are assumed to be evenly distributed in space, which goes against the very nature of cities. Thus, the internal structure of cities has not really been studied before the works of the Chicago School's social geographers. However, years later, the rapid and chaotic urbanization of industrialized countries in the first half of the twentieth century aroused the interest of location theorists for urban residential structure. This interest yielded the birth of Urban Economics.

Urbanization in industrialized countries also generated a demand from the public sector for applied and predictive models, especially to forecast transport demand. Those models stimulated new theoretical developments, based upon an old tradition of Social Physics, which consists in using concepts and models from Physics to solve social problems. This is an alternative strand of theoretical models in quantitative Urban Geography, whose last development is the Complex Systems approach (Fig. 2.1). In the following, historical developments of Location Theory and Complex Systems after the fifties are presented separately. Afterwards, they are compared from the perspective of Urban Morphodynamics.

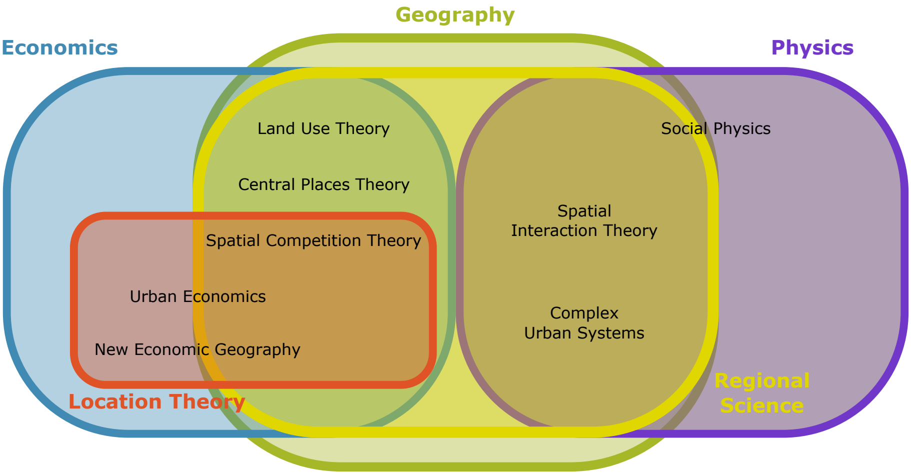

Figure 2.1. Schematic representation of quantitative theories in urban geography. Note that this representations does not aim at drawing a complete Venn diagram. For example, there exist researches at the intersection of Economics and Physics (e.g. Zhang, 1991).

2.2 The heritage of Location Theory

The historical contribution of von Thünen (1826), Christaller (1933), Lösch (1940), Weber (1929) and Hotelling (1929) to Location Theory can be regarded from two perspectives. From a geographical perspective, they have formalized urban geographical theories by starting from their behavioural and economic determinants. From an economic perspective, they have contributed to economic theory by explicitly introducing space as a key determinant of economic activities. In the fifties, this duality has pushed some researchers, and notably the American Economist Walter Isard, to smooth disciplinary cleavages by creating an interdisciplinary social science, the Regional Science (Fig. 2.1), which "*concerns the careful and patient study of social problems with regional or spatial dimensions, employing diverse combinations of analytical and empirical research*" (Isard, 1975, p.2). Thus there is no surprise that regional scientists include "*geographers who find it worthwhile to develop pure theory with extensive use of mathematics and to engage in the construction of large-scale models with considerable analytical content and employing extensive sets of data*", but also "*economists who have become concerned in more than a trifling manner with urban and regional problems and who want to dig into them with more than the tools that economics provide*" (Isard, 1975, p.2). From the fifties, Regional Science has provided a fertile ground for interdisciplinary research, especially involving geographers and economists (see Isard, 1975; Isserman, 1993; Fujita, 1999; Rees, 1999; Waldorf, 2004; Fischer and Nijkamp, 2014; Duranton et al., 2015). It gave birth to two theories that are essential in this thesis: Urban Economics, which appeared in the sixties, and New Economic Geography, which appeared at the beginning of the nineties.[1]

2.2.1 Urban Economics

After having caught the attention of sociological geographers from the School of Chicago, the rapid urbanization of industrialized countries raised new research questions in Location Theory. What can explain the agglomeration of households in cities? What are the determinants of cities' size and shape? Those questions were addressed by the economist William Alonso in a theoretical model that is considered as the seminal model of Urban Economics (Alonso, 1964). Strongly anchored in the fundamentals of Location Theory, Alonso (1964) proposed a model of "Isolated Urban Space" where households are spread in a featureless plain and commute towards a pointwise centre business district (CBD) in order to earn a wage they spend in consuming a composite commodity imported and delivered at their home (his model is detailed in Chapter 3). In this model, he initiated an urban land use theory (Alonso, 1960) and showed that households location results from a trade-off between commuting costs and housing rents, such that the land market structures residential areas in concentric rings around city centres. Thus, from an historical perspective, "*modern urban land use theory, which forms the core of urban economics, is essentially a revival of von Thünen's theory of agricultural land use*" (Fujita, 1989, p.1). Urban Economics is made up with numerous models extending Alonso's work in different ways. In accordance with the perspective of this chapter, the present section exposes the main contributions in terms of urban morphology and urban dynamics.

Urban morphology

The monocentric city model of Alonso (1964) was first extended by Muth (1969). In accordance with the general approach of transport costs used in

[1]The adjective *New* in "New Economic Geography" used to emphasize the link with other fields of Economics (see Krugman, 1998b). In Economics, its use is not systematic any more (see, e.g. Thisse, 2010). Yet as Garretsen and Martin (2010) have noticed, it cannot be dropped without generating a confusion with the older discipline of Economic Geography, which encompass qualitative and quantitative approaches. Thus, the designation "New Economic Geography" is generally maintained since alternative designations such as "Geographical Economics" (Brakman and Garretsen, 2003; Brakman et al., 2009) are not as popular. On the contrary, Urban Economics is an unambiguous formulation, although it used to be called "New Urban Economics" as well (see Richardson, 1977).

Location Theory (Capello, 2014), Muth (1969) introduced households' income into their commuting cost function to account for the fact that the value of one's time rises with his income. The result on the spatial structure is that, according to individual preferences for leisure time, sub-populations of different income levels will be segregated. In particular, high income may locate close to the city centre, and medium income at the periphery, whilst low income are located in-between (Beckmann, 1974; Fujita, 1989). Note that an alternative explanation for this spatial segregation is the presence of urban amenities of all kinds. This process seems more appropriate to describe European cities and has been developed in Urban Economics later on (Brueckner et al., 1999; Wang and Wu, 2011). A second contribution of Muth (1969) was to introduce an explicit housing market, where housing supply is ensured by developers who convert land into housing spaces, whilst in Alonso (1964) households are located on the ground. This contribution enabled the monocentric city model to address additional questions of urban morphology regarding the distribution of open space (Büttler, 1981; Brueckner, 1983) or buildings height (Arnott and MacKinnon, 1977; Grimaud, 1989; Bertaud and Brueckner, 2005). In addition to these two contributions, Muth (1969) also presented the earliest empirical analyses of the monocentric city model (Waldorf, 2004).

A third seminal work in the development of the monocentric city model was made by Mills (1972). His work is often associated to the one of Muth (1969) because they closely followed each other and both used an explicit model of housing production (Brueckner, 1987; Spivey, 2008). Yet the theoretical work of Mills (1972) departed from Muth (1969) by proposing an explicit model of traffic congestion. He also provided an insightful discussion of urban decentralization and on the emergence of sub-centres. Given that the influence of those two processes on urban morphology is still relevant today, Mills' book appears less dated than Muth's one (Waldorf, 2004). In the historical perspective of this chapter, it is worth emphasizing that Mills (1972) extensively used numerical methods to derive his results although it was not common at the early seventies. Contributions of Muth (1969) and Mills (1972) were so crucial that the monocentric city model of Urban Economics is nowadays known as the Alonso-Muth-Mills model (Waldorf, 2004). Further developments of the monocentric city model are out of the scope of this thesis and can be found in general references on Urban Economics (see for example Nijkamp, 1986; Mills, 1987; Fujita, 1989; Cheshire and Mills, 1999; Henderson and Thisse, 2004; Arnott and McMillen, 2008; Glaeser, 2008; Brueckner, 2011; Duranton et al., 2015)

The main drawback of the monocentric city model was the assumption of an exogeneously pre-existing CBD. From its very beginning, Location Theory had emphasized the role of transport costs (and spatial friction in general) on the spatial structure of human activity. This research effort was made at the expense of the study of agglomeration economies, which were nevertheless highlighted by Weber (1909) as an opposing force of equal importance (Capello, 2014). The economic understanding of agglomeration economies has progressed under the seminal works of Marshall (1890), Jacobs (1961), Arrow (1962), Romer (1986) and Porter (1990), who discussed the influence of competition and collaboration in an homogeneous or heterogeneous environment. The modern conception of agglomeration economies in Urban Economics distinguishes localization economies and urbanization economies (Duranton and Puga, 2004). On the one hand, localization economies result from sharing processes, like the share of indivisible facilities, and matching processes, like the increased chance for assorted employers and workers to meet. On the other hand, urbanization economies result from knowledge generation and spillovers, which are complex processes. Although the study of agglomeration economies is out of the scope of this chapter, their historical contributions motivated the apparition of *non-monocentric* city models of Urban Economics.

The first of those models was proposed by Beckmann (1968) who showed that an endogenous urban centre can emerge from social interactions between people. Ten years later, Borukhov and Hochman (1977) and O'Hara (1977) showed that a similar process of agglomeration can be deduced from interactions between firms. Those models of endogeneous urban centres, along with the increasing interest for urban decentralization put forward by Mills (1972), paved the way for *multicentric* city models. The first model of this kind was proposed by Fujita and Ogawa, who introduced agglomeration economies between firms and commuting costs from households to firms (Ogawa and Fujita, 1980; Fujita and Ogawa, 1982; Ogawa and Fujita, 1989). From the perspective of urban morphology, their result can be interpreted as distinct sub-centres on a line, or rings of firms in a two-dimensional monocentric city. Land development *à la* Muth-Mills was introduced in their model by Grimaud (1989) and Liu and Chang (2004) to study non-monotonic spatial variations of buildings height. Given the intractability of the model, further developments of the Fujita-Ogawa setting often require numerical methods, including simulations (Heikkila and Wang, 2009), like it is proposed in this thesis (see Chapter 4). Nevertheless, their work has also initiated a insightful literature of analytical models studying the impact of social and economic interactions on urban morphology, which has refined our understanding of the determinants of non-monocentric urban morphologies (Tauchen and Witte, 1984; Tabuchi, 1986a; Fujita, 1988; Liu and Fujita, 1991; Lucas, 2001; Lucas and Rossi-Hansberg, 2002; Berliant et al., 2002; Rossi-Hansberg, 2004; Carlier and Ekeland, 2007; Helsley and Strange, 2007; Berliant and Wang, 2008; Mossay and Picard, 2011; Picard and Tabuchi, 2013).

For the sake of simplicity, most of the monocentric models were developed assuming a one-dimensional space, which is equivalent to an *assumption of symmetry* around the single urban centre in a two-dimensional space (Lucas, 2001). Yet Urban Economics also proposed non-monocentric city models that question this centrality. Non-monocentric models can also be developed in two-dimensions under the assumption of symmetry, although this restricts the model outputs to circular urban configurations (Ogawa and Fujita, 1989; Lucas, 2001; Lucas and Rossi-Hansberg, 2002; Carlier and Ekeland, 2007). Yet, the set of urban configuration that can emerge from the non-monocentric city model in a two-dimensional space without the assumption of symmetry is potentially huge. Very few studies (if any) in Urban Economics have tried to explore this set, for two reasons. First, this is an intractable mathematical problem which requires the use of numerical methods, whilst Urban Economic theory relies essentially on analytical results. Second, and most importantly, Urban Economics is essentially developed by economists who seek to understand the behavioural and economic determinants of spatial configurations more than the their variety. Understanding the variety of urban morphologies that can result from the same economic assumptions is much more a geographical issue. Consequently, the variety of urban morphologies that can be understood using non-monocentric models of Urban Economics is broader than what may appear at first glance. In this thesis, the model of Fujita and Ogawa (1982) will be integrated in an agent-based simulation model with costly adjustment dynamics. Results will show that short-term and long-term equilibrium configurations emerging in a two-dimensional space vary a lot in terms of their monocentricity and circularity.

To sum up, Urban Economics heavily relies on spatial equilibrium models of a continuum of homogeneous agents locating in a continuous space. This continuous representation of people and space has generated some debate, which seems to have ended up with the consensus that those assumptions are simplifying but consistent (Berliant, 1985; Papageorgiou and Pines, 1990; Asami et al., 1991; Berliant, 1991). The equilibrium approach has also introduced a very specific approach of dynamics as an adjustment procedure, which has been criticized by some geographers (Jin and Wegener, 2013; Simmonds et al., 2013; Anas, 2013). This debate is still going on (White et al., 2015c; Barthelemy, 2016) but no insightful consensus seems close to emerge.

Urban dynamics

Both the monocentric and non-monocentric (including multicentric) city models are static equilibrium urban configurations. In this modelling approach, time is only implicitly considered following three alternative interpretations (Fujita, 1983). First, the "*instant city*" approach (Richardson, 1977) assumes that the city only exists at a single point in time, and that it undergoes no change. Second, the "*malleable city*" approach (Pines, 1976) assumes that the adjustment cost of land use is null or, equivalently, that land uses adjust instantaneously. In that perspective, the state of the city at each point in time is a static equilibrium given the external conditions at that time. This notion is almost identical to the notion of "quasi-static process" in Physics. Third, the "*long-run equilibrium city*" approach (Wheaton, 1977) assumes that the adjustment cost of land use is null at the long-run only. Obviously, assumptions underlying the two first interpretations are so abstract that they simply reflect the lack of consideration of time. That is certainly why, as Fujita (1983) wrote, Alonso (1964) and Muth (1969) adopted the first interpretation but seem to prefer the last. Nevertheless, the assumption that cities become malleable in the long run is not straightforward neither (Anas, 1979; Harrison and Kain, 1974; White, 1977). Moreover, beyond the question of theoretical completeness, important urban phenomenons of the twentieth century such as urban sprawl or leapfrog development, could not be explained by static models (Fujita, 1983; Brueckner, 2000).

In order to address those issues, urban economic models of non-malleable housing appeared in the seventies following two modelling strategies.[2] Some models assumed an irreversible development (or durable capital), which actually consists in modelling urban growth, whilst some others considered the possibility of redevelopment (Brueckner, 2000). On both strands of the literature, a fundamental issue was to model agents' expectations about the future of the city (which is not surprising given the strong commitment of Urban Economics to uncover the behavioural and micro-economic determinants of urban spatial structures). Three strategies were historically used (Fujita, 1983). First, in "*perfect foresight models*", all agents have a perfect knowledge of the future. Second, in "*static foresight models*" (or "*myopic models*"), agents are completely ignorant of the future and they base their decisions on the current state of the city only. Those two modelling strategies correspond to extreme cases that exclude uncertainty from the expectations. Seminal models of urban development under uncertainty used a third assumption called "*rational expectations*", which consists in assuming that agents have the same stochastic expectations about the future, so that their plans are mutually consistent. The literature on dynamic urban models with and without redevelopment is now briefly reviewed.

Firstly, dynamic urban models with durable capital were once more

[2]Some authors also proposed dynamic models with malleable capital in order to address other dynamic issues. For example, Fisch (1977) used Volterra's nonlinear integral equations to study housing filtering from the perspective of the Burgess-Hoyt's direction growth.

proposed by Muth, who studied the dynamic effect of raising income and deteriorating housing capital (Muth, 1973) on urban dynamics. Later on, Evans (1975) studied the effect of population growth in a close region. However, for the seek of simplicity, those models abstracted from spatial and expectations concerns. The earliest and most representative dynamic urban model with explicit space was proposed by Anas (1976, 1978). He showed that under the assumptions of durable housing and myopic foresights, dynamic processes may explain housing filtering and abandonment in central locations (Anas, 1978). Only a few studies extended this setting because it was clear from the beginning that perfect foresight were more desirable (Fujita, 1983). The first model of urban growth with durable housing under perfect foresight was proposed by Fujita (1976), who studied urban sprawl and the fragmentation of the classical von Thünen's rings. This first approach relied on the simplistic assumption that developers do not choose for the structural characteristics of buildings: they are seen as homogeneous housing units (see also Capozza and Helsley, 1989). This assumption was further relaxed by a serial of studies (Fujita, 1982; Wheaton, 1982a; Turnbull, 1988a,b) who revealed how leapfrog urban development, and even "inward" development, may happen in this dynamic context. Within the framework of urban models with durable housing, the only model assuming uncertainty is Capozza and Helsley (1990). They showed, among other things, that the bigger the reluctance of developers to convert land under uncertainty, the more compact the expected city is.

Secondly, models of urban redevelopment appeared in the eighties and essentially focused on two processes: redevelopment because of buildings deterioration or because of changing economic conditions. The first models of this kind abstracted from the pure effect of time by assuming buildings deterioration in a stationary-state economy (hence the problem of expectations disappear, see Brueckner, 1981). Yet this setting is very restrictive and prevents from modelling, for example, urban growth. Thus researchers quickly shifted towards urban models of redevelopment with myopic foresight (Brueckner, 1980b,a; Vousden, 1980; Wheaton, 1982b). These models showed that non-monotonic and sharp variations in housing intensity can result from successive wave of redevelopment. These results contrast with the smooth spatial pattern presented in urban models with malleable capital (Brueckner, 2000). Contrary to urban models with durable housing, very few models of urban redevelopment assume perfect foresight (Brueckner, 1981; Amin and Capozza, 1993; Braid, 2001). This is due to the mathematical intractability of equilibrium computation under the perfect foresight assumption in a non-stationary economy. Some researchers also addressed the problem of urban redevelopment under uncertainty, but most of these studies have abstracted from spatial concerns (O'Flaherty, 1993;

Capozza and Li, 1994; Williams, 1997).

At the end of the twentieth century, the research on dynamic models of Urban Economics was considered as a recent activity with a lot of research perspectives (Miyao, 1996; Brueckner, 2000). The field has slowly evolved since then and dynamic theoretical models have at most one dedicated chapter in modern textbooks of Urban Economics (Fujita, 1989; Cheshire and Mills, 1999; Henderson and Thisse, 2004; Arnott and McMillen, 2008; Duranton et al., 2015). Simultaneously, mathematical methods of nonlinear dynamic systems were quickly spreading among the scientific community (Gleick, 1987; Strogatz, 1994; Barnett et al., 1996). Surprisingly, few works of Urban Economics have focused on nonlinear urban dynamics, although it is a challenging research perspective (Puu, 1989; Capello, 2012). Nonlinear dynamic models did appear in Location Theory during the nineties, but in another a brand new discipline that will quickly rise challenging dynamic questions: the New Economic Geography.

2.2.2 New Economic Geography

Whilst in Urban Economics households compete for land and travel to economic activities (whose location is often fix), in New Economic Geography both households and firms choose where to locate and exchange commodities that are shipped across regions or countries (Thisse, 2010). Contrary to what its name indicates, New Economic Geography is, like Urban Economics, more closely related to Economics than Geography. To the point that some researchers prefer to use alternative designations (Brakman and Garretsen, 2003; Brakman et al., 2009). Although it can be seen as a revival of the trade-off between agglomeration forces and transport costs pushed forward by Weber (1909) and Lösch (1940), its abstract representation of the geographical space and its mechanic modelling of time are important critics addressed by geographers (see Garretsen and Martin, 2010). Thus, after a brief exposition of the driving forces in New Economic Geography models, their spatial and dynamic aspects are discussed in order to complete our forthcoming discussion of Urban Morphodynamics.

The seminal model of New Economic Geography is the core-periphery model of Krugman (1991a). It shows how agglomeration and dispersion forces emerge through (horizontal) demand linkages[3] in a context of Chamberlinian monopolistic competition *à la* Dixit and Stiglitz (1977) with iceberg transport

[3]In contrast, Krugman and Venables (1995) and Venables (1996) studied the influence of *vertical* linkages on agglomeration forces. In Puga (1999), both types of linkages are considered.

cost (for a comprehensive presentation, see Fujita et al., 1999b; Baldwin et al., 2005; Combes et al., 2008). Although its original form was analytically intractable, it has been extended in many ways by a set of analytically tractable models that have collectively improved our understanding of agglomeration and dispersion forces in economic geography. For example, Baldwin (1999) considered that human capital accumulation (instead of migration) is the root of agglomeration forces. In another study, Ottaviano et al. (2002) used a quasi linear utility function with quadratic sub-utility and linear transport cost. In a related work, Pflüger (2004) also used quasi-linear utility function but a sub-utility with constant elasticity of substitution (CES). Obviously, those works depart from the original core-periphery model with Cobb-Douglas utility and CES sub-utility. Thus, looking for the slightest modifications of the original model which make it analytically tractable, Forslid and Ottaviano (2003) proposed to introduce skill heterogeneity between workers and to endow high-skill workers with a bigger interregional mobility. They naturally call this model the "footloose entrepreneur" model. Actually, Robert-Nicoud (2005) showed that all those models share some proximity to an alternative specification of monopolistic competition put forth by Flam and Helpman (1987). Further developments in this encompassing perspective are provided by Ottaviano and Robert-Nicoud (2006) and Pflüger and Südekum (2008). See also the review of Redding (2013).

Most of the analytical results in New Economic Geography are actually based on two-regions settings (see for example Mossay, 2006). Although a better understanding of multiple-regions systems has already been called for (Krugman, 1998a; Fujita et al., 1999b; Neary, 2001; Ottaviano and Thisse, 2004), the consequent increase in the number of equilibria makes it even more difficult to address theoretical questions regarding the selection and stability of those equilibria (Fujita and Thisse, 2009; Behrens and Robert-Nicoud, 2011). An initial effort in developing models with multiple regions has been made by Krugman and Elizondo (1996) who proposed a three-regions model where two regions are subdivisions of a larger one. This setting has been used several times since then (Paluzie, 2001; Behrens, 2011; Commendatore et al., 2014). Another three-regions framework consists in considering equidistant regions (Fujita et al., 1999b; Commendatore and Kubin, 2013). In particular, Fujita et al. (1999b, chap. 6) developed the three-regions case of the classic core-periphery model and showed that for intermediate values of transport cost, both concentration and dispersion of labour are stable distributions of activities. Other spatial settings allowing more than three regions are the linear economy (Ago et al., 2006; Commendatore et al., 2015), the racetrack economy (Fujita et al., 1999b; Castro et al., 2012; Ikeda et al., 2012; Akamatsu et al., 2012) and the hexagonal lattice (Christaller, 1933; Lösch, 1940; Ikeda et al., 2014; Ikeda and Murota, 2014). Note that some analogies can be drawn between those various spatial frameworks (Ikeda et al., 2017a,b).

Both the number of regions and their spatial configuration make it difficult for researchers to deal with the issue of equilibrium multiplicity, which was already present in the original model of Krugman (1991a) and has become a standard feature of New Economic Geography (Fujita et al., 1999b). In order to address this issue, two strategies are particularly important. The first one, which will be discussed in Chapter 6, is to introduce heterogeneity in the setting. The second one, which is of special interest for this review, is to study path-dependency effects in the adjustment dynamics of the regional economy (Arthur, 1994). Thus, time has always been implicitly present in New Economic Geography, but only as a way of discussing issues of equilibrium selection and stability through adjustment dynamics of the system. As a result, in a traditional approach of Economics and in close relationship to Urban Economics, the first models detailed hereafter essentially aimed at studying the influence of agents expectations on equilibrium selection and stability.

The original adjustment dynamics of the core-periphery model of Krugman (1991a) is based on a myopic consideration of wage differentials and it is quite similar to the replicator equation of evolutionary dynamics (see for example Nowak, 2006). While this enables to study numerically the influence of initial (historical) conditions on the selection of the long-term equilibrium, the likeliness of this myopic behaviour assumption is questionable. This has pushed researchers to assume forward-looking expectations, which may generate self-fulfilling prophecies, hence refining the questions of equilibrium selection and stability in dynamic systems with multiple equilibria (Baldwin, 2001). In order to address those issues, researchers in New Economic Geography have started studying the adjustment dynamics using explicit differential equations under various non-myopic expectations. In a seminal effort, Krugman (1991b) and Fukao and Benabou (1993) have proposed a two-regions model described by a set of two linear differential equations including forward-looking expectations. Their setting however departs from the original core-periphery model in that agglomeration results from technological externalities and not from demand linkages. Ottaviano (1999) used an ingenious assumption on inter-temporal elasticity of substitution between consumption goods to show that Krugman (1991b) can be interpreted as a model with pecuniary agglomeration forces, hence matching the initial core-periphery model. In another study, Ottaviano et al. (2002) used a formulation with quadratic utility functions that also yields linear differential equations.

As Baldwin (2001) highlighted, a severe drawback of those linear models is that they loose some important results like, for example, the possibility of simultaneously stable corner and interior solutions. Matsuyama (1991) was the first to address the question of stability in a regional system described by non-linear differential equations. Like Krugman (1991b) and Fukao and Benabou (1993), it is a two-regions model where agglomeration results from technological externalities. Ottaviano (2001) used the methodology of Matsuyama (1991) in a model with pecuniary externalities, which is closer to the original core-periphery setting, and discussed the conditions under which expectations may affect the long-run equilibrium population distribution. Baldwin (2001) pushed the idea even further by coming back to the canonical formulation of the core-periphery model. He combined analytical and simulation tools to show that forward-looking expectation have no incidence on the core-periphery model when migration costs are high.

It has been mentioned that this “logical” approach of time as an adjustment procedure has been criticized, essentially because it prevents from drawing clear analogies with empirical research (Martin, 1999; Boschma and Frenken, 2006; Garretsen and Martin, 2010). This has pushed some researchers to work on an alternative specification of the field, called Evolutionary Economic Geography (Boschma and Martin, 2007, 2010). This young discipline anchors explicitly in natural sciences and uses the mathematical formalization of Darwin’s Evolution Theory that has been developed since the fifties (see Nowak, 2006). It is actually rather ignored by researchers of New Economic Geography (Duranton and Rodríguez-Pose, 2005), because many researchers remain committed to Krugman’s conception that there is no fundamental difference between evolutionary dynamics and the “logical” dynamic approach of New Economic Geography (Fujita et al., 1999b). Nevertheless, Evolutionary Economic Geography is a promising meeting point for researchers from both New Economic Geography and traditional Economic Geography (Garretsen and Martin, 2010). A comprehensive review of Evolutionary Economic Geography is out of the scope of this chapter, especially since the model presented in Chapter 6 of this thesis does not explicitly builds on Evolutionary Economic Geography. Nevertheless, their common motivation makes it likely that formal analogies between both approaches will be uncovered in the future.

2.3 Complex Urban Systems

There exists another set of theoretical models in quantitative Urban Geography, which studies cities through the lens of Complex Systems. This approach results from a historical tradition of using methods and concepts from Physics in Social Sciences, which can be traced back to the seventeenth century. Yet its importance in quantitative Urban Geography appeared during the fifties with the birth of Spatial Interaction Theory. The emergence of this theory was stimulated by the development of large-scale transport models and the increase of numerical computing power. Because of its motivational and methodological differences, Spatial Interaction Theory did not have a huge influence on Location Theory (and on Urban Economics and New Economic Geography in particular). Yet other researchers in Regional Science did notice some complementarity between the approaches,[4] and tried to bridge the gap between them. Unfortunately, the historical context was not in favour of such convergence in geographical theory. After a great success in the fifties, quantitative methods were strongly criticized from the sixties to the eighties by Marxist, Humanist and Post-Modernist approaches,[5] which today are grouped into a consistent discipline called Critical Geography (Castree, 2000; Best, 2009). This critic was echoed in Regional Science as well (Isserman, 1993; Rees, 1999) and by the eighties, post-modernists researchers had constituted a third strand of urban researchers, which had departed from the quantitative methods of Regional Science (Sheppard, 1995; White et al., 2015a). As a result, the historical development of urban modelling was "*turbulent* [with] *substantial shifts*" (Batty, 2008).

Just like in the previous section about Location Theory, this section reviews the Complex Systems approach of cities from the perspective of Urban Morphodynamics. Note that elements of Transport Modelling and Land-Use and Transport Interaction (LUTI) models are mentioned to contextualize the theoretical developments, but their review is out of the scope of this thesis. Modern developments of Transport Modelling can be found in Hensher et al. (2004), Small and Verhoef (2007), Hensher and Button (2008), Ortùzar and Willumsen (2011) and Rodrigue et al. (2013), whilst comprehensive reviews of LUTI models are given by Hunt and Simmonds (1993), Wegener (2004, 2014), Acheampong and Silva (2015), Jones (2016) and Thomas et al. (2018).

2.3.1 Social Physics and Spatial Interaction Models

Social Physics refers to "*the application of the concepts of Physics in the social sciences*" (Wilson, 1969, p.159). Its idea can be traced back to the seventeenth century with the work of Thomas Hobbes (see Ball, 2006, for an historical

[4]The seminal book of Wilson (1970) remains a popular references in Regional Science (Waldorf, 2004).

[5]Brian Berry wrote about Post-Modern scientists that "*the new generation of* [academics] *cultivates a deconstructionist scholarship that dismisses objective realism as unknowable, asserts a hyperrelativism that considers science to be unreliable at best and probably fraudulent, and favors post-structuralist orientations that consider the world to be subject-centered and socially constructed, knowable only through language and text.* [They] *have abandoned space in favor of place, system in favor of individuality and predictability in favor of the unexpected*" (Berry, 1996, p.570).

review). The first application of the gravity model to explain the move of people appeared at the end of the nineteenth century (Carey, 1858) and was followed by two seminal works. First, Ravenstein (1885) used gravity models to study interregional migrations in Britain and he is nowadays regarded as a founder of modern Migration Theory. Second, Reilly (1931) used gravity models to represent consumer fluxes between retail centres, and he formalized the use of those models. During the fifties, gravity models became very popular (especially in Regional Science) as tools to model travel demand.

Indeed, at this time, the rapid urbanization that took place in industrialized countries during the first half of the twentieth century had raised new mobility challenges. Large-scale transport models started being developed in order to support policy decisions of investment in transport infrastructures. The issue was to forecast travel demand, and a first algorithmic framework was proposed, called the four-step model (McNally, 2008). As suggested by its name, it is a sequential procedure that derive travel demand on a road network in four steps (from McNally, 2008):

1. *trip generation*: how many travel movements will be made?
2. *trip distribution*: what are their destination?
3. *model split*: what transport mode will be used?
4. *assignment*: which route will be taken?

Gravity models were widely used to perform the second stage of the algorithm, the trip distribution. In seeking for a good predictive power, researchers modified the canonical gravity law (and especially the distance variable) such that it departed from the Newtonian formulation. Yet the methodological approach has remained the same.[6]

One critic addressed to gravity models is that they were applied mechanistically in the four-step model. More generally, it is commonly considered that Social Physics has always been at risk of having a "Physicalist" approach which, by reducing to much the social processes to match physical law, makes the analogy unsound (see discussions in Berry, 1964; Wilson, 1969; Weidlich, 2006). This risk, along with critics addressed to the four-step model, motivated researchers to explore the micro-foundations of the gravity equation. In a seminal paper, Wilson (1967) proposed to replace the power of distance at the denominator of the canonical gravity equation by an exponential form, which is derived from Statistical Physics (see Wilson, 1970). This functional form has also been derived from utility theory (White, 1976) and discrete choice theory (Timmermans, 1984). Note that the initial

[6]In order to deal with the specific attributes of travel forecasting, *i.e.* its *derived* demand and its *generalized* cost in terms of time, new approaches of transport demaned have been developed. In particular, the *activity-based* approach has become very popular (see Bates, 2008; McNally and Rindt, 2008).

formulation of Wilson's model was static, until Harris and Wilson (1978) showed that it can be embedded in a "slow dynamic" framework that is analogous to the "long-run equilibrium" approach of Urban Economics. Afterwards, Wilson's work was progressively extended by a serial of papers who collectively made up the Spatial Interaction Theory (see Nijkamp and Reggiani, 1988a,b; Fotheringham and O'Kelly, 1988; Wilson, 2010). In turn, Spatial Interaction Theory also influenced back Transport Modelling since the micro-foundations proposed by Wilson and others enabled to enhance interpretation of the results and to develop appropriate statistical tests (Hensher and Button, 2008). Spatial interaction models are still the heart of many transport and LUTI models (Wilson, 2010). Yet during the last fifty years, in accordance with the seek for adaptive methods in Urban Planning, recent urban models started to work at finer spatial and temporal scale in order to formulate short-term predictions. This recent trend went along with the raise of a new paradigm of Social Physics: the Complex Systems.

2.3.2 Complex Systems

The Complex Systems approach is rooted in the General System Theory of Natural Sciences that appeared at the end of the sixties (von Bertalanffy, 1968; Rapoport, 1986). It defines a *system* as a set of components endowed with interactions forming an organized complex whole (Weidlich, 2006). Following this approach, some anticipative geographers have been working on a mathematical theory of geographical systems (Bunge, 1962; Wartz, 1965; Forrester, 1969; Coffey, 1981; Haigh, 1985), but their works have remained marginal for a long time. Note that just like Spatial Interaction Theory, most of these models were static except the seminal system's dynamic model of Forrester (1969). Although complexity seems to be an inherent part of the definition of a system, the modern notion of Complex System has emerged during the eighties from the works of Prigogine and colleagues on dissipative systems in chemical physics (Prigogine and Nicolis, 1967; Prigogine and Lefever, 1968; Nicolis and Prigogine, 1977). Following their works, a complex systems can be defined as a system made up with many interacting subsystems, whose essential characteristic is that its global behaviour depends on its units in a non-trivial way (Vicsek, 2002). Thus Complex Systems are related to a holistic philosophy, although no extreme conception, either holistic or reductionist, seems appropriate to their study (Weidlich, 2006). This approach insists on the role of context and path-dependency in explaining the macro-behaviour of any system, whilst remaining anchored in formal modelling (White et al., 2015c). This plurality has caught the attention of many researchers across a wide range of disciplines, and the study of Complex Systems (sometimes called Complexity Science) is closely related to multidisciplinary research (Nicolis and Rouvas-Nicolis, 2007). The application of Complex Systems to social sciences, and urban studies in particular, went through two schools with significant methodological differences (White et al., 2015c).

Firstly, some researchers built on the work of Prigogine and used mathematical modelling. Addressing issues in Regional Science, Peter Allen and colleagues worked on the emergence of Central Places using dynamic adaptations of Spatial Interaction Models (Allen and Sanglier, 1978, 1979, 1981c,b,a). At the beginning of the eighties, Hermann Haken proposed a mathematical formalization of the Complex System approach in social sciences, which he called Synergetics (Haken, 1983, 2004, 2007). The mathematical tools underlying the development of this field are largely gathered from Statistical Physics. Hence they are not included in the standard formation of most social scientists, what has limited the influence of Synergetics in Regional Sciences. One outstanding exception is the field of Sociodynamics, which results from the common work of Wolfgang Weidlich and Gunter Haag (see Weidlich, 1991, 2003, 2005, 2006; Haag, 2017). Starting from dynamic stochastic models of migration (Weidlich and Haag, 1983; Haag and Dendrinos, 1983; Dendrinos and Haag, 1984; Haag and Weidlich, 1984, 1986; Weidlich and Haag, 1988), they expanded them to address location pattern issues in Regional Science (Weidlich and Haag, 1987; Weidlich and Munz, 1990; Munz and Weidlich, 1990a,b). These developments have stimulated further discussions on decision theory, especially bridging some gaps with nested Logit models of discrete choice theory (Kanaroglou et al., 1986b; Haag, 1989; Haag and Grützmann, 1993), but they were also applied in LUTI models (Haag, 1990; Wegener, 2004; Haag, 2017). Weidlich and Haag's migration model will be further discussed in Chapter 5.

Secondly, another group of researchers explored the Complex Systems aspects of Social Sciences using microsimulations. This research strand was promoted by the Santa Fe Institute, a research centre founded in 1984 by physicists from the Los Alamos Lab (also with with David Pines and Murray Gall-Mann), which has meanwhile become a leading research centre with contributions in Economics (Arthur et al., 1997; Axtell, 1999) and Urban Geography (Pumain, 2004). As White et al. (2015c) notes (with a pedagogical caricature), "*the Prigogine approach investigates the behavior of real systems, whereas the Santa Fe school investigates the algorithmic logic of model systems*" (White et al., 2015c, p.15). In that perspective, and in the context of an increasing computer power, microsimulations became an essential part of their methodology. In this thesis, "microsimulations" models are distinct from the "numerical solving" of mathematical models in that their dynamics is not expressed by a set of equations but by an algorithm (White et al., 2015c). One can thus consider that a pioneer in the use of microsimulation in Geography was Torsten Hägerstrand (Hägerstrand, 1952, 1967) in the development of his "space-time" geography (Hägerstrand, 1970). More recently, two types of microsimulations have become dominant in geographical modelling.[7]

The first one is the cellular automaton, which was formalized and designed in 1940 by the physicist Stanislas Ulam (from the Los Alamos Lab) and the mathematician John von Neumann, in order to study non-linear dynamical systems (von Neumann, 1966). Cellular automata are discrete spaces made up with cells, whose state can change from a time step to another, following transition rules that depend on their spatial neighbourhood (Batty, 2007; White et al., 2015c). Famous examples of cellular automata are the Game of Life by Conway (first published in Gardner, 1970) and the social segregation model of (Schelling, 1969, 1971). The computational power of cellular automata made them popular in Social Sciences, and in Human Geography in particular (see Batty et al., 1997; Batty, 2007). Numerous advanced cellular automata were developed from alternative specifications of cell's shape (Batty and Xie, 1994; Pinto and Antunes, 2010; Moreno et al., 2010), cell's neighbourhood (White and Engelen, 1993; O'Sullivan, 2001; Pinto and Antunes, 2010) or transition rules (Al-Ahmadi et al., 2009). A complete review of the development of cellular automata is out of the scope of this thesis (see for example Benenson and Torrens, 2004; Batty, 2007; White et al., 2015c). However, it is important to note that their flexibility in modelling spatial dynamic phenomena has generated an interest in theoretical models of Urban Morphodynamics. In particular, some researchers bridged gaps between Complex Systems and Urban Economics by using transition rules of cellular automata to represent spatial (economic) externalities. For example, regarding the monocentric city model, Miyao (1978a) showed that mixed land use patterns can appear in the monocentric city model, and Page (1999) studied the influence of positive or negative social externalities on the equilibrium spatial structure. Webster and Wu (2001) studied a segregation model *à la* Schelling between polluting firms and residents. In a serial of works, Geoffrey Caruso studied the influence of green externalities on the spatial and socio-economic outcomes of a sprawling city (Caruso et al., 2005, 2007, 2009, 2015b). All these contributions highlight the potential of cellular automata in research on Urban Morphodynamics.

The second and most recent type of microsimulation method is called agent-based modelling. In geography, agent-based models have appeared from cellular automata, by distinguishing cells from agents that can move between them. Thus, contrary to cellular automata, in agent-based models the basic

[7] Other methods, which are out of the scope of this thesis, include for example Boolean networks or artificial neural networks.

functional unit is the agent and not the cell.[8] Thus they enable to model social systems from the same computational unit as in reality: the individual. This fundamental feature is at the heart of their success in social sciences (Gilbert and Terna, 2000; Bonabeau, 2002; Cederman, 2003; Helbing and Balietti, 2011), including Economics (Tesfatsion, 2001, 2002, 2003; Tesfatsion and Judd, 2006; Axtell, 2008; Farmer and Foley, 2009; Geanakoplos, 2012; Turrell, 2016) and Human Geography (Parker et al., 2003; Liu et al., 2013; Huang et al., 2014; Heppenstall et al., 2016; Heppenstall and Crooks, 2016). Again, the detailed review of these contributions is not relevant here (see the books of Tesfatsion and Judd, 2006; Batty, 2007; Heppenstall et al., 2012). However, just like cellular automata, agent-based models stimulated new theoretical research in Urban Morphodynamics, especially bridging gaps with Urban Economics. For example, they have been used to explore the morphodynamic of the monocentric and non-monocentric city models (Heikkila and Wang, 2009; Lemoy et al., 2010; Delloye et al., 2014, 2015; Olner et al., 2015). An alternative mathematical formalism of agent-based microsimulation models was proposed by Schweitzer (2003); Schweitzer et al. (2005) who used the tools of statistical physics.

In studying the self-organizing dynamics of Complex Systems, many researches have noticed an emergent regularity in dynamic events. Put it roughly, for intensity variables that range from earthquakes strength to words length, large values are comparatively rarer than small values, in proportions that follow regular laws. These laws can be rank-size rules, Pareto laws or power laws depending on that the relationship is expressed in terms of ranking functions, cumulative or probability distribution function (Adamic, 2002). In the context of Complex Systems, Per Bak has proposed a general explanation of these regularities called Self-Organized Criticality (Bak et al., 1987, 1988; Bak and Chen, 1989; Bak, 1996). Note, however, that self-organized criticality is only one among the multiple dynamic processes generating power laws (Newman, 2013). In urban modelling, power laws have been used to relate the size of cities with various functional variables such as their wage levels, land rents or number of patents. Among the authors working in this field, some authors argue that there exists general scaling laws which apply to all cities (Bettencourt et al., 2007; Bettencourt, 2013; Bettencourt and Lobo, 2016), whilst some others underline that power laws are sensitive to measuring techniques and suggest that different cities may have different scaling regimes (Arcaute et al., 2015; Cottineau et al., 2017).

[8] Agent-based models presented in this thesis are somewhere else called "individual-based" models in order, first, to emphasize that agents represent individuals and, second, to avoid confusion with the "agent-based" or "multi-agents" appellations of other disciplines (like computer science), which may designate more general concepts (see Woolridge, 2009).

In a spatial setting, powers laws generate multi-fractal geometrical objects (Sornette, 2006; White et al., 2015c). Fractals are mathematical or geometrical sets that present similar features across an infinite range of scales (see for example Falconer, 1990). They were proposed by the mathematician Benoit Mandelbrot (Mandelbrot, 1967, 1982). Beyond purely mathematical theories, pseudo-fractal (which are self-similar on a limited range of scales only), multi-fractals (which are characterized by multiple scaling behaviours) and stochastic fractals have become popular tools for describing natural phenomenons. Although the generality of fractals as the "geometry of nature" is in debate (Shenker, 1994; Avnir et al., 1998), they have generated a new and fruitful approach of urban morphology (Batty and Longley, 1986, 1994; Frankhauser, 1994; Tannier et al., 2012). They are a powerful tool to measure and characterize urban footprints (Thomas et al., 2008; Tannier et al., 2011, 2012; Thomas et al., 2012; Tannier and Thomas, 2013) and road networks (Sun et al., 2012; Zhang and Li, 2012; Thomas and Frankhauser, 2013; Arcaute et al., 2016). Some researchers have even tried to link those new configurations with classic models of Urban Economics (Cavailhès et al., 2004a, 2010) in order to discuss issues of accessibility to green spaces. More recently, with a fresh presentation of the emphasize of flows in Spatial Interaction Theory, Batty (2013) called for a new approach essentially based on networks, which are the spatial expressions of flows. Power laws, fractals and networks are not essential to this thesis, but they will be in the background of the following discussions (especially in Chapter 3) and they constitute a dynamic research field of Urban Morphodynamics.

2.4 An integrated perspective

To the light of their historical development presented above, Location Theory and Complex Systems appear to share the fundamental conception that spatial friction and spatial interaction are the determinants of the urban spatial structures. Indeed, in the absence of spatial friction, interactions between human activities would not generate any agglomeration or dispersion force. Similarly, in the absence of interactions for sustaining travel demand, spatial friction would have no effect on the spatial structure of human activities. In that perspective, Location Theory and complex systems share a common interest for the *accessibility* of locations, which can be broadly defined as "*a measurement of the spatial distribution of activities about a point, adjusted for the ability and the desire of people or firms to overcome spatial separation*" (Hansen, 1959, p.73). They only difference is that Location Theory focuses on the spatial organisation of activities for given interactions, in Complex Systems, interactions are the object of interest.

The notion of accessibility can be traced back to the early works of

Location Theory (von Thünen, 1826; Weber, 1909; Hotelling, 1929; Christaller, 1933; Lösch, 1940). Initially, they focused on how transport costs influence the location of human activities in the context of predefined interactions. Although their initial emphasize was on the economic meaning of transport costs as "*the rate of discount in space*" (Isard, 1956, p.85), transport have quickly been generalized to a wider interpretation, often referred to as *spatial friction* (Capello, 2014). In the context of spatial interactions (which generate travel demand), spatial frictions produce a variety of competing agglomeration (or centripetal) and dispersion (or centrifugal) forces that shapes the economic space at different geographical scales. This seminal consideration of accessibility has found a renewed interest in New Economic Geography. For example, in the original core-periphery model, interactions are exchanges of goods between regions, and the accessibility is defined with respect to market sizes and iceberg transport costs (Fujita et al., 1999b; Baldwin et al., 2005; Combes et al., 2008).

