

Optimal age specific income taxation

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Abstract

This paper studies optimal earnings taxation in a three period life cycle model where the taxes raised to finance an exogenous amount of public expenditure are allowed to be differentiated across ages. Agents choose their level of education when young and their age of retirement when old. We first look at the problem of optimal taxation when the young can borrow and then turn to the case where young face borrowing constraints. It is shown that, without borrowing constraints, a first best optimum can be decentralized by setting a zero tax rate in the third period and a first period tax lower than the second one. With the borrowing constraint, the government may not be able restore intertemporal efficiency in which case a zero tax rate when old may not be optimal.

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1 Introduction

Should we tax at different rate labor earnings over the life cycle? While there is no evidence of such a tax differentiation at first sight, a proper analysis of the tax system concludes that the implicit tax rate on labor earnings is not constant over the life cycle. The reasons include tax exemptions when young, the progressiveness of the tax system combined with the increasing earning profile (Gervais, 2001) and the tax/social security scheme when old (Gruber & Wise, 1999).

A natural question concerns the optimality of such a tax system. Some recent papers asked what the optimal income tax rates are when it is possible to differentiate them across age.

The first strand of literature is concerned by the optimal Ramsey taxation within a standard life cycle growth model in which agents have age varying tastes for consumption and leisure. This new feature implies a non constant consumption-leisure choice for the representative agent, even in a steady state economy. In this context, an optimizing government always implements an age specific income and consumption tax (Erosa & Gervais, 2001).

A second rationale for age specific earning tax rates is related to the optimum tax theory initiated by Mirrlees (1971) and the use of categorical transfers (Akerlof, 1978; Immonen et al., 1998). When the government cannot observe ability but can observe an exogenous characteristic which is correlated with ability, it can improve the distributional effects of the tax system using lump sum grants. But it can also improve the efficiency of the tax system by conditioning the marginal income tax rates on the observable characteristic. This last criterion is analyzed in Kremer (1997) in the context of age specific income taxation. He concludes that young workers should optimally face lower marginal tax rates than older workers.

The goal of the present paper is to compute optimal age income tax rates within a representative agent model. We thus abstract from any (intra-cohort) redistributive aspect of the tax system. An important aspect of this approach is to give a better understanding of the life cycle decisions using the optimal tax system. In the second approach cited above, the analysis is static while in the first one, age varying taste for consumption and leisure is exogenous. Our model incorporates time varying taste for consumption and labor that is explained by life cycle decisions such as education, retirement and liquidity constraints.

A first concern for age specific tax rates relates to efficiency issues. Two important sources of distortions that arise in life cycle models are the decisions to enter and to leave the labor market. When education is taken into account, the decision to enter the labor market is subject to an opportunity cost, namely the trade-off between higher future income due to the returns of education and the loss of income due to the postponement of

entering the labor market. The second source of distortion has to do with the retirement decision. There is a vast literature studying exit from the labor force and it is generally recognized that both the fiscal and the social security system are likely to distort the retirement decision (e.g Gruber & Wise, 1999). When old, the individual faces a double loss when working an additional year, namely income taxes and foregone retirement benefits partially compensated by an increase in the social security wealth.

A second motivation for age specific income tax rates is the existence of financial constraints. If the capital market is imperfect, there may exist borrowing constraint especially when workers are young and the tax system is likely to affect the strength of this constraint which will in turn affect the individual's decisions. An age specific tax rate could at least partially eliminate this imperfection by imposing some transfers across periods.

The framework adopted in this paper is a simple representative agent model in a partial equilibrium setting. It is assumed that the individual lives for three periods the length of each being normalized to unity for convenience. The individual chooses the amounts of saving in the first and second periods and the fraction of time to devote to education in the first period and to work in the third period. It is further assumed that the individual inelastically supplies one unit of labor in the second period. The government chooses the age specific flat income tax rates which finance an exogenous amount of public expenditure by maximizing the life cycle utility of the representative agent.

The paper proceeds as follows: Section 2 presents in more details our simple life cycle framework. Section 3 and 4 consider respectively the optimal age specific optimal income taxation problem without and with financial constraints. Finally section 5 concludes and discusses the assumptions underlying our model.

2 The model

We use a simple partial equilibrium three period life cycle model. We assume zero interest and discount rate and a zero population growth rate¹. In period 1, the individual supplies a fraction $1 - e$ units of labor and spends a fraction e in education while saving s_1 . In period 2, he supplies inelastically one unit of labor, has return from his first period saving s_1 and decides to save s_2 . In the third period, the individual chooses his retirement date l i.e. the fraction of this period during which he remains at work; his consumption is financed by his earnings and the return from his savings. It is assumed that

¹Implicitly, we assume a stationnary equilibrium within an overlapping generation model. With this assumption, taxing an individual over his life cycle or taxing a society at a given point in time but across generations are identical problems.

