

On the mass, width and coupling constants of the $f_0(980)$

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Abstract

Using the pole approach we determine the mass and width of the $f_0(980)$, in particular we analyze the possibility that two nearby poles are associated to it. We restrict our analysis to a neighborhood of the resonance, using $\pi\pi$ data for the phase shift and inelasticity, and the invariant mass spectrum of the $J/\psi \rightarrow \phi\pi\pi, \phi K\bar{K}$ decays. The formalism we use is based on unitarity and a generalized version of the Breit-Wigner parameterization. We find that a single pole describes the $f_0(980)$, the precise position depending upon the $\pi\pi$ data used (see Eq. (19) of the main text). As a byproduct, values for the $g_{f_0\pi\pi}$ and $g_{f_0K\bar{K}}$ coupling constants are obtained.

1 Introduction

According to the Review of Particles Physics (RPP) [1], the mass of the $f_0(980)$ scalar resonance is 980 ± 10 MeV whereas the width ranges from 40 to 100 MeV. The reasons for such an uncertainty in the width are the great amount and variety of experimental data and the different approaches used to extract the intrinsic properties of the resonance. To these points we could also add the lack of a precise definition of what is meant by mass and width, although there seems to be consensus in using the pole approach, where the mass and width of the resonance are found from the position of the nearest pole in the T -matrix (or equivalently, the S -matrix). However, even if this approach is adopted the final results differ on the number, location and physical interpretation of the poles. This is because the pole approach is not enough to completely fix the framework needed to perform the resonance analysis; in fact there are different formalisms that made use of it. An example is field theory, where a finite imaginary part of the propagator arises after Dyson summation of the one-particle-irreducible diagrams contributing to the two-point function. Unitarity also implies a general complex structure of the T -matrix in terms of which the pole approach becomes relevant to define the mass and width of the resonance. However, the general solution to the unitarity constraint has no implications regarding the number and/or locations of the poles. Thus, most analyses using the pole approach must involve further assumptions.

Far from physical thresholds, the identification of the mass and width of a resonance in terms of the nearest pole in the T -matrix is not ambiguous. However, when the resonance lies in the vicinity of a threshold this identification is not so obvious and more than one single pole can be required for a correct description of the resonance (see for example Ref. [2]). This could be the case for the $f_0(980)$ whose mass is very close to the $K\bar{K}$ threshold. In Ref. [3], Morgan and Pennington (MP) use a formalism general enough to avoid any assumption about the number of poles associated to a resonance. For the particular case of the $f_0(980)$, their exhaustive analysis leads to the conclusion that the $f_0(980)$ is most probably a Breit-Wigner-like resonance—with a narrow width $\Gamma \sim 52$ MeV— which can be described in terms of two nearby poles (in the second and third sheets). Moreover, the precision data coming from $J/\psi \rightarrow \phi(MM)$ decays, where (MM) stands for $\pi\pi$ or $K\bar{K}$, play an essential rôle as a crucial check in favour of the two-pole description of the $f_0(980)$ and disfavour the cases with one or three poles. These results indicate that the description of the $f_0(980)$ using a Breit-Wigner parametrization seems to be appropriate and should give results similar to those in Refs. [3, 5]. However, an existing analysis using a Breit-Wigner

parametrization performed by Zou and Bugg (ZB) [4] concludes that the $f_0(980)$ is most likely a resonance with a large decay width (~ 400 MeV) and a narrow peak width (~ 47 MeV). Later on, MP [5] and ZB [6] have both confirmed their former results, leaving the agreement between the two approaches as an open question.

The pole approach formalism has been successfully applied to hadronic resonances such as the $\rho(770)$ [7, 8, 9] and the $\Delta(1232)$ [10]. An important advantage of this formalism is that it yields process and background independent results. This background independence is only valid if other resonances are not present within the kinematical region under consideration¹. Thus, if one insists on this point, as we will, it is important to restrain the analysis to a neighborhood of the resonance under study. Due to the previous consideration, we exclude from our analysis of the $f_0(980)$ the central production data $pp \rightarrow p(\pi\pi, K\bar{K})p$ [11]–[16], which covers a much wider energy range and whose phenomenological description requires not only $\pi\pi$ and $K\bar{K}$ scattering but also a production mechanism involving many parameters. We also exclude the experimental data on $J/\psi \rightarrow \omega(\pi\pi, K\bar{K})$ decays because the f_0 signal is too weak [17, 18]² and the inclusion of these data in the fit would require further parameters³.

Our purpose in this work is to extract the mass and width of the $f_0(980)$ scalar resonance using the pole approach. As a byproduct, we also obtain values for the coupling constants $g_{f_0\pi\pi}$ and $g_{f_0K\bar{K}}$. We suggest that, for narrow resonances as the $f_0(980)$, the complete one-loop scalar propagator must be used in the pole equation. Thus, our analysis is based on a generalized Breit-Wigner description of a scalar resonance coupled to two channels, not only satisfying unitarity but also including loop effects. As far as the pole is concerned, we pay special attention to the possibility of describing the $f_0(980)$ in terms of more than one pole, a phenomenon which is known to occur when the mass of the resonance is close to a threshold. Nevertheless, it is worth remarking that the need of two poles is not guaranteed, it strongly depends upon the precise value of the renormalized mass of the resonance

¹The amplitude associated to a given resonance is not expected to describe the physics in a large kinematical region (compared to the width of the resonance) where additional resonances can exist.

²After submission of this manuscript the BES Collaboration has confirmed these results [19].

