Epictetusian Rationality*

Gregory Ponthiere†

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Abstract

According to Epictetus, mental freedom and happiness can be achieved by distinguishing between, on the one hand, things that are upon our control (our acts, opinions and desires), and, on the other hand, things that are not upon our control (our body, property and reputation), and by wishing for nothing that is outside our control. This article proposes two accounts of Epictetus's precept, which extend the symmetric factor of the preference relation beyond its boundaries under non-ethical preferences. The I account of Epictetus's precept requires indifference between outcomes differing only on circumstances, whereas the IB account requires indifference between outcomes involving the best replies to circumstances. We study the implications of these precepts on the structure of Epictetusian rationality. The I account is shown to provide ethical foundations for standard consumer theory, and to rule out (counter)adaptive desires, unlike the IB account. Finally, in game-theoretical contexts, the two accounts of Epictetus's precept exclude the existence of prisoner's dilemmas.

Keywords: rationality, Epictetus, preferences, indifference, independence, adaptive preferences.

JEL classification codes: B11, D01, D10.

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†Université catholique de Louvain, Hoover Chair in Economic and Social Ethics and Global Labour Organization (GLO). Address: College Dupriez, 3 Place Montesquieu, 1348 Louvain-la-Neuve, Belgium. E-mail: gregory.ponthiere@uclouvain.be

1 Introduction

Whereas the interest of economists in ethical preferences has been recently renewed (Roemer 2010, 2019, Alger and Weibull 2013, 2016), philosophers have had a long lasting interest in ethical behavior, i.e., in attitudes that persons *should* adopt.[1] Although philosophers's theorizations about ethical behavior did not generally rely on the concept of preferences, their explorations can serve as a starting point for revisiting the content and scope of ethical preferences.

An early attempt to conceptualize and justify some form of ethical behavior can be found in the works of the Stoic philosopher Epictetus (55 AC – 135 AC), in particular in the *Manual* – also known as the *Encheiridion* or the *Handbook* –, which is a short compilation of Epictetus's practical precepts.[2] In a nutshell, the *Manual* compiles a short selection of Epictetus's ethical advice aimed at helping the layman in everyday life, in all – possibly highly adverse – circumstances.

According to Epictetus, mental freedom and happiness can be achieved by distinguishing between, on the one hand, things that are upon our control (our acts, opinions and desires), and, on the other hand, things that are not upon our control (our body, property, offices and reputation), and by wishing for nothing that is outside our control.

The distinction between things upon our control and things not upon our control appears at the very beginning of the *Manual* (I):

> Of things some are in our power, and others are not. In our power are opinion, movement towards a thing, desire, aversion, turning from a thing; and in a word, whatever are our acts. Not in our power are the body, property, reputation, offices (magisterial power), and in a word, whatever are not our own acts. And the things in our power are by nature free, not subject to restraint or hindrance; but the things not in our power are weak, slavish, subject to restraint, in the power of others.

Epictetus's precept consists of a differentiated attitude towards these two classes of things: in order to have a good life, one should reorient one's desires and aversions away from the things outside one's control, and cultivate desires and aversions only for things that are upon control (*Manual*, II):

> Remember that desire contains in it the profession (hope) of obtaining that which you desire; and the profession (hope) in aversion

[1] An earlier economic analysis of ethical preferences through the lenses of Kant can be found in Laffont (1975).

[2] Epictetus is regarded as a major thinker of the philosophical school known as Stoïcism, whose founder was Zeno of Citium (4th century BC). Stoïcism draws its name from the Greek word *stoa* or arcade, the place where Zeno used to teach his philosophy. Epictetus was born in Phrygia, and served part of his life as a slave under the Roman empire. Even after he was freed from slavery, Epictetus kept bodily stigma of bad treatments due to his past master. Epictetus's lectures were transcribed by his pupil Arrian. The *Manual* is often regarded as the first textbook of the history of philosophy (aimed at having thoughts in hand). See Clark (1995).

> (turning from a thing) is that you will not fall into that which you attempt to avoid; and he who fails in his desire is unfortunate; and he who falls into that which he would avoid is unhappy. If then you attempt to avoid only the things contrary to nature which are within your power you will not be involved in any of the things which you would avoid. But if you attempt to avoid disease, or death, or poverty, you will be unhappy.

Epictetus's precept is summarized as follows (*Manual*, II):

> Take away then aversion from all things which are not in our power, and transfer it to the things contrary to nature which are in our power.

Whereas Stoicism is often presented as defending 'Life according to Nature' (Moore 1903, Sidgwick 1907), and, hence, a kind of overall indifference, Epictetus's precept is not about cultivating indifference with respect to *all* things.[3] Epictetus's Stoïcism requires to cultivating indifference only with respect to the things that are not under control (events that are external and independent of one's will). Desires and aversions should not be developed for things that do not depend on us, but well for things that are under our control. From Epictetus's perspective, the good life is reached by being concerned only with things that are up to us, that is, only concerned with our proper self (see Long 2002).

Various aspects of Epictetus's precept can be questioned. First, one can criticize the distinction drawn by Epictetus between things upon our control and things not upon our control. The notion of control is complex, especially in a world of multiple interdependencies. Second, one can also question the logics of Epictetus's rationale. Moore (1903) criticized Stoicism on the ground that it is a metaphysical ethical doctrine, i.e., a doctrine that deduces what should be done from metaphysical premises.[4] But even if one takes Epictetus's distinction for granted, this sole distinction does not imply anything regarding what a person *should* do. Epictetus's precept aims at achieving mental freedom and happiness, but one may not adopt these goals.[5]

In this article, we will leave these issues aside, and focus only on one question: can an Epictetusian decision-maker be rational? The answer depends on how one interprets Epictetus's precept and on how one defines the concept of rationality. In this paper, rationality is defined as the existence of a preference relation R representing the interests of the person in a coherent manner.[6] But

[3] See Moore (1903), p. 93 and Sidgwick (1907), p. 377. These two authors underlined the problems of circularity.for an ethical doctrine recommending 'Life according to Nature': any act can always be interpreted as 'natural'.

[4] See Moore (1903), p. 161.

[5] Epictetus deduces a 'should' from an 'is', a kind of naturalistic fallacy (Moore 1903).

[6] Hence, we define rationality in a sense that differs from how Epictetus himself defined rationality. According to Epictetus, human beings are rational in the sense that they can use their impressions in a reflexive manner, in such a way as to determine whether their impressions are true or false. This feature of human beings was regarded by Epictetus as most promising, and making humans distinct from other animals. See Graver (2009).

even under this definition of rationality, it is far from clear to see whether an Epictetusian decision-maker can be rational. Can one be both Epictetusian *and* rational? If yes, what form Epictetusian rationality would take?

Although Epictetus's *Manual* and modern microeconomic theory belong to two distant domains of thought, one reason for studying the logical connections between these can be found in Sen's criticism of the concept of preferences (Sen 1976). Sen argued that preferences carry too many distinct functions in economic analysis, and encouraged economists to disentangle the various – positive and normative – dimensions of preferences. Formalizing Epictetus's precept about desires and aversions and studying the possibility of Epictetusian rationality contribute to explore some normative aspects of preferences.

This article proposes two accounts of Epictetus's precept in the (acts, circumstances) space, where acts are under control, whereas circumstances are not. These two accounts share a key feature: *each account of Epictetus's precept imposes some extension of the symmetric factor of the preference relation R.* As such, each account of Epictetus's precept extends the 'domain of indifference' beyond its boundaries under standard, non-ethical, preferences. The indifference account (I account) of Epictetus's precept requires indifference between outcomes differing only on circumstances. The indifference between best replies account (IB account) requires indifference only between outcomes involving the best replies to circumstances. We study the implications of these precepts on the preference relation and on the existence of Epictetusian rationality. We show that the I account of Epictetus's precept implies, under reflexivity, completeness and transitivity of the preference relation, that preferences satisfy independence of circumstances. This corollary is robust to weakening transitivity to Suzumura consistency (Suzumura 1976). The IB account of Epictetus's precept implies that, under reflexivity, completeness and transitivity, the preference relation satisfies robustness to dominated alternatives.

In sum, this paper does not only propose formal accounts of Epictetus's precept, but, also, explores the form of preferences that a decision-maker adopting Epictetus's precept could have. In the light of our results, the term 'Epictetusian rationality' is not an oxymoron: if rationality is defined as the existence of a preference ordering, Epictetus's precept is compatible with some form of rationality. Its precise form depends on the account of Epictetus's precept: while the I account implies a preference ordering satisfying independence of circumstances, the IB account requires robustness to dominated alternatives.

Regarding the comparison of the two concepts of Epictetusian rationality, our analysis provides some support for the I account over the IB account. The I account of Epictetusian rationality exhibits three advantages over the IB account. First, when considering the choice of consumption baskets, it appears that the I account provides ethical foundations for the existence of a unique indifference map (defined on baskets) that is invariant to changes in the budget set, unlike the IB account. Second, when considering the implications of our results in the (desires, circumstances) space, it is shown that the I account excludes adaptive and counteradaptive desires, unlike the IB account. Given the problems raised by adaptive desires for normative analysis, this result supports the I account. Third, when considering game-theoretical contexts, we show that both the I account and the IB account of Epictetus's precept rule out the existence of prisoner's dilemmas. However, in games where all players agree on the fact that the outcome where everyone cooperates Pareto-dominates the outcome where everyone defects, Epictetusian players of the I type regard cooperation as a dominant strategy, something that does not necessarily occur under the IB account. Thus the I account of Epictetusian rationality does, in that case, encourage cooperation, something not always true under the IB account.

