Endogenous Timing with Free Entry

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ABSTRACT

A free entry model with linear costs is considered where firms first choose their entry time and then compete in the market according to the resulting timing decisions. Multiple equilibria arise allowing for infinitely many industry output configurations encompassing one limit-output dominant firm and the Cournot equilibrium with free entry as extreme cases. Sequential entry is never observed. Both Stackelberg and Cournot-like outcomes are sustainable as equilibria however. When the number of incumbents is given, entry is always prevented, and industry output is sometimes larger than the entry preventing level.

JEL Classification: **L11, L13**.

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I. INTRODUCTION. This paper considers a model of competition with free entry where firms decide not only whether or not to enter the market, but also their entry time. In the same spirit as the recent literature on endogenous timing,[1] I address the issue of what order of play one should observe when firms are allowed to choose first at which period to enter the market and then, given the resultant timing structure, their output level. A common motivation of many studies in this area is that in the presence of either a first or a later mover advantage[2] the sequence of moves should be dealt with as an equilibrium phenomenon instead of being exogenous, especially when firms are *ex ante* identical. Here I extend the criterion introduced by Hamilton and Slutsky (1990) for the two-period, two-firm case to encompass endogenous number of firms.

The main features of the framework here are that (i) entry takes place until a new entrant expects non positive profits in equilibrium, and (ii) given a sequential pattern of moves in the product market, with possibly many firms moving at the same period, the output decisions of earlier movers are known to subsequent competitors. So, any possible sequential order of play gives rise to a *hierarchical Stackelberg game* where each firm takes the output of earlier and simultaneous movers as given while exactly predicting the strategies of later movers up to the

[1] See, among others, Hamilton and Slutsky (1990) and Mailath (1993). For experimental evidence on endogenous timing see Huck, Müller, and Normann (2002).

[2] As shown by Gal-Or (1985) for a duopoly, first and second mover advantages are usually associated with strategic substitutability as in quantity competition with homogeneous goods and to strategic complementarity as in price competition with differentiated goods, respectively. Also see Amir and Stepanova (2006). With free entry and endogenous number of firms, as in the present paper, the standard taxonomy is preserved and earlier movers have a strategic advantage. See for instance Lemmas 1 and 2 below.

period at which the free entry condition (i) is satisfied.

Three different patterns can *a priori* realize. The first case is *simultaneous entry*, where firms enter the market at the same period and induce a Cournot equilibrium with free entry. The second case is *sequential entry* in the sense of Prescott and Visscher (1977), where firms enter one at time, then play a sequential game with perfect information, and induce a subgame perfect Nash equilibrium (SPNE) with free entry. Beyond these two extremes, the third, more general case is that of a sequential timing structure with some firms moving at the same period. When choosing its entry period, each firm considers its SPNE payoff for the version of the market stage game that would obtain, that is the market stage game with timing structure defined by the firms' timing decisions. The equilibrium of the whole game defines the number of firms active in the market, the production decisions of these firms, and the timing according to which production takes place. I consider the case of linear costs only. The results can be extended to the case of concave costs, but possibly do not hold in the presence of increasing marginal costs.

A key observation in this paper is that, with given number of incumbents, under the free entry assumption, early movers always prevent entry. This is because they anticipate that if entry was accommodated then the equilibrium price would not be larger than in the case of entry preemption. With linear costs these firms can do strictly better by producing the preemptive output themselves at the first stage.

The fact that *one* Stackelberg leader with linear costs might preempt entry is well recognized in the literature on industrial entry deterrence. Vives (1988) makes the interesting point that in a hierarchical Stackelberg model with sequential entry and given number of firms, the leader never accommodates entry of firms which will subsequently block further entry.[3] The mentioned result in the present paper can be actually seen as directly following from Vives' observation. More recently, Etro (2004) studies a free entry Stackelberg model with one leader. With linear costs, the leader prevents entry. Similar intuitions are already in Prescott and Visscher (1977).

