

Nursing Home Choice, Family Bargaining and Optimal Policy in a Hotelling Economy*

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Abstract

We develop a model of family bargaining to study the impact of the distribution of bargaining power within the family on the choice of nursing homes by families, and on the locations and prices chosen by nursing homes in a Hotelling economy. In the baseline (static) model, where the dependent parent cares only about the location of the nursing home, the mark up of nursing homes is increasing in the bargaining power of the dependent parent, and nursing homes are located at the extreme periphery. We compare the laissez-faire with the social optimum (which involves more central locations of nursing homes), and examine its decentralization in first-best and second-best settings. We explore the robustness of our results to introducing a bequest motive in a dynamic OLG model, which allows us to study the joint dynamics of wealth accumulation and nursing home prices. If the bequest motive is strong, the mark up is decreasing in the bargaining power of the dependent. However, wealth accumulation, by reducing interest rates, raises mark up rates and nursing homes prices.

Keywords: Family bargaining, long term care, nursing homes, spacial competition, optimal policy, OLG models.

JEL codes: D10, I11, I18.

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1 Introduction

Due to the ageing process, the provision of long-term care (LTC) to the dependent elderly has become a major challenge for advanced economies. According to the EU (2015), the number of old dependent persons in the Euro Area is expected to grow from about 27 million in 2013 to about 35 million by the year 2060. Although that forecast depends on scenarios concerning mortality and disability trends, it is nonetheless widely acknowledged that, whatever the scenarios are, there will be a substantial rise in LTC needs in the next decades.¹

Nursing homes are important agents in the provision of LTC, especially when serious degrees of dependency are reached. Brown and Finkelstein (2009) show that the probability to enter a nursing home at one point in one's life is large, and lies between 35 % and 50 %. Moreover, because of the increase in the prevalence of serious old-age pathologies, the demand for institutionalized LTC services is expected to grow sharply in the years to come.

At the empirical level, the determinants of nursing home choices were studied by Schmitz and Stroka (2014), on the basis of data from the German sickness fund *Techniker Krankenkasse* on 2,534 people above age 65 who newly moved to a nursing home in 2010. Matching those individual data (on chosen nursing home, zip code before moving, and care level) with institutional data (on prices and reported quality of nursing homes), Schmitz and Stroka (2014) showed that two dimensions drive the choice of nursing homes: location and price. The probability of choosing a nursing home is decreasing in the distance (with respect to the previous household) and in the price.² On the contrary, the reported quality has no significant impact on the choice of nursing homes.

From the perspective of microeconomic theory, the choice of nursing homes raises specific difficulties. A major difficulty lies in the fact that the main beneficiary of the nursing home is dependent, and faces cognitive and/or functional limitations. Because of his limited autonomy, the dependent can hardly be regarded as a "sovereign" consumer selecting the best alternative among the set of affordable nursing homes. Thus one cannot apply standard consumer theory to the choice of nursing home. On the contrary, it is more relevant to consider a model where the family of the dependent parent - including, to an extent varying with the degree of dependency, the dependent himself - will *collectively* choose the nursing home the dependent will be sent to.

The goal of this paper is to try to open the black box of the choice of nursing homes, by developing a microeconomic model where the selected nursing home is the outcome of bargaining between the dependent parent and his family. We study a model of cooperative decision-making where the selected option - i.e. the selected nursing home (defined in terms of price and location) - maximizes a weighted sum of the utilities of family members, the weights reflecting the

¹See Norton (2000) and Cremer et al (2012). Challenges raised by LTC for insurance markets are examined in Brown and Finkelstein (2011).

²Note that there exists significant heterogeneity in the factors driving nursing home choices (in particular concerning the role of nursing home location), as shown by Ramos-Gorand (2016) for the case of France.

bargaining power of each family member. As such, our model is in line with other models of cooperative decision-making concerning LTC arrangements, such as Hoerger et al. (1996), Sloan et al. (1997) and Pezzin et al. (2007).³

Models of family bargaining point to an important determinant of social outcomes: the *distribution of bargaining power* within the family.⁴ As stressed by Sloan et al (1997), the dependent parent and his children can disagree on the kind of supply of LTC (e.g. formal *versus* informal care), because they do not have the same preferences. Hence, the option that will emerge depends on the distribution of bargaining power within the family. Sloan et al (1997) stressed that the bargaining power of the parent depends on three main features: first, his degree of cognitive awareness (which could limit his capacity to take part to the decision); second, his number of children (which can favor competition for gifts); third, his wealth (the strategic bequest motive).

This paper proposes to explore further the consequences of the distribution of bargaining power within the family on LTC outcomes, by considering its impact on the choice of nursing home, and, also, on the characteristics (price and location) of nursing homes that drive that choice. The underlying intuition for considering the impact of family bargaining on nursing homes's characteristics goes as follows. When facing the choice of a nursing home, families take the characteristics of available nursing homes *as given*. However, those characteristics cannot be taken as parameters, but are variables chosen strategically by nursing homes. Thus we need also to explain how family bargaining affects, at the equilibrium, nursing homes price and location.

For that purpose, this paper considers an economy *à la* Hotelling (1929), where a continuum of families, composed of a dependent parent and a child, choose between two nursing homes located along a line, those nursing homes choosing their prices and locations. Two variants of the Hotelling economy will be studied. First, we will consider a baseline static model, where the dependent parent, who has limited capacity to enjoy consumption, wants the nursing home to be as close as possible to his family, whereas his child, being the bearer of LTC spending, cares both about location and price. Second, we will extend this framework to a dynamic overlapping generations model (OLG), in order to introduce a parental bequest motive. This extended model allows us to study the joint dynamics of wealth accumulation and nursing home prices.

There are several reasons why the Hotelling model is relevant for the issue at stake. First, the nursing homes sector exhibits entry barriers and is thus imperfectly competitive. For instance, the opening of a nursing home in France can only be made provided there is an official call for tenders from the French State. The opening of a nursing home requires also a specific authorization, to be signed by national and local authorities. Thus, the number of nursing homes is

³However, our model differs from models of *non-cooperative* decision-making applied to LTC, such as Hiedemann and Stern (1999), Stern and Engers (2002), Konrad et al (2002), Kureishi and Wakabayashi (2007), and Pezzin et al. (2009).

⁴On the impact of the distribution of bargaining power on time allocation, see Konrad and Lommerud (2000). de la Croix and Vander Donkt (2010) and Leker and Ponthiere (2015) studied the impact of bargaining on education outcomes.

limited, as in the Hotelling model. Second, nursing homes are generally left free regarding their geographical location. In the French case, official calls for tenders do not, in general, specify extremely precise locations, so that nursing homes have some room for location choices. Third, nursing homes exhibit positive mark up rates. Martin (2014) estimated that the average mark up rate for French nursing homes ranges in 2011 from 4.2 % to 11.8 %.⁵ Those features make the Hotelling model a natural candidate to study nursing homes.

Anticipating our results, we find, in the baseline model (i.e. without bequest motive), that nursing homes locate at the extremes of the Hotelling line (following the principle of maximum differentiation), while the mark up of nursing homes is increasing in the bargaining power of the dependent parent. The laissez-faire equilibrium is contrasted with the utilitarian social optimum, where nursing homes locate more centrally. If the government can force nursing home locations, the social optimum can be decentralized by merely subsidizing nursing homes to achieve pricing at the marginal cost. However, if locations cannot be forced, the decentralization requires in addition a non-linear tax on location.

Turning now to the dynamic OLG model with wealth accumulation, it is shown that, if the parental bequest motive is strong, the mark up can be decreasing in the bargaining power of the dependent parent, unlike in the baseline model. Moreover, the mark up is shown to be decreasing with the interest rate, since a higher interest rate raises the opportunity cost of LTC expenditures. The existence, uniqueness and stability of stationary equilibria is then studied, and it is shown that, starting from low wealth levels, the convergence towards the stationary equilibrium leads to a rise in the price of nursing homes through higher mark up rates induced by lower interest rates.

Our paper is related to several aspects of the literature on LTC. First, it is related to models of family bargaining, such as Hoerger et al (1996) and Sloan et al (1997), which studied how family bargaining affects the choice of formal *versus* informal LTC provision, as well as the choice of living arrangement. Our contribution is to study how nursing homes react strategically and set prices according to the distribution of bargaining power within the family. Our paper is also related to the literature on location games in the context of LTC, such as Konrad et al (2002) and Kureishi et Wakabayashi (2007). While those papers studied the strategic location of children with respect to a given nursing home location, we do the opposite, and study the strategic location of nursing homes with respect to a given location of children. We also complement IO papers applying Hotelling's model to health issues, such as Brekke et al (2014), who studied competition in prices and quality among hospitals. Our paper complements this IO approach by considering interactions between family bargaining, prices and location outcomes.⁶ We also complement the literature on optimal

⁵Whereas those figures may seem surprising given that the price of LTC formal services in nursing home is regulated in France, it should be reminded that the French authorities do not regulate the price of accomodation services, which can thus give rise to a significant mark up.

⁶Interactions between bargaining and spatial competition are also studied by Bester (1989) in a Hotelling model where prices are the outcome of bargaining between consumers and firms. In our model, the bargaining occurs on the consumer side only.

public policies under LTC, such as Jousten et al (2005), Pestieau and Sato (2008) and Cremer et al. (2016) by exploring the optimal public intervention when nursing homes choose prices and locations strategically. Finally, our paper is also related to studies of long-run dynamics using OLG models with LTC, such as Canta et al (2016) and Pestieau and Ponthiere (2016).

The rest of the paper is organized as follows. Section 2 presents the main assumptions of the baseline model. Section 3 characterizes the laissez-faire, and explores the links between the distribution of bargaining power in the family and the mark up rate of nursing homes. The social optimum and its decentralization are studied in Section 4. Section 5 extends our previous baseline static model to dynamic OLG framework, and examines the robustness of our results to the introduction of a parental bequest motive. Conclusions are drawn in Section 6.

2 The model

There exists a continuum of families composed of a child and of a dependent parent, who are uniformly distributed on a geographical line $[0, L]$ (see Figure 1).⁷ It is assumed that, because of the severity of cognitive / functional limitations, each dependent parent needs to enter a nursing home.

The market for nursing homes is assumed to be imperfectly competitive. Imperfect competition is here due to the existence of entry barriers preventing a free access to that market. Such a limited access to the nursing home market arises, for instance, when the government fixes an arbitrary number of licences to nursing homes, which prevents the entry of new competitors.⁸ For simplicity, we assume that the number of licences for nursing homes is limited to 2. Nursing homes are denoted by $\{A, B\}$, and located on the same $[0, L]$ line as families, at, respectively, a (for nursing home A) and $L - b$ (for nursing home B).

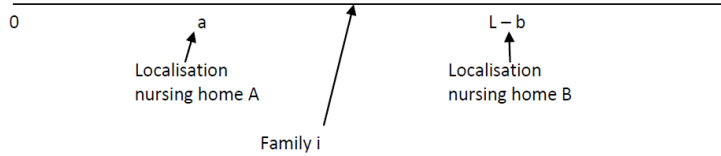


Figure 1: The Hotelling line.

We assume that nursing homes differ on a single dimension - geographical location -, and thus leave aside other dimensions such as the quality of care. This

⁷We suppose here a unique degree of dependence shared by all parents.

⁸Note that, if one assumed, on the contrary, free entry in the nursing home sector, then, in the absence of fixed costs, new nursing homes would enter as long as profits are positive, leading, *in fine*, to zero profits (i.e. pricing at the marginal cost of production).

amounts to assume that nursing homes are of the same quality, or, alternatively, that families do not take quality into account in their choice. Although the recent study by Schmitz and Stroka (2014) provides some support to the latter assumption, it should be stressed here that abstracting from the quality of nursing home may simplify the picture significantly. Clearly, adding the quality dimension to the model would complicate the analysis substantially.

2.1 Preferences: children

The child derives utility from his consumption, which is equal to his exogenous income *minus* the price of the nursing home where his parent is located. This amounts to assume that the child fully supports the cost of nursing home.⁹

The child also derives some disutility from being far from the nursing home where his parent is located. The intuition goes as follows. The child likes visiting his parent, but visiting the parent at the nursing home is costly in terms of time, and this time cost is increasing with the distance between the child and the nursing home of the parent.

For the sake of analytical tractability, the child's utility is quasi-linear:

$$U_c = w - p_i - \gamma x_i^2 \quad (1)$$

where $w > 0$ is the child's income, p_i is the price of the nursing home $i \in \{A, B\}$, $\gamma > 0$ captures the intensity of the disutility of the distance between the nursing home and the child's location. $x_i > 0$ is the distance between the elderly's nursing home $i \in \{A, B\}$ and the child's location.¹⁰

2.2 Preferences: dependent parents

The utility of the dependent parent is assumed to depend only on the distance between the nursing home and his child (i.e. the initial location of the family):¹¹

$$U_d = -\delta x_i^2 \quad (2)$$

where $\delta > 0$ is the intensity of the parent's disutility created by distance between the nursing home and the child's location.

The intuition behind that formulation is the following. Dependent elderly persons have, because of functional and cognitive limitations, a restricted ability to enjoy consumption. Therefore we assume, as a proxy, that their utility is independent from consumption (which is set to zero), and that it depends only on the distance between the nursing home and the family. The underlying

⁹Note that relying on an exogenous income is an obvious simplification. Section 5 develops a dynamic OLG economy where the child's income is endogenous, and where the price of the nursing home is subtracted from the wealth transfer that the child receives from his parent.

