

Fixed-points Method in Robert Maillart's Analysis of Rigid Frame Bridges

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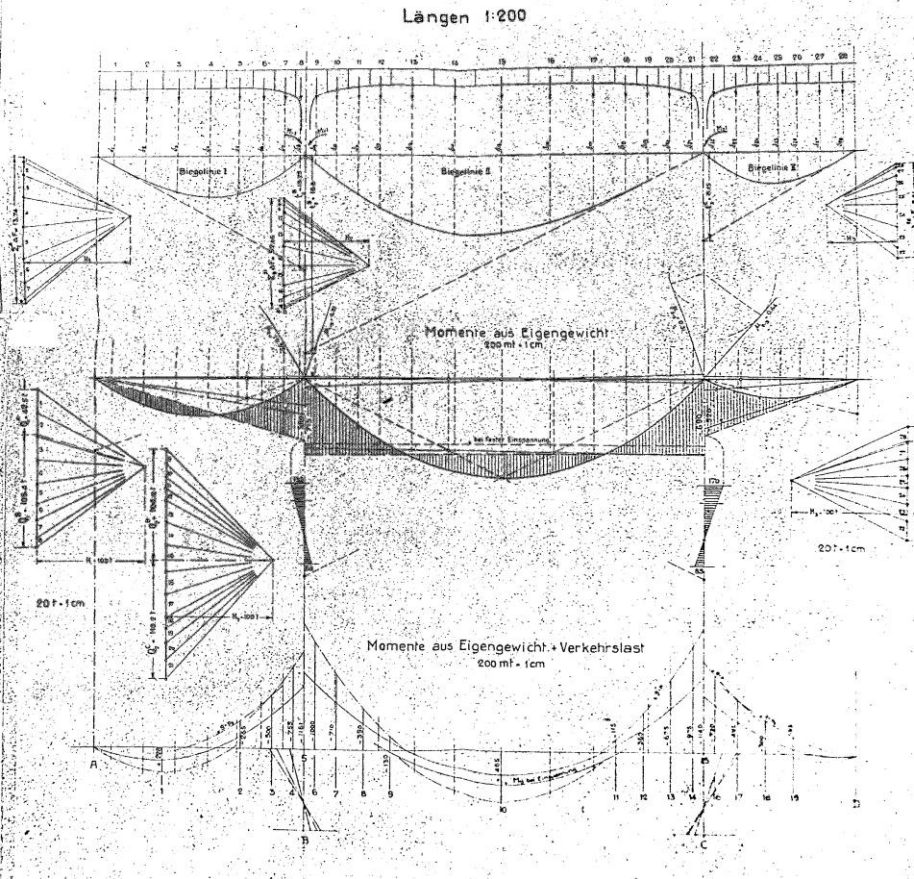
1. Introduction

Background

The **Weissensteinstrasse Overpass** is a **reinforced concrete road bridge** in Bern Switzerland.

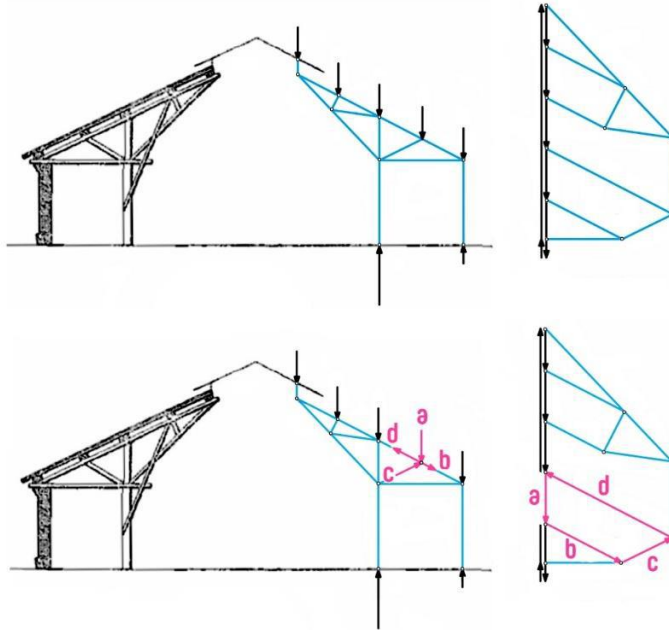
This bridge was designed and engineered by **Robert Maillart** in **1938** and erected in the same year.





the structure was modeled as a three-span rigid frame, analyzed with the fixed-points method.

the fixed-points method is a method of graphic statics for analyzing continuous beams and rigid frames, which are statically indeterminate. It enjoyed some popularity in the later 19th to early 20th century but is almost totally forgotten by the end of 20th century.