As an intermediate result, this paper extends these findings to industries with many established firms, by showing that with linear costs and free entry, *any* given number of incumbents deters entry. What is remarkable is that, with n firms moving simultaneously at the first period and free entry in the subsequent periods, there always exists an equilibrium where only one incumbent produces the entry preventing output, the remaining $(n-1)$ firms produce no output, and the later movers do not enter the market. In addition, other configurations might be sustainable as equilibria, with either m firms producing the entry preventing output in aggregate and the remaining $(n-m)$ firms producing 0, or with the n incumbents producing their Cournot output each and blockaded entry. No other equilibria exist.

So, one firm contemplating entry will never choose to be a late mover, in which

[3] Also see Gilbert and Vives (1986).

case it would get non positive profits (net of the fixed cost of production). This remark is fundamental when treating the number of incumbents as well as the order of moves as endogenously determined by the firms' decisions, which is the main interest of this paper. The corresponding point for the case of endogenous timing is that a candidate follower would prefer either to anticipate its entry time or not to enter the market at all. More specifically, at the SPNE of the endogenous timing game earlier movers always prevent subsequent entry.

An article close to this paper is Anderson and Engers (1992).[4] For given number of firms and a certain class of demand functions allowing for linear demand as a specific case, they consider a game including both Stackelberg with sequential entry and Cournot equilibria as possible outcomes and show that the equilibrium is Stackelberg. The present paper finds that with endogenous timing and free entry, sequential play is never observed. However, both Stackelberg and Cournot-like outcomes are sustainable as equilibria. Namely, the endogenous timing game has multiple SPNE describing a wide range of industry configurations. Whether one configuration with only one firm active in the market producing the entry preventing output, or any configuration with $m \leq n^*$ firms active in the market at period 1, producing an aggregate output which is the largest between the entry preventing output and the Cournot output with m firms, and no firm entering in subsequent periods is a SPNE of the whole game, where n^* is the number of firms active in the market at the Cournot equilibrium with free entry.

[4] Also see Matsumura (1999).

No other configuration is an equilibrium.

The paper is organized as follows: Section II describes the model and some preliminary results; Section III features the equilibria of the game; Section IV provides a discussion and briefly concludes.

II. THE MODEL. This paper characterizes the equilibria of an industry with free entry where firms first choose their entry time and then compete in the market following the sequence of moves resulting from their entry decisions. When entering the market, a firm observes the aggregate output of current incumbents (if any) and decides how much to produce taking the actions of simultaneous entrants as given, while perfectly predicting the strategy of later movers. Entrants that are *active* in the market are identical in that they produce a homogeneous good with inverse demand $p(.)$ and face the same constant marginal cost $c > 0$. More precisely, $p(.)$ is downward sloping and twice continuously differentiable. These firms also bear a setup cost F. Hence the cost function of a firm i is $c(q_i) = cq_i + F$ for $q_i > 0$ and $c(q_i) = 0$ otherwise.

The timing of the game is as follows. In the first or *preplay* stage, there is a large enough number of firms contemplating entry.[5] These firms simultaneously choose whether or not to enter the market and at which period $t \in N$. By entering the market at time t, a firm commits to produce at that time and only at that

[5] See, for instance, Mankiw and Whinston (1986).

time.[6] The action set of firm i at the preplay stage is $A_i = \{enter, not\ enter\} \times N$.[7] Once the timing decisions are taken, they become common knowledge and the market stage game is played according to the timing structure stemming from the preplay stage.

Let n be the number of firms choosing *enter* at the preplay stage. Also let $s \leq n$ be the number of different periods chosen in the preplay stage. Finally, let N_τ be the set of firms choosing $t = \tau$ at the preplay stage. The market stage can be viewed as an s-stage game G whose players are the active firms. When N_τ is a singleton for any τ, G corresponds to the hierarchical Stackelberg game with sequential entry described in Vives (1988) and Anderson and Engers (1992), with the major addition of the free entry assumption. Otherwise there are at least two firms playing simultaneously at some period τ, in which case any firm in N_τ observes the output of all previous movers and makes a simultaneous choice relative to other firms in N_τ. Each firm also predicts the strategy played by subsequent competitors correctly. So, firms which move at the same period are Cournot competitors to each other, Stackelberg leaders for later movers, and Stackelberg followers with respect to earlier entrants.

