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2013 J. Phys.: Conf. Ser. 410 012134

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Two-dimensional quantum dilaton gravity and the quantum cosmological constant

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Abstract. We address the cosmological constant problem in the context of two-dimensional dilaton-Maxwell gravity, coupled to an arbitrary number of scalar matter fields. We are able to quantize the model non-perturbatively; we determine that the realization of the classical symmetries at the quantum level provides a mechanism that fixes the value of the cosmological constant.

1. Introduction

The cosmological constant problem is one of the long-standing issues of modern physics. While we can measure the value of the cosmological constant with great accuracy, we are not able to calculate it in a coherent theoretical framework and the theoretical predictions in quantum field theory are radically different from observations. This disagreement is a hint of the difficult conciliation of Quantum Mechanics and General Relativity in a theory of quantum gravity. Current approaches to the cosmological constant problem, in particular, do not account for the quantum nature of the gravitational interaction and rely on perturbative calculations. In this paper we discuss how quantum gravity can determine the value of the cosmological constant, labelled Λ , and we address the issue in the simplified scenario of two-dimensional dilaton-Maxwell gravity. In Section 2 we discuss a general idea on how quantum gravity, in any space-time dimensions, can provide a mechanism that fixes the cosmological constant. In Section 3 we detail how this ideas apply in the specific case of two-dimensional dilaton-Maxwell gravity, while in Section 4 we briefly discuss our results.

2. The cosmological constant in a quantum theory of gravity

Two interesting examples of a cosmological constant determined by the symmetries in quantum gravity are given by one-dimensional gravity [1] and a two-dimensional toy model [2], both in the presence of scalar matter. We can generalize the principles contained in these works to the case of quantum gravity in any number of space-time dimensions d .

In a completely general way the diffeomorphism invariance, in the Hamiltonian formulation,

provides classical constraint equations, some of which will also include the cosmological constant, $\mathcal{H}^\mu(\Lambda, \dots) = 0$, with $\mu = 1, \dots, d$. Additional gauge symmetries provide additional constraints. When turned into quantum constraint operators, these classical equations become conditions that determine the physical states of the model, $\hat{\mathcal{H}}^\mu(\Lambda)|\psi_{phys}\rangle = 0$, in a way dependent on the value of the cosmological constant and spatial coordinates. This same condition can be seen as a way to determine the value of the cosmological constant required for a given quantum state to be physical. Once a basis is chosen in Hilbert space, one can solve the set of equations $\langle\psi_{phys}|\hat{\mathcal{H}}^\mu(\Lambda)|\psi_{phys}\rangle = 0$ in the parameter space spanned by the cosmological constant itself, the (complex) coordinates which cover the specific (sub)space of states considered and possibly the undetermined parameters of the model. In order to do so the dependence on spatial coordinates has to be integrated out, for instance looking at the Fourier modes of each equation. For each μ this reduces the number of equations from $d - 1$ non countable infinities (an equation per spatial point) to $d - 1$ countable ones (one equation per each Fourier mode). Moreover the commutation relations among the different modes might drastically reduce the number of independent equations. A notable example is the Weyl symmetry in string theory, which requires only three modes of the Virasoro generators to annihilate physical states. This is true in any number of dimensions, as long as diffeomorphism invariance holds. In particular, due to the specific form of the cosmological constant term in the gravitational action, Λ will appear linearly in the constraints. There will then be at most one value of the cosmological constant that allows a given quantum state to be physical.

To turn the formal constraints equations into something able to provide an actual result for Λ it is of course necessary to have fully quantized the theory, so that the explicit form and algebra of $\hat{\mathcal{H}}^\mu$ are known. Therefore we have to turn our attention to those models of gravity that we are able to quantize, also in the presence of additional fields.

