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VOLUNTARY CONTRIBUTIONS WITH UNCERTAINTY: THE ENVIRONMENTAL QUALITY

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Abstract

This paper presents an overlapping generations model of environmental externalities with a depollution technology. Each agent concerned by the environmental degradation can voluntarily contribute in order to reduce it. Contributing to the environmental quality means financing depollution activities and uncertainty concerns its efficiency since it's not always clear or certain what effects will result from a reduction of pollution. We show that if an agent is sufficiently risk averse, voluntary contribution is a decreasing function of average efficiency of depollution technology. If on the contrary, the substitution effect is weaker than the pollution effect, the opposite holds. We show that precaution about environmental quality has two possible consequencies which depend on agent risk aversion. Therefore, understanding the implication of precautionary attitude, leads us to the consideration of the agents' risk aversion characterization and studying the effect of an increase in risk aversion implies a knowledge on prudent attitude. So, we have to conclude that precaution and risk aversion are two complementary and indissociable concepts.

Keywords: Environmental management, Uncertainty, Public goods, Voluntary contributions, Precaution, Risk.
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1 Introduction

In recent years environmental quality has became an important objective of public policy to insure both intergenerational and intragenerational equity. Hence, the intertemporal allocation resources to improve environment has arguably been one of the most debated issues and governments' objectives underline the conflict between positive and negative externalities originating from capital accumulation. The analysis of positive externalities has lead to the theory of endogeneous growth, *e.g.* the positive effect of human capital accumulation on the growth rate of the economy. As for negative externalities, we face the problem of pollution or degradation of environment. Damaging effects of capital accumulation have underline the fact that social welfare does not depend only on capital, but also on the quality of environment. Studying pollution and its negative consequences on individual welfare (through expression of distaste, unpleasantness, distress, concern, anxiety) and capital productivity (ocean or river pollution have effects on fish stocks) calls for an uncertainty framework.

Indeed, one of the problems of reaching very definite conclusions, about the role which environmental quality plays in sustaining economics systems, is that we face considerable lack of basic scientific relationships about that role (see Chichilnisky and Heal [1993]). For example, we do not fully understand how trace gases function in the atmosphere and stratosphere. Then, we are not sure of the ways in which environments function, either internally or in term of their interactions, with the economy. Morever, if we make a mistake, we cannot very often correct it afterwards. The presence of both uncertainty and irreversibility should make us more circumspect about the way to increase environmental quality. But, as it was clearly formulated in an OECD-document [1991] uncertainty and irreversibility should not be used as reason to argue that the protection of the environmental quality is uselessness:

"Where there are threats of serious or irreversible damage, lack of full scientific certainty shall not be used as reason for postponing cost-effective measures to prevent environmental degradation".

Then, environmental modeling should focus on precautionary issues and the OECD statment gives us a good definition of the precautionary principle concerning environmental damage. This definition agrees with the precautionary concept which has been introduced in economics by Leland [1968] and extended by Kimball [1990]. Recently, Cousy [1996] gives some illustrations of this concept with respect to the development of a policy of environmental protection as well as with respect to the development of a general framework. A prudent agent could protect herself against an uncertainty increase (Kimball [1990], Gollier [1994]). The intuition of this prudent attitude is already given in Sandmo [1970] and Drèze and Modigliani [1972].

Concerning environmental problems, precaution is a crucial problem since today's decision concerning environment has an influence on the welfare of both current and future generations (Gollier *et alii* [1997]) . Hence, the issue of environmental quality is intergenerationel as well as intragenerational. Since the works that have been devoted to this problem analyze it in a model of optimal growth or with infinitely-lived agents, they implicitely neglect intergenerational aspects. The overlapping generations models (OLG) seem to be better suited to the analysis of intergenerational aspects, without neglecting the intragenerational aspects. Therefore, there is a growing interest in the overlapping generations approach in the environmental literature (see for example John and Pecchenino [1994] or Marini and Scaramozzino [1995]). In fact, as pointed out by Solow [1986], the allocation of assets across generations is obscured in Ramsey-type models. Solow forcefully argues that discret time overlapping generations models are the natural habitat for these issues.

Furthermore, environmental quality is a public good since individuals are concerned by quality of their environment and are thus often willing to voluntary contribute to maintain it. These agents do not consider the effects of their decisions on future generations' welfare. The social optimum and the competitive outcome are therefore distinct. Agents' behavior, when agents privately decide the resources to allocate to environment quality in a certainty framework, was already studing in Jouvet, Pestieau and Michel [1997]. They shown that the neutrality theorem, due to Warr ([1982], [1983]) and extended by Bernheim and Bagwell [1988], still hold even with heterogenous agents' preferences and efficiency allocation could be restored by using fiscal tools. In an uncertainty framework, a number of writers like Feldstein [1988] or Kimball and Mankiw [1989] pointed out a contradiction with the Ricardian equivalence and then a contradiction with the neutrality theorem. So, one of the main interesting problems concerns the agents contributions behavior in an uncertainty environment. Voluntary contribution with uncertainty raises the following questions: i) How is agent's contribution modified if uncertainty increases? ii) Can we define a "cautiousness" agent's behavior? and iii) What are the implications on the social welfare? In order to tackle these questions one should refer to the precautionary principle as is previously definied in the OECD's statment and used by Cousy [1996].

