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Dependence and Mortality

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Efficient taxation with differential risks of dependence and mortality

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Abstract

The purpose of this note is to analyze the optimal tax and transfer policies that should be conducted in a society where individuals differ according to their productivity and their risk of mortality and dependency. We show that according to the most reasonable estimates of correlation among these three characteristics, an optimal policy should consist of a tax on earning and second period consumption and of a subsidy on long term care spending. The sign of the tax on saving is ambiguous but we can expect a positive tax on saving in reasonable cases.

JEL: H2, H5.

Keywords: long term care, mortality risk, efficient taxation.

1 Introduction

Panel surveys of elderly people such as Survey of Health, Ageing and Retirement in Europe (SHARE) or the U.S. Health and Retirement Study (HRS) consistently point to three correlations: one positive between income and longevity, one also positive between dependence and longevity, and one negative between dependence and income. The purpose of this note is to see the implications of features on the taxation of earnings, saving and long-term care (LTC) insurance. We start with a setting a la Atkinson and Stiglitz (1976) and Cremer et

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al. (2010), in which, without differential risks of mortality or dependence, a tax on earnings suffices to achieve efficiency. Introducing the risks of longevity and dependence makes it desirable to interfere with saving and insurance choices. The setting we adopt is that of a society with two types of individuals differing in their earning capacity and their probability of dependence and of mortality. The government does not know these characteristics and try to influence the choice of labor, saving and LTC consumption through non-linear taxes (or subsidies). We show that the tax structure closely depends on how these characteristics relate to each other.

2 The model

Consider a two-period model, where individuals work and save in the first period and retires in the second. In the second period people face different risks of mortality and dependence. Following Stiglitz (1982) we consider a society comprising two types of individuals that we call unskilled (1) and skilled (2). The proportion of type i ($i = 1, 2$) individuals is denoted by n_i , with $n_1 + n_2 = 1$. Each individual is characterized by three characteristics: (i) w_i (labor productivity in the first period), (ii) π_i (the probability to be alive in the second period), and (iii) p_i (the probability of becoming dependent in the second period). The skilled are more productive so $w_2 > w_1$. As to π_i 's and p_i 's, we assume the following, based on some stylised facts derived from the most recent waves of SHARE:¹

- longevity increases with income: $\pi_2 > \pi_1$;
- conditional upon survival, the probability of dependency decreases with income: $p_2 < p_1$;
- the probability of dependency decreases with income: $\pi_2 p_2 < \pi_1 p_1$;
- the probability of remaining autonomous increases with income: $\pi_2(1 - p_2) > \pi_1(1 - p_1)$.

Type i 's lifetime utility can be written as:

$$U_i = u(c_i) - v(\ell_i) + \pi_i(1 - p_i)u(d_i) + \pi_i p_i H(m_i)$$

where c_i and d_i denote first and second period consumption, m_i , LTC spending, ℓ_i labor supply; both $u(\cdot)$ and $H(\cdot)$ are strictly concave functions, and $v(\cdot)$ is convex. We also assume that $H(x) < u(x)$.

2.1 Laissez faire

We first look at the laissez faire solution for an individual of type i . The problem of an individual of type i is to choose the labor supply, the saving s_i and the insurance premium I_i that maximize:

¹Survey of Health, Ageing and Retirement in Europe. Own calculations.

$$U_i = u(w_i \ell_i - s_i - I_i) - v(\ell_i) + \pi_i(1 - p_i)u(s_i/\pi_i) + \pi_i p_i H(I_i/(p_i \pi_i) + s_i/\pi_i)$$

where we implicitly assume no time preference, a rate of interest equal to 0, an actuarially fair annuity market and LTC insurance.