Form diagram

force diagram

Graphic statics

- a **classic** geometric-based approach for the analysis and design of structures
- **Intuitive** reciprocal diagrams for **relation between form and forces**
- comprises **a variety of graphical methods** for various structure types

Current revive

- has more emphasis on the application of **design**
- Applications limited on structures of **axial-loaded discrete elements**

Research aims

- **Interpreting** Robert Maillart's **application** of the fixed-points method to the Weissensteinstrasse Overpass
- The **principles** of the fixed-points method
- The **history** of the fixed-points method

Research significance

- Fixed-points method as significant **intangible architectural cultural heritage**
- The investigation of the analysis of this bridge could potentially facilitate the study of many historical structures
- Possibility of **revival and reuse** in the context of today's design and engineering

Outline

1. Introduction
2. A brief **history** of fixed-points method
3. The fundamental **principles** of fixed-points method
4. Robert Maillart's **application** in the Weissensteinstrasse Overpass
5. Discussion and ongoing work

2. A brief history of fixed-points method

Historical background

1. Boom of continuous bridges

Britannia Bridge (built in 1850)

2. Birth and dissemination of graphic statics

Die Graphische Statik by Carl Culmann (1864)

The era of Graphic Statics in the later 19th and early 20th Centuries

3. Proliferation of rigid frames

Chicago high-rise boom (1890s)

Hennebique system (1890s)

The origin, birth, and evolution of the fixed-points method

1725
Pierre
Varignon

funicular
polygon



1864
Carl
Culmann

bending
moment
diagram



1868
Otto Mohr

1. elastic
curves
2. fixed-
points
method



1900
Wilhelm
Ritter

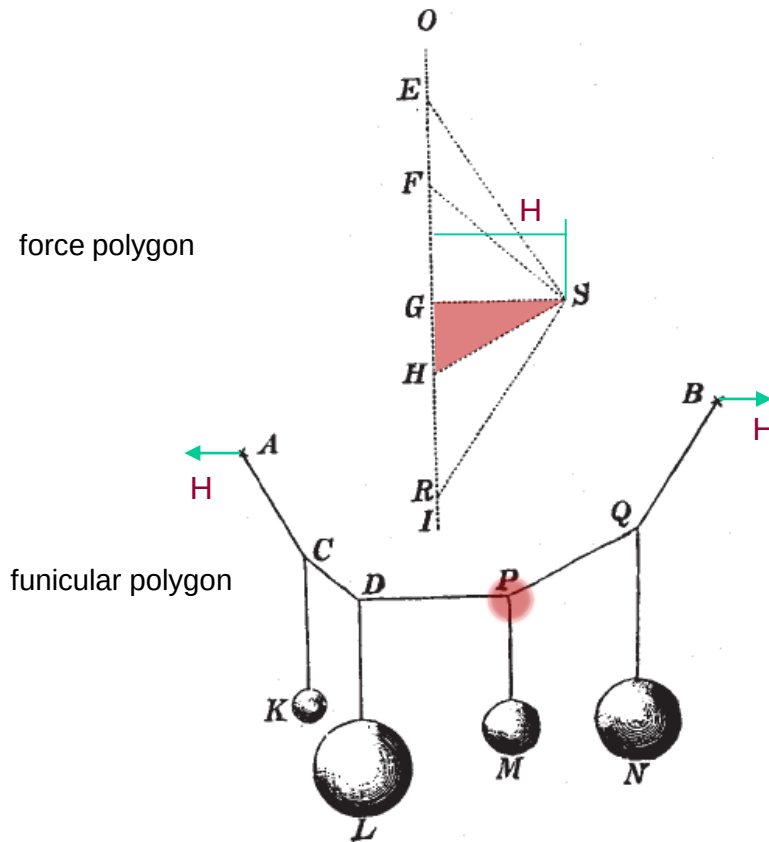
rigid
frame



1916
Ernst
Suter

complex
rigid
frames

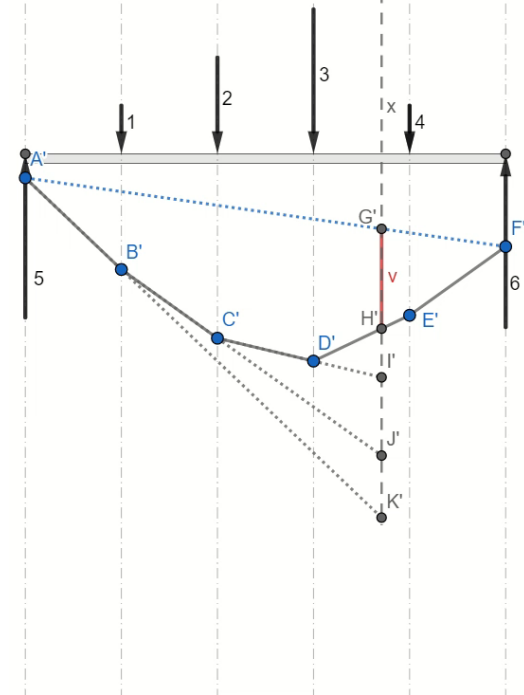
Augustus Du Bois 1875
James Chalmers 1881
Maurice Lévy 1886
Maurice Koechlin 1889
Jules Pillet 1895