In this paper, we consider a society consisting of two periods lived households. We assume that environmental quality degradation results from production activity. The very fact of producing pollutes: pollution is a "bad" joint product of the productive activity *e.g.* CO_2, acid rain, noise, ozon layer depletion. Each agent concerned by the environmental degradation can voluntary contribute in order to reduce it. Contributing to the environmental quality means financing depollution activities and uncertainty concerns its efficiency since it's not always clear or certain what effects will result from a reduction of pollution. Therefore, agents are not certain about the real impact of their contributions to the quality of environment.
In this setting, we study the effect of uncertainty on the agents'contribution behavior. We show that, when average marginal efficacity of depollution technology increases, voluntary contribution variation depends on risk aversion. Clearly, a high risk aversion implies that contribution is a decreasing function of average efficiency and conversly for a less risk averse agent. On the other hand, we show that prudent attitude depends on risk aversion degree. Anticipating our results, a prudent and high risk averse agent increases her contribution to the quality of environment with uncertainty and conversely a prudent but less risk averse one can decreases her contribution. In fact, as it is defined by Kimball [1990], precaution permits a protection against an increase in uncertainty on marginal effciciency but depends on agents'attitude about risk. We also show that an increase in risk aversion has the same impact on optimal individual contribution choice as an increase in uncertainty on depollution marginal efficiency. Finally, a social welfare analysis points out that an increase in expected marginal efficiency of depollution technology improves agents welfare. On an other side, social welfare decreases with uncertainty.

The paper is organized as follows. Section 2 sets out the overlapping generations model with pollution and uncertainty. In section 3 we study the effect of uncertainty on the agents'contribution, while section 4 considers an increase in risk aversion. The social welfare analisisis provided in section 5.

There is then a brief concluding section.

2 The basic model

We consider an infinite-horizon economy consisting of two-period-lived agents and perfectly competitive firms. During each interval of time there are N identical individuals. Each agent works while young and consume while old. Agents preferences are defined over consumption in old age and the index of pollution linked to production activity. Each agent can contribute to environmental quality by financing a linear depollution technology and determinates her own contribution taking as given contributions of all the others; it is a non cooperative subscription equilibrium. The depollution technology is assumed uncertain since agents don't know exactly the real effect of their contribution to environmental quality improvment. The production of a single homogeneous good is given by a neoclassical sector of production.

2.1 Production and pollution

A single homogeneous good is produced in every period using two factors, capital K and labour L. Firms are perfectly competitive profit maximizers using the constant returns production function. At time t, the production level is definied by:

$$Y_t = F(K_t, L_t) \tag{1}$$

where $F(K_t, L_t)$ is a well-behaved CRS production function. We assume that the capital stock totally depreciates during the production process. Accordingly, since firms are perfectly competitive maximizers, both the rate of interest, r_t, and the wage rate, w_t, are equal to the marginal productivity of capital and labor respectively :

$$r_t = F_K^{'}(K_t, L_t) - 1 \tag{2}$$

and

$$w_t = F_L^{'}(K_t, L_t) \tag{3}$$

Equations (2) and (3) are the standard first-order conditions from the firms' maximization problem in period t.
Pollution is considered as a joint product of the productive activity. So,

pollution at time t, P_t, is equal to the amount of pollution at time $t-1$, depleted at the constant rate $\delta \in]0,1]$, plus a porportion $b>0$ of current production, Y_t, minus a proportion, $g>0$, of aggregate contributions :

$$P_t = bY_t + (1-\delta)P_{t-1} - g\sum_{i=1}^{N} z_t^i \tag{4}$$

where z_t^i is the voluntary contribution of agent i to improve environmental quality, g is the marginal efficiency of linear depollution technology and $g-b>0$. This latter condition means that depollution is possible.

In this setting, introducing uncertainty on the depollution technology means that the marginal efficiency of linear depollution technologies depends on a random variable, $\tilde{\epsilon}$. In the following " $\tilde{}$ " denotes randoms variables. We assume that random variable $\tilde{\epsilon}$ is positively related to efficiency:

$$\tilde{g} = a\tilde{\epsilon} + \bar{g} \tag{5}$$

with $a>0$.