¹⁰The child's interests for distance can be interpreted either as a self-oriented concern for being able to visit the parent more often, or, alternatively, as an altruistic concern taking into account that the parent wants to be close to his child (see *infra*).

¹¹Note that the utility of the dependent parent does not depend on the price of the nursing home, since the cost of nursing home is here entirely supported by the child (see above).

intuition is that dependent persons care about keeping a link with their family. But since the number of visits depends on traveling costs, and, thus, on the distance between the dependent and his visitor, it is reasonable to suppose that a shorter distance between the child and the nursing home will raise the number of visits, and, hence, the welfare of the dependent parent.

Note that parents exhibit here no altruism towards children. Section 5 will relax that assumption, by considering a model where parents care about the wealth that they transmit to their children (net of the nursing home price).

2.3 Preferences: the family

Within each family, the interests of the parent and of the child are not perfectly aligned, and there is *a priori* no obvious reason why they should agree on the choice of a nursing home among the two available ones, A and B .

If the parent and the child had to choose a nursing home not among the two nursing homes A and B , but among the set of *all* potential nursing homes (i.e. the set of pairs (p, x) with $0 \leq p, 0 \leq x \leq L$), the fact that parents care only about distance while children care about distance and price has a clear implication: at the family level, the contract curve is a singleton: the nursing home at a zero distance and at a zero price.¹² Thus, if we were considering the entire space of potential nursing homes, the parent and the child would unanimously agree on the nursing home $(p = 0, x = 0)$.

But in the present problem, there are only two nursing homes available, A and B , each of these exhibiting some price $p_A, p_B \geq 0$ and distance $x_A, x_B \geq 0$, and there is no obvious reason why the child and the dependent parent will unanimously prefer one nursing home over the other. For instance, nursing home A may be less distant than nursing home B (so that the parent prefers A to B), but also more expensive than the hypothetical nursing home B' that lies on the same child's indifference curve as B while being at the same distance than A (so that the child prefers B to A). We thus need a way to aggregate the interests of the child and the dependent parent.

Throughout this paper, it is assumed that the family acts cooperatively, that is, the family maximizes a weighted sum of the family members' utilities. Assuming that those weights sum up to 1, we will denote the weight of the child by $\theta \in [0, 1]$, and the weight of the dependent parent by $1 - \theta$. Those weights, which reflect the bargaining power of each family member, are exogenously given. The choice of a nursing home is thus represented here as the outcome of

¹² Any nursing home with a positive distance $x > 0$ is not Pareto optimal, since a Pareto improvement can be achieved by moving along the indifference curve of the child while adopting a less distant nursing home, which increases the well-being of the parent. Thus for any nursing home with positive distance, there exists another nursing home, more expensive, at a lower distance, and which can be preferred unanimously by the parent and the child. Moreover, any nursing home with a strictly positive price $p > 0$ is Pareto dominated by another nursing home at the same distance, but which is cheaper (so that the child is better off).

a family bargaining process. The utility of the family is written as:¹³

$$U_f = \theta U_c + (1 - \theta) U_d \quad (3)$$

The distribution of bargaining power can take *a priori* various forms. The case where $\theta = 1$ arises when the child is the only one who takes part to the decision of choosing a nursing home. On the contrary, when $\theta = 0$, it is the parent who selects the nursing home, and the child obeys to what his parent decides. This case may seem a bit extreme, especially in a context where the child is the one who pays for the nursing home. However, the extent to which this case may arise depends on the prevailing culture within families.

The parameter θ can be interpreted in different ways. As already mentioned, it may reflect the values to which individuals adhere in a society, concerning the extent to which obeying one's parents is regarded as essential or not. But in the context of LTC, it is also possible that θ reflects, to some extent, the degree of cognitive awareness or ability of the dependent parent. Indeed, if the parent is in a situation of weak dependence, he will definitively have a word to say in the choice of a nursing home, unlike what would arise if the parent is in a strong state of dependence (e.g. an advanced Alzheimer condition), in which case the child will choose the nursing home alone ($\theta \rightarrow 1$).

When the family opts for the nursing home $i \in \{A, B\}$, its utility is:

$$U_{f,i} = \theta (w - p_i) - (\theta\gamma + (1 - \theta)\delta)x_i^2. \quad (4)$$

3 The laissez-faire

Let us describe the timing of the model.

1. Nursing homes A and B choose simultaneously their location, a and $L - b$ respectively, on the line $[0, L]$.
2. Nursing homes A and B simultaneously fix the prices (respectively p_A and p_B), anticipating families' demand and taking the price of the other facility as given.
3. Families choose which nursing home $i \in \{A, B\}$ to send the dependent elderly, taking prices and location as given.

As usual, we solve the model backwards, starting from the families' decisions.

¹³Note that we abstract here from participation constraints and from rationality constraints (i.e. agents always choose to play cooperatively), so that both the child and the elderly have an interest in participating to the family decision. Abstracting from outside options is not such a strong assumption in the present context. In many countries, it is prohibited for children to abandon dependent parents. Moreover, the elderly has also no option of changing child and no other option than going to a nursing home which is entirely paid by the child.

3.1 Families' decisions

We solve the demand for each nursing home by first identifying the median family, who, by definition, is indifferent between the two nursing homes. For that family, denoted by m , we have:

$$\begin{aligned} U_{m,A} &= U_{m,B} \\ \theta(w - p_A) - (\gamma\theta + (1 - \theta)\delta)x_{m,A}^2 &= \theta(w - p_B) - (\gamma\theta + (1 - \theta)\delta)x_{m,B}^2 \end{aligned}$$

where $x_{m,A} = m - a$ is the distance between nursing home A and the median family, while $x_{m,B} = L - b - m$ is the distance between nursing home B and the median family. Together with the constraint on distances:

$$x_{m,A} + x_{m,B} = L - a - b$$

we obtain, after some simplifications:

$$x_{m,A} = \frac{\theta(p_B - p_A)}{2(L - a - b)(\gamma\theta + (1 - \theta)\delta)} + \frac{(L - a - b)}{2} \quad (5)$$

An increase (resp. decrease) in price p_A decreases (resp. increases) $x_{m,A}$. Equivalently, the median family is further to the left (resp. right), which implies that the demand for nursing home A decreases (resp. increases). Moreover, we have:

$$x_{m,B} = \frac{\theta(p_A - p_B)}{2(L - a - b)(\gamma\theta + (1 - \theta)\delta)} + \frac{(L - a - b)}{2} \quad (6)$$

Given that all families located on the left of the median family prefer nursing home A over nursing home B , the total demand for nursing home A , denoted by $D_A(p_A, p_B, a, b)$ is equal to $a + x_{m,A}$ and thus, to:

$$D_A(p_A, p_B, a, b) = \frac{\theta(p_B - p_A)}{2(L - a - b)(\gamma\theta + (1 - \theta)\delta)} + \frac{(L + a - b)}{2}.$$

Similarly, the demand for nursing home B , $D_B(p_A, p_B, a, b) = b + x_{m,B}$, is:¹⁴

$$D_B(p_A, p_B, a, b) = \frac{\theta(p_A - p_B)}{2(L - a - b)(\gamma\theta + (1 - \theta)\delta)} + \frac{(L - a + b)}{2}.$$

3.2 Nursing homes' decisions

We now derive nursing homes' decisions. Since there are only two facilities, we assume a duopoly-type of competition.

¹⁴Distances $x_{m,A}$ and $x_{m,B}$ must be positive, which is always verified when the difference in nursing home prices satisfies:

$$-\frac{\gamma\theta + (1 - \theta)\delta}{\theta}(L - a - b)^2 < (p_A - p_B) < \frac{\gamma\theta + (1 - \theta)\delta}{\theta}(L - a - b)^2$$

We check *ex post* that this is effectively the case (see Proposition 1). If the above condition is satisfied, both $D_A(p_A, p_B, a, b)$ and $D_B(p_A, p_B, a, b)$ are positive.

3.2.1 Choice of prices

Nursing homes choose their price given their locations a and $L - b$ and while taking the price of the other nursing home as given. For the sake of simplicity, we suppose that nursing homes have the same linear cost structure so that the average cost by patient is equal to its marginal cost, $c > 0$. We also assume that there is no fixed cost of production.¹⁵

Nursing home i 's profit maximization problem can be written as:

$$\max_{p_i} (p_i - c) D_i(p_i, p_{j \neq i}, a, b) \quad \forall i = \{A, B\}$$

Since the nursing home market is not competitive, each nursing home takes into account that an increase in its price decreases its demand. The first-order conditions (FOC) for equilibrium prices are:

$$a + x_{m,A} + \frac{\partial x_{m,A}}{\partial p_A} (p_A - c) = 0 \quad (7)$$

$$b + x_{m,B} + \frac{\partial x_{m,B}}{\partial p_B} (p_B - c) = 0 \quad (8)$$

where the first two terms on each line account for the marginal effect of increasing the price on the existing demand on the profit and the last term represents the marginal decrease in demand due to the increase in price on the profit. Taking for instance (7), this latter effect only concerns the agents to the right of nursing home A and to the left of the median family, that is, those who are close enough to nursing home B that they may now change for nursing home B following an increase in p_A . An increase in p_A will push the median to the left, inducing more families to use nursing home B .

Solving simultaneously (7) and (8), we obtain the following prices:

$$\begin{aligned} p_A &= -(\gamma\theta + (1 - \theta)\delta) \frac{(a + b - L)(a - b + 3L)}{3\theta} + c \\ p_B &= -(\gamma\theta + (1 - \theta)\delta) \frac{(a + b - L)(-a + b + 3L)}{3\theta} + c \end{aligned}$$

Replacing p_A and p_B in $x_{m,A}$ and $x_{m,B}$, we obtain:¹⁶

$$x_{m,A} = \frac{1}{6} (-5a - b + 3L) \quad \text{and} \quad x_{m,B} = \frac{1}{6} (-5b - a + 3L) \quad (9)$$

3.2.2 Choice of locations

The equilibrium location for nursing homes A and B is obtained by selecting the levels of, respectively, a and b that maximize their own profits, taking into

¹⁵ Assuming no fixed cost does not change our results regarding pricing and location of nursing homes as long as the cost function is linear.

¹⁶ Given that $L - a - b > 0$, a sufficient condition for those distances to be positive is that $L \geq 2a$ and that $L \geq 2b$, which is always verified.

account that both prices and demand depend on a and b and, taking the location of the other nursing home as given.

Nursing home A chooses location a maximizing its profit:

$$\max_a \pi_A = \left[\frac{-(\gamma\theta + (1-\theta)\delta)(a^2 + 2aL + 4bL - b^2)}{+ \frac{c\theta + L^2(\gamma\theta + (1-\theta)\delta)}{\theta} - c} \right] \left[\frac{1}{6} (a - b + 3L) \right]$$

After simplifications, the problem of nursing home A can be rewritten as:

$$\max_a \pi_A = \left[-(\gamma\theta + (1-\theta)\delta) \frac{[a^2 + 2aL + 4bL - b^2 - 3L^2]}{3\theta} \right] \left[\frac{1}{6} (a - b + 3L) \right]$$

Differentiating with respect to a yields after some simplifications:

$$\frac{\partial \pi_A}{\partial a} = \frac{-(\gamma\theta + (1-\theta)\delta)}{18\theta} [3a^2 + 10aL + 3L^2 + 2b(L - a) - b^2] < 0 \quad (10)$$

The expression inside brackets is positive so that the profits of nursing home A are strictly decreasing with location a . Hence, at the laissez faire, the location for nursing home A is at the left extremity of the segment $[0, L]$: $a^{LF} = 0$.

Nursing home B chooses location b maximizing its profit :

$$\max_b \pi_B = \left[\frac{-(\gamma\theta + (1-\theta)\delta)(b^2 + 2bL + 4aL - a^2)}{+ \frac{c\theta + L^2\delta(1-\theta)}{\theta} - c} \right] \left[\frac{1}{6} (b - a + 3L) \right]$$

We obtain the following FOC for b :

$$\frac{-(\gamma\theta + (1-\theta)\delta)}{18\theta} [3b^2 + 10bL + 2a(L - b) + 3L^2 - a^2] < 0 \quad (11)$$

The profits of nursing home B are strictly decreasing with location b . The location for nursing home B is thus at the right extremity (L) of the segment $[0, L]$: $b^{LF} = L$. Hence the principle of the maximum differentiation holds.

Proposition 1 *At the laissez-faire, nursing homes A and B locate at the far extremes of the line $[0, L]$:*

$$a^{LF} = b^{LF} = 0$$

Prices in nursing homes are equal to:

$$p_A^{LF} = p_B^{LF} = c + \frac{(\gamma\theta + (1-\theta)\delta)}{\theta} L^2$$

and demands are $D_A^{LF} = D_B^{LF} = L/2$.

Proof. $a^{LF} = b^{LF} = 0$ have been replaced in the equations for prices. In equilibrium, $D_A^{LF} = x_{m,A}$ and $D_B^{LF} = x_{m,B}$ defined in (9). ■

Proposition 1 states that leaving nursing home choose their locations leads, in an imperfectly competitive economy, to extremely peripheral locations. Moreover, prices exceed the marginal cost, which is a consequence of imperfect competition and of maximum differentiation. Interestingly, the mark up,

$$MarkUp = \frac{\gamma\theta + (1-\theta)\delta}{\theta} L^2$$

depends on the child's bargaining power θ and on the intensity of preferences for distance. Regarding the latter factor, it is clear that if individuals prefer to be closer to each other (i.e. $\gamma, \delta > 0$ increase), the mark up increases.