This notion of accessibility has been refined within the framework of Urban Economics. For example, in the monocentric city model, an interaction between households ans employers is assumed to generate a demand for commuting towards the CBD. Places are then characterized by their distance to the CBD which, along with the transport cost function, contributes to their accessibility to the CBD (Capello, 2014). Then, different accessibility levels, corresponding to differences in attractiveness, are expressed in different bid land rent values. At the end, the land market distributes land uses according to those abilities to pay (Alonso, 1960). Thus, by anchoring in a geometrical space, the notion of accessibility put forth by Urban Economics is intimately related to the land use market, that is, to the spatial structure.

Due to their initial use in transport modelling, spatial interaction models and complex systems emphasize more spatial interactions than spatial frictions. Thus, on the opposite of Location Theory, Spatial Interaction Theory focuses on how spatial interactions emerge from within a given spatial structure in the context of spatial friction. The spatial structure of networks generated by flows is then as important as the spatial structure of the interacting locations, which are the nodes of those networks (see Batty, 2013). Consequently, the two approaches of accessibility are complementary. This complementarity is striking when comparing the mechanics of urban land use theory described by Alonso (1960) to the land use and transport interaction feedback that is used in LUTI models (Fig. 2.2). It turns out that the mechanics of Urban Economics is the reduced form of the LUTI feedback. Whilst Urban Economics has explored the "land use" part of these integrated models, transport models and spatial interaction models have developed the "transport" part. This notion of accessibility as a concept at the intersection of land use and transport, or centres and flows, is important in the modern literature (Morris et al., 1979; Litman,

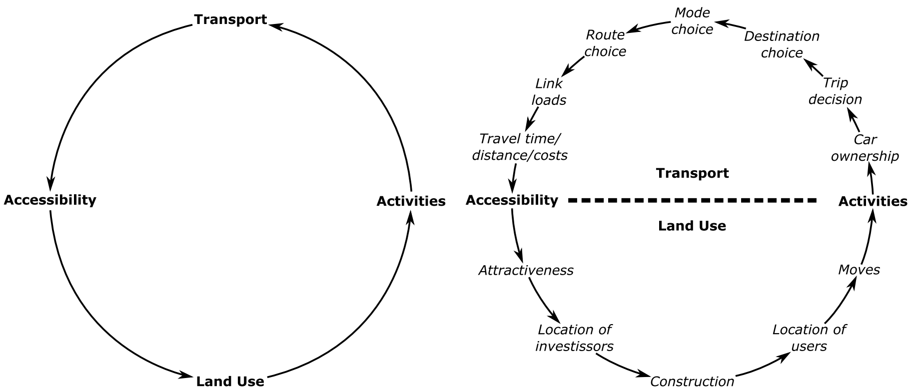

Figure 2.2. **Land use and transport feedback from Wegener and Fuerst (2004).** Schematic representation of the interactions between land use and transport systems in its reduced form (a), as considered in land use theory or urban economics, and its extended form (b), which is used in land use and transport interaction (LUTI) models. Different land uses correspond to different activities, requiring exchanges of goods or people which are the material expressions of spatial interactions by flows. Those flows are channelled through transport networks, have a cost and thus influence the accessibility of a place. Hence, different land uses yield different accessibility levels, corresponding to differences in attractiveness, expressed in different bid rent values. At the end, the land market distributes land uses according to those abilities to pay.

2003; Horner, 2004; Ferreira and Batey, 2007; Banister, 2008; Vandenbulcke et al., 2009; Geurs et al., 2010; Tannier et al., 2012).

Yet Location Theory and Complex Systems also exhibit sharp divergences, especially with respect to their disciplinary backgrounds and motivations. On the one hand, Location Theory aims at explaining the economic rationale for the location of economic activities in space, and hence the functional division of the geographical space into different economic activities (Capello, 2014). Thus it is a question of interest for both economists and geographers, and the field has indeed grown upon contributions from both sides. It has inherited from Economics the seek for a better understanding of theoretical models, which has always been one of its most significant drivers. On the other hand, Complex Systems regard cities as systems made up with many interacting subsystems, whose global behaviour depends on its units in a non-trivial way (Vicsek, 2002). The emphasize on interactions is inherited from Physics, and especially gravitation laws and dissipative systems, which have strongly influenced the historical development summarized above. Scientists working on complex urban systems also inherited from Physics a strong commitment to applied research, which was historically catalysed by transport modelling.

To sum up, one can roughly say that Location Theory is at the intersection of Geography and Economics, whilst Complex Systems are at the boundary of Geography and Physics (Fig. 2.1). These different perspectives have been translated into different methodologies and modelling assumptions that often seem to be unreconcilable in practice (see for example Jin and Wegener, 2013; Simmonds et al., 2013; Anas, 2013; White et al., 2015c; Barthelemy, 2016).

And yet, many researchers presented above have tried to bridge gaps between both disciplines, most often within the community of Regional Science. First, in Urban Economics and New Economic Geography, some authors have tried to introduce nonlinear dynamics that is fundamental in Complex Systems (Puu, 1989; Matsuyama, 1991, 1995; Ottaviano, 2001). Other studies have explored the result of classic urban economic setting in pseudo-fractal environments studied in complex systems (Cavailhès et al., 2004a, 2010). the Second, from the other side, in Spatial Interaction Theory, the seminal work of Wilson (1967) has open new research paths gathering behavioural models from Decision Theory (White, 1976) and Discrete Choice theory (Timmermans, 1984). The “slow dynamic” formalism of Harris and Wilson (1978) also drew clear analogies with the “long-run equilibrium” approach of Urban Economics (Fujita, 1983). Third, Allen and colleagues developed dynamic models trying to reconcile the Central Place Theory with Complex Systems (Allen and Sanglier, 1978, 1979, 1981c,b,a). In the field of Synergetics, some works dedicated to the study of migratory systems were initiator of new analogies between Location Theory and complex Systems. In particular, the dynamic decision theory used in the models of Weidlich and Haag have explicit relationships with the nested Logit models (Kanaroglou et al., 1986b; Haag, 1989; Haag and Grützmann, 1993). More generally, Discrete Choice Theory (Luce, 1959; Manski and McFadden, 1981; Ben-Akiva and Lerman, 1985; Anderson et al., 1992; Train, 2009), which describes individuals’ preferences as more or less deviating from an expected preference, is totally in accordance with the principles of statistical and chemical physics (Grauwin et al., 2009; Lemoy et al., 2011; Lemoy and Bertin, 2012). This theory has already been used to provide behavioural foundations of Spatial Interaction Models (Timmermans, 1984) or migratory systems (Kanaroglou et al., 1986b,a; Anderson and Papageorgiou, 1994a,b). Fourth, some researchers used microsimulations to reinvent classic models of Location Theory from the perspective of Complex Systems. In particular, some authors used the transition rules of cellular automata to represent spatial externalities (Miyao, 1978a; Page, 1999; Webster and Wu, 2001; Caruso et al., 2005, 2007, 2009, 2015b). Some other used agent-based simulations to explore the morphodynamics of urban economic models (Heikkila and Wang, 2009; Lemoy et al., 2010; Delloye et al., 2014, 2015; Olner et al., 2015).

Despite the aforementioned works, bridging theoretical gaps between

Location Theory and Complex systems is not the most common objective of microsimulations models. They are extensively used in Complex systems (Batty, 2007), and they also got interest from Location Theory (Fujita and Mori, 2005; Tesfatsion and Judd, 2006; Farmer and Foley, 2009; Behrens and Robert-Nicoud, 2011) and from applied land use, transport or integrated models (Parry and Bithell, 2012; Tannier et al., 2015; White et al., 2015c; Pumain and Reuillon, 2017). Yet most of the time, they are either used as numerical solving algorithms to address specific theoretical issues, or as a new way of building applied models to forecast the short-term development of a limited geographical area. This diversity of goals, coupled to the flexibility of microsimulations to tackle various (sometimes a-spatial) research questions, is currently generating a huge quantity of models that are hardly comparable (Grimm et al., 2006; Grimm and Railsback, 2012; Angus and Hassani-Mahmooei, 2015). This seriously strives against the establishment of a common domain knowledge. As explained in the introductory chapter, this lack of understanding strives against the ability of urban research to provide urban policy makers with reliable decision support tools, which in turns prevents urban planning to operate the shift toward bottom-up methods that sustainable development goals sorely need.

To sum up, Location Theory and Complex Systems share the fundamental conception that spatial friction and spatial interaction are the determinants of the urban spatial structures. This conception is anchored in the notion of accessibility, which they however approach using different perspectives. On the one hand, Location Theory tends to emphasize the importance of spatial frictions in locating economic activities that are the origin and destinations of pre-supposed spatial interactions. On the other hand, Complex Systems focus spatial interactions that emerge within an heterogeneous space of origin and destinations centres in the presence of spatial friction. This conceptual divergence is minor compared to the differences in disciplinary backgrounds and motivations. Several authors have already tried to smooth these differences by bridging theoretical gaps between both disciplines. This thesis follows on their work and proposes four theoretical models which, from the perspective of urban morphodynamics, merge concepts from Location Theory and Complex systems.

First, Chapter 3 addresses the question of whether scaling regularities in urban monocentric structures can be explained by the classic monocentric city model of Urban Economics. To answer this question, it introduces power laws in the monocentric city model of (Alonso, 1964). Second, Chapter 4 addresses the question of which non-circular and non-monocentric urban configurations can emerge in the non-monocentric city model of Urban Economics in a dynamic context with heterogeneous adjustment speeds. To do so, a dynamic urban model is proposed, based on an agent-based implementation of the non-monocentric city model of Fujita and Ogawa (1982). Third, Chapter 5 proposes a dynamic model of collective discrete choices. This modelling framework gathers from Discrete Choice Theory, Synergetics and Stochastic Calculus in order to reduce the collective dynamics of agents to a set of Itô stochastic differential equations whose diffusion and drift coefficients are explicitly expressed in terms of individual variables. Fourth, Chapter 6 addresses the question of what is the influence of heterogeneous preferences on the adjustment dynamic of New Economic Geography models. In order to answer this question, the developed stochastic modelling framework of is applied to the footloose entrepreneur model of (Forslid and Ottaviano, 2003).

II

Geographical Complexity in Urban Economics

3

Scaling Profiles of Monocentric Cities

The scaling of urban characteristics with total population has become an important research field, yet it is largely disconnected from intra-urban structure. In contrast, the monocentric model of Alonso provides a residential choice-based theory to urban density profiles. However, it is silent about how these profiles scale with population. This chapter bridges this gap by introducing power laws for land, income and transport cost in the Alonso model. From this augmented model, the conditions at which the equilibrium urban structure matches recent empirical findings about the scaling of urban land and population density profiles in European cities are derived. It is shown that the Alonso model is compatible with the observed scaling of population density profiles and satisfactorily represents European cities. The conditions for this compatibility refine current understanding of wage and transport costs elasticities with population. Although they require a scaling power of the profile of the share of urbanised land that is different from what is observed, it is argued that alternatives specifications of transport cost functions could solve this issue.

The research presented in this chapter has been carried out in collaboration with Dr Rémi Lemoy and Prof Geoffrey Caruso (see Delloye et al., 2017, 2018a).

3.1 Introduction

In this increasingly urban World (UN-Habitat, 2016), liaising the social and environmental outcomes of cities to their size is definitely an important question today and for tomorrow. Yet, many outcomes of cities depend crucially on their internal structure, especially on how densely citizens occupy the land they have developed. This occupation emerges from the location decisions of many people interacting in space and is often described or discussed in radial terms, that is how far reaching a city is (the urban fringe distance) and how flat/steep its density profile is. This is a key interest of theoretical and empirical urban economics (see Anas et al., 1998, for a reminder) and the favourite playground of urban planning. The long dispute between compactness or sprawl (e.g. Ewing et al., 2014, for a quick summary) just shows how much this internal structure matters and is worth being studied. Therefore, before summing-up a city as the outcome of a single termed function of population, one needs first to make sure that the internal structure of cities is independent of population, or is at least independent of a simple (well-behaved) transformation of population, and second – particularly if desirable actions need to be made with potential social impacts – one needs to know if this internal structure responds to the same underlying decisional processes independent of size, in other terms that the same urban theory holds across the size distribution of cities.

Nordbeck (1971) provided an intuition to the first need, and opened up a literature strand on allometric urban growth by assuming that cities, similarly to biological objects, keep the same form across sizes. Lemoy and Caruso (2017) recently endorsed this idea and empirically identified the homothetic transformations of density and land profiles with population for European cities. A logical extra step is then to address the second need described above and assess whether models that can generate observed urban radial profiles can also replicate their scaling with population. Finding a valid model that can be applied to any cities after simple rescaling would definitely bear powerful implications for understanding cities and identifying generic planning recipes independent of size. The Alonso-Muth-Mills monocentric framework (Alonso, 1964; Muth, 1969; Mills, 1972) is a perfect candidate because it issues micro-foundations to urban expansion limits and density profiles. It does so after fixing population in its closed equilibrium form, or after fixing its social outcome (utility) in its open form where equilibrium with other cities is then assumed and the population an output.

This paper assesses the theoretical ability and conditions for the Alonso model to replicate the scaling behaviour of urban density and urban land profiles. Given that the Alonso model however assumes a fully urbanised disc, which is inconsistent with the presence of semi-natural land within cities and with a decreasing profile of urbanised land, it proposes a model that exogeneously relaxes this assumption. Then it tests how the standard form of the Alonso model and its relaxed land use form (named "Alonso-LU") empirically perform in Europe after a parsimonious calibration calling only three parameters.

3.2 Background

In the last few decades, and particularly since the advent of the complexity paradigm (Arthur et al., 1997; Vicsek, 2002; Batty, 2007; White et al., 2015c), researchers have reinvested the question of scaling patterns for cities. Most of these investigations, conducted by economists, physicists and geographers, have been dedicated to systems of cities, i.e. the inter-urban scale, with particular attention on rank-size distributions and empirical testing of Zipf's law through space and time (e.g. Pumain, 2004; Bettencourt et al., 2007; Shalizi, 2011; Batty, 2013; Louf and Barthelemy, 2014; Leitão et al., 2016; Cura et al., 2017). Theoretical grounds have been provided along dissipative systems analogies (Bettencourt, 2013) or Gibrat's law of proportionate growth (Pumain, 1982; Gabaix, 1999), ruling out economics of agglomeration. These studies are essentially *a-spatial*, meaning that cities could be reshuffled anywhere (except for instance Pumain and Reuillon, 2017) and, most importantly in light of our objectives, meaning that their intra-urban structure is ignored.

Geographers and physicists have also explored intra-urban scaling, especially Batty and Longley (1994); Frankhauser (1994) have initiated research on fractal geometries and identified their resemblance with land urbanisation patterns. Most of this literature is devoted to identifying irregular urban boundaries (e.g. Tannier et al., 2011) and non-monocentric patterns (Chen, 2013). Apart from two noticeable exceptions by Cavailhès et al. (2004a, 2010), no link is explicitly drawn however in the fractal literature with the fundamental location trade-offs of the urban economic tradition. Even in these particular exceptions, though, densities and rents are output on top of an exogenous land pattern, either multi-fractal or inspired by a Sierpinski carpet. Furthermore, despite fractality implies repeating structures across scales, this literature does not relate to city size distribution and inter-urban research.

In urban economics, the monocentric models arising from Alonso-Muth-Mills explicitly aim at explaining land use patterns, densities and land or housing markets as a function of distance to an exogenous Central Business District (CBD) (Fujita, 1989) and a large theoretical literature has emerged (Fujita and Thisse, 2013; Duranton et al., 2015). Some links have been drawn with inter-city research and the distribution of cities but without addressing population scaling as such. It is rather focused on agglomeration effects and migration costs between cities (e.g. Tabuchi et al., 2005). Empirical studies are less numerous (Cheshire and Mills, 1999; Ahlfeldt, 2008; Spivey, 2008) and again hardly focus on scaling properties with respect to population size. A notable exception is McGrath (2005) who, following Brueckner and Fansler (1983), studied the evolution of city size (measured as the area or radius of urban regions) with different parameters, including population, using data from 33 U.S. cities over five decades. He observed that the sign of the variation of city size is statistically consistent with urban economic models, but did not develop the exact relationship nor the scaling properties of the land or density profiles.

Overall, population scaling in inter-urban research stays strongly disconnected from intra-urban empirics and theory. Scaling laws consider averaged attributes while ignoring the making of urban patterns and their effects on these attributes. They especially ignore the fundamental trade-off between transport and land/housing costs within cities as documented after Alonso, that gives rise to decreasing population and urban land density profiles with distance to the CBD. This chapter attempts to bridge this theoretical gap by integrating recent empirical hints from Lemoy and Caruso (2017) about the scaling of urban profiles into the Alonso model.

Lemoy and Caruso (2017) carried out a radial analysis for over 300 European cities of more than 100 000 inhabitants as of 2006. They analysed the profile of the share of land devoted to housing with distance to the CBD and found that all profiles superpose after their abscissa is rescaled with respect to urban population, thus following a two-dimensional (horizontal) homothetic scaling. Similarly, they analysed population density profiles and found these superpose after a rescaling in abscissae and ordinates, thus following a three-dimensional homothetic scaling. Optimal rescaling is obtained numerically with the square root of population for land use profiles and the cube root of population for population density profiles. This yields the generic profiles shown on Fig. 3.1, with $H_N(r)$ the share of housing land and $\rho_N(r)$ the population density as a function of distance r to the CBD. These are representative profiles which can be rescaled to describe any European city once its population N is given.

The validity of Fig. 3.1 across city sizes cannot be explained by previous geographical research in scaling laws because it has not been linked to a radial intra-urban approach so far. In order to be explained by the standard monocentric theory, one then needs to introduce scaling laws in the Alonso framework before assessing how it suits empirical evidence. This is the work of this chapter, which by doing so actually starts bridging the gap between intra-urban and inter-urban theory.

In addition, one sees from Fig. 3.1 that the land used for housing is far from the constant share (usually 100%) assumed by the canonical version of the Alonso model. In Europe, at the CBD, land for housing is actually about half

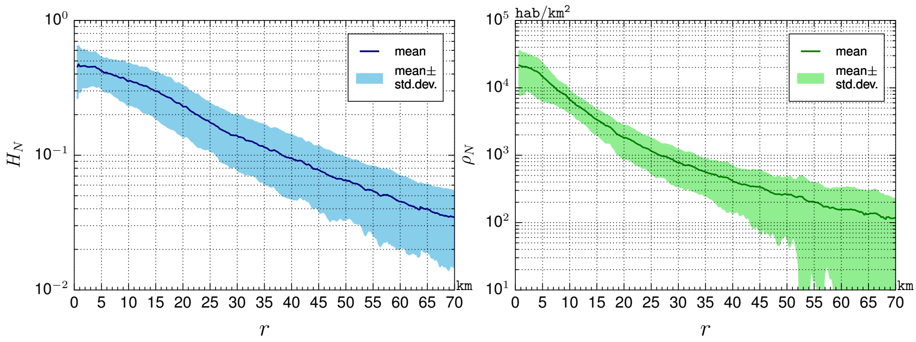

Figure 3.1. Average share of housing land (H_N) and population density (ρ_N) profiles. These profiles have been rescaled without loss of generality to London's population (the largest European Larger Urban Zone in 2006), taken as $N = 1.21\,10^7$ (see Lemoy and Caruso, 2017).

of the land and this share decreases to reach only 10% at 40 km of the CBD for cities like London or Paris. At this stage of the research and given its primary focus on scaling, this chapter opts for an exogenous treatment of the housing land development process. Although some models permit non-urbanised land (agricultural or semi-natural) to be interspersed within the urban footprint because of spatial interactions with residents (Cavailhès et al., 2004b; Caruso et al., 2007), their integration is left to future work.

The remainder of this chapter is organized into fourth sections, the two firsts respectively being theoretical and empirical ones. In the next section, power laws for density and for housing land profiles are introduced in a relaxed version of the Alonso model where housing does not necessarily fully occupy land around the CBD. Then, conditions at which the equilibrium profiles match the scaling exponents of Lemoy and Caruso (2017) are derived. In another, empirical section, their European data are used to calibrate the model, respectively its standard form with constant occupation of land (Alonso) and the relaxed version with exogenously given land profile function (Alonso-LU), thus leaving the model to produce densities within these constraints. Afterwards, Section 3.5 discusses the issue of three-dimensional built-up structures and finally, Section 3.6 concludes.

3.3 Theory: the Alonso-LU model

First, Section 3.3.1 introduces non-housing land uses in the classic monocentric of Alonso (1964). The equilibrium population density profiles is computed using a log-linear utility function to describe households preferences. Second, Section 3.3.2 introduces the scaling laws of the non-housing land use profile, the transport cost function and the wages. It derives the conditions for the equilibrium population density profile to scale homothetically in three dimensions. Third, Section 3.3.3 takes an inter-urban perspective and derives the conditions for cities of different size to coexist at equilibrium. Finally, Section 3.3.4 presents and discusses functional forms of the transport cost function and housing usage profiles that will be used in the empirical work.

3.3.1 Monocentric cities with non-housing land uses

The geographical framework of the Alonso-LU model is an urban version of the *isolated State* inherited from von Thünen (1826). It is a featureless plain, except for a unique Central Business District (CBD), which concentrates all jobs on a point and is accessed by a radial transport system without congestion. Given those assumptions, spatial processes occurring within this plain can be described with respect only to the distance to the CBD (in this chapter, no assumption will break this property). Hence let r be the Euclidean distance to the CBD and $L(r)$ the exogenous land distribution around the CBD. In reality, $L(r)$ is not necessarily a circle of radius r because of, e.g., water bodies (see the *port-city* model in Fujita, 1985 or Koide, 1990). Whatever the form of $L(r)$, the Alonso-LU model departs from the Alonso model by introducing $H(r)$, the share of $L(r)$ that can be used for housing.[1] Hence it is an urban land use augmented model, which is named "Alonso-LU". In the Alonso standard model $H(r) = 1$ (or any other constant), which obviously contrasts with the blue curve in Fig. 3.1. In Alonso-LU, $H(r)$ is an imposed portion of $L(r)$ whose form is provided exogenously. Densities emerge endogenously but are constrained by the available space $H(r)$ which is known to decrease with r (Fig. 3.1). $H(r)$ is only used for housing but its complement $L(r)[1-H(r)]$ cannot (it can, however, be attributed any other land use such as, e.g., natural and semi-natural areas or transport networks).

Each household in the model requires land for housing, works in the CBD and consumes a composite commodity that is produced out of the region and imported at constant price (hence all goods are essentials). Households' residential choice depends on the consumption level of composite commodity they can afford which, given the isolated urban state, is fully determined by the distance to the CBD. It is assumed that households make this choice rationally in the sense of von Neumann and Morgenstern (1944, see e.g. Myerson, 1997). Their choice can be represented by the maximization of a well-behave utility

[1] In this subsection, a single city is considered. Hence the N subscripts are temporarily omitted for the ease of reading.

function[2] $U\Big(z(r), s(r)\Big)$ where $z(r)$ is the amount of composite good (including all consumption goods except housing surface) consumed at distance r from the CBD and s is the housing surface at the same distance.[3]

Each household earns the same wage at the CBD and the budget constraint of each household is binding since the households' utility function is monotonic and does not include any incentive to spend money otherwise. Choosing the composite commodity z as the numeraire, so its price is unity, the budget constraint[4] at distance r from the CBD is

$$z(r) + R(r)s(r) = Y - T(r) \ , \tag{3.1}$$

where $R(r)$ is the housing rent at distance r, Y is the wage of households and $T(r)$ is the total commuting cost at r (it is assumed that $T(r)$ is continuously increasing and differentiable in r). The households' problem consists in maximizing their utility such that the budget constraint (3.1) holds. By continuing to draw inspiration from von Thünen (1826), it is possible to define households' bid rent $\Psi(r, u)$ as "*the maximum rent per unit of land that* [households] *can pay for residing at distance* r [from the CBD] *while enjoying a fixed utility level* u" (Fujita, 1989, p.14). That is,

$$\Psi(r, u) = \max_{z(r), s(r)} \left\{ \frac{Y - T(r) - z(r)}{s(r)} \middle| U\Big(z(r), s(r)\Big) = u \right\} \ , \tag{3.2}$$

where the arguments of the maximum are the bid rent lot size $s(r, u)$ and the bid rent consumption of composite commodity $z(r, u)$. Note that transport cost appear in the budget constraint but not in the utility function. This means that transport is regarded as a *derived* demand, which results from the demand for housing and from the necessity to work. This approach is traditional in transport research as well (Manheim, 1979; Hensher and Button, 2008; Palma et al., 2011; Ortùzar and Willumsen, 2011; Rodrigue et al., 2013).

[2]Formally, U must be twice continuously differentiable, strictly quasi-concave with decreasing marginal rates of substitution, positive marginal utilities and all goods must be essentials. See Fujita (1989, p.311).

[3]In the Alonso model, there is no development of land into housing commodities (land development was introduced into the monocentric theory by Muth, 1969). Hence the housing market is not distinguished from the land market. Throughout this paper, it is referred as the housing market in order to emphasize Alonso's focus on households' choice. Note also that the term "housing" is used in a broad sense without distinguishing, for example, gardens from built space.

[4]This chapter inherits the static feature of Alonso's monocentric model. Hence there is no explicit time unit associated to this model, unlike those of other chapters. Yet the implicit time units of wage, total commuting cost, housing rents and consumptions are assumed to be the same.

In order to derive the equilibrium population density that results from households' interaction on the land market, the closed-city equilibrium is now computed. It consists in considering a closed urban region with a population of mass[5] N and computing the indirect utility level that all households enjoy at equilibrium. The reverse approach, the open-city equilibrium, is an inter-urban perspective that will be discussed in Section 3.3.3. It consists in fixing an indirect utility level u and computing the corresponding equilibrium population N.

In order to express the closed-city equilibrium conditions, let $n(r)$ be the population distribution (the population living between r and $r + \mathsf{d}r$) at distance r from the CBD, which is a continuous and continuously differentiable function. Let also $L(r)$, the land distribution, being continuous and continuously differentiable on $r \geq 0$. Individual location equilibrium requires

$$R(r) = \Psi(r,u) \;:\; n(r) > 0 \;, \tag{3.3}$$

$$R(r) \geq \Psi(r,u) \;:\; \forall r \;. \tag{3.4}$$

Further assuming a default agricultural activity getting at best zero profit yields

$$R(r) = a \;:\; \text{within agricultural area} \;, \tag{3.5}$$

$$R(r) \geq a \;:\; \forall r \;, \tag{3.6}$$

where a is the fixed agricultural land rent. As traditionally in urban economic theory, the agricultural land use is no more than a default land use, that is why the agricultural sector is reduced to its most simple form, represented by a constant rent, although it is not really the case empirically (Chicoine, 1981; Colwell and Dilmore, 1999; Cavailhès et al., 2003). Introducing f as the urban fringe, conditions (3.3) to (3.6) yields

$$R(r) = \begin{cases} \Psi(r,u) \text{ if } r \leq f \\ a \text{ if } r \geq f \end{cases} , \tag{3.7}$$

as well as

$$s(r) = s(r,u) \;:\; r \leq f \;, \tag{3.8}$$

$$L(r)H(r) = n(r)s(r,u) \;:\; r < f \;, \tag{3.9}$$

where $H(r)$, the housing profile of the city, is a departure from the original Alonso model. Equation 3.9 states that land available for housing at a given

[5] In urban economic theory, a population is often regarded as a continuum of individuals. The appropriateness of this assumption has been discussed in Berliant (1985), Papageorgiou and Pines (1990), Asami et al. (1991) and Berliant (1991). This chapter, unlike the next ones, also uses this representation of a population.

commuting distance r within the city is finite and entirely occupied by households. The equilibrium urban fringe is given by the total population condition. This condition results from the fact that the quantity of land $L(r)$ at each commuting distance r is finite. Then, summing the population density over the whole (finite) extent of the city, up to the fringe f, must yield the total population N. Using (3.9), the total population condition writes

$$\int_0^f \frac{L(r)H(r)}{s(r,u)} \mathrm{d}r = N \ . \tag{3.10}$$

Finding values of f and u satisfying equations (3.7) to (3.9) is possible using (3.10) with the boundary rent condition. This condition follows from the competition between urban (i.e. housing) and agricultural (default) land uses, which set the urban fringe f. It is given by

$$\Psi(f,u) = a \ , \tag{3.11}$$

which results from (3.7). Finally, one can compute the population-to-housing density $\phi(r)$ such as

$$\phi(r) = \frac{n(r)}{L(r)H(r)} = \begin{cases} \dfrac{1}{s(r,u)} & \text{if } r \leq f \\ 0 & \text{if } r > f \end{cases} \ , \tag{3.12}$$

and the population density

$$\rho(r) = \phi(r)H(r) \ . \tag{3.13}$$

Alonso-LU model with log-linear utility

Specifying a function form for the model is required for both quantitative predictions and empirical confrontation. In this section, a first step is taken by assuming that households' utility function $U\Big(z(r), s(r)\Big)$ takes the form

$$U\Big(z(r), s(r)\Big) = (1-\beta)\ln\Big(z(r)\Big) + \beta \ln\Big(s(r)\Big) \ , \tag{3.14}$$

where $\beta \in \,]0,1[$ is a parameter representing the share of income (net of transport expenses) devoted to housing, or the relative expenditure in housing. Equation (3.14) is a log-linear utility function, i.e. the logarithmic transformation of the traditional Cobb-Douglas utility function (from Cobb and Douglas, 1928), which gives the same results in the present case since we work with an ordinal utility (this will not be the case any more in Part III). The selection of this functional form is a strong assumption that will be used through the current chapter. The log-linear form was chosen here for several reasons, which also explain why in urban economic literature it is the form of utility function which is used most often. First, it matches the assumption of a well-behaved utility function (see Fujita, 1989, p.12), which is central in the basic monocentric model and ensures that $U\Big(z(r), s(r)\Big)$ is defined only for positive values of $z(r)$ and $s(r)$. Second, it contains only a single parameter β, the share of net income spent in housing, which is available in statistics. Third, β is independent of prices, as found in the empirical literature (Davis and Ortalo-Magné, 2011). Extension to more general representations of preferences, such as utility functions with constant elasticity of substitution (CES), is left for further studies.[6]

With the log-linear utility function (3.14), households' consumption problem writes

$$\max_{r} \left\{ U\Big(z(r), s(r)\Big) = (1-\beta)\ln\Big(z(r)\Big) + \beta\ln\Big(s(r)\Big) \right\} \tag{3.15}$$

$$\text{s.t. } z(r) + R(r)s(r) = Y - T(r) \ . \tag{3.16}$$

To solve this problem, one can compute the marginal rate of substitution

$$\frac{\partial U\Big(z(r), s(r)\Big)}{\partial z(r)} \left(\frac{\partial U\Big(z(r), s(r)\Big)}{\partial s(r)} \right)^{-1} = \frac{(1-\beta)s(r)}{\beta z(r)} \ , \tag{3.17}$$

which can be equalized to the ratio of prices in order to have the optimal choice equation, that is

$$\frac{(1-\beta)s(r)}{\beta z(r)} = \frac{1}{R(r)} \ . \tag{3.18}$$

Simultaneously solving the optimal choice equation (3.18) and the budget constraint (3.16) by appropriate substitutions yields the solution of the households consumption problem,

$$z(r) = (1-\beta)\Big[Y - T(r)\Big] \ , \tag{3.19}$$

$$s(r) = \frac{\beta\Big[Y - T(r)\Big]}{R(r)} \ . \tag{3.20}$$

[6] Actually, the log-linear utility is a homothetic function since it is the logarithmic transformation of the Cobb-Douglas utility, which is itself homogeneous. This is a limit case of the of the utility function with constant elasticities of substitution (CES), which represents homothetic preferences (see for example Varian, 2011).

Substituting back the optimal consumptions (3.19) and (3.20) into the utility function (3.15) yields the indirect utility, which can be set to an arbitrary level u in order to express the bid rent function

$$\Psi(r,u) = e^{-u/\beta}\beta(1-\beta)^{1/\beta-1}\Big[Y - T(r)\Big]^{1/\beta} . \tag{3.21}$$

Finally substituting the bid rent (3.21) into the optimal housing consumption (3.20) yields the bid-max lot size

$$s(r,u) = e^{u/\beta}\Bigg[(1-\beta)\Big[Y - T(r)\Big]\Bigg]^{1-1/\beta} . \tag{3.22}$$

The equilibrium population density profile is found by solving the total population condition (3.10) using the boundary rent condition (3.11). On the one hand, substituting the bid rent function (3.21) into the boundary rent condition (3.11) yields the equilibrium urban fringe

$$f = T^{-1}\big(Y - (1-\beta)^{\beta-1}\beta^{-\beta}a^{\beta}e^{u}\big) . \tag{3.23}$$

On the other hand, consecutively substituting the optimal housing consumption (3.22) into equation (3.9), and the resulting value of population distribution into the population condition (3.10) yields

$$e^{-u/\beta}(1-\beta)^{1/\beta-1}\int_0^f L(r)H(r)\Big[Y - T(r)\Big]^{1/\beta-1}\mathrm{d}r = N . \tag{3.24}$$

In general, an analytical solution for the equilibrium utility u cannot be obtained by substituting the equilibrium urban fringe (3.23) into the expression of total population (3.24). However, with the assumption that the agricultural land rent is null[7] ($a = 0$), the equilibrium urban fringe becomes

$$f = T^{-1}(Y) \;\Leftrightarrow\; Y = T(f) , \tag{3.25}$$

which means that the urban fringe is the distance at which households spend their entire wage in commuting. Equation (3.25) is very powerful since it enables, given a transport cost function $T(r)$, to express the results with respect either to the urban fringe f or to the wage Y. Now, substituting the right-hand-side equation of (3.25) into the population constraint yields the equilibrium

[7]This assumptions is very common in urban economic literature and the it is supported by the relatively low values of agricultural lands compared to those of urban lands (see Chicoine, 1981; Fujita and Thisse, 2013)

utility

$$e^{u/\beta} = N^{-1}(1-\beta)^{1/\beta-1} \int_0^f L(r)H(r)\Big[T(f)-T(r)\Big]^{1/\beta-1} \mathrm{d}r \ , \tag{3.26}$$

which can be consecutively substituted into the optimal housing consumption (3.22) and into the population to housing density (3.12) in order to express the population density function

$$\rho(r) = NH(r)\Big[T(f)-T(r)\Big]^{1/\beta-1} \left[\int_0^f L(r)H(r)\Big[T(f)-T(r)\Big]^{1/\beta-1} \mathrm{d}r\right]^{-1} . \tag{3.27}$$

Note also that the bid rent $\psi(r)$ is given by $\psi(r) = \beta(T(f)-T(r))\rho(r)/H(r)$, that is

$$\psi(r) = N\beta\Big[T(f)-T(r)\Big]^{1/\beta} \left[\int_0^f L(r)H(r)\Big[T(f)-T(r)\Big]^{1/\beta-1} \mathrm{d}r\right]^{-1} . \tag{3.28}$$

To sum up, this section has introduced non-housing land uses in the monocentric model of Alonso (1964) by means of a housing profile $H(r)$ (see equation 3.9). Further assuming a log-linear utility function (equation 3.14), it has derived a general population density profile for a single city of size N (equation 3.27). In the next section, the existence of several cities of different sizes is assumed, and the conditions for the homothetic scaling of their population density profiles (3.27) are derived. The assumption of coexisting cities of different sizes is discussed afterwards, in Section 3.3.3.

3.3.2 Scaling urban profiles

Scaling laws are now introduced. Following a monocentric approach, cities are two-dimensional circular objects whose symmetry can be exploited to describe them along a single horizontal dimension. Thus if one considers, for example, the population density profile with respect to the distance from the city center, it is actually a concise representation of a three dimensional cone with circular basis. The homothetic transformations considered in this paper are the projective transformations of this three dimensional cone with respect to the center of its basis and according to a constant dilation factor. Following Lemoy and Caruso (2017), European population density cones of cities are assumed to be similar to an homothetic transformation whose dilation factor is a unique power law of their respective populations. This unique exponent is called the scaling power or scaling exponent. The abstract case of a city with a single inhabitant, which is useful as a theoretical reference case, is call the unitary city.[8] Still following Lemoy and Caruso (2017), the European cones of the share of land used for housing are assumed to be similar to a non-homothetic transformation, whose dilation factor in the vertical dimension is not the same as in the two horizontal dimensions. Hence the concepts of two-dimensional horizontal and (one-dimensional) vertical scaling exponents.

In accordance with Section 3.2, let $\rho_N(r)$ be the population density profile and $H_N(r)$ be the profile of the share of land used for housing for a city of total population N. Following empirical evidence of Lemoy and Caruso (2017), it is assumed there exists α and γ such that population density profiles $\rho_N(r)$ scale homothetically in three dimensions with the power α of population N, and that housing land radial profiles $H_N(r)$ scale homothetically in the two horizontal dimensions with the power γ of population. These homothetic scaling laws can be formalized as:

$$\rho_N(r) = N^\alpha \rho_1\left(\frac{r}{N^\alpha}\right) \ , \tag{3.29}$$

$$H_N(r) = H_1\left(\frac{r}{N^\gamma}\right) \ , \tag{3.30}$$

where $\rho_1(r)$ and $H_1(r)$ are the population and land use radial profiles of an abstract unitary city of population $N = 1$. According to Lemoy and Caruso (2017), European urban areas obey equations 3.29 and 3.30 (up to some fluctuations which are illustrated later in this work) with the exponents $\alpha \simeq 1/3$ and $\gamma \simeq 1/2$.

In the Alonso-LU model, the scaling of the population density profile relies on two important scaling assumptions: wages and transport costs are assumed to depend on the total population N of the city. Their variations with city size will strive to reproduce the empirical radial profiles of small and large cities. Note that the measure of agglomeration economies and costs through elasticities of wages and transport costs is well established in the empirical economic literature (Rosenthal and Strange, 2004; Combes et al., 2010, 2011, 2012). In the literature, this urban wage premium has been related to the previous discussion of agglomeration economies (see Chapter 2) through mix processes of sorting an learning (Glaeser and Maré, 2001; Yankow, 2006; D'Costa and Overman, 2014; Carlsen et al., 2016; Roca and Puga, 2017). These studies imply power law functions, which are also the backbones of urban scaling laws

[8]Throughout this paper, scaling properties will implicitly refer to scaling with respect to urban population N. Thus, indices "N" are used to indicate exogenous variables or functions that are assumed to vary with N. Accordingly, indices "1" are used to indicate the value of those variables for the unitary city (i.e. $N = 1$).

literature (Bettencourt et al., 2007; Shalizi, 2011; Bettencourt, 2013; Leitão et al., 2016). Hence, following both strands, it is assumed that $Y_N = N^\phi Y_1$, where Y_1 is the wage in a unitary city and ϕ is the elasticity of wage with respect to urban population. Similarly, it is assumed that the transport cost function $T_N(r)$ is a scaling transformation (not necessarily homothetic) of $T_1(r)$, the transport cost function in a unitary city (assumed to be continuously increasing and differentiable in r). The exact form of this transformation will derive from the conditions for a homothetic scaling, as will be clarified hereafter (equation 3.36).

Conditions of homothetic scaling

In order to derive conditions under which the population density function (3.27) respects the homothetic scaling (3.29), one first rescales distances accordingly. Formally, under the following change of variable

$$r_1 = \frac{r}{N^\alpha} \Leftrightarrow r = r_1 N^\alpha \ , \tag{3.31}$$

the population density function (3.27) rewrites

$$\rho_N(r) = \frac{N^{1-\alpha} H_N\big(r_1 N^\alpha\big)\Big[T_N\big(f_1 N^\alpha\big) - T_N\big(r_1 N^\alpha\big)\Big]^{1/\beta - 1}}{\int_0^{f_1} L\big(r_1 N^\alpha\big) H_N\big(r_1 N^\alpha\big)\Big[T_N\big(f_1 N^\alpha\big) - T_N\big(r_1 N^\alpha\big)\Big]^{1/\beta - 1} \mathrm{d}r_1} \ , \tag{3.32}$$

where $r_1 = r/N^\alpha$. Note that the urban fringe f_N has to be rescaled as well, following

$$f_1 = \frac{f_N}{N^\alpha} \Leftrightarrow f_N = f_1 N^\alpha \ . \tag{3.33}$$

This has, due to equation (3.25), important consequences on the scaling properties of Y_N and T_N which are discussed in the inter-urban analysis (Sec. 3.3.3). Finally, assume that $L(r)$ is linearly homogeneous, that γ (the scaling power of H_N, see equation 3.30) is the same as α (the scaling power of ρ_N, see equation 3.29), and that $T_N(r)$ is at least horizontally scaling. This formally writes

$$\forall \lambda \in \mathbb{R}: \quad L(\lambda r) = \lambda L(r) \ , \tag{3.34}$$

$$\gamma = \alpha \ , \tag{3.35}$$

$$\exists \theta \in \mathbb{R}: \quad T_N(r) = N^\theta T_1\left(\frac{r}{N^\alpha}\right) \ . \tag{3.36}$$

The first assumption will add a "$-\alpha$" term to the power of N in the population density function (3.32). The second assumption assumption implies that the horizontal scaling of the housing usage function (3.30) balances the effect of total population. The third assumption, equivalent to $T_N(rN^\alpha) = N^\theta T_1(r)$, enables one to factorize $N^{(1/\beta-1)\theta}$ both in the numerator and the denominator of (3.32), so that they cancel out. Altogether, this yields

$$\rho_N(r) = \frac{N^{1-2\alpha} H_1(r_1)\Big[T_1(f_1) - T_1(r_1)\Big]^{1/\beta-1}}{\int_0^{f_1} L(r_1)H_1(r_1)\Big[T_1(f_1) - T_1(r_1)\Big]^{1/\beta-1} \mathrm{d}r_1} , \tag{3.37}$$

where $r_1 = r/N^\alpha$, which is simply a power function of N. Note that the bid rent can be expressed accordingly as

$$\psi_N(r) = \frac{N^{1/3+\theta}\beta\Big[T_1(f_1) - T_1(r_1)\Big]^{1/\beta}}{\int_0^{f_1} L(r_1)H_1(r_1)\Big[T_1(f_1) - T_1(r_1)\Big]^{1/\beta-1} \mathrm{d}r_1} . \tag{3.38}$$

where $r_1 = r/N^\alpha$. In order to finally get the homothetic scaling (3.29) of the population density function, one has to assume that $1-2\alpha = \alpha$ holds, resulting in

$$\alpha = \frac{1}{3} . \tag{3.39}$$

This result means that the population density profile follows the three-dimensional homothetic scaling (3.29) if and only if $\alpha = 1/3$, which coincidentally matches the empirical evidence of Lemoy and Caruso (2017). This further implies that Alonso's fundamental trade-off between transport and housing is able to explain the observation that cities are similar objects across sizes, provided land profiles, wages and transport costs scale with total population. In other words, a single density profile can be defined from Alonso-LU to match any European city. The main drawback of Alonso-LU is condition (3.35) above, which requires that the scaling exponent of the housing profile is 1/3, instead of the observed value of 1/2 (Lemoy and Caruso, 2017). The possibility of correcting this result by using an explicit houding markt will be discussed in Section 3.5. The results of this subsection are based on the assumption that several cities of different sizes coexist at equilibrium. This assumption is now discussed in an inter-urban analysis.

3.3.3 Inter-urban analysis

Up to now, a closed city of size N has been considered. Yet cities belong to urban systems where households may move from one city to another. This perspective holds two implications. First, since cities of different population size coexist in real urban systems, the equilibrium of the model should be able to reproduce this fact. As a consequence, the benefits and costs of urban agglomeration should vary together when population size changes, so as to compensate each other whatever the size of the city otherwise. If one force would dominate the other, the urban system would either collapse to a single giant city or be peppered into countless unitary cities. Second, since by definition households' location decisions are mutually consistent at equilibrium, the equilibrium utility level has to be the same whatever the city population N. Otherwise, households would have an incentive to move to larger or smaller cities.

First, substituting the scalings of wages, of the urban fringe (3.33) and of the transport cost function (3.36) into the right-hand-side equation of relationship (3.25) yields

$$Y_1 N^{\phi} = T_N(f_N) = T_N(f_1 N^{\alpha}) = N^{\theta} T_1(f_1) = N^{\theta} Y_1 \ , \tag{3.40}$$

which implies

$$\phi = \theta \ . \tag{3.41}$$

Equation 3.41 implies that the elasticity of wages with respect to urban population (ϕ) equals θ, which is the elasticity of the transport cost function once it has been horizontally rescaled. Yet, following the approach of Dixit (1973), ϕ is representative of urban agglomeration economies whilst θ results from agglomeration costs.[9] Hence the condition for several cities of different population to coexist at equilibrium is met. Note that this equality is supported by recent developments in the very limited empirical literature on agglomeration costs (Combes et al., 2012).