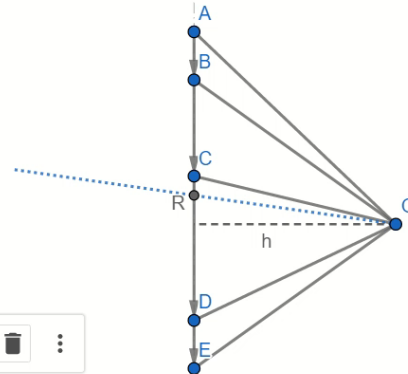
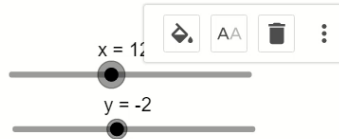
Varignon's method of determining the funicular polygon with forces polygon

Each node of the funicular polygon is represented by a triangle in the force diagram

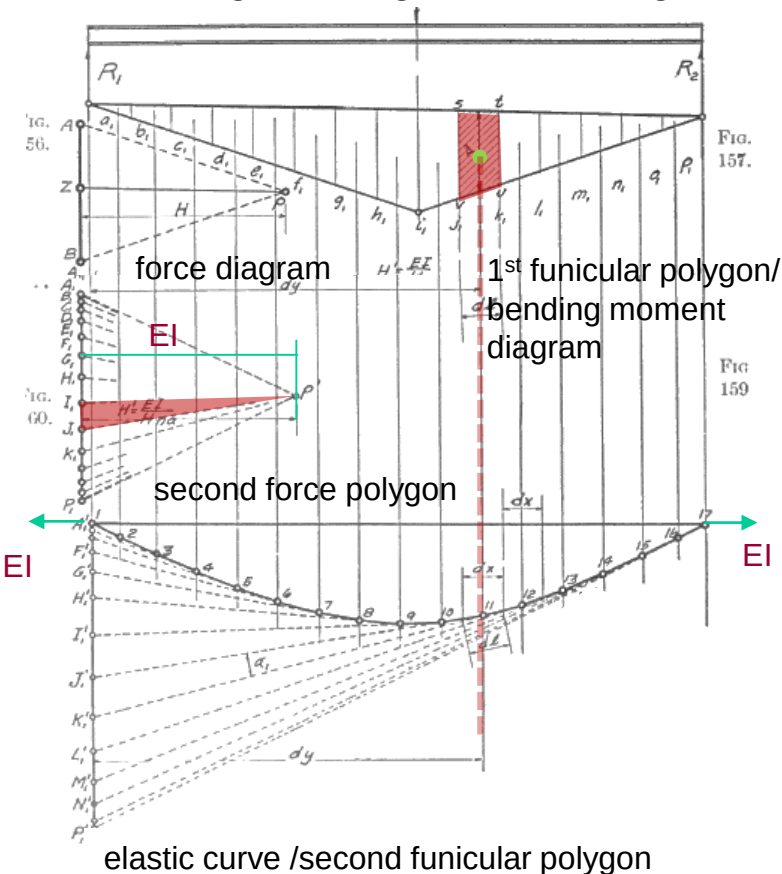
Constructing bending moment diagram as a funicular polygon enclosed with a base-line (closing string)



$$M = v \cdot h$$



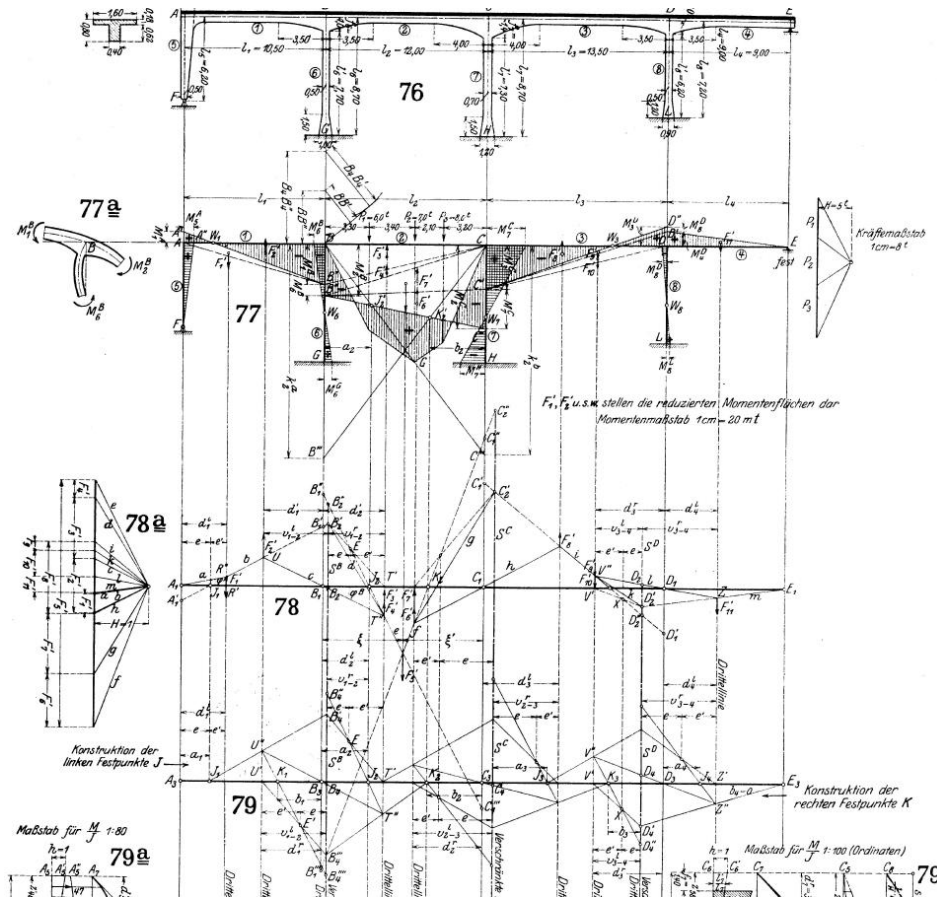
Constructing bending moment diagram as an enclosed funicular polygon