We easily verify the following conditions:

$$\frac{u'(d_i)}{u'(c_i)} = \frac{H'(m_i)}{u'(c_i)} = 1; \quad \frac{v'(\ell_i)}{u'(c_i)} = w_i$$

2.2 Optimum

To obtain the optimality conditions we maximize a weighted sum of individual lifetime utilities subject to two constraints: a resource constraint and a self-selection constraint in which we assume that the parameters are such that type 2 wants to mimick type 1 and not the other way around.² We use the multipliers μ and λ for these two constraints.

$$\begin{aligned} \mathcal{L} = & \sum n_i \{ \alpha_i (u(c_i) - v(\ell_i) + \pi_i(1 - p_i)u(d_i) + \pi_i p_i H(m_i)) - \mu [c_i - w_i \ell_i + \pi_i(1 - p_i)d_i + \pi_i p_i m_i] \} \\ & + \lambda [u(c_2) - v(\ell_2) + \pi_2(1 - p_2)u(d_2) + \pi_2 p_2 H(m_2) - \\ & (u(c_1) - v(w_1 \ell_1/w_2) + \pi_2(1 - p_2)u(d_1) + \pi_2 p_2 H(m_1))] \end{aligned}$$

where the α are individual non negative weights that guarantee a Pareto optimal solution ($\alpha_1 \geq \alpha_2$).

Setting $y_i \equiv w_i \ell_i$, the FOC's are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_2} &= n_2 \alpha_2 u'(c_2) - \mu n_2 + \lambda u'(c_2) = 0 \\ \frac{\partial \mathcal{L}}{\partial d_2} &= [n_2 \alpha_2 u'(d_2) - \mu n_2 + \lambda u'(d_2)] \pi_2 (1 - p_2) = 0 \\ \frac{\partial \mathcal{L}}{\partial m_2} &= [n_2 \alpha_2 H'(m_2) - \mu n_2 + \lambda H'(m_2)] \pi_2 p_2 = 0 \\ \frac{\partial \mathcal{L}}{\partial y_2} &= -n_2 \alpha_2 v'(\ell_2)/w_2 + \mu n_2 - \lambda v'(\ell_2)/w_2 = 0 \end{aligned}$$

²This latter alternative case would occur if the probability of dependence of the skilled were much higher than that of the unskilled. We exclude this case, by simply considering a case in which the skilled individuals result in higher lifetime utility than the unskilled at the laissez faire outcome.

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_1} &= n_1 \alpha_1 u'(c_1) - \mu n_1 - \lambda u'(c_1) = 0 \\
\frac{\partial \mathcal{L}}{\partial d_1} &= [n_1 \alpha_1 u'(d_1) - \mu n_1] \pi_1 (1 - p_1) - \lambda u'(d_1) \pi_2 (1 - p_2) = 0 \\
\frac{\partial \mathcal{L}}{\partial m_1} &= [n_1 \alpha_1 H'(m_1) - \mu n_1] \pi_1 p_1 - \lambda H'(m_1) \pi_2 p_2 = 0 \\
\frac{\partial \mathcal{L}}{\partial y_1} &= -n_1 \alpha_1 v'(\ell_1)/w_1 + \mu n_1 + \lambda v'(y_1/w_2)/w_2 = 0
\end{aligned}$$

From the first set of FOC's we derive the following expressions:

$$\frac{u'(d_2)}{u'(c_2)} = \frac{H'(m_2)}{u'(c_2)} = 1; \quad \frac{v'(\ell_2)}{u'(c_2)} = w_2$$

These equalities express the standard no distortion at the top. In other words, there is no need to distort the choices of saving, LTC and labor of type 2 individuals.

We now turn to the unskilled individuals. With the evidence mentioned in the beginning of this section, we can interpret the tax formulas for the unskilled. Starting with the demand for LTC, we have:

$$\frac{H'(m_1)}{u'(c_1)} - 1 = \frac{\lambda H'(m_1)}{\mu n_1} \left[\frac{\pi_2 p_2}{\pi_1 p_1} - 1 \right] < 0 \iff \frac{\pi_2 p_2}{\pi_1 p_1} < 1$$

Namely, long term care ought to be subsidized as the probability of dependency of the unskilled is higher than that of the skilled individuals. As to second period consumption, we have:

$$\frac{u'(d_1)}{u'(c_1)} - 1 = \frac{\lambda u'(d_1)}{\mu n_1} \left[\frac{\pi_2 (1 - p_2)}{\pi_1 (1 - p_1)} - 1 \right] > 0 \iff \frac{\pi_2 (1 - p_2)}{\pi_1 (1 - p_1)} > 1$$

In words, second period consumption ought to be taxed as the probability of keeping autonomous is higher for the skilled than for the unskilled individuals.