Second, successively applying the two changes of variable (3.31) and (3.33) to the equilibrium utility (3.26), and substituting the conditions of homothetic scaling (3.34) and (3.35) yields

$$e^{u/\beta} = N^{(1/\beta-1)\theta-(1-2\alpha)}(1-\beta)^{1/\beta-1} \int_0^{f_1} L(r_1) H_1(r_1) \Big[T_1(f_1) - T_1(r_1) \Big]^{1/\beta-1} \mathtt{d}r_1 \ . \tag{3.42}$$

Since at equilibrium households have no incentive to move to another city, equilibrium utility (3.42) should not change with N. Thus, equalizing the power of N in equation (3.42) to zero (other terms being already independent of N) and substituting the value of $\alpha = 1/3$ (equation 3.39) yields

$$\theta = \frac{\beta}{3(1-\beta)} \ . \tag{3.43}$$

[9] According to Dixit (1973), urban size is mainly determined by the balance between economies of scale in production and diseconomies in transport. Yet in a competitive labour market, labour is paid to its marginal productivity, so that wage-elasticity is representative of labour productivity, which capitalises itself different effects of urban economies of agglomeration. Similarly, the elasticity of the horizontally-rescaled transport cost function catches agglomeration diseconomies.

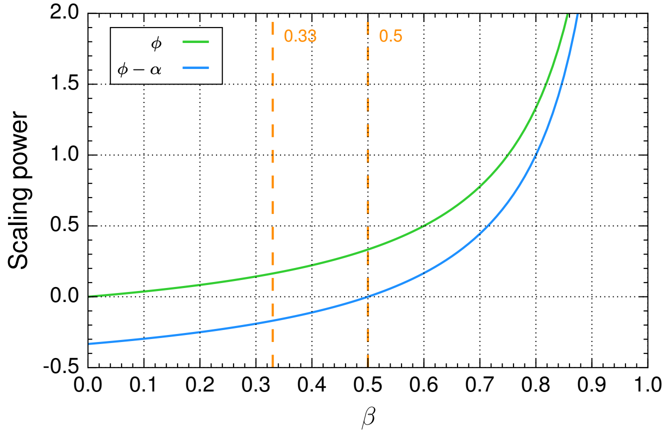

Figure 3.2. Population-elasticities of wage and transport cost with respect to housing relative expenditure. Dashed orange lines highlight values of reference discussed in the text. Recall from equation 3.41 that $\phi = \theta$.

Equation 3.43 provides a relationship between the vertical scaling exponent of the value of transport cost θ (or the population-elasticity of wages ϕ) and households' relative expenditure in housing β. This relation is increasing and suggests that a relative expenditure $\beta = 1/3$, which is in the range of empirically supported values (Accardo and Bugeja, 2009; Davis and Ortalo-Magné, 2011), would be associated to exponents $\phi = \theta = 1/6$ (Fig. 3.2). This value is the same as the superlinearity of socio-economic outputs discussed in Bettencourt (2013); Bettencourt and Lobo (2016). Consequently, the inter-urban perspective inferred by Alonso-LU is definitely compatible with some former theoretical and empirical researches. However, it diverges from urban econometric studies, which consider this elasticity to range from 2% to 5% (Combes et al., 2010, 2011). In addition, following other measures of agglomeration economies that are not only based on wages, the elasticity of productivity with respect to city population is considered to be of maximum 3% to 8% (Rosenthal and Strange, 2004). Alonso-LU does not solve these empirical incompatibilities. More research effort is needed, especially digging into the functional form of the transport cost function as discussed in the next subsection.

3.3.4 Functional form

In this subsection, an operational version of the Alonso-LU model is proposed by selecting appropriate functional forms for the land distribution $L(r)$, the housing profile $H_N(r)$ and the transport cost function $T_N(r)$. The theoretical implications of those forms are discussed as well as their empirical supports. Firstly, the land distribution $L(r)$ and the housing profile $H_N(r)$ are specified by

$$L(r) = 2\pi r \ , \tag{3.44}$$

$$H_N(r) = b \exp\left(\frac{-r}{dN^{1/3}}\right) \ , \tag{3.45}$$

where b is the share of housing land at the CBD and d is the characteristic distance of the housing land profile in a unitary city. One can easily check that the functional forms (3.44) and (3.45) respectively follow the conditions for homotheticity (3.34) and (3.35). The land distribution (3.49) is simply the usual two-dimensional circular framework, and the exponential form (3.50) of the housing land profile has been chosen for its simplicity and goodness of fit, which is discussed in Section 3.4.

Secondly, consider the following form of the transport cost function

$$T_N(r) = cN^{\mu} r^{\sigma} \ , \tag{3.46}$$

where c is the transport cost per unit distance in a unitary city and $\mu, \sigma \in \mathbb{R}^+$. Then the scaling condition (3.36) requires

$$\theta = \alpha\sigma + \mu \ , \tag{3.47}$$

where the elasticity θ of the transport cost function has been broken into two parts. On the one hand, the power effect of distance contributes by $\alpha\sigma$ to the elasticity θ because of the horizontal scaling. On the other hand, the contribution of μ stands for urban population effects like congestion. Further substituting (3.43) and (3.39) into (3.47) yields

$$\mu = \frac{\beta - \sigma(1-\beta)}{3(1-\beta)} \ . \tag{3.48}$$

It appears that the functional form of the transport cost function, and in particular the chosen power σ of the distance to the CBD, imposes a relationship between the elasticity of unitary transport cost with respect to urban population μ and the relative housing expenditure β. In the case of a linear transport cost function ($\sigma = 1$), which is largely practised by urban economists, the elasticity of unitary transport cost with respect to urban population μ equals $\theta - 1/3$ (since $\alpha = 1/3$). It suggests that for $\beta < 1/2$ (which is empirically supported, see Davis and Ortalo-Magné, 2011) the unitary transport cost should decrease with urban population (Fig. 3.2). This strives against Dixit (1973) and the expectation that unitary transport cost is increasing with urban population due to congestion. This result shows that the linear transport is not consistent with the scaling of urban profiles. Concave transport cost functions however fix this shortcoming. For example, in the case of a root-square transport cost function ($\sigma = 0.5$), the elasticity of unitary transport cost with respect to urban population μ equals $\theta - 1/6$, which becomes negative for $\beta < 1/3$.

Thus the Alonso-LU model suggests that a concave transport cost function is more realistic. However, as showed hereunder, such functional form makes the final form of the population density profile difficult to handle. Since the functional form of the Alonso-LU model is intended to be confronted to empirical data, in the remaining of this chapter the linear form of the transport cost function is used, despite its theoretical drawbacks. To sum up, the functional form is given by

$$L(r) = 2\pi r \ , \tag{3.49}$$

$$H_N(r) = b \exp\left(\frac{-r}{dN^{1/3}}\right) \ , \tag{3.50}$$

$$T_N(r) = cN^{\mu} r \ , \tag{3.51}$$

where $\mu = (2\beta - 1)/(3 - 3\beta)$, which follows from (3.48).

Functional population density profile

Substituting the functional form equations (3.49)-(3.51) into the equilibrium population density function (3.37) with $\alpha = 1/3$ yields

$$\rho_N(r) = \frac{N^{1/3}}{2\pi} e^{-r_1/d} (f_1 - r_1)^{\frac{1}{\beta}-1} \left[\int_0^{f_1} r_1 e^{-r_1/d} (f_1 - r_1)^{1/\beta - 1} \, \mathrm{d}r_1 \right]^{-1} \ , \tag{3.52}$$

with $r_1 = r/N^{1/3}$. Now, under the change of variable $y = f_1 - r_1$ the integral in equation (3.52) becomes

$$f_1 e^{-f_1/d} \int_0^{f_1} e^{y/d} y^{1/\beta - 1} \mathrm{d}y - e^{-f_1/d} \int_0^{f_1} e^{y/d} y^{1/\beta} \mathrm{d}y \ , \tag{3.53}$$

that the second change of variable $x = y/d$ turns to

$$f_1 e^{-f_1/d} d^{1/\beta} \int_0^{f_1/d} e^x x^{1/\beta - 1} \mathrm{d}x - e^{-f_1/d} d^{1/\beta + 1} \int_0^{f_1/d} e^x x^{1/\beta} \mathrm{d}x \ . \tag{3.54}$$

The first integral in (3.54) can be integrated by parts using

$$x^{1/\beta-1} = \frac{\partial(\beta x^{1/\beta})}{\partial x} . \tag{3.55}$$

After algebraic simplifications, this yields

$$\beta f_1^{1/\beta+1} - (\beta f_1 + d)\, e^{-f_1/d} d^{1/\beta} \int_0^{f_1/d} e^x x^{1/\beta} \mathtt{d}x , \tag{3.56}$$

which can be finally substituted to the integral into equation (3.52) to give the population density profile

$$\rho_N(r) = \frac{N^{1/3} e^{-r_1/d} (f_1 - r_1)^{\frac{1}{\beta}-1}}{2\pi \left[\beta f_1^{1/\beta+1} - (\beta f_1 + d)\, e^{-f_1/d} d^{1/\beta} \int_0^{f_1/d} e^x x^{1/\beta} \mathtt{d}x\right]} , \tag{3.57}$$

where $r_1 = r/N^{1/3}$, and the associated bid rent profile

$$\psi_N(r) = \frac{N^{1/3+\theta} \beta (f_1 - r_1)^{\frac{1}{\beta}}}{2\pi \left[\beta f_1^{1/\beta+1} - (\beta f_1 + d)\, e^{-f_1/d} d^{1/\beta} \int_0^{f_1/d} e^x x^{1/\beta} \mathtt{d}x\right]} . \tag{3.58}$$

This functional population density profile (3.57) cannot be further simplified, unless one imposes additional constrains on β (see the example 3.3.1). Thus, its final form depends on three parameters: the unitary urban fringe $f_1 = Y_1/c$, the housing expenditure β and the characteristic distance d of the housing land profile in a unitary city. This density profile model is suitable for empirical calibration, which is performed in the next section. Note that this is a daring exercise since all cities in Europe are calibrated at once using only those three parameters. Its success will expose the descriptive power of the homothetic scaling.

Example 3.3.1. *Alonso-LU model with $\beta^{-1} \in \mathbb{N}$.*

By restricting the definition domain of β to $\{x^{-1} | x \in \mathbb{N}_0\}$, so that $\frac{1}{\beta} \in \mathbb{N}$, one gets

$$\int_0^{f_1/d} e^x x^{1/\beta} \mathtt{d}x = \sum_{k=0}^{\frac{1}{\beta}} \left\{ (-1)^{\frac{1}{\beta}-k} \frac{\frac{1}{\beta}!}{k!} \left(\frac{f_1}{d}\right)^k \right\} e^{\frac{f_1}{d}} , \tag{3.59}$$

and (3.57) *becomes, after algebraic manipulation,*

$$\rho_N(r) = \frac{N^{1/3}\,(f_1 - r_1)^{\frac{1}{\beta}-1}\,e^{-\frac{r_1}{d}}}{2\pi d^2 \sum_{k=0}^{\frac{1}{\beta}-1}\left\{(-1)^{\left(\frac{1}{\beta}+1\right)-k}\frac{\left(\frac{1}{\beta}-1\right)!\left(k-\frac{1}{\beta}\right)}{k!}f_1^k\,d^{\left(\frac{1}{\beta}-1\right)-k}\right\}} \,. \tag{3.60}$$

Taking some arbitrary values of β *yields different expansions of the constant term in the population density profile:*

	$\rho_N(r)$
$\beta = \frac{1}{4}$	$\left[2\pi d^2\left(24d^3 - 18f_1d^2 + 6f_1^2d - f_1^3\right)\right]^{-1} N^{1/3}\,(f_1 - r_1)^{\frac{1}{\beta}-1}\,e^{-\frac{r_1}{d}}$
$\beta = \frac{1}{3}$	$\left[2\pi d^2\left(-6d^2 + f_1d - f_1^2\right)\right]^{-1} N^{1/3}\,(f_1 - r_1)^{\frac{1}{\beta}-1}\,e^{-\frac{r_1}{d}}$
$\beta = \frac{1}{2}$	$\left[2\pi d^2\,(2d - f_1)\right]^{-1} N^{1/3}\,(f_1 - r_1)^{\frac{1}{\beta}-1}\,e^{-\frac{r_1}{d}}$

▲

3.4 Empirics: European profiles

In this section, the functional model (3.57) is calibrated to the average European population density profiles of Fig. 3.1 using nonlinear least squares. The calibration procedure is performed in two steps. First, the optimal value of d is calibrated by comparing the share of housing land (3.50) to the average profile for a reference city of population $\bar{N}$. Second, the optimal value of d is substituted into the population density function (3.57), which in turn is calibrated to the average population density profile, once by optimizing the values of f_1 and β, and once by optimizing only the value of f_1 with a fixed $\beta = 1/3$. Results are visualized for four individual cities.

3.4.1 Housing land profile

In this subsection, the share of housing land (3.50) is calibrated to the average profile (Fig. 3.1) for a population of reference $\bar{N}$. This population can be chosen arbitrarily, yet the condition for homothetic scaling (3.35) imposes a scaling power of 1/3, which is different from the empirical one (1/2). As a result, the fit is optimal for the population of reference $\bar{N}$, but rescaling to other population sizes generates an error. Considering the empirical exponents of Lemoy and Caruso (2017), the best model of housing usage is

$$H_N(r) = b\exp\left(\frac{-r}{gN^{1/2}}\right) \,, \tag{3.61}$$

where g is a constant, whereas the approximate model is

$$H'_N(r) = b \exp\left(\frac{-r}{dN^{1/3}}\right) \ . \tag{3.62}$$

The absolute error between the best model (3.61) and the approximate model (3.62) is given by

$$b \exp\left(-\frac{r}{gN^{1/2}}\right)\left[\exp\left(\frac{-r}{dN^{1/3}} - \frac{-r}{gN^{1/2}}\right) - 1\right] \ , \tag{3.63}$$

where the relative error is the term between braces. By definition, $\bar{N}$ is a population size chosen arbitrarily, for which the two characteristic distances are equal, thus annihilating the relative error. That is,

$$d = g\bar{N}^{1/6} \ , \tag{3.64}$$

such that the relative error rewrites

$$\exp\left(\left[\left(\frac{N}{\bar{N}}\right)^{1/6} - 1\right]\frac{-r}{gN^{1/2}}\right) - 1 \ . \tag{3.65}$$

It appears from (3.65) that for any European city with $N > \bar{N}$, the housing share is underestimated and *vice versa* (Fig. 3.3). The relative error is bigger, the bigger the difference between N and $\bar{N}$. Hence a first desirable property is that the relative error for the smallest city is the same as for the largest one. This is equivalent to minimizing the maximal relative error. However, this cannot be true for any value of r since the relative error is increasing in r. This is not the case of the absolute error (3.63), which has a maximum value at

$$\bar{r} = -\frac{gN^{1/2}}{6}\ln\left(\frac{\bar{N}}{N}\right)\left[\left(\frac{N}{\bar{N}}\right)^{1/6} - 1\right]^{-1} \ , \tag{3.66}$$

and at this distance the relative error is simply

$$\left(\frac{\bar{N}}{N}\right)^{\frac{1}{6}} - 1 \ . \tag{3.67}$$

Finally, the critical population $\bar{N}$ is chosen as the value for which the absolute value of the relative error at the critical distance $\bar{r}$ is the same for the smallest city in the database, Derry (UK, $1.03\,10^5$ `hab`), and for the largest, London. This yields

$$\bar{N} = \left(\frac{2}{(1.03\,10^5)^{-1/6} + (1.21\,10^7)^{-1/6}}\right)^6 \simeq 7.03\,10^5. \tag{3.68}$$

Table 3.1. Nonlinear least square results. Calibration are performed on European average profiles made up with 694 points, for a population of reference $\bar{N} = 7.03\,10^5$. Distances d and f_1 are expressed in kilometers. $C(b,d)$ is correlation between parameters. BIC is the Bayesian information criterion.

A. Housing usage			B. Population density				
	Alonso	Alonso-LU		Alonso		Alonso-LU	
b	0.167 (0.005)	0.523 (0.001)	β	0.02	0.34	0.02	0.34
d	∞	5.80 (0.02)	$f_{\bar{N}}$	172.9 (0.5)	12.94 (0.07)	409 (2)	23.1 (0.2)
$C(b,d)$		-0.74					
BIC	$-2\,808$	$-6\,332$	BIC	7 613	8 744	7 544	7 893

For a city with population $\bar{N}$, the best fit suggests that the characteristic distance is $d = 5.8$km (Tab. 3.1). Besides, we see that 52.3% of land is dedicated to housing at the CBD, which slightly offsets the average empirical value (Fig. 3.1). In the Alonso model, the best constant value of housing share is around 17% (Tab. 3.1), which is a poor description of data.

Four cities of different sizes are chosen as illustrations, namely London (Ldn), the largest urban area of the dataset with a population of $N = 1.21\,10^7$ in 2006, Brussels (Bxl), the capital of Belgium with $N = 1.83\,10^6$, Luxembourg (Lux), capital of the country of the same name, with $N = 4.52\,10^5$ and Namur (Nam), the capital of Wallonia in Belgium, with $N = 1.39\,10^5$. Hence, the population of reference $\bar{N}$, for which the error is minimized, is between those of Luxembourg and Brussels. It appears that because of the wrong scaling exponent, the larger the difference between the population N of the considered city and the reference population $\bar{N}$, the larger is the error on housing land share (Fig. 3.3). For $N > \bar{N}$, the housing share is underestimated, and overestimated for $N < \bar{N}$. In the case of the four considered cities, the absolute error does not exceed 12 points (35% in relative terms, see Fig. 3.3).

3.4.2 Population density profile

This subsection performs the calibration of the population density function (3.57) to the average population density profile (Fig. 3.1), using the optimal value d (Tab. 3.1) obtained in the previous subsection. The focus is again on a city of size $\bar{N} = 7.03\,10^5$, this time without loss of generality since the scaling of population density in the model is in agreement with empirical results. The optimal values of the urban fringe $f_{\bar{N}}$ and of the relative expenditure in housing β turn out to be negatively correlated. The best fit is therefore a corner solution with arbitrarily small values of β and arbitrarily high values of $f_{\bar{N}}$ (Fig. 3.4). In the following, an "optimal" model with $\beta = 0.02$ is considered. However,

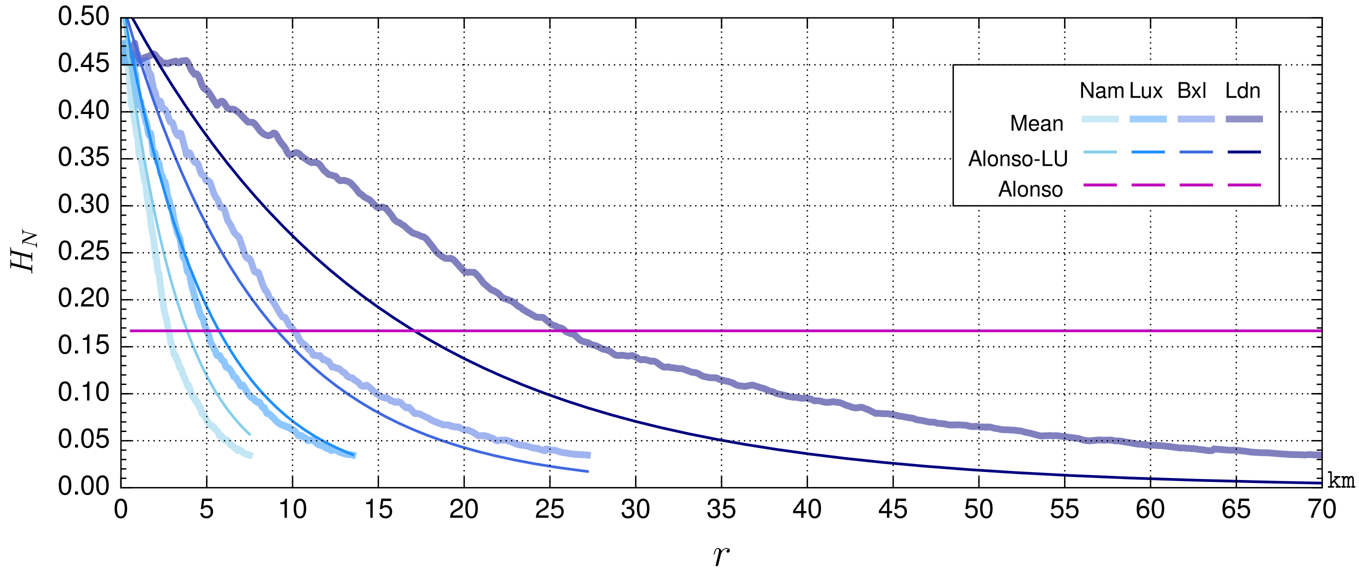

Figure 3.3. Calibration of the average housing usage profile. Mean and best-fit, rescaled to the four example cities.

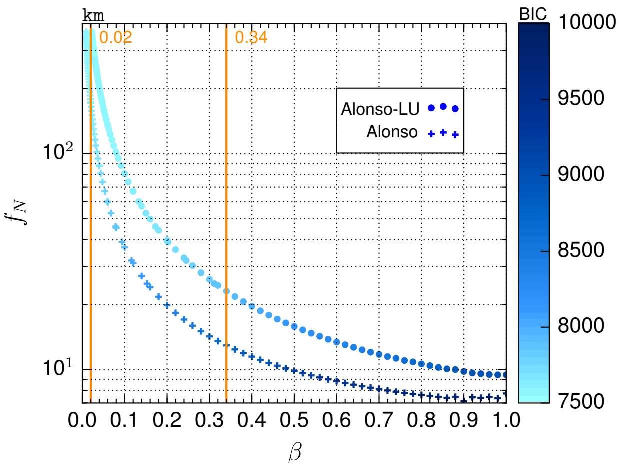

Figure 3.4. Best fit parameters for the average population density profile. The average profile has been rescaled without loss of generality to a reference city of size $\bar{N} = 7.03\,10^5$. Colours represent the Bayesian information criterion (BIC). Orange lines show parameter values of the optimal ($\beta = 0.02$) and constrained ($\beta = 0.34$) models.

this value is unrealistic (Davis and Ortalo-Magné, 2011) and thus questions the ability of monocentric models to describe real cities. As exposed previously (Sec. 3.3.4), this issue could probably be solved by using a concave commuting cost function, but at the expense of mathematical tractability. At this stage, it an intermediate solution is to consider a constrained model with $\beta = 0.34 \simeq 1/3$ as a reference case (Fig. 3.4).

Looking at the best-fit population density profile, Fig. 3.5 focuses on the case of London, knowing that smaller cities are obtained by homothetic rescaling. Note that the relative errors are visually exaggerated because of the semi-logarithmic plot. It appears that the Alonso-LU model outperforms the standard Alonso model, especially for realistic values of β. Both models display densities whose logarithms are concave because density is going to zero at $r = f_{\bar{N}}$. Conversely, the empirical population density profile appears convex. As a result, the best fit model is almost linear in the semi-logarithmic plot (hence almost exponential with linear axes). This form has been long studied empirically in urban economics since Clark (1951). Theoretical justifications for this exponential form have been provided by Mills (1972); Brueckner (1982) after adding building construction in the Alonso model, or by Anas et al. (2000) who used exponential unitary commuting costs. This chapter contributes a different explanation that is parsimonious and works across city sizes.

Using the four cities of reference, Fig. 3.6 shows that the Alonso-LU model

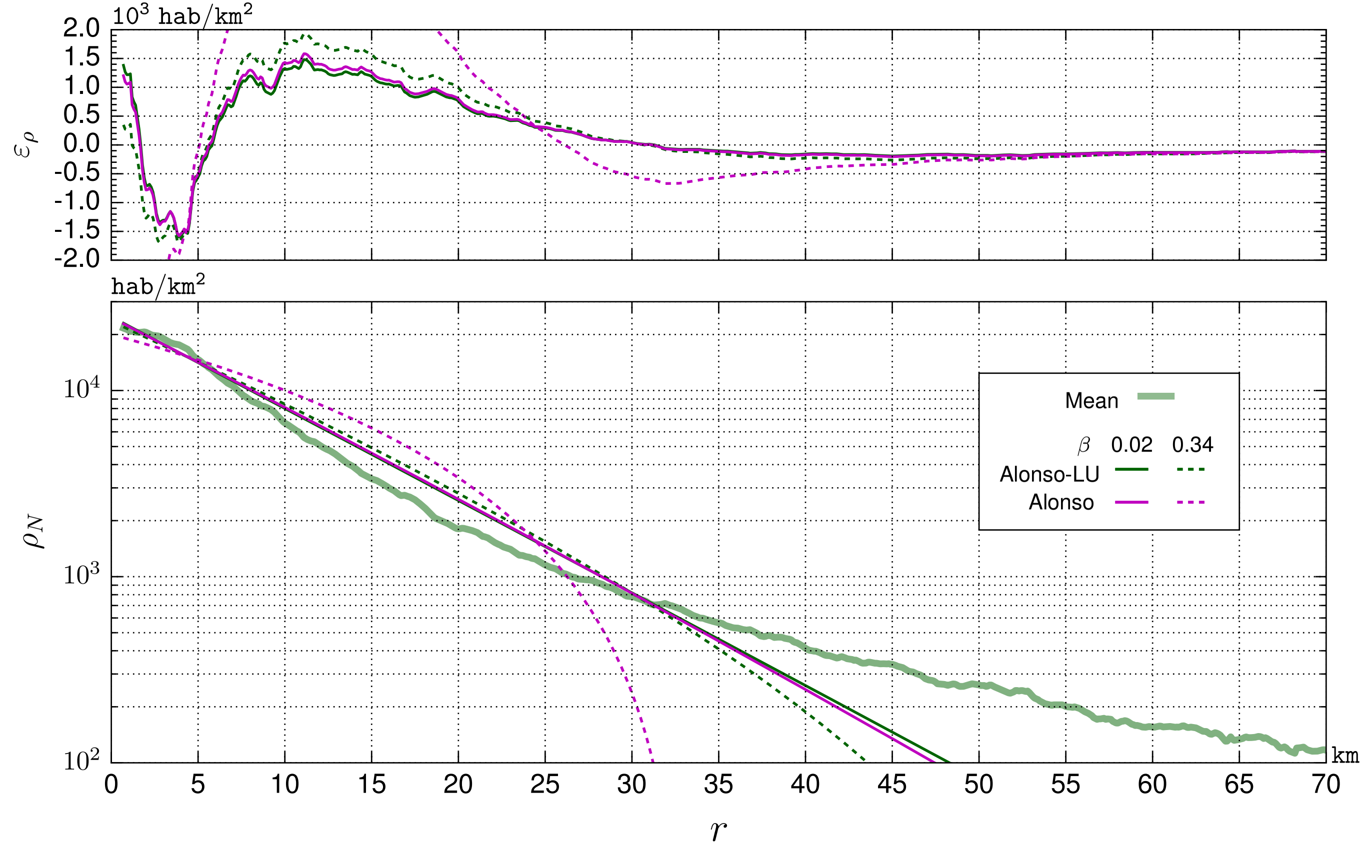

Figure 3.5. Calibration of the average population density profile. Note that the semi-logarithmic scale of the bottom subplot visually exacerbates the error at larger distance from the CBD. On the top subplot, ε_ρ denotes residuals.

gives a good description of population density profiles for European cities, whatever their size. Four additional cities are provided on Fig. 3.7, in order to complement Fig. 3.6: Paris ($N = 1.14\,10^7$), the second biggest city of the database, Wroclaw (Poland, $N = 1.03\,10^6$), Florence (Firenze, in Italy, $N = 6.81\,10^5$) and Varna (Bulgaria, $N = 3.48\,10^5$). Visual inspection reveals that the error is mostly due to deviations of individual data from the average profile, and less to deviations of the model from the average profile (Fig. 3.6 and Fig. 3.7). In that respect, the Alonso-LU model can therefore be considered to be very successful.

Residual variations around the average profiles may have several origins. First, empirical definitions of the urban centre and of the study area are two critical sources of noise (see Parr, 2007; Ahlfeldt, 2011). In the Alonso model, the CBD is the location of jobs, but also the place with the highest population density and the highest rents. Empirically, these three aspects can be used to define the actual CBD. The population density is the most common variable, although Ahlfeldt (2011) carried out a careful calibration of the Alonso model using the density of employment to define the CBD. However, using the rents is harder since they may capitalize the accessibility to other kinds ofurban amenities. An alternative approach, which was used by Lemoy and Caruso (2017), is to use an historical land mark, like a city hall, in search of a compromise. This is actually closer to modern approaches of cities, which regard urban centres are clusters of urban amenities, and not just as employment centres any more (Glaeser et al., 2001).

Second, in addition to the empirical definitions of the CBD and the study area, the relevance of the monocentric assumption also needs to be discussed. The presented empirical profiles depict an obvious monocentric urban structure (Fig. 3.1). However, they represent average values on concentric circles around the CBD. Thus they correspond to a rather coarse-grained description of cities. Although this scale is appropriate here given the level of generality of the analysis, it is well-known that cities exhibit multi-centric patterns at finer scales (Hoyt, 1939; Ogawa and Fujita, 1980; Fujita and Ogawa, 1982; Garreau, 1991; Krugman, 1996). This multicentricity may be due to employment sub-centres, but also to different kinds of urban amenities that are out of the scope of the Alonso-LU model. Anyway, it generates additional noise in the empirical profiles.

Finally, note that neither the values of the income Y_1 nor the unit distance transport cost c in a unitary city are fixed since they do not appear in the expression of the population density (3.57). The calibration is only performed on land use and population density profiles. A more comprehensive calibration including land prices or rents is left for further works. Note that the Alonso-LU model outputs rent profiles that scale non-homothetically with power 1/3

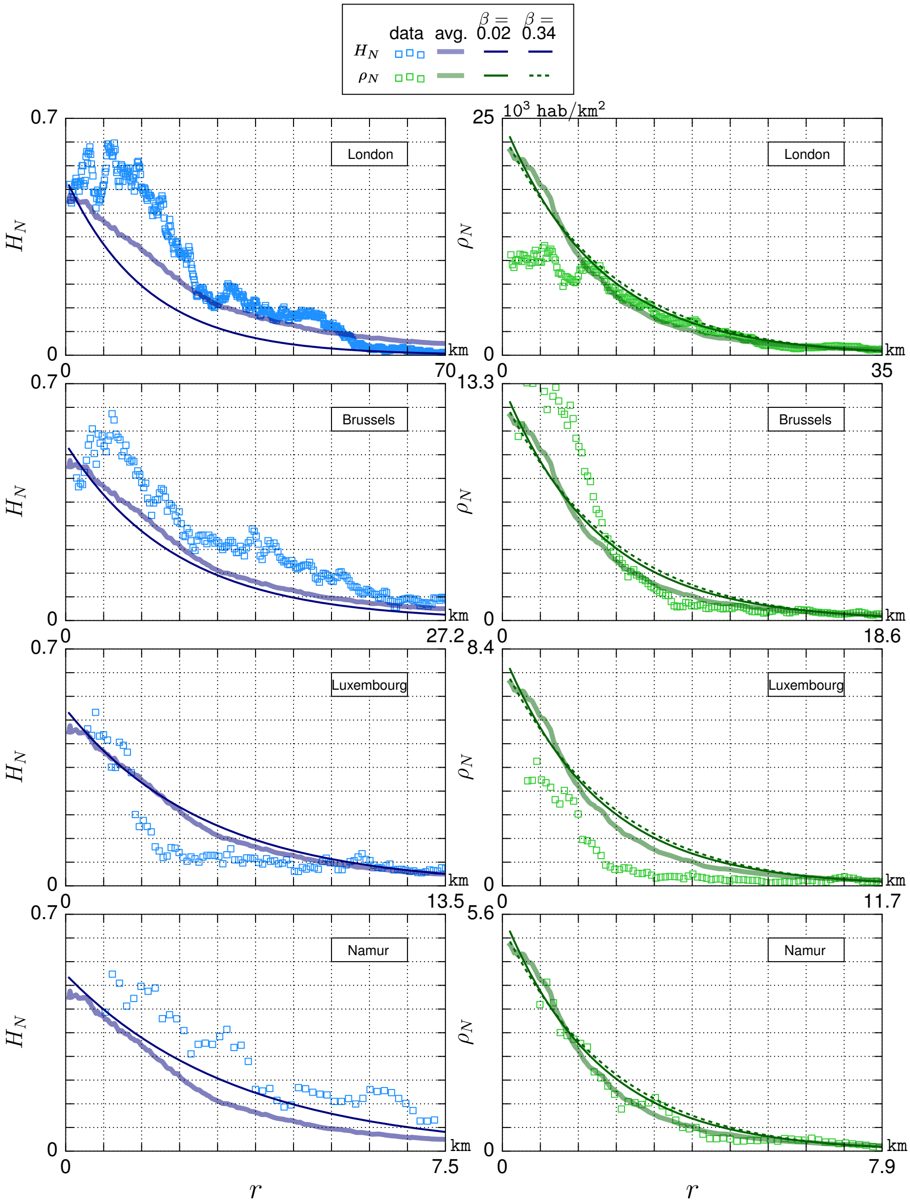

Figure 3.6. First summary plot of the results. Fitted average profiles compared to individual profiles. Left panel: housing share profile. Right panel: population density profile. Axes have been rescaled to maintain the average curves at the same position across subplots.

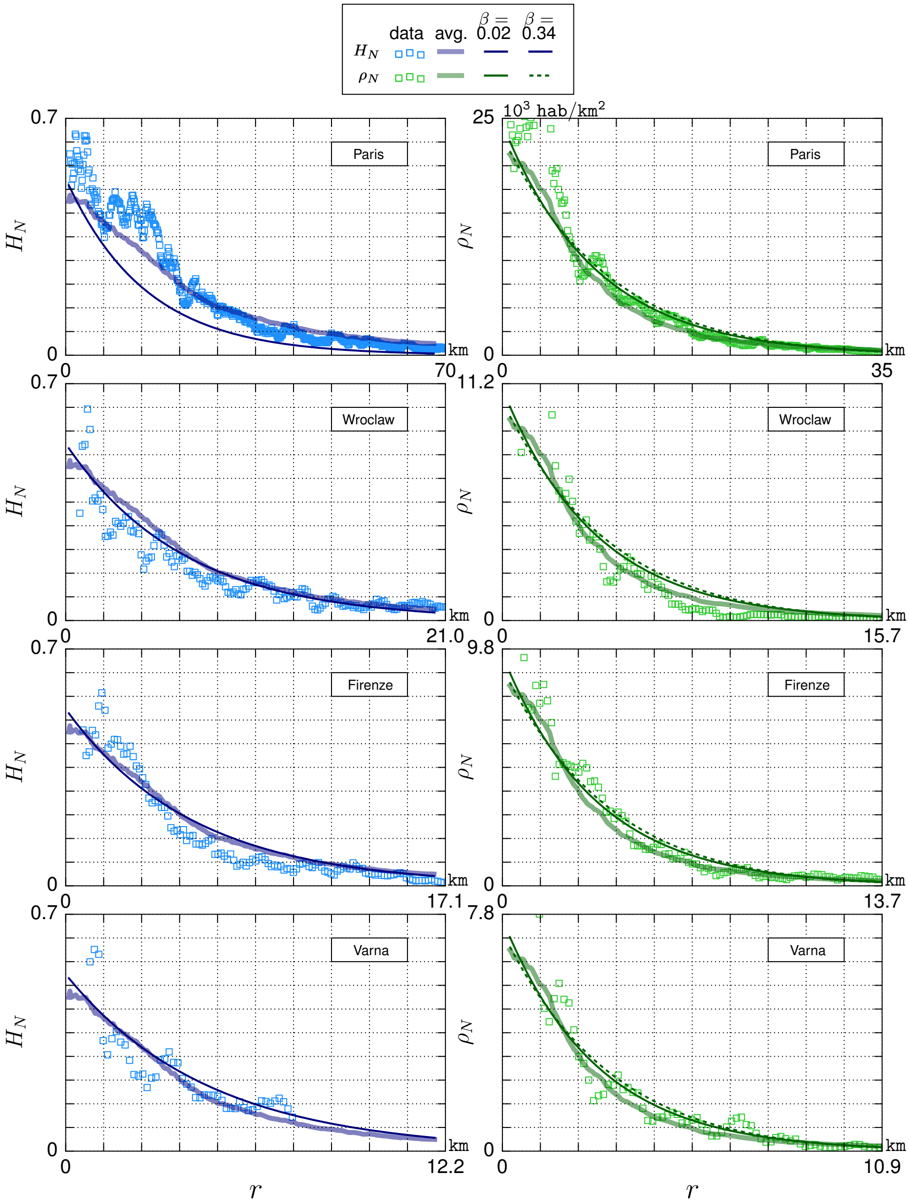

Figure 3.7. Second summary plot of the results. Fitted average profiles compared to individual profiles. Left panel: housing share profile. Right panel: population density profile. Axes have been rescaled to maintain the average curves at the same position across subplots.

in the horizontal dimensions and with power $(1/3 + \theta)$ vertically (see Section 3.58, also comparison with Duranton and Puga, 2015). It is flatter than the density profile because the (exponential) $H_N(r)$ factor present in the density disappears in the equation of rents (3.28). This flatter profile seems realistic. However, radial data for rents across European cities are required to go further.

3.5 Discussion on urban 3D morphology

On of the drawbacks of the Alonso-LU model presented in this chapter is that is infers a scaling exponent of 1/3 for the housing profile ((Sec. 3.3.2)), whilst empirical evidence of Lemoy and Caruso (2017) suggest a value close to 1/2. This failure of the model may be due to the absence of a explicit housing market. Indeed, in the Alonso and Alonso-LU models, households are living “on the ground”: they consume land directly. In reality, land is developed into housing units with more or less intensity. In empirical data of population density, heterogeneity in the intensity of housing development is mixed with the heterogeneity in housing consumptions. Thus, including housing development in the Alonso-LU model would affect the scaling power of the housing profile since in urban centres, tall multi-units buildings produce “artificial” land.

In that perspective, a seminal work was carried out by Muth (1969) and Mills (1972). They extended the monocentric land-use model of Alonso (1964) by assuming that housing land was occupied by residential buildings which are built by developers combining land l and capital k in a production function $H(l, k)$ with constant returns to scale. Consequently, their profit at an abstract location i is given by

$$\pi(i) = R_s(i)H\big(l(i), k(i)\big) - R_l(i)l(i) - rk(i) \ , \tag{3.69}$$

where $R_s(i)$ is the housing rent at location i, $R_l(i)$ is the land rent and location i and r is the constant and homogeneous interest rate of capital. In this model, constant returns in housing production are assumed so that buildings height is approximated by the capital over land ratio (Brueckner, 1987, see Fig. 3.8). This is, of course, an approximation of buildings' geometric attributes. An alternative approach, with explicit treatment of buildings' height, was proposed by Büttler and Beckmann (1980) and Büttler (1981) who use structural analysis in order to express capital expenditures in terms of buildings footprint and height. They concluded that a Cobb-Douglas functional form is a good approximation of the exact cost so that an urban developer profit becomes

$$\pi = R_s(i)\frac{l(i)h(i)}{C} - R_l(i)l(i) - F - Vl(i)^\alpha h(i)^\gamma \ , \tag{3.70}$$

where $h(i)$ is buildings height at i, C is a fixed ceiling height (Fig. 3.8), F is structural fixed cost and V is structural (variable) frame cost. α and γ are

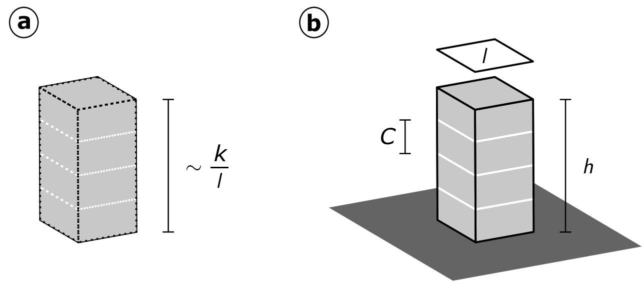

Figure 3.8. Comparison of Muth-Mills' and Büttler's urban development models. Comparison of buildings height modelling in Muth-Mills' model (Muth, 1969; Mills, 1972) on **a** and Büttler's model (Büttler and Beckmann, 1980; Büttler, 1981) on **b**.

coefficients such that $1 < \alpha < \gamma$.

Although the approach of Büttler (1981) is very appealing from a local perspective, his city-wide results are not appropriate for a geometric analysis. This is because of the "extended point" assumption (see Büttler, 1981, footnote 4, p.28) that does not consider buildings' footprint at city-wide scale. That is, they still rely on a density-based description of the urban structure. An example of consequence is that the number of buildings at a particular location is undetermined. A fully geometrical framework would thus require a subdivision of the Geographical space in discrete housing land plots. A preliminary work in this perspective can be found in Appendix A.

It is worth mentioning, though, that buildings are not taller solely in reaction to housing demand. First, from an household perspective, empirical studies showed the impact of the landscape view from a apartment on its housing rent. Indeed, Chau et al. (2007) proposed a theoretical model of developers decision and found a significant influence of the external environment on housing rents. This point was further developed by Liu and Jakus (2014) who measured similar significant influences of amenities inside and outside an urban complex, through both their visibility and accessibility. Secondly, from a business perspective, Koster et al. (2014) led an empirical study in the Netherlands and found that Dutch firms pay on average 4% more for locating in a 10m taller building. They explained this effect by agglomeration economies within buildings and by landmark effect, the later contributing to about 2.8-5.5 points to this result. Thirdly, from a developer perspective, Helsley and Strange (2008) argued that building height decisions often depart from economic rationality by involving competition for prestige among building developers. To put it differently, the relative height of a building (compared to other buildings in the city) may influence the developer decision. Finally, dynamic models of urban growth with durable capital have shown how sharp variations of building height may occur in a city because of different building ages (Brueckner, 1980a,b). All those studies underline the multiple determinants of building heights, whose consideration in a model with explicit residential development is left for further research.

3.6 Conclusion

The internal structure of cities obeys a homothetic scaling relationship with total population, which is important to model and explain in order to bridge intra-urban and inter-urban research, and eventually provide new normative hints for urban planning. This chapter showed that the fundamental trade-off between transport and housing costs is a good behavioural explanation of this internal structure of cities and holds across city sizes. It has proposed an original, augmented version of Alonso's monocentric model (Alonso-LU) that exogenously introduces urban land profile and the scaling of this profile, wages and transport costs. The model succeeds in reproducing the three-dimensional homothetic scaling of the European population density profiles suggested by Nordbeck (1971) and recently uncovered by Lemoy and Caruso (2017). Moreover, the model infers the empirical scaling power of 1/3, and is consistent with an inter-urban perspective, i.e. the coexistence of cities of different sizes. The operational version of the Alonso-LU model performs better than the original Alonso model in reproducing the two empirical average profiles. Not only is the fit good, but it is also very parsimonious in parameters (the urban fringe, the housing expenditure, and the decay of the exponential housing land profile). Moreover, comparison with data from individual cities turns out to be surprisingly good in light of the fact that a single parameter (population) is used to adapt the model to different cities.

The analysis brings those significant new findings but also comes up with three new challenges. First, the inferred scaling power of the land use profile is significantly smaller than the empirical value of Lemoy and Caruso (2017). Second, an explanation of this land use profile, which is exogenous here, is still missing. Third, the proposed model challenges current empirical understanding of wage and transport costs elasticities with population. Further research should address those points. In particular, an endogenous model of housing land development is crucial to explain the presence and increase of non-housing land with distance, as well as the scaling of the housing land profile. Potential candidates to this explanation are models of leapfrog urban land development such as Cavailhès et al. (2004b); Turner (2005); Caruso et al. (2007); Peeters et al. (2014) which invoke interaction with agricultural land, or dynamic models with uncertainty like Capozza and Helsley (1990); Irwin and Bockstael (2002). In the spirit of Muth (1969), the intensity of housing development (including vertical development) within this urban land should also be addressed in order to better describe cities in their vertical dimension. Finally, the implications of using a nonlinear transport cost need to be addressed in order to shed light on urban agglomeration economies and costs across sizes.

4

Morphodynamics of Non-Monocentric Cities

This chapter aims at exploring how individual location decisions affect the shape of a growing city and, more precisely, how they may add up to a configuration that diverges from equilibrium configurations formulated *ex-ante*. To do so, a non-monocentric two-sector city model is provided, merging a static equilibrium analysis with agent-based simulations. Results show that under strong agglomeration effects, urban development is monotonic and ends up with circular, monocentric long-term configurations. For low agglomeration effects however, elongated and multicentric urban configurations may emerge. The occurrence and underlying dynamics of these configurations are also discussed regarding commuting costs and the distance-decay of agglomeration economies between firms. The implicit interpretation of agglomeration economies as knowledge spillovers is discussed, and it is shown that an explicit model of business firms interactions reduces the set of possible configurations. From an applied perspective, this chapter warns urban planning policy makers against the difference that may stand between appropriate long-term perspectives and short-term urban configurations.

The research presented in this chapter has been carried out in collaboration with Prof Dominique Peeters and Prof Isabelle Thomas (see Delloye et al., 2015).