"The deflection curve could (also) be represented by a funicular polygon analogically taking the bending moment as the 'loading' instead of loads."[Mohr, 1868]

$$M(x) = \iint q(x) d^2x$$

$$y(x) = \frac{1}{EI} \iint M(x) d^2x$$

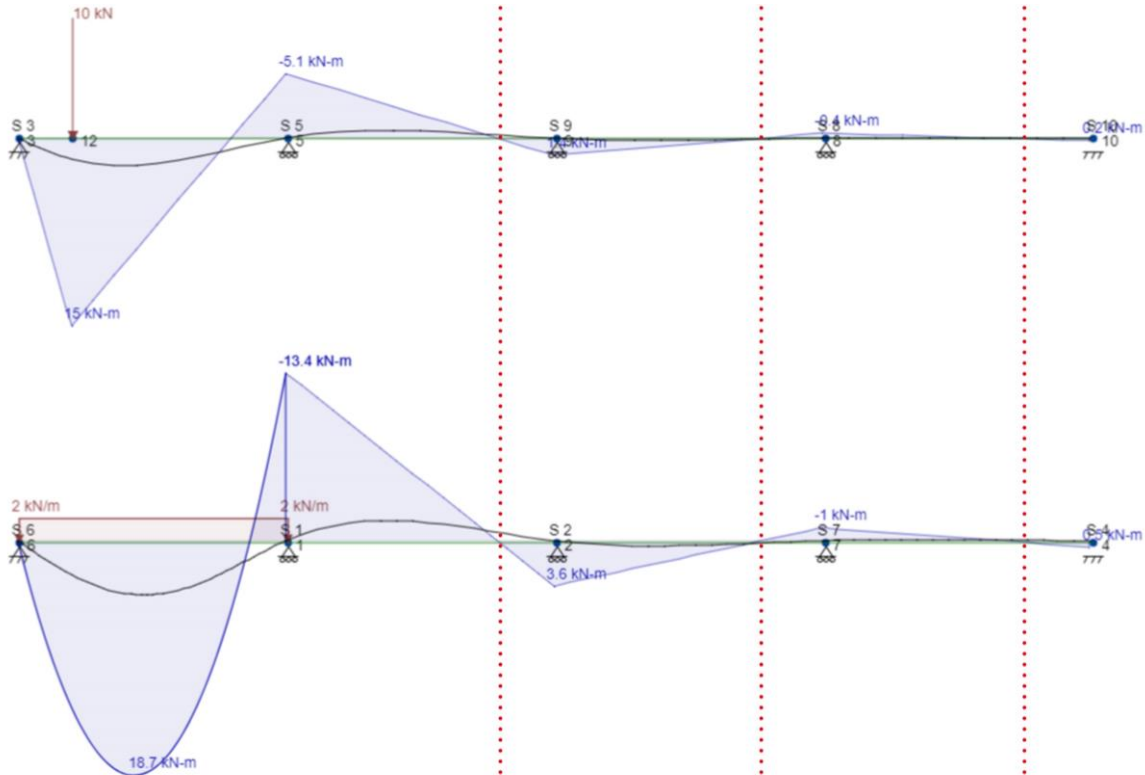


Ernst Suter developed the method to extend it to rigid frames with complex configuration and form

