MONOPOLY EXTRACTION OF AN EXHAUSTIBLE RESOURCE UNDER ALTERNATIVE BACKSTOP TECHNOLOGY ENTRY DATES

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Abstract

It is shown that when a backstop technology is developed guaranteeing negligible post-entry demand for an exhaustible resource, specifically accounting for the inequalities on the positivity restriction and exhaustibility constraint when a resource monopolist faces alternative backstop entry dates results in a number of extraction profiles, each of which is optimal.

I. Introduction

Studies concerned with monopoly extraction of exhaustible resources frequently neglect two important aspects of the problem - (i) the influence of alternative backstop technology entry dates on production, and (ii) the implications of (i) in terms of the inequalities on the positivity restriction and exhaustibility constraint.[1]

There is a tendancy to assume (often implicity) that a single optimal solution satisfies the necessary conditions in the producer's problem (as in Burness (1976) Kamien & Schwartz (1977), Lewis (1976), and Stiglitz (1976)). However, while for specific cases of the monopoly problem suppressing the inequalities yields unique optimal solutions, we show that in the context of uncertain information concerning backstop entry a rigorous formulation of the problem results in a number of extraction profiles, each of which is optimal. Heal (1976) and Hoel (1978), following Nordhaus (1973), study cases in which there is a unique date at which a substitute produced by a backstop technology appears. In contrast we : a) presume that the backstop is developed in order to guarantee negligible post-entry demand for the resource, and b) consider alternative dates at which it is implemented.

The following section sets forth the assumptions of the model. The optimal extraction trajectories are developed in section III. Section IV includes a discussion of the results. Section V concludes.

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(1) The positivity restriction insures that output is non-negative while the exhaustibility constraint states that the sum of depleted reserves cannot exceed the total known stock.

II. The Model

Consider the problem of a cartel which controls the total known stock (S) of a nonrenewable natural resource. The cartel (effectively a monopolist in the market) faces a downward sloping market demand function, $p(Q(t))$, where p is the price at which the flow $Q(t)$ is sold at time t. Assuming away market imperfections in the rest of the economy, the cartel's objective is to maximize the discounted present value of the flow of profits derived from selling the resource.

As the price of an exhaustible resource rises, so does the incentive to develop substitutes and/or alternative technologies. Illustrations of this point (using the example of the recent oil price hikes) include the continued development of nuclear power, consideration of large-scale coal gassification plants, and the exploitation of tar sands, along with a variety of other changes presumably aimed at reducing the dependance of resource using countries on the cartel product.

This factor enters our analysis on two levels :

(i) It suggests that the absolute value of the elasticity facing the resource supplier is likely to be an increasing function of his price (the linear demand curve has this property and is convenient to use in the following analysis);[2] and

(ii) using the concept of a backstop technology characterized earlier, we observe the cartel's output response to a selection of dates at which this substitute might make its appearance.

In order to simplify the analysis, suppose that the numeraire asset earns a constant competitive rate of interest, $r(>0)$, and that the resource can be extracted costlessly. The cartel's policy is then to choose an extraction plan, $Q(t)$, so as to maximize :

$$\int_o^T e^{-rt} \left[p(Q(t),t)Q(t)\right]dt$$

(2) In any case a linear approximation to the "true" demand curve can be taken over some interval. (The choice of demand function is further discussed in section IV).

subject to the positivity restriction, $Q(t) \geq 0$, and the exhaustibility constraint $K(T) \leq S$, where $K(t)$ represents the reserves extracted to time t (such that $K(T) = \int_0^T Q(t)dt$ ie the total amount of reserves recovered in the relevant period).

The current value Hamiltonian for the problem is

$$H = \left[(\alpha - \beta Q)Q - \lambda Q\right]e^{-rt} \qquad (\alpha,\beta) > 0$$

where α and β are stationary parameters of the demand schedule.(3)

The shadow price $\lambda(t)$ represents the marginal value of an additional unit of the stock at time t. For each unit of the resource produced, $\lambda(t)$ is lost from the current price, $p(t)$. Dynamic profits must therefore take into account the loss incurred in extracting the resource (see Hotelling (1931)).