Let τ^* be the first period at which at least one firm chooses to enter the market. Given the timing structure for the market stage game, a strategy $S_{i\tau^*}$ for firm $i \in N_{\tau^*}$ is an output level q_i. For $\tau \neq \tau^*$, a strategy $S_{i\tau}$ for firm

[6] As stressed by Hamilton and Slutsky (1990), this assumption is not restrictive, since the timing of entry rests on firms' decisions and no firm will deviate in equilibrium.

[7] As emphasized in the literature, entry decisions here do not take place in real time. That is, different timing structures leading to the same order of entry can be thought of as equivalent.

$i \in N_\tau$ is a function of the aggregate production of firms in periods preceding τ, $r_i : R_+ \longrightarrow R_+$.

The whole or *extended* game is obtained by adding the preplay stage to the market stage described above. A strategy for firm i in the extended game is a pair (a_i, ϕ_i), where $a_i \in A_i$, and $\phi_i : A_i^n \rightarrow \{S_{iT^*}, S_{iT}\}$.

I assume that a firm enters the market if and only if this firm obtains strictly positive profits in the SPNE of the market stage induced by its timing decision. The profit of firm i is $\Pi_i(Q_{-\,i} + q_i) = (p(Q_{-\,i} + q_i) - c)\, q_i - F$, if $q_i > 0$ and 0 otherwise, where q_i is the output of that firm and $Q_{-\,i}$ is the output of all firms other than i. I also assume that output levels are strategic substitutes, *i.e.* $p^{''} q_i + p^{'} < 0$. It follows that $\Pi_i\,(.)$ is strictly concave in q_i. Finally, I assume that there exists a *free-entry Cournot equilibrium*, that is an integer number of firms n^* and a set of outputs forming a Cournot equilibrium with n^* firms, such that no other firm would obtain positive Cournot equilibrium profits by entry.[8] For further reference, n^* will denote the number of firms active in the market at the Cournot equilibrium with free entry.

As a preliminary step, I now consider two specific versions of the market stage game G, both allowing for free entry and displaying an exogenous timing

[8] Given the assumptions here, if a free-entry equilibrium exists, then it must be unique. In fact, for any number of firms n, there exists a unique and symmetric Cournot equilibrium. In addition, the per firm Cournot equilibrium profit with n firms is strictly decreasing in n. So a sufficient condition for the existence of the (unique) free-entry Cournot equilibrium is that the equilibrium per firm profit tends to 0 when n grows indefinitely. Finally notice that under the free entry condition stated here and since $c\,(0) = 0$, firms active at the free-entry Cournot equilibrium makes strictly positive profits.

structure. The related results in Lemma 1 and 2 below facilitate the analysis for the case of endogenous timing. Nonetheless, they might also be of independent interest to the IO literature on entry deterrence.

Game G^1: In game G^1 only a single firm is in the market at period 1. In this stage, the sole incumbent decides how much to produce taking into account the strategies of potential entrants. At any subsequent period each of possibly many potential entrants observes the aggregate output in the market and then decides whether to enter or not and how much to produce, anticipating the strategies of further entrants correctly and until the free entry condition is satisfied. More precisely, G^1 is meant to capture any version of the market stage game G where $\tau^* = 1$, $N_{\tau^*} = 1$, and $N_\tau \geq 0$ for $\tau > 1$, that is a Stackelberg game with free entry and only one firm in the first period.

Let Y satisfy $(p(Y + q_i^*(Y)) - c)\, q_i^*(Y) = F$, where $q_i^*(q)$ is the unique root of $\frac{\partial \Pi_i(q+q_i)}{\partial q_i} = 0$. Since $q_i^*(Y)$ stems from the first order condition for the maximization of $\Pi_i(Y + .)$, there is no $q_i > 0$ giving firm i positive profits when earlier movers produce Y. Moreover, $(. + q_i^*(.))$ is increasing and $q_i^*(.)$ is decreasing. It follows that for any cumulative output of the incumbents $Z \geq Y$, the potential entrant best reacts by not entering the market.[9] Specifically, $r_i(Y) = 0$, where $r_i(.)$ is the reaction function of any potential entrant i as defined above. Henceforth Y will be referred to as the *entry preventing output.* I assume that entry is

[9] This implies that Y is unique. See Vives (1988) for a complete characterization of the potential entrant's reaction function in the case of linear demand.

not blockaded, that is the monopoly output $y^m < Y$.