Concerning the comparative statics of the mark up with respect to θ , Corollary 1 states that the distribution of bargaining power in the family affects the extent to which nursing homes can extract a large mark up: a higher bargaining power for the dependent implies a higher mark up for nursing homes.

Corollary 1 *The mark up of nursing homes A and B is decreasing with the bargaining power of the child:*

$$\frac{dMarkUp}{d\theta} = -\frac{\delta}{\theta^2} L^2.$$

Proof. Follows from derivating $\frac{(\gamma\theta + (1-\theta)\delta)L^2}{\theta}$ with respect to θ . ■

The intuition behind Corollary 1 goes as follows. The mark up that nursing homes can charge is limited by the child's willingness to pay for the nursing home (since only the child cares about the price). The extent to which this limitation arises depends on the bargaining power of the child. If, for instance, the parent has no bargaining power ($\theta = 1$), the mark-up is equal to γL^2 , which collapses to zero when the child has no preference for location ($\gamma = 0$). Therefore the price cannot deviate from the marginal cost.

When interpreting Corollary 1, it should be stressed that the distribution of bargaining power can reflect various features. First, if the society strongly values the obedience of children to their parents (i.e. low θ), then the choice of a nursing home only reflects parents' preferences, i.e. the concern for location, so that nursing homes can charge a large mark up. If, on the contrary, the society strongly values the democracy within families, then the child's interest for the price will reduce the capacity of nursing homes to extract a large mark up.

Alternatively, if one interprets the distribution of bargaining power as reflecting the degree of cognitive ability or awareness of the dependent parent, then Corollary 1 admits another interpretation. If the elderly's cognitive abilities are strongly limited, then the decision within the family will be made almost entirely by the child (i.e. θ close to 1). This limits the mark up of nursing homes. On the contrary, if the cognitive abilities of the dependent are still important, then the dependent parent will have more power in the nursing home decision, and as a consequence nursing homes will obtain higher margins.

Note that the result stated in Corollary 1 depends crucially on the assumption that only the child cares about the price of nursing home. If, for instance,

the parent was assumed to be altruistic towards his child, he would also care about the price, and the mark up would not necessarily be increasing with the bargaining power of the parent. We would then, under parental altruism, obtain results that are close to the ones of the extended OLG model (see Section 5).

Moreover, Corollary 1 depends on the assumed sharing rule for the cost of LTC between the parent and the child. Throughout this paper, we assume that the child is the ultimate bearer of the LTC cost (either directly or indirectly through reduced bequests). One could, alternatively, assume that the parent and the child share that cost. Then, provided the parent cares about his consumption (unlike in the present model), the impact of the distribution of bargaining power on the mark up of nursing home could be affected, depending on the sharing rule and on the strength of interests of children and parents for consumption.¹⁷

3.3 The family size

Before turning to the social optimum, let us discuss briefly the robustness of our results to relaxing the one child / one parent assumption. Actually, it is easy to show that our results are robust to relaxing that assumption, as long as children share the same location on $[0, L]$.¹⁸

Allowing the parent to have several children (with bargaining power θ_i , incomes w_i and sharing the cost of nursing homes in proportions β_i) does not change our main results. The mark up expression remains identical to the one in Proposition 1, except that it now depends on the family average disutility from distance to the nursing home (in the numerator), and on the sharing rule of LTC spending between children (in the denominator). However, from a quantitative perspective, the mark up may be affected by the family size, through changes in the values of structural parameters.

First, the distribution of bargaining power may be quite different when the size of the family changes. If the rise in the number of children raises the bargaining power of the parent, this leads to larger mark up rates.¹⁹ But if the family takes its decisions on the basis of one person / one vote rule, this may reduce the bargaining power of the parent, leading to lower mark up rates. Second, the sharing rule for LTC costs among children has an important role for the size of the mark up. A dissonance between who has the power and who pays for LTC may raise mark up rates.²⁰ Thirdly, increasing the family size may also

¹⁷It should be stressed, however, that assuming that the young cares more about consumption than the dependent elderly seems to be a weak assumption. Hence, even in this modified setting, it is still likely, under many sharing rules, that there would be a decreasing relation between the bargaining power of the child and the mark up of nursing homes.

¹⁸This extension is available upon request. Note that the assumption that all children live at the *same* place on the line $[0, L]$ is strong. Once several children are present, strategic location choices of children can arise, in line with Konrad et al (2002). However, introducing location choices for nursing homes *and* for children would raise serious modelling difficulties.

¹⁹Sloan et al. (1997) argues that a rise in the number of children may raise the bargaining power of the parent, since this strengthens the threat of leaving no bequest to a child.

²⁰This arises when the child who pays the largest share of LTC spending is not the one who

raise coordination failures: each child may now rely on his brother/sister for the visits of the dependent parent. In that case, the distance becomes irrelevant for each child, which pushes mark up rates down. Hence relaxing the one child / one parent assumption is not neutral from a quantitative perspective.

4 Social optimum

The laissez-faire equilibrium is not satisfactory from a social perspective, since this involves (i) prices higher than marginal costs and (ii) large disutility for children and parents due to the peripheral locations chosen by nursing homes.

This section characterizes the social optimum, and discusses how it can be decentralized by means of policy instruments. Before characterizing the social optimum, let us mention that we focus here on a “constrained” first-best, in the sense that the number of nursing homes is not chosen, and remains fixed to 2.²¹

4.1 The centralized solution

Let us now turn to the social planning problem. For that purpose, we adopt a standard utilitarian social welfare function, where the weights assigned to each individual’s utility (i.e. children and dependent parents) are the same and equal to 1/2.

The social planner chooses locations of nursing homes, a and $L - b$, and consumptions c_A and c_B of young agents (whose parent go to nursing home A or B respectively), so as to maximize total welfare. With a uniform distribution of families on $[0, L]$ (and thus, with a density function $1/L$), the problem is:

$$\begin{aligned} \max_{a, b, c_A, c_B} \quad & W = \int_{j=0}^m \left[\frac{1}{2} (c_A) - \frac{1}{2} (\gamma + \delta) (j - a)^2 \right] \frac{1}{L} dj \\ & + \int_m^L \left[\frac{1}{2} (c_B) - \frac{1}{2} (\gamma + \delta) (L - b - j)^2 \right] \frac{1}{L} dj \\ \text{s.t.} \quad & \int_{j=0}^L w \frac{1}{L} dj \geq \int_{j=0}^m c_A \frac{1}{L} dj + \int_m^L c_B \frac{1}{L} dj + \int_0^L c \frac{1}{L} dj \quad (12) \end{aligned}$$

where $m = m(a, b, c_A, c_B)$, the location of the median family, satisfies:

$$\theta c_A - (\gamma\theta + (1 - \theta)\delta)(m - a)^2 = \theta c_B - (\gamma\theta + (1 - \theta)\delta)(L - b - m)^2. \quad (13)$$

While the bargaining power θ does not appear explicitly in the problem, it is still present through the definition of the median family m . In the Appendix, we show that if a (resp. b) increases, m goes further to the right (resp. to the left). When c_A (resp. c_B) increases, the median family goes further to the right (resp. left), so that more families use nursing home A (resp. B).

has the largest bargaining power (e.g. when children contribute according to their means).

²¹At the (unconstrained) optimum optimum, we would obtain, in the absence of fixed costs, an infinity of nursing homes along $[0, L]$, each child opening a home for his parent.

We now derive the FOCs of the above problem (see the Appendix) and making use of (13), these can be simplified as follows:

$$\frac{\partial \mathcal{L}}{\partial c_A} = \left(\frac{1}{2} - \lambda\right)m + \frac{dm}{dc_A}[c_B - c_A] \left[\lambda - \frac{1}{2} \frac{(1-2\theta)\delta}{\gamma\theta + (1-\theta)\delta} \right] \leq 0 \quad (14)$$

$$\frac{\partial \mathcal{L}}{\partial c_B} = \left(\frac{1}{2} - \lambda\right)(L - m) + \frac{dm}{dc_B}[c_B - c_A] \left[\lambda - \frac{1}{2} \frac{(1-2\theta)\delta}{\gamma\theta + (1-\theta)\delta} \right] \leq 0 \quad (15)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial a} &= (\gamma + \delta) \int_{j=0}^m (j - a) dj \\ &\quad + \frac{dm}{da}[c_B - c_A] \left[\lambda - \frac{1}{2} \frac{(1-2\theta)\delta}{\gamma\theta + (1-\theta)\delta} \right] \leq 0 \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial b} &= (\gamma + \delta) \int_{j=m}^L (L - b - j) dj \\ &\quad + \frac{dm}{db}[c_B - c_A] \left[\lambda - \frac{1}{2} \frac{(1-2\theta)\delta}{\gamma\theta + (1-\theta)\delta} \right] \leq 0 \end{aligned} \quad (17)$$

where λ is the Lagrange multiplier associated with the resource constraint and $dm/dc_A = -dm/dc_B > 0$ as well as $dm/da > 0$ and $dm/db < 0$.

Individual utilities being linear in consumptions, there is an infinity of solutions to this problem. One possible solution consists in setting $c_A^* = c_B^* = \bar{c}$, so that, using the government budget constraint, $\bar{c} = (w - c)$. Replacing for $c_A^* = c_B^*$ in conditions (16) and (17), we obtain:

$$a^* = \frac{m}{2} \text{ and } b^* = \frac{L - m}{2} \quad (18)$$

This is different from what we obtained at the laissez-faire, where $a^{LF} = b^{LF} = 0$. This also implies that using condition (13), the median family locates exactly in the middle of the line $[0, L]$, i.e. $m^* = m(a^*, b^*, \bar{c}, \bar{c}) = L/2$. We thus have $a^* = \frac{1}{4}L$ and $L - b^* = \frac{3}{4}L$. Let us call this specific solution the “symmetric” utilitarian optimum, since it involves equal consumptions for all agents.

However, this is not the only solution. Since utility is linear in consumption, we could also have any pair (c_A, c_B) satisfying the government budget constraint:

$$c_A = \frac{L(w - c) - (L - m)c_B}{m} \text{ with } c_B \in \left[0, \frac{L(w - c)}{m}\right] \quad (19)$$

Using FOCs (16) and (17) as well as the results (18) when $c_A = c_B$, we obtain that if $c_A > c_B$, $a^* < m/2$ and $b^* > (L - m)/2$ so that both a^* , and $L - b^*$ should be distorted to the left (note that there is no reason here for m to be the same as when $c_A = c_B$). To the opposite, if $c_A < c_B$, $a^* > m/2$ and $b^* < (L - m)/2$ so that both a^* and $L - b^*$ should be distorted to the right.²²

²²The precise value of m^* depends on the values taken by (a^*, b^*, c_A^*, c_B^*) which are obtained by solving the system of FOCs for a and b together with the values for c_A^* and c_B^* , the resource constraint of the economy and the condition on the median.

Throughout this paper, we will, when referring to the utilitarian social optimum and its decentralization, focus only the symmetric solution. Our results are summarized in Proposition 2.

Proposition 2 *At the symmetric utilitarian optimum, all young agents enjoy the same consumption, independently from the nursing home of their parents:*

$$c_A^* = c_B^* = w - c$$

At the symmetric utilitarian optimum, nursing homes locate closer on the line $[0, L]$ than at the laissez-faire :

$$a^* = \frac{1}{4}L \text{ and } L - b^* = \frac{3}{4}L$$

and nursing homes A and B equally share demand: $D_A = D_B = m^ = L/2$.*

Proof. See above. ■

The symmetric utilitarian optimum involves a quite different location of nursing homes in comparison to the laissez-faire. Contrary to the laissez-faire, where nursing homes A and B were located at the two extremes of the line (i.e. at 0 and L), nursing homes are located, at the symmetric utilitarian optimum, much more centrally, in the middle of each half of the line (i.e. at $\frac{1}{4}L$ and $\frac{3}{4}L$). Another source of social improvement lies in the increase in consumption in comparison with the laissez-faire: $\bar{c} = w - c > w - p_i$ with $p_i \geq c$. The extent to which the social optimum involves higher consumptions than the laissez-faire depends on the prevailing mark up at the laissez-faire, and, hence, on how the bargaining power is distributed within families. Thus, the symmetric utilitarian social optimum involves welfare gains on two grounds: it makes nursing homes closer (on average) to families, and it increases consumption.

Finally, note that, at the social optimum, families have unequal utilities, because of unequal distance to nursing homes. To minimize these, we could have assumed a social welfare function exhibiting some inequality aversion (Atkinson and Stiglitz 1980). In the decentralization below, one would then need to add lump sum transfers, proportional to the distance to the nursing home, to compensate agents for the disutility due to distance.

4.2 Implementation

Let us study the decentralization of the symmetric utilitarian social optimum. If the government can impose both locations and prices, the social optimum is trivially decentralized. In this section, we will first consider a setting where the government can only impose locations, but not prices. Then, we will consider a setting where the government can impose neither prices, nor locations directly.

