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A Multicountry Model of the Term Structures of Interest Rates with a GVAR

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Abstract

Global interdependencies have caused affine term structure models ($ATSM$s) to adopt a multicountry dimension. Nevertheless, recent referenced $ATSM$s face issues of tractability as the model dimension becomes larger. To close this gap, this paper proposes a $ATSM$ in which the risk factor dynamics follow a global vector-autoregressive ($GVAR$). $ATSM - GVAR$ renders a parsimonious yield curve parametrization, which allows for a fast estimation process, enables meaningful statistical inference of economic relationships, and produces accurate bond yields out-of-sample forecasting. To empirically illustrate our novel $ATSM$, we build a markedly integrated economic system composed of three Latin American economies and China. We find that, consequent to its prominent role in the worldwide economy, China's economic stances have nonnegligible impacts on Latin American yield curve dynamics.

Keywords: Term Structure of Interest Rates, Global Financial Market, GVAR

JEL Classification: C58, E44, G15

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1. Introduction

The interplay between the term structure of interest rates and economic unfolding is of key relevance to economic players. For central banks, the yield curve is an important tool of monetary policy transmission (Evans and Marshall (2007)), whereas for financial investors, the term structure reflects the expectations and risk-return assessments of the future macroeconomic outlook (Ang et al. (2006)). The linkage between macroeconomic fundamentals and the yield curve is at the core of affine term structure models (henceforth $ATSM$s).[1] Recently, $ATSM$s have been developed to accommodate growing countries' interdependence induced by globalization. However, these models fail to properly account for cross-border shocks due to their high complexity and the large numbers of parameters to be estimated. This paper shades new light on this field by means of a novel parsimonious $ATSM$ representation that accurately captures the effects of shocks for globalized economies. This new framework makes important progress toward model tractability and therefore increasing the speed of the estimation process of large-scale $ATSM$s while improving statistical inference and the forecasting abilities of $ATSM$s.

The backbone of our term structure framework is the influential work of Joslin et al. (2014) (henceforth $JPS - ATSM$). In essence, $JPS - ATSM$ assumes the absence of arbitrage opportunities and considers linear state space representations of the yield curve dynamics. In particular, this model representation simultaneously combines the traditional yield curve factors (e.g., level, slope, and curvature) alongside economic and financial variables. The former, labeled spanned factors, grant a good fit for the cross-section of bond yields. The latter, termed unspanned factors, shape the time evolution of the term structure and contribute to explaining the linkage between bond yields and the real economy. With respect to early referenced macrofinance $ATSM$s, $JPS - ATSM$ enables a more tractable

[1] This class of models includes, for instance, the works of Ang and Piazzesi (2003), Bikbov and Chernov (2010), and Wright (2011), Bauer et al. (2014), among many others.

and accurate estimation of the model parameters while preserving good explanatory power of the yield cross-section and providing a clear economic interpretation of their risk factors.

Over the last two decades, many papers have employed $ATSM$s to evaluate the role played by domestic macroeconomic fundamentals (usually inflation and real economic growth) in the evolution of the shape of the U.S. term structure.[2] Recently, $ATSM$s assimilated the effects of a globalized economy. To this end, aside from domestic underlying factors, foreign aspects are considered to steer term structure developments. Based on the $ATSM$ developed by Joslin et al. (2014), Jotikasthira et al. (2015) (henceforth $JLL - ATSM$) extended the unspanned macroeconomic risk setting to accommodate multiple countries. As such, the $JLL - ATSM$ framework enables us to determine the relative contributions of domestic idiosyncratic and foreign shocks to explaining the yield curve dynamics.

As in the traditional $ATSM$ literature, $JLL - ATSM$ assumes that the joint dynamics of the risk factors evolve according to a first-order VAR. This feature of the model raises several important limitations. First, the inclusion of a few supplementary countries in the economic system could quickly render the model extremely large. This issue results, thus, in tractability issues leading to timely (when feasible) estimation. Second, the identification scheme of structural shocks requires a large set of zero restrictions on both the feedback and the variance-covariance matrices. As a consequence, the assessment of the relationship between any two variables of the system becomes undermined. Third, as is standard for reduced form VARs, the nature of the linkages among the units of an economic system are unspecified. Accordingly, these models are silent on the sources of transmission of cross-border shocks.

This paper takes a step toward closing these gaps by proposing an alternative model for multicountry $ATSM$s. In particular, we assume that the dynamics of the risk factors follow

[2]See, for instance, Ang and Piazzesi (2003), Rudebusch and Wu (2008), and Joslin et al. (2014), among others.

a global vector-autoregressive (henceforth, $GVAR$). Along the lines of Chudik and Pesaran (2016), our $GVAR$ can be split into two components. The first, commonly referred to as the marginal model, is a standard $VAR(1)$ that describes the dynamics of global economic factors. The second, denoted by $VARX^*$ models, relates to small-scale country-specific augmented VARs. For each country $VARX^*$, we include its own domestic spanned and unspanned factors, as well as the global unspanned factors, in addition to weighted cross-section averages of foreign variables. These weights capture the degree of interdependence across countries.

The transition matrix is a central pillar of $GVAR$ specifications. It is precisely this peculiarity of the model that enables us to address the key shortcomings of the $JLL - ATSM$. First, the inclusion of an explicit transmission structure allows for transparent control of the mechanism of propagation of shocks, hence providing a normative result. Second, imposing a structure of interdependence among countries greatly reduces the overall number of model parameters and thus boosts the accuracy of the estimates. In addition to promoting econometric tractability, the $GVAR - ATSM$ does not require economic-based controversial restrictions and provides more robust economic interpretations. We also show that, due to this parsimonious feature of our proposed model, the required estimation time is approximately 3 to 5 times shorter than analog multicountry versions of the $JPS - ATSM$ and the $JLL - ATSM$.[3]

To illustrate the new $GVAR - ATSM$, we consider an economic system with substantial interdependence among countries. Consistent with referenced $GVAR$ works, we proxy our measure of country interdependence by trade flow weights.[4] Since geographical proximity

[3]Arguably, the alternative $ATSM$ from Adrian et al. (2013) can yield almost instantaneous model estimates. However, this framework is subject to criticism for overlooking the internal consistency condition of the pricing factors (see Joslin et al. (2011)).

[4]See, for instance, the seminal work of Pesaran et al. (2004). A more formal statistical approach to test

consistently strengthens trading linkages,[5] we include a group of three Latin American countries (Brazil, Mexico, and Uruguay) in our economic cohort. We complete this system with China, the world's leading exporter.[6] [7] Previous studies have investigated the determinant of sovereign bond yields and risk premia in the context of emerging markets.[8] However, to the best of our knowledge, this work is the first on Latin American economies based on a flexible framework that enables the assessment of cross-country interactions.

We appraise our empirical results in three dimensions. First, we compare the out-of-sample forecasting performance of bond yields for both $GVAR-ATSM$ and $JLL-ATSM$. We show that $GVAR-ATSM$ persistently produces more accurate forecast errors in terms of the mean and standard deviation. For instance, for the case of Mexico, the superior forecasting performance of the $GVAR-ATSM$ is observed in almost 100% of the analyzed scenarios. Second, we use generalized impulse response functions (henceforth $GIRF$s) to investigate the yield curve responses of Latin American economies to an economic growth

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for the validity of transition matrices in $GVAR$ setups is described in Candelon et al. (2020).

[5]See, for instance, Disdier and Head (2008).

[6]See the Direction of Trade Statistics (DTS) database released from the International Monetary Fund (IMF).

[7]While it would certainly be of great interest to include the U.S. economy in this economic system, we avoid this choice since the sample period under investigation coincides, to a large extent, with the years of zero short-term rates. Nevertheless, it is important to emphasize that the degree of interconnection between China and the Latin American economies has markedly strengthened in recent years. In terms of trade, China was the largest partner of Brazil and Uruguay and the second largest partner of Mexico in 2019 (DTS-IMF database).

[8]For instance, Chernov et al. (2020) studied five Asian-Pacific economies using a large-scale panel VAR that, nevertheless, requires a fair number of identification restrictions. Iania et al. (2020) employed a $JPS-ATSM$ to assess the role of global economic shocks on the risk premia dynamics of four major emerging markets from different regions of the world.

shock from China. We observe that this same shock induces a flattening in the yield curves of Brazil and Mexico and a steepening in the case of Uruguay. We claim that, for the first two countries, these results reflect the responses of the domestic monetary authorities to inflationary pressures, whereas for the latter, they relate to liquidity reasons. We also show that our model consistently generates narrower confidence intervals and is statistically indistinguishable from those of the $JLL - ATSM$. Third, based on generalized forecast error variance decompositions (henceforth $GFEVD$s), we compute the shares of domestic and foreign shocks that explain the volatility of the yield curve evolutions. Overall, we observe that both $GVAR - ATSM$ and $JLL - ATSM$ point to foreign factors as the main source of bond yield fluctuations, while domestic developments are less important. Moreover, both models suggest that the relative importance of foreign factors increases as the horizon of analysis expands.

This paper proceeds as follows. Section 2 presents the basic methodological features of the single and multicountry $ATSM$s. The $GVAR$ extension is presented in Section 3. The empirical results are presented in Section 4, and Section 5 concludes.

2. Early ATSMs with unspanned economic risks

In this section, we outline the frameworks of $JPS - ATSM$ and $JLL - ATSM$. One convenient aspect of these setups is that they enable a clear split of the yield curves into cross-sectional and time series dimensions. In light of this characteristic of the models, we present one economy $JPS - ATSM$ in Section 2.1. Next, we expose the specific features of the risk factor dynamics from the multicountry $JLL - ATSM$ in Section 2.2. We defer to Section 3 the presentation $JLL - ATSM$'s cross-section representation due to its straight overlap with our $GVAR - ATSM$. In Section 2.3, we point out some limitations of these two setups.

2.1. The seminal ATSM with unspanned macroeconomic risk ($JPS - ATSM$)

2.1.1. Cross-sectional dimension of the term structure

The cross-sectional setting of the $JPS - ATSM$s is based on two central equations. The first assumes that the country i short-term interest rate, $r_{i,t}$, is an affine function of N unobserved (latent) country-specific factors, $X_{i,t}$:

$$r_{i,t} = \delta_{i,0} + \delta'_{i,1} X_{i,t}, \tag{1}$$

where $\delta'_{i,1}$ is a N-dimensional vector with all entries equal to one.

The second equation refers to the state dynamics under the risk-neutral measure (the Q-measure). By assumption, $X_{i,t}$ evolves according to a zero-mean maximally flexible affine $VAR(1)$

$$X_{i,t} = \Phi^Q_{i,X} X_{i,t-1} + \Sigma_{i,X} \varepsilon^Q_{i,t}, \qquad \varepsilon^Q_{i,t} \sim \mathcal{N}(0, I_N), \tag{2}$$

where $\Phi^Q_{i,X}$ is a diagonal matrix, the elements of which are real and distinct eigenvalues, λ^Q_i.