the real wage w is an increasing and concave function of education, that is:

$$w = w(e), w'(e) > 0, w''(e) < 0.$$

The life cycle utility function $U(c_1, c_2, c_3, l)$ is assumed to be time and leisure separable:

$$U(c_1, c_2, c_3, l) = u(c_1) + u(c_2) + u(c_3) + v(1 - l),$$

where c_i denotes period i consumption. $u(\cdot)$ and $v(\cdot)$ satisfy the usual properties: $u'(\cdot), v'(\cdot) > 0, u''(\cdot), v''(\cdot) < 0$ where $u'(\cdot)$ and $u''(\cdot)$ denote respectively the first and second derivatives of $u(\cdot)$. The same holds for $v'(\cdot)$ and $v''(\cdot)$. Finally, $\sigma(c_i)$ denotes the elasticity of the marginal utility that is $\sigma(c_i) = -\frac{c_i u''(c_i)}{u'(c_i)}$ which is also the inverse of the elasticity of intertemporal substitution.

This model implies that labor will be elastic in the first and the third period. The assumption of inelastic labor in period 2 may seem somewhat restrictive but labor elasticity should be thought of in terms of "duration in the labor force" rather than "work hours per week" supplied. Note also that the implication of the model can be comforted by empirical evidence. A recent study by Fenge et.al (2002) showed with german data that labor elasticity was U-shaped over the life cycle reaching 0 for middle-age workers.

2.1 The problem of the individual

The individual has to choose the amount of education e , the first and second period amounts of savings s_1 and s_2 and the age of retirement l maximizing his life cycle utility under the constraint that the first period savings must be non negative. We assume throughout the paper that $s_2 \geq 0$. Denoting t_i the period i proportional income tax rate, the problem of the individual can be written as follows:

$$\begin{aligned} \text{Max}_{e,l,s_1,s_2} \quad & u(c_1) + u(c_2) + u(c_3) + v(1 - l), \\ \text{s.to : } c_1 \quad &= (1 - e)(1 - t_1)w(e) - s_1, \\ c_2 \quad &= (1 - t_2)w(e) - s_2 + s_1, \\ c_3 \quad &= l(1 - t_3)w(e) + s_2, \\ s_1 \quad &\geq 0. \end{aligned}$$

Denoting by μ the Lagrangian multiplier associated with the borrowing constraint on s_1 , the first order conditions with respect to e, l, s_1 and s_2 for an interior solution give respectively:

$$[w'(e)(1-e) - w(e)](1-t_1)u'(c_1) + w'(e)(1-t_2)u'(c_2) + w'(e)l(1-t_3)u'(c_3) = 0, \quad (1)$$

$$w(e)(1-t_3)u'(c_3) - v'(1-l) = 0, \quad (2)$$

$$-u'(c_1) + u'(c_2) + \mu = 0, \quad (3)$$

$$u'(c_2) - u'(c_3) = 0, \quad (4)$$

$$\mu \geq 0.$$

Equation (1) states that education is chosen such that the marginal utility loss due to the foregone income in the first period must equate the marginal utility gain from the higher wage in the future. Equation (2) is the usual condition that labor is chosen so that the marginal rate of substitution between consumption and leisure must equate the net labor productivity. Because we don't impose borrowing constraint between the second and the third period, the individual will always equalize consumption in period 2 and 3 while if the borrowing constraint is binding ($\mu > 0$), the individual will set $c_1 < c_2 = c_3$.

Equations (1), (2), (3) and (4) define the optimal e, l, s_1 and s_2 chosen by the individual as implicit functions of the t_i . The indirect life cycle utility function of the individual is then written as $W = W(t_1, t_2, t_3)$ with derivatives:

$$\begin{aligned} \frac{dW}{dt_1} &= -w(e)(1-e)u'(c_1), \\ \frac{dW}{dt_2} &= -w(e)u'(c_2), \\ \frac{dW}{dt_3} &= -lw(e)u'(c_3). \end{aligned}$$

where we used the envelope theorem.

2.2 The problem of the government

We assume that the government wants to finance an exogenous per capita amount of public expenditure R_0 with flat-rate income taxes and maximizes the representative individual's indirect utility function.

The budget constraint of the government is:

$$R_0 = R \equiv w(e)[(1-e)t_1 + t_2 + lt_3]. \quad (5)$$

where R defines the total tax revenue. The government solves:

$$Max_{t_1, t_2, t_3} W(t_1, t_2, t_3) + \lambda [R - R_0].$$

The different optima are computed in the following manner. First, we derive optimal income tax rates with $\mu = 0$, that is assuming either that the individual can borrow or that his optimal saving s_1 is non negative, which we call problem 1.