The present paper is related to several branches of the literature. First, this is linked to the increasingly large literature on ethical preferences (see Roemer 2010, 2019, Alger and Weibull 2013, 2016). Our analysis of Epictetusian rationality is related to the renewed attention paid by economists to ethical preferences, in particular Kantian preferences.[7] The present article focuses not on generalizability, but on another key ethical dimension of preferences: *the exclusive focus on things under control* (*our* acts, *our* desires, *our* opinions), which implies a particular division of moral labour. In line with the literature on ethical preferences, we examine also how preferences satisfying Epictetus's precept allow to escape from the prisoner's dilemma. Second, the present paper is linked to the literature focusing on the definition of 'rationality' (Sen 1969, 1976, Bossert and Suzumura 2010, Chambers and Echenique 2016). The contribution of this paper lies here in its analysis of the conditions under which Epictetus's precept can be part of human rationality, as defined by the existence of a preference ordering on outcomes. Third, this paper is also related to the literature in economic psychology studying adaptive preferences (Elster 1983, von Weizsacker 2014), because we analyze the relations between several variants of Epictetusian rationality and the occurrence of (counter)adaptive preferences.

The paper is organized as follows. Section 2 presents the model. Section 3 proposes two accounts of Epictetus's precept. Their logical implications on the form of preferences and on the structure of Epictetusian rationality are explored in Section 4. Section 5 revisits consumer theory in the light of Epictetusian rationality. Section 6 studies the relation between Epictetus's precept and adaptive desires. Section 7 explores implications of Epictetus's precept in game-theoretical contexts. Section 8 concludes.

2 The model

A proper formalization of Epictetusian rationality must rely on Epictetus's own representation of the world, that is, on Epictetus's partition of the world between, on the one hand, things under the control of the person (e.g. acts, opinions and desires), and, on the other hand, things that are not under her control (e.g. the body, property, offices and honours).[8] The various elements

[7] The seminal reference is Kant (1785). In economics, see Laffont (1975). Recent works on Kantian preferences include Roemer (2010), Curry and Roemer (2012), Roemer (2019) and De Donder et al (2021).

[8] See the *Manual*, in particular Section I quoted above.

composing these sets would require separate analyses. For the sake of simplicity, our analysis will focus on particular elements. Regarding things under control, we will focus on *acts* of the decision-maker, the set of all possible acts being A. As far as things outside control are concerned, we will focus on *circumstances*, the set of all circumstances being C. An outcome is here defined as a pair (act, circumstance).[9] The (non-empty) set of outcomes is $Y = A \times C$.[10]

In order to explore the relation between Epictetus's precept and rationality, this paper adopts a standard definition of rationality as the existence of a preference ordering defined on the set of outcomes. More formally, we will postulate the existence of a binary relation on the set of outcomes, and we will examine under which conditions this binary relation can be interpreted as a preference relation while satisfying Epictetus's ethical precept. Let $R \subseteq Y \times Y$ be a (binary) relation on Y.

Following Bossert and Suzumura (2010, p. 32-33), we can define the symmetric factor and asymmetric factor of R as follows.

The symmetric factor $I(R)$ of R is:

$$I(R) = \{(x, y) \in Y \times Y \,|(x, y) \in R \text{ and } (y, x) \in R\}$$

The asymmetric factor $P(R)$ of R is:

$$P(R) = \{(x, y) \in Y \times Y \,|(x, y) \in R \text{ and } (y, x) \notin R\}$$

Throughout this paper, we interpret R as a weak preference relation, while $P(R)$ is interpreted as a strict preference relation, $I(R)$ is interpreted as the indifference relation.

In welfare economics, three fundamental properties of binary relations are reflexivity, completeness and transitivity. These are defined as follows.

Defining Δ as the diagonal relation on Y by $\Delta = \{(x, x) \,|x \in Y\}$, reflexivity is defined by:

$$\Delta \subseteq R$$

Completeness is defined as follows:

$$\forall x, y \in Y \text{ such that } x \neq y,\ (x, y) \in R \text{ or } (y, x) \in R$$

Transitivity is defined as follows:

$$\forall x, y, z \in Y, [(x, y) \in R \text{ and } (y, z) \in R] \implies (x, z) \in R$$

[9] Considering all possible outcomes as pairs (act, circumstance) amounts to assume here that all acts are possible under all circumstances.

[10] The partition of the world into acts and circumstances reminds the pioneer work of Savage (1954), who refers to 'states' to describe things not under control that turn out to prevail (Luce and Raiffa 1957). But Savage uses that framework to axiomatize preferences on lotteries (probability distributions on states), something far away from Epictetus's representation of the world, where the idea of probability is absent. Although one could interpret Epictetus's precept as recommending neutrality towards risk, this interpretation is too narrow and Epictetus's precept cannot be reduced to this (even though risk neutrality could follow logically from our accounts of Epictetus's precept if these were studied in Savage's framework).

When R satisfies reflexivity and transitivity, R is a quasi-ordering. When R is a quasi-ordering and satisfies, completeness, R is an ordering.

Finally, note that the transitive closure $tc(R)$ of a binary relation R on Y is defined as follows (see Bossert and Suzumura, 2010, p. 34):

$$tc(R) = \left\{ (x,y) \;\middle|\; \begin{array}{c} \exists K \in \mathbb{N} \text{ and } x^0, ..., x^K \in Y \text{ such that:} \\ \left[x = x^0 \text{ and } \left(x^{k-1}, x^k\right) \in R \text{ for all } k \in \{1, ..., K\} \text{ and } x^K = y\right] \end{array} \right\}$$

3 Two accounts of Epictetus's precept

Epictetus's precept of 'wishing nothing that is outside our control' can be interpreted in several ways. This section proposes two distinct accounts of Epictetus's precept, which can be related to some passages of Epictetus's *Manual*. As we shall see, the proposed accounts of Epictetus's precept are distinct - and can yield contradictory implications - but they share a key feature: *each account of Epictetus's precept imposes an extension of the symmetric factor of the preference relation R.* Whereas these interpretations of Epictetus's precept differ, they both present a view of Stoicism as imposing indifference relations between (some) pairs of outcomes. From this perspective, Stoicism is about defining the proper scope of indifference, and extending the domain of indifference beyond its boundaries under standard, non ethical, preferences.

A first way to formalize Epictetus's precept of 'wishing nothing that is outside our control' consists of the *indifference account* (I account). The indifference account of Epictetus's precept interprets the latter as a particular requirement on the structure of the preference relation R: a requirement of *indifference between outcomes differing only on circumstances, everything else being equal.* According to that interpretation, Epictetus's precept recommends persons to be indifferent between outcomes that differ only on circumstances.

This interpretation of Epictetus's precept has clear support in the *Manual*. Take, for instance, the *Manual*, XXXII:

> For if it is any of the things which are not in our power, it is absolutely necessary that it must be neither good nor bad.

According to Epictetus, things that are not under one's control are 'neither good nor bad'. Circumstances alone cannot make a person better off or worse off. The reason is that circumstances should be regarded merely as giving rise to opportunities to develop particular skills or abilities. From that perspective, there is neither good news nor bad news, but only circumstances that favour the flourishing of the person.[11] Circumstances being neither good nor bad, persons should be indifferent between outcomes differing only on circumstances.

[11] See Epictetus, *Manual*, X:

> On the occasion of every accident (event) that befalls you, remember to turn to yourself and inquire what power you have for turning it to use. If you see a fair man or a fair woman, you will find that the power to resist is temperance (continence). If labor (pain) be presented to you, you will find that it is endurance. If it be abusive words, you will find it to be patience. And if

This requirement of indifference between outcomes that differ only on circumstances can be formalized as follows.

Definition 1 (I account of Epictetus's precept) *For all $x, y \in Y$ such that $x = (a, m)$ and $y = (b, n)$ with $a = b$ and $m \neq n$, we have:*

$$(x, y) \in I(R)$$

The I account of Epictetus's precept does justice to Epictetus's idea that things outside control are neither good nor bad. Under the I account, 'wishing nothing that is outside control' is translated as 'being indifferent with respect to circumstances, everything else being equal'.

Although the indifference account of Epictetus's precept provides a plausible way to interpret Epictetus's ethical thought, this is not the only possible interpretation. There exists - at least - another formalization of Epictetus's precept. Let us read the *Manual*, XXXII:

> But having determined in your mind that everything which shall turn out (result) is indifferent, and does not concern you, and whatever it may be, for it will be in your power to use it well, and no man will hinder this, come then with confidence to the gods as your advisers.

Epictetus encourages persons to adopt the best use of things that are in their power *given the prevailing circumstances*. This second interpretation of Epictetus's precept requires persons to do the best they can given the circumstances that prevail. This interpretation imposes another requirement on the preference relation R: there should be indifference between outcomes resulting from different circumstances, provided the person chooses the best act given each set of circumstances. This alternative interpretation of Epictetus's ethical thought differs from the one under the I account, but is more in line with the handles metaphor, the 'good life' being defined as the most adequate behavior of the self given external constraints (*Manual*, XLIII):

> Everything has two handles, the one by which it may be borne, the other by which it may not. If your brother acts unjustly, do not lay hold of the act by that handle wherein he acts unjustly, for this is the handle which cannot be borne; but lay hold of the other, that he is your brother, that he was nurtured with you, and you will lay hold of the thing by that handle by which it can be borne.