³A phenomenological analysis of the $J/\psi \rightarrow VS$ decays assuming $U(3)$ symmetry predicts that for ideally mixed scalar and vector states the values of the relevant coupling constants are: $g_{J/\psi\phi f_0} = g_{J/\psi\omega\sigma} = 1$ and $g_{J/\psi\omega f_0} = g_{J/\psi\phi\sigma} = 0$. The small departure from ideal mixing of the ϕ and the f_0 states explains, in a first approximation, the importance of the f_0 contribution to the $J/\psi \rightarrow \phi(\pi\pi, K\bar{K})$ decays and, on the contrary, the minor rôle played in the $J/\psi \rightarrow \omega(\pi\pi, K\bar{K})$ decays [20].

and its coupling constants to the two channels.

Values of the renormalized mass m_R and the coupling constants $g_{f_0\pi\pi}$ and $g_{f_0K\bar{K}}$ are obtained from a fit to experimental data including the $\pi\pi$ phase shift and inelasticity as well as the $J/\psi \rightarrow \phi\pi\pi(K\bar{K})$ invariant mass spectra. We then look for poles in the four Riemann sheets associated to a resonance coupled to two channels. Our conclusion is that the $f_0(980)$ can be described in terms of a single pole whose precise position depends upon the $\pi\pi$ data used (see Eq. (19) of the main text for details).

2 Formalism

Before discussing the formalism used for the particular case of the $f_0(980)$, it is convenient to briefly summarize two well-known definitions of mass and width of a given resonance, both widely used in the hadron physics literature (see Ref. [10] and references therein). One definition, known as the *conventional approach*, is based on the behavior of the phase shift of the resonance as a function of the energy, while the other, known as the *pole approach*, is based on the pole position of the resonance, which as discussed in the introduction includes several approaches. We will not consider here the powerful formalism developed in Ref. [3] since it goes beyond a Breit-Wigner-like description of the resonance, to which our analysis is restricted. In this sense, it is worth noticing that these more powerful methods provide further support to the Breit-Wigner description of the $f_0(980)$. Thus, the analysis carried in this paper is more restricted in scope, although it turns out to be general enough for the $f_0(980)$ case.

In the conventional approach, the mass and width of the resonance are defined in terms of the phase shift δ as⁴

$$\delta(s = M_\delta^2) = 90^\circ, \quad \Gamma_\delta = \frac{1}{M_\delta} \left[\frac{d\delta(s)}{ds} \right]_{s=M_\delta^2}^{-1}, \quad (1)$$

respectively. Since the phase shift is extracted from direct comparison with experimental data, the decay width defined in this way is usually called the *visible* or *peak width*. For an elastic Breit-Wigner (BW) resonance the phase shift is chosen as⁵

$$\tan \delta(s) = -\frac{M_\delta \Gamma_\delta(s)}{s - M_\delta^2}, \quad (2)$$

⁴We use the subindexes δ and p to denote the mass and width of the resonance in the conventional and pole approaches respectively.

⁵A s -dependent width $\Gamma_\delta(s)$ in Eq. (2) is mandatory when a background around the resonance is assumed [10].

which leads to the partial-wave amplitude

$$a = \frac{e^{2i\delta} - 1}{2i} = -\frac{M_\delta \Gamma_\delta(s)}{s - M_\delta^2 + iM_\delta \Gamma_\delta(s)} , \quad (3)$$

where s is the center-of-mass energy squared.

In the pole approach (or S -matrix approach), the resonance shows up as a pole in the amplitude

$$a = \frac{R}{s - s_p} + B , \quad (4)$$

where the two terms correspond to the resonant and background contributions separated according to Refs. [21, 22]. Eq. (4) is understood as a power series expansion of the amplitude around s_p , therefore, in order this description to make sense, the background around the pole (which is fixed from the fit to experimental data) should be a smooth function of s affecting minimally the pole position. In this approach, the mass and width of the resonance are defined in terms of the pole position s_p as⁶

$$s_p = m_p^2 - im_p \Gamma_p . \quad (5)$$

The pole approach provides a definition for the parameters of an unstable particle which is *process independent* (independent of the process used to extract them) and also *background independent* (different parametrizations of the background will hardly modify the values obtained for the pole parameters of the resonance).

In the remaining of this section we pursue the pole approach for the case of a resonance coupled to two channels, including furthermore the possibility of a strongly s -dependent width due to the opening of a second two-body threshold. Two ingredients are required in order to build the scattering amplitude to be used in our formalism: unitarity and the complete one-loop scalar propagator.

Concerning the first of the ingredients, unitarity sets stringent constraints on the amplitudes needed for the description of a resonance coupled to several channels. The correct incorporation of these constraints into the S -matrix is compulsory for an adequate analysis of experimental data. The analysis in the general case would require a model independent approach, as in Ref. [3], to determine the number and location of poles associated to the resonance. However, previous analyses [3, 4] have shown that the $f_0(980)$ can be described in terms of a Breit-Wigner-like resonance with two poles associated

⁶The relationship between the mass and width parameters defined in the conventional and pole approaches can be found in Ref. [10].

to it. The Breit-Wigner parameterization is nevertheless a particular case of an amplitude fulfilling unitarity. Indeed, for a relativistic particle, the general solution to the unitarity constraint can be written as [23]

$$T_{ab} = \frac{e^{2i\delta_{a,b}} - 1}{2i\sqrt{\beta_a\beta_b}} \delta_{ab} - \frac{e^{i(\delta_a(s)+\delta_b(s))}}{\sqrt{\beta_a\beta_b}} \frac{\sqrt{G_a G_b}}{F(s) + iG(s)} , \quad (6)$$

where $\delta_{a,b}$ stands for the phase shifts describing the background in channels (a, b) , $\beta_{a,b} = \sqrt{1 - 4m_{a,b}^2/s}$ with $m_{a,b}$ the masses involved in the two-body decays of the two channels, $F(s)$ and $G(s)$ are arbitrary real functions of s , and $G_i(s)$ are positive functions with the property $G(s) = \sum_{i=a,b} G_i(s)$. Identifying $F(s) = s - m_p^2$ and $G(s) = m_p \Gamma_p$, the amplitude (6) reduces in the one channel case to the amplitude (4) up to an overall normalization factor.

