

Integrated Sensing and Communication

From Electromagnetic Modelling to Interference Management in Large-Scale Networks

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Integrated Sensing and Communication: From Electromagnetic Modelling to Interference Management in Large-Scale Networks

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*Never confuse education with intelligence,
you can have a PhD and still be an idiot....*

RICHARD P. FEYNMAN

Abstract

Nowadays, sensing and communication are both critical in numerous applications of everyday life. In future network, joint integration of both functions is promising for multiple motives, including increase of spectral efficiency, lower cost and complexity, and improvement of the overall network performance. In this PhD thesis, such systems have been studied from three different perspectives, especially in vehicular scenarios.

First, at the vehicle level, novel radar propagation and cross-section models are developed from the electromagnetic theory in monostatic configurations. Those models are obtained considering simple flat and curved rectangular targets, and are suited for both near- and far-field applications. They highlight how the received power varies w.r.t. to distance between the antennas and the target, depending on the carrier frequency and the geometry of the scenario. This emphasises the importance of the choice of accurate models. Experimental measurements are also carried out to validate the conclusion drawn from the developed expressions. Besides, expressions of the reflected signal and specific signal processing are developed in multistatic configurations. This shows that the consideration of the spherical wavefronts in the near-field propagation region can lead to radar performance improvement, i.e. an increased resolution and/or accuracy, respectively when orthogonal and non-orthogonal waveforms are transmitted.

Then, within the stochastic geometry framework, novel joint sensing and communication performance metrics are defined, highlighting the impact of one function on the other, and the associated trade-offs. Analytical expressions are computed in a full-duplex scenario at the street level, where the system of interest simultaneously process radar echoes and uplink communication signals. Ray-tracing simulations are also carried out to assess the accuracy of the developed metrics. Furthermore, at the city level, cooperative sensing and communication is analysed, and complex interference mechanism are modelled, namely direct links, reflections, diffractions, and building penetrations. Optimisation of the network parameters is also performed.

Finally, the full-duplex joint sensing and communication scenario is further studied. Two solutions against the interference between both function are proposed, namely interference cancellation and beamforming. In the first case, multiple structures are

proposed, e.g. interference cancellation at the radar function, at the communication function, iterative and dynamic interference cancellation. Such receivers are robust against any scenario, and their complexity may be reduced leveraging on previous measurements. In the second case, optimisation is carried out while ensuring fairness between both functions, and among the targets and communication users. Multiple schemes are designed depending on the scenario, with variable complexity, optimality and required assumptions.

List of Publications

Conference Publications

1. F. De Saint Moulin, C. Oestges, and L. Vandendorpe. “Characterisation and Cancellation of Interference with Multiple Phase-Coded FMCW Dual-Function RADAR Communication Systems”. In: *2022 IEEE 95th Vehicular Technology Conference (VTC2022-Spring)*. 2022.
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3. F. De Saint Moulin, G. Thiran, C. Oestges, and L. Vandendorpe. “Fair Beam-forming Design for MIMO ISAC Systems”. In: *Accepted to 19th International Symposium on Wireless Communication Systems (ISWCS)*. 2024.
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2. F. De Saint Moulin, G. Thiran, C. Craeye, L. Vandendorpe, and C. Oestges. “Near-Field EM-Based Multistatic Radar Range Estimation”. In: 2024.

Letters

F. De Saint Moulin, S. Demey, C. Wiame, L. Vandendorpe, and C. Oestges. “Full-Duplex V2X Integrated Sensing and Communication Scenario: Stochastic geometry, Monte-Carlo, and Ray-Tracing Comparison”. In: *arXiv preprint arXiv:2407.06822*. 2024.

Collaborations

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3. F. De Saint Moulin, G. Thiran, C. Craeye, L. Vandendorpe, and C. Oestges. “Near-Field EM-Based Multistatic Radar Range Estimation”. In: *7th Management Committee and 7th Technical Meeting, Lisbon, Portugal*. 2024.

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Preamble

This PhD thesis has been written targetting engineers graduated in Electricity, with basics in telecommunications and signal processing. As detailed in Section 1.2 of Chapter 1, this work has been divided in three parts of three chapters. The first chapter of each part is dedicated to the description of numerous notions and tools which are useful for the understanding of the next chapters. Thus, experienced readers may skip partially or entirely these chapters, at their convenience. Additionally, it is not required to read the content of the appendices to fully understand the results and conclusions of this work, as they mainly contain mathematical developments and proofs.

Within this manuscript, important definitions from the litterature and significant contributions from this thesis are highlighted in boxes:

Important Definitions from the Literature

Boxes colored in gray summarises important definitions from the litterature, which are used in the thesis.

Significant Result from this Thesis

Boxes colored in green summarises significant results and contributions obtained in this thesis.

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Acronyms

1D	One-Dimensional 155, 171,
2D	Two-Dimensional xvii, 155, 172, 263, 287, 303, 305, 306, 311,
3D	Three-Dimensional 184,
3GPP	3rd Generation Partnership Project 182,
ADC	Analogue-to-Digital Converter 251, 253,
AM	Alternative Maximisation xvii, 302, 303, 305–309, 323, 327,
AP	Access Point 8, 342,
AWGN	Additive White Gaussian Noise 16, 22, 100, 108, 265,
BER	Bit Error Rate xvi, 15, 152, 273–277, 279–281, 284–286, 288, 290–292,
BPP	Binomial Point Process 141,
BPSK	Binary Phase Shift Keying xvi, 273–276,
BS	Base Station 8, 139, 140, 142, 157, 158,
CCDF	Complementary Cumulative Distribution Function 152, 196, 197,
CDF	Cumulative Distribution Function xv, 152, 155, 167, 218–220, 225,
CFAR	Constant False Alarm Rate 263, 281,
CoMP	Cooperative Multi-Point xix, 157, 206,
CPM	Continuous Phase Modulation 258, 342,
CPU	Central Processing Unit 207, 208, 213, 230,
CRB	Cramér-Rao Lower Bound xiv, 15, 21, 98, 116–120, 296,
CUT	Cell Under Test 21, 22,
DFRC	Dual-Function Radar-Communication 2,
DFT	Discrete Fourier Transform 263, 268, 270, 271, 311, 314, 315,
ECC	Error Correcting Codes 273, 275, 277, 292,

EIRP	Effective Isotropic Radiated Power 49, 52, 56, 69, 71, 83,
EM	ElectroMagnetic 1, 7, 8, 10, 31, 40, 42, 45, 49, 50, 53, 55, 64, 67, 68, 98, 99, 101–103, 116, 118, 120, 162, 337,
FMCW	Frequency-Modulated Continuous Wave 3, 11, 226, 247, 253, 258, 262,
GDF	Group-Delay Filter 264,
GO	Geometrical Optics 55, 60, 69, 90, 209, 288, 339,
HCPP	Hard-Core Point Process xiv, 142, 143, 155, 162, 181,
HPPP	Homogeneous Poisson Point Process xiv, 141–146, 218,
IC	Interference Cancellation xv, xvi, 161, 177–179, 265–267, 270, 272–274, 276–278, 282–284, 286, 287, 291–293, 295, 296, 307, 310, 339–342,
ICI	Inter-Carrier Interference 261, 262, 270, 271, 311, 314,
IDFT	Inverse Discrete Fourier Transform 268, 270, 271,
IEEE	Institute of Electrical and Electronics Engineers 60,
IF	Intermediate Frequency 85,
ISAC	Integrated Sensing and Communication xiii, xvi, 2, 3, 5–8, 10, 11, 58, 59, 62, 118, 120, 121, 151, 157, 158, 160–162, 181, 182, 192, 193, 205, 208, 209, 226, 247, 255, 257, 258, 260–270, 273, 275, 277, 278, 282, 284, 288, 291–293, 295, 296, 298–301, 307, 309, 310, 312–315, 320, 321, 327, 337, 339–342,
JCAS	Joint Communication and Sensing xiii, 2, 3, 8, 10, 58, 63, 151, 157, 181, 192, 193, 205, 226, 247, 295, 296, 307,
JRDCCP	Joint Radar Detection and Communication Coverage Probability xiv, xv, 158–160, 166–168, 170, 172, 175, 176, 190–192, 198, 200, 202, 203, 218,
JRSCCP	Joint Radar Success and Communication Coverage Probability xiv, xv, 159, 160, 165, 166, 168–173, 175–179, 190–192, 200, 201, 203, 204, 218, 223–226,
LoS	Line-of-Sight xv, 98, 182, 185, 186, 188, 190, 298,
LPF	Low-Pass Filter 264,
LSE	Least Square Estimation xix, 184, 185, 188,

LT	Laplace Transform 140, 145, 164, 165, 167, 177, 180, 195, 196, 200, 216, 218, 227–229, 234, 236,
MC	Monte-Carlo xv, 139, 167, 184, 188–192, 216, 217, 340,
MCP	Matérn Cluster Process xiv, 142, 143,
MIMO	Multiple-Input Multiple-Output xvi, 157, 253, 258, 262, 263, 293, 295–297, 307, 309, 340,
MOSARIM	MORe Safety for All by Radar Interference Mitigation 6,
MSK	Minimum Shift Keying 258,
MVDR	Minimum Variance Distortionless Response 303, 317,
NC-JT	Non-Coherent Joint Transmission 211,
NLoS	Non Line-of-Sight 98, 182, 185,
NOMA	Non-Orthogonal Multiple Access 206,
NR	New Radio 97,
OFDM	Orthogonal Frequency Division Multiplexing xvi, 3, 11, 98, 226, 247, 258, 259, 262–266, 270, 275, 277, 292, 295, 299, 309–312, 314, 315, 340, 342,
OTFS	Orthogonal Time-Frequency Spacing 4, 258, 342,
PAPR	Peak-to-Average Power Ratio 259,
PC-FMCW	Phase-Coded Frequency Modulated Continuous Wave xvi, 253, 255, 257, 258, 263, 264, 266, 267, 270, 271, 273, 275, 277, 284, 288, 292, 295, 299, 309–312, 315, 340, 342,
PCP	Poisson Cluster Process 142, 162, 206,
PDF	Probability Density Function xv, 20, 164, 165, 177, 178, 180, 188–190, 196, 197, 200,
PEC	Perfect Electric Conductor 40, 42, 68, 69, 103,
PGFL	Probability Generating FunctionL 140, 142, 144, 145, 195, 228, 229, 234, 236,
PLP	Poisson Line Process xiv, xv, xix, 148, 149, 206, 207, 223, 225,
PMC	Perfect Magnetic Conductor 40,
PO	Physical Optics xiii, 40, 41, 63, 67, 79, 104,

PP	Point Process 139–142, 144, 146, 147, 162, 180, 181, 192,
PPP	Poisson Point Process xv, 141, 142, 144–148, 155, 162, 180–182, 184, 195–197, 206, 207, 218–222, 225, 227–230, 234, 236,
PRI	Pulse Repetition Interval 251, 253,
PSK	Phase Shift Keying xvi, 273, 274,
QAM	Quadrature Amplitude Modulation xvi, 273, 274,
QPSK	Quadrature Phase Shift Keying xvi, 273–276, 279, 284, 289,
RCS	Radar Cross-Section xiv, 56, 57, 59–64, 71–73, 76–85, 88–91, 120, 155, 339, 343,
RIS	Reconfigurable Intelligent Surface 98,
ROC	Receiver Operating Characteristic xiii, xv, 22, 23, 222,
RSU	RoadSide Unit xv, 6, 8, 139, 140, 161–163, 167, 170–173, 178, 182, 184, 192, 198, 200, 202, 207–209, 211–219, 223–231, 233–236, 287, 289,
RT	Ray-Tracing xv, xix, 55, 60, 61, 72, 81–83, 151, 163, 181–193, 340,
RV	Random Variable 140, 145, 163, 165, 180, 184, 190, 195, 227–230, 234, 236,
RX	Receiving 184,
SG	Stochastic Geometry xv, 9–11, 15, 63, 81, 83, 90, 139, 151, 155, 167, 181, 182, 184, 188–193, 205, 206, 218, 223, 225, 226, 265, 339, 340,
SINR	Signal-to-Interference-plus-Noise Ratio xvi, 11, 15, 81, 83, 121, 152, 153, 155, 156, 158, 159, 165, 174, 201, 211, 225, 295, 296, 299–301, 304, 305, 307, 309, 311–315, 340, 341,
SIR	Signal-to-Interference Ratio xv, 167, 168, 170–172, 174, 177, 178, 192, 282, 284, 286, 288, 339,
SNR	Signal-to-Noise Ratio xiii, xiv, xvi, 22, 23, 63, 117, 118, 174, 263, 273, 274, 277, 299,
SPA	Stationary Phase Approximation xiv, 64, 65, 68, 74, 75, 79, 90, 91, 93, 94, 104, 112, 118, 120, 123, 125, 127,
TCP	Thomas Cluster Process xiv, 142, 143,
TDMA	Time Division Multiple Access 157,

TE	Transverse Electric 36, 38–40, 42, 68,
TEM	Transverse ElectroMagnetic 36, 42, 68,
TM	Transverse Magnetic 36–38, 40, 42, 68,
TX	Transmitting 184,
UE	User Equipment 8, 139, 140, 158, 208, 295, 296, 300, 301, 303–305, 307, 309, 310, 316, 317, 320, 323, 327, 341,
ULA	Uniform Linear Array 298, 300, 304,
USRP	Universal Radio PeriPheral 343,
V2X	Vehicle-to-Everything xiv–xvi, 5, 8, 10, 160–162, 180, 192, 193, 226, 287, 337, 342,
VNA	Vector Network Analyser xiv, 85, 87,
Wi-Fi	Wireless Fidelity 60, 79, 342,

Notations and Symbols

$\ \cdot \ $	2-Norm of a vector or matrix
$\arg\{ \cdot \}$	Argument of a complex quantity
$\lceil \cdot \rceil$	Ceil Integer Rounding
j	Complex Unity i.e. $j^2 = -1$
$O(\cdot)$	Complexity Order
$\mathbb{E}_{X Y}[\cdot], \mathbb{E}_X[\cdot Y]$	Conditional Expectation over the random variable X given the random variable Y
$\mathbb{P}_{X Y}(\cdot), \mathbb{P}_X(\cdot Y)$	Conditional Probability over the random variable X given the random variable Y
$\cdot^*$	Conjugate Operator of a complex quantity
$\times$	Cross/Cartesian Product between two vectors/two sets
$[x]_{\text{dB}}$	dB Operator i.e. $10 \log_{10} x$
$[x]_{\text{dBm}}$	dBm Operator i.e. $10 \log_{10} x + 30$
δ	Dirac Delta Function
$\cdot$	Dot Product between two vectors
$\{a_i\}$	Enumeration of the multiple elements a_i , by varying the index i (or other indices) of a
$\mathbb{E}_X[\cdot]$	Expectation over the random variable X . If there is no subscript, the expectation is applied on every random variable
$\ \cdot \ _F$	Frobenius Norm of a vector or matrix
$\cdot^H$	Hermitian Operator a matrix or vector
$\text{Im}\{ \cdot \}$	Imaginary Part of a complex quantity
$\mathbb{1}(\cdot)$	Indicator Function
δ_{ij}	Kronecker symbol equal to 1 if $i = j$, and 0 otherwise

∇^2	Laplacian Operator
Q_n	Marcum Q-function of order n
$\mathbf{A}, \mathbf{a}$	Matrix and Vector Notation
$\succeq, \succ$	Matrix Comparison Operators
$[\mathbf{A}]_{ij}$	Matrix Element (i, j) of $\mathbf{A}$
$ \cdot $	Modulus/Norm/Lesbesgue Measure of a complex number/complex vector/set
∇	Nabla Operator
$\Pi(\cdot)$	Non-Centred Rectangular Function which is non-zero between 0 and 1
eigvec	Principal Eigenvector of a matrix
$\mathbb{P}_X(\cdot)$	Probability over the random variable X . If there is no subscript, the probability is applied on every random variable
$\text{Re}\{\cdot\}$	Real Part of a complex quantity
$\cdot^T$	Transpose Operator of a matrix or vector
$\Lambda(\cdot)$	Triangular Function which is non-zero between -1 and 1
vec	Vectorisation Operator of a matrix

Introduction — Sensing and Communication

1

NOWADAY, [ElectroMagnetic \(EM\)](#) wireless systems are designed to fulfil (usually one among) three different purposes, namely radar, communication, and power transfer. The *radar* or *sensing*¹ function aims to gather information about the unknown environment, by transmitting a known signal, receiving echoes of this signal from the numerous objects composing the environment, and processing this signal to extract their parameters, i.e. distances, angles (or directly a position), and speeds. In contrary, The *communication* function objective is to transmit information between devices, in a supposedly known environment. Thus, the transmitted signal is now unknown at the receiver. In practice, the environment is also unknown, but it is estimated at the receiver, most of the time using pilot symbols, i.e. a part of the transmitted signal which is known beforehand. Finally, the *power transfer* aims to transmit electrical energy to devices, to eliminate the need of battery and wire supply.

In this thesis, a focus is made on the sensing and communication functions. Historically, radar and communication systems are operating at distinct carrier frequencies. However, in recent years, owing to the spectrum scarcity, the carrier frequency ranges of both functions tend to increase and get closer, or even overlap. Additionally, both purposes tends to require larger bandwidths and antenna arrays. Therefore, the possibility to operate both functions simultaneously has received an increased attention. This especially raises the following questions:

1. Both the sensing and communication function share the same [EM](#) spectrum. Thus, how to allocate the available resources between both functions?
2. Instead of allocating resources separately between the sensing and communication purpose, could we perform both functions using the same resources, and how to do it efficiently?
3. Owing to the complementarity of the sensing and communication function, could both functions cooperate with each other, in order to improve their respective performance?

Note that, in [\[Liu+22\]](#), the gains of the second and third challenges are respectively named integration and coordination gain.

¹In this thesis, the term *sensing* always refers to radar sensing.

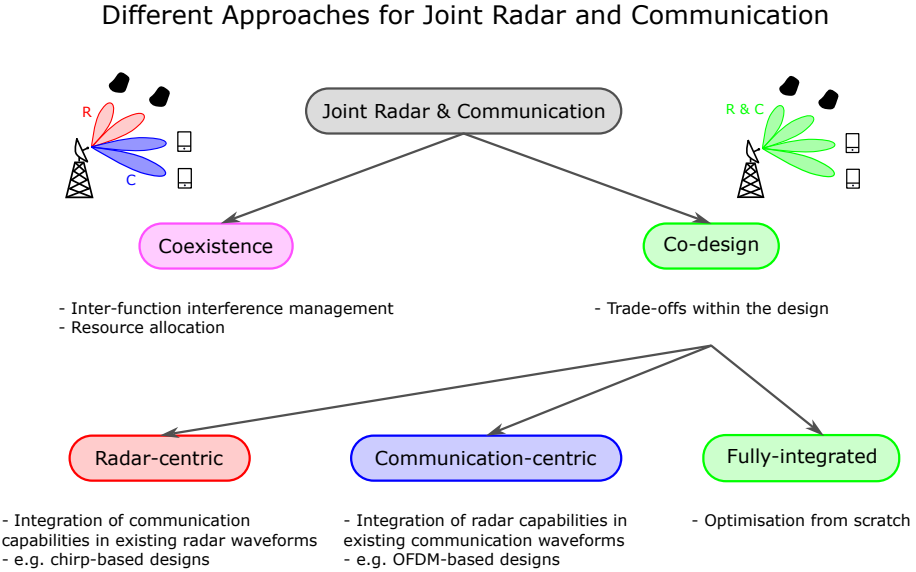


Fig. 1.1 Joint radar and communication under multiple perspectives.

In order to answer to these questions, two approaches have been adopted in the literature: coexistence and co-design. On the one hand, with the *coexistence* perspective, different waveforms are transmitted for each function. The same or different hardware may be used. The main challenges to handle are the interference mitigation and the resource management between both functions. On the other hand, with the *co-design* perspective, a single waveform is transmitted for both functions, using the same hardware. In that case, the main challenge resides in the design of the joint waveform, and more particularly the trade-offs made between both purposes. Integrated design is particularly appealing, as the use of a single hardware could lead to spectral efficiency increase, complexity reduction, and cost reduction. Numerous names are given to co-designed systems, e.g. [Dual-Function Radar-Communication \(DFRC\)](#), [Joint Communication and Sensing \(JCAS\)](#), or [Integrated Sensing and Communication \(ISAC\)](#).

Within the [ISAC](#) literature, three different [ISAC](#) design perspectives are distinguished: radar-centric, communication-centric, or fully-integrated. With the *radar-centric* point of view, one leverages on existing radar waveforms. Then, information is embedded in the waveform, e.g. through phase modulation, amplitude modulation, or index modulation. This includes for instance chirp-modulated waveforms (Section 8.1 of Chapter 8), or frequency hopping systems [Wu+21]. Modest communication capabilities are introduced, usually at the price of slight (or no) radar performance reduction. Contrariwise, with the *communication-centric* point of view, one starts with existing communication waveforms, and modifies the structure to

Joint Radar and Communication: Design Approaches

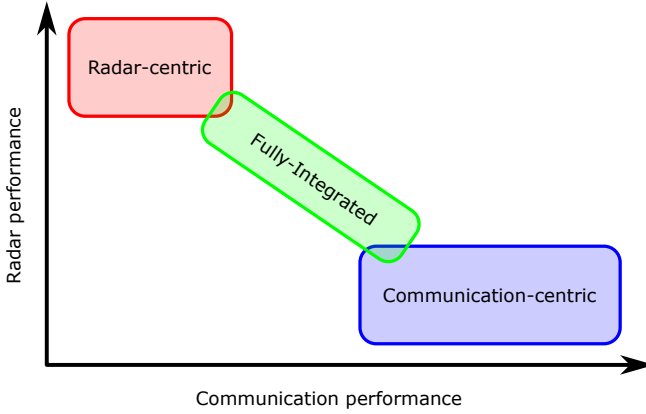


Fig. 1.2 Performance of the three joint radar and communication design approaches (inspired from [Ma+20]).

integrate sensing capabilities. This mainly includes variations of the [Orthogonal Frequency Division Multiplexing \(OFDM\)](#) modulation (Section 8.2 of Chapter 8), but also protocol-based approaches [Kum+18], or spread spectrum techniques [MSJ19]. Small sensing capabilities are introduced, usually at the price of slight (or no) communication performance reduction. Finally, with the *fully-integrated* approach, the [ISAC](#) waveform is designed from scratch through optimisation procedures [Zho+22]. Thus, no function is especially privileged. Though, performance trade-offs are made between both functions, depending on the definition of the problem.

The different perspectives of joint radar and communication are summarised in Figure 1.1, and the performance of the three design approaches are illustrated in Figure 1.2. Yet, the limitations between the different perspectives are not always defined. For instance, even with co-designed systems, coexistence issues, i.e. interference mitigation and resource management, are also challenging when multiple systems are operating simultaneously. Still, these issues are not inherent of [JCAS](#) systems. Other examples are the chirp-modulated [OFDM ISAC](#) waveform [Bod+23], and the multi-carrier [Frequency-Modulated Continuous Wave \(FMCW\)](#) waveform [Wan+18]. Depending on the privileged purpose, both may be considered as radar- or communication-centric approach. Furthermore, multiple perspectives may be combined. A very elegant way to summarise how they may be combined is proposed in [SZA24]. First, the *data lattice* is defined as a mathematical construct that enables data symbols (known or unknown, respectively for radar and communication applications) to be distributed in a multidimensional space. For instance, the time domain is considered for usual complex modulated communications or phase-coded sensing. Instead, the time-frequency space is considered for [OFDM](#). Nonetheless, this

Physical Layer for 6G and Beyond

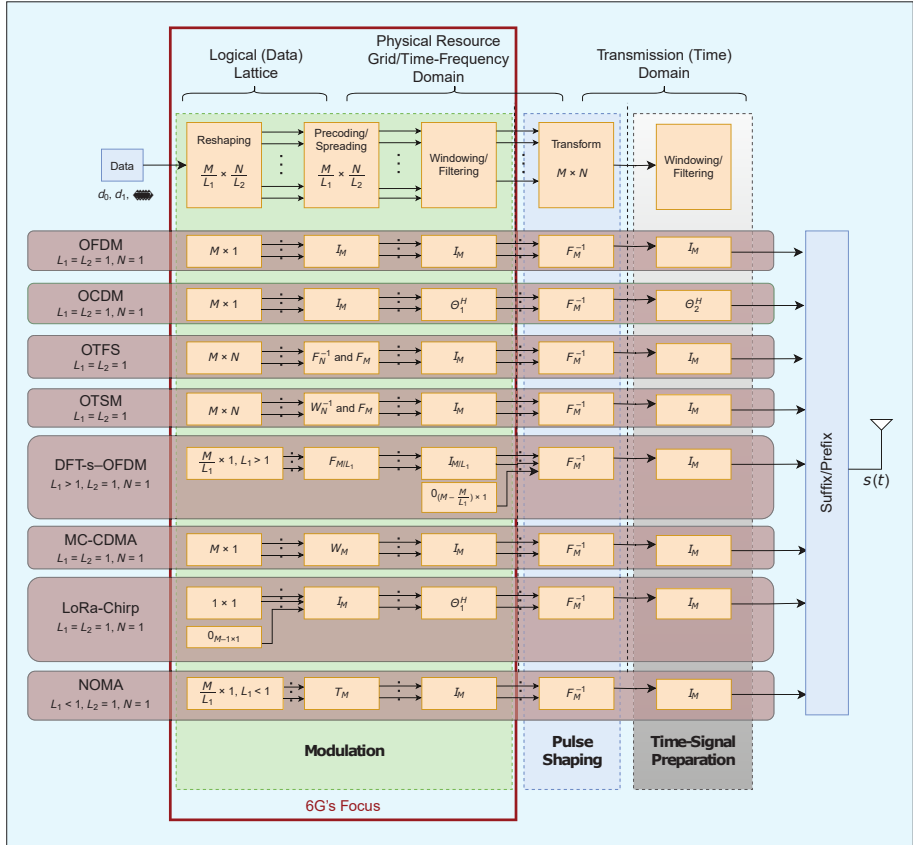


Fig. 1.3 Illustration of different waveforms discussed in the literature for next generation networks [SZA24].

space may include any logical domain. For example, the **Orthogonal Time-Frequency Spacing (OTFS)** modulation uses a delay-Doppler space. This is a mathematical structure in which the symbols are distributed, under any arbitrary form. Then, transformations are performed to reshape the data lattice into the *physical resource grid*, i.e. the time, frequency and space dimensions. Finally, the signal is transmitted after windowing, filtering and suffix/prefix operations. This is illustrated in Figure 1.3.

With such abstraction, the co-design perspective studies how to define the data lattice, windowing, filtering, and suffix/prefix operations, such that both sensing and communication may be performed. In contrary, the co-existence perspective considers divisions of the physical resource grid. Each part may be allocated to sensing, communication or joint waveforms. Thus, one may simultaneously transmit

Applications for Joint Radar and Communication

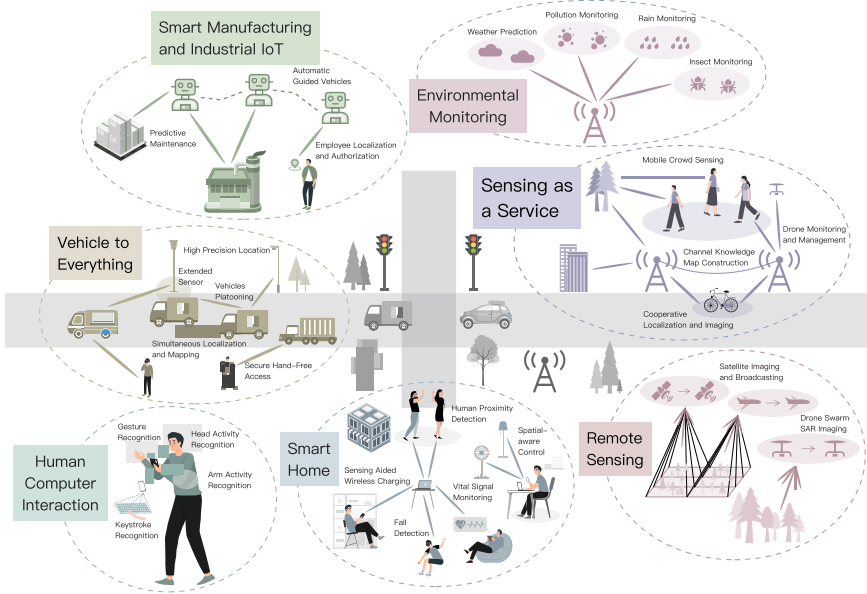


Fig. 1.4 Applications for ISAC systems [Liu+22].

co-designed waveform on different portions of the physical resource grid.

1.1 Scenarios and Applications

Numerous usages of ISAC systems may be found in multiple different scenarios, e.g. **Vehicle-to-Everything (V2X)**, human-computer interaction, smart home and manufacturing, remote sensing, environmental monitoring, as illustrated in Figure 1.4. These scenarios are further detailed in [Liu+22].

Yet, the specific case of the **V2X** scenarios is particularly considered in this thesis, as ISAC systems have raised an increased interest in recent years for such applications. For instance, they may be embedded in vehicles and/or the infrastructure for multiple purposes, such as autonomous driving, cooperative sensing, and **V2X** communications. As a non-exhaustive list, multiple interesting challenges especially related to this application are the interference management between multiple ISAC systems, the embedding of such systems in vehicles, the design of distributed algorithms for resource allocation, or the fusion of information coming from the numerous embedded sensors, e.g. cameras or lidars. In this thesis, as further detailed in Section 1.2, a focus is made on the first challenge, namely the interference management.

1.1.1 Interference Management in Automotive Scenarios

The interference between multiple radar or ISAC systems is a major issue, owing to the constantly increasing number of vehicles and RoadSide Units (RSUs). This challenge has been initially tackled by the *MOre Safety for All by Radar Interference Mitigation* (MOSARIM) project [10]. Different interference mitigation techniques have been developed involving space, time, frequency, code, and polarisation domains. For instance, for coexisting radar systems, [RDP18] has proposed an adaptive digital beamforming technique where the beamforming weights are updated depending on the estimated interference level. Modifications of chirps bandwidth, duration, and pulse repetition interval are implemented in [SM19; Kit+19; Kim16]. Carrier sensing multiple access schemes are adopted in [Ish+19; Ayd+19]. Different multiple access strategies are also compared in [JR21].

However, even if dynamic modifications of the parameters are possible with radar systems, such solutions are not suited for embedded ISAC systems without back-haul link, since communication receivers are not collocated with the transmitter, except when the modification simultaneously carries information and helps for interference mitigation [LS23]. Another approach applicable to both radar and ISAC scenarios is to consider interference suppression algorithms. This can be implemented through iterative interference reduction [Ume+18], subspace error correction [YZ19], short time Fourier transform interpolation [NKY19], sparsity-based [CG19] or other reconstruction methods [Liu+20].

1.1.2 Full-Duplex Integrated Sensing and Communication

In many of the scenarios described in [Liu+22], multiple ISAC systems may coexist in the same environment, either by using orthogonal resources (time, frequency, or space), or they may transmit and receive simultaneously in the same frequency band. By analogy with cellular networks, the latter is denoted as (*inband*) *full-duplex ISAC*.

The full-duplex ISAC scenario is abstracted in Figure 1.5. The ISAC system of interest transmits a joint radar and communication signal. On the one hand, this signal first reaches other transceivers, which can decode the communication carried by the ISAC waveform. This communication link is named *downlink*, by analogy with the cellular denomination. Then, the signal is reflected on the environment, also comprising the other transceivers, and it is received at the ISAC system of interest. On the other hand, the other transceivers simultaneously transmit their own joint radar and communication signals (or simply communication signal if it is compatible with the receiver), which are also received at the ISAC system of interest. This communication link is named *uplink*, again by analogy with the cellular denomination. It is assumed that the same frontend is used for both functions. This applies for monostatic and multistatic configurations. Additionally, especially in

Full-Duplex ISAC Scenario

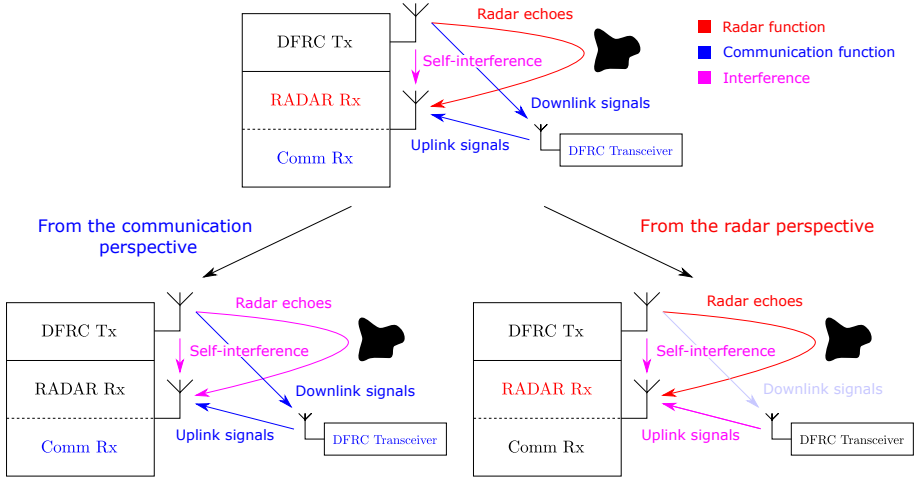


Fig. 1.5 Representation of full-duplex ISAC scenarios. The downlink and uplink transceivers may be the same or different network nodes, or they may also be considered as radar targets. Interference arising from other non-compatible systems is omitted for the sake of clarity.

monostatic configurations, self-interference from the transmitting to the receiving antennas can occur. Additional interference from other non-compatible systems is also present. It is not illustrated for readability.

Consequently, from the radar perspective, the useful signals to decode are the radar echoes from the environment. The interference comprises the self-interference, the interference from other non-compatible systems, and the communication uplink signals. At the opposite, from the communication perspective, the useful signals to decode are the transmitted ISAC waveform. The interference comprises the self-interference, the interference from other non-compatible systems, and the radar echoes. Note that full-duplex scenarios encompass both coexisting and co-designed systems, as the interference between both functions arises from another communication or ISAC system. In this thesis, the self-interference is neglected, as it is not specific to ISAC scenarios, but to full-duplex systems in general. It has also been extensively studied for ISAC, and it is usually mitigated through EM isolation, beamforming, analogue and digital interference cancellation, or other techniques [Baq+19; Mor+22; Liu+23; ZMS18; ZMS18]. The self-interference cancellation problem is even combined with a power allocation problem in [Che+23] to maximise both radar and communication performance.

Three examples of full-duplex scenarios are shortly developed here:

Full-Duplex V2X Scenario: In V2X scenarios, a vehicle equipped with an ISAC system may detect targets, e.g. other vehicles, User Equipments (UEs), e.g. cyclists and pedestrians, or obstacles, while simultaneously receiving communication signals from other vehicles, or from the infrastructure. The opposite is also possible: an RSU may detect targets while simultaneously receiving a communication signals from vehicles, UEs or other RSUs. In both cases, the ISAC system of interest suffers from interference coming from other vehicles, RSUs or UEs. In this thesis, such scenarios are often be considered for illustration, as further detailed in Section 1.2.

Full-Duplex Indoor Scenario: In indoor scenarios [Jia+18; Sto+21], an Access Point (AP) equipped with an ISAC system may serve and track UEs, while receiving simultaneously communication signals from other APs, and the same or/and other UEs. The AP is suffering from interference coming from other APs and/or other UEs.

Full-Duplex Crowd Sensing Scenario: In crowd sensing applications [Sto+23], a Base Station (BS) equipped with an ISAC system may serve and track multiple UEs. The BS is simultaneously receiving data from another macro-BS or from the UEs. The BS is suffering from interference coming from other BSs and/or other UEs.

1.2 Research Questions, Content and Contributions of the Thesis

This thesis is divided in three parts, tackling joint radar and communication challenges at three different scales. V2X scenarios are considered at the vehicle, street or city level, as they well illustrate the faced challenges, as shown in Figure 1.6. Yet, in most of the cases, the obtained results are generalisable to other applications. Numerous research questions, which are subparts of the three questions raised at the start of this Chapter, are considered in this thesis. Those are related either to the modelling or the performance evaluation of JCAS, at the link or system level. This is summarised in Figure 1.7.

Part I focuses on the link level. Leveraging on the EM theory, the radiation and propagation mechanisms involved in ISAC applications are analysed from scratch. This encompasses the transmission from radiative elements, the propagation in free-space, the reflection on obstacles or targets, and the reception by other radiative elements. Owing to the increase of the carrier frequencies and number of antennas, the usual radar and communication models of the literature should be adapted, to encompass additional propagation effects. For instance, with large antenna arrays, the spherical wavefronts cannot be assumed to be planar. Moreover, the dimensions of the obstacles or targets become electrically large w.r.t. the signals wavelength and the considered distances. Thus, such analysis enables to develop accurate radar and communication channel models. More specifically, Chapter 2 starts from

the Maxwell's equation to obtain at the end expressions of the [EM](#) fields radiated by source distributions. The different propagation regions, namely the near- and far-field, are also detailed. Then, Chapter 3 leverages on Chapter 2 to obtain the radar and communication propagation models usually considered in the literature, and to study how they are related to each other. More specifically, for the radar function, the limits of these models are highlighted, and extensions are developed to overcome these restrictions. Experimental measurements are also presented. Besides, as new models for the received power are developed, the associated signal processing must be adapted. This is the focus of Chapter 4, which follows the approach of Chapter 3 to analyse how it is possible to develop accurate expressions of the received signal, and leverage on the propagation effects, which were neglected with usual signal models from the literature, in order to increase the sensing performance. All these results find for instance an application in automotive radar systems: let us assume that two vehicles are following each other. The vehicle behind is equipped with an [ISAC](#) system at its front. Thanks to the novel [EM](#)-based signal models, the back of the next vehicle ahead is abstracted, and the power of the radar echo is accurately estimated, enabling to well predict the achieved performance, and design the system. Then, the distance to the next vehicle ahead is estimated with an improved accuracy.

The estimation of radar and communication performance is critical for the design of [ISAC](#) systems. With the accurate radar and communication propagation models developed in Part I, it is possible to estimate the received radar and communication power levels, given the location of the transmitting and receiving nodes, and the location of the target. Thus, with the help of the [Stochastic Geometry \(SG\)](#) framework, it is possible to evaluate average system performance through averaging over random distributions of network nodes, targets, and/or obstacles. This may be done either at the street level, or considering a complete city. Though, relevant performance metrics are required to perform an accurate assessment. These are the objectives of Part II. First, Chapter 5 summarises the foundations of [SG](#). Then, Chapter 6 details usual performance metrics for both sensing and communication. Yet, a joint evaluation of the capabilities of both functions is interesting to model the interactions between sensing and communication, especially in full-duplex scenarios. Such metrics enable for instance to perform unconstrained optimisation of the network parameters, or to evaluate the performance of one function conditioned on a given requirement for the other. Consequently, [JCAS](#) performance metrics are also defined. In this chapter, those are evaluated in an [V2X](#) scenario at the street level: let us again assume that two vehicles are following each other. The vehicle behind is equipped with an [ISAC](#) system at its front, enabling to simultaneously receive a radar echo from the next vehicle ahead, and a communication signal from the infrastructure. The joint metrics enable to predict how the system will perform, and for instance to optimise the transmitted powers, maximising the achieved performance. Still, the [SG](#) framework requires numerous assumptions in the system model. Hence, ray-tracing

simulations are also performed to assess the accuracy of the obtained results. Finally, rather than considering one single street, Chapter 7 analyses networks in a city scale. At this level, the complete infrastructure may be able to cooperate, and additional complex interference mechanisms must be taken into account. Therefore, the sensing and communication performance metrics defined in Chapter 6 are extended to handle cooperation for both functions between the nodes of the infrastructure, with a complex system model comprising interference from direct links, reflections, diffractions, and building penetrations. Further optimisation is also performed.

Unfortunately, the main drawback of the SG framework is the trade-off between the accuracy of the system model, and the complexity of the mathematical expressions. Either many assumptions are made to simplify the model and obtain tractable results, at the price of less accurate results, or a complex model is considered, and the mathematical results are often intractable. Therefore, Part III makes a step backward, to focus again on full-duplex vehicular scenarios in a street at the link level, but studied thoroughly outside the SG framework. In such applications, the radar and communication functions interfere with each other, and this interference must be mitigated. A first answer to this issue is the interference cancellation. Leveraging on the information carrier by the radar or communication signals, cooperation between both functions may enable to enhance the overall performance of the system. A second answer to the inter-function interference is beamforming, with multi-antenna systems. If the radar targets and communication transmitters are separable in the spatial domain, each function is enabled to focus only on the signal of interest. First, for the development of these two solutions, Chapter 8 details two popular radar- and communication-centric ISAC waveforms, namely the phase-coded FMCW, and the OFDM modulation. Then, Chapter 9 is dedicated to the interference cancellation algorithms. A first insight of the performance improvement achieved with interference cancellation was provided in Chapter 6. However, the high level of abstraction required by the SG framework limited the accuracy of the evaluated radar and communication performance. Hence, leveraging on the specific processing for the waveforms presented in Chapter 8, multiple interference cancellation schemes are proposed, namely at the radar, communication, or at both functions, in static and dynamic scenarios. Finally, regarding the beamforming solution, Chapter 10 analyses multiple beamforming algorithms, depending on the number of radar targets and communication signals. The optimisation of the beamformers is performed for a specific objective function, namely the product of the Signal-to-Interference-plus-Noise Ratios (SINRs) of all radar and communication links, which ensures fairness between both functions, for every target and communication signal.

Figure 1.8 summarises the structure, content, and contributions of this thesis. Before starting the first part, two additional sections are provided in this chapter,

Summary of the Thesis Content and Contributions

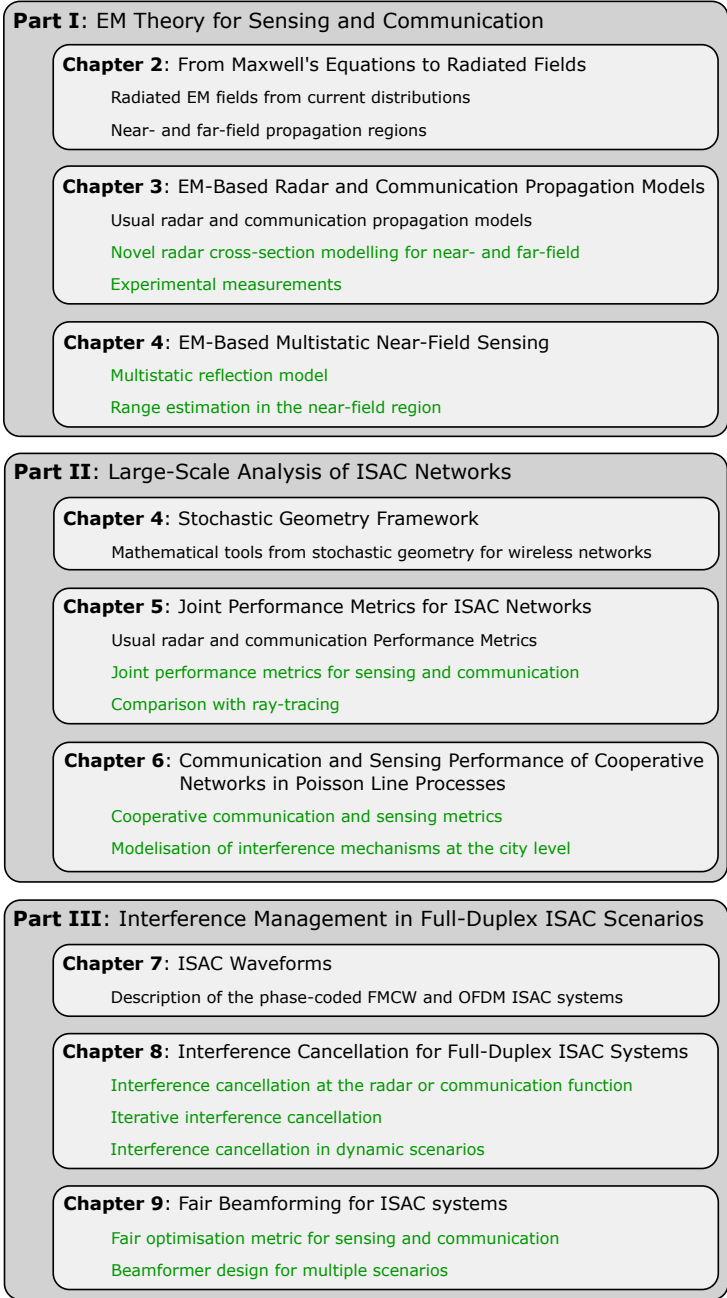


Fig. 1.8 Structure and summary of the thesis content. The contributions are highlighted in green.

acting as useful reminders for the three parts. First, Section 1.3 describes how single-antenna channel models are derived for both functions from a signal processing perspective. Then, Chapter 1.4 quickly reminds numerous usual performance metrics for the radar and communication functions. Some radar metrics are further detailed, as they are often considered in the following chapters.

1.3 Single-Antenna Channel Models for Communication and Sensing

Let us consider a signal s_{RF} in radio-frequencies, such that

$$\exists B, f_c \quad \text{such that} \quad S(f) = 0 \quad \forall f \notin \left[f_c - \frac{B}{2}, f_c + \frac{B}{2} \right],$$

where B and f_c are respectively defined as the *bandwidth* and *carrier frequency* of the signal. For narrowband signals, one can moreover assume that $B \ll f_c$. The *equivalent complex baseband representation* s of such signal is defined as

$$s_{\text{RF}}(t) \triangleq \text{Re} \left\{ s(t) e^{j2\pi f_c t} \right\}.$$

The transmitted signals are usually expressed in their equivalent complex baseband representation to simplify the mathematical analyses.

In this section, usual channel models for communication and sensing are presented. In a first time, single-antenna systems are considered. The multiple-antenna channel models are further discussed in Sections 4.2 of Chapter 4.

1.3.1 Single-Link Channel

Considering in a first time a single link of time-varying delay $\tau_0(t)$, attenuation a_0 , and phase shift ψ_0 , the received signal is written in radio-frequencies as

$$u_{\text{RF}}(t) = \text{Re} \left\{ a_0 e^{j\psi_0} s(t - \tau_0(t)) e^{j2\pi f_c (t - \tau_0(t))} \right\},$$

leading to the following signal in complex baseband representation:

$$u(t) = a_0 e^{j\psi_0} e^{-j2\pi f_c \tau_0(t)} s(t - \tau_0(t)). \quad (1.1)$$

One may develop the time-varying delay $\tau_0(t)$ as

$$\tau(t) = \frac{r_0}{c} - \frac{v_0}{c} t,$$

where r_0 is the total travelled distance, and v_0 is the resultant of the velocities projections on the link. This includes the velocity of the transmitter and receiver, but also the velocity of the obstacles or target. Following the convention of the radar community, the sign of v_0 is supposed to be positive when the link is shortened. Inserting this expression in (1.1) gives

$$u(t) = a_0 e^{j\psi_0} e^{-j2\pi f_c \frac{r_0}{c}} e^{j2\pi f_c \frac{v_0}{c} t} s\left(\left(1 - \frac{v_0}{c}\right)t - \frac{r_0}{c}\right) \approx \alpha e^{j2\pi f_{D0} t} s(t - \tau_0), \quad (1.2)$$

where $f_{D0} \triangleq f_c v_0/c$ is the *Doppler frequency* of the link, $\tau_0 \triangleq r_0/c$ is the (initial) delay, and

$$\alpha_0 = a_0 e^{j\psi_0} e^{-j2\pi f_c \tau_0}.$$

The approximation supposes that the speed v_0 is small compared to the speed of light. This enables to neglect the distortion of the signal induced by the movement of the target. Since the phase shift and attenuation of the channel are usually supposed to be unknown, the additional phase shifts induced by τ_0 cannot be distinguished from ψ_0 . Thus, all the attenuation and phase shift terms are included in the complex coefficient α_0 .

The channel can be modelled as a linear time-variant system, and the corresponding channel impulse response is given in Box 1.1. Equation (1.2) is therefore obtained by convolving the transmitted signal with the channel impulse response:

$$u(t) = \int_{\tau} h(t, \tau) s(t - \tau) d\tau.$$

1.1: Single-Link Single Antenna Channel Model

$$h(t, \tau) = \alpha_0 e^{j2\pi f_{D0} t} \delta(\tau - \tau_0)$$

1.3.2 Multi-Links Channel

Equation (1.2) can be extended by assuming that the environment is modelled as a sum of Q single links, each with their own attenuations, phase shifts and delays. It leads to the following received signal in baseband:

$$u(t) = \sum_{q=0}^{Q-1} \alpha_q e^{j2\pi f_{Dq} t} s(t - \tau_{0,q}). \quad (1.3)$$

The complex coefficient α_q potentially merge the attenuations and phase shifts of multiple clustered links, with comparable delays and Doppler frequencies.

One can again model the channel as a linear time-variant system and write the corresponding channel impulse response as in Box 1.2. In the sensing context, this channel model is named *single-point targets channel model*, as the environment is discretised in a set of Q points, each representing a target.

1.2: Multi-Link Single Antenna Channel Model

$$h(t, \tau) = \sum_{q=0}^{Q-1} \alpha_q e^{j2\pi f_{Dq}t} \delta(\tau - \tau_q)$$

1.4 Communication and Sensing Performance Metrics

Numerous performance metrics have been created to evaluate the performance of communication and sensing systems. On the one hand, for the communication function, one may namely cite from [SBC20] the rate or capacity, energy efficiency, coverage, exposure, latency, and [Bit Error Rate \(BER\)](#). These metrics are often related to the [SINR](#) of the communication link. On the other hand, for the radar function, some performance metrics are the detection, misdetection, and false alarm probability, Cramér-Rao lower bound of the estimates of the targets parameters, resolution in the range, Doppler, and angle domains, ambiguity function, and estimation rate [Bli14]. These are also often related to the [SINR](#) of the radar link, or the characteristics of the radar waveform.

Different definitions of these metrics are encountered, depending on the considered framework. For instance, they are redefined in Chapter 6, specifically for the [SG](#) framework considered in Part II. In the other parts of this thesis, the [BER](#) is considered for the communication function. For the radar function, the detection and false alarm probability, ambiguity function, [CRB](#) and resolutions are analysed. Thus, these metrics are detailed in this section.

1.4.1 Ambiguity Function

The ambiguity function is critical for the design of radar waveforms. It is helpful to characterise many quantities, as for instance the resolution in the range and Doppler domains or the sidelobes level.

1.4.1.1 Single-Point Single-Target Delay-Doppler Estimation

Let us consider a single-point single-target channel model with one target of delay τ , and Doppler frequency f_D . Following (1.2), for a transmitted signal s of energy E_s ,

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the received signal in baseband is written as

$$u(t) = \alpha e^{j2\pi f_D t} s(t - \tau) + w(t),$$

where α is a complex coefficient, w is an **Additive White Gaussian Noise (AWGN)** of variance σ_w^2 . Assuming that the parameter α is unknown, the maximum likelihood estimation problem of the target parameters, jointly with the unknown parameter α , is written as

$$\begin{aligned} \hat{\tau}, \hat{f}_D, \hat{\alpha} &= \arg \min_{\tau, f_D, \alpha} \int_t \left| u(t) - \alpha e^{j2\pi f_D t} s(t - \tau) \right|^2 dt \\ &= \arg \min_{\tau, f_D, \alpha} 2|\alpha|^2 E_s - 2\operatorname{Re} \left\{ \alpha^* \int_t u(t) e^{-j2\pi f_D t} s^*(t - \tau) dt \right\}. \end{aligned} \quad (1.4)$$

First, from the second term, since $\operatorname{Re}\{z\} \leq |z| \forall z \in \mathbb{C}$ with the equality achieved when $z \in \mathbb{R}^+$, the phase of α is estimated as

$$\arg\{\hat{\alpha}\} = \arg \left\{ \int_t u(t) e^{-j2\pi \hat{f}_D t} s^*(t - \hat{\tau}) dt \right\}.$$

Then, minimising (1.4) after inserting the estimate of the phase of α , the delay and Doppler frequency are estimated as

$$\hat{\tau}, \hat{f}_D = \arg \max_{\tau, f_D} \left| \int_t u(t) e^{-j2\pi f_D t} s^*(t - \tau) dt \right|.$$

When the couple of values $(\tilde{\tau}, \tilde{f}_D)$ is tested, neglecting the noise terms, one compute

$$\left| \alpha \int_t s(t - \tau) s^*(t - \tilde{\tau}) e^{-j2\pi(\tilde{f}_D - f_D)t} dt \right| \propto \left| \chi_s(\tilde{\tau} - \tau, \tilde{f}_D - f_D) \right|,$$

where the function χ_s is named *ambiguity function* of the waveform s . Its expression is given in Box 1.3. From the Cauchy-Schwartz inequality, one may show that $|\chi_s(\tau, f_D)| \leq |\chi_s(0, 0)| \forall \tau, f_D$. Therefore, the ambiguity function is maximised at the location of the target in the delay-Doppler map.

1.3: Ambiguity Function

$$\chi_s(\tau, f_D) \triangleq \int_t s(t) s^*(t - \tau) e^{-j2\pi f_D t} dt$$

An ideal ambiguity function would be a Dirac function, i.e. $\chi_s(\tau, f_D) = \delta(\tau)\delta(f_D)$, as it would enable to perfectly distinguish targets in the delay-Doppler map, while achieving a perfect accuracy, thanks to the absence of sidelobes and the infinitely

thin mainlobe. Such function would be achieved by transmitting infinitely a white noise. However, in practical systems, some tolerance w.r.t. the location of the target is required, to be more robust against delay and Doppler mismatches.

1.4.1.2 Ambiguity Function of a Rectangular Pulse

Let us define the normalised rectangular pulse as

$$s(t) = \frac{1}{\sqrt{T}} \Pi\left(\frac{t + \frac{T}{2}}{T}\right),$$

where the function Π is the non-centred rectangular function, defined as

$$\Pi(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

For $0 \leq \tau \leq T$, the ambiguity function of the normalised rectangular pulse is computed as

$$\begin{aligned} \chi_s(\tau, f_D) &= \frac{1}{T} \int_t \Pi\left(\frac{t + \frac{T}{2}}{T}\right) \Pi\left(\frac{t - \tau + \frac{T}{2}}{T}\right) e^{-j2\pi f_D t} dt = \frac{1}{T} \int_{-\frac{T}{2}+\tau}^{\frac{T}{2}} e^{-j2\pi f_D t} dt \\ &= \frac{1}{T} e^{-j\pi f_D \tau} \frac{e^{j\pi f_D (T-\tau)} - e^{-j\pi f_D (T-\tau)}}{j2\pi f_D} \\ &= e^{-j\pi f_D \tau} \left(1 - \frac{\tau}{T}\right) \text{sinc}\left(f_D T \left(1 - \frac{\tau}{T}\right)\right). \end{aligned}$$

Similarly, for $-T \leq \tau \leq 0$, the ambiguity function is computed as

$$\begin{aligned} \chi_s(\tau, f_D) &= \frac{1}{T} \int_t \Pi\left(\frac{t + \frac{T}{2}}{T}\right) \Pi\left(\frac{t - \tau + \frac{T}{2}}{T}\right) e^{-j2\pi f_D t} dt = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}+\tau} e^{-j2\pi f_D t} dt \\ &= \frac{1}{T} e^{-j\pi f_D \tau} \frac{e^{j\pi f_D (T+\tau)} - e^{-j\pi f_D (T+\tau)}}{j2\pi f_D} \\ &= e^{-j\pi f_D \tau} \left(1 + \frac{\tau}{T}\right) \text{sinc}\left(f_D T \left(1 + \frac{\tau}{T}\right)\right). \end{aligned}$$

Therefore, combining the two cases, one obtain Box 1.4. This gives a triangular function w.r.t. the delay, modulated by a sinc function for non-zero Doppler frequencies. The amplitude of Box 1.4 is illustrated in Figure 1.9.

1.4.2 Resolution

The *resolution* is defined as the capacity to distinguish two targets in the delay, Doppler or angle domain. The resolution is directly related to the width of the mainlobe and the level of the sidelobes of the ambiguity function, as the parameters of the targets are extracted from the latter. Similarly to the computation of the bandwidth, the

1.4: Ambiguity Function of a Normalised Rectangular Pulse

$$\chi_s(\tau, f_D) = \begin{cases} e^{-j\pi f_D \tau} \left(1 - \frac{|\tau|}{T}\right) \text{sinc}\left(f_D T \left(1 - \frac{|\tau|}{T}\right)\right) & \text{for } -T \leq \tau \leq T \\ 0 & \text{otherwise} \end{cases}$$

Ambiguity Function of a Rectangular Pulse

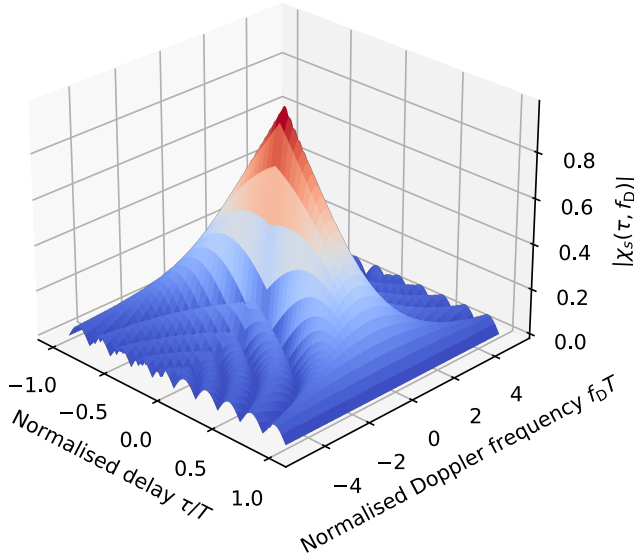


Fig. 1.9 Amplitude of the ambiguity function of the normalised rectangular pulse.

resolution has been defined in multiple ways mathematically. The simplest definition is to consider the location of the first zero of the ambiguity function. This is named the *Rayleigh resolution*.

1.4.2.1 Rayleigh Resolution of a Rectangular Pulse

The delay and Doppler resolution of the normalised rectangular pulse are computed from its ambiguity function, which has been obtained in Box 1.4.

Delay resolution: For the delay, the first zeroes of Box 1.4 along the delay-axis are computed as

$$|\chi_s(\tau, 0)| = 1 - \frac{|\tau|}{T} = 0 \quad \Leftrightarrow \quad \tau = \pm T.$$

Therefore, the Rayleigh resolution of the delay is $\delta_\tau = T$. This result may be related to the Rayleigh bandwidth of the rectangular pulse, i.e. the inverse of its duration. Therefore, the delay and range resolutions are obtained in Box 1.5.

1.5: Delay and Range Resolution

$$\delta_\tau = \frac{1}{B} \quad \leftrightarrow \quad \delta_r = \frac{c}{2B}$$

The expression $c/2B$ is also applicable to many other waveforms. Still, for the rectangular pulse, the pulse bandwidth and duration are related to each other, and the pulse duration also affects the transmitted power. This issue can be overcome thanks to the pulse compression technique, which is presented in Section 8.1.1.1 of Chapter 8.

Doppler resolution: For the Doppler, the first zeroes of Box 1.4 along the Doppler-axis are computed as

$$|\chi_s(0, f_D)| = \text{sinc}(f_D T) = 0 \quad \leftrightarrow \quad f_D = \pm \frac{1}{T}.$$

Thus, the Doppler and speed resolutions are obtained in Box 1.6.

1.6: Doppler and Speed Resolution

$$\delta_{f_D} = \frac{1}{T} \quad \leftrightarrow \quad \delta_v = \frac{c}{2f_c T}$$

1.4.2.2 Limitation of the Rayleigh Resolution

Depending on the shape of the ambiguity function, the Rayleigh resolution may be misleading. For instance, the ambiguity function may quickly drop to a small value, and reach zero only at large distance from the centre of the main lobe, as it will be encountered in Chapter 4. Therefore, looking at the first zero of the ambiguity function is not representative of the capacity to distinguish two targets.

One may instead define the resolution as the width of the main lobe at the location where its amplitude has decreased by x dB. This is named the x dB-resolution. For instance, for the particular case of the rectangular pulse, the Rayleigh resolution is equal to the 3 dB-resolution. Nonetheless, the 3 dB-bandwidth is not equal to the Rayleigh bandwidth, as it is approximately equal to $0.886/T$ for a rectangular pulse of duration T .

In summary, the Rayleigh resolution gives easily a first insight of the capability to distinguish two targets for most ambiguity functions. Otherwise, when the first

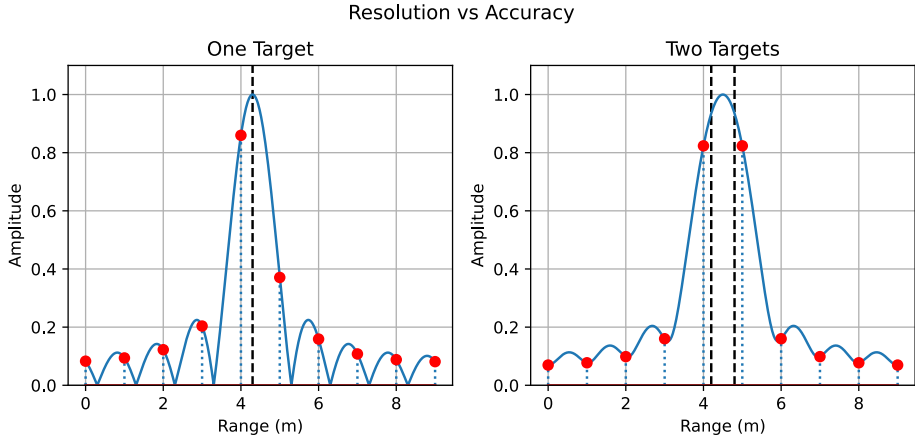


Fig. 1.10 Difference between resolution and accuracy. The positions of the targets are illustrated with black dashed lines. In the left plot, one target is located at $x = 4.3$ m. In the right plot, two targets are located at $x = 4.2$ m and $x = 4.8$ m.

zero is not indicative of the width of the main lobe, one may instead consider the x dB-resolution to better assess this capability.

1.4.2.3 Resolution and Accuracy

The resolution should be distinguished from the accuracy, as they are often mixed in the literature. The *accuracy* denotes the error between the estimated and the true target parameters, whereas the resolution denotes the capacity to distinguish two targets. For a given sampling, increasing the resolution usually also increases the accuracy. Still, the accuracy can also be artificially increased by interpolating the delay-Doppler-angle map, e.g. with zero-padding, but this operation does not affect the resolution. The distinction between resolution and accuracy is highlighted in Figure 1.10. The Rayleigh resolution of waveform is equal to 1 meter. In the left plot, thanks to the interpolation, the accuracy is increased of 0.3 meters. However, in the right plot, the interpolation does not enable to distinguish the two targets.

1.4.3 Cramér-Rao Lower Bound

Let us denote by $f(\mathbf{x}; \theta)$ the **Probability Density Function (PDF)** of an observation vector $\mathbf{x}$, given a set of P parameters $\theta \in \mathbb{R}^P$. The *Fisher information matrix* $\mathbf{I} \in \mathbb{R}^{P \times P}$ is defined as

$$[\mathbf{I}(\theta)]_{ij} \triangleq -\mathbb{E} \left[\frac{\partial^2}{\partial \theta_i \partial \theta_j} \log f(\mathbf{x}; \theta) \right].$$

The *Cramér-Rao Lower Bound* (CRB) is defined as the minimum variance any unbiased estimator $\hat{\theta}_p$ of the parameter $\theta_p = [\theta]_p$ can achieve [Rao92]:

$$\mathbb{E} \left[|\hat{\theta}_p|^2 \right] \geq \left[\mathbf{I}^{-1}(\theta) \right]_{pp}.$$

The maximum likelihood estimator is *efficient*, i.e. it achieves the CRB when the number of observations tends to the infinity.

It is usually supposed that, even if it is not always the case, the lower the CRB of a parameter, the better the accuracy of the estimators of this parameter. Therefore, this is a popular metric in the radar literature, for instance to evaluate the impact of the system parameters on the performance of the target parameters estimation.

1.4.4 Detection, Misdetecction and False Alarm Probability

Let us consider the output of the radar receiver before the detector. The radar detection problem can be translated into a hypothesis testing on such output [Ric05]. For each cell of the delay-Doppler-angle map computed by the receiver, one can define the following two hypotheses:

- Hypothesis H_0 , i.e. there is no target within the cell.
- Hypothesis H_1 , i.e. there is a target within the cell.

The cell under study is named *Cell Under Test* (CUT). Given the output of the radar receiver and the decision which is made, one defines

- The *detection probability* as the probability to suppose that there is a target when there is a target in the CUT.
- The *misdetecction probability* as the probability to suppose that there is no target when there is a target in the CUT.
- The *false alarm probability* as the probability to suppose that there is a target when there is no target in the CUT.

In order to decide which hypothesis is true, one should select a criterion to define the optimal choice. There are multiple options, as for instance the Bayes optimisation criterion, which assigns costs to every possibility. Nonetheless, the most popular option in radar is the Neyman-Pearson criterion [Ric05].

1.4.4.1 Neyman-Pearson Criterion

The *Neyman-Pearson criterion* is a special case of the Bayes optimisation criterion. It maximises the detection probability under the constraint that the false alarm probability does not exceed a given threshold. Therefore, depending on the application,

one may select a low (resp. high) threshold to ensure a low (resp. high) false alarm probability, but at the price of a low (resp. high) detection probability. As it will be confirmed in Section 1.4.4.2, both probabilities are decreasing with an increasing threshold, and a trade-off should be made depending on the system requirements. For the false alarm probability, owing to the high number of cells and measurements, values in the range of 10^{-4} to 10^{-8} are common [Ric05].

1.4.4.2 Additive White Gaussian Noise Model

The simplest model for the value x of the CUT is

$$x = \begin{cases} w & \text{under } H_0 \\ A + w & \text{under } H_1 \end{cases},$$

where A is the complex amplitude generated from the radar echo, and w is assumed to be an AWGN with variance σ_w^2 . Let us denote by γ the required threshold. Looking at the magnitude of the CUT, under H_0 , the value x is Rayleigh-distributed, and the false alarm probability is computed as

$$\mathcal{P}_{\text{FA}} = \mathbb{P}_x (|x| \geq \gamma \mid H_0) = \int_{\gamma}^{\infty} \frac{2v}{\sigma_w^2} e^{-\frac{v^2}{\sigma_w^2}} dv = e^{-\frac{\gamma^2}{\sigma_w^2}}.$$

Therefore, for a given false alarm probability requirement, the threshold γ can be set as

$$\gamma = \sqrt{-\sigma_w^2 \log \mathcal{P}_{\text{FA}}}. \quad (1.5)$$

For the detection probability, the value of x under H_1 is Ricean-distributed. It follows that the detection probability is computed as

$$\begin{aligned} \mathcal{P}_{\text{D}} &= \mathbb{P}_x (|x| \geq \gamma \mid H_1) = \int_{\gamma}^{\infty} \frac{2v}{\sigma_w^2} e^{-\frac{v^2 + |A|^2}{\sigma_w^2}} I_0 \left(\frac{2|A|v}{\sigma_w^2} \right) dv \\ &= Q_1 \left(\sqrt{\frac{2|A|^2}{\sigma_w^2}}, \sqrt{\frac{2\gamma^2}{\sigma_w^2}} \right), \end{aligned}$$

where I_0 is the modified Bessel function of the first kind and order 0, and Q_1 is the Marcum Q-function of order 1. It may also be expressed using (1.5) as a function of the Signal-to-Noise Ratio (SNR) of the CUT, and the false alarm probability, as given in Box 1.7. This function, also named Receiver Operating Characteristic (ROC), is illustrated in Figure 1.11, for different SNRs.

From the properties of the Marcum Q-function, it is known that $Q_1(x, y)$ is a strictly increasing function of x and a decreasing function of y . Therefore, for a given false alarm probability requirement, the higher the SNR, the higher the detection probability. However, decreasing the false alarm probability requirement also decreases

1.7: Detection Probability (AWGN Model)

$$\mathcal{P}_D = Q_1 \left(\sqrt{2 \text{ SNR}}, \sqrt{-2 \log \mathcal{P}_{FA}} \right)$$

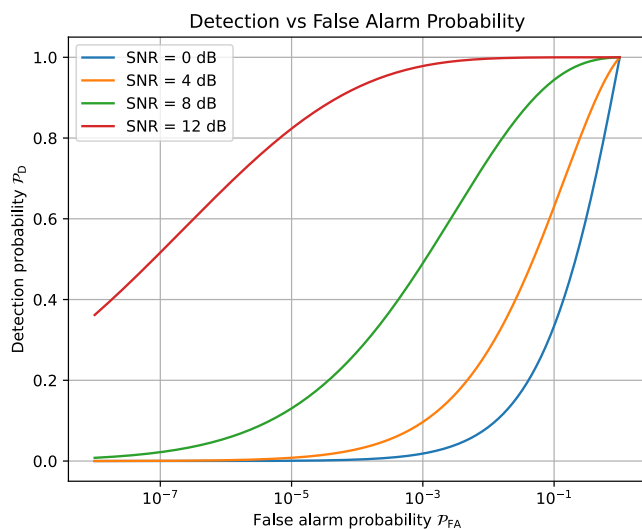


Fig. 1.11 ROC for different SNRs.

the detection probability.

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Part I

Electromagnetic Theory for Sensing and Communication

From Maxwell's Equations to Radiated Fields | 2

This chapter is an introduction to the electromagnetic radiation theory, widely inspired by the electromagnetic-related courses given by the Prof. Oestges and Prof. Craeye, which I thank warmly. Other resources that helped me to write it are [BH11; Bal99; ISU15; Orf16; Kil15], with a special mention for [Orf16] which is a very clear and pedagogic book.

THE EM theory is the foundation of mostly all the electrical engineering disciplines. In this chapter, the fundamental concepts of this theory which are useful for the next chapters are presented. First, the Maxwell's equations are introduced, and the wave equation is derived in free-space. The particular cases of plane and spherical waves are then presented. For plane waves, the reflection and transmission on a planar interface are detailed. Finally, the radiated fields from an arbitrary current source are computed, and the different propagation regions, namely the near- and far-field, are defined.

2.1 Maxwell's Equations

The *Maxwell's equations* are the root of the EM theory. They provide the electric fields $\mathbf{E}$ and $\mathbf{D}$, and magnetic fields $\mathbf{H}$ and $\mathbf{B}$, depending on the charges, currents, and other fields. The electric and magnetic fields are related to each other in an isotropic medium by the *permittivity* ϵ and *permeability* μ of the medium:

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}. \quad (2.1)$$

The Maxwell's equations can be expressed in integral or differential form, and are summarised in Box 2.1. In the electric and magnetic Gauss's laws, $\mathcal{S}$ is a closed surface and $\mathcal{V}$ is the volume enclosed by $\mathcal{S}$. In the Faraday's and Ampere's laws, $\mathcal{C}$ is a closed curve and $\mathcal{S}$ is the surface enclosed by $\mathcal{C}$. The quantities ρ and $\mathbf{J}$ are the total electric charge and current density. The current density can be decomposed into an *imposed current* $\mathbf{J}_{\text{imp}}$ and a *conduction current*:

$$\mathbf{J} = \mathbf{J}_{\text{imp}} + \sigma \mathbf{E},$$

where σ is the *conductivity* of the medium. By combining the Ampere's and electric

2.1: Maxwell's Equations

Gauss's law (D):	$\oiint_S \mathbf{D} \cdot d\mathbf{S} = \iiint_V \rho \, dV$	$\Leftrightarrow \nabla \cdot \mathbf{D} = \rho$
Gauss's law (B):	$\oiint_S \mathbf{B} \cdot d\mathbf{S} = 0$	$\Leftrightarrow \nabla \cdot \mathbf{B} = 0$
Faraday's law:	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \iint_S \mathbf{B} \cdot d\mathbf{S}$	$\Leftrightarrow \nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B}$
Ampere's law:	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \iint_S \mathbf{J} \cdot d\mathbf{S} + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$	$\Leftrightarrow \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t} \mathbf{D}$

Gauss' laws, one obtain the *equation of charge conservation*:

$$\oiint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \iiint_V \rho \, dV \quad \Leftrightarrow \quad \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}.$$

Finally, when there is a change of medium, boundary conditions can be derived from the Maxwell's equations. Denoting by $\hat{\mathbf{e}}_n$ the normal unit vector pointing from medium 1 to medium 2, they are expressed as

$$\begin{aligned} \hat{\mathbf{e}}_n \cdot (\mathbf{D}_2 - \mathbf{D}_1) &= \rho_s, & \hat{\mathbf{e}}_n \times (\mathbf{E}_2 - \mathbf{E}_1) &= 0, \\ \hat{\mathbf{e}}_n \cdot (\mathbf{B}_2 - \mathbf{B}_1) &= 0, & \hat{\mathbf{e}}_n \times (\mathbf{H}_2 - \mathbf{H}_1) &= \mathbf{J}_s. \end{aligned} \quad (2.2)$$

In these conditions, ρ_s and $\mathbf{J}_s$ are the surface charge and current density at the interface between the two media.

2.2: Maxwell's Equations (Frequency Domain)

Gauss's law (D):	$\oiint_S \mathbf{D} \cdot d\mathbf{S} = \iiint_V \rho \, dV$	$\Leftrightarrow \nabla \cdot \mathbf{D} = \rho$
Gauss's law (B):	$\oiint_S \mathbf{B} \cdot d\mathbf{S} = 0$	$\Leftrightarrow \nabla \cdot \mathbf{B} = 0$
Faraday's law:	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -j\omega \iint_S \mathbf{B} \cdot d\mathbf{S}$	$\Leftrightarrow \nabla \times \mathbf{E} = -j\omega \mathbf{B}$
Ampere's law:	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \iint_S \mathbf{J} \cdot d\mathbf{S} + j\omega \iint_S \mathbf{D} \cdot d\mathbf{S}$	$\Leftrightarrow \nabla \times \mathbf{H} = \mathbf{J} + j\omega \mathbf{D}$

It is often convenient to express the Maxwell's equations in the frequency domain. Denoting by j the imaginary unit and $\omega = 2\pi f$ the angular frequency (with f the frequency), the Maxwell's equations in the frequency domain are given by Box 2.2.

The equation of charge conservation becomes

$$\oiint_S \mathbf{J} \cdot d\mathbf{S} = -j\omega \iiint_V \rho \, dV \quad \Leftrightarrow \quad \nabla \cdot \mathbf{J} = -j\omega \rho,$$

and the boundary conditions remains unchanged. Note that, in the frequency domain, the permittivity and permeability in (2.1) could be expressed as frequency-dependent complex quantities, to account for losses in the medium. This will be further discussed in Section 2.3.3. In the following sections, every quantity will be considered in the frequency domain, unless specified otherwise.

2.2 Free-Space Propagation

In free-space, there is no charge or current density, and the Maxwell's equations in differential form can be rewritten as:

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0, \\ \nabla \cdot \mathbf{H} &= 0, \\ \nabla \times \mathbf{E} &= -j\omega\mu_0\mathbf{H}, \\ \nabla \times \mathbf{H} &= j\omega\epsilon_0\mathbf{E}, \end{aligned}$$

with ϵ_0 and μ_0 the free-space permittivity and permeability. Knowing that

$$\nabla \times (\nabla \times \mathbf{Z}) = \nabla(\nabla \cdot \mathbf{Z}) - \nabla^2 \mathbf{Z}, \quad (2.3)$$

the *wave equation* is obtained by combining the four Maxwell's equations in free-space:

$$\begin{aligned} \nabla \times (\nabla \times \mathbf{E}) &= \nabla(\cancel{\nabla \cdot \mathbf{E}}) - \nabla^2 \mathbf{E} = -j\omega\mu_0(\nabla \times \mathbf{H}) = \omega^2\epsilon_0\mu_0\mathbf{E}, \\ \nabla \times (\nabla \times \mathbf{H}) &= \nabla(\cancel{\nabla \cdot \mathbf{H}}) - \nabla^2 \mathbf{H} = j\omega\epsilon_0(\nabla \times \mathbf{E}) = \omega^2\epsilon_0\mu_0\mathbf{H}. \end{aligned}$$

Defining the *wave speed* as $c \triangleq 1/\sqrt{\epsilon_0\mu_0}$ and *wave number* by $k \triangleq \omega/c$, it is rewritten as Box 2.3.

2.3: EM Wave Equation in Free-Space

$$\begin{aligned} (\nabla^2 + k^2) \mathbf{E} &= 0 \\ (\nabla^2 + k^2) \mathbf{H} &= 0 \end{aligned}$$

Two particularly important solutions of the wave equation are the *plane* and *spherical waves*. Their expressions are summarised in Box 2.4, and they are illustrated in Figure 2.1. In these equations, $\mathbf{k} = k \hat{\mathbf{e}}_u$ is the *wave vector*, and $\mathbf{r}$ the position in Cartesian coordinates.

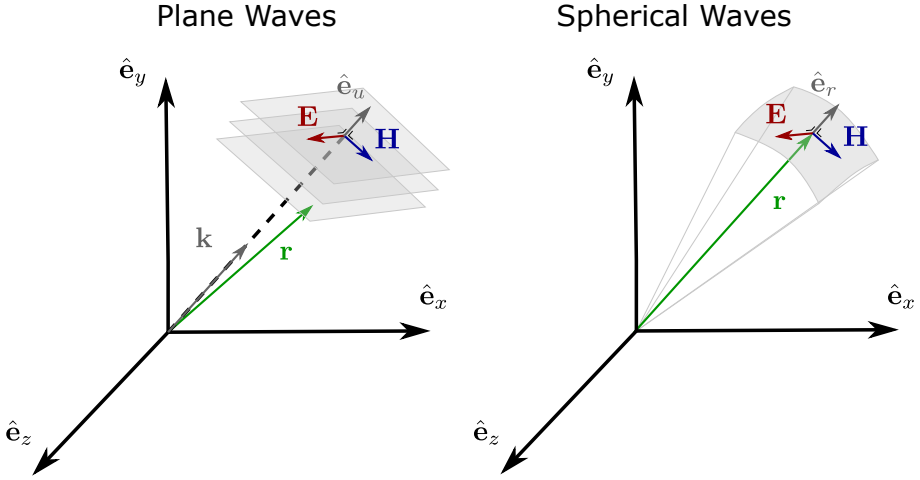


Fig. 2.1 Plane and spherical waves.

2.4: Plane and Spherical Waves

Plane waves:	$\mathbf{E} = \mathbf{E}_0 e^{\pm j \mathbf{k} \cdot \mathbf{r}}, \quad \mathbf{H} = \mathbf{H}_0 e^{\pm j \mathbf{k} \cdot \mathbf{r}}$
Spherical waves:	$\mathbf{E} = \frac{\mathbf{E}_0}{\ \mathbf{r}\ } e^{-j k \ \mathbf{r}\ }, \quad \mathbf{H} = \frac{\mathbf{H}_0}{\ \mathbf{r}\ } e^{-j k \ \mathbf{r}\ }$

Inserting the electric field expression in the Faraday's law, using the vector identity

$$\nabla \times (f\mathbf{Z}) = f\nabla \times \mathbf{Z} + (\nabla f) \times \mathbf{Z}, \quad (2.4)$$

one can find for plane waves that

$$\begin{aligned} \mathbf{H} &= \frac{J}{\omega\mu_0} \left[\left(\nabla e^{\pm j \mathbf{k} \cdot \mathbf{r}} \right) \times \mathbf{E}_0 + e^{\pm j \mathbf{k} \cdot \mathbf{r}} (\nabla \times \mathbf{E}_0) \right] \\ &= \frac{k}{\omega\mu_0} (\mp \hat{\mathbf{e}}_u) \times (\mathbf{E}_0 e^{\pm j \mathbf{k} \cdot \mathbf{r}}) = \frac{1}{\eta} (\mp \hat{\mathbf{e}}_u) \times \mathbf{E}, \end{aligned}$$

where $\eta \triangleq \sqrt{\mu_0/\epsilon_0}$ is the *free-space impedance*. This shows that the magnetic field is perpendicular to the electric field and the direction of propagation. Its amplitude is related to the electric field amplitude through the free-space impedance. The same development can also be performed for spherical waves:

$$\mathbf{H} = \frac{J}{\omega\mu_0} \left[\left(\nabla \frac{e^{\pm j k \|\mathbf{r}\|}}{\|\mathbf{r}\|} \right) \times \mathbf{E}_0 + \frac{e^{\pm j k \|\mathbf{r}\|}}{\|\mathbf{r}\|} (\nabla \times \mathbf{E}_0) \right]$$

$$\begin{aligned}
&= \left(\frac{k}{\omega \mu_0} - \frac{J}{\omega \mu_0 r} \right) \left(\mp \frac{\mathbf{r}}{\|\mathbf{r}\|} \right) \times \left(\frac{\mathbf{E}_0}{\|\mathbf{r}\|} e^{\pm jk\|\mathbf{r}\|} \right) \\
&= \frac{1}{\eta} \left(1 - \frac{J}{kr} \right) (\mp \hat{\mathbf{e}}_r) \times \mathbf{E},
\end{aligned}$$

where $\hat{\mathbf{e}}_r = \mathbf{r}/\|\mathbf{r}\|$ is the radial direction of propagation. This shows that the magnetic field is again perpendicular to the electric field and the direction of propagation. For large kr -products, the J/kr term is negligible, and the same expressions are obtained in both cases. Note that this does not always hold for non-omnidirectional sources, as the fields amplitudes are also functions of the direction of propagation with spherical waves. One can also execute the same development by inserting the magnetic field expression in the Ampere's law, leading to

$$\mathbf{E} = \eta \mathbf{H} \times (\mp \hat{\mathbf{e}}_u),$$

for plane waves, and

$$\mathbf{E} = \eta \left(1 - \frac{J}{kr} \right) \mathbf{H} \times (\mp \hat{\mathbf{e}}_r),$$

for spherical waves. Finally, assuming large kr -products and using the vector identity

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{a} \cdot \mathbf{b}) \mathbf{c}, \quad (2.5)$$

the following equalities are obtained:

$$\begin{aligned}
\mathbf{E} \times \mathbf{H}^* &= \mathbf{E} \times \left(\frac{1}{\eta} (\mp \hat{\mathbf{e}}_u) \times \mathbf{E}^* \right) = \frac{1}{\eta} [(\mathbf{E} \cdot \mathbf{E}^*) (\mp \hat{\mathbf{e}}_u) - \cancel{(\mathbf{E} \cdot (\mp \hat{\mathbf{e}}_u)) \mathbf{E}^*}] = \frac{|\mathbf{E}|^2}{\eta} (\mp \hat{\mathbf{e}}_u) \\
&= (\eta \mathbf{H} \times (\mp \hat{\mathbf{e}}_u)) \times \mathbf{H}^* = \eta [(\mathbf{H} \cdot \mathbf{H}^*) (\mp \hat{\mathbf{e}}_u) - \cancel{(\mathbf{H}^* \cdot (\mp \hat{\mathbf{e}}_u)) \mathbf{H}}] = \eta |\mathbf{H}|^2 (\mp \hat{\mathbf{e}}_u).
\end{aligned}$$

The different relations between the electric field, the magnetic field and the direction of propagation for plane and spherical waves (assuming $kr \gg 1$) are summarised in Box 2.5. In the next sections, the forward propagating solution is considered, and the positive sign of the expressions is used.

2.5: Relations between $\mathbf{E}$, $\mathbf{H}$ and $\hat{\mathbf{e}}_u$ (Plane and Spherical Waves, $kr \gg 1$)

$$\begin{aligned}
\mathbf{H} &= \frac{1}{\eta} (\mp \hat{\mathbf{e}}_u) \times \mathbf{E} \\
\mathbf{E} &= \eta \mathbf{H} \times (\mp \hat{\mathbf{e}}_u) \\
\mathbf{E} \times \mathbf{H}^* &= \frac{|\mathbf{E}|^2}{\eta} (\mp \hat{\mathbf{e}}_u) = \eta |\mathbf{H}|^2 (\mp \hat{\mathbf{e}}_u)
\end{aligned}$$

2.3 Plane Wave at a Planar Interface

In a first time, a lossless dielectric planar interface between two media 1 and 2 is considered, with permittivities, permeabilities, impedances and wave numbers $\epsilon_1, \mu_1, \eta_1, k_1$ and $\epsilon_2, \mu_2, \eta_2, k_2$. The normal unit vector pointing from the medium 1 to the medium 2 is denoted as $\hat{\mathbf{e}}_n$. The different fields and their associated wave vectors are expressed as follows:

$$\begin{aligned} \text{Incident fields:} \quad \mathbf{E}_1^i &= \mathbf{E}_{10}^i e^{-j\mathbf{k}_1^i \cdot \mathbf{r}}, & \mathbf{H}_1^i &= \mathbf{H}_{10}^i e^{-j\mathbf{k}_1^i \cdot \mathbf{r}}, \\ \text{Reflected fields:} \quad \mathbf{E}_1^r &= \mathbf{E}_{10}^r e^{-j\mathbf{k}_1^r \cdot \mathbf{r}}, & \mathbf{H}_1^r &= \mathbf{H}_{10}^r e^{-j\mathbf{k}_1^r \cdot \mathbf{r}}, \\ \text{Transmitted fields:} \quad \mathbf{E}_2 &= \mathbf{E}_{20} e^{-j\mathbf{k}_2 \cdot \mathbf{r}}, & \mathbf{H}_2 &= \mathbf{H}_{20} e^{-j\mathbf{k}_2 \cdot \mathbf{r}}. \end{aligned}$$

Let us define the *incidence plane* as the plane spanned by the incident wave vector $\mathbf{k}_1^i$ and the normal unit vector $\hat{\mathbf{e}}_n$. Two different polarisations are considered:

- The *Transverse Magnetic (TM)* polarisation, also named *p-polarisation*, for which the magnetic field is perpendicular to the incidence plane and the parallel to the interface. The electric field therefore lies in the incidence plane.
- The *Transverse Electric (TE)* polarisation, also named *s-polarisation*, for which the electric field is perpendicular to the incidence plane and the parallel to the interface. The magnetic field therefore lies in the incidence plane.

Other polarisations, named *Transverse ElectroMagnetic (TEM)*, can be decomposed as a linear combination of the *TE* and *TM* polarisations, as shown in Section 2.3.5. Thus, it is not required to study the other cases.

For the following developments, without any loss of generality, let us orientate the Cartesian coordinates such that the incidence plane is the (x, z) -plane, and the normal unit vector $\hat{\mathbf{e}}_n$ is equal to $\hat{\mathbf{e}}_x$. The considered setup is shown in Figure 2.2. The position vector is thus expressed as

$$\mathbf{r} = x \hat{\mathbf{e}}_x + z \hat{\mathbf{e}}_z,$$

and the wave vectors as

$$\begin{aligned} \mathbf{k}_1^i &= k_1 \cos \theta_1^i \hat{\mathbf{e}}_x + k_1 \sin \theta_1^i \hat{\mathbf{e}}_z, \\ \mathbf{k}_1^r &= -k_1 \cos \theta_1^r \hat{\mathbf{e}}_x + k_1 \sin \theta_1^r \hat{\mathbf{e}}_z, \\ \mathbf{k}_2 &= k_2 \cos \theta_2 \hat{\mathbf{e}}_x + k_2 \sin \theta_2 \hat{\mathbf{e}}_z. \end{aligned}$$

In these equations, θ_1^i and θ_1^r are the angles of incidence and reflection, and θ_2 is the angle of transmission. The angles are all defined regarding $-\hat{\mathbf{e}}_x$ for the first medium, and the $\hat{\mathbf{e}}_x$ for the second medium.

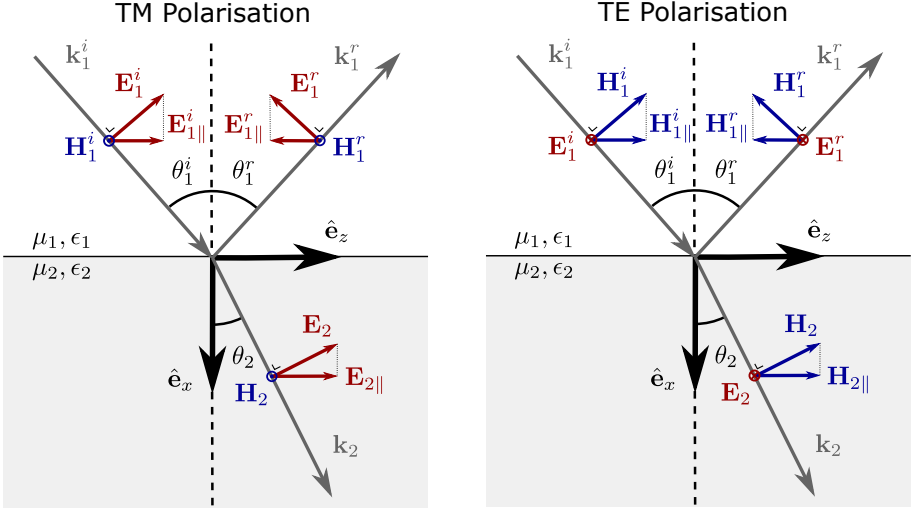


Fig. 2.2 Transverse electric and magnetic polarisations.

2.3.1 Transverse Magnetic Polarisation

For the **TM** polarisation, the electric and magnetic field amplitude vectors are written as

$$\begin{aligned} \mathbf{E}_{10}^i &= -E_{10}^i \sin \theta_1^i \hat{\mathbf{e}}_x + E_{10}^i \cos \theta_1^i \hat{\mathbf{e}}_z, & \mathbf{H}_{10}^i &= -H_{10}^i \hat{\mathbf{e}}_y, \\ \mathbf{E}_{10}^r &= -E_{10}^r \sin \theta_1^r \hat{\mathbf{e}}_x - E_{10}^r \cos \theta_1^r \hat{\mathbf{e}}_z, & \mathbf{H}_{10}^r &= -H_{10}^r \hat{\mathbf{e}}_y, \\ \mathbf{E}_2 &= -E_{20} \sin \theta_2 \hat{\mathbf{e}}_x + E_{20} \cos \theta_2 \hat{\mathbf{e}}_z, & \mathbf{H}_{20} &= -H_{20} \hat{\mathbf{e}}_y. \end{aligned}$$

On the one hand, applying the boundary condition of (2.2) on the tangential electric fields at the interface ($x = 0$), one obtains:

$$E_{20} \cos \theta_2 e^{-jk_2 z \sin \theta_2} - E_{10}^i \cos \theta_1^i e^{-jk_1 z \sin \theta_1^i} + E_{10}^r \cos \theta_1^r e^{-jk_1 z \sin \theta_1^r} = 0.$$

In order to find a solution which holds for any value of z , the phase of the three terms must be equal, leading to the *law of reflection* and the *law of Snell-Descartes* of Box 2.6.

2.6: Law of Reflection and Snell-Descartes

Law of reflection:	$\theta_1^i = \theta_1^r$
Law of Snell-Descartes:	$k_1 \sin \theta_1^i = k_2 \sin \theta_2$

Following these two laws, the boundary condition of (2.2) on the tangential electric

2 | From Maxwell's Equations to Radiated Fields

fields at the interface becomes

$$E_{20} \cos \theta_2 - E_{10}^i \cos \theta_1 + E_{10}^r \cos \theta_1 = 0. \quad (2.6)$$

where $\theta_1 \triangleq \theta_1^i = \theta_1^r$. In another hand, applying the boundary condition of (2.2) on the tangential magnetic fields at the interface ($x = 0$) and leveraging on Box 2.5, one obtains:

$$H_{20} - H_{10}^i - H_{10}^r = 0 \quad \Leftrightarrow \quad \frac{E_{20}}{\eta_2} - \frac{E_{10}^i}{\eta_1} - \frac{E_{10}^r}{\eta_1} = 0, \quad (2.7)$$

Combining (2.6) and (2.7), one obtains the *Fresnel reflection* and *transmission coefficients* of Box 2.7 for the **TM** polarisation. These coefficients can also be expressed in terms of the incidence angle only by computing

$$\cos \theta_2 = \sqrt{1 - \sin^2 \theta_2} = \sqrt{1 - \left(\frac{k_1}{k_2}\right)^2 \sin^2 \theta_1} = \sqrt{1 - \frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2} \sin^2 \theta_1}. \quad (2.8)$$

2.7: Fresnel Reflection and Transmission Coefficients (TM Polarisation)

Reflection:	$\mathcal{R}_{\text{TM}} = \frac{E_{10}^r}{E_{10}^i} = \frac{\eta_1 \cos \theta_1 - \eta_2 \cos \theta_2}{\eta_1 \cos \theta_1 + \eta_2 \cos \theta_2}$
Transmission:	$\mathcal{T}_{\text{TM}} = \frac{E_{20}}{E_{10}^i} = \frac{2\eta_2 \cos \theta_1}{\eta_1 \cos \theta_1 + \eta_2 \cos \theta_2}$

2.3.2 Transverse Electric Polarisation

For the **TE** polarisation, the electric and magnetic field amplitude vectors are written as

$$\begin{aligned} \mathbf{E}_{10}^i &= E_{10}^i \hat{\mathbf{e}}_y, & \mathbf{H}_{10}^i &= -H_{10}^i \sin \theta_1^i \hat{\mathbf{e}}_x + H_{10}^i \cos \theta_1^i \hat{\mathbf{e}}_z, \\ \mathbf{E}_{10}^r &= E_{10}^r \hat{\mathbf{e}}_y, & \mathbf{H}_{10}^r &= -H_{10}^r \sin \theta_1^r \hat{\mathbf{e}}_x - H_{10}^r \cos \theta_1^r \hat{\mathbf{e}}_z, \\ \mathbf{E}_{20} &= E_{20} \hat{\mathbf{e}}_y, & \mathbf{H}_{20} &= -H_{20} \sin \theta_2 \hat{\mathbf{e}}_x + H_{20} \cos \theta_2 \hat{\mathbf{e}}_z. \end{aligned}$$

On the one hand, applying the boundary condition of (2.2) on the tangential electric fields at the interface ($x = 0$), one obtains:

$$E_{20} - E_{10}^i - E_{10}^r = 0. \quad (2.9)$$

On the other hand, applying the boundary condition of (2.2) on the tangential magnetic fields at the interface ($x = 0$), one obtains:

$$H_{20} \cos \theta_2 e^{-jk_2 z \sin \theta_2} - H_{10}^i \cos \theta_1^i e^{-jk_1 z \sin \theta_1^i} + H_{10}^r \cos \theta_1^r e^{-jk_1 z \sin \theta_1^r} = 0.$$

In order to find a solution which holds for any value of z , the phase of the three terms must be equal, leading again to the law of reflection and the law of Snell-Descartes of Box 2.6. Following these two laws and leveraging on Box 2.5, the boundary condition of (2.2) on the tangential electric fields at the interface becomes

$$\begin{aligned} H_{20} \cos \theta_2 - H_{10}^i \cos \theta_1 + H_{10}^r \cos \theta_1 &= 0 \\ \Leftrightarrow \frac{E_{20}}{\eta_2} \cos \theta_2 - \frac{E_{10}^i}{\eta_1} \cos \theta_1 + \frac{E_{10}^r}{\eta_1} \cos \theta_1 &= 0 \end{aligned} \quad (2.10)$$

where $\theta_1 \triangleq \theta_1^i = \theta_1^r$. Combining (2.9) and (2.10), one obtains the Fresnel reflection and transmission coefficients of Box 2.8 for the TE polarisation. Note that $\mathcal{T}_{\text{TE}} = 1 + \mathcal{R}_{\text{TE}}$ in that case.

2.8: Fresnel Reflection and Transmission Coefficients (TE Polarisation)

$$\begin{aligned} \text{Reflection:} \quad \mathcal{R}_{\text{TE}} &= \frac{E_{10}^r}{E_{10}^i} = \frac{H_{10}^r}{H_{10}^i} = \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2} \\ \text{Transmission:} \quad \mathcal{T}_{\text{TE}} &= \frac{E_{20}}{E_{10}^i} = \frac{H_{20}}{H_{10}^i} = \frac{2\eta_2 \cos \theta_1}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2} \end{aligned}$$

2.3.3 Reflection and Transmission with a Good Conductor

With a good conductor of conductivity σ_2 , surface currents generated on the interface should be taken into account in the Ampere's law:

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega \mathbf{D} = (\sigma_2 + j\omega \epsilon_2) \mathbf{E} = j\omega \epsilon_{2,\text{eff}} \mathbf{E},$$

where the *effective permittivity* is defined as

$$\epsilon_{2,\text{eff}} \triangleq \epsilon_2 - j \frac{\sigma_2}{\omega} \approx -j \frac{\sigma_2}{\omega}.$$

This also leads to

$$jk_2 = j\omega \sqrt{\mu_2 \epsilon_{2,\text{eff}}} \approx \sqrt{j\omega \mu_2 \sigma_2}, \quad \text{and} \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_{2,\text{eff}}}} \approx \sqrt{\frac{j\omega \mu_2}{\sigma_2}}.$$

Using the effective permittivity of the second medium, the Fresnel coefficients for both polarisations in Boxes 2.7 and 2.8 are still valid. However, the law of Snell-Descartes given in Box 2.6 is now also a complex equality, involving a complex transmission angle. Equation (2.8) is thus developed as

$$\cos \theta_2 = \sqrt{1 - \frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_{2,\text{eff}}} \sin^2 \theta_1} \approx \sqrt{1 - \frac{j\omega \mu_1 \epsilon_1}{\mu_2 \sigma_2} \sin^2 \theta_1} \xrightarrow{\sigma_2 \uparrow} 1.$$

Therefore, in a good conductor, the Fresnel coefficients are complex values, and the transmitted wave propagates nearly parallel to the interface. Besides, for any reasonably good conductor, the PEC assumption is an excellent approximation, as the Fresnel reflection coefficients are very close to ± 1 , respectively for the TM and TE polarisation.

2.3.4 Physical Optics Approximation

The *Physical Optics (PO) approximation* is a popular method to evaluate the EM fields reflected by an obstacle, enabling in some simple case to obtain tractable analytical results, with a limited complexity. This approximation relies on the surface equivalence theorem and the image theory, which will not be described thoroughly, but summarised in this chapter.

On the one hand, the *surface equivalence theorem* enables to compute the EM fields radiated outside a given volume by defining an equivalent problem. In the equivalent problem, every current distribution comprised within the delimited region are replaced by electric and magnetic surface currents on the surface of the volume, such that the same EM fields are radiated outside. Those surface currents are defined following the boundary conditions (2.2), in which the magnetic surface currents, which are not defined in this thesis, must be introduced. Moreover, since the enclosed volume is not the region of interest, *Perfect Electric Conductors (PECs)* or *Perfect Magnetic Conductors (PMCs)* may be placed infinitely close to the inside surface of the volume, without affecting the EM fields of interest outside the volume.

On the other hand, the *image theory* enables to compute the EM fields radiated near infinite PEC or PMC planes, by instead introducing virtual electric and magnetic source distributions. For instance, if a PMC is considered, the virtual electric current distribution must be identical to the initial electric source, whereas the virtual magnetic current distribution must be the opposite of the initial magnetic source. Combined with the surface equivalence theorem, if a PMC plane is placed infinitely close to the surface of the equivalent volume, the equivalent magnetic surface current is nullified by its opposite image, whereas the equivalent electric surface current is doubled.

Figure 2.3 illustrates the equivalent problems. The PO approximation works well for electrically large scatterers¹, for which the curvature is slowly varying, such that the source can be assumed to be locally planar. Such approximation does not guarantee the reciprocity, since the diffraction and other boundary effects are neglected, and the shadow side of the target is not considered. With the PO approximation, the EM fields reflected by a target can be computed by considering an equivalent electric

¹Usually, the dimension of the scatterer is supposed to be larger than ten times the wavelength.