Together, necessary conditions for a maximum require that :

(1) $\dot{\lambda} = r\lambda$ (or $\dfrac{\dot{\lambda}(t)}{\lambda(t)} = r$)

(2) there exist an optimal extraction profile, $Q^*(t)$, over which H will be maximized with respect to $Q(t)$ $(\forall Q > 0)$, and

(3) a) $\lambda(T) \geq 0$

b) $\lambda(T)\ (S - K^*(T)) = 0$

Condition (1) implies that the initial shadow price rises at the rate of interest or (integrating (1)),

(1') $\lambda(t) = \lambda(0)\ e^{rt}$.(4)

Condition (2) implies the following :

(2') $$Q^*(t) = \begin{cases} \text{a) } \dfrac{1}{2\beta}\left[\alpha - \lambda(0)e^{rt}\right] & \text{if } \alpha > \lambda(0)e^{rt} \\ \text{b) } 0 & \text{if } \alpha \leq \lambda(0)e^{rt}. \end{cases}$$

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(3) An alternative to the stationarity hypothesis would be to assume that the demand curve shifts uniformly over time, or that $p(t) = f(t)\ p(Q(t))$.

(4) This is analogous to what Scott (1967) termed "user costs".

As long as output is nonzero, $\left.\frac{\partial H}{\partial Q}\right|_{Q > 0} = 0$, and the optimal extraction profile is given by (2'a). However, the inequality on the positivity constraint requires $\left.\frac{\partial H}{\partial Q}\right|_{Q=0} < 0$ suggesting that (2'a) is smaller than or equal to zero (ie $\alpha \leq \lambda(0)e^{rt} \Rightarrow Q^*(t) \leq 0$), but since the accumulation of reserves is not permitted in the model, (2'b) holds ie $Q^*(t) = 0$.

Transversality condition (3b) includes the stock constraint $K(T) \leq S$ which states that the sum of optimal extraction over the relevant interval $[0,T]$ cannot exceed the initial endowment, S, ie $S-K^*(T) \geq 0$. Analogous to conventional complementary slackness conditions $\exists a \geq 0$ such that $a(S-K^*(T)) = 0$. Since, from (3a), $a = \lambda(T) \geq 0$, $\lambda(T)\ (S-K(T))$ must be identically equal to zero. If the terminal date of extraction, T, is such that optimal depletion results, then $\lambda(T) = 0$ (since the marginal value placed on an additional unit of the resource when production ceases is zero). However, the inequality on the exhaustibility constraint, $S \geq K^*(T)$, implies that there may be reserves remaining at the end of period T.

III. Optimal Depletion Trajectories

We must now verify all possible trajectories satisfying conditions (1)-(3). The first case we consider is when $\lambda(0) = 0$. This has a number of implications. From (2') we know, $(\forall t < T)\ Q^*(t) = \frac{\alpha}{2\beta} > 0$, which is just the static optimum. The cartel will extract the resource at the static equilibrium level since, at any time t, the marginal value of an additional unit of its stock is identically zero (ie no loss is incurred in extracting the resource). Herfindahl (1965) suggests that this might occur if the future exerts so little pressure on present decisions that user costs are negligible. Another explanation provides that, if there appears to be a fairly abundant resource base a monopolist might ignore depletion in extracting the resource. In the context of our model an alternative explanation appears. A monopolist may suppress the depletion aspect of the problem in the face of backstop entry.

This can be seen as follows. From (3) ; $K^*(T) = \frac{\alpha T}{2\beta} \leq S$ which implies $T \leq T_o = \frac{2\beta S}{\alpha}$. The exhaustion date of the resource is calculated at the optimal rate of depletion, $\frac{\alpha}{2\beta}$. If a backstop technology comes into operation before the resource stock is exhausted, then the problem is reduced to maximizing instantaneous profits at each time t. (An implicit assumption often found in the literature is that $T = T_o$ ie $K^*(T) = S$).

It is now left to verify the necessary conditions with $\lambda(0) > 0$ [5]. Since in this second case $\lambda(T)$ is strictly greater than zero, condition (3b) now implies exhaustion of the resource at T, or (4) $K^*(T) = S$. Interpreting (2') in this context is slightly more difficult, but leads to two interesting subcases.