The second of the ingredients mentioned above will be used in our framework in order to identify the functions $F(s)$ and $G(s)$ to the real and imaginary parts of the complete one-loop propagator respectively. The previous identification has the advantage of incorporating automatically thresholds effects (see below). This procedure requires the use of an effective field theory in order to calculate the full propagator of the $f_0(980)$. In general, effective field theories are of limited use in the description of hadron physics where one expects the interactions to be strong. There are cases, however, where these theories can be used. The treatment of width effects, when the width to mass ratio (taken as an expansion parameter) is small, is an example where effective field theories can be useful for the description of narrow resonances, but not for broad ones. Notice in this respect that for the $f_0(980)$ $\Gamma_{f_0}/m_{f_0} \approx 0.04\text{--}0.1$ [1]. Concerning the final form of the scalar propagator, the use of a simple Breit-Wigner parametrization with constant width, which is applicable only to narrow resonances far from thresholds, is not enough due to the closeness of the $K\bar{K}$ threshold and the f_0 mass. Instead, one could use an energy dependent width, incorporating the kinematic dependences on the energy, but this approach amounts to include only the imaginary part of the self-energy. In our analysis, we prefer to use the fully corrected one-loop propagator, including both the real and imaginary parts of the self-energy, since this approach allows us to have a consistent description of the analytical properties, *i.e.* it provides a proper analytic continuation of the scattering amplitude below the $K\bar{K}$ threshold.

After Dyson summation, the propagator of a scalar particle is [24]

$$\Delta(p^2) = \frac{i}{p^2 - m_0^2 + \Pi(p^2)} , \quad (7)$$

where m_0 is the *bare* or tree-level mass of the resonance and $\Pi(p^2)$ is the one-particle-irreducible (1PI) two-point function. In the *on-shell scheme*, a Taylor expansion of the real part of $\Pi(p^2)$ around the resonance mass allows to rewrite the scalar propagator as

$$\Delta(p^2) = \frac{iZ}{p^2 - m_R^2 + im_R\Gamma_R} + \dots, \quad (8)$$

where the renormalized mass m_R (the so-called *on-shell mass*) and the wave-function renormalization factor Z are defined as

$$\begin{aligned} m_R^2 &= m_0^2 - \text{Re}\Pi(m_R^2), \\ Z^{-1} &= 1 + \text{Re}\Pi'(m_R^2), \end{aligned} \quad (9)$$

with $\text{Re}\Pi'(p^2) = d\text{Re}\Pi(p^2)/dp^2$. By analogy with a Breit-Wigner resonance, the width is defined by⁷

$$\Gamma_R = \frac{1}{m_R} Z \text{Im}\Pi(m_R^2). \quad (10)$$

However, this *on-shell* definition of the resonance width is inadequate since it vanishes when a two-particle s -wave threshold is approached from below [2]. Due to the failure of the Taylor expansion of $\Pi(p^2)$ around m_R^2 , Eq. (10) has not the desired behaviour for a width properly defined. This is precisely the case under consideration since the $K\bar{K}$ threshold lies in the vicinity of the $f_0(980)$ mass.

On the contrary, the *pole approach* provides a consistent definition of the resonance width that behaves sensibly in the threshold region. In this approach, the Taylor expansion of $\Pi(p^2)$ is not performed and the scalar propagator (7) is written as

$$\begin{aligned} \Delta(p^2) &= \frac{i}{p^2 - m_0^2 + \Pi(p^2)} \\ &= \frac{i}{p^2 - m_R^2 + \text{Re}\Pi(p^2) - \text{Re}\Pi(m_R^2) + i\text{Im}\Pi(p^2)}, \end{aligned} \quad (11)$$

where $m_R^2 = m_0^2 - \text{Re}\Pi(m_R^2)$. Within the framework of the pole approach, the scattering amplitude (6) describing a resonance coupled to two channels is obtained by identifying the functions $F(s)$ and $G(s)$ with the denominator of the complete one-loop scalar propagator in Eq. (11): $F(s) = s - m_R^2 + \text{Re}\Pi(s) - \text{Re}\Pi(m_R^2)$ and $G(s) = \text{Im}\Pi(s)$. This procedure leads to

$$\begin{aligned} T_{ab} &= \frac{e^{2i\delta_a(s)} - 1}{2i\sqrt{\beta_a\beta_b}} \delta_{ab} - \frac{e^{i(\delta_a(s) + \delta_b(s))}}{\sqrt{\beta_a\beta_b}} \frac{\sqrt{s}\sqrt{\Gamma_a\Gamma_b}}{s - m_R^2 + \text{Re}\Pi(s) - \text{Re}\Pi(m_R^2) + i\text{Im}\Pi(s)} \\ &= \frac{e^{2i\delta_a(s)} - 1}{2i\sqrt{\beta_a\beta_b}} \delta_{ab} - \frac{e^{i(\delta_a(s) + \delta_b(s))}}{16\pi} \frac{g_a g_b}{s - m_R^2 + \text{Re}\Pi(s) - \text{Re}\Pi(m_R^2) + i\text{Im}\Pi(s)}, \end{aligned} \quad (12)$$