4.1 Introduction

In 2016, 54% of the world's population was urban and this share it is still growing today (UN-Habitat, 2016). On the one hand, this growth partly occurs within existing urban regions, making them denser and denser (World Bank, 2015). On the other hand, undeveloped lands are also converted to urban areas. This may happen through suburbanization (The Economist, 2014), or through the deliberate creation of new cities in low-density regions. The later has actually been intended in many countries across the world. Several recent projects took place in rural areas (such as in China Bai et al., 2014), on artificial island or in desert (*e.g.* Masdar city in Abu Dhabi). New brownfield renewals have also started in United Kingdoms, like in Bicester (Curtin, 2014), and were proposed in Belgium. However, some of these projects failed to attract people, recalling that planning rules should coincide with individual aspirations to achieve urban development (The Economist, 2013). Consequently, understanding how individual interactions may influence the inner properties of a growing urban system is crucial for urban planners to match their plans with those of urban citizens.

The shape or configuration of a city is particularly influenced by both top-down planning rules and bottom-up location decisions (Barthelemy et al., 2013). It is also a major concern since it influences the greenness and productivity of the city (Glaeser and Kahn, 2010; Glaeser, 2011; Wenban-Smith, 2011). Hence, when envisioning a new city, urban planners usually have at least a conceptual objective for its long-run configuration, like the Garden City model in United Kingdom (Howard, 1902; Curtin, 2014). From there, and before implementing any planning rule, important dynamic questions arise. Will the city spontaneously tend to the intended configuration? Will it shape regularly or will it encounter wave-like urbanization? Will activities settle in their definite place or will they move with time? If so, how often will they move?

This chapter aims to address these questions by discussing the sequences of urban configurations emerging during the urbanization of an empty region. To do so, it proceeds in two steps. First, a static equilibrium model is presented whose results are interpreted as long-term equilibrium configurations. Secondly, an agent-based model is developed, relying on the same micro-assumptions, to explore the way cities configurations may diverge from the long-run configurations. So the question is whether within a given micro-setting, basic dynamic assumptions are sufficient to induce unexpected intermediate configurations. Hence, this work anchors in two fields of urban research: urban economics, whose morphological studies traditionally focus on equilibrium configurations (Fujita and Thisse, 2013), and urban quantitative geography, which usually represents cities through dynamic, simulation-based models (Batty, 2007; Heppenstall et al., 2012; White et al., 2015c).

In urban economic literature, many seminal works were static equilibrium models. More precisely, the agricultural land use model of von Thünen (1826) is often recognized as a benchmark for its theoretical approach. It was extended to an urban context by Alonso (1964), with further refinements by Muth (1969) and (Mills, 1972, see Brueckner, 1987), in the one-dimensional monocentric model. From there, Fujita and Ogawa (Ogawa and Fujita, 1980; Fujita and Ogawa, 1982) developed a *multicentric* two-sector city model explaining how subcentres can coexist as firms benefit from agglomeration economies. After them, urban economists pursued by studying two-dimensional configurations but restricted themselves to *circular* (or symmetric) city models (Ogawa and Fujita, 1989; Lucas and Rossi-Hansberg, 2002; Carlier and Ekeland, 2007).

In these models, time is implicitly considered. Thus, their results are traditionally interpreted as long-term equilibria, implicitly assuming that buildings and other urban infrastructures become mobile in the long run (Fujita, 1983). To deal explicitly with this assumption, urban economists also produced dynamic equilibrium models (Fujita, 1983; Miyao, 1987). Yet they still restricted themselves to symmetric configurations. Another approach, which will come back later in the discussion, was used by Krugman (Krugman, 1993, 1996) to explain the *edge cities* depicted by Garreau (1991). Yet his work was limited to a linear city. Thus, in order to build dynamic models that enable to represent non-circular configurations with the same micro-assumptions, other tools are required.

In urban geography, recent developments inspire from the study of complex systems and propose specific methods. Agent-based simulation models for example can reproduce a wide diversity of potentially far-from-equilibrium urban configurations by not assuming equilibrium as a necessary feature (Manson et al., 2012). They are actually the spatial form of *agent-based computational economics*, a recent field of economics that is particularly relevant for heuristic-purposed research (Tesfatsion, 2003; Tesfatsion and Judd, 2006). Combining agent-based simulations with urban economics could thus be a fruitful approach in urban research (Fujita and Mori, 2005), particularly to build simulation models that are theoretically consistent regarding their economic assumptions. Several studies used this association in order to discuss the dynamics of microeconomic urban systems (Heikkila and Wang, 2009; Yang and Ettema, 2012) or regional agglomeration patterns (Li et al., 2013). Note that a few authors have already tried to explore the geographical properties of microeconomic systems by means of cellular automata, which are another popular simulation tool (Caruso et al., 2005, 2007, 2009, 2015a,b). Yet there is almost no study of the morphodynamics of an exogenously growing two-sector city.

Thus the present chapter aims at characterizing the morphology of a simulated, exogenously growing city to see whether it diverges or not from associated static equilibrium configurations. In pursuance of this purpose, it starts by describing the static microeconomic assumptions ruling agents' behaviours, which are largely inspired from the seminal model of Fujita and Ogawa (1982). Starting from these micro-assumptions, equilibrium conditions of four circular configurations are computed. These typical configurations are chosen in the literature so that there is at least one static equilibrium for each parametrization. Only the equilibrium conditions are presented hereafter since computations simply follow the work of Fujita and Ogawa (1982) (see also Delloye et al., 2014). Then, adding the spatio-temporal assumptions, the agent-based model is implemented in NetLogo (Wilensky, 1999), simulations are performed and the morphometric indexes of the resulting configurations are computed in Matlab (MathWorks, 2010). These indexes constitute a second group of results that will be compared to the analytic configurations in order to discuss how the simulated configurations may diverge from circular static equilibria. Please note that the NetLogo implementation of the model can be downloaded from `http://journals.plos.org/plosone/article?id=10.1371/journal.pone.0135871`, although its running time is rather long. A faster version has been developed for the 2017 International Erasmus Program in Geographical Modelling (Rouen, France), whose code is available at `https://github.com/jDelloye/MGM2017/blob/master/FO82model.nlogo`.

4.2 Two-dimensional non-monocentric cities

In this section, the non-monocentric model of Fujita and Ogawa (1982) is extended in two dimensions. The conditions for different urban configurations to be static equilibria are derived for a region of 1 000 households. Those static equilibria will serve as benchmarks in discussing long-term equilibrium configurations of the dynamic model implemented in the next section.

4.2.1 The Fujita-Ogawa model

Let region X be a two-dimensional Euclidean space where every location x is a unitary land plot of soil, such as $X = \{x \,|\, x \in \mathbb{R}^2\}$. The geographical framework of the Fujita-Ogawa model generalizes the isolated state of von Thünen (1826) and Alonso (1964). It is a featureless plain, without exception, where any pair of locations is linked by a transport system without congestion. Note that the spatial footprint of this perfect transport network is not considered, like if it was underground.

Two kinds of similar agents are considered in this model: households, whose number is fixed at N, and business firms that are free to enter or leave the region. They interact in three ways. First, households and business firms interact on the labour market, which is a *between-sector* interaction and second, business firms benefit from agglomeration economies, which is a *within-sector* interaction. Third, all agents are on competition for land, which is at once a *between-sector* and a *within-sector* interaction (see Fujita and Ogawa, 1982). Both the labour market and the land market are supposed to be perfectly competitive. Households occupy a fixed amount of soil s and consume a composite commodity $z(x)$ which is imported from outside the system. Of course in reality the consumption choice of living space does shape urban configurations, yet this process is out of the scope of this model. On the other hand, the consumption of composite good $z(x)$ is variable while its import unitary price is set to 1 (hence the composite commodity is called a *numeraire*). Their preferences on both these goods are supposed to be identical and rational in the sense of von Neumann and Morgenstern (Myerson, 1997) so that their choices can be described by maximization of the utility function $U\Big(z(x), s\Big)$. Preferences are supposed to be monotonous on both goods so that $\partial U/\partial s > 0$ and $\partial U/\partial z > 0$. The households are endowed with on unit of labour and they spend money for their journey to work. Their commuting distance is $d(x, y)$ where x is the household location and y is its employer's business firm location. Thus the budget constraint of a household living at x and working at y is given by

$$z(x) + R(x)s \leq Y(y) - cd(x, y) \ , \tag{4.1}$$

where $R(x)$ the unitary land rent in x and $Y(y)$ is the wage paid by the hiring business firm in y, c is the unitary transport cost and $d(x, y)$ is the (straight line) commuting distance from x to y. Note how this constraint differs from the Alonso model (Equation 3.1): the housing lot size is not variable any more, but now the wage (or income) is. The total transport cost function is not simply a function of the distance to an assumed CBD any more but it depends on the particular worker and business firm locations. In particular, a linear form is used by default. By substituting the budget constraint (4.1) in the utility function $U\Big(z(x), s\Big)$, it comes out that households' choices are described by the maximization of their consumption of composite commodity. This writes

$$\max_{x,y}\Big\{z(x) = Y(y) - cd(x, y) - R(x)s\Big\} \ . \tag{4.2}$$

Business firms use soil (for, say, office space[1]) and labour in fixed quantities, respectively o (for "offices") and l, to produce a good that is

[1] Just like in the Alonso model, there is no development of land into housing commodities or production spaces (land development was introduced into the monocentric theory by Muth, 1969). Although households and business firms are competing on the same land market, through this chapter households' fixed use of land (for housing) is written s, by analogy with notations of the previous chapter, whilst business firms' use of land (for, say, offices) is written o.

exported outside system. Consequently, unevenness among productions is due to different levels of productivity, which are assumed to result from different levels of agglomeration economies. This spatial effect is anchored in a locational potential F such that for any location $x \in X$,

$$F(x) = \int_X b(y) e^{-\alpha\, d(x,y)} dy \tag{4.3}$$

where $b(y)$ is business firms density at y and α is the distance-decay parameter of agglomeration economies. Finally, business firms chose their location in order to maximize their profit $\pi(x)$. That is

$$\max_x \Big\{ \pi(x) = kF(x) - R(x)o - Y(y)l \Big\} \tag{4.4}$$

where k is monetary conversion rate of locational potential. Equation 4.4 implicitly introduces the locational potential in the profit function as a multiplying factor, suggesting that it raises the productivity of business firms. This is a somewhat general form whose exact interpretation depends on the processes that are assumed to underlay agglomeration economies. A comprehensive discussion is provided in Section 4.4. It turns out from the profit function (4.4) that business firms face a spatial dilemma. On the one hand, agglomeration economies (4.3) push them to stay close to each other but on the other hand, to much clustering moves the labour force further away so that higher wages are necessary to enable them to come at work. In order to set the spatial equilibrium conditions, the bid rent function of households Ψ and the bid rent function of business firms Φ are defined by

$$\Psi(x) = \max_y \left\{ \frac{Y(y) - cd(x,y) - z}{s} \,\middle|\, U\Big(z(x), s\Big) = u \right\} \ , \tag{4.5}$$

$$\Phi(x) = \frac{kF(x) - \pi - Y(y)l}{o} \ . \tag{4.6}$$

The bid rent functions highlight the fact that wages have opposite effects on households and business firms ability to pay. Higher wages increase households' bid rent function, whilst they reduce business firms' bid rents. It means that bargaining power in wage negotiation will influence the spatial structure of the city (Heikkila and Wang, 2009). Although this aspect of the model is not discussed in this chapter, a specific tuner was added to the NetLogo model developed for the 2017 International Erasmus Program in Geographical Modelling (Rouen, France), whose code is given at `https://github.com/jDelloye/MGM2017/blob/master/FO82model.nlogo`.

Equilibrium conditions

Land plots are allocated to the agent that proposes the highest bid rent. Moreover, for a particular land plot, the actual rent is the maximal bid rent of the winning agent for that plot. Hence regarding business firms, land market pulls down their profit to zero. This results from the openness of region X since any positive profit would mean that a similar business firm can come from outside and propose a higher bid rent by scarifying this positive profit. Regarding households, a similar competition process takes place. The common knowledge that the city is closed to households (they cannot enter nor leave) gives a strong bargaining power to landlords, and the concurrence among households forces them to reveal their true preferences in order to stay in the auction game. Thus, utility levels are pulled down to zero, which is equivalent to the consumption of no composite good. This counter-intuitive interpretation is purely artificial (it can be removed by assuming a minimal consumption level of composite commodity) and should not confuse the reader.

An equilibrium of the previous setting is a system $\{h(x), b(x), R(x), Y(x), P(x, y), u\}$ where $h(x)$ is households density function, $P(x, y)$ is the share of households locating at x and commuting to y and u is the indirect utility level (Fujita and Ogawa, 1982). Let $\Psi^*(x)$ be the equilibrium households' bid rent function and $\Phi * (x)$ the equilibrium business firms' bid rent function, with $\Psi^*(x) = \Psi(x|u = 0)$ and $\Phi^*(x) = \Phi(x|\pi = 0)$. Finally, let a be the exogenous agricultural land rent and let the urban fringe be the location of points where the agricultural land use is adjacent to another land use.[2] Then the equilibrium necessary and sufficient conditions are given by the following four groups of conditions.

1. Land market equilibrium conditions: at each $x \in X$,

$$R(x) = \max\{\Psi^*(x), \Phi^*(x), a\} \tag{4.7}$$

$$R(x) = \Psi^*(x) \text{ if } h(x) > 0 \tag{4.8}$$

$$R(x) = \Phi^*(x) \text{ if } b(x) > 0 \tag{4.9}$$

$$R(x) = a \text{ on the urban fringe} \tag{4.10}$$

$$sh(x) + ob(x) \leq 1 \tag{4.11}$$

$$sh(x) + ob(x) = 1 \text{ if } R(x) > R_a \tag{4.12}$$

[2] Just like in the Alonso model (Sec. 3.3.1), the agricultural land use is no more than a default land use. However, the absence of predefined city centre prevents one from defining the urban fringe more precisely. This will not be the case any more in Section 4.2.2, where circular configurations are defined with respect to a city centre.

2. Labour market equilibrium conditions: at each $x \in X$,

$$lb(x) = \int_X h(y)P(y,x)dy \tag{4.13}$$

3. Total unit number constraints,

$$\int_X h(x)dx = N \tag{4.14}$$

$$\int_X b(x)dx = \frac{N}{l} \tag{4.15}$$

4. Non-negativity constraints: at each $x \in X$,

$$h(x) \geq 0 \tag{4.16}$$

$$b(x) \geq 0 \tag{4.17}$$

$$R(x) \geq 0 \tag{4.18}$$

$$Y(x) \geq 0 \tag{4.19}$$

$$0 \leq P(x,y) \leq 1 \tag{4.20}$$

$$\int_X P(x,y)dy = 1 \tag{4.21}$$

Before going to the spatial structure of the urban equilibrium configurations, it is worth discussing the properties of equilibrium commuting patterns. Consider two households respectively located in $x = A$ and $x = B$, and respectively commuting to $y = i$ and $y = j$ (with $A \neq B, i \neq j, A \neq i, B \neq j$). Thanks to the perfect transport network assumption, they commute in straight lines. As Ogawa and Fujita (1989) showed, at spatial equilibrium their respective job locations have to be optimal. This writes

$$\begin{cases} Y(i) - cd(A,i) \geq Y(j) - cd(A,j) \\ Y(j) - cd(B,j) \geq Y(i) - cd(B,i) \end{cases}, \tag{4.22}$$

which implies

$$d(A,i) + d(B,j) \geq d(A,j) + d(B,i)\ . \tag{4.23}$$

Condition (4.23) is violated when the segments $[A,i]$ and $[B,j]$ have a single common point which is neither the common starting point nor the common end point (Ogawa and Fujita, 1989). That is, there is no cross-commuting pattern in

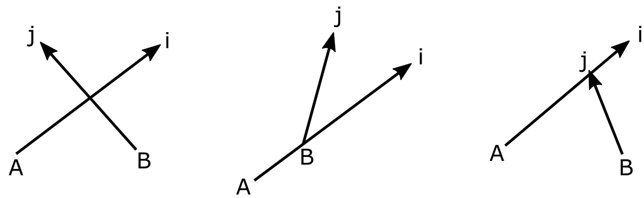

Figure 4.1. Cross commuting patterns in two dimensions. The three cases represented above are cases of cross-commuting that are not possible in equilibrium urban configurations. See the discussion in Ogawa and Fujita (1989).

an equilibrium urban configuration (Fig. 4.1). To put it differently, equilibrium configurations are densely covered by non-intersecting commuting segments. An immediate consequence of this result is that wages are linearly decreasing, at rate c, with distance from the working place (see Ogawa and Fujita, 1980, 1989).

4.2.2 Circular urban configurations

The spatial structure of the equilibrium urban configurations satisfying conditions (4.7) to (4.21) are not easy to explore. This section overcome this difficulty by mimicking the strategy of Fujita and Ogawa (1982), which consists in defining different kinds of circular configurations and to find the conditions under which they satisfy equations (4.7) to (4.21). Those circular configurations will then stand as benchmarks for the agent-based exploration of the model dynamic in Section 4.3. Let us call *business district* the set $\{x \in X | h(x) = 0, b(x) > 0\}$, *residential area* the set $\{x \in X | h(x) > 0, b(x) = 0\}$, and *integrated district* the set $\{x \in X | h(x) > 0, b(x) > 0\}$. Following this, the four circular configurations that are considered in this chapter are the monocentric configuration, the completely mixed configuration, the incompletely mixed configuration (Fig. 4.2), which have been taken from Ogawa and Fujita (1989), and the duocentric rotational configuration (Fig. 4.2), which is introduced here as a circular interpretation of the duocentric city from Fujita and Ogawa (1982). These configurations also prevent one from referring to unrealistic equilibria such as a circular city with a business district at its edge (Lucas and Rossi-Hansberg, 2002).

In those circular configurations, households and business firms densities at location x are one-dimensional functions of the distance to the city centre $r(x)$. Yet even in this simple case, the functional form of the locational potential function (4.3) is hard to manipulate analytically. Indeed, taking the city centre as the origin of the Euclidean framework and using polar coordinates $x =$

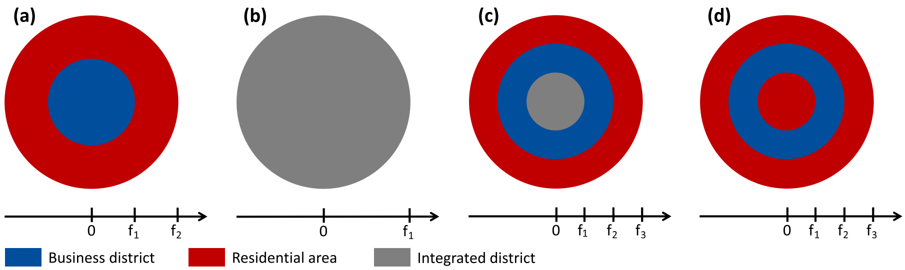

Figure 4.2. The four typical circular configurations of the static equilibrium model. The monocentric configuration **(a)**, the completely mixed configuration **(b)**, the incompletely mixed configuration **(c)** and the rotational duocentric configuration **(d)** are different combinations of areas where only business firms locate (business districts), where only households locate (residential areas) or where both business firms and households colocate (integrated district). The f_i's denote the distances from the respective boundaries to the city centre.

$\Big(\theta(x), r(x)\Big)$, the locational potential writes

$$F(x) = \int_0^\infty \int_0^{2\pi} r(y)b(y)e^{-\alpha d(x,y)} d\theta(y)dr(y) \ , \tag{4.24}$$

where $d(x, y)$ is given by Al-kashi's theorem (law of cosines),

$$d(x,y) = \sqrt{r^2(x) + r^2(y) - 2r(x)r(y)cos\Big(\theta(x) - \theta(y)\Big)} \ . \tag{4.25}$$

Equation 4.24 is not analytically solvable, even if $b(y)$ is a constant function of $r(y)$. Consequently, computing the equilibrium conditions of the circular configurations requires numerical tools, for which an arbitrary parametrization has to be chosen. Mimicking Fujita and Ogawa (1982), it is assumed that $o = 1$ such that a business firm occupies an entire land plot. On the opposite, $h = 0.1$ such that up to 10 households can collocate at the same location. It is also assumed that $l = 10$ so that one business firm and its ten workers occupy at least two locations. Finally, considering a region of $N = 1000$ households yields the following complete parametrization: $\{N, s, o, l\} = \{1000, 0.1, 1, 10\}$. Using those parameter values, the procedure for computing equilibrium conditions of any circular configuration (in terms of the remaining free parameters α, c and k) is as follows. Functional forms of the density functions $h(x)$ and $b(x)$ are substituted into the equilibrium conditions 4.14 and 4.15. This enable to compute the values of inner boundaries between the different districts. Those values enable first, given an arbitrary maximal wage Y_0, to compute the wage $Y(x)$ at any location $x \in X$. Second, they also enable to numerically compute the value of F (in this chapter, approximation has been done using the *dblquad.m* function of Matlab, which is based on a recursive adaptive Simpson's quadrature). Finally, values of $Y(x)$ and $F(x)$ can be introduced in the equilibrium conditions (4.7) to (4.21) in order to express them as conditions on α, c and k. As Fujita and Ogawa (1982) showed, the $\{\alpha, c, k\}$-space can be collapsed into $\{\alpha, c/k\}$-space. In the sequel, the ratio c/k is called the *relative commuting cost.*

This procedure is now applied to the monocentric city for illustration. The monocentric configuration is defined as a business district surrounded by a residential area. It has a single inter-districts boundary f_1 and an urban fringe f_2 (Fig. 4.2). Thus the density functions of households and business firms are

$$h(x) = \begin{cases} 0 & \text{if } x \leq f_1 \\ \dfrac{1}{s} = 10 & \text{if } x > f_1 \end{cases}, \tag{4.26}$$

$$b(x) = \begin{cases} \dfrac{1}{o} = 1 & \text{if } x \leq f_1 \\ 0 & \text{if } x\ f_1 \end{cases}. \tag{4.27}$$

Substituting the density functions (4.26) and (4.27) in the total unit number constraints (4.14) and (4.15) gives the boundary f_1 between the residential area and the business district, and the urban fringe f_2 such that

$$f_1 = \sqrt{\frac{sN}{\pi l}} \simeq 5.64 \tag{4.28}$$

$$f_2 = \sqrt{\frac{(o + sl)N}{\pi l}} \simeq 7.64 \tag{4.29}$$

Arbitrarily setting Y_0 as the value of wages at the city centre, the wage function simply writes $Y(x) = Y_0 - cr(x)$. Substituting this function, as well as the numerical value of the locational potential $F(x)$, into the bid rent functions (4.5) and (4.6) and expending the substitution to the equilibrium conditions yields the equilibrium condition of the monocentric city

$$\frac{c}{k} \leq \min\left\{ \frac{s}{o + sl} \frac{F(0) - F(f_1)}{f_1}, \frac{s}{o + sl} \frac{F(f_1) - F(f_2)}{f_2 - f_1} \right\} \tag{4.30}$$

This procedure can be repeated to the other circular configurations in order to derive their own equilibrium conditions (Fig. 4.3). In prevision of the agent-based model, relative commuting costs have been truncated to value smaller than 0.1 since this condition is necessary for a single production activity to be profitable. That is required by the dynamic assumptions and thus makes other values irrelevant for further comparison with the simulated results. As

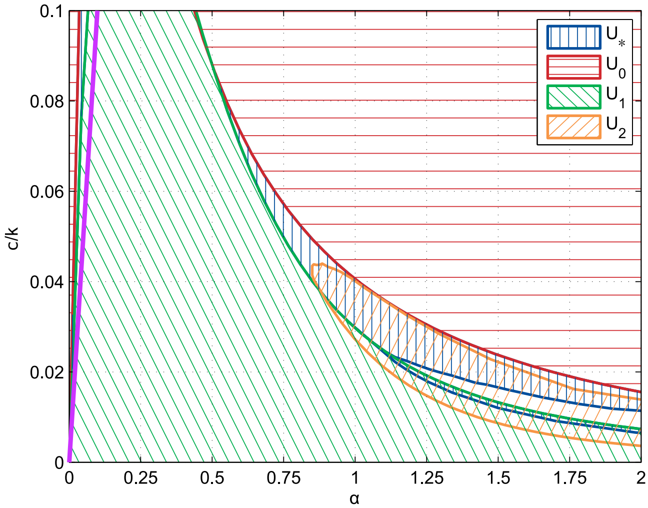

Figure 4.3. Equilibrium conditions of the four typical circular configurations. Equilibrium conditions of the continuous incompletely mixed (U_*), completely mixed (U_0), monocentric (U_1), and rotational duocentric (U_2) urban configurations in the $\{\alpha, c/k\}$-space where α is the distance-decay parameter of the agglomeration economies and c/k is the relative commuting cost. The magenta line is the location of points where $\alpha = c/k$, which is discussed in Section 4.4.

one may have expected, the equilibrium conditions in a $\{\alpha, c/k\}$-space (Fig. 4.3) are qualitatively similar to one-dimensional results of Fujita and Ogawa (1982). More precisely, the duocentric equilibrium area overlaps other equilibrium areas such that in some parts of the figure, multiple equilibria are defined. Although the classic approach in static Urban Economics is to assume that exogenous historical events differentiate these equilibria Fujita et al. (1999b), the dynamic assumptions of the agent-based model presented in the next section will select a particular equilibrium. Yet there is also an area of the state space where the duocentric pattern is the only equilibrium. This is because of the split of the incompletely mixed equilibrium area for $\alpha \geq 1.1$, which due to the constraint of no commuting in the integrated district (see Fujita and Ogawa, 1982, p.176). Among the differences with the one-dimensional equilibrium areas, note the curves' dilation along both the x-axis and the y-axis, simply resulting from the ability of agents to agglomerate more compactly in a two-dimensional space.

4.3 Two-dimensional non-circular cities

One major limitation of the analytical method used by Fujita and Ogawa (1982) is that it requires to formulate urban configurations *ex ante*. Thus one cannot be sure to have exhaustively cover the range of potential equilibrium urban configurations. For example, edge cities (Garreau, 1991) made up with several distinct business districts remain out of the scope of the above analytical method. This section overcome this difficulty by proposing an agent-based implementation of the Fujita-Ogawa model that can model multinuclei equilibrium configurations. The model is based on an *ad hoc* dynamics with costly adjustment that yields path-dependent urban development. From a modelling perspective, an important departure from the previous chapters is the introductions of a discrete space as an approximation of the classic continuous isolated State.

Let us imagine a region facing an exogenous population growth. Growing population generates new households that want to settle down and have a job. Facing the lack of available space in their region of origin, these new households are forced to settle in region X, which is initially empty and rural. This setting of a forced migration may seem rather extreme, yet in reality many processes can lead to an effectively state-led urbanization. See for example the Chinese National New-type of Urbanization Plan (Bai et al., 2014). Anyway, The proposed model could equivalently be interpreted as a region with a pre-existing rural population that will experience a rapid industrialization. Coming back to the model, it is common knowledge that exactly N households will settle in region X. This provides an opportunity for new business activities to be created. In the sequel, all business firms will be assumed to be part of the same industry and to follow the assumptions of Section 4.2. Hence business firms are created in region X following the incentive of labour force availability.

Households and business firms will progressively settle in region X and make up the new city. Let us assume that business firms creation is slower than households settling. There are several reasons for this. First, from an activities' perspective, business creation involves many people thinkings, negotiations and administrative procedures that are time-consuming. Secondly, from a buildings' perspective, offices or industrial buildings usually takes more time to be built than houses (Wegener et al., 1986; Simmonds et al., 2013). Consequently, each iteration will start by the creation and settling of a new business firm and pursue by the faster settling of households and clearing of the labour market. More precisely, households' development rate will be equal to the fixed number of workers employed by a business. By this mean, the spatial structure of the simulated city at a particular time will be analogue to a static-equilibrium pattern for the number of businesses present at that time. This quasi-static, or partially *malleable city* approach (see Fujita, 1983), enables to compare the dynamic model to the static model developed in the previous section. This assumption is crucial since the slow adaptation rate of business firms is an adjustment cost that may prevent intermediate configurations to coincide with the static equilibrium pattern. Without it, the dynamics would likely not bring

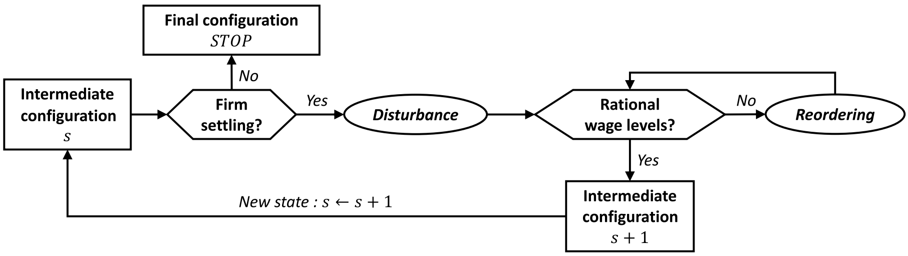

Figure 4.4. Diagram of the system dynamic. Reading starts by the outer-left box, with s representing any state of the model, and follows the black arrows. Hexagonal boxes indicate conditional statements and elliptic boxes stand for adaptation processes.

much insight to analytic results.

Spatial and dynamic settings

Let us discretize X using a two-dimensional square divided in 961 unitary land plots by a regular (31×31)-grid. A Cartesian coordinate system is associated to the grid, with the origin at the centre so that for each land plot $X = x \in [-15, 15] \times [-15, 15] \subset \mathbb{Z}^2$. The initial state of the system, at $s = 0$, is the empty region X. The initial rent of every land plot is the agricultural land rent a that is normalized to 1, thus preventing households from settling for free. Starting from any state s, the new business firm bids for every land plot in region X and settles in the one with the highest bid rent provided that it is higher than the local rent (Fig. 4.4). Region X is assumed big enough to avoid border effects, so that the first business has similar taste for every land plot and the plot (0,0) is chosen without loss of generality. Border effects occurred during simulations but, as we will see further, conclusions are not affected.

The settling of a new business firm changes the locational potential in its neighbourhood (to an extent that depends on α), and may prevent other business firms to pay their rent. Then, it is simply assumed that landowners quickly get them out of the land plot, forcing them to relocate later on. This, in turn, may force several households to leave the city by the same way. This is the *disturbance* phase, going away from the previous state in the sense that agents may leave region X (Fig. 4.4). Then begins the *reordering* phase toward the next state, which starts by the job market adjustment (Fig. 4.4). All business firms in region X start by proposing new wages. They can anticipate the arrival of new households in vacant land plots and see the current wage of settled households. With this information, they propose wages as low as possible provided they attract l workers. Note they can not predict the wage adjustment of other firms. At this step, land rents are also updated. Households are not hired by binding contracts so that after firms proposed new wages, all households in region X simultaneously choose among their own employer plus the business firms with vacant jobs the one that provides them with the highest net income. If they are several of them, then the gross income is maximised. Finally the household associated to the highest net income is the first to get the desired job. One may for example argue that it will be more motivated than others households. This procedure is repeated until no household can find a job and the remaining ones are forced to leave region X.

At the end of this procedure, every household is hired by a business firm and every firm has l workers. It may still happen that a business firm, because of its bounded prediction ability, has proposed wages that are too low to attract l workers although it can support higher wages. In this case, the reordering phase is simply repeated (Fig. 4.4). Each iteration, urban configurations are at short-term equilibrium since profit levels and utility functions are pulled down to zero, so that no agent wants to move unless a new business firm disturbs them. Likewise, the first short-term equilibrium configuration where N households are settled is the long-term equilibrium and stop condition of this model since no agent wants to move, and no business firm can be created without labour availability.

Before going to the results, let us emphasize the similarity of the present work with Krugman's *edge city model* (Krugman, 1996). Indeed, interactions from Fujita and Ogawa's setting provide the breeding ground for Krugman's centripetal and centrifugal forces, whose respective spatial extends will vary according to transport cost c (which will influence employment relationships) and the distance-decay parameter α. Hence this model matches the essential assumptions of Krugman's model (Krugman, 1996), and so one may expect multinuclei configurations to occur. On the other hand, two-dimensional static results are also expected to include multicentric equilibria, just as in the one-dimensional results of Fujita and Ogawa (1982). Next subsection discusses further the matching between the equilibrium area of the multicentric configurations, here restricted to a duocentric one, and the area of occurrence of simulated multinuclei configurations, both measured in the $\{\alpha, c/k\}$-space, as well as their underlying dynamics.

4.3.1 Simulated configurations

Because the dynamic assumptions require a single production activity to be profitable, the relative commuting cost must be lower than 0.1. Otherwise, business activities can not be created, even though population is growing, which lead to the trivial result of a forever empty region. Simulations have thus been performed on the subspace $\{\alpha \times c/k\} = [0, 2] \times [0, 0.098]$, which has been divided in 135 points. More precisely, k was set to 100 whilst c and α were varying between runs. Note that only one run of each parametrization has been realised. This low number results from a trade-off between the number of parametrizations and the number of runs for each of them. Due to the long running time of the model and lack of computer power, any number of runs large enough to proceed with valid statistical tests would have reduced the number of parametrizations to an insignificant value. Since this experiment is an heuristic one, the number of parametrizations was thus given priority. Uncertainty issues are discussed further.

First, let's have a look at extreme parameter results. For $(\alpha, c/k) = (0, 0)$, the simulated long-term equilibrium is a fully random configuration since neither business firms nor households have any incentive to agglomerate nor to disperse (Fig. 4.5). Along the c/k-axis, households locate nearby their hiring business and business firms locate randomly, thus leading to a dispersed configuration. Finally, along the α-axis, business firms cluster and households locate randomly, thus leading to a semi-dispersed configuration. These results show the consistence of this model with basic economic literature.

For $\alpha \neq 0$ and $c/k \neq 0$, long-term configurations present several kind of geometries which will now be discussed regarding to their multicentricity and their circularity. However, for high values of both α and c/k, no long-term configuration was reached (Fig. 4.5). These simulations entered in a cyclic dynamics, coming back to a previous state (sometimes empty). Yet because of the low randomness of the model, they could not escape from this cycle during the experiment. As a result, measures of multicentricity and circularity are not defined for those parts of the $(\alpha, c/k)$-space.

The multicentricity of simulated configurations was measured by the number of distinct business districts, which are themselves defined by 8-cells neighbourhood connectivity among business district land plots. It appears that all long-term configurations under monocentric equilibrium conditions are indeed monocentric ones (Fig. 4.6). Under the equilibrium conditions of the other reference configurations however, the long-term configurations are not always monocentric. Especially when both α and c/k are high enough so that a completely integrated city could be an equilibrium, the dynamic model leads to long-term configurations with 2, 3, up to 6 distinct business districts. Duocentric equilibrium configurations thus emerged in an area of the parameters space that is wider than the expected one. Yet for the (2,0.084)-run, the full settling configuration is monocentric and qualitatively looks like a square-shaped incompletely mixed configuration (Fig. 4.5).

Regarding the dynamic preceding these multicentric configurations, it appears that two processes occurred. On the one hand, the region may develop following the growth of a single city centre until the next incoming business decide to settle a few land plots away from the existing business district, thus creating by itself a second centre which is in turn the starting point of a new urbanization process (Fig. 4.7). On the other hand, the region

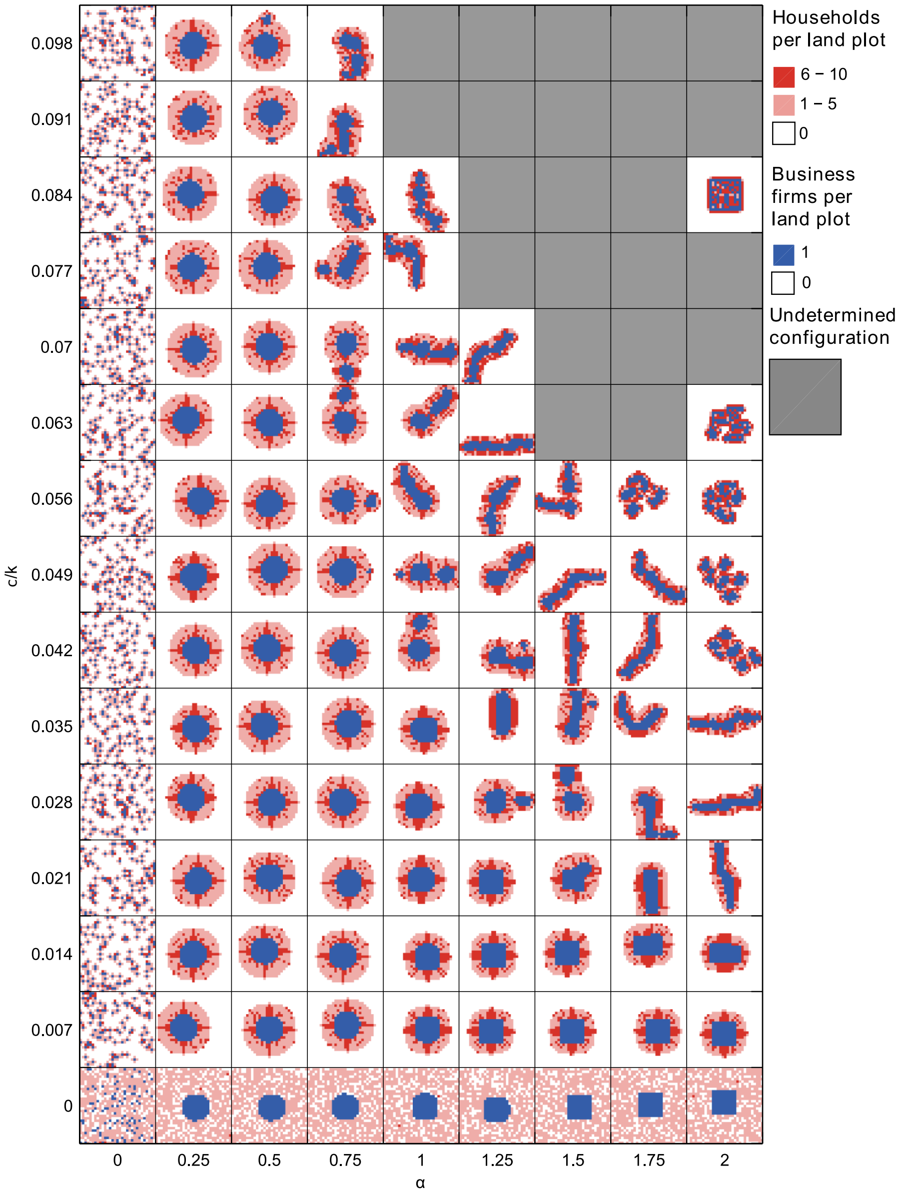

Figure 4.5. Simulated long-run equilibrium configurations. Simulated long-run configurations are given for 135 parametrizations on α and c/k, where α is the distance-decay parameter of the agglomeration economies and c/k is the relative commuting cost. If no long-run equilibrium was reached, the configuration is set undetermined.

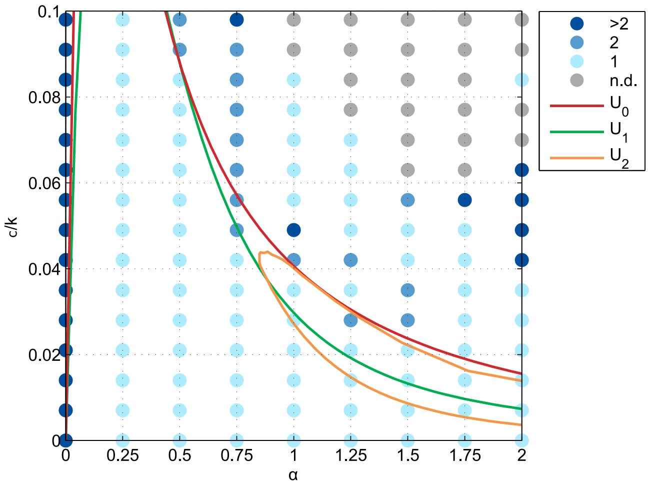

Figure 4.6. Multicentricity of the simulated long-run equilibrium configurations. Blue color scale denotes the number of disconnected business districts in the simulated long-run equilibrium configurations for 135 parametrizations on α and c/k, where α is the distance-decay parameter of the agglomeration economies and c/k is the relative commuting cost. Coloured lines delineate the equilibrium conditions of the continuous completely mixed (U_0), monocentric (U_1), and rotational duocentric (U_2) urban configurations.

may develop following the growth of a single city centre until the next business settling on its border produces a wage rise that forces nearby firms to leave the agglomeration. Some other firms may in turn leave the city so that they break the initial city center in several subcentres. In this case, sudden growths in the number of city subcentres occur along with sudden falls in the number of business firms (Fig. 4.8). Afterwards, the following incoming business firms may thus either re-establish a physical link between the subcentres, or simply develop them separately.

Both paths to multicentricity can be regarded as two parts of the process described by Krugman (1996). The former is the positive part of edge cities' creation, emphasizing the settling of businesses in attractive places, whilst the later emphasizes the negative part of the process, which is the leaving of businesses in unattractive land plots. These processes are clearly distinguished here because of the different agents' displacement rate.

Looking at initial parameter values, it appears that runs where subcentres were created by incoming business firms had parameter values that are close to the limit equilibrium conditions of the completely mixed configuration (Fig. 4.9). Conversely, those where subcentres were created by leaving business firms had parameter values under the completely mixed equilibrium conditions, for $\alpha > 1.5$ and $0.042 \leq c/k \leq 0.063$.

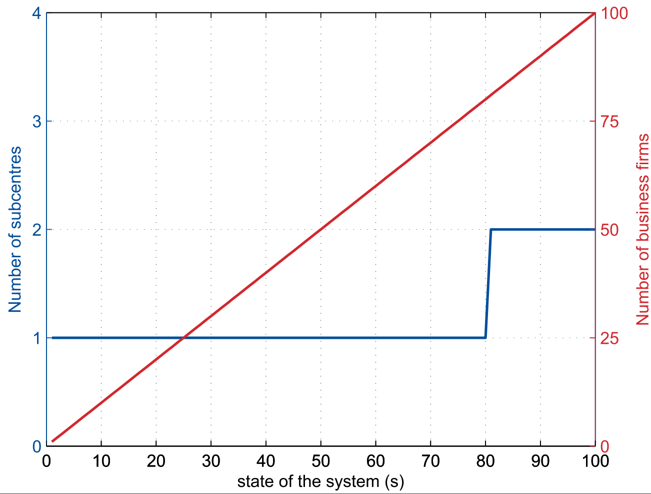

Figure 4.7. Creation of subcentres by incoming firms during the (0.75,0.056)-run. The curves depict the dynamics of the number of firms in the urban agglomeration and of the number of separated business districts during the simulation for $\alpha = 0.75$ and $c/k = 0.056$, where α is the distance-decay parameter of the agglomeration economies and c/k is the relative commuting cost. The plot extends from the first state of the system ($s = 1$) to the long-run equilibrium ($s = 100$).

As previously mentioned, border effects occurred during the simulations because of the unexpected length of several configurations. For 7 runs, the city hit region X boundary before getting its final number of subcentres. However, taking into account the number of subentres in the city before it reached the border does not change the results (Fig. 4.10).

Regarding the monocentric configurations, the circularity index was measured as $(1 - E)$ where E is the eccentricity of the ellipse that has the same second-moments as the city contour. The procedure, implemented in MatLab by the `regionprops` function, consists in computing the covariance matrix of the coordinates of pathes with at least one agent locating on them. Eigenvalues then give the lengths of the major and minor axis of the associate ellipse, from which the focal distance can be computed. The eccentricity of the associate ellipse, which is the ratio of its focal distance and its major axis length, can thus be computed. Once more, it appears that all the long-term configurations under monocentric equilibrium conditions are well-circular ones, with a circularity index larger than 0.5 (Fig. 4.11). Under the equilibrium conditions of the other reference configurations however, the long-term configurations that were monocentric were not circular at all. Instead, they grew marginally to an elongated long-term configuration with a circularity index that is lower than 0.24 for 19 runs out of 27. Yet for the

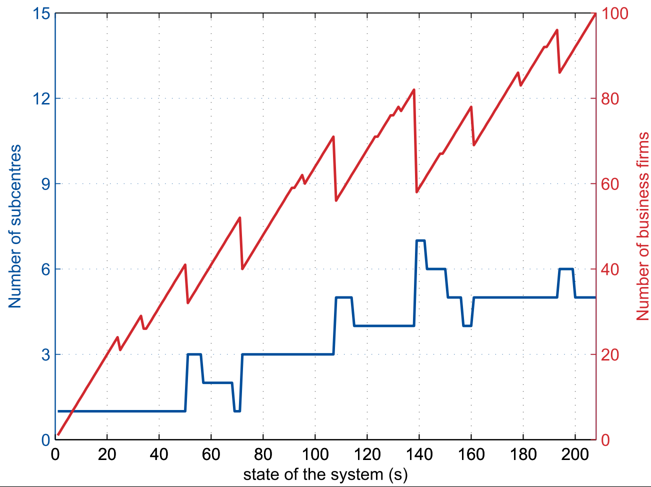

Figure 4.8. Creation of subcentres by moving firms during the $(2, 0.049)$-run. The curves depict the dynamics of the number of firms in the urban agglomeration and of the number of separated business districts during the simulation for $\alpha = 2$ and $c/k = 0.049$, where α is the distance-decay parameter of the agglomeration economies and c/k is the relative commuting cost. The plot extends from the first state of the system $(s = 1)$ to the long-run equilibrium $(s = 208$ here).

