Adaptive Extremum Seeking Control of a Tubular Reactor with Distributed Feed

Pascal Cougnon, Denis Dochain
IMAP, Univ. Cath. Louvain
2 Place Sainte Barbe
1348 Louvain-la-Neuve, Belgium
email : cougnon@imap.ucl.ac.be

Martin Guay
Dept. Chem. Eng.
Queen's University
Kingston K7L 3N6
Canada

Michel Perrier
Dept. Génie Chimique
Ecole Polytechnique
C.P. 6079, Succ. "Centre Ville"
Montréal H3C 3A7, Canada

***Abstract*— This paper presents an adaptive extremum seeking control scheme for a class of nonlinear distributed parameter systems. It addresses the real time optimization of parallel chemical reactions that occurs in a tubular reactor with uniform distributed feed described by a set of hyperbolic partial differential equations for which we assume limited knowledge of the kinetics. An adaptive learning technique is introduced to design a seeking algorithm that drives the system states to the unknown setpoint that maximizes the value of an objective function. Lyapunov's stability theorem is used in the design of the extremum seeking controller structure and the development of the parameter learning laws.**

I. Introduction

In the field of chemical reaction engineering, large activity has been dedicated to the improvement of the parallel reactions selectivity via the development of new catalysts. Such an objective can however be achieved by following an alternative approach. Indeed the concentration of reactants plays a central role in the product distribution of parallel reaction [1]. Moreover, reaction orders in the kinetic relations are of prime importance [2] [3]. A high reactant concentration favors the highest order reaction while a low concentration favors the lowest order reaction. In classical tubular reactor, the reactants are fed at the reactor input and the product is collected at the output. It is obviously difficult, if not impossible, to obtain some desired concentration profile along the reactor by only acting on the inlet concentration. In a tubular reactor with distributed feed, the reactants can be introduced either at the reactor input or at different points along the reactor.

Different research works have investigated the question of the feed distribution in order to increase the selectivity of parallel reactions [4] [5] [6] [7] [8]. The results of these works are based on the perfect knowledge of the structure and the parameters of the reaction kinetics. More specifically, optimal control requires the perfect knowledge of the process model to perform a reliable off-line optimization [9]. However in practice the reaction kinetics may be badly known. In that case, optimal control based on approximate kinetic models results in bad performance of the process.

Since the optimal setpoint depends on the process parameters, we can envisage to consider control approaches that optimize process performance in presence of model uncertainties e.g. adaptive extremum seeking controllers [10] [11] [12].

In this study we extend the previous results of adaptive extremum seeking to a class of problems described by a set of hyperbolic partial differential equations. We investigate an extremum seeking algorithm for tubular reactors with distributed feed. The problem consists of finding the optimal exit concentration of the product of interest resulting from a parallel reactions scheme. The tubular reactors with distributed feed include different feed policies such as discrete feed equally spaced and/or equally distributed or continuous feed [2] [3]. In this paper, we focus on the uniform (continuous) distributed feed which can be realized in practice through e.g. a membrane reactor. The uniform distributed feed can be described with a single variable independent of the spatial coordinate. Since the distribution profile of the feed is uniform along the tubular reactor, the distributed feed is completely defined by its magnitude. This configuration allows to study the behavior of the adaptive extremum seeking controller for systems described by partial differential equations.

The reactor configuration considered in this study is a one-dimensional tubular reactor with two reactants, one being fed along the reactor length and the other being fed at the reactor input. Furthermore the uniform distributed feed is kept within a realistic range by introducing input constraints into the definition of the problem. Indeed a modified cost function is used to account for the input constraints.

Moreover we assume a limited knowledge of the reaction kinetics. Therefore a Lyapunov-based adaptive learning control technique is used to steer the system to its unknown extremum. We also show that a persistence of excitation (PE) condition is necessary to guarantee the convergence of the extremum seeking mechanism.

The paper is organized as follows. Section II describes the problem and provides some preliminary analysis. In section III the parameter estimation algorithm and the controller are designed based on Lyapunov functions. Section IV provides simulation results that illustrate the performance of the controller.