⁷This definition applies only to narrow resonances, $\Gamma_R \ll m_R$, where $\text{Im}\Pi(p^2)$ can be approximated by $\text{Im}\Pi(m_R^2)$ over the width of the resonance. If the resonance is broad, the full energy dependence of $\Pi(p^2)$ must be taken into account.

where $\Gamma_{a,b} = \frac{g_{a,b}^2}{16\pi\sqrt{s}}\beta_{a,b}$ are the partial decay widths of the resonance in channels (a,b) . The common relation $\text{Im}\Pi(p^2) = \sqrt{p^2}\Gamma(p^2)$ is not used to avoid confusion (the width of the resonance in the pole approach is related to the pole position, *it is not* given by the tree level result following from the optical theorem). The renormalized mass m_R and the tree-level coupling constants of the resonance to the two channels $g_{a,b}$ are the parameters to be fitted when confronting the scattering amplitude (12) with data. Once these parameters are extracted from experimental data, the *pole mass* m_p and *pole width* Γ_p of the resonance are obtained from the pole equation

$$D(s_p) = s_p - m_R^2 + \text{Re}\Pi_+(s_p) - \text{Re}\Pi_+(m_R^2) + i\text{Im}\Pi_+(s_p) = 0 , \quad (13)$$

with $s_p = m_p^2 - im_p\Gamma_p$ and $\Pi_+(s) \equiv \Pi(s+i\epsilon)$. The pole equation (13) involves a complex function of a complex variable. If for real s , $\Pi_+(s) = R(s) + iI(s)$, then for arbitrary complex s

$$\Pi_+(s) = \text{Re}R(s) - \text{Im}I(s) + i[\text{Im}R(s) + \text{Re}I(s)] . \quad (14)$$

In order to find *all* the poles associated with a resonance coupled to channels (a,b) we have to look for the poles of Eq. (13) in the four different Riemann sheets defined by the complex channel momenta $p_{a,b}$. Following the conventional classification, the sheets are enumerated according to the signs of $(\text{Im}p_a, \text{Im}p_b)$:

$$\begin{aligned} \text{sheet I} \quad (+ +) : & \quad (\text{Im}p_a > 0, \text{Im}p_b > 0) , \\ \text{sheet II} \quad (- +) : & \quad (\text{Im}p_a < 0, \text{Im}p_b > 0) , \\ \text{sheet III} \quad (- -) : & \quad (\text{Im}p_a < 0, \text{Im}p_b < 0) , \\ \text{sheet IV} \quad (+ -) : & \quad (\text{Im}p_a > 0, \text{Im}p_b < 0) . \end{aligned} \quad (15)$$

We restrict ourselves to the case of two-body channels involving particles of the same mass. Thus, the thresholds for channels (a,b) are $2m_{a,b}$ and the momenta are defined as $p_{a,b}(s) = \sqrt{s - 4m_{a,b}^2}/2$ ($m_a < m_b$ is assumed). Since the complete propagator $D(s)$ is indeed an explicit function of the momenta $p_{a,b}$, $D(s) = D[s, p_a(s), p_b(s)]$, a change in the sign of the imaginary part of the momentum —a change of sheet— is achieved with the replacement of $p_{a,b}$ by $-p_{a,b}$ in the propagator. Therefore, the poles are found solving the following four pole equations:

$$\begin{aligned} D[s, p_a(s), p_b(s)] , \quad & D[s, -p_a(s), p_b(s)] , \\ D[s, -p_a(s), -p_b(s)] , \quad & D[s, p_a(s), -p_b(s)] . \end{aligned} \quad (16)$$

For the case of interest, namely the $f_0(980)$ scalar resonance coupled to a pair of pions and a pair of kaons⁸, the real and imaginary parts of the finite part of the 1PI two-point function $\Pi(s)$ are

$$\begin{aligned} R(s) &= \frac{g_{f_0\pi\pi}^2}{16\pi^2} \left[2 - \beta_\pi \log \left(\frac{1+\beta_\pi}{1-\beta_\pi} \right) \right] + \frac{g_{f_0K\bar{K}}^2}{16\pi^2} \left[2 - \beta_K \log \left(\frac{1+\beta_K}{1-\beta_K} \right) \right] \Theta_K \\ &+ \frac{g_{f_0K\bar{K}}^2}{16\pi^2} \left[2 - 2\bar{\beta}_K \arctan \left(\frac{1}{\beta_K} \right) \right] \bar{\Theta}_K , \\ I(s) &= \frac{g_{f_0\pi\pi}^2}{16\pi} \beta_\pi + \frac{g_{f_0K\bar{K}}^2}{16\pi} \beta_K \Theta_K , \end{aligned} \quad (17)$$

where $\beta_i = \sqrt{1 - 4m_i^2/s}$ for $i = \pi, K$, $\bar{\beta}_K = \sqrt{4m_K^2/s - 1}$, $\Theta_K = \Theta(s - 4m_K^2)$, and $\bar{\Theta}_K = \Theta(4m_K^2 - s)$. It is worth remarking that the step functions Θ are not introduced by hand but result from the present calculation and play a crucial rôle in the determination of the pole structure.