Physical Optics Approximation

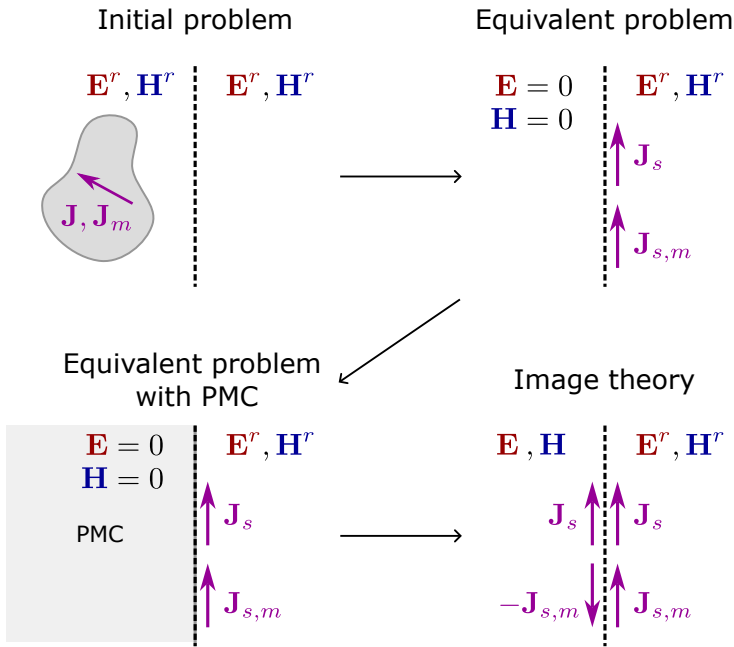


Fig. 2.3 PO approximation. The quantities indexed by the letter m are magnetic currents.

surface current generated at the surface of the equivalent volume surrounding the target, defined as

$$\mathbf{J}_s = 2 \hat{\mathbf{e}}_n \times \mathbf{H}_1^r,$$

with $\hat{\mathbf{e}}_n$ the normal unit vector pointing outside the volume. Yet, since the incident EM fields are known, with the locally planar approximation, the equivalent surface current may instead be related to the incident magnetic field through the Fresnel coefficients of the target, as given in Box 2.9.

2.9: Equivalent Surface Current at the Interface

$$\mathbf{J}_s \approx \begin{cases} 2 \mathcal{R}_{\text{TM}} \hat{\mathbf{e}}_n \times \mathbf{H}_1^i & (\text{TM polarisation}) \\ -2 \mathcal{R}_{\text{TE}} \hat{\mathbf{e}}_n \times \mathbf{H}_1^i & (\text{TE polarisation}) \end{cases}$$

Besides, as described in Section 2.3.3, the PEC assumption is an excellent approximation. In that case, the Fresnel reflection coefficients are very close to ± 1 , respectively for the TM and TE polarisation, and both expressions of Box 2.9 thus merge to give Box 2.10.

2.10: Equivalent Surface Current at the Interface of PECs

$$\mathbf{J}_s = 2 \hat{\mathbf{e}}_n \times \mathbf{H}_1^i.$$

2.3.5 Transverse Electromagnetic Decomposition

One can be interested in decomposing a TEM plane wave into its TE and TM components. To achieve that, let us define a basis $\mathcal{B} \triangleq \{\hat{\mathbf{e}}_{\parallel}; \hat{\mathbf{e}}_{\perp}; \mathbf{k}_1^i\}$ such that

1. the first basis vector $\hat{\mathbf{e}}_{\parallel}$ is parallel to the incidence plane,
2. the second basis vector $\hat{\mathbf{e}}_{\perp}$ is perpendicular to the incidence plane,
3. the third basis vector $\mathbf{k}_1^i$ is the incident wave vector.

Owing to their definitions, the two first basis vectors can respectively be computed as

$$\hat{\mathbf{e}}_{\perp} = \frac{\mathbf{k}_1^i \times \hat{\mathbf{e}}_n}{\|\mathbf{k}_1^i \times \hat{\mathbf{e}}_n\|}, \quad \hat{\mathbf{e}}_{\parallel} = \hat{\mathbf{e}}_{\perp} \times \mathbf{k}_1^i.$$

Using these vectors, the incident electric and magnetic fields can thus be decomposed into the TE and TM polarisations by computing their components in the basis $\mathcal{B}$:

$$\begin{aligned} \mathbf{E}_{10}^i &= (\mathbf{E}_{10}^i \cdot \hat{\mathbf{e}}_{\parallel}) \hat{\mathbf{e}}_{\parallel} + (\mathbf{E}_{10}^i \cdot \hat{\mathbf{e}}_{\perp}) \hat{\mathbf{e}}_{\perp} + (\mathbf{E}_{10}^i \cdot \mathbf{k}_1^i) \mathbf{k}_1^i \\ &\triangleq E_{10\parallel}^i \hat{\mathbf{e}}_{\parallel} + E_{10\perp}^i \hat{\mathbf{e}}_{\perp}, \end{aligned}$$

and

$$\begin{aligned}\mathbf{H}_{10}^i &= (\mathbf{H}_{10}^i \cdot \hat{\mathbf{e}}_{\parallel}) \hat{\mathbf{e}}_{\parallel} + (\mathbf{H}_{10}^i \cdot \hat{\mathbf{e}}_{\perp}) \hat{\mathbf{e}}_{\perp} + (\cancel{\mathbf{H}_{10}^i \cdot \mathbf{k}_1^i}) \mathbf{k}_1^i \\ &\triangleq H_{10\parallel}^i \hat{\mathbf{e}}_{\parallel} + H_{10\perp}^i \hat{\mathbf{e}}_{\perp}.\end{aligned}$$

2.4 Radiation from an Arbitrary Electric Current Source

In order to simplify the derivation of the radiated fields from an arbitrary radiating electric current source, let us introduce the *vector potential* $\mathbf{A}$, defined as

$$\mathbf{H} \triangleq \nabla \times \mathbf{A}.$$

This enables to rewrite the Faraday's law in free-space as

$$\nabla \times (\mathbf{E} + j\omega\mu_0 \mathbf{A}) = 0 \quad \Leftrightarrow \quad \mathbf{E} + j\omega\mu_0 \mathbf{A} = -\nabla\phi,$$

where ϕ is an arbitrary scalar potential. Inserting this result in the Ampere's law when there is an electric current source, using Identity (2.3) one obtains

$$\begin{aligned}\nabla \times \mathbf{H} &= \nabla \times (\nabla \times \mathbf{A}) = \mathbf{J} - j\omega\epsilon_0 (\nabla\phi + j\omega\mu_0 \mathbf{A}) \\ \Leftrightarrow \quad \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} &= \mathbf{J} - j\omega\epsilon_0 \nabla\phi + k^2 \mathbf{A} \\ \Leftrightarrow \quad (\nabla^2 + k^2) \mathbf{A} &= -\mathbf{J} + \nabla (\nabla \cdot \mathbf{A} + j\omega\epsilon_0 \phi).\end{aligned}$$

The scalar and vector potentials are not uniquely defined. One possible choice to set these quantities is the *Lorentz gauge*, defined such that

$$\nabla \cdot \mathbf{A} + j\omega\epsilon_0 \phi = 0 \quad \Leftrightarrow \quad \phi = \frac{j}{\omega\epsilon_0} (\nabla \cdot \mathbf{A}).$$

Using that gauge, the vector potential should satisfy

$$(\nabla^2 + k^2) \mathbf{A} = -\mathbf{J}.$$

First, assuming a current point source $\mathbf{J} = \mathbf{J}_0 \delta(\mathbf{r})$, the solution is given by $\mathbf{A} = G(\mathbf{r}) \mathbf{J}_0$, where G is the *Green's function*:

$$G(\mathbf{r}) \triangleq \frac{e^{-jk\|\mathbf{r}\|}}{4\pi\|\mathbf{r}\|}.$$

Then, let us consider an arbitrary radiation electrical current source, as represented in Figure 2.4. The vector potential generated by an arbitrary current density in a given

Radiation from a Current Source

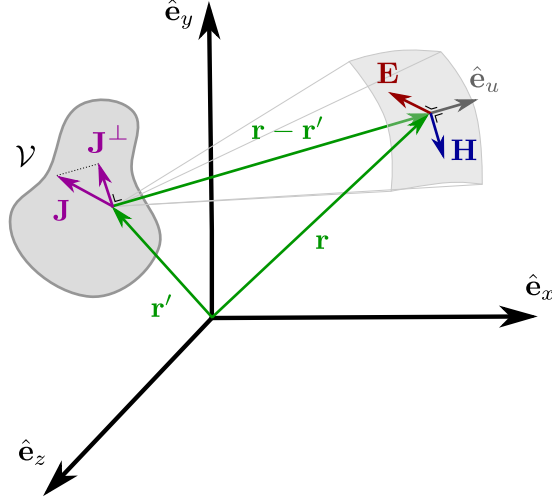


Fig. 2.4 Radiation from an electric current source.

volume $\mathcal{V}$ is given by superposing the Green's functions of each point source:

$$\mathbf{A} = \frac{1}{4\pi} \iiint_{\mathcal{V}} G(\mathbf{r} - \mathbf{r}') \mathbf{J}(\mathbf{r}') d\mathbf{r}' = \frac{1}{4\pi} \iiint_{\mathcal{V}} \frac{e^{-jk\|\mathbf{r} - \mathbf{r}'\|}}{\|\mathbf{r} - \mathbf{r}'\|} \mathbf{J}(\mathbf{r}') d\mathbf{r}'.$$

Finally, using Identity (2.4), the radiated magnetic field is computed as follows:

$$\begin{aligned} \mathbf{H} &= \nabla \times \mathbf{A} \\ &= \frac{1}{4\pi} \iiint_{\mathcal{V}} \left[(\nabla G(\mathbf{r} - \mathbf{r}')) \times \mathbf{J}(\mathbf{r}') + G(\mathbf{r} - \mathbf{r}') (\nabla \times \mathbf{J}(\mathbf{r}')) \right] d\mathbf{r}' \\ &= -\frac{jk}{4\pi} \iiint_{\mathcal{V}} \frac{e^{-jk\|\mathbf{r} - \mathbf{r}'\|}}{\|\mathbf{r} - \mathbf{r}'\|^2} \left(1 - \frac{J}{k\|\mathbf{r} - \mathbf{r}'\|} \right) (\mathbf{r} - \mathbf{r}') \times \mathbf{J}(\mathbf{r}') d\mathbf{r}' \\ &= -\frac{jk}{4\pi} \iiint_{\mathcal{V}} \frac{e^{-jk\|\mathbf{r} - \mathbf{r}'\|}}{\|\mathbf{r} - \mathbf{r}'\|} \left(1 - \frac{J}{k\|\mathbf{r} - \mathbf{r}'\|} \right) \hat{e}_u \times \mathbf{J}(\mathbf{r}') d\mathbf{r}', \end{aligned}$$

where $\hat{e}_u \triangleq (\mathbf{r} - \mathbf{r}')/\|\mathbf{r} - \mathbf{r}'\|$ is the unit vector pointing from the source points to the observation point. Let us define for readability

$$G'(\mathbf{r} - \mathbf{r}') \triangleq \frac{e^{-jk\|\mathbf{r} - \mathbf{r}'\|}}{\|\mathbf{r} - \mathbf{r}'\|} \left(1 - \frac{J}{k\|\mathbf{r} - \mathbf{r}'\|} \right).$$

The radiated electric field is then obtained from the Ampere's law in free-space as

$$\mathbf{E} = -\frac{J}{\omega\epsilon_0} \nabla \times (\nabla \times \mathbf{A})$$

$$\begin{aligned}
 &= -\frac{k}{4\pi\omega\epsilon_0} \iiint_{\mathcal{V}} \nabla \times \left[\frac{G'(\mathbf{r} - \mathbf{r}')}{\|\mathbf{r} - \mathbf{r}'\|} (\mathbf{r} - \mathbf{r}') \times \mathbf{J}(\mathbf{r}') \right] d\mathbf{r}' \\
 &= -\frac{\eta}{4\pi} (\mathcal{E}_1 + \mathcal{E}_2).
 \end{aligned}$$

The integrals $\mathcal{E}_1$ and $\mathcal{E}_2$ are developed as

$$\begin{aligned}
 \mathcal{E}_1 &= \iiint_{\mathcal{V}} \nabla \left(\frac{G'(\mathbf{r} - \mathbf{r}')}{\|\mathbf{r} - \mathbf{r}'\|} \right) \times [(\mathbf{r} - \mathbf{r}') \times \mathbf{J}(\mathbf{r}')] d\mathbf{r}' \\
 &= jk \iiint_{\mathcal{V}} \frac{e^{-jk\|\mathbf{r} - \mathbf{r}'\|}}{\|\mathbf{r} - \mathbf{r}'\|} \left(1 - \frac{3J}{k\|\mathbf{r} - \mathbf{r}'\|} - \frac{3}{k^2\|\mathbf{r} - \mathbf{r}'\|^2} \right) [(\hat{\mathbf{e}}_u \times \mathbf{J}(\mathbf{r}')) \times \hat{\mathbf{e}}_u] d\mathbf{r}',
 \end{aligned}$$

and

$$\begin{aligned}
 \mathcal{E}_2 &= \iiint_{\mathcal{V}} \frac{G'(\mathbf{r} - \mathbf{r}')}{\|\mathbf{r} - \mathbf{r}'\|} \nabla \times [(\mathbf{r} - \mathbf{r}') \times \mathbf{J}(\mathbf{r}')] d\mathbf{r}' \\
 &= \iiint_{\mathcal{V}} \frac{G'(\mathbf{r} - \mathbf{r}')}{\|\mathbf{r} - \mathbf{r}'\|} [(\mathbf{J}(\mathbf{r}') \cdot \nabla) (\mathbf{r} - \mathbf{r}') - \mathbf{J}(\mathbf{r}') (\nabla \cdot (\mathbf{r} - \mathbf{r}'))] d\mathbf{r}' \\
 &\quad - \iiint_{\mathcal{V}} \frac{G'(\mathbf{r} - \mathbf{r}')}{\|\mathbf{r} - \mathbf{r}'\|} [(\nabla \cdot \mathbf{J}(\mathbf{r}')) (\mathbf{r} - \mathbf{r}') - (\mathbf{J}(\mathbf{r}') \cdot \nabla) (\mathbf{r} - \mathbf{r}')] d\mathbf{r}' \\
 &= 2 \iiint_{\mathcal{V}} \frac{e^{-jk\|\mathbf{r} - \mathbf{r}'\|}}{\|\mathbf{r} - \mathbf{r}'\|} \left(\frac{1}{\|\mathbf{r} - \mathbf{r}'\|} - \frac{J}{k\|\mathbf{r} - \mathbf{r}'\|^2} \right) \mathbf{J}(\mathbf{r}') d\mathbf{r}'.
 \end{aligned}$$

For large $k\|\mathbf{r} - \mathbf{r}'\|$ -products, one can further simplify the expressions of the radiated fields, obtaining Box 2.11. In these expressions, the component of the current density $\mathbf{J}^\perp$ perpendicular to the direction of propagation is defined as

$$\mathbf{J}^\perp(\mathbf{r}') \triangleq (\hat{\mathbf{e}}_u \times \mathbf{J}(\mathbf{r}')) \times \hat{\mathbf{e}}_u = \mathbf{J}(\mathbf{r}') - (\hat{\mathbf{e}}_u \cdot \mathbf{J}(\mathbf{r}')) \hat{\mathbf{e}}_u,$$

where the last equality is obtained by applying (2.5). Additionally, for $k\|\mathbf{r} - \mathbf{r}'\| \gg 1$, since the radiated fields are obtained as a superposition of spherical waves, the magnetic field is perpendicular to the electric field, and both fields are perpendicular to the direction of propagation. Therefore, the equalities of Box 2.5 are verified. The radiated EM Fields can also be expressed in the time domain by applying an inverse Fourier transform to the frequency-domain expressions, giving Box 2.12.

2.11: Radiated EM Fields ($k\|\mathbf{r} - \mathbf{r}'\| \gg 1$)

$$\begin{aligned}
 \mathbf{H}(\mathbf{r}) &\cong -\frac{jk}{4\pi} \iiint_{\mathcal{V}} \frac{e^{-jk\|\mathbf{r} - \mathbf{r}'\|}}{\|\mathbf{r} - \mathbf{r}'\|} \hat{\mathbf{e}}_u \times \mathbf{J}(\mathbf{r}') d\mathbf{r}' \\
 \mathbf{E}(\mathbf{r}) &\cong -\frac{jk\eta}{4\pi} \iiint_{\mathcal{V}} \frac{e^{-jk\|\mathbf{r} - \mathbf{r}'\|}}{\|\mathbf{r} - \mathbf{r}'\|} \mathbf{J}^\perp(\mathbf{r}') d\mathbf{r}'
 \end{aligned}$$

2.12: Radiated EM Fields (Time Domain, $k\|\mathbf{r} - \mathbf{r}'\| \gg 1$)

$$\mathbf{H}(\mathbf{r}, t) \cong -\frac{1}{4\pi c} \iiint_V \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \hat{\mathbf{e}}_u \times \frac{\partial}{\partial t} \mathbf{J}\left(\mathbf{r}', t - \frac{\|\mathbf{r} - \mathbf{r}'\|}{c}\right) d\mathbf{r}'$$

$$\mathbf{E}(\mathbf{r}, t) \cong -\frac{\eta}{4\pi c} \iiint_V \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \frac{\partial}{\partial t} \mathbf{J}^\perp\left(\mathbf{r}', t - \frac{\|\mathbf{r} - \mathbf{r}'\|}{c}\right) d\mathbf{r}'$$

Description of the Current Source

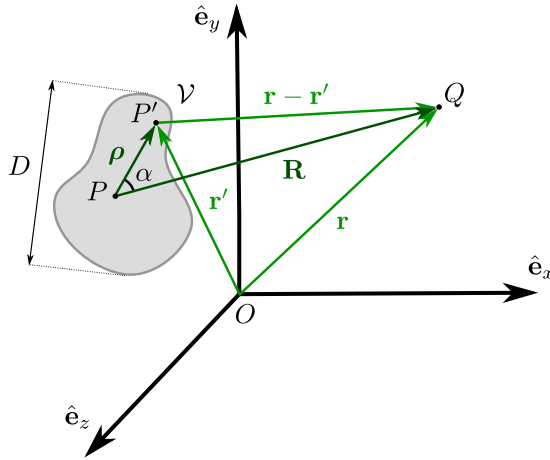


Fig. 2.5 Description of the electric current source.

2.4.1 Near- and Far-Field Propagation Regions

Let us define by O the origin of the Cartesian coordinate system, P' an arbitrary point of the current source, and Q the observation point. The vectors $\mathbf{r}'$ and $\mathbf{r}$ defined previously are respectively pointing from the origin to points P' and Q . One may now define another point P as the *reference point* of the current source. The vector pointing from the reference point P to any point P' of the current source is denoted as $\boldsymbol{\rho}$, and the vector pointing from the reference point P to the observation point Q is denoted as $\mathbf{R}$. Additionally, the largest dimension of the current source is denoted as D . It corresponds to $\max(2\rho)$. These quantities are illustrated in Figure 2.5.

In order to compute the radiated fields in Box 2.11, a Taylor approximation of the distance $\|\mathbf{r} - \mathbf{r}'\|$ can be performed to simplify these expressions. Defining the angle α as the angle between the vectors $\boldsymbol{\rho}$ and $\mathbf{R}$, and denoting by ρ and R the norms of these two vectors, the distance $\|\mathbf{r} - \mathbf{r}'\|$ can be approximated as

$$\|\mathbf{r} - \mathbf{r}'\| = \sqrt{R^2 + \rho^2 - 2R\rho \cos \alpha} = R \sqrt{1 + \left(\frac{\rho}{R}\right)^2 - 2\frac{\rho}{R} \cos \alpha}$$

Near- and Far-Field Propagation

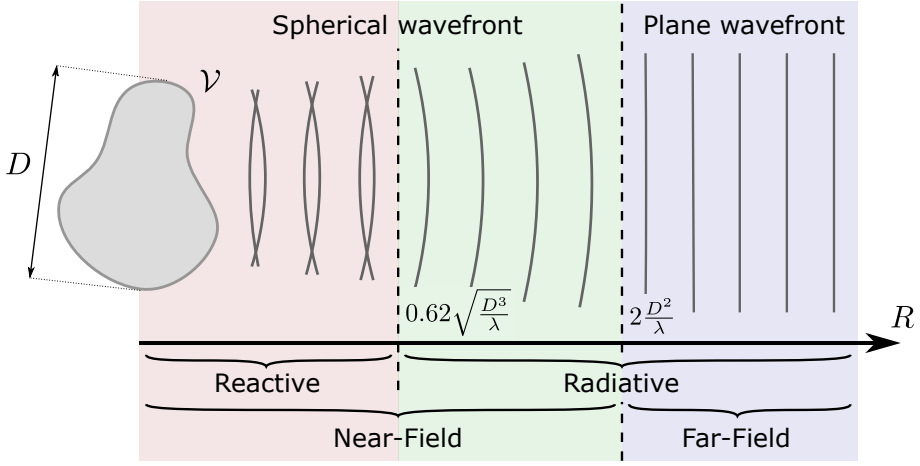


Fig. 2.6 Reactive and radiative near- and far-field regions.

$$\approx R - \rho \cos \alpha + \frac{1}{2} \frac{\rho^2}{R} \sin^2 \alpha + \frac{1}{2} \frac{\rho^3}{R^2} \sin^2 \alpha \cos \alpha + O(\rho^4),$$

Assuming that a phase error up to $\pi/8$ is acceptable in radiated fields:

- One can neglect the third order term if

$$\frac{k}{2} \frac{\rho^3}{R^2} \sin^2 \alpha \cos \alpha \leq \frac{k}{3\sqrt{3}} \frac{\left(\frac{D}{2}\right)^3}{R^2} \leq \frac{\pi}{8} \quad \Leftrightarrow \quad R > 0.62 \sqrt{\frac{D^3}{\lambda}} \triangleq R_{\text{NF}},$$

where R_{NF} is the *Fresnel distance*.

- One can neglect the second order term if

$$\frac{k}{2} \frac{\rho^2}{R} \sin^2 \alpha \leq \frac{k}{2} \frac{\left(\frac{D}{2}\right)^2}{R} \leq \frac{\pi}{8} \quad \Leftrightarrow \quad R > \frac{2D^2}{\lambda} \triangleq R_{\text{FF}}, \quad (2.11)$$

where R_{FF} is the *Fraunhofer distance*.

Based on that, two regions are defined, namely the *near-* and *far-field*, respectively if the distance is smaller or larger than the Fraunhofer distance. In the far-field, it is also assumed for the amplitude that $\|\mathbf{r} - \mathbf{r}'\| \approx R$, and $\hat{\mathbf{e}}_u = \mathbf{R}/R$. This enables to rewrite the expressions of the radiated fields as in Box 2.13.

The near-field region can be further separated into two subregions, namely the *reactive near-field* and the *radiative near-field* (or *Fresnel region*), depending on the predominance of the reactive (i.e. the components proportional to $\|\mathbf{r} - \mathbf{r}'\|^{-3}$) or

2.13: Radiated EM Fields (Far-Field)

$$\begin{aligned}\mathbf{H} &\cong -\frac{jk}{4\pi} \frac{e^{-jkR}}{R} \iiint_{\mathcal{V}} e^{jk\rho \cdot \hat{\mathbf{e}}_u} \hat{\mathbf{e}}_u \times \mathbf{J}(\mathbf{r}') d\mathbf{r}' \\ \mathbf{E} &\cong -\frac{jk\eta}{4\pi} \frac{e^{-jkR}}{R} \iiint_{\mathcal{V}} e^{jk\rho \cdot \hat{\mathbf{e}}_u} \mathbf{J}^\perp(\mathbf{r}') d\mathbf{r}'\end{aligned}$$

radiative components (i.e. the other components) of the fields. The limit between both subregions is usually given by the Fresnel distance, or by $\lambda/2\pi$ for very short radiators. The different propagation regions are summarised in Figure 2.6.

From Box 2.13, one can define in the far-field the *radiation vector* of the antenna by Box 2.14. The radiated fields in the far-field are generated by the transverse components of the radiation vector. Using (2.5), the component of the radiation vector perpendicular to the direction of propagation is defined as

$$\mathbf{F}^\perp \triangleq \iiint_{\mathcal{V}} e^{jk\rho \cdot \hat{\mathbf{e}}_u} \mathbf{J}^\perp(\mathbf{r}') d\mathbf{r}'.$$

This enables to rewrite the radiated fields in far-field following Box 2.15.

2.14: Radiation Vector (Far-Field)

$$\mathbf{F} \triangleq \iiint_{\mathcal{V}} e^{jk\rho \cdot \hat{\mathbf{e}}_u} \mathbf{J}(\mathbf{r}') d\mathbf{r}'$$

2.15: Radiated EM Fields from the Radiation Vector (Far-Field)

$$\begin{aligned}\mathbf{H} &\cong -\frac{jk}{4\pi} \frac{e^{-jkR}}{R} \hat{\mathbf{e}}_u \times \mathbf{F}^\perp \\ \mathbf{E} &\cong -\frac{jk\eta}{4\pi} \frac{e^{-jkR}}{R} \mathbf{F}^\perp\end{aligned}$$

2.4.2 Poynting Vector and Antenna Characteristics

The *Poynting vector* represents the power density of the radiated fields. In the frequency domain, it is defined in Box 2.16. For plane and spherical waves, assuming distances larger than the wavelength, the Poynting vector is developed from Box 2.5

2.16: Poynting Vector

$$\mathbf{S} \triangleq \frac{1}{2} \mathbf{E} \times \mathbf{H}^*$$

as

$$\mathbf{S} = \frac{|\mathbf{E}|^2}{2\eta} \hat{\mathbf{e}}_u = \eta \frac{|\mathbf{H}|^2}{2} \hat{\mathbf{e}}_u$$

Considering the radiated EM fields in the far-field, this expression can be further developed using Box 2.15:

$$\mathbf{S} = \frac{k^2 \eta}{32\pi^2 R^2} |\mathbf{F}^\perp|^2 \hat{\mathbf{e}}_u.$$

The *radiation intensity* is defined as the power radiated by unit solid angle. Denoting the surface element by $dS = R^2 d\Omega$, where $d\Omega = \sin\theta d\theta d\phi$ is the solid angle element, it is defined in Box 2.17, where S is the norm of the Poynting vector, and the dependence of $\mathbf{F}^\perp$ with the angles has been written explicitly. From Box 2.17, the total radiated power is computed as

$$P_{\text{rad}} = \iint_{\theta, \phi} U(\theta, \phi) d\Omega.$$

2.17: Radiation Intensity

$$U(\theta, \phi) \triangleq R^2 S = \frac{k^2 \eta}{32\pi^2} |\mathbf{F}^\perp(\theta, \phi)|^2$$

Based on the radiation intensity, the *directivity* is finally defined as the ratio between the radiation intensity of the antenna compared to the radiation intensity of an isotropic radiator. The *gain* of the antenna is related to the directivity, as it is defined as ratio between the radiation intensity of the antenna compared to the power accepted by the antenna P_t . Both are expressed in Box 2.18. In these expressions, the two powers P_t and P_{rad} are different owing to the losses of the antenna. One can invert the definition of Box 2.18 to compute the power density in any direction:

$$S = \frac{U(\theta, \phi)}{R^2} = \frac{P_t G(\theta, \phi)}{4\pi R^2} = \frac{P_{\text{rad}} D(\theta, \phi)}{4\pi R^2}. \quad (2.12)$$

Owing to that expression, the **Effective Isotropic Radiated Power (EIRP)** is therefore defined as the transmitted power in the maximum direction:

$$\text{EIRP} \triangleq \max_{\theta, \phi} P_t G(\theta, \phi) = \max_{\theta, \phi} P_{\text{rad}} D(\theta, \phi). \quad (2.13)$$

2.18: Antenna Directivity and Gain

$$D(\theta, \phi) \triangleq \frac{4\pi U(\theta, \phi)}{P_{\text{rad}}} = \frac{4\pi |\mathbf{F}^\perp(\theta, \phi)|^2}{\int_\theta \int_\phi |\mathbf{F}^\perp(\theta, \phi)|^2 d\theta d\phi}$$

$$G(\theta, \phi) \triangleq \frac{4\pi U(\theta, \phi)}{P_t}$$

Until now, antennas in transmission mode have been considered. When an antenna is operating in the receiving mode, the *effective area* is defined as the ratio between the received power P_r at the antenna terminal and the arriving power density S_{inc} . For an antenna with a matched load, it is also related to the antenna gain (or directivity gain if losses in the antenna are neglected) in Box 2.19.

2.19: Effective Area (Matched Load)

$$A_e(\theta, \phi) \triangleq \frac{P_r}{S_{\text{inc}}(\theta, \phi)} = \frac{\lambda^2}{4\pi} G(\theta, \phi).$$

One can also compute the voltage and current generated at the receiver from the received electric field $\mathbf{E}_{\text{inc}}$ thanks to the *Lorentz reciprocity theorem*. In particular, this theorem states that the voltage generated at a first antenna by the **EM** fields radiated by a second antenna, fed by a current source is equal to the voltage generated at the second antenna by the **EM** fields radiated by the first antenna, fed by the same current source. This means that the structure of the antennas and the environment affect the transferred **EM** fields in the same way independently of the direction in which they propagate. Therefore, antennas can be analysed in transmitting modes, and reciprocity can be used for the receiving mode. As a consequence, denoting by Z_{ant} and Z_{load} the impedance of the receiving antenna and the load, of the voltage and current generated at the receiver are computed in Box 2.20.

2.20: Voltage and Current in Receiving mode

$$V_r = \frac{1}{I} \mathbf{F}^\perp \cdot \mathbf{E}_{\text{inc}},$$

$$I_r = \frac{1}{(Z_{\text{ant}} + Z_{\text{load}})I} \mathbf{F}^\perp \cdot \mathbf{E}_{\text{inc}}.$$

Radiation from a Linear Antenna

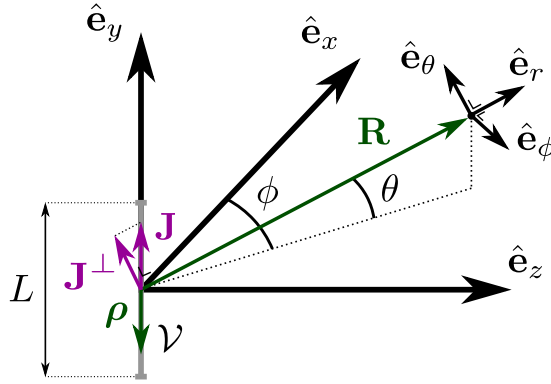


Fig. 2.7 Radiation from a linear antenna.

2.4.3 Hertzian Dipole and Uniform Line Element

Let us consider the linear antenna illustrated in Figure 2.7. The spherical coordinate system is defined as

$$\begin{pmatrix} \hat{\mathbf{e}}_r \\ \hat{\mathbf{e}}_\theta \\ \hat{\mathbf{e}}_\phi \end{pmatrix} = \begin{pmatrix} \cos \theta \cos \phi & \sin \theta & \cos \theta \sin \phi \\ -\sin \theta \cos \phi & \cos \theta & -\sin \theta \sin \phi \\ -\sin \phi & 0 & \cos \phi \end{pmatrix} \begin{pmatrix} \hat{\mathbf{e}}_x \\ \hat{\mathbf{e}}_y \\ \hat{\mathbf{e}}_z \end{pmatrix}.$$

The direction of propagation is $\hat{\mathbf{e}}_u = \hat{\mathbf{e}}_r$.

Multiple models for the current density flowing into the antenna can be considered. The simplest models are the *Hertzian dipole*, which is a point source of electric current, and the *uniform line element*, for which a constant electric current flows in the complete antenna. With such radiative elements, the current density is given by

$$\mathbf{J}(\mathbf{r}') = \begin{cases} LI \delta(\mathbf{r}') \hat{\mathbf{e}}_y & \text{for a Hertzian dipole,} \\ LI \delta(x) \delta(z) \hat{\mathbf{e}}_y & \text{for a uniform line element.} \end{cases}$$

In these expressions, I is the current flowing through the dipole, and L is the length of the dipole. The length is assumed to be a fraction of the wavelength, i.e. $L = \lambda/n$ with $n \in \mathbb{N}$. The Fraunhofer distance of such radiator is computed as

$$R_{\text{FF}} = \frac{2L^2}{\lambda} = \frac{2}{n^2} \lambda.$$

In order to obtain an analytical expression of the radiative fields, it is also assumed that $kR \gg 1$. Therefore, the distance R is larger than the Fraunhofer distance, and

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the wave propagates in the far-field region. Defining $\boldsymbol{\rho} = y \hat{\mathbf{e}}_y$, the radiation vector of the Hertzian dipole is computed as

$$\mathbf{F}_{\text{Hz}} = \iiint_{\mathcal{V}} e^{jk y \sin \theta} LI \delta(\mathbf{r}') \hat{\mathbf{e}}_y d\mathbf{r}' = LI \hat{\mathbf{e}}_y.$$

For the uniform line element,

$$\begin{aligned} \mathbf{F}_{\text{ULE}} &= \iiint_{\mathcal{V}} e^{jk y \sin \theta} LI \delta(y) \delta(z) \hat{\mathbf{e}}_y d\mathbf{r}' \\ &= LI \int_{-\frac{L}{2}}^{\frac{L}{2}} e^{jk y \sin \theta} dy = LI \operatorname{sinc} \left(\frac{L}{\lambda} \sin \theta \right) \hat{\mathbf{e}}_y, \end{aligned}$$

where sinc is the normalised cardinal sine. Using Box 2.17, the radiation intensity of the Hertzian dipole and the uniform line element are respectively computed as

$$\begin{aligned} U_{\text{Hz}}(\theta, \phi) &= \frac{k^2 \eta L^2 I^2}{32 \pi^2} \cos^2 \theta, \\ U_{\text{ULE}}(\theta, \phi) &= \frac{k^2 \eta L^2 I^2}{32 \pi^2} \operatorname{sinc}^2 \left(\frac{L}{\lambda} \sin \theta \right) \cos^2 \theta, \end{aligned}$$

which is maximum for $\theta = 0$. Therefore, in both cases, the EIRP of these antennas is identical, and it is given by

$$\text{EIRP}_{\text{Hz}} = \text{EIRP}_{\text{ULE}} = \frac{k^2 \eta L^2 I^2}{8 \pi}.$$

The electric and magnetic fields are finally computed with the radiation vector following Box 2.15, giving Box 2.21.

2.21: Radiated EM Fields from Linear Antennas (Far-Field)

$$\begin{aligned} \mathbf{H} &\cong -\frac{jkLI}{4\pi} \frac{e^{-jkR}}{R} f(\theta) \hat{\mathbf{e}}_\phi \\ \mathbf{E} &\cong -\frac{jk\eta LI}{4\pi} \frac{e^{-jkR}}{R} f(\theta) \hat{\mathbf{e}}_\theta \end{aligned}$$

In Box 2.21, the function f is the amplitude of the transverse radiation vector $\mathbf{F}^\perp$ for the considered current density distribution, normalised by the amplitude of the current density:

$$f \triangleq \frac{|\mathbf{F}^\perp|}{LI} = \begin{cases} \cos \theta & \text{for a Hertzian dipole,} \\ \operatorname{sinc} \left(\frac{L}{\lambda} \sin \theta \right) \cos \theta & \text{for a uniform line element.} \end{cases} \quad (2.14)$$

2.5 Complex Baseband Representation of Electromagnetic Fields

It is usually convenient to express the transmitted signals in their equivalent complex baseband representation to simplify the mathematical analyses. Therefore, one also want to express the EM fields in their equivalent baseband representations to relate these quantities to baseband signals. Expressions of the time-domain radiated fields in radio-frequency are provided in Box 2.12, involving the derivative of the current density $\mathbf{J}_{\text{RF}}$ in radio-frequency, delayed by a delay τ . Such derivative can further be developed as

$$\begin{aligned}\frac{\partial}{\partial t} \mathbf{J}_{\text{RF}}(t - \tau) &= \frac{\partial}{\partial t} \text{Re} \left\{ \mathbf{J}_{\text{BB}}(t - \tau) e^{J\omega_c(t-\tau)} \right\} \\ &= \text{Re} \left\{ \frac{\partial}{\partial t} \mathbf{J}_{\text{BB}}(t - \tau) e^{J\omega_c(t-\tau)} + J\omega_c \mathbf{J}_{\text{BB}}(t - \tau) e^{J\omega_c(t-\tau)} \right\} \\ &= \text{Re} \left\{ \left(\frac{\partial}{\partial t} \mathbf{J}_{\text{BB}}(t - \tau) + J\omega_c \mathbf{J}_{\text{BB}}(t - \tau) \right) e^{-J\omega_c\tau} e^{J\omega_c t} \right\}.\end{aligned}$$

The baseband representation of the delayed derivative of the current density is therefore given by

$$\left[\frac{\partial}{\partial t} \mathbf{J}_{\text{RF}} \right]_{\text{BB}}(t - \tau) = \left(\frac{\partial}{\partial t} \mathbf{J}_{\text{BB}}(t - \tau) + J\omega_c \mathbf{J}_{\text{BB}}(t - \tau) \right) e^{-J\omega_c\tau}.$$

For narrowband signals, as $B \ll f_c$, the first term with a magnitude in frequency related to the bandwidth, is negligible compared to the second term with a magnitude in frequency related to the carrier frequency:

$$\left[\frac{\partial}{\partial t} \mathbf{J}_{\text{RF}} \right]_{\text{BB}}(f) = J(\omega + \omega_c) e^{-J(\omega + \omega_c)\tau} \mathbf{J}_{\text{BB}}(f) \approx J\omega_c e^{-J(\omega + \omega_c)\tau} \mathbf{J}_{\text{BB}}(f).$$

Defining $k_c \triangleq \omega_c/c$, this enables to rewrite the radiated EM fields in complex baseband representation for narrowband signals as expressed in Box 2.22.

2.22: Radiated EM Fields in Baseband (Time Domain, $k\|\mathbf{r} - \mathbf{r}'\| \gg 1$, $B \ll f_c$)

$$\begin{aligned}\mathbf{H}(\mathbf{r}, t) &\cong -\frac{jk_c}{4\pi} \iiint_{\mathcal{V}} \frac{e^{-jk_c\|\mathbf{r} - \mathbf{r}'\|}}{\|\mathbf{r} - \mathbf{r}'\|} \hat{\mathbf{e}}_u \times \mathbf{J}\left(\mathbf{r}', t - \frac{\|\mathbf{r} - \mathbf{r}'\|}{c}\right) d\mathbf{r}' \\ \mathbf{E}(\mathbf{r}, t) &\cong -\frac{jk_c\eta}{4\pi} \iiint_{\mathcal{V}} \frac{e^{-jk_c\|\mathbf{r} - \mathbf{r}'\|}}{\|\mathbf{r} - \mathbf{r}'\|} \mathbf{J}^\perp\left(\mathbf{r}', t - \frac{\|\mathbf{r} - \mathbf{r}'\|}{c}\right) d\mathbf{r}'\end{aligned}$$

Electromagnetic-Based Radar and Communication Propagation Models

3

Most of the content of this chapter is inspired from [De +24a].

EXPRESSIONS of the EM fields radiated by a given current source, i.e. a transmitting antenna, were provided in Chapter 2, which also defines useful quantities to characterise the power received at another antenna. Thus, it is now possible to compute the link budget of radar and communication links. Yet, for radar applications, different models, namely the radar equation and Geometrical Optics (GO) approximation, are widely used in the literature. At a first glance, they may seem unrelated to each other, as they usually rely on incompatible assumptions, leading to expressions of the received power which are contradictory with each other. Thus, the aim of this chapter is to clarify how these models are related to each other, and to develop novel radar propagation models overcoming their limitations.

In a first time, the usual radar and communication models are thoroughly described in Section 3.1. Then, starting from scratch, the scattered fields by flat and curved square plate are computed in Section 3.2. From, these expressions, the relation between the radar equation and the GO approximation is highlighted, reconciling the two models. Then, novel expressions of the radar cross-sections are obtained, and their impact is analysed numerically in Section 3.3. Finally, experimental measurements in an anechoic chamber are proposed in Section 3.4, to validate the radar cross-section behaviour with the distance, carrier frequency and target geometry.

3.1 Radar and Communication Propagation Models

Multiple propagation models for radar and communication systems are used in the literature. This section aims to summarise the most common models of the literature, and to highlight their differences. On the one hand, for the communication function, the Friis formula is widely used, and extended with complex path-loss models. On the other hand, for the radar function, the radar equation is the most known and extensively used model. Though, from the Ray-Tracing (RT) literature, another propagation model based on the GO approximation is considered for reflections.

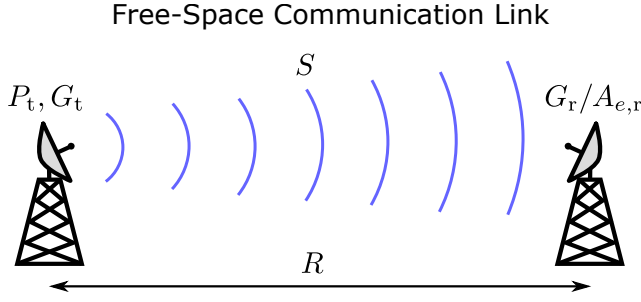


Fig. 3.1 Free-space communication link.

3.1.1 Friis Formula and Radar Equation

Let us first consider a transmitting and a receiving antenna, with respective (maximum) gains G_t and G_r , separated by a distance R . The transmitting antenna is fed by a power P_t . This scenario is illustrated in Figure 3.1. As shown in (2.12), the power density at the receiving antenna is given by

$$S = \frac{P_t G_t}{4\pi R^2} = \frac{\text{EIRP}}{4\pi R^2},$$

where the **EIRP** is defined in (2.13). The received power P_r is then computed using the effective area $A_{e,r}$ of the receiving antenna, given in Box 2.19. Multiplying the incident power density and the effective area of the receiving antenna, the expression of Box 3.1 is obtained. It is known as the *Friis formula* [Fri46].

3.1: Friis Formula

$$P_r = S A_{e,r} = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 R^2}$$

Similar developments can be performed for radar applications. Let us denote by R_t (resp. R_r) the distance between the transmitting (resp. receiving) antenna and the target. The considered scenario is illustrated in Figure 3.2. The incident power density S^i at the target is given by

$$S^i = \frac{P_t G_t}{4\pi R_t^2} = \frac{\text{EIRP}}{4\pi R_t^2}.$$

Similarly to the effective area, the **Radar Cross-Section (RCS)** σ of the target is then defined, as the area intercepting the amount of incident power which would produce the same received power at the radar receiver if it was radiated isotropically. It can be expressed with the incident and reflected power densities S^i and S^r , or equivalently with the incident and reflected electric fields $\mathbf{E}^i$ and $\mathbf{E}^r$, as given in Box 3.2.

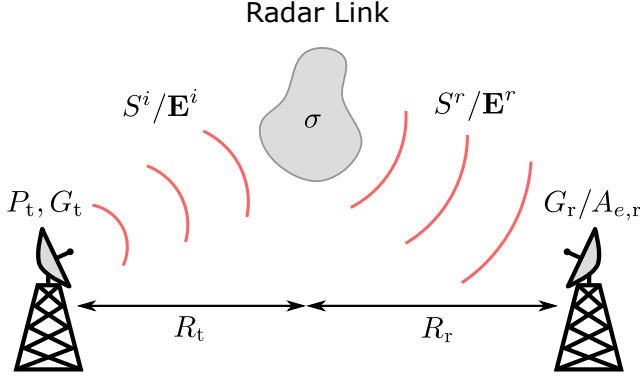


Fig. 3.2 Radar link.

3.2: Radar Cross-Section

$$\sigma \triangleq 4\pi R_r^2 \frac{S^r}{S^i} = 4\pi R_r^2 \frac{|\mathbf{E}^r|^2}{|\mathbf{E}^i|^2}$$

In order to obtain a distance-independent **RCS**, the dependence with the range is often removed by computing instead

$$\sigma_{\text{FF}} = \lim_{R_t, R_r \rightarrow \infty} 4\pi R_r^2 \frac{S^r}{S^i} = \lim_{R_t, R_r \rightarrow \infty} 4\pi R_r^2 \frac{|\mathbf{E}^r|^2}{|\mathbf{E}^i|^2}.$$

However, by taking the limit of the distance to the infinity, one forces the propagation to occur in the far-field region.

The **RCS** is a widely used quantity which is in practice a complex function of many parameters, e.g. as the target geometry, the properties of the material, or the carrier frequency, angle and polarisation of the incident wave. In many applications, the **RCS** is a key quantity to evaluate. For military purposes, aerial and terrestrial vehicles require a low detectability, and the **RCS** is the parameter to minimise. At the opposite, in automotive applications, a high **RCS** is beneficial to improve the detection of other vehicles, pedestrians or obstacles.

The received power P_r is finally obtained by computing the product between the reflected power density S^r at the receiving antenna and its effective area. Using the **RCS** of Box 3.2, Box 3.3 is obtained. This is the well-known *radar equation* [Ric05].

3.3: Radar Equation

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R_t^2 R_r^2}$$

Full-Duplex JCAS Scenario

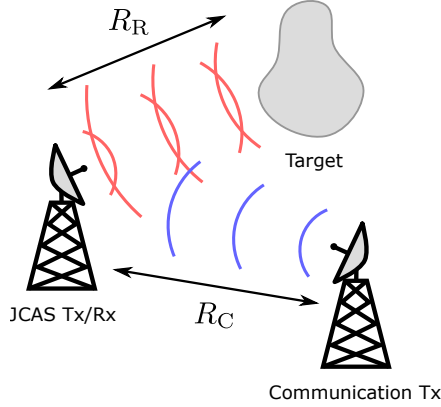


Fig. 3.3 Full-duplex JCAS scenario. The ISAC system simultaneously receives a radar echo from a target, and a communication signal from another system. The downlink communication and interference sources have been omitted for the sake of clarity.

3.1.2 Propagation Losses for Radar and Communication

From Box 3.1, one can isolate the *free-space losses* in communication links as

$$L_{FS,C} = \frac{P_t G_t G_r}{P_r} = \left(\frac{4\pi R}{\lambda} \right)^2 = \frac{4\pi}{\lambda^2} l(R), \quad (3.1)$$

with the so-called *propagation function* equal to

$$l(R) = 4\pi R^2. \quad (3.2)$$

A similar reasoning can be done for radar links in Box 3.3:

$$L_{FS,R} = \frac{P_t G_t G_r}{P_r} = \frac{(4\pi)^3 R_t^2 R_r^2}{\lambda^2 \sigma} = \frac{4\pi}{\lambda^2} l(R_t) l(R_r) \sigma^{-1}. \quad (3.3)$$

Comparing the losses for communication and radar links, the impact of the wavelength is identical for both functions. However, an additional propagation loss affects the radar echoes compared to the communication signals owing to the travelled distance of the reflected wave.

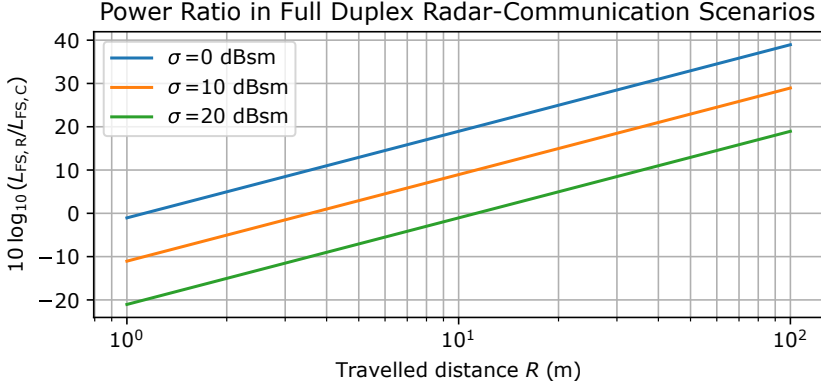


Fig. 3.4 Radar echo power against communication signal power, for the same total travelled distance R .

Let us consider the full-duplex scenario of Figure 3.3. In this scenario, a monostatic ISAC system detects a target at a distance R_R , while receiving simultaneously a communication signal from another ISAC system at a distance R_C . For the same travelled distance R , i.e. $R_C = 2R_R = R$, the power ratio between the radar echo and the communication signal is given by

$$\frac{L_{FS,R}}{L_{FS,C}} = \frac{L\left(\frac{R}{2}\right)^2}{L(R)} \sigma = \frac{L(R)}{16\sigma} = \frac{\pi R^2}{4\sigma}.$$

Therefore, for the radar echo power to be equivalent to the communication signal power for the same travelled distance R , the RCS of the target must be close to $\pi R^2/4$.

Figure 3.4 illustrates this relation for RCS values of 0, 10 and 20 dBsm. Following [Ric05], these values respectively correspond to the order of magnitude of RCSs for humans, automobiles and jet airliners. To obtain equivalent radar and communication power levels, the total travelled distance R must be comprised between 1 and 10 meters depending on the RCS of the target, meaning that in most practical scenarios, the power of the radar echo is much lower than the power of the uplink communication signal.

This conclusion would be valid if the RCS was independent with the distance, which is not always the case in practice. As specified in Section 3.1.1, the RCS of Box 3.2 is also a function of the distance, except in the far-field region. In order to know if the far-field assumption holds, the Fraunhofer distance R_{FF} in (2.11) should be computed. Regarding the dimension of the antennas, as they are often designed w.r.t. the wavelength, the Fraunhofer distance is in most of the cases much smaller than the considered distances between the transmitting and receiving radar antennas, and the targets. This means that the transmission occurs in the far-field region of

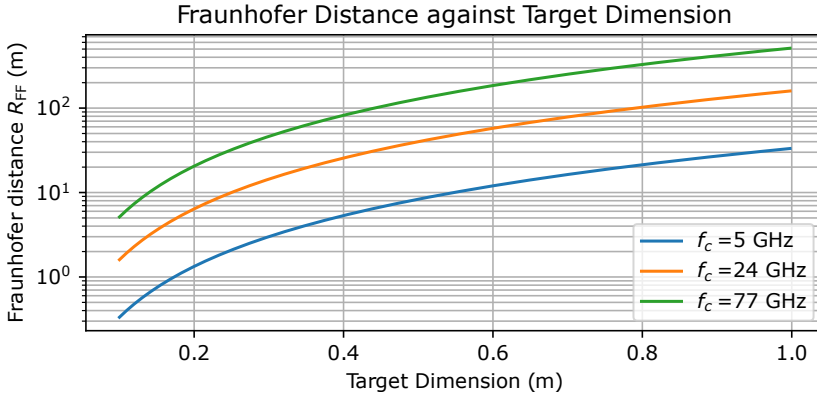


Fig. 3.5 Fraunhofer distance against the target dimension, for different carrier frequencies.

both antennas. However, as the incident power density is absorbed by the target, one should also verify if the Fraunhofer distance of the target is smaller than the distance between the target and the receiving antenna. Thus, since there is no relation between the target dimension and the wavelength, one cannot guarantee that the far-field assumption holds for the reflection.

Figure 3.5 shows the evolution of the Fraunhofer distance with the target dimension, for different carrier frequencies. Frequencies of 5, 24 and 77 GHz respectively correspond to IEEE 802.11ac (and newer) Wi-Fi systems [13], and automotive radar systems [20b]. At the automotive radar carrier frequencies, the Fraunhofer distance is varying from meters to multiple hundred of meters, increasing quickly with the target dimension. For a target dimension of 1 meter, corresponding roughly to the back of an automobile or a truck, the Fraunhofer distance is way larger than the distance between the radar system and the target. This means that, at such frequencies, the propagation occurs in the near-field, and the RCS is highly dependent with the distance! Even at 5 GHz, the Fraunhofer distance increases quickly to a dozen meters, which is comparable to large rooms dimensions, making the far-field assumption questionable.

3.1.3 Geometrical Optics Propagation Model

The RT is a numerical method based on the GO approximation to compute the propagation modelling of complex environment [MPM90]. Rays are obtained when considering the high frequency approximation of the Maxwell's equations. Under that assumption, the propagation always occurs in the near-field region. From the RT literature, the received power at the radar receiver is computed following Box 3.4 [YI15], where $\mathcal{R}$ is the Fresnel reflection coefficient of the target at the specular reflection point, assuming a given polarisation.

3.4: Radar Received Power (GO Approximation)

$$P_r = \frac{P_t G_t G_r \lambda^2 |\mathcal{R}|^2}{(4\pi)^2 (R_t + R_r)^2}$$

In order to be consistent with the radar equation in the near-field, the RCS should be equal to

$$\sigma = 4\pi |\mathcal{R}|^2 \left(\frac{R_t R_r}{R_t + R_r} \right)^2. \quad (3.4)$$

Therefore, in RT applications, the received power is modelled as a function which decays with the *square of the total travelled distance*. This highlights the importance to model correctly the RCS: with a constant RCS, the radar equation gives a received power which decays with *the product of the square of the back and forth distances (fourth power)*.

3.1.4 Extension with other Path-Loss Models

In order to relate the path-losses of (3.1) to real-world measurements, free-space losses have been extended in the literature into different path-loss models. One proposition is the *alpha-beta-gamma model* [Sun+16]. For a distance R in meters and a carrier frequency f in gigahertz, the path-loss is given by Box 3.5, where $R_0 = 1\text{m}$ is a reference distance, $f_0 = 1\text{GHz}$ is a reference frequency, and α , β and γ are three parameters to optimise w.r.t. measurements. In the particular case of $\alpha = 2$, $\gamma = 2$ and $\beta = (4\pi R_0 f_0 \cdot 10^9 / c)^2$, Box 3.5 is expressed in natural values as

$$L(R, f) = \left(\frac{R}{R_0} \right)^2 \left(\frac{4\pi R_0 f_0 \cdot 10^9}{c} \right)^2 \left(\frac{f}{f_0} \right)^2 = \left(\frac{4\pi R}{\lambda} \right)^2,$$

and the free-space losses model of (3.1) is recovered. Another common model is the *floating intercept model* [Mac+13]. It is equivalent to the alpha-beta-gamma model with $\gamma = 2$.

3.5: Alpha-Beta-Gamma Model

$$[L(R, f)]_{\text{dB}} = 10\alpha \log_{10} \left(\frac{R}{R_0} \right) + [\beta]_{\text{dB}} + 10\gamma \log_{10} \left(\frac{f}{f_0} \right)$$

The presented path-loss models are common in the communication literature, but they are less used in the radar literature. One example of such transformation is

3 | Electromagnetic-Based Radar and Communication Propagation Models

presented in [GLC21], which applies a two-way path-loss model to measurements obtained with a monostatic radar to extract path-loss parameters for communication links. Another example is [Cha+23], which performs an empirical path-loss modelling for ISAC at 28 GHz. Additionally, this paper takes into account the variation of the RCS with the distance in the near-field region, following the RCS model developed in [Pou+06].

In this chapter, the Friis and radar equation are extended, based on the floating intercept path-loss model, by replacing only the propagation function l of (3.2) by

$$l(R) = \beta R^\alpha.$$

In that case, the received powers P_C and P_R at the communication and radar receivers, respectively computed with the Friis formula and the radar equations, are rewritten as

$$P_C = \frac{P_t G_t G_r \lambda^2}{4\pi \beta R^\alpha}, \quad P_R = \frac{P_t G_t G_r \lambda^2 \sigma}{4\pi \beta^2 R_t^\alpha R_r^\alpha}.$$

Note that choosing $\beta = 4\pi$ and $\alpha = 2$ leads back to Boxes 3.1 and 3.3. With a modified propagation function, the RCS definition must be adapted as well to take into account the path-loss parameters, giving Box 3.6.

3.6: Radar Cross-Section with Arbitrary Propagation Function

$$\sigma \triangleq \beta R_r^\alpha \frac{(S^r)^{\frac{\alpha}{2}}}{(S^i)^{\frac{\alpha}{2}}} = \beta R_r^\alpha \frac{|\mathbf{E}^r|^\alpha}{|\mathbf{E}^i|^\alpha}$$

By only modifying the propagation function and not the complete path-loss model, the impact of the propagation is isolated from the impact of the target and antenna. Thus, the same path-loss parameters can be used for both the communication and radar functions. Otherwise, if the complete path-loss function is modified for each function, the alpha-beta-gamma model can be used, but different models are required for each function, and the RCS model should also be integrated in the radar path-loss function.

3.1.5 Radar and Communication Power Levels in the Literature

In the literature, when discussing the interference between the communication and radar functions, one can often read multiple variations of the sentence: *"the radar interference power decreases with the fourth power of the distance, whereas the communication signal power decreases with the square of the distance."* This is the con-

sequence of the analysis which has been performed in Section 3.1.2 with constant RCS.

For instance, [Adi+23] makes that assumption while studying the sensing performance of channel-coded communication signals in an interference-limited scenario. The interference mitigation performance in multi-hop cooperative radar-communication vehicular networks is analysed in [AKW21], by defining the radar SNR using the radar equation with constant RCS. The same assumption is made in [Str+19] which uses the fractional Fourier transform coupled with an interference mitigation scheme in a multi-user JCAS automotive scenario. Another framework in which the radar equation is widely used is the sSG framework, which is the main topic of Part II. Many examples which extensively use the radar equation with constant RCSs in this framework are further described in Chapter 6.

Nonetheless, as described in Section 3.1.2, in most cases, a constant RCS could not be assumed, and the radar echoes power levels aren't decreasing with the fourth power of the distance in the near-field. Distance-dependent RCS models are required. This is further discussed in Section 3.1.6.

3.1.6 Near-Field Radar Cross-Section in the Literature

In the XXth century, [TT97] already showed that the RCS of a sphere and a square plate in the near-field region is dependent of the range and orientation of the antennas. Two new definitions of the RCS are proposed in the near-field, as variations of the radar equation. The monostatic and bistatic RCS in the near-field is also analysed with a PO approach in [PD96]. Numerical examples for rectangular and circular plates are also provided. It has been extended in [Pou+06], where complex targets are decomposed into small triangular elements, such that each element propagates in the far-field. A similar approach is proposed in [Leg04].

Following these works, multiple papers have analysed the RCS of targets in the near-field. Namely, [Deb+09] and [Deb+10] compare numerical simulations and measurements for rectangular plates with an oblique incidence at 24 GHz. Other simple targets are analysed in [Td18] and [Cho+21]. Circular plates and cylinder are evaluated numerically for the first. Sphere and cylinder are studied numerically, and compared with measurements at 5.24 GHz for the second. The RCS of complex targets, i.e. the back of a car at 77 GHz, are evaluated in [ER19]. From another perspective, [Cho22] proposes to use the RCS model of [Cho+21] to develop a procedure extracting the scattering matrix coefficients of the targets using analogue beamforming.

Nonetheless, owing to the complexity of the RCS computation, most of these works focus on numerical analysis and measurements. Instead, [Lah+20] proposes

three-dimensional computationally efficient models based on Fresnel integrals to calculate the backscattering model of a rectangular plate. It also compares these models to measurements at 28 GHz. This work is further extended in [Lah+22], which leverages on the proposed models to evaluate the backscattered fields with multiple oriented rectangular surfaces. Therefore, fully analytical expressions for simple geometries are developed in Section 3.2.

3.2 Electromagnetic-Based Monostatic Radar Cross-Section Model

In this section, an analytical model of the RCS of a rectangular plate is developed from the EM theory. In order to obtain this result, the stationary phase approximation is first introduced.

3.2.1 Stationary Phase Approximation

The *Stationary Phase Approximation* (SPA) is a useful method to approximate the integral of a rapidly oscillating function [Bor94]. For the sake of simplicity, the following development is done in one dimension, but it is generalisable for multidimensional integrations. Let us consider the following integral:

$$I = \int_{\Omega} g(x) e^{J\psi(x)} dx,$$

where Ω is the integration domain, and g and ψ are respectively an amplitude and a phase function. The phase function can be developed with its Taylor expansion around a point x_0 as

$$\psi(x) = \psi(x_0) + \left. \frac{d\psi}{dx} \right|_{x=x_0} (x - x_0) + \frac{1}{2} \left. \frac{d^2\psi}{dx^2} \right|_{x=x_0} (x - x_0)^2 + O(x^3).$$

With the SPA, the point x_0 is called *stationary point*, and chosen such that the first derivative of the phase function is equal to zero:

$$\left. \frac{d\psi}{dx} \right|_{x=x_0} = 0.$$

Denoting by ψ_0 the function ψ evaluated at x_0 , and by ψ_0'' the second derivative of ψ evaluated at x_0 , the integral is then approximated as

$$I \approx e^{J\psi_0} \int_{\Omega} g(x) e^{J\frac{1}{2}\psi_0''(x-x_0)^2} dx. \quad (3.5)$$

Next, assuming that the amplitude function g varies slowly compared to the phase function ψ in the vicinity of the stationary point, the amplitude function is taken out of the integral:

$$I \approx g(x_0) e^{J\psi_0} \int_{\Omega} e^{J\frac{1}{2}\psi_0''(x-x_0)^2} dx.$$

Finally, if the integration domain Ω is large enough to contain multiple oscillations of the phase function, or equivalently if ψ'' is large enough w.r.t. the boundaries of Ω , the second-order term of the Taylor expansion is assumed to be oscillating rapidly, leading to successive cancellations. Therefore, expanding the boundaries of Ω to the infinity does not affect the result of the integration, and the integral is approximated as

$$I \approx g(x_0) e^{J\psi_0} \int_{-\infty}^{+\infty} e^{J\frac{1}{2}\psi_0''(x_0)(x-x_0)^2} dx = g(x_0) e^{J\psi_0} \sqrt{\frac{j2\pi}{\psi_0''}}.$$

3.7: Stationary Phase Approximation

$$\begin{aligned} \int_{\Omega} g(x) e^{J\psi(x)} dx &\approx g(x_0) e^{J\psi_0} \int_{\Omega} e^{J\frac{1}{2}\psi_0''(x-x_0)^2} dx \\ &\approx g(x_0) e^{J\psi_0} \sqrt{\frac{j2\pi}{\psi_0''}} \end{aligned}$$

The SPA is summarised in Box 3.7. Note that, even for limited integration domains, the first approximation is still valid if the amplitude function is slowly varying in the integration domain. In that case, the integral can be solved using the *Fresnel integral*, defined as [Abr74]

$$F(x) = \int_0^x e^{J\pi \frac{t^2}{2}} dt, \quad (3.6)$$

and illustrated in Figure 3.6.

Based on that definition, one can easily proof that

$$\int_a^b e^{Jct^2} dt = \sqrt{\frac{\pi}{2c}} \left[F\left(\sqrt{\frac{2c}{\pi}} b\right) - F\left(\sqrt{\frac{2c}{\pi}} a\right) \right]. \quad (3.7)$$

Additionally, $F(-x) = -F(x)$, such that, if $a = -b$,

$$\int_{-b}^b e^{Jct^2} dt = \sqrt{\frac{2\pi}{c}} F\left(\sqrt{\frac{2c}{\pi}} b\right). \quad (3.8)$$

Furthermore, the SPA also requires the stationary point to be located in Ω . Other-

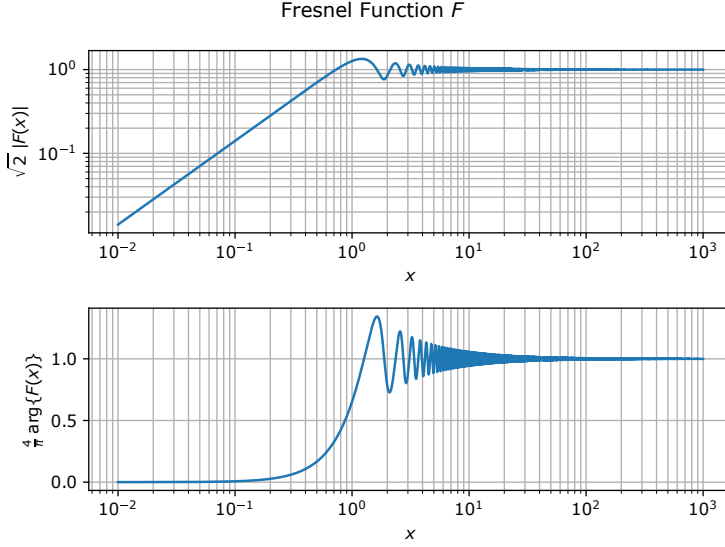


Fig. 3.6 Fresnel function.

wise, the value $g(x_0)$ may not be suited to approximate the behaviour of this function in the integration domain. Still, one may obtain a good approximation of the integral if the function g is well approximated by a Gaussian function:

$$g(x) \approx g_0 e^{\xi x^2}.$$

In that case, (3.5) can be rewritten as

$$I \approx e^{J\psi_0} \int_{\Omega} g_0 e^{\xi x^2} e^{J\frac{1}{2}\psi_0''(x-x_0)^2} dx = g_0 e^{J\psi_0} e^{\frac{J\psi_0''\xi}{2\xi+J\psi_0''} x_0^2} \int_{\Omega} e^{\frac{1}{2}(2\xi+J\psi_0'')\left(x-\frac{J\psi_0''}{2\xi+J\psi_0''} x_0\right)^2} dx,$$

where the second equality is obtained by using the following equality:

$$ax^2 + b(x - x_0)^2 = (a + b) \left(x - \frac{b}{a + b} x_0\right)^2 + \frac{ab}{a + b} x_0^2.$$

That integral can also be solved using the Fresnel integrals and (3.7).

3.2.2 Reflection on a Flat Rectangular Plate

Let us now compute the field reflected by a flat rectangular plate. The system model of Figure 3.7 is considered. A linear antenna located at $(-R, 0, 0)$ is facing a rectangular plate $\mathcal{S}$ of dimensions D_y and D_z , defined as

$$\mathcal{S} \triangleq \left\{ (x, y, z) \in \mathbb{R}^3 \mid x = 0, |y| \leq \frac{D_y}{2}, |z| \leq \frac{D_z}{2} \right\}.$$

Reflection on a Rectangular Plate

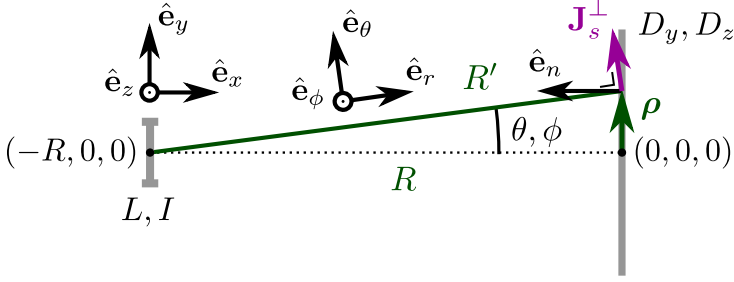


Fig. 3.7 Reflection on a flat rectangular plate (view from the z-axis).

The normal unit vector pointing towards the antennas is given by $\hat{\mathbf{e}}_n = -\hat{\mathbf{e}}_x$. The spherical coordinate system associated with the antenna is defined as

$$\begin{pmatrix} \hat{\mathbf{e}}_r \\ \hat{\mathbf{e}}_\theta \\ \hat{\mathbf{e}}_\phi \end{pmatrix} = \begin{pmatrix} \cos \theta \cos \phi & \sin \theta & \cos \theta \sin \phi \\ -\sin \theta \cos \phi & \cos \theta & -\sin \theta \sin \phi \\ -\sin \phi & 0 & \cos \phi \end{pmatrix} \begin{pmatrix} \hat{\mathbf{e}}_x \\ \hat{\mathbf{e}}_y \\ \hat{\mathbf{e}}_z \end{pmatrix}.$$

One can compute the angle θ and ϕ from the (x, y, z) coordinates as

$$\begin{aligned} \sin \phi &= \frac{z}{\sqrt{(R+x)^2 + z^2}}, & \sin \theta &= \frac{y}{\sqrt{(R+x)^2 + y^2 + z^2}}, \\ \cos \phi &= \frac{R+x}{\sqrt{(R+x)^2 + z^2}}, & \cos \theta &= \frac{\sqrt{(R+x)^2 + z^2}}{\sqrt{(R+x)^2 + y^2 + z^2}}. \end{aligned}$$

3.2.2.1 Scattered Electromagnetic Fields

The characterisation of a Hertzian dipole or a uniform line element is detailed in Section 2.4.3. Assuming that the transmitting antenna is one of these two options, the incident EM fields at the surface of the plate are computed using Box 2.21:

$$\begin{aligned} \mathbf{H}^i &\cong -\frac{jkLI}{4\pi} \frac{e^{-jkR'}}{R'} f(\theta) \hat{\mathbf{e}}_\phi, \\ \mathbf{E}^i &\cong -\frac{jk\eta LI}{4\pi} \frac{e^{-jkR'}}{R'} f(\theta) \hat{\mathbf{e}}_\theta. \end{aligned} \quad (3.9)$$

where the function f is defined in (2.14), and the distance R' is defined as

$$R' = \sqrt{R^2 + y^2 + z^2}.$$

The PO approximation enables to compute an equivalent surface current is generated at the surface $\mathcal{S}$ of the conductor from the incident EM fields following Box

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2.9. Using the **TEM** decomposition detailed in Section 2.3.5, the equivalent surface current density generated at the plate is obtained by summing the surface currents generated by the **TE** and **TM** components of the magnetic field:

$$\mathbf{J}_s \approx 2 \mathcal{R}_{\text{TM}}(\theta, \phi) H_{\perp}^i (\hat{\mathbf{e}}_{\perp} \times \hat{\mathbf{e}}_x) - 2 \mathcal{R}_{\text{TE}}(\theta, \phi) H_{\parallel}^i (\hat{\mathbf{e}}_{\parallel} \times \hat{\mathbf{e}}_x),$$

where $H_{10\perp}^i = \mathbf{H}_1^i \cdot \hat{\mathbf{e}}_{\perp}$, $H_{10\parallel}^i = \mathbf{H}_1^i \cdot \hat{\mathbf{e}}_{\parallel}$, and

$$\hat{\mathbf{e}}_{\perp} = \frac{\hat{\mathbf{e}}_x \times \hat{\mathbf{e}}_r}{\|\hat{\mathbf{e}}_x \times \hat{\mathbf{e}}_r\|}, \quad \hat{\mathbf{e}}_{\parallel} = \frac{\hat{\mathbf{e}}_x \times \hat{\mathbf{e}}_r}{\|\hat{\mathbf{e}}_x \times \hat{\mathbf{e}}_r\|} \times \hat{\mathbf{e}}_r.$$

This expression may be simplified for good conductors by using the **PEC** approximation of Box 2.10:

$$\mathbf{J}_s = 2 \mathbf{H}_1^i \times \hat{\mathbf{e}}_x = -\frac{jkLI}{2\pi} \frac{e^{-jkR'}}{R'} f(\theta) \cos \phi \hat{\mathbf{e}}_y.$$

The perpendicular component of the surface current density is then obtained as

$$\mathbf{J}_s^{\perp} = \mathbf{J}_s - (\mathbf{J}_s \cdot \hat{\mathbf{e}}_r) \hat{\mathbf{e}}_r = -\frac{jkLI}{2\pi} \frac{e^{-jkR'}}{R'} f(\theta) \cos \phi \cos \theta \hat{\mathbf{e}}_{\theta},$$

The reflected electric field is finally computed using Box 2.11 on the surface of the rectangular plate:

$$\begin{aligned} \mathbf{E}^r &\cong -\frac{jk\eta}{4\pi} \iint_S \frac{e^{-jkR'}}{R'} \mathbf{J}_s^{\perp}(\mathbf{r}') d\mathbf{r}' \\ &= -\frac{k^2\eta LI}{8\pi^2} \int_{-\frac{D_z}{2}}^{\frac{D_z}{2}} \int_{-\frac{D_y}{2}}^{\frac{D_y}{2}} \frac{f(\theta) \cos \phi \cos \theta}{(R')^2} e^{-j2kR'} \hat{\mathbf{e}}_{\theta} dy dz, \end{aligned} \quad (3.10)$$

in which θ, ϕ and R' are functions of y and z . Note that the results of Box 2.13 cannot be used, as nothing ensures that the reflected **EM** fields propagates in the far-field of the rectangular plate!

Solution in the near-field: Using the **SPA** of Box 3.7, the developments presented in Appendix 3.A lead to the following approximation of the reflected electric field of (3.10):

$$\mathbf{E}^r \approx -\frac{k^2\eta LI}{8\pi^2} \frac{e^{-j2kR}}{R^2} \int_{-\frac{D}{2}}^{\frac{D}{2}} e^{-j\frac{k}{R}z^2} dz \int_{-\frac{D}{2}}^{\frac{D}{2}} e^{-j\frac{k}{R}y^2} dy \hat{\mathbf{e}}_y \quad (3.11)$$

$$\begin{aligned} &\approx -\frac{k^2\eta LI}{8\pi^2} \frac{e^{-j2kR}}{R^2} \frac{-j\pi R}{k} \hat{\mathbf{e}}_y \\ &= \frac{jk\eta LI}{4\pi} \frac{e^{-2jkR}}{2R} \hat{\mathbf{e}}_y. \end{aligned} \quad (3.12)$$

This approximation holds if the amplitude function $\mathbf{g}$ is slowly varying in the integration domain, and if the rectangular plate is large enough to contain multiple Fresnel zones [And64]. Quantitatively, the *Fresnel zone* radii ρ_m are computed as

$$\frac{k}{R} \rho_m^2 = m \frac{\pi}{2} \quad \Leftrightarrow \quad \rho_m = \frac{1}{2} \sqrt{m \lambda R},$$

Therefore, for a rectangular plate of dimensions D_y and D_z , the radii of the m first Fresnel zones are comprised on the rectangular plate for distances R such that

$$R \leq \frac{\min(D_y, D_z)^2}{m \lambda} \leq \frac{D_y^2 + D_z^2}{m \lambda} = \frac{R_{\text{FF}}}{2m}, \quad (3.13)$$

where R_{FF} is the Fraunhofer distance of the plate. Thus, the distance R is restricted to be a fraction of the Fraunhofer distance of the rectangular plate, meaning that this model is only valid in the near-field region of the plate.

By comparing (3.12) with Box 2.21, one may notice that the scattered electric field in the near-field is equivalent to the electric field which would have been received by an antenna located at a distance $2R$ of the transmitting antenna, with a phase shift of 180° owing to the PEC assumption. The scattered power density is computed following Box 2.16 as

$$S^r = \frac{|\mathbf{E}^r|^2}{2\eta} = \frac{\text{EIRP}}{4\pi(2R)^2},$$

where the EIRP is defined in (2.13). Multiplying this equation by the effective area of the receiving antenna, the resulting power corresponds to the GO approximation in free-space, presented in Section 3.1.3. Moreover, this is also valid if the transmitting and receiving antennas are not aligned with the centre of the plate, as long as the specular reflection point, and multiple Fresnel zones around this point, are still included within the plate.

Solution in the far-field: Instead, in the far-field, for the carrier frequencies usually considered in radar applications, the distance R is large enough such that

$$R \gg R_{\text{FF}} = \frac{2(D_y^2 + D_z^2)}{\lambda} \gg y^2 + z^2, \quad \forall (y, z) \in \mathcal{S}.$$

Thus, the phase function of (3.18) can be approximated with a Taylor series as

$$\psi(y, z) = -2kR \sqrt{1 + \frac{y^2 + z^2}{R^2}} \approx -2kR \left(1 + \frac{y^2 + z^2}{2R^2} \right),$$

giving back (3.19). Additionally, since the plate dimensions are assumed to be small compared to the distance R , one can also assume that $R' \approx R$, $\theta \approx 0^\circ$, and $\phi \approx 0^\circ$ in the

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amplitude function of (3.17), leading again to (3.20). Consequently, this gives back the integral of (3.11), obtained in the near-field. Nonetheless, the condition of (3.13) is not fulfilled. Thus, the integral must be solved using the Fresnel functions.

Solution with Fresnel functions: As described in Section 3.2.1, if the distance R is large such that the first Fresnel zones are not contained in the plate, but the amplitude function g is slowly varying in the integration domain, one can still use the first approximation of Box 3.7 and solve the integral using (3.8):

$$\begin{aligned}
 \mathbf{E}^r &\approx -\frac{k^2 \eta L I}{8\pi^2} \frac{e^{-j2kR}}{R^2} \int_{-\frac{D_z}{2}}^{\frac{D_z}{2}} e^{-j\frac{k}{R}z^2} dz \int_{-\frac{D_y}{2}}^{\frac{D_y}{2}} e^{-j\frac{k}{R}y^2} dy \hat{\mathbf{e}}_y \\
 &\approx -\frac{k^2 \eta L I}{8\pi^2} \frac{e^{-j2kR}}{R^2} \frac{2\pi R}{k} F^* \left(\sqrt{\frac{kD_y^2}{2\pi R}} \right) F^* \left(\sqrt{\frac{kD_z^2}{2\pi R}} \right) \hat{\mathbf{e}}_y \\
 &= -\frac{k\eta L I}{4\pi} \frac{e^{-j2kR}}{2R} \sqrt{\Sigma \left(\frac{R}{R_{FF,y}} \right) \Sigma \left(\frac{R}{R_{FF,z}} \right)} \hat{\mathbf{e}}_y,
 \end{aligned} \tag{3.14}$$

where $R_{FF,y}$ and $R_{FF,z}$ are respectively defined as

$$R_{FF,y} \triangleq \frac{2D_y^2}{\lambda}, \quad \text{and} \quad R_{FF,z} \triangleq \frac{2D_z^2}{\lambda}.$$

The function Σ is defined as

$$\Sigma(x) \triangleq 2 \left(F^* \left(\frac{1}{2x} \right) \right)^2,$$

and it is illustrated in Figure 3.8. Note that, if the transmitting and receiving antennas are not aligned with the centre of the plate, one may use (3.7) instead of (3.8), leading to similar results. Moreover, for tilted plates, a rotation of the coordinate system can be considered, the problem thus reducing to the case where the antennas are unaligned with the centre of the plate.

The solution with Fresnel functions is valid for propagation in both the near- and far-field regions. The function Σ has the following properties:

$$\lim_{x \rightarrow 0} \Sigma(x) = -j, \tag{3.15}$$

$$\lim_{x \rightarrow \infty} \Sigma(x) = \frac{1}{x}. \tag{3.16}$$

Consequently, in the near-field, one may inject (3.15) in (3.14), giving back (3.12).

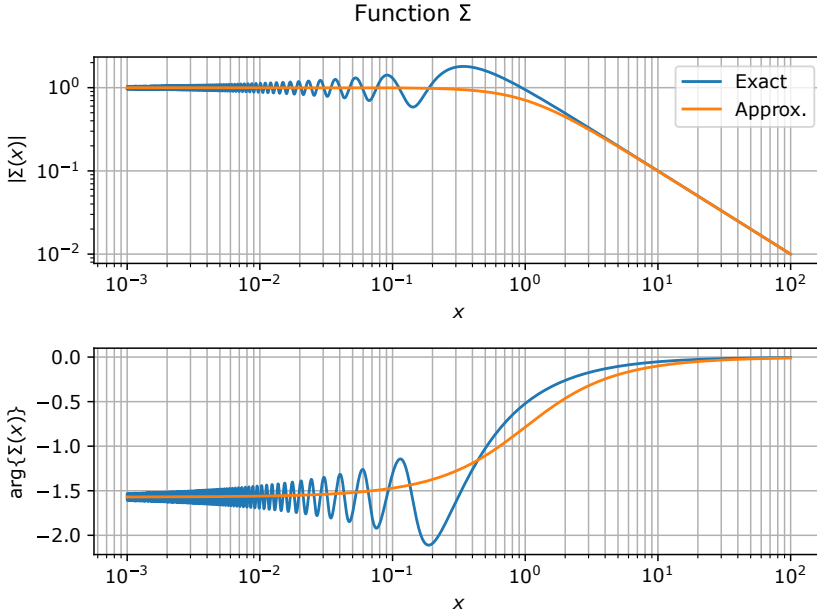


Fig. 3.8 Function Σ and the proposed approximation of Box 3.9.

Contrariwise, in the far-field, one may inject (3.16) in (3.14), leading to

$$\mathbf{E}^r \approx \frac{k\eta LI}{4\pi} \frac{e^{-j2kR}}{2R} \frac{\sqrt{R_{\text{FF},y}R_{\text{FF},z}}}{R} \hat{\mathbf{e}}_y = \frac{k\eta LI}{8\pi} \frac{e^{-j2kR}}{R^2} \sqrt{R_{\text{FF},y}R_{\text{FF},z}} \hat{\mathbf{e}}_y.$$

The corresponding scattered power density is then computed as

$$S^r = \frac{|\mathbf{E}^r|^2}{2\eta} = \frac{\text{EIRP}}{(4\pi R^2)^2} \pi R_{\text{FF},y} R_{\text{FF},z},$$

where the EIRP is again defined in (2.13). Multiplying this equation by the effective area of the receiving antenna, the resulting power corresponds to the radar equation of Box 3.3, assuming that the RCS is modelled as $\sigma \triangleq \pi R_{\text{FF},y} R_{\text{FF},z}$. The expressions of the reflected fields in the near- and far-field regions, and the novel model developed in this section which encompasses both regions, are summarised in Box 3.8.

Approximation of the Fresnel function: In order to approximate the normalised Fresnel integral defined in (3.6), while keeping the asymptotic properties of the function, an approximation of the Fresnel function and the function Σ is proposed in Box 3.9. This approximation is represented in Figure 3.8. Such approximation for the modulus may be extended for any natural number $n \geq 2$ to reduce the transition region, giving Box 3.10.

3.8: Reflected Electric Fields – Flat Rectangular Plate

$$\begin{aligned}
 \text{Near-Field: } \mathbf{E}^r &\approx \frac{j k \eta L I}{4 \pi} \frac{e^{-2 j k R}}{2 R} \hat{\mathbf{e}}_y \\
 \text{Far-Field: } \mathbf{E}^r &\approx \frac{k \eta L I}{8 \pi} \frac{e^{-j 2 k R}}{R^2} \sqrt{R_{\text{FF},y} R_{\text{FF},z}} \hat{\mathbf{e}}_y \\
 \text{New Model: } \mathbf{E}^r &\approx -\frac{k \eta L I}{4 \pi} \frac{e^{-j 2 k R}}{2 R} \sqrt{\Sigma \left(\frac{R}{R_{\text{FF},y}} \right) \Sigma \left(\frac{R}{R_{\text{FF},z}} \right)} \hat{\mathbf{e}}_y
 \end{aligned}$$

3.9: Approximation of the Fresnel Function and the Function Σ

$$\begin{aligned}
 F(x) &\approx \left(\frac{1}{x^2} - 2J \right)^{-\frac{1}{2}} \\
 \Sigma(x) &\approx \frac{x - J}{1 + x^2}
 \end{aligned}$$

3.2.2.2 Radar Cross-Section

Regarding the **RCS**, on the one hand, one can apply the definition of Box 3.2 in the near-field with (3.9) evaluated at the stationary point, and Box 3.8:

$$\sigma \approx 4 \pi R^2 \frac{\left(\frac{k \eta L I}{8 \pi R} \right)^2}{\left(\frac{k \eta L I}{4 \pi R} \right)^2} = \pi R^2,$$

which is consistent with the **RCS** for **RT** applications, given in (3.4). This confirms that a distance-dependent **RCS** model is required in the near-field to accurately model the received radar power. On the other hand, the same can be done in the far-field with (3.9) evaluated at the stationary point:

$$\sigma \approx 4 \pi R^2 \frac{\left(\frac{k \eta L I}{8 \pi R^2} \sqrt{R_{\text{FF},y} R_{\text{FF},z}} \right)^2}{\left(\frac{k \eta L I}{4 \pi R} \right)^2} = \pi R_{\text{FF},y} R_{\text{FF},z}.$$

3.10: Approximation of the Amplitude of the Σ -Function

$$\Sigma(x) \approx (1 + x^n)^{-\frac{1}{n}}$$

In that case, the RCS does not depend on the distance. The model of Box 3.8 is valid in the near- and far-field, and makes the link between the two propagation regions. The RCS is accordingly computed as

$$\sigma \approx 4\pi R^2 \frac{\left(\frac{k\eta LI}{8\pi R}\right)^2 \left| \sqrt{\Sigma\left(\frac{R}{R_{FF,y}}\right) \Sigma\left(\frac{R}{R_{FF,z}}\right)} \right|^2}{\left(\frac{k\eta LI}{4\pi R}\right)^2} = \pi R^2 \left| \Sigma\left(\frac{R}{R_{FF,y}}\right) \Sigma\left(\frac{R}{R_{FF,z}}\right) \right|$$

$$\stackrel{(a)}{\approx} \pi R_{FF,y} R_{FF,z} \left[\left(1 + \left(\frac{R_{FF,y}}{R}\right)^n\right) \left(1 + \left(\frac{R_{FF,z}}{R}\right)^n\right) \right]^{-\frac{1}{n}},$$

where (a) is obtained using the proposed approximation of Box 3.10. The expressions of the RCSs in the near- and far-fields, and the novel model developed in this section, are summarised in Box 3.11.

3.11: RCS Models — Flat Rectangular Plate

$$\begin{aligned} \text{Near-Field: } \sigma &\approx \pi R^2 \\ \text{Far-Field: } \sigma &\approx \pi R_{FF,y} R_{FF,z} \\ \text{New Model: } \sigma &\approx \pi R^2 \left| \Sigma\left(\frac{R}{R_{FF,y}}\right) \Sigma\left(\frac{R}{R_{FF,z}}\right) \right| \\ &\approx \pi R_{FF,y} R_{FF,z} \left[\left(1 + \left(\frac{R_{FF,y}}{R}\right)^n\right) \left(1 + \left(\frac{R_{FF,z}}{R}\right)^n\right) \right]^{-\frac{1}{n}} \end{aligned}$$

The new model is also generalisable with the extended path-loss model proposed in Section 3.1.4: using Box 3.6, one obtains Box 3.12.

3.12: RCS Model — Flat Rectangular Plate and Extended Path-Loss Model

$$\begin{aligned} \sigma &\approx \frac{\beta}{2^\alpha} R^\alpha \left| \Sigma\left(\frac{R}{R_{FF,y}}\right) \Sigma\left(\frac{R}{R_{FF,z}}\right) \right|^{\frac{\alpha}{2}} \\ &\approx \frac{\beta}{2^\alpha} (R_{FF,y} R_{FF,z})^{\frac{\alpha}{2}} \left[\left(1 + \left(\frac{R_{FF,y}}{R}\right)^n\right) \left(1 + \left(\frac{R_{FF,z}}{R}\right)^n\right) \right]^{-\frac{\alpha}{2n}} \end{aligned}$$

3.2.3 Reflection on a Curved Rectangular Plate

In practical scenarios, e.g. automotive applications, targets are complex objects which are difficult to model analytically. In order to generalise the previous results,

Reflection on a Curved Plate

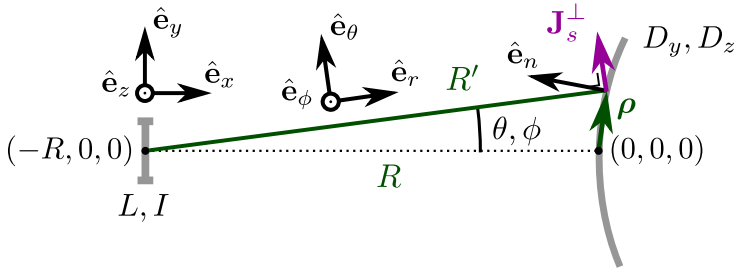


Fig. 3.9 Reflection on a curved rectangular plate (view from the z -axis).

curvature is introduced in both dimensions. The system model of Figure 3.9 is now considered. The curved plate Ω is modelled through a well-behaved arbitrary function $h : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ as

$$\Omega \triangleq \left\{ (x, y, z) \in \mathbb{R}^3 \left| \begin{array}{l} x = h(y, z), \quad |y| \leq \frac{D_y}{2}, \quad |z| \leq \frac{D_z}{2} \\ h(0, 0) = 0, \quad \nabla h(0, 0) = 0, \quad \nabla^2 h(y, z) \geq 0 \end{array} \right. \right\}.$$

The third condition on the function h ensures that the considered surface is convex. It is also assumed that the function h varies slowly w.r.t. the wavelength. Moreover, we suppose that the principal directions of the target are aligned with the y - and z -axis, i.e. that

$$\frac{\partial^2 h}{\partial y \partial z}(0,0) = \frac{\partial^2 h}{\partial z \partial y}(0,0) = 0.$$

For instance, this assumption is verified for the specific shapes analysed in Section 3.2.3.3. If not fulfilled, the integral obtained after applying the SPA may still be solved by performing an adequate change of variables along the principal directions.

Compared to Section 3.2.2, the same spherical coordinate system and angles are used. However, the distance R' is now computed as

$$R' = \sqrt{(R+x)^2 + y^2 + z^2} = \sqrt{(R+h(y,z))^2 + y^2 + z^2}.$$

The normal unit vector pointing outside the curved plate is now given by

$$\hat{\mathbf{e}}_n = \frac{-\hat{\mathbf{e}}_x + \frac{\partial h}{\partial y} \hat{\mathbf{e}}_y + \frac{\partial h}{\partial z} \hat{\mathbf{e}}_z}{\sqrt{1 + \left(\frac{\partial h}{\partial y}\right)^2 + \left(\frac{\partial h}{\partial z}\right)^2}}.$$

3.2.3.1 Scattered Electromagnetic fields

The modification of the normal unit vector impacts the generated surface current density, which is now computed as

$$\mathbf{J}_s \approx 2 \hat{\mathbf{e}}_n \times \mathbf{H}_1^i = -\frac{jkLI}{2\pi} \frac{e^{-jkR'}}{R'} f(\theta) (\hat{\mathbf{e}}_n \times \hat{\mathbf{e}}_\phi),$$

and its perpendicular component w.r.t. the direction of propagation as $\mathbf{J}_s^\perp = \mathbf{J}_s - (\mathbf{J}_s \cdot \hat{\mathbf{e}}_r) \hat{\mathbf{e}}_r$. The integral of the reflected electric field is expressed as

$$\begin{aligned} \mathbf{E}^r &\cong -\frac{k^2 \eta LI}{8\pi^2} \int_{-\frac{Dz}{2}}^{\frac{Dz}{2}} \int_{-\frac{Dy}{2}}^{\frac{Dy}{2}} \frac{f(\theta)}{(R')^2} e^{-j2kR'} [(\hat{\mathbf{e}}_n \times \hat{\mathbf{e}}_\phi) - ((\hat{\mathbf{e}}_n \times \hat{\mathbf{e}}_\phi) \cdot \hat{\mathbf{e}}_r) \hat{\mathbf{e}}_r] dy dz \\ &= \frac{k^2 \eta LI}{8\pi^2} \int_{-\frac{Dz}{2}}^{\frac{Dz}{2}} \int_{-\frac{Dy}{2}}^{\frac{Dy}{2}} \frac{f(\theta)}{(R')^2} e^{-j2kR'} (\hat{\mathbf{e}}_n \cdot \hat{\mathbf{e}}_r) \hat{\mathbf{e}}_\theta dy dz, \end{aligned}$$

which gives back (3.10) for a flat plate, i.e. $h(y, z) = 0$. Using the SPA of Box 3.7, the developments presented in Appendix 3.B lead to the following approximation of the reflected electric field of (3.10) :

$$\begin{aligned} \mathbf{E}^r &\approx -\frac{k^2 \eta LI}{8\pi^2} \frac{e^{-j2kR}}{R^2} \int_{-\frac{D}{2}}^{\frac{D}{2}} e^{-j\frac{k}{R_z} z^2} dz \int_{-\frac{D}{2}}^{\frac{D}{2}} e^{-j\frac{k}{R_y} y^2} dy \hat{\mathbf{e}}_y \\ &\approx -\frac{k^2 \eta LI}{8\pi^2} \frac{e^{-j2kR}}{R^2} \frac{2\pi}{k} \sqrt{R_y R_z} F^* \left(\sqrt{\frac{k D_y^2}{2\pi R_y}} \right) F^* \left(\sqrt{\frac{k D_z^2}{2\pi R_z}} \right) \hat{\mathbf{e}}_y \\ &= -\frac{k \eta LI}{8\pi} \frac{e^{-j2kR}}{R^2} \sqrt{R_y R_z} \sqrt{\Sigma \left(\frac{R_y}{R_{FF,y}} \right) \Sigma \left(\frac{R_z}{R_{FF,z}} \right)} \hat{\mathbf{e}}_y, \end{aligned}$$

where R_y and R_z are defined as

$$R_y \triangleq \frac{R}{1 + R h_y''}, \quad \text{and} \quad R_z \triangleq \frac{R}{1 + R h_z''},$$

and h_y'' and h_z'' are the second derivative of h w.r.t. y and z , evaluated at the stationary point.

3.13: Reflected Electric Field — Curved Rectangular Plate

$$\mathbf{E}^r \approx -\frac{k \eta LI}{8\pi} \frac{e^{-j2kR}}{R^2} \sqrt{R_y R_z} \sqrt{\Sigma \left(\frac{R_y}{R_{FF,y}} \right) \Sigma \left(\frac{R_z}{R_{FF,z}} \right)} \hat{\mathbf{e}}_y$$

The electric field reflected by a curved plate is summarised in Box 3.13. With a curved plate, it is not possible to predict the behaviour of the Σ function in the near-

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and far-field regions, as the distances R_y and R_z adopt different behaviours depending on the value of h_y'' and h_z'' .

3.2.3.2 Radar Cross-Section

Regarding the **RCS**, it is computed by applying the definition of Box 3.2 with (3.9) evaluated at the stationary point, and Box 3.13:

$$\begin{aligned}\sigma &\approx 4\pi R^2 \frac{\left(\frac{k\eta LI}{8\pi R^2}\right)^2 R_y R_z \left| \sqrt{\Sigma\left(\frac{R_y}{R_{FF,y}}\right) \Sigma\left(\frac{R_z}{R_{FF,z}}\right)} \right|^2}{\left(\frac{k\eta LI}{4\pi R}\right)^2} = \pi R_y R_z \left| \Sigma\left(\frac{R_y}{R_{FF,y}}\right) \Sigma\left(\frac{R_z}{R_{FF,z}}\right) \right| \\ &\stackrel{(a)}{\approx} \pi R_{FF,y} R_{FF,z} \left[\left(1 + \left(\frac{R_{FF,y}}{R_y}\right)^n\right) \left(1 + \left(\frac{R_{FF,z}}{R_z}\right)^n\right) \right]^{-\frac{1}{n}},\end{aligned}$$

where (a) is obtained using the proposed approximation of Box 3.10. This result is summarised in Box 3.13.

3.14: RCS Model — Curved Rectangular Plate

$$\begin{aligned}\sigma &\approx \pi R_y R_z \left| \Sigma\left(\frac{R_y}{R_{FF,y}}\right) \Sigma\left(\frac{R_z}{R_{FF,z}}\right) \right| \\ &\approx \pi R_{FF,y} R_{FF,z} \left[\left(1 + \left(\frac{R_{FF,y}}{R_y}\right)^n\right) \left(1 + \left(\frac{R_{FF,z}}{R_z}\right)^n\right) \right]^{-\frac{1}{n}}\end{aligned}$$

Again, the proposed **RCS** model is also generalisable with the extended path-loss model proposed in Section 3.1.4: using Box 3.6, one obtains

$$\begin{aligned}\sigma &\approx \beta R^\alpha \frac{\left(\frac{k\eta LI}{8\pi R^2}\right)^\alpha R_y^{\frac{\alpha}{2}} R_z^{\frac{\alpha}{2}} \left| \sqrt{\Sigma\left(\frac{R_y}{R_{FF,y}}\right) \Sigma\left(\frac{R_z}{R_{FF,z}}\right)} \right|^\alpha}{\left(\frac{k\eta LI}{4\pi R}\right)^\alpha} = \frac{\beta}{2^\alpha} R^\alpha \left| \Sigma\left(\frac{R_y}{R_{FF,y}}\right) \Sigma\left(\frac{R_z}{R_{FF,z}}\right) \right|^{\frac{\alpha}{2}} \\ &\approx \frac{\beta}{2^\alpha} (R_{FF,y} R_{FF,z})^{\frac{\alpha}{2}} \left[\left(1 + \left(\frac{R_{FF,y}}{R_y}\right)^n\right) \left(1 + \left(\frac{R_{FF,z}}{R_z}\right)^n\right) \right]^{-\frac{\alpha}{2n}}.\end{aligned}$$

This result is summarised in Box 3.15.

3.2.3.3 Application to Specific Shapes

Let us consider three specific target shapes, namely the cylinder, the sphere and the paraboloid. The new **RCS** model of Box 3.14 may be considered to evaluate their **RCSs**.

3.15: RCS Model — Curved Rectangular Plate with Extended Path-Loss Model

$$\begin{aligned}\sigma &\approx \frac{\beta}{2^\alpha} (R_y R_z)^{\frac{\alpha}{2}} \left| \Sigma \left(\frac{R_y}{R_{FF,y}} \right) \Sigma \left(\frac{R_z}{R_{FF,z}} \right) \right|^{\frac{\alpha}{2}} \\ &\approx \frac{\beta}{2^\alpha} (R_{FF,y} R_{FF,z})^{\frac{\alpha}{2}} \left[\left(1 + \left(\frac{R_{FF,y}}{R_y} \right)^n \right) \left(1 + \left(\frac{R_{FF,z}}{R_z} \right)^n \right) \right]^{-\frac{\alpha}{2n}}\end{aligned}$$

Cylinder of radius a : With a cylinder of radius $a \gg D_y$, the function h is defined as

$$h(y, z) = a - \sqrt{a^2 - z^2}.$$

The second derivative of h w.r.t. y and z are respectively given by

$$\begin{aligned}\frac{\partial^2 h}{\partial y^2}(y, z) &= 0 \quad \Rightarrow \quad h''_y = 0, \\ \frac{\partial^2 h}{\partial z^2}(y, z) &= \frac{a^2}{(a^2 - z^2)^{\frac{3}{2}}} \quad \Rightarrow \quad h''_z = \frac{1}{a}.\end{aligned}$$

Therefore, the distances R_y and R_z are respectively given by

$$R_y = R, \quad \text{and} \quad R_z = \frac{R}{1 + \frac{R}{a}} = \frac{1}{\frac{1}{R} + \frac{1}{a}},$$

and the RCS is approximated following Box 3.14 as

$$\begin{aligned}\sigma &\approx \pi R_y R_z \left| \Sigma \left(\frac{R}{R_{FF,y}} \right) \Sigma \left(\frac{1}{\frac{R_{FF,z}}{R} + \frac{R_{FF,z}}{a}} \right) \right| \\ &\approx \pi R_{FF,y} R_{FF,z} \left[\left(1 + \left(\frac{R_{FF,y}}{R} \right)^n \right) \left(1 + \left(\frac{R_{FF,z}}{R} + \frac{R_{FF,z}}{a} \right)^n \right) \right]^{-\frac{1}{n}}.\end{aligned}$$

Sphere of radius a : With a sphere of radius $a \gg \max(D_y, D_z)$, the function h is defined as

$$h(y, z) = a - \sqrt{a^2 - y^2 - z^2}.$$

The second derivative of h w.r.t. y and z are respectively given by

$$\begin{aligned}\frac{\partial^2 h}{\partial y^2}(y, z) &= \frac{a^2 - z^2}{(a^2 - y^2 - z^2)^{\frac{3}{2}}} \quad \Rightarrow \quad h''_y = \frac{1}{a}, \\ \frac{\partial^2 h}{\partial z^2}(y, z) &= \frac{a^2 - y^2}{(a^2 - y^2 - z^2)^{\frac{3}{2}}} \quad \Rightarrow \quad h''_z = \frac{1}{a}.\end{aligned}$$

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Therefore, the distances R_y and R_z are respectively given by

$$R_y = R_z = \frac{R}{1 + \frac{R}{a}} = \frac{1}{\frac{1}{R} + \frac{1}{a}},$$

and the RCS is approximated following Box 3.14 as

$$\begin{aligned} \sigma &\approx \pi R_y R_z \left| \Sigma \left(\frac{1}{\frac{R_{FF,y}}{R} + \frac{R_{FF,y}}{a}} \right) \Sigma \left(\frac{1}{\frac{R_{FF,z}}{R} + \frac{R_{FF,z}}{a}} \right) \right| \\ &\approx \pi R_{FF,y} R_{FF,z} \left[\left(1 + \left(\frac{R_{FF,y}}{R} + \frac{R_{FF,y}}{a} \right)^n \right) \left(1 + \left(\frac{R_{FF,z}}{R} + \frac{R_{FF,z}}{a} \right)^n \right) \right]^{-\frac{1}{n}}. \end{aligned}$$

Nonetheless, it may instead be more relevant to consider the integration of the surface current on a disk of radius a instead of a square, to match the projection of the sphere on the plane $(0, y, z)$, and consider the complete visible surface. This is left for future works.