Let now Z^1 be the SPNE output for game G^1. At Z^1 further entry is not profitable, which implies $Z^1 \geq Y$. Firm 1 correctly anticipates that if it produced any $q_1 < Y$ and accommodated entry it would then obtain $\left(p\left(Z^1\right) - c\right) q_1 - F \leq (p(Y) - c) Y - F$. Moreover, since $y^m < Y$ and the profit function is strictly concave in own output, any output level larger than Y would give firm 1 smaller profits given that no other firm is in the market. So, $\Pi_1(0, Y) = \max_{q_1} \Pi_1(r(q_1), q_1)$. That is, firm 1 producing Y and the potential entrants staying out of the market is the unique SPNE of G_1, and $Z^1 = Y$. Moreover, $(p(Y) - c) Y > (p(Y + q_i^*(Y)) - c) q_i^*(Y) = F$, so the incumbent obtains positive profits. This fact is summarized by the following lemma.

Lemma 1: *Game G^1 has a unique SPNE with only one firm in the market, producing at period* 1 *the output Y which satisfies $(p(Y + q_i^*(Y)) - c) q_i^*(Y) = F$, and getting positive profits.*

Etro (2004) obtains the same result for a Stackelberg game with only one leader and many simultaneous followers.[10] While focusing on the special case of a single incumbent as well, the discussion above introduces a general enough argument to be extended to the more realistic case of an industry with many firms at period 1. If the incumbents anticipate that the equilibrium price with

[10] He proceeds by totally differentiating both the first order conditions and the zero profit condition for the followers when treating the number of firms as a continuous variable, and shows that the maximization problem of the leader has a corner solution with entry deterrence if costs are linear.

free entry cannot be larger than the price clearing the market when the aggregate output is Y, then each of them will try to spread the fixed markup at $p(Y)$ over the largest possible market share, that is the incumbents will compete to deter entry. This fact, that is reckoned with by the existing literature, has not received a formal treatment yet. What follows is meant to take a step in this direction.

Game G^n: Game G^n extends G^1 to the case of n firms producing simultaneously at period 1. More precisely, G^n covers any version of the market stage game G with $\tau^* = 1$, $N_{\tau^*} = n$, and $N_\tau \geq 0$ for $\tau > 1$. Consider $n \leq n^*$ only, where n^* is the number of firms in the Cournot equilibrium with free entry as mentioned above. When choosing its output level, each of these firms takes the output of the other $(n-1)$ incumbents as given, while anticipating correctly the best response of new entrants. Since a firm $i \in N_1$ acts as a Cournot competitor with respect to simultaneous movers, its best response to an aggregate output level of the other incumbents $Z \geq Y$ is 0. Given the assumption of linear costs, following the same argument as for game G^1, for $Z < Y$, any $q_i < Y - Z$ is a strictly dominated action for firm i, since by producing q_i it will accommodate further entry and eventually obtain smaller profits than by exactly producing $Y - Z$. So the equilibrium cumulative output of the n incumbents in game G^n is not smaller than Y. More precisely, the reaction function of a firm $i \in N_1$ is

$$r_i^n(Z) = \begin{cases} 0 \text{ if } Z \geq Y \\ \max\{Y - Z, q_i^*(Z)\} \text{ if } Z < Y \end{cases}, \tag{1}$$

where Z is the cumulative output of the $(n-1)$ firms in N_1 different than i, and $q_i^*(.)$ is defined as above.