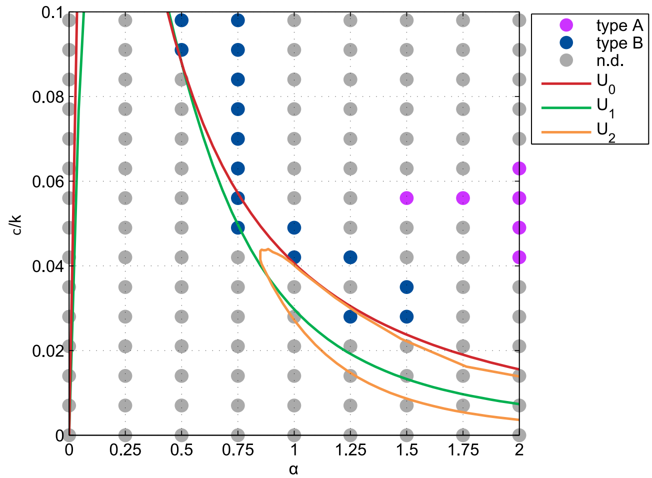

Figure 4.9. Classification of multicentric configurations according to their apparition process. Type A designates configurations where subcentres were created by leaving business firms whilst **type B** points out full settling configurations where subcentres were created by incoming business firms. Coloured lines delineate the equilibrium conditions of the continuous completely mixed (U_0), monocentric (U_1), and rotational duocentric (U_2) urban configurations.

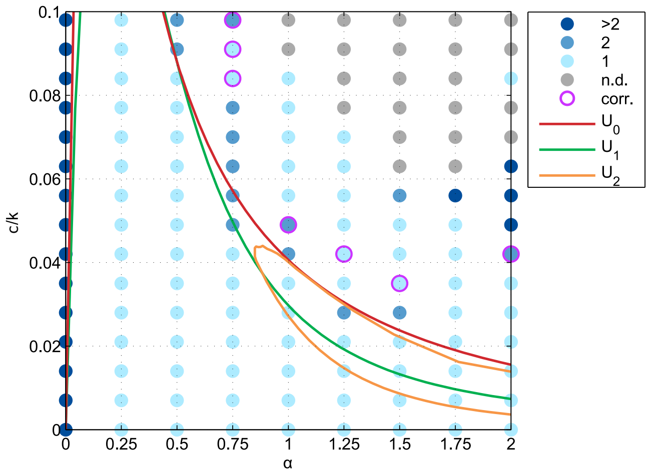

Figure 4.10. Pre-border corrected multicentricity of the simulated long-run equilibrium configurations. Blue color scale denotes the number of disconnected business districts in the simulated long-run equilibrium configurations for 135 parametrizations on α and c/k, where α is the distance-decay parameter of the agglomeration economies and c/k is the relative commuting cost. Pink circles highlight the runs that underwent border effects. Coloured lines delineate the equilibrium conditions of the continuous completely mixed (U_0), monocentric (U_1), and rotational duocentric (U_2) urban configurations.

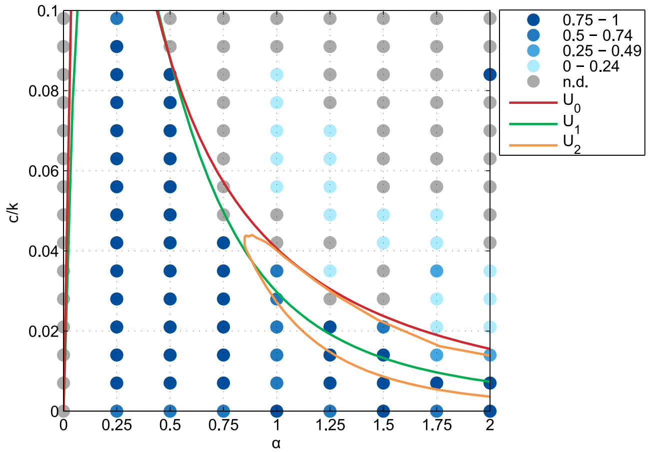

Figure 4.11. Circularity of the simulated long-run equilibrium configurations. Blue color scale denotes the number of disconnected business districts in the simulated long-run equilibrium configurations for 135 parametrizations on α and c/k, where α is the distance-decay parameter of the agglomeration economies and c/k is the relative commuting cost. Coloured lines delineate the equilibrium conditions of the continuous completely mixed (U_0), monocentric (U_1), and rotational duocentric (U_2) urban configurations.

$(2, 0.084)$-run, the full settling configuration is a perfect square and thus presents a circularity of 1.

4.4 Knowledge spillovers

It has been mentioned in Section 4.2 that Equation 4.3 implicitly introduces the locational potential in the profit function as a multiplying factor, suggesting that it raises the productivity of business firms. This is a somewhat general form whose exact interpretation depends on the processes that are assumed to underlay agglomeration economies. This section digs further into the assumption underlying the functional form of $F(x)$ and discusses their consequences.

Agglomeration economies result from different processes which can be split up into productivity increases, e.g. through knowledge spillovers, or cost reduction, e.g. through the share of infrastructures and other kinds of fix costs. One the one hand, productivity increases are described by the multiplicative profit function used in (4.4). In that case, $k = p_o K(o, l)\beta$ where β is the output conversion rate of the locational potential, $K(o, l)$ is the production function of a business firm employing l workers in o areal units of offices and p_o is the unitary export price of the production. On the other hand, cost reductions are better described by an additive form such as

$$\pi(x) = k' + p_p F(x) - R(x)o - Y(x)l \ , \tag{4.31}$$

where p_p is the monetary conversion rate of the locational potential. In that case, $k' = p_o K(o, l)$. Fujita and Ogawa (1982) have noticed that both representations are mathematically equivalent since one may assume $\pi - k' := \pi$ and $p_p := k$. Thus it turns out that the functional form (4.3) of the locational potential can indeed represent different kinds of agglomeration economies, but it requires an appropriate interpretation.

In that perspective, it seems fair to question the interpretation of distance-decay parameter α of the locational potential function. In this section, the focus is on the interpretation of the locational potential as a productivity increase due to knowledge spillovers since it is the most common interpretation and also corresponds to the multiplicative form used in this chapter. The objection raised here is that knowledge spillovers require face-to-face contacts for which it is necessary to move in space. Thus the distance decay parameter α should not be independent of the unitary transport cost c. Although information exchanges are less and less costly, empirical evidence show that numerical information is raw and does not completely substitute to face-to-face contacts. On the opposite, the demand for face-to-face contacts may even increase (Ellison et al., 2007) according to Jevons's paradox.[3] Thus it is important to internalise the decision of face-to-face contacts in the model.

It is assumed that a business firm's productivity can be increased by meeting other business firms for information or intermediate inputs, and that only the business firm who has initiated this meeting benefits from a raise of its productivity. However, the initiator business firm also has to incur the associated transport cost. Formally, let $v(x, y)$ be the number of contacts a business firm in x has with business firms in y, and $V\Big(v(x, y)\Big)$ the contribution of those contacts to the productivity of a business firm at x.

[3] Jevons's paradox refers to any situation in which efficiency improvement of the use of a resource (for example due to technological breakthrough) yields an increase, and not a decrease, of its consumption rate because of an increasing demand. Based on this well-known paradox, Glaeser (2011) proposed a complementary corollary which, applied to ICT's, means that improvements in information technologies increase the demand of face-to-face contacts because face time complements the time spent on communicating electronically.

Then the multiplicative form of the profit function (4.4) rewrites

$$\pi(x) = k\int\limits_X \left\{b(y)V\Big(v(x,y)\Big)\right\} dy - R(x)o - Y(y)l - c\int\limits_X \{b(y)v(x,y)d(x,y)\}\, dy \ . \tag{4.32}$$

where once again $k = p_o K(o,l)\beta$. In the problem of maximizing (4.32), the first stage decision is to choose $v(x,y)$ so that it maximizes, at each location $y \in X$, the net benefit from contacting local business firms. The first order condition of this first stage problem writes

$$k\frac{\partial}{\partial\, v(x,y)}\left\{V\Big(v(x,y)\Big)\right\} - cd(x,y) = 0 \ . \tag{4.33}$$

Let us consider the following functional form, which generalizes the work of Kim (1988),

$$V\Big(v(x,y)\Big) = m\, v(x,y)\,\Big[1 - n\ln\Big(v(x,y)\Big)\Big] \tag{4.34}$$

where $m > 0$ is the contribution of a unit contact between business firms to the productivity of the initiator business firm, and

$$n = \frac{1}{\ln\Big(\bar{v}(x,y)\Big) + 1} \ , \tag{4.35}$$

where $\bar{v}(x,y)$ is the maximal number of profiting contacts between two business firms. Note that the number of visits to another business firm contributes non-linearly to the productivity operator (Fig. 4.12), highlighting the fact that too many contacts per day may be inefficient.

Assuming (4.34), the solution of the first order condition (4.33) is

$$v(x,y) = e^{-\frac{c}{kmn}d(x,y) + \frac{1}{n} - 1} \ , \tag{4.36}$$

which can be substituted back to the profit function (4.32) so as it finally writes

$$\pi(x) = kmne^{1/n-1}\int\limits_X \left\{b(y)e^{-\frac{c}{kmn}d(x,y)}\right\}dy - R(x)o - Y(y)l \ , \tag{4.37}$$

which is mathematically equivalent to the multiplicative profit (4.4) since one may assume $\alpha := c/(kmn)$ and $k'' := kmn\exp(1/n-1)$. Thus the exponential distance decay of the locational potential can be shown to solution a profit maximising business firms' problem of choosing the number of contacts with other business firms, under the condition that the contribution function V of those contacts to the productivity follows (4.34). It appears first that in the

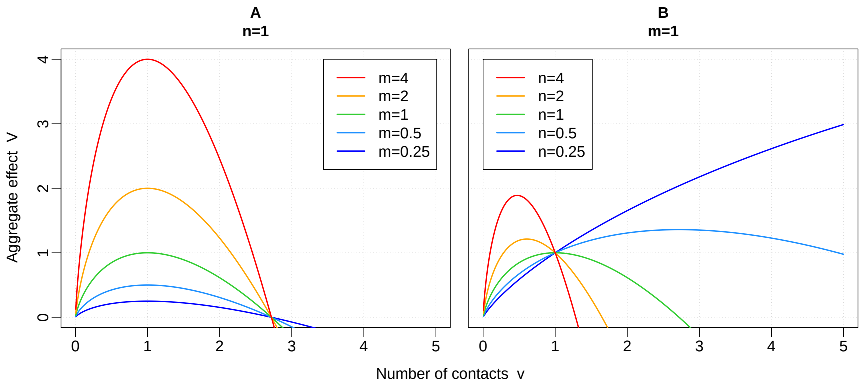

Figure 4.12. Contribution of the number of contacts to a single business firm to the productivity of the initiator business firm. (**A**) The contribution of a unit contact between business firms to the productivity of the initiator m does not influence the maximal number of profitable contacts. (**B**) The parameter n, which is a decreasing function of the maximal number of profitable contacts between two firms, also influences the maximal effect that can obtained from contacting another business firm.

optimal number of contacts (4.36), the parameter m, which describes the contribution of a unit contact between business firms to the productivity of the initiator business firm, reduces the intensity of the distance decay (Fig. 4.13). Larger values of n reduce the distance decay of the optimal number of contact as well, but it also decreases the optimal number of contact with next-door business firms (Fig. 4.13). Second, the locational potential can now be decomposed into a benefit from face-to-face contacts and a necessary transport cost. Whilst the respective effects of m and n on the locational potential are substitutable (the matrix plot Fig. 4.14 is symmetric with respect to the locational potential curve), the underlying values of productivity and total transport costs have radically different orders of magnitude (Fig. 4.14).

This model however imposes that

$$\frac{c}{k''} = \alpha\, e^{1-1/n} \ , \tag{4.38}$$

which is a linear relationship between the relative commuting cost and the distance decay parameter. Kim (1988) originally assumed that $m = n = 1$, such that $k'' = k$ and $\alpha = c/k$. This constraint reduces the relevant area of the $(\alpha \times c/k)$-space to a line which is entirely included in the equilibrium area of the monocentric city (Fig. 4.3). To put it differently, the microfoundation proposed by Kim (1988) reduces the model of Fujita and Ogawa (1982) to a monocentric city model, which is completely at the opposite of their initial motivation. The approach proposed in this section avoids this severe drawback since the linear coefficient between the relative commuting cost and

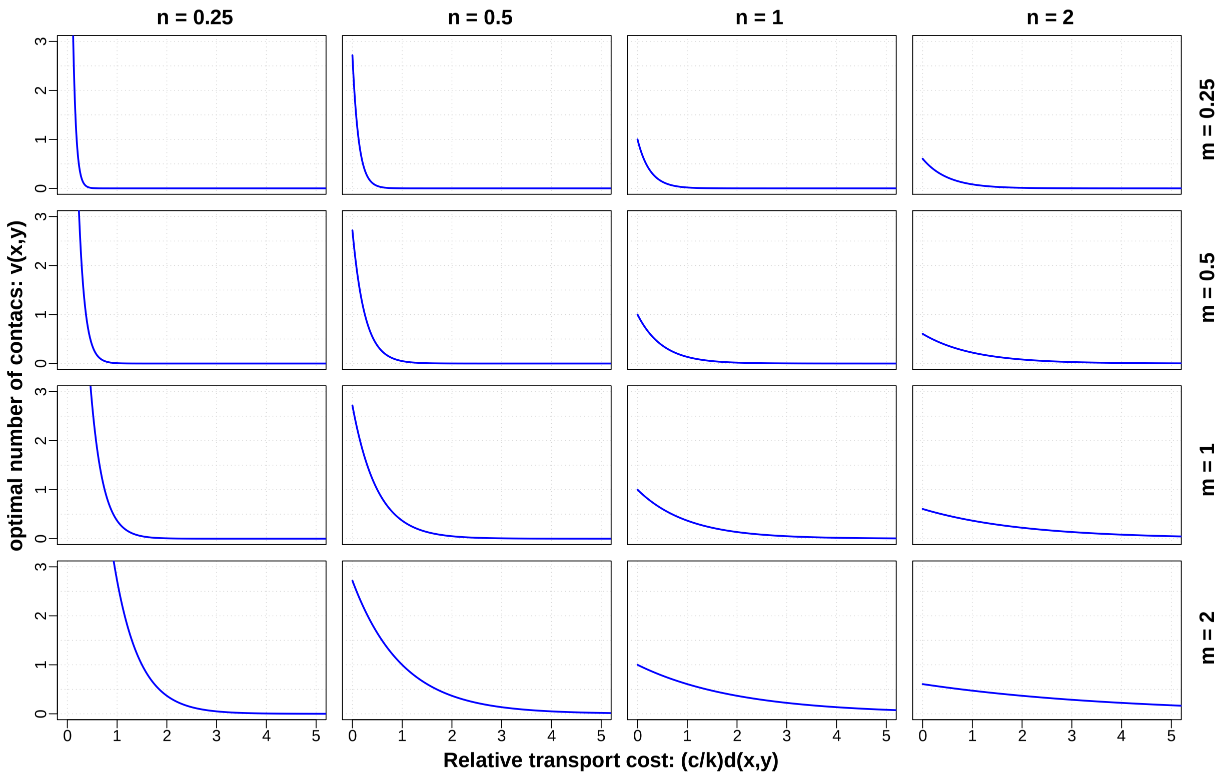

Figure 4.13. Effects of m and n on the optimal number of contacts with a business firm as a function of the relative transport cost $(c/k)d(x,y)$. Increasing values of the contribution of a unit contact between business firms to the productivity of the initiator (m) increase the optimal number of contacts with businesses at every distance but the origin. Increasing values of n increase the optimal number of contacts with distant businesses and reduce the number of contacts with nearby businesses.

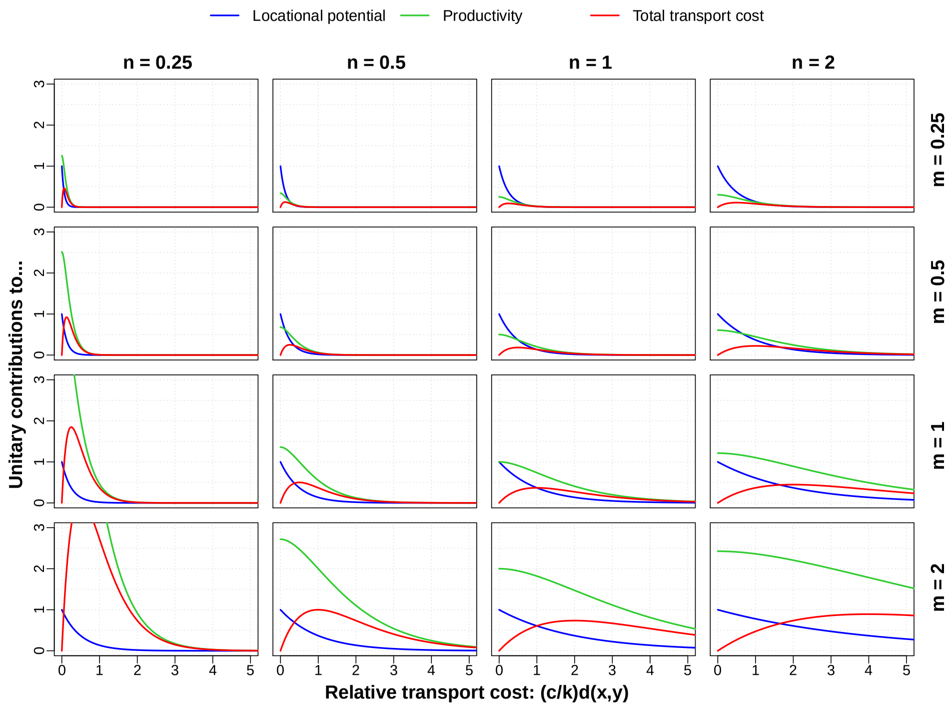

Figure 4.14. Effects of m and n on the locational potential function. Increasing values of the contribution of a unit contact between business firms to the productivity of the initiator (m) increase the productivity and the total transport cost at every distance. Increasing values of $n < 1$ decrease both the productivity and the total transport cost of contacts with nearby business firms, and increase both the productivity and the total transport cost of contacts with distant business firms. Increasing values of $n > 1$ however increase the productivity and the total transport cost at every distance. Note that constants have been omitted so that the blue curve is not exactly equal to the difference between the green curve and the red curve.

the distance decay parameter can take any value in $]0, e[$. An alternative approach would be to assume that experience is a by-product of the production of composite commodity, just as in Arrow (1962) and Helsley (1990) (see also Duranton and Puga, 2004, for a more general discussion). Consequently, "[experience] *acts as a reciprocal public input that entwines production and location decisions*" (Helsley, 1990, p.392) and businesses endogenously choose the number of contacts with other businesses (juast as Tauchen and Witte, 1984). Yet considering those assumptions in the above development would obviously prevents α from being independent of the interacting business firms. Such model would thus depart from the model of Fujita and Ogawa (1982) and its development is left for further studies.

4.5 Discussion

Deducing practical knowledge from the previous experiment requires a good understanding of the involved parameters. Firstly, recall c/k, here named the relative commuting cost, is the ratio of the unitary commuting cost over the monetary conversation rate of the locational potential. Hence, all things otherwise being equal, higher c/k ratio means that commuting costs are high compared to the monetary benefit of the agglomeration economies. Secondly, α describes the distance decay effect of the agglomeration economies. Hence, *ceteris paribus*, higher α value means that agglomeration economies between two business firms are decreasing faster with distance.

In our agent-based simulation model, it appears that under parameter values for which the monocentric configuration stands as an equilibrium, the intermediate configurations never diverge a lot from this equilibrium. It means that for a given relative commuting cost, if agglomeration economies extend far in space, then intermediate configurations always count a single business district and present a circular full settling configuration. This suggests that cities experiencing or implementing strong agglomeration forces undergo less variability in their configurations due to asynchronous development.

Under parameter values for which the other configurations stand as equilibria, which are for higher relative transport cost and distance-decay of the agglomeration economies than the monocentric equilibrium conditions, the configuration is far less stable. Under these conditions, for a given relative commuting cost, agglomeration economies decrease rapidly with distance. Consequently, urban configurations grow marginally and tend to produce elongated configurations with time. Note that since the marginal growth process relies on strong centripetal forces, it can occur either through high relative commuting cost, through high distance-decay of the agglomeration economies or through any equivalent combination of both, which explains their range of occurrence (Fig. 4.5) In this context also, multicentric configurations emerge in two ways.

Firstly, if the distance-decay of agglomeration economies is not too high, which means for a given relative cost that it is just higher than the monocentric conditions would require, subcentres may be created by the arrival of new business firms a few distance away from the previous city centre. This is because agglomeration economies extend far enough for them to benefit from the proximity of the previous city centre, whilst transport costs are too high for them to support wage levels at the boundary of the city centre.

Secondly, if the distance-decay of agglomeration economies is much higher than the monocentric conditions would require, then elongated configurations may break apart in several subcentres. This is because the distance-decay of agglomeration economies is so high that business firms need a great proximity of other firms in order to stay productive. Meanwhile, transport costs are so high that if workers of a firm are pushed further from their employer because of the arrival of a new business firm, the employer can not pay them any more. Thus it is forced to leave its place and relocate, hence changing locational potential in its neighbourhood and forcing in turn other firms to relocate.

These two processes repeat for different values of relative commuting cost, with the following exceptions. For high relative commuting costs, breaks in urban configurations occurred so frequently under high distance-decay of agglomeration economies that a long-run equilibrium configuration could not have been reached by simulations. They often present a cyclic dynamics preventing all the population to settle, although long-term equilibrium may still occur as exemplified by the (2,0.084)-run. In this context, the city seems not stabilizing easily by itself and urban planners should therefore either influence commuting costs and agglomeration economies, or coordinate agents location decisions in order to reach a stable urban configuration.

In order to say more about the cyclic and chaotic dynamics of the model, an interesting experiment would be to run simulations with different ratio of development rates for businesses and households. Indeed, this acts as a transmission delay in the feedbacks conducting agents' location decisions, which is well known to influence the model dynamic as in the classic prey-predator model (Lotka, 1925; Volterra, 1926).

As a cost of its simplicity, this exploratory model presents many technical or conceptual limitations that may be overcome. Although uneven mobility among agents was introduced in the dynamic setting, a more realistic assumption would be to fully distinguish agents from buildings. This would provide a two-dimensional examination of other economic studies (see for example Miyao, 1987 and Anas, 2013).

Although a single run for each parametrization was enough to show how intermediate configurations can differ from expected equilibria, several runs would be necessary to discuss the representativeness of simulated long-term configurations. However, initial conditions are invariable and randomness occurs only when similar land plots have the same value regard to the incoming agent, in which case the winning plot is chosen with even probability. Resulting configurations are thus expected to slightly change in orientation only.

Finally, by taking an analytical equilibrium model as benchmark, this paper does not argue that the study equilibrium conditions of dynamic systems, including for example the convergence conditions, is useless. Actually such *ex-ante* analysis would help to discuss the cyclic dynamics occurring under high-valued parameters. The message here is that agent-based simulation models can easily be designed more consistently with economic literature by this way.

4.6 Conclusion

As a conclusion, this model of urban morphogenesis of a two-sector city with heterogeneous mobility under exogenous population growth shows that urban development is stable and ends up with more homogeneous long-term configurations under strong agglomeration effects. For low agglomeration effects however, urban development is more changing, configurations varying a lot in the number of subcentres and in circularity. More precisely, this chapter has pointed out two different dynamic processes of multicentricity, involving leaving business firms or not, that are well distinguished in terms of commuting cost and agglomeration economies' extents. It has also highlighted a dynamic marginal urbanization leading to elongated long-run configurations. Finally, for extremely low agglomeration forces, repeated moves of agents strive against the stabilization of the urban configuration. To sum up, little variations in parameters and path-dependency effects may conduct cities to very different configurations in the long run. All things otherwise being equal, higher relative commuting cost and distance-decay of agglomeration economies produce more changing urban configurations that require an exogenous intervention to be stabilized. Consequently, this paper warns urban planning policy makers against the difference that may stand between appropriate long-term perspectives, represented here by analytic equilibrium configurations, and short-term urban configurations, simulated here following basic dynamic assumptions.

In the development of this model, two challenges were highlighted. Firstly, the modelling of industrial agglomeration economies is somehow artificial and it has been shown that digging into interpretation of its distance-decay parameter from a knowledge spillovers perspective yields a drastic reduction of the possible urban configurations. This question has to be addressed by proposing a micro-model of business interactions that could be empirically tested independently of the urban economic model. Secondly, some parts of $(\alpha \times c/k)$-space could not be studied carefully because simulations were not converging to a unique configuration. Even if dynamic patterns could have been observed like say, a kind of cyclic behaviour, it would have been impossible, based on simulations only, to say whether those dynamic features would have resulted from the urban intrinsic dynamics or from random perturbations. An obvious research strategy to overcome this difficulty is to expressed the system dynamics by a set of equations from which critical points can at least numerically approximated. This issue is addressed by the stochastic theoretical framework proposed in the next chapter.

III

A Synergetic Approach of Location Theory

5

A Synergetic Model of Dynamic Discrete Choices

In this chapter, a theoretical framework of dynamic collective choices is developed in order to support microsimulation models (like the one used in the previous chapter) with a relevant mathematical formalism. In that purpose, it relies on a discrete representation of space, an continuous dynamic and an explicit set of behavioural assumptions. Starting from a dynamic individual model of discrete choice, an aggregation procedure is applied to describe the collective system dynamics by a master equation, which is in turn analytically approximated by a diffusion process. The chapter is organized in four sections. After a brief introduction in Section 5.1, Section 5.2 sets up the terminology of the framework, Section 5.3 exposes the individual model of dynamic discrete choice and Section 5.4 presents the aggregation procedure and the diffusion approximation which yield the collective system dynamics. The model output is a set of Itô stochastic differential equations that can be used to study non-linear stochastic dynamical systems.

The research presented in this chapter has been carried out in collaboration with Prof Dominique Peeters and Prof Joe Tharakan (see Delloye et al., 2018b).

5.1 Introduction

This chapter introduces the modelling framework that will be used in the next chapter to study noise-induced transitions in dynamic core-periphery models of New Economic Geography. It aims at providing a mathematical description of the collective dynamic choices of individuals, whose behaviours are described using an explicit decision theory, among a set of discrete alternatives in a continuous time. Previous contributions to this formalism have been proposed in the study nonlinear probabilistic migratory systems.

In particular, Moss (1979) proposed a model of migration choice that is based on the discrete choice theory and random utility framework proposed by McFadden (1974) and Manski (1977). This setting has been used in the study of population interactions in migratory systems (Miyao, 1978b; Miyao and Shapiro, 1981; de Palma and Lefevre, 1983, 1985; Ginsburgh et al., 1985; Tabuchi, 1986b). The idea has further been formalized by Ben-Akiva and de Palma (1986), Kanaroglou et al. (1986b,a) and Kanaroglou and Ferguson (1996) using a nested Logit structure (see Ben-Akiva and Lerman, 1985), which enables to decompose the migration decision between a decision to leave (the "*push*" factor) and a choice of destination (the "*pull*" factor). This "push and pull" model has been used by many empirical studies afterwards (Liaw and Ledent, 1987; Liaw, 1990; Anderson and Papageorgiou, 1994a,b; Newbold and Liaw, 1994; Frey et al., 1996; Pellegrini and Fotheringham, 1999; Lee and Waddell, 2010). See also the review of Pellegrini and Fotheringham (2002). Instead of starting from discrete choice theory, Weidlich and Haag (1983) and Haag (1989) developed an alternative dynamic decision theory that is directly inspired from Haken's Synergetics Haken (2004). In those works, individual transition rates from a location to another are mathematically expressed by an exponential of the difference between the so called "*dynamic utilities*" at destination and origin (Weidlich, 2006). Their model has been used in empirical research as well (Haag and Grützmann, 1993; Weidlich and Haag, 1988).

No matter they follow from discrete choice theory or from synergetics, models from de Palma and Lefevre (1983), Ben-Akiva and de Palma (1986), Kanaroglou et al. (1986b) and Haag and Weidlich (1984) all end up with a Master equation that is hardly tractable, and then they use a deterministic approximation of the interregional dynamics. de Palma and Lefevre (1983), Ben-Akiva and de Palma (1986) and Kanaroglou et al. (1986b) rely on a deterministic approximation proposed by Kurtz (1978), whilst Haag and Weidlich (1984) use the condition of detailed balance that results from their particular exponential form of transition rates. Thus they do not provide a satisfactory understanding of the stochasticity that results from aggregated idiosyncratic preferences on the adjustment dynamics of interregional systems. In this chapter, a new stochastic framework is proposed that can be approximated by a continuous diffusion process. Such process can be expressed by a set of stochastic differential equations, which can be numerically simulated in order to study noised-induced properties of the resulting system (see Chapter 5).

5.2 Terminology

This section introduces some terminology and basic relationships that will be used in the remaining of the chapter. Consider an agents set A of N distinct *agents* denoted by the subset of positive integers $\{1, \ldots, N\} \subset \mathbb{Z}^+$, and an alternatives set X of M distinct and mutually exclusive *alternatives* denoted by the subset of positive integers $\{1, \ldots, M\} \subset \mathbb{Z}^+$. Alternatives describe *states* of the agents such that at any time $t \in \mathbb{R}$ there exists a non-injective, non-surjective function $S : A \mapsto X : n \mapsto S(n)$ such that the $S(n)^{\text{th}}$ alternative is the state of the n^{th} agent. Such function can naturally be described by a unique vector $\mathbf{S}$ whose n^{th} component, written $\mathbf{S}_{[n]}$, is $\mathbf{S}_{[n]} = S(n)$. A *state* of the system is a vector $\mathbf{S} \in X^N$, which describes a particular distribution of the N distinct agents to the M distinct alternatives. Accordingly, the *state space* of the system is the hypercube X^N, which contains M^N different states. States can be indexed arbitrarily such that the i^{th} state writes $\mathbf{S}_i$ (note the absence of brackets in the subscript which indicates that this is an indexed state vector and not an element of the state vector). For each state $\mathbf{S}$ there is a unique partition $\bar{A}$ of A such that

$$\forall \bar{a} \in \bar{A} : \exists!\, m \in X : \bar{a} = \{n \in A | \mathbf{S}_{[n]} = m\} \; , \tag{5.1}$$

which consists in grouping agents that are associated to the same alternative. Note that elements of $\bar{A}$ can be ordered with respect to the unique alternative $m \in X$ they refer to. Hence $\bar{a}_m$ is the subset of agents whose state is the m^{th} alternative. For each state $\mathbf{S}$ there is also a unique vector $\mathbf{C}$ such that $\forall m \in X : \mathbf{C}_{[m]} = |\bar{a}_m|$. A *configuration*[1] of the system is a vector $\mathbf{C} \in \mathbb{S}_N^M$ that describes a particular distribution of the N *indistinct* agents to the M distinct alternatives, where $\mathbb{S}_N^M$ is a $(M-1)$-simplex of size N such that

$$\mathbb{S}_N^M := \left\{ \mathbf{C} \in \mathbb{N}^M \middle| \sum_{m=1}^{M} \mathbf{C}_{[m]} = N \right\} \; , \tag{5.2}$$

where $\mathbb{Z}^+$ is the set of non-negative integers. Accordingly, the *configuration space* of the system is the simplex $\mathbb{S}_N^M$, which contains $\mathcal{B}(N+M-1, M)$ different configurations, where $\mathcal{B}(N+M-1, N)$ is the binomial coefficient (Bondarenko,

[1]This terminology is inherited from Weidlich and Haag (1983).

1993)

$$\mathcal{B}(N+M-1,M) := \frac{(N+M-1)!}{M![(N+M-1)-M!} = \frac{(N+M-1)!}{M!(N-1)!} \ . \qquad (5.3)$$

Let $C : X^N \mapsto \$_N^M$ be the application from any state to the corresponding configuration. This application is obviously non-injective since the counting process drops agents' identities. For most applications, including the one in Section 6, agents are similar (or belong to groups of similar agents) so that a particular agent's identity is not a relevant information. Thus, for each application C there is a unique partition $\bar{S}$ of the state space X^N such that

$$\forall \bar{s} \in \bar{S} : \exists! \, \mathbf{C} \in \$_N^M : \bar{s} = \{\mathbf{S} \in X^N | C(\mathbf{S}) = \mathbf{C}\} \ , \qquad (5.4)$$

which consists in grouping states that are associated to the same configuration. An element $\bar{s}$ of $\bar{S}$ can be regarded as a *macro-state* of system, which is nothing more than a group of states. Configurations can be ordered arbitrarily, hence $\mathbf{C}_j$ is the j^{th} configuration. Then macro-states can be ordered as well, with respect to the unique configuration $\mathbf{C}$ they refer to. Hence $\bar{s}_j$ is the j^{th} element of $\bar{S}$, that is, subset of states whose configuration is the j^{th} one. Finally, the number of states in any macro-state $\bar{s} \in \bar{S}$, that is, the number of way one can distribute N distinct agents among M distinct alternatives respecting the configuration $\mathbf{C}$, is given by $\mathcal{M}(N,\mathbf{C})$ where $\mathcal{M}(N,\mathbf{C})$ is the multinomial coefficient (Bondarenko, 1993)

$$\mathcal{M}(N,\mathbf{C}) = \frac{N!}{\prod_{m=1}^{M} \mathbf{C}_{[m]}!} \ . \qquad (5.5)$$

Take for example three agents $A = \{1,2,3\}$ in three regions $X = \{1,2,3\}$ (Fig. 5.1). Then $S(1)$ is the current region's index of agent 1. Suppose that $S(1) = 1$, $S(2) = 1$ and $S(3) = 3$. Then the state of the system is the vector $\mathbf{S} = (1,1,3)$, which is a point in the hypercube of dimension 3 and length 3 (Fig. 5.1), whilst the configuration of the system is the vector $\mathbf{C} = (2,0,1)$, which is a point in the 2-simplex of order 3 (Fig. 5.1). Macro-states of the system are groups of states that are associated to the same configuration. Translating the hypercube of the state space to the origin of the Euclidean framework, they appear as subgroups of the simplexes of different orders that are at the same Hamming distance to the origin.

5.3 Individual dynamics

Take N agents facing M mutually exclusive alternatives, such that $A = \{1, \ldots, N\}$ and $X = \{1, \ldots, M\}$, and consider an arbitrary initial state $\mathbf{S}(t=0) :=$

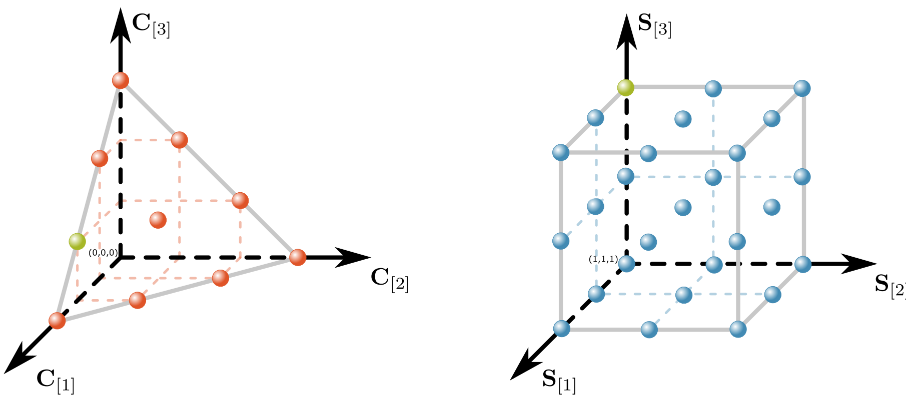

Figure 5.1. **State space and configuration space with $N = 3$ and $M = 3$.** The right hand side plot depicts the state space, which is a cube here whose elements, represented by blue dots, are state vectors. The left hand side plot depicts the configuration space, which is a 2-simplex here whose elements, represented by red dots, are configuration vectors. The green dots on both plots illustrates s state vector and its associated configuration vector .

$\mathbf{S_0} \in X^{|A|}$ of the system. In this section, a first step is made in modelling the system's state dynamic by considering dynamic of a single agent's state (i.e. the change of his alternative). It is argued that a nested Logit structure (see Ben-Akiva and Lerman, 1985, p.285) is naturally indicated to model this class of problem, which is coupled here to dynamic assumptions such that resulting individual dynamics is a continuous-time Markov process. Two functional forms of its transition rates are derived, called *forward* and *backward* transition rates, where the decision of leaving current state respectively precedes and follows the destination state's choice. Those two structures are finally compared to recent researches.

5.3.1 Nested decisions

Agents' choice to change their current alternative follows two assumptions. Firstly, it results from incentives that are observable at the population scale and, idiosyncratic preferences that are non-observable. Secondly, it is alternative-specific because non-observable incentives give a particular status to the current alternative. Consider for example that alternatives are locations. Once trying to model individual moves, a modeller will face the problem that, whatever the deterministic incentives he may consider, the current home of an agent has a particular status that results from unobserved processes like affective attachment. As a result, the decision of leaving home is quite different from the choice of a destination, although it is obviously not

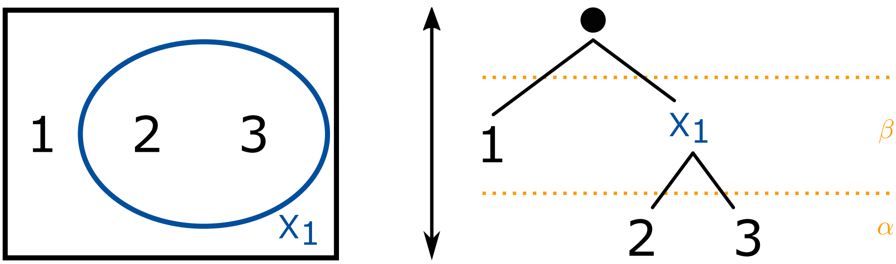

Figure 5.2. Nested structure of the individual decision model with $M = 3$. The box on the left shows the partition of X generated by the particular status of an agent's current alternative, say 1. The three on the right shows how the partition of X decomposes the decision into two subprocesses. Note that contrary to classic nested Logit models, in this chapter it is considered that an agent can goes through the decision three either upward or downward.

independent. Such choice can be modelled as a nested Logit model (Ben-Akiva and Lerman, 1985).

Formally, agents' tastes for alternatives are described by an utility function $\widetilde{U} : X \mapsto \mathbb{R} : i \mapsto \widetilde{U}_i(\mathbf{C}) = U_i(\mathbf{C}) + \varepsilon$ where $U_i(\mathbf{C})$ is the deterministic utility function, which is assumed without loss of generality to depend on the system configuration $\mathbf{C}$, and ε is a random variable standing for idiosyncratic preferences.[2] ε's are Gumbell distributed with zero mean, following the Logit model of discrete choice theory (McFadden, 1974; Anderson et al., 1992). The rationale for the zero mean is that the effects of idiosyncratic preferences, which depend on agent-specific variables such as emotions, subconscious or even genetics, annihilate within large populations. Thus, take an agent with alternative $i \in X$. This agent's current alternative has a complementary set $\bar{X}_i = X \setminus \{i\}$. This partition of the set of alternatives is the support of the nested structure of the decision to change alternative. More precisely, the nested structure decomposes the decision to change alternative into two sub-decisions (note that in a first approach, it is not specified which on takes place first). One sub-decision, between i or $\bar{X}_i$, is the choice to keep his alternative or to change, whilst the other sub-decision, within $\bar{X}_i$, is the choice of a new alternative (Fig. 5.2).

Consider first the sub-decision of a new alternative within $\bar{X}_i$. Following the classic nested Logit approach (Ben-Akiva and Lerman, 1985), the probability

[2]Note that the utility function specified in such way is not ordinal any more but it becomes a cardinal variable (see Manski, 1977; Börsch-Supan, 1990; van Praag, 1991; Koning and Ridder, 1994; Batley, 2008).

for an arbitrary agent to choose alternative $j \in \bar{X}_i$ as his new state writes

$$p(j|i, \mathbf{C}) = e^{\left[U_j(\mathbf{C}) - \mathbb{U}(\bar{X}_i, \mathbf{C})\right]/\alpha} , \tag{5.6}$$

where α is the scale parameters of the Gumbell distribution of the idiosyncratic utilities (Fig. 5.2), and where $\mathbb{U}(\bar{X}_i, \mathbf{C})$ is the attractiveness[3] of the set of possible destinations, such as

$$\mathbb{U}(\bar{X}_i, \mathbf{C}) = \alpha \ln \left(\sum_{j \in \bar{X}_i} e^{U_j(\mathbf{C})/\alpha} \right) . \tag{5.7}$$

Now, consider the agent's sub-decision to abandon his current alternative i. This sub-decision may either precede or follow the choice of a new alternative, and the exact sequence hidden in the global transition from an alternative to another affects the form of the transition probabilities. This influence is now detailed.

forward and backward transition rates

Firstly, consider that the agent decides to abandon (or to keep) alternative i before to potentially choose a new alternative. In that perspective, the agent is going down the decision tree as in a classic nested logit model (Fig. 5.2). Consequently, in a first stage he has to choose between i and $\bar{X}_i$. Since the choice of leaving his current state has the same nature as the destination choice, but differs due to unobserved attributes, its probability follows the same expression but with another scale parameter. This writes

$$p_o^{\mathrm{F}}(i, \mathbf{C}) = \frac{1}{1 + e^{\left[U_i(\mathbf{C}) - \mathbb{U}(\bar{X}_i, \mathbf{C})\right]/\beta}} , \tag{5.8}$$

where β is the scale parameters of the Gumbell distributions of the idiosyncratic utilities at the first stage (Fig. 5.2), such that $0 < \alpha/\beta < 1$ (Ben-Akiva and Lerman, 1985). In that case, the *forward* transition probability writes

$$p_o^{\mathrm{F}}(i, \mathbf{C}) p(j|i, \mathbf{C}) = \frac{e^{\left[U_j(\mathbf{C}) - \mathbb{U}(\bar{X}_i, \mathbf{C})\right]/\alpha}}{1 + e^{\left[U_i(\mathbf{C}) - \mathbb{U}(\bar{X}_i, \mathbf{C})\right]/\beta}} . \tag{5.9}$$

Secondly, consider that the agent starts by choosing a potentially new alternative in $\bar{X}_i$, before he chooses to abandon his current state (or not) by comparing it with the potentially new alternative. That is, the agent is going

[3] The attractiveness, which is also called *inclusive value*, actually corresponds to the expected utility of the set $\bar{X}_i$ (see Ben-Akiva and Lerman, 1985).

upward the decision tree, contrary to the sequence of a classic nested logit model. Thus his first-stage choice is given by (5.6), whilst his second-stage choice, the decision to abandon alternative i (or not), is reduced to a binary choice such that

$$p_o^{\mathrm{B}}(i|j,\mathbf{C}) = \frac{1}{1+e^{\left[U_i(\mathbf{C})-U_j(\mathbf{C})\right]/\beta}} \,, \tag{5.10}$$

which yields the *backward* transition probability

$$p(j|i,\mathbf{C})p_o^{\mathrm{B}}(i|j,\mathbf{C}) = \frac{e^{\left[U_j(\mathbf{C})-\mathbb{U}(\bar{X}_i,\mathbf{C})\right]/\alpha}}{1+e^{\left[U_i(\mathbf{C})-U_j(\mathbf{C})\right]/\beta}} \,. \tag{5.11}$$

Consider now the dynamics of a single agent. It is necessary to define in explicit time units at which rate the decision to change his alternative takes place (whatever it is a forward or backward underlying decision process). To do so, the dynamic process is assumed to be memoryless: the agent considers only the current configuration of the system. For simplicity, it is also assumed that he does not develop any anticipating scenario on the basis of this current configuration. Concretely, the number of times he goes through the nested decision sequence is a Poisson process with (alternative-specific) decision rate $v_i(\mathbf{C}) \in \mathbb{R}^+$. Under those assumptions, an agent's state dynamics is a continuous time Markov chain (Ross, 2009) which is simply a repetition of the decision to change his alternative. Especially, its state set is X and its infinitesimal transition rates (or generator) from an alternative to another depend on the underlying sequence of sub-decisions. In the case of a forward process, the abandonment rate of any alternative $i \in X$ is $w_i(\mathbf{C}) = v_i(\mathbf{C})\, p_o^{\mathrm{F}}(i,\mathbf{C})$ and the transition probabilities, that is, the probabilities to choose any alternative $j \in \bar{X}_i$ for new state given that one has abandoned alternative i, are given by $p(j|i,\mathbf{C})$. In that case, the infinitesimal transition rates are defined by $q^{\mathrm{F}}(j|i,\mathbf{C}) = w_i(\mathbf{C})\, p(j|i,\mathbf{C})$. In the case of a backward process, the infinitesimal transition rates directly write $q^{\mathrm{B}}(j|i,\mathbf{C}) = v_i p(j|i,\mathbf{h}) p_o^{\mathrm{B}}(i|j,\mathbf{C})$.