Paraboloid: With a paraboloid of curvature $c_y \gg \max(D_y, D_z)$ and $c_z \gg \max(D_y, D_z)$ in the y - and z -dimensions, the function h is defined as

$$h(y, z) = \frac{y^2}{c_y} + \frac{z^2}{c_z}.$$

The second derivative of h w.r.t. y and z are respectively given by

$$\begin{aligned} \frac{\partial^2 h}{\partial y^2}(y, z) &= \frac{2}{c_y} \quad \Rightarrow \quad h''_y = \frac{2}{c_y}, \\ \frac{\partial^2 h}{\partial z^2}(y, z) &= \frac{2}{c_z} \quad \Rightarrow \quad h''_z = \frac{2}{c_z}. \end{aligned}$$

Therefore, the distances R_y and R_z are respectively given by

$$R_y = \frac{R}{1 + \frac{2R}{c_y}} = \frac{1}{\frac{1}{R} + \frac{2}{c_y}}, \quad \text{and} \quad R_z = \frac{R}{1 + \frac{2R}{c_z}} = \frac{1}{\frac{1}{R} + \frac{2}{c_z}},$$

and the RCS is approximated following Box 3.14 as

$$\begin{aligned} \sigma &\approx \pi R_y R_z \left| \Sigma \left(\frac{1}{\frac{R_{FF,y}}{R} + \frac{2R_{FF,y}}{c_y}} \right) \Sigma \left(\frac{1}{\frac{R_{FF,z}}{R} + \frac{2R_{FF,z}}{c_z}} \right) \right| \\ &\approx \pi R_{FF,y} R_{FF,z} \left[\left(1 + \left(\frac{R_{FF,y}}{R} + \frac{2R_{FF,y}}{c_y} \right)^n \right) \left(1 + \left(\frac{R_{FF,z}}{R} + \frac{2R_{FF,z}}{c_z} \right)^n \right) \right]^{-\frac{1}{n}}. \end{aligned}$$

Note that, with $c_y \rightarrow \infty$ and $c_z = 2a$, the paraboloid exactly matches the cylinder, and with $c_y = c_z = 2a$, the paraboloid exactly matches the sphere.

3.16: RCS Models — Specific Curved Geometries

$$\begin{aligned}
 \text{Cylinder: } \sigma &\approx \pi R_y R_z \left| \Sigma \left(\frac{R}{R_{FF,y}} \right) \Sigma \left(\frac{1}{\frac{R_{FF,z}}{R} + \frac{R_{FF,z}}{a}} \right) \right| \\
 &\approx \pi R_{FF,y} R_{FF,z} \left[\left(1 + \left(\frac{R_{FF,y}}{R} \right)^n \right) \left(1 + \left(\frac{R_{FF,z}}{R} + \frac{R_{FF,z}}{a} \right)^n \right) \right]^{-\frac{1}{n}} \\
 \text{Sphere: } \sigma &\approx \pi R_y R_z \left| \Sigma \left(\frac{1}{\frac{R_{FF,y}}{R} + \frac{R_{FF,y}}{a}} \right) \Sigma \left(\frac{1}{\frac{R_{FF,z}}{R} + \frac{R_{FF,z}}{a}} \right) \right| \\
 &\approx \pi R_{FF,y} R_{FF,z} \left[\left(1 + \left(\frac{R_{FF,y}}{R} + \frac{R_{FF,y}}{a} \right)^n \right) \left(1 + \left(\frac{R_{FF,z}}{R} + \frac{R_{FF,z}}{a} \right)^n \right) \right]^{-\frac{1}{n}} \\
 \text{Paraboloid: } \sigma &\approx \pi R_y R_z \left| \Sigma \left(\frac{1}{\frac{R_{FF,y}}{R} + \frac{2R_{FF,y}}{c_y}} \right) \Sigma \left(\frac{1}{\frac{R_{FF,z}}{R} + \frac{2R_{FF,z}}{c_z}} \right) \right| \\
 &\approx \pi R_{FF,y} R_{FF,z} \left[\left(1 + \left(\frac{R_{FF,y}}{R} + \frac{2R_{FF,y}}{c_y} \right)^n \right) \left(1 + \left(\frac{R_{FF,z}}{R} + \frac{2R_{FF,z}}{c_z} \right)^n \right) \right]^{-\frac{1}{n}}
 \end{aligned}$$

The RCS models obtained for the cylinder, sphere, and paraboloid are summarised in Box 3.16. Nonetheless, as discussed in Section 2.3.4 of Chapter 2, the curvature or the target must vary slowly for the PO approximation to be accurate. Thus, the approximation may be inaccurate, if the dimensions of the curved rectangular plate are close to $2a$. Moreover, the normal unit vector is fastly varying close to the edges of the sphere and cylinder, which may affect the accuracy of the SPA.

3.3 Numerical Analysis

In this section, simulations are first performed to illustrate the analytical results developed in Section 3.2. Then, an example of the impact of the different propagation models on the evaluation of sensing performance in an automotive scenario is provided.

3.3.1 Evolution of the Radar Cross-Section with the Range, Frequency, and Target Geometry

Figure 3.10 illustrates the evolution of the RCS against the range, frequency, i.e. 5 GHz (Wi-Fi frequencies), 24 GHz, and 77 GHz (automotive radars), and target

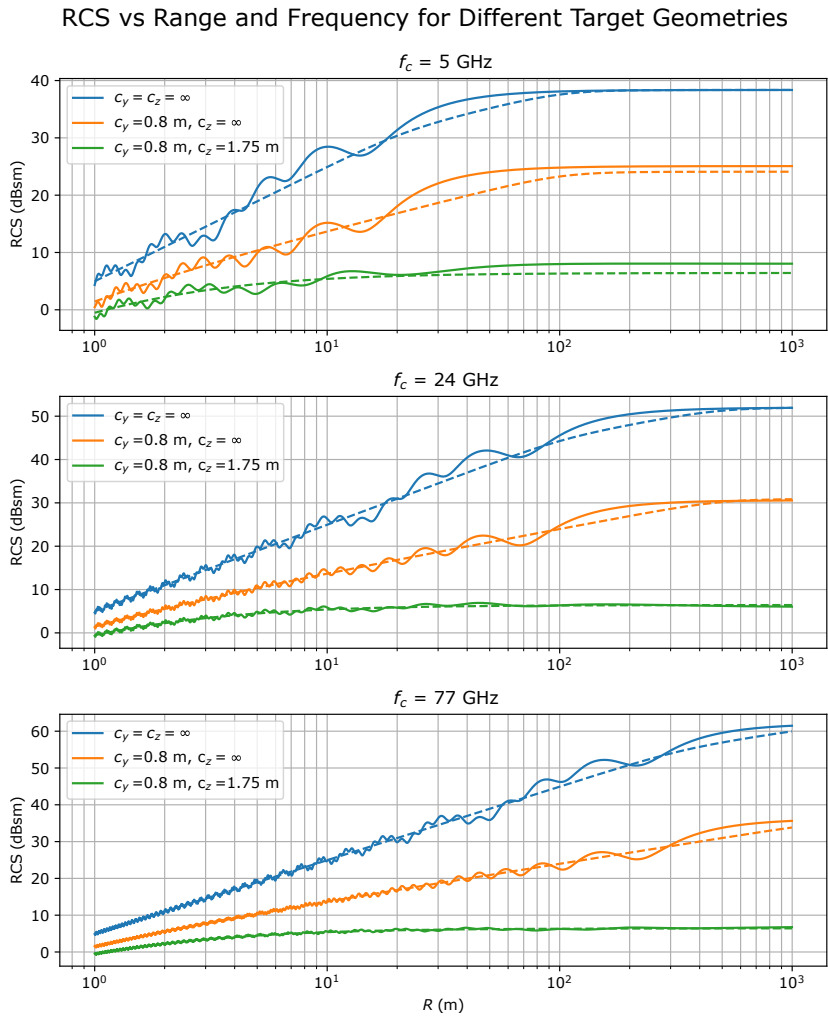


Fig. 3.10 RCS for different ranges, carrier frequencies, and target geometries. The solid and dashed lines respectively represent the RCSs computed using the models of Box 3.16 with $n = 4$.

geometry, with and without the proposed approximation of Σ . The higher the carrier frequency, the better the modulus approximation. It enables to filter out the oscillatory behaviour of the Fresnel functions, while keeping intact the trend of the RCS. However, at low carrier frequencies, when the far-field RCS is reached, an offset is observed for curved targets.

Compared to the flat rectangular plate geometry, introducing curvatures in the y - and z -axis modifies drastically the behaviour of the RCS:

- (i) When $c_y \ll R_{FF,y}$, $c_y \ll R$, $c_z \ll R_{FF,z}$, $c_z \ll R$, the RCS is constant, whatever the Fraunhofer distances.
- (ii) When $c_y \ll R \ll c_z$, or $c_z \ll R \ll c_y$ by symmetry, the RCS is proportional to the distance, until the Fraunhofer distances are reached.
- (iii) When $R \ll c_y$ and $R \ll c_z$, the RCS is proportional to the square of the distance, until the Fraunhofer distances are reached. This is the case corresponding to the rectangular flat plate.

At high carrier frequencies, the Fraunhofer distances are high compared to the considered distances in indoor applications. This is also often the case in automotive scenarios. Therefore, the modelling of the target has a huge impact on the radar propagation model. The RCS is either constant, proportional to the distance, or proportional to the square of the distance, depending on the considered range, frequency, and target geometry.

3.3.2 Application to an Automotive Scenario

As it will be further discussed in Part II, average sensing and communication performance metrics may be analysed using the SG framework. An example of such study is [Fan+20], which analyses the performance of automotive radar systems in a bidirectional multi-lane street. Instead of simply consider a constant RCS, they model fluctuation by introducing a χ^2 -distributed random variable, as suggested in the Swerling I and III models [Swe60]. Random fluctuation models can be coupled with the proposed radar propagation models by using the analytical models of Section 3.2 as the average RCS of the random variable distributions. The performance metric under study is the *radar success probability*. It is defined as the probability that the radar SINR is higher than a given SINR threshold, for a given distance. This metric is further detailed in Section 6.1.2.3 of Chapter 6.

Figure 3.11 illustrates the success probability achieved with Swerling I (exponentially-distributed) and Swerling V (constant) RCS fluctuations models, using as average RCS a constant value, the RT RCS model of (3.4), or the curved plate RCS model of Box 3.14, with and without using the approximation of Box 3.10. One can observe that the selected radar propagation model hugely impacts the radar

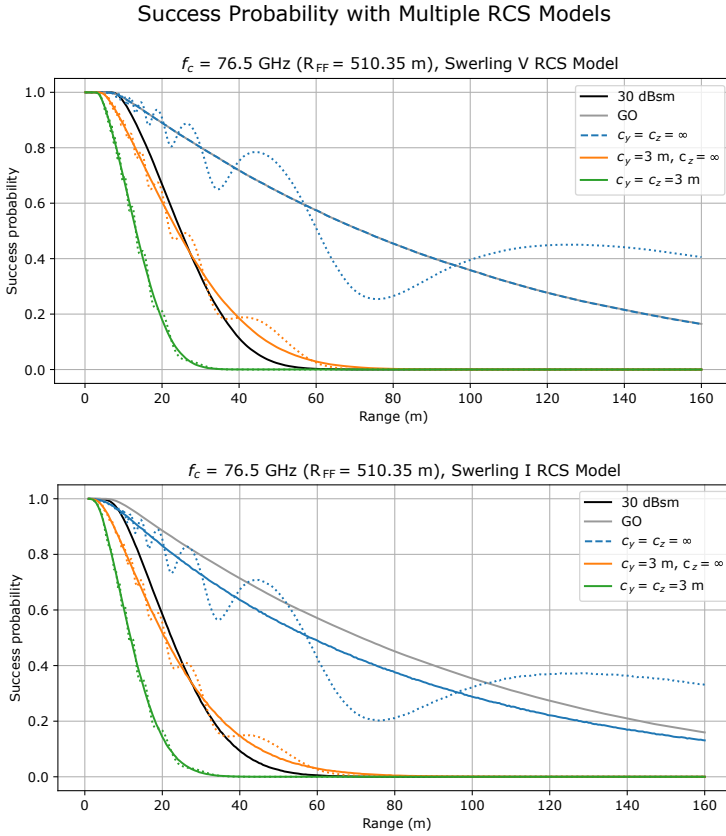


Fig. 3.11 Success probability with multiple RCS Models. Swerling V and I RCS fluctuations models are respectively considered in the top and bottom plots. The black curves consider a constant average RCS, as in [Swe60]. The gray curves use the RT model of (3.4). The blue, orange and green curves are obtained with the model of Box 3.14. Dotted and continuous lines correspond to exact and approximated RCSs.

Parameters	Values
Vehicles density	10 km^{-1}
Plate dimensions	$D_y = D_z = 1 \text{ m}$
Number of lanes	5
Lanes width	3.6 m
Interference probability	0.01
Path-loss parameters	$\alpha = 2, \beta = 4\pi$
Carrier frequency	$f_c = 76.5 \text{ GHz}$
EIRP	10 dB
Receiver gain	$G_r = 30 \text{ dBi}$
Antenna beamwidth	15°
SINR threshold	10 dB

Table 3.1 Parameters of the scenario for Section 3.3.2.

performance owing to the significant differences of received power, related to the dependency of the average RCS with the distance. Additionally, the choice of the curvature radii has also a huge impact on the achieved performance. Whereas a flat square plate seems appropriate for trucks, the introduction of a curvature radius in the y -dimension seems more suited to approximate the back of cars, and leads to a large performance degradation. The RT model is close to the flat square model since a high carrier frequency is considered, leading to a high Fraunhofer distance compared to the radar-to-target distance. Additionally, for the selected configuration, the model with a constant RCS of 30 dBsm also approximates well the model with vertical curvature radius. Finally, the proposed approximation for the Σ -function enables to efficiently filter the oscillation of the average RCS. These oscillations may not be relevant for the evaluation of average performance in the SG framework, owing to the high level of abstraction.

3.4 Experimental Measurements

RCS measurements have been performed in the anechoic chamber of the WELCOME facility at UCLouvain. The five targets of Figure 3.12 have been considered:

1. A square flat plate of dimensions $D_y = D_z = 5 \text{ cm}$.
2. A square flat plate of dimensions $D_y = D_z = 10 \text{ cm}$.
3. A square flat plate of dimensions $D_y = D_z = 1 \text{ m}$.
4. A cylinder of height $D_y = 1 \text{ m}$ and radius $a = 15 \text{ cm}$.
5. A sphere of radius $a = 27.5 \text{ cm}$.



Fig. 3.12 Considered target geometries for RCS measurements.

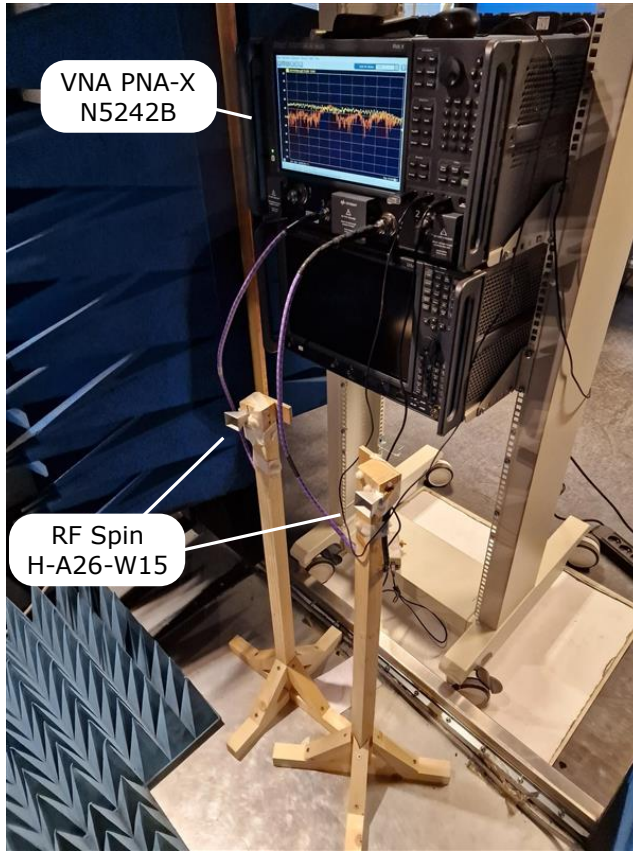


Fig. 3.13 VNA PNA-X N5242B from Keysight, and horn antennas H-A26-W15 from RF Spin, used for RCS measurements.

The measurements have been performed using a **Vector Network Analyser (VNA)** PNA-X N5242B from Keysight, which enables to obtain the transmittance matrix between two inputs at frequencies up to 26.5 GHz, with a transmitted power up to 10 dBm. This power level is sufficient for the considered targets to obtain echoes higher than the noise floor, at the considered **Intermediate Frequency (IF)** bandwidth of 100 Hz. Thus, no amplifier has been used, to avoid any additional source of errors. The antennas are two H-A26-W15 horns from RF Spin, operating at frequencies from 18 GHz to 26.5 GHz. A zoom on the **VNA** and the antennas is shown in Figure 3.13. Note that additional absorbing foam is put between the **VNA** and the antennas during the measurements, as a second reflection on the **VNA** arises for targets with a high **RCS**.

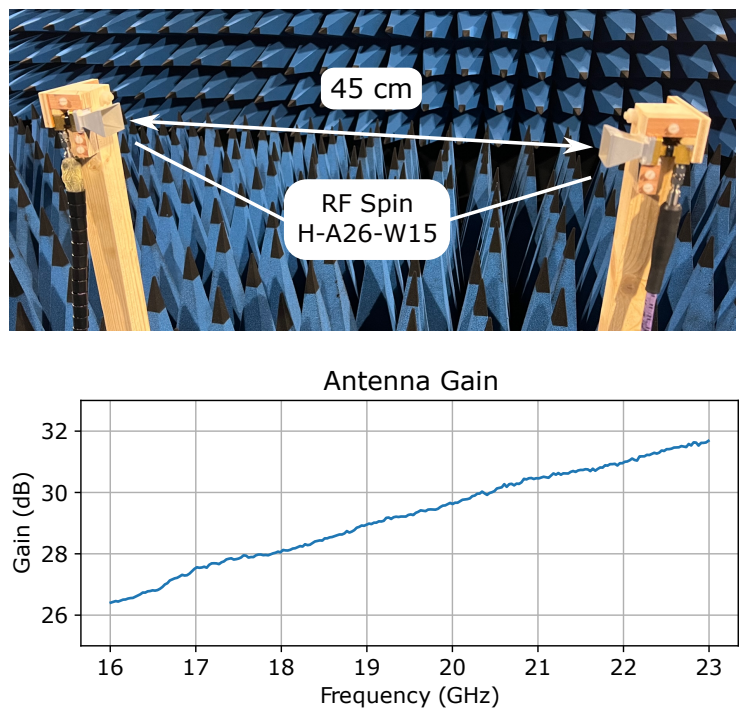


Fig. 3.14 Antenna gain — setup and measurements.

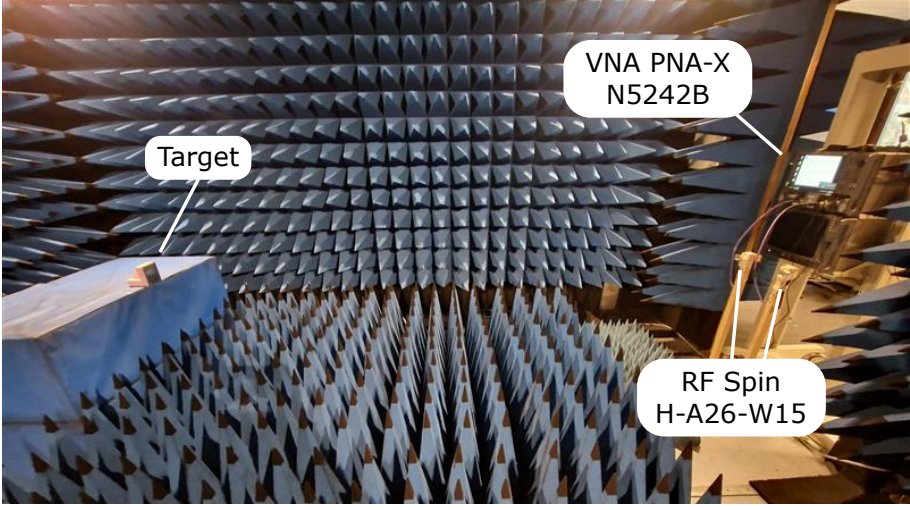


Fig. 3.15 Measurement setup with the square plate of 10 cm. Additional absorbing foam is put in front of the boxes under the targets, and between the VNA and the antennas for large targets, to avoid undesirable reflections.

3.4.1 Measurement of the Antenna Gain

The setup and the measurements of the antenna gain are illustrated in Figure 3.14. Denoting by S_{ant} the transmittance between the output and the input of the VNA, the antenna gain G_{ant} at a given frequency is computed as

$$[G_{\text{ant}}(f)]_{\text{dB}} = 20 \log_{10} |S_{\text{ant}}(f)| - 20 \log_{10} \left(\frac{c}{4\pi f R_{\text{ant}}} \right),$$

following the Friis equation of Box 3.1, with $R_{\text{ant}} = 45$ cm the distance between the two antennas in Figure 3.14, and c the speed of light. A 2-port calibration is carried out with the cables only, so that the cable losses can be excluded.

3.4.2 Measurement of the Radar Cross-Section

The measurement setup is shown in Figure 3.15, with the square plate of 10 cm. Additional absorbing foam is put in front of the boxes under the small targets to avoid undesirable reflections. Moreover, for large targets, the same is done between the antennas and the VNA, as the echo from the target is powerful enough to be reflected a second time on the VNA.

For each target geometry and distance, the clutter transmittance S_{clut} is first measured by performing a measurement with the target removed from the experimental setup, and it is saved in the memory of the VNA. Next, the transmittance S is measured with the target, and the background is removed by the VNA. Thus, the data $S - S_{\text{clut}}$

is extracted for post-processing.

3.4.3 Post-processing

The received data consists on 501 points in the frequency domain, between 15 and 25 GHz (step of 20 MHz). The signal is first converted in time domain. Then, it is filtered with a Hamming window of 41 points, centred on the peak of the dominant path. This enables to further isolate the target response. Next, the RCS σ is obtained from the post-processed data $\tilde{S}$ in the frequency domain by computing

$$[\sigma]_{\text{dB}} = 20 \log_{10} |\tilde{S}(f)| - [G_{\text{ant}}(f)]_{\text{dB}} - [\gamma(f)]_{\text{dB}},$$

with γ defined by the radar equation as

$$\gamma(f) = \frac{c^2}{(4\pi)^3 f^2 R^4},$$

where R is the distance between the target and the centre of the line connecting both antennas.

3.4.4 Results

The RCS measurements performed for the five targets of Figure 3.12 are shown in Figure 3.16. All the measurements are compared with the theoretical models of Boxes 3.11 and 3.14. Less than 0.5 dB of error is achieved for the square plate of 10 cm. Though, for the other square plates, higher discrepancies are observed. For the square plate of 5 cm, the RCS is overestimated, owing to the residual clutter after imperfect cancellation, which is not negligible w.r.t. the useful reflection from the target. Additionally, these differences can also be explained by the difficulty to calibrate the positions and orientations of the antennas and the target. This applies for the three square plates. For the cylinder and the sphere, the results of Box 3.16 are not accurate owing to the high curvature radii of the targets, compared to the considered distances. Additionally, the sphere is only approximated by a curved rectangular plate. Other phenomena like creeping waves are also not taken into account.

Nevertheless, one can observe that the RCS is nearly independent of the distance for the square plate of 5 cm, square plate of 10 cm, and the sphere, whereas it increases with the distance for the square plate of 1 m, and the cylinder. The Fraunhofer distances varies from 27 cm to 38 cm for the square plate of 5 cm, and from 107 cm to 153 cm for the square plate of 10 cm. Thus, owing to the considered distances, this happens in accordance with the theoretical models.

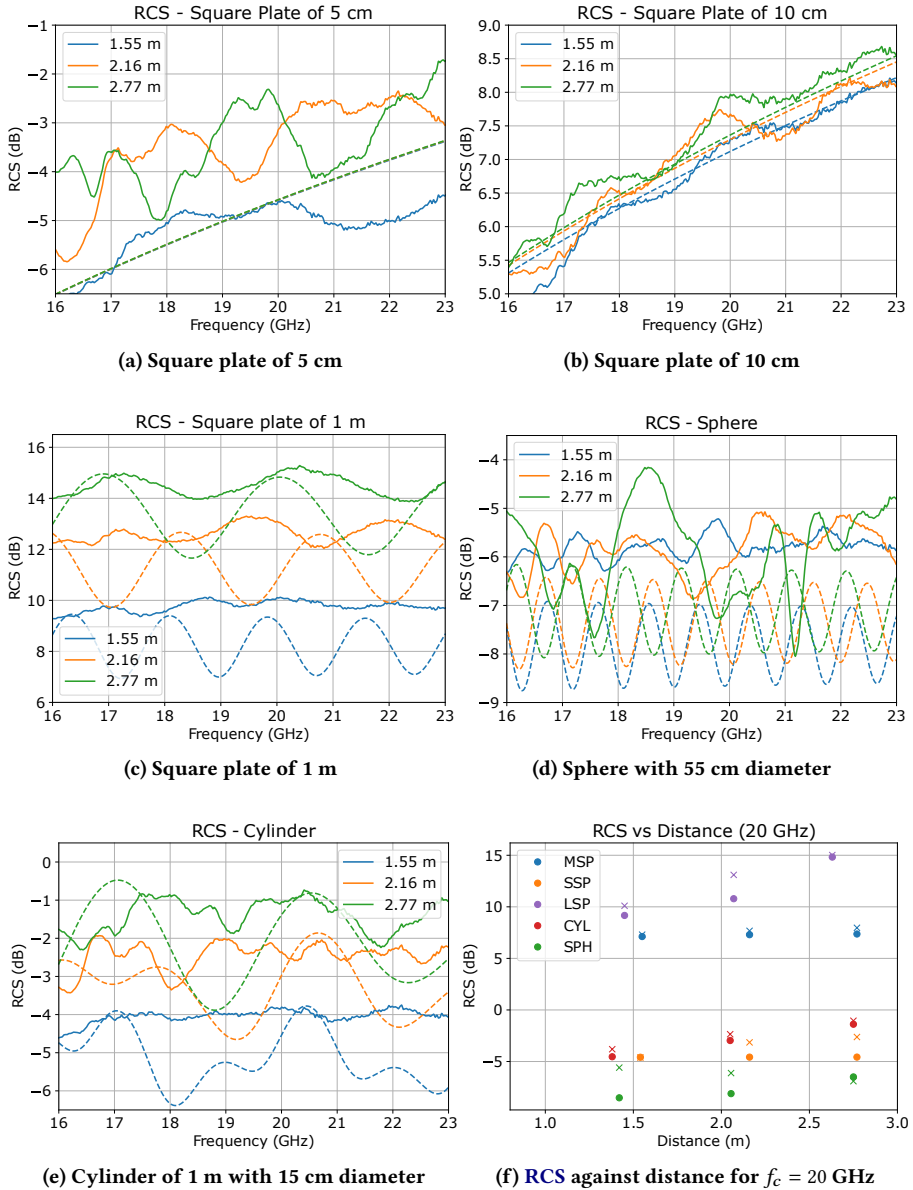


Fig. 3.16 RCS measurements for different, distances, carrier frequencies, and target geometries. The measurements and the theoretical RCSs are respectively represented by solid and dashed lines, or by crosses and dots in Figure 3.16f. The acronyms SSP, MSP, LSP, CYL and SPHR respectively refer to the square plate of 5 cm, square plate of 10 cm, square plate of 1 m, cylinder, and sphere.

3.5 Conclusion

Accurate propagation models are required to evaluate correctly the performance of future radar and communication systems. Whereas the received power is usually evaluated with the Friis equation for communication applications, the radar equation and the GO approximation are two different popular models for sensing applications which seems opposed to each other. In this chapter, these models have first been thoroughly detailed. General path-loss models have also been introduced to enable fitting with real-world measurements. Whereas the received power decreases with the fourth power of the distance using the radar equation, it decreases with the square of the distance using the GO approximation. The key quantity is the RCS in the radar equation. Whereas it is usually supposed to be constant, this is only valid in the far-field propagation region. However, numerous sensing applications, e.g. automotive scenarios, are occurring in the near-field region, and distance-dependent RCS models must be considered. Therefore, a focus has then been made on the sensing application, and more particularly on the evaluation of RCS models in monostatic configurations.

3.5.1 Summary of the Contributions

The contributions of this chapter are summarised as follows:

- Using the SPA, closed-form analytical expressions have been obtained for flat rectangular plate. Depending on the distance to the target, the target dimensions, and the carrier frequency, the developed models unify the radar equation and the GO approximation. The near- and far-field conditions are highlighted mathematically, as they are required to respectively consider distance-dependent and independent RCS.
- The analytical models have been extended for curved rectangular plate geometries. The shape of the target can be arbitrarily defined as a continuous 3D surface. This enables to efficiently model cylinders, paraboloids, and approximate spheres. The obtained results show that the curvature of the target around the specular reflection point, i.e. the second derivatives of the shape function, defines the behaviour of the RCS.
- The importance of good propagation models has been emphasised through numerical analyses. More specifically, within the SG framework, the radar equation has been extensively used with constant RCSs. In comparison with the new models, huge discrepancies have been observed. Moreover, it has been highlighted that the curvature has a huge impact on the RCS trend w.r.t. the distance between the antennas and the target. Even in the near-field region of the target, when curvature is introduced, the RCS is constant, or increases proportionally with the distance, depending on the ratio between the curvature

radii and the distance to the target, whereas it always increases with the square of the distance for flat rectangular plates.

- Experimental measurements of the [RCS](#) have been performed for multiple target geometries. For the square plate of 10 cm, the theoretical [RCS](#) model seems promising, as errors less than 1 dB are committed in the considered carrier frequency range. Nevertheless, even if large discrepancies are observed with the other geometries, the dependency or the independence of the [RCS](#) with the distance has still been highlighted.

3.5.2 Future Works

First, the experimental setup and the associated post-processing may be improved. For instance, the alignment of the antennas and target orientations is difficult for most targets, especially for the curved geometries. Moreover, the actual configuration is quasi-monostatic. Therefore, a correction of the travelled distance may be incorporated in the model, to improve the fitting between the theoretical expressions the measurements. Besides, [RCS](#) measurements of realistic targets, e.g. the back of a real vehicle, may be carried out, to evaluate at which extent the models developed for similar simple target geometries are able to predict the evolution of the [RCS](#) with the distance and carrier frequency.

Then, further investigations may be carried out for the development of analytical expressions with the cylinder and the sphere. The fast variations of the normal unit vector at the edges of the integration boundaries affects the accuracy of the [SPA](#). Moreover, for the sphere, the projection of the sphere on the y - and z -axis is a disk, and not a square. Consequently, this also impacts the fitting of the theoretical expressions with the measurements.

Finally, monostatic configurations have been analysed. Therefore, the extension of the obtained results in multistatic configuration may be studied. This will for instance be performed for flat rectangular plates in Chapter 4, as specified in Section 3.5.3. Yet, with multistatic configurations, the position of the stationary point may be challenging to compute analytically, for curved geometries. Additionally, it may be located outside the target, making difficult the evaluation of the [RCS](#) with the [SPA](#).

3.5.3 What is Next?

This chapter was focused on radar propagation, or equivalently [RCS](#) models. Whereas this is critical for the evaluation of sensing performance, the following approaches can be extended in a signal processing perspective for performance improvements. In this chapter, a highlight has been made on the near-field region, in which the propagation models varies drastically compared to the far-field. Thus, this raises multiple questions:

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1. Instead of considering the received power, could the proposed approach be generalised to obtain accurate closed-form expressions of the received signals, in monostatic or multistatic configurations?
2. How the obtained expressions compares to the usual near- and far-field signal models of the literature? Moreover, if differences are observed, how the model mismatches impact the achieved communication and sensing performance?
3. Is it possible to leverage on the specific near-field propagation effects, emphasised in the developed expressions, to improve the communication and sensing performance? Furthermore, if it is possible, under which conditions this applies?

Consequently, Chapter 4 tries to answer all these questions.

Appendix - Chapter 3

3.A Stationary Phase Approximation for a Flat Rectangular Plate

Relying on the SPA presented in Section 3.2.1, the amplitude and phase functions are respectively defined as

$$\mathbf{g}(y, z) = \frac{f(\theta) \cos \phi \cos \theta}{(R')^2} \hat{\mathbf{e}}_\theta, \quad (3.17)$$

$$\psi(y, z) = -2kR' = -2k\sqrt{R^2 + y^2 + z^2}. \quad (3.18)$$

The stationary point is found by solving

$$\begin{cases} \frac{\partial \psi}{\partial y} = -\frac{2ky}{\sqrt{R^2 + y^2 + z^2}} = 0 \\ \frac{\partial \psi}{\partial z} = -\frac{2kz}{\sqrt{R^2 + y^2 + z^2}} = 0 \end{cases} \Leftrightarrow (y_0, z_0) = (0, 0).$$

The second derivatives of the phase function are then computed as

$$\begin{aligned} \frac{\partial^2 \psi}{\partial y^2} &= -\frac{2k(R^2 + z^2)}{(R^2 + y^2 + z^2)^{\frac{3}{2}}} \Rightarrow \frac{\partial^2 \psi}{\partial y^2}(0, 0) = -\frac{2k}{R}, \\ \frac{\partial^2 \psi}{\partial z^2} &= -\frac{2k(R^2 + y^2)}{(R^2 + y^2 + z^2)^{\frac{3}{2}}} \Rightarrow \frac{\partial^2 \psi}{\partial z^2}(0, 0) = -\frac{2k}{R}, \\ \frac{\partial^2 \psi}{\partial y \partial z} &= \frac{2k y z}{(R^2 + y^2 + z^2)^{\frac{3}{2}}} \Rightarrow \frac{\partial^2 \psi}{\partial y \partial z}(0, 0) = 0. \end{aligned}$$

Therefore, around the stationary point, the phase function is approximated with a Taylor series as

$$\psi(y, z) \approx -2kR - k \frac{y^2 + z^2}{R}. \quad (3.19)$$

Regarding the amplitude function and the polarisation, at the stationary point, $\theta = 0$ and $\phi = 0$, leading to

$$\mathbf{g}(0, 0) = \frac{1}{R^2} \hat{\mathbf{e}}_y. \quad (3.20)$$

3.B Stationary Phase Approximation for a Curved Rectangular Plate

Relying on the SPA presented in Section 3.2.1, the amplitude and phase functions are respectively defined as

$$\mathbf{g}(y, z) = -\frac{f(\theta)}{(R')^2} (\hat{\mathbf{e}}_n \cdot \hat{\mathbf{e}}_r) \hat{\mathbf{e}}_\theta,$$

$$\psi(y, z) = -2kR' = -2k\sqrt{(R + h(y, z))^2 + y^2 + z^2}.$$

The stationary point is found by solving

$$\begin{cases} \frac{\partial \psi}{\partial y} = -2k \frac{y + [R + h(y, z)] \frac{\partial h}{\partial y}(y, z)}{\sqrt{(R + h(y, z))^2 + y^2 + z^2}} = 0 \\ \frac{\partial \psi}{\partial z} = -2k \frac{z + [R + h(y, z)] \frac{\partial h}{\partial z}(y, z)}{\sqrt{(R + h(y, z))^2 + y^2 + z^2}} = 0 \end{cases} \Leftrightarrow \begin{cases} y + [R + h(y, z)] \frac{\partial h}{\partial y}(y, z) = 0 \\ z + [R + h(y, z)] \frac{\partial h}{\partial z}(y, z) = 0 \end{cases}.$$

The solution of this system of equations is not straightforward. Nonetheless, following the Fermat stationary point theorem [MVM09], the stationary point is the one minimising the total travelled distance. Geometrically, it follows that the stationary point is $(y_0, z_0) = (0, 0)$. This is also confirmed mathematically with the system of equations, as it is assumed that $\nabla h(0, 0) = 0$. One can check that, with $h(0, 0) = 0$ and $\nabla h(0, 0) = 0$, the second derivatives of the phase function at the stationary point are given by

$$\begin{aligned} \frac{\partial^2 \psi}{\partial y^2}(0, 0) &= -\frac{2k}{R} \left[1 + R \frac{\partial^2 h}{\partial y^2}(0, 0) \right], \\ \frac{\partial^2 \psi}{\partial z^2}(0, 0) &= -\frac{2k}{R} \left[1 + R \frac{\partial^2 h}{\partial z^2}(0, 0) \right], \\ \frac{\partial^2 \psi}{\partial y \partial z}(0, 0) &= 2k \frac{\partial^2 h}{\partial y \partial z}(0, 0) = 0, \end{aligned}$$

where the last equality is obtained when assuming that the principal directions of the target are aligned with the y - and z -axis. Therefore, denoting by h''_y and h''_z the second derivative of h w.r.t. y and z at the stationary point, the phase function is approximated with a Taylor series as

$$\psi(y, z) \approx -2kR - \frac{k}{R} \left[1 + R h''_y \right] y^2 - \frac{k}{R} \left[1 + R h''_z \right] z^2.$$

Regarding the amplitude function and the polarisation, at the stationary point, $\hat{\mathbf{e}}_n = -\hat{\mathbf{e}}_x$, $\hat{\mathbf{e}}_r = \hat{\mathbf{e}}_x$ and $\theta = 0$, leading back to (3.20):

$$\mathbf{g}(0, 0) = \frac{1}{R^2} \hat{\mathbf{e}}_y.$$

Electromagnetic-Based Multistatic Near-Field Sensing

4

This chapter is a joint work carried out with Guillaume Thiran, which I thanks warmly. Most of the content is inspired from [De +24b]. An extended version of this paper is in preparation [De +24c].

FOR emerging radar and communication systems, high carrier frequencies are considered. For instance, in automotive applications, frequencies in the 77-81 GHz band are now considered [20b]. Additionally, the 5G-New Radio (NR) standard extends previous standards to use frequencies up to 7.125 GHz in a first instance, and then rise up to 24.25 GHz and 71 GHz [20a]. To further improve the performance of both functions, the number of antennas is also increasing, leading to extremely large antenna arrays [Ram+23]. From a signal processing perspective, multiple techniques and algorithms are developed in parallel to support the novel near-field infrastructure. For instance, cooperative nodes are envisaged, such that their capabilities can be aggregated to enhance the overall network performance [AKN20].

Owing to the high carrier frequencies and large antenna arrays, the near-field propagation region is enlarged. Indeed, whereas the Fraunhofer distance of one antenna alone is still in the magnitude of the wavelength, the Fraunhofer distance of the complete array is increasing with the array size. Therefore, the combination of the plane waves transmitted by each antenna forms a spherical wave from the point of view of the receiver. This is further enhanced with cooperative transmissions, as the dimension of the complete network should be considered. For sensing applications, the Fraunhofer distance of the target should also be considered for the reflection. As shown in Chapter 3, at such frequencies, the reflection is nearly always propagating in the near-field region of the target in most applications, e.g. automotive and indoor sensing. Instead of modelling targets as points, one should take into account the targets shapes and dimensions. Such applications therefore call for the development of accurate near-field radar propagation models, and specific signal processing algorithms leveraging on the additional propagation effects which are modelled.

This chapter focuses on the development of novel multistatic near-field prop-

agation models, and specific signal processing leveraging on the near-field effects to enhance the sensing capabilities. First, Section 4.1 summarises existing works on near-field signal processing for communication and sensing. Then, Section 4.2 introduces the usual far-field and near-field signal models from the literature. Next, the EM-based reflected signal model is developed for rectangular plate targets in Section 4.3. Moreover, leveraging on the obtained model, Section 4.4 develops an expression of the maximum likelihood estimator for the range. Finally, the accuracy of the EM model, and the impact of multiple parameters, i.e. the bandwidth, carrier frequency and range, on the modified log-likelihood function are evaluated in Section 4.5. The CRB is also computed numerically.

4.1 Near-field Signal Processing in the Literature

The near-field propagation region raises both challenges and opportunities, which have been studied both for communication and radar system [Con+23; Lu+24]. For a communication perspective, accurate channel estimation and beam splitting are two examples of challenges to tackle [Cui+23]. For instance, the former has been considered in [CD22], exploiting polar-domain sparsity, or in [Liu+24], which proposes a sensing-enhanced channel estimation scheme, reducing the complexity of the process. Line-of-Sight (LoS) and Non Line-of-Sight (NLoS) near-field paths components are modelled in [LD23] to better match actual scenarios. The latter is analysed in [Gao+19], which investigates the wideband beamspace channel estimation problem, or in [MH22], which proposes a low complexity technique to mitigate any misfocus. As opportunities, it enables increase of the single or multi-user capacity owing to the increase of the spatial multiplexing gain in the near-field, compared to far-field transmissions [Ram+23]. For instance, energy-efficient hybrid precoding is proposed in [Gao+16] to improve the spectral efficiency in the near-field. Moreover, the potential of beam focusing for single multi-user communications is analysed in [Zha+22]. Finally, near-field beamforming has also been considered in combination with Reconfigurable Intelligent Surface (RIS), e.g. in [BDS21; Wan+22; Wan+23].

From a radar perspective, considering the near-field propagation as a challenge, [Mei+22] proposes a strategy for compensating the range-dependent phase deviation in near-field coherent radar networks in order to increase the angle-estimation capabilities of such system. On the other hand, considering the near-field propagation as an opportunity, the performance achieved by a large-scale radar array is studied [Sak+22b; Sak+22a], when detecting a target in near-field with an OFDM waveform. By leveraging on the near-field propagation, the authors show numerically and experimentally that the estimation of the position can be improved with respect to traditional far-field propagation models. To evaluate the performance improvement brought by near-field algorithms, the CRB has been evaluated in different scenarios.

In [WXZ24; Sak+21], radar passive targets are localised, the accuracy being linked to waveform parameters. However, the presented works all consider point targets, the near-field effect being only considered from the antenna arrays. In order to consider targets with significant dimensions, it is nonetheless necessary to develop novel propagation models based on the EM theory, to accurately model the near-field effects related to the targets. This is done in [Che+23], where an antenna position is estimated from the signal received over a large surface, relying on a specific EM propagation model. Nevertheless, this work does not consider the detection of a passive radar target.

4.2 Single-Point Multistatic Channel Model in Near- and Far-Field

In Section 1.3 of Chapter 1, single-antenna channel models for communication and sensing have been detailed from a signal processing perspective. One can verify based on Chapter 2 that these channel models are coherent with the expression of the radiated fields in complex baseband representation, given in Box 2.22. In this section, the single-antenna channel models are extended for multi-antenna systems, operating in the near- or far-field region of the antenna arrays for communication links, and/or the target for radar links. As this chapter is focused on the radar function, reflections on targets are considered. Still, the developed models are also valid for communication links, as targets may be considered as scatterers from the communication point of view.

4.2.1 Far-Field Multi-Antenna Channel Model

In the far-field region of a current distribution, as shown in Section 2.4.1 of Chapter 2, the phase variation among the current elements contained in the distribution only varies w.r.t. the distance between these elements. This is valid for the EM fields transmitted by the antenna array, but also for the received EM fields reflected on the target, as long as the receiving antenna array is located in the far-field region of the target. Additionally, in this propagation region, its characteristics, e.g. its shape or dimensions, are all merged into one single complex coefficient, and it is modelled as a single point [Ric05]. This is named the *single-point target model*. Therefore, in the literature, the signal received at the l' th antenna from the reflection on a target of the signal transmitted by the l th antenna is usually modelled as Box 4.1 [TV05].

In Box 4.1, ξ is an unknown complex coefficient, R_t (resp. R_r) is the distance between the centre of the target and the reference point for the transmitting (resp. receiving) antenna array, ρ_l (resp. $\rho_{l'}$) is the distance between the l th transmitting antenna (resp. l' th receiving antenna) and the reference point of the transmitting (resp. receiving) antenna array, ϕ_l (resp. $\phi_{l'}$) is the angle between the vectors starting from the

4.1: Single-Point Target Model (Far-Field)

$$u_{l'l}(t) = \xi e^{-jk(\rho_l \cos \phi_l + \rho_{l'} \cos \phi_{l'})} s_l \left(t - \frac{R_t + R_r}{c} \right) + w_{l'l}(t)$$

Single-Point Target Model

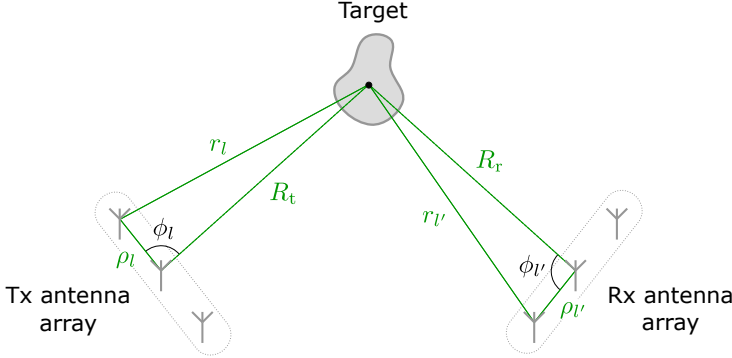


Fig. 4.1 Single-point target model.

reference point of the transmitting (resp. receiving) antenna array, and pointing to the centre of the target and the l^{th} transmitting antenna (resp. l'^{th} receiving antenna), and $w_{l'l}$ is an [AWGN](#). These quantities are illustrated in Figure 4.1. From this model, the range $R_t + R_r$ is usually estimated based on the waveform delay, whereas the phase shifts between the different antennas pairs enable to estimate the angles of departure and arrival.

4.2.2 Near-Field Multi-Antenna Channel Model

To extend the far-field channel model in the near-field region of the transmitting and receiving antenna arrays, in the literature, e.g. [BDS21; Liu+23; Sak+22a], the dependence of the phase shift w.r.t. the exact positions of the antennas and the target has been introduced in Box 4.1, leading to Box 4.2.

4.2: Single-Point Target Model (Near-Field)

$$u_{l'l}(t) = \xi e^{-jk(r_l + r_{l'})} s_l \left(t - \frac{r_l + r_{l'}}{c} \right) + w_{l'l}(t)$$

In Box 4.2, r_l (resp. $r_{l'}$) is the distance between the l^{th} transmitting antenna (resp. l'^{th}

receiving antenna) and the centre of the target. With this model, the impact of the spherical wavefronts is well taken into account in the phase. However, the target is still considered as a single point. This may be accurate in the far-field region, but the impact of the near-field region should also be considered:

1. The impact of the target in the received signal is still modelled as a single complex coefficient ξ , identical for every pair of transmitting and receiving antenna. One may expect this coefficient to vary, even slightly, w.r.t. the antennas positions. For instance, the attenuation of the EM fields is a function of the distance between the antennas and the target.
2. The received waveform is delayed by the distance between the antennas and the centre of the target. Nonetheless, since the targets dimensions and shapes are not negligible w.r.t. the distance between the antennas and the target, the centre of the targets may not be appropriate as reference points. Moreover, one unique point may not be enough to model accurately the target in the near-field region.

Consequently, in Section 4.3, leveraging on the electromagnetic theory, a novel multistatic signal model is developed for a flat rectangular plate. This model is then used in Section 4.4 to estimate the range between the antenna arrays and the target.

4.3 Multistatic Reflection Model for a Rectangular Plate

In this section, analytical expressions of the signal reflected by rectangular plates are developed, in a multistatic near-field configuration. This is first carried out in Section 4.3.1 for a flat rectangular plate, parallel to the antenna array. The cases of oriented flat and curved plates are then discussed in Sections 4.3.2 and 4.3.3.

4.3.1 Parallel Flat Rectangular plate

In order to compute the voltage at a receiving antenna from the EM fields reflected by a flat rectangular plate in a multistatic configuration, the developments of Section 3.2.2 in Chapter 3 are adapted from a monostatic to a multistatic configuration. Let us consider the scenario illustrated in Figure 4.2. An antenna array of N antennas distributed at positions $\{(-R, y_l, z_l)\}$ is facing a rectangular plate $\mathcal{S}$ of dimensions D_y and D_z defined as

$$\mathcal{S} \triangleq \left\{ (x, y, z) \in \mathbb{R}^3 \mid x = 0, |y| \leq \frac{D_y}{2}, |z| \leq \frac{D_z}{2} \right\}.$$

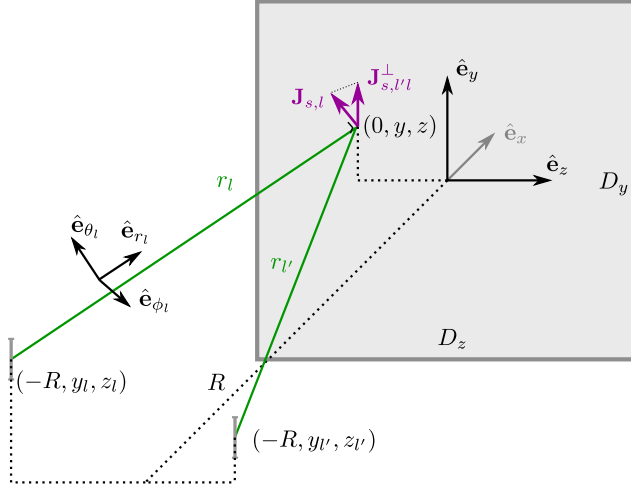


Fig. 4.2 Reflection on a rectangular plate with a planar antenna array.

The normal unit vector pointing towards the antenna array is given by $\hat{\mathbf{e}}_n = -\hat{\mathbf{e}}_x$. The spherical coordinate system associated with the antenna l is also defined as

$$\begin{pmatrix} \hat{\mathbf{e}}_{r_l} \\ \hat{\mathbf{e}}_{\theta_l} \\ \hat{\mathbf{e}}_{\phi_l} \end{pmatrix} = \begin{pmatrix} \cos \theta_l \cos \phi_l & \sin \theta_l & \cos \theta_l \sin \phi_l \\ -\sin \theta_l \cos \phi_l & \cos \theta_l & -\sin \theta_l \sin \phi_l \\ -\sin \phi_l & 0 & \cos \phi_l \end{pmatrix} \begin{pmatrix} \hat{\mathbf{e}}_x \\ \hat{\mathbf{e}}_y \\ \hat{\mathbf{e}}_z \end{pmatrix}.$$

One can compute the angle θ_l and ϕ_l from the (x, y, z) -coordinates as

$$\begin{aligned} \sin \phi_l &= \frac{z - z_l}{\sqrt{(R+x)^2 + (z - z_l)^2}}, & \sin \theta_l &= \frac{y - y_l}{\sqrt{(R+x)^2 + (y - y_l)^2 + (z - z_l)^2}}, \\ \cos \phi_l &= \frac{R+x}{\sqrt{(R+x)^2 + (z - z_l)^2}}, & \cos \theta_l &= \frac{\sqrt{(R+x)^2 + (z - z_l)^2}}{\sqrt{(R+x)^2 + (y - y_l)^2 + (z - z_l)^2}}. \end{aligned}$$

In the following, a focus is made on the signal transmitted by the l^{th} antenna, and received by the l'^{th} antenna.

Assuming that the antenna is either a Hertzian dipole or a uniform line element, following the developments of Section 2.4.3 in Chapter 2, the incident EM fields at the surface of the plate are computed using Box 2.21:

$$\mathbf{H}_l^i \cong -\frac{jkL}{4\pi} I \left(t - \frac{r_l}{c} \right) \frac{e^{-jkr_l}}{r_l} f(\theta_l) \hat{\mathbf{e}}_{\phi_l}, \quad (4.1)$$

$$\mathbf{E}_l^i \cong -\frac{jk\eta L}{4\pi} I \left(t - \frac{r_l}{c} \right) \frac{e^{-jkr_l}}{r_l} f(\theta_l) \hat{\mathbf{e}}_{\theta_l}, \quad (4.2)$$

where the function f is defined in (2.14). W.r.t. the developments of Chapter 3, some differences should be highlighted:

1. A time-varying current is considered, as it is supposed to be modulated by a given signal s of bandwidth B :

$$I(t) = I_0 s(t).$$

Therefore, contrariwise to Chapter 3, the EM fields in this chapter are expressed in the time domain complex baseband representation. Following Box 2.22, (4.1) and (4.2) are obtained by converting the developments of Section 2.4.3 in the time-domain complex baseband representation. These equations are therefore valid only for narrowband signals, i.e. signals such that the bandwidth B is much smaller than the carrier frequency f_c .

2. Owing to the bistatic configuration, different spherical coordinate systems associated with each antenna are defined. Therefore, the EM fields are function of the distance r_l , elevation angle θ_l and azimuth angle ϕ_l related to the antenna l . The distance r_l from the antenna to any point of $\mathcal{S}$ is computed as

$$r_l = \sqrt{R^2 + (y - y_l)^2 + (z - z_l)^2}.$$

Under the PEC approximation, the equivalent surface current at the surface of the plate is computed following Box 2.10:

$$\mathbf{J}_{s,l} \approx 2 \mathbf{H}_l^i \times \hat{\mathbf{e}}_x = -\frac{jkLI_0}{2\pi} s \left(t - \frac{r_l}{c} \right) \frac{e^{-jkr_l}}{r_l} f(\theta_l) \cos \phi_l \hat{\mathbf{e}}_y.$$

Focusing on the l' th receiving antenna, the perpendicular component of the surface current density is then computed as

$$\mathbf{J}_{s,l'l}^\perp = \mathbf{J}_{s,l} - (\mathbf{J}_{s,l} \cdot \hat{\mathbf{e}}_{r_{l'}}) \hat{\mathbf{e}}_{r_{l'}} = -\frac{jkLI_0}{2\pi} s \left(t - \frac{r_l}{c} \right) \frac{e^{-jkr_l}}{r_l} f(\theta_l) \cos \phi_l \cos \theta_{l'} \hat{\mathbf{e}}_{\theta_{l'}}.$$

The reflected EM fields at the l' th receiving antenna are finally computed using Box 2.22 on the surface of the rectangular plate. From these fields, one can compute the received voltage $u_{l'l}$ and current at the l' th receiving antenna using Box 2.20:

$$\begin{aligned} u_{l'l}(t) &\approx -\frac{jk\eta}{4\pi} \iint_{\mathcal{S}} \frac{e^{-jkr_{l'}}}{r_{l'}} \left[\mathbf{F}_{l'}^\perp \cdot \mathbf{J}_{s,l'l}^\perp \left(t - \frac{r_{l'}}{c} \right) \right] d\mathcal{S} \\ &= -\frac{k^2 \eta L^2 I_0}{8\pi^2} \int_{-\frac{D_z}{2}}^{\frac{D_z}{2}} \int_{-\frac{D_y}{2}}^{\frac{D_y}{2}} \frac{e^{-jk(r_l+r_{l'})}}{r_l r_{l'}} s \left(t - \frac{r_l + r_{l'}}{c} \right) f(\theta_l) f(\theta_{l'}) \cos \phi_l \cos \theta_{l'} dy dz, \end{aligned} \quad (4.3)$$

in which $\mathbf{F}_{l'}^\perp$ is the perpendicular radiation vector of the l' th receiving antenna. The angles ϕ_l , θ_l and $\theta_{l'}$, and the distances r_l and $r_{l'}$ are all functions of y and z .

One may already notice that the voltage generated at the receiving antenna l' by the transmitting antenna l , seems to not be equal to the voltage generated at the receiving antenna l by the transmitting antenna l' . This would therefore violate the reciprocity principle. As described in Section 2.3.4 of Chapter 2, the PO does not guarantee the reciprocity, owing to the numerous approximations which are made. Nonetheless, in most scenarios, such conclusion cannot be drawn before solving the integral. Closed-forms expressions of the integral in (4.3) are computed with the SPA in Appendix 4.A, and particularised to linear antenna arrays in Appendix 4.B. As expected, the results obtained in the appendices are reciprocal.

In summary, the signal transmitted by the l^{th} transmitting antenna, reflected by the flat rectangular plate, and received by the l'^{th} receiving antenna, is given in Box 4.3.

4.3: Received Multistatic Near-Field Signal Model

$$u_{l'l}(t) \approx \xi \eta_{l'l} \frac{e^{-j2kr_{l'l}}}{\kappa_{l'l}} s\left(t - \frac{2r_{l'l}}{c}\right) + w_{l'l}(t)$$

In the model of Box 4.3,

- The factor ξ is a constant real coefficient, which does not vary among the pairs of antennas, or the distance R between the antenna array and the target.
- The factor $\eta_{l'l}$ is a complex coefficient, which varies both among the pairs of antennas, and with the distance R between the antenna array and the target. Nonetheless, it is supposed to be nearly constant, as the significant amplitude and phase variations are expected to be brought by the $\kappa_{l'l}$ factor, and the complex exponential term. This is further analysed in Section 4.5.2.
- The factor $\kappa_{l'l}$ is a real coefficient which varies both among the pairs of antennas, and the distance R between the antenna array and the target. It is supposed to model the significant amplitude variations of the signal with the distance.
- The factor $e^{-j2kr_{l'l}}$ is a complex exponential term, which models the significant phase variations of the signal with the distance.
- The distance $r_{l'l}$ is the distance between the l^{th} transmitting and l'^{th} receiving antennas, and the stationary (or specular) point of the reflection on the target. Therefore, what matters is the distance to the stationary point, and not the distance to the centre of the target, as usually considered in the literature.

These factors are summarised in Table 4.1 for the general case and the particular case of the linear antenna array along the z -axis. Yet, note that the general case

Factor	Planar antenna array	Linear antenna array
ξ	$\frac{k\eta L^2 I_0}{8\pi}$	
$\eta_{l'l}$	J	$-\frac{kRA_{l'l}}{\pi r_{l'l}^2}$
$\kappa_{l'l}$	$\frac{4r_{l'l}^3}{4R^2 + (z_l - z_{l'})^2}$	$r_{l'l}$
$r_{l'l}$	$\sqrt{R^2 + \frac{(y_l - y_{l'})^2}{4} + \frac{(z_l - z_{l'})^2}{4}}$	$\sqrt{R^2 + \frac{(z_l - z_{l'})^2}{4}}$

Table 4.1 Quantities in Box 4.3, for the planar antenna array, and the linear antenna array along the z -axis. The expression of $A_{l'l}$ is provided in (4.14).

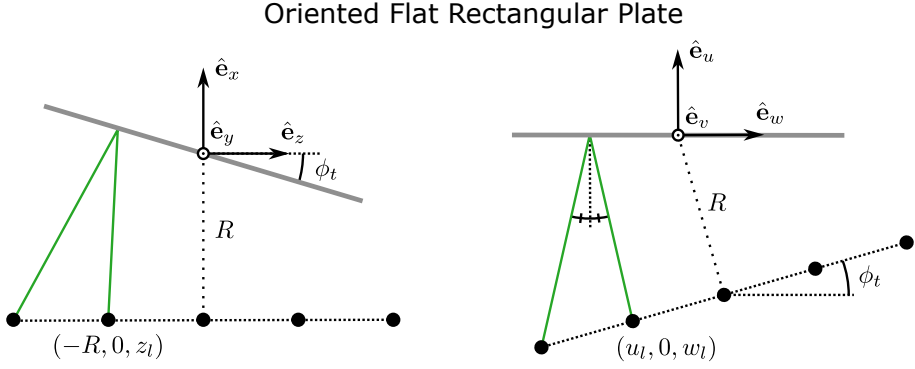


Fig. 4.3 Oriented flat rectangular plate.

holds only if the rectangular plate is large enough to contain multiple Fresnel zones. Contrariwise to Chapter 3, owing to the multistatic configuration, the stationary point can be located anywhere on the target, such that it is not always fulfilled.

Compared to traditional received signal models in the far-field, for which the range of the target can only be estimated from the delay of the transmitted waveform, one may retrieve more information using also the amplitude and phase factors of Box 4.3 to further improve the radar range estimation performance. For instance, the radar resolution is traditionally limited by the bandwidth of the transmitted waveform. Since the phase variation is instead proportional to the carrier frequency, such limit is expected to be overcome in the near-field. This question is analysed in Section 4.4. Before, the cases of the oriented flat and curved plates are first discussed in Section 4.3.2 and 4.3.3.

4.3.2 Oriented Flat Rectangular Plate

If an oriented rectangular plate is instead analysed, the results obtained in Section 4.3.1 are generalisable. For instance, let us consider a rotation of the plate of an angle ϕ_t in azimuth, and a linear antenna array along the z -axis. The antenna array is composed of N antennas distributed at positions $\{(-R, 0, z_l)\}$, and the oriented rectangular plate $\tilde{\mathcal{S}}$ is defined as

$$\tilde{\mathcal{S}} \triangleq \left\{ (x, y, z) \in \mathbb{R}^3 \left| x = -z \tan \phi_t, |y| \leq \frac{D_y}{2}, |z| \leq \frac{D_z}{2} \cos \phi_t \right. \right\}.$$

However, instead of considering this setup, the problem may be instead analysed in another coordinate system $\{\hat{\mathbf{e}}_u; \hat{\mathbf{e}}_v; \hat{\mathbf{e}}_w\}$ aligned with the target:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} \cos \phi_t & 0 & -\sin \phi_t \\ 0 & 1 & 0 \\ \sin \phi_t & 0 & \cos \phi_t \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

In that coordinate system, the target is defined as

$$\tilde{\mathcal{S}} \triangleq \left\{ (u, v, w) \in \mathbb{R}^3 \left| u = 0, |v| \leq \frac{D_y}{2}, |w| \leq \frac{D_z}{2} \right. \right\},$$

and the antennas are now located at positions $\{(u_l, 0, w_l)\}_{l=1, \dots, N}$ such that

$$\begin{cases} u_l = -R \cos \phi_t + z_l \sin \phi_t \\ w_l = R \sin \phi_t + z_l \cos \phi_t \end{cases}. \quad (4.4)$$

Moreover, the angle θ_l and ϕ_l are now computed in the new coordinate system, defined as

$$\begin{aligned} \sin \phi_l &= \frac{w - w_l}{\sqrt{(u - u_l)^2 + (w - w_l)^2}}, & \sin \theta_l &= \frac{v - v_l}{\sqrt{(u - u_l)^2 + (v - v_l)^2 + (w - w_l)^2}}, \\ \cos \phi_l &= \frac{u - u_l}{\sqrt{(u - u_l)^2 + (w - w_l)^2}}, & \cos \theta_l &= \frac{\sqrt{(u - u_l)^2 + (w - w_l)^2}}{\sqrt{(u - u_l)^2 + (v - v_l)^2 + (w - w_l)^2}}. \end{aligned}$$

Therefore, the results developed in Appendix 4.A and 4.B are easily extended by modifying the coordinates x_l and z_l to match the coordinates u_l and w_l of (4.4). The last remaining subtlety is the computation of the specular reflection point: let us denote by z_s the z -coordinate of the specular point. Knowing that the angle of incidence and reflection are equal at the location of the specular reflection, the

following property holds:

$$\frac{x_l}{\sqrt{x_l^2 + (z - z_l)^2}} = \frac{x_{l'}}{\sqrt{x_{l'}^2 + (z - z_{l'})^2}}.$$

Introducing that result in (4.8) gives

$$z_s = \frac{z_l x_{l'} + z_{l'} x_l}{x_l + x_{l'}}.$$

The expression of the received signal with the oriented flat rectangular plate may be interesting to simultaneously estimate the range and the azimuth rotation angle of the plate. Though, this is left for future works.

4.3.3 Curved Rectangular Plate

The analysis of curved geometries is critical for numerous reasons. First, few targets are well approximated by flat plates. Moreover, with the introduction of curvature, slight variations of the specular point position are associated with large variations of the antennas positions. Thus, the number of scenarios the proposed model cannot handle decreases significantly, as the specular point is in most of the cases located within the target. Additionally, the discrepancies between the usual model of Box 4.2 and the developed expressions would probably reduce, as the variations of the specular point positions are smaller.

Nonetheless, curved rectangular plates are challenging in multistatic configurations, as the position of the specular point on the target is often intractable analytically. For instance, the computation of the specular point on a sphere in two dimensions is a well-known challenge, named the *Alhazen problem* [Alh89]. Still, such solution may be computed numerically. Thus, the parametrisation of the developed model on the specular point location for arbitrary targets, and the comparison between the obtained models are left for future works.

4.4 Range Estimation in the Near-Field Region

In this section, a focus is made on the estimation of the distance R between the antenna array and the target, based on the received signal model of Box 4.3. The positions of the N_t transmitting and N_r receiving antennas are assumed to be known.

In a first time, we assume that the signals transmitted by the antennas are received in an orthogonal manner, such that the receiver has access to each signal $u_{l'l}$ independently. Orthogonality could be achieved in the time, frequency or code domain. In the time domain, each transmitting antenna starts the transmission at a given

time, selected such that it is higher than the maximum possible round trip time. In the frequency domain, each transmitting antenna uses a different carrier frequency. Therefore, one may adapt Box 4.3 such that the wave number also varies with the transmitting antenna index. In the code domain, each transmitting antenna transmits a different orthogonal waveform. In that case, it requires adapting Box 4.3 such that the transmitted waveform varies with the transmitting antenna index. Nonetheless, the orthogonality should be ensured whatever the travel time differences between the pairs of antennas, as the delays of the waveforms vary with the antenna indices.

4.4.1 Transmission of Orthogonal Signals

Let us consider that orthogonality is achieved in the time domain. According to Box 4.3, the received signal $u_{l'l}$ at the l' th receiving antenna, generated by the reflection of the signal transmitted by the l th transmitting antenna on the flat rectangular plate, can be written from a signal processing perspective as Box 4.4, where ξ is a constant complex coefficient, identical for every pair of antennas, and assumed to be unknown, and $w_{l'}$ is an AWGN, supposed to be independent for each receiving antenna.

4.4: Received Multistatic Near-Field Signal Model (Signal Processing)

$$u_{l'l}(t) = \xi \mu_{l'l;R}(t) + w_{l'}(t)$$

The function $\mu_{l'l;R}$ in Box 4.4 represents the signal model. For instance, if a full knowledge of the target is available, except for the distance R , one may relate $\mu_{l'l;R}$ to Box 4.3 as

$$\mu_{l'l;R}(t) = \eta_{l'l} \frac{e^{-j2kr_{l'l}}}{\kappa_{l'l}} s \left(t - \frac{2r_{l'l}}{c} \right).$$

One may also assume that the parameter $\eta_{l'l}$ does not vary much among the pairs of antennas, such that it can be considered to be constant. In that case, it is merged with the unknown constant coefficient ξ , leading to

$$\mu_{l'l;R}(t) = \frac{e^{-j2kr_{l'l}}}{\kappa_{l'l}} s \left(t - \frac{2r_{l'l}}{c} \right).$$

One step further is assuming that the parameter $\kappa_{l'l}$ is not varying as much as the phase term among the pairs of antennas, such that it does not bring any information on the range. Under that assumption, the parameter $\kappa_{l'l}$ is also merged with ξ , and the signal model is expressed as

$$\mu_{l'l;R}(t) = e^{-j2kr_{l'l}} s \left(t - \frac{2r_{l'l}}{c} \right).$$

The three considered signal models are summarised in Box 4.5.

4.5: Approximated Signal Models

$$\begin{aligned}
 \text{Full Knowledge:} \quad & \mu_{l'l;R}(t) = \eta_{l'l} \frac{e^{-j2kr_{l'l}}}{\kappa_{l'l}} s \left(t - \frac{2r_{l'l}}{c} \right) \\
 \text{Constant } \eta_{l'l}: \quad & \mu_{l'l;R}(t) = \frac{e^{-j2kr_{l'l}}}{\kappa_{l'l}} s \left(t - \frac{2r_{l'l}}{c} \right) \\
 \text{Constant } \eta_{l'l} \text{ and } \kappa_{l'l}: \quad & \mu_{l'l;R}(t) = e^{-j2kr_{l'l}} s \left(t - \frac{2r_{l'l}}{c} \right)
 \end{aligned}$$

With independent noises at each receiving antenna, the joint maximum likelihood estimation problem of the distance R and the unknown coefficient ξ is developed as

$$\begin{aligned}
 \hat{R}, \hat{\xi} &= \arg \min_{\hat{R}, \hat{\xi}} \sum_{l'=0}^{N_t-1} \sum_{l=0}^{N_t-1} \int_t |u_{l'l}(t) - \xi \mu_{l'l;\hat{R}}(t)|^2 dt \\
 &= \arg \min_{\hat{R}, \hat{\xi}} |\hat{\xi}|^2 \sum_{l'=0}^{N_t-1} \sum_{l=0}^{N_t-1} \int_t |\mu_{l'l;\hat{R}}(t)|^2 dt - 2\text{Re} \left\{ \hat{\xi}^* \sum_{l'=0}^{N_t-1} \sum_{l=0}^{N_t-1} \int_t u_{l'l}(t) \mu_{l'l;\hat{R}}^*(t) dt \right\}.
 \end{aligned} \tag{4.5}$$

First, from the second term of the objective function, since $\text{Re}\{z\} \leq |z| \forall z \in \mathbb{C}$ with the equality achieved when $z \in \mathbb{R}^+$, the phase of ξ is estimated as

$$\arg \left\{ \hat{\xi} \right\} = \arg \left\{ \sum_{l'=0}^{N_t-1} \sum_{l=0}^{N_t-1} \int_t u_{l'l}(t) \mu_{l'l;\hat{R}}^*(t) dt \right\}.$$

Then, minimising (4.5) after inserting the estimate of the phase of ξ , the amplitude of ξ is estimated as

$$|\hat{\xi}| = \frac{\left| \sum_{l'=0}^{N_t-1} \sum_{l=0}^{N_t-1} \int_t u_{l'l}(t) \mu_{l'l;\hat{R}}^*(t) dt \right|}{\sum_{l'=0}^{N_t-1} \sum_{l=0}^{N_t-1} \int_t |\mu_{l'l;\hat{R}}(t)|^2 dt}.$$

Combining the phase and amplitude of ξ , this leads to the following estimate:

$$\hat{\xi} = \frac{\sum_{l'=0}^{N_t-1} \sum_{l=0}^{N_t-1} \int_t u_{l'l}(t) \mu_{l'l;\hat{R}}^*(t) dt}{\sum_{l'=0}^{N_t-1} \sum_{l=0}^{N_t-1} \int_t |\mu_{l'l;\hat{R}}(t)|^2 dt}.$$

Inserting this result into (4.5), the maximum likelihood estimation problem of the distance R is written as Box 4.6, where the function Υ is named the *modified log-likelihood function*.

Therefore, to estimate the distance R , the correlations of all the received signals with their corresponding signal models are all computed and summed in a coherent

4.6: Multistatic Near-Field Range Estimation (Orthogonal Case)

$$\hat{R} = \arg \max_{\tilde{R}} \Upsilon(\tilde{R}) \quad \text{with} \quad \Upsilon(\tilde{R}) \triangleq \frac{\left| \sum_{l'=0}^{N_t-1} \sum_{l=0}^{N_t-1} \int_t u_{l'l}(t) \mu_{l'l;\tilde{R}}^*(t) dt \right|^2}{\sum_{l'=0}^{N_t-1} \sum_{l=0}^{N_t-1} \int_t |\mu_{l'l;\tilde{R}}(t)|^2 dt}$$

manner. Then, the result is normalised with the sum of the energies of all the signal models. Though, this should be evaluated for every candidate $\tilde{R}$ of the distance, making this solution computationally expensive.

4.4.2 Transmission of Non-Orthogonal Signals

If there is no orthogonality between the transmitted signals, according to Box 4.3, the received signal $u_{l'}$ at the receiving antenna l' is generated by the reflection of all the signals transmitted simultaneously by the transmitting antennas on the rectangular plate:

$$u_{l'}(t) = \xi \sum_{l=0}^{N_t-1} \mu_{l'l;\tilde{R}}(t) + w_{l'}(t),$$

where ξ , $\mu_{l'l;\tilde{R}}$ and $w_{l'}$ are defined similarly as in Section 4.4.1.

With independent noises at each receiving antenna, the joint maximum likelihood estimation problem of the distance R and the unknown coefficient ξ is developed the same way as in Section 4.4.1:

$$\begin{aligned} \hat{R}, \hat{\xi} &= \arg \min_{\tilde{R}, \xi} \sum_{l'=0}^{N_t-1} \int_t \left| u_{l'}(t) - \xi \sum_{l=0}^{N_t-1} \mu_{l'l;\tilde{R}}(t) \right|^2 dt \\ &= \arg \min_{\tilde{R}, \xi} |\xi|^2 \sum_{l'=0}^{N_t-1} \int_t \left| \sum_{l=0}^{N_t-1} \mu_{l'l;\tilde{R}}(t) \right|^2 dt - 2\text{Re} \left\{ \xi^* \sum_{l'=0}^{N_t-1} \int_t u_{l'}(t) \sum_{l=0}^{N_t-1} \mu_{l'l;\tilde{R}}^*(t) dt \right\}. \end{aligned} \quad (4.6)$$

Following exactly the same developments as in Section 4.4.1, the following estimates of ξ and R are obtained:

$$\begin{aligned} \hat{\xi} &= \frac{\sum_{l'=0}^{N_t-1} \int_t u_{l'}(t) \sum_{l=0}^{N_t-1} \mu_{l'l;\hat{R}}^*(t) dt}{\sum_{l'=0}^{N_t-1} \int_t \left| \sum_{l=0}^{N_t-1} \mu_{l'l;\hat{R}}(t) \right|^2 dt}, \\ \hat{R} &= \arg \max_{\tilde{R}} \frac{\left| \sum_{l'=0}^{N_t-1} \int_t u_{l'}(t) \sum_{l=0}^{N_t-1} \mu_{l'l;\tilde{R}}^*(t) dt \right|^2}{\sum_{l'=0}^{N_t-1} \int_t \left| \sum_{l=0}^{N_t-1} \mu_{l'l;\tilde{R}}(t) \right|^2 dt}, \end{aligned}$$

and the estimation of Box 4.6 is rewritten as Box 4.7.

Parameters	Values
Number of antennas	$N = 13$
Antenna spacing	$\Delta = 0.125$ m
Antenna length/current	$L^2 I_0 = 1$ m ² A
Signal bandwidth	$B = 100$ MHz
Carrier frequency	$f_c = 77$ GHz
Target dimensions	$D_y = 0.8$ m, $D_z = 1.75$ m
Target Distance	$R = 4$ m

Table 4.2 Parameters of the scenarios for Section 4.5.

4.7: Multistatic Near-Field Range Estimation (Non-Orthogonal Case)

$$\hat{R} = \arg \max_{\tilde{R}} \Upsilon(\tilde{R}) \quad \text{with} \quad \Upsilon(\tilde{R}) \triangleq \frac{\left| \sum_{l'=0}^{N_t-1} \int_t u_{l'}(t) \sum_{l=0}^{N_t-1} \mu_{l'l;\tilde{R}}^*(t) dt \right|^2}{\sum_{l'=0}^{N_t-1} \int_t \left| \sum_{l=0}^{N_t-1} \mu_{l'l;\tilde{R}}(t) \right|^2 dt}$$

Compared to the orthogonal case, the same operations are performed, except that the contributions of the transmitted signals cannot be separated, as they are all summed together at the receiver.

4.5 Numerical Analysis

In order to validate the proposed signal models and analyse the sensing performance of the proposed maximum likelihood range estimator, a scenario which mimics an automotive setting is proposed, in which a first vehicle equipped with a linear antenna array estimates the distance R to the next car driving ahead. The N antennas are spread linearly and centred at the front of the vehicle, with a spacing of Δ . The position of the l^{th} antenna is thus given by

$$\mathbf{p}_l = \left(-R, 0, \left(-\frac{N-1}{2} + l \right) \Delta \right).$$

The same signal is transmitted at all antennas, orthogonally in time, or simultaneously. The waveform s is defined as a sinc function of bandwidth B :

$$s(t) = \frac{\sin(\pi B t)}{\pi B t}.$$

Unless stated otherwise, the parameters of the scenario are summarised in Table 4.2.

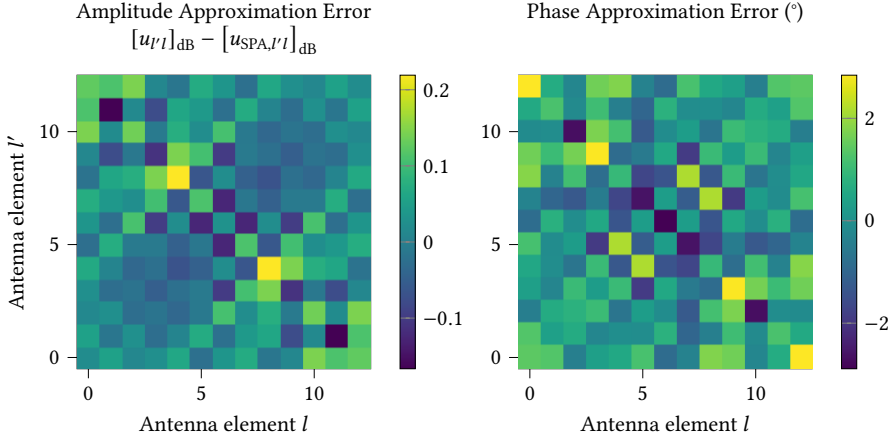


Fig. 4.4 Amplitude and phase errors committed by the SPA ($u_{SPA,l'l}$), compared to numerical integration of (4.3) ($u_{l'l}$), for a constant transmitted signal.

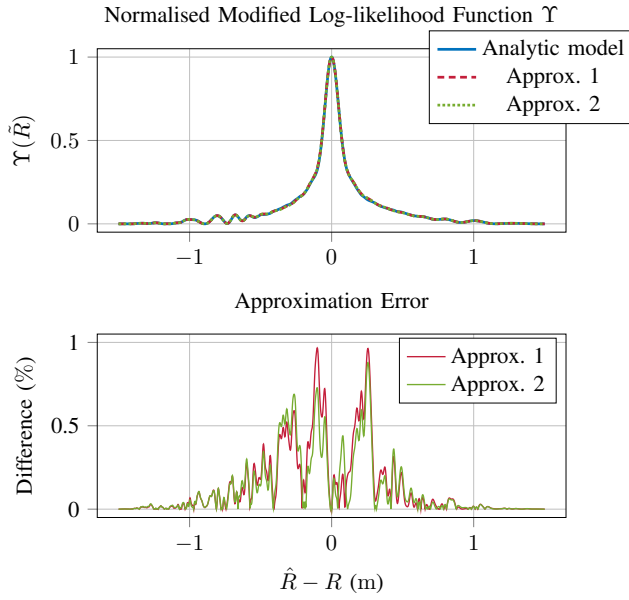
4.5.1 Validation of the Multistatic Reflection Model

First, let us assess the accuracy of the SPA. Figure 4.4 illustrates the amplitude and phase errors committed by the SPA, w.r.t. the numerical integration of (4.3), for a constant transmitted signal, i.e. $s(t) = 1$. Differences up to 0.3 dB in amplitude and 3° in phase are observed, whatever the considered transmitting and receiving antenna pair. These low errors suggest that the signal model of Box 4.5 with full knowledge is an accurate model on which the maximum likelihood range estimation can be performed.

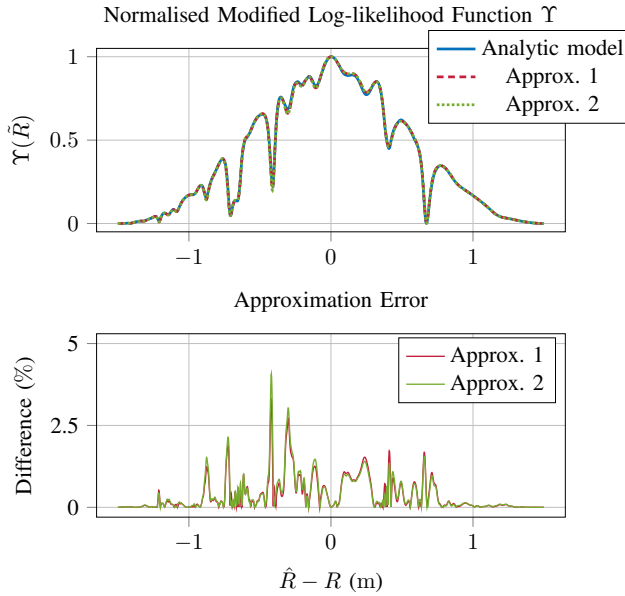
4.5.2 Impact of the Signal Model on the Modified Log-likelihood Function

The maximum likelihood estimator designed in Section 4.4.1 and 4.4.2 in the orthogonal and non-orthogonal case are both functions of the signal model $\mu_{l'l,R}$. This model can be chosen arbitrarily depending on the knowledge of the scenario available at the receiver. It therefore also integrates any mismatch between the actual and supposed scenario. The three propositions of Box 4.5, and detailed in Section 4.4.1, are analysed in this section.

The maximum likelihood range estimator can be analysed through its ambiguity function, i.e. the modified log-likelihood function Υ in noiseless scenarios. Figure 4.5 evaluates the impact of the selected signal model on Υ , in the orthogonal and non-orthogonal case, in the neighbourhood of the distance R . More precisely, the exact signals $u_{l'l}$ are computed numerically, and fed to the modified log-likelihood function. Then, for each possible value of $\tilde{R}$, the value of the modified log-likelihood



(a) Orthogonal



(b) Non-orthogonal

Fig. 4.5 Impact of the selected signal model on the modified log-likelihood function Υ , in the orthogonal and non-orthogonal case. The analytic signal is given by Box 4.3. The first and second approximations are respectively corresponding to the two simplifications proposed in Box 4.5.

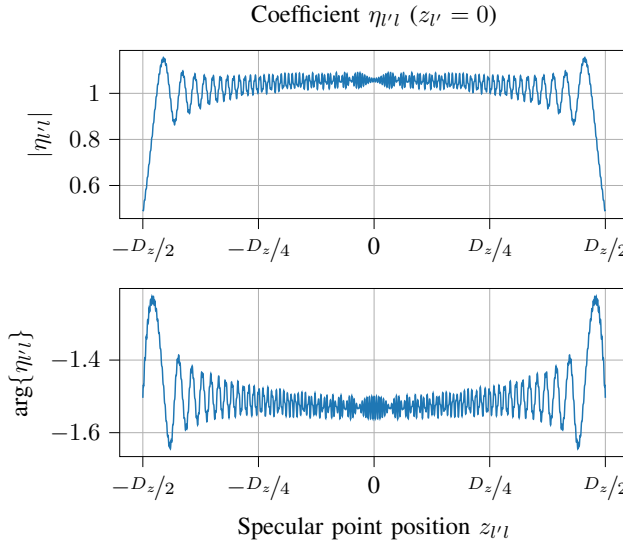


Fig. 4.6 Coefficients $\eta_{l'l}$ as a function of the specular point position, assuming $z_{l'} = 0$.

function is computed with each signal model. These values, normalised to a unitary maximum, are represented in the upper panels of the figure.

As it can be observed, the maximum of the ML function indicates the exact target location for the three models, even if they are only analytically tractable approximations of the received signal. Still, in the non-orthogonal case, an increased sidelobes level is observed. This will be further discussed in Section 4.5.3. Looking at discrepancies between the full analytical model and the two simplifications of Box 4.5, they can difficultly be observed in the upper panels, since the differences between the ML values are shown to remain below 1% and 5%, respectively in the bottom panel of the orthogonal and non-orthogonal case. This is particularly convenient, as the exact modelling of $\eta_{l'l}$ is highly dependent on the target geometry, and thus unlikely to be available at the receiver. This is explained by Figure 4.6, which illustrates the variation of the coefficients $\eta_{l'l}$ w.r.t. the position of the specular point on the plate. Except when the specular point is located at the edges of the rectangular plate, the coefficients do not vary significantly, neither in amplitude, nor in phase. As the plate is large w.r.t. the signal wavelength, the surrounding of the specular point is similar for each pair of antennas.

4.5.3 Impact of the System Parameters on the Modified Log-likelihood Function

Still analysing the modified log-likelihood function Υ , the impact of the bandwidth, carrier frequency, and range are illustrated in Figure 4.7 for the orthogonal and

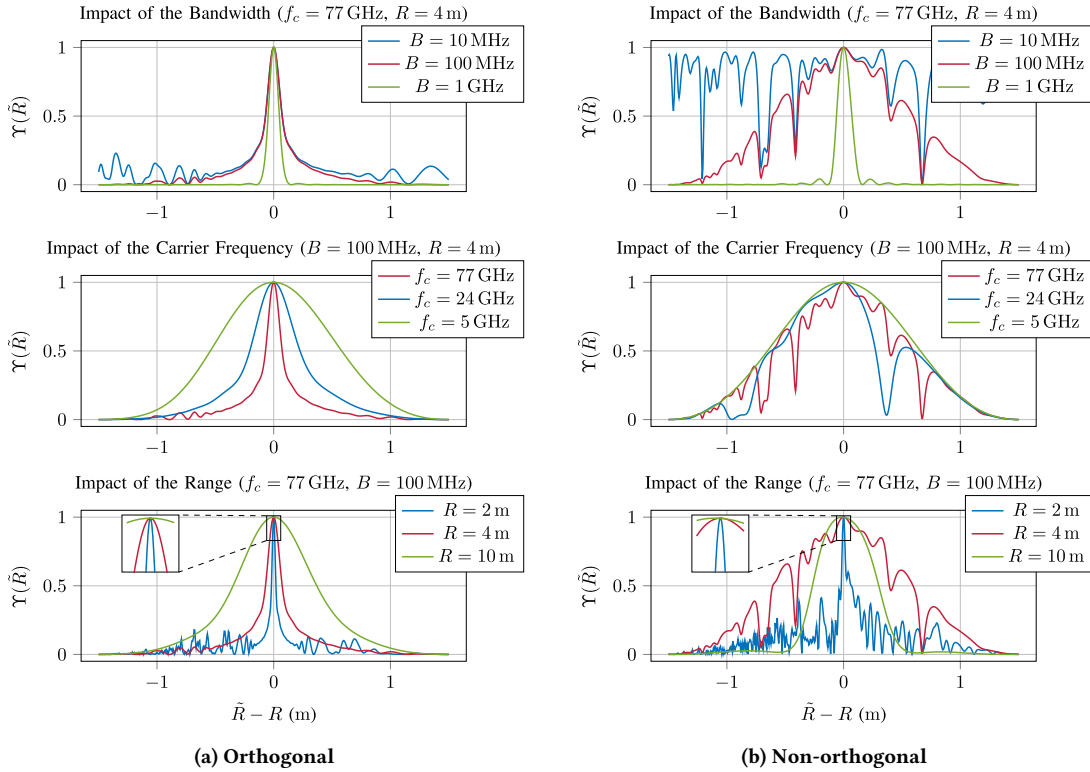


Fig. 4.7 Impact of the system parameters on the modified log-likelihood function, in the orthogonal and non-orthogonal case.

non-orthogonal case, in the neighbourhood of the distance R . As for the impact of the signal model, the exact signals $u_{l'l}$ are first computed numerically, and fed to the modified log-likelihood function. Then, owing to the negligible discrepancies introduced by the simplified models of Box 4.5, the modified log-likelihood function has been evaluated assuming $\eta_{l'l}$ constant.

Let us first focus on the orthogonal case. In the upper panel, one can observe that increasing the bandwidth has two effects. First, it reduces the sidelobes level, thus reducing the ambiguities. Then, for high bandwidths, it also reduces the width of the mainlobe. It can be checked that, for $B = 1$ GHz, the 3-dB pulse width is equal to the traditional resolution, i.e. $c/2B = 0.15$ m. On the middle panel, the impact of the carrier frequency is visible. For a carrier frequency of $f_c = 5$ GHz, the pulse width is again equal to the traditional resolution $c/2B = 1.5$ m. Therefore, it shows that the developed estimation method gives in the worst case the same performance as classical range estimation methods which only leverage on the delay of the waveform. Otherwise, the resolution is improved thanks to the phase information brought by the novel EM-based near-field signal model. Finally, the lower panel illustrates the impact of the range. It can be observed that better resolutions are achieved, but only at close ranges. Nonetheless, even for distances still in the near-field region of the target, increasing the distance leads to a degradation of the resolution, up to $c/2B$. Indeed, the further the target, the smaller the differences of phase among the pairs of antennas.

In the non-orthogonal case, it is difficult to distinguish the mainlobe and the sidelobes. Whereas the increase of the carrier frequency helps improve the resolution in the orthogonal case, this only impacts the accuracy of the estimation in the non-orthogonal case. Yet, the bandwidth still limits the mainlobe and sidelobes level, and the developed estimation method also gives in the worst case the same performance as classical range estimation methods with non-orthogonal waveforms. Nonetheless, the performance improvement is unpredictable, as it varies quickly with the parameters of the scenario. For instance, in the lower panel, a range of 4 meters gives a larger envelope than a range of 10 meters, but it can be shown that this is not the case at 9 and 11 meters. This will be further illustrated in Section 4.5.4, where the CRB is analysed.

4.5.4 Cramér-Rao Lower Bound

To complement the analysis of the parameters impact, the CRB is numerically evaluated. More specifically, from [Thi+24], it can be proven that, assuming the unknown complex coefficient ξ to be a nuisance parameter, the CRB of the range R is proportional to the inverse of the second derivative of the modified log-likelihood function:

$$\text{CRB}_R = - \left(\frac{1}{\sigma_w^2} \frac{\partial^2 \Upsilon_{\text{noise-free}}(R)}{\partial R^2} \right)^{-1}, \quad (4.7)$$

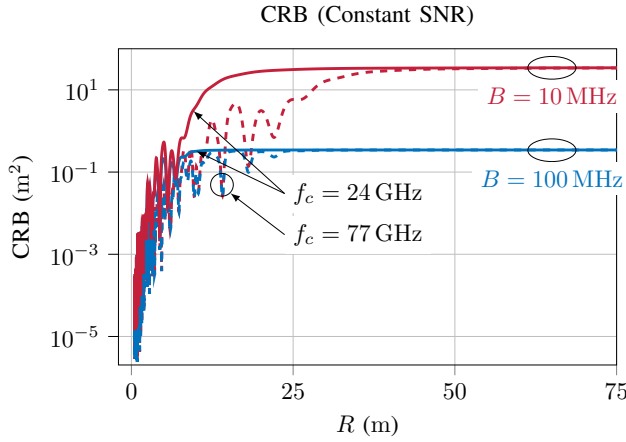
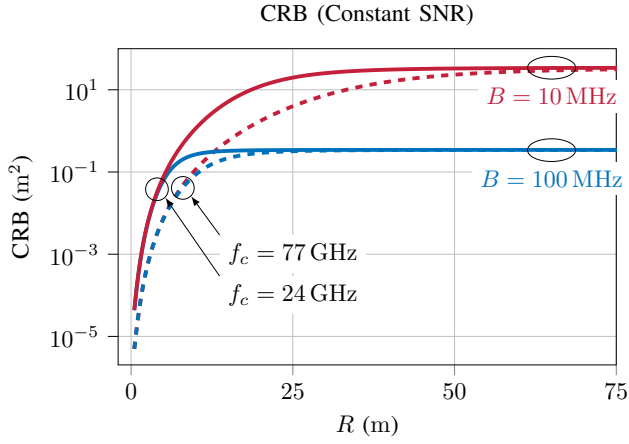


Fig. 4.8 CRB against the distance R , for different carrier frequencies f_c and bandwidths B , with a constant SNR, in the orthogonal and non-orthogonal case.

where σ_w^2 is the noise power, and $\Upsilon_{\text{noise-free}}$ is respectively given by Box 4.6 or Box 4.7 for the orthogonal and non-orthogonal case, with the signals $u_{l'}$ replaced by $\xi\mu_{l';R}$. The complete demonstration is provided in the PhD thesis of Guillaume Thiran.

Figure 4.8 shows the CRB as a function of the distance R , for different carrier frequencies and bandwidths, with a constant SNR at the receiving antennas, in the orthogonal and non-orthogonal case. As it can be observed, the estimation method always leverages on the most informative value among the waveform delay (related to the bandwidth), and the phase shift (related to the carrier frequency). Whereas for small ranges, the phase information dominates, the CRB saturates to a constant value related to the bandwidth when the distance increases. Note that the saturation already occurs in the near-field region. Nevertheless, in the non-orthogonal case, the different transmitted waveforms add up unpredictably in phase, leading to rapid oscillations of the performance with the distance. Yet, the performance are never worsened compared to usual signal processing, leveraging only on the delay of the waveform.

4.6 Conclusion

The near-field propagation region has raised an increased interest for both communication and sensing in recent years, owing to the higher carrier frequencies, larger antenna arrays, and smaller range considered for future ISAC applications. In this chapter, a focus has been made on near-field multistatic sensing. First, the far- and near-field single point target models have been presented. Whereas the phase shifts of the far-field model only depend on the angles of departure and arrival, the exact distances between the transmitting and receiving antennas, and the centre of the target are considered in the near-field model. However, even if it already enables to grasp some insight about the benefits which may be extracted from the spherical wavefronts, such point target model is inaccurate when considering electrically and physically large targets at short distances.

4.6.1 Summary of the Contributions

The contributions of this chapter are summarised as follows:

- Following the same approach as Chapter 3, the signal received from a reflection on a flat rectangular plate has been computed in a multistatic configuration, leveraging on the EM theory and the SPA. The antenna arrays are supposed to be either in a planar or linear configuration, within a plane parallel to the target. The proposed models work either in the near- or far-field region, under the condition that the specular reflection point is located within the target for every pair of antennas. Moreover, the accuracy of the SPA has been assessed by comparing the channel coefficients obtained with numerical integrations

and the developed expressions. It has shown that this approximation works very well for multistatic reflections on flat rectangular plates when the specular point is located within the target. The obtained expression has highlighted the impact of the spherical wavefronts, as a function of the distance between the transmitting and receiving antennas, and the specular point, instead of the centre of the target. Moreover, this phase shift set aside, the remaining complex coefficient has been shown to slightly vary for each pair of antennas, instead of being constant. Yet, it does not impact significantly the estimation performance.

- Leveraging on the developed expressions, the maximum likelihood estimator of the distance between the antenna array and the target has been computed. Three different signal models have been considered for the estimator, depending on the level of knowledge of the target. Namely, either the complete expression of the received signal is used, or simplified expressions can be instead considered, assuming some coefficients to be constant among the pairs of antennas. Furthermore, other expressions of the received signals may be introduced to analyse the impact of other model mismatches. Additionally, the discrepancies between the different signal models for the maximum likelihood estimator have been analysed numerically. Negligible differences are observed in the modified log-likelihood function between the two proposed approximations and the complete analytical model, respectively in the orthogonal and non-orthogonal case. This motivates the use of the simplest signal model, as it requires less information about the target. More specifically, it only depends on the position of the specular point, which is assumed to be located within the target.
- The impact of the bandwidth, carrier frequency, and range of the target on the modified log-likelihood function has been analysed numerically. It has shown that, in the orthogonal case, at high carrier frequencies and for close targets, the resolution of the range estimation is improved, further than the usual resolution of $c/2B$ obtained only with the delay of the waveform. Moreover, even in the non-orthogonal case, the accuracy of the estimation is enhanced, and leveraging on the phase shift in the near-field region never worsens the performance. Furthermore, the CRB has been evaluated, which confirms the observations made from the study of the impact of the system parameters. It also highlights that the target should be close w.r.t. the dimension of the antenna arrays to observe a performance improvement, even within the near-field region.

4.6.2 Future Works

The developed multistatic model for flat rectangular plates has shown some similarities with the near-field single point target model of Box 4.2. The main difference which has the most impact resides in the specific position of the reflection point. In the model of Box 4.2, the centre of the target is considered, whereas the new model of Box 4.3 considers instead the position of the specular reflection point.

Nonetheless, the flat rectangular plate is one of the worst case for the model of Box 4.2, as the flatness of the plate induces large variations of the specular point depending on the considered pair of antennas. Instead, as described in Section 4.3.3, the introduction of curvature should modify significantly the discrepancies between the two models, as a slight variation of the specular point location is associated with a large variation of the antennas positions. Therefore, such target geometries should be analysed, even if it is difficult to obtain tractable results for the positions of the specular points with these shapes.

Then, the main limitation of the SPA is the specular point, which must be located within the target for the result to be accurate. With curved geometries, this is less of an issue, owing to the latter explanation. Still, for flat geometries, the SPA may give a good approximation when the specular point is not located within the target, if the amplitude function is well approximated by a Gaussian function, as proposed in Section 3.2.1 of Chapter 3. Thus, such approximation may be investigated to enlarge the scenarios for which the proposed model holds.

Next, as discussed in Section 4.3.2, a joint estimation of the range and orientation of the target may be possible in near-field multistatic scenarios. Thus, this should be further investigated. Moreover, the development of such a model could also help evaluate the impact of a model mismatch related to the orientation of the target. Furthermore, one may also estimate the angle, and thus the position of the target. This is performed by extending the estimators of Boxes 4.7 and 4.6 to jointly estimate the range and angle.

Finally, the impact of the number of antennas and the antenna spacing should also be studied. Moreover, the impact of other model mismatches, for instance the use of a point target instead of an extended target model, and the complete analytical computation of the CRB require further analyses. Yet, such questions are answered in the PhD thesis of Guillaume Thiran. A first insight of his work on the CRB for extended target models is available in [Thi+24].

4.6.3 What is Next?

This chapter concludes the first part of this PhD thesis. Starting from the EM theory, novel RCS and received signal models have been developed, accounting for the near- and far-field propagation regions. Specific signal processing has been developed, and performance improvements have been highlighted consequently.

In the next part, ISAC systems will be analysed in a larger scale. A first glimpse of such analysis has been provided in Section 3.3.2 of Chapter 3. One may be interested in the performance of an ISAC system, interacting with other nodes modelled at the street level, or even at the city level. Leveraging on the developed radar and

communication propagation models, the power levels of the received radar echoes and communication signals can be assessed accurately. Therefore, the performance of ISAC systems may be inferred based on the radar and communication SINRs at the system of interest. Still, this requires the definition of relevant metrics to assess such performance. Additionally, they may be very sensitive to the scenario and the positions of the nodes. Thus, the stochastic geometry framework seems to be a good candidate to carry this study.

Appendix - Chapter 4

4.A Stationary Phase Approximation for a Parallel Flat Rectangular Plate

As done in Chapter 3, one can again rely on the SPA of Section 3.2.1. The amplitude and phase functions are defined as

$$\begin{aligned}
 g(y, z) &= s \left(t - \frac{r_l + r_{l'}}{c} \right) \frac{f(\theta_l) f(\theta_{l'}) \cos \phi_l \cos \theta_{l'}}{r_l r_{l'}} \\
 &\stackrel{(a)}{=} s \left(t - \frac{r_l + r_{l'}}{c} \right) \frac{R}{R^2 + (y - y_l)^2 + (z - z_l)^2} \frac{R^2 + (z - z_{l'})^2}{(R^2 + (y - y_{l'})^2 + (z - z_{l'})^2)^{\frac{3}{2}}}, \\
 \psi(y, z) &= -k(r_l + r_{l'}) \\
 &= -k \left[\sqrt{R^2 + (y - y_l)^2 + (z - z_l)^2} + \sqrt{R^2 + (y - y_{l'})^2 + (z - z_{l'})^2} \right].
 \end{aligned}$$

where (a) is obtained with the Hertzian dipole assumption, i.e. $f(\theta_l) = \cos \theta_l$. The stationary point is found by solving

$$\begin{cases} \frac{\partial \psi}{\partial y} = -2k \left[\frac{y - y_l}{\sqrt{R^2 + (y - y_l)^2 + (z - z_l)^2}} + \frac{y - y_{l'}}{\sqrt{R^2 + (y - y_{l'})^2 + (z - z_{l'})^2}} \right] = 0 \\ \frac{\partial \psi}{\partial z} = -2k \left[\frac{z - z_l}{\sqrt{R^2 + (y - y_l)^2 + (z - z_l)^2}} + \frac{z - z_{l'}}{\sqrt{R^2 + (y - y_{l'})^2 + (z - z_{l'})^2}} \right] = 0 \end{cases} \quad (4.8)$$

The solution of this system of equations is not straightforward. Nonetheless, following again the Fermat's stationary point theorem [MVM09], the stationary point $(y_{l'l}, z_{l'l})$ is the one minimising the total travelled distance, which is obtained as

$$y_{l'l} = \frac{y_l + y_{l'}}{2}, \quad z_{l'l} = \frac{z_l + z_{l'}}{2}.$$

Note that the following developments are valid if and only if the stationary point is comprised in the rectangular plate, i.e. $(y_{l'l}, z_{l'l}) \in \mathcal{S}$.