It first follows from (1) that any output configuration with only one firm producing the entry preventing output Y and the other $(n-1)$ firms in N_1 producing 0 is a SPNE of G^n.[11] (1) also says that when firm i's *Cournot reaction function* $q_i^*(Z)$ lies above the line $q_i = Y - Z$, firm i behaves as a Cournot competitor irrespective of the strategies of the potential entrants. Let now q_i^{Cn} denote the Cournot equilibrium output of firm i when there are n firms active on the market, *i.e.* the equilibrium output of the game when the n firms move simultaneously and no further entry is allowed. It is well known that for the case at hand, this equilibrium is unique and symmetric, so that $q_i^{Cn} = q^{Cn}$, for all i.[12] Therefore, if $nq^{Cn} \geq Y$, then each incumbent producing q^{Cn} and no other firm entering the market in subsequent periods is a SPNE of G^n. In this case, for any $Z > (n-1)\,q^{Cn}$, $r_i^n(Z) = q_i^*(Z)$ for any i.[13] This, together with the uniqueness of the Cournot equilibrium with n firms, implies that when $nq^{Cn} \geq Y$, there is no other SPNE. Analogously, if $nq^{Cn} < Y$, then the equilibrium output q_i^n of any firm $i \in N^1$ in game G^n must satisfy both

$$(a)\;\; q_i^n \geq q_i^*\left(\sum_{j\neq i} q_j^n\right) \;\text{ and }\; (b)\;\; q_i^n = Y - \sum_{j\neq i} q_j^n. \tag{2}$$

[11] Remind that $q_i^*(0) = y^m < Y$, which implies $Y = r_i^n(0)$.

[12] See, for instance, Amir and Lambson (2000).

[13] This follows from $q_i^*(0) = y^m < Y$, the assumption that $nq^{Cn} \geq Y$, and the fact that the Cournot equilibrium with n firms is *stable* under best reply dynamics. In particular, the latter implies that the curves $q_i = q_i^*(Z)$ and $q_i = Y - Z$ cross only once.

There are possibly infinitely many such equilibria. In particular, for $m \leq n$, any output configuration satisfying $(2.a)$ above, with m firms producing the aggregate output Y and the remaining $(n-m)$ firms in N^1 producing 0, is a SPNE of G^n. In this case, any incumbent producing a strictly positive output makes strictly positive profits.[14]

It is worth noting that game G^n encompasses game G^1 as a special case, *i.e.* $n = 1$. Note also that the aggregate output $n^* q^{Cn^*}$ at the Cournot equilibrium with free entry is larger than the entry preventing output Y. To see that, observe that the number of firms at the Cournot equilibrium with free entry n^* satisfies $\left[p\left(n^* q^{Cn^*+1} + q^*\left(n^* q^{Cn^*+1}\right)\right) - c\right] q^*\left(n^* q^{Cn^*+1}\right) \leq F$. This means that even the monopoly output relative to the residual demand of the potential entrant gives this firm non positive profits. So output $n^* q^{Cn^*+1}$ would also block entry in the hierarchical Stackelberg model, which implies $n^* q^{Cn^*+1} \geq Y$. Since $q^{Cn^*+1} < q^{Cn^*}$, it follows that $n^* q^{Cn^*} > Y$. These facts are summarized in Lemma 2, where use is made of the following *definition*:

$$\text{let } \widehat{n} \in R \text{ satisfy } nq^{Cn} \gtrless Y \text{ if and only if } n \gtrless \widehat{n}. \tag{3}$$

Note that $\widehat{n}$ defined in (3) exists for the reason that $y^m < Y$ (*i.e.* $q^{C1} < Y$), $n^* q^{Cn^*} > Y$, and the fact that nq^{Cn} is increasing in n.[15]

Lemma 2: *At any SPNE of G^n only $m \leq n$ firms produce a positive output,*

[14] In fact, any firm i would get $(p(Y) - c)\, q_i^n > (p(Y + q^*(Y)) - c)\, q^*(Y) = F$.

[15] See, for instance, Amir and Lambson (2000).

all moving at period 1, *producing aggregate output* $Z^n \geq Y$ *and obtaining strictly positive profits. There always exists a SPNE with one firm producing* Y *and the remaining* $(n-1)$ *firms producing* 0. *In addition, there exist a real number* $\widehat{n}$ *such that, if and only if* $n < \widehat{n}$, *any configuration where* m *firms produce the aggregate output* Y, *while the other* $(n-m)$ *firms produce* 0, *and condition* $(2.a)$ *is satisfied is also a SPNE. If and only if* $n \geq \widehat{n}$, *there also is a SPNE with* n *firms producing* q^{Cn} *each. There is no other SPNE.*