What matters is not the circumstances that prevail, but what the person can do of these circumstances. From Epictetus's perspective, the good life requires to take things beyond our control by means of the 'good handle', that is, the

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> you have been thus formed to the (proper) habit, the appearances will not carry you along with them.

'good part' of these things. Taking the 'good part' of circumstances requires to use circumstances to make skills flourish.

This alternative account of Epictetus's precept, which can be called indifference between best replies to circumstances (or IB account), can be defined formally as follows.

Definition 2 (IB account of Epictetus's precept) *Suppose that, for a circumstance m, we have that:*

$$\exists w = (a, m) \in Y : (w, x) \in P(R) \text{ for all } x = (b, m) \in Y \text{ with } b \neq a$$

Suppose that, for a circumstance $n \neq m$, we have that:

$$\exists y = (c, n) \in Y : (y, z) \in P(R) \text{ for all } z = (d, n) \in Y \text{ with } d \neq c$$

Then one has:

$$(w, y) \in I(R)$$

The IB account requires not indifference between outcomes involving the same acts but different circumstances, but, only, between outcomes involving the best acts under different circumstances. This account of Epictetus's precept is close to the handles metaphor: there is always a good way to take circumstances, and once this good way is followed, circumstances do not matter. The IB account does justice to that intuition, by requiring indifference between outcomes including the best replies to given circumstances.

To illustrate the distinction between these two accounts of Epictetus's precept, let us consider the following example, involving two circumstances (either bodily comfort or discomfort), and two acts (either gardening or playing the flute), leading to four possible outcomes w, x, y and z.[12]

	Circumstances	
Acts	bodily comfort	bodily discomfort
gardening	w = (gardening, comfort)	x = (gardening, discomfort)
playing the flute	y = (flute, comfort)	z = (flute, discomfort)

Table 1: Four outcomes in the (acts, circumstances) space.

What does Epictetus's precept tell us about preferences in this example? From Epictetus's perspective, things outside our control are neither good nor bad. Thus whether the person suffers from bodily discomfort or not is neither a good news nor a bad news for her. The reason is that each circumstance is favorable to the enhancement of some skills or abilities. Thus bodily discomfort should not be regarded as an inevitable obstacle making gardening or playing the flute less desirable. Actually, the core of Epictetusian rationality is to highlight

[12] Bodily discomfort is a circumstance that Epictetus has personally experienced, and that is mentioned several times in the *Manual*.

that *circumstances are no inevitable obstacle in life*: circumstances only invite the enhancement of alternative skills, but should not make persons better off or worse off. Only things under control can make persons better off or worse off.

One way to capture this view is to require that, for a given act, circumstances should not matter. The I account of Epictetus's precept requires thus indifference between outcomes w (gardening under bodily comfort) and x (gardening under bodily discomfort), and between outcomes y (playing the flute under bodily comfort) and z (playing the flute under bodily discomfort), that is, $(w, x), (y, z) \in I(R)$.

Another way of interpreting Epictetus's precept requires indifference between the outcomes involving the best act for given circumstances (IB account). If, for instance, gardening is preferred under bodily comfort, and playing the flute under bodily discomfort, the IB account implies indifference between the outcomes w (gardening under bodily comfort) and z (playing the flute under bodily discomfort): $(w, z) \in I(R)$.

This simple example suffices to illustrate that the two accounts of Epictetus's precept - the I account and the IB account - are not logically equivalent, in the sense that it is possible to be Epictetusian under one account of Epictetus's precept without being Epictetusian under the other account.[13] To see this, suppose that the person in Table 1 has preferences satisfying: $(w, y) \in P(R)$ and $(z, x) \in P(R)$. Then the IB account requires $(w, z) \in I(R)$. This contradicts the I account: indeed, the I account requires $(w, x), (y, z) \in I(R)$, which, together with $(w, y) \in P(R)$, implies $(w, z) \in P(R)$, which contradicts $(w, z) \in I(R)$ required under the IB account. We thus face here two distinct - not logically equivalent - accounts of Epictetus's precept.

4 Implications on Epictetusian rationality

Let us now examine some implications of the two accounts of Epictetus's precept on the form of the preference relation R, and, hence, on the possibility - or impossibility - of Epictetusian rationality. For that purpose, let us first define the following property of the relation R.

Definition 3 (Independence of circumstances) *The relation R on Y satisfies independence of circumstances if and only if, for any outcomes (a, n), (b, n), (a, m), $(b, m) \in A \times C$ with $a \neq b$, $n \neq m$, we have:*

$$((a, n), (b, n)) \in P(R) \iff ((a, m), (b, m)) \in P(R)$$

and

$$((a, n), (b, n)) \in I(R) \iff ((a, m), (b, m)) \in I(R)$$

[13] In other words, among all possible relations R on Y, some are neither Epictetusian in the I sense nor in the IB sense, while others can be Epictetusian only in either the I sense or the IB sense, or be Epictetusian under both the I sense and the IB sense. But the latter case does not concern all preferences that are Epictetusian in at least one sense. There is no equivalence between the two accounts of Epictetus's precept.

Independence of circumstances requires that the ranking of outcomes involving different acts but same circumstances should be invariant to changing circumstances.

We can now consider a first implication of the I account of Epictetus's precept on the form of Epictetusian rationality.

Proposition 1 *If the relation R on Y satisfies the I account of Epictetus's precept, and is complete, reflexive and transitive, R must satisfy independence of circumstances.*

Proof. See the Appendix. $\blacksquare$

Proposition 1 states that the I account of Epictetus's precept, if imposed on R together with standard requirements of reflexivity, completeness and transitivity, has strong implications on the structure of R: R must satisfy independence of circumstances. Thus, under the I account, the ranking of acts for given circumstances should be robust across all possible circumstances.[14]

As such, Proposition 1 provides an important result for the existence of Epictetusian rationality. It suggests that an Epictetusian decision-maker (in the sense of the I account) can only be coherent provided he has preferences that satisfy independence of circumstances, that is, preferences over acts that are robust to changes of circumstances. Otherwise, the Epictetusian decision-maker would lack coherence. There would then be a conflict between Epictetus's precept and basic requirements of rationality, making Epictetusian rationality an oxymoron. Independence of circumstances is thus a logical corollary of the I account of Epictetus's precept and standard requirements of R.

In order to give the intuition behind that result, it is useful to turn back to the example of Table 1. As stated above, the I account of Epictetus's precept requires indifference between outcomes w and x and between outcomes y and z, that is, $(w,x),(y,z) \in I(R)$. What Proposition 1 tells us is that, if the relation R on $Y = \{w,x,y,z\}$ satisfies the I account of Epictetus's precept, and is complete, reflexive and transitive, then R must also satisfy independence of circumstances, that is, we must have: either $(w,y),(x,z) \in P(R)$ or $(y,w),(z,x) \in P(R)$ or $(w,y),(x,z) \in I(R)$, but we cannot have $(w,y),(z,x) \in P(R)$ or $(y,w),(x,z) \in P(R)$. Indeed, if independence of circumstances were not satisfied by R, there would be a preference cycle leading to a contradiction. For instance, if we had $(w,x),(y,z) \in I(R)$ and $(y,w),(x,z) \in P(R)$, we would deduce the following:

$$\begin{aligned}(y,w) &\in R \text{ and } (w,x) \in R \implies (y,x) \in R \text{ (by transitivity)} \\ (z,y) &\in R \text{ and } (y,x) \in R \implies (z,x) \in R \text{ (by transitivity)}\end{aligned}$$

a contradiction with $(x,z) \in P(R)$.

When looking at Proposition 1, one may wonder whether this result is due to coherence requirements. Proposition 1 assumes that R satisfies a strong

[14] Note that Proposition 1 does not state an equivalence result: it is possible for a relation R to satisfy completeness, reflexivity, transitivity and independence of circumstances *without* satisfying the I account of Epictetus's precept.

coherence requirement: transitivity. One may be curious to know whether the result of Proposition 1 still prevails once one weakens the coherence requirement on R. For that purpose, we will now consider replacing transitivity of R by either quasi-transitivity (Sen 1969) or Suzumura consistency (1976).

A relation R is quasi-transitive if and only if $P(R)$ is transitive:

$$\forall x, y, z \in Y, (x, y) \in P(R) \text{ and } (y, z) \in P(R) \implies (x, z) \in P(R)$$

A relation R is Suzumura consistent if and only if

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Proposition 2 summarizes our results regarding the weakening of coherence requirement of the relation R under the I account of Epictetus's precept.

Proposition 2
- *A relation R on Y that satisfies the I account of Epictetus's precept, is complete, reflexive and quasi-transitive does not necessarily satisfy independence of circumstances.*
- *A relation R on Y that satisfies the I account of Epictetus's precept, is complete, reflexive and Suzumura consistent must necessarily satisfy independence of circumstances.*

Proof. See the Appendix. ■

Proposition 2 suggests that weakening transitivity to quasi-transitivity does not preserve the result of Proposition 1, but that weakening transitivity to Suzumura consistency preserves that result: a relation R satisfying reflexivity, completeness, Suzumura consistency and the I account of Epictetus's precept must also satisfy independence of circumstances.