3. Two-dimensional dilaton gravity and the quantized cosmological constant

Two-dimensional generalized dilaton theories represent a very interesting class of models, widely studied in the last two decades. They proved to be very useful in the understanding of classical and quantum gravity, allowing to face conceptual issues also relevant to higher dimensions. Most of the results are well summarized in [3, 4]. In order to generalize the results of [2], we consider the most general action principle of dilaton gravity in the presence of a Maxwell field:

$$S_{DM} = \int_{\mathcal{M}} d^2x \sqrt{-g} \left(XR - U(X)X_{,\mu}X^{,\mu} - 2V(X) - \frac{1}{4}G(X)F_{\mu\nu}F^{\mu\nu} \right), \quad (1)$$

where X is the dilaton, U , V and G are arbitrary functions of X , and $F_{\mu\nu}$ is the usual field strength for the vector gauge field A_μ . We will consider all fields to vanish at infinity so no boundary terms appear. In [5] it was realized that, with specific restrictions on the U and G potentials, there exists a dual formulation of (1) in terms of two decoupled Liouville fields on flat space-time. In particular the dual action principle takes the form:

$$S_{dual} = \int d^2x \sqrt{-g_b} \left[\frac{1}{2} (Z_{,\mu}Z^{,\mu} - Y_{,\mu}Y^{,\mu}) - 2\Lambda e^{Z/\xi} - e^{-Y/\xi} \frac{\vartheta}{2\Lambda} F_{\mu\nu}F^{\mu\nu} + \xi (Z - Y) R_b \right], \quad (2)$$

where Z and Y are two scalar fields, Λ is the cosmological constant, ϑ is an arbitrary constant and ξ plays the role of a coupling constant. The space-time metric gives a line element $dx^2 = -\lambda_0\lambda_1 dt^2 + (\lambda_0 - \lambda_1)dt ds + ds^2$, which is in fact pure gauge. The subscript b indicates gravitational quantities that are calculated with respect to this metric. The inclusion of scalar matter is trivial and it is simply done by adding an arbitrary number D of terms in the form:

$$\mathcal{S}_\phi = -\frac{1}{2} \int d^2x \sqrt{-g_b} \phi_{,\mu} \phi^{,\mu} \quad (3)$$

To deal with the gauge invariances it is useful to look at the BRST formulation of the model [6]; after the introduction of ghost degree of freedom and the BRST charge, the BRST extended constraints are readily calculated and, as expected from the properties of the duality, it is easy to check that their algebra is first class. In particular the (smeared) Poisson brackets read:

$$\begin{aligned} \{L^\pm(f), L^\pm(g)\} &= \pm L^\pm(g, sf - f, sg) , \\ \{L^\pm(f), L^\emptyset(g)\} &= 0 , \\ \{L^\pm(f), L^\mp(g)\} &= -\frac{1}{4\Lambda_G} (e^{Y/\xi} \Pi_1 L^\emptyset)(fg) , \end{aligned} \quad (4)$$

where $\pm$ label the two Virasoro generators and $\emptyset$ indicates the constraint related to the $U(1)$ gauge invariance. As expected these brackets are vanishing on the constraints surface. In the following we will choose to work in the conformal gauge for the gravitational sector, $\lambda_0 = \lambda_1 = 1$, and the Coulomb gauge in the $U(1)$ sector, $A_0 = A_{1,s} = 0$.

In order to quantize this model in the canonical formalism we can compactify the space dimension to a circle, in a way equivalent to the standard box quantization in quantum field theory. We can set the compactification scale to 1 with a suitable choice of units. Since Newton's constant is dimensionless in two dimensions, we are always entitled to this choice. In the following we are also considering $c = \hbar = 1$. Therefore we consider $s \in [0, 2\pi)$, so that all classical fields will be required to be periodic along the compact dimension, i.e. for any field F we have $F(t, 0) = F(t, 2\pi)$. Quantization follows the usual prescriptions of Dirac's approach to constrained systems and field operators are expressed in terms of creation/annihilation operators. Normal ordering is chosen when needed for composite operators. The decoupling between Z , Y and scalar matter fields plays a fundamental role in allowing us to quantize the fields, and therefore the classical constraints, by working in each sector separately. In particular the two BRST extended Virasoro constraints are a sum of terms from each field $L^\pm = L^{\pm,Z} + L^{\pm,Y} + L^{\pm,g} + \sum_{n=1}^D L^{\pm,\phi}$, where g labels the contribution from the ghosts. We will require the classical symmetries to be unbroken at the quantum level: the quantum operators corresponding to the L 's will have to exhibit a closed algebra. In particular we will make sure that no central term appears in the commutators. The $L^\emptyset$ constraint has identically vanishing brackets with the two Virasoro generators and itself, hence the corresponding operator is quantum mechanically trivial. The Virasoro algebra, on the other hand will exhibit a central extension. The ghosts and scalar matter degrees of freedom reproduce the usual results known from string theory. Looking at their Fourier modes the contributions to the total central extension are $c_g = -\frac{13}{6}r^3 - \frac{1}{6}r$ and $c_\phi = \frac{1}{12}r^3 + \frac{1}{6}r$. The two Liouville fields on the other hand require the presence of a quantum correction to the coupling constant ξ which appears in front of the term containing R_b in (2) [7] and in particular one has $\xi_Z = \xi_Y = \xi - \frac{1}{8\pi\xi}$. Interestingly, as Z and Y appear with opposite signs in (2), the central extensions of their Virasoro contributions cancel each other. The total quantum Virasoro algebra will therefore contain a central extension $c = \frac{1}{12}(D - 26)r^3 + \frac{D+1}{6}r$.