Considering the case where $a=0$ means there is no uncertainty on depollution technology (this case was analyzed by Jouvet, Pestieau and Michel [1997]). We assume that $\epsilon \in I = [\underline{\epsilon}, \bar{\epsilon}]$, with a probability distribution $G(\tilde{\epsilon})$, with a nil expected value of ϵ, $E(\tilde{\epsilon}) = 0$, and given variance,$Var(\tilde{\epsilon}) = \sigma^2$. Consequently, $E(\tilde{g}) = \bar{g}$ denotes the average marginal efficiency of depollution technology and $Var(\tilde{g}) = a^2\sigma^2$ denotes the variance of this technology. The parameter a is interpreted as an index of uncertainty level. An increase in a implies an increase in unefficiency risk. Morever, agents' contribution reduces the environmental damage for all value of ϵ if and only if:

$$a\underline{\epsilon} + \bar{g} - b > 0 \Leftrightarrow \underline{\epsilon} > \frac{b-\bar{g}}{a} \tag{6}$$

Note that, since we consider the production of the homogeneous good as the only source of pollution, condition (6) must hold. Indeed, contributing to environmental quality means that some units of homogeneous good are allocated to depollution and the very fact of producing thoses units is already a source of pollution. Assuming that parameter a is strictly positive, pollution becomes uncertain and defined by:

$$\tilde{P}_t = bY_t + (1-\delta)P_{t-1} - \tilde{g}\sum_{i=1}^{N} z_t^i \tag{7}$$

This equation defines a stochastic stock of pollution in the economy. Production is non-stochastic since agents' contribution decision doesn't change the homogenous good production level. To see this we present now the agents' behavior.

2.2 Agents' behavior and intertemporal equilibrium conditions

We assume a constant population of size N consisting of identical agents. Each agent lives two periods, working while young and consuming while old. Pollution is a public bad which enters in the utility of all agents. Agents born at date t have preferences defined over the consumption in old age, c_{t+1}, and the index of the pollution, P_{t+1}. These preferences are represented by a utility function $U(c_{t+1}, P_{t+1})$. Assume that $U(.)$ is three times continuously differentiable and, $U_c > 0, U_P < 0, U_{cc} < 0, U_{PP} \leq 0$ and $U_{cP} \leq 0$. This latter condition means that pollution has a negative impact on consumption since an increase in pollution induces a decrease in marginal utility in consumption (Michel and Rotillon [1995]).
In the first period, agents born at time t inelastically supply the firm with one unit of labor each, $L_t = N$, and receive the competitive wage w_t. Their income is entirely devoted to savings, s_t:

$$w_t = s_t \tag{8}$$

When old, agents receive their savings with a gross return, $R_{t+1} = (1 + r_{t+1})$. Then, each agent divides this amount between consumption, c_{t+1}, and a voluntary contribution to the environmental quality, z_{t+1}, while taking as given the contribution of all the others:

$$R_{t+1} s_t = c_{t+1} + z_{t+1} \tag{9}$$

Therefore, the contribution choice does not influence the production level, even if, as long as $\delta < 1$, the individual contribution to environmental quality, z_{t+1}, influences the utility of agents born at date t and the welfare of agents born at date $t + 1$. Since we consider a non cooperative subscription equilibrium, equation (7) for an agent i becomes :

$$\tilde{P}_{t+1} = bY_{t+1} + (1 - \delta)P_t - \tilde{g} z^i_{t+1} - \tilde{g} Z^{-i}_{t+1} \tag{10}$$

where z^i_{t+1} is the own contribution of agent i and Z^{-i}_{t+1} is the sum of other agents' contributions.
Each agent maximizes her expected utility function subject to constraints (8),(9), (10) and the non negativity condition on voluntary contribution, $z_t \geq 0$, $\forall t$.

$$\begin{cases} \max\limits_{c_{t+1}, z_{t+1}} & E(U(c_{t+1}, P_{t+1})) \\ \\ \text{s.c.} & w_t = s_t \\ & R_{t+1} s_t = c_{t+1} + z_{t+1} \\ & \tilde{P}_{t+1} = bY_{t+1} + (1-\delta)P_t - \tilde{g} z^i_{t+1} - \tilde{g} Z^{-i}_{t+1} \end{cases}$$

where $E(.)$ is the expected utility conditional on time t with respect to the distribution function G. The solution to this problem satisfies the following first order condition:

$$E(-U^{'}_{c_{t+1}} - (a\tilde{\epsilon} + \bar{g})U^{'}_{P_{t+1}}) = 0 \; if \; z^i_{t+1} > 0$$

or,

$$\int_{\underline{\epsilon}}^{\bar{\epsilon}} (U^{'}_{c_{t+1}} + (a\tilde{\epsilon} + \bar{g})U^{'}_{P_{t+1}}) dG(\tilde{\epsilon}) = 0 \; if \; z^i_{t+1} > 0 \tag{11}$$

The trade-off between consumption and contribution depends on the average of marginal efficiency of the depollution technology, $\bar{g}$ and the risk level, a. We note $H(z, \bar{g}, a)$ this condition.
Intertemporal equilibrium of this economy is defined by the following relations. First, equilibrium conditions between investment and savings imply:

$$K_{t+1} = N s_t \tag{12}$$

Second, total amount of contributions is defined by:

$$Z_t = \sum_{i=1}^{N} z^i_t \tag{13}$$

and firm equilibrium conditions:

$$r_t = F^{'}_K(K_t, N) - 1 \; , \; w_t = F^{'}_L(K_t, N)$$

Finally, output market equilibrium is given by,

$$f(k_t) = c_t + z_t + k_{t+1}$$

where $f(k_t) = F(K_t/N, 1)$ since $F(.)$ is a CRS production function and $k_t = K_t/N$ is the capital-labor ratio.