In the light of Propositions 1 and 2, it appears that, under the I account of Epictetus's precept, Epictetusian rationality can hardly be separated from independence of circumstances, that is, robustness of the ranking of acts to the prevailing circumstances. Independence of circumstances is thus a central component of Epictetusian rationality under the I account of Epictetus's precept.

Let us now examine the implications of the IB account of Epictetus's precept. For that purpose, we need first to define robustness to dominated alternatives.

Definition 4 (Robustness to dominated alternatives) *The relation R on Y satisfies robustness to dominated alternatives if and only if, for any outcomes (a, m), (b, m), (a, n), $(b, n) \in Y$ with $a \neq b$, $n \neq m$, we have:*

$$\begin{aligned}
\{((a,m),(b,m)),((a,n),(b,n))\} &\subset P(R) \iff \{((a,n),(b,m)),((a,m),(b,n))\} \subset P(R) \\
\{((a,m),(b,m)),((b,n),(a,n))\} &\subset P(R) \iff \{((a,m),(a,n)),((b,n),(b,m))\} \subset P(R)
\end{aligned}$$

Robustness to dominated alternatives states that, when comparing four outcomes involving two distinct acts and two distinct circumstances, if the asymmetric part of R includes two pairs of outcomes, the asymmetric part of R must also include the two pairs of outcomes obtained by shifting the dominated outcomes across the dominant outcomes. The property of robustness to dominated alternatives is close, but distinct from the double cancellation axiom.

Proposition 3 *If the relation R on Y satisfies the IB account of Epictetus's precept, and is complete, reflexive and transitive, R must satisfy robustness to dominated alternatives.*

Proof. See the Appendix. ■

Proposition 3 suggests that if one adopts the IB account of Epictetus's precept, then, under standard completeness and coherence requirements of R, it must be the case that R satisfies robustness to dominated alternatives.[15]

Proposition 4 examines the robustness of that result to relaxing coherence requirements for the relation R.

Proposition 4

- *A relation R on Y that satisfies the IB account of Epictetus's precept, is complete, reflexive and quasi-transitive does not necessarily satisfy robustness to dominated alternatives.*
- *A relation R on Y that satisfies the IB account of Epictetus's precept, is complete, reflexive and Suzumura consistent must necessarily satisfy robustness to dominated alternatives.*

Proof. See the Appendix. ■

The results of Propositions 3 and 4 suggest that robustness to dominated alternatives is a key component of Epictetusian rationality under the IB account.

In sum, this section highlights that the two accounts of Epictetus's precepts have different implications concerning the structure of the preference relation R, and, hence, concerning the form of Epictetusian rationality. The I account implies that R satisfies independence of circumstances, whereas the IB account implies that R satisfies robustness to dominated alternatives.

Which variant of Epictetusian rationality is the most appealing? To compare the merits of these accounts of Epictetusian rationality, the next sections examine their implications in three general contexts: (i) the context of a person's preference on consumption baskets (Section 5), (ii) the context of a person's preferences over her desires (Section 6), and (iii) the context of strategic interactions between persons (Section 7).

5 Consumer theory

What are the implications of the two accounts of Epictetus's precept in basic economic environments, such as the choice of a consumption basket under a budget constraint? To answer that question, this section revisits consumer theory in the light of Epictetus's precept.

From the perspective of consumption theory, the consumer's acts consist in purchasing some consumption baskets among the set of affordable baskets. Circumstances take here the form of a particular income level and a particular

[15] Note that Proposition 3 does not state an equivalence result: it is possible for a relation R to satisfy completeness, reflexivity, transitivity and robustness to dominated alternatives *without* satisfying the IB account of Epictetus's precept.

set of prices for the different goods, both factors determining which basket is affordable or not. From a static, short-run perspective, these factors affecting the budget constraint can be regarded as circumstances. Table 2 illustrates the choice of a consumption basket involving quantities of two goods, denoted goods 1 and 2. The two baskets are $q \equiv (q_1, q_2)$ and $q' \equiv (q'_1, q'_2)$, with $q_1 \neq q'_1$ and $q_2 \neq q'_2$. Let us suppose that the consumer's income is fixed to a given level, and abstract from this, so that circumstances take here the form of two sets of prices: $p \equiv (p_1, p_2)$ and $p' \equiv (p'_1, p'_2)$. As above, we assume that all acts under study are feasible under each set of circumstances, that is, that consumption baskets are affordable under each set of prices.[16]

	Circumstances	
Acts	prices p	prices p'
purchasing basket q	w = (q, p)	x = (q, p')
purchasing basket q'	y = (q', p)	z = (q', p')

Table 2: A choice between two consumption baskets.

In this example, the I account of Epictetus's precept states that the person should be indifferent between outcomes where she enjoys the same consumption basket, but where prices are different, that is, $(w, x), (y, z) \in I(R)$. In other words, changes in prices cannot make the person better off or worse off as long as she enjoys the same consumption basket.

This implication of the I account is important, because this establishes that how well-off a consumer is depends only on the consumption basket enjoyed, independently from prices. Thus the I account implies here the existence of a unique indifference map defined on consumption baskets, which is invariant to the prevailing budget set. One can thus regard the I account of Epictetus's precept as providing some ethical foundations for a key feature of consumer theory: the existence of a unique indifference map on consumption baskets that does not vary with the budget set.

Consider now the IB account of Epictetus's precept. Several cases should be distinguished here. If, for instance, purchasing the basket q is the best reply to prices p and p', that is, if $(w, y), (x, z) \in P(R)$, the IB account implies that the person should be indifferent between these two outcomes, that is, $(w, x) \in I(R)$. Given that, in each case, the same consumption basket is enjoyed, this implication is in line with basic economic theory: changes in the budget set should not affect the indifference map defined on baskets. However, if purchasing the basket q is the best reply to prices p, but purchasing the basket q' is the best reply to prices p', implying, under the IB account, that $(w, z) \in I(R)$, then the consumer does no longer satisfy the economic theory of the consumer. Indeed, in that case, the consumer does not have a unique indifference map that is invariant to budget sets.

[16] The reason is that, if some acts are not feasible, Epictetus's precept cannot say anything about these acts. Ethical thoughts - about what *should* be - cannot concern impossible acts.

This example illustrates that, whereas the I account of Epictetus's precept provides ethical foundations to the existence of a unique indifference map on consumption baskets that does not depend on the budget set, the IB account does not provide such foundations.

Note that similar results would be obtained if circumstances involved different income levels, but same prices. In that case, the I account implies indifference between consumption baskets enjoyed under different income levels, and, hence, implies the existence of a unique indifference map that does not vary with the budget set. On the contrary, the IB account does not guarantee that the consumer is necessarily indifferent between enjoying the same basket under different income levels, and, hence, does not provide ethical foundations for the existence of a unique indifference map invariant to budget sets.

Finally, it is worth underlining the relation that exists between, on the one hand, the property of independence of circumstances implied by the I account, and, on the other hand, the Weak axiom of revealed preferences (WARP) (Samuelson 1938, Chambers and Echenique 2016). The WARP is an observable implication of the existence of a unique indifference map on consumption baskets, which is invariant to changing the budget set. The WARP states that, if a basket q is chosen when another basket q' is also available under prices p, the choice of q' under prices p' cannot take place when q is still available under p'.

As stated in Proposition 1, the I account, together with basic richness and consistency requirements on R, implies that R satisfies independence of circumstances. In the context of consumer's choice, this implies that, if a consumer prefers strictly a basket q over a basket q' under prices p, he should also prefer strictly q over q' under other prices p', that is, it cannot be the case that basket q' is chosen when q is also available. This requirement is equivalent to WARP: any violation of independence of circumstances implies a violation of WARP and any violation of WARP implies a violation of independence of circumstances.

Proposition 5 summarizes our results.

Proposition 5 *Suppose that two consumption baskets $q \equiv (q_1, q_2)$ and $q' \equiv (q'_1, q'_2)$ are affordable under two sets of prices $p \equiv (p_1, p_2)$ and $p' \equiv (p'_1, p'_2)$, the consumer's income being left constant.*

- *Unlike the IB account, the I account of Epictetus's precept implies that there exists a unique indifference map defined on consumption baskets, which is invariant to changes in the budget set.*
- *Independence of circumstances is equivalent to WARP.*

Proof. See above. ■

Proposition 5 states that the two variants of Epictetus's precept have different implications when examining consumption choices. Whereas the I account provides ethical foundations for the standard theory of the consumer, by implying the existence of a unique indifference map (defined on baskets) that is invariant to changes in the budget set, the same is not true for the IB account.

6 Adaptive desires

When considering the relation between Epictetus's ethical doctrine and rationality, a key issue concerns the existence of adaptive or counteradaptive preferences. As studied in Elster (1982), adaptive preferences occur when agents modify their tastes or their desires in such a way as to reduce cognitive dissonance between what they want and what they can obtain given their constraints. The adaptation of tastes makes agents desire less things that they cannot obtain, and desire more things that they can obtain. On the contrary, counteradaptive preferences occur when persons desire more things that they cannot afford.

What is the relation between Epictetus's precept and adaptive preferences? What are the implications of Epictetus's precept on the structure of preferences on desires? Does Epictetus's Stoicism imply adaptive or counteradaptive preferences? How robust are the answers to the account of Epictetus's precept?