The second derivatives of the phase function are then computed as

$$\frac{\partial^2 \psi}{\partial y^2} = -k \left[\frac{R^2 + (z - z_l)^2}{(R^2 + (y - y_l)^2 + (z - z_l)^2)^{\frac{3}{2}}} + \frac{R^2 + (z - z_{l'})^2}{(R^2 + (y - y_{l'})^2 + (z - z_{l'})^2)^{\frac{3}{2}}} \right],$$

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$$\begin{aligned}\frac{\partial^2 \psi}{\partial z^2} &= -k \left[\frac{R^2 + (y - y_l)^2}{(R^2 + (y - y_l)^2 + (z - z_l)^2)^{\frac{3}{2}}} + \frac{R^2 + (y - y_{l'})^2}{(R^2 + (y - y_{l'})^2 + (z - z_{l'})^2)^{\frac{3}{2}}} \right], \\ \frac{\partial^2 \psi}{\partial y \partial z} &= k \left[\frac{(y - y_l)(z - z_l)}{(R^2 + (y - y_l)^2 + (z - z_l)^2)^{\frac{3}{2}}} + \frac{(y - y_{l'})(z - z_{l'})}{(R^2 + (y - y_{l'})^2 + (z - z_{l'})^2)^{\frac{3}{2}}} \right].\end{aligned}$$

Let us define by $r_{l'l}$ the distance between the stationary point and the transmitting or receiving antenna:

$$r_{l'l} \triangleq \sqrt{R^2 + \frac{(y_l - y_{l'})^2}{4} + \frac{(z_l - z_{l'})^2}{4}} = \frac{1}{2} \sqrt{4R^2 + (y_l - y_{l'})^2 + (z_l - z_{l'})^2}.$$

Around the stationary point, one obtains

$$\begin{aligned}\frac{\partial^2 \psi}{\partial y^2}(y_{l'l}, z_{l'l}) &= -\frac{k}{2} \frac{4R^2 + (z_l - z_{l'})^2}{r_{l'l}^3}, \\ \frac{\partial^2 \psi}{\partial z^2}(y_{l'l}, z_{l'l}) &= -\frac{k}{2} \frac{4R^2 + (y_l - y_{l'})^2}{r_{l'l}^3}, \\ \frac{\partial^2 \psi}{\partial y \partial z}(y_{l'l}, z_{l'l}) &= \frac{k}{2} \frac{(y_l - y_{l'})(z_l - z_{l'})}{r_{l'l}^3}.\end{aligned}$$

Therefore, the phase function is approximated with a Taylor series as

$$\psi(y, z) \approx -2kr_{l'l} - \frac{k}{r_{l'l}^3} (\mathbf{x} - \mathbf{x}_{l'l})^T \mathbf{Z}_{l'l} (\mathbf{x} - \mathbf{x}_{l'l}), \quad (4.9)$$

in which

$$\mathbf{x} = \begin{bmatrix} y \\ z \end{bmatrix}, \quad \mathbf{x}_{l'l} = \begin{bmatrix} y_{l'l} \\ z_{l'l} \end{bmatrix},$$

and

$$\mathbf{Z}_{l'l} = \begin{bmatrix} R^2 + \frac{(z_l - z_{l'})^2}{4} & -\frac{(y_l - y_{l'})(z_l - z_{l'})}{4} \\ -\frac{(y_l - y_{l'})(z_l - z_{l'})}{4} & R^2 + \frac{(y_l - y_{l'})^2}{4} \end{bmatrix}.$$

One may show that, given a 2×2 symmetric matrix $\mathbf{Z}$ with elements $z_{ij} \triangleq [\mathbf{Z}]_{ij}$, the eigenvalues and eigenvectors of $\mathbf{Z}$ are respectively computed as

$$\lambda_{1,2} = \frac{z_{11} + z_{22} \pm \Delta}{2},$$

with $\Delta = \sqrt{(z_{22} - z_{11})^2 + 4z_{12}^2}$, and

$$\mathbf{v}_{1,2} = \frac{1}{\sqrt{2\Delta(\Delta \pm (z_{22} - z_{11}))}} \begin{bmatrix} 2z_{12} & z_{22} - z_{11} \pm \Delta \end{bmatrix}^T.$$

Applying that results to $\mathbf{Z}_{l'l}$, one obtains

$$\Delta_{l'l} = \frac{(y_l - y_{l'})^2}{4} + \frac{(z_l - z_{l'})^2}{4},$$

leading to

$$\lambda_1 = R^2 + \frac{(y_l - y_{l'})^2}{4} + \frac{(z_l - z_{l'})^2}{4} = r_{l'l}^2, \quad \lambda_2 = R^2,$$

and

$$\begin{aligned} \mathbf{v}_1 &= \frac{1}{\sqrt{(y_l - y_{l'})^2 + (z_l - z_{l'})^2}} \begin{bmatrix} z_l - z_{l'} & -(y_l - y_{l'}) \end{bmatrix}^T, \\ \mathbf{v}_2 &= \frac{1}{\sqrt{(y_l - y_{l'})^2 + (z_l - z_{l'})^2}} \begin{bmatrix} y_l - y_{l'} & z_l - z_{l'} \end{bmatrix}^T. \end{aligned}$$

This enables to rewrite the phase function as

$$\psi(\mathbf{x}) \approx -2kr_{l'l} - \frac{k}{r_{l'l}^3} (\mathbf{x} - \mathbf{x}_{l'l})^T \mathbf{Q}_{l'l} \mathbf{D}_{l'l} \mathbf{Q}_{l'l}^T (\mathbf{x} - \mathbf{x}_{l'l}),$$

with

$$\mathbf{Q}_{l'l} = \frac{1}{\sqrt{(y_l - y_{l'})^2 + (z_l - z_{l'})^2}} \begin{bmatrix} z_l - z_{l'} & y_l - y_{l'} \\ -(y_l - y_{l'}) & z_l - z_{l'} \end{bmatrix}, \quad \mathbf{D}_{l'l} = \begin{bmatrix} r_{l'l}^2 & 0 \\ 0 & R^2 \end{bmatrix}.$$

Using the change of variable $\tilde{\mathbf{x}} = \mathbf{Q}_{l'l}^T (\mathbf{x} - \mathbf{x}_{l'l})$, the phase function is now expressed in term of its principal components as

$$\psi(\tilde{\mathbf{x}}) \approx -2kr_{l'l} - \frac{k}{r_{l'l}^3} \tilde{\mathbf{x}}^T \mathbf{D}_{l'l} \tilde{\mathbf{x}} = -2kr_{l'l} - \frac{k}{r_{l'l}} \tilde{y}^2 - \frac{kR^2}{r_{l'l}^3} \tilde{z}^2,$$

where $\tilde{y}$ and $\tilde{z}$ are the two components of $\tilde{\mathbf{x}}$.

Regarding the amplitude function, at the stationary point,

$$g(y_{l'l}, z_{l'l}) = s \left(t - \frac{2r_{l'l}}{c} \right) \frac{R [4R^2 + (z_l - z_{l'})^2]}{4r_{l'l}^5}, \quad (4.10)$$

Using the SPA of Box 3.7, the received voltage of (4.3) is finally computed as

$$u_{l'l}(t) \approx -\frac{k^2 \eta L^2 I_0}{8\pi^2} s \left(t - \frac{2r_{l'l}}{c} \right) \frac{R [4R^2 + (z_l - z_{l'})^2]}{4r_{l'l}^5} e^{-j2kr_{l'l}} A_{l'l}, \quad (4.11)$$

with

$$A_{l'l} = \iint_S e^{-j \frac{k}{4r_{l'l}^3} (\mathbf{x} - \mathbf{x}_{l'l})^T \mathbf{Z}_{l'l} (\mathbf{x} - \mathbf{x}_{l'l})} dydz.$$

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The integral of the phase on S can be developed following the principal components of ψ as

$$\begin{aligned} A_{l'l} &= \iint_{\tilde{S}} e^{-j\frac{k}{r_{l'l}}\tilde{y}^2} e^{-j\frac{kR^2}{r_{l'l}^3}\tilde{z}^2} d\tilde{y}d\tilde{z} \\ &\approx \sqrt{-\frac{j\pi r_{l'l}}{k}} \sqrt{-\frac{j\pi k r_{l'l}^3}{R^2}} = -\frac{j\pi r_{l'l}^2}{kR}. \end{aligned} \quad (4.12)$$

where $\tilde{S}$ is the transformation of S in the principal components space, leading to

$$u_{l'l}(t) \approx \frac{j k \eta L^2 I_0}{8\pi} \frac{4R^2 + (z_l - z_{l'})^2}{4r_{l'l}^3} e^{-j2kr_{l'l}} s\left(t - \frac{2r_{l'l}}{c}\right).$$

However, the approximation made in (4.12) holds only if the amplitude of the function g is slowly varying in the integration domain, and if the rectangular plate is large enough to contain multiple Fresnel zones [And64]. Owing to the multistatic configuration, the stationary point can be located anywhere on the target, such that the second condition is not fulfilled in many cases. This issue is circumvented by considering a linear antenna array distributed along the z -axis.

4.B Received Signal with a Linear Antenna Array

In the particular case of a linear antenna array distributed along the z -axis, $y_l = y_{l'} = 0$ for every transmitting and receiving antenna. Thus, the stationary point $(y_{l'l}, z_{l'l})$ is obtained as

$$y_{l'l} = 0, \quad z_{l'l} = \frac{z_l + z_{l'}}{2}.$$

The distance $r_{l'l}$ between the stationary point and the transmitting or receiving antenna is then computed as

$$r_{l'l} = \sqrt{R^2 + \frac{(z_l - z_{l'})^2}{4}} = \frac{1}{2} \sqrt{4R^2 + (z_l - z_{l'})^2}.$$

Next, the phase function of (4.9) is rewritten as

$$\psi(y, z) \approx -2kr_{l'l} - \frac{k}{r_{l'l}} y^2 - \frac{kR^2}{r_{l'l}^3} z^2.$$

In that particular case, the principal components of the phase functions are aligned with the y and z axes. Regarding the amplitude function of (4.10), it is rewritten as

$$g(0, z_{l'l}) = \frac{R}{r_{l'l}^3}.$$

Using the SPA of Box 3.7, the received voltage of (4.12) is finally computed as

$$u_{l'l}(t) \approx -\frac{k^2 \eta L^2 I_0}{8\pi^2} \frac{R}{r_{l'l}^3} e^{-j2kr_{l'l}} A_{l'l} s\left(t - \frac{r_l + r_{l'}}{c}\right), \quad (4.13)$$

with

$$\begin{aligned} A_{l'l} &= \int_{-\frac{D_y}{2}}^{\frac{D_y}{2}} e^{-j\frac{k}{r_{l'l}} y^2} dy \int_{-\frac{D_z}{2} - z_{l'l}}^{\frac{D_z}{2} - z_{l'l}} e^{-j\frac{kR^2}{r_{l'l}^3} z^2} dz \\ &= \frac{\pi r_{l'l}^2}{kR} F^*\left(\sqrt{\frac{2k}{r_{l'l}}} \frac{D_y}{2}\right) \left[F^*\left(\sqrt{\frac{2kR^2}{r_{l'l}^3}} \left(\frac{D_z}{2} - z_{l'l}\right)\right) + F^*\left(\sqrt{\frac{2kR^2}{r_{l'l}^3}} \left(\frac{D_z}{2} + z_{l'l}\right)\right) \right], \end{aligned} \quad (4.14)$$

where the last equality is obtained using (3.7), and $F(x) = -F(-x)$. Inserting that result in (4.13), the received signal is finally rewritten as

$$u_{l'l}(t) \approx -\frac{k\eta L^2 I_0}{8\pi} \frac{e^{-j2kr_{l'l}}}{r_{l'l}} \eta_{l'l} s\left(t - \frac{r_l + r_{l'}}{c}\right),$$

with $\eta_{l'l} = kRA_{l'l}/\pi r_{l'l}^2$.

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Part II

Large-Scale Analysis of Integrated Sensing and Communication Networks

Stochastic Geometry Framework

5

This chapter introduces the stochastic geometry framework, and is based on my Master thesis [DOV20]. Related reference books are [BB10a; BB10b].

THE *Stochastic Geometry* (SG) Framework is a branch of mathematics which provides mathematical models and appropriate statistical methods to study and analyse random spatial patterns based on Point Processes (PPs), or the random shape theory [Di 16]. This tool is well suited for the analysis of large scale wireless networks since they can be viewed as a collection of nodes, located randomly in space. These nodes represent transmitters or receivers of different types depending on the considered network topology, as UEs, BSs, RSUs or vehicles. It enables estimation of the average performance for radar and communication networks through closed-form (or quasi closed-form) analytical expressions. The most common alternative to obtain insights about the performance of networks is Monte-Carlo (MC) simulation. They do not require any restrictive assumption, but they are very time and computationally expensive. Nonetheless, they provide an estimation of the complete statistics of the network.

Using SG, one expects to obtain simple analytical expressions, such that the impact of some key parameters of the network can be extracted from the SG analytic expressions, to draw conclusions and optimise the network, which is not possible with MC simulations [WOV17]. However, depending on the system model and the considered performance metric, the complexity of the SG expressions can rise significantly, making the analysis of the network performance difficult. This can be leveraged by the use of additional assumptions, at the price of a system model which reflects less accurately real-world applications. Therefore, the relevance of the SG framework must also be analysed in comparison to MC simulations or other alternatives in terms of complexity.

In this chapter, the SG framework is presented, and useful mathematical tools and models are detailed. First, the point process theory is introduced, and the most common point processes used to model in wireless networks are presented. Then, the properties of the Poisson point processes are detailed. Finally, the Poisson line process is introduced.

5.1 Point Process Theory

A **PP** Φ is a countable random collection of points that resides in some measurable space, typically $\mathbb{R}^n$. It is usually used to model the positions of network nodes, as **UEs**, **BSs**, **RSUs**, vehicles or other targets. It is uniquely defined in Box 5.1 as a set of **Random Variables** (RVs) $\mathbf{X}_i \in \mathbb{R}^n$ describing the coordinates of the nodes in the **PP**, or through its *random counting measure* Ψ_Φ , a function which returns the number of points of Φ in any subset $\mathcal{A} \subset \mathbb{R}^n$.

5.1: Point Process and Random Counting Measure

$$\Phi \triangleq \{\mathbf{X}_i, i \in \mathbb{N}\} \quad \leftrightarrow \quad \Psi_\Phi(\mathcal{A}) \triangleq \sum_{\mathbf{X}_i \in \mathcal{A}} \mathbb{1}(\mathbf{X}_i \in \mathcal{A})$$

Three important statistical measures of **PPs** are:

- The *intensity measure*, also known as *expectation measure* or *mean measure*, is the mean of the random counting measure on a given subset $\mathcal{A} \subset \mathbb{R}^n$. It is defined in Box 5.2.

5.2: Intensity Measure

$$\Lambda_\Phi(\mathcal{A}) \triangleq \mathbb{E}_{\Psi_\Phi} [\Psi_\Phi(\mathcal{A})]$$

- The *void probability* is the probability that a given subset $\mathcal{A} \in \mathbb{R}^n$ does not contain any points of the **PP**. It is defined in Box 5.3.

5.3: Void Probability

$$V_\Phi(\mathcal{A}) \triangleq \mathbb{P}_{\Psi_\Phi} (\Psi_\Phi(\mathcal{A}) = 0)$$

- The **Probability Generating Functional** (PGFL) of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}^+$ is the mean of the product of f evaluated at every point of the **PP**. It is defined in Box 5.4.

The **PGFL** is especially useful to compute the *Laplace Transform* (LT) of a **RV** Z which can be expressed as $Z = \sum_{\mathbf{X}_i \in \Phi} f(\mathbf{X}_i)$:

$$\mathcal{L}_Z(s) \triangleq \mathbb{E}_\Phi [e^{-sZ}] = \mathbb{E}_\Phi \left[\prod_{\mathbf{X}_i \in \Phi} e^{-sf(\mathbf{X}_i)} \right] = \Pi_\Phi(e^{-sf}). \quad (5.1)$$

5.4: Probability Generating Functional

$$\Pi_{\Phi}(f) \triangleq \mathbb{E}_{\Phi} \left[\prod_{\mathbf{X}_i \in \Phi} f(\mathbf{X}_i) \right]$$

5.1.1 Point Processes for Wireless Networks

In the context of wireless networks, some PPs are especially relevant to model the topology of networks [Hma+21]. Among them, popular options are the binomial point process, the Poisson point process, the Poisson cluster process, and the hard-core point process.

5.1.1.1 Binomial Point Process

A *Binomial Point Process* (BPP) on a bounded set $\mathcal{A} \subset \mathbb{R}^n$ is a PP such that a fixed number N of points are distributed uniformly in $\mathcal{A}$. Consequently, the number of points comprised in any subset $\mathcal{B} \subset \mathcal{A}$ follows a Binomial distribution:

$$\mathbb{P}_{\Psi_{\Phi}}(\Psi_{\Phi}(\mathcal{B}) = k) = \binom{N}{k} \left(\frac{|\mathcal{B}|}{|\mathcal{A}|} \right)^k \left(1 - \frac{|\mathcal{B}|}{|\mathcal{A}|} \right)^{N-k}, \quad (5.2)$$

where $|\cdot|$ denotes the Lebesgue measure, corresponding to the surface of the subset in $\mathbb{R}^2$.

5.1.1.2 Poisson Point Process

A *Poisson Point Process* (PPP) in $\mathbb{R}^n$ is a PP such that

1. The numbers of points in any disjoint subsets $\mathcal{A}_i \subset \mathbb{R}^n$ are independent.
2. The number of points in any subset $\mathcal{A}$ follows a Poisson distribution:

$$\mathbb{P}_{\Psi_{\Phi}}(\Psi_{\Phi}(\mathcal{A}) = k) = \frac{(\Lambda_{\Phi}(\mathcal{A}))^k}{k!} e^{-\Lambda_{\Phi}(\mathcal{A})}.$$

The PPP is an extension of the BPP, as the number of points in the subset is not fixed, but follows a Poisson distribution. It is completely characterised by its intensity function Λ . However, it can also be characterised by its *density* or *intensity function* λ . The density is defined in Box 5.5 as the intensity measure of the PPP for an infinitesimal volume $d\mathbf{x} \subset \mathbb{R}^n$ located at $\mathbf{x} \in \mathbb{R}^n$. The product $\lambda(\mathbf{x})d\mathbf{x}$ can be interpreted as the infinitesimal probability that there is a point of the PPP in an infinitesimal area $d\mathbf{x}$ located at $\mathbf{x}$.

A *Homogeneous Poisson Point Process* (HPPP) is a particular case of the PPP for which the density is uniform, such that its intensity measure is expressed as $\Lambda(\mathcal{A}) =$

5.5: Density of a Poisson Point Process

$$\Lambda(d\mathbf{x}) \triangleq \lambda(\mathbf{x})d\mathbf{x} \quad \leftrightarrow \quad \Lambda(\mathcal{A}) \triangleq \int_{\mathcal{A}} \lambda(\mathbf{x})d\mathbf{x}$$

$\lambda|\mathcal{A}|$ for any subset $\mathcal{A} \subset \mathbb{R}^n$. For the **HPPP**, the density is therefore also defined as the mean number of points in $\mathcal{A}$. In opposition with **HPPPs**, **PPPs** which are not homogeneous are often referred as *inhomogeneous PPPs*. Examples of **HPPPs** are shown in Figures 5.1 and 5.2.

5.1.1.3 Poisson Cluster Process

A *Poisson Cluster Process* (PCP) is a **PP** composed of clusters of points, centred at the positions of parent points distributed following a **HPPP**. This is useful to model scenarios where points are susceptible to be grouped in limited volumes, for instance during particular events, e.g. hotspots [GKD19], or in disaster scenarios [Hay+18].

In $\mathbb{R}^2$, two particular **PCPs** are the *Matérn Cluster Process* (MCP) and the *Thomas Cluster Process* (TCP) [Hay+18]. For the first, a fixed or random number of points is uniformly distributed in circular areas, located around each parent point. For the second, these points are normally distributed around each parent point. These **PPs** are illustrated in Figure 5.1.

5.1.1.4 Hard-Core Point Process

A *Hard-Core Point Process* (HCPP) with *repulsive distance* $d \in \mathbb{R}^n$ is a **PP** for which all the points are mutually separated by a distance greater than d . It enables to avoid unrealistic scenarios in which nodes are close to each other, as it is for instance unlikely for **BSs** in wireless networks.

Two particular cases of **HCPPs** are the *Matérn HCPP of type I and II* [Hae11]. For both types, the points are first distributed following a **HPPP**. Then, in the first case, the points mutually located at a distance lower than d are all removed. In the second case, a random mark is assigned to each point of the **HPPP**, and a point is deleted if there exists another point with a smaller mark located at a distance closer than d . These two **HCPPs** are illustrated in Figure 5.2.

Unfortunately, with **HCPPs**, the mathematical developments are often intractable, owing to the **PGFL** of such **PPs** which does not have a closed-form expression. Nonetheless, following [Hae11], the **PPP** is a good approximation for **HCPPs** if the densities of the **PPPs** are adapted accordingly.

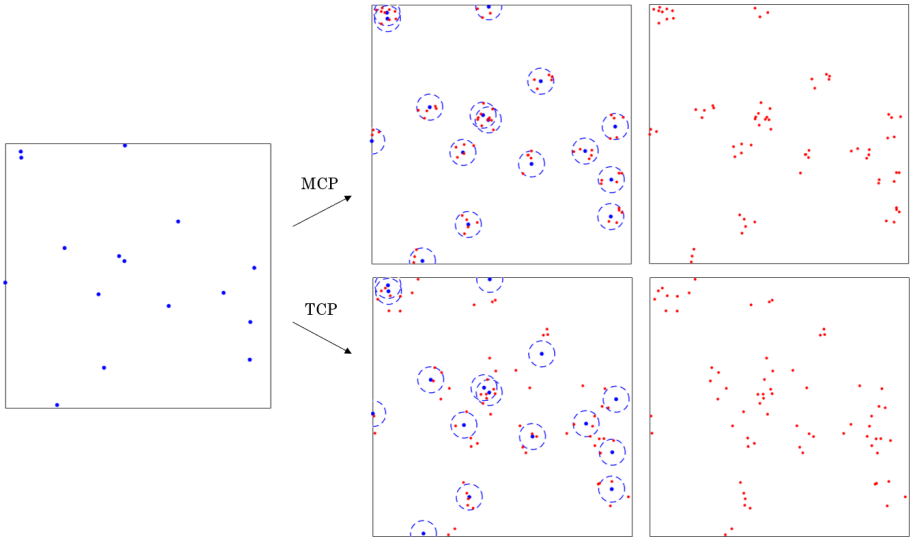


Fig. 5.1 Generation of a **MCP** and a **TCP** in $\mathbb{R}^2$. The blue points represent the parent **HPPP**. The dashed circles show the radius of the clusters (resp. the standard deviation of the clusters distribution) for the **MCP** (resp. **TCP**). The red points represent the **MCP** and **TCP** generated around the parent points.

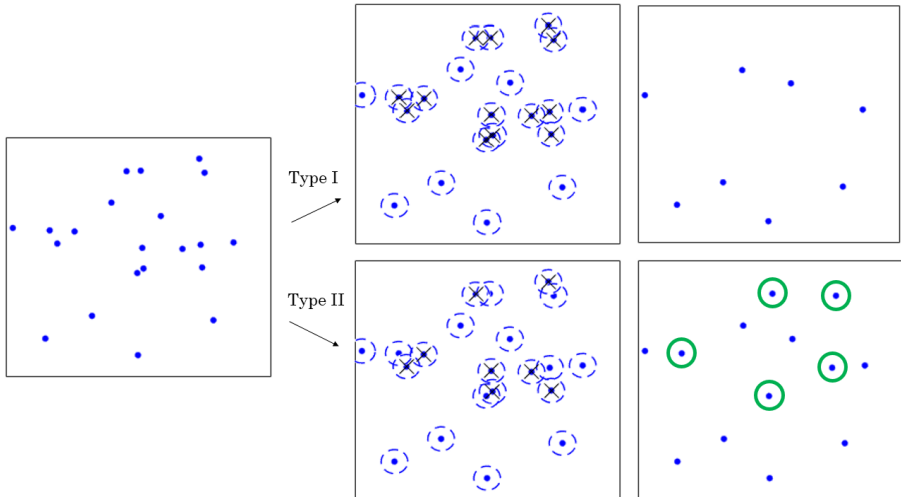


Fig. 5.2 Generation of Matérn **HCPPs** of type I and II in $\mathbb{R}^2$ from an **HPPP**. The dashed circles illustrate the repulsive distance around each point of the parent **HPPP**. The green circles highlight the points which are kept in the Matérn **HCPP** of type II, but removed in the Matérn **HCPP** of type I.

5.1.2 Palm Theory

The *Palm theory* formalises the notion of conditional distribution for **PPs**, given some points at arbitrary locations. Two particular distributions of this theory are the Palm distribution and the reduced Palm distribution. On the one hand, the *Palm distribution* is the conditional distribution of the **PP** given that one point is located at an arbitrary location. On the other hand, the *reduced Palm distribution* is the Palm distribution of the **PP**, for which the arbitrary point is assumed to be excluded from the distribution of the **PP**.

5.2 Properties of Poisson Point Processes

The **PPP**, introduced in Section 5.1.1.2, is the most popular **PP** to model the spatial distribution of wireless networks nodes, owing to its simplicity and mathematical tractability [Hma+21]. Those provide quite realistic distributions of the nodes compared to real networks, and it has a lot of useful properties, which lead to tractable analytical results. Some of these properties are summarised in this section.

5.2.1 Void Probability

The void probability of a **PPP** is given in Box 5.6. It is directly obtained from (5.2) by setting $k = 0$. For the particular case of **HPPPs** with density λ , it can be further simplified as

$$V_{\Phi}(\mathcal{A}) = e^{-\lambda|\mathcal{A}|}.$$

5.6: Void Probability of a Poisson Point Process

$$V_{\Phi}(\mathcal{A}) = e^{-\Lambda_{\Phi}(\mathcal{A})}$$

5.2.2 Campbell's Theorem

The Campbell's theorem is detailed in Box 5.7. For the particular case of **HPPPs** with density λ , it can be further simplified as

$$\mathbb{E}_{\Phi} \left[\sum_{\mathbf{X}_i \in \Phi} f(\mathbf{X}_i) \right] = \lambda \int_{\mathbb{R}^n} f(\mathbf{x}) \, d\mathbf{x}.$$

5.2.3 Probability Generating Functional

The **PGFL** of a **PPP** is detailed in Box 5.8.

5.7: Campbell's Theorem

Let us consider a function $f : \mathbb{R}^n \rightarrow \mathbb{R}^+$, and a PPP $\Phi \in \mathbb{R}^n$ of intensity measure Λ_Φ . The expectation of the sum of f , evaluated at every point of the PPP, is given by

$$\mathbb{E}_\Phi \left[\sum_{\mathbf{X}_i \in \Phi} f(\mathbf{X}_i) \right] = \int_{\mathbb{R}^n} f(\mathbf{x}) \Lambda_\Phi(d\mathbf{x}).$$

5.8: Probability Generating Functional of a Poisson Point Process

The PGFL of a PPP $\Phi \in \mathbb{R}^n$ with intensity measure Λ_Φ of a function $f : \mathbb{R}^n \rightarrow [0, 1]$ is given by

$$\Pi_\Phi(f) = \exp \left(- \int_{\mathbb{R}^n} (1 - f(\mathbf{x})) \Lambda_\Phi(d\mathbf{x}) \right).$$

For the particular case of HPPPs with density λ , it can be further simplified as

$$\Pi_\Phi(f) = \exp \left(-\lambda \int_{\mathbb{R}^n} (1 - f(\mathbf{x})) d\mathbf{x} \right).$$

This also implies that, with an HPPP, the LT of a RV Z which can be expressed as $Z = \sum_{\mathbf{X}_i \in \Phi} f(\mathbf{X}_i)$ is given by

$$\mathcal{L}_Z(s) = \exp \left(-\lambda \int_{\mathbb{R}^n} (1 - e^{-sf(\mathbf{x})}) d\mathbf{x} \right). \quad (5.3)$$

5.2.4 Displacement Theorem

The *displacement theorem* for PPPs is detailed in Box 5.9.

5.9: Displacement Theorem

The transformation of a PPP $\Phi \in \mathbb{R}^n$ with intensity measure Λ_Φ by a probability kernel $p : \mathbb{R}^n \times \mathcal{A} \subset \mathbb{R}^{n'} \rightarrow [0, 1]$ is a PPP $\Phi' \in \mathbb{R}^{n'}$ with intensity measure $\Lambda_{\Phi'}$, such that

$$\Lambda_{\Phi'}(\mathcal{A}) = \int_{\mathbb{R}^n} p(\mathbf{x}, \mathcal{A}) \Lambda_\Phi(d\mathbf{x}).$$

5 | Stochastic Geometry Framework

For the particular case of **HPPPs** with density λ , it can be further simplified as

$$\Lambda_{\Phi'}(\mathcal{A}) = \lambda \int_{\mathbb{R}^n} p(\mathbf{x}, \mathcal{A}) \, d\mathbf{x}.$$

When expressed in terms of density instead of intensity measure, the displacement theorem is rewritten as Box 5.10. This formulation is more easily interpreted, as it directly involves the probability density function of the displacement vector between the two spaces, for every point of the initial space.

5.10: Displacement Theorem (Densities)

The transformation of a **PPP** $\Phi \in \mathbb{R}^n$ with density λ following the probability density function of the displacement vector $f_{\mathbf{x}'|\mathbf{x}} : \mathbb{R}^{n'} \times \mathbb{R}^n \rightarrow [0, 1]$ from $\mathbf{x} \in \mathbb{R}^n$ to $\mathbf{x}' \in \mathbb{R}^{n'}$ is a **PPP** $\Phi' \in \mathbb{R}^{n'}$ with density λ' , such that

$$\lambda'(\mathbf{x}') = \int_{\mathbb{R}^n} f_{\mathbf{x}'|\mathbf{x}}(\mathbf{x}'|\mathbf{x}) \lambda(\mathbf{x}) \, d\mathbf{x}.$$

The deterministic displacement by a bijection $g : \mathbb{R}^n \rightarrow \mathbb{R}^{n'}$, also named *mapping*, is a particular case of the displacement theorem where a point $\mathbf{x} \in \mathbb{R}^n$ is transformed into a point $g(\mathbf{x}) \in \mathbb{R}^{n'}$ with probability 1. In that case, the resulting **PP** is a **PPP** $\Phi' \in \mathbb{R}^{n'}$ with intensity measure $\Lambda_{\Phi'}$ expressed as

$$\Lambda_{\Phi'}(\mathcal{A}') = \Lambda_{\Phi}(g^{-1}(\mathcal{A}')), \quad \mathcal{A}' \subset \mathbb{R}^{n'}.$$

Thanks to that property, it can be shown that any **HPPP** is *stationary* and *isotropic* (i.e. *motion-invariant*), namely that any translation or rotation of an **HPPP** results in another **HPPP** with the same density [Di 16].

5.2.5 Thinning Theorem

The *thinning* operation of a **PPP** $\Phi \in \mathbb{R}^n$ is an operation for which each point $\mathbf{X}_i \in \Phi$ is kept in the **PP** with a probability $p : \mathbb{R}^n \rightarrow [0, 1]$. The thinning theorem is detailed in Box 5.11. Note that the thinning theorem is also a consequence of the displacement theorem of Box 5.9 for which the points of the hyperplane $\mathbb{R}^n$ are displaced independently out of the hyperplane in $\mathbb{R}^{n+1}$ with probability $1 - p$. The generated **PPP** Φ_p is then composed of the points which are still in the initial hyperplane.

5.2.6 Superposition Theorem

The *superposition* operation of k **PPPs** $\Phi_i \in \mathbb{R}^n$, $i = 1, \dots, k$ is an operation which combines all the **PPPs** to form the **PP** $\Phi = \bigcup_{i=1}^k \Phi_i$. The superposition theorem is detailed in Box 5.12.

5.11: Thinning Theorem

The thinning operation of a PPP $\Phi \in \mathbb{R}^n$ with intensity measure Λ_Φ by a probability $p : \mathbb{R}^n \rightarrow [0, 1]$ generates a PPP $\Phi_p \in \mathbb{R}^n$ with intensity measure Λ_{Φ_p} , such that

$$\Lambda_{\Phi_p}(\mathcal{A}) = \int_{\mathcal{A}} p(\mathbf{x}) \Lambda_\Phi(d\mathbf{x}), \quad \mathcal{A} \in \mathbb{R}^n.$$

Furthermore, the PP $\Phi_{1-p} \in \mathbb{R}^n$ composed of the points which have been independently removed from Φ with probability $1-p$ is also a PPP, independent of Φ_p , with intensity measure $\Lambda_{\Phi_{1-p}}$ expressed as

$$\Lambda_{\Phi_{1-p}}(\mathcal{A}) = \int_{\mathcal{A}} (1-p(\mathbf{x})) \Lambda_\Phi(d\mathbf{x}), \quad \mathcal{A} \in \mathbb{R}^n.$$

5.12: Superposition Theorem

The superposition operation of K PPPs $\{\Phi_k\} \in \mathbb{R}^n$, $k = 1, \dots, K$ with intensity measures Λ_{Φ_k} generates a new PPP $\Phi = \cup_{i=1}^K \Phi_i$ with intensity measure Λ_Φ expressed as

$$\Lambda_\Phi(\mathcal{A}) = \sum_{k=1}^K \Lambda_{\Phi_k}(\mathcal{A}), \quad \mathcal{A} \in \mathbb{R}^n.$$

5.2.7 Slivnyak-Mecke's Theorem

The *Slivnyak-Mecke's theorem*, based on the Palm theory introduced in Section 5.1.2, is detailed in Box 5.13.

5.13: Slivnyak-Mecke's Theorem

The reduced Palm distribution of a PPP $\Phi \in \mathbb{R}^n$ is equal to its original distribution.

This theorem means that a PPP $\Phi \in \mathbb{R}^n$ keeps the same distribution, even if one of its points is removed. It is a consequence of the independence between the number of points of the PPP in any disjoint set: removing the point $\mathbf{X}_i \in \Phi$ is equivalent to suppress from the PPP an infinitesimally small ball $\mathcal{B}(\mathbf{X}_i, \epsilon) \in \mathbb{R}^n$ centred at $\mathbf{X}_i$ with radius $\epsilon \rightarrow 0$. This does not affect the distribution of the points in $\mathbb{R}^n \setminus \mathcal{B}(\mathbf{X}_i, \epsilon)$ by definition of the PPP, and therefore does not affect the initial PPP as $\epsilon \rightarrow 0$ [AGD16].

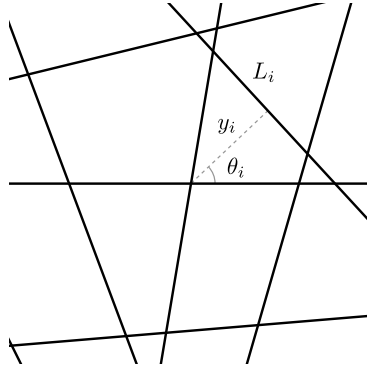


Fig. 5.3 Illustration of a PLP in $\mathbb{R}^2$.

5.3 Poisson Line Process

The *Poisson Line Process* (PLP) generalises the two-dimensional PPP by distributing lines in $\mathbb{R}^2$ instead of points [Dem23]. These lines are fully characterised with two parameters:

1. the radial distance y_i from the origin;
2. the angle θ_i measured in the counter-clockwise direction between an axis of reference (i.e. the horizontal axis), and the line formed by the projection of the origin on the line of the PLP.

The PLP can therefore be generated from the following process:

$$\Phi = \{(y_i, \theta_i) \in \mathbb{R}^+ \times [0, 2\pi[, i \in \mathbb{N}\}.$$

The process Φ can represent any planar line process, as the radial distances and angles are generated arbitrarily. The PLP is the particular case for which the radial distances are generated following a PPP in $\mathbb{R}^+$. An example of PLP is illustrated in Figure 5.3.

The Cox PLP is obtained by associating a PPP with each line of the PLP, modelling node locations along the lines. Regarding the angles θ_i , they are usually distributed uniformly in $[0, 2\pi[$. If the angles are drawn from the discrete set of angles $\{0, \pi/2, \pi, 3\pi/2\}$ with a uniform probability, one obtains the particular case of the *Manhattan PLP*, containing only vertical and horizontal lines. Note that the Manhattan PLP can also be characterised with two independent PPPs in $\mathbb{R}$. Each PPP respectively represents the coordinates of the crossings of the lines with the horizontal and vertical axis.

Cox PLPs are especially useful to model stochastically a city in a large scale. Contrariwise to the two-dimensional PPP, the Cox PLP conditions the node locations by the fact that they belong to a given line. One may also use Manhattan PLPs depending

Abstraction of Cities with Poisson Line Processes

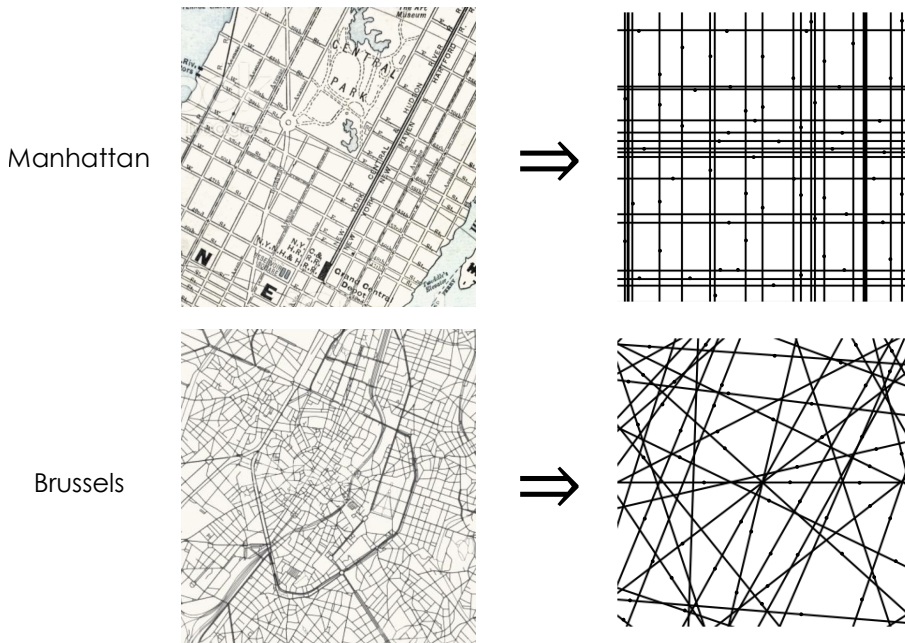


Fig. 5.4 Abstraction of Brussels and Manhattan with Cox PLPs and Manhattan PLPs.

on the layout of the city. Figure 5.4 illustrates these PLPs, used as abstractions for the cities of Manhattan and Brussels.

Joint Performance Metrics for Integrated Sensing and Communication Networks

6

This chapter is a work carried out with the help of Charles Wiame, which I thank warmly. I also thank Simon Demey for providing ray-tracing simulation results. Most of the content is inspired from [De +24a] and [De +24c].

THE SG framework, presented in Chapter 5, has been widely used to evaluate the performance of wireless networks at a large-scale level. For that purpose, relevant performance metrics are required, enabling to assess accurately the performance of the network, and to perform optimisation of the parameters of the network. Such metrics are defined in the literature for communication and sensing applications. Still, since scenarios involving simultaneously both functions are more and more analysed for future networks, a joint evaluation of the radar and communication performance is required.

Therefore, this chapter focuses on the definition of joint performance metrics in full-duplex ISAC scenarios. In such scenario, the analysis of the interactions between both functions is critical for an accurate performance evaluation. First, Section 6.1 details usual radar and communication metrics from the literature, within the SG framework. Then, novel joint metrics are proposed, and analytical expressions are obtained in Section 6.2 for a vehicular scenario. Finally, results obtained with the SG framework are compared to RT simulations in Section 6.3, in order to assess the relevancy of the developed analytical expressions.

6.1 Performance Metrics in Stochastic Geometry

As described in Section 1.4 of Chapter 1, numerous performance metrics have been defined in the literature, both for sensing and communication. In this section, common metrics are presented for both functions within the SG framework. Additionally, new joint metrics are proposed for JCAS applications.

6.1.1 Communication Metrics

For communication applications, several performance metrics which are evaluated include the coverage probability, the spectral efficiency, the energy efficiency and the average exposure. Most of them are functions of the SINR at the receiver. Many papers analysing wireless communication scenarios through these performance metrics are presented in [Hma+21].

6.1.1.1 Coverage and Outage Probability

The *coverage probability* (resp. *outage probability*) is defined as the probability that the SINR χ at the communication receiver is higher (resp. lower) or equal than a predefined SINR threshold η . The coverage and outage probability are therefore respectively equal to **Complementary Cumulative Distribution Function (CCDF)** and **Cumulative Distribution Function (CDF)** of the SINR. Expressions of these metrics are provided in Box 6.1.

6.1: Coverage and Outage Probability

Coverage probability:	$C(\eta) \triangleq \mathbb{P}_{\chi}(\chi \geq \eta)$
Outage probability:	$O(\eta) \triangleq \mathbb{P}_{\chi}(\chi < \eta) = 1 - C(\eta)$

The coverage probability is directly related to the rate and spectral efficiency, as it will be detailed in Section 6.1.1.2. It is also related to the BER, as it is also a function of the SINR. For instance, to achieve a BER of 10^{-3} with a QPSK modulation at a given probability, the SINR threshold should be set to at least 9.8 dB.

6.1.1.2 Rate and Spectral Efficiency

The *rate* or *capacity* is defined as the maximum number of bits per second that the communication receiver is able to decode without any error in a given bandwidth B . Similarly, the *spectral efficiency* is defined as the rate divided by the bandwidth. Denoting by χ the SINR of the communication receiver, the CCDFs of these quantities are given in Box 6.2.

6.2: Rate and Spectral Efficiency

Rate:	$\mathcal{R}(\tau) \triangleq \mathbb{P}_{\chi}(B \log_2(1 + \chi) \geq \tau)$
Spectral efficiency:	$\mathcal{R}_e(\tau) \triangleq \mathbb{P}_{\chi}(\log_2(1 + \chi) \geq \tau)$

Both metrics can be computed from the coverage (or outage) probability, and vice

versa:

$$\begin{aligned}\mathcal{R}(\tau) &= \mathbb{P}_\chi (B \log_2(1 + \chi) \geq \tau) = \mathbb{P}_\chi \left(\chi \geq 2^{\frac{\tau}{B}} - 1 \right) = C \left(2^{\frac{\tau}{B}} - 1 \right), \\ \mathcal{R}_e(\tau) &= \mathbb{P}_\chi (\log_2(1 + \chi) \geq \tau) = \mathbb{P}_\chi (\chi \geq 2^\tau - 1) = C(2^\tau - 1).\end{aligned}\quad (6.1)$$

In other words, requiring the SINR to be higher than a given threshold η with a given probability is equivalent to requiring the rate to be higher than a threshold $\tau = B \log_2(1 + \eta)$ with the same probability, and vice versa.

Under standard modelling assumptions [Di +18], one can also compute the average rate and spectral efficiency, following Box 6.3.

6.3: Average Rate and Spectral Efficiency

Average rate:	$\bar{\mathcal{R}} \triangleq B \mathbb{E}_\chi [\log_2(1 + \chi)]$
Average spectral efficiency:	$\bar{\mathcal{R}}_e \triangleq \mathbb{E}_\chi [\log_2(1 + \chi)]$

Again, both average metrics can be computed by integrating the coverage (or outage) probability. For the average spectral efficiency,

$$\bar{\mathcal{R}}_e = \int_0^\infty \mathbb{P}_\chi (\log_2(1 + \chi) \geq \tau) d\tau = \int_0^\infty C(2^\tau - 1) d\tau = \frac{1}{\ln 2} \int_0^\infty \frac{C(\eta)}{1 + \eta} d\eta.$$

Yet, in some works, another definition of the average rate (or spectral efficiency), also named *potential throughput*, is considered, as defined in Box 6.4. With the definition of Box 6.3, an average over the possible realisations of the SINR is computed. Therefore, it assumes that the channel statistics remain unchanged (i.e., stationarity is assumed) for every realisation of the network, and ergodism (i.e. short channel coherence time) is required. Contrariwise, in Box 6.4, no average is computed. Nonetheless, the rate which would be achieved if a SINR threshold η is required to decode successfully the signal is pondered by coverage probability at this threshold.

6.4: Average Rate and Spectral Efficiency (Potential Throughput)

Average rate:	$\tilde{\mathcal{R}}(\eta) \triangleq B \log_2(1 + \eta) C(\eta)$
Average spectral efficiency:	$\tilde{\mathcal{R}}_e(\eta) \triangleq \log_2(1 + \eta) C(\eta)$

6.1.1.3 Energy Efficiency

In order to evaluate the rate with regard to the consumed power, the *energy efficiency* is defined in Box 6.5 as the rate divided by the total power consumption P of the

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network. One can also compute the average energy efficiency as Box 6.6, following either the definition of Box 6.3 or Box 6.4.

6.5: Energy Efficiency

$$\mathcal{E}(\nu) \triangleq \mathbb{P}_{\chi} \left(\frac{B}{P} \log_2(1 + \chi) \geq \nu \right)$$

6.6: Average Energy Efficiency

Following Box 6.3: $\bar{\mathcal{E}} \triangleq \frac{B}{P} \mathbb{E}_{\chi} [\log_2(1 + \chi)]$

Following Box 6.4: $\tilde{\mathcal{E}}(\eta) \triangleq \frac{B}{P} \log_2(1 + \eta) C(\eta)$

Some works define the average energy efficiency based on the average spectral efficiency and the total power consumption per unit area [Moz+16; QCK11; CQK12]. Nonetheless, the total noise power is a function of the bandwidth, making these definitions less relevant.

6.1.1.4 Exposure

The *exposure* is defined as the probability for the total power to exceed a given power threshold γ at a given location. If this location corresponds to the position of a communication receiver, one can distinguish the power S of the desired signal, and the interference I from other transmitters. Otherwise, there is no desired signal, and every source is considered as an interference. This is summarised in Box 6.7. One can also compute the average exposure as Box 6.8.

6.7: Exposure

With a desired signal: $\mathcal{U}(\gamma) \triangleq \mathbb{P}_{S,I} (S + I \geq \gamma)$

With interference only: $\mathcal{U}(\gamma) \triangleq \mathbb{P}_I (I \geq \gamma)$

6.1.2 Sensing Metrics

Sensing performance metrics are mostly derived from communication performance metrics. They include the detection probability, the false alarm probability, the success probability, and the estimation rate. They are either functions of the received powers,

6.8: Average Exposure

With a desired signal: $\overline{U} \triangleq \mathbb{E}_{S,I} [S + I]$

With interference only: $\overline{U} \triangleq \mathbb{E}_I [I]$

or [SINR](#) at the radar receiver. Section 6.1.2.1 provides some examples of radar scenarios analysed in the literature.

6.1.2.1 Sensing Metrics in the Stochastic Geometry Literature

In the [SG](#) literature, [[MSP18](#)] analyses the [CDF](#) of the interference with a population of pulsed-radar devices, [[RSG20](#); [RSG21](#); [RG22a](#)] consider the radar performance in a cluttered environment, and [[PH18](#)] evaluates the detection probability when multiple obstacles are distributed around the radar systems. Many automotive scenarios have also been studied. Namely, [[Fan+20a](#)] compares the performance obtained with different random [RCS](#) models. To model the position of the vehicles, instead of using [PPP](#), [[Al+18](#)] proposes to use a [1D](#) lattice, and [[MSO20](#)] uses [2D](#) Matérn [HCPPs](#). In [[Fan+20b](#)], an arbitrary number of lanes in both directions is considered, and the reflected interference from vehicles on neighbouring lanes is taken into account. In [[Chu+20](#)], a multiple lane scenario is also considered, with front- and side-mounted radars with directional antenna patterns. To obtain a fine-grained analysis, [[Gha+21](#); [GKG22](#)] evaluate the meta distribution of the [SINR](#), which enables to analyse the reliability of the detection at each individual vehicle.

6.1.2.2 Detection and False Alarm Probability

The *detection probability* is defined as the probability that the total received power at the radar receiver, i.e. the radar echoes U from the targets and the interference I from other transmitters, is higher than a predefined threshold γ , given the presence of targets. Contrariwise, the *false alarm probability* is defined as the probability that the total received power at the radar receiver, i.e the interference from other transmitters only, is higher than the threshold γ , given the absence of targets. These metrics are summarised in Box 6.9.

6.9: Detection and False Alarm Probability

Detection probability: $\mathcal{D}(\gamma) \triangleq \mathbb{P}_{S,I} (S + I \geq \gamma)$

False alarm probability: $\mathcal{F}(\gamma) \triangleq \mathbb{P}_I (I \geq \gamma)$

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6.1.2.3 Success Probability

The *success probability* is defined similarly as the coverage probability. Namely, it is the probability that the SINR χ at the radar receiver is higher than a predefined threshold η . Its expression is provided in Box 6.10.

6.10: Success Probability

$$\mathcal{S}(\eta) \triangleq \mathbb{P}_{\chi}(\chi \geq \eta)$$

In many works presented in Section 6.1.2.1, this metric is also named detection probability. The relation between both metrics has been clarified in Section 1.4.4 of Chapter 1. For a given false alarm probability requirement, the SINR threshold is set to achieve a given detection probability. For instance, as illustrated in Figure 1.11, a detection probability of 50% is achieved with a false alarm requirement of 10^{-7} , for a SINR threshold of 12 dB. Again, it can also be related to the radar estimation rate, as detailed in Section 6.1.2.4.

6.1.2.4 Estimation Rate

The *estimation rate* is defined similarly to the communication rate. It is a measure of the information about the target, retrieved from radar measurements [Bli14]. For a given radar with pulse repetition interval T_{PRI} and SINR χ at the receiver, it is defined in Box 6.11.

6.11: Estimation Rate

$$\mathcal{R}_r(\tau) \triangleq \mathbb{P}_{\chi} \left(\frac{1}{2 T_{\text{PRI}}} \log_2(1 + \chi) \geq \tau \right)$$

The estimation rate can be computed from the success probability, and vice versa:

$$\mathcal{R}_r(\tau) = \mathbb{P}_{\chi} \left(\frac{1}{2 T_{\text{PRI}}} \log_2(1 + \chi) \geq \tau \right) = \mathbb{P}_{\chi} \left(\chi \geq 2^{2 T_{\text{PRI}} \tau} - 1 \right) = \mathcal{S} \left(2^{2 T_{\text{PRI}} \tau} - 1 \right).$$

In other words, requiring the SINR to be higher than a given threshold η with a given probability is equivalent to requiring the estimation rate to be higher than a threshold $\tau = \frac{1}{2 T_{\text{PRI}}} \log_2(1 + \eta)$ with the same probability.

One can also compute the average estimation rate, following Box 6.12.

Again, the average estimation rate can instead be computed by integrating the

6.12: Average Estimation Rate

$$\overline{\mathcal{R}}_r \triangleq \frac{1}{2 T_{\text{PRI}}} \mathbb{E}_{\chi} [\log_2(1 + \chi)]$$

success probability. For instance,

$$\begin{aligned} \overline{\mathcal{R}}_r &= \frac{1}{2 T_{\text{PRI}}} \int_0^{\infty} \mathbb{P}_{\chi} (\log_2(1 + \chi) \geq \tau) d\tau \\ &= \frac{1}{2 T_{\text{PRI}}} \int_0^{\infty} \mathcal{S}(2^{\tau} - 1) d\tau = \frac{1}{2 T_{\text{PRI}} \ln 2} \int_0^{\infty} \frac{\mathcal{S}(\eta)}{1 + \eta} d\eta. \end{aligned}$$

6.1.3 Joint Radar and Communication in the Stochastic Geometry Literature

Considering JCAS applications, many works have been published, regarding either coexisting radar and communication or co-designed systems. In the former, with coexisting systems, [RMP19; RMP20] evaluate the radar range and the communication coverage probability of a JCAS network with **Time Division Multiple Access (TDMA)**. The interference from a rotating radar sharing the same spectrum as a cellular system is considered in [KII21], whereas [Rao+21] analyses the interference of a massive **Multiple-Input Multiple-Output (MIMO)** cellular network on a radar system. Finally, the development of **Cooperative Multi-Point (CoMP)** detection is presented in [SPK20; SPK22]. In the latter, with co-designed systems, [Fan+19] evaluates the cooperative detection volume of an **ISAC** system where targets are detected with the main beam, and different **BSs** cooperate by communicating through the secondary beams. The increase of communication rate with **ISAC** systems is evaluated in [RG22b], and an analytical framework to optimise the system parameters is proposed. Regarding automotive applications, the cooperative detection range is evaluated in [Ma+19] and [Gho+21], respectively with spectrum allocation between both functions or with an **ISAC** system. This range is defined as the total range in which targets are detected by multiple vehicles communicating with each other.

6.1.4 Joint Performance Metrics

In order to evaluate the performance of multiple links or functions simultaneously, joint metrics have recently been defined by combining existing communication metrics. For instance, [SZA15] considered jointly the achieved rate in the uplink and downlink. Another example is simultaneous wireless information and power transfer, analysed in [DL17].

Nevertheless, joint statistics for both the radar and communication functions is

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also interesting:

1. Both functions mutually interfere, requiring a joint evaluation of the performance achieved by both functions, as they are highly correlated.
2. In addition to recovering the univariate metrics, conditional metrics can also be defined from joint metrics, enabling to study the performance of one function, given a requirement on the other function.
3. Joint metrics enable to provide a single objective function, which takes into account both functions simultaneously to optimise the parameters of ISAC systems. No additional constraint is required, or they can be integrated into the objective function using the conditional metrics.

For instance, recently, in the literature, [Gan+24] has analysed the ISAC performance through a joint metric in a cooperative scenario, where multiple BSs participate for the positioning of UEs, while it receives data from the nearest BS. In this thesis, two novel joint metrics are proposed: the joint radar detection and communication coverage probability, and the joint radar success and communication coverage probability.

6.1.4.1 Joint Radar Detection and Communication Coverage Probability

The *Joint Radar Detection and Communication Coverage Probability* (JRDCCP) is defined as an extension of the communication coverage probability, detailed in Section 6.1.1.1, and the radar detection probability, detailed in Section 6.1.2.2. It is defined as the probability to jointly achieve a communication SINR χ_C higher than a predefined communication SINR threshold η_C , and receive a total power, i.e. the radar echoes S_R from the target, the interference I_R for the radar function, and the noise W_R , higher than a predefined radar power threshold γ_R . Its expression is given in Box 6.13. The JRDCCP metric is always defined in pair with the false alarm probability (Section 6.1.2.2), as the radar power threshold γ_R is set to achieve a given false alarm probability requirement:

$$\mathcal{F}(\gamma_R) = \mathbb{P}_{I_R} (I_R + W_R \geq \gamma_R).$$

6.13: Joint Radar Detection and Communication Coverage Probability

$$\mathcal{J}_D(\eta_C, \gamma_R) \triangleq \mathbb{P}_{\chi_C, S_R, I} (\chi_C \geq \eta_C, S_R + I_R + W_R \geq \gamma_R)$$

From the JRDCCP, one can compute the communication coverage, and the radar detection probability as

$$C(\eta_C) = \lim_{\gamma_R \rightarrow 0} \mathcal{J}_D(\eta_C, \gamma_R), \quad (6.2)$$

$$\mathcal{D}(\gamma_R) = \lim_{\eta_C \rightarrow 0} \mathcal{J}_D(\eta_C, \gamma_R). \quad (6.3)$$

6.1.4.2 Joint Radar Success and Communication Coverage Probability

The *Joint Radar Success and Communication Coverage Probability* (JRSCCP) is defined as an extension of the communication coverage probability, detailed in Section 6.1.1.1, and the radar success probability, detailed in Section 6.1.2.3. It is defined as the probability to jointly achieve a communication SINR χ_C higher than a predefined communication SINR threshold η_C , and a radar SINR χ_R higher than a predefined radar SINR threshold η_R . Its expression is given in Box 6.14.

6.14: Joint Radar Success and Communication Coverage Probability

$$\mathcal{I}_S(\eta_C, \eta_R) \triangleq \mathbb{P}_{\chi_C, \chi_R} (\chi_C \geq \eta_C, \chi_R \geq \eta_R)$$

From the JRSCCP, one can compute the communication coverage probability and the radar detection probability as

$$C(\eta_C) = \lim_{\eta_R \rightarrow 0} \mathcal{I}_S(\eta_C, \eta_R), \quad (6.4)$$

$$S(\eta_R) = \lim_{\eta_C \rightarrow 0} \mathcal{I}_S(\eta_C, \eta_R). \quad (6.5)$$

6.1.4.3 Fréchet Upper and Lower Bounds

The *Fréchet inequalities* are the simplest relations to obtain upper and lower bounds for joint metrics directly from their associated univariate metrics [Fré35]. They are summarised in Box 6.15.

6.15: Fréchet Inequalities

$$\mathbb{P}_{X,Y} (X \geq x, Y \geq y) \geq \max (0, \mathbb{P}_X (X \geq x) + \mathbb{P}_Y (Y \geq y) - 1)$$

$$\mathbb{P}_{X,Y} (X \geq x, Y \geq y) \leq \min (\mathbb{P}_X (X \geq x), \mathbb{P}_Y (Y \geq y))$$

Applying the Fréchet to the JRDCCP and JRSCCP metrics of Boxes 6.13 and 6.14, one obtains the upper and lower bounds given in Box 6.16.

6.16: Fréchet Upper and Lower Bounds for the JRDCCP and JRSCCP

$$\max \{0, C(\eta_C) + \mathcal{D}(\eta_R) - 1\} \leq \mathcal{I}_D(\eta_C, \eta_R) \leq \min \{C(\eta_C), \mathcal{D}(\eta_R)\}$$

$$\max \{0, C(\eta_C) + S(\eta_R) - 1\} \leq \mathcal{I}_S(\eta_C, \eta_R) \leq \min \{C(\eta_C), S(\eta_R)\}$$

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6.1.4.4 Conditional Metrics

One can be interest in the performance achieved by one function, given a requirement on the other function. More specifically, on the one hand for the communication, the communication coverage probability can be evaluated given a fulfilled radar detection or success probability requirement. Conditioning the [JRDCCP](#) and [JRSCCP](#) of Boxes 6.13 and 6.14 w.r.t. the radar function, the conditional metrics of Box 6.17 are obtained.

6.17: Communication Conditional Metrics

$$\begin{aligned}\mathcal{J}_{\text{D}|\text{C}}(\eta_{\text{C}}|\gamma_{\text{R}}) &\triangleq \mathbb{P}_{\chi_{\text{C}}|S_{\text{R}},I}(\chi_{\text{C}} \geq \eta_{\text{C}} \mid S_{\text{R}} + I_{\text{R}} \geq \gamma_{\text{R}}) = \frac{\mathcal{J}_{\text{D}}(\eta_{\text{C}}, \gamma_{\text{R}})}{\mathcal{D}(\gamma_{\text{R}})} \\ \mathcal{J}_{\text{C}|\text{S}}(\eta_{\text{C}}|\eta_{\text{R}}) &\triangleq \mathbb{P}_{\chi_{\text{C}}|\chi_{\text{R}}}(\chi_{\text{C}} \geq \eta_{\text{C}} \mid \chi_{\text{R}} \geq \eta_{\text{R}}) = \frac{\mathcal{J}_{\text{S}}(\eta_{\text{C}}, \eta_{\text{R}})}{S(\eta_{\text{R}})}\end{aligned}$$

On the other hand, for the radar function, the radar detection or success probability can be evaluated given a fulfilled communication coverage probability requirement. By conditioning the [JRDCCP](#) and [JRSCCP](#) of Boxes 6.13 and 6.14 w.r.t. the communication function, the conditional metrics of Box 6.18 are obtained.

6.18: Sensing Conditional Metrics

$$\begin{aligned}\mathcal{J}_{\text{D}|\text{C}}(\gamma_{\text{R}}|\eta_{\text{C}}) &\triangleq \mathbb{P}_{S_{\text{R}},I|\chi_{\text{C}}}(S_{\text{R}} + I_{\text{R}} \geq \gamma_{\text{R}} \mid \chi_{\text{C}} \geq \eta_{\text{C}}) = \frac{\mathcal{J}_{\text{D}}(\eta_{\text{C}}, \gamma_{\text{R}})}{C(\eta_{\text{C}})} \\ \mathcal{J}_{\text{S}|\text{C}}(\eta_{\text{R}}|\eta_{\text{C}}) &\triangleq \mathbb{P}_{\chi_{\text{R}}|\chi_{\text{C}}}(\chi_{\text{R}} \geq \eta_{\text{R}} \mid \chi_{\text{C}} \geq \eta_{\text{C}}) = \frac{\mathcal{J}_{\text{S}}(\eta_{\text{C}}, \eta_{\text{R}})}{C(\eta_{\text{C}})}\end{aligned}$$

6.2 Application to a V2X Scenario

In this section, the evaluation of the different metrics presented in Section 6.1 is performed in a full-duplex [V2X](#) scenario, which belongs to the category of scenarios presented in Section 1.1.2 of Chapter 2.

6.2.1 System Model

Let us consider the two-lane [V2X](#) scenario illustrated in Figure 6.1. The vehicles are all equipped at the front side with an [ISAC](#) system. The transmitted powers, and the antenna beamwidths at the transmitter and receiver of the vehicles are respectively denoted as P_{V} , ϕ_{Vt} and ϕ_{Vr} . The typical vehicle simultaneously receives a radar echo reflected by the first vehicle in front and driving in the same lane, and a communication

Full-Duplex ISAC Automotive Scenario

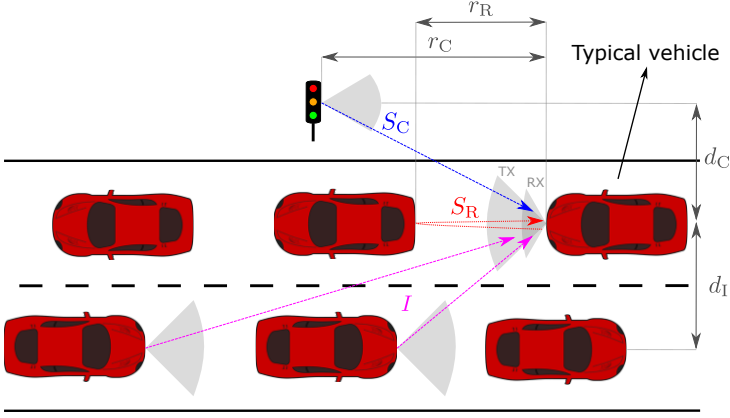


Fig. 6.1 Considered two-lane V2X scenario.

signal from the nearest RSU ahead. The transmitted power and antenna beamwidth of the RSU at the transmitter are respectively denoted as P_L and ϕ_{L_t} . The distance perpendicular to the road direction between the typical vehicle and the RSU (resp. the centre of the opposite lane) is denoted d_C (resp. d_I). The minimum and maximum detectable range of the radar function, the minimum distance parallel to the road of the communication signal, and the minimum distance parallel to the road of the interfering signals, are respectively denoted as r_{Rmin} , r_{Rmax} , r_{Cmin} and r_{Imin} . Note that r_{Cmin} and r_{Imin} are directly related to the antenna beamwidths of the different systems:

$$r_{Cmin} = \frac{d_C}{\tan(\min(\phi_{L_t}, \phi_{V_r})/2)}, \quad r_{Imin} = \frac{d_I}{\tan(\min(\phi_{V_t}, \phi_{V_r})/2)}. \quad (6.6)$$

The echoes arising from vehicles driving in the opposite direction are not considered in this work. This assumption is motivated by the stronger attenuation affecting the radar echoes compared to the interfering signals, and the use of directive beamforming at the transmitter. Interference Cancellation (IC) of the interfering echoes at the communication function based on the acquired radar information could also be implemented following the approach presented in Chapter 9, but this is not studied in this thesis. Nevertheless, it is still assumed that this scenario is interference-limited owing to the high interference arising from the vehicles driving in the opposite direction. Therefore, the noise is neglected in this Chapter 6.2.2.

As detailed in Section 1.1.2 of Chapter 1, this V2X application is a variation of the full-duplex ISAC scenario. For the communication function, the radar echo generated by the typical vehicle is considered as an interference. Contrariwise, for the radar function, the communication signal transmitted by the RSU is considered as an interference. Moreover, the signals transmitted by the vehicles driving in the opposite

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lane are interfering with both functions. The self-interference is not considered in this work, as this issue is not specific to **ISAC** scenarios, but to full-duplex systems in general. It is assumed that it has been mitigated through **EM** isolation, beamforming, self-interference cancellation, or other techniques. The extension of the two-lane **V2X** scenario into a multi-lane scenario is proposed in Section 6.2.4.2.

6.2.1.1 Nodes Distribution using Point Processes

Assuming that the typical vehicle is located at $r = 0$ m, the vehicles driving in the direction of the typical vehicle, the interfering vehicles on the opposite way, and the **RSUs** are distributed on three parallel lines following three independent **PPs** Φ_R , Φ_I and Φ_C , of intensities respectively given by

$$\begin{aligned}\lambda_R(r) &= \lambda_R \mathbb{1}(r_{R\min} \leq r \leq r_{R\max}), \\ \lambda_I(r) &= \lambda_I \mathbb{1}(r_{I\min} \leq r), \\ \lambda_C(r) &= \lambda_C \mathbb{1}(r_{C\min} \leq r),\end{aligned}$$

where λ_R , λ_I and λ_C are respectively the densities of vehicles driving on the same lane as the typical vehicle, the density of interfering vehicles, and the density of **RSUs**.

PPs have been selected to model all the node locations, owing to their tractability and analytical flexibility. Other existing **PPs**, e.g. **HCPPs** and **PCPs**, presented in Section 5.1.1 of Chapter 5, are by contrast more accurate for modelling real world scenarios, at the price of a higher complexity [Hma+21]. The integration of these **PPs** in the proposed models is discussed in Section 6.2.5.

6.2.1.2 Propagation Models

Following the models detailed in Section 3.1.4 of Chapter 3, linear path-loss models are considered:

$$[L_k(r)]_{\text{dB}} = [\beta_k]_{\text{dB}} + 10 \alpha_k \log_{10}(r),$$

where $k \in \{R, C, I\}$ designates the considered link, α_k is the path-loss exponent, and β_k the intercept.

Communication link: For the communication link, the received power S_C is given by the Friis formula of Box 3.1:

$$S_C(r) = P_L G_C \frac{c^2}{4\pi f_c^2} L_C^{-1} \left(\sqrt{r^2 + d_C^2} \right) = \rho_C (r^2 + d_C^2)^{-\frac{\alpha_C}{2}},$$

where

$$\rho_C \triangleq \frac{P_L G_C c^2}{4\pi f_c^2 \beta_C}.$$

In this equation, r is the distance between the two nodes parallel to the road, f_c is the carrier frequency, and G_C is the gain of the complete communication link. The small-scale fading has been neglected for the sake of tractability. However, it is assumed to be highly Ricean [Stü13], since the **RSU** is located on the same side of the road than the typical vehicle, and high frequencies, i.e. 24 or 77 GHz, are considered in automotive scenarios.

Interfering links: For the interfering links, the total interfering power is also computed with the Friis formula:

$$I = \sum_{i \mid x_i \in \Phi_I} P_V G_I \frac{c^2}{4\pi f_c^2} L_I^{-1} \left(\sqrt{x_i^2 + d_I^2} \right) |h_i|^2 = \sum_{i \mid x_i \in \Phi_I} \rho_I (x_i^2 + d_I^2)^{-\frac{\alpha_I}{2}} |h_i|^2,$$

where

$$\rho_I \triangleq \frac{P_V G_I c^2}{4\pi f_c^2 \beta_I}.$$

The beamforming gain of the complete interfering link is denoted as G_I , and $|h_i|^2$ are small-scale fading **RVs**, also supposed to be Ricean-distributed.

Radar link: Regarding the radar link, following the discussions of Chapter 3, the received power S_R is modelled following the **RT** propagation model, presented in Section 3.1.3 of Chapter 3:

$$S_R(r) = P_V G_R \frac{c^2}{4\pi f_c^2} L_R^{-1}(2r) = \rho_R r^{-\alpha_R}, \quad (6.7)$$

where

$$\rho_R \triangleq \frac{P_V G_R c^2}{4\pi f_c^2 \beta_R}.$$

The gain of the complete radar link is denoted as G_R . The small-scale fading is also neglected, but it is again supposed to be highly Ricean. The **RT** model is motivated by the high carrier frequency considered in automotive scenarios. For example, the Fraunhofer distance for a target with a maximum dimension of 1 meter is higher than 100 meters for carrier frequencies of both 24 and 77 GHz.

To simplify the notations of the next sections, the following definitions are introduced:

- The distance between the typical vehicle and the next vehicle driving ahead is denoted as r_R .
- The distance between the typical vehicle and the nearest **RSU** parallel to the road is denoted as r_C .

Relation between Metrics and Preliminary Results

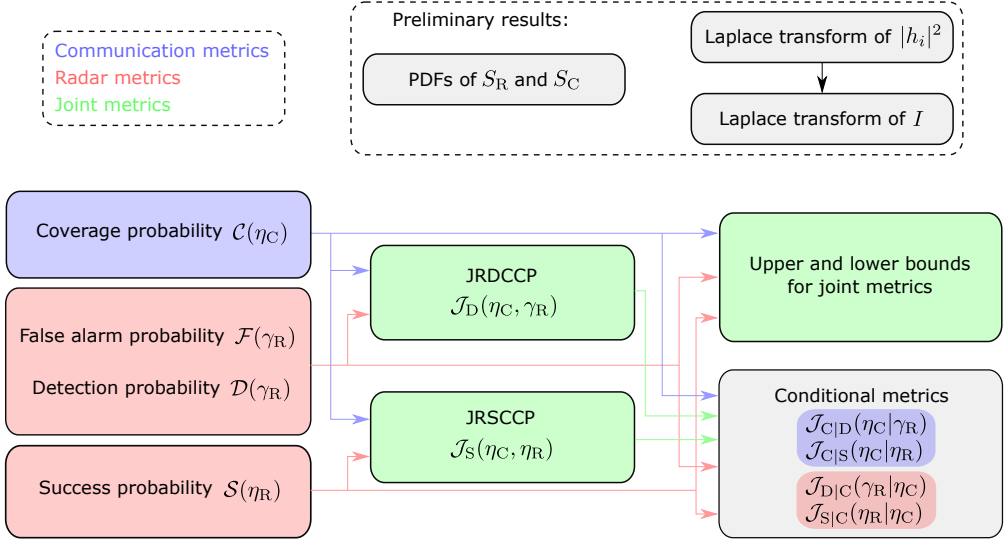


Fig. 6.2 Preliminary results and metrics computed in Section 6.2.2.

- The received useful power for the radar link is rewritten as $S_R \equiv S_R(r_R)$.
- The received useful power for the communication link is rewritten as $S_C \equiv S_C(r_C)$.
- The minimum and maximum useful power for the radar link are rewritten as $S_{Rmin} \equiv S_R(r_{Rmax})$ and $S_{Rmax} \equiv S_R(r_{Rmin})$.
- The minimum and maximum useful power for the communication link are rewritten as $S_{Cmin} \equiv S_C(r_{Cmax})$ and $S_{Cmax} \equiv S_C(r_{Cmin})$.

6.2.2 Development of the Metrics

In order to obtain closed-form expressions of the univariate, joint and conditional metrics, preliminary results are first required, namely the PDFs of S_R and S_C , the LT of the Ricean small scale fading, and the LT of the interference I . These results are computed in Appendices 6.A and 6.B. Then, the Gil-Pelaez theorem and two useful operators are defined in Section 6.2.2.1. Finally, all the metrics are computed in Appendices 6.C to 6.H, and the results are summarised in Section 6.2.2.3. Figure 6.2 illustrates the relations between the different considered metrics, and the required preliminary results.

6.2.2.1 Gil-Pelaez Theorem and Operators $\mathfrak{E}$ and $\mathfrak{P}$

The Gil-Pelaez theorem enables to compute the cumulative distribution function of a RV X based on its characteristic function ϕ_X [Gil51]. It is summarised in Box 6.19.

6.19: Gil-Pelaez Theorem

$$\mathbb{P}_X(X \leq x) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{1}{\tau} \operatorname{Im} \{ \phi_X(\tau) e^{-j\tau x} \} d\tau$$

6.2.2.2 Operators $\mathfrak{E}$ and $\mathfrak{P}$

Two operators are defined to simplify the expressions of the metrics. First, the operator $\mathfrak{E}$ enables to rewrite the expectation of a function $g : \mathbb{R}^n \rightarrow \mathbb{R}^m$ with $n, m \in \mathbb{N}$, in a given domain $\Omega \subseteq \mathbb{R}^n$:

$$\mathfrak{E}_\Omega [g(\mathbf{X})] \triangleq \mathbb{E}_X [g(\mathbf{X}) \mathbb{1}(\mathbf{X} \in \Omega)] .$$

Then, the operator $\mathfrak{P}$ is derived from the Gil-Pelaez theorem. Defining two functions $g_1 : \mathbb{C} \rightarrow \mathbb{C}$ and $g_2 : \mathbb{C} \times \mathbb{R}^n \rightarrow \mathbb{C}$, for a given domain $\Omega \subseteq \mathbb{R}^n$, it is defined as

$$\mathfrak{P}(g_1, g_2; \Omega) \triangleq \mathfrak{E}_\Omega \left[\frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{1}{\tau} \operatorname{Im} \{ g_1(-j\tau) g_2(-j\tau, \mathbf{X}) \} d\tau \right] .$$

6.2.2.3 Summary

A summary of the expressions obtained for the computation of the different metrics is provided in Table 6.1. In order to evaluate these metrics, averaging on the variables S_R and S_C is performed, using the PDFs of (6.13) and (6.14). The computation of the LT of the interference I given in (6.11) is also required. It itself depends on the LT of the small-scale fading, obtained in (6.12).

For the JRSCCP to be non-zero, the condition of Box 6.20 has been obtained. This result highlights a trade-off between the communication and radar functions: it is impossible to achieve simultaneously arbitrarily high radar and communication SINR thresholds requirements, the useful power of one function being detrimental for the other. This condition seems too restrictive to enable both functions to work simultaneously. Nonetheless, one may instead consider the SINR of the decision variable in the JRSCCP. In that case, the threshold of 0 dB is shifted by the processing gains of both functions, enabling to achieve correct performance for both functions.

For each point of the metrics, four integrations are required, respectively over S_R and S_C in the operator $\mathfrak{E}$, τ in the operator $\mathfrak{P}$, and r in the LT of the interference. For the false alarm probability, as it does not depend on S_R , it is limited to three

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Metric	Function ψ	Region Ω
Coverage probability $C(\eta_C) = \mathbb{P}(\mathcal{L}_I, \psi_C; \Omega_C)$	$\psi_C(s) = \exp\left(s\left(\frac{S_C}{\eta_C} - S_R\right)\right)$	$\Omega_C = \left\{S_{Rmin} \leq S_R \leq S_{Rmax} \atop \eta_C S_R \leq S_C \leq S_{Cmax}\right\}$
False alarm probability $\mathcal{F}(\gamma_R) = 1 - \mathbb{P}(\mathcal{L}_I, \psi_F; \Omega_F)$	$\psi_F(s) = \exp(s(\gamma_R - S_C))$	$\Omega_F = \{0 \leq S_C \leq \min(\gamma_R, S_{Cmax})\}$
Detection probability $\mathcal{D}(\gamma_R) = 1 - \mathbb{P}(\mathcal{L}_I, \psi_D; \Omega_D)$	$\psi_D(s) = \exp(s(\gamma_R - S_R - S_C))$	$\Omega_D = \left\{S_{Rmin} \leq S_R \leq S_{Rmax} \atop 0 \leq S_C \leq \min(\gamma_R - S_R, S_{Cmax})\right\}$
Success probability $S(\eta_R) = \mathbb{P}(\mathcal{L}_I, \psi_S; \Omega_S)$	$\psi_S(s) = \exp\left(s\left(\frac{S_R}{\eta_R} - S_C\right)\right)$	$\Omega_S = \left\{S_{Rmin} \leq S_R \leq S_{Rmax} \atop 0 \leq S_C \leq \min(S_R/\eta_R, S_{Cmax})\right\}$
JRDCCP $\mathcal{J}_D(\eta_C, \gamma_R) = \mathbb{P}(\mathcal{L}_I, \psi_{D1}; \Omega_{D1})$ $+ \mathbb{P}(\mathcal{L}_I, \psi_{D2}; \Omega_{D2}) - \mathfrak{E}_{\Omega_{D2}}\left[\frac{1}{2}\right]$	$\psi_{D1}(s) = \psi_C(s)$ $\psi_{D2}(s) = \psi_C(s) - \psi_D(s)$	$\Omega_{D1} = \left\{S_{Rmin} \leq S_R \leq S_{Rmax} \atop \max(\eta_C S_R, \gamma_R - S_R) \leq S_C \leq S_{Cmax}\right\}$ $\Omega_{D2} = \left\{S_{Rmin} \leq S_R \leq S_{Rmax} \atop \frac{\eta_C \gamma_R}{\eta_C + 1} \leq S_C \leq \min(\gamma_R - S_R, S_{Cmax})\right\}$
JRSCCP $\mathcal{J}_S(\eta_C, \eta_R) = \mathbb{P}(\mathcal{L}_I, \psi_{S1}; \Omega_{S1})$ $+ \mathbb{P}(\mathcal{L}_I, \psi_{S2}; \Omega_{S2})$	$\psi_{S1}(s) = \psi_C(s)$ $\psi_{S2}(s) = \psi_S(s)$	$\Omega_{S1} = \left\{S_{Rmin} \leq S_R \leq S_{Rmax} \atop \eta_C S_R \leq S_C \leq \min\left(\frac{\eta_C(\eta_R + 1)}{\eta_R(\eta_C + 1)} S_R, S_{Cmax}\right)\right\}$ $\Omega_{S2} = \left\{S_{Rmin} \leq S_R \leq S_{Rmax} \atop \frac{\eta_C(\eta_R + 1)}{\eta_R(\eta_C + 1)} S_R \leq S_C \leq \min\left(\frac{S_R}{\eta_R}, S_{Cmax}\right)\right\}$

Table 6.1 Computation of the different metrics. The regions Ω and functions ψ are obtained in appendices 6.C to 6.H. Moreover, the regions have here been merged with the minimum and maximum values of S_R and S_C .

6.20: Requirement Trade-off for the JRSCCP

$$\eta_C \eta_R \leq 1 \quad \Leftrightarrow \quad [\eta_C]_{\text{dB}} + [\eta_R]_{\text{dB}} \leq 0 \text{ dB}$$

Parameters	Values
Transmitted powers	$P_V = P_L = 20 \text{ dBm}$
Beamforming gains	$G_R = G_C = G_I = 1$
Minimum detectable range	$r_{\text{Rmin}} = 5 \text{ m}$
Antennas beamwidths	$\phi_{Vt} = 22.5^\circ, \phi_{Vr} = \phi_{Lt} = 45^\circ$
Path-loss coefficients	$\beta_R = \beta_C = \beta_I = 4\pi$
Path-loss exponents	$\alpha_R = 2, \alpha_C = \alpha_I = 3$
Nodes densities	$\lambda_V = 0.02 \text{ m}^{-1}, \lambda_L = 0.01 \text{ m}^{-1},$ $\lambda_I = 0.002 \text{ m}^{-1}$
Street topology	$d_C = 2.5 \text{ m}, d_I = 3 \text{ m}$

Table 6.2 Parameters of the scenario for Section 6.2.3.

integrations. In order to reduce the complexity, one may first pre-compute the **LT** of the interference, as it is used in every metrics, and does not vary with S_R and S_C . Then, using the Gil-Pelaez theorem, its **CDF** may also be pre-computed. Therefore, having computed the **CDF** beforehand, only two integrations must be performed for each point of the metrics, except for the false alarm probability, which requires only one integration.

6.2.3 Numerical Analysis

In this section, numerical analyses are performed based on the scenario defined in Table 6.2. First, the different metrics are presented. **MC** simulations are performed to validate the analytical models. Then, optimisation w.r.t. the **RSUs** transmitted power and density is performed.

6.2.3.1 Analysis of the Metrics

Figure 6.3 illustrates the false alarm probability, the detection probability and the **JRDCCP**, obtained with **SG** and **MC** simulations. As expected from their definitions given in Box 6.9, the false alarm probability is lower than the detection probability at any threshold. The false alarm and detection probabilities decrease to zero when the radar threshold increases. Therefore, the number of false alarms is lowered at the price of fewer detections. For low values of the radar threshold γ_R , corresponding to high false alarm probabilities, the detection probability is close to one, and the **JRDCCP** tends to be equal to the communication coverage probability. Contrariwise, for low values of the communication **Signal-to-Interference Ratio (SIR)** threshold η_C ,

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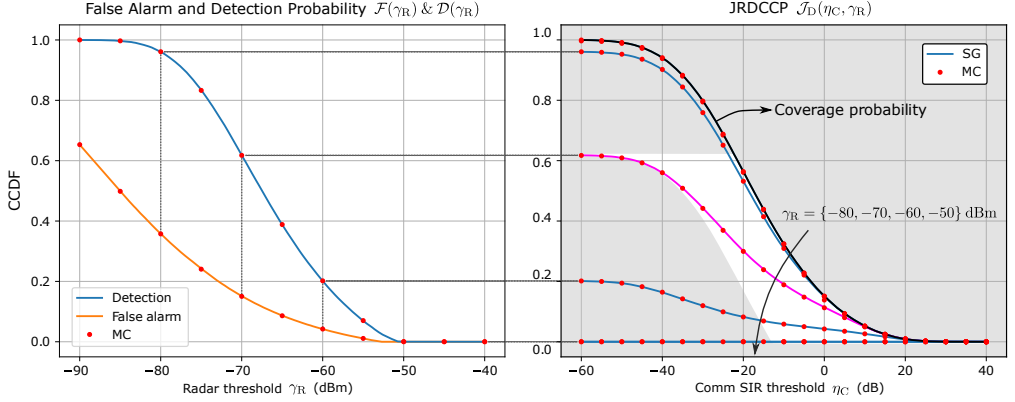


Fig. 6.3 False alarm and detection probability (left), and JRDCCP (right). The white area corresponds to the region delimited by the Fréchet upper and lower bounds of the magenta curve, given in Box 6.16.

the communication coverage probability is close to one, and the JRDCCP tends to be equal to the detection probability.

The same behaviour is observed with the JRSCCP, shown in Figure 6.4. For low values of the radar SIR threshold η_R , the radar success probability is close to one, and the JRSCCP tends to be equal to the coverage probability. By contrast, for low values of η_C , the coverage probability is close to one, and the JRSCCP tends to be equal to the radar success probability. Additionally, the condition given in Box 6.20 is verified: whatever the values of η_R and η_C , the metric decreases to zero when the sum of the SIR thresholds in dB exceeds 0 dB. This condition illustrates directly that a trade-off should be achieved between the radar and the communication function. Owing to the negative impact of the communication signal on the radar function and vice-versa, arbitrarily high performance cannot be achieved simultaneously for both functions.

Two conditional metrics are also illustrated in Figure 6.3d, namely $\mathcal{J}_{C|S}$ and $\mathcal{J}_{S|C}$, defined in Boxes 6.17 and 6.18. For these metrics, the same condition applies: among the vehicle achieving a communication (resp. radar) SIR of at least η_C (resp. η_R) in dB, there is a zero probability of achieving a radar (resp. communication) SIR higher than $-\eta_R$ (resp. $-\eta_C$) in dB. This illustrates again the trade-off between the radar and the communication function. It is not possible to achieve arbitrarily high performance with one function given an arbitrary high requirement for the other function.

The Fréchet upper and lower bounds are illustrated in both Figure 6.3 and 6.4. In each case, the bounds become tighter as the joint metrics are evaluated for lower thresholds. At these thresholds, the joint metrics tend to be equal to radar or communication metrics (detection, success and coverage probability), and the bounds merge

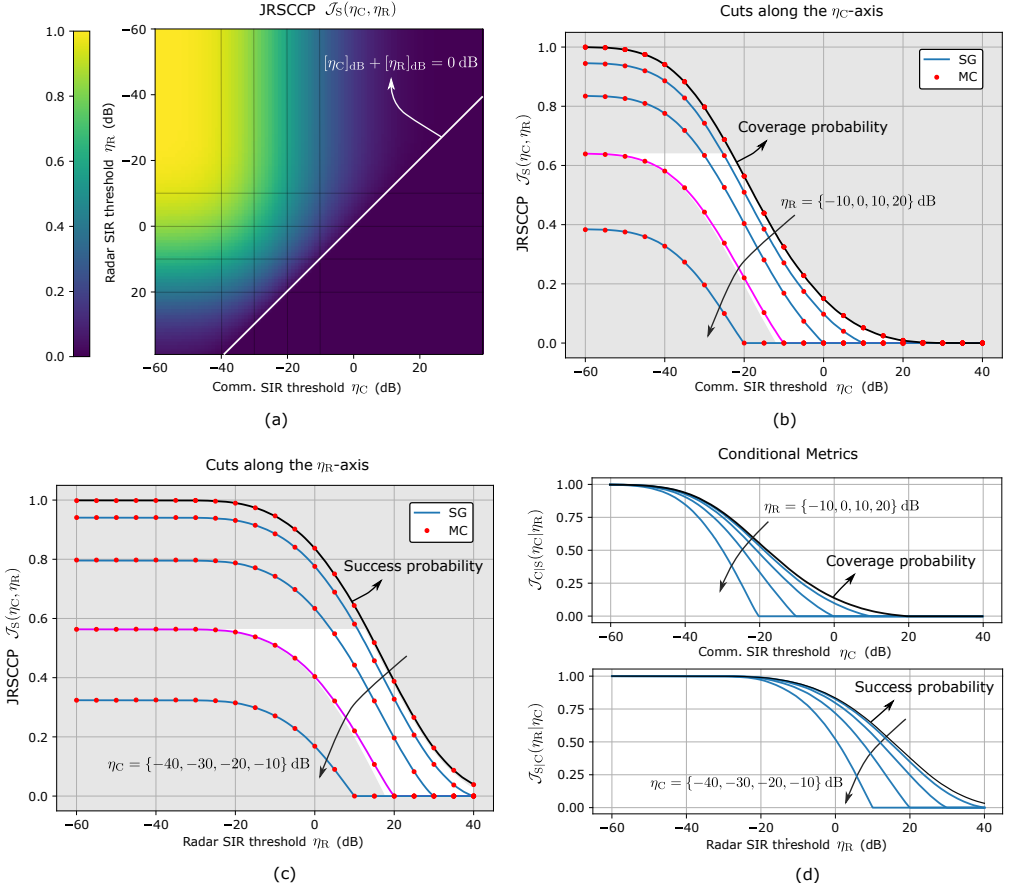


Fig. 6.4 JRSCCP (a), cut along the η_C -axis (b), cut along the η_R -axis (c), and conditional metrics (d). The white areas in (b) and (c) correspond to the regions delimited by the Fréchet upper and lower bounds of the magenta curves, given in Box 6.16.

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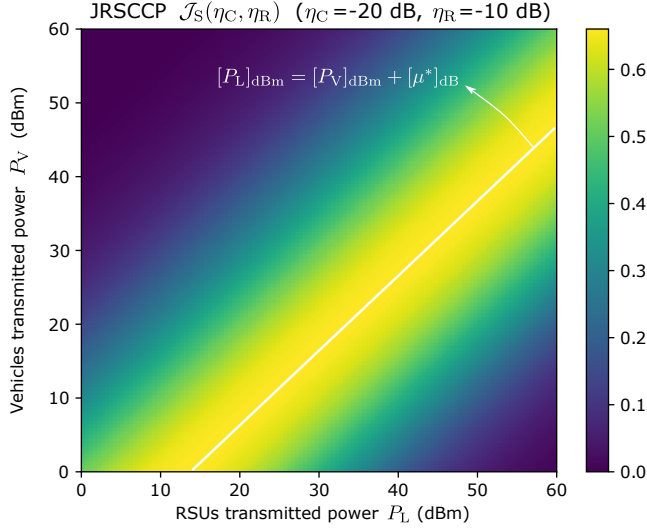


Fig. 6.5 JRSCCP against transmitted powers, with $\eta_R = -10$ dB and $\eta_C = -20$ dB. The optimal power ratio of (6.9) is denoted as μ^* .

with the univariate metrics.

6.2.3.2 Optimisation of the Network Parameters

One may use the joint performance metrics to optimise the transmitted power of the vehicles and the RSUs, and the density of RSUs. First, regarding the JRSCCP, the optimisation problem is formulated for given radar and communication SIR thresholds η_R and η_C as

$$\begin{aligned} P_L^*, P_V^* &= \arg \max_{P_L, P_V} \mathcal{J}_S(\eta_C, \eta_R; P_L, P_V) \\ \text{s.t.} \quad & P_{Lm} \leq P_L \leq P_{LM}, \\ & P_{Vm} \leq P_V \leq P_{VM}. \end{aligned} \quad (6.8)$$

The minimum and maximum values for both the RSUs and vehicles powers are respectively denoted as P_{Lm} , P_{LM} , P_{Vm} and P_{VM} . The problem can be formulated similarly with the JRDCCP.

Figure 6.5 illustrates how the JRSCCP evolves for different vehicles and RSUs transmitted powers P_V and P_L , at given SIR thresholds η_C and η_R . Under the interference-limited scenario assumption, the JRSCCP can be expressed as a function of the ratio $\mu \triangleq P_L/P_V$ between the RSUs and vehicles transmitted powers. The power ratio should be therefore set such that neither the radar nor the communication function is disadvantaged, relatively to the required thresholds. Defining by $\tilde{S}_C \triangleq S_C/P_L$,

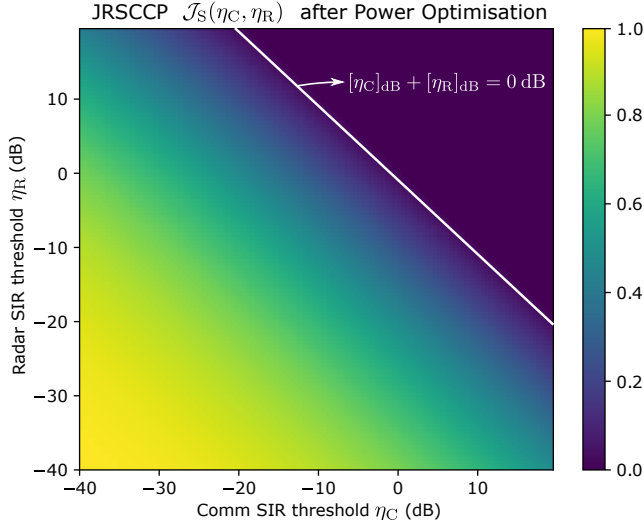


Fig. 6.6 JRSCCP for different radar and communication SIR thresholds η_R and η_C , with the transmitted powers of the RSUs and vehicles optimised for each pair of thresholds. The minimum and maximum powers are $P_{Lm} = P_{LM} = 0$ dBm and $P_{Vm} = P_{VM} = 60$ dBm.

$\tilde{S}_R \triangleq S_R/P_V$ and $\tilde{I} \triangleq I/P_V$, (6.8) can be rewritten as

$$\begin{aligned} \mu^* &= \arg \max_{\mu} \mathcal{J}_S(\eta_C, \eta_R; \mu) \\ \text{s.t.} \quad &\mu_m \leq \mu \leq \mu_M, \end{aligned} \quad (6.9)$$

with $\mu_m \triangleq P_{Lm}/P_{Vm}$, $\mu_M \triangleq P_{LM}/P_{VM}$, and

$$\mathcal{J}_S(\eta_C, \eta_R; \mu) = \mathbb{P}_{\tilde{S}_R, \tilde{S}_C, \tilde{I}} \left(\mu \frac{\tilde{S}_C}{\tilde{S}_R + \tilde{I}} \geq \eta_C, \frac{\tilde{S}_R}{\mu \tilde{S}_C + \tilde{I}} \geq \eta_R \right).$$

This enables to perform a univariate optimisation to solve (6.9), for instance with a 1D grid search, or a golden-section search [Kie53]. When μ^* is found, one can select the lowest power levels satisfying both $\mu^* = P_L^*/P_V^*$, and the transmitted powers constraints.

The achieved JRSCCP after power optimisation is illustrated in Figure 6.6, for power levels between 0 and 60 dBm. For each pair of thresholds, the joint metric is optimised w.r.t. the RSUs and vehicles transmitted powers, and the achieved performance is shown. Indeed, the JRSCCP is higher when low SIR thresholds are required. Nevertheless, even if the metric is increased for every pair of thresholds, condition Box 6.20 is still verified. Even with power optimisation, arbitrarily high SIR requirements cannot be set for both functions simultaneously.

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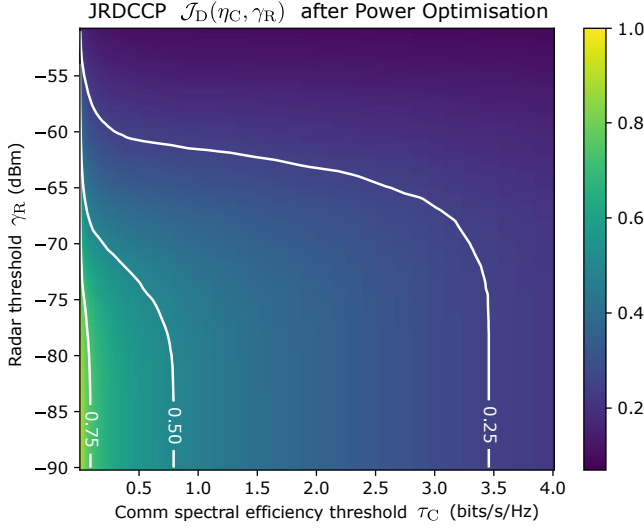


Fig. 6.7 JRDCCP for different radar thresholds γ_R and communication spectral efficiency thresholds $\tau_C = 2^{\eta_C} - 1$, with the transmitted powers of the RSUs and vehicles optimised for each pair of thresholds. The minimum and maximum powers are $P_{Lm} = P_{Vm} = 0$ dBm and $P_{Lm} = P_{Vm} = 60$ dBm.

The same analysis is done in Figure 6.7 for the JRDCCP, in which the change of variable of (6.1) has been performed to compute the joint radar detection and communication spectral efficiency. Again, higher performance are achieved for low radar thresholds γ_R , but it also induces a higher false alarm probability. For the considered scenario, radar thresholds up to -80 dBm give similar performance, ensuring a communication rate of 0.8 bits/s/Hz or higher with a probability of at least 50%.

One can also be interested to optimise the transmitted power and density of the RSUs to design the RSUs infrastructure. For instance, for given SIR thresholds requirements, with the JRSCCP, the optimisation problem is formulated as

$$\begin{aligned} P_L^*, \lambda_L^* &= \arg \max_{P_L, \lambda_L} \mathcal{J}_S(\eta_C, \eta_R; P_L, \lambda_L) \\ \text{s.t.} \quad & P_{Lm} \leq P_L \leq P_{Lm}, \\ & \lambda_{cm} \leq P_V \leq \lambda_{cm}. \end{aligned}$$

Minimum and maximum values for the density of the RSUs are respectively denoted as λ_{cm} and λ_{cm} . The problem is solved in Figure 6.8 using a 2D grid search for transmitted powers between 0 and 60 dBm, and densities between 10^{-3} and $5 \cdot 10^{-1} \text{ m}^{-2}$. Conclusions similar to Figure 6.5 are drawn. An increased density leads to a

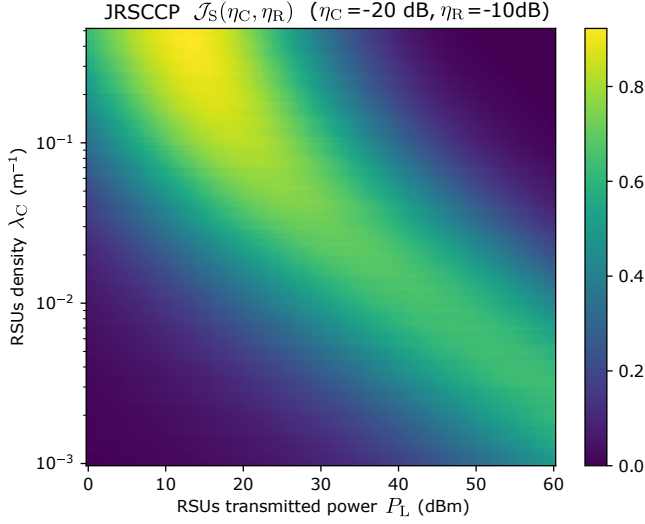


Fig. 6.8 JRSCCP against RSUs transmitted power and density, with $\eta_R = -10$ dB, $\eta_C = -20$ dB, and $P_V = 30$ dBm.

higher probability for the nearest RSU to be close, increasing the received power for the communication function, but also the interference at the radar function, similarly to the impact of an increase of the transmitted power. Consequently, increasing the density also enables to reduce the transmitted power while keeping good performance. Finally, one can observe that a higher JRSCCP is achieved for high densities and low transmitted powers, compared to the one achieved for high transmitted powers and low densities. At low density, the distance between the RSU and the vehicle is large with a high probability, and a high communication power does not enable to compensate the path-loss for every network realisation.

6.2.4 System Model Extensions

In this section, extensions of the system model are proposed, namely the introduction of interference cancellation capabilities, the generalisation to a multi-lane road scenario, and the introduction of other point processes for modelling of the nodes locations.

6.2.4.1 Introduction of Interference Cancellation

As discussed in Section 1.1.2 of Chapter 1, on the one hand, from the radar point of view, any signal arriving at the receiver other than the radar echoes is an interfering signal. On the other hand, from the communication point of view, any signal arriving at the receiver other than communication signals of interest is considered as interference. Enabling some cooperation between both functions, communication signals which are decoded at the communication receiver

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could be (partially) suppressed at the radar receiver, and radar echoes which are processed at the radar receiver could be (partially) suppressed at the communication receiver. This could also handle the self-interference if the residual power of the direct link is not saturating the frontend of the receiver, as the direct link can be considered either as a radar echo, or a communication signal with a known payload.

In the considered scenario, we assume that the signals transmitted by the interfering vehicles are not decoded by the typical vehicle. Therefore, the interference arising from the opposite lane is not cancelled, but the cooperation of both functions enables to mitigate the impact of S_R on the communication function, and S_C on the radar function.

Interference cancellation is modelled in [TH15] by multiplying the interference term by a factor $\zeta \in [0, 1]$, defining at which extent the interference has been cancelled. Perfect interference cancellation is achieved when $\zeta = 0$, whereas no interference cancellation is performed when $\zeta = 1$. Another approach is followed in [BJ09], which defines the coverage probability as the probability to simultaneously have the interference-to-signal-plus-noise and the SNR higher than a given SINR threshold. Instead, we assume in this section that the higher the SINR at the communication (resp. radar) receiver, the better the interference cancellation at the radar (resp. communication) receiver. Thus, leveraging on both approaches, we introduce a ζ factor which depends on the performance of the decoding at the other function.

Definition of the residuals: The residual interference ζ_R is supposed to be high when the radar SIR is low, as the radar receiver has difficulties to estimate correctly the parameters of the radar echoes. The same applies for the residual interference ζ_C , which is supposed to be high when the communication SIR is low, as the communication receiver has difficulties to estimate correctly the symbols and parameters of the communication signals. Moreover, the interference level could even be increased owing to an imprecise signal cancellation. Conversely, when these SIRs are high, the interference is supposed to be efficiently cancelled, leading to nearly zero residuals.