Based on equations (1) and (2) , Dai and Singleton (2000) showed that the country-specific yield of a zero-coupon bond with maturity of n periods, $y^{(n)}_{i,t}$, is affine on $X_{i,t}$. For notational simplicity, we collect J yields into the vector $Y_{i,t} = [y^{(1)}_{i,t}, ..., y^{(J)}_{i,t}]'$ such that

$$Y_{i,t} = A_X(\delta_{i,0}, \lambda^Q_i, \Sigma_{i,X}) + B'_X(\lambda^Q_i) X_{i,t}, \tag{3}$$

where the forms of $A_X(\delta_{i,0}, \lambda^Q_i, \Sigma_{i,X})$ and $B_X(\lambda^Q_i)$ are restricted to preclude arbitrage opportunities in the bond market of country i.

Joslin et al. (2011) showed that a rotation from latent factors $X_{i,t}$ to portfolios of yields, the spanned factors $P_{i,t}$, leads to an observationally equivalent term structure representation. This invariant transformation assumes that N portfolios of yields are perfectly priced and observed without errors, while the remaining $J - N$ portfolios are priced and observed imper-

fectly. Specifically, the spanned factors are computed as $P_{i,t} = V_i Y_{i,t}$, where V_i is a full-rank matrix. Accordingly, $Y_{i,t}$ is an affine function of $P_{i,t}$ as follows:

$$Y_{i,t} = A_P(\delta_{i,0}, \lambda_i^Q, \Sigma_{i,X}, V_i) + B_P'(\lambda_i^Q, V_i) P_{i,t}. \tag{4}$$

2.1.2. State dynamics of the term structure

The model state dynamics incorporate N spanned factors, M financial-economic variables, the unspanned factors $M_{i,t}$. Formally, the model state vector, $Z_{i,t}$, is formed as $Z_{i,t} = [M_{i,t}', P_{i,t}']'$, and it follows a standard unrestricted Gaussian $VAR(1)$ under the physical measure (the P-measure)

$$Z_{i,t} = C_i^{\mathbb{P}} + \Phi_i^{\mathbb{P}} Z_{i,t-1} + \Sigma_i \varepsilon_{Z,t}^{\mathbb{P}}, \qquad \varepsilon_{Z,t}^{\mathbb{P}} \sim \mathcal{N}(0, I_K), \tag{5}$$

where $K = M + N$ is simply the dimension of $Z_{i,t}$.

It is worth specifying the role played by unspanned factors in term structure developments. Although unspanned factors do not directly appear as an element of the pricing setting, they impinge on the dynamics of the spanned factors and ultimately affect bond yields through equation (4).

2.2. $JLL - ATSM$ state dynamics

In $JLL - ATSM$, the economic system comprises the global economy, one worldwide large (dominant) economy, and another set of smaller economies. We denote by C the number of countries in this system, and we index the dominant country by D.

2.2.1. Risk factor construction

$JLL - ATSM$ resorts to a series of projections to build the risk factors present in the state dynamics. This approach aims at forming domestic variables, the dynamics of which are trimmed from the influence of either the other countries, the global economy, or both.

The domestic spanned factors are built in two steps. First, for all countries of the economic system, $P_{i,t}$ is projected on $M_{i,t}$ in this same country. Second, for the nondominant economies, the residuals of the regressions obtained in the first stage are further projected onto the orthogonalized spanned factors of the dominant country. Stated simply, we have

$$P_{i,t} = b_i M_{i,t} + P^e_{i,t}, \tag{6}$$

$$P^e_{i,t} = c^D_i P^e_{D,t} + P^{e*}_{i,t}, \tag{7}$$

where $P^e_{i,t}$ ($P^{e*}_{i,t}$) corresponds to the (non-)dominant country i time series residuals.

The composition of the unspanned factors pursues a logic similar to that of the spanned factors. For the dominant economy, $M_{D,t}$ is projected onto the G global economic factors, M^W_t, and for the other economies, the residuals of the first regression are used in the orthogonalization process of the nondominant economies

$$M_{D,t} = a^W_D M^W_t + M^e_{D,t}, \tag{8}$$

$$M_{i,t} = a^W_i M^W_t + a^D_i M^e_{D,t} + M^{e*}_{i,t}. \tag{9}$$

2.2.2. Risk factor dynamics and model restrictions

The risk factors of the economic system comprise G global economic factors and K country-specific (including both spanned and unspanned) factors for each of the C countries. Accordingly, the total number of variables of this cohort total to $F = KC + G$.

The risk factors of interest include orthogonalized spanned and unspanned factors. More specifically, the state vector is formed by $Z^e_t = (M^{W'}_t, M^{e'}_{D,t}, P^{e'}_{D,t}, M^{e*'}_{2,t}, P^{e*'}_{2,t} ... M^{e*'}_{C,t}, P^{e*'}_{C,t})'$, and its dynamics under the P-measure evolve as a $VAR(1)$

$$Z^e_t = C^e_Y + \Phi^e_Y Z^e_{t-1} + \Sigma^e_Y \varepsilon^e_{Z,t}, \qquad \varepsilon^e_{Z,t} \sim \mathcal{N}(0, I_F). \tag{10}$$

The feedback matrix, Φ^e_Y, and the variance-covariance matrix, $\Sigma^e_Y \Sigma^{e'}_Y$, are formed by

$(KC + G)^2$ entries each. It is therefore clear that the inclusion of supplementary countries and/or domestic factors can dramatically increase the number of model parameters. To mitigate the overparametrization issue, Jotikasthira et al. (2015) imposed a set of zero-restrictions on the feedback matrix, Φ_Y^e, and on the Cholesky factor of the variance-covariance, Σ_Y^e. We briefly comment below on the features of these two matrices.[9]

The structure of Φ_Y^e allows the global and dominant economies to directly feed back to each other, and both also directly impact the dynamics of the other economies. In contrast, shocks from smaller economies are assumed not to spread across borders by any means. Moreover, within each country, spanned and unspanned factors can freely respond to each other.

For the matrix Σ_Y^e, no cross-country shock correlations are possible, but the correlations are possible from the dominant country to the smaller economies and from the world to the rest of the system. Furthermore, the shocks across groups of spanned and unspanned factors of the same economy are bound to be uncorrelated.

2.3. Limitations of JPS-ATSM and JLL-ATSM

A well-known shortcoming of VAR models is that the dimensionality of the system can largely increase with the inclusion of a few additional factors. The relevance of this drawback is amplified in the context of $ATSM$s estimated along the lines of Joslin et al. (2014). The reason is that the estimation process of the variance-covariance matrix requires the use of numerical optimization techniques (see Section 3.3).

To illustrate the required estimation computational burden of several $ATSM$s, we present in Table 1 the number of parameters in the variance-covariance matrix that require estimation for specifications containing different numbers of countries. The entries of this same table are computed for models featuring two global unspanned factors ($G = 2$), two domestic unspanned factors ($M = 2$), and three domestic spanned factors ($N = 3$). The label $JPS-$

[9]See Appendix A for a more detailed design of Φ_Y^e and Σ_Y^e.

$ATSM$ refers to a model in which the risk factor dynamics are completely unrestricted, whereas $JLL - ATSM$ contemplates a restricted VAR along the lines previously described. The $GVAR - ATSM$ is our setup detailed in Section 3.

Table 1: Number of estimated parameters in the variance-covariance matrix

	Number of countries			
Model label	3	4	5	10
JPS-ATSM	153	253	378	1378
JLL-ATSM	81	133	198	835
GVAR-ATSM	48	63	78	153

Note: The entries in the table are computed for an economic system containing two global unspanned factors ($G = 2$), two domestic unspanned factors ($M = 2$), and three domestic spanned factors ($N = 3$).

It is possible to verify that $JPS-ATSM$ and $JLL-ATSM$ quickly become intractable as the number of countries of the economic system increases.[10] The construction of the variance-covariance of a five-country model requires the (numerical) estimation of 378 parameters in the $JPS - ATSM$ and almost 200 in the $JLL - ATSM$, more than double of the $GVAR - ATSM$. It is also worth noting that a ten-country $GVAR - ATSM$ depends upon the estimation of as many parameters as a $JPS - ATSM$ formed by three economies.

In addition to building on a heavily parameterized VAR system, we consider that $JLL - ATSM$'s set of restrictions imposed in this same VAR are open to criticism in two further dimensions. First, the authors resort to Cholesky decomposition as the identification strategy of the structural shocks of the model. This approach has been widely criticized because of the dependence of the results on the ordering of the countries in the VAR (see, e.g., Bikbov and Chernov (2010)). Moreover, this choice implies the unrealistic assumption that all of the variables of the dominant economy are more exogenous than any other factor from the other economies in the system. Second, the structure of matrices Φ_Y^e and Σ_Y^e fully disrupts the

[10] We further illustrate the degree of complexity of these models by presenting the required estimation time for a four-country system in Section 4.2.

transmission channel of cross-border shocks among nondominant economies – an undesirable feature of models designed to study multicountry specifications.

3. Multicountry $ATSM$ with unspanned economic risk and a $GVAR$ ($GVAR-ATSM$)

We divide the exposition of our multicountry $GVAR-ATSM$ into four sections. In Section 3.1, we introduce the joint yield curve cross-sections, whereas in Section 3.2, we display the $GVAR$ risk factor dynamics. In Section 3.3, we describe the model estimation procedure of our entire $GVAR-ATSM$. Finally, in Section 3.4, we highlight the $GVAR-ATSM$ sources of computational efficiency gain compared to the other $ATSM$s.

3.1. The cross-section of the multicountry yield curves

The cross-sectional dimension of our multicountry $ATSM$ follows the same structure present in $JLL-ATSM$. Specifically, we encompass the sets of country-specific yields, spanned factors, and intercepts from equation (4) into, respectively, $Y_t = [Y'_{1,t}, Y'_{2,t}, ..., Y'_{C,t}]'$, $P_t = [P'_{1,t}, P'_{2,t}, ..., P'_{C,t}]'$, and $A_P(\Theta) = [A'_P(\Theta_1), A'_P(\Theta_2), ..., A'_P(\Theta_C)]'$, where $\Theta_i = \{\delta_{i,0}, \lambda_i^Q, \Sigma_{i,X}, V_i\}$ for each country i. Furthermore, we assume that the spanned factors of one country have no contemporaneous effect on the term structure of any other country. In this sense, $B_P(\Theta)$ is block diagonal such that $B_P(\Theta) = diag(B'_P(\Theta_1), B'_P(\Theta_2), ..., B'_P(\Theta_C))$. Therefore, we represent the multicountry term structure for all C countries as

$$Y_t = A_P(\Theta) + B_P(\Theta)P_t. \tag{11}$$

3.2. The $GVAR$ model

3.2.1. The $VARX^*$ and the marginal model

The $VARX^*$ builds on standard small-scale local VAR models augmented by foreign-specific variables, the *star* variables. These variables are a weakly exogenous weighted average of foreign variables. Formally, the star variables from country i are formed into $Z^{*'}_{i,t} = WZ'_{j,t}$,

where each entry of W, $w_{i,j}$, measures the degree of interdependence of country i with country j.[11] In this setting, the choice of W is crucial since it determines the transmission channels among countries.