To answer these questions, this section examines the implications of Epictetus's precept in the (desires, circumstances) space. The (non empty) set of all desires is denoted by D, so that the (non empty) set of outcomes is $Z = D \times C$. We will study the implications of Epictetus's precept on the binary relation $R \subseteq Z \times Z$. Focusing on that alternative space is justified in the light of the *Manual*, where desires are regarded as key elements under the control of a person.[17] Hence, Epictetus's precept can also cast light on the structure of preferences on desires.

It should be stressed that all results of Section 4, which were derived for a binary relation R defined on $Y = A \times C$, can also be shown to be true for a binary relation R defined on $Z = D \times C$. A relation R on Z that is complete, reflexive and transitive (or Suzumura consistent) and satisfies the I account of Epictetus's precept (defined on Z) satisfies independence of circumstances. Moreover, a relation R on Z that is complete, reflexive and transitive (or Suzumura consistent) and satisfies the IB account of Epictetus's precept (defined on Z) satisfies robustness to dominated alternatives.

In that setting, (counter)adaptive desires can be defined as follows.

Definition 5 ((counter)adaptive desires) *Consider two circumstances m and $n \neq m$, with m being more favorable for the satisfaction of a desire d rather than for the one of a desire $e \neq d$, and n being more favorable to the satisfaction of the desire e rather than for the one of the desire d.*

- *Adaptive desires occur iff:*

$$((d,m),(e,m)) \in P(R) \text{ and } ((e,n),(d,n)) \in P(R)$$

- *Counteradaptive desires occur iff:*

$$((d,n),(e,n)) \in P(R) \text{ and } ((e,m),(d,m)) \in P(R)$$

[17] See, in particular, sections VII, XXI, XL, XLII and L of the *Manual*.

Adaptive desires (resp. counteradaptive desires) take place when there is a variation in the ranking over desires when circumstances change, in the direction of desiring less (resp. more) things that are less reachable under the new circumstances.

Let us illustrate the notion of adaptive and counteradaptive desires by means of a simple example in the (desires, circumstances) space.

	Circumstances	
Desires	elected to public office	not elected to public office
desire for social recognition	w = (desire recognition, office)	x = (desire recognition, no office)
desire for tranquility	y = (desire tranquility, office)	z = (desire tranquility, no office)

Table 3: Four outcomes in the (desires, circumstances) space.

In Table 3, adaptive desires take place when the person prefers desiring social recognition over desiring tranquility when being elected to public office, and prefers desiring tranquility over desiring social recognition when not being elected. Counteradaptive desires occur when the person prefers desiring social recognition over desiring tranquility when not being elected, but prefers desiring tranquility over desiring social recognition when being elected.

When examining the implications of Epictetus's precept in the (desires, circumstances) space, a key question is whether or not Epictetus's precept allows for adaptive desires, that is, whether or not a binary relation R on Z that is complete, reflexive and transitive and satisfies Epictetus's precept is compatible with adaptive or counteradaptive desires.

The answer to that question depends on the precise account of Epictetus's precept. As stated in Proposition 6, the I account of Epictetus's precept, by implying, under mild richness and coherence conditions, that R satisfies independence of circumstances, rules out adaptive and counteradaptive desires.

Proposition 6 *When R is complete, reflexive, transitive and satisfies the I account of Epictetus's precept, there can be no (counter)adaptive desires.*

Proof. By Proposition 1, the relation R satisfies independence of circumstances. This property requires, in the (desires, circumstances) space, that the ranking of outcomes involving different desires but the same circumstances should be invariant to the postulated circumstances. This invariance excludes adaptive and counteradaptive desires. For instance, in Table 3, adaptive preferences arise when, for instance, $(w, y), (z, x) \in P(R)$. But this violates independence of circumstances requiring either $(w, y), (x, z) \in P(R)$ or $(y, w), (z, x) \in P(R)$. The same kind of argument holds for counteradaptive desires. ■

To see the intuition behind that result, let us consider Table 3. According to Epictetus, being elected to public office (or not) is a circumstance, and, hence, should be regarded as neither good nor bad. From Proposition 1, we know that R, if it is complete, reflexive, transitive and satisfies the I account of Epictetus's precept, must also satisfy independence of circumstances. This implies, in the present case, that the ranking over desires should be invariant to the prevailing circumstances. Under the I account of Epictetus's precept, if the person prefers desiring social recognition over desiring tranquility when she is elected to public office, she should also prefer desiring social recognition over desiring tranquility when she is not elected to public office. Thus the fact of not being elected should not make the person adapt her desires towards desiring tranquility rather than social recognition. This rules out adaptive or counteradaptive desires.

Note, however, that the IB account of Epictetus's precept does not rule out (counter)adaptive desires, unlike the I account.

Proposition 7 *When R is complete, reflexive, transitive and satisfies the IB account of Epictetus's precept, there can be (counter)adaptive desires.*

Proof. See Table 3. The IB account is compatible with, for instance, the case of adaptive desires where the person prefers desiring tranquility over desiring social recognition when not being elected to public office, and prefers desiring social recognition over desiring tranquility when being elected to public office. The IB account is also compatible with the case of counteradaptive desires where the person prefers desiring tranquility over desiring social recognition when being elected to public office, and prefers desiring social recognition over desiring tranquility when not being elected to public office. ■

In sum, the relation between Epictetusian rationality and adaptive or counteradaptive desires depends on the adopted account of Epictetus's precept. While the I account rules out adaptive and counteradaptive desires, the IB account allows for these adaptation of desires to circumstances.

From the perspective of supporting one account of Epictetus's precept over the other, these results can be interpreted in different ways. From a positive point of view, the fact that the IB account allows, unlike the I account, for adaptive desires can be regarded as supporting that account. Indeed, if one interprets Epictetus's doctrine as justifying a reduction of cognitive dissonance between what one wants and what one can have, the IB account seems superior to the I account. But from a normative point of view, the fact that the I account of Epictetus's precept rules out (counter)adaptive desires can be interpreted as a strength of that account in comparison to the IB account. Indeed, the (counter)adaptive desires phenomenon is usually regarded as disqualifying welfarist normative approaches. This tends to give some support for the I account, on the grounds of its prohibition of (counter)adaptive desires.

7 Game theory

Let us finally consider implications of Epictetusian rationality in the context of strategic interactions between persons. As stated by Epictetus, whereas the acts of a person should be the object of her desires, the acts of *others* are mere circumstances for her, with respect to which she should be indifferent.[18] This motivates us to study corollaries of Epictetusian rationality for game theory.

[18] See, in particular, sections XI, XXI, XXVII, XXXIII and XLIV of the *Manual.*

For that purpose, let us consider a simple simultaneous cooperation game involving two players (A and B) and two strategies (defect or cooperate). In Table 4, the letters in brackets indicate the payoffs for each player under the associated outcome, the first letter corresponding, as usual, to the payoff for player A, while the second letter indicates the payoff for player B.

	Player B	
Player A	defect	cooperate
defect	(a, w)	(b, x)
cooperate	(c, y)	(d, z)

Table 4. A two-person cooperation game.

What are the implications of the I account and the IB account of Epictetus's precept in this game? An implication concerns the existence of a prisoner's dilemma, that is, a game where a Nash equilibrium is Pareto-dominated by an outcome of the game that is not a Nash equilibrium.

Proposition 8 *Under the I account and the IB account of Epictetus's precept, Epictetusian rationality rules out the existence of Prisoner's dilemma.*

Proof. See the Appendix. ■

Proposition 8 states that Epictetusian rationality, whether one formalizes it under the I account or the IB account of Epictetus's precept, rules out the existence of prisoner's dilemmas. In other words, when individuals have Epictetusian rationality, the situation where a Nash equilibrium is Pareto-dominated by an outcome of the game that is not a Nash equilibrium cannot occur.

What is the intuition behind that result? Consider first the I account. The I account implies indifference between outcomes that involve the same acts but different circumstances. This implies the equalities: $a = b$, $c = d$, $w = y$ and $x = z$. Under these pay-offs, it is logically impossible that an outcome is both a Nash equilibrium and Pareto-dominated by another outcome of the game. Consider now the IB account of Epictetus's precept. Under all possible assumptions about the levels of payoffs under the four outcomes of the game, the IB account implies indifference relations between outcomes involving the best reply of each player. These indifference relations tend, jointly with transitivity, to exclude the case where a Nash equilibrium can be Pareto-dominated by an outcome that is not a Nash equilibrium.

Proposition 8 states that Epictetusian rationality rules out the existence of prisoner's dilemmas. However, it does not tell us how Epictetusian players play the cooperation game of Table 4. In order to study the implications of Epictetusian rationality on the outcome of the cooperation game, let us suppose that each player knows that the outcome where all players cooperate Pareto-dominates the outcome where all players defect, and examine the implications of Epictetusian rationality in that context.

Proposition 9 *Consider the game of Table 4. Suppose that each player knows that the outcome where all players cooperate Pareto-dominates the outcome* *where all players defect, that is: $d > a$ and $z > w$. Then cooperating is a dominant strategy for Epictetusian players under the I account, but not necessarily under the IB account.*

Proof. See the Appendix. ■

Proposition 9 points to a major difference between the I account and the IB account of Epictetus's precept. If the outcome where everyone cooperates Pareto-dominates the outcome where everyone defects, cooperating must be a dominant strategy for Epictetusian players under the I account, but not necessarily for Epictetusian players under the IB account.