3 Aversion and precaution about environment degradation

In this section, we consider a change in the average marginal efficiency of the linear depollution technology and a change in the uncertainty level of this technology, a. Firstly, we analyze the impact of the average marginal efficiency of the linear depollution technology, $E(\tilde{g}) = \bar{g}$, on agents' contribution. Secondly, we study the implications of precautionary attitude on agents' contribution choice.

3.1 Contributions sensibility on the average efficiency of linear technology

This analysis leads us to compare the substitution effect between consumption and contribution and the pollution effect on agents' utilities. Indeed, agents'contribution choice depends on this two effects and contribution could increase or decrease with the average marginal efficiency of depollution technology. Morever, the comparaison between substitution and pollution requires the study of agents' risk aversion degree. The univariate risk case has already been deal by Khilstrom and Mirman [1974], and, Diamond and Stiglitz [1974]. Following those authors, risk aversion is defined by introducing a premium, θ, that is interpreted as the maximum amount the individual is willing to pay in order to decrease risk, that is:

$$E(U(c_{t+1}, \tilde{P}_{t+1})) = U(c_{t+1}, bY_{t+1} + (1-\delta)P_t - \bar{g}(1-\theta)\sum_{i=1}^{N} z_{t+1}^i) \qquad (14)$$

Firstly, let us consider the second order Taylor expansion of function $U(c_{t+1}, P_{t+1}(g))$, around $g = \bar{g}$. The expectation of this function is given by:

$$E(U(c_{t+1}, \tilde{P}_{t+1})) \sim U(c_{t+1}, bY_{t+1}+(1-\delta)P_t-\bar{g}\sum_{i=1}^{N} z_{t+1}^i)+\frac{Var(\tilde{g})}{2}U''_{PP}(\sum_{i=1}^{N} z_{t+1}^i)^2$$

Secondly, we consider the first order Taylor expansion of function $U(c_{t+1}, P_{t+1}(g(1-\theta)))$ around $\theta = 0$:

$$U(c_{t+1}, bY_{t+1}+(1-\delta)P_t-\bar{g}(1-\theta)\sum_{i=1}^{N} z_{t+1}^i) \sim U(c_{t+1}, bY_{t+1}+(1-\delta)P_t-\bar{g}\sum_{i=1}^{N} z_{t+1}^i)+\theta U'_P\sum_{i=1}^{N} z_{t+1}^i$$

Therefore, the premium admits by a risk averse agent is approximately,

$$\theta \sim \frac{a^2\sigma^2}{2}\frac{U''_{PP}}{U'_P}\sum_{i=1}^{N} z_{t+1}^i \tag{15}$$

with $Var(\tilde{g}) = a^2\sigma^2$ and $\sum_{i=1}^{N} z_{t+1}^i$ is lead to the subscription equilibrium framework.
Then, absolute risk aversion A is defined as follow,

$$A = \frac{U''_{PP}}{U'_P} \tag{16}$$

And relative risk aversion,A_r is defined as $A_r = gA\sum_{i=1}^{N} z_t^i$. We consider that an agent is "sufficiently risk averse" if $A_r > 1$. Then, we can state our first proposition,

Proposition 1 *For an agent sufficiently risk averse against environmental degradation, $A_r > 1$, if the substitution effect is greater than the pollution effect, then voluntary contribution to the quality of environment is a decreasing function of average efficiency of depollution technology for all state of nature, $\frac{dz}{d\bar{g}} < 0$.*
For a less risk averse agent $A_r < 1$, if the substitution effect is lower than the pollution effect, then voluntary contribution is an increasing function of average efficiency of depollution technology, $\frac{dz}{d\bar{g}} > 0$.

Proof : Starting form the first order condition (11),

$$H(z,\bar{g},a) = \int_{\underline{\epsilon}}^{\bar{\epsilon}} (U'_{c_{t+1}} + (a\tilde{\epsilon}+\bar{g})U'_{P_{t+1}})dG(\tilde{\epsilon}) = 0 \; if \; z^i_{t+1} > 0$$

and using the implicit function theorem, we obtain:

$$\frac{dz}{d\bar{g}} = -\frac{H'_g}{H'_z}$$

where H'_z is the second order condition,

$$H'_z = \int_{\underline{\epsilon}}^{\bar{\epsilon}} (U''_{cc} + 2(a\tilde{\epsilon}+\bar{g})U''_{cP} + (a\tilde{\epsilon}+\bar{g})^2 U''_{PP})dG(\tilde{\epsilon}) \tag{17}$$

where $U''_{cc} + (a\tilde{\epsilon}+\bar{g})U''_{cP} + (a\tilde{\epsilon}+\bar{g})^2 U''_{PP} < 0$, $\forall \epsilon \in I$ under the assumption that pollution has a negative impact on consumption ($U''_{cP} \leq 0$). We can note that assumption $U_{cP} \leq 0$ is in fact too strong. It would be sufficient to assume, $\forall \epsilon \in I$:

$$-\frac{U''_{cc} + (a\epsilon+\bar{g})^2 U''_{PP}}{2(a\epsilon+\bar{g})} > U''_{cP}$$