In addition to $Z_{i,t}^{*'}$, the $VARX^*$ model for country i includes the K domestic risk factors and the G unspanned global factors. We assume that $Z_{i,t}$ follows a $VARX^*(1,1,1)$ under the P-measure of the form

$$Z_{i,t} = C_i^X + \Phi_i^X Z_{i,t-1} + \Phi_i^{X^*} Z_{i,t-1}^* + \Phi_i^{X^W} M_{t-1}^W + \Sigma_i^X \varepsilon_{i,t}^X, \qquad \varepsilon_{i,t}^X \sim \mathcal{N}(0, I_K). \tag{12}$$

Without loss of generality, we do not include contemporaneous terms for ease of exposition. The marginal model describes the joint dynamics of the global factors. We consider that M_t^W evolves according to a standard $VAR(1)$

$$M_t^W = C^W + \Phi^W M_{t-1}^W + \Sigma^W \varepsilon_t^W, \qquad \varepsilon_t^W \sim \mathcal{N}(0, I_G). \tag{13}$$

In this arrangement, the dynamics of the global variables are limited to their own developments. To more closely match $JLL - ATSM$, this current setting can be extended to accommodate the influence of a dominant country in the world economy. One possible way to pursue this implementation is to incorporate the risk factors from the dominant country into the system (13).[12]

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[11] The $w_{i,j}$s are normalized by $\sum_{j=1}^{C} w_{i,j} = 1$ with w_{ii}=0 for all i.

[12] In the marginal model, any number of dominant countries can be included simultaneously. Clearly, the decision about the set of relevant dominant economies should be motivated by the specific matter under investigation.

3.2.2. The GVAR representation

Solving the $GVAR$ model for the system as a whole requires the construction of country-specific link matrices, W_i. To this end, we collect country i's domestic and foreign variables in a $2K-$dimension vector $d_{i,t}$, $d_{i,t} = [Z'_{i,t},\ Z^{*'}_{i,t}]'$ and all endogenous country-specific variables of the system from the $VARX^{*}$'s in a $KC-$dimension vector x_t, $x_t = [Z'_{1,t},\ Z'_{2,t}, \dots Z'_{C,t}]'$. Based on these inputs, W_i satisfies the identity $d_{i,t} \equiv W_i x_t$.

The state vector contains both the global economic variables and the country-specific risk factors as follows $Z_t = [M_t^{W'},\ Z'_{1,t},\ Z'_{2,t}, \dots Z'_{C,t}]'$. In our setup, Z_t follows a $GVAR(1)$ under the physical measure as:

$$Z_t = C_y + \Phi_y Z_{t-1} + \Sigma_y \varepsilon_{y,t}, \qquad \varepsilon_{y,t} \sim \mathcal{N}(0, I_F), \tag{14}$$

where $C_y = [C^W,\ C_1^X,\ C_2^X, \dots C_C^X]$, $\varepsilon_{y,t} = [\varepsilon_t^{W'},\ \varepsilon'_{1,t},\ \varepsilon'_{2,t}\ \dots\ \varepsilon'_{C,t}]'$ and $\Sigma_y = diag(\Sigma^W,\ \Sigma_1^X,\ \Sigma_2^X\ \dots\ \Sigma_C^X)$. The matrix Φ_y is a composite of the feedback matrices from both the marginal and the $VARX^{*'}$s models of all countries, more specifically, $\Phi_y = \begin{bmatrix} I_G & 0_{G\times CK} \\ \Phi^{X^W} & G_1 \end{bmatrix}$, where $\Phi^{X^W} = [\Phi_1^{X^W},\ \Phi_2^{X^W},\ \dots\ \Phi_C^{X^W}]'$, and $G_1 = [\Phi_1 W_1,\ \Phi_2 W_2, \dots\ \Phi_C W_C]'$ for $\Phi_i = [\Phi_i^X,\ \Phi_i^{X^*}]$ and $i = 1, 2, \dots C$.

3.3. Estimation procedure

Our estimation strategy builds on the single-country methodology of $JPS - ATSM$. This approach offers some important advantages compared to earlier reference macrofinance term structure models, such as that of Ang and Piazzesi (2003). In particular, the framework of Joslin et al. (2014) allows for a simplified estimation procedure that, nevertheless, ensures efficient parameter estimates. The reason is that the $JPS - ATSM$ setting enables a convenient split of the likelihood function between the parameters governing the P- and Q-measures. As a result, the estimation of the physical and risk-neutral parameters can be performed rather independently, largely reducing the required computational burden during

the estimation process. Our estimation approach, hence, follows a two-step procedure.

In the first step, we obtain the parameters of the $GVAR$ model presented in equation (14). For this purpose, we use ordinary least squares to separately estimate each country's $VARX^*(1,1,1)$ and the marginal model. Subsequently, we use the estimates of C_i^X, Φ_i^X, $\Phi_i^{X^*}$, Φ_i^W, C^W, Φ^W, and Σ^W to build the $GVAR$ parameters C_y and Φ_y.

In the second step, we use maximum likelihood to convene the country-specific risk-neutral parameters $\Phi_{i,X}^Q$, $\delta_{i,0}$, and $\Sigma_{i,X}$, present in $A_P(\Theta)$ and $B_P(\Theta)$ of equation (11).[13] Joslin et al. (2014) showed that the estimation of equation (5) by least squares generates variance-covariance estimates close to the global optimum. As such, on the basis of the estimates from the first estimation step, we employ the $N \times N$ matrices at the bottom right of Σ_i^X as the starting values of $\Sigma_{i,X}$ in the second stage.[14] Due to the absence of closed-form expressions, $\Phi_{i,X}^Q$ and $\Sigma_{i,X}$ must be evaluated numerically.

3.4. $GVAR$: source of computational advantages

The source of computational advantage from the $GVAR - ATSM$ results from the dimension of a $GVAR$ being invariant to the number of countries present in the cohort. More specifically, $GVAR$ is built from small-scale setups (the individual countries $VARX^*$s and the marginal models); hence, its dimension depends exclusively on the quantity of global and each country's risk factors, i.e., $K + G$. In contrast, in $JLL - ATSM$, the full dimension of the system is the length state vector, $F = CK + G$. In this sense, it is clear that the $GVAR - ATSM$ bears substantial estimation efficiency gains over the $JLL - ATSM$ once the number of countries of the system becomes large, as we illustrated in Table 1.

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[13]The $JPS - ATSM$ setting also requires the estimation of the variance of the errors from the country-specific $J - N$ portfolio of yields observed with errors. Like Jotikasthira et al. (2015), we assume that these parameters are equal at the country level, but they can differ across countries.

[14]Note that, by definition, the bottom right $N \times N$ part of Σ_i^X is equivalent to $\Sigma_{i,X}$.

4. $GVAR - ATSM$: an empirical illustration for Latin American countries

We study an economic system consisting of China and three Latin American economies: Brazil, Mexico, and Uruguay. For the estimation of $JLL - ATSM$, we choose China as the dominant economy due to its recent prominent economic role in this region.[15] For the sake of generality, we opt to assign no dominant country for the $GVAR - ATSM$.

We choose the country-specific state vector ($Z_{i,t}$) to contain three spanned ($N = 3$) and two unspanned domestic factors ($M = 2$). The spanned factors can thus be interpreted as level ($L_{i,t}$), slope ($S_{i,t}$), and curvature ($C_{i,t}$).[16] As unspanned factors, we include the domestic rates of inflation ($\pi_{i,t}$) and of economic activity growth ($g_{i,t}$), in addition to their global counterparts, π_t^W and g_t^W. Therefore, the complete state vector Z_t is formed by $Z_t = [M_t^{W'}, M_{1,t}', P_{1,t}', M_{2,t}', P_{2,t}' \ldots M_{C,t}', P_{C,t}']'$ such that $M_t^W = [g_t^{W'}, \pi_t^{W'}]'$, $M_{i,t} = [g_{i,t}', \pi_{i,t}']'$ and $P_{i,t} = [L_{i,t}', S_{i,t}', C_{i,t}']'$. For $JLL - ATSM$, the variables with the index $i = 1$ represent those of the dominant economy.

It is worth noting that the use of this dataset is particularly convenient for the estimation of term structure models based on the framework of Joslin et al. (2014). In fact, it is widely documented that the performance of standard $ATSM$s in terms of model fit and forecasting is undermined when nominal short-term rates are constrained to the zero level (see, e.g., Christensen and Rudebusch (2016)). To illustrate the adequacy of our data to the proposed modeling framework, we show, in Figure 1, that each country's short-term interest rates were far from zero throughout the period under study.

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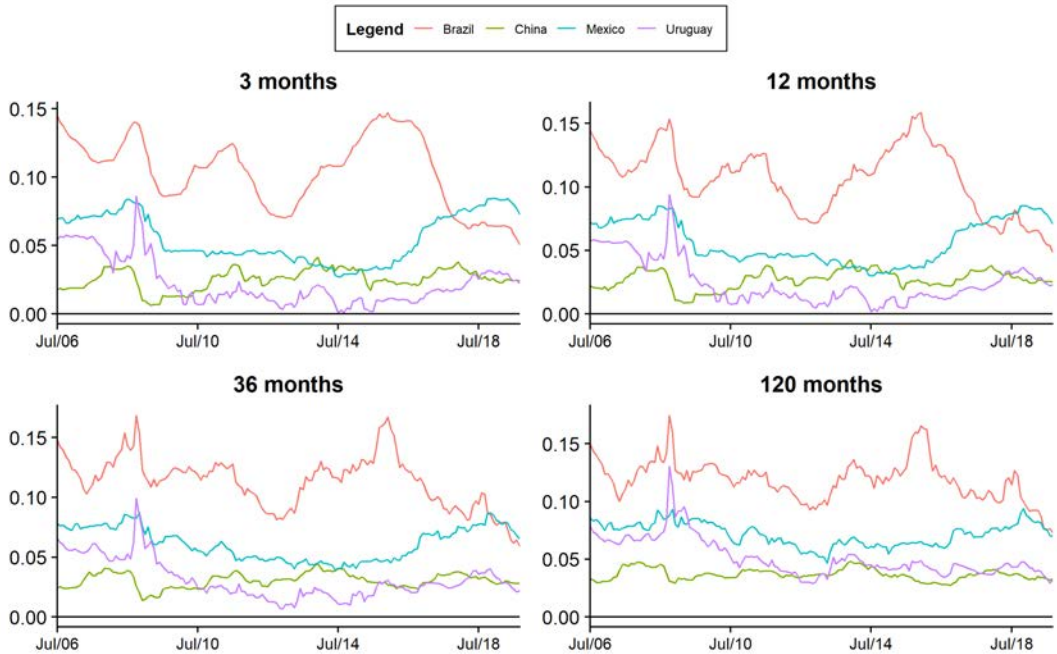
[15] In comparison to the U.S. economy, China currently enjoys more preponderant bilateral trade relations with two of the three Latin American economies in our sample. By way of illustration, according to the DTS-IMF database, in 2019, China's (U.S. 's) exports and imports accounted for approximately 10% (62%), 25% (15%), and 22% (8%) of the total trade of Mexico, Brazil, and Uruguay, respectively.

