CHOKING OF PARALLEL COMPRESSIBLE STREAMS WITH SHEAR AND WALL FRICTION

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Summary We present the extension of a 1D framework for parallel compressible streams, also known as compound flows. According to the original isentropic theory, a choked compound flow is sonic in the throat. However, it has recently been observed that the sonic section can move up- and downstream. In this work, we show analytically and numerically that shear and wall friction are the responsible physical mechanisms. The resulting framework accurately captures choking, but also provides the dividing streamline and the axial evolution of relevant flow variables such as the Mach number and the static pressure. Moreover, these predictions have a minor computational cost, offering an excellent compromise between accuracy and costs compared to 0D models and 2D or 3D simulations. We therefore believe that the extended framework opens perspectives in modelling of applications with compound flows, such as ejectors.

INTRODUCTION

Compound flows are at the heart of several applications, most notably in the exhaust of jet engines for external flows and in ejectors for internal flows. We concentrate on the latter in this work, where the contour of the duct generally has the shape of a convergent-divergent nozzle (cf. figure 1). In that case, choking occurs if the back pressure is reduced sufficiently. Pearson et al. [1] were the first to observe that the sonic section of a compound flow can consist of a subsonic and a supersonic component. Their 1D model formed the base for further development of the compound flow theory [2, 3], but all assumed an isentropic flow and hence the sonic section was always located at the throat. However, Lamberts et al. [4] and Kracik et al. [5] observed choking in different sections. In this work, we extend the isentropic compound flow theory with shear between the streams and friction at the wall [6]. The resulting 1D model captures the displacement of the sonic section, providing a new tool for in-depth analysis and modelling of internal compound flows.

GOVERNING EQUATIONS

We consider the case of two streams which have a different stagnation pressure at the inlet, but share the same static pressure and total temperature. The main assumption is that the static pressure is equal in both streams at *any* axial position x: $p_p(x) = p_s(x) = p(x)$, where p denotes the static pressure. The evolution of the individual cross-sections A_p and A_s is unknown a priori and should be calculated. They are subject to a geometrical constraint imposed by the known contours of the channel $A(x) = A_p(x) + A_s(x)$. The governing equations consist of conservation of mass, momentum and energy in each stream $i \in [p, s]$. For a steady, quasi-1D flow, these read as follows:

$$\frac{d}{dx}(\rho_i A_i V_i) = 0, \qquad \frac{d}{dx}(\rho_i A_i V_i^2) = -A_i \frac{dp_i}{dx} + F_i, \qquad \frac{d}{dx}(\rho_i A_i V_i h_{t,i}) = 0, \tag{1}$$

where ρ denotes the density, V is the velocity, F is a force per unit length and h_t is the total enthalpy. The shear and wall friction forces are defined through equations (2)-(5) and are schematically shown on figure 1.

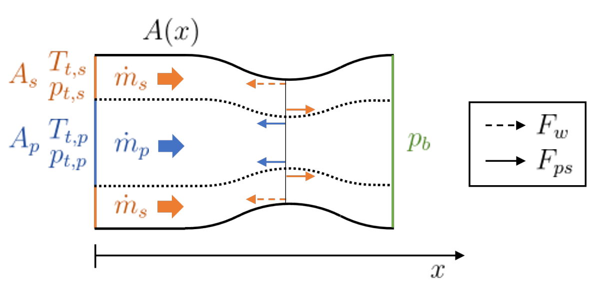

Figure 1. We consider a compound flow with 2 streams.

$$F_w = -\frac{1}{2} f_w \rho_s V_s^2 l_w \tag{2}$$

$$F_{ps} = \frac{1}{2} f_{ps} \left(\rho_p V_p^2 - \rho_s V_s^2\right) l_{ps} \tag{3}$$

$$F_p = -F_{ps} \tag{4}$$

$$F_s = F_{ps} + F_w. \tag{5}$$

The equations above eventually lead to the following equation for the static pressure, where we introduce the Mach number *Ma* and the compound choking indicator β (cf. [3]):