We then identify the condition for which the individual will choose a negative s_1 given these optimal income tax rates. When this condition is fulfilled and assuming that the individual cannot borrow, we turn to problem 2. We first compute the optimum considering $\mu > 0$ in the individual's problem. If the resulting outcome of this optimum leads to $c_2 > c_1$, the optimum is Regime 1 where the government does not restore intertemporal consumption smoothing. Else, the optimum is Regime 2 with $c_1 = c_2$

3 Problem 1: The problem of the government with no (binding) borrowing constraint

Let consider first that $\mu = 0$ in the individual's problem, that is either the capital market is perfect or the individual chooses a non negative s_1 (in the next section we give condition for the optimal s_1 to be non negative). From (1) to (4), the optimal choices of e and l by the individual are given by the two following equations:

$$\begin{aligned} w'(e) [2 - e + l] - w(e) &= \\ t_1 [(1 - e)w'(e) - w(e)] - w'(e) [t_2 + lt_3] &= 0, \end{aligned} \quad (1')$$

$$w(e)(1 - t_3)u'(c) - v'(1 - l) = 0. \quad (2')$$

where $c = c_i \equiv \frac{1}{3}[lw(e)(1 - t_3) + w(e)(1 - t_2) + (1 - e)w(e)(1 - t_1)]$. Given that interest and discount rates are equal to zero, the individual chooses to equalize consumption $c_i = c$ across the three periods. (1') states that the individual chooses the fraction of time in education that maximizes his life-cycle income.

Let us now turn to the government's problem. A first best allocation (that can be attained with the use of a lump-sum instrument if available) is described by the following equations:

$$\begin{aligned} w'(e) [2 - e + l] - w(e) &= 0, \\ w(e)u'(c) - v'(1 - l) &= 0, \\ R &= R_0. \end{aligned}$$

The first equation states that education is chosen such that the gross return of education equates its cost. The second implies that the age of retirement is chosen such that the marginal rate of substitution between consumption and leisure is equal to labor productivity.

Substituting these first best expressions in (1') and (2'), such an allocation can be decentralized by setting (t_1, t_2, t_3) such that:

$$w'(e) [t_1(1 - e) + t_2 + lt_3] - w(e)t_1 = 0, \quad (6)$$

$$t_3 = 0, \quad (7)$$

$$R = R_0. \quad (5)$$

As it can be inferred by (1'), equation (6) is the condition for education to be not distorted. Clearly, the decision to invest in education is not distorted as long as the tax bill on additional earnings associated with education (first term) is equal to the subsidies to this investment (second term). Note also for future reference that (6) and (7) imply that the individual's decision variables have no marginal effect on tax revenues that is

$$\frac{dR}{de} = w'(e) [t_1(1 - e) + t_2 + lt_3] - w(e)t_1 = 0, \quad (6')$$

$$\frac{dR}{dl} = wt_3 = 0. \quad (7')$$

Combining equation (6) and the fact that the first best education is described by $w'(e) [2 + l - e] = w(e)$, it is straightforward to show that:

$$t_1(1 + l) = t_2 + lt_3.$$

This equation means that the first period tax must be equal to the average income tax in the second and the third period. A uniform proportional tax fulfills this condition but since one must not tax the third period income, the tax rates will be differentiated among the periods. Using the budget constraint of the government (5), we are now able to characterize the optimum:

Proposition 1 *When there is no (binding) borrowing constraint, the use of age dependent income tax rates leads to a first best optimum with:*

$$\begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \begin{pmatrix} \frac{R_0}{w(e)(2+l-e)} \\ t_1(1+l) \\ 0 \end{pmatrix}$$

The first period tax is then the ratio of the per capita public expenditure to the gross working-life wage income. The second period tax will never be lower than the first period tax : the special case of a uniform taxation between the first and the second period arises when the fraction of time devoted to work in the third period is 0. The fact that the first period tax is lower than the second one comes from the introduction of an endogenous retirement age. Because one must not tax the third period income since it would distort the choice of l , income that is not taxed in this period will be taxed in the second period in order not to distort education.

4 Problem 2: The constrained problem of the government

Until now, we considered that the individual was not subject to a borrowing constraint in the first period. We now consider the optimal three period tax structure under the assumption that the individual is liquidity-constrained in period 1 at the solution of problem 1. In the individual's problem:

$$s_1 = \begin{cases} \frac{2}{3}(1-e)(1-t_1)w(e) - \frac{1}{3}(1-t_2)w(e) - \frac{1}{3}lw(e)(1-t_3) & \text{if } \mu = 0, \\ 0 & \text{if } \mu > 0. \end{cases}$$

Using the first order condition (1') of the individual problem, one knows that education is set such that the marginal gain of income in period 2 and 3 must equate the marginal loss of income in period 1. Using this, one can rewrite the optimal saving as follow:

$$s_1 = \begin{cases} \frac{w(e)(1-t_1)}{3w'(e)} [3w'(e)(1-e) - w(e)] & \text{if } \mu = 0, \\ 0 & \text{if } \mu > 0. \end{cases}$$

Defining e_c as the level of education such that $3w'(e_c)(1-e_c) - w(e_c) = 0$, we are able to characterize the condition for which the individual is constrained in the first period.