The intuition is that, for Epictetusian players of the I type, there must be two indifference relations: indifference between the outcome (cooperate, cooperate) and the outcome (cooperate, defect), and, also, indifference between (defect, defect) and (defect, cooperate). Hence, if the payoff is higher under (cooperate, cooperate) than under (defect, defect), these two indifference relations imply that *cooperation is a dominant strategy*. But the same rationale does not hold when Epictetusian players satisfy the IB account, because in that case they are fewer induced indifference relations.

It should be stressed here that the rationale leading to cooperation under Epictetusian rationality (I account) differs from the one that prevails under Kantian rationality. Kantian rationality makes agents focus only on generalizable acts, that is, on the diagonal of the matrix of the game (Curry and Roemer 2012). Under the assumption that (cooperate, cooperate) Pareto-dominates (defect, defect), Kantian rationality thus implies that agents choose to cooperate. The reason for cooperation is different under Epictetusian rationality: it is the fact that all outcomes where the agent cooperates are *equally good*, and the fact that all outcomes where the agent defects are *equally bad*, which, together with the assumption (cooperate, cooperate) Pareto-dominates (defect, defect), imply that agents cooperate. *Epictetusian rationality under the I variant, by extending the scope of indifference, provides another path out of the prisoner's dilemma, distinct from the one suggested by Kantian rationality.*

8 Conclusions

The term 'Epictetusian rationality' is not an oxymoron: it is possible to adopt Epictetus's precept while having a well-defined preference relation on outcomes. Our analyses show that the structure of Epictetusian rationality depends on how one formalizes Epictetus's precept, that is, on the imposed extension of the symmetric factor of the preference relation R, i.e., the 'scope of indifference'. Under the I account, a preference relation that is complete, reflexive and transitive (or Suzumura consistent) must satisfy independence of circumstances. Under the IB account, a preference relation that is complete, reflexive and transitive (or Suzumura consistent) must satisfy robustness to dominated alternatives.

Regarding the comparison of the two variants of Epictetusian rationality, our analyses give a threefold advantage to the I account over the IB account. First, when examining the link between Epictetus's precept and consumer theory, it appears that the I account provides, unlike the IB account, ethical foundations to a key feature of consumer theory: the existence of a unique indifference map (defined on baskets) that is invariant to changes of the budget set. Second, when studying the (desire, circumstances) space, we find that the I account excludes (counter)adaptive desires, unlike the IB account. Given the problems caused by (counter)adaptive desires for normative analysis, this result supports the I account. Third, both accounts rule out the existence of prisoner's dilemmas, but the I variant encourages cooperation more than the IB account. When players agree on the fact that the outcome where everyone cooperates Pareto-dominates the outcome where everyone defects, cooperation is a dominant strategy for Epictetusian players of the I type, but not necessarily under the IB variant.

This latter result provides the strongest case for Epictetusian rationality under the I variant, especially at a time where the tragedy of commons is exemplified by climate change and other global disorders that emerge under the standard form of rationality. The I variant of Epictetusian rationality, by imposing that agents are indifferent between all outcomes of the game where they defect, and, also, indifferent between all outcomes of the game where they cooperate, provides a path to get out of the prisoner's dilemma. *Extending the domain of indifference allows us to avoid prisoner's dilemmas.* It should be stressed, however, that avoiding prisoner's dilemmas arises also under other forms of rationality, such as Kantian rationality.

This observation leads us to a final remark. While the present paper casts some light on what Epictetus's precept means, as well as on its implications for the possibility of Epictetusian rationality, this does not provide an answer to the question: *should* a person adopt Epictetusian rationality? As for Kantian rationality or any other type of rationality, this question remains without answer. Epictetusian rationality is just one way of representing the world among many other frameworks, and the justification of this particular type of rationality goes beyond the scope of this paper.

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10 Appendix

10.1 Proof of Proposition 1

Let us prove this proposition by contradiction. For that purpose, let us take the following example with two acts (a and b) and two circumstances (m and n). The table below shows the four possible outcomes.

	Circumstances	
Acts	m	n
a	(a,m)	(a,n)
b	(b,m)	(b,n)

The account I of Epictetus's precept implies $((a,m),(a,n)) \in I(R)$ as well as $((b,m),(b,n)) \in I(R)$. Suppose that independence of circumstances is not satisfied. For instance, one has $((a,m),(b,m)) \in P(R)$ and $((b,n),(a,n)) \in P(R)$.

$((a,m),(b,m)) \in R$ and $((a,m),(a,n)) \in R$ imply, by transitivity,

$$((a,n),(b,m)) \in R$$

$((a,n),(b,m)) \in R$ and $((b,m),(b,n)) \in R$ imply, by transitivity,

$$((a,n),(b,n)) \in R$$

But $((a,n),(b,n)) \in R$ contradicts $((b,n),(a,n)) \in P(R)$.

Similar contradictions would be reached under other violations of independence of circumstances. For instance, if one has $((a,m),(b,m)) \in I(R)$ and $((b,n),(a,n)) \in P(R)$, one deduces, by means of transitivity, that $((a,n),(b,n)) \in R$, which contradicts $((b,n),(a,n)) \in P(R)$.

Finally, note that similar preference cycles and contradictions would be obtained in examples involving a larger number of acts and circumstances. Indeed, the I account of Epictetus's precept implies indifference relations between all outcomes with same acts and different circumstances, which leads to preference cycles and contradictions when the independence of circumstances is not satisfied by R.

10.2 Proof of Proposition 2

Let us first consider quasi-transitivity. Proposition 2 can be proved by finding a single example of relation R that satisfies account I, reflexivity, completeness, quasi transitivity and that violates independence of circumstances. Such an example is given by the following example,

	Circumstances	
Acts	m	n
a	$u=(a,m)$	$v=(a,n)$
b	$w=(b,m)$	$x=(b,n)$
c	$y=(c,m)$	$z=(c,n)$

Account I implies $\{(u,v),(w,x),(y,z)\} \subset I(R)$. We assume also a violation of independence of circumstances: $\{(u,w),(x,v)\} \subset P(R)$. We also assume that $\{(w,y),(u,y),(z,x),(z,v)\} \subset P(R)$.

We assume that R includes also other pairs, so that:

$$R = \left\{ \begin{array}{c} (u,u),(v,v),(w,w),(x,x),(y,y),(z,z),(u,v),(v,u),(w,x),(x,w),(y,z),(z,y),(y,v),(v,y), \\ (u,y),(z,v),(w,z),(z,w),(u,x),(x,u),(u,z),(z,u),(w,v),(v,w),(x,y),(y,x),(w,y),(z,x) \end{array} \right\}$$

R satisfies reflexivity, since $\{(u,u),(v,v),(w,w),(x,x),(y,y),(z,z)\} \subset R$.

R satisfied account I of Epictetus's precept by construction.

R satisfies also completeness.

R violates independence of circumstances by construction.

But R satisfies also quasi-transitivity. Indeed, we have: $\{(u,w),(w,y),(u,y)\} \subset P(R)$ as well as $\{(z,x),(x,v),(z,v)\} \subset P(R)$, so that $P(R)$ is transitive.

The second part of Proposition 2 can be proved by showing that a relation R that satisfies account I, reflexivity, completeness, and Suzumura consistency must necessarily satisfy independence of circumstances.

Let us prove this by contradiction and show that, when R that satisfies account I, reflexivity, completeness, and violates independence of circumstances, it violates Suzumura consistency.

Consider first the following 2x2 case.

	Circumstances	
Acts	m	n
a	$u=(a,m)$	$v=(a,n)$
b	$w=(b,m)$	$x=(b,n)$

Account I implies $\{(u,v),(w,x)\} \subset I(R)$. We assume also a violation of independence of circumstances: $\{(u,w),(x,v)\} \subset P(R)$.

Let us assume that R is reflexive, so that: $\{(u,u),(v,v),(w,w),(x,x)\} \subset R$.

Given the completeness of R, there are 9 possible cases:

Case 1: $(u,x) \in P(R)$ and $(v,w) \in P(R)$

Case 2: $(u,x) \in P(R)$ and $(w,v) \in P(R)$

Case 3: $(u,x) \in P(R)$ and $(v,w) \in I(R)$

Case 4: $(x,u) \in P(R)$ and $(v,w) \in P(R)$

Case 5: $(x,u) \in P(R)$ and $(w,v) \in P(R)$

Case 6: $(x,u) \in P(R)$ and $(v,w) \in I(R)$

Case 7: $(u,x) \in I(R)$ and $(v,w) \in P(R)$

Case 8: $(u,x) \in I(R)$ and $(w,v) \in P(R)$

Case 9: $(u,x) \in I(R)$ and $(v,w) \in I(R)$

Let us now show that in each case, R violates Suzumura consistency.

In case 1, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u),(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (u,x),(v,w) \\ (v,x),(w,v),(x,u) \end{array} \right\} \\ &= R \cup \{(v,x),(w,v),(x,u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x,y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(v,w) \in P(R)$ and $(w,v) \in tc(R)$, thus a violation of Suzumura consistency.

In case 2, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (u,x),(w,v) \\ (v,w),(v,x),(x,u) \end{array} \right\} \\ &= R \cup \{(v,x),(v,w),(x,u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(w,v) \in P(R)$ and $(v,w) \in tc(R)$, thus a violation of Suzumura consistency.