It should be noted that when $n \geq \widehat{n}$, the hierarchical Stackelberg game G^n has one equilibrium where the incumbents produce as in Cournot. In addition, when $n^* > n \geq \widehat{n}$, the Cournot output deters entry of followers only, while it is not enough to prevent that of a simultaneous competitor. A remark of independent interest relates to the effects of a change of F. Given the assumptions here, $p\left(Y + q_i^*\left(Y\right)\right) q_i^*\left(Y\right)$ is strictly decreasing in Y. So a rise of F causes the entry preventing output Y to reduce. Since a change of F leaves the *Cournot reaction functions* $q_i^*\left(Y\right)$ unaffected while reducing Y, a larger set up cost will also reduce $\widehat{n}$.

Example: Consider game G^2 with linear demand $a - (q_1 + q_2)$ and $c = 0$. I assume $a > 4\sqrt{F}$, which implies that entry is not blockaded. The entry preventing output is $Y = a - 2\sqrt{F}$. Given this setup, at the Cournot equilibrium with two firms, each firm produces $q^{C2} = \frac{a}{3}$. The reaction function of firm 1 is:

$$r_1^2(q_2) = \begin{cases} 0 \text{ if } q_2 \geq a - 2\sqrt{F} \\ \frac{1}{2}(a - q_2) \text{ if } a - 2\sqrt{F} > q_2 \geq a - 4\sqrt{F} \\ a - 2\sqrt{F} - q_2 \text{ if } a - 4\sqrt{F} > q_2 \end{cases},$$

and similarly for firm 2. So G^2 has the following equilibria: if $6\sqrt{F} > a > 4\sqrt{F}$, (a) only one firm active in the market, producing output $a - 2\sqrt{F}$ at period 1; (b) only two firms active in the market, producing output $\frac{a}{3}$ each. If $a \geq 6\sqrt{F}$, (c) only one firm active in the market, producing output $a - 2\sqrt{F}$ at period 1; (d) only two firms active in the market, producing any output configuration satisfying both $q_1 = a - 2\sqrt{F} - q_2$ and $a - 4\sqrt{F} \geq q_2 \geq 2\sqrt{F}$.

Figure 1 describes the reaction curves for the game at hand; the thicker line is the reaction curve of firm 1. The equilibria of type (d) lie on the segment line AB.