4.2.1 Case A: locations can be forced

A first way to decentralize the optimum consists in forcing locations of nursing homes A and B at a^* and at $L - b^*$ and in giving nursing homes a subsidy for

each patient entering their facility so that the price they charge is $p_A = p_B = c$ and, thus $c_A = c_B = w - c$. Let us denote S_i the subsidy received by nursing home i for each patient who is located in this facility. Households decisions do not change. Only the problem faced by nursing homes is now different. The problems of Section 3.2 can thus be rewritten as:

$$\max_{p_i} (p_i + S_i - c) D_i(p_i, p_{j \neq i}, a, b), \forall i = \{A, B\}$$

with p_i the prices faced by patients going to nursing homes i . FOCs are:

$$\begin{aligned} a + x_{m,A} + \frac{\partial x_{m,A}}{\partial p_A} (p_A + S_A - c) &= 0 \\ b + x_{m,B} + \frac{\partial x_{m,B}}{\partial p_B} (p_B + S_B - c) &= 0 \end{aligned}$$

Solving this system, one gets that (d stands for decentralization):

$$p_A^d = -(\gamma\theta + (1-\theta)\delta) \frac{(a+b-L)(a-b+3L)}{3\theta} + \frac{3c-2S_A-S_B}{3} \quad (20)$$

$$p_B^d = -(\gamma\theta + (1-\theta)\delta) \frac{(a+b-L)(-a+b+3L)}{3\theta} + \frac{3c-2S_B-S_A}{3} \quad (21)$$

Equalizing these with the optimal prices levels, one obtains:

$$S_A(a, b) = (\gamma\theta + (1-\theta)\delta) \frac{(L-a-b)(a-b+L)}{\theta} \quad (22)$$

$$S_B(a, b) = (\gamma\theta + (1-\theta)\delta) \frac{(a+b-L)(a-b-L)}{\theta} \quad (23)$$

which yield the following optimal values of the subsidies

$$S_A(a^*, b^*) = S_B(a^*, b^*) = (\gamma\theta + (1-\theta)\delta) \frac{L^2}{2\theta}. \quad (24)$$

The subsidy is the same for each nursing home. This level is different from the mark up in Proposition 1, because locations are forced at different places than at the laissez faire. Since prices chosen by the facility depend on its location, these prices (in the absence of subsidization) would anyway be different from those at the laissez faire because location is different. The level of the subsidy is fixed so as to equalize prices to marginal costs, given the forced location.

Under $\delta > 0$, the subsidy is decreasing in θ . The intuition is similar to that of the variation of the mark up with θ (see section 3.2.2). When θ is higher, i.e. the child has more bargaining power, the ability of the nursing home to deviate from the marginal cost and to charge high prices is more limited (since the child cares about the price). The optimal subsidy can therefore be smaller.

4.2.2 Case B: locations cannot be forced

When the government cannot force the locations of nursing homes at a^* and $L - b^*$, we need additional fiscal instruments to make sure that nursing homes

choose the optimal locations. Like before, the problem of the family does not change, so that we will only consider the modified problems of nursing homes. The government still sets two subsidies, $S_A(a, b)$ and $S_B(a, b)$, so that prices are equalized to marginal costs, $p_A^d = p_B^d = c$ and thus $c_A = c_B = w - c$. Those subsidies are identical to those defined in equations (22) and (23). We then assume that the government taxes nursing homes if they deviate from their optimal location, by means of non-linear tax functions $t_A(a - a^*, b)$ and $t_B(b - b^*, a)$ satisfying $\partial t_i(x, \cdot)/\partial x > 0 \forall x \in [0, L]$.²³ Under that taxation scheme, nursing home A chooses its location so as to maximize its profit, taking into account that it will be taxed if its location differs from the optimal one:

$$\begin{aligned} \max_a \pi^{A,d} &= (p_A^d + S_A(a, b)) - c) D_A(a, b) - t_A(a - a^*, b) \\ &= S_A(a, b) D_A(a, b) - t_A(a - a^*, b) \end{aligned}$$

with $D_A(a, b)$ obtained when $p_A^d = p_B^d = c$ and thus equal to $(L + a - b)/2$ at the decentralized allocation. Note that the nursing home may have an interest in choosing a location a different from the optimal one so as to maximize $S_A(a, b) D_A(a, b)$. The FOCs of this problem is:

$$\frac{\partial [S_A(a, b) D_A(a, b)]}{\partial a} - t'_A(a - a^*, b) \leq 0$$

so that the optimal marginal tax at $a = a^*$ should be equal to

$$t'_A(0, b) = \frac{(\gamma\theta + (1 - \theta)\delta)}{2\theta} (3a^* + b - L)(-a^* + b - L). \quad (25)$$

In this decentralized framework, such a tax level therefore enables to obtain the optimal location since at a^* , the profit of the firm is maximum given the location of firm B at $L - b$. Using the same reasoning for nursing home B , we obtain that the optimal level for the marginal tax at $b = b^*$ should be equal to:

$$t'_B(a, 0) = \frac{(\gamma\theta + (1 - \theta)\delta)}{2\theta} (a - b^* - L)(a + 3b^* - L). \quad (26)$$

The non-linear tax faced by each facility depends on the location of the other facility. Hence, if a nursing home deviates from its optimal location, this influences the non-linear tax faced by the other facility. Note that outside the optimum, facilities can either be taxed or subsidized depending on the values of a and $L - b$. Of course, at the optimal values a^* and b^* , both taxes are null, $t'_A(0, 0) = t'_B(0, 0) = 0$, since both nursing homes choose the optimal locations.

Our results on the decentralization of the symmetric utilitarian optimum are summarized in the proposition below:

Proposition 3 *The decentralization of the symmetric utilitarian social optimum can be attained through the following instruments:*

²³Note that directly controlling nursing home locations or imposing a non-linear tax are identical. We derive those two cases as these correspond to different instruments that the government can use depending on the political constraints it may face.

a) If location can be forced at a^* and $(L - b^*)$, the government only needs to subsidize nursing homes by means of a subsidy per patient equal to (24).

b) If location cannot be forced, the government needs to set a non-linear tax on the nursing homes if they deviate from their optimal location, in addition to the above subsidies. Marginal taxes are equal to (25) and (26).

Proof. See above. ■

Proposition 3 states that it is possible, thanks to adequate policy instruments, to decentralize the symmetric utilitarian social optimum, and to induce nursing homes to choose the optimal locations, and to charge the optimal prices. Note that, due to the fact that the mark up prevailing at the laissez-faire depends on the distribution of bargaining power within the family, it is also the case that the optimal values for subsidies depend on how the bargaining power is distributed within families, that is, on the level of θ (see (24), (25) and (26)).

5 Wealth accumulation and LTC price dynamics

Up to now, we considered, for simplicity, a static economy with given resources. In order to study the joint dynamics of wealth accumulation and nursing home prices, this section extends the baseline model to a three-period OLG model.

5.1 The OLG economy

Each cohort is a continuum of agents of size L . Fertility is at the replacement level (one child per young agent). Period 1, whose duration is normalized to 1, is childhood, during which the child takes no decision. In period 2 (also of length 1), the agent is a young adult. He works in the production of goods, has one child, and saves an exogenous fraction $s \in]0, 1[$ of his resources, while he consumes a fraction $1 - s$ of his resources. He bargains with his father concerning the nursing home choice. In period 3, whose duration is $\lambda \in]0, 1[$, the individual is old and dependent, and is sent to a nursing home chosen by the dependent and his child through bargaining. When the parent dies, the saved resources that are not spent in a nursing home are transmitted to his child.²⁴

5.1.1 Production of LTC and of goods

The economy is composed of two sectors: on the one hand, the production of LTC by nursing homes (which takes place over a period $\lambda < 1$); on the other hand, the production of goods (which takes place over a period of length 1).

The nursing home market is assumed to have the same duopolistic structure as in the baseline model.²⁵ We suppose that the nursing home activity, which takes place only over a subperiod of size $\lambda < 1$, requires a quantity of good

²⁴We assume that $\lambda < 1$ so that the young adult always inherit while he is autonomous.

²⁵Assuming a fixed number of licences is a stronger assumption in this dynamic setting than in the baseline static model. This amounts to assume that lobbying groups are strong enough to sustain entry barriers over long periods of time.

equal to c for each dependent person, as in the baseline model.²⁶ This quantity is purchased on the goods market, during a subperiod of size λ .

The production of goods occurs in a perfectly competitive sector. Production involves capital K_t and labor $L_t = L$ following a Cobb-Douglas process:

$$Y_t = \phi K_t^\alpha L^{1-\alpha} \quad (27)$$

where Y_t denotes the output, and $\alpha \in [0, 1]$. In intensive terms, we have:

$$y_t = \phi k_t^\alpha \quad (28)$$

where $y_t \equiv \frac{Y_t}{L}$ and $k_t \equiv \frac{K_t}{L}$ are the output and the capital per young adult.

We suppose a full depreciation of capital after one period of use. Factors are paid at their marginal productivity:

$$w_t = \phi(1 - \alpha)k_t^\alpha \quad (29)$$

$$R_t = \phi\alpha k_t^{\alpha-1} \quad (30)$$

where w_t is the wage rate, and R_t is one plus the interest rate.

5.1.2 Budget constraints

Total available resources of each young individual are his wage w_t plus what he receives at the death of his parent (i.e. after a fraction of time λ), *minus* the price he paid for the nursing home of his parent. Thus the available resources of the young are:

$$w_t + g_t - \lambda p_{it} \quad (31)$$

where g_t is the raw intergenerational transfer from the dead parent to the child, while λp_{it} is the total price of LTC paid by the child for his dependent parent. Given that the young saves a fraction s of his resources, his consumption is:

$$(1 - s)(w_t + g_t - \lambda p_{it}) \quad (32)$$

Consumption is then decreasing in the nursing home (either A or B) price and in the duration of dependency λ .

The raw intergenerational transfer g_t coming from the parent is equal to the interest factor times the savings of the parent:

$$g_t = R_t s [w_{t-1} + g_{t-1} - \lambda p_{it-1}] \quad (33)$$

The descending transfer from the parent to the child g_t depends on the transfer that the parent received, when he was young, from his own parent, i.e. g_{t-1} . Thus the model describes a dynamic of wealth accumulation. Obviously LTC expenditures tend to limit the scope of accumulation across generations.

²⁶Note that, if one wanted to make the LTC sector employ also labor and capital, the fact that the LTC activity stops when the dependent elderly die (i.e. after a period λ) would create a period of length $1 - \lambda$ during which those factors would either be unemployed or reallocated towards the goods sector. By supposing that the LTC sector requires c units of good per dependent person, we abstract from those modelling difficulties.

5.1.3 Preferences

As in the baseline model, we suppose that individuals care, at the young age, about their consumption, and about how far they are from the nursing home of their parent, and that, at the old age, individuals care only about the distance between their nursing home and the location of their child. However, we introduce here a parental *bequest motive*. We assume, unlike in the baseline model, that old individuals now care about the wealth they transmit to their child net of the price paid for the nursing home.

The lifetime utility of a young adult at time t is given by:

$$(1-s)(w_t + g_t - \lambda p_{it}) - \gamma \lambda x_{it}^2 + \mu (R_{t+1}^e s (w_t + g_t - \lambda p_{it}) - \lambda p_{it+1}^e) - \delta \lambda x_{it+1}^e \quad (34)$$

where the preference parameter $\mu \in [0, 1]$ reflects the parent's interest in giving some wealth to his child net of the (expected) price paid for the nursing home. Note that the future interest factor is written in expectation terms, i.e. R_{t+1}^e . The same remark holds for the price of the future nursing home in which the agent will be at $t+1$, i.e. p_{it+1}^e , and its distance from his children, i.e. x_{it+1}^e .²⁷

Using the same parameter θ to represent the bargaining power of the child, the lifetime utility of the family is now given by:

$$\begin{aligned} & \theta \left[(1-s)(w_t + g_t - \lambda p_{it}) - \gamma \lambda x_{it}^2 \right. \\ & \quad \left. + \mu (R_{t+1}^e s (w_t + g_t - \lambda p_{it}) - \lambda p_{it+1}^e) - \delta \lambda x_{it+1}^e \right] \\ & + (1-\theta) \left[(1-s)(w_{t-1} + g_{t-1} - \lambda p_{jt-1}) - \gamma \lambda x_{jt-1}^2 \right. \\ & \quad \left. + \mu (R_t^e s (w_{t-1} + g_{t-1} - \lambda p_{jt-1}) - \lambda p_{it}) - \delta \lambda x_{it}^2 \right] \quad (35) \end{aligned}$$

Because of the OLG structure, the utility of the family depends on three nursing home choices. First, the wealth accumulated in $t-1$ by the parent depends on the nursing home j where his own parent was sent. Second, the consumption in t of the young agent depends on the nursing home price p_i where his parent is sent. Third, the net transfer that the young agent will leave to his child in $t+1$ depends on the nursing home price p_l where he will be sent once old.

Note that in our baseline model, the unique opportunity cost of LTC expenditures was to reduce consumption of the young. Here, however, in the more general case where $\mu, s > 0$, the opportunity cost of LTC is threefold, and involves not only a reduction of the consumption of the young to an extent $1-s$, but also two other effects related to wealth transmission. First, LTC spending reduces, proportionally to s , the amount of wealth that can be transmitted from the child of the dependent to his own child. The extent to which LTC spending reduce the size of intergenerational transfers depends on how large the interest factor R_{t+1}^e is. Second, the price of LTC also reduces the wealth transfer that the old gave to his child, which matters for the elderly to an extent $\mu(1-\theta)$.

²⁷Note also that, given that the duration of the old age is $\lambda < 1$, the disutility of distance is normalized by λ .