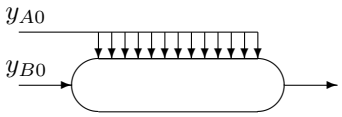

Fig. 1. Tubular reactor with uniform distributed feed

II. Problem Formulation

Consider the following liquid phase parallel chemical reactions in a tubular reactor :

$$\begin{aligned} A + B &\overset{k_1}{\rightarrow} C \\ A + B &\overset{k_2}{\rightarrow} D \end{aligned} \tag{1}$$

where the reaction kinetics are given by the following equations :

$$r_C = k_1' C_A^{\alpha_1} C_B \tag{2}$$

$$r_D = k_2' C_A^{\alpha_2} C_B \tag{3}$$

with $\alpha_1 < \alpha_2$. This general case can be illustrated by the elimination-substitution competition in organic reactions [13] [14].

Here the objective is to maximize the concentration of C at the reactor output. Due to the reaction orders with respect to A, it is obvious that the selectivity of C is improved by maintaining a low concentration of A [2] [3]. This can be achieved by using a tubular reactor with uniform distributed feed of A (Fig. 1). Such reactor can be described by a set of dimensionless partial differential equations (PDE).

$$\begin{aligned} \frac{\partial y_A}{\partial \theta} &= -y_\nu \frac{\partial y_A}{\partial \xi} - k_1\, y_A^{\alpha_1}\, y_B - k_2\, y_A^{\alpha_2}\, y_B \\ &\quad + q\,(y_{A0} - y_A) \\ \frac{\partial y_B}{\partial \theta} &= -y_\nu \frac{\partial y_B}{\partial \xi} - k_1\, y_A^{\alpha_1}\, y_B - k_2\, y_A^{\alpha_2}\, y_B \\ &\quad - q\, y_B \\ \frac{\partial y_C}{\partial \theta} &= -y_\nu \frac{\partial y_C}{\partial \xi} + k_1\, y_A^{\alpha_1}\, y_B - q\, y_C \\ \frac{\partial y_\nu}{\partial \xi} &= q \end{aligned} \tag{4}$$

where $\theta \in \mathbf{R}^+$ and $\xi \in [0,1]$ are the time and space variables ; y_A, y_B and $y_C \in S_{Y1} \subset \mathbf{R}^+$ are the concentrations of A, B and C, respectively ; $y_\nu \in S_{Y2} \subset \mathbf{R}^+$ is the flow rate ; and $q \in S_q \subset \mathbf{R}^+$ is the distributed feed (control action). The boundary conditions are given by :

$$y_A(\theta,0) = 0 \ ; \ y_B(\theta,0) = y_{B0} \ ; \ y_C(\theta,0) = 0. \tag{5}$$

Here we assume that the measurements of the concentration of C at the reactor output are available on-line and that the reaction rates are unknown. The objective is to find the value of the uniform distributed feed q that maximizes the reactor exit concentration of C at steady-state.

Note that at steady-state, exit concentration C (y_C^{ss}) denoted $\pi(q)$ is a function only of the distributed feed all other parameters being constant.

Assumption 2.1: The function $\pi(q)$ is continuously differentiable and admits a maximum on S_{Y1}.

Since the reaction rates are unknown, an analytical expression for $\pi(q)$ cannot be derived. Let us therefore choose to parameterize this unknown function by using a neural network approximation technique [10] [12], i.e. :

$$\pi(q) = W^T S(q) + \mu_l(\theta) \tag{6}$$

with $\mu_l(\theta)$, the NN approximation error and the basis function vector

$$S(q) = [s_1(q), s_2(q), \ldots, s_l(q)]^T \tag{7}$$

$$s_i(q) = exp\left(\frac{-(q-\varphi_i)^2}{\sigma_i^2}\right) - \frac{(q-\varphi_i)^2}{\sigma^2} \tag{8}$$

for $i = 1, 2, \ldots, l$. The basis weights parameters W are assumed to take values in a compact set of $\mathbb{R}^l$. The functional approximation of the equilibrium manifold is dependent on the center of the receptive field φ_i and on the width of the Gaussian function σ_i. The convexity of the functions $s_i(u)$ is ensured by choosing an appropriate value for σ. Universal approximation results stated in [15] and [16] indicate that the approximation error is bounded and if l is chosen sufficiently large, then $W^T S(q)$ can approximate any continuous function to any desired accuracy on a compact set.