[16] See, for instance, Litterman and Scheinkman (1991).

Figure 1: Time series of yields



Note: The graphs show the monthly historical evolution of domestic yields corresponding to maturities of 3, 12, 36 and 120 months. Rates are annualized. The sample period is from July 2006 to September 2019.

4.1. Data

We use monthly data ranging from July 2006 to September 2019. Our dataset contains four categories of series: *(i)* global economic indicators; *(ii)* domestic economic indices; *(iii)* country-specific term structure of interest rates; and *(iv)* bilateral trade flows.

Our measure of global inflation is the year-over-year variation in the Organisation for Economic Cooperation and Development ($OECD$) aggregated Consumer Price Index (CPI) containing all of its member countries. As a proxy variable of worldwide economic growth, we use Lutz Kilian's index of global real economic activity, available at the author's webpage.[17]

At the country level, we retrieve the country-specific CPIs from the IMF to build the domestic inflation factors. For the composition of the economic growth variables, we collect

[17]See Kilian (2009) and Kilian and Zhou (2018) for a detailed methodological explanation about the compilation of this index.

the monthly GDP leading indicator compiled by the OECD and available at the FRED.[18] To construct all domestic unspanned factors, $\pi_{i,t}$ and $g_{i,t}$, we calculate the year-on-year changes of the respective indicator.

The term structure of each country comprises end-of-month zero-coupon yields from maturities of 3, 6, 12, 36, 60 and 120 months. To broaden the sample span, we adopt different sorts of datasets. For China, Mexico, and Uruguay, we use government bond yields. For the two former countries, data are denominated in domestic currency and are collected from Bloomberg, whereas for the latter, the yields are U.S. dollars denominated and are retrieved from the Uruguayan stock exchange (BEVSA). For Brazil, we collect swap fixed-DI contracts[19], quoted in domestic currency from Brazil's Stock Exchange and Over-the-Counter Market ($B3$).

We build the transition matrix, W, from the trade flows between each pairwise combination of countries in the sample. Specifically, we define the strength of interdependence of country i with respect to country j, w_{ij}, as $w_{ij} = \frac{I_{ij}+E_{ij}}{\sum_{j=1}^{C} I_{ij}+E_{ij}}$, where I_{ij} (E_{ij}) corresponds to the value of goods imports (exports) from economy i to j. Further, to compute I_{ij} and E_{ij}, we total over the entire sample period the yearly series of bilateral trade flows released by the IMF.[20] Table 2 reports the complete transition matrix.

Figure 2 represents the time series of all of the built-up risk factors used in our model. This group of variables includes the global economic factors and country-specific spanned (level, slope, and curvature) and unspanned (inflation and economic growth) factors.

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[18]The latest index is not available for Uruguay. In such cases, we construct a monthly time series from the cubic interpolation of quarterly GDP data, to which we apply the Hodrick-Prescott filter to remove the GDP trend (see Hodrick and Prescott (1997)).

[19]Swap fixed-DI contracts are derivative securities indexed to the overnight rate of Brazil's interbank loan market.

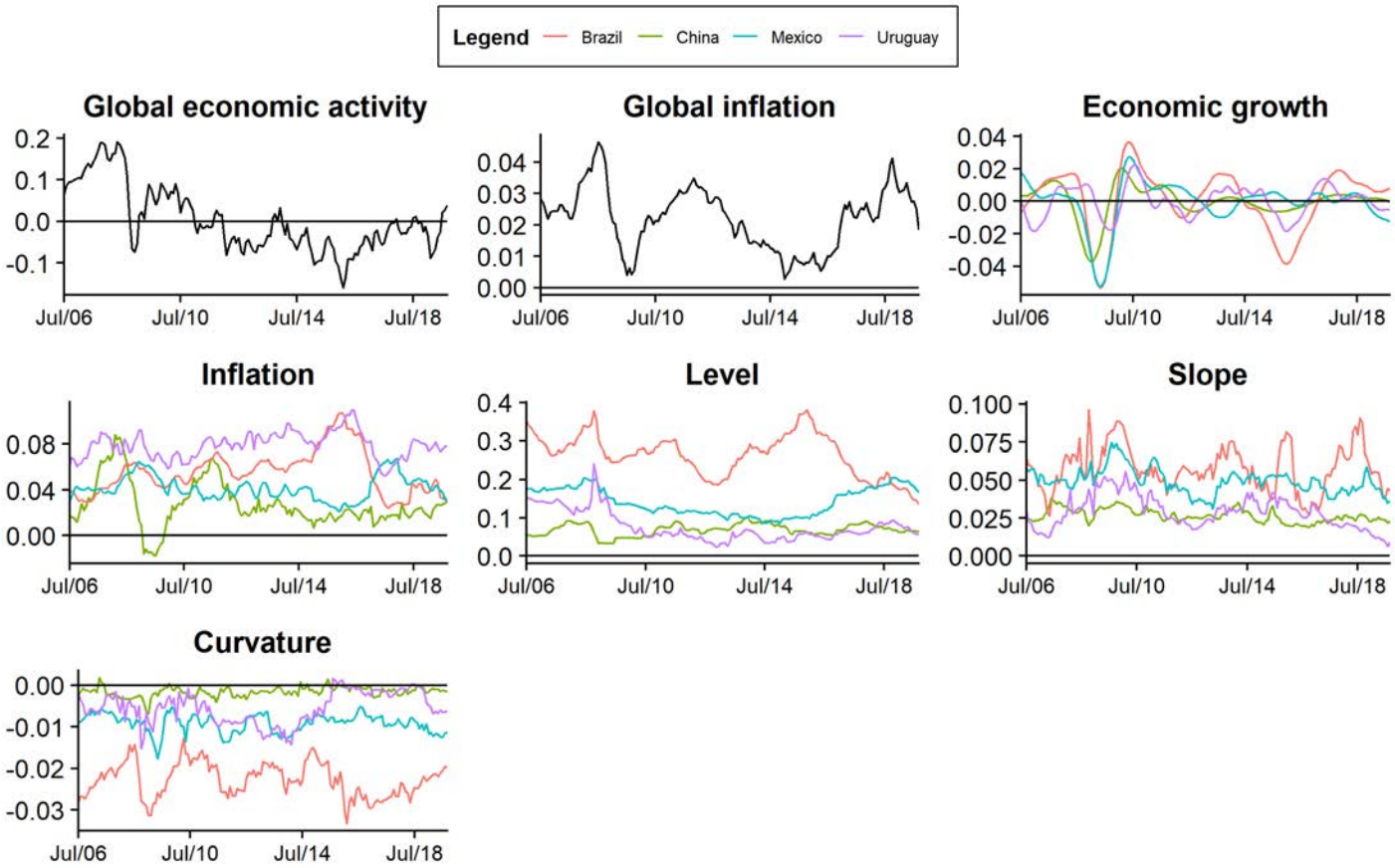
[20]The values of goods, imports and exports are free on board in U.S. dollars.

Table 2: Transition matrix (W)

	China	Brazil	Mexico	Uruguay
China	-	0.655	0.316	0.030
Brazil	0.827	-	0.123	0.050
Mexico	0.860	0.133	-	0.008
Uruguay	0.381	0.550	0.069	-

Note: Each entry corresponds to the share of the bilateral trade flows between any two countries over the total amount of one country's trade flows with the other economies in the system. Trade flows are computed as the yearly sum of exports and imports from the period spanning from 2006 to 2019. Trade flow values are free on board and quoted in U.S. dollars. All rows total one.

Figure 2: Time series of risk factors



Note: The graphs show the historical evolution of the spanned and unspanned factors. Global economic activity is the index compiled by Kilian (2009). Global inflation is the year-over-year variation of the *OECD*'s aggregated CPI containing all of its member countries. Domestic inflation and economic growth are, respectively, the year-over-year variation of each country's CPI from the IMF and the GDP leading indicator from the *OECD*. For Uruguay, economic growth is the yearly growth rate of a monthly index built using cubic interpolation of the quarterly GDP and the Hodrick-Prescott filter. Level, slope, and curvature are the first three principal components of each country's yield set. The sample period is from July 2006 to September 2019.

4.2. Empirical results

4.2.1. Estimation time

In the preliminary investigation presented in Table 2, $GVAR-ATSM$ requires the estimation of fewer model parameters than the other frameworks. Accordingly, it is reasonable to expect that the estimation time of our framework is shortened. To illustrate this fact, in Table 3, we present a comparison of the necessary estimation burden for several $ATSM$s. In Appendix A, we summarize some general features of each of the models in Table A2.

In Table 3, all of the listed setups incorporate the complete set of risk factors, $F = 22$ (five domestic factors for each of the four economies and the two global ones). Columns λ^Q and Σ present the overall number of eigenvalues and the distinct entries of the variance-covariance matrix estimated under each representation, respectively. Each model is estimated 5 times using a standard computer.[21] The estimation time intervals correspond to the minimum and maximum numbers of minutes necessary to estimate each of these models.

In this same table, we construct two versions of the $JPS-ATSM$. In the variant labeled $JPS-ATSM$, the parameters governing the Q-dynamics are obtained for each of the four countries separately. In the other version, designated $JPS-ATSM$ *adapted*, we accommodate the estimates of these same parameters for all of the economies jointly so that we can measure the time savings in the estimation.

In the original $JLL-ATSM$ form, the elements of the Σ matrix are exclusively computed alongside the other parameters that are part of the $P-$dynamics. This procedure greatly reduces the computational estimation time since Σ does not undergo the numerical optimization process from the risk-neutral dynamics part of the likelihood function.[22] As such, for the purpose of rendering the computational time comparable across the other classes of

[21]The processor used for estimation is an Intel(R) Core(TM) $i5-8365U$ CPU cadenced at 1.6 GHz.

[22]Jotikasthira et al. (2015) argued that, although this procedure is not fully efficient, it has little impact on the authors' main empirical findings.