In case 3, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (u,x),(w,v),(v,w) \\ (v,x),(x,u) \end{array} \right\} \\ &= R \cup \{(v,x),(x,u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(u,x) \in P(R)$ and $(x,u) \in tc(R)$, thus a violation of Suzumura consistency.

In case 4, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (x,u),(v,w) \\ (w,v),(u,x),(x,w) \end{array} \right\} \\ &= R \cup \{(w,v),(u,x),(x,w)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(v,w) \in P(R)$ and $(w,v) \in tc(R)$, thus a violation of Suzumura consistency.

In case 5, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (x,u),(w,v) \\ (v,w)\,,(w,u),(u,x) \end{array} \right\} \\ &= R \cup \{(v,w),(w,u),(u,x)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(w,v) \in P(R)$ and $(v,w) \in tc(R)$, thus a violation of Suzumura consistency.

In case 6, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (x,u),(w,v),(v,w) \\ (w,u),(u,x) \end{array} \right\} \\ &= R \cup \{(w,u),(u,x)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x,y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(x,u) \in P(R)$ and $(u,x) \in tc(R)$, thus a violation of Suzumura consistency.

In case 7, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (x,u),(u,x),(v,w) \\ (v,x),(w,u),(w,v) \end{array} \right\} \\ &= R \cup \{(v,x),(w,u),(w,v)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x,y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(v,w) \in P(R)$ and $(w,v) \in tc(R)$, thus a violation of Suzumura consistency.

In case 8, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (x,u),(u,x),(w,v) \\ (v,w),(v,x),(w,u) \end{array} \right\} \\ &= R \cup \{(v,x),(w,u),(v,w)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x,y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(w,v) \in P(R)$ and $(v,w) \in tc(R)$, thus a violation of Suzumura consistency.

In case 9, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (x,u),(u,x),(w,v),(v,w) \\ (v,x),(w,u) \end{array} \right\} \\ &= R \cup \{(v,x),(w,u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here $(u, w) \in P(R)$ and $(w, u) \in tc(R)$, thus a violation of Suzumura consistency.

Together, these 9 cases cover all possible cases in the 2x2 example. In each of these cases, the relation R satisfying reflexivity, completeness, I account of Epictetus's precept and violating independence of circumstances also violate Suzumura consistency.

The previous proof can be extended to all cases involving more than 2 acts and 2 circumstances. The reason why this proof can be extended lies in the fact that the I account of Epictetus's precept imposes indifference between all outcomes differing only along circumstances. Thus, the addition of a number of circumstances does not make the preference cycle vanish, because the indifference relations across all circumstances just increase the width of the preference cycle. Hence, when independence of circumstances is violated, the I account, by requiring all these indifference relations, makes R violate Suzumura consistency. In a similar way, adding a number of acts does not prevent the occurrence of a preference cycle when independence of circumstances is violated. Thus the previous proof can be extended to cases of all dimensions concerning the number of acts and circumstances.

10.3 Proof of Proposition 3

Let us prove this proposition by contradiction. For that purpose, let us take the following example with two acts (a and b) and two circumstances (m and n). The table below shows the four possible outcomes.

	Circumstances	
Acts	m	n
a	(a, m)	(a, n)
b	(b, m)	(b, n)

Note that if $P(R) = \varnothing$, the property of robustness to dominated alternatives is trivially satisfied. Same thing if $P(R) \neq \varnothing$ but $P(R)$ includes a single pair of outcomes or two pairs of outcomes sharing the same circumstances.

Suppose now that $\{((a, m), (b, m)), ((b, n), (a, n))\} \subset P(R)$. The account IB of Epictetus's precept implies $((a, m), (b, n)) \in I(R)$. Suppose that robustness to dominated alternatives is not satisfied. For instance, one has $((a, n), (a, m)) \in P(R)$.

In that case, we have that $((a, n), (a, m)) \in R$ and $((a, m), (b, n)) \in R$, so that, by transitivity:

$$((a, n), (b, n)) \in R$$

a contradiction with $((b, n), (a, n)) \in P(R)$.

Alternatively, if one had another violation of robustness to dominated alternatives, such as $((b,m),(b,n)) \in P(R)$, we would have that $\{((b,m),(b,n)),((b,n),(a,m))\} \subset R$, which implies, by transitivity;

$$((b,m),(a,m)) \in R$$

a contradiction with $((a,m),(b,m)) \in P(R)$.

Finally, note that similar preference cycles and contradictions would be obtained in examples involving a larger number of acts and circumstances. Indeed, the IB account of Epictetus's precept implies indifference relations between all outcomes with the best act for given circumstances, which leads to preference cycles and contradictions when the robustness to dominated alternatives is not satisfied by R.

10.4 Proof of Proposition 4

Regarding the first bullet list item, this can be proved by showing an example of case where a preference relation R satisfying the IB account of Epictetus's precept, completeness, reflexivity and quasi-transitivity but violates robustness to dominated alternatives. For that purpose, let us take the following example with three acts (a, b and c) and two circumstances (m and n). The table below shows the six possible outcomes.

	Circumstances	
Acts	m	n
a	$u=(a,m)$	$v=(a,n)$
b	$w=(b,m)$	$x=(b,n)$
c	$y=(c,m)$	$z=(c,n)$

Let us assume that $\{(u,w),(w,y),(u,y),(v,x),(x,z),(v,z)\} \subset P(R)$.

Account IB implies $(u,v) \in I(R)$.

We assume also a violation of robustness to dominated alternatives: $(x,u) \in I(R)$.

We assume that R includes also other pairs, so that:

$$R = \left\{ \begin{array}{c} (u,u),(v,v),(w,w),(x,x),(y,y),(z,z),(u,v),(v,u), \\ (u,w),(w,y),(u,y),(v,x),(x,z),(v,z),(u,z),(z,u), \\ (x,u),(u,x),(w,x),(x,w),(y,z),(z,y),(w,v),(v,w),(w,z),(z,w), \\ (y,v),(v,y),(y,x),(x,y) \end{array} \right\}$$

R satisfies reflexivity, since $\{(u,u),(v,v),(w,w),(x,x),(y,y),(z,z)\} \subset R$.

R satisfies the account IB of Epictetus's precept by construction.

R satisfies also completeness.

R violates robustness to dominated alternatives by construction.

But R satisfies also quasi-transitivity. Indeed, we have: $\{(u,w),(w,y),(u,y)\} \subset P(R)$ as well as $\{(z,x),(x,v),(z,v)\} \subset P(R)$, so that $P(R)$ is transitive.

The second part of Proposition 4 can be proved by showing that a relation R that satisfies the account IB, reflexivity, completeness, and Suzumura consistency must necessarily satisfy robustness to dominated alternatives.

Let us prove this by contradiction and show that, when R that satisfies account IB, reflexivity, completeness, and violates robustness to dominated alternatives, it violates Suzumura consistency.

Consider first the following 2x2 case.

Acts	Circumstances m	n
a	$u = (a, m)$	$v = (a, n)$
b	$w = (b, m)$	$x = (b, n)$

Suppose that $P(R) = \varnothing$. Then robustness to dominated alternatives is trivially satisfied. Suppose that $P(R)$ includes only one element, or elements only for a given circumstance. Here again, robustness to dominated alternatives would be trivially satisfied.

Thus the relevant examples concern cases where we have, for instance, $\{(u, w), (v, x)\} \subset P(R)$.

Account IB implies $(u, v) \in I(R)$.

Let us assume that R is reflexive, so that: $\{(u, u), (v, v), (w, w), (x, x)\} \subset R$.

Given the completeness of R, we also assume $(w, x) \in I(R)$, but a similar proof could be obtained under either $(w, x) \in P(R)$ or $(x, w) \in P(R)$.

There are 8 possible cases concerning the violations of robustness to dominated alternatives:

Case 1: $(x, u) \in P(R)$ and $(v, w) \in P(R)$
Case 2: $(x, u) \in P(R)$ and $(w, v) \in P(R)$
Case 3: $(x, u) \in P(R)$ and $(v, w) \in I(R)$
Case 4: $(u, x) \in P(R)$ and $(w, v) \in P(R)$
Case 5: $(u, x) \in P(R)$ and $(w, v) \in I(R)$
Case 6: $(u, x) \in I(R)$ and $(v, w) \in I(R)$
Case 7: $(u, x) \in I(R)$ and $(v, w) \in P(R)$
Case 8: $(u, x) \in I(R)$ and $(w, v) \in P(R)$

Let us now show that in each case, R violates Suzumura consistency.

In case 1, the transitive closure of R is:

$$
\begin{aligned}
tc(R) &= \left\{ \begin{array}{c} (u, u), (v, v), (w, w), (x, x), (u, v), (v, u), (w, x), (x, w), (u, w), (x, v), \\ (x, u), (v, w), \\ (w, v), (u, x) \end{array} \right\} \\
&= R \cup \{(w, v), (u, x)\} \supseteq R
\end{aligned}
$$

Suzumura consistency requires that:

$$\forall x, y \in Y, (x, y) \in tc(R) \implies (y, x) \notin P(R)$$

Here $(v, w) \in P(R)$ and $(w, v) \in tc(R)$, thus a violation of Suzumura consistency.

In case 2, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (x,u),(w,v) \\ (v,w)\,,(w,u) \end{array} \right\} \\ &= R \cup \{(v,w),(w,u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x,y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(w,v) \in P(R)$ and $(v,w) \in tc(R)$, thus a violation of Suzumura consistency.