For the sake of tractability, even if the total interference should be considered to assess the efficiency of the interference cancellation scheme, we assume that these functions do not depend on the interfering signals I , but only on the power of the radar echoes and communication signals. In most of the simulated scenarios, it has been observed that the powers S_R and S_C are large w.r.t. the interference, motivating this assumption. Three different case are evaluated:

1. No interference cancellation is performed, leading to $\zeta_R(S_C, S_R) \triangleq S_R$ and $\zeta_C(S_C, S_R) \triangleq S_C$. This gives back the results of Section 6.2.2.

2. A perfect interference cancellation is performed at both functions, leading to $\zeta_R(S_C, S_R) \triangleq 0$ and $\zeta_C(S_C, S_R) \triangleq 0$. This is an ideal case, in which the radar and communication functions do not interfere with each other after cancellation.
3. An imperfect interference cancellation is performed at both functions, depending on the ratio between the powers of the radar echoes and communication signals, leading to the following residuals:

$$\zeta_R(S_C, S_R) \triangleq S_R \zeta\left(\frac{S_R}{S_C}\right), \quad \zeta_C(S_C, S_R) \triangleq S_C \zeta\left(\frac{S_C}{S_R}\right).$$

The function ζ should ideally describe the performance of the interference cancellation scheme. In view of all the arguments set out above, an option is to define it following Box 6.21, with a and b two fitting parameters.

6.21: Residual Interference Function

$$\zeta(x) \triangleq \frac{a}{1 + x^b}$$

This function is illustrated in Figure 6.9 for different values of a and b . On the one hand, choosing $a > 1$ enables to model the maximum increase of the interference for low power ratios, respectively equal to $a S_C$ or $a S_R$ for the residual interference at the radar and communication functions. On the other hand, the higher the value of b , the higher the efficiency of the cancellation when the power ratio is higher than 1, but the quicker the increase of interference in the opposite case. It can be shown that such shape is relevant to model the residual interference, from the interference cancellation schemes proposed in Chapter 9. Still, this definition is arbitrary and constitutes an example. Yet, any residual functions can be introduced in the framework, as long as they do not depend on the interfering power I .

Definition of the metrics: Introducing the residuals, the JRDCCP and JRSCCP of Boxes 6.13 and 6.14 are respectively redefined as

$$\mathcal{J}_D^{\text{IC}}(\eta_C, \gamma_R) \triangleq \mathbb{P}_{S_C, S_R, I} \left(\frac{S_C}{\zeta_R(S_C, S_R) + I} \geq \eta_C, S_R + \zeta_C(S_C, S_R) + I \geq \gamma_R \right),$$

$$\mathcal{J}_S^{\text{IC}}(\eta_C, \eta_R) \triangleq \mathbb{P}_{S_C, S_R, I} \left(\frac{S_C}{\zeta_R(S_C, S_R) + I} \geq \eta_C, \frac{S_R}{\zeta_C(S_C, S_R) + I} \geq \eta_R \right).$$

Starting from these expressions, all the metrics with interference cancellation are obtained following the same developments as presented in Appendices 6.C to 6.H. This leads to the results summarised in Table 6.3. For readability, the residuals have been shortened as $\zeta_C \triangleq \zeta_C(S_C, S_R)$ and $\zeta_R \triangleq \zeta_R(S_C, S_R)$. One may also obtain the

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Metric	Function ψ	Region Ω
Coverage probability $\mathcal{C}^{\text{IC}}(\eta_C) = \mathbb{P}(\mathcal{L}_I, \psi_C^{\text{IC}}; \Omega_C^{\text{IC}})$	$\psi_C^{\text{IC}}(s) = \exp\left(s\left(\frac{S_C}{\eta_C} - \zeta_R\right)\right)$	$\Omega_C^{\text{IC}} = \left\{ \begin{array}{l} 0 \leq S_C \leq S_{C\max} \\ S_{R\min} \leq S_R \leq S_{R\max} \\ S_C - \eta_C \zeta_R \geq 0 \end{array} \right\}$
False alarm probability $\mathcal{F}^{\text{IC}}(\gamma_R) = 1 - \mathbb{P}(\mathcal{L}_I, \psi_F^{\text{IC}}; \Omega_F^{\text{IC}})$	$\psi_F^{\text{IC}}(s) = \exp(s(\gamma_R - \zeta_C))$	$\Omega_F^{\text{IC}} = \left\{ \begin{array}{l} 0 \leq S_C \leq S_{C\max} \\ S_{R\min} \leq S_R \leq S_{R\max} \\ \zeta_C \geq \gamma_R \end{array} \right\}$
Detection probability $\mathcal{D}^{\text{IC}}(\gamma_R) = 1 - \mathbb{P}(\mathcal{L}_I, \psi_D^{\text{IC}}; \Omega_D^{\text{IC}})$	$\psi_D^{\text{IC}}(s) = \exp(s(\gamma_R - S_R - \zeta_C))$	$\Omega_D^{\text{IC}} = \left\{ \begin{array}{l} 0 \leq S_C \leq S_{C\max} \\ S_{R\min} \leq S_R \leq S_{R\max} \\ \gamma_R - S_R - \zeta_C \geq 0 \end{array} \right\}$
Success probability $\mathcal{S}^{\text{IC}}(\eta_R) = \mathbb{P}(\mathcal{L}_I, \psi_S^{\text{IC}}; \Omega_S^{\text{IC}})$	$\psi_S^{\text{IC}}(s) = \exp\left(s\left(\frac{S_R}{\eta_R} - \zeta_C\right)\right)$	$\Omega_S^{\text{IC}} = \left\{ \begin{array}{l} 0 \leq S_C \leq S_{C\max} \\ S_{R\min} \leq S_R \leq S_{R\max} \\ \zeta_C - \frac{S_R}{\eta_R} \geq 0 \end{array} \right\}$
JRDCCP $\mathcal{J}_D^{\text{IC}}(\eta_C, \gamma_R) = \mathbb{P}(\mathcal{L}_I, \psi_{D1}^{\text{IC}}; \Omega_{D1}^{\text{IC}})$ $+ \mathbb{P}(\mathcal{L}_I, \psi_{D2}^{\text{IC}}; \Omega_{D2}^{\text{IC}}) - \mathfrak{E}_{\Omega_{D2}^{\text{IC}}} \left[\frac{1}{2} \right]$	$\psi_{D1}^{\text{IC}}(s) = \psi_C^{\text{IC}}(s)$ $\psi_{D2}^{\text{IC}}(s) = \psi_C^{\text{IC}}(s) - \psi_D^{\text{IC}}(s)$	$\Omega_{D1}^{\text{IC}} = \left\{ \begin{array}{l} 0 \leq S_C \leq S_{C\max} \\ S_{R\min} \leq S_R \leq S_{R\max} \\ \gamma_R - S_R - \zeta_C \leq 0 \\ S_C/\eta_C - \zeta_R \geq 0 \end{array} \right\}$ $\Omega_{D2}^{\text{IC}} = \left\{ \begin{array}{l} 0 \leq S_C \leq S_{C\max} \\ S_{R\min} \leq S_R \leq S_{R\max} \\ \gamma_R - S_R - \zeta_C \geq 0 \\ S_C/\eta_C - \zeta_R \geq \gamma_R - S_R - \zeta_C \end{array} \right\}$
JRSCCP $\mathcal{J}_S^{\text{IC}}(\eta_C, \eta_R) = \mathbb{P}(\mathcal{L}_I, \psi_{S1}^{\text{IC}}; \Omega_{S1}^{\text{IC}})$ $+ \mathbb{P}(\mathcal{L}_I, \psi_{S2}^{\text{IC}}; \Omega_{S2}^{\text{IC}})$	$\psi_{S1}^{\text{IC}}(s) = \psi_C^{\text{IC}}(s)$ $\psi_{S2}^{\text{IC}}(s) = \psi_S^{\text{IC}}(s)$	$\Omega_{S1}^{\text{IC}} = \left\{ \begin{array}{l} 0 \leq S_C \leq S_{C\max} \\ S_{R\min} \leq S_R \leq S_{R\max} \\ 0 \leq S_C/\eta_C - \zeta_R \leq S_R/\eta_R - \zeta_C \end{array} \right\}$ $\Omega_{S2}^{\text{IC}} = \left\{ \begin{array}{l} 0 \leq S_C \leq S_{C\max} \\ S_{R\min} \leq S_R \leq S_{R\max} \\ 0 \leq S_R/\eta_R - \zeta_C \leq S_C/\eta_C - \zeta_R \end{array} \right\}$

Table 6.3 Computation of the different metrics with interference cancellation.

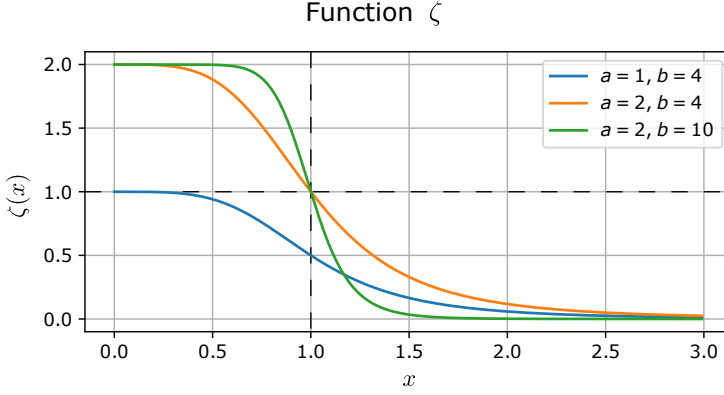


Fig. 6.9 Function ζ of Box 6.21 for different parameters a and b .

Fréchet upper and lower bounds of the joint metrics, or the conditional metrics directly following Section 6.1.4.3 and 6.1.4.4.

Figure 6.10 and 6.11 illustrate the impact of partial and perfect interference cancellation on the JRSCCP. The parameters of the ζ -function have been set for instance to $a = 2$, and $b = 4$. With this arbitrary choice, the residual interference ζ_R (resp. ζ_C) at the communication function (resp. radar function) is increased when the power ratio S_R/S_C (resp. S_C/S_R) is smaller than 1, to a maximum of $2S_R$ (resp. $2S_C$). Moreover, around 85% of the interference is cancelled when $S_R/S_C = 2$ (resp. $S_C/S_R = 2$), as illustrated in Figure 6.9.

With perfect IC, the performance drastically increases since the interfering power I is in most of the cases lower than the radar and communication powers S_R and S_C . Instead, with imperfect interference cancellation, at high radar (resp. communication) SIR thresholds, it is likely that the radar (resp. communication) signal is dominant for the metric to be non-zero. In that situation, the IC performs well at the communication (resp. radar) function, but the interference level is potentially increased at the other function since $a > 1$. Therefore, depending on the considered pair of thresholds, the performance may either be improved or worsened by the IC scheme. Nevertheless, in the considered scenario, for most pairs of SIR thresholds, this leads to performance improvement. Finally, when partial or perfect IC is introduced, condition Box 6.20 does not apply. The coupling of both functions becomes limited. It therefore enables to reach much higher radar and communication requirements.

6.2.4.2 Multi-Lane V2X Scenario

Following the system model of [Fan+20a], one may generalise the two-lane scenario to an M -lane scenario, by adapting the PDFs of S_R and S_C , and the LT of the interference I . The new scenario is illustrated in Figure 6.12. Let us now denote by d

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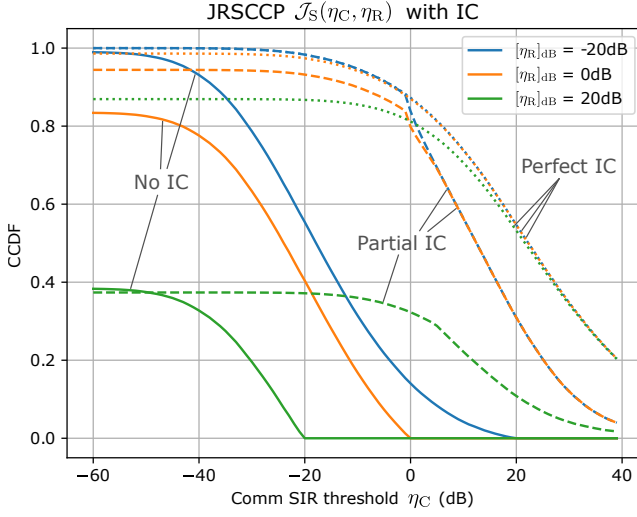


Fig. 6.10 JRSCCP without IC, with partial IC ($a = 2$ and $b = 4$) and with perfect IC, for fixed radar SIR thresholds $\eta_R = \{-20, 0, 20\}$ dB.

the width of one lane, and by d_L the perpendicular distance between the RSUs and the centre of the nearest lane. In the two-lane scenario, $d_l = d$, and $d_C = d_L$. The typical vehicle is assumed to be located in the m^{th} lane starting from the centre of the road.

First, for the communication signal transmitted by the nearest RSUs, the perpendicular distance d_C is now computed as

$$d_C = d_L + (M - m)d.$$

The minimum distance $r_{C\min}$ and the PDF of S_C are then adapted by plugging the new distance d_C in (6.6) and (6.14).

Then, for the interference, the perpendicular distance $d_{l,n}$, starting from the centre of the road to the n^{th} lane, is given by

$$d_{l,n} = (m + n - 1)d.$$

The minimum distance $r_{l\min,n}$ from the centre of the street to the n^{th} interfering lane is then obtained by plugging the new distance $d_{l,n}$ in (6.6). The total interference at the typical vehicle is thus given by

$$I = \sum_{n=1}^M \sum_{i \mid x_i \in \Phi_{l,n}} \rho_l (x_i^2 + d_{l,n}^2)^{-\frac{\alpha_l}{2}} |h_{i,n}|^2,$$

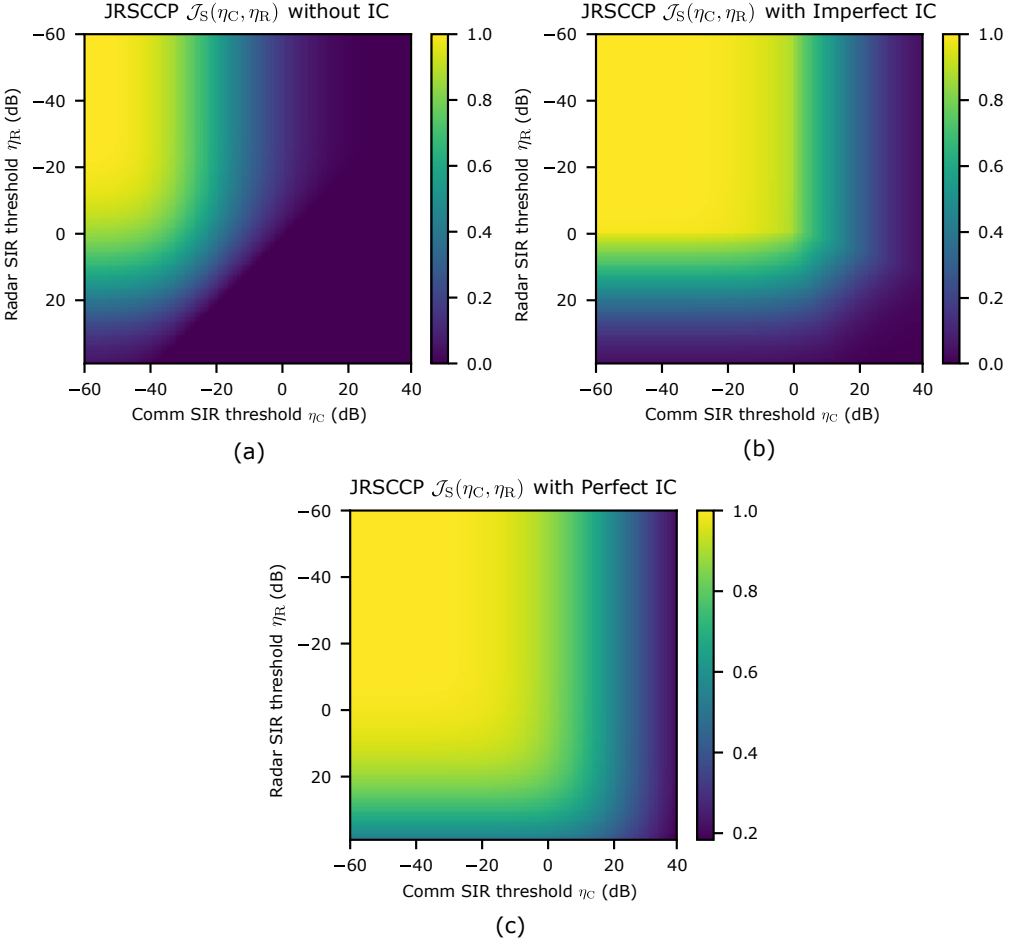


Fig. 6.11 JRSCCP without IC (a), with partial IC (b, $a = 2$ and $b = 4$), and with perfect IC (c).

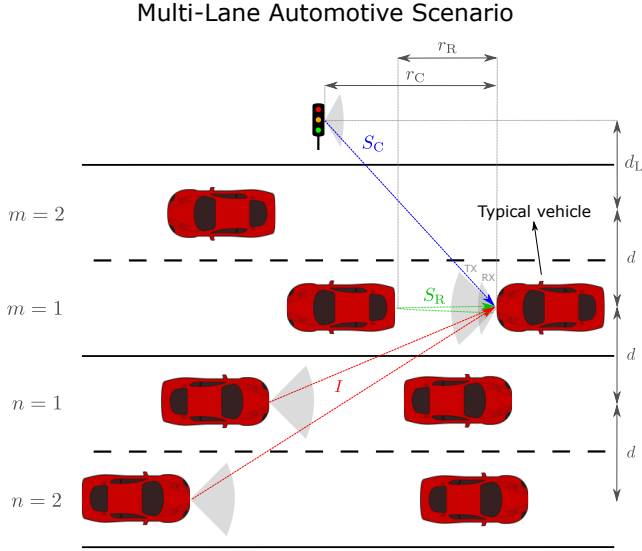


Fig. 6.12 Multi-lane V2X scenario.

where $\Phi_{l,n}$ are independent PPPs modelling the interfering nodes on each lane, and $|h_{i,n}|^2$ are the independent small-scale fading associated with each interfering link. Based on that, the LT of the total interference is computed following the development of (6.11):

$$\mathcal{L}_I(s) = \exp \left(-\lambda_I \sum_{n=1}^M \int_{r_{\min,n}}^{\infty} \left[1 - \mathcal{L}_{|h_{i,n}|^2} \left(s \rho_I(r^2 + d_{l,n})^{-\frac{\alpha_I}{2}} \right) \right] dr \right).$$

This result is obtained owing to the independence between the different interfering PPPs and small-scale fading RVs.

Additional improvements of the model could include numerous other sources of interference, as considered in [Fan+20b; KHF21], and the blockage induced by the vehicles, which is analysed in [CB22].

6.2.5 Generalisation to other Point Processes

In order to generalise the framework to integrate other PPs, the PDFs of S_R , S_C and the LT of I should be adapted to the considered PP. Owing to the high mathematical complexity potentially introduced by other PPs, these functions can be approximated if needed. For instance, [Mar+16] approximates the LT of the interference using a sigmoidal approximation, or following a transformed distribution approach. Another interesting proposal is presented in [DWX18] to approximate any PP as two inhomogeneous PPPs through inhomogeneous double thinning. In that case, an equivalent

network model is introduced based on two independent inhomogeneous PPPs to approximate the original PP and obtain tractable results. The introduction of inhomogeneous PPPs in the proposed system model in Section 6.2.1 is then straightforward.

6.2.6 Generalisation to Other Scenarios

The different metrics presented in Section 6.1 can be applied to any JCAS scenario. Moreover, the developments of Section 6.2.2 are straightforwardly translated to any full-duplex scenario, as the ones detailed in Section 1.1.2 of Chapter 1, i.e. scenarios in which an ISAC system simultaneously receives a radar and an uplink communication signal while suffering from interference.

6.3 Comparison with Ray-Tracing Simulations

In order to assess the relevancy of the proposed system model, RT simulations have been carried out. The RT framework is assumed to be the closest framework to the ground truth. Therefore, both the comparison of the received powers, and the evaluation of sensing and communication metrics, are interesting to assess the relevancy of the SG system model and the developed expressions.

First, the differences between the system model of the two frameworks are highlighted in Section 6.3.1. Then, to reduce the discrepancies between the two system models, extensions of the SG system model are proposed In Section 6.3.2. Next, a parameters fitting is performed in Section 6.3.3 with the generated RT dataset to extract the parameters of the system model for the SG framework. Finally, the power statistics and the sensing and communication performance metrics computed in the different frameworks are compared in Sections 6.3.4 and 6.3.5.

6.3.1 Generation of the Environment

Following the system model proposed in Section 6.2.1, the environment, illustrated in Figure 6.13, has been generated in the RT simulations. The main differences in the environment are the following:

- Buildings are modelled along the streets as rectangular blocks. Perpendicular streets are generated at coordinates following a PPP. This generates additional multipath in the RT simulations. Thus, for the radar link, these should be filtered out to isolate the useful radar echo, arising from the next vehicle ahead, w.r.t. the other radar echoes from the environment.
- Vehicles are modelled as rectangular blocks. Stacking vehicles are removed from the street. Thus, the position of the vehicles is better modelled by HCPPs instead of PPPs. Still, as discussed in Section 5.1.1.4, HCPPs are often intractable, and the PPP is a good approximation if the nodes density is adapted accordingly.

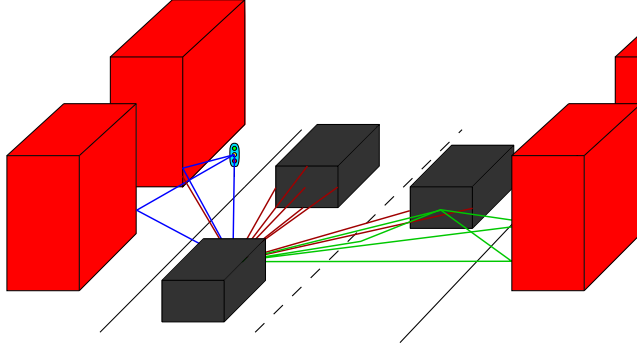


Fig. 6.13 Illustration of the **RT** scenario. Vehicles are represented in black, and buildings in red. The **RSU** is represented by a traffic light. Blue, red and green rays respectively correspond to communication, radar, and interference links.

- Vehicles are positioned in both lanes on the street following two **PPPs** with the same density. The interfering vehicles are selected by performing a thinning with a given probability of interference. Therefore, the vehicles in the opposite lane are not all interfering, but they may still act as obstacles.
- The transmitting and receiving antennas are located at non-zero heights. For the **ISAC** systems of the vehicles, they are both located at the front of the vehicles, but slightly separated from each other.

Three examples of simulated scenarios are shown in Figure 6.14. The parameters of the **RT** simulations are summarised in Table 6.4.

6.3.2 Monte-Carlo System Model

To further improve the **SG** system model of Section 6.2.1, two additional features are added:

- **LoS** and **NLoS** propagations are considered. Owing to the presence of the numerous vehicles on both lanes, and buildings acting like obstacles, both **LoS** and **NLoS** propagations occur for the communication and interfering links. Therefore, for these links, the propagation parameters $\{\alpha_{v,l}\}$ and $\{\beta_{v,l}\}$ with $v \in \{C, I\}$ and $l \in \{L, N\}$ are instead defined for **LoS** or **NLoS** links. The probability for the considered link to be in **LoS** is denoted as $p_{L,v}(r)$, with r the distance between the two nodes. To determine which nodes are in **LoS** or **NLoS**, independent thinnings are performed on the **PPPs** of the **RSUs** and interfering vehicles, with probability $p_{L,v}(r)$. The **LoS** probability model considered in this chapter is the **3GPP** d_1/d_2 model [3GP15]:

$$p_{L,v}(r) = \min\left(\frac{d_{1,v}}{r}, 1\right) \left[1 - \exp\left(-\frac{r}{d_{2,v}}\right)\right] + \exp\left(-\frac{r}{d_{2,v}}\right), \quad (6.10)$$

Examples of Ray-Tracing Simulated Scenarios

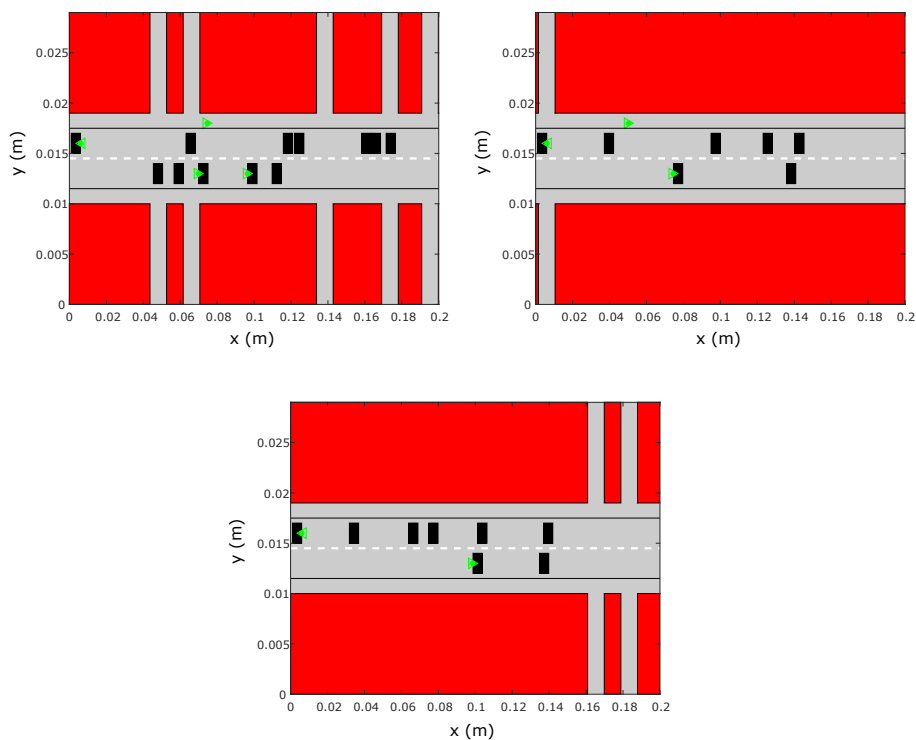


Fig. 6.14 Three examples of simulated RT scenarios. Vehicles are represented in black, buildings in red, and antennas in green.

Parameters	Values
3D position of the radar TX antenna	(6.1, 16, 0.5) m
3D position of the radar RX antenna	(6.1, 16.2, 0.5) m
3D position of the RSUs TX antenna	(6.1 + r_C , 18, 2.5) m
3D position of the interfering TX antennas	(6.1 + r_I , 13, 0.5) m
Streets length	2 km
Streets width	9 m
Perpendicular streets density	10 km ⁻¹
RSUs density λ_C	10 km ⁻¹
Vehicles density λ_R	20 km ⁻¹
Interfering vehicles density λ_I	2 km ⁻¹
Carrier frequency f_c	26 GHz
Transmitted powers P_V and P_L	20 dBm
Beamforming gains G_R , G_C and G_I	1

Table 6.4 Parameters of the RT simulations. The distances 6.1 + r_C and 6.1 + r_I are computed from the PPPs of the RSUs and interfering vehicles.

with $\{d_{1,v}\}$ and $\{d_{2,v}\}$ four fitting parameters.

- Small-scale fading is considered for every link. Even if it is supposed that the small-scale fading is highly Ricean, the introduction of such RV helps improve the accuracy of the models fitting. It is therefore assumed that Ricean small-scale fading affects both the radar and communication links, in addition to the interfering links, with K -factors to determine from the RT simulations.

Since these features affects the mathematical tractability of the models, we name the enhanced framework the *Monte-Carlo system model*, in opposition to the SG system model, which designates the one detailed in Section 6.2.1.

6.3.3 Generated Data and Parameters Fitting

From the dataset generated with the RT simulations, the parameters of the system model for the SG and MC framework are extracted. To perform this estimation, the different nodes densities are first estimated by averaging the number of nodes over the different network realisations. Instead of obtaining the densities of Table 6.4, the densities extracted from the dataset are equal to 18.6 km⁻¹ for the vehicles, and 1.79 km⁻¹ for the interfering nodes. As expected, owing to the removal of overlapping vehicles, the vehicles density is reduced, leading to lower λ_R and λ_I .

Then, for the path-loss parameters, **Least Square Estimation (LSE)** in the log-log domain is applied. More specifically, denoting by $\{S_{v,n}^{RT}\}$ with $v \in \{R, C, I\}$ the received

		Mixed	LoS	NLoS
Radar	$\hat{\alpha}_R$	1.74	—	—
	$[\hat{\beta}_R]_{\text{dB}}$	3.72	—	—
Comm.	$\hat{\alpha}_C$	2.20	1.53	2.32
	$[\hat{\beta}_C]_{\text{dB}}$	0.42	3.37	0.40
Interference	$\hat{\alpha}_I$	4.64	1.12	4.35
	$[\hat{\beta}_I]_{\text{dB}}$	0	14.41	0

Table 6.5 Propagation parameters obtained through **LSE** fitting of the ray tracing data, for the radar, communication and interfering links. Either **LoS** and **NLoS** links are separated, or they are mixed. The parameters of the radar links have been computed considering only the power of the radar echo of the next vehicle ahead, whereas the environment echoes are filtered out of the dataset.

powers for each link, associated with the distances $\{r_{v,n}\}$, the **LSE** problem is written for the considered linear path-loss model as

$$\hat{\alpha}_v, \hat{\beta}_v = \arg \min_{\alpha_v, \beta_v} \sum_{n=0}^{N-1} \left([S_{v,n}^{\text{RT}}]_{\text{dB}} - [\beta_v]_{\text{dB}} - \alpha_v [r_n]_{\text{dB}} \right)^2,$$

where N denotes the number of simulated **RT** scenarios. This problem is solved by computing

$$\begin{bmatrix} \hat{\alpha}_v \\ [\beta_v]_{\text{dB}} \end{bmatrix} = \begin{bmatrix} \sum_{n=0}^{N-1} [r_n]_{\text{dB}} & N \\ \sum_{n=0}^{N-1} [r_n]_{\text{dB}}^2 & \sum_{n=0}^{N-1} [r_n]_{\text{dB}} \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n=0}^{N-1} [S_{v,n}^{\text{RT}}]_{\text{dB}} \\ \sum_{n=0}^{N-1} [S_{v,n}^{\text{RT}}]_{\text{dB}} [r_n]_{\text{dB}} \end{bmatrix}.$$

Note that, for the interference, the power $S_{I,n}^{\text{RT}}$ stands for the power of the interference coming from one single vehicle. The total interference I is obtained by summing the power from each interfering vehicle within a simulated scenario. Furthermore, **LSE** can be applied after separating the **LoS** and **NLoS** links, or mixing all the links together. The estimated parameters are summarised in Table 6.5. Low path-loss exponents are estimated for the radar and communication links, owing to the canyon-like structure of the street, even with the presence of perpendicular streets. Contrariwise, the path-loss exponent of the interfering link is very high, owing to the strong blockage and high **NLoS** probability induced by the vehicles driving on both lanes, as later illustrated in Figure 6.18.

Figure 6.15 to 6.17 illustrates the received powers for the radar, communication, and interfering links, when **LoS** and **NLoS** links are separated or not. In order to compute the received powers, all the paths comprising up to two interactions with the environment are considered. Diffraction on the edges of the buildings and vehicles is only considered at the last interaction. For the radar link, two cases are

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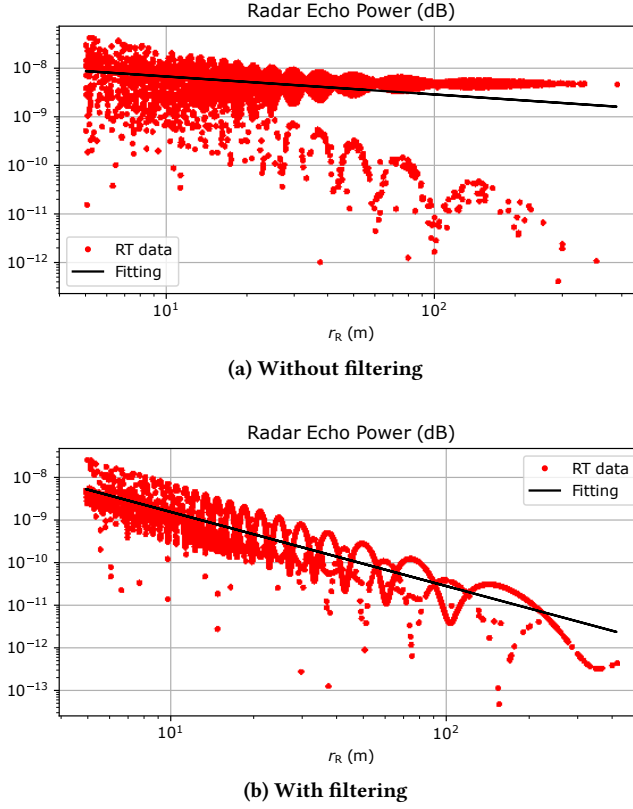


Fig. 6.15 Received radar echo power with **RT** simulations, and fitting, with and without the clutter filtered out.

illustrated: either the echoes from the environment are kept, or they are filtered out. The echoes from the environment comprise all the rays which does not interact with the next vehicle ahead. This encompasses diffraction on building edges, or reflection and diffraction on other vehicles. Since these rays do not interact with the next vehicle ahead, this corresponds to all the high power data points in Figure 6.15a which do not decrease with the distance r_R . Cancelling these echoes leads to Figure 6.15b, which illustrates the power of the radar echo of the next vehicle ahead. Oscillations of the power with the distance r_R are explained by diffractions on the edges of the vehicle, with or without reflections on the buildings, which generates the observed interference pattern. Such interference patterns are also observed in Figures 6.16 and 6.17 generated by the same cause.

Regarding the LoS probability models, the distances $\{d_{1,v}\}$ with $v \in \{C, I\}$ are set equal to the minimum distances $r_{v\min}$ of the communication and interfering links.

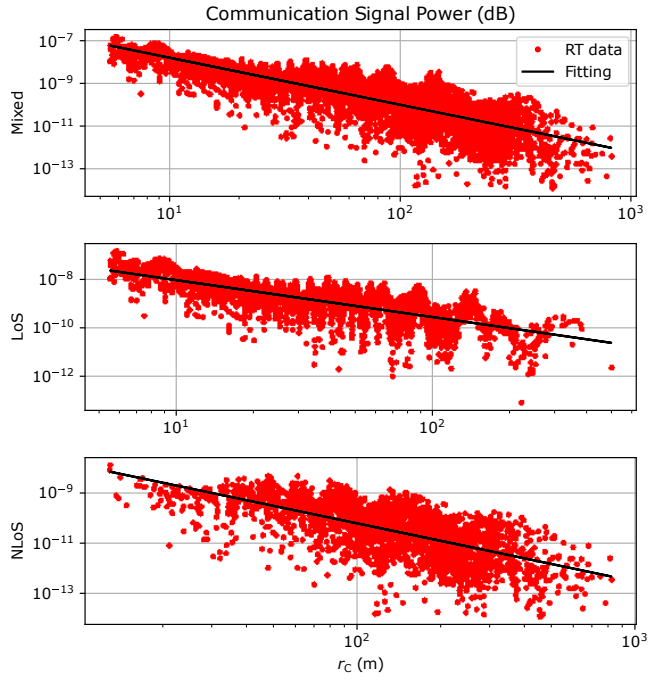


Fig. 6.16 Received communication signal power with RT simulations, and fitting.

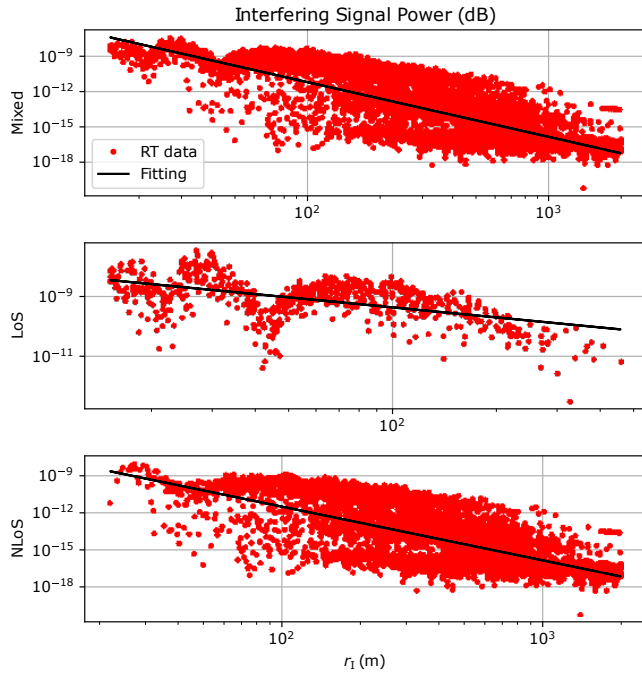


Fig. 6.17 Received interfering signals power with RT simulations, and fitting.

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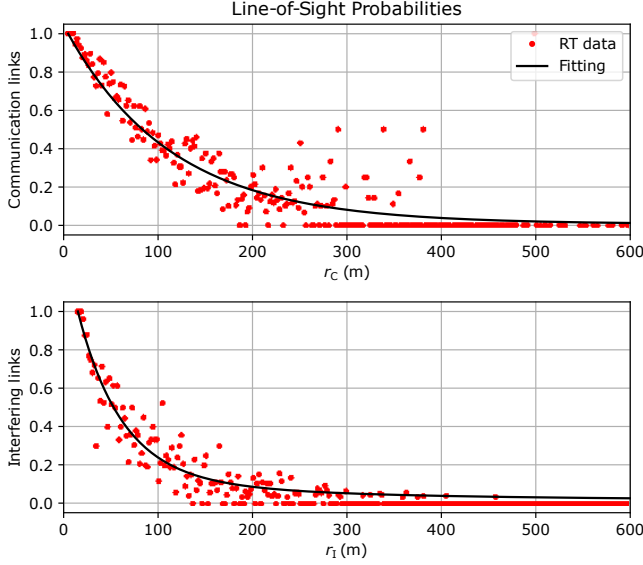


Fig. 6.18 LoS probabilities for the communication and interfering links.

The distances $\{d_{2,v}\}$ are instead computed by solving another LSE problem:

$$\hat{d}_{2,v} = \arg \min_{d_{2,v}} \sum_{n=0}^{N-1} \left(p_{Lv,n}^{\text{RT}} - p_{Lv}(r_{v,n}; d_{1,v}, d_{2,v}) \right)^2,$$

where $\{p_{Lv,n}^{\text{RT}}\}$ are the LoS probabilities of each link, estimated from the RT data, and associated with the distances $\{r_{v,n}\}$, and p_{Lv} is the LoS probability model of (6.10). The fitting illustrated in Figure 6.18 leads to $\hat{d}_{2,C} = 110.44$ m for the communication link, and $\hat{d}_{2,I} = 43.95$ m for the interfering links. Indeed, the estimated distance is shorter for the interfering links, since the interfering signals encounters vehicles driving in both lanes, while the communication link is only blocked by vehicles driving on the same lane as the vehicle of interest.

Finally, for the small-scale fading K -factors, they are estimated by computing the ratio between the powers from the RT data and the estimated path-loss models at the same distance. This enables to isolate the fading component of the link. Performing this operation has shown that the small-scale fading of the three links can be modelled with Rayleigh fading, i.e. $K \approx 0$.

6.3.4 Comparison of the Powers Distributions

Figure 6.19 compares the PDF of the radar, communication and interfering powers, obtained with the RT simulations, the SG system model of Section 6.2.1, and the MC system model of Section 6.3.2. First, the same order of magnitudes are observed for

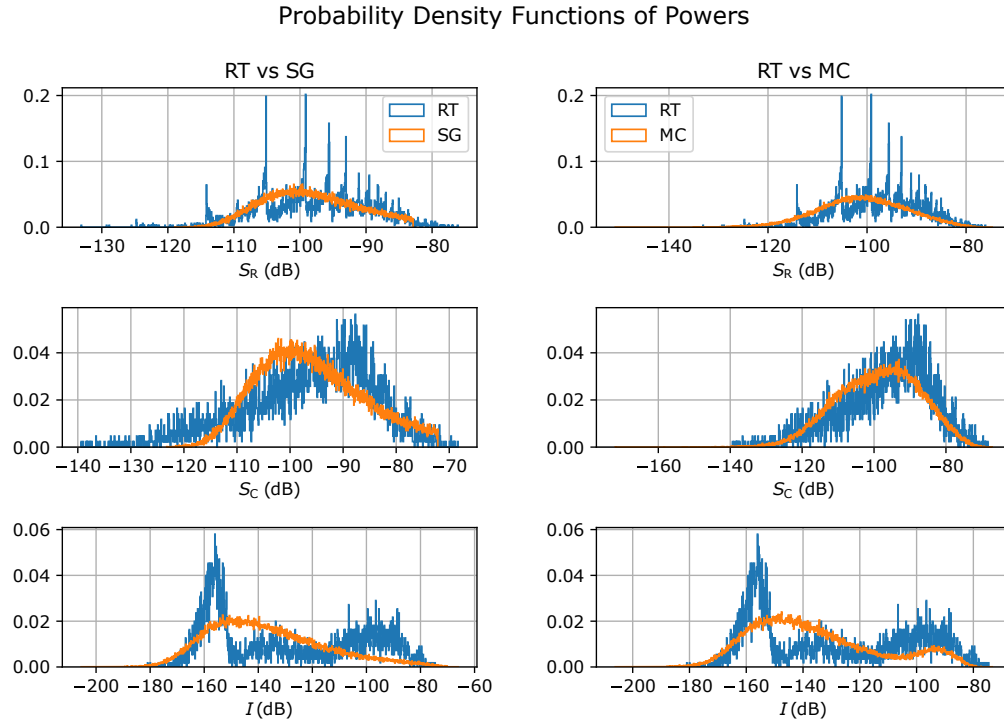


Fig. 6.19 PDFs of the radar, communication and interfering powers, with the RT, SG and MC system models.

the three powers with the three models. Regarding the radar power, the shape of the PDF is consistent in the three cases, except for the multiple peaks observed in the RT simulations. This is less accurate for the communication link, owing to the blockage of the vehicles which are not taken into account in the SG model. Nonetheless, the introduction of fading RVs and LoS probability models enable the MC model to achieve a more accurate shape. The discontinuities related to the maximum power levels of the SG models vanish with the introduction of the fading RVs. Unfortunately, neither the SG nor MC model succeeds to reproduce the shape of the interfering power. With the MC model, the introduction of the LoS probability enables to model accurately the high power peak in the PDF, but the blockage of the vehicles narrows the low power peak around -160 dB in the RT simulations, whereas the low power peak of the SG and MC models are very wide. A decrease of the vehicles density could therefore help better model the interfering power, as it would reduce the blockages.

6.3.5 Comparison of the Metrics

First, univariate metrics, namely the communication coverage, radar success and radar detection probabilities, are compared with the three system models in Figure 6.20. For the coverage and success probabilities, a good fitting is achieved, even with the differences in the interference modelling. The metrics obtained with the RT and MC system models are very close to each other, whereas the SG system model overestimates slightly the slope. However, for the detection probability, larger differences are observed. In that case, both the SG and the MC model underestimates the detection probability. Contrariwise to the other metrics, the MC model gives lower detection probabilities than the two other models.

Unfortunately, contrariwise to the univariate metrics, larger discrepancies between the three system models are observed in Figure 6.21 both for the JRDCCP and the JRSCCP. Still, for the JRSCCP, the performance achieved with the MC system model seems closer to the ones achieved with the RT system model, as expected. Moreover, whatever the considered system model, the condition of Box 6.20 is still observed.

6.4 Conclusion

The evaluation of relevant performance metrics is critical for the design of future wireless networks, including both sensing and communication features. For that purpose, the SG framework provides mathematical tools enabling to compute analytically the average performance in given scenarios, at the price of additional assumptions in the system model. In this chapter, popular performance metrics from the SG literature have been presented for both functions. Yet, the performance of the radar and communication functions are highly correlated. This is encompassed by the definition of joint metrics, evaluating simultaneously the performance of both functions.

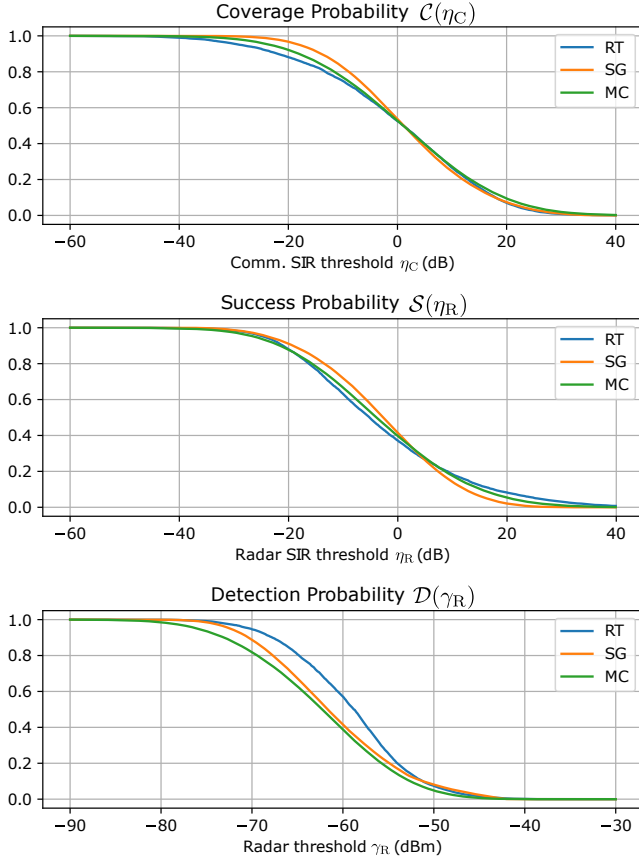


Fig. 6.20 Coverage, success and detection probabilities.

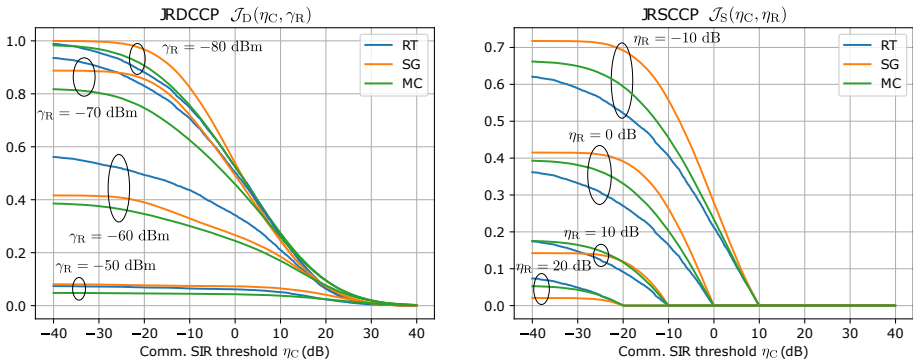


Fig. 6.21 JRDCCP and JRSCCP, with the RT, SG and MC system models.

6.4.1 Summary of the Contributions

The contributions of this chapter are summarised as follows:

- Two novel joint metrics have been proposed, i.e. the [JRDCCP](#) and [JRSCCP](#), enabling to jointly evaluate the radar and communication performance. Moreover, conditional metrics have also been detailed, enabling to evaluate radar or communication performance, conditioned on the requirement for the other function. Upper and lower bounds, function of the univariate metrics, have also been also proposed. Closed form expressions of the joint and univariate metrics have been obtained for a full-duplex [V2X JCAS](#) scenario, where a vehicle simultanelously detects the next vehicle ahead, while receiving a communication signal from the infrastructure, and interference from other vehicles. From the expressions of the [JRSCCP](#), a trade-off between both functions has been highlighted: increasing the [SIR](#) threshold requirement for one function requires to decreases the threshold of the other function, for the metric to be non-zero. Nevertheless, even if a vehicular scenario has been analysed in this chapter, the obtained expressions are easily extended for other full-duplex [ISAC](#) scenarios, as detailed in Section 1.1.2 of Chapter 1. Through the analysis of the joint metrics, performance trade-offs between the radar and communication functions have been highlighted. Optimisation on the vehicles and [RSUs](#) transmitted powers, and [RSUs](#) densities, has also been performed.
- The proposed definitions have been extended to include interference cancellation capabilities. The framework is compatible with any model of residual interference at the radar or communication function, provided that it depends only on the powers of the radar echo and communication signal. The introduction of interference cancellation enables to further increase the [SIR](#) threshold requirements of both functions, without nullifying the metric. Apart from interference cancellation, other generalisations have also been discussed, i.e. the use of other [PPs](#), or the extension to multi-lane scenarios.
- In order to compare the communication and sensing performance evaluated with different system models, [RT](#) simulations have been carried out. Instead of only evaluating the performance achieved with the [SG](#) expressions, extensions of the system model have also been proposed as a third option, to compute more accurately the performance through [MC](#) simulations. After performing a fitting of the densities and propagation parameters, it has been shown that the [SG](#) and [MC](#) system models seem relevant w.r.t. [RT](#) simulations for the evaluation of univariate sensing and communication metrics. Nonetheless, for the proposed joint metrics, the results obtained with the two other system models are rather inaccurate, but the same trends are observed with the three models.

6.4.2 Future Works

The comparison between the **SG** framework and the **RT** simulations has shown small discrepancies for the evaluation of the average univariate performance, but larger errors have been observed for the joint metrics. Therefore, the model mismatches between the two frameworks must first be further investigated. For instance, the blockage by other vehicles, the reflections on the ground or the diffraction mechanisms are not taken into account. It is maybe possible to slightly complexify the **SG** model, in order to improve the accuracy of the evaluated **JCAS** performance, with an acceptable increase of complexity.

Then, the residual function for the interference cancellation model has been inferred from logical deductions, and it can be shown that it is coherent with the results obtained for the interference cancellation schemes proposed in Chapter 9. Nonetheless, the achieved performance are difficult to analyse, as the definition of the residual seems sensitive to the considered waveform and scenarios. This abstraction is difficult to model accurately, motivating further investigations of the performance gains at each function obtained thanks to the interference cancellation, with specific **ISAC** systems, outside the **SG** framework. Such analysis is done in Chapter 9. Besides, deeper analyses of the residuals for different **ISAC** systems could be performed, to further develop the residual functions. For instance, different functions may be considered for sensing and communication, or it may depend on the considered scenario.

Finally, three generalisations have been detailed in Section 6.2.4, namely the extension to roads with multiple lanes, the introduction of other PPs, and the generalisation to other full-duplex scenarios. Further investigations of these proposals may be considered.

6.4.3 What Is Next?

The performance of **ISAC** systems has been evaluated in this chapter, in a full-duplex **V2X** scenario, using the **SG** framework. Such application corresponds to what could happen in the streets of future smart cities, where vehicles and the infrastructure may gather information about the environment, and communicate. Thus, this raises multiple questions:

1. In such scenario, multiple nodes are equipped with **ISAC** systems. In that case, since the nodes of the infrastructure embedding such capabilities are mutually connected through backhaul links, would they be able to cooperate with each other? How would that impact the achieved communication and sensing performance?
2. Simple interference models are considered in this chapter, within one street under study. How to model a complete city in a larger scale within the **SG**

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framework? In such case, which additional interference mechanisms should be modelled?

This is the main topic of Chapter 7, which tries to answer these two questions.

Appendix - Chapter 6

6.A Laplace Transform of the Interference

The **LT** of the interference I is related to the **PGFL** of the **PPP** Φ_I . Following (5.1),

$$\begin{aligned}
 \mathcal{L}_I(s) &= \mathbb{E}_{\Phi_I, \{|h_i|^2\}_{i \in \mathbb{N}}} \left[\exp \left(-s \sum_{i \mid x_i \in \Phi_I} \rho_I (x_i^2 + d_I^2)^{-\frac{\alpha_I}{2}} |h_i|^2 \right) \right] \\
 &= \mathbb{E}_{\Phi_I, \{|h_i|^2\}_{i \in \mathbb{N}}} \left[\prod_{i \mid x_i \in \Phi_I} \exp \left(-s \rho_I (x_i^2 + d_I^2)^{-\frac{\alpha_I}{2}} |h_i|^2 \right) \right] \\
 &\stackrel{(a)}{=} \mathbb{E}_{\Phi_I} \left[\prod_{i \mid x_i \in \Phi_I} \mathbb{E}_{|h_i|^2} \left[\exp \left(-s \rho_I (x_i^2 + d_I^2)^{-\frac{\alpha_I}{2}} |h_i|^2 \right) \right] \right] \\
 &\stackrel{(b)}{=} \exp \left(-\lambda_I \int_{r_{\min}}^{\infty} \left[1 - \mathcal{L}_{|h_i|^2} \left(s \rho_I (r^2 + d_I^2)^{-\frac{\alpha_I}{2}} \right) \right] dr \right), \quad (6.11)
 \end{aligned}$$

where (a) is obtained using the independence of the small-scale fading **RVs**, and (b) is obtained using the definition of the **PGFL** in Box 5.8. In this expression, the **LT** of the small-scale fading $|h_i|^2$ appears.

The small-scale fading is supposed to be Ricean-distributed. Let us denote by ν^2 the power of the dominant path, and σ^2 the power of the scattered paths. The small-scale fading is expressed in Cartesian coordinates as $|h_i|^2 = X_i^2 + Y_i^2$, where $X_i \sim \mathcal{N}(\nu \cos \theta, \sigma^2)$ and $Y_i \sim \mathcal{N}(\nu \sin \theta, \sigma^2)$ are two independent and normally distributed **RVs**, with $\theta \in [0, 2\pi[$. By the properties of the normal distribution, X_i^2/σ^2 and Y_i^2/σ^2 follow two non-centralised χ^2 distributions with one degree of freedom [Pat49]. Therefore, using the **LT** of the χ^2 distribution, the **LT** of X^2 and Y^2 are computed as

$$\begin{aligned}
 \mathcal{L}_{X^2}(s) &= \exp \left(-\frac{\nu^2 \cos^2 \theta s}{1 + 2\sigma^2 s} \right) (1 + 2\sigma^2 s)^{-\frac{1}{2}}, \\
 \mathcal{L}_{Y^2}(s) &= \exp \left(-\frac{\nu^2 \sin^2 \theta s}{1 + 2\sigma^2 s} \right) (1 + 2\sigma^2 s)^{-\frac{1}{2}},
 \end{aligned}$$

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and the **LT** of $|h_i|^2$ is finally obtained as

$$\mathcal{L}_{|h_i|^2}(s) = \mathcal{L}_{X^2}(s) \mathcal{L}_{Y^2}(s) = \exp\left(-\frac{v^2 s}{1 + 2\sigma^2 s}\right) (1 + 2\sigma^2 s)^{-1}.$$

To express it as a function of the K -factor, when the small-scale fading power is normalised,

$$v^2 \triangleq \frac{K}{K+1}, \quad 2\sigma^2 \triangleq \frac{1}{K+1},$$

leading to

$$\mathcal{L}_{|h_i|^2}(s) = \exp\left(-\frac{K s}{K+1+s}\right) \frac{K+1}{K+1+s}. \quad (6.12)$$

6.B Probability Density Functions of S_R and S_C

To obtain the **PDFs** of S_R and S_C , the first step is to obtain the **PDFs** of r_R and r_C .

Probability density function of r_R : The probability to have at least one vehicle in the radar detectable range is given by

$$\mathbb{P}_{\psi_{\Phi_R}}(\psi_{\Phi_R}([r_{Rmin}, r_{Rmax}]) > 0) = 1 - V_{\Phi_R}([r_{Rmin}, r_{Rmax}]) = 1 - e^{-\lambda_R(r_{Rmax} - r_{Rmin})},$$

where V_{Φ_R} is the void probability of the **PPP**, given in Box 5.6. Next, using the Bayes rule, the **CCDF** of r_R assuming that there is at least one vehicle in the radar detection range is developed as

$$\begin{aligned} \bar{F}_{r_R}(r) &= \mathbb{P}_{r_R|\psi_{\Phi_R}}(r_R > r \mid \psi_{\Phi_R}([r_{Rmin}, r_{Rmax}]) > 0) \\ &= \mathbb{P}_{r_R|\psi_{\Phi_R}}(\psi_{\Phi_R}([0, r]) = 0 \mid \psi_{\Phi_R}([r_{Rmin}, r_{Rmax}]) > 0) \\ &= \frac{\mathbb{P}_{r_R, \psi_{\Phi_R}}(\psi_{\Phi_R}([0, r]) = 0, \psi_{\Phi_R}([r_{Rmin}, r_{Rmax}]) > 0)}{\mathbb{P}_{\psi_{\Phi_R}}(\psi_{\Phi_R}([r_{Rmin}, r_{Rmax}]) > 0)}. \end{aligned}$$

three cases can be distinguished:

- When $r \leq r_{Rmin}$, the two subsets $[0, r]$ and $[r_{Rmin}, r_{Rmax}]$ are disjoint, and by the disjoint sets property of **PPPs**, one obtains

$$\bar{F}_{r_R}(r) = \mathbb{P}_{\psi_{\Phi_R}}(\psi_{\Phi_R}([0, r]) = 0) = 1.$$

- When $r \geq r_{Rmax}$, then $[r_{Rmin}, r_{Rmax}] \subseteq [0, r]$. As the two events of the numerator are incompatible, one obtains

$$\bar{F}_{r_R}(r) = 0$$

- When $r_{Rmin} \leq r \leq r_{Rmax}$, the expression of $\bar{F}_{r_R}$ is developed as

$$\begin{aligned} \bar{F}_{r_R}(r) &= \frac{\mathbb{P}_{r_R, \psi_{\Phi_R}}(\psi_{\Phi_R}([r_{Rmin}, r]) = 0, \psi_{\Phi_R}([r, r_{Rmax}]) > 0)}{1 - e^{-\lambda_R(r_{Rmax} - r_{Rmin})}} \\ &\stackrel{(a)}{=} \frac{\mathbb{P}_{r_R, \psi_{\Phi_R}}(\psi_{\Phi_R}([r_{Rmin}, r]) = 0) \mathbb{P}_{r_R, \psi_{\Phi_R}}(\psi_{\Phi_R}([r, r_{Rmax}]) > 0)}{1 - e^{-\lambda_R(r_{Rmax} - r_{Rmin})}} \\ &= \frac{e^{-\lambda_R(r - r_{Rmin})} (1 - e^{-\lambda_R(r_{Rmax} - r)})}{1 - e^{-\lambda_R(r_{Rmax} - r_{Rmin})}} = \frac{e^{-\lambda_R(r - r_{Rmin})} - e^{-\lambda_R(r_{Rmax} - r_{Rmin})}}{1 - e^{-\lambda_R(r_{Rmax} - r_{Rmin})}}, \end{aligned}$$

where (a) is obtained by the disjoint sets property of PPPs.

The PDF of r_R (assuming that there is a vehicle in the radar detectable range) is finally obtained by deriving the CCDF:

$$f_{r_R}(r) = -\frac{d\bar{F}_{r_R}}{dr_R}(r) = \frac{\lambda_R e^{-\lambda_R(r - r_{Rmin})}}{1 - e^{-\lambda_R(r_{Rmax} - r_{Rmin})}} \mathbb{1}(r_{Rmin} \leq r \leq r_{Rmax}).$$

Probability density function of r_C : The PDF of r_C is obtained similarly as the PDF of r_R , except that there is no maximum communication range, i.e.

$$\mathbb{P}_{\psi_{\Phi_C}}(\psi_{\Phi_C}([r_{Cmin}, \infty[) > 0) = 1.$$

The CCDF of r_C is developed as

$$\bar{F}_{r_C}(r) = \mathbb{P}_{r_C}(\psi_{\Phi_C}([0, r]) = 0).$$

Two cases are distinguished:

- When $r \leq r_{Cmin}$, one obtains $\bar{F}_{r_C}(r) = 1$.
- When $r \geq r_{Cmin}$, the expression of $\bar{F}_{r_C}$ is developed as

$$\bar{F}_{r_C}(r) = \mathbb{P}_{r_C}(\psi_{\Phi_C}([r_{Cmin}, r]) = 0) = e^{-\lambda_C(r - r_{Cmin})}.$$

The PDF of r_C is then obtained by deriving the CCDF:

$$f_{r_C}(r) = -\frac{d\bar{F}_{r_C}}{dr_C}(r) = \lambda_C e^{-\lambda_C(r - r_{Cmin})} \mathbb{1}(r \geq r_{Cmin}).$$

Probability density functions of S_R and S_C : Now that the PDFs of r_R and r_C have been computed, the PDFs of S_R and S_C can be obtained using the change of variable formula for a univariate PDF with a monotonic and invertible transformation [Eva+08].

For S_R , by inverting (6.7), the transformation g_R is defined as

$$g_R(s) \triangleq \rho_R^{\frac{1}{\alpha_R}} s^{-\frac{1}{\alpha_R}},$$

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one obtains

$$\begin{aligned} f_{S_R}(s) &= f_{r_R}(g_R(s)) \left| \frac{dg_R}{ds}(s) \right| \\ &= \frac{g_R(s)}{\alpha_R s} \frac{\lambda_R e^{-\lambda_R(g_R(s)-r_{Rmin})}}{1 - e^{-\lambda_R(r_{Rmax}-r_{Rmin})}} \mathbb{1}(S_{Rmin} \leq s \leq S_{Rmax}). \end{aligned} \quad (6.13)$$

For S_C , by inverting (6.2.1.2), the transformation g_C is defined as

$$g_C(s) = \sqrt{\rho_C^{\frac{2}{\alpha_C}} s^{-\frac{2}{\alpha_C}} - d_C^2},$$

one obtains

$$\begin{aligned} f_{S_C}(s) &= f_{r_C}(g_C(s)) \left| \frac{dg_C}{ds}(s) \right| \\ &= \frac{g_C^2(s) + d_C^2}{\alpha_C g_C(s) s} \lambda_C e^{-\lambda_C(g_C(s)-r_{Cmin})} \mathbb{1}(0 \leq s \leq S_{Cmax}). \end{aligned} \quad (6.14)$$

6.C Joint Radar Detection and Communication Coverage Probability

In the considered scenario, the **JRDCCP** of Box 6.13 is rewritten as

$$\mathcal{F}_D(\eta_C, \gamma_R) = \mathbb{P}_{S_C, S_R, I} \left(\frac{S_C}{S_R + I} \geq \eta_C, S_R + S_C + I \geq \gamma_R \right). \quad (6.15)$$

For the communication function, the interference comprises signals from vehicles driving in the opposite lane, and the radar echo from the next vehicle ahead. For the radar function, a detection occurs when the total received power, comprising the communication signal transmitted by the **RSU**, is higher than a predefined threshold. It can be developed as

$$\begin{aligned} \mathcal{F}_D(\eta_C, \gamma_R) &= \mathbb{P}_{S_C, S_R, I} \left(\frac{S_C}{S_R + I} \geq \eta_C, S_R + S_C + I \geq \gamma_R \right) \\ &= \mathbb{E}_{S_C, S_R} \left[\mathbb{P}_{I|S_C, S_R} \left(\frac{S_C}{S_R + I} \geq \eta_C, S_R + S_C + I \geq \gamma_R \right) \right] \\ &= \mathbb{E}_{S_C, S_R} \left[\mathbb{P}_{I|S_C, S_R} \left(\gamma_R - S_R - S_C \leq I \leq \frac{S_C}{\eta_C} - S_R \right) \right]. \end{aligned}$$

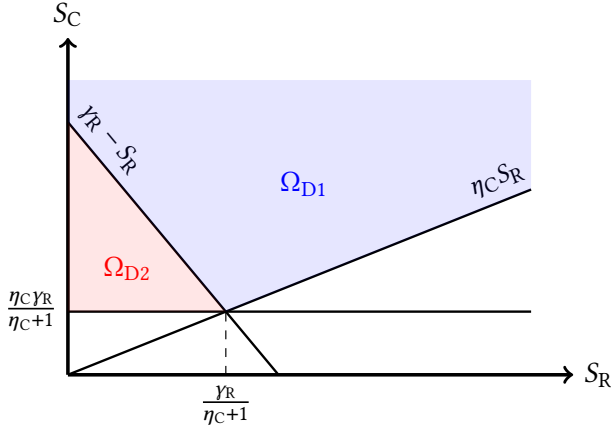


Fig. 6.22 Illustration of the regions Ω_{D1} and Ω_{D2} .

This probability is non-zero only when the pairs (S_C, S_R) are comprised in the domains Ω_{D1} and Ω_{D2} , defined as

$$\Omega_{D1} \triangleq \left\{ (S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad \begin{array}{l} \gamma_R - S_R - S_C \leq 0 \\ \frac{S_C}{\eta_C} - S_R \geq 0 \end{array} \right\},$$

$$\Omega_{D2} \triangleq \left\{ (S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad \begin{array}{l} \gamma_R - S_R - S_C \geq 0 \\ \frac{S_C}{\eta_C} - S_R \geq \gamma_R - S_R - S_C \end{array} \right\},$$

which can also be developed as

$$\Omega_{D1} = \left\{ (S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad S_C \geq \max(\eta_C S_R, \gamma_R - S_R) \right\},$$

$$\Omega_{D2} = \left\{ (S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad \frac{\eta_C \gamma_R}{\eta_C + 1} \leq S_C \leq \gamma_R - S_R \right\}.$$

These regions are illustrated in Figure 6.22.

Applying the Gil-Pelaez theorem of Box 6.19 in Ω_{D1} , one obtains

$$\mathbb{P}_{I|S_C, S_R} \left(I \leq \frac{S_C}{\eta_C} - S_R \right) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{1}{\tau} \text{Im} \left\{ \phi_I(\tau) e^{-j\tau \left(\frac{S_C}{\eta_C} - S_R \right)} \right\} d\tau,$$

where ϕ_I is the characteristic function of the interference. Similarly, in Ω_{D2} , one

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obtains

$$\begin{aligned}
 & \mathbb{P}_{I|S_C, S_R} \left(\gamma_R - S_R - S_C \leq I \leq \frac{S_C}{\eta_C} - S_R \right) \\
 &= \mathbb{P}_{I|S_C, S_R} \left(I \leq \frac{S_C}{\eta_C} - S_R \right) - \mathbb{P}_{I|S_C, S_R} (I \leq \gamma_R - S_R - S_C) \\
 &= \frac{1}{\pi} \int_0^\infty \frac{1}{\tau} \operatorname{Im} \left\{ \phi_I(\tau) \left[e^{-j\tau(\gamma_R - S_R - S_C)} - e^{-j\tau\left(\frac{S_C}{\eta_C} - S_R\right)} \right] \right\} d\tau.
 \end{aligned}$$

Therefore, knowing that the characteristic function of the interference is related to its [LT](#) as $\phi_I(\tau) = \mathcal{L}_I(-j\tau)$, the [JRDCCP](#) metric is computed using the operators of [6.2.2.1](#) as

$$\mathcal{J}_D(\eta_C, \gamma_R) = \mathfrak{P}(\mathcal{L}_I, \psi_{D1}; \Omega_{D1}) + \mathfrak{P}(\mathcal{L}_I, \psi_{D2}; \Omega_{D2}) - \mathfrak{E}_{\Omega_{D2}} \left[\frac{1}{2} \right],$$

where the functions ψ_{D1} and ψ_{D2} are defined as

$$\begin{aligned}
 \psi_{D1}(s) &\triangleq e^{s\left(\frac{S_C}{\eta_C} - S_R\right)}, \\
 \psi_{D2}(s) &\triangleq e^{s\left(\frac{S_C}{\eta_C} - S_R\right)} - e^{s(\gamma_R - S_R - S_C)}.
 \end{aligned}$$

Note that the operator $\mathfrak{E}$ is computed the expectation over S_R and S_C , requiring the [PDFs](#) of [\(6.13\)](#) and [\(6.14\)](#).

6.D Joint Radar Success and Communication Coverage Probability

In the considered scenario, the [JRSCCP](#) of [Box 6.14](#) is rewritten as

$$\mathcal{J}_S(\eta_C, \eta_R) = \mathbb{P}_{S_C, S_R, I} \left(\frac{S_C}{S_R + I} \geq \eta_C, \frac{S_R}{S_C + I} \geq \eta_R \right). \quad (6.16)$$

For the communication function, the interference comprises signals from vehicles driving in the opposite lane, and the radar echo from the next vehicle ahead. For the radar function, the interference comprises signals from vehicles driving in the opposite lane, and the communication signal transmitted by the [RSU](#). It can be developed as

$$\begin{aligned}
 \mathcal{J}_S(\eta_C, \eta_R) &= \mathbb{P}_{S_C, S_R, I} \left(\frac{S_C}{S_R + I} \geq \eta_C, \frac{S_R}{S_C + I} \geq \eta_R \right) \\
 &= \mathbb{E}_{S_C, S_R} \left[\mathbb{P}_{I|S_C, S_R} \left(\frac{S_C}{S_R + I} \geq \eta_C, \frac{S_R}{S_C + I} \geq \eta_R \right) \right] \\
 &= \mathbb{E}_{S_C, S_R} \left[\mathbb{P}_{I|S_C, S_R} \left(I \leq \frac{S_C}{\eta_C} - S_R, I \leq \frac{S_R}{\eta_R} - S_C \right) \right]
 \end{aligned}$$

$$= \mathbb{E}_{S_C, S_R} \left[\mathbb{P}_{I|S_C, S_R} \left(I \leq \min \left(\frac{S_C}{\eta_C} - S_R, \frac{S_R}{\eta_R} - S_C \right) \right) \right].$$

This probability is non-zero only when the pairs (S_C, S_R) fulfil the following conditions:

$$\begin{cases} \frac{S_C}{\eta_C} - S_R \geq 0 \\ \frac{S_R}{\eta_R} - S_C \geq 0 \end{cases} \Leftrightarrow \eta_C S_R \leq S_C \leq \frac{S_R}{\eta_R}.$$

This can only be fulfilled if

$$\eta_C \eta_R \leq 1 \quad \Leftrightarrow \quad [\eta_C]_{\text{dB}} + [\eta_R]_{\text{dB}} \leq 0 \text{ dB}. \quad (6.17)$$

Equation (6.17) highlights a trade-off between the communication and radar functions: it is impossible to achieve simultaneously arbitrarily high radar and communication SINR thresholds requirements, the useful power of one function being detrimental for the other. This condition seems too restrictive to enable both functions to work simultaneously. Nonetheless, one may instead consider the SINR of the decision variable in the JRSCCP. In that case, the threshold of 0 dB is shifted by the processing gains of both functions, enabling to achieve correct performance for both functions.

If (6.17) is fulfilled, one can define the domains Ω_{S1} and Ω_{S2} as

$$\begin{aligned} \Omega_{S1} &\triangleq \left\{ (S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad 0 \leq \frac{S_C}{\eta_C} - S_R \leq \frac{S_R}{\eta_R} - S_C \right\}, \\ \Omega_{S2} &\triangleq \left\{ (S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad 0 \leq \frac{S_R}{\eta_R} - S_C \leq \frac{S_C}{\eta_C} - S_R \right\}, \end{aligned}$$

which can also be rewritten as

$$\begin{aligned} \Omega_{S1} &\triangleq \left\{ (S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad \eta_C S_R \leq S_C \leq \frac{\eta_C(\eta_R + 1)}{\eta_R(\eta_C + 1)} S_R \right\}, \\ \Omega_{S2} &\triangleq \left\{ (S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad \frac{\eta_C(\eta_R + 1)}{\eta_R(\eta_C + 1)} S_R \leq S_C \leq \frac{S_R}{\eta_R} \right\}. \end{aligned}$$

These regions are illustrated in Figure 6.23.

Applying the Gil-Pelaez theorem of Box 6.19 in Ω_{S1} , one obtains

$$\mathbb{P}_{I|S_C, S_R} \left(I \leq \frac{S_C}{\eta_C} - S_R \right) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{1}{\tau} \text{Im} \left\{ \phi_I(\tau) e^{-j\tau \left(\frac{S_C}{\eta_C} - S_R \right)} \right\} d\tau.$$

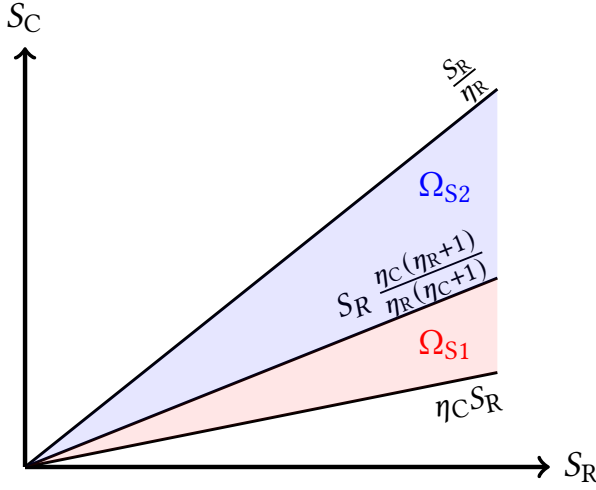


Fig. 6.23 Illustration of the regions Ω_{S1} and Ω_{S2} .

Similarly, in Ω_{S2} , one obtains

$$\mathbb{P}_{I|S_C, S_R} \left(I \leq \frac{S_R}{\eta_R} - S_C \right) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{1}{\tau} \text{Im} \left\{ \phi_I(\tau) e^{-j\tau \left(\frac{S_R}{\eta_R} - S_C \right)} \right\} d\tau.$$

Therefore, the JRDCCP metric is computed using the operators of 6.2.2.1 as

$$\mathcal{J}_S(\eta_C, \eta_R) = \mathfrak{P}(\mathcal{L}_I, \psi_{S1}; \Omega_{S1}) + \mathfrak{P}(\mathcal{L}_I, \psi_{S2}; \Omega_{S2}),$$

where the functions ψ_{S1} and ψ_{S2} are defined as

$$\begin{aligned} \psi_{S1}(s) &\triangleq e^{s \left(\frac{S_C}{\eta_C} - S_R \right)}, \\ \psi_{S2}(s) &\triangleq e^{s \left(\frac{S_R}{\eta_R} - S_C \right)}. \end{aligned}$$

6.E False Alarm Probability

In the considered scenario, the radar false alarm probability of Box 6.9 is rewritten as

$$\mathcal{F}(\gamma_R) = \mathbb{P}_{S_C, S_R, I} (S_C + I \geq \gamma_R). \quad (6.18)$$

This corresponds to the total received power at the radar function when there is no radar echo, comprising the communication signal transmitted by the RSU and the

interfering signals. It can be developed as

$$\begin{aligned}\mathcal{F}(\gamma_R) &= \mathbb{P}_{S_C, S_R, I} (S_C + I \geq \gamma_R) \\ &= \mathbb{E}_{S_C, S_R} [\mathbb{P}_{I|S_C, S_R} (S_C + I \geq \gamma_R)] \\ &= 1 - \mathbb{E}_{S_C, S_R} [\mathbb{P}_{I|S_C, S_R} (I \leq \gamma_R - S_C)].\end{aligned}$$

This probability is non-zero only when the pairs (S_C, S_R) are comprised in the domain Ω_F defined as

$$\Omega_F \triangleq \{(S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad S_C \leq \gamma\}.$$

Applying the Gil-Pelaez theorem of Box 6.19 in Ω_F , one obtains

$$\mathbb{P}_{I|S_C, S_R} (I \leq \gamma_R - S_C) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{1}{\tau} \text{Im} \left\{ \phi_I(\tau) e^{-j\tau(\gamma_R - S_C)} \right\} d\tau.$$

Therefore, the false alarm probability is computed using the operators of 6.2.2.1 as

$$\mathcal{F}(\gamma_R) = 1 - \mathfrak{P}(\mathcal{L}_I, \psi_F; \Omega_F),$$

where the function ψ_F is defined as

$$\psi_F(s) \triangleq e^{s(\gamma_R - S_C)}.$$

6.F Coverage Probability

The communication coverage probability can be obtained from either the [JRDCCP](#) or [JRSCCP](#) following respectively (6.2) or (6.4). In both cases, the domains Ω_{D1} and Ω_{S1} tends to be equal to Ω_C defined as

$$\Omega_C \triangleq \{(S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad \eta_C S_R \leq S_C\}.$$

Defining the function

$$\psi_C(s) \triangleq \psi_{D1}(s) = \psi_{S1}(s),$$

the communication coverage probability is computed as

$$C(\eta_C) = \mathfrak{P}(\mathcal{L}_I, \psi_C; \Omega_C).$$

6.G Detection Probability

The radar detection probability can be obtained from the [JRDCCP](#) following (6.3). The domain Ω_{D2} tends to be equal to Ω_D defined as

$$\Omega_D \triangleq \{(S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad S_C \leq \gamma_R - S_R\}.$$

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Following the same approach as for the false alarm probability, the radar detection probability is computed as

$$\mathcal{D}(\gamma_R) = 1 - \mathfrak{P}(\mathcal{L}_I, \psi_D; \Omega_D),$$

where the function ψ_D is defined as

$$\psi_D(s) \triangleq e^{s(\gamma_R - S_R - S_C)}.$$

6.H Success Probability

The radar success probability can be obtained from the JRSCCP following (6.5). The domain Ω_{S_2} tends to be equal to Ω_S defined as

$$\Omega_S \triangleq \left\{ (S_C, S_R) \in \mathbb{R}^+ \times \mathbb{R}^+ \quad \text{s.t.} \quad S_C \leq \frac{S_R}{\eta_R} \right\}.$$

Defining the function

$$\psi_S(s) \triangleq \psi_{S_2}(s),$$

the radar success probability is computed as

$$\mathcal{S}(\eta_R) = \mathfrak{P}(\mathcal{L}_I, \psi_S; \Omega_S).$$

Communication and Sensing Performance of Cooperative Networks in Poisson Line Processes

7

This chapter is a work carried out with the help of Charles Wiame, which I again thanks warmly. Most of the content is inspired from [De +23]. An extended version of this paper is in preparation [De +24b].

RADAR and communication performance have been thoroughly investigated in Chapter 6, in which univariate and joint metrics are proposed for the analysis of both functions. Whereas this analysis has been performed at the street level, one may be interested in the evaluation of radar and communication performance at the city scale. Indeed, ISAC systems are a promising technology for future smart cities, e.g. for the safety of road users (vehicles, cyclists or pedestrians), or for autonomous driving [Zha+22]. Nevertheless, at the city level, multiple additional aspects must be considered. For instance, the nodes of the infrastructure, connected through backhaul links, may cooperate with each other to improve the overall communication and sensing performance. Furthermore, complex interference mechanisms should be modelled, e.g. diffraction on the streets, reflections on obstacles, or building penetrations. The configuration of the streets is also affecting the useful and undesired power levels at the node of interest. Therefore, the SG framework seems appealing for the modelling of the cities infrastructure and the nodes locations, also enabling an analytical evaluation of average sensing and communication performance.

This chapter analyses the performance of JCAS cooperative networks at a city scale, within the SG framework. First, Section 7.1 summarises existing works related to cooperative networks and cities modelling in SG. Then, Section 7.2 details all the models and metrics considered in this chapter. These metrics are next developed in Section 7.3. Finally, the achieved sensing and communication performance with cooperation are evaluated in Section 7.4. Optimisation of the network parameters is also performed.

Works	Function	Vehicular	Processes	Cooperation
[EAA20; Pap+21; Che+22; PD23; GSK23]	Comm.	✗	PPP	CoMP
[Sun+24]	Comm.	✗	PCP	CoMP
[Sun+20; JH21; Wan+20; PW22a; QKA24]	Comm.	✓	PLP	✗
[PW22b; CB23]	Comm.	✓	PLP (+ PPP)	Relaying
[SDD21]	Comm.	✓	PLP	CoMP
[SPK20]	Radar + Comm.	✗	PPP	CoMP
[SPK22]	Radar + Comm.	✗	β -Ginibre	CoMP
This chapter	Radar + Comm.	✓	PLP	CoMP

Table 7.1 Non-exhaustive list of multiple papers considering **PLPs** and/or **CoMP** systems in the literature, compared to this chapter.

7.1 Poisson Line Processes and Cooperative Networks in the Literature

On the one hand, as detailed in Section 5.3 of Chapter 5, **PLP** are especially suited for the modelisation of complete cities. On the other hand, **Cooperative Multi-Point (CoMP)** systems have also raised an increase interest in recent years, especially with the **Non-Orthogonal Multiple Access (NOMA)** protocol, and for cell-free networks. Numerous works have been published in these fields within the **SG** framework. Some recent publications have been summarised in Table 7.1. Among these, [SPK20] and [SPK22] introduce multiple cooperation rules for the radar function, which will be considered in this chapter. They are further described in Section 7.2.4.2.

7.2 System Model

This section presents the considered streets topology, cooperation rules, propagation models, and the cooperative sensing and communication performance metrics which will be developed in Section 7.3.

7.2.1 Streets Topology

The street deployment is modelled by means of a **PLP**, as also done in many other works presented in Section 7.1. The **PLP** can be generated from the following process:

$$\Xi_S = \{(y_i, \theta_i) \in \mathbb{R}^+ \times [0, 2\pi[, i \in \mathbb{N}\},$$

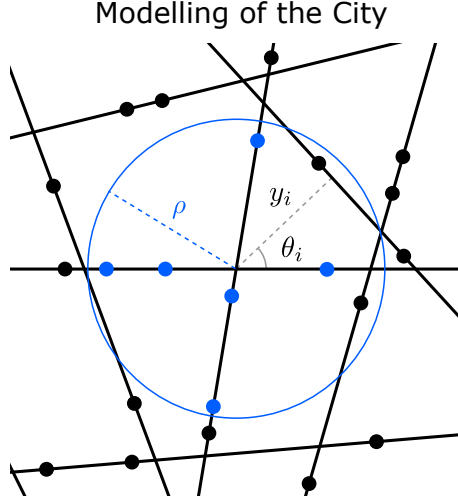


Fig. 7.1 Illustration of a PLP modelling the streets topology.

where y_i are the radial distances from the origin, and θ_i are the counter-clockwise angles between the horizontal axis, and the line formed by the projection of the origin on the street, as illustrated in Figure 5.3. Nonetheless, the performance of a typical node is strongly impacted by its location in the city, e.g. along a street, or in a crossroad. Thus, one may compute overall metrics with a weighted average, depending on the probability to be located in a crossroad of $N_S + 1$ lanes.

For a crossroad of $N_S + 1$ streets, the first $N_S + 1$ streets are associated with radial distances $y_i = 0$, $i = 0, \dots, N_S$. The other radial distances are distributed following a PPP Φ_S of density λ_S . The angle θ_0 is equal to $\pi/2$, such that the first street is always oriented horizontally. The other angles θ_i , $\forall i \geq 0$ are uniformly distributed in $[0, 2\pi[$ to form a Cox PLP. As described in Section 5.3 of Chapter 5, the Manhattan PLP is obtained as a particular case of the Cox PLP. Finally, nodes are distributed along the streets following one-dimensional PPPs Φ_{vi} of densities λ_{vi} . These nodes may model vehicles, RSU, pedestrians, or other devices located in the streets. Such model is illustrated in Figure 7.1. Denoting by $x \in \Phi_{vi}$ the position of a node in the i^{th} street, the absolute position $\mathbf{p}$ of the node in the city is computed as

$$\mathbf{p} = \begin{pmatrix} \cos \theta_i & \sin \theta_i \\ \sin \theta_i & -\cos \theta_i \end{pmatrix} \begin{pmatrix} y_i \\ x \end{pmatrix}. \quad (7.1)$$

7.2.2 Cooperation Rule

In this chapter, we assume that the nodes in Ξ_S model RSUs, which are connected to a Central Processing Unit (CPU) through the backhaul network. These RSUs are

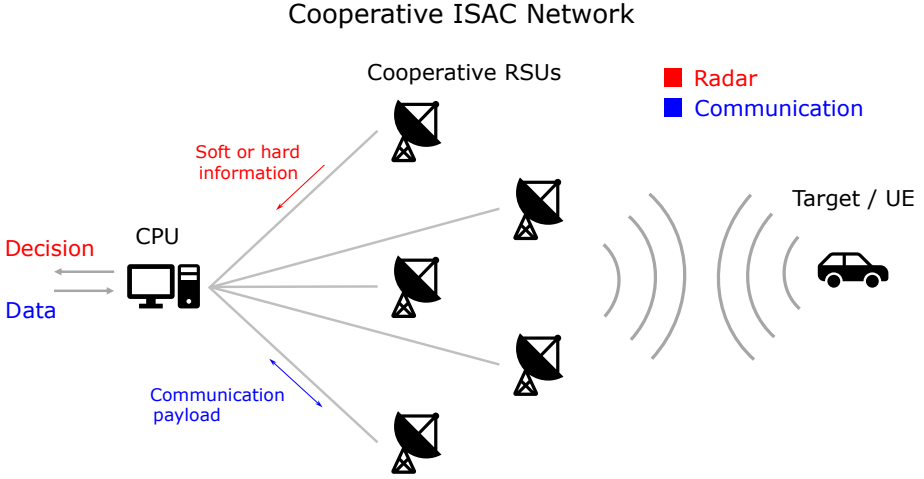


Fig. 7.2 Illustration of the cooperative sensing and communication process. Information is transmitted towards UEs from all the RSUs, whereas a decision is drawn at the CPU based on the RSUs radar outputs about the presence of targets.

equipped with ISAC systems, and all RSUs located at a distance lower than ρ in the streets of the crossroad are assumed to cooperate. The set of cooperative RSUs is denoted as $\mathcal{C} = \bigcup_{j=0}^{N_s} C_j$, with $C_j \triangleq \{x \in \Phi_{vj} \mid |x| \leq \rho\}$ the set of cooperative RSUs in the j^{th} street of the crossroad. This is also shown in Figure 7.1. From a communication perspective, they all transmit information to a typical vehicle located at the origin. From a sensing perspective, they receive radar echoes from the vehicle, and transmit soft or hard information to the CPU, which draws a conclusion on the presence of the vehicle. This cooperative ISAC process is illustrated in Figure 7.2.

7.2.3 Propagation Models

We assume a linear path-loss model, for instance the alpha-beta-gamma model of Box 3.5, presented in Section 3.1.4 of Chapter 3. The crossroad of interest is located at $(0, 0)$. Depending on the considered link and propagation mechanism, the received power at a node located at position $\mathbf{p}_1$ from a transmitted located at position $\mathbf{p}_2$, potentially located in two different roads forming an angle ψ , with an intersection at position χ , can be written in the form:

$$P = \kappa g(\mathbf{p}_1, \mathbf{p}_2, \chi, \psi) \gamma,$$

where γ is an exponentially distributed random variable modelling the small-scale fading of the link. The coefficient κ and functions g vary depending on the considered link, i.e. communication, radar, or interference.

7.2.3.1 Communication links

For the communication link between a cooperative **RSUs** at position $\mathbf{p}_1$ and the typical vehicle at position $\mathbf{p}_2 = (0, 0)$, the function g is defined as

$$g_C(\mathbf{p}_1) \triangleq \|\mathbf{p}_1\|^{-\alpha_C}, \quad (7.2)$$

where α_C denotes the path-loss exponent of the communication link.

7.2.3.2 Radar links

For the radar link between a cooperative **RSUs** at position $\mathbf{p}_1 = \mathbf{p}_2 = \mathbf{p}$ and the typical vehicle at position $(0, 0)$, the function g is defined as

$$g_R(\mathbf{p}) \triangleq \|2\mathbf{p}\|^{-\alpha_R}, \quad (7.3)$$

where α_R denotes the path-loss exponent of the radar link. A **GO** model is considered, as presented in Section 3.1.3 of Chapter 3, owing to the high frequencies and small distances considered in future **ISAC** networks.

7.2.3.3 Interfering links

Multiple interference sources may be modelled in the considered scenario. In this chapter, four different types of interference are considered. The path-loss of the interfering links is denoted as α_I .

Interference of type I: The interference of type I comprises all the direct interfering links between nodes located in the same street. As for the communication link, the function g is defined as

$$g_{I,I}(\mathbf{p}_1, \mathbf{p}_2) \triangleq \|\mathbf{p}_1 - \mathbf{p}_2\|^{-\alpha_I}. \quad (7.4)$$

Interference of type II: The interference of type II comprises all the unwanted reflection on scatterers located in a crossroad, reflected towards a **RSU**. The function g is defined as

$$g_{I,II}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{x}, \psi) \triangleq \left(\frac{1 + \cos \psi}{2} \right)^{v_r} (\|\mathbf{p}_1 - \mathbf{x}\| + \|\mathbf{p}_2 - \mathbf{x}\|)^{-\alpha_I}, \quad (7.5)$$

following the reflection model proposed in [Man12]. The parameter v_r defines the angular beamwidth of the reflective contribution.

Interference of type III: The interference of type III comprises all the unwanted diffraction on the corners of the crossroad, towards a **RSU**. The function g is defined

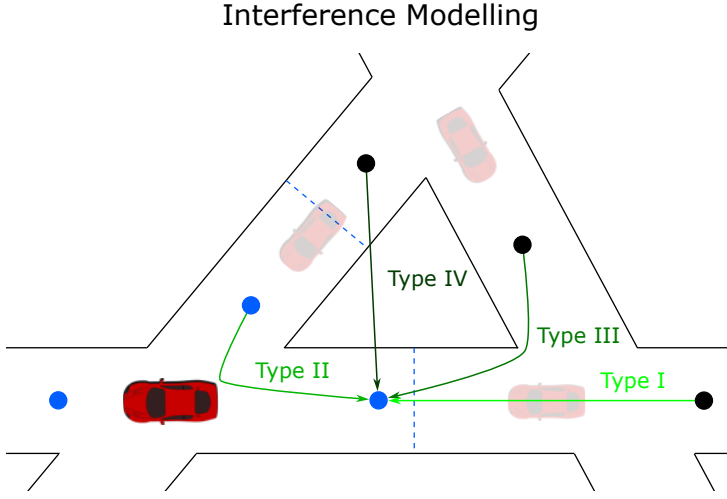


Fig. 7.3 Illustration of the four types of interference.

as

$$g_{\text{I,III}}(\mathbf{p}_1, \mathbf{p}_2, \boldsymbol{\chi}, \psi) \triangleq \left[\|\mathbf{p}_1 - \boldsymbol{\chi}\| + \|\mathbf{p}_2 - \boldsymbol{\chi}\| + k_b \left(\frac{2\psi}{\pi} \right)^{v_b} \|\mathbf{p}_1 - \boldsymbol{\chi}\| \|\mathbf{p}_2 - \boldsymbol{\chi}\| \right]^{-\alpha_1}, \quad (7.6)$$

following the Berg diffraction model proposed in [Ber95]. The parameter k_b is defined as $k_b = \sqrt{q_b f_c}/c$, and q_b and v_b are two empirical parameters.

Interference of type IV: The interference of type IV comprises all the indirect interference through penetration in the buildings. The function g is defined as

$$g_{\text{I,IV}}(\mathbf{p}_1, \mathbf{p}_2) \triangleq 10^{-\frac{2\delta}{10}(1+2\lambda_s\|\mathbf{p}_1-\mathbf{p}_2\|)} \|\mathbf{p}_1 - \mathbf{p}_2\|^{-\alpha_1}, \quad (7.7)$$

following the model proposed in [Dem23]. The parameter δ is the average penetration loss from the external walls of the buildings, and $2\lambda_s\|\mathbf{p}_1 - \mathbf{p}_2\|$ is the average number of walls crossed.

These four types of interference are illustrated in Figure 7.3. Note that the models proposed for the interference of type I and III have been compared to ray-tracing simulations in [Wia+23], showing similar results when the propagation parameters of these models, i.e. the coefficient κ_1 and the path-loss exponent α_1 , are fitted to the ray-tracing simulations.

Whatever the type of link, based on the angles θ_j, θ_l and radial distances y_j, y_l of the two crossing streets, the positions $\mathbf{p}_1$ and $\mathbf{p}_2$ can first be computed as function of the one-dimensional positions $z_j \in \Phi_{vj}$ and $z_l \in \Phi_{vl}$ within the streets using (7.1).

Next, the position of the crossroad χ can be computed from (7.1) by solving w.r.t. χ_j and χ_l the following system of equations:

$$\begin{cases} y_j \cos \theta_j + \chi_j \sin \theta_j = y_l \cos \theta_l + \chi_l \sin \theta_l, \\ y_j \sin \theta_j - \chi_j \cos \theta_j = y_l \sin \theta_l - \chi_l \cos \theta_l. \end{cases}$$

Then, using again (7.1):

$$\chi = \begin{pmatrix} \cos \theta_j & \sin \theta_j \\ \sin \theta_j & -\cos \theta_j \end{pmatrix} \begin{pmatrix} y_j \\ \chi_j \end{pmatrix} = \begin{pmatrix} \cos \theta_l & \sin \theta_l \\ \sin \theta_l & -\cos \theta_l \end{pmatrix} \begin{pmatrix} y_l \\ \chi_l \end{pmatrix}.$$

Finally, The angle ψ can be computed from the positions of the RSUs and the crossroad:

$$\psi = \arccos \frac{(\mathbf{p}_1 - \chi)^T (\mathbf{p}_2 - \chi)}{\|\mathbf{p}_1 - \chi\| \|\mathbf{p}_2 - \chi\|}.$$

Thus, the received power can be rewritten as

$$P = \kappa g(x_j, x_l, \theta_j, \theta_l, y_j, y_l) \gamma,$$

which explicitly involves the positions of the nodes within the streets, and streets topology. This formulation is therefore more suited for the following developments.

7.2.4 Cooperative Performance Metrics

As described in Section 7.2.2, multiple RSUs cooperate with each other to improve both radar and communication performance. Therefore, one may extend the radar and communication metrics detailed in Section 6.1 to cooperative networks.

7.2.4.1 Cooperative Communication

For the communication function, a **Non-Coherent Joint Transmission (NC-JT)** cooperation scheme is considered. Such scheme is for instance considered in [Tan+14]. In other words, the same information is transmitted from all the cooperative RSUs in a non-coherent manner.

Cooperative Coverage Probability: With the NC-JT scheme, one may extend the definition of Box 6.1, presented in Section 6.1.1.1 of Chapter 6, by taking into account the total useful communication power in the communication SINR. This leads to the following expression for the cooperative communication coverage probability of the typical vehicle located in the crossroad:

$$\mathcal{P}_C(\eta_C) \triangleq \mathbb{P} \left(\frac{\sum_{j=0}^{N_S} \sum_{i \in C_j} S_{Cji}}{I_C + W_C} \geq \eta_C \right), \quad (7.8)$$

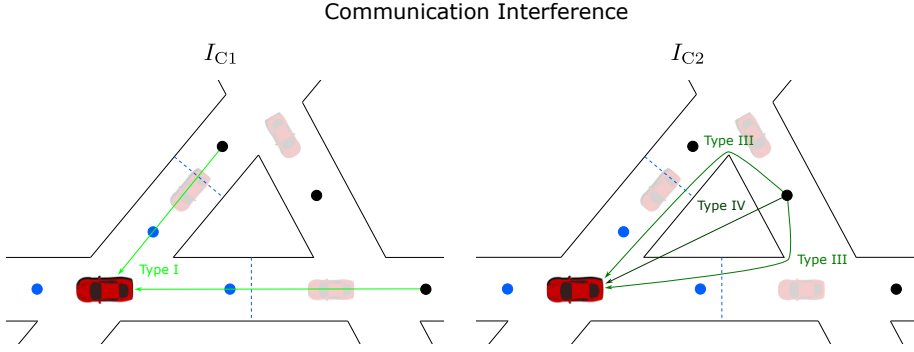


Fig. 7.4 Interference sources for the communication function.

where S_{Cji} is the received power from the i^{th} cooperative RSU in the j^{th} street, I_C is the interference power at the typical vehicle, and W_C is the constant noise power. Following (7.2), the useful communication power is expressed as

$$S_{Cji} = \kappa_C |x_{ji}|^{-\alpha_C} \gamma_{Cji},$$

with x_{ji} the position of the i^{th} cooperative RSU in the j^{th} street. The total communication interference power aggregates two interference sources, denoted as I_{C1} and I_{C2} , and illustrated in Figure 7.4:

1. The interference of type I from the RSUs in the streets belonging to the crossroad, which are not cooperating. Following (7.4), the first interference source is expressed as

$$I_{C1} = \sum_{j=0}^{N_S} \sum_{k|z_{jk} \in \Phi_{vj} \setminus C_j} \kappa_I |z_{jk}|^{-\alpha_I} \gamma_{C1I,jk}.$$

2. The interference of type III and IV from the RSUs in the streets not belonging to the crossroad. Following (7.6) and (7.7), the second interference source is expressed as

$$I_{C2} = \sum_{l=N_S+1}^{\infty} \sum_{k|z_{lk} \in \Phi_{vl}} \left[\sum_{j=0}^{N_S} \kappa_I g_{I,III}(0, z_{lk}, \theta_j, \theta_l, 0, y_l) \gamma_{C2III,jlk} + \kappa_I g_{I,IV}(0, z_{lk}, 0, \theta_l, 0, y_l) \gamma_{C2IV,jlk} \right].$$

Diffraction may arise from any street belonging to the crossroad of interest. Since the vehicle of interest is located at $(0, 0)$, multiple parameters of the function g are not required, and are thus set to zero.

7.2.4.2 Cooperative Sensing

For the radar function, we assume that each cooperative **RSU** draws a binary decision on the presence of the vehicle in the crossroad. Then, the **CPU** gathers all the decisions, and determine if there is a target or not depending on the selected decision rule.

Success Probability of one **RSU:** The success probability of Box 6.10, presented in Section 6.1.2.3 of Chapter 6, is used to evaluate the probability that the i^{th} cooperative **RSU** of the j^{th} street belonging to the crossroad successfully detects the vehicle:

$$\mathcal{P}_{S,ji}(\eta_R) \triangleq \mathbb{P} \left(\frac{S_{Rji}}{I_{Rji} + W_{Rji}} \geq \eta_R \right),$$

where S_{Rji} is the power of the received radar echo, I_{Rji} is the interference power at the **RSU**, and W_{Rji} is the constant noise power. Following (7.3), the useful radar power is expressed as

$$S_{Rji} = \kappa_R |2x_{ji}|^{-\alpha_R} \gamma_{Rji},$$

with x_{ji} the position of the i^{th} cooperative **RSU** in the j^{th} street. The total radar interference power aggregates five interference sources, denoted as $I_{R1,ji}$ to $I_{R5,ji}$, and illustrated in Figure 7.5:

1. The interference of type I and II from the **RSUs** in the j^{th} street, which are not cooperating. Following (7.4) and (7.5), the first interference source is expressed as

$$I_{R1,ji} = \sum_{k|z_{jk} \in \Phi_{vj} \setminus C_j} \left[\kappa_I |x_{ji} - z_{jk}|^{-\alpha_I} \gamma_{R1I,jik} + \kappa_I (|x_{ji}| + |z_{jk}|)^{-\alpha_I} \gamma_{R1II,jik} \mathbb{1}(x_{ji}z_{jk} \geq 0) \right].$$

The interference of type II is assumed to only occur when the cooperative and interfering **RSUs** are both on the same side of the crossroad.

2. The interference of type I and II from the **RSUs** in the j^{th} street, which are cooperating. Following (7.4) and (7.5), the second interference source is expressed as

$$I_{R2,ji} = \sum_{k|x_{jk} \in C_j \setminus \{x_{ji}\}} \left[\kappa_I |x_{ji} - x_{jk}|^{-\alpha_I} \gamma_{R2I,jik} + \kappa_I (|x_{ji}| + |x_{jk}|)^{-\alpha_I} \gamma_{R2II,jik} \mathbb{1}(x_{ji}x_{jk} \geq 0) \right].$$

The interference of type II is again assumed to only occur when the cooperative and interfering **RSUs** are both on the same side of the crossroad.

3. The interference of type II, III and IV from the **RSUs** in the other streets of the crossroad, which are cooperating. Following (7.5), (7.6) and (7.7), the third

interference source is expressed as

$$I_{R3,ji} = \sum_{l=0}^{N_S} \sum_{k|z_{lk} \in \Phi_{vl} \setminus C_l, l \neq j} \left[\kappa_I g_{I,II} (x_{ji}, z_{lk}, \theta_j, \theta_l, 0, 0) \gamma_{R3II,jilk} \right. \\ \left. + \kappa_I g_{I,III} (x_{ji}, z_{lk}, \theta_j, \theta_l, 0, 0) \gamma_{R3III,jilk} \right. \\ \left. + \kappa_I g_{I,IV} (x_{ji}, z_{lk}, \theta_j, \theta_l, 0, 0) \gamma_{R3IV,jilk} \right].$$

Since both the j^{th} and l^{th} streets belong to the crossroad, $y_j = y_l = 0$, and the position of the crossroad χ is $(0, 0)$.