So far we have discussed the framework needed for the description of a resonance coupled to two channels. However, there are not current experiments that allow for a direct comparison of two-particle scattering amplitudes with experimental data. Therefore, in order to carry out the numerical analysis one has to rely on production processes such as $J/\psi \rightarrow \phi(\pi\pi, K\bar{K})$ and $J/\psi \rightarrow \omega(\pi\pi, K\bar{K})$ or central production in proton-proton scattering $pp \rightarrow p(\pi\pi, K\bar{K})p$. As stated in the Introduction, we will perform our analysis using only the former $J/\psi \rightarrow \phi(\pi\pi, K\bar{K})$ decays as a mechanism for producing pairs of pions and kaons. In this respect, we follow Ref. [3] to relate the production amplitude F , also constrained by unitarity, to the scattering amplitudes T_{ab} in Eq. (12). The corresponding amplitudes for $J/\psi \rightarrow \phi(\pi\pi, K\bar{K})$ are then written as

$$\begin{aligned} F_\pi &\equiv F(J/\psi \rightarrow \phi\pi^+\pi^-) = \sqrt{\frac{2}{3}} [\alpha_\pi(s)T_{\pi\pi} + \alpha_K(s)T_{K\pi}] , \\ F_K &\equiv F(J/\psi \rightarrow \phi K^+K^-) = \sqrt{\frac{1}{2}} [\alpha_\pi(s)T_{\pi K} + \alpha_K(s)T_{KK}] , \end{aligned} \quad (18)$$

where the real coupling functions $\alpha_{\pi,K}(s)$ are parametrized as $\alpha_i(s) = \gamma_{i0} + \gamma_{i1}s$ and the γ_i are obtained from the fit. Note that the theoretical expression for the $\pi\pi$ phase shift and inelasticity are obtained from the $T_{\pi\pi}$ scattering amplitude in Eq. (6). The $\pi\pi$ data used in our fits are extracted from three different experimental analyses, two of them [25, 26] based on data from the reaction $\pi^-p \rightarrow \pi^+\pi^-n$ [29] and the third one [27, 28] from $\pi^-p_\uparrow \rightarrow \pi^+\pi^-n$ [30] and $\pi^-p \rightarrow \pi^0\pi^0n$ [31].

⁸In our analysis, we work in the isospin limit and therefore the mass difference between K^0 and K^+ is not taken into account for the $K\bar{K}$ threshold. The inclusion of this mass difference would deserve a more refined 3-channel analysis with eight Riemann sheets that is beyond the scope of the present work.

3 Numerical analysis

Before proceeding with the numerical analysis we should keep in mind that our method is based on the pole approach and thus, in order to obtain a background independent fit to the data, we need to restrain ourselves to a neighborhood of the $f_0(980)$ resonance. For this reason we have chosen to work with experimental data on $J/\psi \rightarrow \phi\pi^+\pi^-$ and $J/\psi \rightarrow \phi K^+K^-$ decays [18, 32] and on the $\pi\pi$ phase shift and inelasticity [25, 26, 27, 28] in the range $0.8 \leq \sqrt{s} \leq 1.1$ GeV. It is worth noticing that within the kinematical region between 0.8 GeV and the point where the rising of the $\pi\pi$ phase shift starts due to the appearance of the $f_0(980)$, the contribution of the σ scalar resonance to the phase shift in this region can be reasonably described in terms of an energy polynomial⁹ (see below).

For the $\pi\pi$ phase shift we have used three different sets of data. The first two sets differ mainly in the point lying just around 980 MeV and correspond to solutions B [25] (including this controversial point) and D [26] (not including it) of Ref. [29]. The last set of data corresponds to the “down-flat” solution of Refs. [27, 28] that seems to be the most preferable solution after a joint analysis of the S -wave $\pi^+\pi^-$ and $\pi^0\pi^0$ data [30, 31]. In the following we will denote these three sets of $\pi\pi$ phase shift data as sets B, D and DF respectively. Part of the differences concerning the pole parameters of the $f_0(980)$ resonance reported in the literature could be due to the use of different data. In this work, in order to quantify the influence of the $\pi\pi$ phase shift used, we have performed fits using data-sets B, D and DF.

In the data fitting, a background term can be introduced for one or both channels and furthermore different energy dependences of the phase shifts can be considered. In our analysis we have included a background for the $\pi\pi$ and $K\bar{K}$ channels both with an energy dependence ranging from constant ($\delta = b_0$) to quadratic ($\delta = b_0 + b_1s + b_2s^2$). For the $\pi\pi$ case, the contribution of the σ resonance to the background term is included in this way. For the $K\bar{K}$ case, only b_1 and b_2 are independent parameters since the background term must vanish by continuity below the kaon threshold. So then, the maximum number of parameters of our fits is 12: the renormalized mass m_R , the coupling constants $g_{f_0\pi\pi}$ and $g_{f_0K\bar{K}}$, the parameters b_0^π, b_1^π and b_2^π for the background term for pions, b_1^K and b_2^K for the kaon background, and the constants $\gamma_{\pi 0}, \gamma_{\pi 1}$ and $\gamma_{K 0}, \gamma_{K 1}$ parametrizing the F_π and F_K amplitudes.

The results of the different fits performed show that *i)* the χ^2 improves

⁹Moreover, the choice of using experimental data only around 980 MeV avoids the need of describing the broad bump seen in the $\pi\pi$ phase shift around 600 MeV for which a simple polynomial parametrization is not adequate.