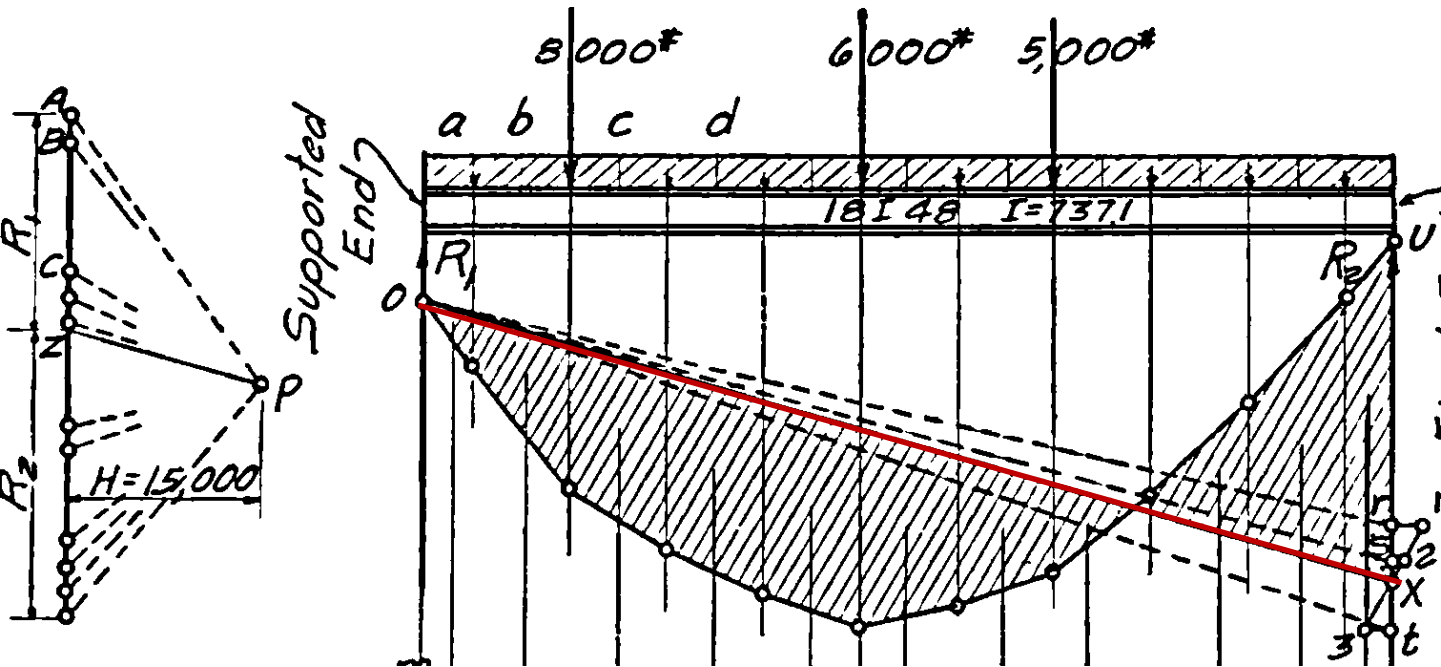
Maillart quoted Suter's book in his design document

3. Fundamental principles of fixed-points method

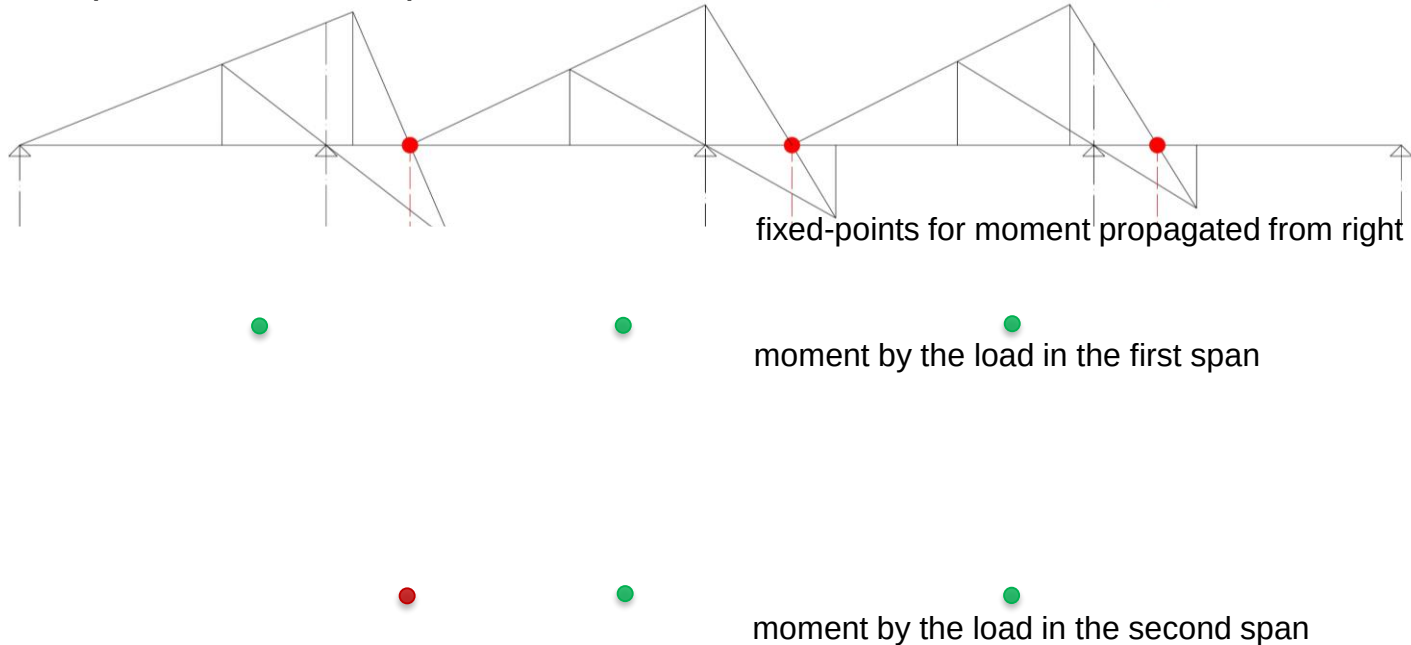
The fixity of the points of contraflexure and inflection