Proposition 2 *When s_1 cannot be negative, for e evaluated at the optimal vector of taxes described in proposition 1, the individual is constrained if and only if $e > e_c$ where e_c is unique and defined by*

$$3w'(e_c)(1-e_c) - w(e_c) = 0$$

When the individual chooses $e > e_c$, the first period income is lower than future income since the first period labor supply is small relative to the second and third period ones. This makes the individual borrow from future period income.

4.1 Individual optimization

Assume now that $e > e_c$ in the unconstrained problem where e is evaluated at optimal first best taxes presented in proposition 1. The first order conditions for e and l become:

$$\begin{aligned} U_e &= (1 - t_1)[w'(e)(1 - e) - w(e)]u'(c_1) + \\ &\quad w'(e)[(1 - t_2) + l(1 - t_3)]u'(c) = 0, \end{aligned} \quad (1'')$$

$$U_l = w(e)(1 - t_3)u'(c) - v'(1 - l) = 0. \quad (2'')$$

with

$$\begin{aligned} c_1 &= (1 - e)w(e)(1 - t_1), \\ c &= c_2 = c_3 = \frac{1}{2}w(e)[(1 - t_2) + l(1 - t_3)]. \end{aligned}$$

U_e and U_l denote respectively the first order derivatives of the life cycle utility U with respect to e and l . Note that since the constraint $s_1 \geq 0$ is binding, we have $c_1 < c$. The main difference of this setup is that the individual is not able to smooth consumption between the first period and the later ones. Instead of equating the marginal loss and gain of life cycle income in money terms, the individual chooses education by equating the marginal loss and gain of utilities in (1'').

4.2 Comparative statics

In this particular problem, the individual chooses two variables. In analyzing the effect of t_i on these two decision variables, it is useful to distinguish from their direct effects, the indirect effect via the change in the other decision variable. Formally, $\frac{de}{dt_i}$ and $\frac{dl}{dt_i}$ can be decomposed as follows:

$$\begin{aligned} \frac{de}{dt_i} &= C \left[\left. \frac{de}{dt_i} \right|_{U_e=0} + \left(\left. \frac{de}{dl} \right|_{U_e=0} \right) \left. \frac{dl}{dt_i} \right|_{U_l=0} \right], \\ \frac{dl}{dt_i} &= C \left[\left. \frac{dl}{dt_i} \right|_{U_l=0} + \left(\left. \frac{dl}{de} \right|_{U_l=0} \right) \left. \frac{de}{dt_i} \right|_{U_e=0} \right], \end{aligned}$$

where $\left. \frac{de}{dt_i} \right|_{U_e=0}$ and $\left. \frac{dl}{dt_i} \right|_{U_l=0}$ represent the direct uncorrected effects of taxes on e and l . The corrective term $C = \left[1 - \left(\left. \frac{de}{dl} \right|_{U_e=0} \right) \left(\left. \frac{dl}{de} \right|_{U_l=0} \right) \right]^{-1} = \frac{1}{U_{ee}U_{ll} - U_{el}^2}$ is greater than zero given that the second order conditions of the individuals problem holds.

(1'') defines the optimal e for a given l while (2'') defines the optimal l for a given e . Differentiations of (1'') and (2'') give:

$$\begin{aligned}\left.\frac{de}{dl}\right|_{U_e=0} &= -\frac{U_{el}}{U_{ee}} = -\frac{w'(e)(1-t_3)u'(c)(1-\sigma(c))}{U_{ee}} \gtrless 0 \text{ if } \sigma(c) \lesseqgtr 1, \\ \left.\frac{dl}{de}\right|_{U_l=0} &= -\frac{U_{le}}{U_{ll}} = -\frac{w'(e)(1-t_3)u'(c)(1-\sigma(c))}{U_{ll}} \gtrless 0 \text{ if } \sigma(c) \lesseqgtr 1,\end{aligned}$$

where U_{ee} and U_{ll} are strictly negative.

The sign of these effects depends upon $\sigma(c)$. To understand this, observe in equation (1'') that increasing l has a positive substitution effect on e since it increases the marginal gain of income in period 3 by $w'(e)(1-t_3)$. It also has a negative income effect since it increases disposable income in period 3 by $w(e)(1-t_3)$. Recall that $\sigma(c)$ is a measure of the utility cost of variability in consumption. When $\sigma(c) > 1$, the income effect dominates since this utility cost is relatively high so that e reacts negatively to an increase in l . Otherwise, the substitution effect dominates and e reacts positively to an increase in l . By construction, the relation is symmetric for the effect of l on e .