Fit	set B	set D	set DF
m_R^2 (GeV ²)	0.966 ± 0.003	0.982 ± 0.001	0.978 ± 0.003
$g_{f_0\pi\pi}^2/16\pi$ (GeV ²)	0.071 ± 0.007	0.065 ± 0.005	0.11 ± 0.01
$g_{f_0K\bar{K}}^2/16\pi$ (GeV ²)	0.25 ± 0.03	0.16 ± 0.01	0.31 ± 0.04
b_0^π	1.89 ± 0.09	1.47 ± 0.03	0.49 ± 0.05
b_1^π (GeV ⁻²)	-1.1 ± 0.2	1.47 ± 0.09	4.6 ± 0.1
b_2^π (GeV ⁻⁴)	0.8 ± 0.1	-0.50 ± 0.06	-2.83 ± 0.09
b_1^K (GeV ⁻²)	28.7 ± 0.6	15.5 ± 0.3	26.6 ± 0.4
b_2^K (GeV ⁻⁴)	-12.6 ± 0.2	-6.8 ± 0.1	-11.5 ± 0.2
$\gamma_{\pi 0}$	5.1 ± 0.4	-0.2 ± 0.9	5.0 ± 1.0
$\gamma_{\pi 1}$ (GeV ⁻²)	-1.5 ± 0.5	8.2 ± 1.0	1.7 ± 1.0
$\gamma_{K 0}$	-27.2 ± 0.5	-39.5 ± 0.7	-21.7 ± 0.5
$\gamma_{K 1}$ (GeV ⁻²)	29.7 ± 0.5	45.0 ± 0.7	26.2 ± 0.6
$\chi^2/\text{d.o.f}$	1.10	0.87	1.10

Table 1: Values for the parameters obtained from a joint fit to the $J/\psi \rightarrow \phi(\pi\pi, K\bar{K})$ decays and the $\pi\pi$ phase shift and inelasticity. Set B, D or DF refers to the set of $\pi\pi$ phase shift data used in the fit.

when a $K\bar{K}$ background is included, although changing its energy dependence makes no relevant difference; *ii*) the values obtained for the physically relevant parameters (m_R , $g_{f_0\pi\pi}$ and $g_{f_0K\bar{K}}$) change only a few percent when different backgrounds are considered. The outcome of the fit for the case of quadratic backgrounds for pions and kaons is written in Table 1 and shown in Fig. 1. The values for the renormalized mass m_R , the coupling constants $g_{f_0\pi\pi}$ and $g_{f_0K\bar{K}}$, the χ^2 per degree of freedom and the other fitted parameters are presented for sets B, D and DF of $\pi\pi$ data. Concerning the values of the coupling constants obtained from the fit, we observe that the coupling of the $f_0(980)$ to kaons is stronger than the coupling to pions: $g_{f_0K\bar{K}}^2/g_{f_0\pi\pi}^2 = 3.52, 2.46, 2.82$ for sets B, D and DF respectively. Model dependent values for these coupling constants have been recently reported by the SND [33], CMD-2 [34] and KLOE [35] Collaborations. Their analyses, based on the study of the $\phi \rightarrow \pi^0\pi^0\gamma$ radiative process, give $g_{f_0K^+K^-}^2/g_{f_0\pi^+\pi^-}^2 = 4.6 \pm 0.8$ (SND), 3.61 ± 0.62 (CMD-2) and 4.00 ± 0.14 (KLOE)¹⁰. Other analyses, based either on $pp \rightarrow p(\pi\pi, K\bar{K})p$ central production [11] or on $f_0(980)$ production in hadronic Z_0 decay [36], suggest the

¹⁰The coupling constants $g_{f_0\pi\pi(K\bar{K})}$ used in our analysis are related to the more common coupling constants $g_{f_0\pi^+\pi^-(K^+K^-)}$ and $g_{\pi(K)}$ used in experimental analyses by $\frac{2}{3} \frac{g_{f_0\pi\pi}^2}{4\pi} = \frac{g_{f_0K^+K^-}^2}{4\pi} = \frac{4}{3} g_\pi m_{f_0}^2$ and $\frac{1}{2} \frac{g_{f_0K\bar{K}}^2}{4\pi} = \frac{g_{f_0K^+K^-}^2}{4\pi} = g_K m_{f_0}^2$.

same behaviour for the coupling constants and obtain $g_K/g_\pi = 2.1 \pm 0.6$ and $g_K/g_\pi \simeq 10$ respectively. On the contrary, the analysis by the E791 Collaboration [37] on the $f_0(980)$ production in $D_s \rightarrow 3\pi$ decays gives $g_K/g_\pi = 0.2 \pm 0.6$. The values we obtain for both coupling constants and for their ratio are smaller than model predictions [38, 39, 40] and also than previous determinations [41].

Let us now proceed to determine the mass and width of the f_0 resonance within the pole approach. Fits to the data sets B, D and DF lead to values for the renormalized mass of $m_R = 983 \pm 2$ MeV, $m_R = 991 \pm 1$ MeV and $m_R = 989 \pm 2$ MeV respectively. The pole parameters of the resonance are determined once the values of the renormalized mass and the ones regarding the coupling constants (see Table 1) are included in the pole equation (13). The numerical solution of the pole equation yields for data sets B, D and DF:

$$\begin{aligned} m_p^B &= 987 \pm 3 \text{ MeV} , & \Gamma_p^B &= 42 \pm 9 \text{ MeV} , \\ m_p^D &= 999 \pm 2 \text{ MeV} , & \Gamma_p^D &= 39 \pm 8 \text{ MeV} , \\ m_p^{DF} &= 1001 \pm 6 \text{ MeV} , & \Gamma_p^{DF} &= 52 \pm 16 \text{ MeV} . \end{aligned} \tag{19}$$

