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1 Maria Bevilacqua

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Coalgebraic models of a Lawvere theory

Abstract. Objects in the categories of bialgebras, Hopf algebras or Hopf braces have in common an underlying coalgebra structure. When this structure is cocommutative, they can be interpreted as product-preserving functors from a Lawvere theory to the category **Coalg** of (cocommutative) coalgebras. The idea of passing from models in **Set** to models in **Coalg** was considered in [1] where a “linearization process” was used to study some non associative algebraic structures. This process associates a classical term in the algebraic theory with a linearized one, which is a homomorphism of coalgebras, thereby describing, for instance, how group identities transfer to Hopf algebra identities. Several analogies between classical and coalgebraic models hold. For instance, protomodularity in categories of coalgebraic models can be characterized by means of linearized terms, in the same spirit as the characterization of protomodular varieties of universal algebra. Furthermore, it is possible to construct limits and free functors between categories of coalgebraic models, generalizing some of the constructions provided in [2] and [3].

In light of the semi-abelian nature of **Hopf_{coc}** [4] and **HBR** [5], it is natural to investigate protomodularity, regularity or semi-abelianness in these categories of coalgebraic models more broadly. In fact, it turns out that, if a category of coalgebraic models admits a forgetful functor to **Hopf_{coc}**, it is semi-abelian. This result allows us to construct new examples of semi-abelian categories. In particular, considering the Lawvere theories of radical rings and digroups, we take their models in coalgebras defining the categories **HRadRng** of *Hopf radical rings* and **HDiGrp** of *Hopf digroups*. We obtain the chain of functors

$$\mathbf{HRadRng} \hookrightarrow \mathbf{HSKB} \hookrightarrow \mathbf{HDiGrp} \rightarrow \mathbf{Hopf}_{\text{coc}}$$

where all the involved categories turn out to be semiabelian.

References

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- [4] M. Gran, F. Sterck and J. Vercruysse, *A semi-abelian extension of a theorem by Takeuchi*, Journal of Pure and Applied Algebra 223 (2019), no. 10, 4171–4190.
- [5] M. Gran and A. Sciandra, *Hopf braces and semi-abelian categories*, arXiv:2411.19238, 2024.

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2 Davide Lai

🏠 Desy, Hamburg

Algebraic Structures and Open Problems in AdS/CFT Integrability

Abstract. Since the early 2000s, the discovery of integrable sectors in the AdS/CFT correspondence has established a deep connection between string theory, quantum groups, and exactly solvable models. In this framework, the $\mathcal{R}$ -matrix in specific representations encodes the scattering properties of interacting strings through quantum group symmetries. While integrability has led to major successes in computing the energy spectrum, a complete algebraic formulation in terms of a universal $\mathcal{R}$ -matrix is still needed to fully understand scattering amplitudes. In this talk, I will review the known structures and highlight key open problems, along with some recent ideas and potential directions that merit further investigation. $\square$

3 Manuel Mancini

🏠 University of Palermo

On weak action representability, action accessibility and varieties of non-associative algebras

Abstract. It is well known that in the semi-abelian category of groups, internal actions are represented by automorphisms. This means that the category of groups is action representable with the actor of a group being its group of automorphisms. The notion of action representability has proven to be quite restrictive: for instance, it was shown that the only non-abelian variety of non-associative algebras which is action representable is the variety of Lie algebras. More recently G. Janelidze introduced the concept of weakly action representable category, which includes a wider class of categories, such as the variety of associative algebras and the variety of Leibniz algebras.

This notion was studied in the context of varieties of algebras: it was shown that every object X of an algebraically coherent variety of non-associative algebras admits an external weak representation, which consists of a partial algebra $E(X)$ together with a monomorphism of functors $\text{Act}(-, X) \rightarrow \text{Hom}(U(-), E(X))$, where U denotes the forgetful functor.

The aim of this talk is to investigate the relationship between action accessibility and weak action representability in the frame of varieties of non-associative algebras. Using an argument of J. R. A. Gray in the setting of groups, we prove that the varieties of k -nilpotent ($k > 2$) and n -solvable ($n > 1$) Lie algebras are not weakly action representable. These are the first known examples of action accessible varieties of non-associative algebras that fail to be weakly action representable, establishing that a subvariety of a (weakly) action representable variety of non-associative algebras need not be weakly action representable.

We then aim to study the representability of actions in the context of categories of unitary non-associative algebras, which are ideally exact in the sense of G. Janelidze. After describing the monadic adjunction associated with any category of unitary algebras, we prove that the categories of unitary associative algebras, unitary alternative algebras and unitary Poisson algebras are action representable.

This is joint work with Xabier García Martínez (Universidade de Santiago de Compostela, Spain) and Federica Piazza (Università degli Studi di Messina, Italy). $\square$

4 Mariano Messori

🏠 University of Milan

Prenormal categories

Abstract. In this talk we will discuss the notion of (pointed) *prenormal category*, modelled after regular categories, but with the key notions of coequaliser and kernel pair replaced by those of cokernel and kernel. Prenormality is a weaker condition than normality à la Z. Janelidze, as the former is devised to be completely independent from regularity. We will display the fundamental properties of prenormal categories, including a characterisation in terms of a factorisation system involving normal epimorphisms. We will also present a range of examples, with the category of commutative monoids constituting a central one. Finally, we will show how this notion can be generalised to a non-pointed context in a way which is particularly well-suited to study *pretorsion theories*.

Joint work with Sandra Mantovani.