Table 3: Time required for the model estimation

Model label	F	λ^Q	Σ	$\lambda^Q + \Sigma$	Required estimation time (interval, in minutes)
JPS-ATSM	22	3	253	256	14.5 - 15
JPS-ATSM adapted	22	12	253	265	10.5 - 12
GVAR-ATSM	22	12	63	75	4 - 4.5
JLL-ATSM	22	12	133	145	3.8 - 4.2
JLL-ATSM adapted	22	12	133	145	20 - 22

Note: F represents the dimension (overall sum of domestic and foreign factors) of the system. λ^Q corresponds to the number of eigenvalues governing the Q-dynamics. For the $JPS-ATSM$, λ^Q is the number of country-specific eigenvalues, whereas for the other models λ^Q is the overall sum of eigenvalues from all countries in the economic system. Σ denotes the required number of parameter estimates for the variance-covariance matrix. The model label $JLL-ATSM$ *adapted* refers to a model in which Σ from $JLL-ATSM$ is estimated in the two steps described in Section 3.3. All models are estimated for four countries (China, Brazil, Mexico, and Uruguay), two global unspanned factors (economic growth, and inflation), two domestic unspanned factors (economic growth, and inflation), and three domestic spanned factors (level, slope, and curvature). The sample covers the period from July 2006 to September 2019. Each model is estimated 5 times. The bounds of the required estimation time intervals correspond to the minimum and maximum numbers of minutes necessary to estimate each one of the models. A personal computer with a processor Intel(R) Core(TM) $i5-8365U$ CPU cadenced at 1.6 GHz is used for the estimations.

models, we build an alternative version of the $ATSM-JLL$ in which the maximization of the variance-covariance matrix is part of the two estimation steps described in Section 3.3. We assign the label $JLL-ATSM$ *adapted* to this variant of the model,[23] whereas the model identified simply by $JLL-ATSM$ follows the same estimation procedure as in the original paper.

This experiment shows a significant gain in terms of time of the $GVAR-ATSM$ compared to the selected alternative specifications. Interestingly, the estimation time is shortened by approximately a factor of 3 to 5 times relative to the adapted multicountry $JPS-ATSM$ and $JLL-ATSM$ counterparts. Notably, the time-saving benefit is further amplified once statistical inference is performed through simulation methods in which use multiple model estimations, such as bootstrapping or Monte Carlo. Furthermore, it is worth emphasizing that the necessary estimation times for both $GVAR-ATSM$ and $JLL-ATSM$ are reasonably

[23]In the general $JLL-ATSM$, the estimation of Σ under the physical dynamics also requires the use of numerical optimization techniques (see Appendix C). For this estimation step, we interrupt the optimization of Σ once the absolute tolerance on the function value can no longer be improved by 0.01.

similar, although the latter model incorporates a substantially greater number of estimated parameters.

4.2.2. Fit of the models

In Table 4, we present the general goodness of fit from both $GVAR-ATSM$ and $JLL-ATSM$. For each model, we assess the first two moments (mean and standard deviation) according to two measures of model accuracy. First, we compute the fitted model that results exclusively from the spanned factors, as portrayed in equation (11). Accordingly, this method provides the fit from the parameters governing the Q-dynamics. Second, we build the fit of the overall models that include the dynamics of both physical and risk-neutral parameters. More specifically, this measure is jointly formed from equation (11) and the state dynamics of each $ATSM$. We illustrate both measures of goodness of fit for the short (3 months) and long (120 months) ends of the maturity spectrum for the four countries in the sample.[24]

From Table 4, we observe that $GVAR-ATSM$ and $JLL-ATSM$ closely match the actual data. In terms of volatility, $JLL-ATSM$ underestimates the true data realizations in some cases under the model-implied form. Nevertheless, these differences are certainly economically modest.

[24]Further, in Appendix B, we report the fitted time series of the $GVAR-ATSM$ and the $JLL-ATSM$ for these same bond yield maturities.

Table 4: Goodness of fit

	China		Brazil		Mexico		Uruguay	
	3 months	120 months	3 months	120 months	3 months	120 months	3 months	120 months
Mean								
Data	0.025	0.037	0.104	0.119	0.055	0.072	0.023	0.052
Fitted $GVAR$	0.026	0.037	0.104	0.119	0.055	0.072	0.023	0.052
Fitted JLL	0.026	0.037	0.104	0.119	0.055	0.072	0.023	0.052
Model-implied $GVAR$	0.026	0.037	0.104	0.119	0.055	0.072	0.022	0.051
Model-implied JLL	0.026	0.037	0.104	0.119	0.055	0.072	0.022	0.051
Standard deviation								
Data	0.008	0.005	0.026	0.018	0.018	0.010	0.017	0.017
Fitted $GVAR$	0.008	0.005	0.026	0.018	0.018	0.010	0.017	0.017
Fitted JLL	0.008	0.005	0.026	0.018	0.018	0.010	0.017	0.017
Model-implied $GVAR$	0.007	0.005	0.026	0.016	0.018	0.009	0.017	0.016
Model-implied JLL	0.007	0.005	0.025	0.014	0.018	0.009	0.016	0.015

Note: The table compares the first two moments (means and standard deviations) of the actual data and the model-fitted bond yields. Fitted values are computed based exclusively on the risk-neutral parameters, whereas the model-implied statistics are estimated including the parameters governing both the P- and Q-dynamics. The labels $GVAR$ and JLL refer, respectively, to the $GVAR - ATSM$ and $JLL - ATSM$. The entries in the tables are expressed in per annum amounts and are computed for the period spanning from July 2006 and September 2019.

4.3. Out-of-sample forecasting performance

We now investigate whether our less parameterized framework can render more accurate bond yield forecasts than the $JLL - ATSM$. Our general metrics of forecasting performance are the root-mean-square error ($RMSE$) and the standard deviation of the forecast errors ($STD\ FE$).

We report the model forecast in Table 5. To generate the properties of the out-of-sample forecast, we proceed as follows. First, we estimate each of the $GVAR - ATSM$ and $JLL - ATSM$ conditional on a restricted information set that spans the period from July-2006 to December-2013. Next, for each country, we produce the bond yield forecasts for all six observable maturities at forecast horizons varying between 1 and 12 periods ahead. Finally, we compare these predictions with the actual observed values to obtain the forecast errors.

We repeat this procedure iteratively, including the observation of the next month.[25] Once this process is completed, we compute, for each country, the average $RMSE$ and the $STD\ FE$ across all observable maturities and forecast horizons. These measures are the summary measures reported in Table 5. Furthermore, for all 72 country-specific forecasting outputs (12 forecasting horizons for each of the six observed yields), we measure the percentage of cases in which the $GVAR - ATSM$ displays smaller $RMSE$ and $STD\ FE$ than $JLL - ATSM$. The outcomes of these calculations are presented in the column labeled $\frac{GVAR}{JLL} < 1$.

An overall analysis of Table 5 highlights that both $ATSM$s capture more accurately the future bond yield fluctuations in China and least precisely in Brazil. It is also obvious that $GVAR - ATSM$ exhibits better predictive capabilities than $JLL - ATSM$ in terms of the first two moments of the forecasting errors: among all 288 cases under investigation, $GVAR - ATSM$ produces more accurate forecasts in approximately 74% of the cases and less volatile forecast errors in approximately 70% of them. The comparative performance of the $GVAR - ATSM$ is best for Mexico and Brazil, where the favorable ratios of both forecasting indicators are on the order of 100% of the cases for the first country and of 80% for the last country. It is therefore clear that $GVAR - ATSM$ provides superior out-of-sample yield curve predictions.

[25] To ensure that the global number of forecast errors is identical across all forecast horizons, the last array of yield forecasts is generated for the information set containing data until September 18.

Table 5: Bond yields out-of-sample forecast

	RMSE			STD FE		
	GVAR	JLL	$\frac{GVAR}{JLL} < 1$	GVAR	JLL	$\frac{GVAR}{JLL} < 1$
China	0.006	0.006	0.569	0.006	0.006	0.319
Brazil	0.018	0.021	0.792	0.018	0.021	0.792
Mexico	0.009	0.013	0.972	0.007	0.013	1.000
Uruguay	0.009	0.011	0.639	0.009	0.011	0.611
Overall	0.010	0.013	0.743	0.010	0.013	0.681

Note: The table presents the out-of-sample root mean square errors ($RMSE$) and the standard deviation of the forecast errors ($STD\ FE$) for the six observed country-specific bond yields. The forecast horizons range from 1 to 12 months. The information set of the first (last) forecasts contains data from July/2006 to December/2013 (September/2018). The columns $GVAR$ and JLL report the simple average of all of these forecasts for the $GVAR - ATSM$ and $JLL - ATSM$, respectively. The column $\frac{GVAR}{JLL} < 1$ shows the proportion of cases for which the $RMSE$ and the $STD\ FE$ are smaller for the $GVAR - ATSM$.

4.3.1. GIRFs and GFEDVs analyses

We qualitatively assess our $GVAR - ATSM$ based on $GIRF$s and $GFEDV$s.[26] In contrast to the Cholesky identification restrictions employed in $JLL - ATSM$, $GIRF$s and $GFEDV$s provide empirical results that are invariable to the ordering of the variables (see, e.g., Pesaran and Shin (1998)). This feature of $GIRF$s and $GFEDV$s is imperative in a fairly large system like the one under study, in which the risk factors can be ordered in more than 10^{21} alternative forms.

The current setting enables the appraisal of 22 risk factor shocks. Accordingly, we are able to address a rich set of factor responses to myriad domestic and foreign shocks. To save space, we focus on the $GIRF$s of the term structures of Brazil, Mexico, and Uruguay to an

[26] We present in Appendix D the analytical derivations of the $GIRF$s and $GFEDV$s.

economic growth shock in China.[27] We display these results in Figure 3.

From Figure 3, we observe that a shock to Chinese output growth has similar impacts on the bond yields of Brazil and Mexico. Specifically, we observe that all yields shift upwards and that short maturities are more responsive than long maturities, implying a flattening of the yield curves of these countries. This effect could result from active monetary policy responses to inflationary pressures arising from a positive foreign demand shock.

The impact of the Chinese economic growth shock on the Uruguayan term structure has a distinct pattern: the long end of the yield curve drops less than the short end, resulting in a steeper yield curve. Since Uruguay yields data are denominated in U.S. dollars, it is less likely that they reflect the reaction of the monetary authority to demand shocks. Rather, U.S. dollar inflows might push bond yields downward through the risk compensation channel in two dimensions. First, since Uruguay is a small economy, modest amounts of capital inflows might substantially improve the capability of the government to commit to its debt repayment, leading to a reduction in default risk (Reinhart et al. (2003)). Second, because Uruguay's bond market is fairly illiquid,[28] financial investors might require a smaller liquidity premium to hold Uruguayan bonds when the country experiences inflows of U.S. dollars.