This determination of the pole mass and width of the $f_0(980)$ resonance together with its couplings constants to the $\pi\pi$ and $K\bar{K}$ channels constitute the main result of this work. Our results in Eq. (19) are in fair agreement with several values of the $f_0(980)$ pole parameters appeared recently in the literature. The values $m_p = 994$ MeV and $\Gamma_p = 28$ MeV are obtained from an analysis of meson-meson interactions in a nonperturbative chiral approach [42]. Similar analyses give $m_p = 987$ MeV and $\Gamma_p = 28$ MeV [43] or $m_p = 981.4$ MeV and $\Gamma_p = 44.8$ MeV [44]. Other analysis based on the study of meson-meson interactions in different coupled channel unitarity models give $m_p = 1015 \pm 15$ MeV and $\Gamma_p = 86 \pm 16$ MeV [45], $m_p = 991 \pm 3$ MeV and $\Gamma_p = 71 \pm 14$ MeV [41], $m_p = 1008$ MeV and $\Gamma_p = 54$ MeV [46], $m_p = 993.2 \pm 6.5 \pm 6.9$ MeV and $\Gamma_p \sim 100$ MeV [47] or $m_p = 1006$ MeV and $\Gamma_p = 34$ MeV [48].

In addition to this result we have also analyzed the variation of the pole position as a function of the renormalized mass m_R , *i.e.* the coupling constants $g_{f_0\pi\pi}$ and $g_{f_0K\bar{K}}$ are kept fixed to their values for set D in Table 1. In Fig. 2, we show m_p^2 versus $m_p\Gamma_p$, which are related to the real and imaginary parts of the pole s_p , for values of the renormalized mass in the range $941 \leq m_R \leq 1027$ MeV. Thus, each point on the plot corresponds to a solution of the pole equation (13) —in terms of the values obtained for m_p and Γ_p — for a given value of m_R . Only the physically relevant pole is shown in Fig. 2; complex conjugate poles or any other kind of poles are not included. In order to generate Fig. 2, we looked for solutions of the pole equation in

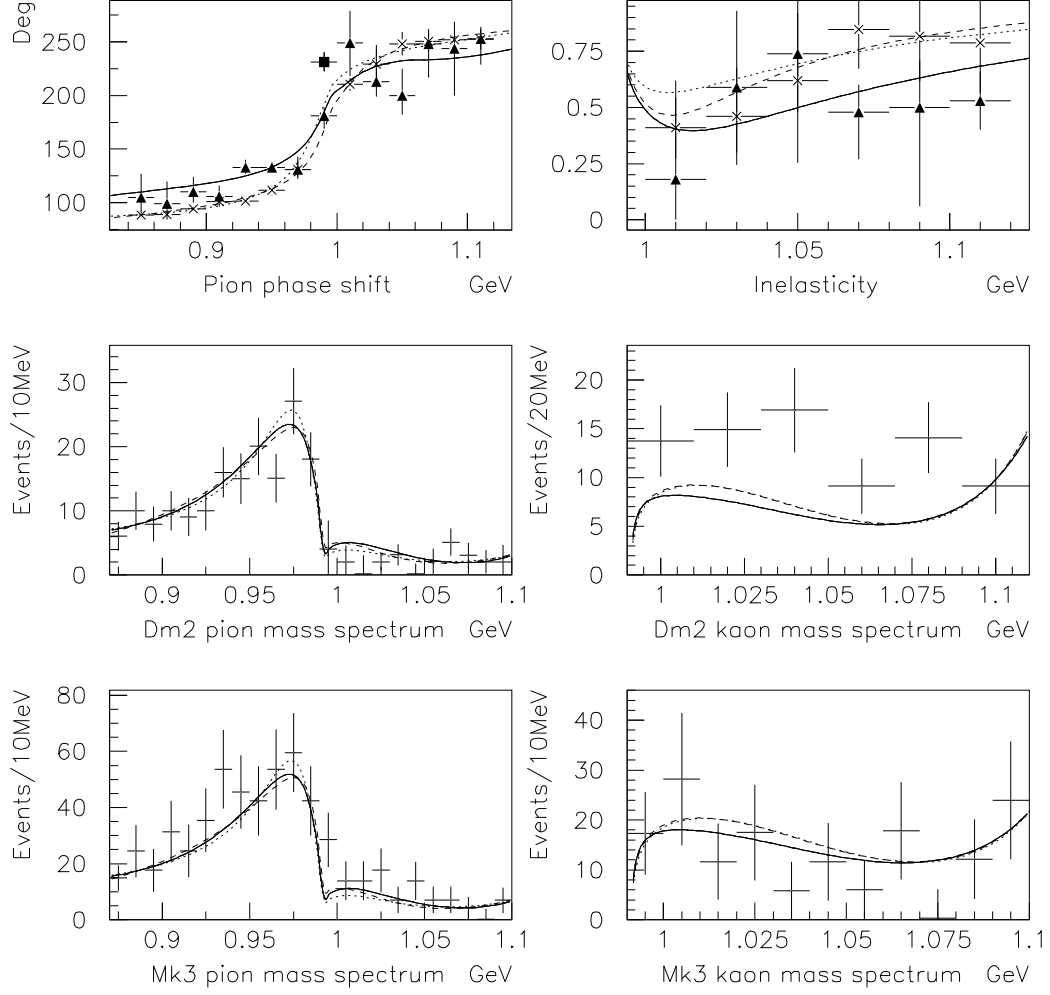


Figure 1: Fit to $\pi\pi$ data on the phase shift and inelasticity (cross [26], solid square [25], solid triangle [27, 28]) and to data on $J/\psi \rightarrow \phi(\pi\pi, K\bar{K})$ production [18, 32]. Fits to the data sets B, D and DF are shown with dotted, dot-dashed and solid lines respectively.