From these formulas one can obtain the implicit tax on saving (which is the capital income tax in the New Dynamic Public Finance).

$$\pi_1 \left(p_1 \frac{H'(m_1)}{u'(c_1)} + (1 - p_1) \frac{u'(d_1)}{u'(c_1)} - 1 \right) = \pi_1 \left(\frac{p_1}{1 - \tilde{\lambda} \left(\frac{\pi_2 p_2}{\pi_1 p_1} - 1 \right)} + \frac{1 - p_1}{1 - \tilde{\lambda} \left(\frac{\pi_2 (1 - p_2)}{\pi_1 (1 - p_1)} - 1 \right)} - 1 \right)$$

where $\tilde{\lambda} = \frac{\lambda u'(c_1)}{\mu n_1}$. This expression is complicated, but when $\pi_1 - \pi_2$ is close to zero, the implicit tax on saving is positive.³

³The reason is as follows. $\frac{H'(m_1)}{u'(c_1)} = \left(1 - \tilde{\lambda} \left(\frac{\pi_2 p_2}{\pi_1 p_1} - 1 \right) \right)^{-1}$,

Finally, we have the tax formula for labor:

$$1 - \frac{v'(\ell_1)}{u'(c_1)w_1} = \tilde{\lambda} \left[\frac{v'(\ell_1)}{u'(c_1)w_1} - \frac{v'(y_1/w_2)}{u'(c_1)w_2} \right] > 0$$

As in the conventional optimal taxation problem, $y_1/w_2 < \ell_1$ and $1/w_2 < 1/w_1$ imply the positive marginal income tax rate for the unskilled individuals.

To decentralize this optimum one can use a tax on earnings and saving and a subsidy on the insurance premium that imply the same distortions as that found in the above inequalities. Note that in the absence of private insurance for long term care, the above policy would consist of a LTC public benefit different for the two types that would be financed by a tax on the saving and the earnings of the unskilled and a lump sum tax paid by the skilled individuals.

3 Conclusion

We have shown in this note that under the assumption of higher probability of survival for the skilled and a lower probability of turning dependant, assumptions that are verified in most societies, the optimal policy towards long term care is to subsidize long-term care insurance and tax earnings. The sign on the tax on saving is ambiguous as saving is also used for dependence. However, we can expect a positive tax on saving in reasonable cases. In the case where there is no market for private insurance, the government can supply long-term care benefits that would vary between the two types of households.

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$\frac{u'(d_1)}{u'(c_1)} = \left(1 - \tilde{\lambda} \left(\frac{\pi_2(1-p_2)}{\pi_1(1-p_1)} - 1 \right) \right)^{-1}$. We then have $\pi_1 p_1 - \tilde{\lambda}(\pi_2 p_2 - \pi_1 p_1) > 0$ and $\pi_1(1-p_1) - \tilde{\lambda}(\pi_2(1-p_2) - \pi_1(1-p_1)) > 0$. When $\pi_1 = \pi_2$, the expression of (1) becomes $\frac{\pi_1^3 \tilde{\lambda}^2 (p_1 - p_2)^2}{(\pi_1 p_1 - \tilde{\lambda}(\pi_2 p_2 - \pi_1 p_1))(\pi_1(1-p_1) - \tilde{\lambda}(\pi_2(1-p_2) - \pi_1(1-p_1)))} > 0$: the implicit tax on saving is positive (regardless of $p_1 \geq p_2$). By continuity, (1) is positive when $\pi_1 - \pi_2$ is close to zero. One can show that (1) is also positive when $p_1 - p_2$ is close to zero and $\pi_1 < \pi_2$. Needless to say, when $\pi_1 = \pi_2$ and $p_1 = p_2$, the Atkinson-Stiglitz theorem holds so that the tax on saving is zero.

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