$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\beta}\left(\frac{dA}{dx} + \frac{1}{2} f_{ps} l_{ps} \frac{\left(Ma_p^2 - Ma_s^2\right)^2}{Ma_p^2 Ma_s^2} - \frac{1}{2} f_w l_w \left(1 + (\gamma - 1) Ma_s^2\right)\right), \tag{6}$$

$$\beta = \sum_{i \in [p,s]} A_i \frac{1 - Ma_i^2}{\gamma Ma_i^2}. \tag{7}$$

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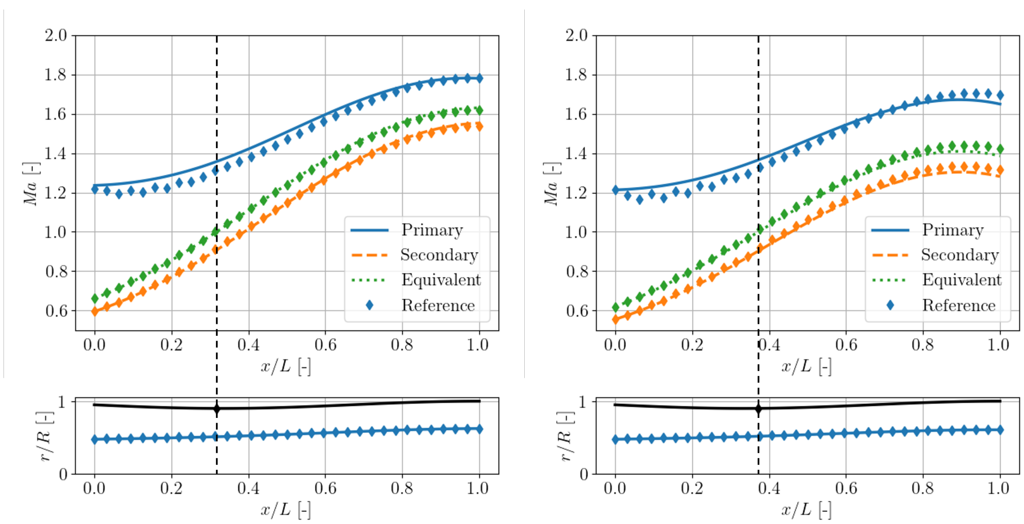
Equation (6) is singular if $\beta = 0$. This condition corresponds to a sonic compound flow [3]. The flow is compound-subsonic if $\beta > 0$ (e.g., low Mach numbers in equation (7)) and compound-supersonic if $\beta < 0$. The numerator of equation (6) also equals zero in the sonic section to keep the static pressure gradient bounded (the flow expands continuously through the sonic point). In absence of forces, this corresponds to a zero gradient of the cross-section, i.e., the throat. The shear term is always positive, so the area gradient should become negative. Hence, shear tends to move the sonic section upstream in a convergent section. The opposite is true for the friction term, which is always negative.

RESULTS

Figure 2 shows the predictions of the Mach number in a choked nozzle in a case with shear but no friction (2a) and with both forces (2b). The equivalent Mach number is directly related to β (cf. [7]) and equals unity if $\beta = 0$. The sonic section is thus located upstream of the throat (located at $x/L = 0.333$) in the first case and downstream in the second case, which supports the analytical observations in the previous section. The reference curves on figure 2 have been obtained by post-processing axisymmetric RANS simulations on the same geometry, performed with the rhoCentralFoam solver in OpenFOAM and the $k - \omega$ turbulence model. The excellent match with the 1D model indicates good accuracy of the 1D predictions, which have been obtained in a few seconds, as opposed to a few days for the CFD.



(a) Shear without wall friction, sonic at $x/L = 0.317$.

(b) Shear and wall friction, sonic at $x/L = 0.372$.

Figure 2. Comparison of predictions of the extended 1D compound flow theory with post-processed CFD. Shear pushes the sonic section upstream, friction pushes it downstream. The dividing streamline is shown below and is also in excellent agreement.

CONCLUSIONS

We have extended the isentropic compound flow theory with shear and wall friction forces. Besides better agreement with post-processed CFD results, the novel framework captures the up- and downstream displacement of the sonic section of a compound flow. To the knowledge of the authors, our model is the first to propose an explanation for this observed displacement. We therefore believe that this study paves the way to new in-depth and fine analysis of the choking mechanisms in supersonic ejectors.

References

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