In the following, we present and comment separately the direct uncorrected effects of taxes on the decision variables. Direct uncorrected effects of t_1 are given by:

$$\begin{aligned}\left.\frac{de}{dt_1}\right|_{(U_e=0)} &= \frac{[w'(e)(1-e) - w(e)][1 - \sigma(c_1)]u'(c_1)}{U_{ee}} \gtrless 0 \text{ if } \sigma(c_1) \lesseqgtr 1, \\ \left.\frac{dl}{dt_1}\right|_{(U_l=0)} &= 0.\end{aligned}$$

Increasing t_1 has a positive substitution effect on e since it decreases the marginal loss of income in period 1 by $-(w'(e)(1-e) - w(e))$. It also has a negative income effect on e since it decreases period 1 disposable income by $w(e)(1-e)$. If the individual wants to keep this income constant, his only way for doing so is to reduce e . The positive (resp. negative) effect dominates if $\sigma(c_1)$ is lower (resp. higher) than 1. As explained above, when $\sigma(c_1) > 1$, the utility cost of variability in consumption is relatively high so that income effects dominate.

Similarly, one obtains:

$$\begin{aligned}\left.\frac{de}{dt_2}\right|_{(U_e=0)} &= \frac{w'(e)u'(c)[1 - \sigma(c)]}{U_{ee}} \gtrless 0 \text{ if } \sigma(c) \gtrless 1, \\ \left.\frac{dl}{dt_2}\right|_{(U_l=0)} &= \frac{\left[\frac{1}{2}(w(e))^2(1-t_3)u''(c)\right]}{U_{ll}} > 0.\end{aligned}$$

The direct effects of t_2 on e also includes a substitution and an income effect whose signs are however the opposite of those of the effects of a rise in t_1 . An increase in t_2 has a negative substitution effect since it decreases the marginal gain of income in period 2 by $w'(e)$. It has also a positive income effect since it decreases period 2 disposable income by $w(e)$. The income effect will dominate if $\sigma(c) > 1$. t_2 has only a positive income effect on l since it decreases disposable income in each of period 2 and 3 by $\frac{1}{2}w(e)$

$$\begin{aligned}\left.\frac{de}{dt_3}\right|_{(U_e=0)} &= l \left.\frac{de}{dt_2}\right|_{(U_e=0)}, \\ \left.\frac{dl}{dt_3}\right|_{(U_e=0)} &= l \left.\frac{dl}{dt_2}\right|_{(U_l=0)} + S_{l3}.\end{aligned}$$

where $S_{l3} = \frac{w(e)u'(c)}{U_{ll}} < 0$. Since, by assumption, the individual smooths consumption in period 2 and 3, the effect of t_3 on e and l is the same effect as that of t_2 but multiplied by l . There is however an additional negative substitution effect of t_3 on l measured by S_{l3} since increasing t_3 decreases the marginal productivity of labor by $w(e)$.

4.3 The two regimes

In this particular problem, the government faces two possible regimes. In regime 2, it restores intertemporal equality of consumptions in which case $c_1 = c$ (or equivalently $\mu = 0$). In regime 1, $c_1 < c$ (or equivalently $\mu > 0$). Quite obviously, the optimum is in regime 1 if equating consumptions by the use of the available instruments implies an efficiency loss. We first detail and present in a first part the optimum in regime 1 and then identify the set of parameters for which the optimum is effectively the one in regime 1. The second part details the optimum in regime 2 and the third part illustrates the optimum with a CES utility function.

4.3.1 Regime 1

When $\mu > 0$, the government program leads to first order conditions with respect to t_1 t_2 and t_3 given by:

$$\begin{aligned}-(1-e)w(e)u'(c_1) + \lambda \left[w(e)(1-e) + \frac{dR}{de} \frac{de}{dt_1} + \frac{dR}{dl} \frac{dl}{dt_1} \right] &= 0, \\ -w(e)u'(c) + \lambda \left[w(e) + \frac{dR}{de} \frac{de}{dt_2} + \frac{dR}{dl} \frac{dl}{dt_2} \right] &= 0, \\ -lw(e)u'(c) + \lambda \left[lw(e) + \frac{dR}{de} \frac{de}{dt_3} + \frac{dR}{dl} \frac{dl}{dt_3} \right] &= 0,\end{aligned}$$

where $\frac{dR}{de}$ and $\frac{dR}{dl}$ are given by the left hand sides of (6') and (7').

Combining first order conditions with respect to t_2 and t_3 , one obtains the choice of t_3 relative to the one of t_2 :

$$\frac{dR}{de} \left(l \frac{de}{dt_2} - \frac{de}{dt_3} \right) + \frac{dR}{dl} \left(l \frac{dl}{dt_2} - \frac{dl}{dt_3} \right) = 0. \quad (8)$$

Noting that $-l = \frac{dW/dt_2}{dW/dt_3}$ deals with a welfare balanced change of t_2 and t_3 , this equation means that the effect of a welfare balanced change in t_2 and t_3 on R must equate to zero. Using the comparative static results enables us to write the above equation as²:

$$S_{l3} \left[\frac{dR}{de} \left(\frac{de}{dl} \Big|_{U_e=0} \right) + \frac{dR}{dl} \right] = 0.$$

This final expression simply says that a change in l taking into account the indirect effect on e has no impact on tax revenue. In other words, given that $S_{l3} < 0$, the optimum is to neutralize the deadweight loss due to the substitution effect of t_3 on l by setting

$$\frac{dR}{de} \left(\frac{de}{dl} \Big|_{U_e=0} \right) + \frac{dR}{dl} = 0. \quad (9)$$

Remember that in the first best optimum where the liquidity constraint is not binding, it is optimal to set $\frac{dR}{de}$ and $\frac{dR}{dl}$ equal to zero in (6') and (7'). Now, these first best allocations are not sustainable given the liquidity constraint so that t_3 may be different from zero.