This condition means that compensation between consumption and pollution is not too strong (see Michel and Rotillon [1995]). $sign(\frac{dz}{d\bar{g}})$ is given by $sign(H'_g)$.

$$H'_g = \int_{\underline{\epsilon}}^{\bar{\epsilon}} (\sum_{i=1}^{N} z^i_{t+1}(U''_{cP} + (a\tilde{\epsilon}+\bar{g})U''_{PP}) - U'_P)dG(\epsilon) \tag{18}$$

We denote $h(\epsilon) = \sum_{i=1}^{N} z^i_{t+1}(U''_{cP} + (a\epsilon+\bar{g})U''_{PP}) - U'_P$. If $h(\epsilon) > 0$, for all states of nature, an increase in the average efficiency of the depollution technology induces an increase in the agent's contribution and, if $h(\epsilon) < 0$, contribution is a decreasing function in $\bar{g}$. We obtain the following condition:

$$\forall \epsilon \in I \;,\; h(\epsilon) < 0 \iff \sum_{i=1}^{N} z^i_{t+1}(\frac{U''_{cP} + gU''_{PP}}{U'_P}) > 1$$

$\square$

Under assumption $U''_{cP} \leq 0$ and $U'_P < 0$, condition $A_r > 1$ is sufficient to determinate the total effect of average efficiency variation since if $g\frac{U''_{PP}}{U'_P}\sum_{i=1}^{N} z^i_t > 1$ then $\sum_{i=1}^{N} z^i_{t+1}(\frac{U''_{cP} + gU''_{PP}}{U'_P}) > 1$.

It seems natural to call the term $U''_{cP} + gU''_{PP}$ the substitution effect since this term denotes the variation of the trade-off between consumption and contribution (11) when pollution varies (Sandmo [1970]), this term represents the undirect effect. The term U'_P refers to pollution disutility and represents the direct effect. Then, existence of risk aversion is a sufficient condition for the substitution effect being positive and the total effect can be determinated with a threshold on agents' risk aversion degree. A high relative risk aversion, $A_r > 1$ is a sufficient condition for defining the total effect of an increase in average efficiency of depollution technology. In fact, high relative risk aversion implies that the substitution effect is stronger than the disutility pollution effect. In this case, contribution is a decreasing function of average efficiency, $\bar{g}$. A more intuitive interpretation of this results is as follow: for a given level of contributions, to increase average efficiency implies to increase agents utility since the expected pollution level decreases. The trade-off between consumption and contribution depends on the marginal disutility of pollution and the marginal utility of consumption. Therefore, if the substitution effect between consumption and contribution is upper than the marginal pollution disutility, an agent choses to increase her consumption when expected efficiency of depollution increases. In terms of risk aversion, an increase in risk aversion makes the agent less inclined to expose her ressources to the possibility of inefficiency, hence this agent prefers to increase her consumption rather than her contribution. Indeed, faced with an increasing efficiency of depollution technology, a very risk averse agent prefers to increase the 'certainty part' of her utility by increasing her consumption and a less risk averse agent should prefer to increase her contribution (riskier choice) to the environmental quality.

3.2 Contributions and precautionary principle

As is defined by Gollier [1994] and Cousy [1996] precaution is the prudent attitude and answer to be adopted in a situation of uncertainty about risk.

Here, we have to analyze the change of contribution due to an increase or decrease in risk. A shift in risk is represented by a shift in parameter a (since a shift in a implies an increase or a decrease in $Var(\tilde{g}) = a^2\sigma^2$). In other words, studying prudence behavior induces analyzing contribution variation when initial risk level $a > 0$ becomes $\bar{a} > a$.
We define a prudent agent as an agent characterized by a positive absolute prudence degree. This absolute prudence is defined with a positive premium, π, such as $E(k(\tilde{g})) = k(\bar{g}(1-\pi))$. Hence, the compensating precautionary premium is the addition to expected wealth required by an individual in order to be willing to keep current utility unchanged while taking a higher risk. As previously, with $k(g) \equiv -U'_{c_{t+1}}(c_{t+1}, P_{t+1}(g)) - gU'_{P_{t+1}}(c_{t+1}, P_{t+1}(g))$, using Taylor expansions around $g = \bar{g}$ and around $\pi = 0$, we obtain the following equations of equivalence,

$$E(k(\tilde{g})) \sim k(\bar{g}) + \frac{Var(\tilde{g})}{2}k''(\bar{g}) \tag{19}$$

and,

$$k(\bar{g}(1-\pi)) \sim k(\bar{g}) - \pi\bar{g}k'(\bar{g}) \tag{20}$$

Therefore, precautionary premium is given by:

$$\pi \sim \frac{Var(\tilde{g})k''(\bar{g})}{2\bar{g}k'(\bar{g})} \tag{21}$$

Then, we have finally,

$$\pi \sim \sum_{i=1}^{N} z_{t+1}^{i} \frac{a^2\sigma^2}{2\bar{g}} \frac{\sum_{i=1}^{N} z_{t+1}^{i}(U'''_{cPP} + U'''_{PPP}) - 2U''_{PP}}{\sum_{i=1}^{N} z_{t+1}^{i}(U''_{cP} + gU''_{PP}) - U'_{P}} \tag{22}$$