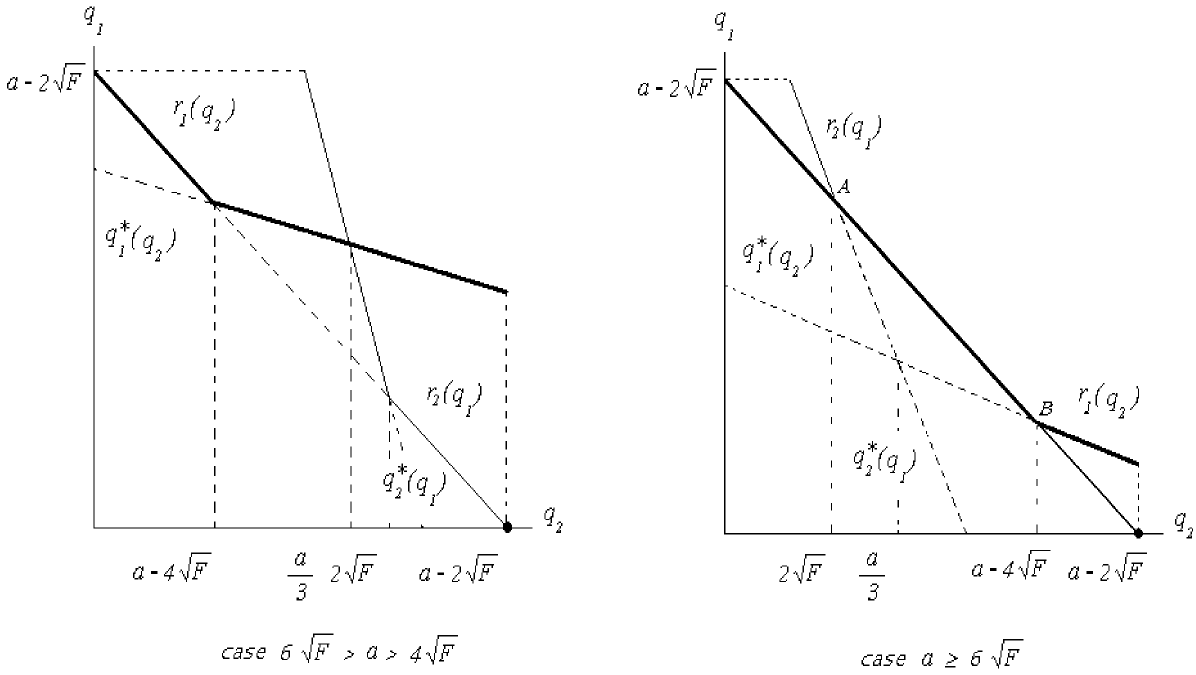

Figure 1: Game G^2 with linear demand

III. EQUILIBRIA. A SPNE of the extended game is a set N^E of n firms active in the market stage game, together with a vector of timing decisions $\mathbf{t} \in N^n$, and a vector of output decisions $\mathbf{q} \in R^n$, such that, (.) the output decisions of the active firms constitute a SPNE of the market stage game with timing structure $\mathbf{t}$, and (..) given $\mathbf{t}$ and n, no firm $i \in N^E$ can profitably deviate from $\mathbf{t}$, and no firm $i \notin N^E$ can profitably enter the market.

Given Lemmas 1 and 2 above, in a SPNE equilibrium of the extended game, at least one firm is active at period 1. Indeed if no firm was entering at that time, then there would be at least one firm obtaining a SPNE profit strictly smaller than the profit going to the Stackelberg leader in game G^1 described in the previous section, and then being strictly better off by choosing to enter at period 1. As a consequence, any vector $\mathbf{t}$ substantiable in the SPNE of the extended game gives rise to a market stage game of the form G^n discussed above. It then follows that in a SPNE of the market stage, no firm enters at period $t \geq 2$. This is because, as argued in Lemma 2 above, at any SPNE of G^n the aggregate output of the incumbents is not smaller than the entry preventing output Y. So sequential entry is never observed.

Since for $n > 1$, any game G^n has multiple equilibria, a firm deviating from its current entry decision must consider any of these equilibria separately, as alternative standard for the resultant market stage game. The SPNE configurations of the extended game are summarized in the following proposition.

PROPOSITION: *Let n^* and $\widehat{n}$ be defined as in Section II and in* (3) *above, respectively. Also, let Y be defined as in Lemma* 1 *above. At any SPNE of the extended game there are m firms entering the market at period* 1 *and no firm entering at subsequent periods, where $m \leq n^*$.*

Any of the following configurations is a SPNE of the extended game:

(1) *Only one firm enters the market at period* 1 *and produces Y.*

(2) *Only m firms, with $m < \widehat{n}$, enter the market at period* 1 *and produce a strictly positive quantity each and an aggregate output level Y, under condition*

(2.a) *mentioned in the previous section.*

(3) *Only* m *firms, with* $\widehat{n} \leq m \leq n^*$, *enter the market at period* 1 and *produce* q^{Cm} *each, with* $mq^{Cm} > Y$.

No other configuration is sustainable as SPNE of the extended game.

PROOF: The first part of the proposition follows from the previous discussion. The fact that $m \leq n^*$ follows from the definition of n^*. Given this, the remainder of the proof just considers the problem of a firm e contemplating entry.

(1) From Lemma 1 in the previous section, game G^1 has a unique SPNE with the only incumbent producing the entry preventing output Y. Given this equilibrium, entry by firm e would induce a market stage game G^2. Irrespective of 2 being larger or smaller than $\widehat{n}$, game G^2 has a SPNE where the former incumbent produces Y while the entrant produces 0. This follows from Lemma 2. Taking this equilibrium as the benchmark for game G^2, entry would yield firm e no profit. So no firm e enters the market and the configuration in (1) is an equilibrium.