From a physical point of view all variables in system (4) are continuous functions of space and time. The continuity of the approximation error is then guaranteed by the continuity of basis functions. The continuity and the bound of the approximation error justify the following assumption :

Assumption 2.2: The NN approximation error satisfies $\left|\frac{\partial \mu_l(\theta)}{\partial \theta}\right| < \bar{\mu}_l$ with constant $\bar{\mu}_l > 0$ over a compact set.

III. Controller Design

In this section we design a control law to steer the system to its unknown maximum. Moreover constraints are imposed in the optimization problem to maintain the control action in a realistic range :

$$\begin{aligned} &\max_q \ y_C^{ss} \\ &\text{s.t. } q_{min} < q < q_{max}. \end{aligned} \tag{9}$$

Thus, the objective of the control strategy is to track the optimum exit steady-state concentration of C at the reactor output. To solve this problem, let us define a cost function with log-barrier functions [17] to account for the input constraints :

$$\begin{aligned} J(q) &= y_C^{ss} + \mu_1 \log(q - q_{min} - \epsilon_1) \\ &\quad + \mu_2 \log(q_{max} - q - \epsilon_2) \end{aligned} \tag{10}$$

where $\mu_1, \mu_2, \epsilon_1, \epsilon_2$ are strictly positive constants. An appropriate choice of these constants allows to preserve the shape of the function y_C^{ss} on the available range of the control action while the cost function will sharply decrease close to the boundaries.

The cost function is maximized when

$$\frac{\partial J}{\partial q}(q^*) = 0. \tag{11}$$

By using the parametrization (6), the cost function becomes :

$$\begin{aligned} J(q) &= W^T S(q) + \mu_1 \log(q - q_{min} - \epsilon_1) \\ &\quad + \mu_2 \log(q_{max} - q - \epsilon_2) \end{aligned} \tag{12}$$

Since the true parameters W are assumed to be unknown, the following cost function is considered :

$$\begin{aligned} \hat{J}(q) &= \hat{W}^T S(q) + \mu_1 \log(q - q_{min} - \epsilon_1) \\ &\quad + \mu_2 \log(q_{max} - q - \epsilon_2) \end{aligned} \tag{13}$$

where $\hat{W}$ is the estimate W. Consequently, the optimum $\hat{q}^*$ is the solution of :

$$\frac{\partial \hat{J}}{\partial q}(\hat{q}^*) = 0. \tag{14}$$

The gradient of $\hat{J}(q)$ with respect to q is equal to :

$$\begin{aligned} \Gamma_1 &= \left(\hat{W}^T \frac{\partial S(q)}{\partial q}\right) + \frac{\mu_1}{q - q_{min} - \epsilon_1} \\ &\quad - \frac{\mu_2}{q_{max} - q - \epsilon_2} \end{aligned} \tag{15}$$

while the Hessian of $\hat{J}(q)$ with respect to q is written as follows :

$$\begin{aligned} \Gamma_2 &= \left(\hat{W}^T \frac{\partial^2 S(q)}{\partial q^2}\right) - \frac{\mu_1}{(q - q_{min} - \epsilon_1)^2} \\ &\quad - \frac{\mu_2}{(q_{max} - q - \epsilon_2)^2}. \end{aligned} \tag{16}$$

Let us define :

$$z_s = \Gamma_1 - d(\theta) \tag{17}$$

where $d(\theta)$ is a dither signal.

First of all let us define a control law that guarantees the convergence of z_s to zero .