5.3.2 Decision rates and transition rates

In this section, the forward and backward transition rates are compared to the exponential and logistic forms that are used in the literature. It is shown that the exponential form is compatible with the forward transition rate but that the logistic transition rate, however, cannot be derived from the nested structure of individual decision presented in this chapter.

In their development of Sociodynamics (Weidlich and Haag, 1983, 1988; Haag, 1989), Weidlich and Haag assume that transition rates have the following exponential form

$$q^{\mathrm{WH}}(j|i,\mathbf{C}) = \frac{w}{2} e^{\frac{U_j(\mathbf{C})-U_i(\mathbf{C})}{\gamma}} \,, \tag{5.12}$$

where w is a *time-scale parameter*[4] and γ is a *social temperature*, those appellations being inspired by analogy with statistical physics. They argue that this is "*the most plausible and simultaneously the most simple form*"(Weidlich, 1991, p.34) for the transition rates. Although this form is indeed rather simple, its plausibility is not straightforward, especially with respect to the proposed nested structure of individual decisions. Both approaches are reconciled hereafter.

First, consider that the time-scale parameter w of the exponential form is equal a constant decision rate v of the forward and backward transition rates. In that case, the theories are incompatible. Intuitively, the problem lies in the positive values of $U_j(\mathbf{C}) - U_i(\mathbf{C})$. Indeed, the exponential function is not bounded in that domain and thus allows for infinite transition rates as limit cases. Both the forward and backward transition rates, however, are bounded by the decision rate v, because probabilities in (5.9) and (5.11) are bounded by one. Formally, equalizing (5.12) with either (5.9) or (5.11) yields two specific forms of β whose denominator is in both cases

$$2\ln\left(e^{\frac{U_j(\mathbf{C})-\mathbb{U}(\bar{X}_i,\mathbf{C})}{\alpha}}e^{\frac{U_i(\mathbf{C})-U_j(\mathbf{C})}{\gamma}}-1\right) = 2\ln\left(p(j|i,\mathbf{C})e^{\frac{U_i(\mathbf{C})-U_j(\mathbf{C})}{\gamma}}-1\right) , \tag{5.13}$$

with the necessary condition for existence $U_j(\mathbf{C}) - U_i(\mathbf{C}) < 0$, which confirm the previous intuition that, if the time-scale parameter w of the exponential form is equal to the decision rate v of the nested Logit forms, then at the best the forms are compatible only for randomly-driven transitions, which is undesirable.

Second, consider as previously that the decision rate $v_i(\mathbf{C})$ is alternative-specific. Since this rate has to be specified prior to any decision, it can only depends on $U_i(\mathbf{C})$ or $\mathbb{U}(\bar{X}_i,\mathbf{C})$, but not on $U_j(\mathbf{C})$. This condition cannot be respected in the backward case. Regarding the forward case however, one gets

$$v_i^{\text{WH}}(\mathbf{C}) = \frac{w}{2}\, e^{\frac{U_j(\mathbf{C})-U_i(\mathbf{C})}{\gamma}} e^{\frac{\mathbb{U}(\bar{X}_i,\mathbf{C})-U_j(\mathbf{C})}{\alpha}} \left(1+e^{\frac{U_i(\mathbf{C})-\mathbb{U}(\bar{X}_i,\mathbf{C})}{\beta}}\right) , \tag{5.14}$$

where $U_j(\mathbf{C})$ can be removed by assuming $\gamma = \alpha$, in which case

$$v_i^{\text{WH}}(\mathbf{C}) = \frac{v}{2}\big[1+\mathbb{O}_i(\mathbf{C})\big]\mathbb{O}_i(\mathbf{C})^{\beta/\alpha-1} , \tag{5.15}$$

where

$$\mathbb{O}_i(\mathbf{C}) = e^{\left[\mathbb{U}(\bar{X}_i,\mathbf{C})-U_i(\mathbf{C})\right]/\beta} , \tag{5.16}$$

is the odds of outmigration and where $v \in \mathbb{R}^+$ is the decision rate of an individual who evaluates his current location as good as the expected value of

[4]The factor 1/2 is not usual but was included here, without loss of generality, for convenience. It will enhance the following interpretation of the time-scale parameter.

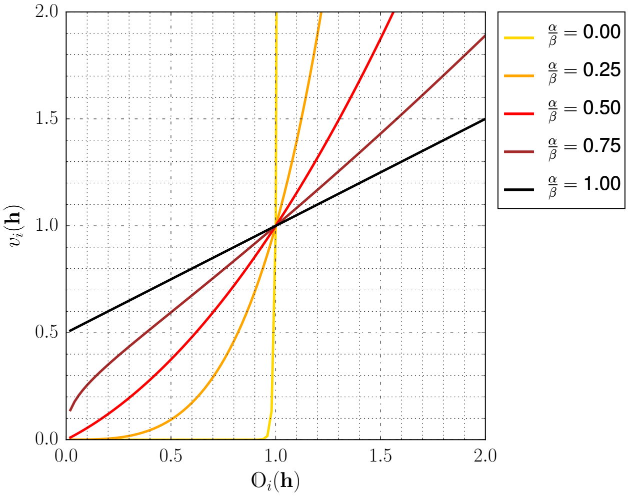

Figure 5.3. Relationship between the origin-specific individual decision rate and the odds of outmigration. Equation 5.15, which relates the origin-specific individual decision rate $v_i(\mathbf{h})$ to the odds of outmigration $\mathbb{O}_i(\mathbf{h})$, is plot for five different values of the ratio α/β.

the potential destinations. Writing (5.15) in terms of the odds of outmigration shows the intuition this relationship. The decision rate (5.15) increases with the odds of outmigration, and this increase is stronger for low values of the ratio α/β (Fig. 5.3). From Ben-Akiva and Lerman (1985), if $\alpha = \beta$ then the nested structure disappears, which would mean in this case that there is no "home sweet home" effect. At the opposite, lower values of α/β strengthen this effect. Hence the rationale for (5.15) is that the stronger the "home sweet home" effect, the more sensitive the decision rate is with respect to the odds of outmigration. To sum up, the exponential transition rate used in sociodynamics (Weidlich, 2006) is compatible with the forward transition rate proposed in the nested structure of individual decision, providing that the social temperature is equal to the second-stage scale parameter and that the decision rate varies with the odds of outmigration according to (5.15).

In another collection of works, Grauwin et al. (2009, 2012) and Lemoy et al. (2011) used a logistic form for the transition rates, that is

$$q^{\mathrm{L}}(j|i, \mathbf{C}) = \frac{w}{1 + e^{\frac{[U_i(\mathbf{C}) - U_j(\mathbf{C})]}{\gamma}}} \ , \tag{5.17}$$

and they explicitly refer to the multinomial Logit model (see for example Anderson et al., 1992). The key point is that this transition rate is given *after* that the destination state x has been randomly chosen. By doing so, the transition is reduced to a binary choice problem whose multinomial Logit form is obviously equivalent to a logistic equation.[5] Hence by construction, the logistic approach of Grauwin et al. (2009, 2012) and Lemoy et al. (2011) is closer to the backward transition rate developed here. However, an formal analogy cannot be drawn using the above development. First, consider again that the time-scale parameter w of the logistic form is equal to a constant decision rate v of the forward and backward transition rates. Although the all forms are bounded by the same value, they are still not compatible. Indeed, there is no assumption on γ that allows to equalize the logistic form with the forward or backward transition rates. Second, consider as previously that the decision rate $v_i(\mathbf{C})$ is alternative-specific. Then, once again, since this rate has to be specified prior to any decision, it can only depends on $U_i(\mathbf{C})$ or $\mathbb{U}(\bar{X}_i, \mathbf{C})$, but not on $U_j(\mathbf{C})$. This condition cannot be respected neither in the forward nor in the backward case. Consequently, the logistic transition rate used by Grauwin et al. (2009, 2012) and Lemoy et al. (2011) does not seem to be compatible with the nested structure of individual decisions proposed in this chapter.

5.4 Collective dynamics

In this section, individual dynamic decisions are firstly aggregated into a collective dynamic system described by a Master equation. Secondly, the master equation is approximated by a diffusion process.

5.4.1 Aggregation procedure

Considering that the N agents in A can decide to change their alternative, one is interested in the configuration dynamic $\mathbf{C}(t)$. It is assumed that agents' decisions are independent Poisson processes (although their rates are related to each others). Thus, each agent independently follows the individual dynamics described above, and the probability for two agents to act simultaneously is negligible. This enables to specify, for each state of the system, which the other states are reachable in an infinitesimal time step $\mathtt{d}t$. Regarding the configurations, the infinitesimal change vector $\boldsymbol{\delta}(t) = \mathbf{C}(t+\mathtt{d}t) - \mathbf{C}(t)$ can only take $K = M(M-1)$ different values, each one describing the change of a single agent, from one alternative for another, along one of the K possible abandoned-new couples of alternatives. The components of the change vector describing the change of an agent from alternative $m \in X$ for alternative $m' \in X$ are all zero, except the m^{th} component that is -1 and the $m'^{\text{ th}}$ component that is 1. All instances of the change vector can be ordered arbitrarily and

[5]The reader may convince himself by multiplying the numerator and the denominator of the equation on the right hand side of (5.17) by $e^{U_j(\mathbf{C})/\alpha}$.

grouped into a $(M \times K)$ matrix $\mathbf{J}$ called the jump matrix,[6] such that the k^{th} change vector is the k^{th} column of $\mathbf{J}$, which notes $\mathbf{J}_{[.k]}$. For convenience, let $O : \{\mathbf{J}_{[.1]}, \ldots \mathbf{J}_{[.K]}\} \mapsto X$ and $I : \{\mathbf{J}_{[.1]}, \ldots, \mathbf{J}_{[.K]}\} \mapsto X$ be respectively the origin and destination functions such that if $\mathbf{J}_{[.k]}$ is the change vector describing an agent change from alternative m for alternative m', then $O(\mathbf{J}_{[.k]}) = m$ and $I(\mathbf{J}_{[.k]}) = m'$. Actually, under the assumption of independent departure times, the collective spatial dynamics of the N agents is a continuous-time Markov chain as well (see the demonstration below). Especially, its state set is $\mathbb{S}_N^M$ and its transition rates from any configuration $\mathbf{C}$ to any other $\mathbf{C}'$ are given by

$$Q(\mathbf{C}'|\mathbf{C}) = \begin{cases} \mathbf{C}_{[O(\mathbf{C}'-\mathbf{C})]}\ q\Big(D\big(\mathbf{C}'-\mathbf{C}\big)\Big|O\big(\mathbf{C}'-\mathbf{C}\big),\mathbf{C}\Big), \\ \qquad\qquad\qquad\qquad\qquad \text{if } \mathbf{C}'-\mathbf{C} \in \{\mathbf{J}_{[.1]},\ldots,\mathbf{J}_{[.K]}\} \\ 0, \quad \text{otherwise.} \end{cases} \tag{5.18}$$

Aggregate transition rates: demonstration

In order to derive the dynamics of $\mathbf{C}(t)$, one starts from the most detailed description of the system, which is in terms of states $\mathbf{S}$. The strategy is to build the Markov process $\mathbf{C}(t)$ by grouping the states of the Markov process $\mathbf{S}(t)$ by the macro-states defined in the partition $\bar{S}$ (see Section 5.2). Since agents decisions are independent, the infinitesimal transition rate of the collective system, that is, the probability per time unit that the system goes from one state $\mathbf{S}$ to another state $\mathbf{S}'$, can be expressed in terms of individual transition rates $q(j|i,\mathbf{C})$ between alternatives (regardless of their functional form). Formally, the transition rate $Q(\mathbf{S}'|\mathbf{S})$ from state $\mathbf{S} \in X^N$ to state $\mathbf{S}' \in X^N$ is

$$Q(\mathbf{S}'|\mathbf{S}) = \begin{cases} q\Big(D\big(C(\mathbf{S}')-C(\mathbf{S})\big)\Big|O\big(C(\mathbf{S}')-C(\mathbf{S})\big),C(\mathbf{S})\Big), & \text{if } d_H(\mathbf{S},\mathbf{S}')=1 \\ \qquad\qquad\qquad\qquad\qquad 0, & \text{otherwise,} \end{cases} \tag{5.19}$$

where $d_H(\mathbf{S},\mathbf{S}')$ is the Hamming distance between $\mathbf{S}$ and $\mathbf{S}'$, such that condition $d_H(\mathbf{S},\mathbf{S}')=1$ simply says that only one agent has moved during the infinitesimal time step. Note that the value of non-zero transition rates from a state to another can be expressed in terms of origin and destination configurations only, but the condition for not being null cannot. Since the states have been indexed, the transition rates can be grouped into a (M^N, M^N)-matrix $\mathbf{Q}$ such that for all $i, i' \in \{1, ..., M^N\}$: $\mathbf{Q}_{[i,i']} = Q(\mathbf{S}_{i'}|\mathbf{S}_i)$ Now, consider the partition $\bar{S}$ defined in Section 5.2. The goal here is to build

[6]This terminology is taken from stochastic modelling (see for example Gardiner, 2004; Allen, 2007; Fuchs, 2013).

the Markov process $\mathbf{C}(t)$ by grouping the states of the Markov process $\mathbf{S}(t)$ into macro-states. Following Kemeny and Snell (1976) and Tian and Kannan (2006), let $\mathbf{U}$ be the $\mathcal{B}(N+M-1,M) \times M^N$ matrix whose j^{th} row is the probability vector having equal components for micro-states in $\bar{s}_i$ and 0 elsewhere (Tian and Kannan, 2006, p.687). This yields $\forall j \in \{1,\dots,\mathcal{B}(N+M-1,M)\} : \forall i \in \{1,\dots,M^N\}$:

$$\mathbf{U}_{[j,i]} := \begin{cases} \dfrac{1}{\mathcal{M}(N,\mathbf{C}_j)}, & \text{if } \mathbf{S}_i \in \bar{s}_j \\ 0, & \text{otherwise.} \end{cases} \tag{5.20}$$

where $\mathcal{M}$ is, again, the multinomial coefficient (Bondarenko, 1993). Still following Kemeny and Snell (1976) and Tian and Kannan (2006), let $\mathbf{V}$ be the $M^N \times \mathcal{B}(N+M-1,M)$ matrix whose j^{th} column is a vector with 1's in the components corresponding to micro-states in $\bar{s}_j$, and 0 elsewhere. This yields $\forall i \in \{1,\dots,M^N\} : j \in \{1,\dots,\mathcal{B}(N+M-1,M)\}$:

$$\mathbf{V}_{[i,j]} := \begin{cases} 1, & \text{if } \mathbf{S}_i \in \bar{s}_j \\ 0, & \text{otherwise.} \end{cases} \tag{5.21}$$

From (5.20) and (5.21), one gets $\forall i,i' \in \{1,\dots,M^N\}$:

$$(\mathbf{VU})_{[i,i']} = \sum_{j=1}^{\mathcal{B}(N+M-1,M)} \left\{\mathbf{V}_{[k,l]}\mathbf{U}_{[l,k']}\right\} = \begin{cases} \dfrac{1}{\mathcal{M}(N,\mathbf{C}_j)}, & \text{if } \mathbf{S}_i, \mathbf{S}_{i'} \in \bar{s}_j \\ 0, & \text{otherwise,} \end{cases} \tag{5.22}$$

and $\forall i \in \{1,\dots,M^N\} : \forall j \in \{1,\dots,\mathcal{B}(N+M-1,M)\}$:

$$\begin{aligned} \Big(\mathbf{QV}\Big)_{[i,j]} &= \sum_{i'=1}^{M^N} \left\{\mathbf{Q}_{[i,i']}\mathbf{V}_{[i',j]}\right\} \\ &= \sum_{\mathbf{S}_{i'} \in \bar{s}_j} \left\{\mathbf{Q}_{[i,i']}\right\} \ . \end{aligned} \tag{5.23}$$

The last sum in (5.23) is, for a given state $\mathbf{S}_i$, the sum of the transition rates to all other states $\mathbf{S}_{i'}$ in a given destination macro-state $\bar{s}_j$. First, since all the destinations states $\mathbf{S}_{i'}$ are in the same macro-state $\bar{s}_j$, they correspond to the same configuration $\mathbf{C}_j$. Hence it appears from (5.19) that the non-zero transition rates from $\mathbf{S}_i$ to any of those states have the same value, which is

$$q\Big(D\big(\mathbf{C}_j - C(\mathbf{S}_i)\big)\Big|O\big(\mathbf{C}_j - C(\mathbf{S}_i)\big), C(\mathbf{S}_i)\Big) \ , \tag{5.24}$$

Second, note that the transition rate from $\mathbf{S}_i$ to another state $\mathbf{S}_{i'}$ in $\bar{s}_j$ may also be null, because of the condition in (5.19), which specifies that only one agent can move in an infinitesimal time step. As a result, some states in $\bar{s}_j$ may not be reachable from $\mathbf{S}$, whilst some other may be (see Fig. 5.4). The number of reachable states in $\bar{s}_j$ starting from $\mathbf{S}_i$ is easy to derive. Indeed, it was mentioned previously that for any couple of configuration, like $C(\mathbf{S}_i)$ and $\mathbf{C}_j$, there is at most one couple of origin and destination alternatives, respectively given by $O(\mathbf{C}_j - C(\mathbf{S}_i))$ and $D(\mathbf{C}_j - C(\mathbf{S}_i))$. Thus for any achievable transition between configurations, the number of way the unique underlying change of alternative can occur is necessarily equal to the number of agents whose initial state is the origin alternative. Consequently, (5.23) writes

$$\Big(\mathbf{QV}\Big)_{[i,j]} = \begin{cases} C(\mathbf{S}_i)_{[O(\mathbf{C}_j - C(\mathbf{S}_i))]}\; q\Big(D\big(\mathbf{C}_j - C(\mathbf{S}_i)\big)\Big| O\big(\mathbf{C}_j - C(\mathbf{S}_i)\big), C(\mathbf{S}_i)\Big), \\ \qquad\qquad \text{if } \mathbf{C}_j - C(\mathbf{S}_i) \in \{\mathbf{J}_{.1}, \ldots, \mathbf{J}_{.K}\} \\ 0, \quad \text{otherwise,} \end{cases} \tag{5.25}$$

where the constraint for non-zero transitions is now expressed in terms of configurations as well. From (5.22) and (5.23), one gets $\forall i \in \{1, \ldots, M^N\}$: $\forall j \in \{1, \ldots, \mathcal{B}(N + M - 1, M)\}$:

$$\begin{aligned} (\mathbf{VUQV})_{[i,j]} &= \sum_{i'=1}^{M^N} \big\{(\mathbf{VU})_{[i,i']}(\mathbf{QV})_{[i',j]}\big\} \\ &= \sum_{\mathbf{S}_{i'} \in \bar{s}_k} \left\{\frac{1}{\mathcal{M}(N, \mathbf{C}_k)}(\mathbf{QV})_{[i',j]}\right\} , \end{aligned} \tag{5.26}$$

where $\bar{s}_k \ni \mathbf{S}_i$. Substituting (5.25) into (5.26) yields

$$\Big(\mathbf{VUQV}\Big)_{[i,j]} = \begin{cases} (\mathbf{C}_i)_{[O(\mathbf{C}_j - \mathbf{C}_i)]}\; q\Big(D\big(\mathbf{C}_j - \mathbf{C}_i\big)\Big| O\big(\mathbf{C}_j - \mathbf{C}_i\big), \mathbf{C}_i\Big), \\ \qquad\qquad \text{if } \mathbf{C}_j - \mathbf{C}_i \in \{\mathbf{J}_{.1}, \ldots, \mathbf{J}_{.K}\} \\ 0, \quad \text{otherwise,} \end{cases} \tag{5.27}$$

where $\mathbf{C}_i = C(\mathbf{S}_i)$, so that $(\mathbf{VUQV}) = (\mathbf{QV})$. According to (Tian and Kannan, 2006, lemma 2.8, p.689), this equality is sufficient to say that the Markov process with state space X^N and transition probability matrix $\mathbf{Q}$ is lumpable with respect to partition $\bar{S}$, and that the associated transition matrix

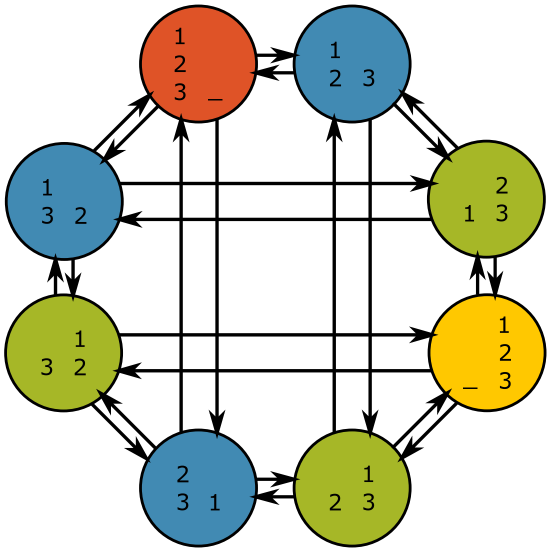

Figure 5.4. **Partition of the states into macro-states in the case $N = 3$ and $M = 2$.** Each circle represent a state. Colours depict the different items of the partition, and arrows show which state can be related to another by the move of a single agent.

$\bar{\mathbf{Q}}$ is $\forall j, j' \in \{1, \ldots, \mathcal{B}(N+M-1, M)\}$:

$$\begin{aligned} \bar{\mathbf{Q}}_{[j,j']} &:= (\mathbf{UQV})_{[j,j']} \\ &= \sum_{i=1}^{M^N} \left\{ \mathbf{U}_{[j,i]} (\mathbf{QV})_{[i,j']} \right\} \\ &= \begin{cases} (\mathbf{C}_j)_{[O(\mathbf{C}_{j'} - \mathbf{C}_j)]} \; q\Big(D\big(\mathbf{C}_{j'} - \mathbf{C}_j\big) \Big| O\big(\mathbf{C}_{j'} - \mathbf{C}_j\big), \mathbf{C}_j \Big), \\ \qquad\qquad\qquad\qquad \text{if } \mathbf{C}_{j'} - \mathbf{C}_j \in \{\mathbf{J}_{.1}, \ldots, \mathbf{J}_{.K}\} \\ 0, \quad \text{otherwise.} \end{cases} \end{aligned} \tag{5.28}$$

This last expression corresponds to the transition rate given in (5.18). These transition rates formally describe the collective dynamics in terms of individual variables. In the next subsection, they are used to explore the dynamic the transition probability function, which of first importance in operational research.

5.4.2 Transition probability function

One is interested in describing the transition probability function $P(\mathbf{C}, t \,|\, \mathbf{C_0}, N)$, denoting the probability that a M alternatives system with

N agents that is in configuration $\mathbf{C_0}$ at time $t = 0$ will be in configuration $\mathbf{C}$ a time t later. Its derivative with respect to time, written $\dot{P}(\mathbf{C}, t \,|\, \mathbf{C_0}, N)$, is described by the well-known discrete Kolmogorov-forward equation (or Master equation, see Gardiner, 1985, 2004), that is

$$\dot{P}(\mathbf{C}, t \,|\, \mathbf{C_0}, N) = \sum_{\mathbf{C}' \in \$_N^M} \left\{ Q(\mathbf{C} \,|\, \mathbf{C}', N) P(\mathbf{C}', t \,|\, \mathbf{C_0}, N) - Q(\mathbf{C}' \,|\, \mathbf{C}, N) P(\mathbf{C}, t \,|\, \mathbf{C_0}, N) \right\}, \tag{5.29}$$

whose solution has no general closed-form expression, so that numerical methods are required (Ross, 2009). Yet before shifting to numerical tools, this chapter proposes to use an analytical approximation of the discrete Kolmogorov's forward equation as a continuous diffusion process (Fuchs, 2013), which can in turn be expressed as a set of Itô stochastic differential equations. The diffusion approximation is based on the idea that infinitesimal changes describing the evolution of the configuration $\mathbf{C}$ are somehow small compared to the agents population's size N. Thus for a large population, the dynamic change of $\mathbf{C}$ can be approximated by a continuous process.

Diffusion approximation

The following approximation is inspired from Allen (2007) and Fuchs (2013). Let the j^{th} *intensive* configuration be $\mathbf{c}_j = \mathbf{C}_j / N \in \$^M$, where $\$^M = \$_1^M$ is the standard $(M-1)$-simplex. Using (5.18), the transition rate between two intensive configurations $\mathbf{c}, \mathbf{c}' \in \$^M$ is given by $w(\mathbf{c}'|\mathbf{c}, N) = Q(\mathbf{C}'|\mathbf{C})$

$$w(\mathbf{c}'|\mathbf{c}, N) = \begin{cases} N\mathbf{c}_{[O(N\mathbf{c}'-N\mathbf{c})]} \; q\Big(D\big(N\mathbf{c}' - N\mathbf{c}\big)\Big| O\big(N\mathbf{c}' - N\mathbf{c}\big), N\mathbf{c}\Big), \\ \qquad\qquad\qquad \text{if } \mathbf{c}' - \mathbf{c} \in \{\mathbf{j}_{[.1]}, \dots, \mathbf{j}_{[.K]}\} \\ 0, \quad \text{otherwise}, \end{cases} \tag{5.30}$$

where $\mathbf{j} := \mathbf{J}/N$. Intensive transition rates can be grouped into a $(K \times 1)$ matrix $\mathbf{w}(\mathbf{c}, N)$ such that $\mathbf{w}_{[k]}(\mathbf{c}, N) = w(\mathbf{c} + \mathbf{j}_{[.k]}|\mathbf{c}, N)$. With this notation, the intensive transition probability function writes

$$\dot{P}(\mathbf{c}, t \,|\, \mathbf{c_0}, N) = \sum_{k=1}^{K} \left\{ \mathbf{w}_{[k]}(\mathbf{c} - \mathbf{j}_{[.k]}, N) P(\mathbf{c} - \mathbf{j}_{[.k]}, t \,|\, \mathbf{c_0}, N) - \mathbf{w}_{[k]}(\mathbf{c}, N) P(\mathbf{c}, t \,|\, \mathbf{c_0}, N) \right\}, \tag{5.31}$$

which can be approximated by a particular Taylor expansion around $\mathbf{c}$, written $\mathcal{T}_{\mathbf{c}}$, which is called a Kramers-Moyal expansion. This yields

$$\mathcal{T}_{\mathbf{c}}\left(\dot{P}(\mathbf{c},t\,|\,\mathbf{c_0},N)\right) = \sum_{k=1}^{K}\left\{\mathcal{T}_{\mathbf{c}}\left(\mathbf{w}_{[k]}(\mathbf{c}-\mathbf{j}_{[.k]},N)P(\mathbf{c}-\mathbf{j}_{[.k]},t\,|\,\mathbf{c_0},N)\right)-\mathbf{w}_{[k]}(\mathbf{c},N)P(\mathbf{c},t\,|\,\mathbf{c_0},N)\right\}, \tag{5.32}$$

where

$$\begin{aligned}\mathcal{T}_{\mathbf{c}}\left(\mathbf{w}_{[k]}(\mathbf{c}-\mathbf{j}_{[.k]},N)P(\mathbf{c}-\mathbf{j}_{[.k]},t\,|\,\mathbf{c_0},N)\right) &= \mathbf{w}_{[k]}(\mathbf{c},N)P(\mathbf{c},t\,|\,\mathbf{c_0},N)\\ &-\sum_{j=1}^{M}\left\{\frac{\partial}{\partial\mathbf{c}_{[j]}}\left\{\mathbf{w}_{[k]}(\mathbf{c},N)P(\mathbf{c},t\,|\,\mathbf{c_0},N)\right\}\mathbf{j}_{[j,k]}\right\}\\ &+\frac{1}{2}\sum_{j,j'=1}^{M}\left\{\frac{\partial^2}{\partial\mathbf{c}_{[j]}\partial\mathbf{c}_{[j']}}\left\{\mathbf{w}_{[k]}(\mathbf{c},N)P(\mathbf{c},t\,|\,\mathbf{c_0},N)\right\}\mathbf{j}_{[j,k]}\mathbf{j}_{[j',k]}\right\}\\ &+\text{H.O.T.}\,,\end{aligned} \tag{5.33}$$

Thus the transition probability function writes

$$\begin{aligned}\dot{P}(\mathbf{c},t\,|\,\mathbf{c_0},N) \simeq &-\sum_{j=1}^{M}\left\{\frac{\partial}{\partial\mathbf{c}_{[j]}}\left\{\sum_{k=1}^{K}\left\{\mathbf{q}_{[k]}(\mathbf{c},N)\mathbf{j}_{[j,k]}\right\}P(\mathbf{c},t\,|\,\mathbf{c_0},N)\right\}\right\}\\ &+\frac{1}{2}\sum_{j,j'=0}^{M}\left\{\frac{\partial^2}{\partial\mathbf{c}_{[j]}\partial\mathbf{c}_{[j']}}\left\{\sum_{k=0}^{K}\left\{\mathbf{q}_{[k]}(\mathbf{c},N)\mathbf{j}_{[j,k]}\mathbf{j}_{[j',k]}\right\}P(\mathbf{c},t\,|\,\mathbf{c_0},N)\right\}\right\}\\ &+\text{H.O.T.}\,,\end{aligned} \tag{5.34}$$

or more concisely, truncating to the second order term,[7]

$$\dot{P}(\mathbf{c},t\,|\,\mathbf{c_0},N) \simeq -\nabla\cdot\Big(\boldsymbol{\mu}(\mathbf{c},N)P(\mathbf{c},t\,|\,\mathbf{c_0},N)\Big)+\frac{1}{2}\nabla^2\Big(\boldsymbol{\Sigma}(\mathbf{c},N)P(\mathbf{c},t\,|\,\mathbf{c_0},N)\Big)\,, \tag{5.35}$$

where $\boldsymbol{\mu}(\mathbf{c},N)$ and $\boldsymbol{\Sigma}(\mathbf{c},N)$ are respectively the expected change and covariance matrix of the infinitesimal intensive change vector $\boldsymbol{\delta}(t)/N$, where ∇ is the nabla (or Del) operator and where the central dot $\cdot$ stands for the scalar product. Finally, assume that individual transition rates of the agents from

[7]Note that the accuracy of the approximation does not necessarily improve when truncating after higher degrees since according to Pawula's theorem, the Kramers-Moyal expansion either terminates after the first or second order term, or it contains an infinity of terms (Pawula, 1967).

one alternative to another follow

$$\forall j, j' \in X : q(j'|j, \mathbf{C}) = q(j'|j, \mathbf{c}) \;, \tag{5.36}$$

which can be substituted into (5.18) so that

$$\forall \mathbf{c}, \mathbf{c}' \in \mathbb{S}^M : w(\mathbf{c}'|\mathbf{c}, N) = N w(\mathbf{c}'|\mathbf{c}) \;, \tag{5.37}$$

where $w(\mathbf{c}'|\mathbf{c}) := w(\mathbf{c}'|\mathbf{c}, 1)$. Similarly, let us write $\mathbf{w}(\mathbf{c}) := \mathbf{w}(\mathbf{c}, 1)$. Substituting (5.37) into (5.35) yields

$$\dot{P}(\mathbf{c}, t \,|\, \mathbf{c_0}, N) \simeq -\nabla \cdot \Big(\boldsymbol{\mu}(\mathbf{c}) P(\mathbf{c}, t \,|\, \mathbf{c_0}, N)\Big) + \frac{1}{2N} \nabla^2 \Big(\boldsymbol{\sigma}^2(\mathbf{c}) P(\mathbf{c}, t \,|\, \mathbf{c_0}, N)\Big) \;, \tag{5.38}$$

describing a diffusion process where

$$\boldsymbol{\mu}(\mathbf{c}) = \mathbf{j} \times \mathbf{Q}(\mathbf{C}) \tag{5.39}$$

$$\boldsymbol{\sigma}^2(\mathbf{c}) = N\boldsymbol{\Sigma} = \mathbf{j} \times \mathbb{D}\Big(\mathbf{w}(\mathbf{c})\Big) \times \mathbf{j}^{\mathrm{tr}} \;, \tag{5.40}$$

where $\mathbf{j}^{\mathrm{tr}}$ is the transpose of $\mathbf{j}$ and where $\mathbb{D}\Big(\mathbf{w}(\mathbf{c})\Big)$ is the $(K \times K)$-diagonal matrix whose main diagonal's components are the elements of $\mathbf{w}(\mathbf{c})$.

Equation 5.38 is an approximation since N is still in the right-hand side expression, at the denominator of the diffusion term. Yet the approximation becomes exact for the limit $N \to \infty$, which corresponds to a deterministic model where agents are perfectly rational. Besides, Fuchs (2013) has shown that equation (5.37) makes the method more robust since a van Kampen expansion (see van Kampen, 1992) would not yield different results than the Kramers-Moyal expansion. Finally, the probability distribution exactly solving the right-hand side of (5.38) is identical to the distribution of solutions to

$$\begin{cases} \mathtt{d}\mathbf{c}(t) = \boldsymbol{\mu}(\mathbf{c}, t)\mathtt{d}t + \dfrac{\boldsymbol{\sigma}(\mathbf{c}, t)}{\sqrt{N}} \mathtt{d}\mathbf{W}(t) \\ \mathbf{c}(0) = \mathbf{c_0} \end{cases} \tag{5.41}$$

which is a system of Itô stochastic differential equations where $\mathbf{W}(t)$ is a Wiener process and $\boldsymbol{\sigma}$ is the Cholesky decomposition of $\boldsymbol{\sigma}^2$. Note that any other square root of $\boldsymbol{\sigma}^2$ can be used (see Fuchs, 2013, p.40, referring to Stroock and Varadhan, 1997).

The set of stochastic differential equations in (5.41) provides an interesting description of the collective dynamics of the individuals. Indeed, the drift and diffusion coefficient distinguish the deterministic dynamic of the system, which is related to the observable average preferences of the population, from the stochastic dynamic, which results from the unobservable idiosyncratic preferences of individuals. Hence this form echoes the decomposition of the individual utility function in Discrete Choice Theory. Moreover, its a well-known mathematical object for which efficient simulations have been developed. Its ability to enhance the predictive ability of current theoretical models will be demonstrated in the next Chapter, with an application to New Economic Geography.

5.5 Empirical perspectives

It has been mentioned in the introductory chapter of this thesis that calibration, validation and verification issues are among the major barriers to the application of microsimulation models in supporting urban planning decisions (White et al., 2015b). A promising solution is to calibrate and validate these models on the basis of their intrinsic mechanics, which results from interactions between individuals (White et al., 2015b, p.217). Social, economic and travel behaviours of individuals can indeed be observed. This can be done using classic census and other survey-based datasets, or using modern data that are by-products of people consumption and travel behaviours (like the GPS signal of their mobilephone or the expenditure data of their fidelity program cards). Note, however, that working with individual data raises specific challenges. First, privacy issues limit the ability of researchers to share their works and this may contribute a suspicious reception of their results by other scholars. Moreover, by-product data of the digital age came up with specific new challenges of cleaning and interpretation since they do not result from a rigorous sampling procedure (Graham and Shelton, 2013; Kitchin, 2013, 2014; Heppenstall and Crooks, 2016; Adam et al., 2017). To sum up, there are substantial operational problems to be solved in order to use individual data to calibrate microsimulation (or general micro-based) models.

In addition to the issue of data availability, this calibration strategy requires, first, a good understanding of the relationships between the urban dynamic and individual behaviours and, second, reliable statistical methods to test behavioural models. Since it combines Discrete Choice Theory and Synergetics in a consistent way, the mathematical modelling framework described above is certainly a good candidate for digging into emerging properties of urban dynamical systems. Thus in this section, the ability of its behavioural model to be calibrated on empirical data is discussed.

The synergetic model of discrete choice developed in this chapter has been inspired from research on migratory systems carried out by Kanaroglou et al. (1986b), Kanaroglou et al. (1986a), Anderson and Papageorgiou (1994a) and Anderson and Papageorgiou (1994b). Those researchers did not only make a substantial modelling progress in combining the nested Logit model of Williams (1977, see also Ben-Akiva and Lerman, 1985) with the migratory model of Weidlich and Haag (1983, 1988), but they also confronted their model to interregional migration data from Canada during the second half of the twentieth century. Based on their results, this section starts by exposing how individual choices can be statistically tested by maximum likelihood estimation. Afterwards, it outlines the challenges of applying the methodology of this chapter to a spatial decision problem.

Firstly, consider the calibration of the destination choice model at the second stage of the "forward" nested decision model presented in Section 5.3.1. Assume that for any couple of distinct alternatives $i, j \in X$, the destination probability depends on a vector of parameters $\mathbf{B}$ and on the initial configuration vector $\mathbf{C}$, such that it rewrites $p(j|i, \mathbf{B}, \mathbf{C})$. Let n_i^{L} be the observed number of agents abandoning alternative $i \in X$ during the time interval Δt, and n_{ij} be the corresponding number of agents who chose alternative j for new state. Let also $\mathbf{N}$ be the observed origin-destination matrix for the same time interval, such as $\mathbf{N}_{[i,j]} = n_{ij}$. Then, Kanaroglou et al. (1986a) have demonstrated that the probability of the origin-destination matrix $\mathbf{N}$ to occur is

$$\Phi(\mathbf{B}, \mathbf{C}) = \prod_{i \in X} \left\{ n_i^{\mathrm{L}}! \left(\prod_{j \in \bar{X}_i} \left\{ n_{ij}! \right\} \right)^{-1} \prod_{j \in \bar{X}_i} \left\{ p(j|i, \mathbf{B}, \mathbf{C})^{n_{ij}} \right\} \right\} , \tag{5.42}$$

and that its log-likelihood function is

$$\ln\Big(\Phi(\mathbf{B}, \mathbf{C})\Big) = \sum_{i \in X} \left\{ \sum_{j \in \bar{X}_i} \left\{ n_{ij} \ln \Big(p(j|i, \mathbf{B}, \mathbf{C}) \Big) \right\} \right\} . \tag{5.43}$$

Note that neither the existence nor the uniqueness of the solution to maximizing (5.43) with respect to $\mathbf{B}$ is guaranteed (see Kanaroglou et al., 1986a).

Secondly, consider the calibration of the departure choice model at the first stage of the "forward" nested decision model presented in Section 5.3.1. Assume that for any alternative $i \in X$, the destination probability depends on a vector of parameters $\mathbf{A}$ and on the initial configuration vector $\mathbf{C}$, such that it rewrites $p_o(i, \mathbf{A}, \mathbf{C})$. Let n_i be the initial number of agents with alternative i, and $\mathbf{n}^{\mathrm{L}}$ be the vector of departures such as $\mathbf{n}^{\mathrm{L}}_{[i]} = n_i^{\mathrm{L}}$. Then, Kanaroglou et al. (1986a) have demonstrated that the probability of the departure vector $\mathbf{n}^{\mathrm{L}}$ to occur is

$$\Psi(\mathbf{A}, \mathbf{C}) = \prod_{i \in X} \left\{ \mathcal{B}(n_i, n_i^{\mathrm{L}}) \; p_o(i, \mathbf{A}, \mathbf{C})^{n_i^{\mathrm{L}}} \; \Big(1 - p_o(i, \mathbf{A}, \mathbf{C}) \Big)^{n_i - n_i^{\mathrm{L}}} \right\} , \tag{5.44}$$

where $\mathcal{B}$ is the binomial coefficient, and that its log-likelihood function is

$$\ln\Big(\Psi(\mathbf{A},\mathbf{C})\Big) = \sum_{i\in X}\Big\{n_i^{\mathrm{L}}\ln\Big(p_o(i,\mathbf{A},\mathbf{C})\Big)\Big\} + \sum_{i\in X}\Big\{(n_i - n_i^{\mathrm{L}})\ln\Big(1-p_o(i,\mathbf{A},\mathbf{C})\Big)\Big\}\ . \tag{5.45}$$

Note, again, that neither the existence nor the uniqueness of the solution to maximizing (5.45) with respect to $\mathbf{B}$ is guaranteed (see Kanaroglou et al., 1986a). Nevertheless, both (5.43) and (5.45) provide statistical tools to calibrate the behavioural model using an origin-destination matrix $\mathbf{N}$ and classic algorithms described in Kanaroglou et al. (1986a).

The time interval used to collect the dataset is not mentioned in the above development, although the observed number of departures is expected to increase with the length of the time interval during which observations are collected. The departure probability is actually calibrated with respect to a reference time step that can be adapted to the time interval of the data. Following the decision model of this chapter, $v_i(\mathbf{C})^{-1}$ is the time interval at which only one decision is expected to occur. It means that if one wants to estimate the abandonment probability $p_o(i,\mathbf{A},\mathbf{C})$ from empirical data, he has to count the number of abandonments within a time interval $v_i(\mathbf{C})^{-1}$. Unfortunately, available data may have been reported within a time interval Δt that is not necessarily equal to $v_i(\mathbf{C})^{-1}$. Anderson and Papageorgiou (1994a) solved this problem by assuming the proportionality of the odds of outmigration with time but here, the Poisson process yields a simpler solution. Indeed, one easily shows that the empirical abandonment probability $p_o^{\mathrm{E}}(i)$, measured by counting the number of abandonments of alternative i within a time interval Δt, is related to the theoretical probability $p_o(i,\mathbf{A},\mathbf{C})$, defined in a time interval $v_i(\mathbf{C})^{-1}$, by $p_o^{\mathrm{E}}(i) = [\Delta t/v_i(\mathbf{C})^{-1}]p_o(i,\mathbf{A},\mathbf{C})$. Thus the empirical time interval is not an issue for calibration, as long as it is small enough (see below).

Once the behavioural model has been calibrated, the diffusion approximation proposed in this chapter enables to simulate the evolution of the collective system with explicit measures of the uncertainty due to idiosyncratic preferences. This method is a promising tool for operational research. Yet in the case of migratory systems, its implementation has to address specific issues.

The first issue is the identification of alternatives, which is a classic problem in spatial choice models (Tardiff, 1979). In particular, the well-known Modifiable Areal Unit Problem (MAUP) applies here since a relatively coarse spatial resolution raises two difficulties. First, it averages the spatial attributes of locations over an area that may be too large in comparison with the scale considered by individuals when deciding to migrate. For example, individuals may be interested in having green spaces they can walk to on weekends. If their walking range is much smaller than the size of the basic spatial unit, the heterogeneity of the spatial distribution of green amenities within the basic spatial unit influences a lot their decision. In that case, the average number (or area) of green amenities within the basic spatial unit is a poor proxy of the accessibility to green amenities as it is perceived by the decision maker. Second, even if individuals only consider their location as a dot on a map, without caring about their neighbourhood, the basic spatial unit is an aggregate of the actual alternatives considered by individuals. Then, the coarser the spatial resolution, the larger the heterogeneity within the basic spatial unit. How to use the nested Logit model to deal with this heterogeneity has been discussed by Kanaroglou and Ferguson (1996), Ferguson and Kanaroglou (1997) and Kanaroglou and Ferguson (1998). Regarding the temporal resolution of the dataset, a similar kind of problem occurs. If the temporal resolution is too coarse, migration events that are considered as simultaneous may actually be following each other, the first event having influenced the occurrence of the second one. That is, the temporal resolution may not correspond to the response time of the decision maker to an environmental change. This issue has to be handle cautiously.

The second issue in applying the modelling framework presented in this chapter to migratory systems is the presence of regional effects. Regardless of alternative locations' attractivity, there exist barriers and drivers to migrations: a changing macroeconomic or political context, cultural or language regional differences, etc. For example, in a recent study, Thomas et al. (in press) have shown that in Belgium, provincial boundaries have a surprisingly strong influence on the migration pattern. In order to catch those regional effects, Anderson and Papageorgiou (1994a) have introduced an *impedance* term in the calibration of the destination probabilities, which is a vector of variables discouraging migrations. Similarly, they introduced a *propensity* term in the calibration of the departure probabilities, which is a vector of variable encouraging migrations. Choosing the variables to include in those two terms, however, remains a crucial empirical task.

To sum up, the modelling framework presented in this chapter opens up new perspectives for operational research. Yet its success depends on the empirical calibration of its behavioural model. Dedicated statistical methods of calibration have been developed in studies on migratory systems (Kanaroglou et al., 1986b,a; Anderson and Papageorgiou, 1994a,b; Kanaroglou and Ferguson, 1996; Ferguson and Kanaroglou, 1997; Kanaroglou and Ferguson, 1998). However, spatial decisions problems raise specific issues that must always be addressed. In particular, this section has highlighted the modifiable areal unit problem and the presence of regional barriers or drivers to migration as two major issues in future empirical works.