Let $T^*\big|_{\lambda(0) > 0}$ (representing some time at which production is shut down) satisfy $\alpha = \lambda(0)e^{rT^*}$. Then for $t < T^*$, production will occur as in (2'a) ie $Q^*(t) = \frac{1}{2\beta}(\alpha - \lambda(0)e^{rt}) > 0$, while for $t \geq T^*$, (2'b) will hold and $Q^*(t) = 0$. The problem is to know whether $T^* \gtreqless T$ (where T satisfies (4) ie exhaustion). We will consider in turn i) $T > T^*$ and ii) $T \leq T^*$.

Consider the first of these two subcases ie where $T > T^*\big|_{\lambda(0) > 0}$. From (3) we have $S = K^*(T) = \int_0^{T^*} Q^*(t)dt + \int_{T^*}^{T} Q^*(t)dt = \int_0^{T^*} \frac{1}{2\beta}(\alpha - \lambda(o)e^{rt})dt$ (since production ceases $\forall t > T^*$).

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(5) Since transversality condition (3a) implies $\lambda(T) = \lambda(0)e^{rt} \geq 0$, we are not concerned with the case of $\lambda(0) < 0$.

Substituting $\lambda(0) = \alpha e^{-rT^*}$ into the above yields $S = \frac{\alpha T^*}{2\beta} - \frac{\alpha e^{-rT^*}}{2\beta r}(e^{rT^*} - 1)$, or (4) $rT^* + e^{-rT^*} = \frac{2\beta rS}{\alpha} + 1$.

In order to solve for T^*, consider a function $f(x) = x + e^{-x}$, where $f'(x) = 1 - e^{-x} > 0$ (for $x > 0$). Define the inverse of the function as $g = f^{-1}$ on $+ [1,\infty[$.

Then, $f(r\hat{T}^*) = 1 + \frac{2\beta rS}{\alpha}$ and $rT^*\big|_{\lambda(0)} = g(1 + \frac{2\beta rS}{\alpha})$ such that $T^*\big|_{\lambda(0)} = \hat{T} \equiv \frac{1}{r}\, g\,(1 + \frac{2\beta rS}{\alpha})$, a constant. Since by assumption $T > T^*$, $T > \hat{T}$. Thus there exist trajectories verifying the necessary conditions with $\lambda(0) > 0$ such that $T^*\big|_{\lambda(0)} < T$ iff $T > \hat{T}$ and $\lambda(0) = \alpha e^{-r\hat{T}}$ $(\hat{T} \le t \le T)$.

The second subcase $T \le T^*\big|_{\lambda(0)} > 0$ implies $Q(t) = \frac{1}{2\beta}(\alpha - \lambda(0)e^{rt}) > 0$ for $t < T$. Since $K(T) = S$, we can write

$S = \int_0^T \frac{1}{2\beta}(\alpha - \lambda(0)e^{rt})dt = \frac{\alpha T}{2\beta} - \frac{\lambda(0)}{2\beta r}(e^{rt} - 1)$. Solving for the initial value of the costate variable yields $\lambda(0) = \frac{1}{e^{rt}-1}(\alpha rt - 2\beta rS)$. But $\lambda(0) > 0 \Leftrightarrow \alpha rT > 2\beta rS$, $T > T_o$ and since by assumption $T < T^*\big|_{\lambda(0)}$ $\lambda(0)e^{rT} \le \lambda(0)e^{rT^*} = \alpha$. Thus $\alpha e^{-rT} \ge \lambda(0) = \frac{1}{e^{rT}-1}(\alpha rT - 2\beta rS)$ or, analogous to the previous subcase, (5) $rT + e^{-rT} \le \frac{2\beta rS}{\alpha} + 1$.

The inverse of the function $f(rT)$ is $rT \le g(\frac{2\beta rS}{\alpha} + 1)$. So $\hat{T} \ge T$. Furthermore, if $r\hat{T} = g(1+rT_o)$ and $f(r\hat{T}) = 1 + rT_o$, then $1 + rT_o = r\hat{T} + e^{-r\hat{T}}$ $< r\hat{T} + 1$ and $T_o < \hat{T}$.