Let us consider the Lyapunov function candidate :

$$V_1 = \frac{1}{2} z_s^2. \tag{18}$$

By taking the time derivative of V_1, we obtain :

$$\dot{V}_1 = z_s \left(\Gamma_2 \dot{q} + \dot{\hat{W}}^T \frac{\partial S}{\partial q} - \dot{d}(\theta)\right). \tag{19}$$

In order to achieve the negative definiteness of $\dot{V}_1$, let us consider the following dynamical control law :

$$\dot{q} = \frac{1}{\Gamma_2} \left(-k_z z_s - \dot{\hat{W}}^T \frac{\partial S}{\partial q} + \dot{d}(\theta)\right) \tag{20}$$

with $k_z > 0$. Indeed the time derivative of V_1 is then equal to :

$$\dot{V}_1 = -k_z z_s^2. \tag{21}$$

The control law described by (20) guarantees the convergence to the maximum of the cost function (13). Since the kinetic parameters are assumed to be unknown, an update law for these parameters is needed in order to reconstruct the unknown function $\pi(q)$. We know that the concentration y_C is the sum between the steady state (y_C^{ss}) and the transient concentration (Δy_C).

$$y_C(\theta, 1) = y_C^{ss}(q(\theta), 1) + \Delta y_C(\theta). \tag{22}$$

By using the neural network approximation, the variation of C at the reactor output can be expressed by :

$$\dot{y}_C(\theta, 1) = W^T \frac{\partial S}{\partial q} \dot{q} + \frac{\partial \mu_l}{\partial \theta} + \frac{\partial \Delta y_C(\theta)}{\partial \theta}. \tag{23}$$

Notice that the exit concentration of C is the described by the finite dimensional system (23) compared to the infinite dimensional system (4).

Even if y_C is measured at the reactor output, its time evolution is unknown because W is unknown. The estimate $\hat{y}_C$ is generated by the following estimation law :

$$\dot{\hat{y}}_C(\theta, 1) = \hat{W}^T \frac{\partial S}{\partial q} \dot{q} + c(\theta)^T \dot{\hat{W}} + k_y (y_C - \hat{y}_C) \tag{24}$$

where $c(\theta)$ are time-varying vector valued functions and k_y is a strictly positive constant.

The dynamics of the estimation error ($\tilde{y}_C = y_C - \hat{y}_C$) is given by :

$$\dot{\tilde{y}}_C = \tilde{W}^T \frac{\partial S}{\partial q} \dot{q} + \frac{\partial \mu_l}{\partial \theta} + \frac{\partial \Delta y_C(\theta)}{\partial \theta} - c(\theta)^T \dot{\hat{W}} - k_y \tilde{y}_C \tag{25}$$

where $\tilde{W} = W - \hat{W}$ is the vector of parameter estimation errors. Note that in the rest of the paper, $\tilde{y}_C(\theta, 1)$ will be denoted by $\tilde{y}_C$.

The next step is to prove that the estimation error $\tilde{y}_C$ and the parameter estimation errors $\tilde{W}$ converge to zero. In that case, the optimum of the approximate function reached in the optimization step will be in the neighborhood of the real optimum of the system.

Let us define :

$$\eta = \tilde{y}_C - c(\theta)^T \tilde{W} \tag{26}$$

and the following Lyapunov function :

$$V_2 = \frac{1}{2} \eta^2. \tag{27}$$

Its time derivative is given by :

$$\begin{aligned} \dot{V}_2 &= \eta \left(\tilde{W}^T \frac{\partial S}{\partial q} \dot{q} + \frac{\partial \mu_l}{\partial \theta} + \frac{\partial \Delta y_C(\theta)}{\partial \theta} \right. \\ &\quad \left. - k_y \tilde{y}_C - \dot{c}(\theta)^T \tilde{W}\right). \end{aligned} \tag{28}$$

Substitution of eq. (26) into eq. (28) yields :

$$\begin{aligned} \dot{V}_2 &= \eta \left(\tilde{W}^T \frac{\partial S}{\partial q} \dot{q} + \frac{\partial \mu_l}{\partial \theta} + \frac{\partial \Delta y_C(\theta)}{\partial \theta} \right. \\ &\quad \left. - k_y \eta - k_y c(\theta)^T \tilde{W} - \dot{c}(\theta)^T \tilde{W}\right). \end{aligned} \tag{29}$$