5.6 Conclusion

This chapter has proposed an original framework of individual discrete choice dynamics. This individual model infers the exponential form of sociodynamics from the nested Logit structure of discrete choice theory, and it can be empirically tested. Secondly, the master equation describing the collective dynamics of individual decision makers has been deduced from the micro-model and, using a diffusion approximation, it has been reduced to a set of nonlinear stochastic differential equations. Those equations distinguish the deterministic dynamic of the system, which is related to the observable average preferences of the population, from the stochastic dynamic, which results from the unobservable idiosyncratic preferences of individuals. Thus they open new perspectives for theoretical modelling in collective location choices, which will be exposed in the next chapter. Finally, the modelling framework presented in this chapter enables to explore new calibration and validation methods of collective dynamics that rely on the individual decision model. Those methods, however, will have to overcome the operational difficulty of getting and working with individual data. In the perspective of applying this modelling framework to location decisions, calibration and validation procedures will also have to deal with the specificities of spatial choice models.

6

Noise-Induced Transitions in Core-Periphery Models

Heterogeneous agents have recently been introduced in core-periphery models of New Economic Geography. Yet those models treat equilibrium stability locally using *ad hoc* deterministic adjustment procedures based on expected migrations that cannot catch the influence of agents heterogeneity on the collective dynamics. However, the stochastic modelling framework developed in the previous chapter can expose the effect of idiosyncratic preferences on the dynamic of interacting location choices. Thus, this chapter presents an application of this modelling framework to a dynamic footloose entrepreneur core-periphery model of New Economic Geography, with heterogeneous agents. Results show that heterogeneous preferences generate noise-induced transitions in the interregional distribution dynamic of high-skilled workers. Those transitions require the use of adapted notions of equilibrium stability and selection, which have been numerically discussed using numerical simulations. These refined theoretical concepts finally enable to formulate new predicted trajectories for the interregional system.

The research presented in this chapter has been carried out in collaboration with Prof Dominique Peeters and Prof Joe Tharakan (see Delloye et al., 2018b).

6.1 Introduction

The geographic distribution of human population and economic activities is closely related to regional development (Williamson, 1965; Wheaton and Shishido, 1981; Fujita and Thisse, 2003; Desmet and Rossi-Hansberg, 2010). This is why it has become an important political issue, as exemplified by international economic integration policies such as the North American Free Trade Agreement or the European Economic Area. The distribution of human activities results from migration decisions that depend, *inter alia*, on market incentives, such as wage differentials or costs of living. Market incentives to migrate are fundamental in New Economic Geography, which essentially studies how trade costs interact with demand and input-output linkages in economic production (Krugman, 1991a; Venables, 1996; Fujita, 1999; Combes et al., 2008; Baldwin et al., 2005). Models of New Economic Geography mainly use representative agents, although migrations also depend on idiosyncratic preferences for market or non-market attributes of locations (Jacobs, 1961; Hicks, 1963; Rosen, 1979; Roback, 1982; Greenwood, 1985; Glaeser, 2008). It is only recently that researchers have started to study the effect of agents heterogeneity on these standard models.

Firms heterogeneity has been introduced in international trade by Jean (2002) and Melitz (2003). Their contribution has been followed by several models of economic geography studying, *inter alia*, how more productive firms cluster in larger markets (Baldwin and Okubo, 2006; Nocke, 2006; Melitz and Ottaviano, 2008). Heterogeneous households' preferences for locations have been introduced in new economic geography by Tabuchi and Thisse (2002).[1] Several papers have followed, showing how taste heterogeneity acts as a dispersion force (Mossay, 2003; Murata, 2003, 2007; Zeng, 2008; Candau and Fleurbaey, 2011).

However, those studies use *ad hoc* deterministic adjustment procedures based on expected migrations under myopic behaviours. Those deterministic approximations are problematic since they do not consider the uncertainty on adjustment dynamic trajectories that results from aggregating individual shocks generated by idiosyncratic preferences for locations. Yet this aggregate uncertainty turns the dynamic of the population distribution to a stochastic dynamical system (Longtin, 2010), in which case non-trivial effects such as noise-induced transitions (Horsthemke and Lefever, 2006) may occur, especially in the nonlinear cases proposed by Matsuyama (1991, 1995), Ottaviano (2001) and Baldwin (2001). From an applied perspective, those *ad hoc* adjustment dynamics also suffer from the lack of explicit time unit that

[1] See Amiti and Pissarides (2005) and Mori and Turrini (2005) for heterogeneous skills among households. See also Nocco (2009) for a combined treatment of taste and skills heterogeneity.

prevents adjustment trajectories to be expressed as dynamic scenarios. This shortcoming strives against the use of New Economic Geography models to support policy recommendations.

This chapter distinguishes from the previous literature by studying the aggregate effect of idiosyncratic preferences on the adjustment dynamics of core-periphery models. Especially, idiosyncratic preferences are added to the footloose entrepreneur model (Forslid and Ottaviano, 2003; Baldwin et al., 2005), which is taken as an illustrative case for its analytical tractability and its proximity to the original core-periphery model of Krugman (1991a). The geographical setting is made up with three equidistant regions. It is done for simplicity but the proposed methodology does not impose any limit neither on the number nor on the spatial structure of regions. The dynamic assumptions are inspired by the nested Logit approach of Ben-Akiva and de Palma (1986), Kanaroglou et al. (1986b,a) and Kanaroglou and Ferguson (1996), but they are also consistent with the exponential transition rate of Weidlich and Haag (1983) and Haag (1989). An original contribution of this chapter is to overcome the problem of aggregating idiosyncratic preferences by using an analytical diffusion approximation of the continuous-time Markov process (Gardiner, 1985; Allen, 2003, 2007; Fuchs, 2013). This enables to express the dynamics of the interregional population distribution by a system of nonlinear *stochastic* differential equations, hence following the nonlinear studies of Matsuyama (1991, 1995), Ottaviano (2001) and Baldwin (2001), but emphasizing the influence of taste heterogeneity (instead of forward-looking expectations) on equilibrium selection and stability. Using those equations, the issues of equilibrium selection and stability are discussed in stochastic terms (Khasminskii, 2012). These refined notions finally enable to formulate new predicted trajectories for the interregional system, which is a substantial improvement of the capacity of the footloose entrepreneur model to support policy recommendations.

The remaining of this chapter is structured in three sections. Section 6.2 presents the footloose entrepreneur model of Forslid and Ottaviano (2003) in a three-regions setting. Afterwards, Section 6.3 discusses equilibrium selection and stability in probabilistic terms. Using these refined notions, Section 6.4 proposed predicted dynamic trajectories of the interregional system and argues that they improve the trajectories inferred from deterministic models. Finally, Section 6.5 concludes.

6.2 A dynamic footloose entrepreneur model

Consider a population of workers spread among three regions, denoted by the subset of positive integers $\{1, 2, 3\}$. They consume an horizontally differentiated manufacturing good and a so called agricultural good, both being traded between regions. Each worker is endowed with one unit of labour, of either high-skilled or low-skilled type. High-skilled workers are employed in the manufactoral sector whilst low-skill workers are employed in both agricultural and manufactural sectors. The regional high-skill and low-skill endowments of the i^{th} region at time t respectively write $h_i(t)$ and $l_i(t)$, such that at any time t, $h_1(t) + h_2(t) + h_3(t) = H$ and $l_1(t) + l_2(t) + l_3(t) = L$. High-skilled workers are perfectly mobile whilst low-skilled workers are immobile and evenly distributed. Thus at any time, low-skill endowment of region i is $l_i(t) = L/3$, and the distribution of high-killed workers is given by the vector $\mathbf{h}(t)$ whose components are the $h_i(t)$'s.

In this subsection, time is fixed (hence the variable t is omitted for ease of reading) and the distribution of high-skilled workers is given. One derives the clearing-market values of quantities and prices under the given distribution of high-skilled workers. Note that most of the computations are skipped. The reader is invited to consult Forslid and Ottaviano (2003) or Baldwin et al. (2005) for any detail that would remain unclear.

On the demand side, considering workers as customers, high-skilled and low-skilled workers only differ by their wage, whose regional values respectively write y_i and y_i^{L}. Their consumption preferences are represented by the utility function $u(x_i, a_i) = \mu \ln(x_i) + (1 - \mu) \ln(a_i)$ with

$$x_i = \left(\int_M d_i(m)^{(\sigma-1)/\sigma} \mathrm{d}m \right)^{\sigma/(\sigma-1)}, \tag{6.1}$$

where $\mu \in [0, 1]$ is a constant, x_i is individual consumption of manufactures in the i^{th} region, a_i is individual consumption of agricultural products in the i^{th} region, M is the set of all varieties of x, $d_i(m)$ is the individual consumption of variety m in the i^{th} region and finally, σ is both the demand-elasticity of any variety and the elasticity of substitution between any two varieties.

Turning now to the supply side, on the one hand, firms of the agricultural sector produce an homogeneous good under perfect competition and constant return to scale using low-skilled labour. Units are freely chosen so that one labour unit produces one output unit. The agricultural good is freely traded and it is chosen as the numeraire.[2] On the other hand, firms of the manufactural sector produce differentiated varieties, they are monopolistically competitive

[2] Note that the chosen unit of the homogeneous agricultural good a_i, the assumption of perfect competition and the choice of a_i as the numeraire altogether imply $y_i^{\text{L}} = 1$, $\forall i \in \{1, 2, 3\}$. This holds as long as the agricultural good is produced in all regions, which requires $\mu < \sigma/(2\sigma - 1)$ (Forslid and Ottaviano, 2003). This assumption is assumed to hold for now on.

and employ both low-skill and high-skill labour under increasing returns to scale. More precisely, a one-to-one relationship exists between firms and variety, and the total production cost of the firm producing $x_i(m)$ units of variety m in region i is given by

$$c_i(m) = \gamma\, y_i + \eta\, x_i(m) y_i^{\mathrm{L}} \ , \tag{6.2}$$

where γ is the fixed input requirement of high-skill labour and $\eta\, x_i(m)$ is the marginal input requirement of low-skill labour. The manufactural good endures trade barriers, which are modelled as iceberg costs. Assuming that the 3 regions are equidistant, $\tau \in [1, +\infty[$ is the amount of manufactural good that has to leave the origin region in order to deliver one unit at the region of destination.

Forslid and Ottaviano (2003) have shown that maximizing consumers utility, firms profit and clearing markets yields the indirect utility of high-skilled workers in any region, given their interregional distribution. More precisely, freely indexing the three regions by i, j and k, the indirect utility in the i^{th} region writes

$$U_i(\mathbf{h}) = \ln\left(\mu^{\mu}(1-\mu)^{1-\mu} \left[\frac{(\sigma-1)\gamma^{1/(1-\sigma)}}{\sigma\eta} \right]^{\mu} \left[h_i + \tau^{1-\sigma}\Big(h_j + h_k\Big)\right]^{\mu/(\sigma-1)} y_i^* \right) \ , \tag{6.3}$$

where y_i^* is the local equilibrium wage of high-skilled workers, implicitly given by

$$y_i^* = \theta \left[\frac{y_i^* h_i + L/3}{h_i + \phi\Big(h_j + h_k\Big)} + \frac{\phi\Big(y_j^* h_j + L/3\Big)}{h_j + \phi\Big(h_i + h_k\Big)} + \frac{\phi\Big(y_k^* h_k + L/3\Big)}{h_k + \phi\Big(h_i + h_j\Big)} \right] \ , \tag{6.4}$$

with $\phi = \tau^{1-\sigma}$ and $\theta = \mu/\sigma$. Since the three regions are perfectly symmetric, indirect utilities in other regions write accordingly. Thus $\{y_1^*, y_2^*, y_3^*\}$ is a system of linear equations that can be solved to obtain the equilibrium wages as functions of the spatial distribution of high-skilled workers. Substituting (6.3) into (5.12) finally yields the individual transition rates from any region i to any other $j \in R_i$,

$$q(j \,|\, i, \mathbf{h}) = \frac{v}{2} \left[\frac{h_j + \phi\Big(h_i + h_k\Big)}{h_i + \phi\Big(h_j + h_k\Big)} \right]^{\frac{\mu}{\alpha(\sigma-1)}} \left[\frac{Ah_j^2 + B(h_i^2 + h_k^2) + Ch_j(h_i + h_k) + Dh_i h_k}{Ah_i^2 + B(h_j^2 + h_k^2) + Ch_i(h_j + h_k) + Dh_j h_k} \right]^{\frac{1}{\alpha}} , \tag{6.5}$$

where

$$A = 3\phi \ , \tag{6.6}$$
$$B = \theta(2\phi^2 - \phi - 1) + (\phi^2 + \phi + 1) \ , \tag{6.7}$$
$$C = \theta(\phi^2 + \phi - 2) + 2(\phi^2 + \phi + 1) \ , \tag{6.8}$$
$$D = \theta^2(2\phi^2 - 3\phi + \phi^{-1}) + 2\theta(\phi^2 - \phi^{-1}) + (2\phi^2 + 3\phi + \phi^{-1}) \ , \tag{6.9}$$

are constants.

6.3 Stochastic discussion of equilibria

This section starts by presenting the equilibrium stability, selection and dynamic trajectories of the deterministic system (the limit case $H \to \infty$) before discussing those concepts in the stochastic case. It follows from (6.5) that differential equations of the system (5.41) have complex nonlinear forms, hence the results are obtained numerically. Through this section, the parameters are $\alpha = 10^{-2}$, $\mu = 0.5$, $\sigma = 5$, $\tau = 1.1504$ and $v = 1/365$. Those parameters were chosen to reproduce the intermediate case of (Fujita et al., 1999b, section 6.1), where even and uneven distributions of high-skilled workers are possible equilibria. Note that one parametrization is enough to support the results and so a comprehensive sensitivity analysis is out of the scope of this paper. Numerical resolution of the stochastic differential equations are performed using the Euler method, on a daily basis, over 20 years.

Results are presented in a simplex whose summits are $\mathbf{S}_1 = (1, 0, 0)$, $\mathbf{S}_2 = (0, 1, 0)$ and $\mathbf{S}_3 = (0, 0, 1)$ (Fig. 6.1). Just like in the intermediate case of (Fujita et al., 1999b, section 6.1), in the deterministic case the regional system has four stable equilibria: one corresponding to the even dispersal of high-skilled workers ($\mathbf{s_0^*}$) and three corresponding to their agglomeration in each region ($\mathbf{s_1^*}$, $\mathbf{s_2^*}$ and $\mathbf{s_3^*}$, see Fig. 6.1). Note that contrary to Krugman (1991) and Fujita (1999), the agglomeration is never complete and a small number of high-skilled workers remains in the other regions. This results from the dispersion strength of idiosyncratic preferences described by Tabuchi and Thisse (2002) and Murata (2003). There are also three saddle points which are crucial to understand dynamic trajectories of the system. Indeed, starting from e.g. $\mathbf{s} = (0.5, 0.1, 0.4)$, the dynamic trajectory will first be attracted to the saddle point before reaching its unstable manifold and ending up in equilibrium $\mathbf{s_1^*}$ (see the violet trajectory on Fig. 6.1). Note that in the deterministic case, this is the only trajectory starting from $\mathbf{s} = (0.5, 0.1, 0.4)$, such that by extension there is a bijective correspondence between each point of the simplex and the associated long-term equilibrium. Thus equilibrium selection is not a issue.

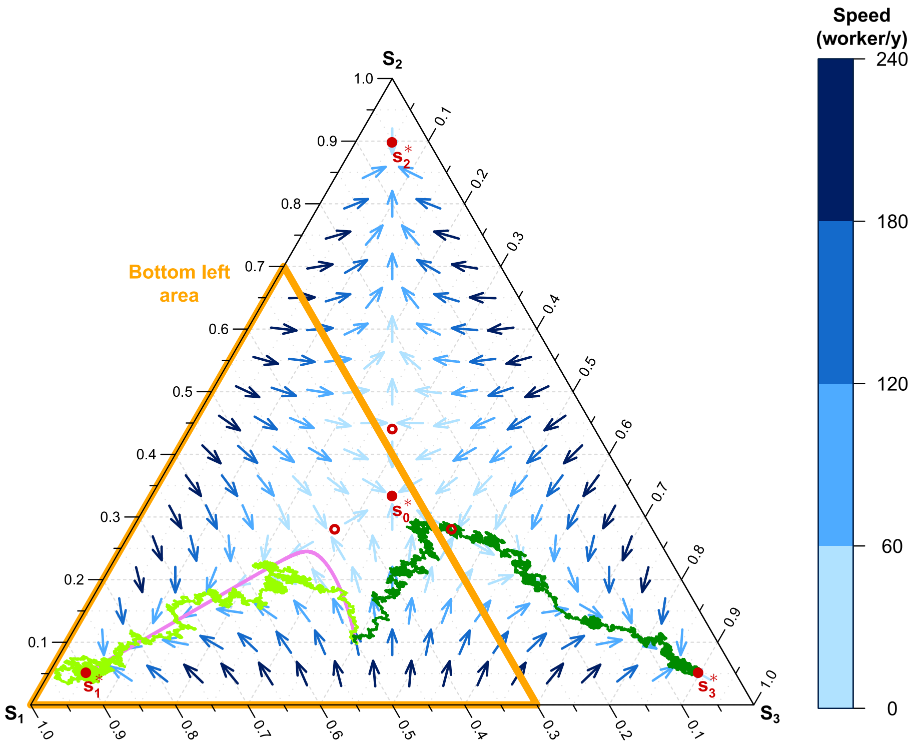

Figure 6.1. **Noise-induced transitions in the footloose entrepreneur model with three regions.** The black triangle is the simplex of the population distribution among the three regions, with summits $\mathbf{S}_1 = (1, 0, 0)$, $\mathbf{S}_2 = (0, 1, 0)$ and $\mathbf{S}_3 = (0, 0, 1)$. Blue arrows depict the expected values of the instantaneous change vector $\boldsymbol{\delta}(t)$. Red dots are stable equilibria and red circles are saddle points. Violet line is the unique deterministic trajectory starting from $\mathrm{s} = (0.5, 0.1, 0.4)$, whilst green lines are two sample paths starting from $\mathrm{s} = (0.5, 0.1, 0.4)$. The orange triangle is the area that is explored for discussing equilibrium selection.

In the stochastic case however, this bijective correspondence does not hold any more. In the simplest case, idiosyncratic preferences simply add noise around the deterministic trajectory such that the regional system finally oscillates around its deterministic equilibrium. Yet in other cases, they may also push the system towards another equilibrium. For example, a system in initial state $\mathbf{s} = (0.5, 0.1, 0.4)$ may end gravitating around state $\mathbf{s_3^*}$ instead of state $\mathbf{s_1^*}$ (Fig. 6.1). Oscillations around equilibrium states require an adequate notion of equilibrium stability, and noise-induced transitions confuse the equilibrium selection pattern. Those two problems are now addressed. Note that due to the symmetry of the simplex, results will focus on its bottom left area (Fig. 6.1).

6.3.1 Equilibrium stability

Idiosyncratic preferences induce oscillating moves of the system state around the equilibria. Although their probability is negligible, large deviations from the expected trajectory may occur. In that case, the only meaningful definition of stability is that "*at any fixed time, the sample function should lie in the neighborhood of the origin with sufficiently high probability*" (Khasminskii, 2012, p.27). To assess the stabilities of equilibria $\mathbf{s_0^*}$ and $\mathbf{s}_1^*$ according to this definition, the following procedure is applied. Starting from an equilibrium, say $\mathbf{s_0^*}$, 1000 simulations are performed. For each simulation, the root mean square distance between $\mathbf{s_0^*}$ and the temporary state of the system is computed at the beginning of each year. This finally enables to plot, for each year, the estimated probability for the regional system to be closer than a given distance to the $\mathbf{s_0^*}$.

As expected, the estimated probability is an increasing function of the maximal root mean square distance to the equilibrium, may it be either $\mathbf{s_0^*}$ (Fig. 6.2) or $\mathbf{s_1^*}$ (Fig. 6.3). Starting from the dispersed equilibrium $\mathbf{s_0^*}$, one sees that the probability of the interregional system not being in the neighbourhood of $\mathbf{s_0^*}$ becomes negligible for a radius larger than 0.4 units (Fig. 6.2). This radius actually includes the other equilibria in the neighbourhood. For smaller radius, increasing the time interval decreases the probability that the system remains in the neighbourhood. The rationale is that a larger time interval increases the probability of large deviations from the expected trajectory to occur and to push the interregional system towards another equilibrium. The agglomerated equilibrium $\mathbf{s_1^*}$ is much more stable since starting from it, the probability for the interregional system to not remain in the neighbourhood of $\mathbf{s_1^*}$ becomes negligible for a radius larger than 0.04 units only (Fig. 6.3). Moreover, the time interval has no effect on this relationship. The explanation is that the agglomerated equilibrium $\mathbf{s_1^*}$ is further from the saddle points than the dispersed equilibrium $\mathbf{s_0^*}$ (Fig. 6.1). As a result, its basin of attraction is wider and deviations from the equilibrium are not amplified.

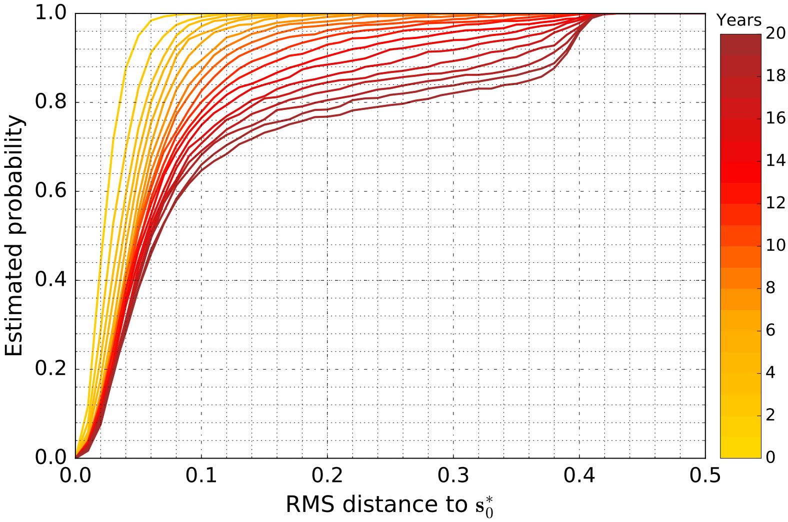

Figure 6.2. Probabilistic discussion of the stability of equilibrium $\mathbf{s_0^*}$. Starting from equilibrium $\mathbf{s_0^*}$, 1 000 simulations were used to estimate the probability (on ordinate) to be closer than a given root mean square distance (on abscissa) to $\mathbf{s_0^*}$. Those probabilities were computed on a yearly basis during 20 years (see the color scale).

6.3.2 Equilibrium selection

Complementary to equilibrium stability is the question of equilibrium selection. In the stochastic case, the question is: starting from an initial state that is not an equilibrium, what is the probability to lie, after a fixed time, in a given neighbourhood of the different equilibria? To study this question, the following procedure is applied. Exploiting the symmetry of the vector field, the bottom-left area of the simplex is evenly covered by 33 initial conditions. For each starting state, 1000 simulations are performed and for each simulation, the root mean square distances from $\mathbf{s_0^*}$ and $\mathbf{s_1^*}$ to the temporary state of the system are computed every five years. This finally enables to plot, for each starting initial state, the estimated probability for the regional system to be, at five years time intervals, in the vicinity of $\mathbf{s_0^*}$ of $\mathbf{s_1^*}$.

Fig. 6.4 depicts the probability for the interregional system to be, after five years, within 0.3 units from $\mathbf{s_0^*}$. The probability function has been estimated by linear interpolation between the 33 initial states. Although the probability is globally decreasing with the distance to $\mathbf{s_0^*}$, it decreases faster along the unstable manifold of the saddle point. Hence the saddle point does influence equilibrium selection and the effect of distance to $\mathbf{s_0^*}$ is not isotropic. Increasing the time interval reduces the probability to lie in the neighbourhood of $\mathbf{s_0^*}$

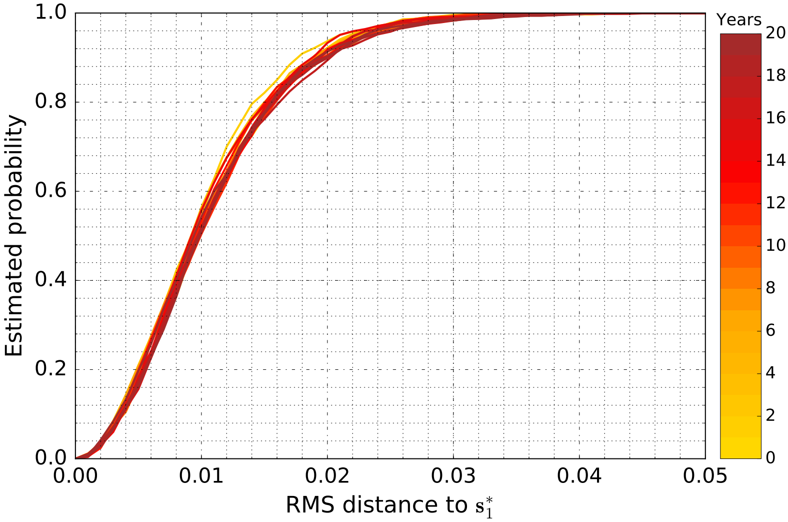

Figure 6.3. Probabilistic discussion of the stability of equilibrium $\mathbf{s_1^*}$. Red dots are stable equilibria and red circles are saddle points. Starting from equilibrium $\mathbf{s_1^*}$, $1\,000$ simulations were used to estimate the probability (on ordinate) to be closer than a given root mean square distance (on abscissa) to $\mathbf{s_1^*}$. Those probabilities were computed on a yearly basis during 20 years (see the color scale).

(Fig. 6.5, 6.6 and 6.7). It also shows that isolines of the estimated probability function tend to be convex with respect to the saddle point (see for example 6.6). The agglomerated equilibrium $\mathbf{s_1^*}$ turns out to be more often selected since in most of the state space, the probability to end up in the neighbourhood of $\mathbf{s_1^*}$ is higher than 0.8 (Fig. 6.8). Isolines of the estimated probability function are globally concentric around $\mathbf{s_1^*}$, and they slowly move further away as the time interval growths (Fig. 6.9, 6.10 and 6.11).

6.4 Application: Dynamic Trajectories

Thanks to the explicit time unit of the model, which can be adapted to empirical data (Sec. 5.5), dynamic trajectories can be analysed. It appears that along with equilibrium selection and stability, dynamic trajectories are also strongly influenced by noise induced transitions. To see this, consider again the initial state $\mathbf{s} = (0.5, 0.1, 0.4)$ (Fig. 6.1), and take the viewpoint of region 1. According to the deterministic model, its share of high-skilled workers will slightly decrease before increasing such that after 20 years, region 1 contains 89.8% of the high-skilled workers (Fig. 6.12). The stochastic model however conjectures that starting from this state, which is at 0.3 units from $\mathbf{s_1^*}$, the probability for the interregional system to be within, say, 0.1 units from $\mathbf{s_1^*}$ after 20 years is only about 0.7 (Fig. 6.11). If one takes the expected trajectory of all the sample paths that have actually ended up in this neighbourhood, it appears that the decrease in $\mathbf{s}_1$ is smaller, and that growth starts sooner than in the deterministic case (Fig. 6.12). The 90% confidence interval estimated by bootstrapping shows that this difference is significant. It results from the fact that the deterministic value aggregate the likeliness of different scenarios for region 1 (that it becomes the dominant region, that one of the two other regions become dominant or that the high-skilled workers spread evenly), whist the expected trajectory is only based on sample paths that actually ends up with the agglomeration of high-skilled workers in region 1. Thus the stochastic model provides a more accurate description of the different possible scenarios, and for this reason it constitutes a better tool for decision making.

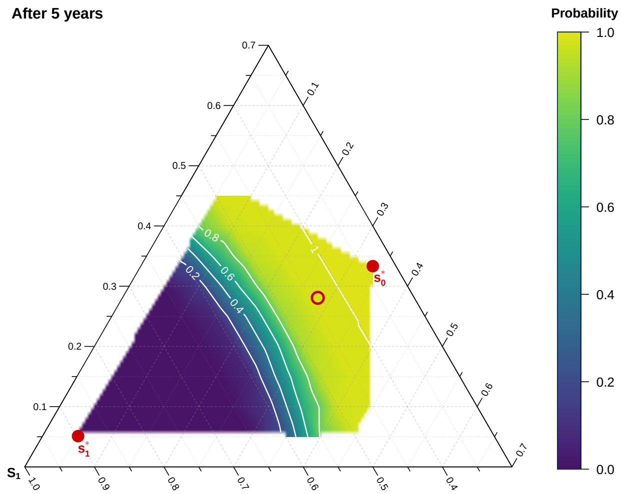

Figure 6.4. Probability of selecting equilibrium s_0^* after 5 years. Taking any point within the triangle as a starting point, the color scale depicts the probability of the system to be closer than 0.3 units to s_0^* after 5 years. The area was sampled using 33 starting points with $1\,000$ simulations for each, and the probability functions was obtained by linear interpolation.

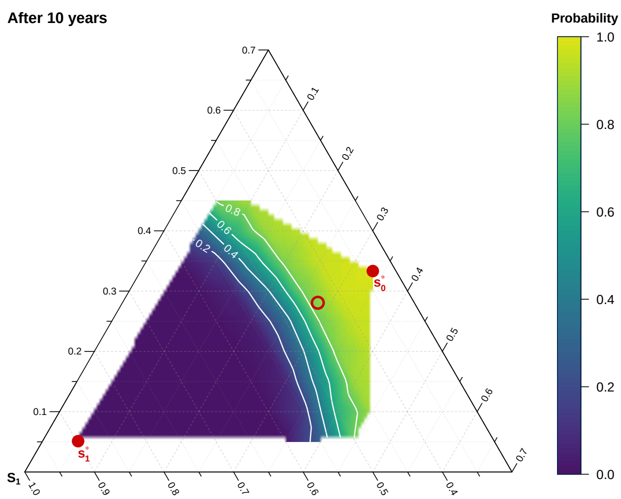

Figure 6.5. Probability of selecting equilibrium s_0^* after 10 years. Taking any point within the triangle as a starting point, the color scale depicts the probability of the system to be closer than 0.3 units to s_0^* after 10 years. The area was sampled using 33 starting points with $1\,000$ simulations for each, and the probability functions was obtained by linear interpolation.

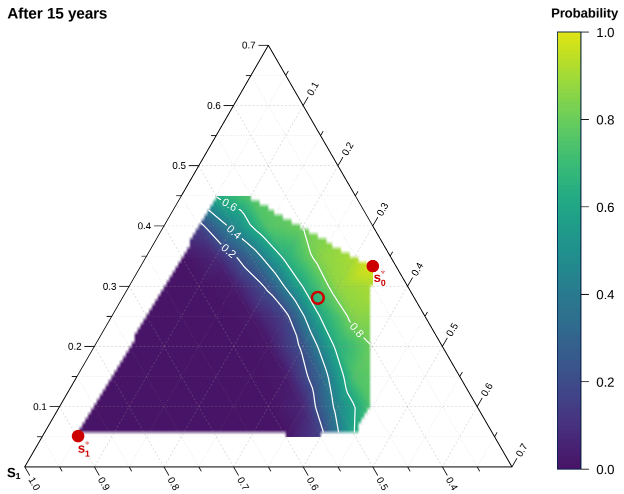

Figure 6.6. Probability of selecting equilibrium s_0^* after 15 years. Taking any point within the triangle as a starting point, the color scale depicts the probability of the system to be closer than 0.3 units to s_0^* after 15 years. The area was sampled using 33 starting points with $1\,000$ simulations for each, and the probability functions was obtained by linear interpolation.

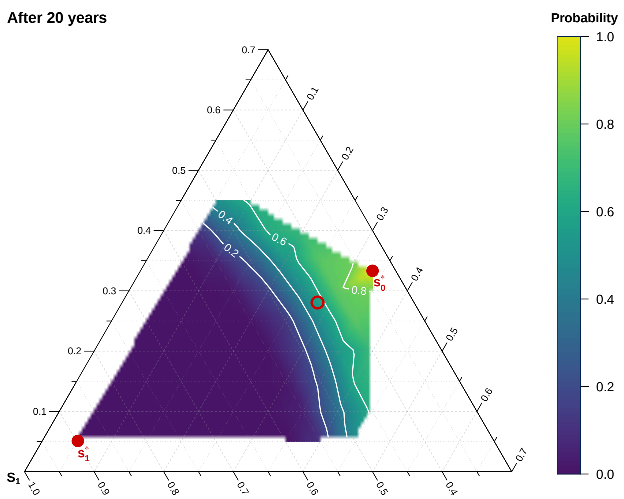

Figure 6.7. Probability of selecting equilibrium s_0^* after 20 years. Taking any point within the triangle as a starting point, the color scale depicts the probability of the system to be closer than 0.3 units to s_0^* after 20 years. The area was sampled using 33 starting points with $1\,000$ simulations for each, and the probability functions was obtained by linear interpolation.

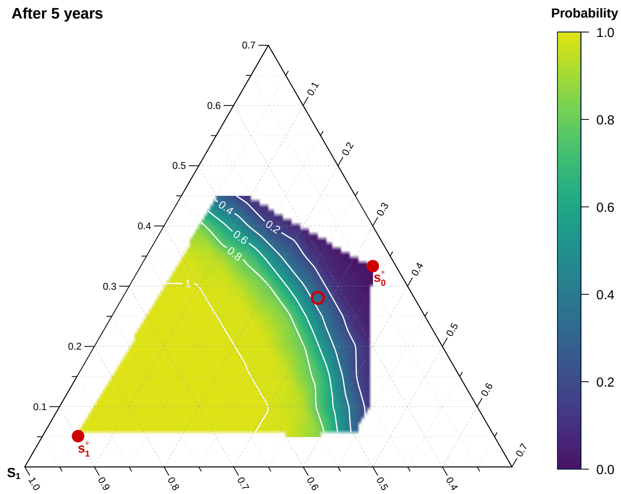

Figure 6.8. Probability of selecting equilibrium s_1^* after 5 years. Taking any point within the triangle as a starting point, the color scale depicts the probability of the system to be closer than 0.3 units to $\mathbf{s}_1^*$ after 5 years. The area was sampled using 33 starting points with $1\,000$ simulations for each, and the probability functions was obtained by linear interpolation.

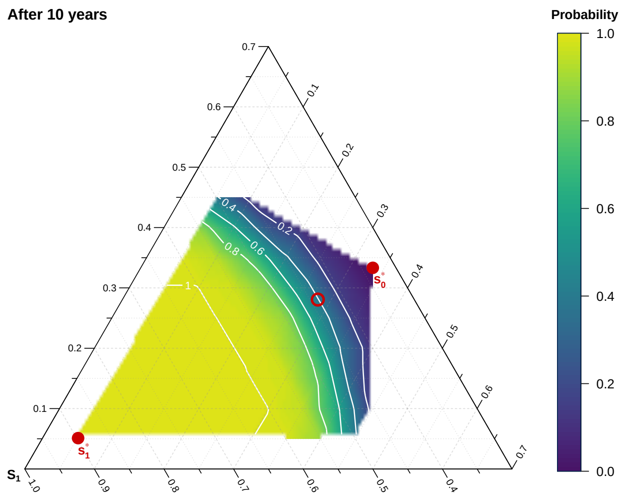

Figure 6.9. Probability of selecting equilibrium s_1^* after 10 years. Taking any point within the triangle as a starting point, the color scale depicts the probability of the system to be closer than 0.3 units to $\mathbf{s}_1^*$ after 10 years. The area was sampled using 33 starting points with $1\,000$ simulations for each, and the probability functions was obtained by linear interpolation.

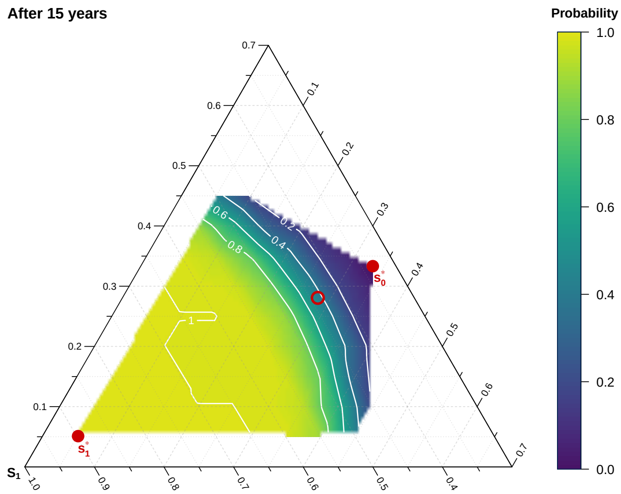

Figure 6.10. Probability of selecting equilibrium s_1^* after 15 years. Taking any point within the triangle as a starting point, the color scale depicts the probability of the system to be closer than 0.3 units to s_1^* after 15 years. The area was sampled using 33 starting points with $1\,000$ simulations for each, and the probability functions was obtained by linear interpolation..

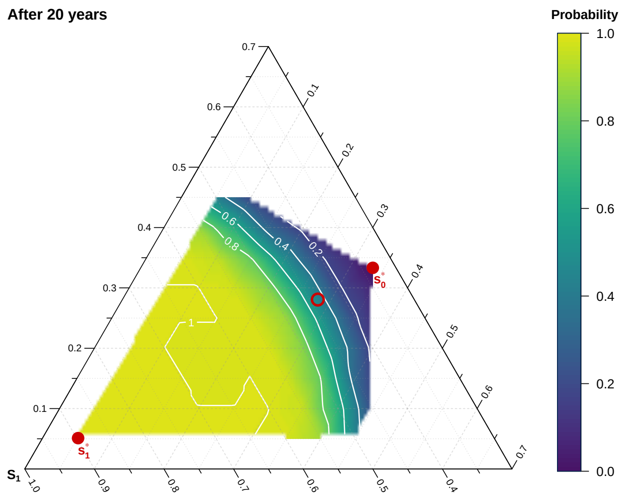

Figure 6.11. Probability of selecting equilibrium s_1^* after 20 years. Taking any point within the triangle as a starting point, the color scale depicts the probability of the system to be closer than 0.3 units to s_1^* after 20 years. The area was sampled using 33 starting points with $1\,000$ simulations for each, and the probability functions was obtained by linear interpolation.

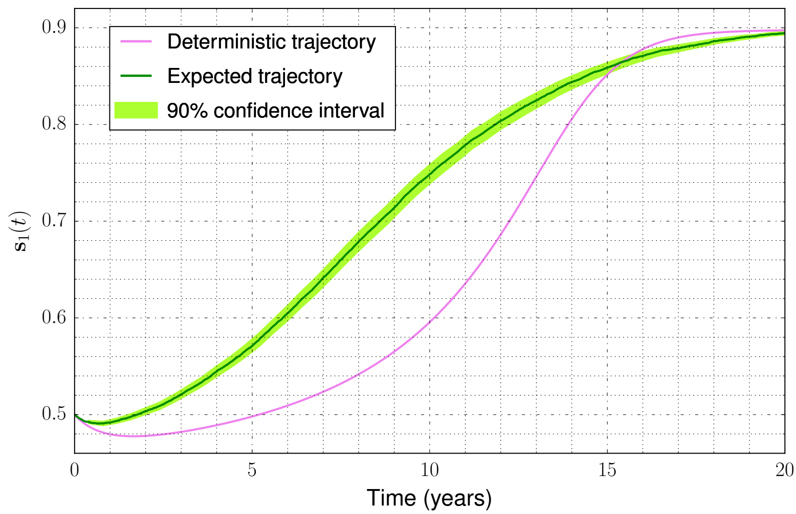

Figure 6.12. Expected and deterministic trajectories. Starting from the initial state $\mathbf{s} = (0.5, 0.1, 0.4)$, the violet line depicts the deterministic evolution of $\mathbf{s}_1(t)$ for the next 20 years. The green line is the expected trajectory of the sample paths ending up closer than 0.3 units to $\mathbf{s}_1^*$ after 20 years, and the lime buffer around it is the 90% confidence interval.

6.5 Conclusion

In this chapter, the modelling framework developed in the previous chapter has been applied to the footloose entrepreneur core-periphery model of New Economic Geography. By those means, it has uncovered the existence of noise-induced transitions in the spatial dynamics of the population of high-skilled workers. Those transitions affect three essential properties of the footloose entrepreneur model. First, it requires the notion of equilibrium stability to be discussed in stochastic terms. It has been shown that the dispersed equilibrium is less stable than the agglomerated ones in the sense that starting from this equilibrium, the probability for the interregional system to leave its neighbourhood becomes negligible for larger radius than for the agglomerated equilibria. Second, equilibrium selection does not follow a bijective relationship between initial conditions and equilibria any more. It has been shown that for most of the possible initial states, the probability for the interregional system to end up in the vicinity of the agglomerated equilibria is larger than the probability to end up in the vicinity of the dispersed equilibrium. Besides, increasing the time interval reduces the probability to lie in the neighbourhood of the dispersed equilibrium, but increases the probability to lie in the neighbourhood of the agglomerated one.

However, the configuration of the state space, and especially the location of saddle points, has to be considered in evaluating this probability. Third, even if the interregional system ends up in the vicinity of the equilibrium that is selected by the deterministic model, the adjustment trajectory inferred from the stochastic model is significantly different from the deterministic path. This difference is lower, the larger the urban population is. This results from the ability of the stochastic model to differentiate scenarios. This constitutes, along with the use of explicit time units, a substantial improvement of the capacity of the footloose entrepreneur model (and other models of New Economic Geography) to support policy recommendations.

IV

Concluding Remarks

7

Towards an Urban Morphodynamic Theory

This chapter brings a new perspective on the work carried out in this thesis. Instead of viewing the five last chapters as distinct contributions to the study of Urban Morphodynamics, they can be regarded as progressive steps towards a new formalization of this discipline, which is the global objective of this thesis. This chapter starts by arguing, on the basis of the elements presented in Chapter 2, that theoretical frameworks of Location Theory and Complex Systems have converged to a common formalization of Urban Morphodynamics using discrete spaces, natural (opposed to "logical") dynamics and behavioural micro-foundations. Second, it shows that the theoretical models presented in chapters 3 to 6 have taken substantial steps in overcoming the remaining conceptual barriers between them. In doing so, they have contributed to the development of a common modelling framework. Thus, this chapter ends by presenting the challenges that remain before this framework constitutes an operational urban morphodynamic theory.

7.1 Contributions to an emergent formalism

In the previous chapters, the works carried out during this thesis have been presented as distinct theoretical contributions to Urban Morphodynamics, addressing different research questions using different tools (see Fig. 1.2). However, collectively, they have supported the global objective of this thesis, formulated in its introductory chapter, which is to contribute to a new theoretical approach of Urban Morphodynamics. Indeed, this section argues, first, that on the basis of the elements presented in Chapter 2, theoretical frameworks of Location Theory and Complex Systems have converged to a formalization of Urban Morphodynamics using discrete spaces, natural (opposed to "logical") dynamics and behavioural micro-foundations. Second, it shows that the theoretical models presented in chapters 3 to 6 have contributed to the development of a theoretical framework that relies on this formalism.

Urban Morphodynamics designates the evolution of the spatial structure of an urban system from elementary interactions between its components. Thus, three fundamental modelling aspects have to be considered: (1) the explicit consideration of space, (2) the explicit consideration of time and (3) the explicit consideration of individuals, either isolated or grouped into subsystems. Regarding these three modelling aspects, recent developments in Location Theory and Complex Systems seem to converge.

Firstly, with respect to space, a discrete approach has been adopted in both disciplines. In Location Theory, this shift was initiated in New Economic Geography. In this discipline, the geographical space is not an Euclidean continuum any more but it has been substituted to a set of discrete locations, or "regions", whose spatial configuration varies from a line to an hexagonal lattice. Such representation corresponds to the discrete representation of space that has been dominant in transport modelling, LUTI models and Spatial Interactions Models since the work of Lowry (1964). Secondly, the conception of time in Location Theory has also become closer to an explicit dynamic in the sense of General System Theory. Starting from static models where time was merely implicitly considered, dynamic models arose from the discussion of equilibrium stability. Although this "logical" time approach (Garretsen and Martin, 2010) remains dominant in Urban Economics and New Economic Geography, alternative models with a "real" dynamic have been proposed in both Urban Economics (Fujita, 1983; Miyao, 1987; Brueckner, 2000) and Economic Geography (Krugman, 1991b; Matsuyama, 1995; Baldwin, 2001; Ottaviano, 2001).

Thirdly, regarding the focus on micro-behaviours, convergence comes from a shift in Social Physics initiated by Complex Systems. The transportation and

LUTI models developed after the fifties, which fostered the growth of Spatial Interaction Theory, were aggregate models which only had a limited success in application. Many critics arose, the most exemplar being Lee's "*Requiem for Large Scale Models*" (Lee, 1973). In reaction, urban planners became more and more interested in short-term predictions and urban researchers focused more and more on the behavioural and economic drivers of urban dynamics. In that context, the emergence of Complex systems and microsimulations yielded a paradigm shift from "*Marco-Statics to Micro-Dynamics*" (Batty, 2008). This renewed interest for the behavioural determinants of urban dynamics breaks the barriers that used to be between, for example, Urban Economics and Spatial Interaction Theory (see Fujita, 1983, p.412).