The three cases considered are characterized by

(A) $\lambda(0) = 0$, $T \le T_o$
(B) $\lambda(0) > 0$, $T > \hat{T}$
(C) $\lambda(0) > 0$, $T_o < T \le \hat{T}$.

The necessary conditions give for the first case

(6) $Q^*(t) = \frac{\alpha}{2\beta}$; $K^*(T) \leq S$.

For case (B),

(7) $Q^*(t) = \frac{1}{2\beta}(\alpha - \lambda(0)e^{rt})\Big|_{\lambda(0)=\alpha e^{-r\hat{T}}}$ or $Q^*(t) = \left[\frac{\alpha}{2\beta}\left(1-e^{-r(\hat{T}-t)}\right)\right]$

$\forall\ 0 \leq t < \hat{T}$,

and

$Q^*(t) = 0$

$\forall\ t > \hat{T}$.

Finally, case (C) implies

(8) $Q^*(t) = \frac{1}{2\beta}(\alpha - \lambda(0)e^{rt})\Big|_{\lambda(0)=\frac{1}{e^{rT}-1}(\alpha rT - 2\beta rS)}$

and $K^*(T) = S$.

For each of the three cases there exists a unique trajectory verifying the necessary conditions. Furthermore, due to the concavity of the problem, the trajectory in each case <u>is</u> the optimal solution.

IV. Results of the Model

In order to illustrate the preceeding analysis suppose that a backstop technology exists which is currently unavailable, but which will replace the cartel product at some uncertain date, T.[6] Recorded in Figure 1 are the cartel's alternative production responses to information giving the date of implementation as :

(A) $T_1 \leq T_o$, (B) $T_2 \geq \hat{T}$, or (C) $T_o < T_3 < \hat{T}$.

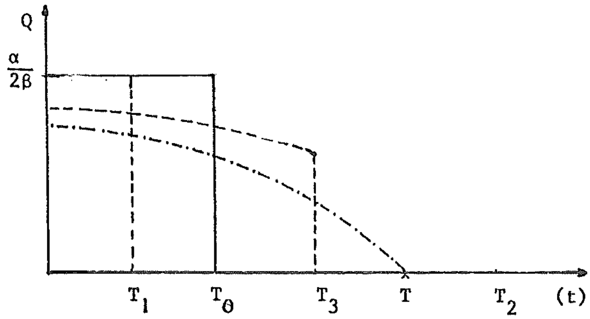

Figure 1

If the backstop makes its appearance prior to T_o, then the problem is a static one, and output will be constant over time at the level $Q(t) = \frac{\alpha}{2\beta}$ until production ceases at T_1. Furthermore the resource will only be exhausted if T_1 happens to identically coincide with T_o.

On the other hand, if the backstop occurs sometime after $\hat{T}$ output will strictly decrease until exhaustion at $\hat{T}$, and is zero beyond. Thus resource owners (the cartel) may, under conditions implied by the model, find it optimal to deplete their reserves prior to the appearance of a backstop technology.

The final possibility is that the backstop enters at some intermediate period between T_o and $\hat{T}$. Complete exhaustion will be the optimal policy throughout this range and output, with an initial value between that of the previous cases ((A) and (B)), will be strictly decreasing until it is achieved. This is the standard result most often cited in the literature.

(6) As alluded to earlier, following implementation of the backstop, at no price will there be significant demand for the resource.

The first result, in which the static optimum appears comes from a failure to account for depletion of the stock. In contrast to explanations put forward by Herfindahl (1965) and others, the model suggests that a monopolist suppresses the depletion aspect of the problem due to the imminence of backstop entry. If implementation of the backstop is thought to occur close enough to the starting date of production, the cartel monopolist will pursue a static policy of extraction until production is shut down ie with the event of backstop entry (which will only coincidentally be linked with exhaustion).