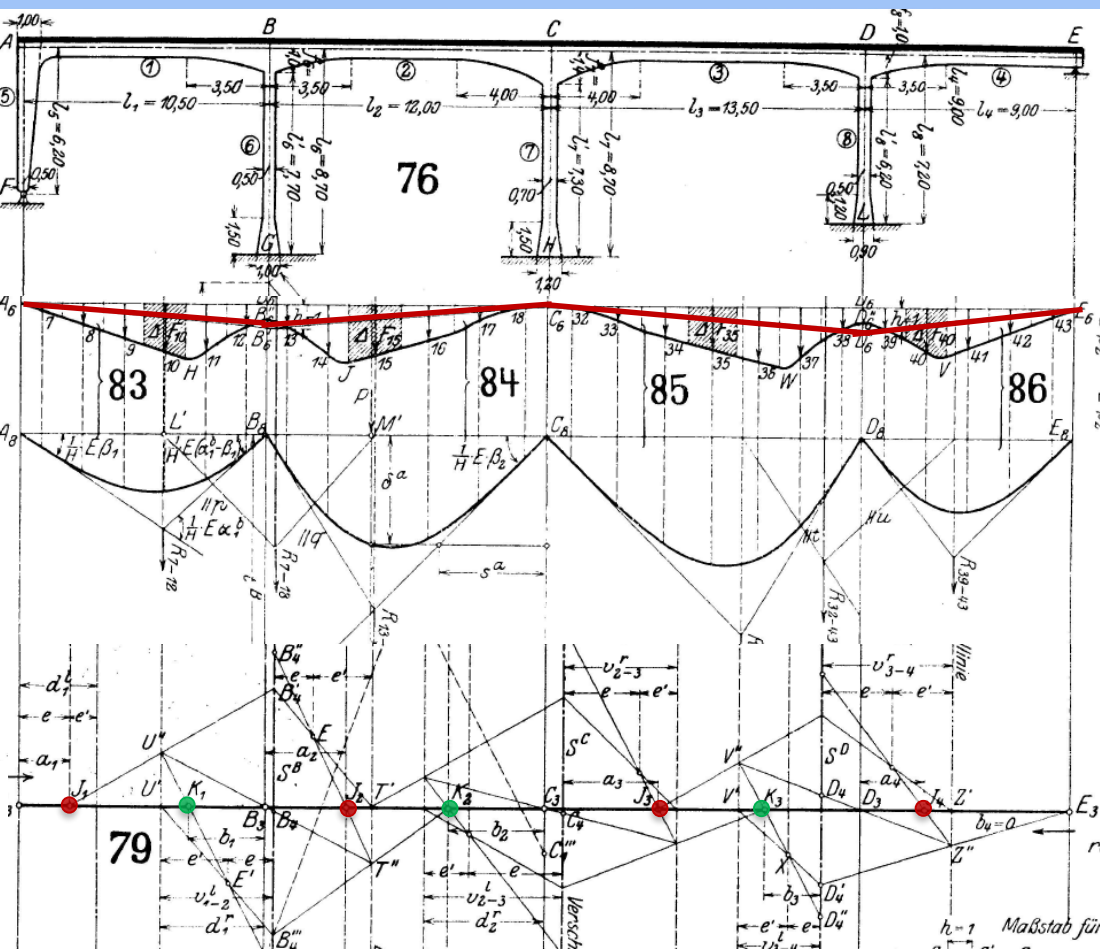


Decompose the bending moment diagram into the funicular polygon and its baseline (closing string)