5.2 Temporary equilibrium

At each period, nursing homes A and B choose their locations on $[0, L]$ and their prices. Moreover, each family chooses in which nursing home the dependent parent is sent. The timing is the same as above, and the problem is also solved by backward induction. The only difference with respect to the baseline model is that all decisions are conditional on the available resources and production factor prices, and also conditional on expectations regarding future prices.

5.2.1 Family decision

As above, we solve the demand for each nursing home by first identifying the median family in period t , for whom the following equality prevails:

$$\begin{aligned} & \left[\begin{aligned} & \theta \left[\begin{aligned} & (1-s)(w_t + g_t - \lambda p_{At}) - \gamma \lambda x_{m,At}^2 \\ & + \mu (R_{t+1}^e s (w_t + g_t - \lambda p_{At}) - \lambda p_{lt+1}^e) - \delta \lambda x_{lt+1}^{e2} \end{aligned} \right] \\ & + (1-\theta) \left[\begin{aligned} & (1-s)(w_{t-1} + g_{t-1} - \lambda p_{jt-1}) - \gamma \lambda x_{jt-1}^2 \\ & + \mu (R_t s (w_{t-1} + g_{t-1} - \lambda p_{jt-1}) - \lambda p_{At}) - \delta \lambda x_{m,At}^2 \end{aligned} \right] \end{aligned} \right] \\ = & \left[\begin{aligned} & \theta \left[\begin{aligned} & (1-s)(w_t + g_t - \lambda p_{Bt}) - \gamma \lambda x_{m,Bt}^2 \\ & + \mu (R_{t+1}^e s (w_t + g_t - \lambda p_{Bt}) - \lambda p_{lt+1}^e) - \delta \lambda x_{lt+1}^{e2} \end{aligned} \right] \\ & + (1-\theta) \left[\begin{aligned} & (1-s)(w_{t-1} + g_{t-1} - \lambda p_{jt-1}) - \gamma \lambda x_{jt-1}^2 \\ & + \mu (R_t s (w_{t-1} + g_{t-1} - \lambda p_{jt-1}) - \lambda p_{Bt}) - \delta \lambda x_{m,Bt}^2 \end{aligned} \right] \end{aligned} \right] \end{aligned}$$

which simplifies to:

$$\begin{aligned} & \theta [(1-s)(-p_{At}) - \gamma x_{m,At}^2 + \mu (R_{t+1}^e s (-p_{At}))] + (1-\theta) [\mu (-p_{At}) - \delta x_{m,At}^2] \\ = & \theta [(1-s)(-p_{Bt}) - \gamma x_{m,Bt}^2 + \mu (R_{t+1}^e s (-p_{Bt}))] + (1-\theta) [\mu (-p_{Bt}) - \delta x_{m,Bt}^2]. \end{aligned}$$

Hence

$$(x_{m,Bt}^2 - x_{m,At}^2)(\gamma\theta + \delta(1-\theta)) = (p_{At} - p_{Bt}) [\theta(1-s) + \theta\mu s R_{t+1}^e + (1-\theta)\mu] \quad (36)$$

Using $x_{m,At} + x_{m,Bt} = L - a_t - b_t$, we have:

$$x_{m,it} = \frac{(p_{jt} - p_{it}) [\theta(1-s) + \theta\mu s R_{t+1}^e + (1-\theta)\mu]}{2(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))} + \frac{(L - a_t - b_t)}{2}$$

with $\{i, j\} = \{A, B\}$. Obviously, if $\mu = s = 0$, those expressions are the same as in the baseline model without wealth transmission. Assuming myopic anticipations (i.e. $R_{t+1}^e = R_t$), the demands for the two nursing homes are:

$$D_{At}(p_{At}, p_{Bt}, a_t, b_t) = \frac{(p_{Bt} - p_{At}) [\theta(1-s) + \theta\mu s R_t + (1-\theta)\mu]}{2(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))} + \frac{(L + a_t - b_t)}{2} \quad (37)$$

$$D_{Bt}(p_{At}, p_{Bt}, a_t, b_t) = \frac{(p_{At} - p_{Bt}) [\theta(1-s) + \theta\mu s R_t + (1-\theta)\mu]}{2(L - a_t - b_t)(\gamma\theta + \delta(1-\theta))} + \frac{(L - a_t + b_t)}{2} \quad (38)$$

A higher interest factor makes the demand more reactive to prices. Indeed, it fosters wealth transmission to the next generation (up to the preference for transmission μ) and, therefore, raises the opportunity cost of LTC spending. As a consequence, a high interest rate will limit the capacity of nursing homes to extract large rents as we show below.²⁸

5.2.2 Nursing homes decisions

This section considers nursing homes choices of prices and locations. In order to avoid a too large departure with respect to our baseline model, we assume that the discount factor of nursing homes is equal to 1, so that their objective at a given period is to maximize their profits at that same period.²⁹

Setting prices Let us first consider the choice of prices conditionally on the nursing homes' location.³⁰ The problem faced by nursing home $i = \{A, B\}$ at time t is:

$$\max_{p_{it}} (p_{it} - c) D_{it}(p_{it}, p_{j \neq i}, a_t, b_t) \forall i = \{A, B\}$$

with $D_{At}(p_{At}, p_{Bt}, a_t, b_t)$ and $D_{Bt}(p_{At}, p_{Bt}, a_t, b_t)$ given by (37) and (38). The FOC yields:

$$\begin{aligned} p_{At} &= \frac{c}{2} + \frac{p_{Bt}}{2} + \frac{(L - a_t - b_t)(\gamma\theta + \delta(1 - \theta))}{[\theta(1 - s) + \theta\mu s R_t + (1 - \theta)\mu]} \frac{(L + a_t - b_t)}{2} \\ p_{Bt} &= \frac{c}{2} + \frac{p_{At}}{2} + \frac{(L - a_t - b_t)(\gamma\theta + \delta(1 - \theta))}{[\theta(1 - s) + \theta\mu s R_t + (1 - \theta)\mu]} \frac{(L - a_t + b_t)}{2} \end{aligned}$$

Hence, solving for optimal prices, we have:

$$p_{At} = c + \frac{(L - a_t - b_t)(\gamma\theta + \delta(1 - \theta))}{3[\theta(1 - s) + \theta\mu R_t s + (1 - \theta)\mu]} [3L + a_t - b_t] \quad (39)$$

$$p_{Bt} = c + \frac{(L - a_t - b_t)(\gamma\theta + \delta(1 - \theta))}{3[\theta(1 - s) + \theta\mu R_t s + (1 - \theta)\mu]} [3L - a_t + b_t] \quad (40)$$

where the last terms on the RHS of the above equations are the mark up rates imposed by nursing homes. The higher the interest factor is, the lower the mark up charged by nursing homes is. As mentioned previously, the intuition is that a higher interest factor raises the opportunity cost of LTC spending, makes the demand more reactive to prices and, therefore, limits the mark up nursing homes can charge.

²⁸Note that our assumption of myopic anticipations regarding future production factor prices is made for analytical tractability. Actually, if one assumed, on the contrary, rational anticipations, then the demand for nursing homes A and B would not depend on the current interest rate, but, rather, on the interest rate prevailing at the next period. This would significantly complicate the analysis of the dynamic system (see *infra*).

²⁹It is implicitly assumed that the nursing homes are run by some capital holders who do not lie on the $[0, L]$ line. Those capital holders entirely spend their profits on the goods market, so that the market for goods clears at the temporary equilibrium (see below).

³⁰Here LTC prices are prices expressed in terms of goods.

Substituting for those prices in the location of the nursing homes with respect to the median family, we obtain:

$$x_{m,At} = \frac{3L - 5a_t - b_t}{6} \text{ and } x_{m,Bt} = \frac{3L - a_t - 5b_t}{6} \quad (41)$$

Choosing locations Let us now consider the location choices of the two nursing homes A and B at time t , conditionally on the optimal prices derived above. Using (39), the problem of nursing home A is:

$$\max_{a_t} \frac{(L - a_t - b_t)(\gamma\theta + \delta(1 - \theta))}{18[\theta(1 - s) + \theta\mu R_t s + (1 - \theta)\mu]} [3L + a_t - b_t]^2$$

The FOC is:

$$\frac{(\gamma\theta + \delta(1 - \theta)) [3L + a_t - b_t]}{18[\theta(1 - s) + \theta\mu R_t s + (1 - \theta)\mu]} [-L - 3a_t - b_t] < 0$$

Thus it is optimal for nursing home A to choose $a_t = 0$, that is, to locate at the extreme left of the segment $[0, L]$.

Using (40), the problem of nursing home B is:

$$\max_{b_t} \frac{(L - a_t - b_t)(\gamma\theta + \delta(1 - \theta))}{18[\theta(1 - s) + \theta\mu R_t s + (1 - \theta)\mu]} [3L - a_t + b_t]^2$$

The FOC is:

$$\frac{(\gamma\theta + \delta(1 - \theta)) [3L - a_t + b_t]}{18[\theta(1 - s) + \theta\mu R_t s + (1 - \theta)\mu]} [-L - a_t - 3b_t] < 0$$

Thus the nursing home B chooses $b_t = 0$, that is, to locate at L . The following proposition summarizes our results.

Proposition 4 *At the temporary equilibrium under myopic anticipations, the two nursing homes locate at the far extreme of the line $[0, L]$, independently of the distribution of bargaining power within the family.*

Prices in the two nursing homes are equal to:

$$p_{At} = p_{Bt} = c + \frac{(\gamma\theta + \delta(1 - \theta))}{[\theta(1 - s) + \theta\mu R_t s + (1 - \theta)\mu]} L^2$$

The demand for each nursing home is $D_{At} = D_{Bt} = L/2$.

Proof. See above. ■

At the temporary equilibrium, the principle of maximum differentiation holds. Nursing homes maximize their profits by being located at the two ex-

extremes of the segment $[0, L]$, as in the baseline model.³¹ However, in comparison with the baseline model, the formulae for prices are here different, and depends on how large the propensity to save s is, on how much parents care about transmitting wealth (i.e. the level of μ), and on the interest factor R_t . The reason why those factors influence prices lies in the fact that wealth accumulation matters for individuals. The price of nursing homes determines LTC expenditures, which limit wealth accumulation and wealth transmission.

For a given distribution of bargaining power, the price of nursing homes is decreasing with μ , that is, with the intensity of the parental bequest motive. Thus, individual's willingness to transmit wealth limits the extent to which nursing homes can realize a high mark-up. This limitation in the mark up is even larger when the interest factor R_t is larger.

Another difference with respect to the baseline model concerns the effect of the bargaining power distribution on the mark up, as stated in Corollary 2.

Corollary 2 *At the temporary equilibrium with myopic anticipations, the mark up of nursing homes varies non-monotonically with the bargaining power of the child in the family:*

$$\frac{dMarkup}{d\theta} = \frac{-\delta(1-s) - s\delta\mu R_t + \gamma\mu}{[\theta(1-s) + \theta\mu R_t s + (1-\theta)\mu]^2} L^2$$

We have

$$\frac{dMarkup}{d\theta} < 0 \iff \delta[-(1-s) - s\mu R_t + \frac{\gamma}{\delta}\mu] < 0$$

Proof. This is obtained by taking the derivative of $\frac{(\gamma\theta + \delta(1-\theta))}{[\theta(1-s) + \theta\mu R_t s + (1-\theta)\mu]} L^2$ with respect to θ . ■

That result is quite different from what prevailed in the baseline model, where a larger bargaining power for children had the unambiguous effect to reduce the margins of nursing homes. Indeed, when $s = \mu = 0$, we have $\frac{dMarkup}{d\theta} = \frac{-\delta}{\theta^2} L^2 < 0$ (Corollary 1). This is not necessarily the case here. Indeed, there are now three effects at play. On the one hand, increasing prices reduces consumption of the young, up to their propensity to consume. It also reduces the amount of resources that he can transmit to his child. For these two reasons, θ limits the capacity of nursing homes to charge high prices, so that when θ increases, the mark up charged by nursing homes has to decrease.

³¹Note also that, at the temporary equilibrium, the market for goods clears. To see this, note that the demand for goods is equal to:

$$C + I_g + \lambda Lc + \lambda L(Markup)$$

where C is consumption of the young, I_g is the investment in the goods sector, λLc are the goods used as inputs in nursing homes, and $\lambda L(Markup)$ are the profits of nursing home holders spent on the goods market. The demand for goods can be rewritten as

$$L(w_t + g_t - \lambda p_t) + L\lambda p_t = L(\phi(1-\alpha)k_t^\alpha + \phi\alpha k_t^\alpha)$$

which is equal to the supply of goods, $L\phi k_t^\alpha$.

This is reflected by the two first terms, $-(1-s) - s\mu R_t$ in the above inequality. On the other hand, the parent gets lower net utility from transmitting wealth if LTC prices increase. But as θ increases, his bargaining power decreases and this allows nursing homes to charge higher prices. This last effect is reflected through $\gamma\mu/\delta$ in the above equality and is positive. Depending on the magnitude of these three effects, it may then be the case that the mark up increases with θ (for instance if γ/δ is high).

Corollary 2 suggests that the results obtained in the baseline model are not fully robust to how we specify the interests of the dependent parents. True, when these are only concerned about the distance between the nursing home and the child, a higher bargaining power for parents raises the mark up for nursing homes. However, once a parental bequest motive is introduced, this result does not necessarily hold anymore and, we may have that, for a high level of μ , an increase in the bargaining power of the parent decreases the mark up.