Some references:

- Z. Janelidze. ‘The pointed subobject functor, 3×3 lemmas, and subtractivity of spans’. In: *Theory Appl. Categ.* 33.11 (2010)
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5 Matteo Misurati

🏠 University of Ferrara

Quasi-Hopf algebras of dimension 6

Abstract. The classification of quasi-Hopf algebras is in an early age, with most advancements focusing on the semisimple or basic case. With respect to the semisimple case, this is due to the fact that semisimple quasi-Hopf algebras characterize fusion categories with integer Frobenius-Perron dimensions of simple objects. Thus, the classification of such fusion categories provides for free the classification of the semisimple quasi-Hopf algebras and vice versa. For instance, this is the case in dimension p and pq , see [3] and [2] respectively.

We completed the classification of the 6-dimensional quasi-Hopf algebras, by proving that any such algebra is semisimple; this is achieved by classifying braided Hopf algebras in Yetter-Drinfeld module categories over a quasi-Hopf algebra, which are categorically equivalent to biproduct quasi-Hopf algebras in the sense of Radford. If time permits, we will also see how tools employed to obtain this result can be used to produce various examples of quasi-Hopf algebras.

References

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- [2] P. Etingof, S. Gelaki, V. Ostrik, *Classification of fusion categories of dimension pq* , Int Math Res Notices **57** (2004), 3041-3056.
- [3] P. Etingof, D. Nikshych, V. Ostrik, *On fusion categories*, Annals of Mathematics **162** (2005), 581-642.

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6 Andrea Rivezzi

🏠 Charles University in Prague

Quantization of infinitesimal braidings and pre-Cartier quasi-bialgebras

Abstract. An infinitesimal braiding for a braided monoidal category is an additional natural transformation satisfying the so-called infinitesimal hexagon relations. Those structures play a fundamental role in the theory of deformations of k -linear braided monoidal categories. Namely, P. Cartier proved that any symmetric braided monoidal category with an infinitesimal braiding can be non-trivially deformed after choosing a Drinfeld associator.

Very recently, further steps on this theory were made. First, A. Ardizzoni, L. Bottegoni, A. Sciandra, and T. Weber (ABSW) developed the theory of pre-Cartier categories, where the braiding is not necessarily commuting with the infinitesimal braiding. Next, L. Bottegoni, F. Renda, and A. Sciandra (BRS) provided numerous examples through the representation categories of pre-Cartier bialgebras, the latter being the algebraic counterpart of pre-Cartier categories. In (ABSW), it was asked whether one can quantize the infinitesimal braiding of a pre-Cartier bialgebra H , that is, given the topological bialgebra associated to H , one can find a (topological) $\mathcal{R}$ -matrix such that its linear term is the infinitesimal $\mathcal{R}$ -matrix of H . The answer for some classes of bialgebras was proved to be positive in (BRS).

In this talk, I will present further recent developments on this theory. We consider a non-strict version of pre-Cartier categories, leading to the notion of a pre-Cartier quasi-bialgebra. Then, I will present a generalization of Cartier's theorem to the non-symmetric setting. This allows us to answer the quantization problem in two settings: the one of Cartier bialgebras (where the output is a quasitriangular quasi-bialgebra) and the one of pre-Cartier bialgebras with an additional commutation hypothesis (where the output is an honest bialgebra). If there is enough time, I will also discuss the infinitesimal quantum Yang-Baxter equation and the Cartier rings, the latter being a generalization of braid groups and being represented by modules of a pre-Cartier quasi-bialgebra.

This is joint work with C. Esposito, J. Schnitzer, and T. Weber.

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7 Thomas Weber

🏠 Charles University in Prague

Infinitesimal braidings and pre-Cartier bialgebras

Abstract. There is a well-known correspondence between quasitriangular bialgebras and braided monoidal categories, where the action of the universal $\mathcal{R}$ -matrix induces a braiding on the representation category of the bialgebra. In this talk we are describing the first-order deformation of the above picture. Namely, we introduce infinitesimal braidings on pre-Cartier categories and pre-Cartier bialgebras with infinitesimal $\mathcal{R}$ -matrices as their algebraic counterparts.

It turns out that these pre-Cartier bialgebra structures correspond to Hochschild 2-cocycles which satisfy a deformed version of the quantum Yang-Baxter equation, while they give rise to Hochschild 2-coboundaries in the Cartier triangular Hopf algebra framework. We discuss explicit examples on the $E(n)$ Hopf algebras and q -deformed $GL(2)$. As main results we provide an infinitesimal FRT construction and a Tannaka-Krein reconstruction theorem for pre-Cartier coquasitriangular bialgebras. The former admits canonical non-trivial solutions and thus induces non-trivial infinitesimal $\mathcal{R}$ -forms on all FRT bialgebras.

The talk is based on a collaboration with A. Ardizzoni, L. Bottegoni and A. Sciandra. It further serves as a preparation for the presentation of A. Rivezzi, which concerns the quantization of infinitesimal braidings. $\square$

8 Zhenbang Zuo

🏠 University of Turin

Quotient Category of a Multiring Category

Abstract. P. Gabriel defined the (Serre) quotient category $\mathcal{A}/\mathcal{C}$ of an abelian category $\mathcal{A}$ by a Serre subcategory $\mathcal{C}$, and he also proved that the Serre quotient category of an abelian category is still an abelian category (1962). The aim of this talk is to introduce a tensor structure for the Serre quotient category of an abelian monoidal category with biexact tensor product to make the canonical functor a monoidal functor. In this tensor product, the Serre quotient category of a multiring category (resp. a multitensor category) by a two-sided Serre tensor-ideal is still a multiring category (resp. a multitensor category).

This talk is based on a joint work with Gongxiang Liu (Nanjing University, China).

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