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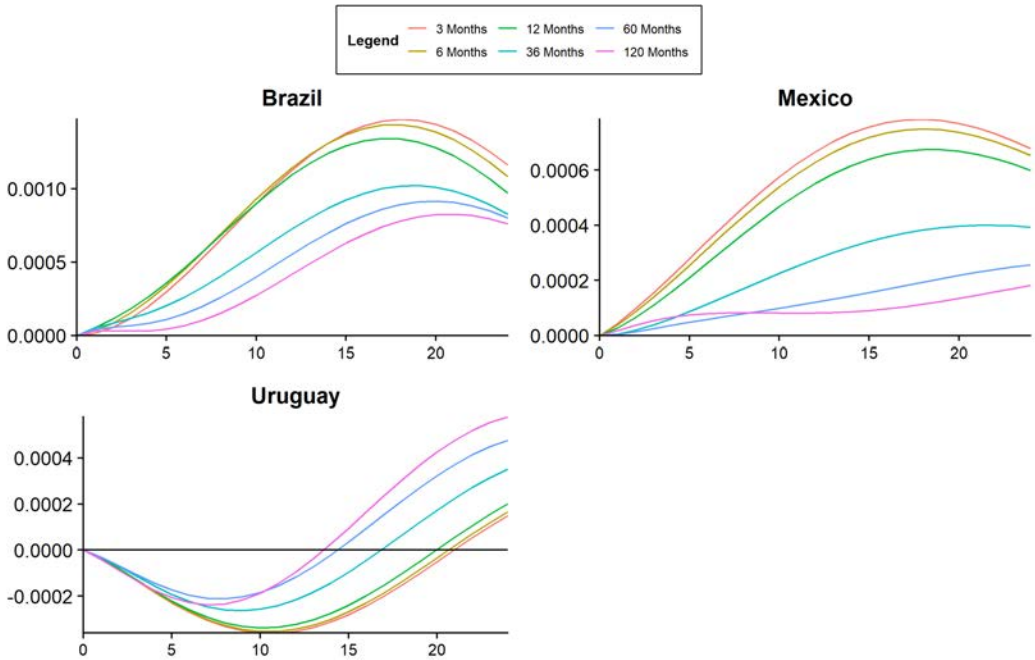
[27] In Appendix B, we further report the cross-border yield curve responses to the economic growth shocks of the other countries. All of the remaining outcomes of the other shocks are available from the authors upon request.

[28] For the sake of illustration, in 2013, Uruguay was ranked 98th out of 183 countries on the IMF's Financial Market Depth index, which measures the development level of domestic financial markets in terms of size and liquidity (see Svirydzenka (2016)).

Figure 3: Generalized impulse responses from the $GVAR - ATSM$ – Shock to economic growth in China



Note: The size of the innovation is equal to one standard deviation.

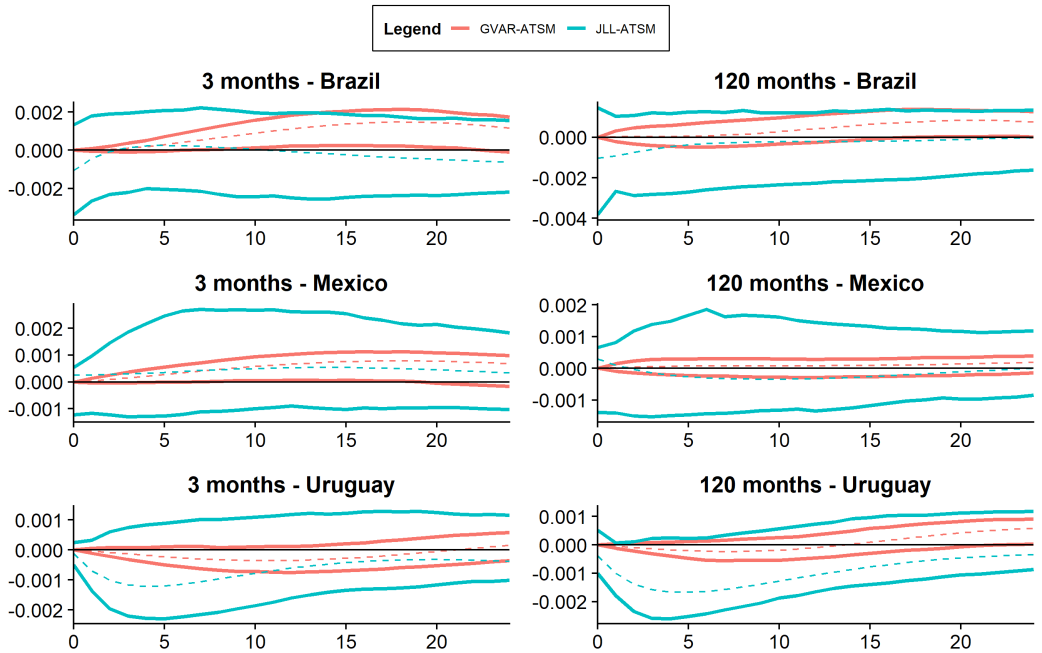
In Figure 4, we qualitatively compare the $GIRF$s from the $GVAR - ATSM$ and the $JLL - ATSM$ at different ranges of the maturity spectrum. We use a bootstrap procedure to estimate the confidence bounds at a level of significance of 95%.[29] For the $JLL - ATSM$, the $GIRF$s are computed for the orthogonalized Chinese economic growth shock.

Three points deserve attention. First, for all cases, the $GIRF$s obtained with the $GVAR - ATSM$ are statistically indistinguishable from those of the $JLL - ATSM$ at this level of significance. Second, we show that the confidence intervals of $GVAR - ATSM$ are substantially narrower than those obtained from $JLL - ATSM$. A similar pattern emerges from the $GIRF$s of the three other domestic economic shocks reported in Appendix B. These results corroborate that our more parsimonious representation produces more accurate model outputs, while $JLL - ATSM$'s bulky model parametrization leads to vague statistical inferences

[29] We detail our bootstrap methodology in Appendix E. An alternative recent method to perform statistical inference for structural impulse response in high-dimensional VARs was proposed by Krampe et al. (2021).

about variable responses to shocks in general. Third, $GVAR - ATSM$ indicates that the Chinese output growth shock is, at times, statistically significant for Brazil and Mexico. This finding constitutes an interesting result since it sheds new light on the relevant international channels of shock transmission to these countries. While previous studies have documented U.S. economy shocks as an important driver of economic fluctuation in Latin America,[30] less attention has been paid to investigating spillover effects from China. In this sense, it is somewhat surprising to verify that, despite the recent strengthening of the trade relations among these countries, China's economic growth might already have nontrivial effects on the asset price dynamics of Latin American economies.

Figure 4: Generalized impulse responses of the $GVAR - ATSM$ and the $JLL - ATSM$ – Shock to economic growth in China



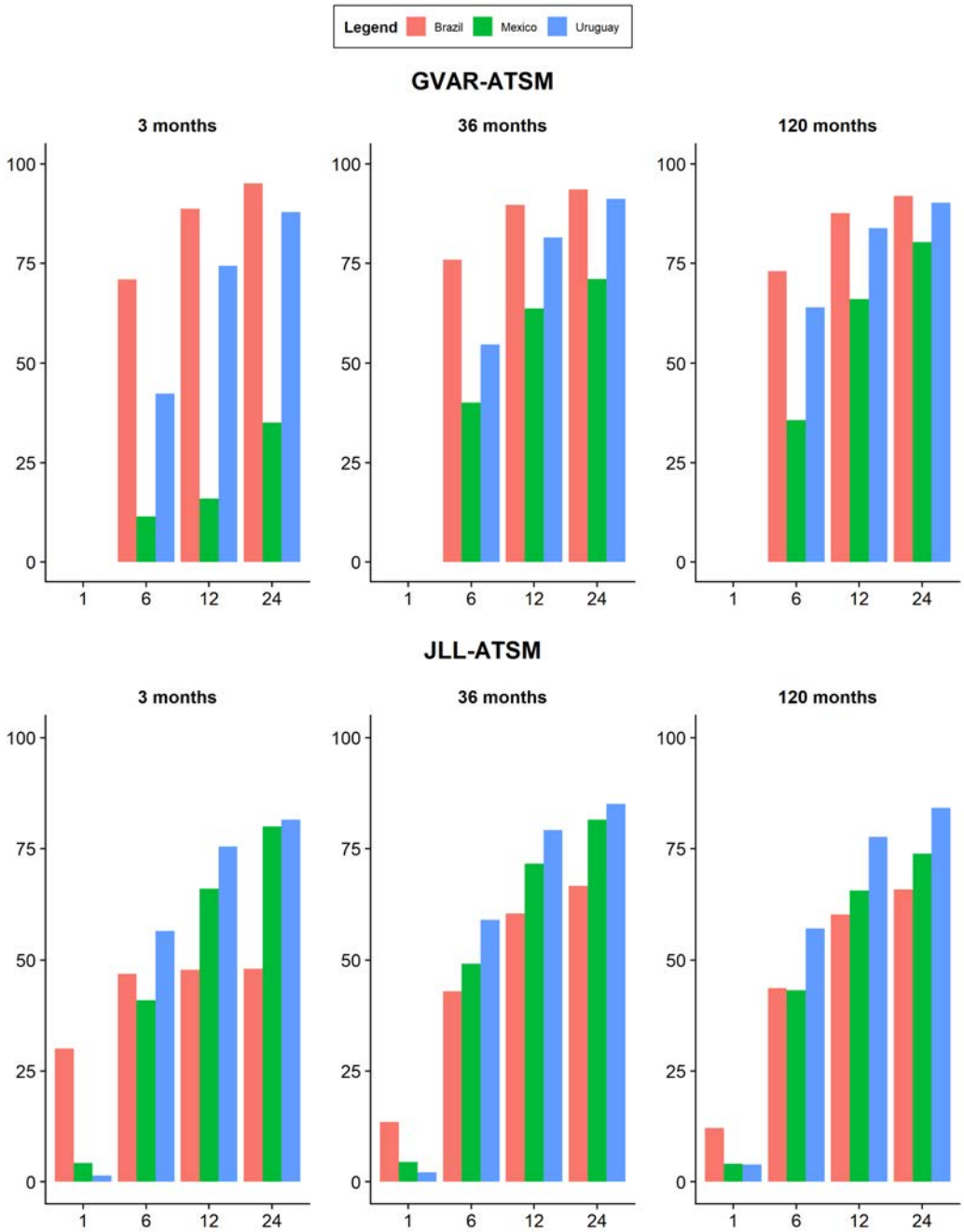
Note: The dashed lines correspond to the point estimates based on the real data. The continuous lines are the 95% confidence bounds computed by bootstrapping. For $JLL-ATSM$, the yield responses are computed for the orthogonalized version of the Chinese economic growth shock. The size of the innovation is equal to one standard deviation.

In Figure 5, we report the $GFEVD$s. In particular, we compute the shares of contributions of domestic and foreign shocks that explain the volatility of bond yield shocks for

[30]See, for instance, Canova (2005) for a referenced work.

different segments of the maturity spectrum. Two aspects are worth emphasizing. First, overall, we observe that both models indicate that foreign shocks are the main drivers of bond yield volatility for the three Latin American economies. This feature of the models emphasizes that cross-border developments play a key role in explaining the dynamics of domestic term structures. Second, for both models and all of the countries, the importance of foreign shocks increases with the forecast horizon. Among all cases, the largest shares of foreign shocks are captured by the $GVAR-ATSM$ after 24 months for Brazil and Uruguay. In such cases, the shares of foreign shocks reach values greater than 90%.

Figure 5: Generalized forecast error variance decomposition of yields – Share of foreign shocks



Note: The bars represent the sum of the proportion of the cross-border shocks (global and foreign economies' spanned and unspanned factors) for each economy. The x-axes show the number of forecast periods ahead in months. The y-axes are expressed in percentages. For $JLL-ATSM$, the outputs are computed based on the orthogonal version of the risk factors.