Finally, it should be stressed that the relation between the distribution of bargaining power in the family and the mark up of nursing homes depends on the level of the interest factor R_t . When this is high, it is likely that the mark up remains decreasing with the bargaining power of the children. However, when it is low, the mark up may increase with the bargaining power of the children (this is actually the case when $\gamma\mu > \delta(1-s) + s\delta\mu R_t$). Hence, in order to study the dynamics of nursing home prices, it is necessary to study its links with the dynamics of wealth accumulation as we do in the following section.

5.3 Intertemporal equilibrium

The capital accumulation follows the law:

$$k_{t+1} = s(w_t + g_t - \lambda p_t) \quad (42)$$

where $g_t = R_t s_{t-1} = R_t s[w_{t-1} + g_{t-1} - \lambda p_{t-1}]$. The capital accumulation equation shows that high nursing home prices prevent capital accumulation given the fixed propensity to save.³² Similarly the equation for transfer g_t shows that high nursing home prices prevent wealth accumulation.

Hence the economy can be described by the following system:

$$\begin{aligned} k_{t+1} &= s(w_t + g_t - \lambda p_t) \\ g_{t+1} &= R_{t+1} s[w_t + g_t - \lambda p_t] \\ p_{t+1} &= c + \frac{(\gamma\theta + \delta(1-\theta))}{[\theta(1-s) + \theta\mu s R_{t+1} + (1-\theta)\mu]} L^2 \end{aligned}$$

Note that, from the first two relations, it appears that $g_{t+1} = R_{t+1} k_{t+1} = \phi\alpha(k_{t+1})^{\alpha-1} k_{t+1} = \phi\alpha k_{t+1}^\alpha$. Hence, substituting for $g_t = \phi\alpha k_t^\alpha$, $R_t = \phi\alpha k_t^{\alpha-1}$

³²Note that this result follows from the assumption that all profits of the nursing homes are spent on the good market. Alternatively, if we had supposed that all the profits were saved, then high nursing homes prices would have favoured capital accumulation.

and $w_t = \phi(1 - \alpha)k_t^\alpha$, the system can be reduced to a two-dimensional system:³³

$$k_{t+1} = s(\phi(1 - \alpha)k_t^\alpha + \phi\alpha k_t^\alpha - \lambda p_t) = s(\phi k_t^\alpha - \lambda p_t) \quad (43)$$

$$p_{t+1} = c + \frac{(\gamma\theta + \delta(1 - \theta))}{\left[\theta(1 - s) + \theta\mu s\phi\alpha [s(\phi k_t^\alpha - \lambda p_t)]^{\alpha-1} + (1 - \theta)\mu\right]} L^2 \quad (44)$$

A stationary equilibrium is defined as a situation where the economy perfectly reproduces itself over time. Formally, a stationary equilibrium is a pair (k_t, p_t) such that $k_{t+1} = k_t = k$ and $p_{t+1} = p_t = p$.

Proposition 5 examines the issues of existence and uniqueness of a stationary equilibrium in our economy.

Proposition 5 Denote $\Lambda \equiv (\gamma\theta + \delta(1 - \theta)) L^2$ and denote $\Omega \equiv \theta(1 - s) + (1 - \theta)\mu$. Denote the equation:

$$\Phi(x) \equiv \theta\mu (s\phi)^2 \alpha x^{2\alpha-1} - \Omega x + s\phi(\Omega - \mu\theta\alpha)x^\alpha - \lambda s^2 c \mu \theta \phi \alpha x^{\alpha-1} - \Omega \lambda s c - \lambda s \Lambda$$

Regarding the existence and uniqueness of a stationary equilibrium, three cases can arise:

- If there exists no $x \geq 0$ such that $\Phi(x) = 0$, then there exists no stationary equilibrium.
- If there exists only one $x \geq 0$ such that $\Phi(x) = 0$, then there exists one stationary equilibrium (k, p) .
- If

$$c + \frac{\Lambda}{[\theta(1 - s) + \mu]} < (s\phi)^{\frac{1}{1-\alpha}} \frac{\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}}{\lambda s}$$

, then there exists at least $x_1, x_2 > 0$ with $x_1 \neq x_2$ such that $\Phi(x_1) = \Phi(x_2) = 0$, and there exist at least two stationary equilibria (k_1, p_1) and (k_2, p_2) with

$$\begin{aligned} 0 &< k_1 < (s\alpha\phi)^{\frac{1}{1-\alpha}} < k_2 < (s\phi)^{\frac{1}{1-\alpha}} \\ p_1 &< p_2 \end{aligned}$$

Proof. See the Appendix. ■

Proposition 5 states that three distinct cases can arise. Either there exists no stationary equilibrium, or there exists only one stationary equilibrium, or there is a multiplicity (an even number) of stationary equilibria. The first case (non-existence) arises when the productivity parameter ϕ is too low (i.e. the kk

³³Note that, if one assumed rational expectations instead of myopic expectations, the mark up would then depend on the future interest rate, so that p_{t+1} would depend on R_{t+2} , and, hence, on k_{t+2} . The resolution would then have required the introduction of another variable, and it would not have been possible to reduce the system to a two-dimensional system. Our reliance on myopic anticipations thus facilitates the analysis in the present context.

locus is low), or, alternatively, when the size of the line L is too large (i.e. the pp relation is high), which pushes towards non-sustainable mark up. That case is illustrated on Figure 2 below.³⁴ The second case (uniqueness) arises under quite particular parametrization, but cannot be excluded *a priori* (Figure 3).

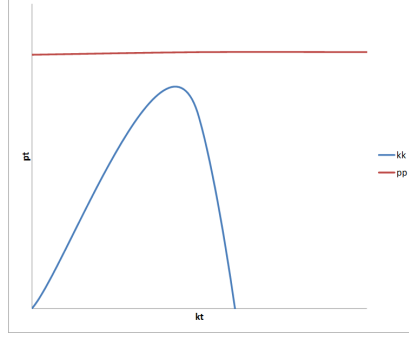


Figure 2: Non existence of a stationary equilibrium.

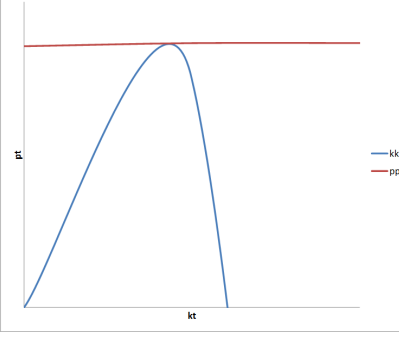


Figure 3: Existence of a unique stationary equilibrium.

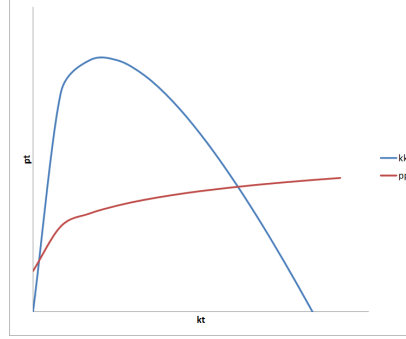


Figure 4: Existence of two stationary equilibria

The most general case is the third one, where there exists a multiplicity (an even number) of stationary equilibria (see Figure 4). We know for sure that, when there exist only two equilibria, there is a positive correlation between, on the one hand, the sustainable level of capital, and, on the other hand, the price of nursing home associated to those stationary equilibria. Proposition 6 states that richer stationary economies are also characterized by higher nursing home prices. The intuition behind that result lies in the fact that the mark up can be higher in richer economies, where the interest factor is lower.

³⁴On Figures 2 to 4, the kk relation is the kk locus, while the pp relation coincides with the price equation giving p_t as a function of k_t (see the Appendix).

Whereas Proposition 5 informs us about the existence and uniqueness of a stationary equilibrium in our economy, Proposition 6 examines the stability. For that purpose, we focus on the most general case (case 3), where there exists an even number of stationary equilibria.

Proposition 6 *Considering the stability of stationary equilibria (k_1, p_1) and (k_2, p_2) , two cases can arise:*

- If, for a stationary equilibrium level k_r , we have

$$\left| s\phi\alpha k_r^{\alpha-1} + \frac{\Lambda\lambda\mu s^\alpha\theta\phi\alpha(\alpha-1)k_r^{\alpha-2}}{[\mu s^\alpha\theta\phi\alpha k_r^{\alpha-1} + \Omega]^2} \right| < 1$$

then the stationary equilibrium (k_r, p_r) is locally stable.

- If, for a stationary equilibrium level k_r , we have

$$\left| s\phi\alpha k_r^{\alpha-1} + \frac{\Lambda\lambda\mu s^\alpha\theta\phi\alpha(\alpha-1)k_r^{\alpha-2}}{[\mu s^\alpha\theta\phi\alpha k_r^{\alpha-1} + \Omega]^2} \right| > 1$$

then the stationary equilibrium (k_r, p_r) is a saddle point (thus unstable).

- Assuming $\alpha = \frac{1}{2}$ and $c = 0$, the condition for local stability for (k_1, p_1) is:

$$\left| \frac{\Omega s\phi}{-\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) - \Psi} - \frac{1}{4} \frac{\Lambda\mu s^2\theta\phi \left[\frac{2\Omega}{\left[-\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) - \Psi\right]} \right]^3}{\left[\frac{\Omega\mu s^{1/2}\theta\phi}{\left[-\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) - \Psi\right]} + \Omega \right]^2} \right| < 1$$

whereas the condition for stability for (k_2, p_2) is:

$$\left| \frac{\Omega s\phi}{-\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) + \Psi} - \frac{1}{4} \frac{\Lambda\mu s^2\theta\phi \left[\frac{2\Omega}{\left[-\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) + \Psi\right]} \right]^3}{\left[\frac{\Omega\mu s^{1/2}\theta\phi}{\left[-\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right) + \Psi\right]} + \Omega \right]^2} \right| < 1$$

$$\text{where } \Psi \equiv \sqrt{\left(\frac{\mu s\theta\phi}{2} - s\phi\Omega\right)^2 - 4\Omega\left(\lambda s\Lambda - \frac{(s\phi)^2\mu\theta}{2}\right)}.$$

Proof. See the Appendix. ■

It is hard, without imposing further constraints on parameters, to assess the plausibility of the stability conditions, since these depend on the level of k at the stationary equilibrium, for which there is no closed form solution. Depending on the parametrization, which can lead to a more or less large k at the stationary equilibrium, the steady-state can be either locally stable or

unstable (it then takes the form of a saddle path equilibrium). This difficulty to interpret stability conditions is the reason why the second part of Proposition 6 imposes further parametric restrictions (i.e. $\alpha = 1/2$ and $c = 0$), and derives explicit stability conditions for the two stationary equilibrium, conditions that depend only on the structural parameters of the economy.

The rest of this section proposes to explore numerically the issue of convergence towards a stationary equilibrium. For that purpose, we rely on the values of structural parameters as shown in Table 1. The initial level of k is set to 1 for convenience. When combined with the values of α and ϕ , this yields, for a period of 35 years, an initial interest rate equal to about 3.2 %. The savings rate is fixed to 20 %. Without any prior information on the distribution of bargaining power within the family, we fix $\theta = 0.5$. The value of λ leads to a duration of stay in the nursing home equal to 3.5 years. Values for c and L are fixed in such a way that, for the chosen values of γ , δ and μ , the ratio of the price of nursing home per period over the wage per period fits the data. The average annual wage in France equals, according to INSEE, about 2202 euros per month, implying 26424 euros per year, whereas the average price for one year in a nursing home equals 35000 euros. This yields a ratio $p/w = 1.3245$. Moreover, according to Martin (2014, p. 128), mark up rates in the nursing sector are, on average, equal to about 11.76 % (of the price) for firms including a large network of nursing homes.³⁵ Given that the initial wage equals 7, and the ratio $\frac{p}{w} = 1.3245$, this leads to $c = 8.2$ and $L = 1.6$. Figures 5, 6 and 7 show, respectively, the dynamics of the capital to labor ratio, the interest rate and the mark up rate of nursing homes.

Parameters	α	ϕ	γ	δ	μ	θ	λ	s	c	L
Values	0.3	10.0	0.5	0.5	1.0	0.5	0.1	0.2	8.2	1.6

Table 1: Calibration of parameters.

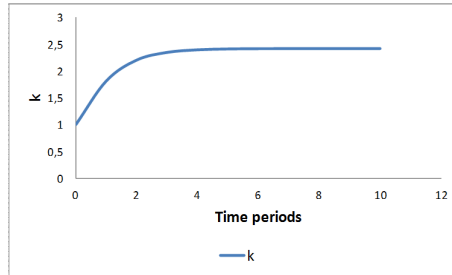


Figure 5: Dynamics of k .

³⁵That average mark up rate is computed for firms including more than 50 nursing homes. Martin (2014) shows that the average mark up rate equals 6.9 % for firms including between 5 and 49 nursing homes, and to 4.2 % for firms including less than 5 nursing homes.

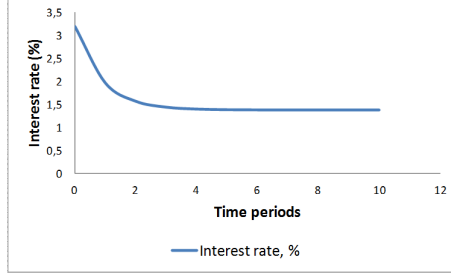


Figure 6: Dynamics of interest rate.