4. The interference of type II, III and IV from RSUs in the other streets of the crossroad, which are cooperating. Following (7.5), (7.6) and (7.7), the fourth interference source is expressed as

$$I_{R4,ji} = \sum_{l=0}^{N_S} \sum_{k|x_{lk} \in C_l, l \neq j} \left[\kappa_I g_{I,II} (x_{ji}, x_{lk}, \theta_j, \theta_l, 0, 0) \gamma_{R4II,jilk} \right. \\ \left. + \kappa_I g_{I,III} (x_{ji}, x_{lk}, \theta_j, \theta_l, 0, 0) \gamma_{R4III,jilk} \right. \\ \left. + \kappa_I g_{I,IV} (x_{ji}, x_{lk}, \theta_j, \theta_l, 0, 0) \gamma_{R4IV,jilk} \right].$$

Since both the j^{th} and l^{th} streets again belong to the crossroad, $y_j = y_l = 0$, and the position of the crossroad χ is $(0, 0)$.

5. The interference of type II, III, and IV from RSUs in the l^{th} street which does not belong to the crossroad. Following (7.5), (7.6) and (7.7), the fifth interference source is expressed as

$$I_{R5,ji} = \sum_{l=N_S+1}^{\infty} \sum_{k|z_{lk} \in \Phi_{vl}} \left[\kappa_I g_{I,II} (x_{ji}, z_{lk}, \theta_j, \theta_l, 0, y_l) \gamma_{R5II,jilk} \right. \\ \left. + \kappa_I g_{I,III} (x_{ji}, z_{lk}, \theta_j, \theta_l, 0, y_l) \gamma_{R5III,jilk} \right. \\ \left. + \kappa_I g_{I,IV} (x_{ji}, z_{lk}, \theta_j, \theta_l, 0, y_l) \gamma_{R5IV,jilk} \right].$$

Since the j^{th} street belongs to the crossroad, $y_j = 0$.

Detection probability of one RSU: Instead of the success probability, one may consider the detection probability of Box 6.9, presented in Section 6.1.2.2 of Chapter 6. In that case, the detection probability that the i^{th} cooperative RSU of the j^{th} street belonging to the crossroad successfully detects the vehicle is expressed as

$$\mathcal{P}_{D,ji}(\zeta_R) \triangleq \mathbb{P} (S_{Rji} + I_{Rji} + W_{Rji} \geq \zeta_R),$$

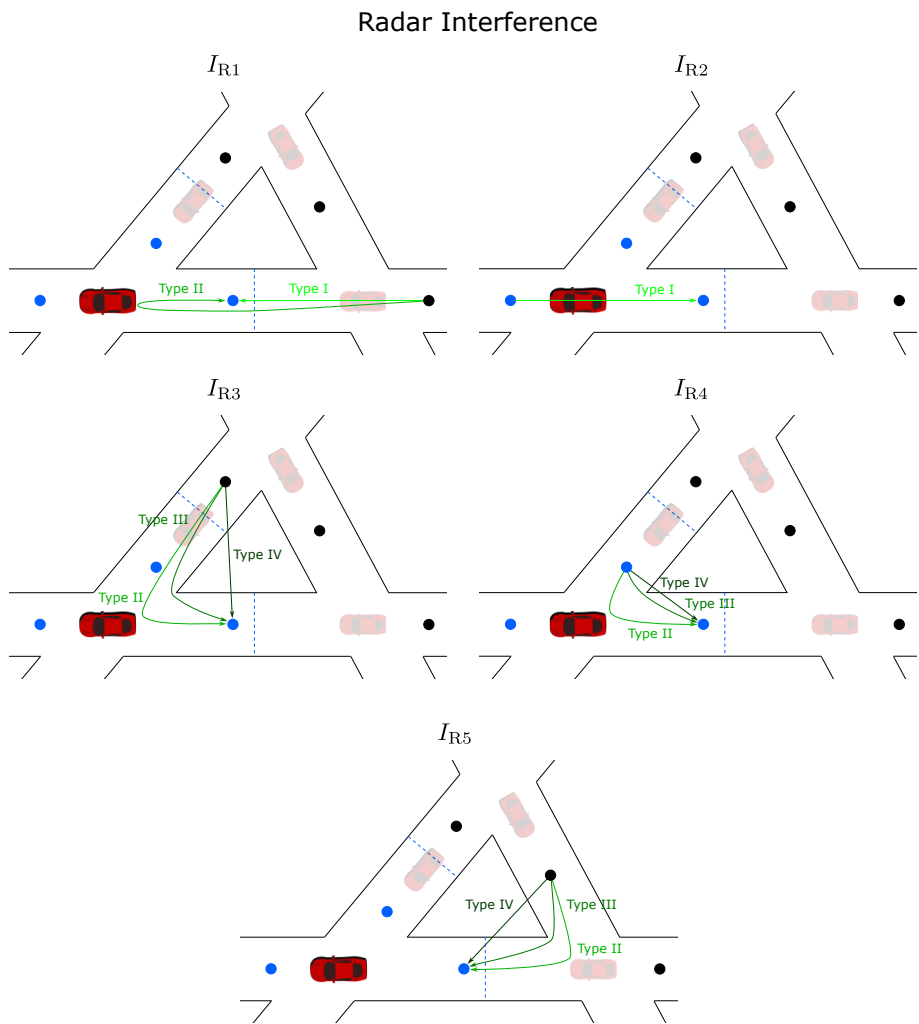


Fig. 7.5 Interference sources for the radar function. For the second interference source, there is no interference of type II in the illustrated scenario, as the interfering RSU is located on the other side of the crossroad.

where ζ_R is the detection threshold. The useful radar echo and the interference power are identical to the ones presented for the success probability.

Decision rule: Multiple decision rule are analysed in this chapter:

- **The K rule:** A vehicle is present if at least K RSUs among the cooperative RSUs have detected the target.
- **The OR rule:** A vehicle is present if at least one RSU among the cooperative RSUs have detected the target. This corresponds to the K rule with $K = 1$.
- **The $x\%$ -MAJ rule:** A vehicle is present if at least $x\%$ of the cooperative RSUs have detected the target. If the percentage is omitted, the 50%-MAJ rule is considered. If the number n_T of cooperative RSUs is known, the $x\%$ -MAJ rule is equivalent to the K rule with $K = \lceil x n_T \rceil$.
- **The AND rule:** A vehicle is present if all the cooperative RSUs have detected the target. If the number n_T of cooperative RSUs is known, this corresponds to the K rule with $K = n_T$.

As every rule can be derived as a particular case of the K rule when the total number of cooperative RSUs is known, we focus on the K rule in the following. The metrics can then be averaged with the probability density function of n_T .

7.3 Development of the Cooperative Metrics

The cooperative sensing and communication metrics are developed in Appendices 7.A and 7.B, with the system model proposed in Section 7.2. These results are summarised in Section 7.3.1 and 7.3.2, assuming that the positions $\mathbf{x}$ of the cooperative RSUs, and the angles θ of the streets belonging to the crossroad, are known beforehand.

7.3.1 Cooperative Communication

For the communication function, the cooperative coverage probability is computed in Box 7.1 using the Gil-Pelaez theorem of Box 6.19, where the random variable U is defined as

$$U = \sum_{j=0}^{N_S} \sum_{i \in C_j} S_{Cji} - \eta_C I_C.$$

The computation of the LT of U is detailed in Section 7.A. It requires the computation of the LTs of I_{C1} and I_{C2} .

The average over the positions $\mathbf{x}$ may be lifted easily, as the average of the LT of U over $\mathbf{x}$ is straightforward. However, an average over the angles θ is still required to consider random crossroad geometries. Thus, this is only analysed through MC simulations.

7.1: Cooperative Coverage Probability

$$\mathcal{P}_C(\eta_C; \mathbf{x}, \theta) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{1}{\tau} \text{Im} \{ \mathcal{L}_{U|\mathbf{x},\theta}(-j\tau) e^{-j\tau\eta_C W_C} \} d\tau$$

Communication	Integrations	Sensing	Integrations
$\mathcal{L}_{I_{C1}}$	1	$h_{13}^{(m)}$	1
		$h_2^{(m)}, h_4^{(m)}$	0
$\mathcal{L}_{I_{C2}}$	3	$h_5^{(m)}$	3
		$h_W^{(m)}, h_\zeta$	0

Table 7.2 Number of numerical integrations in the quantities required for the computation of the cooperative coverage, success and detection probabilities.

7.3.2 Cooperative Sensing

For the radar function, the number of cooperative RSUs is known, as their locations are assumed to be known. Under that assumption, every decision rule can be expressed as a variation of the K rule, and both the cooperative success and detection probabilities are computed following Box 7.2. The coefficients $a^{(m)}$ and the functions h are detailed in Section 7.B.

7.2: Cooperative Success and Detection Probabilities

$$\begin{aligned} \mathcal{P}_S^{(K)}(\eta_R; \mathbf{x}, \theta) &= \sum_{m=1}^{2^{n_T}} a^{(m)} h_{13}^{(m)}(\eta_R; \mathbf{x}, \theta) h_2^{(m)}(\eta_R; \mathbf{x}) h_4^{(m)}(\mathbf{x}, \theta) \\ &\quad \cdot h_5^{(m)}(\eta_R; \mathbf{x}, \theta) h_W^{(m)}(\eta_R; \mathbf{x}) \\ \mathcal{P}_D^{(K)}(\zeta_R; \mathbf{x}, \theta) &= \sum_{m=1}^{2^{n_T}} a^{(m)} h_{13}^{(m)}(-1; \mathbf{x}, \theta) h_2^{(m)}(-1; \mathbf{x}) h_4^{(m)}(-1; \mathbf{x}, \theta) \\ &\quad \cdot h_5^{(m)}(-1; \mathbf{x}, \theta) h_W^{(m)}(-1; \mathbf{x}) h_\zeta(1; \mathbf{x}) \end{aligned}$$

Both averages over the positions $\mathbf{x}$ and angles θ are required to consider random RSUs positions and crossroad geometries. Additionally, averaging over the number of cooperative RSUs is also required. Thus, this is only analysed through MC simulations.

7.3.3 Complexity Analysis

On the one hand, the evaluation of the cooperative coverage probability requires the computation of the **LT** of the interference I_{C1} and I_{C2} . On the other hand, the evaluation of the cooperative success probability requires the computation of the functions h . Table 7.2 details the number of numerical integrations required for each quantity, enabling to compute the cooperative communication and sensing metrics. One can observe that most integrations involve the **LT** of the interference I_{C2} , and the function $h_s^{(m)}$, which is a function of the interference I_{R5} . Therefore, the interference arising from the roads outside the crossroads is the most difficult to evaluate. Still, it will be shown in Section 7.4 that this source of interference may be negligible compared to the interference arising from the streets belonging to the crossroad.

7.3.4 Other Sensing and Performance Metrics

Numerous other popular communication and sensing metrics were presented in Section 6.1 of Chapter 6. Similarly to the cooperative coverage, detection and success probabilities, they may be generalised to cooperative scenarios. For instance, the radar false alarm probability is often evaluated in pair with the detection probability. Additionally, two novel joint metrics have been defined, i.e. the **JRDCCP** and **JRSCCP**. Unfortunately, these other metrics are mathematically intractable within the **SG** framework. Still, they may be evaluated using Monte-Carlo simulations.

7.4 Numerical Analysis

In this section, the impact of the different interference sources is first analysed. A focus is made on the model of the city with Cox processes in Section 7.4.1, compared to usual **PPPs**. The equivalent density for the **HPPP** is computed as

$$\lambda_{s,PPP} = \frac{N_S + 1}{\pi\rho} \lambda_s,$$

such that the same average number of cooperative nodes is achieved with the two models. Then, the cooperative communication and sensing metrics are evaluated, and the impact of the nodes distribution is again highlighted. Finally, the **JRSCCP** is used in Section 7.4.2, in order to perform optimisation of the **RSUs** density and the radius of association. The parameters of the scenario are summarised in Table 7.3. Random city topologies are considered. Thus, the presented results are obtained through Monte-Carlo simulations.

7.4.1 Cox Processes versus Poisson Point Processes

Figure 7.6 illustrates the **CDFs** obtained with Cox processes and **PPPs**, for the communication and sensing functions. First, one can notice that the interference

Parameters	Values
Roads density	$\lambda_s = 0.034 \text{ m}^{-1}$
RSUs density	$\lambda_v = 0.1 \text{ m}^{-1}$
Cooperative distance	$\rho = 20 \text{ m}$
Transmitted power	$P_t = 29 \text{ dBm}$
Beamforming gains	$G_t = G_r = 18 \text{ dBi}$
Carrier frequency	$f_c = 24 \text{ GHz}$
Path-loss parameters	$\alpha_R = \alpha_C = 1, \alpha_I = 2.8$ $[\beta_R]_{\text{dB}} = [\beta_C]_{\text{dB}} = 55 \text{ dB},$ $[\beta_I]_{\text{dB}} = 46.7 \text{ dB}$
Reflection parameters	$v_r = 4$
Diffraction parameters	$k_b = 1.58 \text{ m}^{-1}, v_b = 1.5$
Wall losses	$\delta = 20 \text{ dB}$

Table 7.3 Parameters of the scenario for Section 7.4.

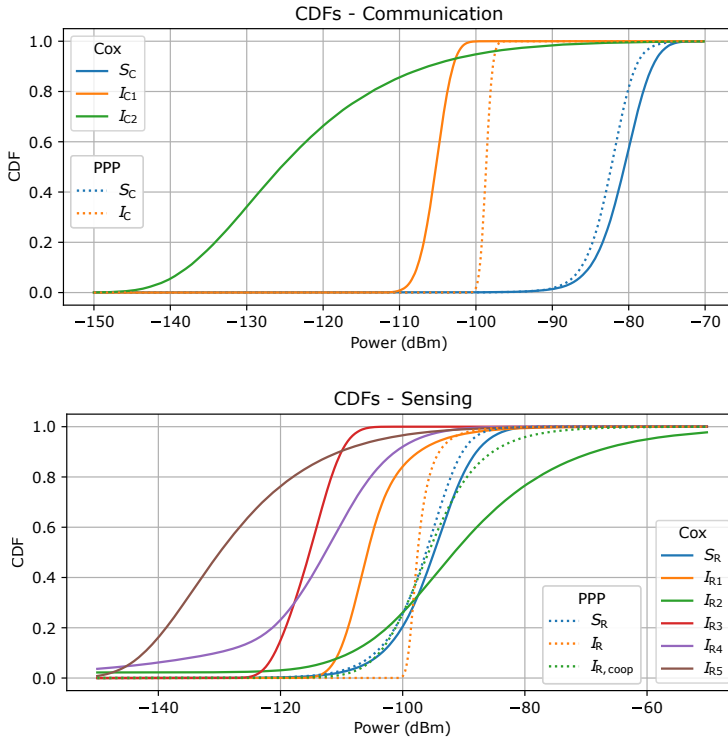


Fig. 7.6 Communication (top) and sensing (bottom) powers CDFs, with Cox processes and PPPs. The quantity $I_{R,coop}$ denotes the interference arising from the other cooperative nodes with PPPs.

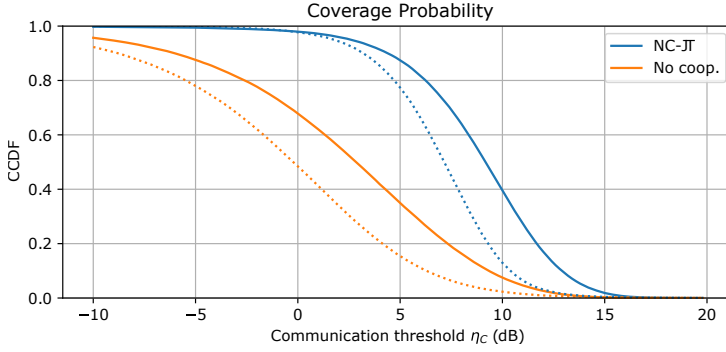


Fig. 7.7 Coverage probability, using Cox processes (solid lines) or PPPs (dotted lines), with and without cooperation.

arising from the streets which do not belong to the crossroad, namely I_{C2} and I_{R5} , is negligible for both functions in most scenarios, except in the few cases where the interfering street is crossing the streets of the crossroad close to the centre. Therefore, as specified in Table 7.2, since both quantities require three integrations, they may be neglected to hugely lower the mathematical complexity of the provided expressions.

Then, focusing first on the communication function, compared to the results obtained with Cox processes, the useful signal is underestimated with PPPs, whereas the interference level is overestimated. Thus, PPPs should provide lower bounds for the cooperative coverage probability obtained with Cox processes. This is indeed observed in Figure 7.7, illustrating the coverage probability obtained using the two models, with and without cooperation. Moreover, it also shows that the cooperation scheme enables to improve the coverage probability. Contrariwise, for the radar function, comparing the two models, the useful signals CDFs are similar, but the PPPs overestimates the interference power, except for the interference I_{R2} between the cooperative nodes of the same street, which dominates all the other interference sources. This leads to different tendencies in the top plot of Figure 7.8, showing the success probability obtained using the two models, with and without cooperation. Rules with low majority percentages give lower performance with PPPs, whereas the opposite is observed for rules with high majority percentages. This is not the case anymore when the cooperative nodes use orthogonal resources to perform the detection, as illustrated in the bottom plot of Figure 7.8. In that case, the interference I_{R2} is zero, and the PPPs provide lower bounds for the radar success probability.

With regard to the achieved sensing performance, as expected, the OR rule maximises the cooperative success probability. This is not the case for the cooperative detection probability, evaluated for different cooperative false alarm probabilities in Figure 7.9. Surprisingly, the AND rule enables to maximise the former, whatever the

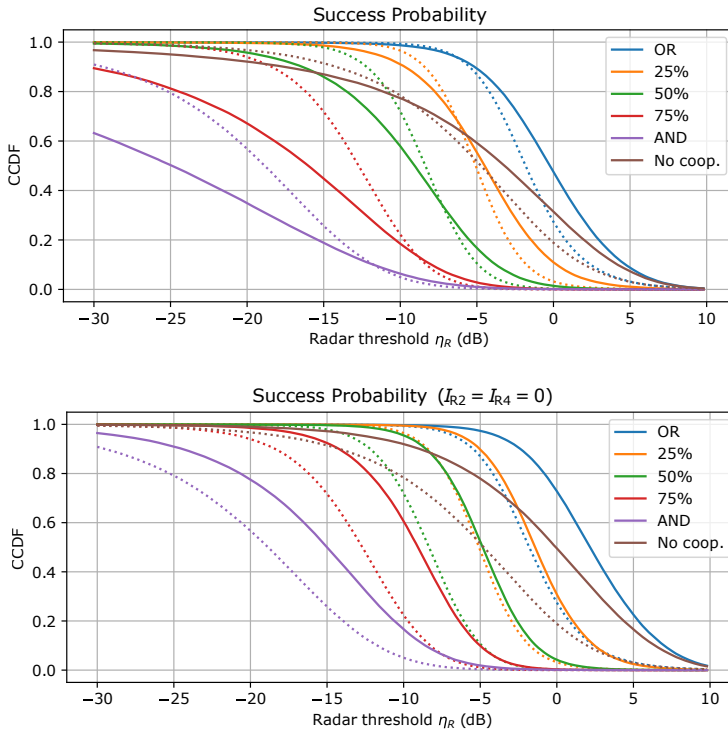


Fig. 7.8 Success probabilities, using Cox processes (solid lines) or PPPs (dotted lines), with and without cooperation. The bottom plot illustrates the case where the interference arising from other cooperative nodes is neglected, thus corresponding to a scenario with orthogonal resources.

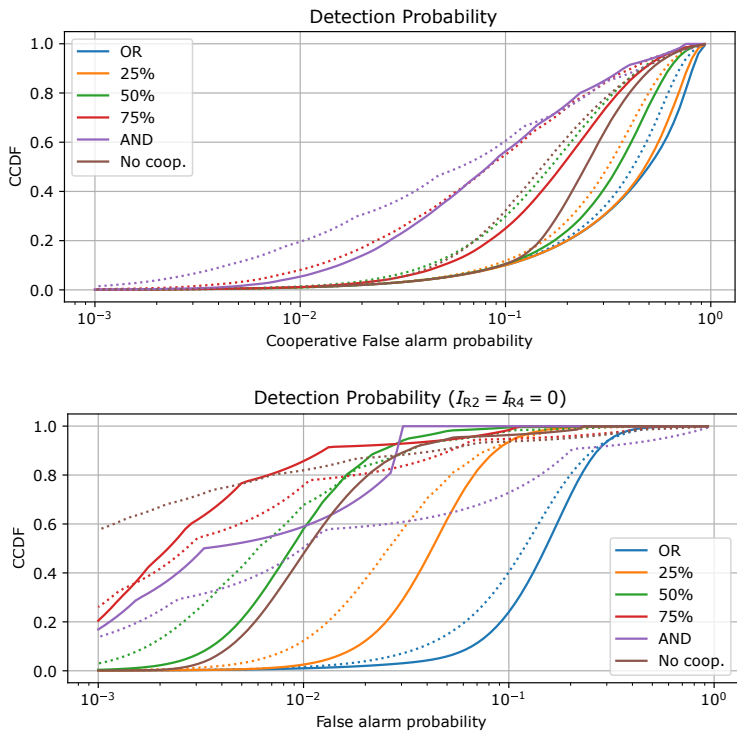


Fig. 7.9 ROC using Cox processes (solid lines) or PPPs (dotted lines), with and without cooperation. The bottom plot illustrates the case where the interference arising from other cooperative nodes is neglected, thus corresponding to a scenario with orthogonal resources.

selected false alarm probability. Furthermore, cooperation enables to improve the achieved performance for majorities higher or equal to 50% and 75%, respectively when there the interference between the cooperative **RSUs** is neglected or not. Still, additional simulations should be performed, as the results with the 75% MAJ and the AND rule are unexploitable in the bottom figure.

7.4.2 Optimisation of the Network Parameters

As specified in Section 7.3.4, the evaluation of the cooperative joint metrics is not mathematically tractable. Still, the Fréchet bounds of Box 6.15 may be computed from the univariate metrics. Figure 7.10 illustrates the optimisation of the **JRSCCP** (and its upper and lower bounds) w.r.t. the **RSUs** density and the radius of the cooperative region. Even if the Fréchet bounds (especially the lower bound) are quite loose, the maximum of the **JRSCCP** is close to the maximum of the upper and lower bounds. Thus, network optimisation may be carried out based on the Fréchet bounds, which only depend on the univariate cooperative metrics. Besides, the higher the majority percentages, the lower the optimal **RSUs** density and cooperation radius. Indeed, it is easier to obtain high majorities when the number of cooperative nodes is small. Finally, for the OR rule and the case without cooperation, the cooperation radius can be increased as much as required without affecting the joint metric, as the coverage probability keeps increasing, whereas the sensing performance are only affected by the closest nodes using these rules.

7.5 Conclusion

Cooperative sensing and communication networks are promising to enhance the performance of both functions at the city scale. Yet, the performance analysis of such scenario raises new challenges for the modelling of the network, and the development of tractable analytical expressions. In this chapter, the radar and communication performance achieved by cooperative **RSUs** in a **PLP** have been analysed within the **SG** framework.

7.5.1 Summary of the Contributions

The contributions of this chapter are summarised as follows:

- Numerous interference mechanisms, specific to the evaluation of the performance at the city scale, have been modelled. This comprises interference from direct links, reflections on obstacles, diffractions on the edges of the streets, and building penetrations. Based on that, the cooperative communication coverage, radar success, and radar detection probabilities have been defined. For the radar function, multiple decision rules have been proposed from the literature. Sadly, the joint metrics developed in Chapter 6 are analytically intractable, but they may be evaluated with Monte-Carlo simulations.

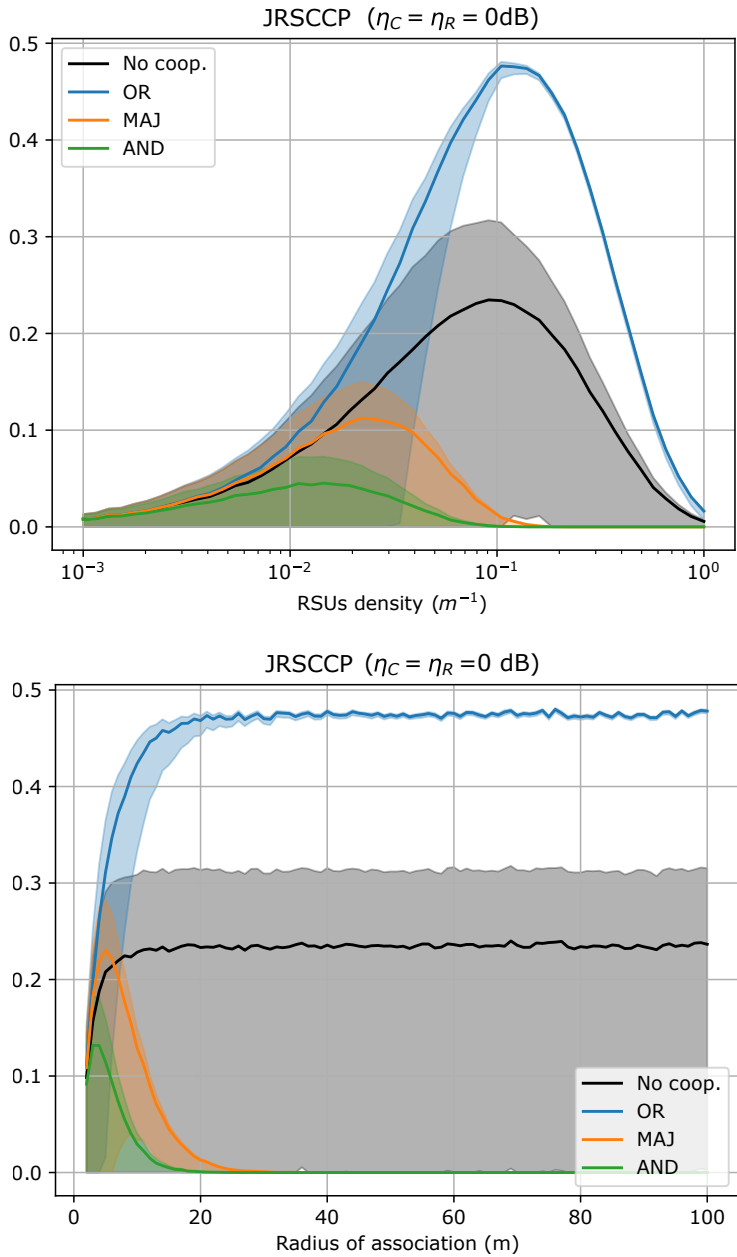


Fig. 7.10 Optimisation of the cooperative JRSCCP against the RSUs density (top), and radius of the cooperation region (bottom).

- Analytical expressions of the cooperative metrics have been developed. Unfortunately, their complexity is high, such that Monte-Carlo simulations are more efficient to evaluate the network performance. Yet, the interference arising from the streets outside the crossroad of interest introduces most of the complexity. Thus, if this source of interference is neglected, these expressions are quickly evaluated for scenarios in which the number of cooperative **RSUs**, their locations on the crossroad, and the angles of the streets belonging to the crossroad are known.
- Numerical analyses have been carried out for random city topologies:
 1. The **CDFs** of the useful signals and interference obtained with **PLP** and **PPP** to model the topology of the city have been compared for both functions. Whereas the **CDFs** of the useful signals are similar, the **PPP** overestimates the total interference in most cases, except when cooperative **RSUs** within the same streets are interfering with each other for sensing.
 2. The sensing and communication performance metrics have been evaluated, for both **PLP** and **PPP** models. Since the interference is overestimated in most cases with **PPPs**, the obtained metrics act as lower bounds of the **PLP** for the communication function. This is also valid for the sensing function, when the interference between cooperative nodes is neglected, as if the sensing was performed with orthogonal resources. Moreover, looking at the achieved performance, the OR rule maximises the cooperative success probability with given **SINR** threshold requirements, whereas the AND rule maximises the cooperative detection probability, with a given cooperative false alarm probability.
 3. The **JRSCCP** metrics has been optimised w.r.t. the **RSUs** density and radius of association. Even if this metric is not tractable mathematically, the optimal **RSUs** density and radius of association achieved by the Fréchet bounds are close to the optimal values of the joint metrics. This enables to perform optimisation with the analytical expressions. Moreover, the higher the majority of the decision rule, the lower the optimal **RSUs** density and radius of association.

7.5.2 Future Works

Owing to the high complexity of the analytical expressions, they may be evaluated only for scenarios in which the number of cooperative **RSUs**, their locations on the crossroad, and the angles of the streets belonging to the crossroad are known. Thus, discrete optimisation could first be performed with **SG**, to find the optimal locations of the **RSUs** around the crossroad, for a given crossroad topology.

Then, if the locations of the **RSUs** around the crossroad is known, one may replicate this configuration for every crossroad in the city, and evaluate the

inter-crossroads interference in this situation. This generalises the proposed system model, especially in Manhattan scenarios where the crossroads topology is identical. In that case, performing optimisation of the [RSUs](#) locations would also impact directly the interference arising from the other crossroads.

Next, even if the Fréchet bounds are helpful to perform optimisation of the [JRSCCP](#), tractable approximations of the joint metrics should be further investigated, to precise the value of the optimal network parameters. For instance, the similarities with the product of the cooperative communication and success probabilities may be analysed.

Finally, as it has been done in Chapter 6, ray-tracing simulations may be performed to assess the relevance of the developed framework. This has been performed in [[Wia+23](#)] for joint communication metrics, but it does not enable to assess the behaviour of the [JCAS](#) metrics. Indeed, the analysis of Chapter 6 has shown a good matching for the univariate metrics, but higher discrepancies have been observed for the joint metrics.

7.5.3 What is Next?

This chapter concludes the second part of this PhD thesis. The [SG](#) framework has been leveraged to analyse the performance of [ISAC](#) systems in networks at a street or city scale. Whereas the performance of both sensing and communication can be evaluated separately, joint metrics enable to grasp all the interactions between both functions, and perform optimisation of the network parameters. Moreover, at the city level, cooperation enables to improve the network performance for both functions. Though, the complexity of the developed analytical expressions is high, and the [SG](#) framework furthermore requires to perform multiple approximations in the system model to obtain tractable results.

Therefore, in the next part, performance of [ISAC](#) systems are analysed at the street scale, outside the [SG](#) framework. More specifically, the full-duplex [V2X](#) scenario is further investigated, for specific [FMCW](#) and [OFDM ISAC](#) receivers. Two solutions are detailed to mitigate the interference between the radar echoes and the uplink signals, namely interference cancellation and beamforming.

Appendix - Chapter 7

7.A Cooperative Communication

The communication coverage probability of (7.8) can be developed using the Gil-Pelaez of Box 6.19 as

$$\begin{aligned}\mathcal{P}_C(\eta_C) &= \mathbb{P}_{\{S_{Cji}\}, I_C} \left(\frac{\sum_{j=0}^{N_S} \sum_{i \in C_j} S_{Cji}}{I_C + W_C} \geq \eta_C \right) \\ &= \mathbb{P}_{\{S_{Cji}\}, I_C} \left(\sum_{j=0}^{N_S} \sum_{i \in C_j} S_{Cji} - \eta_C I_C \geq \eta_C W_C \right) \\ &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{1}{\tau} \text{Im} \left\{ \mathcal{L}_U(-j\tau) e^{-j\tau\eta_C W_C} \right\} d\tau,\end{aligned}$$

where the random variable U is defined as

$$U = \sum_{j=0}^{N_S} \sum_{i \in C_j} S_{Cji} - \eta_C I_C.$$

Denoting by $\mathbf{x} \in C$ the positions of the cooperative RSUs, and θ the angles of the streets belonging to the crossroad, the LT of U is then computed as

$$\mathcal{L}_U(s) = \mathbb{E}_{\mathbf{x}} \left[\prod_{j=0}^{N_S} \prod_{i \in C_j} \mathcal{L}_{S_{Cji}|x_{ji}}(s) \right] \mathcal{L}_{I_{C1}}(s) \mathbb{E}_{\theta} [\mathcal{L}_{I_{C2}|\theta}(s)].$$

This expression is obtained as the small-scale fading RVs and the PPPs $\{\Phi_{vj}\}$ are independent with each other. This assumes random cooperative RSUs locations, and random crossroad geometries. One may instead consider that these parameters are known beforehand:

$$\mathcal{L}_{U|\mathbf{x},\theta}(s) = \prod_{j=0}^{N_S} \prod_{i \in C_j} \mathcal{L}_{S_{Cji}|x_{ji}}(s) \mathcal{L}_{I_{C1}}(s) \mathcal{L}_{I_{C2}|\theta}(s).$$

7.A.1 Laplace Transform of the Total Communication Power

The set of cooperative nodes C_j in the j^{th} street of the crossroad is generated by performing a thinning of the PPP Φ_{vj} , with a probability function p defined as

$$p(x) = \begin{cases} 1 & \text{if } |x| \leq \rho, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, following the thinning theorem presented in Box 5.11, C_j is also a PPP, with density

$$\lambda_{C_j}(x) = \lambda_{vj} \mathbb{1}(|x| \leq \rho). \quad (7.9)$$

Thus, using the PPPs PGFL of Box 5.8, the LT of the total communication power is expressed as

$$\begin{aligned} \mathbb{E}_{\mathbf{x}} \left[\prod_{j=0}^{N_S} \prod_{i \in C_j} \mathcal{L}_{S_{C_{ji}}|x_{ji}}(s) \right] &= \prod_{j=0}^{N_S} \mathbb{E}_{C_j} \left[\prod_{i \in C_j} \mathcal{L}_{S_{C_{ji}}|x_{ji}}(s) \right] \\ &= \prod_{j=0}^{N_S} \exp \left(-2\lambda_{vj} \int_0^\rho (1 - \mathcal{L}_Y(s \kappa_C x^{-\alpha_C})) dx \right) \\ &= \exp \left(-2\lambda_v(N_S + 1) \int_0^\rho \frac{s \kappa_C x^{-\alpha_C}}{1 + s \kappa_C x^{-\alpha_C}} dx \right), \end{aligned}$$

where the last equality is obtained assuming a constant RSUs density for all the streets, and using the LT of the exponentially distributed small-scale fading RVs. If the positions of the cooperative RSUs is instead assumed to be known beforehand, the LT of the total communication power is then computed as

$$\prod_{j=0}^{N_S} \prod_{x_{ji} \in C_j} \mathcal{L}_{S_{C_{ji}}|x_{ji}}(s) = \prod_{j=0}^{N_S} \prod_{x_{ji} \in C_j} \frac{1}{1 + s \kappa_C |x_{ji}|^{-\alpha_C}}.$$

7.A.2 Laplace Transform of I_{C1}

The set of interfering nodes $\Phi_{vj} \setminus C_j$ in the j^{th} street of the crossroad is generated following the thinning theorem of Box 5.11 as the complementary set of C_j . Therefore, $\Phi_{vj} \setminus C_j$ is also a PPP, with density

$$\lambda_{\Phi_{vj} \setminus C_j}(x) = \lambda_{vj} \mathbb{1}(|x| > \rho).$$

Similarly as the [LT](#) of the total communication power, the [LT](#) of the first interference source is obtained as

$$\mathcal{L}_{I_{C1}}(s) = \exp \left(-2\lambda_v(N_S + 1) \int_{\rho}^{\infty} \frac{s \kappa_C x^{-\alpha_C}}{1 + s \kappa_C x^{-\alpha_C}} dx \right).$$

7.A.3 Laplace Transform of I_{C2}

Conditioned on the angles θ of the streets belonging to the crossroad, the [LT](#) of the second interference source is developed as

$$\begin{aligned} \mathcal{L}_{I_{C2|\theta}}(s) &= \mathbb{E} \left[\prod_{l=N_S+1}^{\infty} \prod_{k|z_{lk} \in \Phi_{vl}} \prod_{j=0}^{N_S} \exp \left(-s \kappa_I g_{I,III} (0, z_{lk}, \theta_j, \theta_l, 0, y_l) \right) \right. \\ &\quad \cdot \exp \left(-s \kappa_I g_{I,IV} (0, z_{lk}, 0, \theta_l, 0, y_l) \right) \left. \right] \\ &\stackrel{(a)}{=} \mathbb{E}_{\Phi_S, \{\Phi_{vl}\}, \{\theta_l\}} \left[\prod_{l=N_S+1}^{\infty} \prod_{k|z_{lk} \in \Phi_{vl}} \iota_{C2}(z_{lk}, \theta_l, y_l) \right] \\ &\stackrel{(b)}{=} \mathbb{E}_{\Phi_S, \{\theta_l\}} \left[\prod_{l=N_S+1}^{\infty} \exp \left(-\lambda_{vl} \int_{-\infty}^{\infty} [1 - \iota_{C2}(z, \theta_l, y_l)] dz \right) \right] \\ &\stackrel{(c)}{=} \mathbb{E}_{\Phi_S} \left[\prod_{l=N_S+1}^{\infty} \frac{1}{2\pi} \int_0^{2\pi} \exp \left(-\lambda_{vl} \int_{-\infty}^{\infty} [1 - \iota_{C2}(z, \theta, y_l)] dz \right) d\theta \right] \\ &\stackrel{(d)}{=} \exp \left(-\lambda_s \int_0^{\infty} \left[1 - \frac{1}{2\pi} \int_0^{2\pi} \exp \left(-\lambda_v \int_{-\infty}^{\infty} [1 - \iota_{C2}(z, \theta, y)] dz \right) d\theta \right] dy \right), \end{aligned}$$

with the function ι_{C2} defined as

$$\begin{aligned} \iota_{C2}(z, \theta, y) &\triangleq \prod_{j=0}^{N_S} \mathcal{L}_{\gamma} (s \kappa_I g_{I,III} (0, z, \theta_j, \theta, 0, y)) \mathcal{L}_{\gamma} (s \kappa_I g_{I,IV} (0, z, 0, \theta, 0, y)) \\ &= \prod_{j=0}^{N_S} \frac{1}{1 + s \kappa_I g_{I,III} (0, z, \theta_j, \theta, 0, y)} \frac{1}{1 + s \kappa_I g_{I,IV} (0, z, 0, \theta, 0, y)}. \end{aligned}$$

Equalities (a) to (d) are respectively obtained using the definition of the [LT](#) of the exponentially distributed small-scale fading [RVs](#), the [PGFL](#) of the [RSUs PPPs](#) of [Box 5.8](#), averaging over the angles of the streets not belonging to the crossroad, and using the [PGFL](#) of the streets [PPP](#)s again with [Box 5.8](#). Additionally, it is assumed that the [RSUs](#) density is the same for every street.

7.B Cooperative Sensing

As described in Section 7.2.4.2, multiple decision rules may be considered at the CPU to draw a decision about the presence of the vehicle in the crossroad. Nonetheless, if the number of cooperative node is known, all the presented rules brings down to particular cases of the K rule. It has been shown in Section 7.A.1 that the cooperative sets $\{C_j\}$ follow PPPs of density given by (7.9). Therefore, the number of cooperative RSUs in each street is a Poisson RV, and the joint probability mass function of the number of cooperative RSUs in each street is given by

$$f_{n_0, \dots, n_{N_S}}(n_0, \dots, n_{N_S}) = \frac{(2\lambda_v \rho)^{n_0 + \dots + n_{N_S}}}{n_0! \cdots n_{N_S}!} e^{-2(N_S+1)\lambda_v \rho}.$$

The cooperative success or detection probability may then be computed as a weighted average of the cooperative probabilities conditioned with the number of nodes in each street, with the K rule. The parameter K varies with the number of nodes to fit the required decision rule.

7.B.1 Cooperative Sensing with the K Rule

Following [SPK22], assuming that the number of cooperative RSUs is known for each street of the crossroad, the cooperative probability obtained with the K rule can be written as

$$\mathcal{P}_X^{(K)}(\eta_R) = \mathbb{E}_{-\backslash \mathcal{Y}_R} \left[\sum_{k=K}^{n_T} \sum_{m=1}^{\binom{n_T}{k}} \prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathcal{P}_{X,ji}^{\delta_{ji}^{(k,m)}}(\eta_R) [1 - \mathcal{P}_{X,ji}(\eta_R)]^{1-\delta_{ji}^{(k,m)}} \right], \quad (7.10)$$

where $\mathbb{E}_{-\backslash \mathcal{Y}_R}$ denotes the expectation over every RV, except the small-scale fading RVs of the useful radar echoes. The binary values $\delta_{ji}^{(k,m)}$ define if the cooperative i^{th} cooperative RSU in the j^{th} street of the crossroad successfully performs or not a detection, for each combination indexed by m . For example, with $K = 2$, $n_T = 3$ and $N_S = 1$, discarding the index j , the expectation and the threshold η_R for readability, the probability is computed as

$$\mathcal{P}_X^{(K)} = \mathcal{P}_{X,0} \mathcal{P}_{X,1} (1 - \mathcal{P}_{X,2}) + \mathcal{P}_{X,0} (1 - \mathcal{P}_{X,1}) \mathcal{P}_{X,2} + (1 - \mathcal{P}_{X,0}) \mathcal{P}_{X,1} \mathcal{P}_{X,2} + \mathcal{P}_{X,0} \mathcal{P}_{X,1} \mathcal{P}_{X,2}.$$

The corresponding vectors $[\delta^{(k,m)}]_i = \delta_i^{(k,m)}$ are written as

$$\begin{aligned} \delta^{(2,1)} &= [1 \quad 1 \quad 0], \\ \delta^{(2,2)} &= [1 \quad 0 \quad 1], \\ \delta^{(2,3)} &= [0 \quad 1 \quad 1], \\ \delta^{(3,3)} &= [1 \quad 1 \quad 1]. \end{aligned}$$

In order to further develop (7.10), one may notice from the example that this expression can be developed as a sum of products of the probabilities of the n_T cooperative RSUs:

$$\sum_{k=K}^{n_T} \sum_{m=1}^{\binom{n_T}{k}} \prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathcal{P}_{X,ji}^{\delta_{ji}^{(k,m)}}(\eta_R) [1 - \mathcal{P}_{X,ji}(\eta_R)]^{1-\delta_{ji}^{(k,m)}} = \sum_{m=1}^{2^{n_T}} a^{(m)} \prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathcal{P}_{X,ji}^{b_{ji}^{(m)}}(\eta_R),$$

where $\mathbf{b}^{(m)}$ are the $2^{n_T} - 1$ non-zero binary words of size n_T , listing all the possible combinations of the cooperative RSUs which detects a target. Denoting by $c^{(m)} \triangleq \|\mathbf{b}^{(m)}\|_1$ the cardinality of these vectors, the coefficients $a^{(m)}$ are computed as

$$a^{(m)} = \begin{cases} (-1)^{c^{(m)}-K} \frac{c^{(m)} - K + 1}{c^{(m)}} \binom{c^{(m)}}{K-1} & \text{if } c^{(m)} \geq K, \\ 0 & \text{otherwise.} \end{cases}$$

This expression is obtained by noticing that every term of the sum associated with a word $\mathbf{b}^{(m)}$ with the same cardinality $c^{(m)}$ is associated with the same coefficient $a^{(m)}$, computed as

$$a^{(m)} = \sum_{p=0}^{c^{(m)}-K} (-1)^p \binom{c^{(m)}}{c^{(m)}-p}.$$

When there isn't a sufficient number of cooperative RSUs which detect the target, i.e. $c^{(m)} \geq K$, then $a^{(m)} = 0$. For example, developing the result obtained for $K = 2$, $n_T = 3$ and $N_S = 1$, discarding the index j , the expectation and the threshold η_R for readability, it gives

$$\begin{aligned} \mathcal{P}_X^{(K)} &= \mathcal{P}_{X,0} \mathcal{P}_{X,1} (1 - \mathcal{P}_{X,2}) + \mathcal{P}_{X,0} (1 - \mathcal{P}_{X,1}) \mathcal{P}_{X,2} + (1 - \mathcal{P}_{X,0}) \mathcal{P}_{X,1} \mathcal{P}_{X,2} + \mathcal{P}_{X,0} \mathcal{P}_{X,1} \mathcal{P}_{X,2} \\ &= \mathcal{P}_{X,0} \mathcal{P}_{X,1} + \mathcal{P}_{X,0} \mathcal{P}_{X,2} + \mathcal{P}_{X,1} \mathcal{P}_{X,2} - 3 \mathcal{P}_{X,0} \mathcal{P}_{X,1} \mathcal{P}_{X,2} + \mathcal{P}_{X,0} \mathcal{P}_{X,1} \mathcal{P}_{X,2} \\ &= \mathcal{P}_{X,0} \mathcal{P}_{X,1} + \mathcal{P}_{X,0} \mathcal{P}_{X,2} + \mathcal{P}_{X,1} \mathcal{P}_{X,2} - 2 \mathcal{P}_{X,0} \mathcal{P}_{X,1} \mathcal{P}_{X,2}. \end{aligned}$$

The binary words with cardinality $c^{(m)} \geq 2$ are the following:

$$\begin{aligned} \mathbf{b}^{(4)} &= [1 \quad 1 \quad 0], \\ \mathbf{b}^{(5)} &= [1 \quad 0 \quad 1], \\ \mathbf{b}^{(6)} &= [0 \quad 1 \quad 1], \\ \mathbf{b}^{(7)} &= [1 \quad 1 \quad 1]. \end{aligned}$$

The words $\mathbf{b}^{(4)}$ to $\mathbf{b}^{(7)}$ have a cardinality of 2, and their associated coefficients are equal to 1. The word $\mathbf{b}^{(7)}$ has a cardinality of 3, and its associated coefficient is equal to -2.

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To summarise, the cooperative sensing metrics are computed as

$$\mathcal{P}_X^{(K)}(\eta_R) = \sum_{m=1}^{2^{n_T}} a^{(m)} \mathbb{E}_{-\backslash \mathcal{Y}_R} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathcal{P}_{\mathbf{x},ji}^{b_{ji}^{(m)}}(\eta_R) \right].$$

The expectation is developed the same way for both the success and detection probability. For the success probability, one obtains

$$\begin{aligned} \mathbb{E}_{-\backslash \mathcal{Y}_R} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathcal{P}_{S,ji}^{b_{ji}^{(m)}}(\eta_R) \right] &= \mathbb{E}_{-\backslash \mathcal{Y}_R} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathbb{P}_{\mathcal{Y}_R} \left(\frac{S_{Rji}}{I_{Rji} + W_{Rji}} \geq \eta_R \right)^{b_{ji}^{(m)}} \right] \\ &= \mathbb{E}_{-\backslash \mathcal{Y}_R} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathbb{P}_{\mathcal{Y}_R} \left(\gamma_{Rji} \geq \frac{\eta_R I_{Rji} + \eta_R W_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right)^{b_{ji}^{(m)}} \right] \\ &= \mathbb{E}_{-\backslash \mathcal{Y}_R} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \exp \left(-\frac{b_{ji}^{(m)} \eta_R I_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \exp \left(-\frac{b_{ji}^{(m)} \eta_R W_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \right] \\ &= \mathbb{E}_{\mathbf{x},\theta} \left[\mathbb{E}_{\{I_{Rji}\}|\mathbf{x},\theta} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \exp \left(-\frac{b_{ji}^{(m)} \eta_R I_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \exp \left(-\frac{b_{ji}^{(m)} \eta_R W_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \right] \right]. \end{aligned}$$

For the detection probability, one obtains

$$\begin{aligned} \mathbb{E}_{-\backslash \mathcal{Y}_R} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathcal{P}_{D,ji}^{b_{ji}^{(m)}}(\zeta_R) \right] &= \mathbb{E}_{-\backslash \mathcal{Y}_R} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathbb{P}_{\mathcal{Y}_R} (S_{Rji} + I_{Rji} + W_{Rji} \geq \zeta_R)^{b_{ji}^{(m)}} \right] \\ &= \mathbb{E}_{-\backslash \mathcal{Y}_R} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathbb{P}_{\mathcal{Y}_R} \left(\gamma_{Rji} \geq \frac{\zeta_R - I_{Rji} - W_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right)^{b_{ji}^{(m)}} \right] \\ &= \mathbb{E}_{-\backslash \mathcal{Y}_R} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \exp \left(\frac{b_{ji}^{(m)} I_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \exp \left(\frac{b_{ji}^{(m)} W_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \exp \left(-\frac{b_{ji}^{(m)} \zeta_R}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \right] \\ &= \mathbb{E}_{\mathbf{x},\theta} \left[\mathbb{E}_{\{I_{Rji}\}|\mathbf{x},\theta} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \exp \left(\frac{b_{ji}^{(m)} I_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \exp \left(\frac{b_{ji}^{(m)} W_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \exp \left(-\frac{b_{ji}^{(m)} \zeta_R}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \right] \right]. \end{aligned}$$

Thus, decomposing the expectation, the cooperative success and detection probabilities can both be rewritten as

$$\begin{aligned} \mathcal{P}_S^{(K)}(\eta_R) &= \sum_{m=1}^{2^{n_T}} a^{(m)} \mathbb{E}_{\mathbf{x},\theta} \left[h_{13}^{(m)}(\eta_R; \mathbf{x}, \theta) h_2^{(m)}(\eta_R; \mathbf{x}) h_4^{(m)}(\eta_R; \mathbf{x}, \theta) \right. \\ &\quad \left. \cdot h_5^{(m)}(\eta_R; \mathbf{x}, \theta) h_W^{(m)}(\eta_R; \mathbf{x}) \right], \end{aligned}$$

$$\mathcal{P}_D^{(K)}(\zeta_R) = \sum_{m=1}^{2^{n_T}} a^{(m)} \mathbb{E}_{\mathbf{x}, \theta} \left[h_{13}^{(m)}(-1; \mathbf{x}, \theta) h_2^{(m)}(-1; \mathbf{x}) h_4^{(m)}(-1; \mathbf{x}, \theta) \right. \\ \left. \cdot h_5^{(m)}(-1; \mathbf{x}, \theta) h_W^{(m)}(-1; \mathbf{x}) h_\zeta(\zeta_R; \mathbf{x}) \right].$$

The functions h are detailed in the following sections.

7.B.2 First and Third Interference Source

The first and third interference sources are both functions of the positions of the RSUs in the streets belonging to the crossroad, which are not cooperating. Therefore, they must be handled together. The function h_{13} , modelling the impact of these two sources of interference, is developed as

$$\begin{aligned} h_{13}^{(m)}(u; \mathbf{x}, \theta) &= \mathbb{E}_{\{I_{R1,ji}\}, \{I_{R3,ji}\} | \mathbf{x}, \theta} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \exp \left(-\frac{u b_{ji}^{(m)} I_{R1,ji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \exp \left(-\frac{u b_{ji}^{(m)} I_{R3,ji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \right] \\ &\stackrel{(a)}{=} \mathbb{E}_{\{\Phi_{vq}\}_{q \leq N_S}} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \prod_{k | z_{jk} \in \Phi_{vj} \setminus C_j} \iota_{R1}(x_{ji}, z_{lk}) \prod_{\substack{l=0 \\ l \neq j}}^{N_S} \prod_{k | z_{lk} \in \Phi_{vl} \setminus C_l} \iota_{R3}(x_{ji}, z_{lk}, \theta_j, \theta_l) \right] \\ &\stackrel{(b)}{=} \prod_{j=0}^{N_S} \mathbb{E}_{\Phi_{vj}} \left[\prod_{k | z_{jk} \in \Phi_{vj} \setminus C_j} \prod_{i=0}^{n_j-1} \iota_{R1}(x_{ji}, z_{lk}) \prod_{\substack{l=0 \\ l \neq j}}^{N_S} \prod_{i=0}^{n_l-1} \iota_{R3}(x_{li}, z_{jk}, \theta_l, \theta_j) \right] \\ &\stackrel{(c)}{=} \prod_{j=0}^{N_S} \exp \left(-\lambda_{vj} \int_{|z| \geq \rho} \left[1 - \prod_{i=0}^{n_j-1} \iota_{R1}(x_{ji}, z) \prod_{\substack{l=0 \\ l \neq j}}^{N_S} \prod_{i=0}^{n_l-1} \iota_{R3}(x_{li}, z, \theta_l, \theta_j) \right] dz \right), \end{aligned}$$

where the functions ι_{R1} and ι_{R3} are defined as

$$\begin{aligned} \iota_{R1}(x, z) &\triangleq \mathcal{L}_Y \left(\frac{u b_{ji}^{(m)} \kappa_I |x - z|^{-\alpha_I}}{\kappa_R |x|^{-\alpha_R}} \right) \mathcal{L}_Y \left(\frac{u b_{ji}^{(m)} \kappa_I (|x| + |z|)^{-\alpha_I}}{\kappa_R |x|^{-\alpha_R}} \mathbb{1}(xz \geq 0) \right) \\ &= \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I |x - z|^{-\alpha_I}}{\kappa_R |x|^{-\alpha_R}}} \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I (|x| + |z|)^{-\alpha_I}}{\kappa_R |x|^{-\alpha_R}} \mathbb{1}(xz \geq 0)}, \\ \iota_{R3}(x, z, \theta_0, \theta_1) &\triangleq \mathcal{L}_Y \left(\frac{u b_{ji}^{(m)} \kappa_I g_{I,II}(x, z, \theta_0, \theta_1)}{\kappa_R |x|^{-\alpha_R}} \right) \mathcal{L}_Y \left(\frac{u b_{ji}^{(m)} \kappa_I g_{I,III}(x, z, \theta_0, \theta_1)}{\kappa_R |x|^{-\alpha_R}} \right) \\ &\quad \cdot \mathcal{L}_Y \left(\frac{u b_{ji}^{(m)} \kappa_I g_{I,IV}(x, z, \theta_0, \theta_1)}{\kappa_R |x|^{-\alpha_R}} \right) \\ &= \frac{1}{1 + \frac{u \kappa_I g_{I,II}(x, z, \theta_0, \theta_1)}{\kappa_R |x|^{-\alpha_R}}} \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I g_{I,III}(x, z, \theta_0, \theta_1)}{\kappa_R |x|^{-\alpha_R}}} \frac{1}{1 + \frac{u b_{ji}^{(m)} b_{ji}^{(m)} \kappa_I g_{I,IV}(x, z, \theta_0, \theta_1)}{\kappa_R |x|^{-\alpha_R}}}. \end{aligned}$$

Equalities (a) to (c) are respectively obtained by using the definition of the LT of the exponentially distributed small-scale fading RVs, by rearranging the product operators, and using the PGFL of the RSUs PPPs of Box 5.8.

7.B.3 Second Interference Source

With known cooperative RSUs positions, using the definition of the LT of the exponentially-distributed small-scale fading RVs, the function h_2 , modelling the impact of the second interference source, is developed as

$$\begin{aligned} h_2^{(m)}(u; \mathbf{x}) &= \mathbb{E}_{\{I_{R2,ji}\}|\mathbf{x}} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \exp \left(-\frac{u b_{ji}^{(m)} I_{R2,ji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \right] \\ &= \prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \prod_{k|x_{jk} \in C_j \setminus \{x_{ji}\}} \iota_{R2}(x_{ji}, x_{jk}), \end{aligned}$$

with the function ι_{R2} defined as

$$\begin{aligned} \iota_{R2}(x_0, x_1) &\triangleq \mathcal{L}_Y \left(\frac{u b_{ji}^{(m)} \kappa_I |x_0 - x_1|^{-\alpha_I}}{\kappa_R |x_0|^{-\alpha_R}} \right) \mathcal{L}_Y \left(\frac{u b_{ji}^{(m)} \kappa_I (|x_0| + |x_1|)^{-\alpha_I}}{\kappa_R |x_0|^{-\alpha_R}} \mathbb{1}(x_{ji}x_{jk} \geq 0) \right) \\ &= \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I |x_0 - x_1|^{-\alpha_I}}{\kappa_R |x_0|^{-\alpha_R}}} \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I (|x_0| + |x_1|)^{-\alpha_I}}{\kappa_R |x_0|^{-\alpha_R}} \mathbb{1}(x_{ji}x_{jk} \geq 0)}. \end{aligned}$$

7.B.4 Fourth Interference Source

With known cooperative RSUs positions and angles of the crossroad streets, using the definition of the LT of the exponentially-distributed small-scale fading RVs, the function h_4 , modelling the impact of the fourth interference source, is developed as

$$\begin{aligned} h_4^{(m)}(u; \mathbf{x}) &= \mathbb{E}_{\{I_{R4,ji}\}|\mathbf{x}} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \exp \left(-\frac{u b_{ji}^{(m)} I_{R4,ji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \right] \\ &= \prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \prod_{\substack{l=0 \\ l \neq j}}^{N_S} \prod_{k|x_{lk} \in C_l} \iota_{R4}(x_{ji}, x_{lk}, \theta_j, \theta_l), \end{aligned}$$

with the function ι_{R4} defined as

$$\iota_{R4}(x_0, x_1, \theta_0, \theta_1) \triangleq \mathcal{L}_Y \left(\frac{u b_{ji}^{(m)} \kappa_I g_{I,\Pi}(x_0, x_1, \theta_0, \theta_1, 0, 0)}{\kappa_R |x_0|^{-\alpha_R}} \right)$$

$$\begin{aligned}
& \cdot \mathcal{L}_\gamma \left(\frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,III}}(x_0, x_1, \theta_0, \theta_1, 0, 0)}{\kappa_R |x_0|^{-\alpha_R}} \right) \mathcal{L}_\gamma \left(\frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,IV}}(x_0, x_1, \theta_0, \theta_1, 0, 0)}{\kappa_R |x_0|^{-\alpha_R}} \right) \\
&= \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,II}}(x_0, x_1, \theta_0, \theta_1, 0, 0)}{\kappa_R |x_0|^{-\alpha_R}}} \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,III}}(x_0, x_1, \theta_0, \theta_1, 0, 0)}{\kappa_R |x_0|^{-\alpha_R}}} \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,IV}}(x_0, x_1, \theta_0, \theta_1, 0, 0)}{\kappa_R |x_0|^{-\alpha_R}}}.
\end{aligned}$$

7.B.5 Fifth Interference Source

The fifth interference source is function of the positions of the **RSUs** in the streets not belonging to the crossroad. The function h_5 , modelling the impact of this source of interference, is developed as

$$\begin{aligned}
h_5^{(m)}(u; \mathbf{x}, \boldsymbol{\theta}) &= \mathbb{E}_{\{I_{\text{R5},ji}\} | \mathbf{x}, \boldsymbol{\theta}} \left[\prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \exp \left(-\frac{u b_{ji}^{(m)} I_{\text{R5},ji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \right] \\
&\stackrel{(a)}{=} \mathbb{E}_{\Phi_S, \{\Phi_{vq}\}_{q>N_S}, \{\theta_q\}_{q>N_S} | \mathbf{x}, \boldsymbol{\theta}} \left[\prod_{l=N_S+1}^{\infty} \prod_{k | z_{lk} \in \Phi_{v,l}} \iota_{\text{R5}}(z_{lk}, \theta_j, \theta_l, y_l) \right] \\
&\stackrel{(b)}{=} \mathbb{E}_{\Phi_S, \{\theta_q\}_{q>N_S} | \mathbf{x}, \boldsymbol{\theta}} \left[\prod_{l=N_S+1}^{\infty} \exp \left(-\lambda_{vl} \int_{-\infty}^{\infty} [1 - \iota_{\text{R5}}(z, \theta_j, \theta_l, y_l)] dz \right) \right] \\
&\stackrel{(c)}{=} \mathbb{E}_{\Phi_S | \mathbf{x}, \boldsymbol{\theta}} \left[\prod_{l=N_S+1}^{\infty} \frac{1}{2\pi} \int_0^{2\pi} \exp \left(-\lambda_{vl} \int_{-\infty}^{\infty} [1 - \iota_{\text{R5}}(z, \theta_j, \theta, y_l)] dz \right) d\theta \right] \\
&\stackrel{(d)}{=} \exp \left(-\lambda_s \int_0^{\infty} \left[1 - \frac{1}{2\pi} \int_0^{2\pi} \exp \left(-\lambda_v \int_{-\infty}^{\infty} [1 - \iota_{\text{R5}}(z, \theta_j, \theta, y)] dz \right) d\theta \right] dy \right).
\end{aligned}$$

with the function ι_{R5} defined as

$$\begin{aligned}
\iota_{\text{R5}}(z, \theta_0, \theta_1, y) &\triangleq \prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \mathcal{L}_\gamma \left(\frac{u \mathcal{L}_\gamma \left(\frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,III}}(x_0, x_1, \theta_0, \theta_1, 0, 0)}{\kappa_R |x_0|^{-\alpha_R}} \right) \kappa_I g_{\text{I,II}}(x_{ji}, z, \theta_0, \theta_1, 0, y)}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \\
&\cdot \mathcal{L}_\gamma \left(\frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,III}}(x_{ji}, z, \theta_0, \theta_1, 0, y)}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \mathcal{L}_\gamma \left(\frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,IV}}(x_{ji}, z, \theta_0, \theta_1, 0, y)}{\kappa_R |x_{ji}|^{-\alpha_R}} \right) \\
&= \prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,II}}(x_{ji}, z, \theta_0, \theta_1, 0, y)}{\kappa_R |x_{ji}|^{-\alpha_R}}} \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,III}}(x_{ji}, z, \theta_0, \theta_1, 0, y)}{\kappa_R |x_{ji}|^{-\alpha_R}}} \\
&\cdot \frac{1}{1 + \frac{u b_{ji}^{(m)} \kappa_I g_{\text{I,IV}}(x_{ji}, z, \theta_0, \theta_1, 0, y)}{\kappa_R |x_{ji}|^{-\alpha_R}}}.
\end{aligned}$$

Equalities (a) to (d) are respectively obtained using the definition of the **LT** of the exponentially distributed small-scale fading **RVs**, the **PGFL** of the **RSUs PPPs** of Box 5.8, averaging over the angles of the streets not belonging to the crossroad, and using the **PGFL** of the **RSUs PPPs** again with Box 5.8. Additionally, it is assumed that the **RSUs** density is the same for every street.

7.B.6 Noise and Threshold Functions

As the noise power and the detection threshold are supposed to be constant, the two functions h_W and h_ζ are directly obtained as

$$h_W^{(m)}(u; \mathbf{x}) = \prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \exp \left(-\frac{u W_{Rji}}{\kappa_R |x_{ji}|^{-\alpha_R}} \right),$$

$$h_\zeta(u; \mathbf{x}) = \prod_{j=0}^{N_S} \prod_{i=0}^{n_j-1} \exp \left(-\frac{u}{\kappa_R |x_{ji}|^{-\alpha_R}} \right).$$

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Part III

Interference Management in Full-Duplex Integrated Sensing and Communication Scenarios

Integrated Sensing and Communication Waveforms | 8

This chapter introduces popular ISAC systems from the literature, based on the FMCW and OFDM waveforms.

IN recent years, numerous JCAS waveforms design have been proposed in the literature. Among the multiple candidates proposed to fulfil simultaneously both functions, two designs stand out, namely the FMCW and OFDM ISAC systems. The former is radar-centric, and leverages on the chirp waveform to integrate communication capabilities in multiple manners. Contrariwise, the latter is communication-centric, and leverages on the OFDM modulation to perform sensing.

This chapter is dedicated to the complete description of such waveforms. FMCW systems are first analysed in Section 8.1, whereas a focus is made on OFDM systems in Section 8.2. Finally, in this part of the thesis, a focus is made on full-duplex scenarios. Thus, the structures of the FMCW and OFDM receivers are detailed in Section 8.3 for full-duplex applications.

8.1 Frequency Modulated Continuous Wave

The FMCW is the a popular waveform in automotive radar applications [WHM21]. This waveform is composed of a succession of *chirps*, i.e. single tones signals with a linearly increasing or decreasing frequency during time. From such systems, multiple variations have been proposed in the literature to integrate communication capabilities. This section first presents the chirp waveform, and how it is used for sensing. Then, one popular variation of this waveform for ISAC, i.e. phase-coded FMCW, is detailed. Finally, other variations of the phase-coded FMCW from the literature are introduced.

8.1.1 Chirp Waveform

A chirp of duration T , bandwidth B , and transmitted at a carrier frequency f_c is defined as a signal with a linearly increasing or decreasing instantaneous frequency, and phase given in Box 8.1. This is also known as *linear frequency modulation*. The chirp is renamed *upchirp* or *downchirp* respectively if the frequency is increasing or decreasing. When it is not specified, one usually consider upchirps.

8.1: Instantaneous Frequency and Phase of a Chirp

$$\begin{aligned} f(t) &= f_c \pm \frac{B}{T}t \\ \psi(t) &= 2\pi \int_0^t f(t') dt' = 2\pi f_c t \pm \pi \frac{B}{T}t^2 \end{aligned} \quad \text{for } 0 \leq t \leq T$$

One may notice that the instantaneous frequency of the chirps varies from f_c to f_c+B . Instead, the chirp may also be centred in frequency by defining the instantaneous frequency and phase as

$$\begin{aligned} f(t) &= f_c - \frac{B}{2} \pm \frac{B}{T}t \\ \psi(t) &= 2\pi \left(f_c - \frac{B}{2} \right) t \pm \pi \frac{B}{T}t^2 \end{aligned} \quad \text{for } 0 \leq t \leq T.$$

8.1.1.1 Ambiguity Function of a Chirp-Modulated Signal

Let us denote by s an arbitrary waveform in complex baseband representation, with ambiguity function χ_s . By modulating this waveform with an (up)chirp, one obtains

$$\tilde{s}(t) = s(t) e^{j\pi \frac{B}{T}t^2}.$$

The ambiguity function of the chirp-modulated signal is then computed as

$$\begin{aligned} \chi_{\tilde{s}}(\tau, f_D) &= \int_t \tilde{s}(t) \tilde{s}^*(t - \tau) e^{-j2\pi f_D t} dt \\ &= \int_t s(t) s^*(t - \tau) e^{j\pi \frac{B}{T}t^2} e^{-j\pi \frac{B}{T}(t - \tau)^2} e^{-j2\pi f_D t} dt \\ &= e^{-j\pi \frac{B}{T}\tau^2} \int_t s(t) s^*(t - \tau) e^{j2\pi \frac{B}{T}\tau t} e^{-j2\pi f_D t} dt \\ &= e^{-j\pi \frac{B}{T}\tau^2} \chi_s \left(\tau, f_D - \frac{B}{T}\tau \right). \end{aligned}$$

This result is summarised in Box 8.2. This shows that the ambiguity function of a chirp-modulated signal is a skewed version of the ambiguity function of the original signal in the delay-Doppler plane. Therefore, this modulation introduces a coupling between the delay and Doppler dimensions.

Applying this operation to a normalised rectangular pulse, from Box 1.4, one obtains Box 8.3. Regarding the delay resolution, the first zero of the ambiguity

8.2: Ambiguity Function of a Chirp-Modulated Signal

$$\chi_s(\tau, f_D) = e^{-j\pi \frac{B}{T} \tau^2} \chi_s\left(\tau, f_D - \frac{B}{T} \tau\right)$$

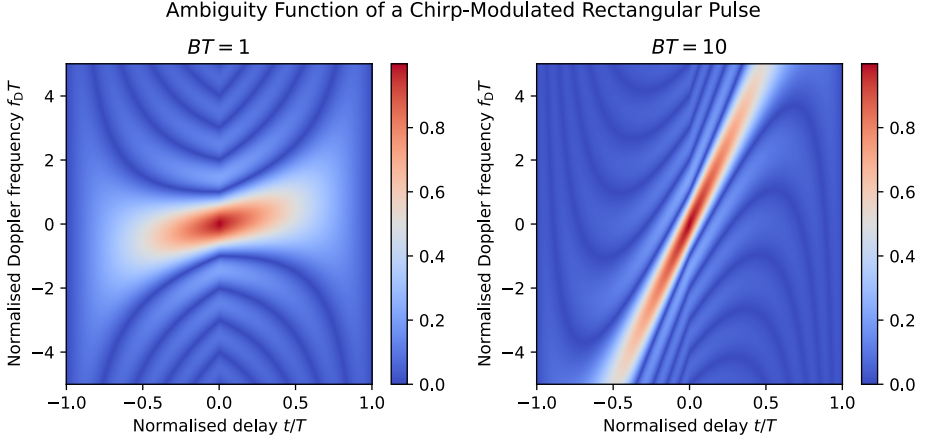


Fig. 8.1 Ambiguity function of a chirp-modulated rectangular pulse for different BT -products.

function along the delay-axis is computed as

$$|\chi_s(\tau, 0)| = B\tau \left(1 - \frac{|\tau|}{T}\right) = 1 \quad \xRightarrow{\tau > 0} \quad \tau = \frac{T}{2} \left(1 - \sqrt{1 - \frac{4}{BT}}\right) \quad \xrightarrow{BT \gg 1} \quad \frac{1}{B}.$$

Therefore, for high BT -products, the Rayleigh resolution of the range with a chirp-modulated rectangular pulse is still equal to $c/2B$, as in Box 1.5. However, the chirp bandwidth is not related to the pulse duration, meaning that one may increase the resolution while keeping constant the pulse duration and energy. This technique is named *pulse compression*. The ambiguity function of the chirp-modulated rectangular pulse is illustrated in Figure 8.1 for different BT -products.

8.3: Ambiguity Function of a Chirp-Modulated Rectangular Pulse

$$|\chi_s(\tau, f_D)| = \begin{cases} \left(1 - \frac{|\tau|}{T}\right) \operatorname{sinc}\left((f_D T - B\tau) \left(1 - \frac{|\tau|}{T}\right)\right) & \text{for } -T \leq \tau \leq T \\ 0 & \text{otherwise} \end{cases}$$

8.1.1.2 Stretch Processing

The *stretch processing*, also known as *dechirping*, is a special technique for matched filtering of chirp waveforms. Let us assume that the following signal is received in baseband (the noise is neglected here):

$$u(t) = \xi s(t - \tau) = \xi e^{j\pi \frac{B}{T}(t-\tau)^2},$$

where ξ is an unknown complex coefficient, containing for instance the response of the target, and τ is the delay of the reflection on the target. The dechirp operation consists by mixing the received signal by the conjugate of the transmitted waveform:

$$\begin{aligned} y(t) &= u(t) s^*(t) \\ &= \xi e^{j\pi \frac{B}{T}(t-\tau)^2} e^{-j\pi \frac{B}{T}t^2} \\ &= \xi e^{j\pi \frac{B}{T}\tau^2} e^{-j2\pi \frac{B}{T}\tau t}. \end{aligned}$$

Therefore, at the output of this operation, an additional constant phase shift is added, and the output signal oscillates at a frequency named *beat frequency*, equal to $f_B = B\tau/T$. This enables to estimate the delay of the target by performing a Fourier analysis, and estimating the beat frequency. One may observe that this processing is equivalent to the computation of an ambiguity function.

If the dechirp operation is performed in radio-frequency, one obtains

$$\begin{aligned} y(t) &= r_{\text{RF}}(t) s_{\text{RF}}(t) \\ &= \xi \cos\left(2\pi f_c t + \pi \frac{B}{T}(t - \tau)^2\right) \cos\left(2\pi f_c t + \pi \frac{B}{T}t^2\right) \\ &= \frac{\xi}{2} \cos\left(4\pi f_c t + \pi \frac{B}{T}[(t - \tau)^2 + t^2]\right) + \frac{\xi}{2} \cos\left(\pi \frac{B}{T}[(t - \tau)^2 - t^2]\right) \\ &= \frac{\xi}{2} \cos\left(4\pi f_c t + \pi \frac{B}{T}[2t^2 + \tau^2 + 2t\tau]\right) + \frac{\xi}{2} \cos\left(2\pi \frac{B}{T}\tau t\right). \end{aligned}$$

Therefore, after filtering the high-frequency component of the signal, one obtains again a signal oscillating at the beat frequency.

When the received signal is also affected by a Doppler shift, one obtains

$$y(t) = \xi e^{j\pi \frac{B}{T}\tau^2} e^{-j2\pi(\frac{B}{T} - f_D)\tau t}.$$

As shown in the ambiguity function of a chirp-modulated signal, the delay and Doppler frequency are coupled with each other. Therefore, one may estimate first the Doppler frequency in another way, and then compensate its effect on the beat frequency to estimate the Doppler. This will be further described in Section 8.1.2.

In any case, the beat frequency is usually equal to a small fraction of the bandwidth. Therefore, if the dechirp operation can be performed in the analogue domain, the [Analogue-to-Digital Converter \(ADC\)](#) sampling rate is limited compared to other waveforms, for which the sampling rate should at least be equal to the bandwidth. This is a significant advantage in terms of power consumption and cost of the system.

8.1.2 Frequency Modulated Continuous Wave for Sensing

Let us now assume that the transmitter emits multiple successive centred chirps, such that the instantaneous frequency of the complete signal is given by [Box 8.4](#).

8.4: Instantaneous Frequency of the FMCW Waveform

$$f(t) = f_c + \sum_{p=0}^{P-1} \left[-\frac{B}{2} + \frac{B}{T}(t - pT_{\text{PRI}}) \right] \Pi \left(\frac{t - pT_{\text{PRI}}}{T} \right)$$

In [Box 8.4](#), P is the number of transmitted pulses, T is the chirp duration, $T_{\text{PRI}} > T$ is the [Pulse Repetition Interval \(PRI\)](#), and Π is the non-centred rectangular function:

$$\Pi(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

The guard interval T_g is set such that $T_g = T_{\text{PRI}} - T \geq \max_k \{\tau_k\}$, to avoid inter-pulse interference. Note that, by introducing the shift of $-B/2$, the chirp is centred around the carrier frequency. The instantaneous phase at time $t \in [pT_{\text{PRI}}, pT_{\text{PRI}} + T]$ is then computed as

$$\begin{aligned} \psi(t) &= 2\pi f_c t + 2\pi \sum_{p'=0}^{p-1} \int_0^T \left[-\frac{B}{2} + \frac{B}{T}t' \right] dt' + 2\pi \int_0^{t-pT_{\text{PRI}}} \left[-\frac{B}{2} + \frac{B}{T}t \right] dt' \\ &= 2\pi f_c t + 2\pi \left[-\frac{BT}{2} + \frac{BT}{2} \right] + 2\pi \left[-\frac{B}{2}(t - pT_{\text{PRI}}) + \frac{1}{2} \frac{B}{T}(t - pT_{\text{PRI}})^2 \right] \\ &= 2\pi f_c t - \pi B(t - pT_{\text{PRI}}) + \pi \frac{B}{T}(t - pT_{\text{PRI}})^2, \end{aligned}$$

leading to the waveform given in [Box 8.5](#).

Following the single-point targets channel model of (1.3), the received signal in baseband (neglecting the noise) is given by

$$u(t) = \int_{\tau} h(t, \tau) s(t - \tau) d\tau$$

8.5: FMCW Waveform (Complex Baseband Representation)

$$s(t) = \sum_{p=0}^{P-1} e^{-j\pi B(t-pT_{\text{PRI}})} e^{j\pi \frac{B}{T}(t-pT_{\text{PRI}})^2} \Pi\left(\frac{t-pT_{\text{PRI}}}{T}\right)$$

$$= \sum_{k=0}^{K-1} \alpha_k e^{j2\pi f_{\text{D}k} t} \sum_{p=0}^{P-1} e^{-j\pi B(t-\tau_k-pT_{\text{PRI}})} e^{j\pi \frac{B}{T}(t-\tau_k-pT_{\text{PRI}})^2} \Pi\left(\frac{t-\tau_k-pT_{\text{PRI}}}{T}\right).$$

In this equation, K is the number of single point targets, τ_k is the delay, and $f_{\text{D}k}$ is the Doppler frequency of the k^{th} target. For the sake of readability, let us define the *pulse notation* of a signal x as

$$x(p, t) = x(t - pT_{\text{PRI}}), \quad \text{for } 0 \leq t \leq T_{\text{PRI}}.$$

The received signal in pulse notation is written as

$$u(p, t) = \sum_{k=0}^{K-1} \alpha_k e^{j2\pi f_{\text{D}k} T_{\text{PRI}} p} e^{j2\pi f_{\text{D}k} t} e^{-j\pi B(t-\tau_k)} e^{j\pi \frac{B}{T}(t-\tau_k)^2} \Pi\left(\frac{t-\tau_k}{T}\right).$$

After dechirping, one obtains

$$\begin{aligned} y(p, t) &= u(p, t) s^*(p, t) \\ &= u(p, t) e^{j\pi B t} e^{-j\pi \frac{B}{T} t^2} \Pi\left(\frac{t}{T}\right) \\ &= \sum_{k=0}^{K-1} \tilde{\alpha}_k e^{j2\pi f_{\text{D}k} T_{\text{PRI}} p} e^{-j2\pi \left(\frac{B}{T} \tau_k - f_{\text{D}k}\right) t} \Pi\left(\frac{t-\tau_k}{T-\tau_k}\right), \end{aligned} \quad (8.1)$$

with

$$\tilde{\alpha}_k = \alpha_k e^{j\pi B \tau_k} e^{j\pi \frac{B}{T} \tau_k^2}.$$

From (8.1), one may observe that the Doppler frequencies of the targets can be estimated from a Fourier analysis performed over the successive pulses (also called *slow-time*), whereas the delays can be estimated from a Fourier analysis performed within the pulses (also called *fast-time*). Either the Doppler frequency is supposed to be large compared to the beat frequency, or it is compensated by skewing back the delay-Doppler map by the right angle. The delay/range and Doppler/speed resolutions are summarised in Box 8.6 where $T_{\text{CPI}} = PT_{\text{PRI}}$ is the coherent processing interval.

For the maximum non-ambiguous detectable Doppler frequency, assuming that both positive and negative Doppler frequencies are estimated, one obtains

$$f_{\text{D max}} = \frac{1}{2T_{\text{PRI}}} \quad \leftrightarrow \quad v_{\text{max}} = \frac{c}{4f_c T_{\text{PRI}}}.$$

8.6: Delay/Range and Doppler/Speed Resolutions of the FMCW Radar

$$\begin{aligned} \delta_\tau &= \frac{1}{B} & \leftrightarrow & \delta_r = \frac{c}{2B} \\ \delta_{f_D} &= \frac{1}{T_{\text{CPI}}} & \leftrightarrow & \delta_v = \frac{c}{2f_c T_{\text{CPI}}} \end{aligned}$$

Regarding the maximum non-ambiguous detectable range, it depends on the configuration of the radar system. After dechirping, a low pass filter is usually applied to remove all the frequency components higher than the highest beat frequency the radar should be able to process, i.e. BT_g/T if the guard interval is set depending on the maximal detectable range. Otherwise, it is limited by the maximum sampling rate of the [ADC](#). Figure 8.2 illustrates the dechirping and low-pass filtering for [FMCW](#) radars.

8.1.3 Phase-Coded Frequency Modulated Continuous Wave for Integrated Sensing and Communication

Multiple methods have been proposed in the literature to integrate communication capabilities in [FMCW](#) systems. One of the simplest modifications is to transmit an upchirp for a bit equal to 1, and a downchirp for a bit equal to 0. The cross-correlation of an up and downchirp is equal to

$$\left| \frac{1}{T} \int_0^T e^{j2\pi \frac{B}{T} t^2} dt \right|^2 = \frac{1}{4BT} \left| F\left(\sqrt{4BT}\right) \right|^2 \xrightarrow{BT \gg 1} \frac{1}{2BT},$$

where F is the Fresnel function of (3.6). The cross-correlation is small for high BT -products, compared to the auto-correlation of the up- and downchirp, equal to 1. This good cross-correlation property enables to use the same waveform, unaltering the radar function while transmitting information. This approach is adopted in [MS20] in association with Alamouti space-time coding. However, the drawback of this method is the achievable rate, limited by the [PRI](#). The same applies for instance to [Fag+23], which uses frequency shift keying to modify the starting frequency of the chirps for communication.

In order to integrate multiple symbols, another method is to modulate the usual communication payload with a chirp. This has been first proposed by [BEJ07], with amplitude modulation. Then, it has been extended in phase with different designs in [Ala+19; Uys20; Bar+21] for higher order modulations, and in [Bek+21] for MIMO systems. Hardware implementations have also been demonstrated in [McC+19; UO20; Kum+21]. Finally, the performance achieved with different receiver structures are compared in [Kum+22; Kum+23]. This method is named *Phase-Coded Frequency*

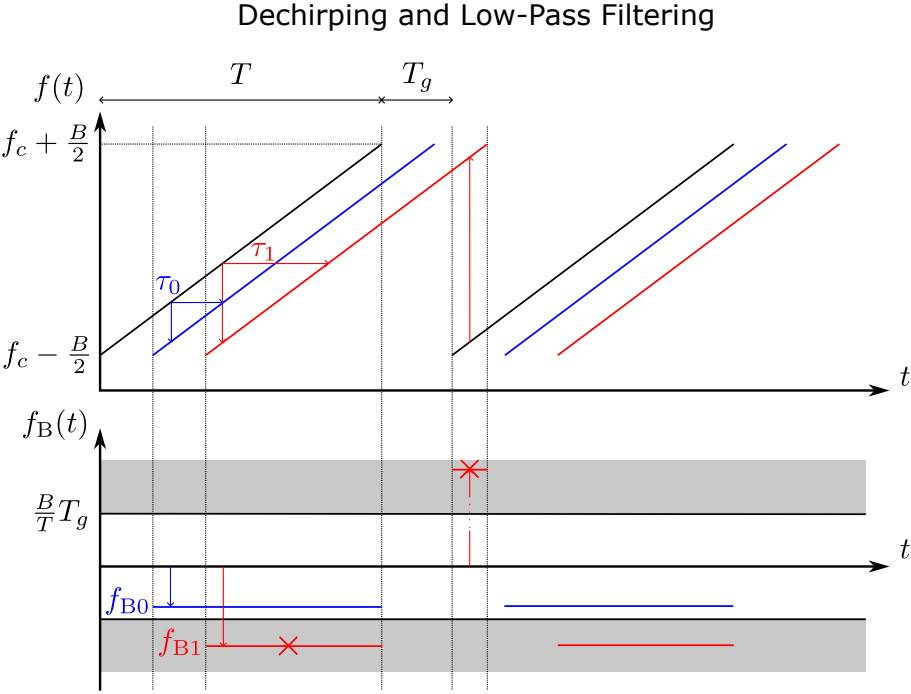


Fig. 8.2 Dechirping and low-pass filtering operation. Two echoes with delays τ_1 and τ_2 are illustrated, with $\tau_1 < T_g < \tau_2$. Therefore, the low-pass filter remove the beat frequency f_{B2} of the second echo.

Modulated Continuous Wave (PC-FMCW), and it is detailed in this section.

Let us consider a classical complex pulse modulation scheme, where symbols I_m are transmitted with a rectangular pulse shaping filter. The transmitted signal is written as

$$s_C(t) = \sum_{n=0}^{N-1} I_n \Pi\left(\frac{t - nT_C}{T_C}\right),$$

where T_C is the communication symbol duration. In a PC-FMCW ISAC system, this signal is modulated by a chirp, written as

$$s_R(t) = e^{-j\pi B_R t} e^{j\pi \frac{B_R}{T} t^2} \Pi\left(\frac{t}{T}\right),$$

where $T = NT_C$ is the chirp duration, which is also the duration of the communication symbol train, and B is the chirp bandwidth. When multiple pulses are transmitted, the PC-FMCW waveform of Box 8.7 is obtained, where $I_{p,n}$ are the communication symbols transmitted during the p^{th} pulse.

8.7: PC-FMCW ISAC Waveform (Complex Baseband Representation)

$$s(t) = \sum_{p=0}^{P-1} \left[\sum_{n=0}^{N-1} I_{p,n} \Pi\left(\frac{t - pT_{\text{PRI}} - nT_C}{T_C}\right) \right] e^{-j\pi B(t - pT_{\text{PRI}})} e^{j\pi \frac{B}{T}(t - pT_{\text{PRI}})^2}$$

8.1.3.1 Radar Processing

At the ISAC receiver, neglecting the noise, the received signal reflected by K targets, with complex coefficients $\{\alpha_k\}$, delays $\{\tau_k\}$ and Doppler frequencies $\{f_{Dk}\}$, is written as

$$u(t) = \sum_{k=0}^{K-1} \alpha_k e^{j2\pi f_{Dk} t} s(t - \tau_k),$$

which is written in pulse notation as

$$u(p, t) = \sum_{k=0}^{K-1} \alpha_k e^{j2\pi f_{Dk} T_{\text{PRI}} p} e^{j2\pi f_{Dk} t} \sum_{n=0}^{N-1} I_{p,n} \Pi\left(\frac{t - \tau_k - nT_C}{T_C}\right) e^{-j\pi B(t - \tau_k)} e^{j\pi \frac{B}{T}(t - \tau_k)^2}.$$

At the radar receiver, a dechirp operation is first performed. Assuming that $\max_k \{\tau_k\} \leq T_{\text{PRI}} - T$, the processed signal is written as

$$y(p, t) = u(p, t) e^{j\pi B t} e^{-j\pi \frac{B}{T} t^2} \Pi\left(\frac{t}{T}\right)$$

$$= \sum_{k=0}^{K-1} \beta_k e^{j2\pi f_{\text{D}k} T_{\text{PRI}} p} e^{-j2\pi \left(\frac{B}{T} \tau_k - f_{\text{D}k}\right) t} \sum_{n=0}^{N-1} I_{p,n} \Pi \left(\frac{t - \tau_k - nT_{\text{C}}}{T_{\text{C}}} \right),$$

where

$$\beta_k = \alpha_k e^{j\pi B \tau_k} e^{j\pi \frac{B}{T} \tau_k^2}.$$

Compared to classical chirp-based radars, the presence of the communication symbols will induce a dispersion of the echo power on the delay-Doppler map. This is for instance studied in [Uys20], and it will be further analysed in Section 9.1.1. One method to circumvent this problem is the use of a group delay filter.

Let us denote by H_{gd} a group delay filter, i.e. $|H_{\text{gd}}| = 1$. The *group delay* is defined as

$$\tau_{\text{gd}} \triangleq -\frac{1}{2\pi} \frac{\text{d arg} \{H_{\text{gd}}(f)\}}{\text{d}f}.$$

A signal with envelope s and carrier frequency f_0 passing through the group delay filter is transformed in radio frequency as [Opp+97]

$$\tilde{s}(t) \approx \text{Re} \left\{ s(t - \tau_{\text{gd}}) e^{j2\pi f_0 (t - \tau_{\text{pd}})} \right\},$$

where τ_{pd} is named *phase delay* and defined as

$$\tau_{\text{pd}} \triangleq -\frac{1}{2\pi} \frac{\arg \{H_{\text{gd}}(f)\}}{f}.$$

This supposes that the envelope is slowly varying compared to the carrier frequency f_0 :

$$\frac{1}{2\pi} \left| \frac{\text{d}}{\text{d}t} \log s(t) \right| \ll f_0.$$

Assuming that the beat frequencies of the targets are large compared to their Doppler frequencies, one may assume that $B\tau_k/T - f_{\text{D}k} \approx B\tau_k/T$. Therefore, the echo of the target associated with the beat frequency $f_{\text{B}} = -B\tau_k/T$ should be delayed by a group delay $\tau_{\text{gd}}(f) = Tf/B$. This leads to the following phase response:

$$\arg \{H_{\text{gd}}(f)\} = \phi_0 - 2\pi \int_0^f \tau_{\text{gd}}(f) \text{d}f = \phi_0 - 2\pi \int_0^f \frac{T}{B} f \text{d}f = \phi_0 - \pi \frac{T}{B} f^2,$$

with ϕ_0 being a constant phase offset. The phase delay is thus equal to $\tau_{\text{pd}} = Tf/2B$. Applying this filter to the received signal, one obtains

$$y_{\text{gd}}(p, t) \approx \sum_{k=0}^{K-1} \beta_k e^{-j\pi \frac{B}{T} \tau_k^2} e^{j2\pi f_{\text{D}k} T_{\text{PRI}} p} e^{-j2\pi f_{\text{D}k} t} \sum_{n=0}^{N-1} I_{p,n} \Pi \left(\frac{t - nT_{\text{C}}}{T_{\text{C}}} \right)$$

Group Delay Filtering

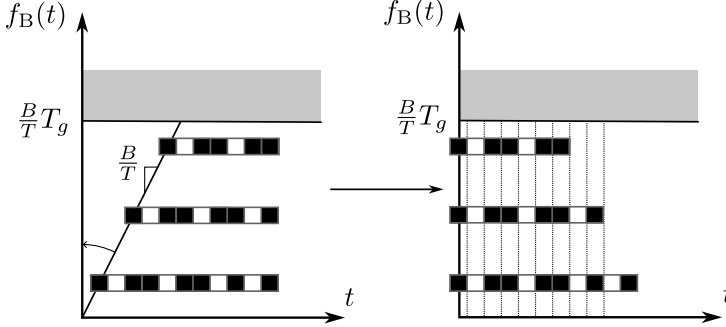


Fig. 8.3 Group delay filtering after the dechirp and low-pass filter operations. The black and white squares represent the communication symbols.

$$= \sum_{n=0}^{N-1} I_{p,n} \Pi \left(\frac{t - nT_C}{T_C} \right) \sum_{k=0}^{K-1} \gamma_k e^{j2\pi f_{Dk} T_{PRI} p} e^{-j2\pi f_{Bk} t}, \quad (8.2)$$

where γ_k is defined as

$$\gamma_k = \beta_k e^{-j\pi \frac{B}{T} \tau_k^2} = \alpha_k e^{j\pi B \tau_k}.$$

Thanks to the group delay filter, the multiple radar echoes associated with different targets are now aligned in time, as illustrated in Figure 8.3. This enables to compensate the known communication symbols, allowing to perform the Fourier analysis in the slow- and fast-time domain:

$$z(p, t) = \sum_{n=0}^{N-1} y_{gd}(p, t) \frac{I_{np}^*}{|I_{np}|^2} \Pi \left(\frac{t - nT_C}{T_C} \right) \approx \sum_{k=0}^{K-1} \gamma_k e^{j2\pi f_{Dk} T_{PRI} p} e^{-j2\pi f_{Bk} t}. \quad (8.3)$$

8.1.3.2 Communication Processing

At the communication receiver, which may also be another **ISAC** or communication system compatible with the **PC-FMCW** waveform, the received signal from a Q -paths channel with complex coefficients $\{\alpha_q\}$, delays $\{\tau_q\}$, and Doppler frequencies $\{f_{Dq}\}$, is written as

$$u(t) = \sum_{q=0}^{Q-1} \alpha_q e^{j2\pi f_{Dq} t} s(t - \tau_q),$$

which is rewritten in pulse notation as

$$u(p, t) = \sum_{q=0}^{Q-1} \alpha_q e^{j2\pi f_{Dq} T_{PRI} p} e^{j2\pi f_{Dq} t} \sum_{n=0}^{N-1} I_{p,n} \Pi \left(\frac{t - \tau_q - nT_C}{T_C} \right) e^{-j\pi B(t - \tau_q)} e^{j\pi \frac{B}{T}(t - \tau_q)^2}.$$

First, time synchronisation is performed to align the signal with the start of the most powerful path. Then a dechirp operation is performed. Denoting by τ_M this delay, one obtains

$$\begin{aligned} y(p, t) &= u(p, t + \tau_M) e^{j\pi B t} e^{-j\pi \frac{B}{T} t^2} \Pi\left(\frac{t}{T}\right) \\ &= \sum_{q=0}^{Q-1} \beta_{pq} e^{-j2\pi(\frac{B}{T}\tilde{\tau}_q - f_{Dq})t} \sum_{n=0}^{N-1} I_{p,n} \Pi\left(\frac{t - \tilde{\tau}_q - nT_C}{T_C}\right). \end{aligned} \quad (8.4)$$

This corresponds to a symbol train passing through a time-variant channel with delays $\tilde{\tau}_q = \tau_q - \tau_M$, Doppler frequencies of $B\tilde{\tau}_q/T - f_{Dq}$, and complex coefficients β_{pq} given by

$$\beta_{pq} = \alpha_q e^{-j2\pi f_{Dq}\tau_M} e^{j2\pi f_{Dq}T_{PR1}p} e^{j\pi B\tau_M q} e^{j\pi \frac{B}{T}\tilde{\tau}_q^2}.$$

Therefore, usual communication processing techniques can be applied to recover the transmitted symbols, i.e. matched filtering, frequency offset estimation and correction, and equalisation.

8.1.4 Extensions of the Frequency Modulated Continuous Wave Integrated Communication and Sensing System

Other variations of the [PC-FMCW ISAC](#) waveforms have been proposed in the literature. For instance, a first issue of the [PC-FMCW](#) waveform is the bandwidth increase compared to the chirp bandwidth, consequence of the introduction of the communication symbols. This problem is overcome in [\[Zha+17a\]](#), which separates the chirp in a three-section waveform to limit the excess bandwidth. For instance, [\[Ma+20; Ma+21\]](#) have extended the waveform in a multi-carrier multi-antenna design, leveraging on index modulation. Another solution proposed by [\[Ni+19\]](#) which avoid bandwidth increases is to modify the transmission rate during the chirp duration.

In order to reduce the distortions induced by phase changes at the power amplifier, [Continuous Phase Modulation \(CPM\)](#) and [Minimum Shift Keying \(MSK\)](#) have been introduced for the communication function in [\[DZZ17; Lam+19\]](#). An experimental setup implementing this waveform has been presented in [\[BZH21\]](#). This has been further extended in [\[Zha+17b\]](#) for multi-user applications with [MIMO](#) systems. Besides, to further increase the data rate, the [CPM FMCW ISAC](#) waveform is coupled with low density parity check codes in [\[Li+19\]](#).

More recently, to keep a low complexity at the radar receiver, [\[ZRA22\]](#) has combined the [OTFS](#) modulation with [FMCW](#) waveforms. It is also combined with [OFDM](#) in [\[Bod+23\]](#), using the [FMCW](#) waveforms to simultaneously perform for radar sensing and channel estimation, acting like pilot signals for the communication function.

8.2 Orthogonal Frequency Division Multiplexing

Another popular modulation for communication applications is the **OFDM**. The **OFDM** modulation transmits multiple symbols on different orthogonal subcarriers. This enables to perform easily the equalisation of frequency selective channels at the communication receiver. Nonetheless, its extension to radar applications is also straightforward [SZW09; TJX14]. Experimental results with such implementation are for example obtained in [Sch+19]. Another processing for the radar function which is not detailed in this chapter consists to directly compute the ambiguity function of the **OFDM** symbols. Such approach is for instance proposed in [FNG06; SW11; BHS17]. The main drawbacks of the **OFDM** modulation are the high **Peak-to-Average Power Ratio (PAPR)** and sensitivity to frequency offsets, which should be accommodated for both radar and communication functions [HL05].

An **OFDM** symbol is generated in baseband from the symbols I_n as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} I_n e^{j2\pi n \Delta_f (t - T_{CP})} \Pi\left(\frac{t}{T_{PRI}}\right),$$

where Δ_f is the subcarrier spacing, T_{CP} is the cyclic prefix duration, and $T_{PRI} = T + T_{CP}$, with T the **OFDM** symbol duration. To ensure the orthogonality between the subcarriers, the following condition should be fulfilled:

$$\frac{1}{T} \left| \int_0^T e^{j2\pi(n-m)\Delta_f t} dt \right|^2 = \delta_{nm} \Leftrightarrow \text{sinc}^2((n-m)\Delta_f T) = \delta_{nm} \Leftrightarrow \Delta_f = \frac{1}{T},$$

where sinc is the normalised cardinal sine, and δ_{nm} is the Kronecker symbol. When multiple **OFDM** symbols are transmitted, the signal of Box 8.8 is obtained.

8.8: OFDM ISAC Waveform (Complex Baseband Representation)

$$s(t) = \frac{1}{\sqrt{N}} \sum_{p=0}^{P-1} \sum_{n=0}^{N-1} I_{p,n} e^{j\frac{2\pi}{T} n(t - pT_{PRI} - T_{CP})} \Pi\left(\frac{t - pT_{PRI}}{T_{PRI}}\right)$$

One may relate the continuous-time to the discrete-time **OFDM** implementation by sampling this expression at a rate MN/T , with M the oversampling factor. Looking at the sampled pulse notation of this signal, one obtains

$$s(p, l) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} I_{p,n} e^{j\frac{2\pi}{NM} n(l - N_{CP})},$$

with $N_{CP} = T_{CP}/NM$. This corresponds to the inverse discrete Fourier transform of the

input symbols, where the input symbol sequence has been zero-padded with $N(M-1)$ zeros. The parametrisation of such waveform for ISAC is detailed in [Bra+09].

8.2.1 Communication Processing

At the communication receiver, neglecting the noise, the received signal in base-band from a Q -paths channel with complex coefficients α_q , delays τ_q and Doppler frequencies f_{Dq} is written as

$$u(t) = \sum_{q=0}^{Q-1} \alpha_q e^{j2\pi f_{Dq}t} \frac{1}{\sqrt{N}} \sum_{p=0}^{P-1} \sum_{n=0}^{N-1} I_{p,n} e^{j\frac{2\pi}{T}n(t-\tau_q-pT_{\text{PRI}}-T_{\text{CP}})} \Pi\left(\frac{t-\tau_q-pT_{\text{PRI}}}{T_{\text{PRI}}}\right).$$

First, time synchronisation is performed to align the signal with the start of the most powerful path. Still, thanks to the Fourier transform properties, it is only required to start the processing in a window of duration T_{CP} before the start of the first received path. Such timing may be achieved through a correlation with the preamble, or with the Schmidl & Cox algorithm [SC97]. Then, the signal is sampled at rate MN/T and the cyclic prefix is discarded. Denoting by τ_M the delay of the most powerful path, one obtains in pulse notation

$$\begin{aligned} u(p, l) &= r\left(l\frac{T}{MN} + \tau_M + pT_{\text{PRI}} + T_{\text{CP}}\right) \\ &= \sum_{q=0}^{Q-1} \alpha_q e^{j2\pi f_{Dq}(\tau_M + pT_{\text{PRI}} + l\frac{T}{MN} + T_{\text{CP}})} \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} I_{p,n} e^{j\frac{2\pi}{T}n(l\frac{T}{MN} - \tilde{\tau}_q)} \Pi\left(\frac{l\frac{T}{MN} + T_{\text{CP}} - \tilde{\tau}_q}{T_{\text{PRI}}}\right) \\ &\approx \sum_{q=0}^{Q-1} \beta_q e^{j2\pi f_{Dq}T_{\text{PRI}}p} \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} I_{p,n} e^{-j\frac{2\pi}{T}\tilde{\tau}_qn} e^{j\frac{2\pi}{NM}(n+f_{Dq}T)l}, \end{aligned}$$

where $\tilde{\tau}_q = \tau_q - \tau_M$, and

$$\beta_q = \alpha_q e^{j2\pi f_{Dq}(\tau_M + T_{\text{CP}})}.$$

The last approximation is obtained assuming that N_{CP} is designed such that it is larger than the channel length. Finally, an NM -discrete Fourier transform is applied over the fast-time, giving the following signal:

$$\begin{aligned} y(p, m) &= \frac{1}{\sqrt{N}} \sum_{l=0}^{NM-1} u(p, l) e^{-j\frac{2\pi}{NM}lm} \\ &= \sum_{q=0}^{Q-1} \beta_q e^{j2\pi f_{Dq}T_{\text{PRI}}p} \sum_{n=0}^{N-1} I_{p,n} e^{-j\frac{2\pi}{T}\tilde{\tau}_qn} \left(\frac{1}{N} \sum_{l=0}^{NM-1} e^{j\frac{2\pi}{NM}(n+f_{Dq}T-m)l} \right) \\ &= I_{p,m} \sum_{q=0}^{Q-1} \beta_q \zeta(f_{Dq}T) e^{j2\pi f_{Dq}T_{\text{PRI}}p} e^{-j\frac{2\pi}{T}\tilde{\tau}_qm} + \text{ICI}_{p,m}, \end{aligned} \quad (8.5)$$

where the **Inter-Carrier Interference** (ICI) is given by

$$\text{ICI}_{p,m} = \sum_{\substack{n=0 \\ n \neq m}}^{N-1} I_{p,n} \sum_{q=0}^{Q-1} \beta_q \zeta(n - m + f_{Dq}T) e^{j2\pi f_{Dq}T_{\text{PRI}}p} e^{-j\frac{2\pi}{T} \tilde{\tau}_q n},$$

and

$$\zeta(x) \triangleq \frac{1}{N} \sum_{l=0}^{NM-1} I_{p,n} e^{j\frac{2\pi}{NM}lx} = \frac{1}{N} \frac{\sin(\pi x)}{\sin(\frac{\pi}{NM}x)} e^{j\pi \frac{NM-1}{NM}x}.$$

Whereas the first term of (8.5) contains the symbol to decode, the second term is **ICI** induced by the Doppler frequencies of the channel. Depending on their values, a Doppler frequency estimation and correction can be performed before or simultaneously with the timing estimation to reduce their impacts. Finally, after performing a channel estimation based on pilot symbols, one may simply divide $y(p, m)$ by the channel estimate to obtain $I_{p,m}$.