By fixing $D = 26$ we can eliminate the cubic contribution, while by shifting the zero modes as $\hat{L}_0^\pm \rightarrow \hat{L}_0^\pm - 27/12$ we can reabsorb the linear term, giving a Virasoro algebra free of central extensions. The quantum constraint equations (dropping the ghost contributions by virtue of BRST invariance) are therefore given by:

$$\begin{aligned} \langle \hat{L}^+ + \hat{L}^- \rangle &= -\frac{1}{2} \langle P_Z^2 + Z_{,s}^2 \rangle + \frac{1}{2} \langle P_Y^2 + Y_{,s}^2 \rangle + \sum \frac{1}{2} \langle P_i^2 + \phi_{i,s}^2 \rangle + 2\xi_Z \langle Z_{,ss} \rangle - 2\xi_Y \langle Y_{,ss} \rangle + \\ &\quad + \Lambda \left[2 \langle e^{Z/\xi} \rangle + \frac{1}{4\Lambda_G} \langle e^{Y/\xi} P_{A_1}^2 \rangle \right] - 27/6 \\ \langle \hat{L}^+ - \hat{L}^- \rangle &= \langle P_Z Z_{,s} \rangle - 2\xi_Z \langle P_{Z,s} \rangle + \langle P_Y Y_{,s} \rangle - 2\xi_Y \langle P_{Y,s} \rangle + \sum \langle P_i \phi_{i,s} \rangle \\ \langle L^\emptyset \rangle &= \langle P_{A_1,s} \rangle , \end{aligned} \quad (5)$$

where the P 's are conjugate momenta. These equations can be solved for the cosmological constant and its explicit value calculated with a suitable choice for the representation of the states and the operators themselves.

4. Results and outlook

A first interesting result is that the Z field contributes to Λ with a sign opposite with respect to the contributions of the Y and scalar matter fields. This allows excitations of the Z field to provide cancellations which can in principle generate arbitrarily small values for Λ , in contrast with the usual QFT picture in which Λ is naturally of the order of the Planck scale.

Furthermore, as Λ appears linearly in the quantum constraints equations, once a quantum state is required to be physical, only one specific value of the cosmological constant is allowed, as it is expected from the considerations made in Section 2. On the other hand for a given value of Λ different quantum states can be physical.

In addition a series of quantum corrections appear: besides the shift of the zero modes of the Virasoro generators, which determines a non vanishing and purely quantum cosmological constant in the vacuum, the coupling constants ξ_Z and ξ_Y require additional contributions. These corrections will influence the value of Λ , suggesting that quantum gravity might play a very important role in building up the value of the cosmological constant.

With a suitable choice of representation for quantum states and operators it is also possible to study the spectrum of the cosmological constant for the lower excitations of this model. To this purpose the natural choice for the states are Fock excitations, while we can employ a coherent states representation for the operators [8]. With this combination it is possible to turn (5) into gaussian integrals over complex variables and to solve them explicitly for Λ in terms of the parameters that define the quantum states (see [2, 9] for a detailed description of the procedure applied to a slightly different model). Let us stress that the resulting value of the cosmological constant is determined in a completely non-perturbative way and to our knowledge this is the first case, although two-dimensional, in which the quantum nature of the gravitational interaction is taken into account.

Acknowledgments

SZ benefits from a PhD research grant of the Institut Interuniversitaire des Sciences Nucléaires (IISN, Belgium). This work is supported by the Belgian Federal Office for Scientific, Technical and Cultural Affairs through the Interuniversity Attraction Pole P6/11.

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