Let us consider that $\dot{c}(\theta)$ is given by :

$$\dot{c}(\theta) = -k_y c(\theta) + \dot{q}\frac{\partial S}{\partial q} \tag{30}$$

with $k_y > 0$. Let us substitute eq. (30) into eq. (29). This gives :

$$\dot{V}_2 = -k_y\eta^2 + \frac{\partial \mu_l}{\partial \theta}\eta + \frac{\partial \Delta y_C(\theta)}{\partial \theta}\eta. \tag{31}$$

Assumption 3.1: The time derivative of the transient satisfies $\left|\frac{\partial \Delta y_C(\theta)}{\partial \theta}\right| < d\bar{\Delta}y_C$ with $d\bar{\Delta}y_C > 0$.
From a physical point of view, all variables in Eq. (4) are bounded functions of time that verifies Assumption 3.1.

By using the upper bounds $\left|\frac{\partial \mu_l}{\partial \theta}\right| < d\bar{\mu}_l$ and $\left|\frac{\partial \Delta y_C(\theta)}{\partial \theta}\right| < d\bar{\Delta}y_C$, we get :

$$\dot{V}_2 \leq -k_y|\eta|\left(|\eta| - \frac{d\bar{\mu}_l}{k_y} - \frac{d\bar{\Delta}y_C}{k_y}\right). \tag{32}$$

Consequently, it follows from standard Lyapunov theory arguments that η is bounded and enter the region $\left\{\eta \left| |\eta| < \frac{d\bar{\mu}_l}{k_y} + \frac{d\bar{\Delta}y_C}{k_y}\right.\right\}$ as $\theta \rightarrow \infty$.

Eq. (32) shows that the state η converges to a small neighborhood of the origin. It remains to show that the original state variable $\tilde{y}_C$ and the parameter estimation errors $\tilde{W}$ converge to a small neighborhood of the origin. This can be achieved by using an appropriate choice of the parameter update law. Since $\tilde{y}_C$ is known, we can choose the parameter update law as follows :

$$\dot{\hat{W}} = \gamma_w c(\theta)\tilde{y}_C. \tag{33}$$

To guarantee the negative definiteness of the Hessian Γ_2, the following projection algorithm is used :

$$\dot{\hat{W}} = \begin{cases} \gamma_w c(\theta)\tilde{y}_C & \text{if } \hat{W}^T dS_2 < -w_m \text{ or} \\ & \text{if } \hat{W}^T dS_2 = -w_m \\ & \text{and } dS_2^T c(\theta)\tilde{y}_C \leq 0 \\ \gamma_w\left(I - \frac{dS_2 dS_2^T}{dS_2^T dS_2}\right)c(\theta)\tilde{y}_C & \text{otherwise} \end{cases} \tag{34}$$

with $dS_2 = \frac{\partial^2 S(q)}{\partial q^2}$ and w_m a small positive constant. The projection algorithm (34) ensures that our parameters remain in a compact set Ω_w defined by :

$$\Omega_w = \left\{\hat{W} | \hat{W}^T dS_2 \leq -w_m\right\}, \tag{35}$$

In the following we show that under suitable assumptions, the parameter update law (34) guarantees the convergence of the parameter estimation errors to zero.

Lemma 3.1: Consider the differential equation :

$$\dot{z}(\theta) = -\phi(\theta)\phi(\theta)^T z(\theta) \tag{36}$$

and assume that there exists a $T > 0$ and a $k > 0$ such that :

$$\int_\theta^{\theta+T} \phi(\tau)\phi(\tau)^T \geq kI \tag{37}$$

then the origin is a globally exponentially stable equilibrium of the system.

The proof of this lemma can be found in [18].

Assumption 3.2: The dither signal $d(\theta)$ is such that the time-varying vector $c(t)$ satisfies :

$$\int_\theta^{\theta+T} c(\tau)c(\tau)^T \geq k_N I_N$$

for positive constants T and k_N.