Equivalently, combining the optimality conditions of t_1 and t_2 , one has:

$$\begin{aligned} & \frac{u'(c_1)(1-e)}{u'(c)} \left(\frac{dR}{de} \frac{de}{dt_2} + \frac{dR}{dl} \frac{dl}{dt_2} + w(e) \right) \\ & - \left(\frac{dR}{de} \frac{de}{dt_1} + \frac{dR}{dl} \frac{dl}{dt_1} + (1-e)w(e) \right) = 0. \end{aligned} \quad (10)$$

As before, a welfare balanced change in t_1 and t_2 , has no effect on total tax revenue. Given that period 1 and 2 consumptions are different, this equation reflects the trade-off between efficiency and the balance between period 1 and other periods consumptions. Since l is chosen not to have any (net) impact on R by (9), every effects on R via a change in l will vanish. Therefore, using comparative statics, (10) can be written as² :

$$\begin{aligned} & w(e) \left(\frac{(1-e)u'(c_1)}{u'(c)} - (1-e) \right) \\ & + \frac{dR}{de} \left(\frac{(1-e)u'(c_1)}{u'(c)} \left(\frac{de}{dt_2} \Big|_{U_e=0} \right) - \left(\frac{de}{dt_1} \Big|_{U_e=0} \right) \right) = 0. \end{aligned} \quad (11)$$

²See the appendix for a more detailed computation.

The first term of this equation represents the tax revenue gain associated with an increase in t_2 combined with a balanced welfare decrease in t_1 . The second term represents the revenue loss associated with such a change via the effect on education. Again, remember that in the first best optimum presented in section 2.1, one has $\frac{dR}{de} = 0$ and $c_1 = c$ so that the two terms of equation (13) are equal to zero. Of course, in regime 2, $c_1 < c$, so that the first term is positive. Therefore, a necessary and sufficient condition to be in regime 2 is that the second term of (11) be negative, that is that a decrease in t_1 combined with a welfare balanced increase of t_2 has a negative effect on tax revenue. Moreover, optimality requires necessarily that $\frac{dR}{de} > 0$ ³. The condition for which regime 2 is the optimum is then

$$\frac{(1-e)u'(c_1)}{u'(c)} \left(\frac{de}{dt_2} \Big|_{U_e=0} \right) - \left(\frac{de}{dt_1} \Big|_{U_e=0} \right) < 0.$$

Multiplying both sides of the inequality by $u'(c)$ and using the comparative static results, this is equivalent to:

$$w'(e)(1-e)(1-\sigma(c)) - (w'(e)(1-e) - w(e))(1-\sigma(c_1)) > 0. \quad (12)$$

Inequality (12) describes the condition for which an increase in t_2 combined with a balanced welfare decrease in t_1 decreases the level of education.

When (12) is not fulfilled, then the optimum is in regime 2 which we now characterize.

4.3.2 Regime 2

In this regime, the government implicitly makes the individual chooses his optimal s_1 to be equal to 0. It implies that education is given by e_c where $3w'(e_c)(1-e_c) - w(e_c) = 0$. In other words, first order conditions of the individual's problem can be rewritten as

$$\begin{aligned} 3w'(e_c)(1-e_c) - w(e_c) &= 0, \\ w(e_c)(1-t_3)u'(c) - v'(1-l) &= 0, \\ c_i &= c. \end{aligned}$$

The way to reach the optimum is straightforward: the government taxes the second period income and subsidizes the first period income to the point where $c_1 = c$. Moreover, since t_3 has a distortive effect via the substitution effect on l , t_3 is set to 0 (there is no indirect effect on e since education is given by the binding borrowing constraint).

We are now able to fully characterize the constrained optimum of the government:

³Assume instead that (1''), (2''), (5), (9) and (11) describes an optimum with $c_1 < c$ and that $\frac{dR}{de} < 0$. Then a marginal decrease of e would create a budget surplus and relax the borrowing constraint. This contradicts the fact that it is an optimum.