5. Conclusions

This paper builds a multicountry $ATSM$ in which the dynamics of the risk factors evolve according to a $GVAR$ model. Compared to extant multicountry frameworks, $GVAR - ATSM$ better captures the interdependence among countries within economic systems since it requires the specification of a cross-border transmission channel. Our setting also offers a more parsimonious term structure model representation, which grants a faster estimation process while providing more economically meaningful empirical results and better predictive power.

Our $GVAR-ATSM$ is estimated on a system composed of four emerging markets (Brazil, China, Mexico, and Uruguay) for which the trading linkages are relevant. Based on the analysis of impulse responses and variance decompositions, we show that the Chinese output growth shock, alongside other cross-border developments, has an important influence on the yield curve dynamics of these three Latin American countries. This result underscores the ever-increasing dependence of Latin America on worldwide economic events. Furthermore, our findings support that China has recently assumed a major role in the emerging markets of Latin America. While Canova (2005), among other earlier studies, pointed to the U.S. economy as an important source of shock transmission to Latin America, we present evidence that financial investors seem to account for economic developments in China as a relevant factor of risk. We also show that $GVAR - ATSM$ outperforms the referenced multicountry $JLL - ATSM$ in terms of out-of-sample forecasting performance of bond yields. The $GVAR - ATSM$ therefore paves the way to a broad range of empirical questions related to the interactions among term structures and the macroeconomy around the globe.

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Appendix A Supplementary Tables

Table A1: Parameters structure under the P-dynamics (JLL-ATSM)

	Φ_Y^e							Σ_Y^e						
	M^W	M_D^e	P_D^e	M_i^{e*}	P_i^{e*}	M_j^{e*}	P_j^{e*}	M^W	M_D^e	P_D^e	M_i^{e*}	P_i^{e*}	M_j^{e*}	P_j^{e*}
M^W	x	x	x					x						
M_D^e	x	x	x					x	x					
P_D^e	x	x	x							x				
M_i^{e*}	x	x	x	x	x			x	x		x			
P_i^{e*}	x	x	x	x	x					x		x		
M_j^{e*}	x	x	x			x	x	x	x				x	
P_j^{e*}	x	x	x			x	x			x				x

Note: Relationships among the risk factors from the $JLL-ATSM$ as of the feedback matrix (Φ_Y^e) and the Cholesky factorization term (Σ_Y^e). The variable labels M and P correspond to the unspanned and spanned factors, respectively. The superscripts e and e^* indicate orthogonalized variables according to the procedure described in the Section 2.2.1. The table shows the responses from the factors listed in the rows to shocks from the factors placed in the columns. The entries marked with X indicate the absence of restrictions, whereas empty entries denote zero restrictions. Factors indexed by W and D correspond to the global economy and the dominant country, respectively. The remaining variables indexed by i and j represent any two smaller economies.

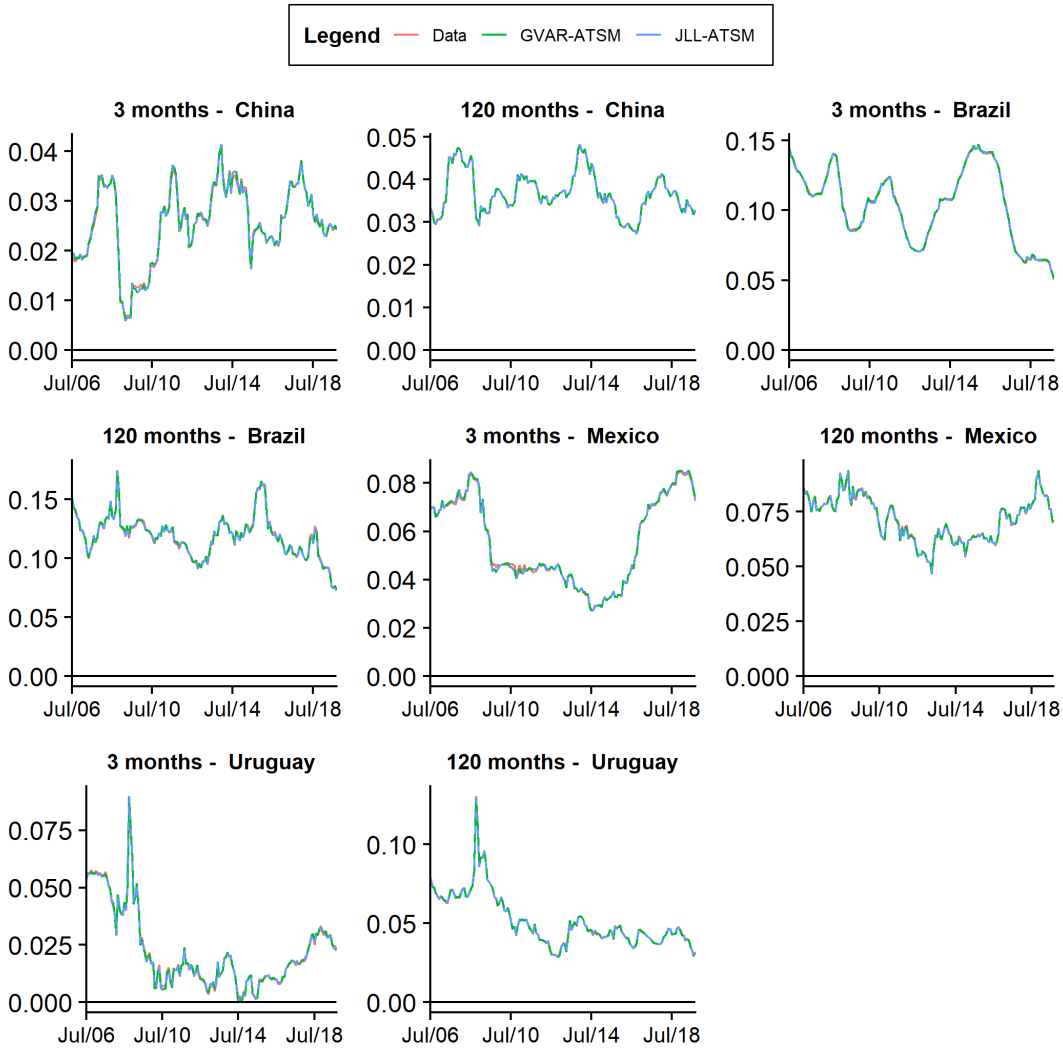
Table A2: Summary of model estimation features

	P-dynamics					Q-dynamics		Σ		**Dom. Eco.**
	Individual		Joint			Individual	Joint	P	P and Q	
	UR	R	UR	R						
				JLL	*GVAR*					
JPS-ATSM	x					x			x	
JPS-ATSM adapted			x				x		x	
JLL-ATSM				x			x	x		x
JLL-ATSM adapted				x			x		x	x
GVAR-ATSM					x		x		x	

Note: The table reports features of the several $ATSM$s with regard to: *(i)* the estimation of the P-dynamics parameters (*P-dynamics* columns); *(ii)* the estimation of the Q-dynamics parameters (*Q-dynamics* columns); *(iii)* the estimation of the variance-covariance matrix (Σ columns); and *(iv)* the presence of a dominant economy (*Dom. Eco.* column). Under the P- and Q-dynamics, the $VAR(1)$ part of each $ATSM$ can be estimated either on a country-by-country basis (*Individual* columns) or jointly for all countries in the economic system (*Joint* columns). Furthermore, under the P-measure, the dynamics of the risk factors can be unrestricted (*UR* columns) or restricted (*R* columns). For the restricted cases, the constraints are those present in the $JLL - ATSM$ and in the $GVAR - ATSM$. The estimation of the Σ matrix can be performed exclusively along with the other parameters of the P-dynamics (P column) or jointly under both P- and Q-parameters (*P and Q* column). The entries marked with X indicate that the mentioned characteristic is present, whereas empty entries denote the absence of this same feature.

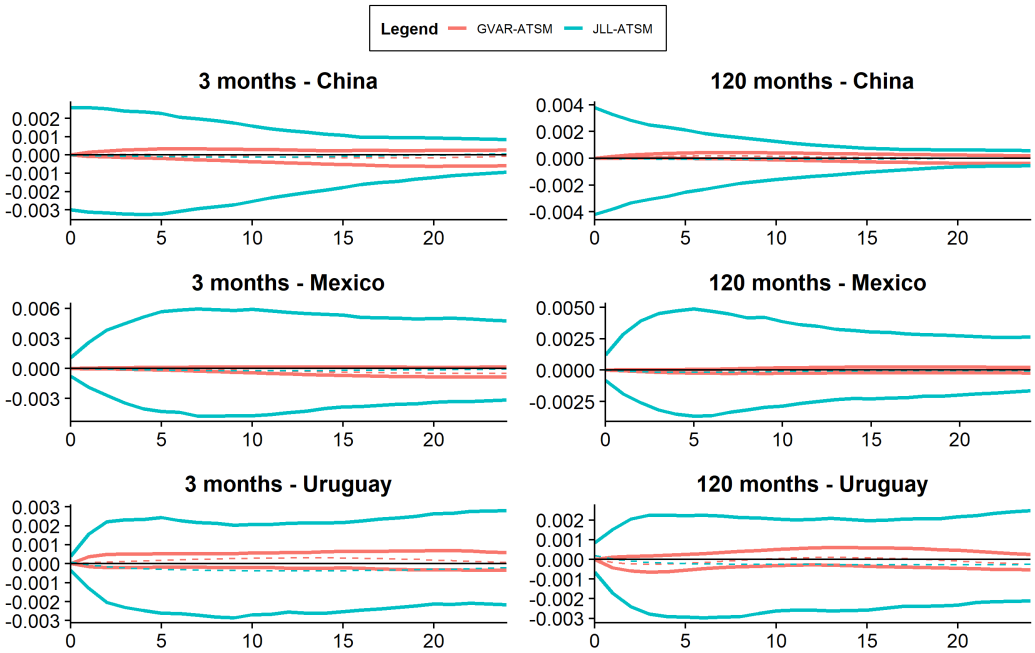
Appendix B Supplementary Graphs

Figure B1: Fitted model comparison



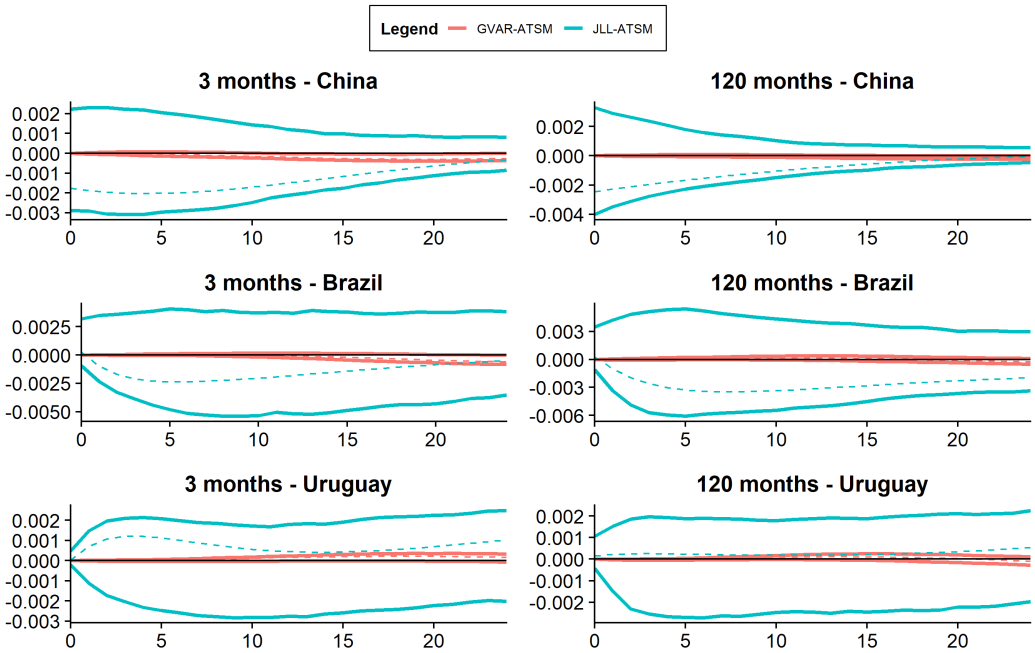
Note: Time series of the bond yields with maturities of 3 and 120 months and their respective fitted values for both $GVAR-ATSM$ and $JLL-ATSM$. The fitted model values are computed based exclusively on the risk-neutral parameters. Rates are annualized. The sample period is from July 2006 to September 2019.