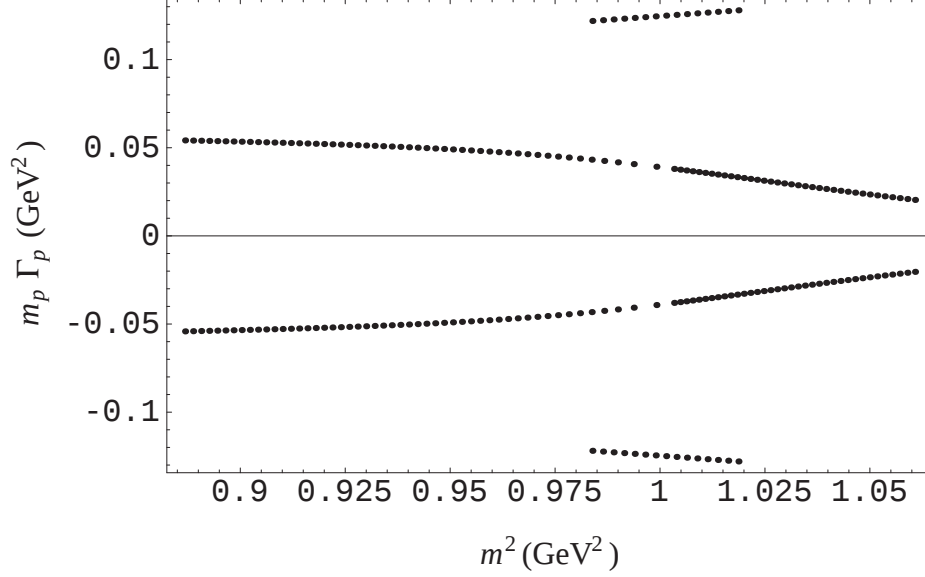


Figure 2: Behavior of the pole position (m_p^2 versus $m_p \Gamma_p$) as a function of the renormalized mass m_R in the range $941 \leq m_R \leq 1027$ MeV. The set of points with larger values of $|m_p \Gamma_p|$ appearing above 0.975 GeV^2 correspond to values of $m_R > 1020$ MeV and are associated to poles in sheet III.

each of the four Riemann sheets reaching the following conclusions:

1. We did not find a pole neither in sheet I nor IV in the vicinity of 980 MeV (we looked for poles in the range 960–1020 MeV).
2. We find poles in sheet II in the range $941 \leq m_R \leq 1027$ MeV. From these solutions we see that the pole mass is always larger than the renormalized mass, *i.e.* $m_p^{II} > m_R$. When $4m_\pi^2 < m_R^2 < 4m_K^2$, we obtain a pole width Γ_p that has to be identified with the $f_0 \rightarrow \pi\pi$ decay width. The width so found does not coincide with the decay width calculated from the tree level expression $\Gamma_{f_0 \rightarrow \pi\pi} = g_{f_0 \pi\pi}^2 \beta_\pi / 16\pi \sqrt{s}$, the difference arising from the contribution of the $\text{Im}R(s)$ term in Eq. (14) once s takes a complex value.
3. We find poles in sheet III *only* for $m_R > 1020$ MeV. In this case, the pole mass is always of the order of 20 MeV smaller than the renormalized mass, *i.e.* $m_p^{III} < m_R$, and Γ_p should be identified with the tree level width $\Gamma_{f_0 \rightarrow \pi\pi} + \Gamma_{f_0 \rightarrow K\bar{K}}$. Again, this decay width does not coincide with the pole width for the same reasons as before.

From points 2 and 3 above we conclude that only one pole in sheet II will be necessary to describe the $f_0(980)$ resonance, since poles in sheet III only

appear for $m_R > 1020$ MeV while our fit to data always yields $m_R < 1$ GeV. Moreover, these points also indicate that the *overlapping of poles is not possible*. The idea of the overlapping of poles states that values of the renormalized mass m_R close but below the kaon threshold ($m_R < 2m_K$) may lead to values of the pole mass m_p above the threshold ($m_p > 2m_K$) and that values of m_R close but above the threshold ($m_R > 2m_K$) may lead to values of m_p below the threshold ($m_p < 2m_K$). We stress that our conclusion *is not* general, it depends upon the values used for the coupling constants, in particular on the ratio $g_{f_0 K \bar{K}}^2 / g_{f_0 \pi \pi}^2$ [49].

4 Conclusions

Using a generalized version of the Breit-Wigner parametrization based upon unitarity and a propagator obtained from effective field theory including loop contributions, we have performed a fit to experimental data on the $\pi\pi$ phase shift, the inelasticity and the $J/\psi \rightarrow \phi\pi\pi$ and $J/\psi \rightarrow \phi K \bar{K}$ decays. The fit has been restricted to a neighborhood of 980 MeV ($0.8 \leq \sqrt{s} \leq 1.1$ GeV) thus providing a process and background independent way of extracting the intrinsic properties (pole mass and width) of the $f_0(980)$ scalar resonance.

The solution of the pole equation for the values of the parameters resulting from the fit allows us to conclude that the $f_0(980)$ is described in terms of a *single pole in sheet II* and yields the values $m_{f_0}^B = 987 \pm 3$ MeV and $\Gamma_{f_0}^B = 42 \pm 9$ MeV, $m_{f_0}^D = 999 \pm 2$ MeV and $\Gamma_{f_0}^D = 39 \pm 8$ MeV, and $m_{f_0}^{DF} = 1001 \pm 6$ MeV and $\Gamma_{f_0}^{DF} = 52 \pm 16$ MeV for the data-sets B, D and DF respectively. We also analyzed the behaviour of the pole position as a function of the renormalized mass m_R and found that a pole in sheet III only arises when $m_R > 1020$ MeV while our fit to data yields always values $m_R < 1$ GeV.

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