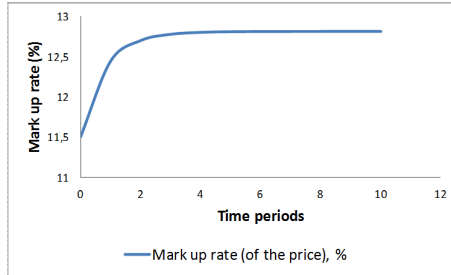


Figure 7: Dynamics of mark up rate.

During the convergence towards the stationary equilibrium, capital accumulates, which pushes interest rates down from 3.2 % towards 1.4 % (Figure 6). That drop in the interest rate is relatively small, but it has non negligible effects on the mark up rate charged by nursing homes, and, hence, on the price of nursing homes. As shown on Figure 7, the mark up rate increases by about 1.3 percentage points during the transition towards the stationary equilibrium.

Although non exhaustive, our numerical simulations show that capital accumulation tends, by reducing the interest rate, to increase mark up rates. The underlying intuition is that, as the interest rate goes down, the opportunity cost of spending in LTC is reduced, which makes families less sensitive to the price, and, hence, allows nursing homes to extract a higher mark up. Thus our OLG model provides a simple explanation for the rise of LTC spending in the nursing home sector and suggests that this rise may be a simple corollary of wealth accumulation, which pushes interest rates down and, hence, mark up rates up.

Finally, note that the possible multiplicity of stationary equilibria and their potential instability makes the general problem of the characterization and the decentralization of the long-run utilitarian social optimum quite complex. The Appendix solves that social planning problem for the case where the structural parameters are such that there exists a unique stable stationary equilibrium.

6 Conclusions

The rise of LTC needs constitutes a challenge not only for policy-makers, but, also, for economic theorists. The main beneficiary of LTC is a dependent person, who, by definition, suffers from cognitive and / or functional limitations. Given that a dependent person is far from a standard "sovereign" consumer, one cannot apply standard demand theory to study the demand for nursing homes.

This paper explored a simple model of intrafamily bargaining where the demand for nursing homes is the outcome of a family bargaining process between the dependent parent and his child. In that framework, the bargaining power of the dependent parent can reflect, in part, the extent to which his cognitive / functional capacities are limited. In particular, we focused on interactions between, on the one hand, the distribution of bargaining power within the family, and, on the other hand, the characteristics of nursing homes: price and location. For that purpose, we studied two variants - one static and one dynamic - of the Hotelling model of spacial competition between nursing homes.

We showed that the laissez-faire equilibrium is far from satisfactory from a social perspective, since this involves extremely peripheral locations of nursing homes (following the principle of maximal differentiation), as well as prices above the marginal cost of production. We showed also that the mark up of nursing homes is, in the baseline model without bequest motive, increasing with the bargaining power of the dependent parent (since the child is the unique bearer of LTC spending). Note, however, that this result does not necessarily hold once a bequest motive is introduced. In a dynamic OLG model with wealth accumulation, a sufficiently strong bequest motive can make the mark up of nursing homes decreasing in the bargaining power of the dependent parent.

Another important finding concerns the comparison of the laissez-faire equilibrium with the utilitarian social optimum. In comparison with the laissez-faire, the utilitarian social optimum involves nursing homes located in a more central manner, which leads to a lower disutility of the distance for families. The social optimum involves also lower prices than the laissez-faire. We showed that the social optimum can be decentralized by forcing locations and subsidizing nursing homes (on a per patient basis). In a second-best world where locations cannot be forced, the decentralization requires, in addition, a non-linear tax of nursing homes based on how far they locate from the socially optimal locations.

Our extension to an OLG economy emphasized that the dynamics of wealth accumulation and of LTC prices are related through various channels. First, high LTC prices, by reducing, under a fixed propensity to save, the size of descending wealth transfers, prevent capital accumulation. But the relationship goes also in the other way: a higher capital stock reduces the interest rate, which decreases the opportunity cost of LTC spending. Hence, wealth accumulation tends, by lowering interest rates, to raise the price of nursing homes through higher mark up rates. Our numerical analysis showed that, as the interest rate falls from 3.2 % to 1.4 %, the mark up rate increases by 1.3 percentage points. As such, the extension of our baseline static model to a dynamic OLG economy allows us to point to a possible explanation for the increase in nursing home

prices, which could be a simple corollary of wealth accumulation, which lowers interest rates and thus the opportunity cost of LTC expenditures.

In sum, this paper suggests that, when considering the dynamics of nursing home prices, one should pay particular attention to the interplay between microeconomic and macroeconomic determinants. At the micro level, nursing home prices vary with the ability of nursing homes to charge high mark up rates, which depends on the preferences of family members, and on the distribution of bargaining power within the family. However, since the opportunity cost of LTC spending depends on the return on capital, the dynamics of nursing home prices cannot be separated from the dynamics of wealth accumulation, which is itself influenced by LTC spending (and, thus, by microeconomic factors).

Finally, it should be stressed that our study suffers also from some limitations. First, our study focused on two determinants of nursing home choices (price and location) and abstracted from the quality of nursing homes. Although Schmitz and Stroka (2014) show that quality is not a crucial determinant of nursing home choices, assuming that quality does not matter at all constitutes a strong assumption. Introducing quality in the model may affect the picture substantially, by influencing the level and the dynamics of nursing homes prices. The robustness of our results to introducing quality remains to be examined.³⁶ A second limitation concerns the spatial structure: this paper studies nursing home locations while assuming that families are distributed along a line. However, one could, alternatively, have considered families distributed along a circle (as in Salop 1979) or along any other geometrical form. Considering optimal nursing home choices and locations under those alternative spatial structures goes beyond the scope of this paper, and is thus left for future research. A third limitation concerns the postulated absence of informal care in our model. One could add, for families, another option, which would consist in providing informal care to the elderly parent at home. Provided the disutility of providing informal LTC for children is low (so that some families decide to keep their parents at home), adding that third option could affect the demand for nursing homes, and, hence, the mark up and locations of nursing homes. All those limitations suggest that the present paper is only a starting point for a deeper theoretical exploration of nursing home choices and locations.

7 References

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³⁶See Gonzalez et al (2016) on a model of pharmaceutical innovation with both horizontal and vertical differentiation of drugs.

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8 Appendix

8.1 The social optimum

Fully differentiating condition (13) with respect to (a, b, c_A, c_B) , one can show that:

$$\begin{aligned}\frac{dm}{da} &= \frac{m-a}{L-b-a} > 0 \\ \frac{dm}{db} &= \frac{m-(L-b)}{L-b-a} < 0 \\ \frac{dm}{dc_A} &= \frac{\theta}{2(\gamma\theta + (1-\theta)\delta)(L-b-a)} > 0 \\ \frac{dm}{dc_B} &= -\frac{\theta}{2(\gamma\theta + (1-\theta)\delta)(L-b-a)} < 0\end{aligned}$$

Problem (12) yields the following rearranged first order conditions.

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_A} &= \int_{j=0}^m (\frac{1}{2} - \lambda) dj + \frac{1}{2} \frac{dm}{dc_A} \\ &\quad \times [c_A - c_B + (\gamma + \delta)((L-b-m)^2 - (m-a)^2)] \\ &\quad - \lambda(c_A - c_B) \frac{dm}{dc_A} \leq 0\end{aligned}\tag{45}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_B} &= \int_{j=m}^L (\frac{1}{2} - \lambda) dj + \frac{1}{2} \frac{dm}{dc_B} \\ &\quad \times [c_A - c_B + (\gamma + \delta)((L-b-m)^2 - (m-a)^2)] \\ &\quad - \lambda(c_A - c_B) \frac{dm}{dc_B} = 0\end{aligned}\tag{46}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial a} &= (\gamma + \delta) \int_{j=0}^m (j-a) dj + \frac{1}{2} \frac{dm}{da} \\ &\quad \times [(c_A - c_B) + (\gamma + \delta)((L-b-m)^2 - (m-a)^2)] \\ &\quad - \lambda(c_A - c_B) \frac{dm}{da} \leq 0\end{aligned}\tag{47}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial b} &= (\gamma + \delta) \int_{j=m}^L (L-b-j) dj + \frac{1}{2} \frac{dm}{db} \\ &\quad \times [(c_A - c_B) + (\gamma + \delta)((L-b-m)^2 - (m-a)^2)] \\ &\quad - \lambda(c_A - c_B) \frac{dm}{db} \leq 0\end{aligned}\tag{48}$$

8.2 Proof of Proposition 5

The dynamics of the economy are described by equations (43) and (44). Let us define the kk locus, along which capital per worker is constant. This is defined by the relation:

$$k_t = s(\phi k_t^\alpha - \lambda p_t)$$

Isolating p_t , we obtain:

$$p_t = \frac{s\phi k_t^\alpha - k_t}{\lambda s} \quad (49)$$

Let us focus on the (k_t, p_t) space. The kk locus intersects the x axis at $k_t = 0$ and when $s\phi k_t^\alpha - k_t = 0$, that is, when $k_t = (s\phi)^{\frac{1}{1-\alpha}}$. Let us denote this level of k as $\hat{k} \equiv (s\phi)^{\frac{1}{1-\alpha}}$.

The kk locus reaches its maximum when $s\phi\alpha k_t^{\alpha-1} - 1 = 0$, that is, when $k = (s\alpha\phi)^{\frac{1}{1-\alpha}}$. Let us denote this level of k by $\bar{k} \equiv (s\alpha\phi)^{\frac{1}{1-\alpha}}$.

The kk locus, starting from $(0, 0)$, is increasing for $k_t < \bar{k}$ and decreasing for $k_t > \bar{k}$. Note that, for $k_t > \hat{k}$, only negative prices could sustain a positive capital intensity.

Let us now consider the pp locus. This is defined by the equality:

$$p_{t+1} = c + \frac{(\gamma\theta + \delta(1-\theta))}{\left[\theta(1-s) + \mu s\theta\phi\alpha [s(\phi k_t^\alpha - \lambda p_t)]^{\alpha-1} + (1-\theta)\mu\right]} L^2 \quad (50)$$

Reminding that: $s(\phi k_t^\alpha - \lambda p_t) = k_{t+1}$, and backwaring this expression by one period, one can deduce from the pp locus expression what we will call the pp relation. Unlike the pp locus, the pp relation does not provide the future level of price as a function of the past level of the price, but is merely a simple price equation, which shows how the current price of nursing homes depends on the current interest rate:

$$p_t = c + \frac{(\gamma\theta + \delta(1-\theta))}{\left[\theta(1-s) + \mu s\theta\phi\alpha k_t^{\alpha-1} + (1-\theta)\mu\right]} L^2 \quad (51)$$

The pp relation cannot replace the pp locus when considering the issue of stability (see below), but as far as the existence and uniqueness is concerned, the reliance on the pp relation simplifies the picture significantly.

The RHS of the pp relation is clearly increasing in k_t . Moreover, we have:

$$\lim_{k_t \rightarrow 0} p_t = c \text{ and } \lim_{k_t \rightarrow +\infty} p_t = c + \frac{(\gamma\theta + \delta(1-\theta))}{[\theta(1-s) + (1-\theta)\mu]} L^2$$

From the first expression, we have that the pp relation lies above the kk locus when $k_t \rightarrow 0$.

Moreover, at $\hat{k} = (s\phi)^{\frac{1}{1-\alpha}}$, the kk locus crosses the x axis, while the pp relation takes the value:

$$p_t = c + \frac{(\gamma\theta + \delta(1-\theta))}{[\theta(1-s) + \mu\theta\alpha + (1-\theta)\mu]} L^2 > 0$$

Thus, when we consider very low or very high values of k_t , the pp relation lies above the kk locus.

Three cases can arise: (1) no stationary equilibrium exists; (2) there exists only one stationary equilibrium; (3) there exists at least two stationary equilibria.

Case (1) arises only when the pp relation is strictly above the kk locus for any $k_t \in \left[0, (s\phi)^{\frac{1}{1-\alpha}}\right]$. There exists no stationary equilibrium when the following equality never holds:

$$c + \frac{(\gamma\theta + \delta(1-\theta))}{\left[\theta(1-s) + \mu s\theta\phi\alpha k_t^{\alpha-1} + (1-\theta)\mu\right]} L^2 = \frac{s\phi k_t^\alpha - k_t}{\lambda s}$$

That expression can be rewritten as follows:

$$\Phi(k_t) \equiv \theta\mu(s\phi)^2 \alpha k_t^{2\alpha-1} - \Omega k_t + s\phi k_t^\alpha (\Omega - \mu\theta\alpha) - \lambda s^2 c \mu\theta\phi\alpha k_t^{\alpha-1} - \Omega\lambda s c - \lambda s\Lambda = 0 \quad (52)$$

Thus the non existence of a stationary equilibrium arises when there exists no $k_t \geq 0$ such that $\Phi(k_t) = 0$. In that case, the pp curve is always strictly above the kk locus.

Case (2) arises when there exists a unique solution, $k \geq 0$ to $\Phi(k_t) = 0$. In that case, the pp curve is tangent to the kk locus at one point.