Proposition 3 *If (12) is fulfilled, the optimum is regime 1 given by equations (1)'', (2)'', (5) and:*

$$w(e)t_3 = -\frac{dR}{de} \frac{de}{dl} \Big|_{U_e=0} , \quad (9)$$

$$w(e) \left(\frac{(1-e)u'(c_1)}{u'(c)} - (1-e) \right) = \frac{dR}{de} \left(\left(\frac{de}{dt_1} \Big|_{U_e=0} \right) - \frac{(1-e)u'(c_1)}{u'(c)} \left(\frac{de}{dt_2} \Big|_{U_e=0} \right) \right) , \quad (11)$$

where $\frac{dR}{de} > 0$.

Otherwise, the optimum is regime 2 given by (1)', (2)', (5) and:

$$c_i = c \text{ for every } i, \quad (13)$$

$$t_3 = 0 \quad (14)$$

In regime 1, the aim is to neutralize the deadweight loss due to the substitution effect of t_3 on l (equation (9)), and to redistribute income from the second to the first period at the cost of distorting downward the level of education (equation (11)). Condition (12) describes the condition for which the redistributive objective goes against this efficiency consideration. In analyzing inequality (12), when both $\sigma(c_1)$ and $\sigma(c)$ are lower than 1, (c_1 and c are substitutes), regime 1 is optimal since relaxing the borrowing constraint by reducing t_1 and increasing t_2 reduces the level of education. On the contrary, if $\sigma(c_1)$ and $\sigma(c)$ are greater than 1 (c_1 and c are complements), regime 2 is optimal. When $\sigma(c_1) > 1$ and $\sigma(c) < 1$, things are more complicated since a rise in t_2 has a negative effect on education and a decrease in t_1 has a positive effect on education. The total effect of a welfare balanced change of t_1 and t_2 on education depends upon the difference between $\sigma(c_1)$ and $\sigma(c)$ but also upon the relative return of education in period 1. When the return of education in period 1 $\frac{w'(e)(1-e)-w(e)}{w'(e)(1-e)}$ is high in absolute value (i.e. an increase in education implies a high loss in period 1 income), the positive effect on e caused by the decrease of t_1 dominates the negative one caused by the welfare balanced increase in t_2 and regime 2 is the optimum. A symmetric argument applies to the case where $\sigma(c_1) < 1$ and $\sigma(c) > 1$.

When (12) is not fulfilled, the government can use first and second period taxes to erase the liquidity constraint without efficiency loss and does not tax the third period income.

The third period tax is nil in regime 1 while it may be either positive or negative in regime 2. By (9), it is immediate to see that the sign of t_3 is the opposite one of $\frac{de}{dl} \Big|_{U_e=0}$. The following corollary identifies the sign of t_3 in regime 1:

Corollary 4 When (12) is fulfilled (regime 1), $t_3 \lesseqgtr 0$ if $\sigma(c) \lesseqgtr 1$.

When $\sigma(c) = 1$, l has no indirect effect on R via a change in e . In this case, a zero third period tax rate eliminates the deadweight loss due to S_{l3} . When $\sigma(c) < 1$, consumptions are relatively high substitutes so that e and l are positively correlated. It means that a decrease in l due to S_{l3} decreases indirectly education and hence tax revenues. The way to correct this distortion is to implement a negative third period tax. A symmetric reasoning concludes that the third period tax should be positive in the opposite case where $\sigma(c) > 1$.

4.4 The CES utility function example

A clear cut example arises when one considers a CES type utility function. In this case, the elasticity of marginal utility is constant that is $\sigma(x) = \sigma$ for every x . Optimum in Regime 2 is then described by (1''), (2''), (5) and

$$w(e)t_3 = \frac{w'(e)(1-t_3)u'(c)(1-\sigma)}{U_{ee}}$$

$$w(e) \left[\frac{(1-e)u'(c_1)}{u'(c)} - (1-e) \right] = -\frac{dR}{de} \frac{(1-\sigma)w(e)u'(c_1)}{U_{ee}}$$

When $\sigma < 1$, consumptions are relatively high substitutes. Relaxing the borrowing constraint by decreasing the disposable income in period 2 and increasing the one in period 1 makes the individual choose a lower level of education since the substitution effects dominate. This implies an efficiency loss which goes against the redistributive objective so that regime 2 is optimal. Moreover, the government implements a negative income tax rate in the third period in order to neutralize the substitution effect of t_3 on l .

On the contrary, when $\sigma \geq 1$, the relative high complementarity between consumptions does not imply a decrease of education when the government relaxes the borrowing constraint by the use of t_1 and t_2 . The optimum is then regime 1 where the government uses first and second period tax rates in order to eliminate the liquidity constraint and does not tax the third period.

5 Conclusion

This paper has studied optimal flat rate income taxation when the rates are allowed to vary with the individual's age. A three period life cycle model was built in which the decision to educate in the first period and retire in the third period were endogenous. Without liquidity constraint, the optimal income tax rates are such that the third period tax should be 0 and that the first period tax should be positive but lower than the

second one. With borrowing constraints, it was shown that under some circumstances, the government could use only the two first period tax rates to restore intertemporal equity without efficiency loss. Whereas, for certain values of the elasticity of marginal utility and return of education, restoring intertemporal equity is too costly and a tax or a subsidy on the third period income may be desirable in order to eliminate the compensated deadweight loss implied by the age of retirement decision.