Under assumption 3.2, Lemma 3.1 shows that the origin of the differential equation :

$$\dot{\tilde{W}} = -\gamma_w c(\theta)c(\theta)^T\tilde{W} \tag{38}$$

is an exponentially stable equilibrium. Moreover, the proof of Lemma 3.1 shows that :

$$-\tilde{W}^T c(\theta)c(\theta)^T\tilde{W} \leq -k_w\|\tilde{W}\|^2 \tag{39}$$

with k_w a positive constant.

Let us define the Lyapunov function :

$$V_w = \frac{1}{2\gamma_w}\tilde{W}^T\tilde{W} \tag{40}$$

The rate of change of V_w along the trajectories of the system is given by :

$$\begin{aligned} \dot{V}_w &= -\tilde{W}^T c(\theta)c(\theta)^T\tilde{W} - \tilde{W}^T c(\theta)\eta \\ &+ \begin{cases} 0 & \text{if } \hat{W}^T dS_2 < -w_m \text{ or} \\ & \text{if } \hat{W}^T dS_2 = -w_m \\ & \text{and } dS_2^T c(\theta)\tilde{y}_C \leq 0 \\ \tilde{W}^T\frac{dS_2 dS_2^T}{dS_2^T dS_2}\left(c(\theta)c(\theta)^T\tilde{W} + c(\theta)\eta\right) & \text{otherwise} \end{cases} \end{aligned} \tag{41}$$

The property of the projection algorithm [19] shows that :

$$\dot{V}_w \leq -\tilde{W}^T c(\theta)c(\theta)^T\tilde{W} - \tilde{W}^T c(\theta)\eta \tag{42}$$

By completing the squares we get :

$$\dot{V}_w \leq -\frac{1}{2}\tilde{W}^T c(\theta)c(\theta)^T\tilde{W} + \frac{1}{2}\eta^2 \tag{43}$$

Introducing eq.(39) into eq.(43) leads to :

$$\begin{aligned} \dot{V}_w &\leq -\frac{k_w}{2}\|\tilde{W}\|^2 + \frac{1}{2}\eta^2 \\ &\leq -k_w\gamma_w V_w + \frac{1}{2}\eta^2 \end{aligned} \tag{44}$$

Since η is bounded and converges to a small neighborhood of zero, $\tilde{W}$ also converges to a small neighborhood of zero.

The convergence of the estimated gradient z_s and the estimation error $\tilde{y}_C$ provides the convergence of the closed-loop system to an adjustable neighborhood of the unknown steady-state optimum.

Theorem 3.1: Consider the parallel reactions (1) taking place in a tubular reactor with uniform distributed feed. The control law (20) with the parameter update law (34) and the persistent excitation condition such that :

$$\int_\theta^{\theta+T} c(\tau)c(\tau)^T \geq k_N I_N$$

for positive constants T and k_N, drive the system (4) to the maximum exit concentration of the product of interest.

Proof : straightforward from the above arguments.

IV. Simulation Results

This section presents simulation results of the proposed adaptive extremum-seeking controller. The dimensionless parallel reactions kinetic parameters listed in Table I are calculated from arbitrary values in a realistic range of liquid-phase reaction rate constants [20] [21].

TABLE I
KINETIC PARAMETERS OF THE PARALLEL REACTIONS

Parameters	Value
k_1	13.2
k_2	10.0
α_1	0.4
α_2	1

The initial conditions $y_A(0,\xi),\ y_B(0,\xi),\ y_C(0,\xi)$ are set equal to 0 while we consider the boundary conditions $y_{A0} = 5$, $y_{B0} = 1$ and $y_{\nu 0} = 1$.

The centers of neural network approximation functions are evenly spaced points on the interval [0, 1] and the width σ_i is set to 2 for $1 \leq i \leq 6$ while σ is set to 1.