Figure B2: Generalized impulse responses of the $GVAR - ATSM$ and the $JLL - ATSM$ – Shock to economic growth in Brazil



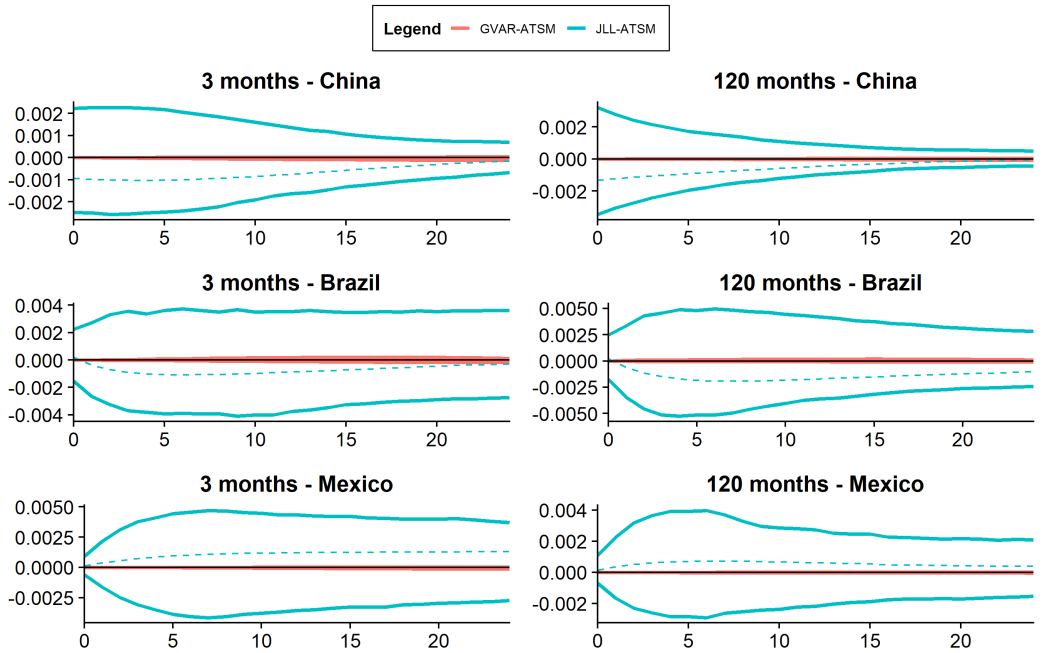
Note: The dashed lines correspond to the point estimates based on the real data. The continuous lines are the 95% confidence bounds computed by bootstrapping. For $JLL-ATSM$, the yield responses are computed for the orthogonalized version of the Brazilian economic growth shock. The size of the innovation is equal to one standard deviation.

Figure B3: Generalized impulse responses of the $GVAR - ATSM$ and the $JLL - ATSM$ – Shock to economic growth in Mexico



Note: The dashed lines correspond to the point estimates based on the real data. The continuous lines are the 95% confidence bounds computed by bootstrapping. For $JLL-ATSM$, the yield responses are computed for the orthogonalized version of the Mexican economic growth shock. The size of the innovation is equal to one standard deviation.

Figure B4: Generalized impulse responses of the $GVAR - ATSM$ and the $JLL - ATSM$ – Shock to economic growth in Uruguay



Note: The dashed lines correspond to the point estimates based on the real data. The continuous lines are the 95% confidence bounds computed by bootstrapping. For $JLL-ATSM$, the yield responses are computed for the orthogonalized version of the Uruguayan economic growth shock. The size of the innovation is equal to one standard deviation.

Appendix C P-dynamics model estimation for the $JLL - ATSM$

In $JLL-ATSM$, the risk factor dynamics under the P-measure follow a standard structural VAR with restrictions on both the feedback and the variance-covariance matrices. As such, the estimation is performed in two steps, preserving the constraints detailed in Table A1.

In the first step, we use restricted least squares to obtain C_Y^e and Φ_Y^e. Conditional on these estimates, the second step involves the maximum-likelihood estimation of Σ_{Y^e} assuming that v_t^e follows a Gaussian distribution of the form $\mathcal{N} \sim (0, \Sigma_{Y^e}\Sigma'_{Y^e})$. At this stage, due to the absence of a closed-form solution for Σ^e, the estimation requires the use of numerical optimization methods.

Appendix D *GIRF*s and *GFEVD*s

To construct the $GIRF$s and $GFEVD$s, we recast the $JLL - ATSM$ state-space representation in two stages. First, we re-express equation (11) as an affine function of the set of non-orthogonal risk factors

$$Y_t = A_Z(\Theta) + B_Z(\Theta)Z_t,$$

where $A_Z(\Theta) = A_P(\Theta)$ and $B_Z(\Theta)$ have nonzero entries for the coefficients of the country-specific yields associated with their respective spanned factors and have zero entries elsewhere. Second, we rewrite the state dynamics of equation (10) as:

$$Z_t^e = C_Y^e + \Phi_Y^e Z_{t-1}^e + v_t^e.$$

where $v_t^e = \Sigma_Y^e \varepsilon_{Z,t}^e$ and $\varepsilon_{Z,t}^e \sim \mathcal{N}(0, I_F)$. Denoting the variance-covariance matrix of this VAR process by Σ^e, it follows that $\Sigma^e = \Sigma_Y^e \Sigma_Y^{e'}$ and that $v_t^e \sim N(0, \Sigma^e)$. To complete the setup, the Π matrix links Z_t to Z_t^e from $Z_t = \Pi Z_t^e$, where Π contains the loadings of the orthogonalization procedure of Z_t (equations (6) through (9)).

To compute the $GIRF$s and $GFEVD$s of the country yields, we extend the results of Pesaran and Shin (1998) for a $VAR(1)$ to the state-space form of $JLL - ATSM$. For the $GIRF$s, the h-period in the future responses of all factors to a one-standard deviation shock to the risk factor j is stored in the vector:

$$\psi_j^{Z,e}(h) = \frac{(\Phi_Y^e)^h \Sigma^e e_j}{\sigma_j},$$

where e_j is a $F \times 1$ vector containing unity on the jth entry and zeros elsewhere, and σ_j is the standard deviation of the jth variable. Using $Z_t = \Pi Z_t^e$, it is straightforward to see that the responses of the yields to a one-standard deviation shock to the orthogonalized risk factor j are

$$\psi_j^{Y,e}(h) = B_Z(\Theta)\Pi\psi_j^{Z,e}(h).$$

For the $GFEVD$s, within the $VAR(1)$ setting, the contribution of the shock in the jth variable to explain the h-step ahead forecast error variance of the ith variable is

$$\theta_{ij}^{Z,e}(h) = \sigma_i^{-2} \sum_{l=0}^{h} \frac{(e_i'(\Phi_Y^e)^l \Sigma^e e_j)^2}{e_i'(\Phi_Y^e)^l \Sigma^e (\Phi_Y^e)^{l'} e_i}.$$

Similarly, defining $\Psi_l = B_Z(\Theta)\Pi(\Phi_Y^e)^l$, we use

$$\theta_{kj}^{Y,e}(h) = \sigma_k^{-2} \sum_{l=0}^{h} \frac{(e_k' \Psi_l \Sigma^e e_j)^2}{e_k' \Psi_l \Sigma^e \Psi_l' e_k}$$

to determine the contribution of the shock in the jth variable to the variance of the h-step ahead forecast error of the kth yield. To render the results comparable across countries and different forecast horizons, we normalize $\sum_{k=1}^{CJ} \theta_{kj}^{Y,e}(h) = 1$ for every h and k.

Appendix E Bootstrap procedure

We build on a standard residual bootstrap procedure to construct the confidence intervals of the $GIRF$s. Our approach features four main steps described as follows. First, holding fixed the P-dynamics real-data estimates for the intercepts, K_0, and for the feedback matrix, K_1, we draw and resample a set of shocks, ε_t^{**}, from their historical distribution to compute some artificial time series of the risk factors, Z_t^{**}:

$$Z_t^{**} = K_0 + K_1 Z_{t-1}^{**} + \varepsilon_t^{**}$$

where Z_t^{**} contains all applicable unspanned (global, domestic, and/or foreign) and spanned factors $Z_t^{**'} = [M_t^{**'}, P_t^{**'}]'$. The parameters from the P-dynamics are then estimated on Z_t^{**}. We discard the draws that lead to nonstationary systems or otherwise proceed to the next step. We set $Z_1^{**} = Z_1$ to initialize the autoregressive process.

Second, holding fixed the real-data estimates of the intercepts, A_i, and the slope coefficients, B_i, from the yield-affine equation, we build the country-specific artificial time series

of bond yields, $Y_{i,t}^{**}$, from its country-specific artificial time series of spanned factors, $P_{i,t}^{**}$, and some resampled shock, $u_{i,t}^{**}$:

$$Y_{i,t}^{**} = A_i + B_i P_{i,t}^{**} + u_{i,t}^{**}$$

where A_i and B_i are derived from no-arbitrage conditions.

Third, we compute the country-specific weight matrix V_i^{**} compatible with the generated artificial series as $P_{i,t}^{**} = V_i^{**} Y_{i,t}^{**}$. Finally, on the basis of Z_t^{**}, Y_t^{**} and every country's V_i^{**}, we re-estimate the entire model to obtain the new set of estimates for K_0^{**}, K_1^{**}, A_i^{**}, and B_i^{**} and the remaining parameters of a traditional $ATSM$. We repeat this same procedure multiple times for different draws.