Interpreting the second case, where exhaustion occurs prior to backstop entry, requires a closer look at the restrictions placed on the demand function. The results obtained are linked with the choice of the demand equation and may be valid over a limited domain. In particular, as exhaustion is approached it is unlikely that a simple linear specification will continue to hold. Demand may prove to be very inelastic around this point (eg price would likely approach infinity if a total embargo on oil were suddenly announced). Consequently, interpreting the model with no substitute until backstop entry, and complete substitution beyond that date, departs from the domain in which the linear specification is valid.

However, what one observes (at least with respect to oil) is a number of alternative energy sources whose share in satisfying world demand is gradually on the increase. Found alongside this existing bundle are anticipated ultimate backstops (such as controlled nuclear fusion). Now consider the problem of a cartel monopolist facing both aspects of the substitute. Suppose,

(i) There exists a perfect substitute which is currently available at a given constant price, $\bar{p}$,[7] and

(ii) there exists a date T at which another substitute - the ultimate backstop - becomes available.

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[7] Alternatively, allowing for technological advances, we might consider $\bar{p}$ to be a decreasing function of time.

The production cost of the existing substitute, $\bar{p}$, plays the role of a limit price somewhat analogous to that in Gaskin's (1971) treatment of barriers to entry.

Demand for the cartel product (corresponding to its share of total demand eg for energy sources), is assumed proportional to the spread between the limit price and the market price of the resource.

In order to allow the existing substitute to play a role even if its production cost, $\bar{p}$, is greater than the cartel price, we specify the following demand function.

$$Q(p(t)) = a(\bar{p} - p(t)) \qquad 0 < a < \infty$$ [8].

If we further suppose that use of the alternative energy source increases with a reduction in the spread between p and $\bar{p}$ (such that the substitute is fully utilized when the two are identical) this insures that, while total energy consumption is nondecreasing, the proportion supplied by the cartel diminishes.

Finally, inverting the stationary market demand curve yields,

$$p(t) = \bar{p} - 1/a\ Q(t)$$

which is simply our original specification with $\alpha = \bar{p}$ and $\beta = 1/a$.

Thus returning to case (2), where exhaustion occurs prior to backstop entry, it is now clear that α represents the price of some "carryover" technology (or other existing substitute) which is capable of replacing the exhausted resource at $\hat{T}$ until a backstop appears at T_2.

[8] Use of a more expensive substitute is likely to occur for both political and strategic reasons, for example : (i) in order to satisfy policies of energy independence, (ii) to reverse current account imbalances, or (iii) in the interest of National Security.

V. Final Remarks

Clearly a collusive cartel's impression of the strategy adopted by non-cartel countries concerning the development of substitutes (and/or backstop technologies) will help shape the extractive groups depletion policy.(9) Specifically, rather than choosing alternative (deterministic) backstop technology entry dates one might wish to formulate the problem stochastically.

However, assuming date T to be nonstochastic is equivalent to a random date $\tilde{T}$ occuring with certainty at least at time T, and attaching a low associated exponential probability. Thus maximizing the expected value of the cartel's discounted profits over an uncertain interval 0 to $\tilde{T}$ (ie max $E\left[\int_0^{\tilde{T}} e^{-\delta t}\, \Pi(t)dt\right]$) is identical to the deterministic case where the interest rate incorporates an element of risk (ie since $\max \int_0^T E\left[t < \tilde{T}\right] e^{-\delta t}\, \Pi(t)dt = \max \int_0^T e^{-\rho t}\, e^{-\delta t}\, \Pi(t)dt$).

It has been shown that specifically accounting for the inequalities on the positivity restriction and exhaustibility constraint when a resource monopolist faces uncertain information concerning backstop entry yields a number of extraction profiles, each of which is optimal. This result suggests the existance of a dual problem, facing resource-importing countries developing a backstop technology which guarantees negligible post-entry demand for the resource. Indeed it would be desirable to link the supply side partial equilibrium analysis presented here, with a demand side analysis in which the date of backstop entry is a choice variable.

(9) In which case perhaps technologically advanced resource-using countries might do well to strategically announce (or conceal) the entry date of its backstop technology in order to obtain a desired production response from a "less advanced" group of resource-producing countries.

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