The process of fixed-points method



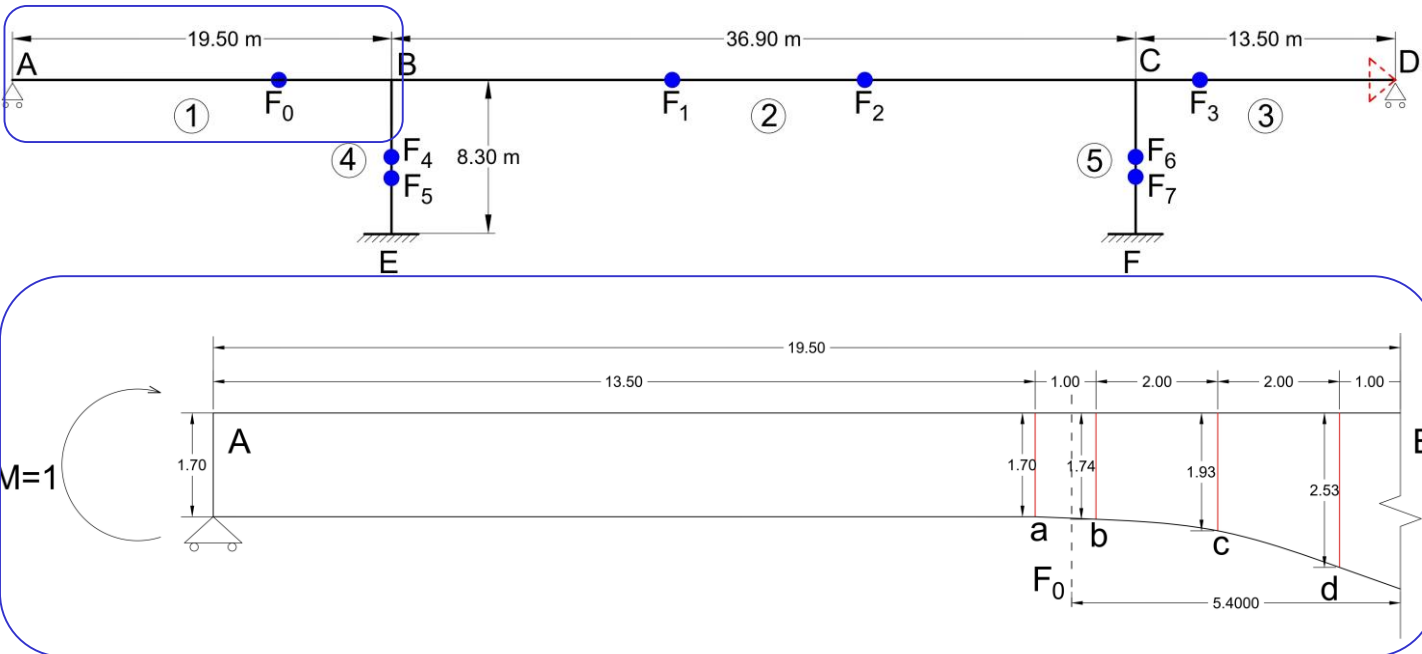


members with
varying section

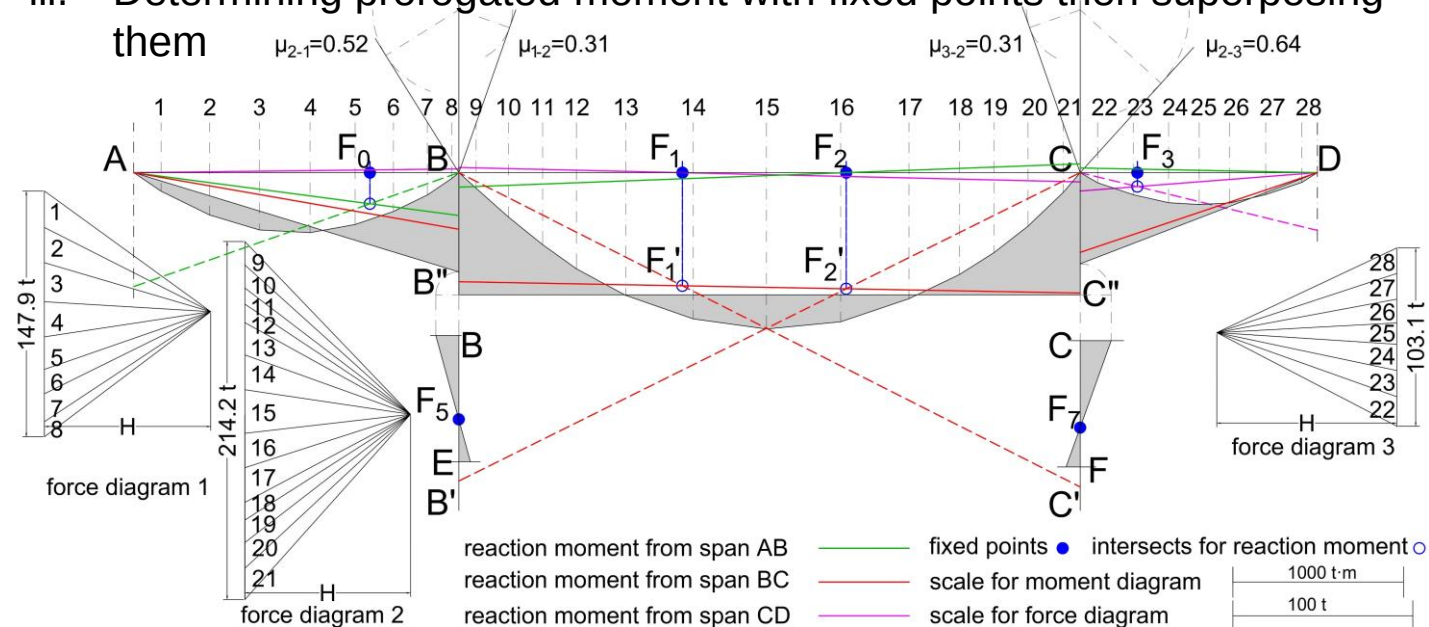
the fixed-points
are determined
with the slice
method