To ensure sufficient excitation, the dither signal is defined as follows [10] [11] [12] :

$$\dot{d}(\theta) = -k_d d(\theta) + a(\theta) \tag{45}$$

with k_d a strictly positive constant and :

$$a(\theta) = \sum_{i=1}^{5} (A_{1i} sin(\omega_i \theta) + A_{2i} cos(\omega_i \theta)) \tag{46}$$

where A_{1i} and A_{2i} are chosen as unit random numbers and ω_i are different frequencies.

The design parameters are presented in Table II. The

TABLE II
CONTROLLER PARAMETERS

Parameters	Value
q_{min}, q_{max}	0, 1
μ_1, μ_2	0.001, 0.001
ϵ_1, ϵ_2	0.001, 0.001
k_d	5
k_z	1
k_y	1
γ_w	10000
w_m	0.1
$q(0)$	0.5
$d(0)$	0
$c(0)$	$[0\ 0\ 0\ 0\ 0\ 0]^T$
$\hat{W}(0)$	$[1\ 1\ 1\ 1\ 1\ 1]^T$

simulation results are shown in Figures 2 to 4. Figure 2 shows the research path of the adaptive extremum seeking algorithm. We see that the control action q is kept within the constraints. Figures 3 and 4 present the evolution of the corresponding control input and exit concentration of C, respectively. The simulations show that the adaptive controller recovers the unknown optimum with only measurements of the exit concentration of C.

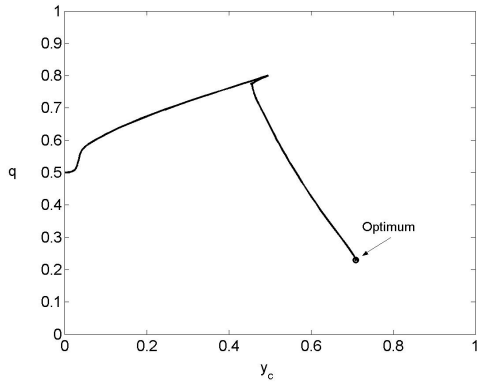

Fig. 2. Research path of the adaptive extremum seeking algorithm.

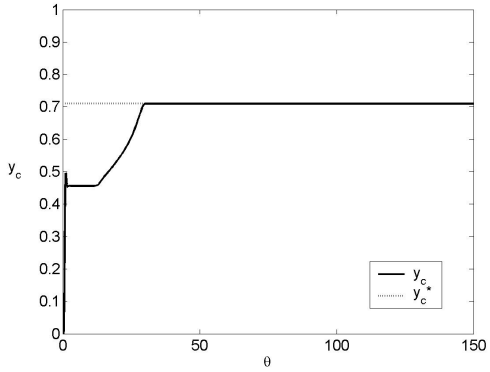

Fig. 3. Concentration of the product of interest at the reactor output y_C ("—") and its maximum y_C^* ("- -").

V. Conclusions

We have solved a class of extremum seeking control problems for tubular reactor with uniform distributed feed represented by unknown reaction kinetic rate expressions subject to control action constraints. An adaptive learning technique was used to derive a relation between the exit concentration of the product of interest and the distributed feed. The adaptive extremum seeking algorithm converges to a small neighborhood of the unknown optimum when the dither signal satisfies a persistent excitation condition.

Future works will concentrate on the case where the product of interest is not available for on-line measurement and the more general case where the distributed feed is not uniform.

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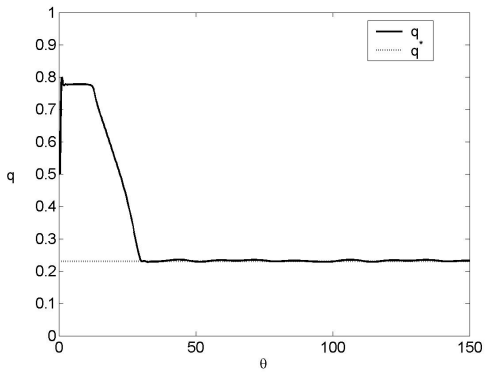

Fig. 4. Control input q ("—") and the optimal control input q^* ("- -").

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