8.2.2 Radar Processing

At the **ISAC** receiver, neglecting the noise, the received signal reflected by K targets with complex coefficients α_k , delays τ_k and Doppler frequencies f_{Dk} is written as

$$u(p, t) = \sum_{k=0}^{K-1} \alpha_k e^{j2\pi f_{Dk}T_{\text{PRI}}p} e^{j2\pi f_{Dk}t} \sum_{n=0}^{N-1} I_{p,n} \Pi\left(\frac{t - \tau_k - nT_C}{T_C}\right) e^{-j\pi B(t - \tau_k)} e^{j\pi \frac{B}{T}(t - \tau_k)^2}.$$

The radar receiver follows the same structure as the communication receiver. Either it is supposed that the delays of the reflected echoes are smaller than the cyclic prefix duration, or a time synchronisation is performed to align the signal with the start of a reference path, supposed to be the most powerful one. In that case, knowing the parameters of the reference path, one may extract the parameters of the other paths. Following the developments of Section 8.2.1, after sampling, discarding the cyclic prefix, and applying a NM -discrete Fourier transform, one obtains the following signal:

$$y(p, m) = I_{p,m} \sum_{k=0}^{K-1} \beta_k \zeta(f_{Dk}T) e^{j2\pi f_{Dk}T_{\text{PRI}}p} e^{-j\frac{2\pi}{T} \tilde{\tau}_k m} + \text{ICI}_{p,m}, \quad (8.6)$$

where the **ICI** is given by

$$\text{ICI}_{p,m} = \sum_{\substack{n=0 \\ n \neq m}}^{N-1} I_{p,n} \sum_{k=0}^{K-1} \beta_k \zeta(n - m + f_{Dk}T) e^{j2\pi f_{Dk}T_{\text{PRI}}p} e^{-j\frac{2\pi}{T} \tilde{\tau}_k n},$$

and

$$\zeta(x) \triangleq \frac{1}{N} \sum_{l=0}^{NM-1} I_{p,n} e^{j\frac{2\pi}{NM}lx} = \frac{1}{N} \frac{\sin(\pi x)}{\sin(\frac{\pi}{NM}x)} e^{j\pi \frac{NM-1}{NM}x}.$$

After dividing the output by the transmitted symbols, a Fourier analysis in the slow- and fast-time enables to extract from the useful part of (8.6) the delays and Doppler frequencies of the targets.

8.9: Delay/Range and Doppler/Speed Resolutions of the OFDM Radar

$$\begin{aligned} \delta_r &= \frac{1}{B} & \leftrightarrow & \delta_r = \frac{c}{2B} \\ \delta_{f_D} &= \frac{1}{T_{\text{CPI}}} & \leftrightarrow & \delta_v = \frac{c}{2f_c T_{\text{CPI}}} \end{aligned}$$

One may observe from (8.6) that, similarly to (8.1), the achieved resolutions are identical for the **FMCW** and **OFDM** radar. Those are summarised in Box 8.9. Nonetheless, both waveforms have different pros and cons. For instance, the **FMCW** radar outputs can be sampled at low rate (up to the maximal beat frequency), whereas the **OFDM** radar must be sampled at the full bandwidth. Still, contrariwise to the **FMCW** radar, the **OFDM** radar does not suffer from delay-Doppler coupling.

8.2.3 Extensions of the Orthogonal Frequency Division Multiplexing Integrated Sensing and Communication Systems

Numerous other variations of the **OFDM ISAC** waveform have been proposed in the literature. As a non-exhaustive list, to handle the negative impact of **ICI**, [HY18] proposes a method to suppress the **ICI** for **OFDM** radars. From another point of view, [KWK21a] and [KWK21b] develop specific **ICI**-aware algorithms for **MIMO OFDM ISAC** systems. A **MIMO** configuration with frequency division multiplexing is considered in [LLY16]. More complex structures are proposed in [HY16] based on antennas multiplexing with subcarrier interleaving, and in [XP22; Kni+19] based on random antennas multiplexing with compressive sensing. The use of the Alamouti coding scheme is also analysed in [Ley+20].

On the one hand, to reduce the high peak-to-average power ratio, which is one of the main drawback of the **OFDM** modulation, constant envelope structures are introduced in [Tho+08; LTZ18; Mul21]. On the other hand, to decouple the radar bandwidth from the communication bandwidth, chirp modulation of the **OFDM** waveform has been proposed in [Wan+18]. This has been extended for **MIMO** configurations in [Oli+21]. Leveraging on the advantages of both structures, [Did+16] merged both constant envelope and chirp-modulated **OFDM** waveforms.

To further improve the data rate, from a communication perspective, index modulation of the subcarrier indices or antennas is considered in [AH09; TSH11; Bař+13].

In contrary, from a radar perspective, high resolutions are achieved in [Pod+19] using a wideband OFDM structure for range, velocity and direction-of-arrival estimation. Finally, for complexity reduction, stepped OFDM has been considered in [LMI15; Sch+18] to achieve high resolutions with lower bandwidths. The sampling rate of the digital-to-analogue converters and vice-versa has been reduced to the extreme in [Zhu+21], which proposes a low complexity MIMO OFDM scheme with 1-bit sampling.

8.3 Full-Duplex Integrated Sensing and Communication Receivers

In summary, the radar and communication processing chains for PC-FMCW and OFDM ISAC receivers are illustrated in Figures 8.4 and 8.5. The PC-FMCW waveform enables to transmit large bandwidths, but the communication rate is limited, whereas it is the contrary for the OFDM waveform. In both cases, the signal is supposed to be already sampled, as in software defined radio structures. Still, one may consider other architectures for the receivers, and adapt the block diagrams accordingly. For instance, performing the dechirping of the PC-FMCW waveform in the analogue domain may enable to sample the beat frequencies at low rates, but specific synchronisation structures are required [Lam+20].

On the one hand, at the radar function, both systems works with 2D matrices to distinguish the slow- and fast-time. First, (8.3) and (8.6) are obtained with waveform-specific processing. Then, a 2D-Discrete Fourier Transform (DFT) is applied to recover the delays and Doppler frequencies of the targets in the Fourier domain. Finally, a Constant False Alarm Rate (CFAR) detector is applied on the delay-Doppler map to extract the number of targets and their parameters. The CFAR detectors enable to estimate the noise (and interference) power level, and to set the detection threshold of Section 1.4.4 accordingly, at the price of a small SNR loss. More information can be found in [Ric05]. On the other hand, at the communication function, both systems starts by locating the beginning of the payload, before working with 2D in the slow- and fast-time. Then, waveform-specific processing is applied to extract the transmitted symbols.

Still, with such receiver, the interference between the radar and communication functions, detailed in Section 1.1.2, is not handled at all. Therefore, in the Chapters 9 and 10, interference cancellation and beamforming are introduced to mitigate the interference between both functions with such receivers.

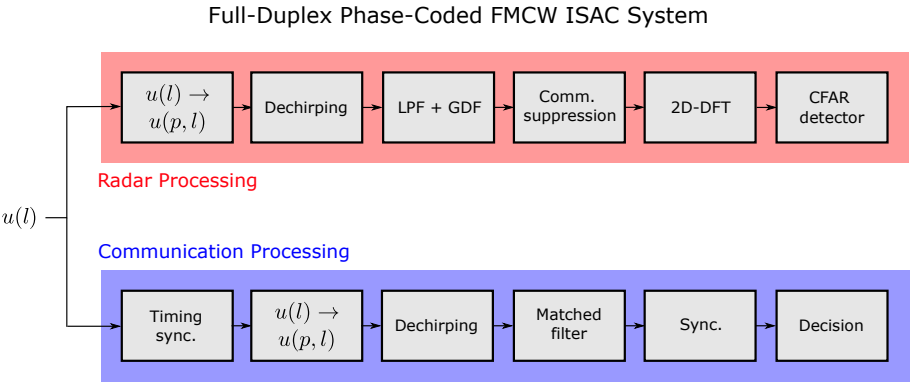


Fig. 8.4 Block diagram of a full-duplex **PC-FMCW ISAC** receiver. The low-pass and group delay filtering are respectively denoted as **LPF** and **GDF**. The synchronisation block performs the carrier frequency estimation and correction, and the channel estimation and equalisation.

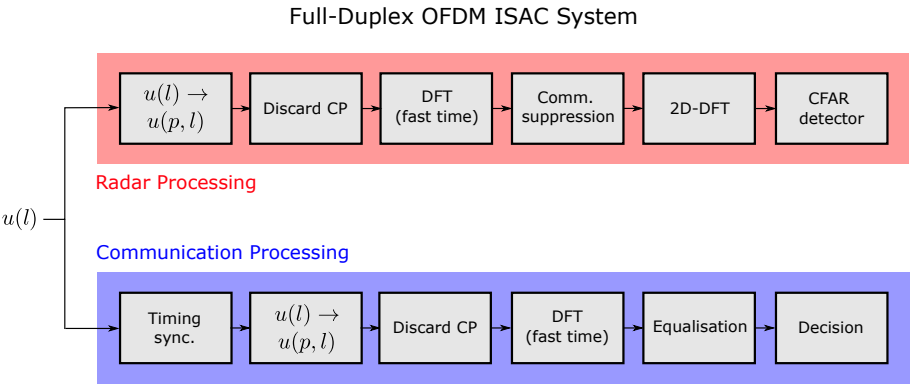


Fig. 8.5 Block diagram of a full-duplex **OFDM ISAC** receiver. The equalisation block also performs the channel estimation. For high mobility applications, an additional carrier frequency estimation and correction block can be integrated.

Interference Cancellation Schemes for Full-Duplex Integrated Sensing and Communication Systems

9

Most of the content of this chapter is inspired from [DOV22] and [DOV24].

IN Section 1.1.2 of Chapter 1, the full-duplex ISAC scenario has been presented. Two issues have been raised, i.e. the self-interference from the transmitter to the receiver of the ISAC system, and the interference between the radar and communication functions. As specified in Section 1.1.2, the self-interference will not be studied in this thesis. Nonetheless, analyses of the second issue are still lacking in the literature. On the one hand, from the radar point of view, the communication signal transmitted by another ISAC (or compatible communication) system is an interference. On the other hand, from the communication point of view, the radar echoes are considered as interference. Therefore, enabling some cooperation between both functions, one may (partially) suppress the communication signal at the radar function, and/or the radar echoes at the communication function. Such IC has for instance been considered in [Sit+11], with OFDM ISAC systems. Within the SG framework, a first insight of the expected performance improvement has been presented in Section 6.2.4.1. However, the achieved performance are computed from a residual function which has been arbitrarily defined. Thus, it may not be accurate. Moreover, the SG framework is an abstraction in which numerous details of the processing cannot be easily taken into account, e.g. an inaccurate synchronisation.

In this chapter, the IC between both functions is further investigated by fully modelling the full-duplex ISAC scenario, including the complete transmitter and receiver processing chains and the channel impairments. It is assumed that every ISAC system in the scenario shares the same parameters. Following the single-point channel model of (1.3), the received signal at the ISAC system of interest is given in Box 9.1. The number of targets and communication links are denoted as K and Q , and w is an AWGN. The signal s_R denotes the signal transmitted by the ISAC system of interest. This signal carries communication symbols, for the downlink communication function. Moreover, the reflections of this signal on the environment

Structure of the Chapter

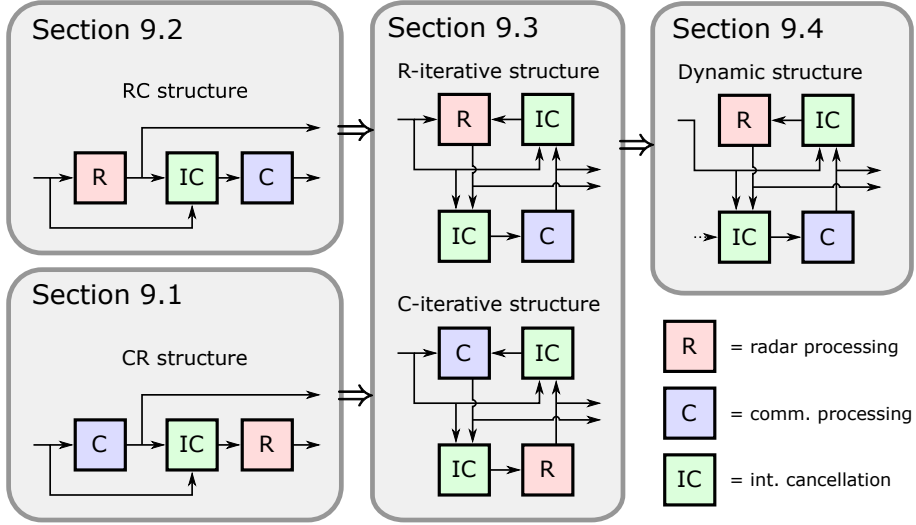


Fig. 9.1 Organisation of the chapter, and representation of the different IC structures. The first solid arrow entering the blocks from the left represents the received signal. The solid arrows at the output of the radar and communication blocks represents the targets number and parameters, the communication channel parameters, and the decoded symbols. The solid arrows at the output of the IC blocks represents the received signal in which the radar or communication interference has been cancelled. The dotted arrow in the dynamic structure represents radar information forwarded from previous measurements.

are received and processed by the radar function. Simultaneously, other ISAC systems are transmitting different signals $\{s_{Cq}\}$. Multipath channels may be included within the summation over the index q . These signals are also received at the ISAC system of interest, which extracts the uplink communication information. Note that these signals are not used at the radar function to gather additional information about the environment. The waveforms s_R and s_{Cq} may be any ISAC waveform, such as the PC-FMCW or OFDM waveforms, detailed in Chapter 8. The complex coefficients, Doppler frequencies and delays of the radar and communication paths are denoted as $\{\alpha_v\}$, $\{f_{Dv}\}$ and $\{\tau_v\}$, where $v \in \{R, C\}$.

9.1: Single-Antenna Full-Duplex Received Signal

$$u(t) = \sum_{k=0}^{K-1} \alpha_{Rk} e^{j2\pi f_{DRk}t} s_R(t - \tau_{Rk}) + \sum_{q=0}^{Q-1} \alpha_{Cq} e^{j2\pi f_{DCq}t} s_{Cq}(t - \tau_{Cq}) + w(t)$$

Multiple IC schemes are detailed in this chapter, as summarised in Figure 9.1.

First, the impact of the interference on the radar and communication functions is analysed, respectively in Sections 9.1 and 9.2. Two structures are then proposed to cancel the interference at one function, in cooperation with the other. Next, in Section 9.3, iterative IC structures are introduced, further improving the performance and the robustness of the receiver. Finally, in Section 9.4, the application of the iterative structure in dynamic scenarios is discussed, leveraging on previous radar measurements.

9.1 Interference Cancellation at the Radar Function

At the radar function, the signals of interest are the radar echoes, while the uplink communication signals are interfering. First, the impact of the communication signals on the delay-Doppler map is analysed. Then, IC is introduced at the radar function, based on the output of the communication function.

9.1.1 Interference between Phase Coded Frequency Modulated Continuous Wave Systems

If PC-FMCW ISAC systems are considered, the communication signals are filtered at the radar function after dechirping if the total Doppler frequency, i.e. the carrier frequency offsets, communication channel Doppler frequencies, and beat frequencies generated by the dechirp operation, is higher than the cut-off frequency of the receiver low pass filter. However, if the relative delay between the interferer and the receiver is lower than the maximum detectable delay of the radar receiver, the delay-Doppler map will be corrupted by the interfering communication signals. Fortunately, the transmitted data are different for the PC-FMCW ISAC system of interest and the other interfering systems, enabling an inherent interference mitigation [Uys20].

After the dechirp operation, group-delay filtering and sampling, assuming that the communication paths have not been filtered out, the q^{th} processed interfering signal is written following (8.2) as

$$y_{Cq,gd}(p, l) \approx \sum_{n=0}^{N-1} I_{Cq,p,n} \Pi\left(\frac{l - nM}{M}\right) \gamma_{Cq} e^{j2\pi f_{DCq} T_{PRI} p} e^{-j2\pi \frac{f_{BCq} T_C}{M} l},$$

where M is the oversampling factor, T_{PRI} is the pulse repetition interval, γ_{Cq} is a complex coefficient associated with the q^{th} path of variance $\mathbb{E}[|\gamma_{Cq}|^2] = \sigma_{Cq}^2$, I_{Cq} are the symbols transmitted by the q^{th} ISAC system, and f_{BCq} is the beat frequency generated for the q^{th} path. Since symbols of the communication signals are different

9 | Interference Cancellation Schemes for Full-Duplex Integrated Sensing and Communication Systems

for each ISAC system, they are not well compensated at the radar function:

$$z_{Cq}(p, l) = \gamma_{Cq} e^{j2\pi f_{DCq} T_{PRI} p} \sum_{n=0}^{N-1} \frac{I_{Cq,p,n} I_{R,p,n}^*}{|I_{R,p,n}|^2} \Pi\left(\frac{l - nM}{M}\right) e^{-j2\pi \frac{f_{BCq} T_C}{M} l}.$$

To compute the impact of such interference in the delay-Doppler map while accounting for the random path coefficients and symbols, the power spectral density of z_{Cq} cannot be computed directly, as the signal is not stationary. One may instead compute the expectation of the autocorrelation of z_{Cq} over the fast- and slow-time, and then apply DFT and Inverse Discrete Fourier Transform (IDFT) over both dimensions to observe the distribution of the signal power in the delay-Doppler map. The expectation of the autocorrelation of z_{Cq} over both dimensions is computed as

$$\begin{aligned} C_{z_{Cq}}(\Delta_p, \Delta_l) &= \mathbb{E} \left[\sum_{p=0}^{P-1} \sum_{l=0}^{MN-1} z_{Cq}(p, l) z_{Cq}^*(p - \Delta_p, l - \Delta_l) \right] \\ &\stackrel{(a)}{=} \sigma_{Cq}^2 P e^{j2\pi f_{DCq} T_{PRI} \Delta_p} \delta(\Delta_p) e^{-j2\pi \frac{f_{BCq} T_C}{M} \Delta_l} \sum_{l=0}^{MN-1} \sum_{n=0}^{N-1} \Pi\left(\frac{l - nM}{M}\right) \Pi\left(\frac{l - \Delta_l - nM}{M}\right) \\ &= \sigma_{Cq}^2 MNP \delta(\Delta_p) e^{-j2\pi \frac{f_{BCq} T_C}{M} \Delta_l} \Lambda\left(\frac{\Delta_l}{M}\right), \end{aligned} \quad (9.1)$$

where (a) is obtained assuming that independent symbols are drawn from normalised constellations at every system, such that

$$\mathbb{E} \left[\frac{I_{Cq,p,n} I_{Cq,p',n'}^* I_{Rq,p,n} I_{Rq,p',n'}^*}{|I_{Rq,p,n}|^2 |I_{Rq,p',n'}|^2} \right] = \delta(n - n') \delta(p - p'), \quad (9.2)$$

and Λ is the triangular function, i.e.

$$\Lambda(x) = \begin{cases} 1 - |x| & \text{if } |x| \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Finally, applying a DFT on the slow-time and an IDFT on the fast-time, one obtains an insight on the power dispersion of the q^{th} communication signal in the delay-Doppler map:

$$\mathcal{P}_{z_{Cq}}(\Delta_f, \Delta_\tau) = \sigma_{Cq}^2 PN \frac{\sin^2\left(\frac{\pi}{N} (\Delta_\tau - f_{BCq} T)\right)}{\sin^2\left(\frac{\pi}{NM} (\Delta_\tau - f_{BCq} T)\right)}. \quad (9.3)$$

This shows that the power of the communication signal is spread uniformly in the Doppler dimension, and follows a sin/sin shape in the delay dimension. The maximum value of the power is located around the normalised beat frequency $f_{BCq} T$, and it is equal to $\sigma_{Cq}^2 PNM^2$.

Power Distribution of the Communication Signal

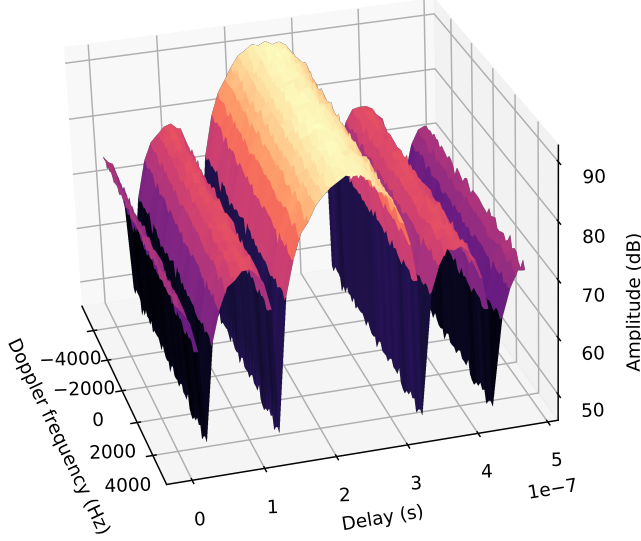


Fig. 9.2 Illustration of the power distribution of the communication signals interference in the delay-Doppler map. The parameters are $\alpha_C = 1$, $\tau_C = 0.25 \mu\text{s}$, $f_{DC} = -300 \text{ Hz}$, $P = 50$, $N = 10$, $M = 2000$, $T = 100 \mu\text{s}$, $T_{PRI} = 100.5 \mu\text{s}$, $B = 100 \text{ MHz}$.

That result can be compared with the power distribution of the radar echoes, for which the communication symbols are well compensated. After the dechirp operation, group delay filter and sampling, the processed echoes are written following (8.2) as

$$y_{Rk,gd}(p, l) \approx \sum_{n=0}^{N-1} I_{Rp,n} \Pi\left(\frac{l - nM}{M}\right) \gamma_{Rk} e^{j2\pi f_{DRk} T_{PRI} p} e^{-j2\pi \frac{f_{BRk} T_C}{M} l},$$

where M is the oversampling factor, T_{PRI} is the pulse repetition interval, γ_{Ck} is a complex coefficient associated with the k^{th} echo of variance $\mathbb{E}[|\gamma_{Rk}|^2] = \sigma_{Rk}^2$, I_R are the symbols transmitted by the ISAC system of interest, and f_{BRk} is the beat frequency generated for the k^{th} echo. The communication symbols of the joint waveform are next compensated at the radar function:

$$z_{Rk}(p, l) = \gamma_{Rk} e^{j2\pi f_{DRk} T_{PRI} p} e^{-j2\pi \frac{f_{BRk} T_C}{M} l},$$

The expectation of the autocorrelation of z_{Rk} over the fast- and slow-time is obtained as

$$\begin{aligned} C_{z_{Rk}}(\Delta p, \Delta l) &= \mathbb{E} \left[\sum_{p=0}^{P-1} \sum_{l=0}^{MN-1} z_{Rk}(p, l) z_{Rk}^*(p - \Delta p, l - \Delta l) \right] \\ &= \sigma_{Rk}^2 MNP e^{j2\pi f_{DRk} T_{PRI} \Delta p} e^{-j2\pi \frac{f_{BRk} T_C}{M} \Delta l}. \end{aligned}$$

9 | Interference Cancellation Schemes for Full-Duplex Integrated Sensing and Communication Systems

Finally, applying a **DFT** on the slow-time and an **IDFT** on the fast-time, one obtains

$$\mathcal{P}_{z_{rk}}(\Delta_f, \Delta_\tau) = \sigma_{Cq}^2 MNP \frac{\sin(\pi(\Delta_f - f_{DRk}T_{CPI}))}{\sin(\frac{\pi}{P}(\Delta_f - f_{DRk}T_{CPI}))} \frac{\sin(\pi(\Delta_\tau - f_{BRk}T))}{\sin(\frac{\pi}{NM}(\Delta_\tau - f_{BRk}T))},$$

where T_{CPI} is the coherent processing interval. This shows that the power of the radar echoes follows a sin/sin shape in both the delay and Doppler dimension. The maximum value of the power is located around the normalised beat frequency $f_{BCq}T$ and normalised Doppler frequency $f_{DRk}T_{CPI}$, and it is equal to $\sigma_{Cq}^2 P^2 N^2 M^2$. Comparing this result with the maximum value of (9.3), one observes that, for the same variances of the radar and communication channels, a power gain of NP is obtained for the radar echoes compared to the interfering communication signals. Note that, if the radar echoes were not aligned with the group delay filter, the communication symbols would not be well compensated, and the power of the radar echoes would also be dispersed in the delay-Doppler map.

When the number of communication symbols N is large, the main lobe of (9.3) is widened. Nearly constant interference may be observed on the complete delay-Doppler map if this number is sufficient. Nonetheless, the power of the communication signals is only dispersed and not suppressed. Therefore, for high communication signals power, the equivalent noise floor is increased, which severely degrades the radar detection performance. This motivates the introduction of **IC** at the radar function.

9.1.2 Interference between Orthogonal Frequency Division Multiplexing Systems

If **OFDM ISAC** systems are instead considered, similarly to **PC-FMCW ISAC** systems, the symbols of the communication signals, which are different for each **ISAC** system, are not well compensated at the radar function. Starting from (8.6), neglecting the **ICI**, the following signal is obtained for the q^{th} path after performing an erroneous compensation:

$$z_{Cq}(p, l) = \beta_{Cq} \zeta(f_{DCq}T) \frac{I_{Cq,p,l} I_{R,p,l}^*}{|I_{R,p,l}^*|^2} e^{j2\pi f_{DCq} T_{PRI} p} e^{-j\frac{2\pi}{T} \tilde{\tau}_{Cq} l},$$

with T the **OFDM** pulse duration, and β_{Cq} a complex coefficient associated with the q^{th} path of variance $\mathbb{E}[|\beta_{Cq}|^2] = \sigma_{Cq}^2$. The expectation of the autocorrelation of z_{Cq} over both dimensions is then computed as

$$C_{z_{Cq}}(\Delta_p, \Delta_l) = \mathbb{E} \left[\sum_{p=0}^{P-1} \sum_{l=0}^{N-1} z_{Cq}(p, l) z_{Cq}^*(p - \Delta_p, l - \Delta_l) \right]$$

$$\begin{aligned}
 &= \sigma_{Cq}^2 |\zeta(f_{DCq}T)|^2 PN e^{j2\pi f_{DCq}T_{PRI}\Delta_p} \delta(\Delta_p) e^{-j\frac{2\pi}{T}\tilde{\tau}_{Cq}\Delta_l} \delta(\Delta_l) \\
 &= \sigma_{Cq}^2 |\zeta(f_{DCq}T)|^2 PN \delta(\Delta_p) \delta(\Delta_l),
 \end{aligned}$$

where the last equality is again obtained assuming that independent symbols are drawn from normalised constellations at every system, leading to (9.2). Finally, applying a **DFT** on the slow-time domain and an **IDFT** on the fast-time domain, one obtains an insight on the power dispersion of the q^{th} communication signal in the delay-Doppler map:

$$\mathcal{P}_{z_{Cq}}(\Delta_f, \Delta_\tau) = \sigma_{Cq}^2 |\zeta(f_{DCq}T)|^2 PN. \quad (9.4)$$

This shows that the power of the communication signal is spread uniformly in the both the delay and Doppler dimensions.

That results can be compared with the power distribution of the radar echoes, for which the communication symbols are well compensated. Starting again from (8.6), the following signal is obtained for the k^{th} target after compensating correctly the symbols:

$$z_{Rk}(p, l) = \beta_{Rk} \zeta(f_{DRk}T) e^{j2\pi f_{DRk}T_{PRI}p} e^{-j\frac{2\pi}{T}\tilde{\tau}_{Rk}l},$$

with β_{Rk} a complex coefficient associated with the k^{th} target of variance $\mathbb{E}[|\beta_{Rk}|^2] = \sigma_{Rk}^2$. The expectation of the autocorrelation of z_{Rk} over both dimensions is then computed as

$$\begin{aligned}
 C_{z_{Rk}}(\Delta_p, \Delta_l) &= \mathbb{E} \left[\sum_{p=0}^{P-1} \sum_{l=0}^{N-1} z_{Rk}(p, l) z_{Rk}^*(p - \Delta_p, l - \Delta_l) \right] \\
 &= \sigma_{Rk}^2 |\zeta(f_{DRk}T)|^2 PN e^{j2\pi f_{DRk}T_{PRI}\Delta_p} e^{-j\frac{2\pi}{T}\tilde{\tau}_{Rk}\Delta_l},
 \end{aligned}$$

Finally, applying a **DFT** on the slow-time domain and an **IDFT** on the fast-time domain, one obtains an insight on the power dispersion of the q^{th} communication signal in the delay-Doppler map:

$$\mathcal{P}_{z_{Cq}}(\Delta_f, \Delta_\tau) = \sigma_{Cq}^2 |\zeta(f_{DRk}T)|^2 PN \frac{\sin(\pi(\Delta_f - f_{DRk}T_{CPI}))}{\sin(\frac{\pi}{P}(\Delta_f - f_{DRk}T_{CPI}))} \frac{\sin(\pi(\Delta_\tau - \tilde{\tau}_{Rk}B))}{\sin(\frac{\pi}{N}(\Delta_\tau - \tilde{\tau}_{Rk}B))}. \quad (9.5)$$

This shows that the power of the radar echoes follows a sin/sin shape in both the delay and Doppler dimensions. The maximum value of the power is located around the normalised delay $\tilde{\tau}_{Rk}B$ and Doppler frequency $f_{DRk}T_{CPI}$, and is equal to $\sigma_{Cq}^2 |\zeta(f_{DRk}T)|^2 P^2 N^2$. Comparing this result with the maximum value of (9.4), one observes that, for the same variances of the radar and communication channels, and the impact of the **ICI** being neglected, a power gain of NP is obtained for the radar echoes compared to the interfering communication signals, as with **PC-FMCW**

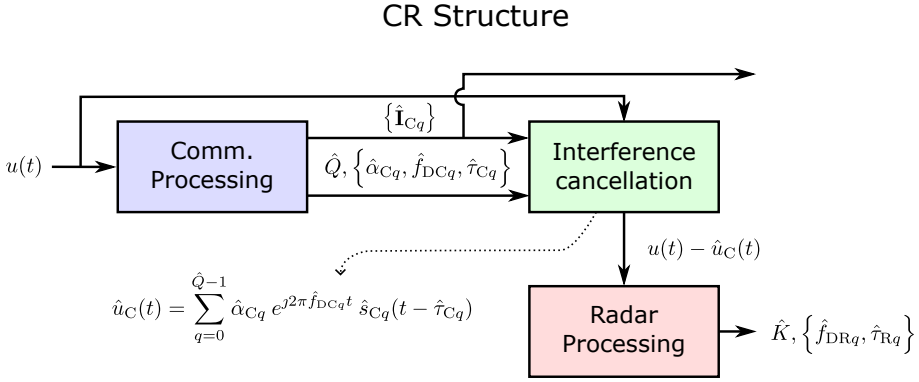


Fig. 9.3 CR structure. The communication function forwards the communication synchronisation parameters and the decoded symbols $\hat{\mathbf{I}}_C$ to the IC block.

Parameters	Values
Bandwidth	$B = 20 \text{ MHz}$
Chirp duration	$T = 100 \text{ } \mu\text{s}$
Guard interval	$T_g = 0.5 \text{ } \mu\text{s}$
Pulse number	$P = 50$
Symbols number	$L_c = 100$
False alarm probability	$\mathcal{P}_{FA} = 10^{-4}$
Radar channel parameters	$ \alpha_R = 0.1, \tau_R = 0.1 \text{ } \mu\text{s}, f_{DR} = 1000 \text{ Hz}$
Comm. channel parameters	$ \alpha_C = 1, \tau_C = 0.25 \text{ } \mu\text{s}, f_{DC} = -300 \text{ Hz}$

Table 9.1 Parameters of the scenario for Section 9.1.3.

systems.

9.1.3 Interference Cancellation

In scenarios where the power of the communication signals is assumed to be larger than the power of the radar echoes, IC can be applied at the radar function. The communication signals are first decoded at the communication function. Then, the retrieved information, i.e. the received symbols $\{\hat{\mathbf{I}}_{Cq}\}$, and synchronisation parameters $\{\alpha_{Cq}, f_{DCq}, \tau_{Cq}\}$, is transferred to the radar function. Leveraging on these parameters, the communication signals are reconstructed, and subtracted from the received signal. This receiver is named *CR structure*, based on the order in which the radar and communication functions are processed. It is illustrated in Figure 9.3.

9.1.3.1 Sensing and Communication Performance

To evaluate the performance of the CR structure, a scenario with **PC-FMCW ISAC** systems is defined in Table 9.1, with one radar echo and communication link. Figure 9.4 illustrates the probability of detection obtained for different modulation indices, i.e. **Binary Phase Shift Keying (BPSK)**, **Quadrature Phase Shift Keying (QPSK)**, **16-Phase Shift Keying (PSK)** and **16-Quadrature Amplitude Modulation (QAM)**, after **IC** at the radar function, compared to the probability of detection obtained when there is no interference, or when interference is not cancelled. In the considered scenario, the communication signal power is 20 dB higher than radar echoes. Even if the interference power is spread in the delay-Doppler map, owing to the presence of uncompensated communication symbols, the radar echoes are hidden below the interference noise floor.

Thanks to the **IC**, large performance improvement is achieved, depending on the **BER** obtained at the communication function (and equivalently on the modulation indices). This is shown in Figure 9.5. At low communication **SNR**, the **BER** is high, leading to multiple symbol errors at the cancellation. The higher the modulation index, the higher the number of errors, leading to a decrease of the probability of detection. In that case, the obtained performance is even worse than the performance obtained when there is no **IC** with high modulation indices. However, at higher communication **SNR**, the **BER** decreases, and the cancellation efficiency increases as well. Therefore, the probability of detection increases to one. The increase is slower for high modulation indices since the **BER** decreases more slowly in the considered **SNR** range. Regarding the 16-**QAM** constellation, the detection performance is worse than the one achieved with the 16-**PSK** for **SNRs** up to around 13 dB, even with a lower **BER**, owing to the non-constant envelope of the transmitted **ISAC** waveform.

9.1.3.2 Introduction of Error Correcting Codes

In order to further improve the **IC** capabilities, **Error Correcting Codes (ECC)**, e.g. convolutional codes, are introduced in the structure. We assume that bits are coded at the **ISAC** transmitters using the systematic convolutional code $\mathbf{G}(D) = \left[1 \quad \frac{1+D}{1+D+D^2}\right]$ of rate $R_c = 1/2$, with D the delay operator. At the receiver, soft Viterbi decoding is performed.

First, regarding the inherent interference mitigation of both **ISAC** waveforms discussed in Section 9.1.1 and 9.1.2, the auto-correlation of z_{Cq} presented in (9.1) remains valid only if the condition of (9.2) is still fulfilled after coding. Apart from repetition coding, even if convolutional codes integrate redundancy in the coded bits, pairs of symbols remain empirically independent, except at the start of the sequence. This is illustrated in Figure 9.6. Correlation between the first bits of the sequence is observed owing to the structure of the code. Still, with a **QPSK** mapping, this does

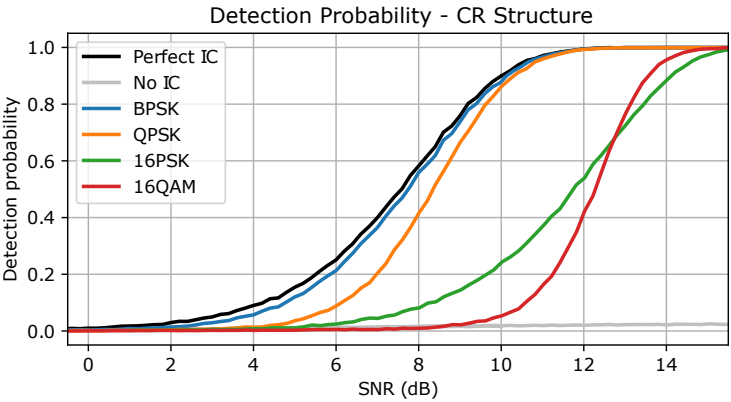


Fig. 9.4 Probability of detection against SNR with BPSK, QPSK, 16-PSK and 16-QAM modulation, with and without IC.

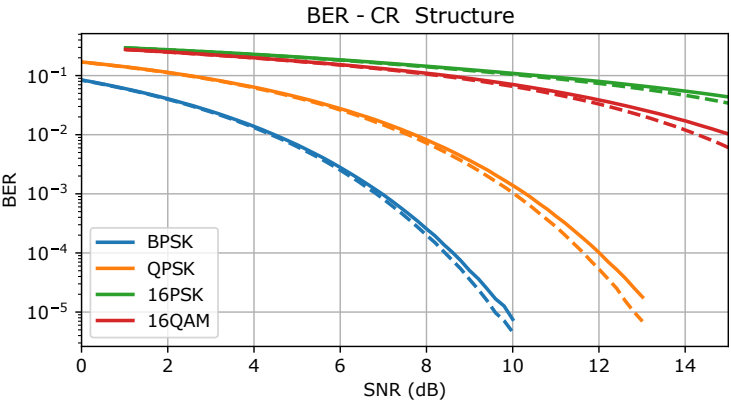


Fig. 9.5 BER against SNR with BPSK, QPSK, 16-PSK and 16-QAM modulation, with the CR structure (solid lines) and without any interference (dashed lines).

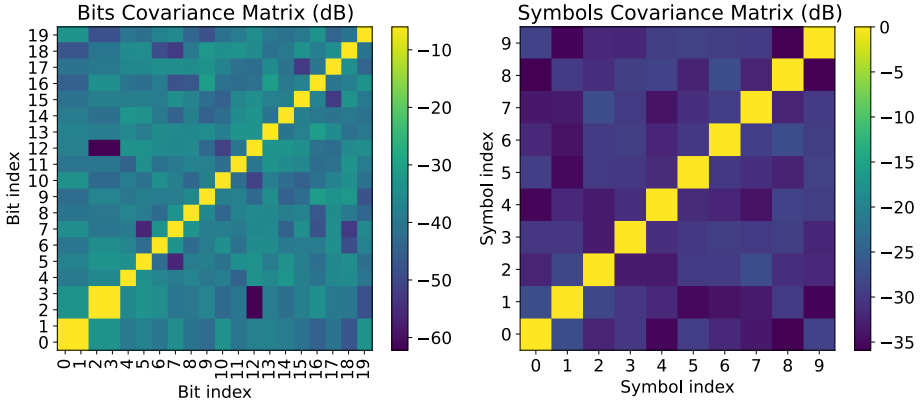


Fig. 9.6 Bits and symbols covariance matrices after coding, with QPSK mapping.

not induce correlation of the first symbols. Nevertheless, interleaving can also be introduced to ensure no covariance. Therefore, with the considered convolutional code, the dispersion of the interfering communication signal in the Delay-Doppler remains unchanged.

To ensure a fair comparison is performed between the uncoded and coded transmissions, coded bits are transmitted with half power in the coded transmission in order to keep a constant energy per bit E_b . The uncoded transmission uses BPSK constellation, whereas the coded transmission uses QPSK constellation in which the redundancy bits are transmitted in quadrature, ensuring the same transmission time. This also enables to conserve the same $\text{SNR} = MR_c E_b/N_0$ in both transmissions, with M the modulation index ($M = 1$ for BPSK, and $M = 2$ for QPSK). Figure 9.7 illustrates the achieved BER between uncoded BPSK and coded QPSK transmissions. Thanks to the presence of ECC in the ISAC waveform, a lower BER is achieved with the coded transmission, except for low E_b/N_0 ratios. This improvement is directly translated in Figure 9.8, showing that a higher detection probability is achieved with a coded QPSK transmission compared to uncoded BPSK. The performance improvement is small since the detection probability is already close to the performance achieved without any interference in that scenario, which acts as an upper bound. Note that a further increase of the code memory would also help improve the detection probability to some extent. Moreover, the same conclusions can be drawn with higher modulation indices if ECCs with higher correction capabilities are considered.

9.1.4 Summary

PC-FMCW and OFDM systems are inherently robust to interference from other ISAC systems at the radar function. Nonetheless, when the communication component is dominant compared to the radar echoes, the increased noise floor is still an issue.

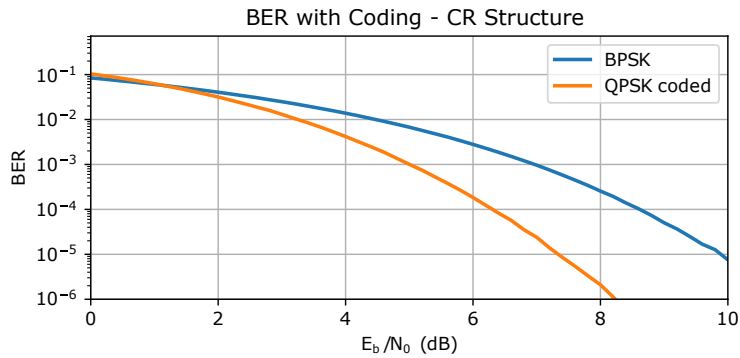


Fig. 9.7 BER against E_b/N_0 for uncoded BPSK and coded QPSK communication symbols.

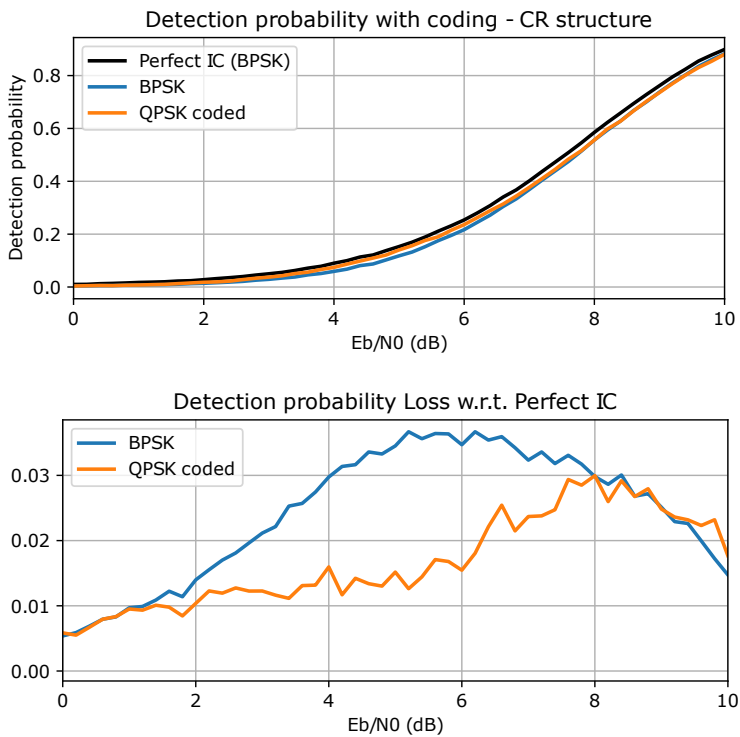


Fig. 9.8 Probability of detection against E_b/N_0 after IC, for uncoded BPSK and coded QPSK communication symbols.

Therefore, **IC** can be introduced before the radar function, leveraging on the outputs of the communication function to estimate and suppress the communication signals. **IC** performs well when the **BER** obtained at the communication output is sufficiently low. The modulation index of the communication signal can be increased at the price of a lower detection probability for a given **SNR** at the communication function. Introducing **ECCs** helps improve the achieved performance at both functions, as lower **BERs** are achieved, which also leads to better **IC** at the radar function.

9.2 Interference Cancellation at the Communication Function

At the communication function, the signals of interest are the uplink communication signals, while the radar echoes are interfering. First, the impact of the radar echoes on the communication signals is discussed. Then, **IC** is introduced at the communication function, based on the output of the radar function.

9.2.1 Interference between Phase Coded Frequency Modulated Continuous Wave Systems

If **PC-FMCW ISAC** systems are considered, as shown in Figure 8.4, dechirping is also applied at the communication function. Hence, the interfering components rotate at frequencies equal to the sum of the Doppler and beat frequencies associated with the radar echoes, computed w.r.t. the time reference given by the timing synchronisation block. Thus, low-pass filtering can be applied after dechirping to filter echoes with large beat frequencies and limit the interference. This still assumes that the Doppler frequencies of the communication signals are negligible compared to the beat frequencies of the radar echoes. Nonetheless, except that, there is no inherent interference mitigation mechanism at the communication function. Therefore, **IC** can be applied to increase the communication performance when interference occurs.

9.2.2 Interference between Orthogonal Frequency Division Multiplexing Systems

If **OFDM ISAC** systems are considered, all the received signal are processed similarly, and there is no inherent interference mitigation mechanism at the communication function. Therefore, **IC** can be applied to increase the communication performance when interference occurs.

9.2.3 Interference Cancellation

In scenarios where the power of the radar echoes is assumed to be larger than the power of the communication signals, **IC** can be applied at the communication

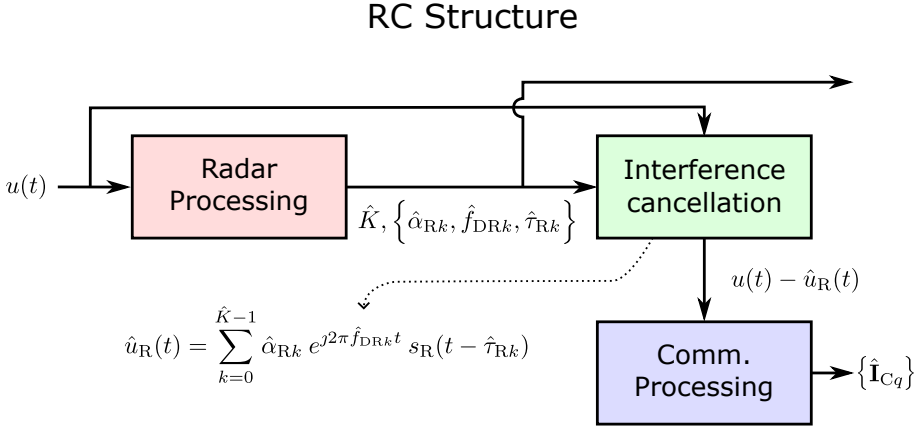


Fig. 9.9 RC structure. The radar function forwards the number of targets and their parameters to the IC block.

function. The radar echoes are first processed at the radar function. Then, the retrieved information, i.e. the number of targets and their parameters $\{\alpha_{Rk}, f_{DRk}, \tau_{Rk}\}$, is transferred to the communication function. Leveraging on these parameters, the radar echoes are reconstructed, and subtracted from the received signal. This receiver is named *RC structure*. It is illustrated in Figure 9.9.

Note that the complex coefficients $\{\alpha_{Rk}\}$ can be estimated from the delay-Doppler map, but this may be inaccurate, owing to the inter-target interference. Instead, owing to the joint waveform structure which is shared by every system, radar echoes can be processed as communication signals, for which the delays and the Doppler frequencies have already been estimated at the radar function. Additionally, the transmitted symbols are also known. Another option is to estimate all the parameters of each radar echo successively based on the estimated number of targets of the radar receiver, but this solution does not leverage on all the information gathered at the radar function.

9.2.3.1 Sensing and Communication Performance

The performance achieved by the RC structure depends on the accuracy of the radar estimates, which is directly related to the delay and Doppler resolutions of the radar function. In order to increase the Doppler resolution, one can increase the total duration of the payload. Nevertheless, this also impacts the radar processing gain, and the sensitivity to the frequency offset. Otherwise, the accuracy can be improved by interpolating the delay-Doppler map with zero-padding. Two scenarios with phase-coded ISAC systems are defined in Table 9.2, with one radar echo and communication link, leading to different Doppler frequency resolutions and accuracies.

Parameters	Values
Bandwidth	$B = 20 \text{ MHz}$
Chirp duration	$T = 100 \text{ } \mu\text{s}$
Guard interval	I: $T_g = 0.5 \text{ } \mu\text{s}$ II: $T_g = 5 \text{ } \mu\text{s}$
Pulse number	$P = 50$
Symbols number	$L_c = 100$
False alarm probability	$\mathcal{P}_{\text{FA}} = 10^{-4}$
Modulation	QPSK
Radar channel parameters	$ \alpha_{\text{R}} = 1$, I: $\tau_{\text{R}} = 0.1 \text{ } \mu\text{s}$, $f_{\text{DR}} = 1000 \text{ Hz}$ II: $\tau_{\text{R}} = 1 \text{ } \mu\text{s}$
Comm. channel parameters	$ \alpha_{\text{C}} = 1$, I: $\tau_{\text{R}} = 0.25 \text{ } \mu\text{s}$, $f_{\text{DC}} = -300 \text{ Hz}$ II: $\tau_{\text{R}} = 2.5 \text{ } \mu\text{s}$

Table 9.2 Parameters of the scenario for Section 9.2.3. Two scenarios are defined, and respectively denoted as I and II.

Number of pulses P	Resolution ($1/T_{\text{CPI}}$)		Accuracy	
	Scenario I	Scenario II	Scenario I	Scenario II
50	199 Hz	190.5 Hz	5 Hz	47.6 Hz
100	99.5 Hz	85.2 Hz	5 Hz	47.6 Hz
150	66.3 Hz	63.5 Hz	5 Hz	15.9 Hz
200	49.8 Hz	47.6 Hz	5 Hz	0 Hz

Table 9.3 Resolutions and accuracies achieved for both scenarios of Table 9.2.

The higher the number of transmitted pulses, the better the Doppler resolution. Yet, this does not imply that a higher accuracy is achieved. The latter depends on the Doppler frequency of the target, w.r.t. the discrete delay-Doppler grid. For instance, Table 9.3 summarises the achieved resolutions and accuracies in both scenarios defined in Table 9.2. On the one hand, with the parameters of the first scenario, even if the resolution is improving with the considered number of pulse, the accuracy remains constant. Yet, the length of the payload also increases, leading to many errors at the reconstruction of the end of the signal, owing to the phase rotation of the symbols. This is translated in Figure 9.10, showing the BER achieved with the RC structure in the first scenario, for different numbers of pulse. Increasing the number of transmitted pulses leads to an increase of BER. Note that the detection probability is not shown in that case, as it is always very close to one in this scenario.

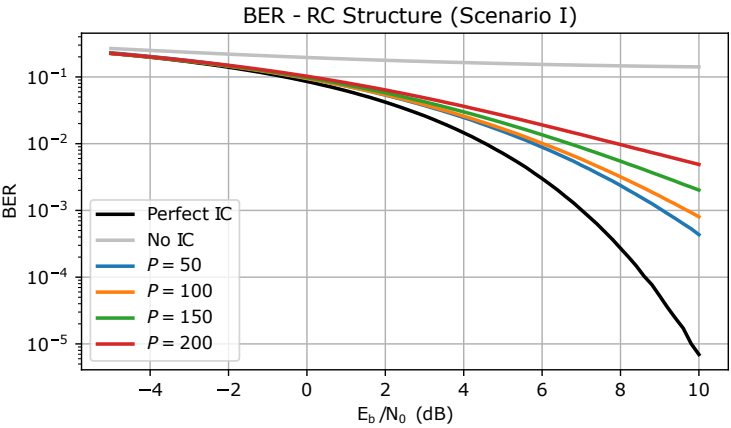


Fig. 9.10 BER against E_b/N_0 for multiple numbers of pulse.

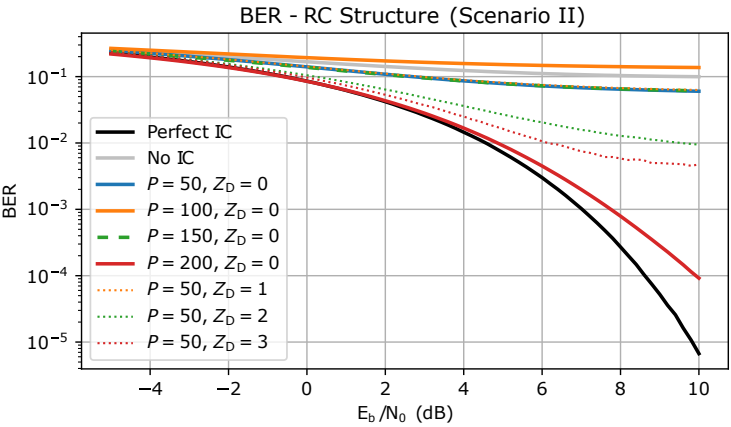


Fig. 9.11 BER against E_b/N_0 for multiple numbers of pulse and zero-padding factors (Z_D = number of padded zeros).

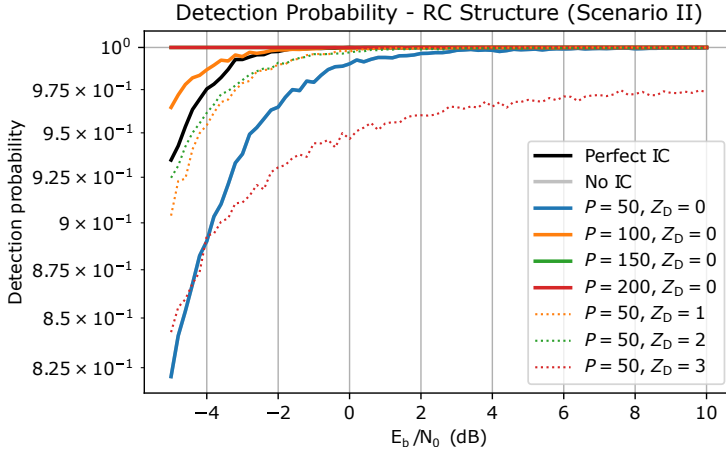


Fig. 9.12 Detection probability against E_b/N_0 for multiple numbers of pulse and zero-padding factors (Z_D = number of padded zeros).

On the other hand, with the parameters of the second scenario, the accuracy remains constant with 50 and 100 transmitted pulses, but decreases for higher number of pulse. The achieved BER and detection probability are shown in Figure 9.11 and 9.12, for different numbers of pulses and zero-padding factors.

- When the number of pulse is increased from 50 to 100 pulses, worse performance is achieved, as the same accuracy is achieved, with a doubled payload length.
- When the number of pulse is increased from 100 to 150 pulses, the improvement of accuracy enhances the BER. Yet, the increase of payload duration worsens the performance, leading to a limited improvement w.r.t. the transmission of 100 pulses.
- When the number of pulse is increased from 150 to 200 pulses, the Doppler frequency is estimated perfectly, leading to significant performance improvement, regardless of the increased payload duration.

With zero-padding, the accuracy is increased without affecting the payload length. For instance, with 50 transmitted pulses, a zero-padding factor of 1 does not increase the accuracy, but the payload length being unaffected, performance losses are avoided. With higher zero-padding factors, the accuracy is increased, and better performance is achieved. Note that a reduction of the detection probability is observed with a zero-padding factor of 3, which is translated by an increase of the BER, and lower performance compared to the transmission of 200 pulses. This is a consequence of the CFAR detector, which is not able to estimate accurately the noise power in the delay-Doppler map with high zero-padding factors, owing to the interpolation of the noise and target response.

9.2.3.2 Summary

Contrariwise to the radar function, the communication function is more sensitive to interference from other ISAC systems. Nonetheless, when the radar echoes are dominant compared to the communication signals, IC can be applied efficiently to the communication receiver, leveraging on the outputs of the radar function to suppress the radar echoes. The estimation of the echoes parameters is either based on the radar outputs, or it is aided by the communication synchronisation algorithms, leveraging on the waveform structure shared by all systems. In the former, the radar delay and Doppler accuracies should be sufficiently high in order to ensure accurate estimations of the delays and Doppler frequencies, leading to an efficient cancellation. The resolutions and accuracies can be increased by modifying the system parameters. Otherwise, zero-padding of the delay-Doppler map is helpful to increase the accuracy of the delay and Doppler estimations, without modifying the bandwidth nor the payload duration.

9.3 Iterative Interference Cancellation

In Sections 9.1 and 9.2, two IC structures at the radar or communication function, namely the CR and RC structures, have been detailed. However, each receiver only works well, respectively at high communication or radar SIR. Thus, IC should be integrated at the radar (resp. communication) function only if the communication signals (resp. radar echoes) are dominant compared to the radar echoes (resp. communication signals). To solve this issue, if the power levels of the different signals are estimated beforehand, one may switch between both structures to use the appropriate IC scheme. Another solution which does not require any a priori information is to perform iterative cancellation.

Iterative structures are illustrated in Figure 9.13. In such structures, the received signal is forwarded to the first radar or communication block, and to IC blocks. The radar block estimates the number of targets and their parameters, while the communication function estimates the synchronisation parameters and the decoded symbols of the communication signals. These receivers are named *R- and C-iterative structures*, respectively when they start with the radar or communication function. Such structures work well irrespective of the considered scenario. If the power of the radar echoes (resp. communication signals) is dominant, even if the signal at the input of the radar (resp. communication) function is corrupted by a poor IC output, the radar (resp. communication) function still performs well owing to the high power of the radar echoes (resp. communication signal). Thus, the next IC block efficiently suppress the interference for the other function, and good performance are achieved. When the powers of the radar echoes and communication signals are equivalent, the structure still achieves good performance thanks to the high radar processing gain of

Iterative Structures

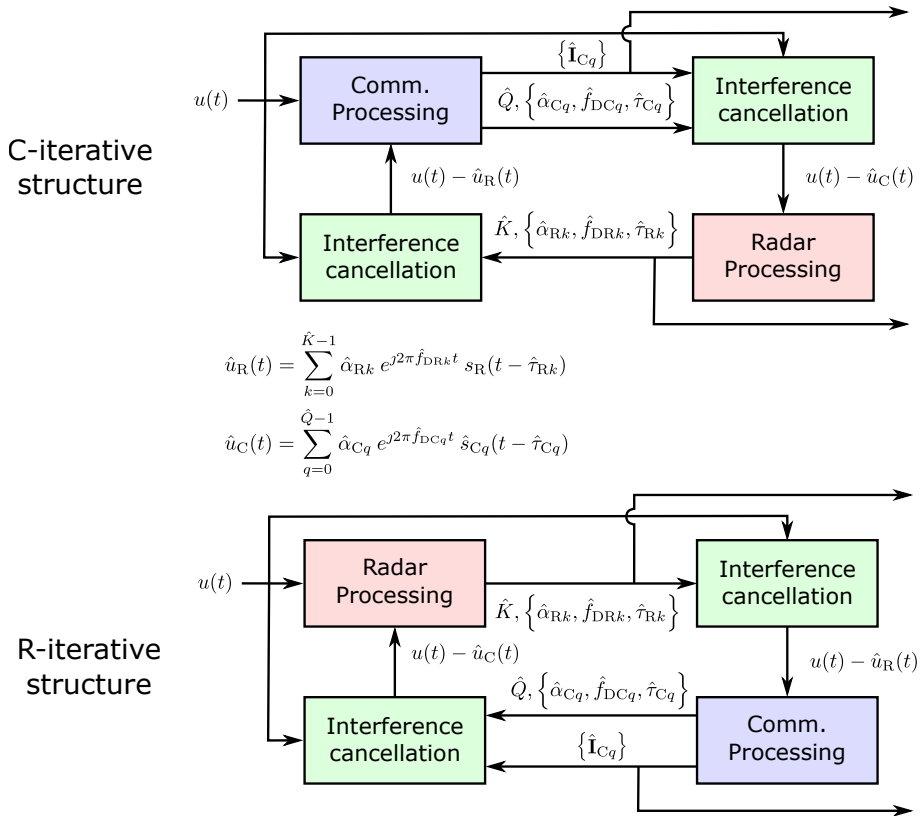


Fig. 9.13 Iterative structures. The radar function forwards the number of targets and their parameters to one IC block, while the communication function forwards the communication synchronisation parameters and the decoded symbols $\hat{\mathbf{I}}_C$ to another IC block.

Parameters	Values
Bandwidth	$B = 20$ MHz
Chirp duration	$T = 100$ μ s
Guard interval	$T_g = 5$ μ s
Ratio E_b/N_0	$E_b/N_0 = 10$ dB
Pulse number	$P = 50$
Symbols number	$L_c = 100$
False alarm probability	$\mathcal{P}_{FA} = 10^{-4}$
Modulation	QPSK
Zero-padding factor	$Z_D = 3$
Radar channel parameters	$ \alpha_R = 0.1 \rightarrow 10$, $\tau_R = 1$ μ s, $f_{DR} = 1000$ Hz
Comm. channel parameters	$ \alpha_C = 1$, $\tau_C = 2.5$ μ s, $f_{DC} = -300$ Hz

Table 9.4 Parameters of the scenario for Section 9.3.1.

the radar function.

Multiple specific iterative IC receivers are defined in this thesis, namely the *RCR structure* (extending the CR structure with one additional radar block), *CRC structure* (extending the RC structure with one additional communication block), and *RCRC structure* (R-iterative receiver with two complete stages). Note that there is no benefit to perform more than two complete stages, since there are no new operations performed w.r.t. the two-stage iterative structure. Parallel IC receivers can also be defined by combining simple or iterative structures.

9.3.1 Sensing and Communication Performance

In order to evaluate the performance achieved by the iterative structure, a scenario with PC-FMCW ISAC systems is defined in Table 9.4, with one radar echo and communication link. The power of the radar echo varies such that radar to communication power ratios from -20 dB to 20 dB are considered. Figures 9.14 and 9.15 illustrate the detection probability and BER achieved for different SIRs, i.e. radar echo to communication signal power ratios. In Figures 9.14 and 9.15, three regions can be identified:

For radar to communication SIRs between -20 dB and -10 dB: The communication signal is dominant compared to the radar echoes. Therefore, low detection probabilities are achieved without IC before the radar function, as with the RC structure. The other simple and iterative structures nonetheless achieve good detection probabilities since there is at least one IC stage before the radar function. Regarding the BER, the CR, RC and RCR structures achieve similar performance, with a slight

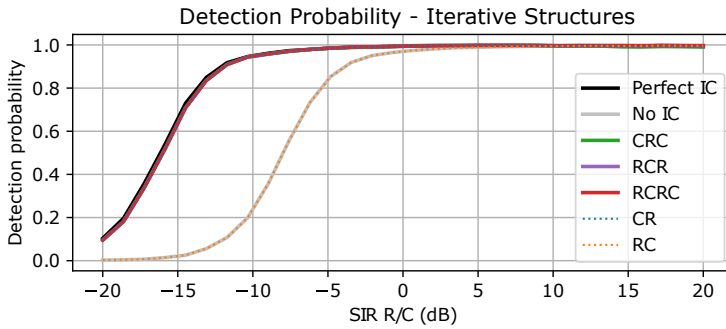


Fig. 9.14 Detection probability against radar to communication power ratio, with simple and iterative structures.

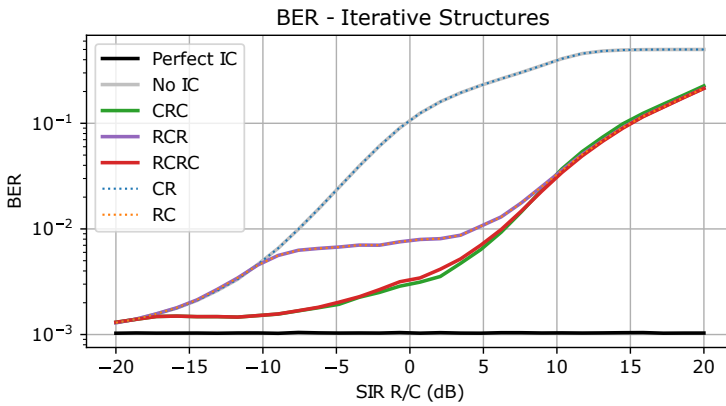


Fig. 9.15 BER against radar to communication power ratio, with simple and iterative structures.

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increase of BER for the RC structure, owing to the poor radar and IC performed at the first stage. However, the CRC and RCRC structures achieve better performance with an increasing radar echo power since the radar function is improved by the first communication and IC stage, and then the communication performance is improved at the second stage by the radar function and IC block.

For radar to communication SIRs between -10 dB and 0 dB: The radar echo power increases to reach power levels equivalent to the communication signal. Owing to the high radar processing gain, the RC and all the iterative structures perform well compared the CR structure, even if the SIR is still negative. The BERs achieved by the CRC and RCRC structures starts to increase with the radar echo power since the IC is not perfect, and the residual radar interference is increasing. Still, the RCR structure achieves lower performance compared to the other iterative structures because of a poorer radar detection and IC at the first stage. This first stage is nonetheless improving with the radar echo power, and the performance does not decrease as fast as for the other structures.

For radar to communication SIRs between 0 dB and 20 dB: The radar echo power is dominant compared to the communication signal. All the structures efficiently suppress the radar interference at the communication function, but the residual interference continues to increase with the radar echo power, and the achieved BERs thus increase. Furthermore, at SIRs close to 20 dB, the detection probabilities of the CR and CRC structures decreases slightly owing to the poor communication and IC performed at the first stage.

Finally, note that the RCRC structure achieves slightly lower performance compared to the CRC structure between -5 dB and 5 dB, owing to the additional IC residual from the first stage, which propagates through the complete structure.

9.3.2 Summary

Compared to simple structures, iterative structures enable to achieve good performance at the radar and communication function, irrespective of the considered scenario, at the price of a higher complexity. IC is iteratively performed before each function, to handle scenarios where the powers of the radar echoes and communication signals are comparable, or unbalanced. Multiple iterative structures have been defined with different complexity and performance, i.e. the RCR, CRC and RCRC receivers. The CRC receiver seems to achieve the best radar and communication performance. Still, even with iterative structures, the IC is not perfect. Therefore, the residual interference of the radar echo on the communication signal, increasing with the radar echo power, is detrimental for the communication function at high SIRs.

Dynamic Automotive Scenario

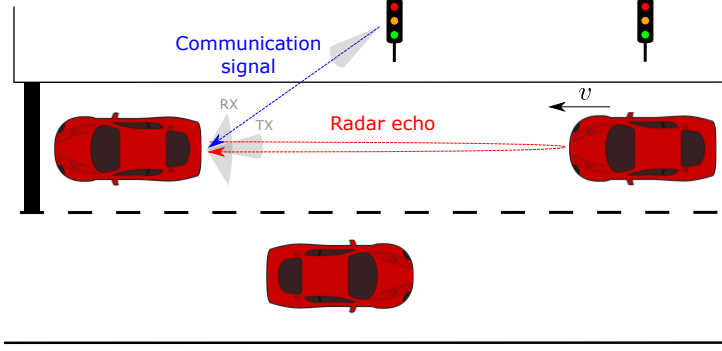


Fig. 9.16 Considered dynamic V2X scenario. A vehicle is receiving a communication signal from a RSU, illustrated by traffic lights, while detecting the next vehicle behind, which is braking.

9.4 Interference Cancellation in Dynamic Scenarios

The iterative IC structures presented in Section 9.3 are efficient irrespective of the scenario (which is not the case for the simple RC and CR structures). Additionally, they do not require any a priori estimation of the signals power levels. However, the price to pay is an increased complexity.

In dynamic scenarios, one may leverage on previous radar measurements to estimate the power levels of the signals, or to reduce the complexity of the iterative structures. This is done for instance in [Yua+21], which predicts the vehicles states based on their estimated kinematic parameters, and infers delays and Doppler frequencies associated with different channel paths. In this section, a focus is made on the R-iterative structures, following the same approach, the first radar function is replaced by an estimation of the radar echo parameters, computed as an update of previous radar processing. These receivers are respectively named *dynamic CR structure* and *dynamic CRC structure*.

9.4.1 System Model

The vehicular scenario of Figure 9.16 is considered, with one radar echo and communication signal. The vehicle of interest, located in $(0, 0)$, is stopped at a crossroad. It receives simultaneously a communication signal from a RSU, and detects the vehicle behind, which is braking. Denoting by $\mathbf{p}_C$ and $\mathbf{p}_R(n)$ the 2D positions of the RSU and the moving vehicle at the n^{th} transmission, the links parameters are computed as

$$\tau_C = \frac{\|\mathbf{p}_C\|}{c}, \quad \tau_R(n) = \frac{2\|\mathbf{p}_R(n)\|}{c}, \quad f_{DR}(n) = \frac{2f_c v(n)}{c},$$

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$$\alpha_C(n) = \frac{c e^{-j2\pi f_c \tau_C}}{4\pi f_c \|\mathbf{p}_C\|}, \quad \alpha_R(n) = \frac{c \mathcal{R} e^{-j2\pi f_c \tau_R(n)}}{4\pi f_c (2\|\mathbf{p}_R(n)\|)}, \quad (9.6)$$

where $v(n)$ is the speed of the moving vehicle at time index n , f_c is the carrier frequency and c the speed of light. The considered radar propagation model is a GO-based model, with $\mathcal{R}$ being the Fresnel reflection coefficient of the target. Assuming a good conductor, it is approximated as $\mathcal{R} \approx 1$. As discussed in Chapter 3, this model is suited for automotive radars owing to the high carrier frequencies and dimensions of the target, meaning that a near-field propagation model should be considered. Note that the same analysis can be performed with other propagation models, and fading on the links can also be introduced without discarding the following discussions and conclusions.

With the coefficients of (9.6), the SIR is computed as

$$\text{SIR}(n) = \frac{\|\mathbf{p}_C\|^2}{4\|\mathbf{p}_R(n)\|^2}.$$

Let us assume a time interval Δ_t between two radar measurements. In the considered scenario, the estimates of the delays and Doppler frequencies at the next transmission are computed as

$$\begin{cases} \hat{f}_{\text{DR}}(n+1) = \hat{f}_{\text{DR}}(n), \\ \hat{\tau}_{\text{R}}(n+1) = \hat{\tau}_{\text{R}}(n) + \frac{\Delta_t}{2c} \hat{v}(n) = \hat{\tau}_{\text{R}}(n) + \frac{\Delta_t}{f_c} \hat{f}_{\text{DR}}(n). \end{cases}$$

This simple estimation update process could be improved, for instance using complex tracking mechanisms as extended Kalman filtering [Liu+20]. Nonetheless, the proposed process is sufficient to achieve good performance with dynamic structures.

9.4.2 Sensing and Communication Performance

Table 9.5 depicts the scenario parameters with PC-FMCW ISAC systems, and their evolutions is shown in Figure 9.17. The delay between two transmissions is set as $\Delta_t = 66.6$ ms. At the start, the communication signal power dominates. As time goes by, the moving vehicle approaches the motionless vehicle, leading to an increased radar echo power $|\alpha_R(n)|^2$, and a decreased delay $\tau_R(n)$. Figures 9.18 and 9.19 illustrate the achieved BER and detection probability over time. First, regarding the detection probability, oscillations are observed with time owing to the finite accuracy for the estimation of the delay at the radar function. Except for the RC structure, all other simple, iterative, and dynamic receivers achieve a detection probability close to one, thanks to the interference cancellation of the communication signal. Next, regarding the BER, two cases can be distinguished:

- Until $t \approx 0.7$ s, the RC, RCR, and dynamic RCR structures achieve a slightly

Parameters	Values
Bandwidth	$B = 20 \text{ MHz}$
Chirp duration	$T = 100 \text{ }\mu\text{s}$
Guard interval	$T_g = 5 \text{ }\mu\text{s}$
Ratio E_b/N_0	$E_b/N_0 = 10 \text{ dB}$
Pulse number	$P = 50$
Symbols number	$L_c = 100$
False alarm probability	$\mathcal{P}_{\text{FA}} = 10^{-4}$
Modulation	QPSK
Zero-padding factor	$Z_D = 3$
Vehicle Parameters	$\mathbf{p}_R(0) = (30, 0) \text{ m}, v(n) = v = 15 \text{ m/s}$
RSU parameters	$\mathbf{p}_C = (14, 2.5) \text{ m}$

Table 9.5 Parameters of the scenario for Section 9.4.2.

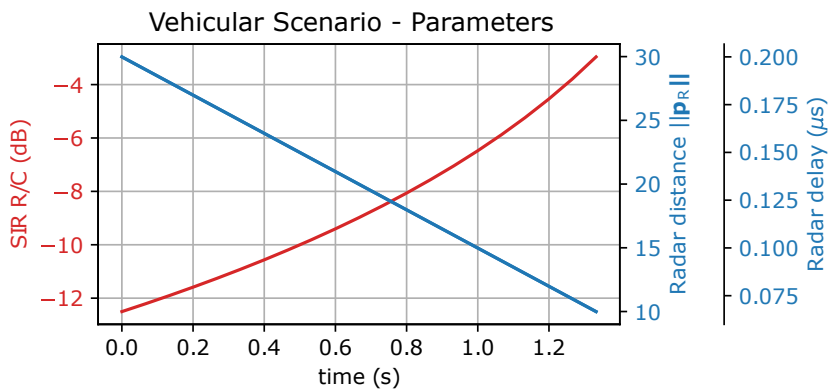


Fig. 9.17 Parameters of the automotive scenario against time.

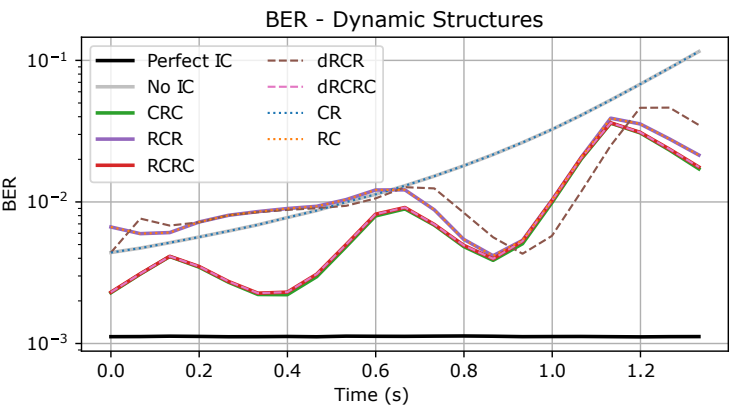


Fig. 9.18 BER over time, with iterative and dynamic structures. The lowercase *d* in the legend stands for dynamic structures.

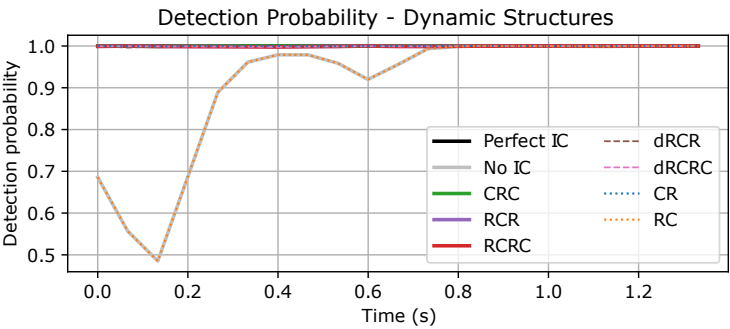


Fig. 9.19 Detection probability over time, with iterative and dynamic structures. The lowercase *d* in the legend stands for dynamic structures.

higher BER than the other structures, which is also higher than the one achieved with the structure without any interference cancellation of the radar echo. At low radar echo to communication signal power ratios, these structures are hugely impacted by the communication signal when performing the first radar and interference cancellation stage. The reconstruction and cancellation of the radar echo is thus inefficient, leading to an increased interference level at the communication function.

- For higher times, the performance of all structures are similar or better than the performance of the CR structure. The radar echo power is still lower than the communication signal, but it becomes sufficient to perform an efficient radar and interference cancellation stage, even without suppressing the communication signal beforehand. Thus, at such times, starting by the radar function is no more detrimental.

Regarding the dynamic structures, on the one hand, the dynamic RCRC receiver achieves the same radar and communication performance as the RCRC structure. The first radar and interference cancellation stage of this structure has nearly no impact on the last communication block, as shown by the CRC structure which achieves the same performance. On the other hand, this is not the case with the dynamic CRC structure. The performance observed with this structure are shifted in time compared to the performance achieved by the CRC structure. This is a consequence of the update step, which relates the efficiency of the previous to the actual time instant. When the radar function performs well (resp. poorly), the estimation of the delay and Doppler frequency at the next time instant is accurate (resp. inaccurate), leading to good (resp. low) performance for the next transmission.

9.4.3 Summary

Leveraging on multiple successive transmissions in dynamic scenarios, the complexity of R-iterative structures is reduced by replacing the first radar block by an update of previous radar estimations, at the price of a slight performance reduction. One can still improve the performance by using more complex tracking mechanisms, or by enhancing the radar resolution.

9.5 Conclusion

Even with co-designed systems, the interference between the sensing and communication of two ISAC systems in full-duplex is still an issue. A first solution to overcome this problem is the IC. Leveraging on the cooperation between both functions, sharing their processing outputs may help the other function to reconstruct and suppress the interfering signal.

9.5.1 Summary of the Contributions

The contributions of this chapter are summarised as follows:

- The impact of the uplink communication signals (resp. radar echoes) on the radar function (resp. communication function) has first been analysed mathematically, for **PC-FMCW** or **OFDM ISAC** systems. It has been shown that the communication interference on the radar function is inherently mitigated, as its power is spread on the delay-Doppler map at the output of the receiver. Still, the noise floor is increased, and the targets may be drawn in the interference. Moreover, for the communication function, there is no inherent interference mitigation mechanism.
- **IC** has been introduced at either the sensing or the communication function. On the one hand, when the power of the uplink communication signals is dominant, these signals are first processed, reconstructed and cancelled from the input of the radar function. Accordingly, it has been shown that the detection probability is drastically increased after cancellation. Moreover, the higher the communication constellation index, the lower the performance at the radar function, owing to the signal reconstruction errors. **ECCs** also help increase the performance of both functions. On the other hand, when the power of the radar echoes is dominant, they are processed, reconstructed, and cancelled from the input of the communication function. In that case, it has been shown that the achieved **BER** is highly dependent on the accuracy of the estimated radar parameters, i.e. the Doppler frequencies and delays of the radar echoes. The accuracy may be improved by transmitting longer payloads, at the price of a higher sensitivity to the Doppler frequency, or by introducing zero-padding.
- Iterative **IC** structures have been proposed. By iterating between both functions, the proposed structures are robust against any scenario, at the price of a high complexity. Among the proposed receivers, the best trade-off between performance and complexity is achieved by the **CRC** structure, i.e. when the communication is processed in a first stage, then the radar function, and finally the communication function in a second stage.
- In order to reduce the complexity of the iterative receivers, dynamic scenarios have been introduced, in which the system can leverage on the previous radar estimation. Using that information, the delay and Doppler frequencies of the radar echo at the next transmission is estimated beforehand. This enables to remove the first radar block of the **R**-iterative structure, and reduce the complexity of the system, at the price of a slight performance reduction.

9.5.2 Future Works

In dynamic scenarios, it has been proposed to leverage on the previous radar measurements to predict the radar echoes parameters at the next transmission.

Nevertheless, the approach may be generalised for the communication function as well. If the other **ISAC** system is also tracked by the radar function, similarly to what has been done with the radar echo, the C-iterative structure may also first be adapted to reduce the complexity of this receiver.

Next, the tracking algorithm could be improved, as a simple linear uniform movement has been supposed. For instance, variations of the Doppler frequency may be predicted based on multiple previous measurements. Besides, extended Kalman filtering seems to be a promising approach to better leverage on the past information.

Finally, **IC** schemes work well in simulation, but experimental measurements may be carried out to assess the relevancy of such approach. Especially, the impact of the clutter on the communication function has been neglected. This assumption may hold if the uplink communication signals have non-zero Doppler frequencies, such that it may be isolated from the other signals.

9.5.3 What is Next?

This chapter has investigated a first solution, namely the **IC**, against the inter-function interference generated by two **ISAC** systems in full-duplex mode. Another solution to tackle this problem with **MIMO** systems is the design of specific beamformers, to isolate the signal of interest in the spatial domain. Nonetheless, the design may favour one of the two functions. Even among the radar or communication function, some users or targets may be emphasised. Therefore, the design of the objective function for the optimisation problem is also critical. This is the topic of Chapter 10.

Fair Beamforming for Integrated Sensing and Communication Systems

10

This chapter is a joint work carried out with Guillaume Thiran, with equal contributions, which I again thanks warmly. Most of the content is inspired from [De +24].

IN Chapter 9, IC has been considered to improve the communication and sensing performance of single-antenna ISAC systems in full-duplex scenarios. With MIMO systems, another way to tackle the interference issue, which may be complementary to IC, is to perform beamforming, enabling isolation of the signals of interest in the spatial domain. Nevertheless, the beamformers design must be based on a relevant optimisation metric, defined specifically for the considered application. For instance, in full-duplex ISAC scenarios, it should not favorise the sensing or communication purpose, and not favorise any of the targets or UEs, except in a controlled manner if priority is introduced, either w.r.t. the function, or the targets and UEs. This topic is analysed in this chapter.

First, Section 10.1 summarises existing works in the design of JCAS beamformers. Next, the system model is detailed in 10.2. Namely, expressions of the communication and radar SINRs are obtained in MIMO configurations for the PC-FMCW and OFDM ISAC systems presented in Chapter 8. Moreover, a fair objective function is proposed, to ensure that both functions achieve good performance. Then, solutions of the optimisation problem are provided in Section 10.3 for different scenarios, and numerical analyses are performed in Section 10.4.

10.1 Joint Communication and Sensing Beamforming in the Literature

The JCAS beamforming has already been widely studied in the literature. For instance, an uplink communication signal is received together with a radar echo in [Has+18; HHA18]. The designed beamformers are able to nullify the interference between both systems. Motivated by complexity reduction, [NWH22] and [Don+20] provide suboptimal beamformers in a downlink scenarios, with a maximisation of the SINR

of the radar links, constrained by the SINR of the communication links. In [Liu+21], the radar CRB is minimised with SINR constraints on the UEs. The beamformers are either obtained in closed-form in the single-user case, or the problem is relaxed and solved with numerical methods for multi-user systems. The radar CRB is also considered in [Zhu+23], and minimised under a communication sum-rate constraint. In [Liu+23], an iterative algorithm optimises both the radar power and communication rates, while ensuring a low self-interference in a full-duplex scenario.

In order to optimise the performance related to all communication users and radar targets, one possible paradigm is to consider one metric as an objective with the others set as constraints. This is the approach adopted by [NWH22; Liu+21; Zhu+23]. Still, choosing adequate constraint thresholds is challenging and may require multiple iterations to ensure sufficient performance for both functions. Alternatively, a weighted-sum of the radar and communication objectives can be performed [Liu+23]. However, such formulation might totally neglect one objective in favour of the others. This issue is tackled in [Che+21], in which a fair optimisation of the downlink radar and communication performance is performed. The proposed approach nonetheless requires to solve a sequence of convex optimisation problems and the computational burden this implies might be incompatible with the real-time tracking mechanisms usually associated with radar beamforming schemes.

In summary, following the numerous works on the JCAS beamforming, a focus is made in this chapter on a beamforming scheme which is able to fairly optimise the transmit and receive beamformers in full-duplex multi-user multi-target scenarios, detailed in Section 1.1.2 of Chapter 1. Moreover, the numerical complexity of the proposed scheme should be controlled.

10.2 System Model

In this chapter, the full-duplex scenario detailed in Section 1.1.2 of Chapter 1 is considered. The scenario is illustrated in Figure 10.1. The ISAC system of interest simultaneously receives radar echoes from K targets, and uplink communication signals from Q ISAC systems. All the ISAC systems are equipped with N_t and N_r antennas at the transmitter and receiver, respectively. As in Chapter 9, the self-interference is again neglected, as it remains are supposed to be negligible compared to the power of the uplink communication signals and radar echoes. Instead of performing IC, leveraging on the MIMO configuration of the ISAC systems, beamforming is applied at the ISAC transmitters and receivers. The beamformers at the transmission and reception are respectively denoted as $\mathbf{f}$ and $\mathbf{g}$.

Full-Duplex MIMO Scenario

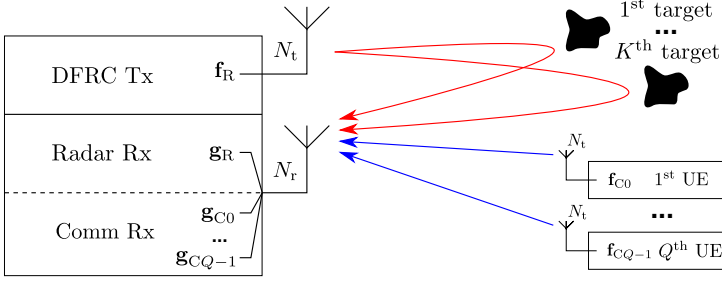


Fig. 10.1 Full-duplex **MIMO** scenario, with beamformers $\mathbf{f}_R$ and $\mathbf{f}_C$ at the transmitters, and $\mathbf{g}_R$ and $\mathbf{g}_C$ at the receivers.

10.2.1 Multiple-Antennas Channel Model

In Section 4.2 of Chapter 4, **MIMO** channel models for sensing and communication, i.e. Boxes 4.1 and 4.2, are presented from the literature. Conveniently, when the noise is neglected, they may both be rewritten in a matrix form as follows:

$$\mathbf{u}(t) = \sum_{l=1}^L \alpha_l e^{j2\pi f_{Dl}t} \mathbf{a}_{rl} \mathbf{a}_{tl}^H \mathbf{s}(t - \tau_l), \quad (10.1)$$

where L is the number of radar targets or communication paths, and $\{\alpha_l, f_{Dl}, \tau_l\}$ are complex coefficients, Doppler frequencies and delays associated with the radar targets or communication paths. This corresponds to the **MIMO** channel impulse response given in Box 10.1¹, valid for both the sensing and communication function. Denoting by N_t and N_r the number of transmitting and receiving antennas, the vector $\mathbf{s} \in \mathbb{C}^{N_t}$ are the transmitted signals at each transmitting antenna, and the vectors $\mathbf{a}_{tl} \in \mathbb{C}^{N_t}$ and $\mathbf{a}_{rl} \in \mathbb{C}^{N_r}$ are named the *transmitter* and *receiver steering vectors*.

10.1: MIMO Multi-Link Channel Model

$$\mathbf{h}(t, \tau) = \sum_{l=1}^L \alpha_l e^{j2\pi f_{Dl}t} \mathbf{a}_{rl} \mathbf{a}_{tl}^H \delta(\tau - \tau_l)$$

The steering vectors respectively contain the phase shifts associated with the specific antenna index w.r.t. to given reference points at the transmitter and receiver. For instance, in the far-field region, following Box 4.1, the steering vectors are given

¹The channel impulse response is also a function of the targets or scatterers absolute positions, or the angles of departure and arrival in the far-field region of the antenna arrays. Still, it has been hidden in the vectors $\mathbf{a}_{tl}$ and $\mathbf{a}_{rl}$ for the sake of readability.

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by

$$[\mathbf{a}_{tl}]_n = e^{jk\rho_{tn} \cos \phi_{tl}}, \quad [\mathbf{a}_{rl}]_n = e^{-jk\rho_{tn} \cos \phi_{tl}},$$

where ρ_{vn} with $v \in \{t, r\}$ is the distance between the l^{th} transmitting or receiving antenna, and the reference point of the transmitting or receiving antenna array, and ϕ_{vln} is the angle between the vectors starting from the reference point of the transmitting or receiving antenna array, and pointing to the centre of the l^{th} target and the l^{th} transmitting antenna (resp. l^{th} receiving antenna). These quantities are illustrated in Figure 4.1 of Chapter 4. For ULAs, taking the first antenna of the arrays as reference point, it is further simplified as

$$[\mathbf{a}_{tl}]_n = e^{jkn\Delta \cos \phi_{tl}}, \quad [\mathbf{a}_{rl}]_n = e^{-jkn\Delta \cos \phi_{tl}},$$

where Δ is the antenna spacing, and the angle ϕ is constant for each antenna array. Equation 10.1 also works in the near-field region for radar links, or for communication links without LoS components. Following Box 4.2, the steering vectors are obtained as

$$[\mathbf{a}_{tl}]_n = e^{jkr_{ln}}, \quad [\mathbf{a}_{rl}]_n = e^{-jkr_{ln}},$$

where r_{ln} are the distances between the n^{th} transmitting or receiving antennas and the l^{th} target or obstacle.

Still, in order to obtain (10.1), it is assumed that the complex coefficients $\{\alpha_l\}$, Doppler frequencies $\{f_{Dl}\}$ and the delays $\{\tau_l\}$ does not vary significantly, whatever the considered pair of antennas. This is a reasonable assumption in the far-field region of the antenna arrays and the targets, but this is not always true in the near-field region. This should be kept in mind when using this model and the results which will be developed this chapter for near-field propagation.

10.2.2 Radar and Communication Processing

Following (10.1), supposing for simplicity flat fading channels for the communication links, the received signal at the ISAC system of interest can be written as Box 10.2, where the subscripts R and C refer to the radar and communication signals, and $\mathbf{n}$ is the noise vector.

10.2: Multi-Antenna Full-Duplex Received Signal

$$\mathbf{u}(t) = \sum_{k=0}^{K-1} \alpha_{Rk} e^{j2\pi f_{DRk} t} \mathbf{a}_{Rtk} \mathbf{a}_{Rtk}^H \mathbf{s}_R(t - \tau_{Rk}) + \sum_{q=0}^{Q-1} \alpha_{Cq} e^{j2\pi f_{DCq} t} \mathbf{a}_{Ctq} \mathbf{a}_{Ctq}^H \mathbf{s}_C(t - \tau_{Cq}) + \mathbf{n}(t)$$

It is assumed that the same signal is transmitted at all antennas, i.e.

$$\mathbf{s}_R(t) = \mathbf{f}_R s_R(t), \quad \mathbf{s}_{Cq}(t) = \mathbf{f}_{Cq} s_{Cq}(t), \quad \forall q.$$

The signals s_R and s_{Cq} are assumed to be **PC-FMCW** or **OFDM ISAC** waveforms. Expressions of the **SINRs** for every target and communication signal are developed for both waveforms in Appendices 10.A and 10.B. Note that, these developments may be straightforwardly adapted to other waveforms.

In summary, whatever the adopted **ISAC** waveform, the **SINRs** of the radar targets and communication signals can be written as

$$\text{SINR}_{Rk} = \frac{\kappa_{RRkk} |\mathbf{f}_R^H \mathbf{a}_{Rtk}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rrk}|^2}{\sum_{\substack{k'=0 \\ k' \neq k}}^{K-1} \kappa_{RRk'k} |\mathbf{f}_R^H \mathbf{a}_{Rtk'}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rrk'}|^2 + \sum_{q=0}^{Q-1} \kappa_{CRqk} |\mathbf{f}_{Cq}^H \mathbf{a}_{Ctq}|^2 |\mathbf{g}_R^H \mathbf{a}_{Crk}|^2 + 1},$$

$$\text{SINR}_{Cq} = \frac{\kappa_{CCqq} |\mathbf{f}_{Cq}^H \mathbf{a}_{Ctq}|^2 |\mathbf{g}_{Cq}^H \mathbf{a}_{Ccq}|^2}{\sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} \kappa_{CCq'q} |\mathbf{f}_{Cq'}^H \mathbf{a}_{Ctq'}|^2 |\mathbf{g}_{Cq'}^H \mathbf{a}_{Ccq'}|^2 + \sum_{k=0}^{K-1} \kappa_{RCkq} |\mathbf{f}_R^H \mathbf{a}_{Rtk}|^2 |\mathbf{g}_{Cq}^H \mathbf{a}_{Rrk}|^2 + 1},$$

On the one hand, for the radar echoes, the interference comprises the echoes of the other targets, and the communication signals. On the other hand, for the uplink signals, the interference comprises the other communication signals, and the radar echoes. Every term is pondered by a coefficient κ , equal to the ratio between the power of the considered quantity, excluding the beamforming gains, and the noise at the receiver. Thus, they are equivalent to the **SNRs** of the useful and interfering signals. One may therefore optimise the beamformers $\mathbf{f}_R$, $\mathbf{g}_R$, $\{\mathbf{f}_{Cq}\}$, and $\{\mathbf{g}_{Cq}\}$, depending on the criterion of interest.

10.2.3 Fair Integrated Sensing and Communication Metric

In order to design the $Q + 1$ beamforming vectors at the transmitters and receivers, the product of the radar and communication **SINRs** [Bjo+14] is maximised:

$$\mathcal{M} \triangleq \prod_{k=0}^{K-1} \text{SINR}_{Rk} \prod_{q=0}^{Q-1} \text{SINR}_{Cq}. \quad (10.2)$$

The optimisation problem to solve is thus written as

$$\begin{aligned} \mathbf{f}_R^*, \mathbf{g}_R^*, \{\mathbf{f}_{Cq}^*, \mathbf{g}_{Cq}^*\} = \max_{\mathbf{f}_R, \mathbf{g}_R, \{\mathbf{f}_{Cq}, \mathbf{g}_{Cq}\}} \quad & \mathcal{M} \\ \text{s.t.} \quad & \|\mathbf{f}_R\|_2 = 1, \quad \|\mathbf{g}_R\|_2 = 1, \\ & \|\mathbf{f}_{Cq}\|_2 = 1, \quad \|\mathbf{g}_{Cq}\|_2 = 1, \quad \forall q = 0, \dots, Q-1. \end{aligned} \quad (10.3)$$

Parameters	Values
Antennas numbers	$N_t = N_r = 8$
Targets angles	$\theta_{R0} = 43^\circ, \theta_{R1} = -43^\circ$
Coefficients κ	$\kappa_{RR} = \begin{bmatrix} \kappa & 0.1\kappa \\ 0.1 & 1 \end{bmatrix}$

Table 10.1 Parameters of the scenario for Section 10.2.3, with $\kappa > 0$.

This problem is either solved for ISAC systems in tracking mode, or discrete codebook design. Therefore, the parameters of the targets and communication systems are supposed to be available for optimisation. The impact of a model mismatch on the performance is briefly discussed in Section 10.4.1.

The proposed metric offers the advantage to fairly consider all targets and communication users, since they all impact in the same manner the objective function. Moreover, any low factor heavily penalises such a product. This consequently prevents the targets and UEs with low SINRs to be set aside, without requiring to arbitrarily define any weight or threshold. Additionally, at high SINR, it is equivalent to a sum-rate maximisation [Liu+17].

In order to illustrate the benefits of the SINR-product metric, a sensing scenario with two targets and no UE is defined, with parameters given in Table 10.1. The antenna arrays of the transmitter and receiver are modelled as Uniform Linear Arrays (ULAs). The targets are supposed to be in the far-field, such that the two steering vectors can be written as

$$\mathbf{a}_{Rvk} = \left[1 \quad e^{-jk\Delta \cos \theta_{Rvk}} \quad \dots \quad e^{-jk\Delta(N_v-1) \cos \theta_{Rvk}} \right]^T, \quad (10.4)$$

with $v \in \{t, r\}$, Δ the antenna spacing assumed to be equal to $\lambda/2$, and θ_{Rvk} the angle of departure/arrival of the k^{th} target. The coefficient κ in Table 10.1 models the path-loss ratio between the first and second radar target. A coefficient κ lower than 1 means that the first target is in less favourable conditions than the second.

Figure 10.2 illustrates the Pareto-optimal SINR pairs $(\text{SINR}_{R0}, \text{SINR}_{R1})$, as well as the point selected by the SINR-product and sum metrics. As it can be observed, the SINR-product metric always selects a Pareto-optimal point, and fairly considers both targets, even if one suffers from severe channel degradation. In contrast, the SINR-sum metric drastically favours the target with the best channel condition, at the detriment of the other. As a result, the SINR-sum metric is extremely sensitive to channel changes, as it can be observed by comparing the results with $\kappa = 0.99$ and $\kappa = 1.01$. Whereas the Pareto frontier remains almost unmodified, the SINR of the

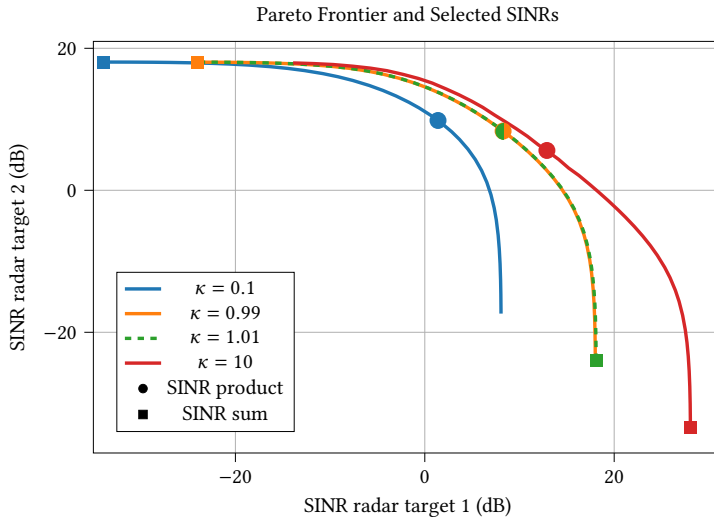


Fig. 10.2 Pareto frontier and selected point for the **SINR**-product and sum metrics, in a scenario with two radar targets. The factor κ is the ratio between the path loss of the first radar target and the second.

targets jumps from around -23 dB to 18 dB.

10.3 Transmitters and Receivers Beamformers Design

This section focuses on the development of beamformers for the **ISAC** transmitters and receivers, with the **SINR**-product metric. The optimisation problem of (10.3) is solved in Appendices 10.C to 10.F for different scenarios, i.e. with different numbers of targets and **UEs**. For that extent, a dimensionality reduction approach is first detailed in Section 10.3.1. It is helpful to reduce the complexity of scenarios in which large antenna arrays are considered. Then, different solutions are summarised in Section 10.3.2.

10.3.1 Dimensionality Reduction

As shown in [Liu+21], the optimal beamforming vectors $\mathbf{f}_R$ and $\{\mathbf{f}_{Cq}\}$ (resp. $\mathbf{g}_R$ and $\{\mathbf{g}_{Cq}\}$) belong to the spaces generated by the transmitters (resp. receivers) steering vectors, named the steering spaces. Additional vectors, perpendicular to the steering spaces, are also required, as the optimisation of the transmitted powers is not included in (10.3). Thus, this allows solutions in which power is transmitted in directions orthogonal to the steering spaces. Yet, it is possible to extend the optimisation problem of (10.3) by introducing power constraints, such that these

K	Q	Assumption	Solution	Details	Optimality	Complexity $O(\cdot)$
1	1	-	$\mathbf{f}_R^* = \mathbf{a}_{Rt}, \quad \mathbf{f}_C^* = \mathbf{a}_{Ct},$ $\mathbf{g}_R^* = \mathbf{a}_{Rr} - \frac{\mathbf{a}_{Cr}^H \mathbf{a}_{Rr}}{N_r} \frac{N_t N_r}{\kappa_{CR}^{-1} + N_t N_r} \mathbf{a}_{Cr},$ $\mathbf{g}_C^* = \mathbf{a}_{Cr} - \frac{\mathbf{a}_{Rr}^H \mathbf{a}_{Cr}}{N_r} \frac{N_t N_r}{\kappa_{RC}^{-1} + N_t N_r} \mathbf{a}_{Rr},$	-	Global	N
2	0	-	$\mathbf{f}_R^* = \lambda_t \mathbf{a}_{Rt0} + (1 - \lambda_t) e^{J \arg\{\mathbf{a}_{Rt1}^H \mathbf{a}_{Rt0}\}} \mathbf{a}_{Rt1}$ $\mathbf{g}_R^* = \lambda_r \mathbf{a}_{Rr0} + (1 - \lambda_r) e^{J \arg\{\mathbf{a}_{Rr1}^H \mathbf{a}_{Rr0}\}} \mathbf{a}_{Rr1}$	Grid search on (λ_t, λ_r)	Global	$N + M^2$
≤ 2	≤ 2	$\kappa_{RRk'k} = 0$ $\mathbf{g}_{Cq} = \mathbf{a}_{Ctq}$	(10.11) and (10.12)	AM on (λ_t, λ_r)	Local	$N + S_1 \log(M)$
				AM on $\mathbf{f}_R, \{\mathbf{g}_{Cq}\}$	Relaxed local	$S_2 N^6$
≤ 2	≤ 2	$\kappa_{RRk'k} = 0$ $\mathbf{g}_{Cq} = \mathbf{a}_{Ctq}$	(10.11) and (10.12)	AM on $\tilde{\mathbf{f}}_R, \{\tilde{\mathbf{g}}_{Cq}\}$	Relaxed local	$N + S_3$
				AM	Local	$N + S_4 M^2$

Table 10.2 Transmitters and receivers beamforming methods against the number of targets and users. For the sake of readability, the normalisation constants of all beamformers have been omitted. Yet, they should be normalised to a unit norm. For the complexity analysis, the maximum number of antennas is denoted $N \triangleq \max(N_t, N_r)$. The letter M denotes the number of points selected to discretise the $[0, 1]$ interval. The numbers S_1, S_2, S_3 and S_4 are the number of iterations performed in the alternating maximisation scheme, which depends on the variables on which the alternation takes place, and the desired accuracy. The scaling of the complexity w.r.t. K and Q has not been included in the table, as these numbers are expected to remain small w.r.t. the other quantities in practical scenarios.

perpendicular vectors are not required. This is done in Guillaume Thiran's PhD thesis.