4. Robert Maillart's application in the Weissensteinstrasse Overpass

Determining all the fixed points analytically with Suter's method



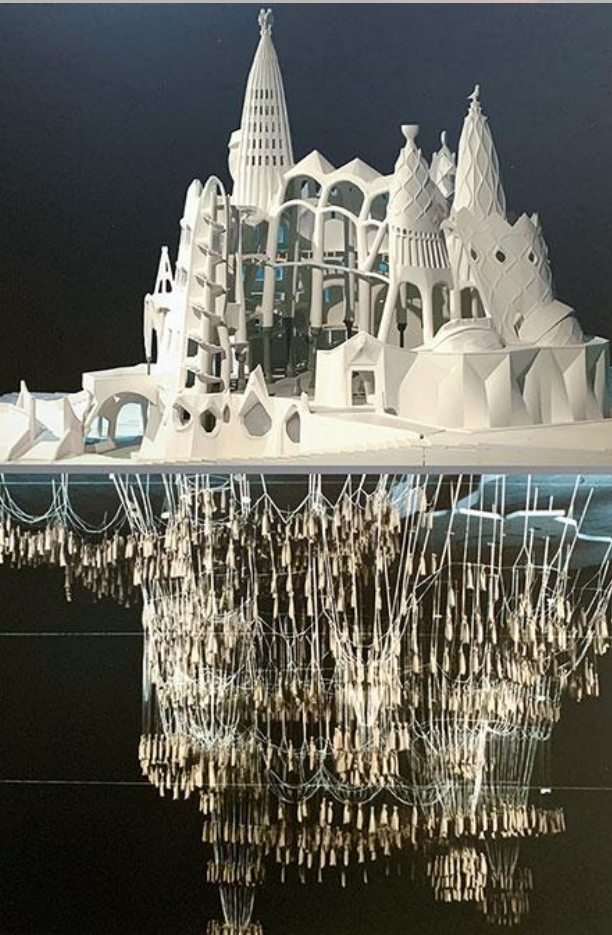
- i. Determining the funicular polygons of bending moment diagram with force diagrams
- ii. Determining the preliminary base-lines of bending moment diagram with crossing-lines
- iii. Determining prorogated moment with fixed points then superposing them



5. Discussion and ongoing work

- **Advantage of the fixed-points method:**
 - a) The relation between form and bending moment is intuitive
 - b) Especially advantageous for members with varying section

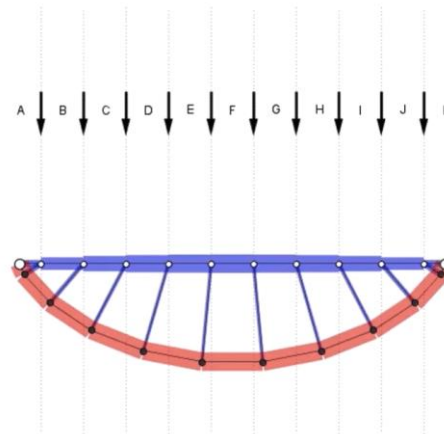
- **Disadvantage:**
 - a) Somewhat abstruse for understanding
 - b) Efficiency limited by hand-drawing
 - c) Less compatible for computerization



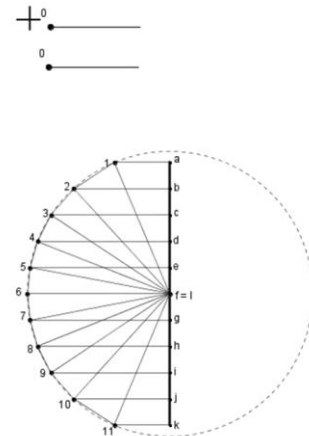
“Creativity is a result of the real perception of the problem.”

——Félix Candela

Form diagram



Force diagram



Questions:

- a) Can fixed-points method be reused for a more visual /interactive design ?
- b) Can this method integrate with current method?
- c) Can this method be applied to plastic analysis?

Ongoing work

- Deepening the historical study of the graphical methods for continuous structures

(Graphical methods for continuous beams and frames- history, methods and Robert Maillart's application)

- Simplifying the principles of the fixed-points method
- parameterization

THANK YOU

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Japan.
Meetings
& Events

New ideas start here