However, still regarding these three modelling aspects, some conceptual divergences remain that have to be overcome in order to build an operational urban morphodynamic theory. Firstly, with respect to space, the discrete approach used in New Economic Geography is barely used to describe the discretization of a continuous space. Instead, it describes abstract locations organized, e.g., around a circle or on a hexagonal lattice. It has generated the common idea that Urban Economics studies cities whilst New Economic Geography deals with regional issues, although it is a rather arbitrary distinction (Thisse, 2010; Fujita and Thisse, 2013). Indeed, there is no obvious technical barrier preventing them from representing the cells of a grid or the administrative units of a map. The model simply needs to be adapted.

Secondly, with respect to time, the dynamic models of Urban Economics (Fujita, 1983; Miyao, 1987; Brueckner, 2000), New Economic Geography (Krugman, 1991b; Matsuyama, 1995; Baldwin, 2001; Ottaviano, 2001) and the dynamic central place models (White, 1977; Allen and Sanglier, 1978; White, 1978; Allen and Sanglier, 1979, 1981c,b,a) have shown that there is a common interest for explicit dynamic models. It turns out, then, that the problem is not that much the opposition of static versus dynamic modelling than the opposition of equilibrium versus disequilibrium analysis. The initial motivation of researchers in Location Theory for equilibrium stability makes them prone to focus on "near-to-equilibrium" dynamic processes. On the opposite, the "out-of-equilibrium" dynamic behaviour of systems is a fundamental concept in Complex Systems (Nicolis and Prigogine, 1977).

Thirdly, with respect to micro-behaviours, a major conceptual difference remains: the deterministic nature of many models of Location Theory sharply contrasts with the stochastic methodology put forth by Complex Systems. In Location Theory, individual behaviours are preferably deterministic, for the seek of simplicity, and stochastic behaviours (for example due to idiosyncratic preferences) are only introduced if they have an anticipated effect on the model's equilibrium. This approach has been strongly influenced by the static perspective taken by most of these models. In Complex Systems however, models are essentially dynamic and often non-linear. As a result, stochastic processes always play a significant role in shaping the system behaviour. Thus in Complex Systems, stochastic models are the rule and not the exception.

To sum up, from the perspective of Urban Morphodynamics, three conceptual oppositions remain between Location Theory and Complex Systems. The abstract or geometrical representation of discrete space, the near-to-equilibrium or far-from-equilibrium approach of dynamics and the deterministic or stochastic foundation of micro-behaviours. Those three issues are actually addressed by the models presented in this thesis.

Firstly, in Chapter 3, a static model in continuous space is used to tackle an important question of micro-behaviour: can the scaling laws of the monocentric shape of European cities be explained by micro-behaviours, especially the economic trade-off between transport costs and housing space put forward by Alonso (1964)? The proposed model is deterministic, and it succeeds in reproducing the scaling behaviour of the population density profiles that has been observed in European cities (Lemoy and Caruso, 2017). Yet, there is quite a lot of variation among these scaling profiles, which were treated as noise. Section 3.4.2 has discussed the most likely sources of variations, one of them being the multicentricity at finer scale. This multicentricity may be partly due to subcentres of employment, but they are also likely to correspond to alternative centres generated by different kinds of urban amenities. Since neither the Alonso nor the Alonso-LU model initially aims at describing cities with that level of details, they should, however, consider the influence of these external effects. One way of doing it, which has been used in Chapters 5 and 6, is to use discrete choice models.

Secondly, in Chapter 4, the micro-assumptions are taken from the non-monocentric model of Fujita and Ogawa (1982), but they are used in a setting with discrete space and discrete time. Regarding space, the mathematical model treats it as a continuous set, but in the agent-based implementation, the discrete grid is used as a numerical approximation. This is a first approach of providing the discrete space approach with a clear geometrical interpretation. Regarding time, the mathematical model uses a static approach that almost abstracts from it. Dynamic assumptions are then introduced by an algorithmic way in the agent-based model. This alternative enables a partial approach of out-of-equilibrium dynamics. Indeed, the short-run equilibria are actually partial equilibria with respect to the current number of inhabitants. The stop condition, on the other hand, is a long-run equilibrium that is comparable to a static equilibrium. It diverges, however, from equilibrium configurations formulated *ex ante* because of path-dependency effects.

Finally, in Chapter 5 and Chapter 6, a mathematical model is developed which explicitly considers a discrete space, an out-of-equilibrium dynamics and a stochastic decision model. Thus, contrary to the two previous chapters, it aims at proposing an integrated formalism. To do so, it uses a different mathematical setting that gathers from Discrete Choice Theory, Synergetics and Stochastic Calculus. Regarding space, the method deals with an abstract set of discrete locations, and its application to New Economic Geography assumes three equidistant regions. Hence the application to a grid or a map is left for further research. Regarding dynamics, an explicit time approach enables to shed a new light on equilibrium selection and stability. In particular, the application to the footloose entrepreneur model demonstrates the ability of the stochastic modelling framework to reconcile near-to-equilibrium and far-from-equilibrium dynamic approaches. Regarding mirco-behaviours, the discrete choice theory used to introduce stochasticity in the behavioural model. Moreover, Synergetics enables to a keep track of this stochasticity at the collective scale. Finally, the diffusion approximation of stochastic calculus enables to formulate a set of equations that open new simulation perspectives.

In conclusion, the three models of this thesis depict progressive steps towards a integrated formalism of Urban Morphodynamics based on discrete space, natural dynamics and micro-behavioural assumptions. Since this formalism is also used by many microsimulation models, it opens the perspective of supporting predictive microsimulation models with explicative theoretical models. However, several challenges still remain before this framework constitutes an operational urban morphodynamic theory.

7.2 The challenges of an operational urban morphodynamic theory

In order to reach an operational status, the proposed modelling framework should be able to support predictive microsimulations models. In this section, the next-step challenges in modelling the geographical space, the natural out-of-equilibrium dynamics and the micro-behaviours of agents are briefly presented.

First of all, a remaining challenge in modelling the geographical space is to use the discrete set of locations to describe the discretization of a continuous geometrical space, such as the cells of a grid or the administrative units of a map. Although it has already been mentioned, such models has not been develop in this thesis. Doing so will introduce additional attributes of locations (such as their area, compactness, etc.) that are usually put aside by theoretical models, which require some modelling adaptations. For example, distances between locations need to be adapted to the structure of the transport networks. Neighbourhood effects, which are so important in urban models, cannot be defined in terms of Euclidean distance any more, like in Urban Economics, but they must be integrated as discrete mathematical mappings. These two examples enable to imagine the substantial role that networks will have to play in the development of an operational urban morphodynamic theory (see also Batty, 2013).

Secondly, regarding time, probably the most important issue to be addressed is the heterogeneity of the time-scales involved in the evolution of urban structures. In brief, the problem is that built infrastructures evolve more slowly than economic and demographic variables, which are themselves slower than individual moves and purchases (Wegener et al., 1986; Simmonds et al., 2013). As a result, asynchronous dynamic processes may produce unexpected results (see Chapter 4). Hence those various time scales have to be considered in an operational theory. Regarding the proposed modelling framework, the difficulty is that the use of a diffusion approximation requires the system's variables to change "slowly", whilst some urban processes, like the opening of a new shopping centre, are better modelled as sudden shocks. If the microsimulation models can deal with these shocks rather easily, it is not the case of the proposed mathematical framework. A perspective is to explore the study of generalized Markov Processes (see Gardiner, 2004) that can combine jump and diffusion processes.

Thirdly, the micro-foundations in terms of behavioural models can certainly be improved in many different ways. First, on the basis of Urban Economics and New Economic Geography models, it seems crucial to explore the influence of forward-looking expectations in agents' decisions. At first glance, there is no fundamental opposition in developing them in the modelling framework of this thesis. The only restriction is that these anticipations have to be built upon the present state of the system only, and not on its history, in order not to violate the Markovian assumption. The possibility to build non-Markovian models is actually a second research direction to explore. Although Markov processes have demonstrated their ability to catch path-dependency effects in economic development (Arthur, 1994; Arthur et al., 1997), there is no doubt that a greater emphasize on individual behaviours will, at some point, require to dig deeper into the effect of their past experience on their decisions outcomes. Finally, the integration of social networks is another issue. Modern developments of information and communication technologies have exacerbated the role of social networks in triggering social events. In the Synergetic perspective of this thesis, a innovative approach proposed by Helbing (2010) consists in using Boltzmann-like equations in order to take into account the pairwise interactions between individuals, which are out of the scope of the Sociodynamic developed by Weidlich and Haag (Weidlich, 2006) and used in this thesis.

In conclusion, Location Theory and Complex systems have all the elements required to build consistent theoretical models of urban morphodynamics in support of the predictive microsimulation models that are increasingly used in urban planning. As a result, urban researchers will have all the necessary tools to develop a better understanding of urban dynamics and of its complexity. It has been argued that such knowledge would help urban planners to depart from top-down approach based on static plans to become an inclusive process. Ultimately, this shift is a necessary condition to address modern challenges of sustainable urban development worldwide. As a result, the quest of an encompassing urban geographical theory is definitely not a fantasy of selfish intellectuals, it is a scientific duty. In that perspective, there is no doubt that "*there are real gains to be made from uniting the two approaches in order to bring the richness of domain knowledge embodied in location theory into the powerful and flexible modeling framework of the complex self-organizing systems approach*" (White et al., 2015c, p.54). Yet, how to combine both modelling traditions in consistent mathematical models, in a "*new social physics*" (Batty, 2008, p.15), is a challenging question. Yet formalisms of Urban Morphodynamics in Location Theory and Complex Systems appear to have naturally converged to a common formalism using discrete spatial units, explicit dynamic and explicit micro-behaviours. This convergence is an opportunity to build an operational urban morphodynamic theory. This thesis has tried to sketch out a way to do it, not by building new tools, but by using the existing ones in innovative ways. That fundamental intention is better understood using this metaphor which regards scientific disciplines as distinct islands in a archipelago, more or less close to each other. In that regard, the establishment of an operational urban morphodynamic theory is not so much about building bridges, but instead about pumping water, in order to reveal the sea floor in which Location Theory and Complex systems are nothing more than landmarks in a common landscape.

V

Appendices

A

A Model of 3D Built-Up Morphology

This appendix aims at sketching a way of explaining distributions of building geometric attributes within cities. To do so, it considers a dynamic choice model of location decisions by households within a monocentric city, which yields a stochastic growth process of populations at different locations. Assuming that households are accommodated in buildings constructed by rational developers, the distribution of geometric attributes are derived from the total population size and the preferences of individuals. Thus, this appendix provides an original quantitative description of how economic variables influence the evolution of urban geometry, which is a necessary step toward better understanding and management of urban structures. Moreover, resulting distributions are suitable for empirical testing. Finally, the method used is likely to adapt to more complex models, thus opening the way to further research in urban geography and urban economics.

A.1 Model

This method anchors in urban economics by deriving city-wide spatial distributions from assumptions on households, developers and landlords behaviours. The first step is to define the urban space into an Euclidean framework and to divide it into urban blocks.

In this section, functional forms of households utility and developers profit are provided. From there, equilibrium housing rent and land rent are derived for a single urban block. The resulting values of buildings height, footprint and number are deduced for any given i and $n_{[i]}$.

Consequently, it is first possible to derive building geometric attributes within a block for any given population and distance to city centre. From there, both equilibrium steady-state and out-of-equilibrium dynamics of building geometric attributes can be deduced from population dynamics. Population dynamics, on its part, can be derived from additional dynamic assumptions, although the resulting (master) equation is not really tractable. That is why a large city approximation is used, which results in a set of continuous stochastic differential equations of Itô type. From there, respective influences of the deterministic part and the stochastic part of households location decision on the urban structure can be discussed.

Urban space

The geographical framework of is a long and thin urban agglomeration divided in M residential square blocks, adjacent to the CBD (Fig. A.1). Blocks are of length L, such that the length of the residential area is ML. Blocks are indexed by $i \in \{0, \dots, M-1\}$ and ordered by increasind distance to the CBD. Residents commute toward the CBD using a unique road with unitary transport cost of τ-with no congestion (Fig. A.1). Transport within blocks is assumed to be costless and each block has one road access, located at the corner which is the closest to the CBD. Consequently, iL is the commuting distance from block i to the CBD.

The city is closed and occupied by N households. They may move between blocks, yielding to $K = M(M-1)$ distinct potential population fluxes. Blocks belong to absentee landowners and are rented at rent $R_l(i)$. Their local populations at time t (in number of households) are denoted by $n(i,t)$. The city state is described by the population vector $\mathbf{n}(t)$ whose i^{th} component is $\mathbf{n}_{[i]}(t) = n(i,t)$. The state space of the system is the $(M-1)$-simplex of size N, which writes $\mathbb{S}_N^M$, such that

$$\mathbb{S}_N^M := \left\{ \mathbf{n} \in \mathbb{N}^M \,\middle|\, \sum_{m=1}^{M} \mathbf{n}_{[m]} = N \right\} , \tag{A.1}$$

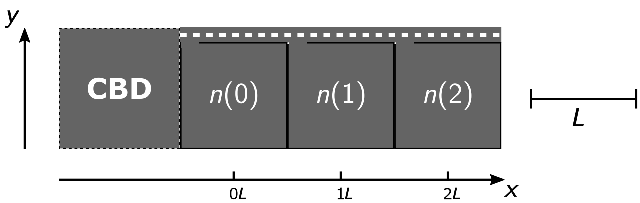

Figure A.1. **schematic map of the system.** The long and thin city is represented here in the case of $M = 3$ urban blocks.

where $\mathbb{N}$ is the set of non-negative integers (or natural numbers).

Housing market

Following the monocentric city model detailed in Chapter 3, each household requires land for housing, works in the CBD and consumes a composite commodity that is produced out of the region and imported at constant price (hence all goods are essentials). Households' residential choice depends on the consumption level of composite commodity they can afford which, given the urban space, is fully determined by the distance iL to the CBD (hence by the block's index i). It is assumed that households make this choice rationally in the sense of von Neumann and Morgenstern (1944, see e.g. Myerson, 1997). Their choice can be represented by the maximization of a well-behave utility function $U\Big(z(i), s(i)\Big)$ where $z(i)$ is the amount of composite good consumed at i and $s(i)$ is the housing area consumption at i. Contrary to Chapter 3, the land has to be developed into housing areas. This activity is carried out by developers whose behaviour is describes further. Back to households' consumption behaviour, log-linear utility is assumed so that

$$U\Big(z(r), s(r)\Big) = (1-\beta)\ln\Big(z(r)\Big) + \beta\ln\Big(s(r)\Big) , \tag{A.2}$$

where $\beta \in]0,1[$ is a parameter representing the share of income (net of transport expenses) devoted to housing, or the relative expenditure in housing.

Each household earns the same wage at the CBD and the budget constraint of each household is binding since the households' utility function is monotonic and does not include any incentive to spend money otherwise. Choosing the composite commodity z as the numeraire, so its price is unity, the budget constraint at i is

$$z(i) + R_s(i)s(i) = Y - iL\tau , \tag{A.3}$$

where Y is the wage of all households. The households' problem consists in maximizing their utility such that the budget constraint (3.1) holds.

Substituting (A.3) in (A.2) yields

$$U(i) := U\big(z(i), s(i)\big) = (1-\beta)\ln\big(Y - R_s(i)s(i) - iL\tau\big) + \beta\ln\big(s(i)\big) \ , \quad \text{(A.4)}$$

which can be maximized with respect to housing area consumption, yielding the individual housing demand

$$s_D(i) = \frac{\beta(Y - iL\tau)}{R_s(i)} \ , \qquad \text{(A.5)}$$

and so the total housing demand at i is

$$S_D\big(i, \mathbf{n}(t)\big) = n(i,t)s_D(i) = \frac{n(i,t)\beta(Y - iL\tau)}{R_s(i)} \ . \qquad \text{(A.6)}$$

Households live in buildings that are assumed to be parallelepiped rectangles with a square basis and a height h. Buildings are built by developers who are assumed perfectly rational. They rent land from absentee landowners and builds B residential buildings on it. Multiplying the number of building increases the unitary demand of land, for example because buildings cannot touch each other. Note that in this first model, this technical constraint is the only reason for empty spaces to remain between buildings since households do not value empty spaces. This will strongly influence the results. Thus, for a development of b buildings extending on l, the footprint of a single building is

$$l_b(i) := \frac{l(i)}{[b(i)]^{\alpha_b}} \ , \qquad \text{(A.7)}$$

where α_b is the wasted land coefficient such that $1 < \alpha_b$. From a purely geometrical process, the total housing production at block i is

$$H(i) := \frac{\big(b(i)\big)^{1-\alpha_b} l(i)h(i)}{C} \ , \qquad \text{(A.8)}$$

where C is the fixed ceiling height. Note that in the following, b is treated as a real number and h is not necessarily a multiple of C. Following Büttler (1981), interest rate r and depreciation rate d of capital are constant. Developers also expect constant real revenues over time, with an infinite time horizon. Finally, maintenance preserves the capital invested in the residential development. Consequently, the daily revenue of a residential development at block i is simply $R_s(i)H(i)$. Regarding buildings costs, a pseudo cost function expressing construction costs in terms of $l(i)$ and $h(i)$ is used. Following the structural analysis of Büttler and Beckmann (1980), a Cobb-Douglas formulation with fix cost is assumed. Additionally, it is considered that this fix cost raises sublinearly with b. Thus, the construction cost writes

$$F' b(i)^{\alpha_F} + V b(i)^{1-\alpha_b\alpha} l(i)^{\alpha} h(i)^{\gamma} \ , \qquad \text{(A.9)}$$

where F' is the fix construction cost of a single building development, α_F is a technical coefficient[1] such that $0 < \alpha_F < 1$, V' is the structural frame cost coefficient, α is the cost coefficient of land and γ is the cost coefficient of height such as $1 < \alpha < \gamma$. Finally, the land cost of the residential development at block i is given by $R_l(i)l(i)$, where $R_l(i)$ is land rent at block i.

The building cost, multiplied by the sum of interest rate and depreciation rate, and the land cost are compared to the revenues, the difference being the periodical profit (Büttler, 1981). Consequently, developers daily profit is given by

$$\pi(i) := R_s(i)H(i) - Fb(i)^{\alpha_F} - Vb(i)^{1-\alpha_b\alpha}l(i)^{\alpha}h(i)^{\gamma} - R_l(i)l(i)\ , \tag{A.10}$$

where $F := F'(r + d)$ is the actualized fix construction cost of a single building development and $V := V'(r + d)$ is the actualized structural frame cost coefficient. Deriving (A.10) gives the profit-maximizing pseudo-demands for l, h and b, respectively written l_D, h_D and b_D, such as

$$l_D(i) = \left[V^{-\alpha_F}\left(\frac{\gamma-\alpha}{R_l(i)}\right)^{\alpha_F(\gamma-1)-\beta}\left(\frac{R_s(i)}{\gamma c}\right)^{\gamma\alpha_F}\left(\frac{\lambda}{F\alpha_F}\right)^{\lambda}\right]^{\frac{1}{\alpha_F(\alpha-1)-\lambda}}\ , \tag{A.11}$$

$$h_D(i) = \left[V^{1-\alpha_b}\left(\frac{\gamma-\alpha}{R_l(i)}\right)^{(1-\alpha_b)(\alpha-1)}\right. \cdot\left.\left(\frac{R_s(i)}{\gamma C}\right)^{-\alpha_F(\alpha-1)+\alpha_b\alpha-1}\left(\frac{\lambda}{F\alpha_F}\right)^{(\alpha-1)(\alpha_b-1)}\right]^{\frac{1}{\alpha_F(\alpha-1)-\beta}}\ , \tag{A.12}$$

$$b_D(i) = \left[V^{-1}\left(\frac{\gamma-\alpha}{R_l(i)}\right)^{\gamma-\alpha}\left(\frac{R_s(i)}{\gamma C}\right)^{\gamma}\left(\frac{\lambda}{F\alpha_F}\right)^{\alpha-1}\right]^{\frac{1}{\alpha_F(\alpha-1)-\lambda}}\ , \tag{A.13}$$

where

$$\lambda := \gamma - 1 - \alpha_b(\gamma-\alpha)\ . \tag{A.14}$$

Substituting (A.11), (A.12) and (A.13) in (A.8) yields the total housing supply at b_i,

$$S_S(i) = \left[(VC^{\gamma})^{-\alpha_F}\left(\frac{\gamma-\alpha}{R_l(i)}\right)^{\alpha_F(\gamma-\alpha)}\right. \cdot\left.\left(\frac{R_s(i)}{\gamma}\right)^{\alpha_F(\gamma-\alpha+1)+\lambda}\left(\frac{\lambda}{F\alpha_F}\right)^{\lambda}\right]^{\frac{1}{\alpha_F(\alpha-1)-\lambda}}\ . \tag{A.15}$$

[1] Equivalently, one could have defined a fixed development construction cost F'' such that the whole fix cost reads $F'' + Fb(i)$. The equivalence holds for $\alpha_F = \ln\left(\frac{F''}{F'} + b\right) / \ln(F')$.

In every block, housing supply must equal housing demand. Thus, equalizing (A.6) and (A.15) gives the equilibrium housing rent as a function of the land rent. That yields

$$\begin{aligned} R_s^*\big(i,\mathbf{n}(t)\big) = &\Big(n(i,t)\beta(Y-iL\tau)\Big)^{\frac{\alpha_F(\alpha-1)-\lambda}{\gamma\alpha_F}} CV^{\frac{1}{\gamma}} \\ &\cdot\left[\left(\frac{R_l(i)}{\gamma-\alpha}\right)^{\alpha_F(\gamma-\alpha)} \gamma^{\alpha_F(\gamma-\alpha+1)+\lambda}\left(\frac{\lambda}{F\alpha_F}\right)^{-\lambda}\right]^{\frac{1}{\gamma\alpha_F}} . \end{aligned} \tag{A.16}$$

Land market

Substituting (A.16) into (A.11) gives local demand for land,

$$l_D^*\big(i\,;\mathbf{n}(t)\big) = \frac{n(i,t)\beta(Y-iL\tau)(\gamma-\alpha)}{\gamma R_l(i)} , \tag{A.17}$$

which is provided by absentee landowners. It is assumed that in addition to developers, a rural sector is competing for land. More precisely, it is assumed that the agricultural demand for land is totally inelastic at rent a which, unlike Chapter 3, is a linear function of the local population such that

$$a\big(i\,;\mathbf{n}(t)\big) := \alpha_a n(i,t) . \tag{A.18}$$

Since landowners behave as monopoly on their own block, they will provide land to the maximal bidder. However, it is assumed that at equal rent, they are forced to provide developers with land to house local households. From (A.17) and (A.18), this yields

$$R_l^*\big(i,\mathbf{n}(t)\big) = \max\left\{\alpha_a n(i,t)\,, \frac{n(i,t)\beta(Y-i\tau)(\gamma-\alpha)}{\gamma L^2}\right\} , \tag{A.19}$$

so that there are potentially two regions, the urban area where $R_l^* = l_D^*$ and the suburban area where $R_l^* = a$. Yet, note that(A.19) rewrites

$$R_l^*\big(i\,;\mathbf{n}(t)\big) = n(i,t)\rho(i) , \tag{A.20}$$

where $\rho(i)$ is the per capita land rent at i, such that

$$\rho(i) := \max\left\{\alpha_a\,, \frac{\beta(Y-iL\tau)(\gamma-\alpha)}{\gamma L^2}\right\} . \tag{A.21}$$

Consequently, it is possible to find the boundary between the two regions by equalizing the two members of (A.21). This yields,

$$f := \frac{1}{\tau}\left(Y - \frac{\alpha_a\gamma L^2}{\beta(\gamma-\alpha)}\right) . \tag{A.22}$$

Blocks equilibrium

First, in the urban area, one gets

$$(R_l^*)_{\mathrm{u}}\big(i,\mathbf{n}(t)\big) = \frac{n(i,t)\beta(Y-iL\tau)(\gamma-\alpha)}{\gamma L^2}\ , \tag{A.23}$$

$$l_{\mathrm{u}}^*(i) = L^2\ , \tag{A.24}$$

so that

$$(r_s^*)_{\mathrm{u}}\big(i\,;\mathbf{n}(t)\big) = CV^{\frac{1}{\gamma}}\left[\Big(n(i,t)\beta(Y-iL\tau)\Big)^{\alpha_F(\gamma-1)-\lambda}L^{-2\alpha_F(\gamma-\alpha)}\gamma^{\lambda+\alpha_F}\left(\frac{\lambda}{F\alpha_F}\right)^{-\lambda}\right]^{\frac{1}{\gamma\alpha_F}}\ , \tag{A.25}$$

from which on gets the equilibrium housing rent,

$$s_{\mathrm{u}}^*\big(i,\mathbf{n}(t)\big) = C^{-1}V^{\frac{-1}{\gamma}}\left[n(i,t)^{\lambda-\alpha_F(\gamma-1)}\left(\frac{\beta(Y-iL\tau)}{\gamma}\right)^{\lambda+\alpha_F}L^{2\alpha_F(\gamma-\alpha)}\left(\frac{\lambda}{F\alpha_F}\right)^{\lambda}\right]^{\frac{1}{\gamma\alpha_F}}\ , \tag{A.26}$$

the equilibrium buildings height,

$$h_{\mathrm{u}}^*\big(i,\mathbf{n}(t)\big) = V^{\frac{-1}{\gamma}}L^{\frac{-2\alpha}{\gamma}}\gamma^{\frac{1-\alpha_F-\alpha_b\alpha}{\alpha_F\gamma}}\left(\frac{\lambda}{F\alpha_F}\right)^{\frac{\alpha_b\alpha-1}{\alpha_F\gamma}}\Big(n(i,t)\beta(Y-iL\tau)\Big)^{\frac{\alpha_F+\alpha_b\alpha-1}{\alpha_F\gamma}}\ , \tag{A.27}$$

the equilibrium building footprint,

$$(l_b^*)_{\mathrm{u}}\big(i,\mathbf{n}(t)\big) = L^2\left(\frac{\gamma F\alpha_F}{n(i,t)\beta(Y-iL\tau)\lambda}\right)^{\frac{\alpha_b}{\alpha_F}}\ , \tag{A.28}$$

and the equilibrium utility,

$$U_{\mathrm{u}}^*\big(i,\mathbf{n}(t)\big) = \ln\left[(1-\beta)^{1-\beta}C^{-\beta}V^{\frac{-\beta}{\gamma}}\left[\Big(n(i,t)\Big)^{\lambda-\alpha_F(\gamma-1)}\Big(Y-iL\tau\Big)^{\frac{\alpha_F(\gamma-\beta)-\beta\lambda}{\beta}}\cdot\left(\frac{\beta}{\gamma}\right)^{\lambda+\alpha_F}L^{2\alpha_F(\gamma-\alpha)}\left(\frac{\lambda}{F\alpha_F}\right)^{\lambda}\right]^{\frac{\beta}{\gamma\alpha_F}}\right]\ . \tag{A.29}$$

On the other hand, in the suburban area one gets

$$(R_l^*)_{\rm a}\big(i,\mathbf{n}(t)\big) = \alpha_a n(i,t) \ , \tag{A.30}$$

$$l_{\rm a}^*(i) = \frac{\beta(Y-iL\tau)(\gamma-\alpha)}{\gamma\alpha_a} \ , \tag{A.31}$$

so that

$$\begin{aligned}(R_s^*)_{\rm a}\big(i,\mathbf{n}(t)\big) = n(i,t)^{\frac{\alpha_F(\gamma-1)-\lambda}{\gamma\alpha_F}} \Big(\beta(Y-iL\tau)\Big)^{\frac{\alpha_F(\alpha-1)-\beta}{\gamma\alpha_F}} CV^{\frac{1}{\gamma}} \\ \cdot \left[\left(\frac{\alpha_a}{\gamma-\alpha}\right)^{\alpha_F(\gamma-\alpha)} \gamma^{\alpha_F(\gamma-\alpha+1)+\lambda} \left(\frac{\lambda}{F\alpha_F}\right)^{-\lambda}\right]^{\frac{1}{\gamma\alpha_F}} \ ,\end{aligned} \tag{A.32}$$

from which on gets the equilibrium housing rent,

$$\begin{aligned}&s_{\rm a}^*\big(i,\mathbf{n}(t)\big) = \\ &C^{-1}V^{\frac{-1}{\gamma}}\Bigg[n(i,t)^{\lambda-\alpha_F(\gamma-1)} \\ &\qquad \cdot \left(\frac{\beta(Y-iL\tau)}{\gamma}\right)^{\lambda+\alpha_F(\gamma-\alpha+1)} \frac{\gamma-\alpha}{\alpha_a}^{\alpha_F(\gamma-\alpha)} \left(\frac{\lambda}{F\alpha_F}\right)^{\lambda}\Bigg]^{\frac{1}{\gamma\alpha_F}} \ ,\end{aligned} \tag{A.33}$$

the equilibrium buildings height,

$$\begin{aligned}h_{\rm a}^*\big(i,\mathbf{n}(t)\big) = V^{\frac{-1}{\gamma}} \frac{\gamma-\alpha}{\alpha_a}^{\frac{-\alpha}{\gamma}} \gamma^{\frac{-1}{\alpha_F\gamma}} \left(\frac{\lambda}{F\alpha_F}\right)^{\frac{\alpha_b\alpha-1}{\alpha_F\gamma}} \\ \Big(n(i,t)\Big)^{\frac{\alpha}{\gamma}+\frac{\alpha_b-1}{\alpha_F[\alpha_F(\alpha-1)-\lambda]}} \Big(\beta(Y-iL\tau)\Big)^{\frac{\alpha_b-1}{\alpha_F[\alpha_F(\alpha-1)-\lambda]}} \ ,\end{aligned} \tag{A.34}$$

the equilibrium building footprint,

$$(l_b^*)_{\rm a}\big(i,\mathbf{n}(t)\big) = n(i,t)^{-\frac{\alpha_b}{\alpha_F}} \left(\frac{\beta(Y-iL\tau)}{\gamma}\right)^{1-\frac{\alpha_b}{\alpha_F}} \frac{\gamma-\alpha}{\alpha_a} \left(\frac{\lambda}{F\alpha_F}\right)^{-\frac{\alpha_b}{\alpha_F}} \ , \tag{A.35}$$

Table A.1. Parametrization of the model.

Notation	Value	Unit
α_a	$2.5\,10^{-5}$	.
α_{c_b}	0.9	.
α_ε	1	.
α_h	1.9	.
α_l	1.1	.
α_{l_b}	1.05	.
α_s	0.35	.
c_b	100	¢d^{-1}
c_o	$1\,10^{-3}$	$\frac{¢}{\mathrm{m}^{2\alpha_l+\alpha_h}\,\mathrm{d}}$
h_s	2.5	m
L_b	800	m
N	$1\,10^5$	.
τ	$5\,10^{-4}$	¢m^{-1}
v	0.33/365	.
w	120	¢d^{-1}

and the equilibrium utility,

$$U^*_{\mathrm{a}}\big(i,\mathbf{n}(t)\big) = \ln\left[(1-\beta)^{1-\beta}C^{-\beta}V^{\frac{-\beta}{\gamma}}\left[\Big(n(i,t)\Big)^{\lambda-\alpha_F(\gamma-1)}\Big(Y-iL\tau\Big)^{\frac{\alpha_F}{\beta}\big(\gamma-\beta(\alpha-1)\big)+\lambda}\right.\right.$$
$$\left.\left.\cdot\left(\frac{\beta}{\gamma}\right)^{\lambda+\alpha_F(\gamma-\alpha+1)}\frac{\gamma-\alpha}{\alpha_a}^{\alpha_F(\gamma-\alpha)}\left(\frac{\lambda}{F\alpha_F}\right)^{\lambda}\right]^{\frac{\beta}{\gamma\alpha_F}}\right]. \tag{A.36}$$

Considering parametrization 1 (Tab. A.1), it appears that buildings height is increasing with population density and decreasing with distance to CBD, just as expected (Fig. A.2). Relationship with population is convex in urban area and concave in suburban area. In both areas, buildings footprint is decreasing in population density, following a convex curve (Fig. A.3), whilst the effect of distance is relatively small. Finally, The number of buildings is a similar increasing convex function of the population in both urban and suburban areas (Fig. A.4). The closer an urban block is to the CBD, the bigger the growth is.

A.2 Preliminary results and discussion

Still considering parametrization 1, the vector field of the deterministic mean dynamics indicates a single equilibrium (Fig. A.5). Thus the dynamics is much

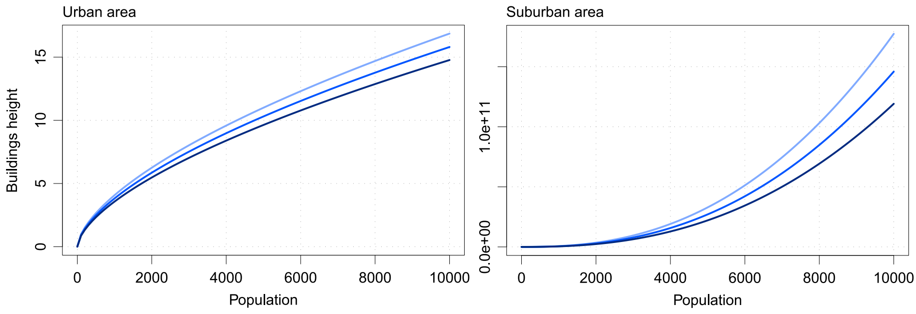

Figure A.2. Equilibrium buildings height in urban and suburban areas. Variations of buildings height with respect to urban block's population. Darker curves are further from the CBD.

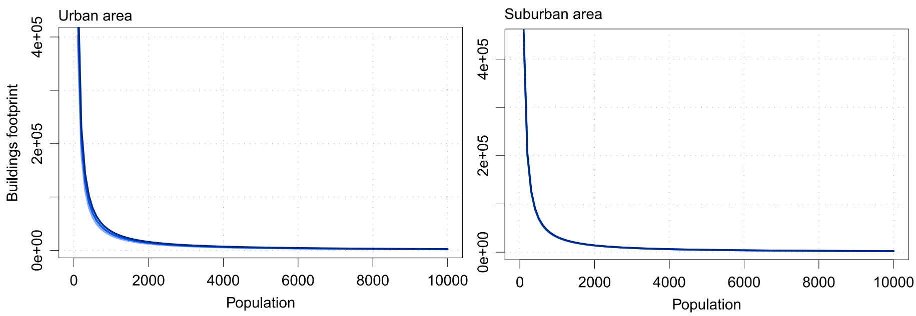

Figure A.3. Equilibrium buildings footprint in urban and suburban areas. Variations of buildings footprint with respect to urban block's population. Darker curves are further from the CBD.

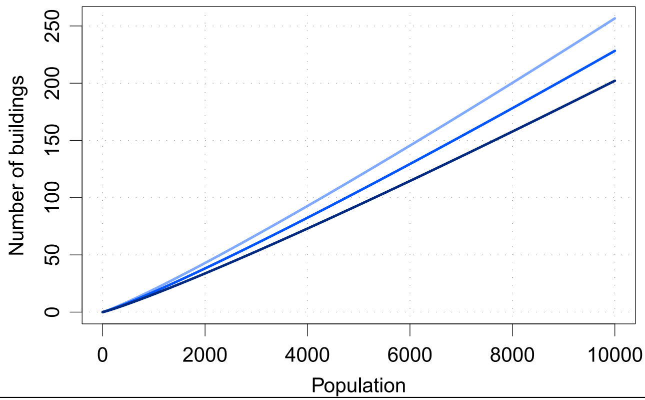

Figure A.4. Equilibrium number of buildings in urban and suburban areas. Variations of the number of buildings with respect to urban block's population. Darker curves are further from the CBD.

less rich than in the core-periphery model Section 6.3 or in non-monocentric city models Chapter 4. This is a direct consequence of the absence of endogenous agglomeration forces that considerably reduces the interactions at work in this model. Just as in the classic Alonso model, the population distribution at steady-state is a decreasing function of distance to CBD. However it is almost linear, with a break at urban-suburban boundary (Fig. **??**). The resulting buildings geometric attributes, as well as the equilibrium economic variables, are drawn for information.

To sum up, this work as proposed a monocentric city model in Muth-Mills style. Starting from a definition of urban space that anchors in a Euclidean space, and adding some microeconomic assumptions on the way a residential development is split up into several buildings, this works succeeds in discussing the variations of the numbers of buildings, their footprint and height with respect to distance from CBD and local population density. The dynamic framework of the previous chapter highlighted the uniqueness of the equilibrium population density profile, from which equilibrium geometric attributes were derived. Preliminary results show that different profiles behave as expected, that is with an increasing number of building with larger footprint and smaller height. Yet it seems necessary to introduce recreational open spaces in the setting in order to model villas, which have significantly contributed to urban morphologies in cities shaped by urban sprawl. This perspective is left for future research.

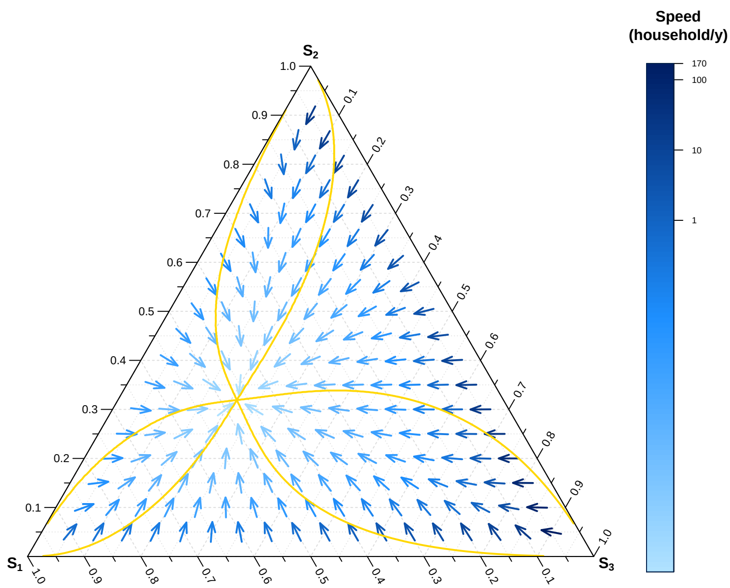

Figure A.5. Unique stable equilibrium of the Alonso model adapted to the stochastic framework of this thesis The black triangle is the simplex of the population distribution among the three regions, with summits $\mathbf{S}_1 = (1,0,0)$, $\mathbf{S}_2 = (0,1,0)$ and $\mathbf{S}_3 = (0,0,1)$. Blue arrows depict the expected values of the instantaneous change vector. The red dot depicts the unique stable equilibrium. Orange lines are the nullclines, which intersect at equilibrium.

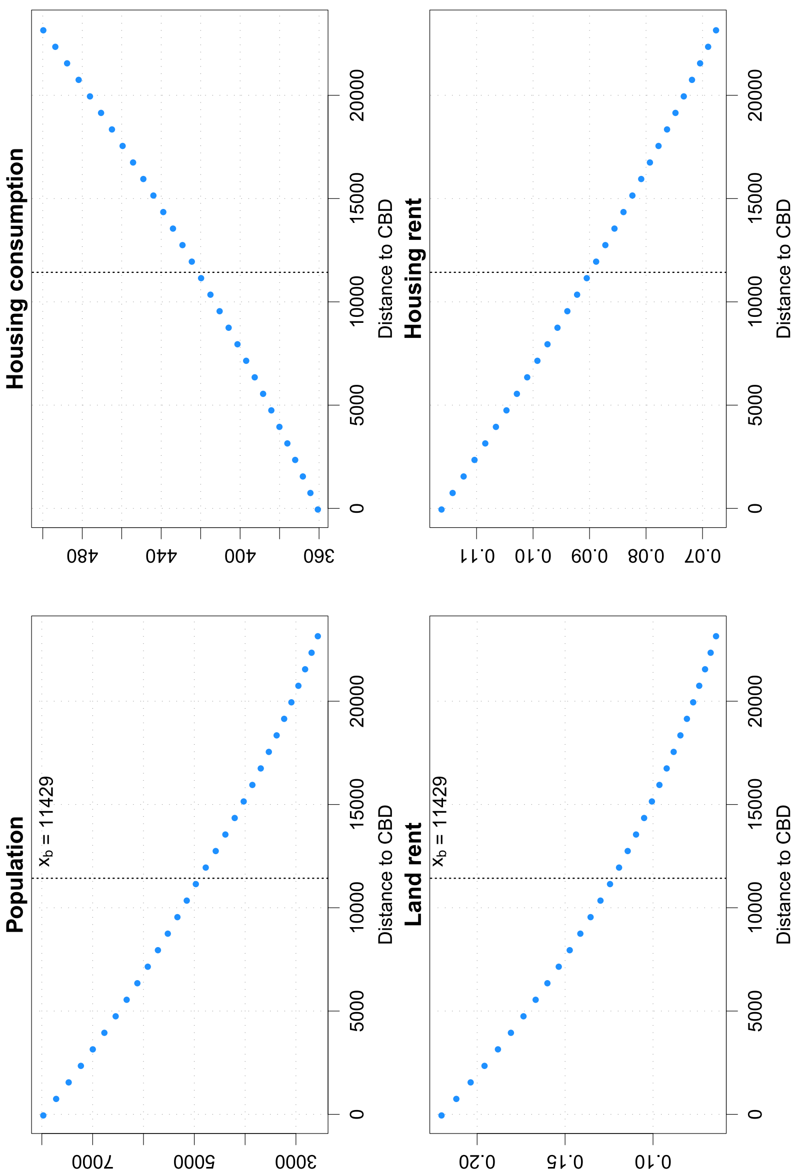

Figure A.6. Equilibrium values population distribution, housing consumption and rents. Those profiles depict the same trends as in classic monocentric theory, although they are less curvy. Note that this may be a bias of the chosen parametrization si that further sensitivity analyses are required.

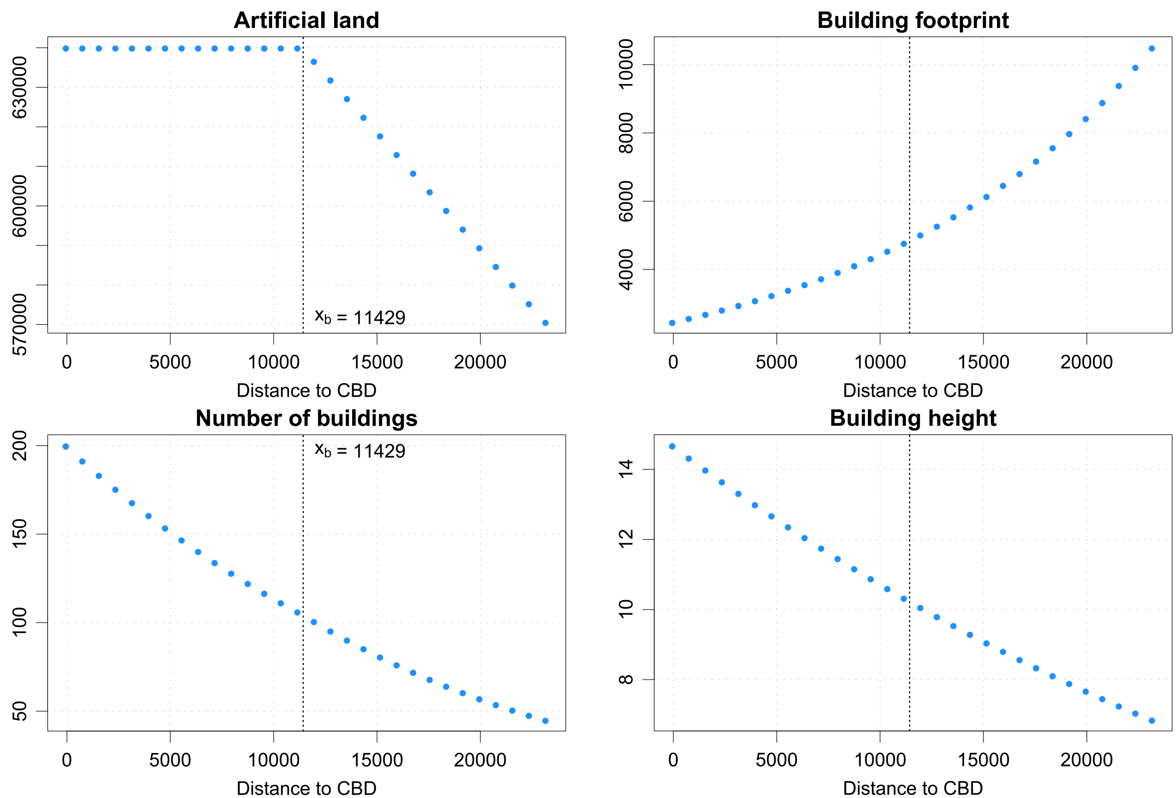

Figure A.7. Equilibrium values buildings geometrical attributes. The profile of buildings geometric attributes correspond to the expected decrease of urban density with increasing distance to the CBD.

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