Following this transformation, the optimal solutions can be written in the form

$$\begin{aligned} \mathbf{f}_R &= \mathbf{S}_{Rt} \tilde{\mathbf{f}}_R, & \mathbf{g}_R &= \mathbf{S}_r \tilde{\mathbf{g}}_R, \\ \mathbf{f}_{Cq} &= \mathbf{S}_{Ctq} \tilde{\mathbf{f}}_{Cq}, & \mathbf{g}_{Cq} &= \mathbf{S}_r \tilde{\mathbf{g}}_{Cq}, \quad \forall q = 0, \dots, Q-1. \end{aligned}$$

where $\tilde{\mathbf{f}}_R \in \mathbb{C}^{K+1}$, $\{\tilde{\mathbf{f}}_{Cq}\} \in \mathbb{C}^2$, and $\tilde{\mathbf{g}}_R, \{\tilde{\mathbf{g}}_{Cq}\} \in \mathbb{C}^{K+Q+1}$. The matrices $\mathbf{S}_{Rt}$, $\{\mathbf{S}_{Ctq}\}$, and $\mathbf{S}_r$ are defined as

$$\begin{aligned} \mathbf{S}_{Rt} &\triangleq \begin{bmatrix} \mathbf{a}_{Rt0} & \cdots & \mathbf{a}_{RtK-1} & \mathbf{a}_{Rt}^\perp \end{bmatrix} && \in \mathbb{C}^{N_t \times (K+1)}, \\ \mathbf{S}_{Ctq} &\triangleq \begin{bmatrix} \mathbf{a}_{Ctq} & \mathbf{a}_{Ctq}^\perp \end{bmatrix} && \in \mathbb{C}^{N_t \times 2}, \\ \mathbf{S}_r &\triangleq \begin{bmatrix} \mathbf{a}_{Rr0} & \cdots & \mathbf{a}_{RrK-1} & \mathbf{a}_{Cr0} & \cdots & \mathbf{a}_{CrQ-1} & \mathbf{a}_r^\perp \end{bmatrix} && \in \mathbb{C}^{N_r \times (K+Q+1)}. \end{aligned}$$

The vectors $\mathbf{a}_{Rt}^\perp \in \mathbb{C}^{N_t}$, $\{\mathbf{a}_{Ct}^\perp\} \in \mathbb{C}^{N_t}$, and $\mathbf{a}_r^\perp \in \mathbb{C}^{N_r}$ are the vectors perpendicular to the steering spaces, respectively generated by the matrices $\mathbf{S}_{Rt} \setminus \{\mathbf{a}_{Rt}^\perp\}$, $\{\mathbf{a}_{Ctq}\}$, and $\mathbf{S}_r \setminus \{\mathbf{a}_r^\perp\}$. This formulation reduces the dimension of the vectors to optimise from N_t or N_r elements to $K+1$, 2, and $K+Q+1$ elements, i.e. from the antennas spaces to the steering spaces. In the scenario analysed in next sections, the two formulations will be considered depending on the cases. Note that, for $\mathbf{f}_R$ to be normalised to a unit power, the vector $\tilde{\mathbf{f}}_R$ should be normalised to $\tilde{\mathbf{f}}_R / \|\mathbf{S}_{Rt} \tilde{\mathbf{f}}_R\|$, and similarly for the others vectors.

10.3.2 Summary

Table 10.2 summarises the different scenarios, the associated beamforming schemes, and their assumptions. The optimality and complexity of the solutions are also provided. Based on that, methods can be selected depending on the number of radar targets and UEs.

When there is only one target and UE, the optimal beamformers are obtained as the **Minimum Variance Distortionless Response (MVDR)** solutions. These closed-form solutions have the lowest complexity among all the methods, while being globally optimal. For two radar targets and no UE, a global optimum can be found through a **2D** grid search. If this complexity is too high, either an **Alternative Maximisation (AM)** on (λ_t, λ_r) can be performed, or the method proposed for $K \leq 2$ and $Q \leq 2$ can be applied in the steering space. For the other cases with $K \leq 2$ and $Q \leq 2$, the **AM** method should be operated in the steering space, as it has a much lower complexity than in the antenna space. Finally, the complexity of the **AM** proposed for general K and Q makes it a relevant method only if $K \geq 3$ or $Q \geq 3$, owing to the lack of analytical solution in these cases.

10.3.3 Extension for Downlink Communication

The metric proposed in (10.2) can be extended to take into account downlink communication links. Similarly to the development proposed in Appendix 10.B, as there is inter-function interference in the downlink, denoting by L the number of downlink UEs, the SINR of the l^{th} downlink UE would then be written as

$$\text{SINR}_{Dl} = \kappa_{Dl} |\mathbf{f}_R^H \mathbf{a}_{Dtl}|^2 |\mathbf{g}_D^H \mathbf{a}_{Dtl}|^2,$$

where κ_{Dl} is a coefficient equal to the ratio between the power of the link excluding the beamforming gains and the noise at the receiver, $\mathbf{a}_{Dtl}$ and $\mathbf{a}_{Drl}$ are respectively the transmitter and receiver steering vectors, and $\mathbf{g}_D$ is the receiver beamformer associated with the l^{th} downlink UE. These SINRs can be introduced in the product metric to take into account these downlink UEs.

Since the SINRs of the downlink UEs involve the transmitter beamformer $\mathbf{f}_R$, the methods proposed in Table 10.2 are still valid, considering $K + L$ targets and downlink UEs instead of K targets only. Yet, note that these UEs may already be handled if they are considered as radar targets.

10.4 Numerical Analysis

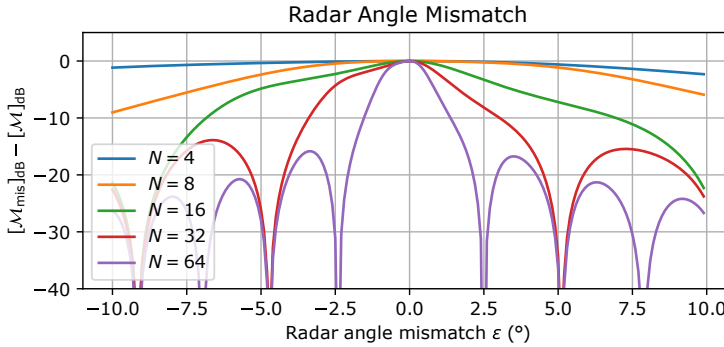
In this section, multiple numerical analyses are performed in the different scenarios of Section 10.3. As in Section 10.2.3, the antenna arrays of the transmitters and receivers are modelled with ULAs, with the target and the UEs supposed to be in the far-field region. The steering vectors are therefore written as in (10.4). The antenna spacing is assumed to be equal to $\Delta = \lambda/2$. First, the impact of an angle mismatch on the metric is investigated, in a one-target one-user scenario. Then, the performance obtained with the full and Hermitian relaxations are analysed in a two-target no-user scenario. Finally, the beam patterns obtained with the Hermitian relaxation are illustrated in a two-target two-user scenario.

10.4.1 Impact of Angle Mismatch

Since the parameters of the scenario are assumed to be known beforehand, the impact of a model mismatch is first analysed. Table 10.3 defines parameters for a scenario with one target one UE. Assuming a mismatch ϵ in the radar angle θ_R , Figure 10.3 illustrates the impact of ϵ on the metric. The higher the number of antennas, the higher the sensitivity against the mismatch. Particularly large losses arise when the mismatch is sufficiently large to deviate the target outside the mainlobe. In that scenario, the mismatch does not affect significantly the communication SINR, owing to the large difference between the target and user angles. In scenarios where the radar target and communication user are close to each other, the mismatch is even

Parameters	Values
Target angle	$\theta_R = 43^\circ$
UE angle	$\theta_C = -43^\circ$
Radar coefficients κ	$\kappa_{RR} = 1, \kappa_{CR} = 0.1$
Comm. coefficients κ	$\kappa_{CC} = 1, \kappa_{RC} = 0.1$

Table 10.3 Parameters of the scenario for Section 10.4.1.


 Fig. 10.3 One-target one-user scenario — Impact of the radar angle mismatch on the metric for different number of antennas $N_t = N_r = N$.

more detrimental, as it has a significant effect on the other function as well.

10.4.2 Performance of the Full and Hermitian Relaxations

Table 10.4 defines parameters for a sensing scenario with two radar targets and no UE. Figure 10.4 illustrates the beamforming gains $G_{f_R}(\theta) \triangleq |\mathbf{f}_R^H \mathbf{a}(\theta)|$ of the radar transmitter, where $\mathbf{a}$ is the steering vector in the far-field region for a given angle θ , and the vector $\mathbf{f}_R$ is obtained through 2D grid search, AM with full relaxation, and AM with Hermitian relaxation. All three solutions direct two mainlobes towards the targets. Both relaxations provide very similar solutions, whereas the optimal solution increases slightly the power directed towards the target suffering the most from interference. This enables to ensure fairness between both targets, achieving a higher SINR-product.

In order to better compare the solutions, Figure 10.5 illustrates the performance gaps between the metrics obtained with both relaxations and the optimal beamforming vector, obtained through 2D grid search, for different noise powers. For low and high noise-to-interference ratios, the relaxations tend to reach the optimal solution. Between these extremes, a small loss in performance is observed, being reduced by the Hermitian relaxation, at the price of a slightly higher complexity.

Parameters	Values
Antennas numbers	$N_t = N_r = 8$
Targets angles	$\theta_{R0} = 12^\circ, \theta_{R1} = 43^\circ$
Coefficients κ	$\kappa_{RR} = \begin{bmatrix} 1 & 0.5 \\ 0.1 & 1 \end{bmatrix}$

Table 10.4 Parameters of the scenario for Section 10.4.2.

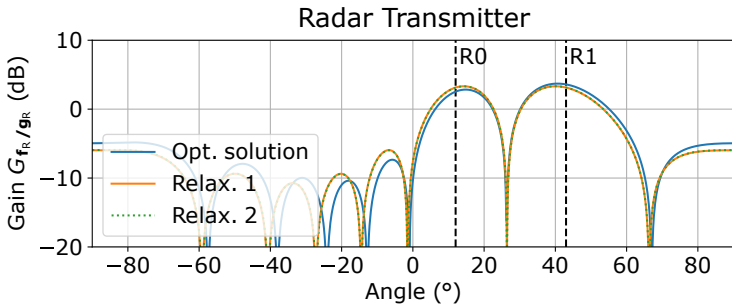


Fig. 10.4 Two-target no-user sensing scenario – Beamforming gains of the radar transmitter, obtained with 2D grid search, AM with full relaxation (Relax. 1), and AM with Hermitian relaxation (Relax. 2).

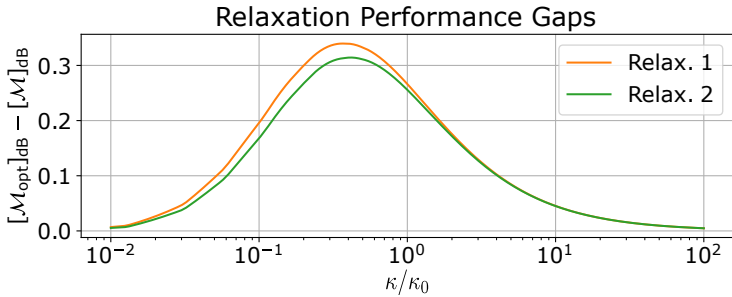


Fig. 10.5 Two-target no-user sensing scenario – Performance gap between the 2D grid search, AM with full relaxation (Relax. 1), and AM with Hermitian relaxation (Relax. 2). The parameter κ_0 corresponds to the initial values of κ_{RR} , defined in Table 10.4.

Parameters	Values
Targets angles	$\theta_R = \{12, 43\}^\circ$
UEs angles	$\theta_C = \{21, -67\}^\circ$
Radar coefficients κ	$\kappa_{RR} = \begin{bmatrix} 1 & 0.5 \\ 0.1 & 1 \end{bmatrix}, \kappa_{CR} = \begin{bmatrix} 1 & 0.1 \\ 0.2 & 1 \end{bmatrix}$
Comm. coefficients κ	$\kappa_{CC} = \begin{bmatrix} 1 & 0.1 \\ 0.2 & 1 \end{bmatrix}, \kappa_{RC} = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & 0.1 \end{bmatrix}$

Table 10.5 Parameters of the scenario for Section 10.4.3.

10.4.3 Beamformers in a Two-Target Two-User Scenario

Table 10.5 defines parameters for a scenario with two radar targets and UEs. Figure 10.6 illustrates the beamforming gains $G_{\mathbf{f}_R}(\theta) \triangleq |\mathbf{f}_R^H \mathbf{a}(\theta)|$, $G_{\mathbf{g}_R}(\theta) \triangleq |\mathbf{g}_R^H \mathbf{a}(\theta)|$, and $G_{\mathbf{g}_{Cq}}(\theta) \triangleq |\mathbf{g}_{Cq}^H \mathbf{a}(\theta)|$, of the radar transmitter, radar receiver, and communication receivers, obtained through AM with full relaxation, AM with Hermitian relaxation, and AM on $\tilde{\mathbf{f}}_R$, $\tilde{\mathbf{g}}_R$ and $\{\tilde{\mathbf{g}}_{Cq}\}$ in the steering space.

First, focusing on the radar transmitter, two mainlobes are directed in the direction of the radar targets, while the sidelobes are allowed to send power in the direction of the UEs. Indeed, the power transmitted towards the UEs is not penalised in the metric. This is not the case for the radar receiver, as the communication signals interfere with the radar function. Therefore, the power received from the UEs is minimised. In contrary, for the communication receivers, this is the power of radar echoes which is minimised, while the mainlobes are directed towards the UEs. These conclusions are valid for the three optimisation schemes. Whereas the two relaxations gives very similar results, the alternative optimisation on $\tilde{\mathbf{f}}_R$, $\tilde{\mathbf{g}}_R$ and $\{\tilde{\mathbf{g}}_{Cq}\}$ in the steering space shows some differences. Since the solutions obtained through the relaxations must still be projected into the space of the rank-one matrices, the alternative optimisation in the steering space achieves slightly better performance. Still, none of the methods ensures to find a global optimum of the SINR-product metric.

10.5 Conclusion

IC has been proposed in Chapter 9 to handle the interference arising when multiple ISAC systems are transmitting in full-duplex. When MIMO systems are considered, another promising solution is to perform beamforming. In this chapter, numerous works about the beamforming for JCAS scenarios have been summarised, highlighting the lack of a fair solution with an acceptable computational complexity.

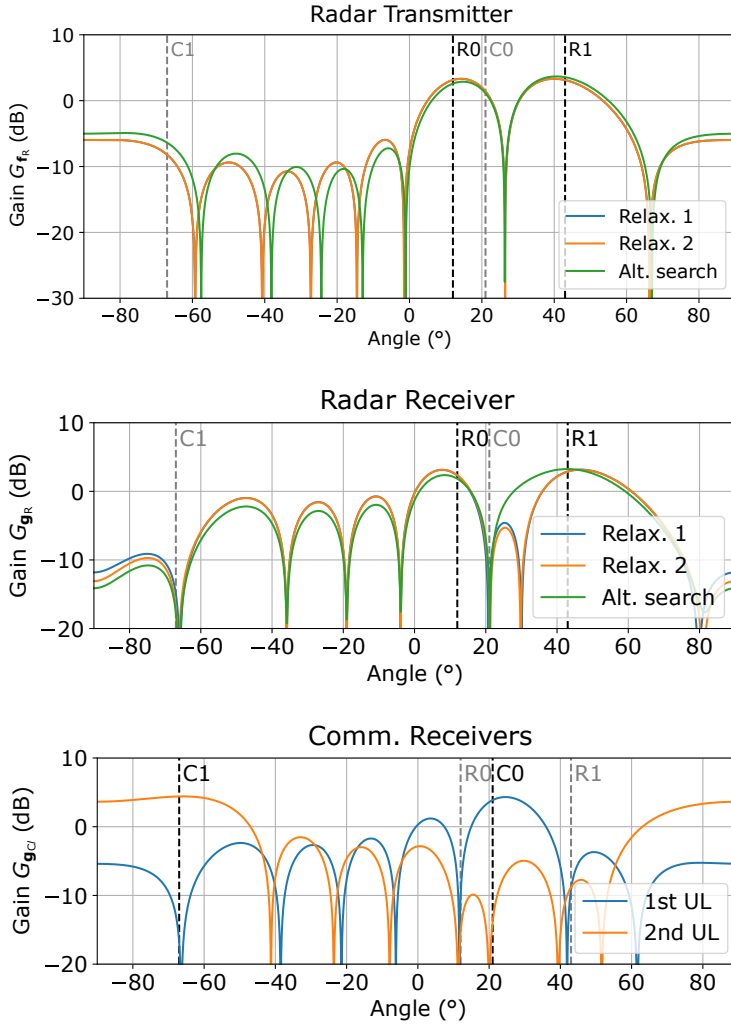


Fig. 10.6 Two-target two-user scenario – Beamforming gains of the radar transmitter (top), radar receiver (middle), and communication receivers (bottom). The beamformers of the radar transmitter and receiver are synthesised either through **AM** with full relaxation (Relax. 1), **AM** with Hermitian relaxation (Relax. 2), or **AM** on $\tilde{f}_R$, $\tilde{g}_R$ and $\{\tilde{g}_{Cq}\}$ in the steering space (Alt. search).

10.5.1 Summary of the Contributions

The contributions of this chapter are summarised as follows:

- Analytical expressions of the SINRs associated with each target and UEs have then been developed for MIMO PC-FMCW and OFDM ISAC systems. Leveraging on that, a fair and threshold-free objective function has been proposed for the optimisation, namely the product of the SINRs of the targets and UEs. Such metric has shown to ensure fairness, as the performance of every target and UE impacts similarly the metric, and any low factor heavily reduces the product. Moreover, the obtained solution is robust against channel changes, in contrary to SINR-sum metrics.
- Based on the SINR-product metric, numerous scenarios have been considered, in which the number of targets or UEs varies. The different solutions which have been obtained vary in optimality, complexity, and required assumptions. For instance, the higher is the number of targets and UEs, the higher is the complexity of the proposed scheme. Yet, when the number of targets or UEs are lower or equal to 2, the proposed schemes provide lower complexities compared to the AM method in the steering space, which is the only solution left for the other scenarios. Though, for two-target two-user scenarios, the original problem must be relaxed, leading to sub-optimal, but still satisfactory solutions.
- Numerous numerical analyses have been performed. Firstly, the impact of an angle mismatch has been evaluated, to investigate the impact of tracking errors and codebook discretisation. It has shown that, as expected, the higher the number of antennas, the higher the sensitivity w.r.t. the mismatch. Secondly, the performance of the relaxations have been analysed, showing small discrepancies with the optimal solution. Thirdly, the optimal beamformers obtained in a two-target two-user scenarios have been illustrated. As expected, it is observed that the radar (resp. communication) function tries to nullify the power of the uplink communication signals (resp. radar echoes).

10.5.2 Future Works

The need of a fair optimisation objective metric has been highlighted throughout this chapter. However, depending on the application, it may be interesting to be able to prioritise the sensing or communication function. Priority among the targets and UEs may even be considered. Thus, further research may first analyse how to extend the SINR-product metric to add such capability. This should be carried out, while keeping an acceptable complexity. Otherwise, the other metrics of the literature may be considered again.

Then, another weakness is the limitation on the number of targets, downlink and uplink UEs. Further investigations should be carried out to extend the proposed solu-

tions to more general scenarios. One way to handle this problem would be to consider different information streams for downlink **UEs**, sent with different beamformers, such that the downlink information streams would be decoupled from uplink echoes. Additionally, the radar receiver may process separately each radar target, instead of processing all the targets jointly.

10.5.3 What is Next?

This chapter concludes the third and final part of this PhD thesis. Two solutions, namely the **IC** and beamforming, have been proposed to mitigate the interference issue in full-duplex **ISAC** scenarios. For that purpose, **PC-FMCW** and **OFDM ISAC** systems have been considered. Both the **IC** and beamforming are promising to improve both the sensing and communication performance.

Appendix - Chapter 10

10.A Radar Processing

Both for the [PC-FMCW](#) and the [OFDM](#) waveform, following the processing detailed in Sections 8.1.3.1 and 8.2.2 of Chapter 8, neglecting the noise (and the [ICI](#) for the [OFDM](#) waveform), one obtains from (8.3) and (8.6) the following radar output:

$$\begin{aligned} \mathbf{z}(p, n) = & \sum_{k=0}^{K-1} \gamma_{RRk} \mathbf{a}_{Rrk} \mathbf{a}_{Rtk}^H \mathbf{f}_R e^{J \frac{2\pi}{P} f_{DRk} T_{CPI} p} e^{J \frac{2\pi}{N} B \tau_{Rk} n} \\ & + \sum_{q=0}^{Q-1} \gamma_{CRq} \mathbf{a}_{Crq} \mathbf{a}_{Ctq}^H \mathbf{f}_{Cq} \tilde{I}_{C,p,n} e^{J \frac{2\pi}{P} f_{DCq} T_{CPI} p} e^{J \frac{2\pi}{N} B \tau_{Cq} n}, \end{aligned}$$

where γ_v with $v \in \{RR, CR\}$ are complex coefficients, $\tilde{I}_{C,p,n}$ represent the impact of the communication symbols of the uplink communication signals, after they have not been well compensated, T_{CPI} is the coherent processing interval, and B is the waveform bandwidth. After applying a [2D-DFT](#), one obtains

$$\begin{aligned} \mathbf{Z}(f, \tau) = & \sum_{k=0}^{K-1} \gamma_{RRk} \mathbf{a}_{Rrk} \mathbf{a}_{Rtk}^H \mathbf{f}_R \chi_{RRk}((f - f_{DRk}) T_{CPI}, (\tau - \tau_{Rk}) B) \\ & + \sum_{q=0}^{Q-1} \gamma_{CRq} \mathbf{a}_{Crq} \mathbf{a}_{Ctq}^H \mathbf{f}_{Cq} \tilde{I}_{C,p,n} \chi_{CRq}((f - f_{DCq}) T_{CPI}, (\tau - \tau_{Cq}) B), \end{aligned}$$

where χ_{RR} and χ_{CR} are respectively the ambiguity function of the radar echo, and the cross-ambiguity between the communication signal and the radar waveform.

One may finally combine the signals of all antennas with the beamforming vector $\mathbf{g}_R$. This leads to the following [SINR](#) for the k^{th} target at the radar function:

$$\text{SINR}_{Rk} = \frac{\kappa_{RRk} |\mathbf{f}_R^H \mathbf{a}_{Rtk}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rrk}|^2}{I_{RRk} + I_{CRk} + 1},$$

where I_{RRk} is the interference from the ambiguity function of the other targets on the

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cell of the target of interest:

$$I_{RRk} = \sum_{\substack{k'=0 \\ k' \neq k}}^{K-1} \kappa_{RRk'k} \left| \mathbf{f}_R^H \mathbf{a}_{Rtk'} \right|^2 \left| \mathbf{g}_R^H \mathbf{a}_{Rtk'} \right|^2,$$

and I_{CRk} is the interference from the communication signals on the cell of the target of interest:

$$I_{CRk} = \sum_{q=0}^{Q-1} \kappa_{CRqk} \left| \mathbf{f}_{Cq}^H \mathbf{a}_{Ctq} \right|^2 \left| \mathbf{g}_R^H \mathbf{a}_{Ctq} \right|^2.$$

The coefficients κ contains the impact of the complex coefficients, ambiguity functions and noise on the SINR:

$$\kappa_{RRk'k} \triangleq \mathbb{E} \left[|\gamma_{Rk}|^2 \right] \frac{\left| \chi_{RRk} \left([f_{DRk'} - f_{DRk}] T_{CPI}, (\tau_{Rk'} - \tau_{Rk}) B \right) \right|^2}{\sigma_R^2},$$

$$\kappa_{CRqk} \triangleq \mathbb{E} \left[|\gamma_{Cq}|^2 \right] \mathbb{E} \left[|\tilde{I}_{C,p,n}|^2 \right] \frac{\left| \chi_{CRk} \left([f_{DCq} - f_{DRk}] T_{CPI}, (\tau_{Cq} - \tau_{Rk}) B \right) \right|^2}{\sigma_R^2},$$

where σ_R^2 is the noise power at the output of the radar function. If the condition of (9.2) is also fulfilled, then $\mathbb{E} \left[|\tilde{I}_{C,p,n}|^2 \right] = 1$.

10.B Communication Processing

The communication processing is quite different for PC-FMCW and OFDM systems. Moreover, contrariwise to the radar processing, each communication signal is handled separately, the other signals being considered as interference, similarly to the radar echoes. Still, similar expressions of the communication SINRs can be obtained.

PC-FMCW ISAC: Following the processing detailed in Section 8.1.3.2 of Chapter 8, based on (8.4), the signal obtained after timing synchronisation and dechirping channel is computed as

$$\begin{aligned} \mathbf{y}_q(p, t) = & \beta_{CCpq} \mathbf{a}_{CRq} \mathbf{a}_{Ctq}^H \mathbf{f}_{Cq} e^{j2\pi \tilde{f}_{DCq} t} \sum_{n=0}^{N-1} I_{Cq,p,n} \Pi \left(\frac{t - nT_C}{T_C} \right) \\ & + \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} \beta_{CCpq'} \mathbf{a}_{CRq'} \mathbf{a}_{Ctq'}^H \mathbf{f}_{Cq'} e^{j2\pi \tilde{f}_{DCq'} t} \sum_{n=0}^{N-1} I_{Cq',p,n} \Pi \left(\frac{t - \tilde{\tau}_{Cq'q} - nT_C}{T_C} \right) \\ & + \sum_{k=0}^{K-1} \beta_{RCpk} \mathbf{a}_{RRk} \mathbf{a}_{Rtk}^H \mathbf{f}_R e^{j2\pi \tilde{f}_{DRk} t} \sum_{n=0}^{N-1} I_{R,p,n} \Pi \left(\frac{t - \tilde{\tau}_{Rkq} - nT_C}{T_C} \right), \end{aligned}$$

where β_v with $v \in \{\text{CC}, \text{RC}\}$ are complex coefficients, $\tilde{f}_{Dw}$ with $w \in \{\text{R}, \text{C}\}$ are frequencies containing the Doppler frequencies of the links, the carrier frequency offsets between the different **ISAC** systems, and the beat frequencies generated by the dechirping², I_w are the communication symbols from all the **ISAC** systems, and $\tilde{\tau}_w = \tau_w - \tau_{Cq}$ are the delays of the links, w.r.t. the time reference obtained from the timing synchronisation algorithm. Focusing on the q^{th} signal, after sampling, carrier frequency estimation and correction, and matched filtering, one obtains

$$\begin{aligned} \mathbf{z}_q(p, m) = & \beta_{\text{CC}pq} \mathbf{a}_{\text{Cr}q} \mathbf{a}_{\text{Ct}q}^H \mathbf{f}_{\text{C}q} \chi_{\text{CC}qq}(p, m) \\ & + \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} \beta_{\text{CC}pq'} \mathbf{a}_{\text{Cr}q'} \mathbf{a}_{\text{Ct}q'}^H \mathbf{f}_{\text{C}q'} e^{j2\pi(\tilde{f}_{\text{DC}q'} - \tilde{f}_{\text{DC}q})t} \chi_{\text{CC}q'q}(p, m) \\ & + \sum_{k=0}^{K-1} \beta_{\text{RC}pk} \mathbf{a}_{\text{Rr}k} \mathbf{a}_{\text{Rt}k}^H \mathbf{f}_{\text{R}} e^{j2\pi(\tilde{f}_{\text{DR}k} - \tilde{f}_{\text{DC}q})t} \chi_{\text{RC}kq}(p, m), \end{aligned}$$

where the functions χ_v with $v \in \{\text{CC}, \text{RC}\}$ are defined as

$$\begin{aligned} \chi_{\text{CC}q'q}(p, m) & \triangleq \sum_{n=0}^{N-1} I_{\text{C}q', p, n} E_{\Pi} \Lambda \left(\frac{(m-n)T_C - \tilde{\tau}_{\text{C}q'}}{T_C} \right), \\ \chi_{\text{RC}kq}(p, m) & \triangleq \sum_{n=0}^{N-1} I_{\text{R}, p, n} E_{\Pi} \Lambda \left(\frac{(m-n)T_C - \tilde{\tau}_{\text{R}k}}{T_C} \right), \end{aligned}$$

with E_{Π} the energy of the pulse shaping filter. Note that, for the q^{th} signal, one obtains

$$\chi_{\text{CC}qq}(p, m) = \sum_{n=0}^{N-1} I_{\text{C}q', p, n} \Lambda(m-n) = I_{\text{C}q', p, m} E_{\Pi}.$$

The signals of all antennas may finally be combined with the beamforming vector $\mathbf{g}_{\text{C}q}$. This leads to the following **SINR** for the q^{th} uplink signal at the communication function:

$$\text{SINR}_{\text{C}q} = \frac{\kappa_{\text{CC}qq} \left| \mathbf{f}_{\text{C}q}^H \mathbf{a}_{\text{Ct}q} \right|^2 \left| \mathbf{g}_{\text{C}q}^H \mathbf{a}_{\text{Cr}q} \right|^2}{I_{\text{CC}q} + I_{\text{RC}q} + 1}, \quad (10.5)$$

where $I_{\text{CC}q}$ is the interference from the matched filtering of the other communication signals:

$$I_{\text{CC}q} = \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} \kappa_{\text{CC}q'q} \left| \mathbf{f}_{\text{C}q'}^H \mathbf{a}_{\text{Ct}q'} \right|^2 \left| \mathbf{g}_{\text{C}q}^H \mathbf{a}_{\text{Cr}q'} \right|^2,$$

²For the communication signal of interest, the beat frequency generated by the dechirping is equal to zero if the timing synchronisation is perfect.

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and I_{RCq} is the interference from the matched filtering of the radar echoes:

$$I_{RCk} = \sum_{k=0}^{K-1} \kappa_{RCkq} \left| \mathbf{f}_R^H \mathbf{a}_{Rtk} \right|^2 \left| \mathbf{g}_{Cq}^H \mathbf{a}_{Rrk} \right|^2.$$

The coefficients κ contains the impact of the complex coefficients, matched filters, and noise on the SINR:

$$\begin{aligned} \kappa_{CCq'q} &\triangleq \mathbb{E} \left[|\beta_{CCpq'}|^2 \right] \frac{\mathbb{E} \left[|\chi_{CCq'q}(p, m)|^2 \right]}{\sigma_{Cq}^2}, \\ \kappa_{RCkq} &\triangleq \mathbb{E} \left[|\beta_{RCpk}|^2 \right] \frac{\mathbb{E} \left[|\chi_{RCkq}(p, m)|^2 \right]}{\sigma_{Cq}^2}. \end{aligned}$$

OFDM ISAC: Following the processing detailed in Section 8.2.1 of Chapter 8, based on (8.5) with negligible ICI, the signal obtained after the DFT operation is computed as

$$\begin{aligned} \mathbf{y}_q(p, m) &= \beta_{CCq} \zeta(f_{Dq}T) \mathbf{a}_{Cq} \mathbf{a}_{Ctq}^H \mathbf{f}_{Cq} e^{j2\pi f_{Dq} T_{PRI} p} e^{-j \frac{2\pi}{T} \tilde{\tau}_q m} I_{Cq,p,m} \\ &+ \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} \beta_{CCq'} \mathbf{a}_{Cq'} \mathbf{a}_{Ctq'}^H \mathbf{f}_{Cq'} e^{j2\pi f_{Dq'} T_{PRI} p} \chi_{CCq'q}(p, m) \\ &+ \sum_{k=0}^{K-1} \beta_{RCk} \mathbf{a}_{Rtk} \mathbf{a}_{Rtk}^H \mathbf{f}_R e^{j2\pi f_{DRk} T_{PRI} p} \chi_{RCkq}(p, m), \end{aligned}$$

where the functions χ_v with $v \in \{CC, RC\}$ are defined as

$$\begin{aligned} \chi_{CCq'q}(p, m) &\triangleq \frac{1}{N} \sum_{n=0}^{N-1} I_{C,q',p,n} e^{j \frac{2\pi}{T} \tilde{\tau}_{Cq'} n} \sum_{l=0}^{NM-1} e^{j \frac{2\pi}{NM} (n + f_{DCq'} T - m) l} \Pi \left(\frac{l \frac{T}{MN} + T_{CP} - \tilde{\tau}_{Cq'}}{T_{PRI}} \right), \\ \chi_{RCkq}(p, m) &\triangleq \frac{1}{N} \sum_{n=0}^{N-1} I_{R,k,p,n} e^{j \frac{2\pi}{T} \tilde{\tau}_{Rk} n} \sum_{l=0}^{NM-1} e^{j \frac{2\pi}{NM} (n + f_{DRk} T - m) l} \Pi \left(\frac{l \frac{T}{MN} + T_{CP} - \tilde{\tau}_{Rk}}{T_{PRI}} \right). \end{aligned}$$

Note that the rectangular functions have been kept in the definitions of the functions χ_v , as there is no guarantee that the delays of the interfering signals are smaller than the cyclic prefix duration. In these equations, the quantities β_v are complex coefficients, f_{Dw} with $w \in \{R, C\}$ are frequencies containing the Doppler frequencies of the links, and the carrier frequency offsets between the different ISAC systems, I_w are the communication symbols from all the ISAC systems, and $\tilde{\tau}_w$ are the delays of the links, w.r.t. the time reference obtained from the timing synchronisation algorithm. If perfect synchronisation is achieved, $\tilde{\tau}_w = \tau_w - \tau_{Cq}$ and $\tilde{\tau}_{Cq} = 0$. Nevertheless, with the Schmidl & Cox algorithm, one may start the processing in a

window of duration T_{CP} before the start of communication signal of interest.

The signals of all antennas may finally be combined with the beamforming vector $\mathbf{g}_{Cq}$. This leads to the following **SINR** for the q^{th} uplink signal at the communication function:

$$\text{SINR}_{Cq} = \frac{\kappa_{CCq} \left| \mathbf{f}_{Cq}^H \mathbf{a}_{Ctq} \right|^2 \left| \mathbf{g}_{Cq}^H \mathbf{a}_{Crq} \right|^2}{I_{CCq} + I_{RCq} + 1}, \quad (10.6)$$

where I_{CCq} is the interference from the matched filtering of the other communication signals:

$$I_{CCq} = \sum_{\substack{q'=0 \\ q' \neq q}}^{Q-1} \kappa_{CCq'q} \left| \mathbf{f}_{Cq'}^H \mathbf{a}_{Ctq'} \right|^2 \left| \mathbf{g}_{Cq}^H \mathbf{a}_{Crq'} \right|^2,$$

and I_{RCq} is the interference from the matched filtering of the radar echoes:

$$I_{RCq} = \sum_{k=0}^{K-1} \kappa_{RCkq} \left| \mathbf{f}_R^H \mathbf{a}_{Rtk} \right|^2 \left| \mathbf{g}_{Cq}^H \mathbf{a}_{Rrk} \right|^2.$$

The coefficients κ contains the impact of the complex coefficients, **DFTs** of other signals, and noise on the **SINR**:

$$\begin{aligned} \kappa_{CCq'q} &\triangleq \mathbb{E} \left[|\beta_{CCq'}|^2 \right] \frac{\mathbb{E} \left[\left| \chi_{CCq'q}(p, m) \right|^2 \right]}{\sigma_{Cq}^2}, \\ \kappa_{RCkq} &\triangleq \mathbb{E} \left[|\beta_{RCk}|^2 \right] \frac{\mathbb{E} \left[\left| \chi_{RCkq}(p, m) \right|^2 \right]}{\sigma_{Cq}^2}, \end{aligned}$$

Note that, for the communication signal of interest,

$$\chi_{CCq}(p, m) = \frac{1}{N} \sum_{n=0}^{N-1} I_{C,q,p,n} e^{j \frac{2\pi}{NM} (n + f_{DCq} T - m)l} = \zeta(f_{DCq} T) I_{C,q,p,m}.$$

The **SINR** of (10.6) obtained for the **OFDM ISAC** system is identical to the **SINR** of (10.5) obtained for the **PC-FMCW ISAC** system. The only difference is the definition of the weights κ_v with $v \in \{CC, RC\}$, which should be adapted depending on the considered waveform.

10.C One-Target One-User Scenario

With one target and one UE, discarding the indices $k = 0$ and $q = 0$ for readability, the objective metric of (10.2) can be rewritten as

$$\begin{aligned} \mathcal{M} &= \frac{\kappa_{RR} |\mathbf{f}_R^H \mathbf{a}_{Rt}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rr}|^2}{\kappa_{CR} |\mathbf{f}_C^H \mathbf{a}_{Ct}|^2 |\mathbf{g}_R^H \mathbf{a}_{Cr}|^2 + 1} \frac{\kappa_{CC} |\mathbf{f}_C^H \mathbf{a}_{Ct}|^2 |\mathbf{g}_C^H \mathbf{a}_{Cr}|^2}{\kappa_{RC} |\mathbf{f}_R^H \mathbf{a}_{Rt}|^2 |\mathbf{g}_C^H \mathbf{a}_{Cr}|^2 + 1} \\ &\propto \frac{|\mathbf{g}_R^H \mathbf{a}_{Rr}|^2}{\kappa_{CR} |\mathbf{g}_R^H \mathbf{a}_{Cr}|^2 + \frac{1}{|\mathbf{f}_C^H \mathbf{a}_{Ct}|^2}} \frac{|\mathbf{g}_C^H \mathbf{a}_{Cr}|^2}{\kappa_{RC} |\mathbf{g}_C^H \mathbf{a}_{Rr}|^2 + \frac{1}{|\mathbf{f}_R^H \mathbf{a}_{Rt}|^2}}. \end{aligned} \quad (10.7)$$

Focusing first on the beamforming vectors of the transmitters $\mathbf{f}_R$ and $\mathbf{f}_C$, the metric is only increasing with $|\mathbf{f}_C^H \mathbf{a}_{Ct}|$ and $|\mathbf{f}_R^H \mathbf{a}_{Rt}|$. Therefore, the optimal beamformers for the transmitters are computed as

$$\mathbf{f}_R^* = \frac{\mathbf{a}_{Rt}}{\sqrt{N_t}}, \quad \mathbf{f}_C^* = \frac{\mathbf{a}_{Ct}}{\sqrt{N_t}}.$$

Interestingly, this shows that, even when the interference of the other function is taken into account, the steering vectors at the transmitters, the optimal beamforming vectors are aligned with the transmitters, as in the interference-free case.

Plugging the optimal beamforming vectors into (10.7), one obtains

$$\begin{aligned} \mathcal{M} &\propto \frac{|\mathbf{g}_R^H \mathbf{a}_{Rr}|^2}{\kappa_{CR} N_t |\mathbf{g}_R^H \mathbf{a}_{Cr}|^2 + 1} \frac{|\mathbf{g}_C^H \mathbf{a}_{Cr}|^2}{\kappa_{RC} N_t |\mathbf{g}_C^H \mathbf{a}_{Rr}|^2 + 1} \\ &= \frac{\mathbf{g}_R^H \mathbf{a}_{Rr} \mathbf{a}_{Rr}^H \mathbf{g}_R}{\mathbf{g}_R^H (\kappa_{CR} N_t \mathbf{a}_{Cr} \mathbf{a}_{Cr}^H + \mathbf{I}_{N_r}) \mathbf{g}_R} \frac{\mathbf{g}_C^H \mathbf{a}_{Cr} \mathbf{a}_{Cr}^H \mathbf{g}_C}{\mathbf{g}_C^H (\kappa_{RC} N_t \mathbf{a}_{Rr} \mathbf{a}_{Rr}^H + \mathbf{I}_{N_r}) \mathbf{g}_C}, \end{aligned} \quad (10.8)$$

where the matrix $\mathbf{I}_N$ is the identity matrix of size $N \times N$. The above shows that the radar and communication optimisation problems are decoupled, as two independent *Rayleigh quotients*. The optimisation of Rayleigh quotient is a well-known problem, which is written as

$$\mathbf{x}^* = \arg \max_{\mathbf{x} \in \mathbb{C}^N} \frac{\mathbf{x}^H \mathbf{A} \mathbf{x}}{\mathbf{x}^H \mathbf{B} \mathbf{x}},$$

with $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{N \times N}$, and $\mathbf{A} \succ 0, \mathbf{B} \succeq 0$. As the matrix $\mathbf{B}$ is positive semi-definite, it may be decomposed following the *Cholesky decomposition* as $\mathbf{B} = \mathbf{L}^H \mathbf{L}$. Thus, the problem can be rewritten as

$$\tilde{\mathbf{x}}^* = \arg \max_{\tilde{\mathbf{x}} \in \mathbb{C}^N} \frac{\tilde{\mathbf{x}}^H \mathbf{L}^{-H} \mathbf{A} \mathbf{L}^{-1} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^H \tilde{\mathbf{x}}},$$

and the solution $\mathbf{x}^*$ is then computed as $\mathbf{x}^* = \mathbf{L}^{-1} \tilde{\mathbf{x}}^*$. For such problem, the optimal vector is the principal eigenvector of the matrix $\mathbf{L}^{-H} \mathbf{A} \mathbf{L}^{-1}$ [Den09, Lemma 8.4.3],

denoted here by the operator eigvec:

$$\mathbf{x}^\star = \mathbf{L}^{-1} \text{eigvec} \left\{ \mathbf{L}^{-H} \mathbf{A} \mathbf{L}^{-1} \right\}.$$

This can be further developed if the matrix $\mathbf{A}$ is a rank-one matrix, i.e. $\mathbf{A} = \mathbf{a} \mathbf{a}^H$, with $\mathbf{a} \in \mathbb{C}^N$. In this case,

$$\mathbf{x}^\star = \mathbf{L}^{-1} \text{eigvec} \left\{ \mathbf{L}^{-H} \mathbf{a} \left(\mathbf{L}^{-H} \mathbf{a} \right)^H \right\} \propto \mathbf{L}^{-1} \mathbf{L}^{-H} \mathbf{a} = \mathbf{B}^{-1} \mathbf{a}. \quad (10.9)$$

This solution is also named the **MVDR** beamforming vectors [NWH22]. Applied to (10.8), it gives

$$\begin{aligned} \mathbf{g}_R^\star &= \mu_R \left(\kappa_{CR} N_t \mathbf{a}_{Cr} \mathbf{a}_{Cr}^H + \mathbf{I}_{N_r} \right)^{-1} \mathbf{a}_{Rr} = \mu_R \left(\mathbf{a}_{Rr} - \eta \frac{N_t N_r}{\kappa_{CR}^{-1} + N_t N_r} \mathbf{a}_{Cr} \right), \\ \mathbf{g}_C^\star &= \mu_C \left(\kappa_{RC} N_t \mathbf{a}_{Rr} \mathbf{a}_{Rr}^H + \mathbf{I}_{N_r} \right)^{-1} \mathbf{a}_{Cr} = \mu_C \left(\mathbf{a}_{Cr} - \eta^* \frac{N_t N_r}{\kappa_{RC}^{-1} + N_t N_r} \mathbf{a}_{Rr} \right), \end{aligned}$$

where the coefficients μ_v with $v \in \{R, C\}$ are normalisation factors, and $\gamma = \mathbf{a}_{Cr}^H \mathbf{a}_{Rr} / N_r$ is the correlation between the radar and communication steering vectors. This shows that the optimal receivers beamforming vectors are aligned with their receivers steering vectors, plus a skewing depending on the level of interference, noise, and correlation between the steering vectors of both functions.

10.D Two-Target No-User (Sensing) Scenario

In the sensing scenario with two targets and no UE, the objective metric of (10.2) can be rewritten similarly to the one-target one-user scenario as

$$\begin{aligned} \mathcal{M} &= \frac{\kappa_{RR00} |\mathbf{f}_R^H \mathbf{a}_{Rt0}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rr0}|^2}{\kappa_{RR10} |\mathbf{f}_R^H \mathbf{a}_{Rt1}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rr1}|^2 + 1} \frac{\kappa_{RR11} |\mathbf{f}_R^H \mathbf{a}_{Rt1}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rr1}|^2}{\kappa_{RR01} |\mathbf{f}_R^H \mathbf{a}_{Rt0}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rr0}|^2 + 1} \\ &\propto \left(\kappa_{RR10} + \frac{1}{|\mathbf{f}_R^H \mathbf{a}_{Rt1}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rr1}|^2} \right)^{-1} \left(\kappa_{RR01} + \frac{1}{|\mathbf{f}_R^H \mathbf{a}_{Rt0}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rr0}|^2} \right)^{-1}. \quad (10.10) \end{aligned}$$

Owing to the symmetry of the metric in this scenario, the solution is found the same way for the beamforming vector of the transmitter and receiver. Following Section 10.3.1, the optimal beamforming vectors can be written in the steering vectors space as

$$\mathbf{f}_R^\star = \frac{1}{\sqrt{N_t}} \frac{\lambda_t \mathbf{a}_{Rt0} + (1 - \lambda_t) e^{j\phi_t} \mathbf{a}_{Rt1}}{\sqrt{\lambda_t^2 + (1 - \lambda_t)^2 + 2\lambda_t(1 - \lambda_t) \text{Re} \{ e^{-j\phi_t} \gamma_t \}}},$$

$$\mathbf{g}_R^* = \frac{1}{\sqrt{N_r}} \frac{\lambda_r \mathbf{a}_{Rr0} + (1 - \lambda_r) e^{j\phi_r} \mathbf{a}_{Rr1}}{\sqrt{\lambda_r^2 + (1 - \lambda_r)^2 + 2\lambda_r(1 - \lambda_r) \operatorname{Re} \{e^{-j\phi_r} \gamma_r\}}},$$

where $\lambda_v \in [0, 1]$ and ϕ_v with $v \in \{t, r\}$ are two weights and phases to compute, and $\gamma_v = \mathbf{a}_{Rv1}^H \mathbf{a}_{Rv0} / N_v$ are the correlation between the steering vectors of the two targets.

Let us focus the developments on the beamforming vector of the transmitter, as it is identical for the beamforming vector of the receiver. First, developing the beamforming gains, one obtains

$$\begin{aligned} |\mathbf{f}_R^{*H} \mathbf{a}_{Rr0}|^2 &= N_t \frac{\lambda_t^2 + (1 - \lambda_t)^2 |\gamma_t|^2 + 2\lambda_t(1 - \lambda_t) \operatorname{Re} \{e^{-j\phi_t} \gamma_t\}}{\lambda_t^2 + (1 - \lambda_t)^2 + 2\lambda_t(1 - \lambda_t) \operatorname{Re} \{e^{-j\phi_t} \gamma_t\}}, \\ |\mathbf{f}_R^{*H} \mathbf{a}_{Rt1}|^2 &= N_t \frac{\lambda_t^2 |\gamma_t|^2 + (1 - \lambda_t)^2 + 2\lambda_t(1 - \lambda_t) \operatorname{Re} \{e^{-j\phi_t} \gamma_t\}}{\lambda_t^2 + (1 - \lambda_t)^2 + 2\lambda_t(1 - \lambda_t) \operatorname{Re} \{e^{-j\phi_t} \gamma_t\}}. \end{aligned}$$

In both cases, to find the optimal phase ϕ_t , it may be noticed that these quantities can be rewritten as the following function:

$$f(x) = \frac{c_0 + x}{c_1 + x}, \quad \text{with } c_0 \leq c_1.$$

As this function is an increasing function of x , maximising (10.10) is equivalent to maximise x w.r.t. ϕ_t , which is achieved when $\phi_t = \arg\{\gamma_t\}$. This leads to

$$\begin{aligned} \mathbf{f}_R^* &= \frac{1}{\sqrt{N_t}} \frac{\lambda_t \mathbf{a}_{Rt0} + (1 - \lambda_t) e^{j\arg\{\gamma_t\}} \mathbf{a}_{Rt1}}{\sqrt{\lambda_t^2 + (1 - \lambda_t)^2 + 2\lambda_t(1 - \lambda_t) |\gamma_t|}}, \\ \mathbf{g}_R^* &= \frac{1}{\sqrt{N_r}} \frac{\lambda_r \mathbf{a}_{Rr0} + (1 - \lambda_r) e^{j\arg\{\gamma_r\}} \mathbf{a}_{Rr1}}{\sqrt{\lambda_r^2 + (1 - \lambda_r)^2 + 2\lambda_r(1 - \lambda_r) |\gamma_r|}}. \end{aligned}$$

There is no closed form solution for λ_v with $v \in \{t, r\}$. Nevertheless, λ_v is comprised in $[0, 1]$, and it can be shown that the optimisation of $-\log \mathcal{M}$ w.r.t. λ_r or λ_t is a convex problem. Still focusing on the beamforming vector of the transmitter, the logarithm of (10.10) is developed as

$$\begin{aligned} -\log \mathcal{M} &= C(\mathbf{g}_R^*) + \log \left(\kappa_{RR10} |\mathbf{g}_R^{*H} \mathbf{a}_{Rr1}|^2 + \frac{1}{|\mathbf{f}_R^{*H} \mathbf{a}_{Rt1}|^2} \right) \\ &\quad + \log \left(\kappa_{RR01} |\mathbf{g}_R^{*H} \mathbf{a}_{Rr0}|^2 + \frac{1}{|\mathbf{f}_R^{*H} \mathbf{a}_{Rt0}|^2} \right) \\ &= C(\mathbf{g}_R^*) + \log \left(\kappa_{RR10} |\mathbf{g}_R^{*H} \mathbf{a}_{Rr1}|^2 + \frac{1}{N_t} \frac{\lambda_t^2 + (1 - \lambda_t)^2 + 2\lambda_t(1 - \lambda_t) |\gamma_t|}{\lambda_t^2 |\gamma_t|^2 + (1 - \lambda_t)^2 + 2\lambda_t(1 - \lambda_t) |\gamma_t|} \right) \end{aligned}$$

$$\begin{aligned}
& + \log \left(\kappa_{RR01} |\mathbf{g}_R^{\star H} \mathbf{a}_{Rr0}|^2 + \frac{1}{N_t} \frac{\lambda_t^2 + (1 - \lambda_t)^2 + 2\lambda_t(1 - \lambda_t)|\gamma_t|}{\lambda_t^2 + (1 - \lambda_t)^2|\gamma_t|^2 + 2\lambda_t(1 - \lambda_t)|\gamma_t|} \right) \\
& = C(\mathbf{g}_R^{\star}) + \log \left(\kappa_{RR10} |\mathbf{g}_R^{\star H} \mathbf{a}_{Rr1}|^2 + \frac{1}{N_t} + \frac{1 - |\gamma_t|^2}{N_t} \frac{\lambda_t^2}{[1 - (1 - |\gamma_t|)\lambda]^2} \right) \\
& + \log \left(\kappa_{RR01} |\mathbf{g}_R^{\star H} \mathbf{a}_{Rr0}|^2 + \frac{1}{N_t} + \frac{1 - |\gamma_t|^2}{N_t} \frac{(1 - \lambda_t)^2}{[1 - (1 - |\gamma_t|)(1 - \lambda_t)]^2} \right) \\
& = \tilde{C}(\mathbf{g}_R^{\star}) + \log \left(\frac{\kappa_{RR10} N_t |\mathbf{g}_R^{\star H} \mathbf{a}_{Rr1}|^2 + 1}{1 - |\gamma_t|^2} + \frac{\lambda_t^2}{[1 - (1 - |\gamma_t|)\lambda_t]^2} \right) \\
& + \log \left(\frac{\kappa_{RR01} N_t |\mathbf{g}_R^{\star H} \mathbf{a}_{Rr0}|^2 + 1}{1 - |\gamma_t|^2} + \frac{(1 - \lambda_t)^2}{[1 - (1 - |\gamma_t|)(1 - \lambda_t)]^2} \right).
\end{aligned}$$

Let us define the function h as

$$h(x) \triangleq \log \left(c_0 + \frac{x^2}{(1 - c_1 x)^2} \right), \quad c_0 \geq 1, \quad c_1 \in [0, 1].$$

The function h is convex if the sign of its second derivative is always positive. Additionally, if $h(x)$ is convex, then $h(1 - x)$ is also convex. As both terms depending on λ_t can be rewritten in the form of $h(x)$ or $h(1 - x)$, analysing the convexity of h is sufficient to ensure that the optimisation of λ_t is a convex problem. The same reasoning can be applied to λ_r .

The second derivative of h is computed as

$$\frac{d^2}{dx^2} h(x) = 2 \frac{x^2(2c_1 x - 1) + c_0(c_1 x - 1)^2(2c_1 x + 1)}{(1 - c_1 x)^2 [x^2 + c_0(c_1 x - 1)^2]^2}.$$

The denominator of this function is always positive. The numerator can be developed as

$$\begin{aligned}
x^2(2c_1 x - 1) + c_0(c_1 x - 1)^2(2c_1 x + 1) &= 2c_1(1 + c_0 c_1^2)x^3 - (1 + 3c_0 c_1^2)x^2 + c_0 \\
&= c_0(2c_1^3 x^3 - 3c_1^2 x^2 + 1) + 2c_1 x^3 - x^2 \\
&= c_0(c_1 x - 1)^2(2c_1 x + 1) + x^2(2c_1 x - 1) \\
&\geq (c_1 x - 1)^2(2c_1 x + 1) + x^2(2c_1 x - 1),
\end{aligned}$$

where the equality is achieved when $c_0 = 1$. Thus, one may study the sign of the function $\tilde{h}$ defined as

$$\tilde{h}(x) \triangleq 2c_1(1 + c_1^2)x^3 - (1 + 3c_1^2)x^2 + c_0.$$

Using the Descartes' rule of signs [Cur18], there is 0 or 2 roots such that $x \geq 0$, and one 1 root such that $x \leq 0$. Additionally, $\tilde{h}(0) = 1$ and $\tilde{h}'(0) = 0$. This implies that the

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next minimum $x_{\min}$ of $\tilde{h}$ is larger than 0. Looking at the position of this minimum, the first derivative of $\tilde{h}$ is computed as

$$\frac{d}{dx} \tilde{h}(x) = 6c_1(1 + c_1^2)x^2 - 2(1 + 3c_1^2)x = 6c_1x(1 + c_1^2) \left(x - \frac{1 + 3c_1^2}{3c_1(1 + c_1^2)} \right),$$

giving the following root:

$$x_{\min} = \frac{1 + 3c_1^2}{3c_1(1 + c_1^2)}.$$

The function $\tilde{h}$ is always positive either if $x_{\min} \geq 1$ and $\tilde{h}(1) \geq 0$, or if $x_{\min} < 1$ and $\tilde{h}(x_{\min}) \geq 0$. When $x_{\min} \geq 1$,

$$c_1 \leq 0.4425 \Rightarrow \tilde{h}(1) = c_1(2c_1^2 - 3c_1 + 2) \geq 0.$$

When $x_{\min} < 1$,

$$c_1 > 0.4425 \Rightarrow \tilde{h}(x_{\min}) = 1 - \frac{(1 + 3c_1^2)^3}{27c_1^2(1 + c_1^2)^2} > 0.$$

This implies that the function $\tilde{h}$ is always positive, and therefore that the optimisation of $-\log \mathcal{M}$ w.r.t. λ_t or λ_r is a convex problem.

In summary, since the optimisation of $-\log \mathcal{M}$ w.r.t. λ_t or λ_r is a convex problem, one may perform an alternating maximisation on λ_t and λ_r to find a local extremum. To find the unique optimum at each step, efficient methods such as golden-section search [Kie53] can be employed, with a complexity in $\mathcal{O}(\log M)$ where M is the number of points in the interval $[0, 1]$.

10.E Two-Target Two-User Scenario

When there is two targets and two UEs in the considered scenario, for the sake of tractability, two assumptions are considered:

1. the radar waveform is designed such that the targets do not interfere with each other in the delay-Doppler map, i.e. $\kappa_{RRk'k} = 0$ for $k \neq k'$. This is a reasonable assumption for well-designed radar waveforms, and targets far away in the delay-Doppler map.
2. For the beamforming vectors of the UEs transmitters, an uncoordinated design has to be carried out, as they are not collocated with the ISAC system under study. Therefore, each UE optimises the transmitted power in the direction of

the ISAC system by setting

$$\mathbf{f}_{Cq} = \frac{1}{\sqrt{N_t}} \mathbf{a}_{Ctq}, \quad \forall q = 0, \dots, Q-1.$$

Thus, it remains to design the beamforming vectors $\mathbf{f}_R$, $\mathbf{g}_R$, and $\{\mathbf{g}_{Cq}\}$.

With the two assumptions above, the metric of (10.2) can be rewritten in the form of

$$\begin{aligned} \mathcal{M} = & \frac{\kappa_{RR00} |\mathbf{f}_R^H \mathbf{a}_{Rt0}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rr0}|^2}{\sum_{q=0}^1 \kappa_{CRq0} N_t |\mathbf{g}_R^H \mathbf{a}_{Crq}|^2 + 1} \frac{\kappa_{RR11} |\mathbf{f}_R^H \mathbf{a}_{Rt1}|^2 |\mathbf{g}_R^H \mathbf{a}_{Rr1}|^2}{\sum_{q=0}^{Q-1} \kappa_{CRq1} N_t |\mathbf{g}_R^H \mathbf{a}_{Crq}|^2 + 1} \\ & \cdot \frac{\kappa_{CC00} N_t |\mathbf{g}_{C0}^H \mathbf{a}_{Cr0}|^2}{\kappa_{CC10} N_t |\mathbf{g}_{C0}^H \mathbf{a}_{Cr1}|^2 + \sum_{k=0}^{K-1} \kappa_{RCk0} |\mathbf{f}_R^H \mathbf{a}_{Rtk}|^2 |\mathbf{g}_{C0}^H \mathbf{a}_{Rrk}|^2 + 1} \\ & \cdot \frac{\kappa_{CC11} N_t |\mathbf{g}_{C1}^H \mathbf{a}_{Cr1}|^2}{\kappa_{CC01} N_t |\mathbf{g}_{C1}^H \mathbf{a}_{Cr0}|^2 + \sum_{k=0}^{K-1} \kappa_{RCk1} |\mathbf{f}_R^H \mathbf{a}_{Rtk}|^2 |\mathbf{g}_{C1}^H \mathbf{a}_{Rrk}|^2 + 1}. \end{aligned}$$

For the optimisation of $\mathbf{f}_R$, using [Den09, Fact 7.4.9], one obtains

$$\begin{aligned} \mathcal{M} \propto & \frac{|\mathbf{f}_R^H \mathbf{a}_{Rt0}|^2}{\kappa_{CC10} N_t |\mathbf{g}_{C0}^H \mathbf{a}_{Cr1}|^2 + \sum_{k=0}^{K-1} \kappa_{RCk0} |\mathbf{f}_R^H \mathbf{a}_{Rtk}|^2 |\mathbf{g}_{C0}^H \mathbf{a}_{Rrk}|^2 + 1} \\ & \cdot \frac{|\mathbf{f}_R^H \mathbf{a}_{Rt1}|^2}{\kappa_{CC01} N_t |\mathbf{g}_{C1}^H \mathbf{a}_{Cr0}|^2 + \sum_{k=0}^{K-1} \kappa_{RCk1} |\mathbf{f}_R^H \mathbf{a}_{Rtk}|^2 |\mathbf{g}_{C1}^H \mathbf{a}_{Rrk}|^2 + 1} \\ = & \frac{\mathbf{f}_R^H \mathbf{A}_{Rt} \mathbf{f}_R \mathbf{f}_R^H \mathbf{B}_{Rt} \mathbf{f}_R}{\mathbf{f}_R^H \mathbf{C}_{Rt} \mathbf{f}_R \mathbf{f}_R^H \mathbf{D}_{Rt} \mathbf{f}_R} = \frac{\text{tr} \{ \mathbf{f}_R \mathbf{f}_R^H \mathbf{A}_{Rt} \mathbf{f}_R \mathbf{f}_R^H \mathbf{B}_{Rt} \}}{\text{tr} \{ \mathbf{f}_R \mathbf{f}_R^H \mathbf{C}_{Rt} \mathbf{f}_R \mathbf{f}_R^H \mathbf{D}_{Rt} \}} = \frac{\text{vec}\{\mathbf{F}_R\}^H (\mathbf{A}_{Rt}^* \otimes \mathbf{B}_{Rt}) \text{vec}\{\mathbf{F}_R\}}{\text{vec}\{\mathbf{F}_R\}^H (\mathbf{C}_{Rt}^* \otimes \mathbf{D}_{Rt}) \text{vec}\{\mathbf{F}_R\}}, \end{aligned}$$

where the Hermitian matrices $\mathbf{A}_{Rt}$, $\mathbf{B}_{Rt}$, $\mathbf{C}_{Rt}$, $\mathbf{D}_{Rt} \in \mathbb{C}^{N_t \times N_t}$ are defined as

$$\begin{aligned} \mathbf{A}_{Rt} & \triangleq \mathbf{a}_{Rt0} \mathbf{a}_{Rt0}^H, & \mathbf{B}_{Rt} & \triangleq \mathbf{a}_{Rt1} \mathbf{a}_{Rt1}^H, \\ \mathbf{C}_{Rt} & \triangleq \sum_{k=0}^{K-1} \kappa_{RCk0} |\mathbf{g}_{C0}^H \mathbf{a}_{Rrk}|^2 \mathbf{a}_{Rtk} \mathbf{a}_{Rtk}^H + \left(\kappa_{CC10} N_t |\mathbf{g}_{C0}^H \mathbf{a}_{Cr1}|^2 + 1 \right) \mathbf{I}_{N_t}, \\ \mathbf{D}_{Rt} & \triangleq \sum_{k=0}^{K-1} \kappa_{RCk1} |\mathbf{g}_{C1}^H \mathbf{a}_{Rrk}|^2 \mathbf{a}_{Rtk} \mathbf{a}_{Rtk}^H + \left(\kappa_{CC01} N_t |\mathbf{g}_{C1}^H \mathbf{a}_{Cr0}|^2 + 1 \right) \mathbf{I}_{N_t}. \end{aligned}$$

The same is obtained for the optimisation of $\mathbf{g}_R$:

$$\mathcal{M} \propto \frac{|\mathbf{g}_R^H \mathbf{a}_{Rr0}|^2}{\sum_{q=0}^1 \kappa_{CRq0} N_t |\mathbf{g}_R^H \mathbf{a}_{Crq}|^2 + 1} \frac{|\mathbf{g}_R^H \mathbf{a}_{Rr1}|^2}{\sum_{q=0}^{Q-1} \kappa_{CRq1} N_t |\mathbf{g}_R^H \mathbf{a}_{Crq}|^2 + 1}$$

$$= \frac{\mathbf{g}_R^H \mathbf{A}_{Rr} \mathbf{g}_R \mathbf{g}_R^H \mathbf{B}_{Rr} \mathbf{g}_R}{\mathbf{g}_R^H \mathbf{C}_{Rr} \mathbf{g}_R \mathbf{g}_R^H \mathbf{D}_{Rr} \mathbf{g}_R} = \frac{\text{tr} \{ \mathbf{g}_R \mathbf{g}_R^H \mathbf{A}_{Rr} \mathbf{g}_R \mathbf{g}_R^H \mathbf{B}_{Rr} \}}{\text{tr} \{ \mathbf{g}_R \mathbf{g}_R^H \mathbf{C}_{Rr} \mathbf{g}_R \mathbf{g}_R^H \mathbf{D}_{Rr} \}} = \frac{\text{vec}\{\mathbf{G}_R\}^H (\mathbf{A}_{Rr}^* \otimes \mathbf{B}_{Rr}) \text{vec}\{\mathbf{G}_R\}}{\text{vec}\{\mathbf{G}_R\}^H (\mathbf{C}_{Rr}^* \otimes \mathbf{D}_{Rr}) \text{vec}\{\mathbf{G}_R\}},$$

with the Hermitian matrices $\mathbf{A}_{Rr}, \mathbf{B}_{Rr}, \mathbf{C}_{Rr}, \mathbf{D}_{Rr} \in \mathbb{C}^{N_r \times N_r}$ defined as

$$\begin{aligned} \mathbf{A}_{Rr} &\triangleq \mathbf{a}_{Rr0} \mathbf{a}_{Rr0}^H, & \mathbf{B}_{Rr} &\triangleq \mathbf{a}_{Rr1} \mathbf{a}_{Rr1}^H, \\ \mathbf{C}_{Rr} &\triangleq \sum_{q=0}^1 \kappa_{CRq0} N_t \mathbf{a}_{Crq} \mathbf{a}_{Crq}^H + \mathbf{I}_{N_r}, & \mathbf{D}_{Rr} &\triangleq \sum_{k=0}^{K-1} \sum_{q=0}^{Q-1} \kappa_{CRq1} N_t \mathbf{a}_{Crq} \mathbf{a}_{Crq}^H + \mathbf{I}_{N_r}. \end{aligned}$$

Finally, for the optimisation of $\{\mathbf{g}_{Cq}\}$, one obtains

$$\begin{aligned} \mathcal{M} &\propto \frac{|\mathbf{g}_{C0}^H \mathbf{a}_{Cr0}|^2}{\kappa_{CC10} N_t |\mathbf{g}_{C0}^H \mathbf{a}_{Cr1}|^2 + \sum_{k=0}^{K-1} \kappa_{RCk0} |\mathbf{f}_R^H \mathbf{a}_{Rtk}|^2 |\mathbf{g}_{C0}^H \mathbf{a}_{Rrk}|^2 + 1} \\ &\quad \cdot \frac{|\mathbf{g}_{C1}^H \mathbf{a}_{Cr1}|^2}{\kappa_{CC01} N_t |\mathbf{g}_{C1}^H \mathbf{a}_{Cr0}|^2 + \sum_{k=0}^{K-1} \kappa_{RCk1} |\mathbf{f}_R^H \mathbf{a}_{Rtk}|^2 |\mathbf{g}_{C1}^H \mathbf{a}_{Rrk}|^2 + 1} \\ &= \frac{\mathbf{g}_{C0}^H \mathbf{A}_{Cr0} \mathbf{g}_{C0}}{\mathbf{g}_{C0}^H \mathbf{B}_{Cr0} \mathbf{g}_{C0}} \frac{\mathbf{g}_{C1}^H \mathbf{A}_{Cr1} \mathbf{g}_{C1}}{\mathbf{g}_{C1}^H \mathbf{B}_{Cr1} \mathbf{g}_{C1}}, \end{aligned}$$

with the matrices $\{\mathbf{A}_{Crq}\}, \{\mathbf{B}_{Crq}\} \in \mathbb{C}^{N_r \times N_r}$ defined as

$$\begin{aligned} \mathbf{A}_{Crq} &= \mathbf{a}_{Crq} \mathbf{a}_{Crq}^H, \\ \mathbf{B}_{Crq} &= \kappa_{CCq'q} N_t \mathbf{a}_{Crq'} \mathbf{a}_{Crq'}^H + \sum_{k=0}^{K-1} \kappa_{RCkq} |\mathbf{f}_R^H \mathbf{a}_{Rtk}|^2 \mathbf{a}_{Rrk} \mathbf{a}_{Rrk}^H + \mathbf{I}_{N_r}, \end{aligned}$$

where $q' = 1$ (resp. $q' = 0$) when $q = 0$ (resp. $q = 1$). The optimisations of the beamforming vectors of the communication receivers are decoupled. Thus, the optimal beamformers are obtained by maximising Rayleigh quotients. Following (10.9), the solutions are given by

$$\mathbf{g}_{Cq}^* = \mu_{Cq} \mathbf{B}_{Crq}^{-1} \mathbf{a}_{Crq}, \quad \forall q = 0, 1, \quad (10.11)$$

where the coefficients $\{\mu_{Cq}\}$ are normalisation factors.

In summary, for both radar beamformers, the following problem must be solved:

$$\begin{aligned} \mathbf{X}^* &= \arg \max_{\mathbf{X}} \frac{\text{vec}\{\mathbf{X}\}^H (\mathbf{A}^* \otimes \mathbf{B}) \text{vec}\{\mathbf{X}\}}{\text{vec}\{\mathbf{X}\}^H (\mathbf{C}^* \otimes \mathbf{D}) \text{vec}\{\mathbf{X}\}} \\ \text{s.t. } &\mathbf{X} = \mathbf{X}^H, \\ &\mathbf{X} \geq \mathbf{0}, \\ &\text{rank}(\mathbf{X}) = 1. \end{aligned} \quad (10.12)$$

The optimal beamformer $\mathbf{x}^*$ is then obtained as $\mathbf{X}^* = \mathbf{x}^* \mathbf{x}^{*H}$. Note that this approach also works for less than two targets and UEs. However, the constraint set is non convex. Thus, obtaining a solution is challenging. To overcome this issue, two relaxations are proposed. First, all the constraints of (10.12) are relaxed in Section 10.E.1. Then, a symmetry constraint is introduced in Section 10.E.2 to improve the relaxation. Finally, once the vectors $\mathbf{f}_R^*$ and $\mathbf{g}_R^*$ are computed, one obtains the optimal beamformers for the communication receivers $\{\mathbf{g}_{Cq}^*\}$ using (10.11). AM between the optimal beamformers $\mathbf{f}_R^*$ and $\{\mathbf{g}_{Cq}^*\}$ is then performed until a local extremum is reached.

10.E.1 Full Relaxation

When all the constraints of (10.12) are relaxed, the optimal matrix $\mathbf{X}^*$ can be rewritten as $\mathbf{X} = \mathbf{x}_2 \mathbf{x}_1^H$, with $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{C}^N$. Using again [Den09, Fact 7.4.9], the objective function of (10.12), one obtains

$$\frac{\mathbf{x}_1^H \mathbf{A} \mathbf{x}_1 \mathbf{x}_2^H \mathbf{B} \mathbf{x}_2}{\mathbf{x}_1^H \mathbf{C} \mathbf{x}_1 \mathbf{x}_2^H \mathbf{D} \mathbf{x}_2} = \frac{\text{tr} \{ \mathbf{x}_2 \mathbf{x}_1^H \mathbf{A} \mathbf{x}_1 \mathbf{x}_2^H \mathbf{B} \}}{\text{tr} \{ \mathbf{x}_2 \mathbf{x}_1^H \mathbf{C} \mathbf{x}_1 \mathbf{x}_2^H \mathbf{D} \}} = \frac{\text{vec}\{\mathbf{X}\}^H (\mathbf{A}^* \otimes \mathbf{B}) \text{vec}\{\mathbf{X}\}}{\text{vec}\{\mathbf{X}\}^H (\mathbf{C}^* \otimes \mathbf{D}) \text{vec}\{\mathbf{X}\}}.$$

Therefore, the vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ can be optimised independently:

$$\mathbf{x}_1^* = \arg \max_{\mathbf{x}} \frac{\mathbf{x}^H \mathbf{A} \mathbf{x}}{\mathbf{x}^H \mathbf{C} \mathbf{x}}, \quad \mathbf{x}_2^* = \arg \max_{\mathbf{x}} \frac{\mathbf{x}^H \mathbf{B} \mathbf{x}}{\mathbf{x}^H \mathbf{D} \mathbf{x}}.$$

This is the optimisation of two Rayleigh quotients, for which the solutions are computed following (10.9):

$$\mathbf{x}_1^* \propto \mathbf{C}^{-1} \mathbf{a}, \quad \mathbf{x}_2^* \propto \mathbf{D}^{-1} \mathbf{b},$$

where the vectors $\mathbf{a}$ and $\mathbf{b}$ are defined such that $\mathbf{A} = \mathbf{a} \mathbf{a}^H, \mathbf{B} = \mathbf{b} \mathbf{b}^H$. The optimal matrix $\mathbf{X}^*$ is finally given by

$$\mathbf{X}^* \propto \mathbf{x}_2^* \mathbf{x}_1^{*H} = \mathbf{D}^{-1} \mathbf{b} \mathbf{a}^H \mathbf{C}^{-H}.$$

Nevertheless, even if the matrix $\mathbf{X}^*$ is rank-one, it is neither Hermitian, nor positive definite. To find the optimal beamforming vector, the closest admissible solution is selected, as the projection of $\mathbf{X}^*$ on the set

$$\mathcal{S} \triangleq \{ \mathbf{X} = \zeta \mathbf{x} \mathbf{x}^H \mid \zeta \in \mathbb{C}, \mathbf{x} \in \mathbb{C}^N, \|\mathbf{x}\|_2 = 1 \}.$$

Thus, the final beamforming vector $\mathbf{x}^*$ is obtained by minimising the following Frobenius norm:

$$\mathbf{x}^* = \arg \min_{\mathbf{x}, \zeta} \|\mathbf{X}^* - \zeta \mathbf{x} \mathbf{x}^H\|_F^2.$$

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This norm can be developed as

$$\begin{aligned}\|\mathbf{X}^\star - \zeta \mathbf{x}\mathbf{x}^H\|_F^2 &= \text{tr} \left\{ \left(\mathbf{X}^\star - \zeta \mathbf{x}\mathbf{x}^H \right)^H \left(\mathbf{X}^\star - \zeta \mathbf{x}\mathbf{x}^H \right) \right\} \\ &= \text{tr} \left\{ \mathbf{X}^{\star H} \mathbf{X}^\star - \zeta \mathbf{X}^{\star H} \mathbf{x}\mathbf{x}^H - \zeta^* \mathbf{x}\mathbf{x}^H \mathbf{X}^\star + |\zeta|^2 \right\} \\ &= |\zeta|^2 + \text{tr} \left\{ \mathbf{X}^{\star H} \mathbf{X}^\star \right\} - 2\text{Re} \left\{ \zeta^* \mathbf{x}^H \mathbf{X}^\star \mathbf{x} \right\}.\end{aligned}$$

To maximise the Frobenius norm, the phase of ζ should be chosen such that

$$\arg \{ \zeta \} = \arg \left\{ \mathbf{x}^{\star H} \mathbf{X}^\star \mathbf{x} \right\}.$$

Finally, knowing that $\mathbf{X}^\star \propto \mathbf{x}_2^\star \mathbf{x}_1^{\star H}$, the optimal $\mathbf{x}^\star$ is obtained as

$$\mathbf{x}^\star = \arg \min_{\mathbf{x}} |\mathbf{x}^H \mathbf{X}^\star \mathbf{x}| = \arg \min_{\mathbf{x}} |\mathbf{x}^H \mathbf{x}_1^\star| |\mathbf{x}^H \mathbf{x}_2^\star|. \quad (10.13)$$

Thus, the solution may span in the space of the vectors $\mathbf{x}_1^\star$ and $\mathbf{x}_2^\star$. Owing to the symmetry of the problem, following the similar developments in Section 10.D, the optimal beamforming vector $\mathbf{x}^\star$ is obtained as

$$\mathbf{x}^\star = \mu \left(0.5 \frac{\mathbf{x}_1^\star}{\|\mathbf{x}_1^\star\|_2} + 0.5 e^{j \arg \{ \mathbf{x}_2^{\star H} \mathbf{x}_1^\star \}} \frac{\mathbf{x}_2^\star}{\|\mathbf{x}_2^\star\|_2} \right),$$

where the coefficient μ is a normalisation factor.

10.E.2 Hermitian Relaxation

When considering the Hermitian constraint only, two operators and matrices must be defined for the following developments:

- The *complex vectorisation* operator $\text{vec}_{\mathbb{C}}$ is defined as

$$\text{vec}_{\mathbb{C}} : \mathbb{C}^{N \times N} \rightarrow \mathbb{R}^{2N^2 \times 1} : \mathbf{E} \rightarrow \begin{bmatrix} \text{vec}\{\text{Re}\{\mathbf{E}\}\} \\ \text{vec}\{\text{Im}\{\mathbf{E}\}\} \end{bmatrix}.$$

The inverse operator is denoted as $\text{vec}_{\mathbb{C}}^{-1}$.

- The *complex half-vectorisation* operator $\text{vech}_{\mathbb{C}} : \mathbb{C}^{N \times N} \rightarrow \mathbb{R}^{N^2 \times 1}$ is defined for Hermitian matrices similarly as the complex vectorisation operator, except for the duplicate entries, which are discarded. In other words, given a matrix $\mathbf{E} \in \mathbb{C}^{N \times N}$, $\text{vech}_{\mathbb{C}}\{\mathbf{E}\}$ is the vector of size $N^2 \times 1$, whose first $N(N+1)/2$ elements are the stacked columns of the upper triangular part of $\text{Re}\{\mathbf{E}\}$, including the diagonal. The last $N(N-1)/2$ elements are the stacked columns of the upper triangular part of $\text{Im}\{\mathbf{E}\}$.

- The *complex duplication matrix* $\mathbf{T} \in \mathbb{R}^{2N^2 \times N^2}$ is the unique matrix fulfilling

$$\mathbf{T} \text{vech}_{\mathbb{C}}\{\mathbf{E}\} = \text{vec}_{\mathbb{C}}\{\mathbf{E}\},$$

for any Hermitian matrix $\mathbf{E} \in \mathbb{C}^{N \times N}$.

- Given a matrix $\mathbf{E} \in \mathbb{C}^{N \times N}$, the matrix $\mathbf{M}_{\mathbf{E}} \in \mathbb{R}^{2N \times 2N}$ is defined as

$$\mathbf{M}_{\mathbf{E}} \triangleq \begin{bmatrix} \text{Re}\{\mathbf{E}\} & -\text{Im}\{\mathbf{E}\} \\ \text{Im}\{\mathbf{E}\} & \text{Re}\{\mathbf{E}\} \end{bmatrix}.$$

As an illustration, let us consider the 2×2 matrix

$$\mathbf{E} = \begin{bmatrix} a_r + ja_i & b_r + jb_i \\ c_r + jc_i & d_r + jd_i \end{bmatrix},$$

where $a_r, b_r, c_r, d_r, a_i, b_i, c_i, d_i \in \mathbb{R}$. First, the associated 4×4 matrix $\mathbf{M}_{\mathbf{E}}$ is given by

$$\mathbf{M}_{\mathbf{E}} = \begin{bmatrix} a_r & b_r & -a_i & -b_i \\ c_r & d_r & -c_i & -d_i \\ a_i & b_i & a_r & b_r \\ c_i & d_i & c_r & d_r \end{bmatrix}.$$

Let us now assume that $a_i = 0, d_i = 0, b_r = c_r$ and $b_i = -c_i$, such that the matrix $\mathbf{E}$ is Hermitian. Applying the complex vectorisation and half-vectorisation operators, one obtains

$$\begin{aligned} \text{vec}_{\mathbb{C}}\{\mathbf{E}\} &= [a_r \quad b_r \quad c_r \quad d_r \quad a_i \quad b_i \quad c_i \quad d_i]^T \\ &= [a_r \quad b_r \quad -b_r \quad d_r \quad 0 \quad b_i \quad -b_i \quad 0]^T, \\ \text{vech}_{\mathbb{C}}\{\mathbf{E}\} &= [a_r \quad b_r \quad d_r \quad b_i]^T. \end{aligned}$$

Finally, the unique duplication matrix $\mathbf{T}$ for 2×2 matrices is given by

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}^T.$$

Instead of optimising all the elements of $\mathbf{X} \in \mathbb{C}^{N \times N}$, since the matrix $\mathbf{X}$ must be Hermitian, one may optimise the vector $\tilde{\mathbf{x}} \in \mathbb{R}^{N^2 \times 1}$ defined as $\tilde{\mathbf{x}} \triangleq \text{vech}_{\mathbb{C}}\{\mathbf{X}\}$. Using the complex duplication matrix, it follows that

$$\text{vec}_{\mathbb{C}}\{\text{vec}\{\mathbf{X}\}\} = \text{vec}_{\mathbb{C}}\{\mathbf{X}\} = \mathbf{T} \text{vech}_{\mathbb{C}}\{\mathbf{X}\} = \mathbf{T}\tilde{\mathbf{x}}. \quad (10.14)$$

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In order to rewrite the objective function of (10.12) as a function of $\mathbf{x}$, one may first develop the product $\mathbf{v}^H \mathbf{X} \mathbf{v}$, for a vector $\mathbf{v} = \mathbf{v}_r + j\mathbf{v}_i \in \mathbb{C}^{N \times 1}$ and a matrix $\mathbf{E} = \mathbf{E}_r + j\mathbf{E}_i \in \mathbb{C}^{N \times N}$, with $\mathbf{v}_r, \mathbf{v}_i \in \mathbb{R}^{N \times 1}$ and $\mathbf{E}_r, \mathbf{E}_i \in \mathbb{R}^{N \times N}$:

$$\begin{aligned} \mathbf{v}^H \mathbf{E} \mathbf{v} &= (\mathbf{v}_r^T - j\mathbf{v}_i^T) (\mathbf{E}_r + j\mathbf{E}_i) (\mathbf{v}_r + j\mathbf{v}_i) \\ &= \mathbf{v}_r^T \mathbf{E}_r \mathbf{v}_r + \mathbf{v}_i^T \mathbf{E}_r \mathbf{v}_i - \mathbf{v}_r^T \mathbf{E}_i \mathbf{v}_i + \mathbf{v}_i^T \mathbf{E}_i \mathbf{v}_r \\ &\quad + j \left(\mathbf{v}_r^T \mathbf{E}_i \mathbf{v}_r + \mathbf{v}_i^T \mathbf{E}_i \mathbf{v}_i - j\mathbf{v}_i^T \mathbf{E}_r \mathbf{v}_r + j\mathbf{v}_r^T \mathbf{E}_r \mathbf{v}_i \right). \end{aligned}$$

If the matrix $\mathbf{E}$ is Hermitian, then $\mathbf{E}_r = \mathbf{E}_r^T$, and $\mathbf{E}_i = -\mathbf{E}_i^T$. Thus,

$$\begin{aligned} \mathbf{v}_r^T \mathbf{E}_i \mathbf{v}_r &= (\mathbf{v}_r^T \mathbf{E}_i \mathbf{v}_r)^T \Leftrightarrow \mathbf{v}_r^T \mathbf{E}_i \mathbf{v}_r = -\mathbf{v}_r^T \mathbf{E}_i \mathbf{v}_r \Rightarrow \mathbf{v}_r^T \mathbf{E}_i \mathbf{v}_r = 0, \\ \mathbf{v}_i^T \mathbf{E}_i \mathbf{v}_i &= (\mathbf{v}_i^T \mathbf{E}_i \mathbf{v}_i)^T \Leftrightarrow \mathbf{v}_i^T \mathbf{E}_i \mathbf{v}_i = -\mathbf{v}_i^T \mathbf{E}_i \mathbf{v}_i \Rightarrow \mathbf{v}_i^T \mathbf{E}_i \mathbf{v}_i = 0, \\ \mathbf{v}_r^T \mathbf{E}_r \mathbf{v}_i &= (\mathbf{v}_r^T \mathbf{E}_r \mathbf{v}_i)^T = \mathbf{v}_i^T \mathbf{E}_r \mathbf{v}_r, \end{aligned}$$

leading finally to

$$\mathbf{v}^H \mathbf{E} \mathbf{v} = \mathbf{v}_r^T \mathbf{E}_r \mathbf{v}_r + \mathbf{v}_i^T \mathbf{E}_r \mathbf{v}_i - \mathbf{v}_r^T \mathbf{E}_i \mathbf{v}_i + \mathbf{v}_i^T \mathbf{E}_i \mathbf{v}_r = \text{vec}_{\mathbb{C}}\{\mathbf{v}\}^H \mathbf{M}_{\mathbf{E}} \text{vec}_{\mathbb{C}}\{\mathbf{v}\}. \quad (10.15)$$

Inserting (10.14) and (10.15) into the objective function of (10.12), one obtains

$$\begin{aligned} \frac{\text{vec}\{\mathbf{X}\}^H (\mathbf{A}^* \otimes \mathbf{B}) \text{vec}\{\mathbf{X}\}}{\text{vec}\{\mathbf{X}\}^H (\mathbf{C}^* \otimes \mathbf{D}) \text{vec}\{\mathbf{X}\}} &= \frac{\text{vec}_{\mathbb{C}}\{\text{vec}\{\mathbf{X}\}\}^H \mathbf{M}_{\mathbf{A}^* \otimes \mathbf{B}} \text{vec}_{\mathbb{C}}\{\text{vec}\{\mathbf{X}\}\}}{\text{vec}_{\mathbb{C}}\{\text{vec}\{\mathbf{X}\}\}^H \mathbf{M}_{\mathbf{C}^* \otimes \mathbf{D}} \text{vec}_{\mathbb{C}}\{\text{vec}\{\mathbf{X}\}\}} \\ &= \frac{\tilde{\mathbf{x}}^T \mathbf{T}^T \mathbf{M}_{\mathbf{A}^* \otimes \mathbf{B}} \mathbf{T} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^T \mathbf{T}^T \mathbf{M}_{\mathbf{C}^* \otimes \mathbf{D}} \mathbf{T} \tilde{\mathbf{x}}}. \end{aligned}$$

In summary, when keeping only the Hermitian constraint, the optimisation problem of (10.12) is rewritten as

$$\tilde{\mathbf{x}}^* = \arg \max_{\mathbf{x} \in \mathbb{R}^{N^2}} \frac{\tilde{\mathbf{x}}^T \mathbf{T}^T \mathbf{M}_{\mathbf{A}^* \otimes \mathbf{B}} \mathbf{T} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^T \mathbf{T}^T \mathbf{M}_{\mathbf{C}^* \otimes \mathbf{D}} \mathbf{T} \tilde{\mathbf{x}}},$$

and the optimal Hermitian matrix $\mathbf{X}^*$ is obtained by computing $\mathbf{X}^* = \text{vec}_{\mathbb{C}}^{-1}\{\mathbf{T} \tilde{\mathbf{x}}^*\}$. Still, the obtained matrix is not rank-one. Therefore, the projection of $\mathbf{X}^*$ on the set $\mathcal{S}$ proposed in Section 10.E.1 is again performed. Following (10.13), the optimal beamforming vector $\mathbf{x}^*$ is obtained as

$$\mathbf{x}^* = \arg \min_{\mathbf{x}} |\mathbf{x}^H \mathbf{X}^* \mathbf{x}|.$$

Using again [Den09, Lemma 8.4.3], the solution is obtained by computing the principal eigenvector of $\mathbf{X}^*$.

10.F Other Scenarios

For more than two radar targets and UEs, obtaining additional analytical results is extremely challenging. Following Section 10.3.1, still assuming that each UE optimises the transmitted power in the direction of the ISAC system as they are not colocated, locally optimal solutions can nevertheless be obtained by performing AM between the beamforming vectors of the transmitters and receivers $\tilde{\mathbf{f}}_R$, $\tilde{\mathbf{g}}_R$, and $\{\tilde{\mathbf{g}}_{Cq}\}$, in the steering space. Another AM is carried out to compute the elements of these vectors.

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Conclusion and Future Works

11

INTEGRATED sensing and communication systems is a promising technology to improve the performance of existing networks, and provides two functions for future multi-services infrastructures. In this thesis, ISAC systems have been studied under multiple facets. Three parts have been distinguished, considering V2X applications at the vehicle, street, or city scale, and focusing either on the performance or modelling, at the link or system level. Their content is first summarised in Section 11.1, highlighting the contributions of this thesis. Then, Section 11.2 provides future research questions related to the topic of joint sensing and communication. Research questions, contributions and future works are summarised in Figure 11.1, which extends Figure 1.7 by including the different contributions of the three parts, helping to answer these questions. For the questions remaining unanswered, future works are also presented.

11.1 Take-Home Messages and Contributions

First, at the link level, Part I has leveraged on the EM theory to analyse the behaviour of the transmitted waveforms at the vehicle level. More specifically, the reflection of the EM waves on flat and curved objects has been thoroughly studied. On the one hand, such analysis has enabled to develop novel radar propagation and cross-section models in Chapter 3. These models highlight specificities of the near-field propagation region, which applies in more and more applications, especially in vehicular scenarios. Namely, depending on the shape of the target, the received power may decrease with the square, third power, or fourth power of the distance to the target. They also make the bond between the usual sensing models of the literature. Numerical simulations have emphasised the significance of selecting a good model for the considered application. Experimental measurements have also been carried out to validate the developed expressions. On the other hand, novel near-field signal models have been obtained for multistatic radar configurations. The obtained expressions highlight that point target models are not accurate when the target dimensions are electrically and physically large, w.r.t. to the geometry of the scenario and the wavelength. Based on the obtained analytical expression, a maximum likelihood estimator of the distance between the antenna array and the target has been developed. Through numerical analyses, it has been shown that one can capitalise on the phase information in addition to the waveform delay to enhance

Summary of Research Questions, Contributions and Future Works

	Link	System
Modelling	✓ How to bridge existing models? ✓ When should they be used? Monostatic NF/FF RCS model Experimental RCS measurements Multistatic NF/FF signal model	✓ How to model interference at a large scale? ✓ When is the SG relevant? Interference modelling at the city scale
Performance	How does near-field propagation impact the performance? ✓ How to handle the interference in full-duplex scenarios? ✗ When should full-duplex or half-duplex be considered? NF Range estimation Full-duplex beamforming Fair R&C optimisation metrics Experimental ISAC Data/sensor fusion	Validation with RT ✓ How cooperation does impact the performance? Joint R&C performance metrics Full-duplex interference cancellation Cooperative R&C performance metrics Comparisons • Waveforms • Performance • Complexity • Cost

Part I

Part II

Part III

Future works

Fig. 11.1 Summary of research questions, contributions, and future works of this thesis. The acronyms NF and FF respectively refer to near- and far-field, and the letters R and C to radar and communication. The red cross mark indicates that the research question has not been addressed.

the resolution and/or accuracy of the estimation, respectively with orthogonal and non-orthogonal transmissions. Take-home messages for Part I are summarised in Box 11.1.

11.1: Take-home Messages — Part I

- The radar equation is suited for both near- and far-field applications. Yet, the RCS must be modelled as a function of the distance in the near-field region.
- The GO propagation model is only accurate in the near-field region, and it assumes locally planar targets around the specular point. Thus, it is suited for high carrier frequencies applications, e.g. automotive sensing.
- In the near-field region, extended target models should be considered. When considering point target models, one should ensure that the reference point of the target is the specular reflection point, instead of the centre of the target.
- The radar range resolution and accuracy are increased in the near-field region, when estimators which leverage both on the waveform delay and phase information are considered.

Next, at the system level, Part II has leveraged on the SG framework to assess the joint sensing and communication performance of vehicular networks, at the street or city scale. In Chapter 6, relevant univariate performance metrics have been summarised, and joint ISAC metrics have been defined, enabling to fully grasp the impact of the interactions of both functions on each other. For instance, a performance trade-off regarding the SIR requirements has been highlighted in full-duplex ISAC scenarios. Optimisation of the network parameters has also been carried out, and the developed framework has moreover been extended to include IC capabilities. Additionally, the accuracy of the obtained results has been assessed by performing a comparison with ray-tracing simulations. In Chapter 7, cooperative sensing and communication and complex interference mechanisms are introduced at the city scale. Analytical expressions of the cooperative performance metrics have been developed, but their mathematical complexity is high, showing the limits of the SG framework. Yet, comparisons of the proposed models have been analysed through numerical analyses, and the cooperation schemes have shown to improve the performance. Moreover, optimisation of the network parameters has again been carried out. Take-home messages for Part II are summarised in Box 11.2.

11.2: Take-home Messages – Part II

- Joint radar and communication metrics enable to evaluate the performance of both functions with a single metric, while taking into account the impact of one function on the other.
- In full-duplex ISAC scenarios, the radar and communication functions interfere with each other. Thus, the design is constrained by a trade-off w.r.t. the system requirements. Interference mitigation techniques can be considered to alleviate this constraint.
- For the considered automotive scenarios, the SG framework gives relevant results w.r.t. RT simulations for the univariate metrics, with a lower complexity. Nonetheless, higher discrepancies are observed for joint metrics.
- At the city scale, cooperation between the network nodes can help improve the radar and communication performance of the network. Yet, the cooperation rule should be carefully designed depending on the requirements.
- When considering a complete city with complex interference mechanisms, the SG framework is less relevant owing to the high mathematical complexity of the models, compared to the MC framework.

Finally, back at the link level, Part III has focused on vehicular full-duplex ISAC scenario, with PC-FMCW and OFDM systems. In such applications, the radar echoes and uplink communication signals interfere with each other. Thus, two solutions have been proposed in Chapter 9 and 10, respectively IC and beamforming. In the former, the specific signal processing of the considered systems has been thoroughly analysed, showing that the radar function has inherent interference mitigation capabilities, that does not exist at the communication function. Though, targets may still be drowned under the higher noise floor. Thus, IC has been integrated at the radar or communication function, for the system to work respectively in scenarios where the radar echoes or uplink communication signals are dominant. In order to increase the robustness of the developed receivers, iterative IC has been introduced. With such structure, good performance are achieved whatever the scenario, at the price of a high complexity. Consequently, the complexity has been reduced in dynamic applications, by leveraging on previous radar measurements to predict the position and speed of the targets at the next measurement. Instead of considering the IC, the beamforming has been investigated with MIMO systems. The SINR-product metric has been selected to ensure fairness between both functions, and among the targets

and uplink communication users. Multiple schemes have been designed depending on the number of targets and users, with variable complexity, optimality and required assumptions. The beamformers have been analysed through numerical analyses, to study the impact of channel mismatches, and assess the quality of the proposed solutions. Take-home messages for Part III are summarised in Box 11.3.

11.3: Take-home Messages — Part III

- With full-duplex ISAC systems, IC and beamforming are two solutions to mitigate the interference between the radar and communication functions on each other.
- Different IC structures have been proposed. The higher the robustness of the system, the higher the complexity.
- Contrariwise to the SINR-sum metric, the SINR-product metric ensures fairness between the radar and communication functions, and among the targets and UEs.
- The higher the number of targets and UEs, the higher the complexity of the proposed beamforming schemes.
- The choice between full-duplex or orthogonal systems should still be analysed. This requires comparisons between popular waveforms, and evaluations of the achieved performance, complexity, and cost of the system.

11.2 Future Works

Numerous extensions of the results provided in this thesis have been proposed in the conclusion of each chapter. Instead, this section focuses on research questions which have not been exploited in this work.

11.2.1 Full-Duplex versus Half-Duplex Integrated Sensing and Communication Systems

In this thesis, a particular focus has been made on the interference modelling between multiple ISAC systems, and full-duplex ISAC scenarios. However, one should also consider in parallel the evaluation of the performance achieved by equivalent half-duplex systems. Especially for the third part of the PhD thesis, the radar and communication performance achieved by full-duplex ISAC systems implementing the proposed IC schemes should be compared to the performance achieved by half-duplex

ISAC systems, orthogonalised in time or frequency, which are not interfering with each other. Yet, such analysis should be made for different waveforms, and with various metrics, as they are not all affected identically by modifications of the systems parameters, e.g. the bandwidth, or the coherent processing interval.

11.2.2 Communication/Sensing-aided Sensing/Communication

In Chapter 9, the benefits of introducing cooperation between both functions has been emphasised, as IC at one function leverages on the information provided by the other. More generally, the complementarity of the sensing and communication function is interesting for the improvement of their respective performance. As the communication function requires channel information to be able to recover the transmitted information, the most intuitive solution is to leverage on the sensing function, as its purpose is to gather information about the environment. Nonetheless, the opposite may also be considered, as this information can be transmitted between the network nodes using the communication function. Besides, the received information must be aggregated to the radar output in some way. Thus, this research question is directly related to the challenge of the data fusion.

11.2.3 Data Fusion

Owing to the increasing number of sensors carried by nowadays vehicles, and the capability to share information between multiple network nodes, the data fusion is a critical topic to investigate. Indeed, one may capitalise on the redundant information generated by the multiple sensors of the vehicles to enhance the sensing function. At a specific sensing node, this information may be considered as a prior knowledge, or it may be merged with the output of the node of interest. This is the topic of Martin Willame's PhD thesis. For instance, in the communication-aided sensing perspective, the beamforming feedback information shared by multiple Wi-Fi APs carries information about the angles of arrival, which may be merged with the sensing performed at each AP [Wil+24].

11.2.4 Fully-Integrated Designs and Other Waveforms

The PC-FMCW and OFDM ISAC systems have been considered in Part III to analyse full-duplex V2X scenarios. Nonetheless, these waveforms are either radar- or communication-centric. Thus, fully-integrated waveforms should be investigated. As the design is executed from scratch, constraints specific to the application may be integrated. Additionally, other joint interesting ISAC waveforms, e.g. frequency-hopping, OTFS, or CPM-based designs, have not been described in this thesis. Still, their different pros and cons may be interesting for the considered full-duplex ISAC scenarios, or for other applications.

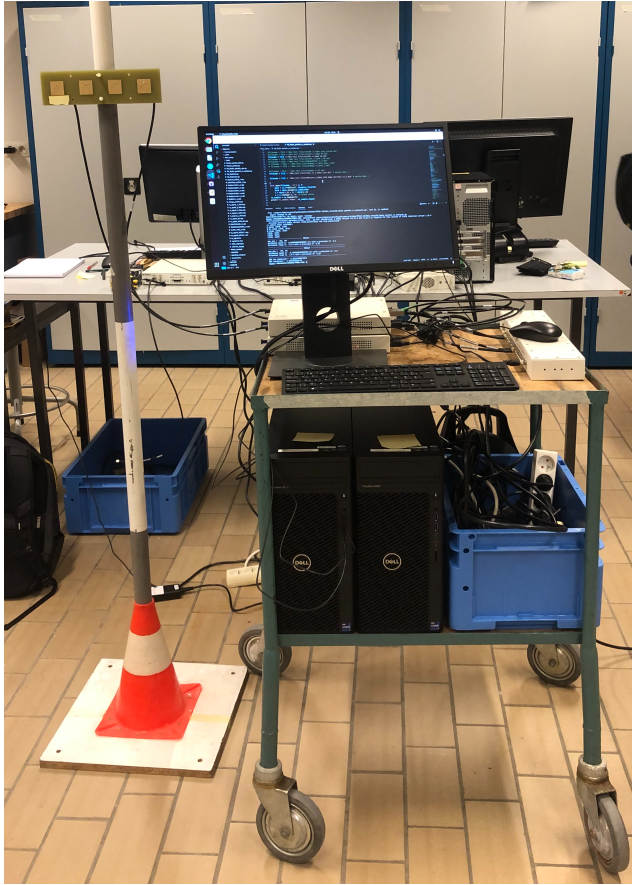


Fig. 11.2 Mobile experimental setup under development at UCLouvain.

11.2.5 Experimental Integrated Sensing and Communication

Experimental measurements have been performed in Chapter 3 to validate the novel RCS models which have been developed. Otherwise, ray-tracing simulations have been carried out in Chapter 6 to assess the accuracy of the analytical expressions of the metrics. Nonetheless, the results of the other chapters are all theoretical, and additional experimental analyses would have been interested to complete this work. Besides, the development of an experimental setup raises new practical challenges which may be tackled, as synchronisation or non-idealities. For that purpose, an experimental setup based on [Universal Radio Peripherals \(USRPs\)](#) is developed at UCLouvain. A mobile version of the setup is illustrated in Figure 11.2. This setup should be able to carry out joint sensing and communication experimentations, enabling to assess the results presented in this thesis, and open the lead to new challenges.

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