The premium is an increasing function of the marginal efficiency variance. This implies that a prudent agent demands a higher premium when risk increases. Futhermore, equation (22) provides the absolute prudence degree, C_A:

$$C_A = \frac{\sum_{i=1}^{N} z_{t+1}^{i}(U'''_{cPP} + U'''_{PPP}) - 2U''_{PP}}{\sum_{i=1}^{N} z_{t+1}^{i}(U''_{cP} + gU''_{PP}) - U'_{P}} \tag{23}$$

Then, two kinds of prudent agents could be formaly defined. First, a sufficiently risk averse agent, as is defined in proposition 1 (the substitution effect upper than the pollution effect), is a prudent one if and only if

$$\sum_{i=1}^{N} z_{t+1}^i (U_{cPP}^{'''} + U_{PPP}^{'''}) - 2U_{PP}^{''} > 0$$

Second, a less averse agent, with $\sum_{i=1}^{N} z_{t+1}^i (U_{cP}^{''} + gU_{PP}^{''}) - U_P^{'} < 0$, is a prudent one if and only if

$$\sum_{i=1}^{N} z_{t+1}^i (U_{cPP}^{'''} + U_{PPP}^{'''}) - 2U_{PP}^{''} < 0$$

Considering a risk in the marginal efficiency of the depollution technology means that contribution is the only way to prevent against risk. Assuming a positive contribution to environmental quality for an initial risk level $a > 0$ and considering our definition of a prudent agent, we obtain the following proposition:

Proposition 2 *If agents are sufficiently risk averse and precautionuous, $(C_A > 0)$, toward pollution then an increase in risk of technology inefficiency implies an increase in contribution to the quality of environment:*
If $\forall \epsilon \in I$, $A_r > 1$ *and* $C_A > 0$, *then* $\dfrac{dz}{da} > 0$.
If an agent is not sufficiently risk averse (the substitution effect is lower than the pollution effect) but precautionuous, then an increase in risk implies a decrease in her contribution:
If $\forall \epsilon \in I$, $\sum_{i=1}^{N} z_{t+1}^i (U_{cP}^{''} + gU_{PP}^{''}) - U_P^{'} < 0$ *and* $C_A > 0$, *then* $\dfrac{dz}{da} < 0$.

Proof: As previously, using the theorem of implicite functions, we deduce from equation (11):

$$\frac{dz}{da} = -\frac{H_a^{'}}{H_z^{'}}$$

where the sign of $(\dfrac{dz}{da})$ is given by the sign of $H_a^{'}$.

$$H_a^{'} = \int_{\underline{\epsilon}}^{\bar{\epsilon}} (\tilde{\epsilon}[\sum_{i=1}^{N} z_{t+1}^i (U_{cP}^{''} + (a\tilde{\epsilon} + \bar{g})U_{PP}^{''}) - U_P^{'}])dG(\tilde{\epsilon})$$

To obtain the sign of this expression, we have to study the derivative of function

$$h(\epsilon) = \sum_{i=1}^{N} z_{t+1}^{i}(U_{cP}^{''} + (a\epsilon + \bar{g})U_{PP}^{''}) - U_{P}^{'}$$

If $\forall \epsilon \in I$, $h^{'}(\epsilon) > 0$ then

$$\begin{cases} \forall \epsilon < 0 \, ; h(\epsilon) < h(0) \Leftrightarrow \epsilon h(\epsilon) > \epsilon h(0) \\ \forall \epsilon > 0 \, ; h(\epsilon) > h(0) \Leftrightarrow \epsilon h(\epsilon) > \epsilon h(0) \end{cases}$$

Assuming that $h(\epsilon)$ is a continuous function, we obtain:

$$\int_{\underline{\epsilon}}^{\bar{\epsilon}} \tilde{\epsilon} h(\tilde{\epsilon}) dG(\tilde{\epsilon}) > \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon h(0) dG(\tilde{\epsilon}) = h(0)E(\tilde{\epsilon}) = 0$$

So, if $h(\epsilon)$ is an increasing function of ϵ, $H_{a}^{'} > 0$ and consequently $\dfrac{dz}{da} > 0$.

$$h^{'}(\epsilon) = a \sum_{i=1}^{N} z_{t+1}^{i}(-(\sum_{i=1}^{N} z_{t+1}^{i}(U_{cPP}^{'''} + gU_{PPP}^{'''}) + 2U_{PP}^{''})$$