In case 3, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (x,u),(w,v),(v,w) \\ (w,u) \end{array} \right\} \\ &= R \cup \{(w,u)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x,y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(u,w) \in P(R)$ and $(w,u) \in tc(R)$, thus a violation of Suzumura consistency.

In case 4, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (u,x),(w,v) \\ (v,w),(v,x),(x,u) \end{array} \right\} \\ &= R \cup \{(v,w),(x,u),(v,x)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x,y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(w,v) \in P(R)$ and $(v,w) \in tc(R)$, thus a violation of Suzumura consistency.

In case 5, the transitive closure of R is:

$$\begin{aligned} tc(R) &= \left\{ \begin{array}{c} (u,u)\,,(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (u,x),(w,v),(v,w) \\ (v,x) \end{array} \right\} \\ &= R \cup \{(v,x)\} \supseteq R \end{aligned}$$

Suzumura consistency requires that:

$$\forall x,y \in Y, (x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(x,v) \in P(R)$ and $(v,x) \in tc(R)$, thus a violation of Suzumura consistency.

In case 6, the transitive closure of R is:

$$\begin{array}{rcl} tc(R) & = & \left\{ \begin{array}{c} (u,u),(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (u,x),(x,u),(w,v),(v,w) \\ (w,u) \end{array} \right\} \\ & = & R \cup \{(w,u)\} \supseteq R \end{array}$$

Suzumura consistency requires that:

$$\forall x,y \in Y,(x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(u,w) \in P(R)$ and $(w,u) \in tc(R)$, thus a violation of Suzumura consistency.

In case 7, the transitive closure of R is:

$$\begin{array}{rcl} tc(R) & = & \left\{ \begin{array}{c} (u,u),(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (x,u),(u,x),(v,w) \\ (v,x) \end{array} \right\} \\ & = & R \cup \{(v,x)\} \supseteq R \end{array}$$

Suzumura consistency requires that:

$$\forall x,y \in Y,(x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(x,v) \in P(R)$ and $(v,x) \in tc(R)$, thus a violation of Suzumura consistency.

In case 8, the transitive closure of R is:

$$\begin{array}{rcl} tc(R) & = & \left\{ \begin{array}{c} (u,u),(v,v),(w,w),(x,x),(u,v),(v,u),(w,x),(x,w),(u,w),(x,v), \\ (x,u),(u,x),(w,v) \\ (v,w),(v,x),(w,u) \end{array} \right\} \\ & = & R \cup \{(v,x),(w,u),(v,w)\} \supseteq R \end{array}$$

Suzumura consistency requires that:

$$\forall x,y \in Y,(x,y) \in tc(R) \implies (y,x) \notin P(R)$$

Here $(w,v) \in P(R)$ and $(v,w) \in tc(R)$, thus a violation of Suzumura consistency.

Together, these 8 cases cover all possible cases in the 2x2 example. In each of these cases, the relation R satisfying reflexivity, completeness, I account of Epictetus's precept and violating independence of circumstances also violate Suzumura consistency.

The previous proof can be extended to all cases involving more than 2 acts and 2 circumstances. The reason why this proof can be extended lies in the fact that the IB account of Epictetus's precept imposes indifference between best replies to circumstances. Thus, the addition of a number of circumstances does not make the preference cycle vanish, because the indifference relations across all best replies to circumstances just increase the width of the preference cycle. Hence, when robustness to dominated alternatives is violated, the IB account, by requiring all indifference relations between best replies to circumstances, makes R violate Suzumura consistency. In a similar way, adding a number of acts does not prevent the occurrence of a preference cycle when robustness to dominated alternatives is violated. Thus the previous proof can be extended to cases of all dimensions concerning the number of acts and circumstances.

10.5 Proof of Proposition 8

	Player B	
Player A	defect	cooperate
defect	(a, w)	(b, x)
cooperate	(c, y)	(d, z)

Table 4: A simple cooperation game.

A prisoner's dilemma consists of the existence of a Nash equilibrium that is Pareto-dominated by an outcome that is not a Nash equilibrium. Let us show that the I account of Epictetus's precept rules out this case. If persons take the behavior of others as a circumstance, the I account implies:

$$\begin{aligned} a &= b \text{ and } c = d \\ w &= y \text{ and } x = z \end{aligned}$$

These equalities are incompatible with the existence of a prisoner's dilemma. Indeed, under these equalities, the Table 4 becomes:

	Player B	
Player A	defect	cooperate
defect	(a, w)	(a, x)
cooperate	(c, w)	(c, x)

Table 5: A simple cooperation game.

Given this matrix of payoffs, it is hard to see how a Nash equilibrium could be Pareto-dominated by an outcome that is not a Nash equilibrium. Suppose, for instance, that (defect, defect) is a Nash equilibrium. Then $a > c$ and $w > x$. These two inequalities prevent any other outcome of the game to Pareto-dominate (defect, defect). The same conclusion would hold for any other Nash equilibrium given the payoff matrix. Thus the I account of Epictetus's precept rules out the existence of prisoner's dilemmas.

Let us now consider the IB account of Epictetus's precept. Several cases should be distinguished.

Consider first the case where defect is the best reply to cooperate or defect. We then have: $a > c$, $b > d$ and $w > x, y > z$ and the outcome (defect, defect) is the Nash equilibrium. The IB account requires: $a = b$ and $w = y$.

Under the IB account, the payoff matrix becomes:

	Player B	
Player A	defect	cooperate
defect	(a, w)	(a, x)
cooperate	(c, w)	(d, z)

Table 6: A simple cooperation game.

Given the inequalities $a > c$ and $w > x$, the Nash equilibrium cannot be Pareto-dominated by the outcomes where one player cooperates and the other defects. Moreover, given $a = b > d$ and $w = y > z$, the outcome (cooperate, cooperate) cannot Pareto-dominate the Nash equilibrium (defect, defect). Thus no prisoner's dilemma exists.

Consider now the case where the best reply to defect is defect, and the best reply to cooperation is cooperation. We thus have: $a > c$, $w > x$, as well as $d > b$ and $z > y$. The IB account implies that: $a = d$ and $z = w$. Hence the payoff matrix becomes:

	Player B	
Player A	defect	cooperate
defect	(a, w)	(b, x)
cooperate	(c, y)	(a, w)

Table 7: A simple cooperation game.

Given the payoff inequalities $a > c$, $w > x$, the outcome (cooperate, defect) cannot Pareto-dominate (defect, defect). Similarly, the outcome (cooperate, defect) cannot Pareto-dominate the outcome (defect, defect). Moreover, the outcome (cooperate, cooperate) does not Pareto-dominate (defect, defect).

Consider now the case where defect is the best reply to cooperate, and cooperate is the best reply to defect. We thus have: $c > a, b > d$ as well as $y > z, x > w$. From the IB account, we have then: $c = b$ and $y = x$, and the payoff matrix becomes:

	Player B	
Player A	defect	cooperate
defect	(a, w)	(b, x)
cooperate	(b, x)	(d, z)

Table 8: A simple cooperation game.

Given $c = b > a$, the outcome (defect, defect) cannot Pareto-dominate (cooperate, defect). Moreover, given $b > d$, the outcome (cooperate, cooperate) cannot Pareto-dominate (cooperate, defect). Finally, the outcome (defect, cooperate) does not Pareto-dominate (cooperate, defect). Similarly, the outcome (defect, cooperate) is not Pareto-dominated by (defect, defect), since $x > w$. Moreover, the outcome (defect, cooperate) is not Pareto-dominated by (cooperate, cooperate), since $b > d$. Thus, there can be no prisoner's dilemma under the IB account of Epictetus's precept.

10.6 Proof of Proposition 9

Let us consider Table 4 again, with $d > a$ and $z > w$.

	Player B	
Player A	defect	cooperate
defect	(a, w)	(b, x)
cooperate	(c, y)	(d, z)

Table 4: A simple cooperation game.

Under the I account, we must have $a = b$ and $c = d$. Together with $d > a$, these inequalities imply $c > a$ and $d > b$. Thus cooperation is a dominant strategy for player A. Concerning player B, the I account implies $w = y$ and $x = z$. Together with $z > w$, these inequalities imply: $x > w$ and $z > y$. Thus cooperation is a dominant strategy for player B. Cooperation is thus a dominant strategy under the I account.

Let us now consider the IB account. For player A, the inequality $d > a$ rules out the case where cooperation is the best reply to cooperation and defect is the best reply to defect. Indeed, if that case were true, the IB account would imply $d = a$. Same result for player B. Suppose now that defect is a dominant strategy. Then the IB account would imply that $a = b$. But since $d > a$, we would have also $d > b$, which contradicts the idea that defect is a dominant strategy. A similar contradiction can be reached for player B. Consider now the case where cooperating is a dominant strategy. We then have $c > a$ and $d > b$. The IB account implies $c = d$. These inequalities are compatible with $d > a$. Cooperation can also be a dominant strategy for player B. Consider now the case where cooperate is the best reply to defect ($c > a$), and defect is the best reply to cooperate ($b > d$). The IB account would then imply $b = c$. But then $b = c$ and $c > a$ imply $b > a$. Moreover, $b > d$ and $b = c$ imply $c > d$. These inequalities are compatible with $d > a$, so this case cannot be ruled out. Thus it is not necessarily the case that cooperation is a dominant strategy under the IB account.