(2) From Lemma 2 in the previous section, game G^m has a SPNE with m firms active in the market and producing the aggregate output Y. Given this equilibrium, entry by firm e would induce a market stage game G^{m+1}. Irrespective of $m+1$ being larger or smaller than $\widehat{n}$, game G^{m+1} has a SPNE where only one firm $i \neq e$ produces Y while the remaining m firms produce 0. Again, this follows from Lemma 2. Taking this equilibrium as the benchmark for game G^{m+1}, entry would yield firm e no profit. So no firm e enters the market and the configuration

in (2) is an equilibrium.

(3) From Lemma 2 in the previous section, for $m \geq \widehat{n}$, game G^m has a SPNE with m firms active in the market and producing q^{C_m} each. Since $m \leq n^*$, each incumbent makes positive profits. Given this equilibrium, entry by firm e would induce a market stage game G^{m+1}. Again, from lemma 2, G^{m+1} has one equilibrium with only one firm $i \neq e$ producing Y and the remaining m firms producing 0. This equilibrium gives firm e zero profit. So taking it as the benchmark for game G^{m+1}, no firm e would enter the market, and the configuration in (3) is an equilibrium.

To see that there are no other SPNE remind that any output vector sustainable as a SPNE of the extended game must be a SPNE of a related market stage game, together with the fact that any market stage game which is sustainable in a SPNE of the extended game is of the form G^n described in the previous section. ■

IV. CONCLUSION. This paper has examined a model of market competition with both free entry and endogenous timing for entry decisions. Under the assumption of linear production costs, there are multiple equilibria allowing for many industry output configurations that encompass one limit-output dominant firm and the Cournot equilibrium with free entry as extreme cases. Infinitely many other configurations in between, with $m < \widehat{n}$ firms producing the entry preventing output Y in aggregate, or any configuration with $\widehat{n} < m < n^*$ firms producing their Cournot output each is an equilibrium. At any of these equilibria entry is deterred by first movers, that is sequential entry is never observed. No other equilibria exist.

The result here rests on equilibrium properties of industrial contexts where many established incumbents cope with an infinite number of potential entrants as described by game G^n in section II. Previous studies argued that in the case of a given number of entrants, entry prevention is a *public good* to the extent that the benefit from preventing entry is not perfectly appropriable by the larger incumbent.[16] This might lead to underinvestment in entry deterrence. This paper shows that with free entry and linear costs, underinvestment is never the case. Instead, each incumbent has strong incentives to produce the entry preventing output by itself, and any configuration with $m \leq n$ firms producing this output and the remaining ones producing 0 is an equilibrium. With a larger number of incumbents, *i.e.* $n > \widehat{n}$, entry is blockaded by the n-firms Cournot output.[17] A related point is that unlike the standard setting for endogenous timing with two firms, here simultaneous play does not necessarily means a Cournot-like output.

While the results here hold with concave costs,[18] they need not hold in the case of increasing marginal costs. However, the analysis is still valid with U-shaped average cost curves whenever the entry preventing output is smaller than the incumbents' minimum efficient scale. A welfare analysis is not immediate. When total output is Y, welfare is decreasing in the equilibrium number of firms, even in the specific case of linear costs, and the socially efficient number of firms is

[16] See for instance, Bernheim (1984) and Waldman (1987).
[17] A similar result is reported in Gilbert and Vives (1986).
[18] In this case the further assumption $-p' + c'' > 0$ is needed. See Amir and Lambson (2000).

1. As demonstrated by Mankiw and Whinston (1986), when $n = n^*$, there are too many firms operating in the market. But, for $n > \widehat{n}$, the loss in industrial profits must be contrasted against the benefits to consumers from the larger aggregate output, which is something beyond the scope of this paper.

Finally, a natural consideration is that removing barriers to entry in an industry could cause the number of firms to reduce. This point may be of some interest when looking at industries with too many inefficient firms. The paper might suggest that despite the large number of firms in the market, inefficiencies in these industries are caused by barriers to entry which dissuade incumbents from investing in entry prevention and in efficient scale.[19]

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[19] This is, for instance, the case of many industries in Italy, where a large number of small firms produces at high average costs. See Banca d'Italia (2005). Another example is the retail industry in India with many little shops accounting for 6-7% of national employement and a land-ceiling legislation preventing entry at efficient scale in many states. See the Economist (2006).

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