This analysis, while having an endogenous time structure, is quite restrictive in that tax instruments are limited to wage tax rates. This analysis has to be part of a Ramsey tax problem in a general equilibrium framework. A result of Erosa and Gervais (2001) is that an interest income tax combined with an age specific wage tax rate is always desirable when the consumption-leisure path is not constant over the life cycle. Combining their approach with the present model would be very instructive to see how their result holds within a model that incorporates better individuals decisions over the life cycle.

Second, the model could be extended to the case where one allows redistribution between agents of the same cohort. This time, one expects that the third period tax is different from 0 given the desirability to redistribute income within generations.

Finally, the model could serve as a basis to study optimum taxation à la Mirrlees but this must await a dynamic version of it.

Appendix

A Derivation of expression (9)

Using (8) and decomposing direct and indirect effects, one has:

$$\begin{aligned} & \frac{dR}{de} \left(\begin{aligned} & l \frac{de}{dt_2} \Big|_{U_e=0} - \frac{de}{dt_3} \Big|_{U_e=0} \\ & + \left(\frac{de}{dl} \Big|_{U_e=0} \right) \left(l \frac{dl}{dt_2} \Big|_{U_l=0} - \frac{dl}{dt_3} \Big|_{U_l=0} \right) \end{aligned} \right) \\ & + \frac{dR}{dl} \left(\begin{aligned} & l \frac{dl}{dt_2} \Big|_{U_l=0} - \frac{dl}{dt_3} \Big|_{U_l=0} \\ & + \left(\frac{dl}{de} \Big|_{U_l=0} \right) \left(\frac{de}{dt_2} \Big|_{U_e=0} - \frac{de}{dt_3} \Big|_{U_e=0} \right) \end{aligned} \right) = 0. \end{aligned} \quad (A_1)$$

(16)

By the comparative statics, we have:

$$\begin{aligned} l \frac{de}{dt_2} \Big|_{U_e=0} &= \frac{de}{dt_3} \Big|_{U_e=0}, \\ l \frac{dl}{dt_2} \Big|_{U_l=0} - \frac{dl}{dt_3} \Big|_{U_l=0} &= -S_{l3}. \end{aligned}$$

Thus, (A₁) simplifies to:

$$\frac{dR}{de} \left(\frac{de}{dl} \Big|_{U_e=0} \right) S_{l3} + \frac{dR}{dl} S_{l3} = 0$$

Factorizing by S_{l3} leads to equation (9).

B Derivation of expression (11)

Let decompose the effect of the taxes on tax revenue via the decision variables. One has:

$$\begin{aligned}
& \frac{dR}{de} \frac{de}{dt_i} + \frac{dR}{dl} \frac{dl}{dt_i} = \\
& C \frac{dR}{de} \left(\frac{de}{dt_i} \Big|_{U_e=0} + \frac{de}{dl} \Big|_{U_e=0} \frac{dl}{dt_i} \Big|_{U_l=0} \right) \\
& + C \frac{dR}{dl} \left(\frac{dl}{dt_i} \Big|_{U_l=0} + \frac{dl}{de} \Big|_{U_l=0} \frac{de}{dt_i} \Big|_{U_e=0} \right) + = \\
& C \frac{de}{dt_i} \Big|_{U_e=0} \left(\frac{dR}{de} + \frac{dR}{dl} \frac{dl}{de} \Big|_{U_l=0} \right) \\
& + C \frac{dl}{dt_i} \Big|_{U_l=0} \left(\frac{dR}{de} \frac{de}{dl} \Big|_{U_e=0} + \frac{dR}{dl} \right) = \\
& C \frac{de}{dt_i} \Big|_{U_e=0} \left(\frac{dR}{de} + \frac{dR}{dl} \frac{dl}{de} \Big|_{U_l=0} \right)
\end{aligned}$$

where we used (9). Using again (9) for the expression in brackets, we can rewrite:

$$\frac{dR}{de} \frac{de}{dt_i} + \frac{dR}{dl} \frac{dl}{dt_i} = C \frac{de}{dt_i} \Big|_{U_e=0} \frac{dR}{de} \left(1 - \frac{de}{dl} \Big|_{U_e=0} \frac{dl}{de} \Big|_{U_l=0} \right)$$

Given that $C = \frac{1}{1 - \frac{de}{dl} \Big|_{U_e=0} \frac{dl}{de} \Big|_{U_l=0}}$, we finally obtain:

$$\frac{dR}{de} \frac{de}{dt_i} + \frac{dR}{dl} \frac{dl}{dt_i} = \frac{de}{dt_i} \Big|_{U_e=0} \frac{dR}{de} \quad (A_2)$$

Inserting this expression in (10) and rearranging the terms leads to equation (11).

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