Case (3) arises when the pp curve is below the kk locus when the kk locus reaches its maximum, that is, at $\bar{k} = (s\alpha\phi)^{\frac{1}{1-\alpha}}$. In that case, and given that the pp curve is above the kk locus for low values of k_t and for high values of k_t , it must be the case that the kk locus and the pp curve intersect at least twice. The condition such that the pp curve is below the kk locus when the kk locus reaches its maximum is:

$$c + \frac{(\gamma\theta + \delta(1-\theta))}{\left[\theta(1-s) + \mu s\theta\phi\alpha \left((s\alpha\phi)^{\frac{1}{1-\alpha}}\right)^{\alpha-1} + (1-\theta)\mu\right]} L^2 < \frac{s\phi (s\alpha\phi)^{\frac{\alpha}{1-\alpha}} - (s\alpha\phi)^{\frac{1}{1-\alpha}}}{\lambda s}$$

This can be rewritten as:

$$c + \frac{(\gamma\theta + \delta(1-\theta))}{[\theta(1-s) + \mu]} L^2 < (s\phi)^{\frac{1}{1-\alpha}} \frac{\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}}{\lambda s}$$

When that condition holds, we know for sure that $\Phi(k_t) = 0$ admits at least two solutions. In that case, there exists at least two stationary equilibria. Thus, if we denote the two stationary equilibria as (k_1, p_1) and (k_2, p_2) , we have $k_1 < k_2$ and $p_1 < p_2$.

8.3 Proof of Proposition 6

Let us now consider the stability of those stationary equilibria. We have:

$$\begin{aligned} k_{t+1} &= s(\phi k_t^\alpha - \lambda p_t) \equiv F(k_t, p_t) \\ p_{t+1} &= c + \frac{\Lambda}{\left[\mu s\theta\phi\alpha [s(\phi k_t^\alpha - \lambda p_t)]^{\alpha-1} + \Omega\right]} \equiv G(k_t, p_t) \end{aligned}$$

The Jacobian matrix is:

$$J \equiv \begin{pmatrix} \frac{\partial F(k_t, p_t)}{\partial k_t} & \frac{\partial F(k_t, p_t)}{\partial p_t} \\ \frac{\partial G(k_t, p_t)}{\partial k_t} & \frac{\partial G(k_t, p_t)}{\partial p_t} \end{pmatrix}$$

We have:

$$\begin{aligned}\frac{\partial F(k_t, p_t)}{\partial k_t} &= s\phi\alpha k_t^{\alpha-1}, & \frac{\partial F(k_t, p_t)}{\partial p_t} &= -s\lambda \\ \frac{\partial G(k_t, p_t)}{\partial k_t} &= \frac{-\Lambda \left[\mu s^\alpha \theta \phi^2 \alpha^2 (\alpha-1) [(\phi k_t^\alpha - \lambda p_t)]^{\alpha-2} k_t^{\alpha-1} \right]}{\left[\mu s \theta \phi \alpha [s(\phi k_t^\alpha - \lambda p_t)]^{\alpha-1} + \Omega \right]^2} \\ \frac{\partial G(k_t, p_t)}{\partial p_t} &= \frac{\Lambda \left[\lambda \mu s^\alpha \theta \phi \alpha (\alpha-1) [(\phi k_t^\alpha - \lambda p_t)]^{\alpha-2} \right]}{\left[\mu s \theta \phi \alpha [s(\phi k_t^\alpha - \lambda p_t)]^{\alpha-1} + \Omega \right]^2}\end{aligned}$$

The determinant of the Jacobian matrix is equal to 0, whereas the trace is, at the stationary equilibrium:

$$s\phi\alpha k^{\alpha-1} + \frac{\Lambda \lambda \mu s^\alpha \theta \phi \alpha (\alpha-1) k^{\alpha-2}}{[\mu s^\alpha \theta \phi \alpha k^{\alpha-1} + \Omega]^2}$$

The Jacobian matrix has two eigen values. One is zero and the other is

$$s\phi\alpha k^{\alpha-1} + \frac{\Lambda \lambda \mu s^\alpha \theta \phi \alpha (\alpha-1) k^{\alpha-2}}{[\mu s^\alpha \theta \phi \alpha k^{\alpha-1} + \Omega]^2}$$

Hence, two cases can arise:

- If

$$\left| s\phi\alpha k^{\alpha-1} + \frac{\Lambda \lambda \mu s^\alpha \theta \phi \alpha (\alpha-1) k^{\alpha-2}}{[\mu s^\alpha \theta \phi \alpha k^{\alpha-1} + \Omega]^2} \right| < 1$$

then the stationary equilibrium is locally stable.

- If

$$\left| s\phi\alpha k^{\alpha-1} + \frac{\Lambda \lambda \mu s^\alpha \theta \phi \alpha (\alpha-1) k^{\alpha-2}}{[\mu s^\alpha \theta \phi \alpha k^{\alpha-1} + \Omega]^2} \right| > 1$$

then the stationary equilibrium is a saddle point (and thus unstable).

To go further in the investigation, we need to be able to have closed form solutions for equilibrium levels of k and p . For that purpose, let us suppose that $\alpha = \frac{1}{2}$ and $c = 0$.

Using (52), we have, at the equilibrium, that k is a solution to

$$-\frac{(s\phi)^2 \mu \theta}{2} + k\Omega + k^{\frac{1}{2}} \left[\frac{\mu s \theta \phi}{2} - s\phi\Omega \right] + \lambda s\Lambda = 0$$

Denoting $x \equiv k^{1/2}$, this condition can be written as:

$$\Omega x^2 + x \left(\frac{\mu s \theta \phi}{2} - s\phi\Omega \right) + \lambda s\Lambda - \frac{(s\phi)^2 \mu \theta}{2} = 0.$$

We have

$$\Delta = \left(\frac{\mu s \theta \phi}{2} - s\phi\Omega \right)^2 - 4\Omega \left(\lambda s\Lambda - \frac{(s\phi)^2 \mu \theta}{2} \right)$$

and thus, the solutions to the above polynomial are:

$$x_{1,2} = \frac{-\left(\frac{\mu s \theta \phi}{2} - s\phi\Omega \right) \pm \sqrt{\left(\frac{\mu s \theta \phi}{2} - s\phi\Omega \right)^2 - 4\Omega \left(\lambda s\Lambda - \frac{(s\phi)^2 \mu \theta}{2} \right)}}{2\Omega}$$

implying

$$k_{1,2} = \left[\frac{-\left(\frac{\mu s \theta \phi}{2} - s \phi \Omega\right) \pm \sqrt{\left(\frac{\mu s \theta \phi}{2} - s \phi \Omega\right)^2 - 4 \Omega \left(\lambda s \Lambda - \frac{(s \phi)^2 \mu \theta}{2}\right)}}{2 \Omega} \right]^2.$$

Hence prices satisfy

$$p_{1,2} = c + \frac{\Lambda}{\left[\mu s \theta \phi \alpha k_{1,2}^{\alpha-1} + \Omega\right]}$$

In that case, the stability condition becomes, in case of k_1 :

$$\left| \frac{\Omega s \phi}{-\left(\frac{\mu s \theta \phi}{2} - s \phi \Omega\right) + \Psi} - \frac{1}{4} \frac{\Lambda \mu s^2 \theta \phi \left[\frac{2 \Omega}{-\left(\frac{\mu s \theta \phi}{2} - s \phi \Omega\right) + \Psi} \right]^3}{\left[\frac{\Omega \mu s^{1/2} \theta \phi}{-\left(\frac{\mu s \theta \phi}{2} - s \phi \Omega\right) + \Psi} + \Omega \right]^2} \right| < 1$$

where $\Psi \equiv \sqrt{\left(\frac{\mu s \theta \phi}{2} - s \phi \Omega\right)^2 - 4 \Omega \left(\lambda s \Lambda - \frac{(s \phi)^2 \mu \theta}{2}\right)}$.

In the case of k_2 , the condition for stability is:

$$\left| \frac{\Omega s \phi}{-\left(\frac{\mu s \theta \phi}{2} - s \phi \Omega\right) - \Psi} - \frac{1}{4} \frac{\Lambda \mu s^2 \theta \phi \left[\frac{2 \Omega}{-\left(\frac{\mu s \theta \phi}{2} - s \phi \Omega\right) - \Psi} \right]^3}{\left[\frac{\Omega \mu s^{1/2} \theta \phi}{-\left(\frac{\mu s \theta \phi}{2} - s \phi \Omega\right) - \Psi} + \Omega \right]^2} \right| < 1$$

8.4 Long-run utilitarian optimum and implementation

In order to study the long-run utilitarian optimum in our OLG framework, we assume that the structural parameters of our economy are such that there exists a unique stable stationary equilibrium. We then consider the problem of a utilitarian social planner who selects locations, consumptions and capital that maximize social welfare at the stationary equilibrium.

The problem of the utilitarian social planner can be rewritten as follows:³⁷

$$\begin{aligned} \max_{a, b, c_A, c_B, k} \quad & W = \int_{j=0}^m \left[\frac{1}{2} c_A - \frac{1}{2} \lambda (\gamma + \delta) (j - a)^2 \right] \frac{1}{L} dj \\ & + \int_m^L \left[\frac{1}{2} c_B - \frac{1}{2} \lambda (\gamma + \delta) (L - b - j)^2 \right] \frac{1}{L} dj \\ \text{s.t.} \quad & \int_{j=0}^m c_A \frac{1}{L} dj + \int_m^L c_B \frac{1}{L} dj + \int_0^L c \frac{1}{L} dj + k \leq f(k) \end{aligned} \quad (53)$$

with c_A and c_B , the consumption levels of an agent using the nursing home A or B respectively and, where $m = m(a, b, c_A, c_B)$ is the location of the median family, satisfies the condition:

$$\theta c_A - \lambda (\gamma \theta + (1 - \theta) \delta) (m - a)^2 = \theta c_B - \lambda (\gamma \theta + (1 - \theta) \delta) (L - b - m)^2. \quad (54)$$

³⁷Note that in the objective function of the social planner, we used the lifetime utility of a young specified in equation (34), where we removed the part on the utility of giving so as to avoid double counting.

FOCs are:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_A} &= \int_{j=0}^m \left(\frac{1}{2} - \eta\right) dj + \frac{1}{2} \frac{dm}{dc_A} \\ &\quad \times [c_A - c_B + \lambda(\gamma + \delta)((L - b - m)^2 - (m - a)^2)] \\ &\quad + \eta(c_B - c_A) \frac{dm}{dc_A} \leq 0\end{aligned}\tag{55}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_B} &= \int_{j=m}^L \left(\frac{1}{2} - \eta\right) dj + \frac{1}{2} \frac{dm}{dc_B} \\ &\quad \times [c_A - c_B + \lambda(\gamma + \delta)((L - b - m)^2 - (m - a)^2)] \\ &\quad + \eta(c_B - c_A) \frac{dm}{dc_B} = 0\end{aligned}\tag{56}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial a} &= \lambda(\gamma + \delta) \int_{j=0}^m (j - a) dj + \frac{1}{2} \frac{dm}{da} \\ &\quad \times [c_A - c_B + \lambda(\gamma + \delta)((L - b - m)^2 - (m - a)^2)] \\ &\quad - \eta(c_A - c_B) \frac{dm}{da} \leq 0\end{aligned}\tag{57}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial b} &= \lambda(\gamma + \delta) \int_{j=m}^L (L - b - j) dj + \frac{1}{2} \frac{dm}{db} \\ &\quad \times [c_A - c_B + \lambda(\gamma + \delta)((L - b - m)^2 - (m - a)^2)] \\ &\quad - \eta(c_A - c_B) \frac{dm}{db} \leq 0\end{aligned}\tag{58}$$

$$\frac{\partial \mathcal{L}}{\partial k} = \eta(f'(k) - k) \leq 0\tag{59}$$

with η , the Lagrange multiplier associated with the resource constraint, $dm/dc_A = -dm/dc_B$ and $dm/da > 0$ while $dm/db < 0$. Like in Section 4, one solution (the symmetric utilitarian optimum) consists in setting $c_A = c_B$ and are thus equal to $f(k^*) - k^* - c$ with k^* obtained from $f'(k^*) = 1$.

Replacing for $c_A = c_B$ in the FOCs for a and b , we therefore obtain that $a^* = m/2$, $b^* = (L - m)/2$ and therefore that $m^* = L/2$ which implies that $a^* = L/4$, $b^* = 3L/4$. Thus we have:

$$\begin{aligned}c_A^* &= c_B^* = f(k^*) - k^* - c \\ a^* &= L/4 \text{ and } b^* = 3L/4 \\ m^* &= L/2 \\ f'(k^*) &= 1\end{aligned}$$

The optimal capital level satisfies the standard Golden Rule condition.

Let us now turn to the implementation mechanism. First, note that the above optimality conditions imply that at the decentralized equilibrium, one necessarily has $p_A = p_B = c$.³⁸ Therefore, using the results of Section 4.2.1, we obtain that the optimal subsidy levels should be equal to (24) when location can be forced and that it is independent of the time period. In the same way, if location can only be indirectly affected through non linear taxation, we obtain the same results as in Section 4.2.2. At each time period, the non linear taxes should be equal to $t'_A(0, b_t)$ and $t'_B(a_t, 0)$ defined by (25) and (26) respectively. In both cases, we now also need to introduce intergenerational transfers so as to obtain the optimal level of capital, i.e. satisfying $f'(k^*) = 1$.

³⁸First, $c_i = w - p_i$ but since $c_A = c_B$, necessarily, $p_i = p \forall i$. Second, replacing for $c_A = c_B = w - p$ and for $w = f(k) - kf'(k)$ in the intertemporal budget constraint, one obtains that $-p + c = k(f'(k) - 1) = 0$.