A necessary condition for $h^{'}(\epsilon) > 0$ is:

$$\sum_{i=1}^{N} z_{t+1}^{i} \frac{U_{cPP}^{'''} + U_{PPP}^{'''}}{U_{PP}^{''}} > 2$$

□

If we refer to Leland [1968], Drèze and Modigliani [1972] and Kimball [1990] results, precautionary behavior means that agent changes herself own contribution to prevent her against pollution risk. Characterizing a prudent agent by a positive absolute prudence implies that a prudent agent can contribute more or less when risk increases. This approach in term of prudence is similar to Kimball'one [1990] with a risk income. Then, prudent and due care were obligatory to compense a disutility of a risk averse agent due to an increasing risk.

An important point of our context is that precautionary doesn't means that an agent must over contribute to prevent risk. In fact, if an agent is very risk averse, prudent attitude implies to increase the contribution and conversly

if she is not enough risk averse. Then, taking care must be understood as a compensation of the risk aversion effect. An intuition of this result is the following: a very risk averse agent choses to increase her consumption rather her contribution, a prudent attitude consists to compensed her natural preference for certainty by increasing the "uncertainty part of utility". On the other hand, a low risk averse agent contributes more to the environmental quality and therefore a prudent attitude means to reduce her contribution for diminishing the part of uncertainty of her utility.
In our framework, precaution is defined as a prudent answer to be adpoted in a uncertain situation about dangers of a less marginal efficiency of depollution technology. Agents well know the negatif impact of pollution on their utility and this danger is sufficiently known to be guarded against. Here, there is two opposite effects. First, an increase in contribution leads to a decrease in consumption and so, contribution could be interpreted as an income reduction. In this way, risk in marginal efficiency could be interpreted as an *income risk*. Second, contribution could be viewed as an investment and so, a risk in marginal efficiency could be interpreted as an *interest risk*. Nervertheless, prudent behavior arises to protect oneself against a decrease in utility caused by a potential high pollution level and trade-off between voluntary contribution and consumption is regarding as a trade-off between an ecologically prudent decision and a consumption decision. In that sence, if agents are sufficiently risk averse, prudent attitude leads them to decrease their consumption in order to insure an expected depollution level. In this case, we define contribution as a risk reducing variable.

4 Increase in risk aversion

In this section, we examine the effect of a marginal increase in risk aversion, where the initial attitude is risk neutrality. In general, we expect a more risk averse decision maker to select a less risky position. Indeed, we find that an increase in risk aversion will induce an increase in the contribution whenever contribution is risk reducing. In other worlds, according to proposition 2, an increase in risk aversion induces an increase in agents' contribution if precaution implies an increase in contribution.

Proposition 3 *An increase in risk aversion (inclination) leads to a higher (lower) contribution to environmental quality level, if contribution is risk* *reducing, $dz/da > 0$.*

Proof: Considering two agents, one is risk neutral and the other is risk averse. For risk neutral one, the first order condition (11) is:

$$-U'_{c_{t+1}}(z^*) - E[\tilde{g}]U'_{P_{t+1}}(z^*) = 0$$

where z^* denotes the optimal contribution level of this agent. A risk averse agent first order condition is defined by:

$$E[-U'_{c_{t+1}}(\hat{z}) - \tilde{g}U'_{P_{t+1}}(\hat{z})] = 0$$

where $\hat{z}$ denotes her optimal contribution. $\hat{z} > z^*$ implies under the second order condition (17):

$$0 = -U'_{c_{t+1}}(z^*) - E[\tilde{g}]U'_{P_{t+1}}(z^*) > -U'_{c_{t+1}}(\hat{z}) - E[\tilde{g}]U'_{P_{t+1}}(\hat{z})$$

and

$$E[-U'_{c_{t+1}}(z^*) - \tilde{g}U'_{P_{t+1}}(z^*)] > 0 = E[-U'_{c_{t+1}}(\hat{z}) - \tilde{g}U'_{P_{t+1}}(\hat{z})]$$

Considering $k(z,g) = -U'_{c_{t+1}}(z) - gU'_{P_{t+1}}(z)$ those two previous conditions means function $k(.)$ is convex with respect to its second argument. In fact, $\hat{z} > z^*$ implies that $E[k(z^*,\tilde{g}) > k(z^*,E[\tilde{g}])$. In the proof of proposition 2, we have already shown that a sufficient condition for the convexity is given by:

$$\sum_{i=1}^{N} z^i_{t+1} \frac{U'''_{cPP} + U'''_{PPP}}{U''_{PP}} > 2$$

This last condition means that contribution is risk reducing.

□

In fact, considering that initial attitude is risk neutrality acts in the same way as considering two utility functions, U and V where V, by assumtion represents greater risk aversion than U. That is, there exists an increasing and strictly concave function f such that $V = f(U)$. The concave transformation serves to increase the index of absolute risk aversion as Diamond and Stiglitz [1974] showned. The result is due to the fact that optimal behavior for an agent consists of balancing two opposite effects of an increase in the contribution. The first effect is that an increase in the contribution reduces the consumption level. The second effect concerns the fact that an increase in the contribution leads to a lower level of pollution. In a certainty framework, the equilibrium is achieved when disutility of pollution equal consumption utility. Under uncertainty and risk aversion, an agent wants a greater value (compared to certainty) on the reduction of pollution since allocation effect to depollution is uncertain.

5 Welfare

In this section, we study welfare consequencies of uncertainty. Welfare function, W, is the sum of individual expected lifetime utilities. The social welfare problem can thus be written as:

$$\begin{cases} \max \quad W = \sum_{i=1}^{N} E(U^i(c_t, P_t)) \\ \\ \text{s.t.} \quad c_t^i = f'(k_t)k_t - z_t^* \\ \qquad \tilde{P}_t = bY_t + (1-\delta)P_{t-1} - \tilde{g}\sum_{i=1}^{N} z_t^{i*} \end{cases}$$

where z^* denotes the optimal individual contribution to the environment quality.
Then, we obtain the following not surprizing proposition on social welfare:

Proposition 4 *Welfare is an increasing function of the average contribution efficiency and a decreasing function of the uncertainty.*

$$\frac{dW}{d\bar{g}} = \sum_{i=1}^{N} E[-U_P^i \sum_{i=1}^{N} z_t^{i*}] > 0$$

and

$$\frac{dW}{da} = \sum_{i=1}^{N} E[-\tilde{\epsilon} U_P^i \sum_{i=1}^{N} z_t^{i*}] < 0$$

Proof: The impact of the average marginal efficiency of depollution technology, $\bar{g}$, on social welfare is given by:

$$\frac{dW}{d\bar{g}} = \sum_{i=1}^{N} E[\frac{dU^i}{d\bar{g}}] = \sum_{i=1}^{N} E[-U_P^i \sum_{i=1}^{N} z_t^{i*}] \tag{24}$$

Under assumption $U^i_P < 0$, for all states of nature (ϵ),

$$\frac{dW}{d\bar{g}} > 0$$

And the impact of uncertainty variation is given by:

$$\frac{dW}{da} = \sum_{i=1}^{N} E[\frac{dU^i}{da}] = \sum_{i=1}^{N} E[-\tilde{\epsilon} U^i_P \sum_{i=1}^{N} z_t^{i*}] \tag{25}$$

Similary to demonstration of proposition 2, under assumption $U''_{PP} \leq 0$ for all states of nature (ϵ), we obtain:

$$\frac{dW}{da} < 0$$

□

The analysis of social welfare leads us to a standard result. Clearly, an increase in average efficiency of depollution technology implies a higher expected depollution and then a lower expected level of pollution. Social welfare increases with consumption and depollution, therefore a greater level of depollution induces a welfare increase. Considering a risk increase, variance of marginale efficiency increases and welfare decreases since uncertain rises to a high potential pollution level. Note that the analysis of social welfare do not take account of prudence or risk aversion, since agents have already chose their optimal contribution to the quality of environment level. Futhermore, in a Nash non cooperative equilibrium, since agents chose individualy their contribution, they never take account of the positive externality of the other provided by prudent attitude. Therefore, even if agents due care of environmental degradation by increasing their contribution, the negative effect on welfare is always higher than the positive effect due to greater contribution level.

6 Concluding remarks

This paper has presented an analysis of agents' behavior in their contribution to quality of environment when marginal efficiency of depollution technology

is uncertain. In this framework, we first show that if an agent is sufficiently risk averse (the substitution effect is greater than the pollution effect), then voluntary contribution is a decreasing function of average efficiency of depollution technology. If on the contrary, the substitution effect is weaker than the pollution effect, the opposite holds.
The second result, and the main one, is directly linked to our definition of prudence: a prudent agent is an agent who admits a positive premium. Considering this definition, we show that a prudent attitude in contribution has two possible consequencies. These consequencies depend on agent risk aversion: i) if the agent is sufficiently risk averse, an increase in the uncertainty of depollution technology implies an increase in contribution, ii) If an agent is not sufficiently risk averse then an increase in risk implies a decrease in her contribution. Therefore, understanding the implication of precautionary attitude, leads us to the consideration of the agents' risk aversion characterization and studying the effect of an increase in risk aversion implies a knowledge on prudent attitude. So in our framework, we have to conclude that precaution and risk aversion are two complementary and indissociable concepts.
Finally, social welfare analysis gives us some topics for future research. Indeed, refering to Cousy' intuition [1996], if a government could signal a higher risk in order to develop prudent agents' attitude, this signal could allow to significatively reduce the gap between the sub-optimal equilibrium (due to a Nash-non cooperative decisions process and the fact that agents do not take into account the effect of production on pollution) and a social optimal contribution.
Moreover, we can remark that we have discussed our model in terms of environmental quality but the analysis is more general since we consider environment as a public good.

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