A Calderon preconditioner for large ground planes with arbitrary contours

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***Abstract*—Many applications involving antenna and antenna arrays require the presence of a large planar ground plane. The size of the ground plane usually scales in the tens of wavelengths, which represents a heavy burden in terms of computational resources. In this paper we apply the Method of Moments (MoM) combined with a Calderón preconditioner. We ensure the regularity of the refined mesh by using an initial regular mesh of rooftop basis functions. The regularity of the refined mesh allows for the application of the Calderón preconditioner in an iterative solver which exhibits linear and $N \log N$ complexities regarding memory and matrix-vector multiplication, respectively. These complexities are enabled by the Toeplitz-Block Block-Toeplitz (TBBT) structure of the MoM matrices which allows for the use of the fast Fourier Transform (FFT) in the matrix-vector multiplications. The operations are done at the level of half the basis function in order to include the half basis functions that the preconditioner requires on the edges of open structures. An additional simple algebraic multiplication allows to modify the mesh in order to simulate ground planes with arbitrary contours.**

***Index Terms*—Method of Moments (MoM), Calderón preconditioner, fast Fourier transform (FFT), Toeplitz matrices.**

I. Introduction

In modern computational electromagnetics, large planar structures such as ground planes are often responsible for slowing down simulations while also requiring important memory resources. This is notably the case for large antenna arrays scaling in the tens of wavelengths and thus requiring just as large planar ground planes. While efficient techniques are available for simulating arrays made of identical elements, evaluating the impact of a finite size ground plane is often challenging. These simulations are required for instance when considering antenna arrays for radioastronomy [1], [2]. In applications such as this one, the level of accuracy required from the simulations is such that the finite aspect of the ground plane has to be considered. This in order to estimate its effect on the radiation pattern and to quantify the thermal noise due to the soil located beneath the ground plane [3], [4]. The shape of the ground plane can be arbitrary and, when using the Method of Moments (MoM), it needs to be described with fine elements scaling in the tenth of the wavelength. Hence, large ground plane described with small elements lead to heavy simulations and this calls for iterative solver where the role of the preconditioner is of paramount importance. Different techniques have been used to reduce the number of iterations required by the iterative solver. These techniques include block diagonal preconditioners [5] and shield-block preconditioners [6]. Besides, another class of preconditioners based on Calderón identities have shown their ability to maintain good condition number with decreasing mesh density [7].

In this paper we use the Method of Moments (MoM) combined with the Generalized Minimal RESidual (GMRES) iterative solver [8] while including a Calderón preconditioner. In this case, we use rooftop basis functions such that the regularity of the primary and the refined meshes lead to a Toeplitz-Block Block-Toeplitz (TBBT) structure of the refined mesh MoM matrix. The TBBT structure enables the use of the fast Fourier Transform (FFT) to compute the matrix-vector multiplications. The basis functions outside the domain of interest are artificially canceled by suppressing columns and rows by simple multiplication with a diagonal matrix. Thus, linear and $N \log N$ complexities are obtained regarding memory and matrix-vector multiplication. This represents an ideal tool to analyze large planar structures such as large ground planes and can be incorporated along MoM-based techniques for antenna arrays [9], [10].

II. MoM formulation

An arbitrarily-shaped domain is meshed with rooftop basis functions (red rooftops in Fig. 1). In order to use the Calderón preconditioner, the initial mesh has to be refined into quadrilateral barycentric basis functions [11]. Therefore we refined the initial mesh of rooftop basis functions into a regular mesh of rooftop basis functions (green rooftops in Fig. 1) which are twice smaller than the ones of the initial mesh. Since we deal with an open structure, it is required to add half basis functions on the edge of the domain (shown in black in Fig. 1) [12]. The preconditioned system of equations reads [12]:

$$\Big(\mathbf{P}^T\ \mathbf{Z_b}\ \mathbf{Q}\ \mathbf{Z_b}\ \mathbf{R}\Big)\mathbf{I} = \Big(\mathbf{P}^T\ \mathbf{Z_b}\ \mathbf{Q}\Big)\mathbf{V_b} \qquad (1)$$

where $\mathbf{P}, \mathbf{R}$ are sparse matrices, $\mathbf{V_b}$ is the excitation vector on the refined mesh and $\mathbf{I}$ is the vector of current coefficients. The superscript T denotes the matrix transpose operator. $\mathbf{Z_b}$ is

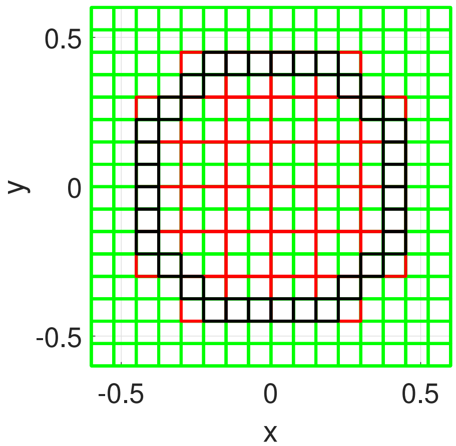

Fig. 1. Initial domain meshed with red rooftop basis functions. The refined mesh is shown in green and the half basis functions required on the edges of the domain are shown in black.

the MoM matrix of the refined mesh located inside the domain of interest. The matrix $\mathbf{R}$ realizes the mapping between the inital mesh and the refined mesh. The matrix $\mathbf{P}$ maps the initial mesh with dual basis functions [11]. The matrix $\mathbf{Q}$ is defined in [12].

The multiplication with matrix $\mathbf{Z_b}$ can be done using the FFT. Thanks to the translational symmetries of the half basis functions composing the green mesh, the MoM matrices are TBBT. In order to restrain the interactions to the domain of interest, rows and columns are suppressed by simple multiplications with diagonal matrices, as done in [13].

Thanks to this, an iterative solver using a Calderon preconditioner with linear memory complexity and $O(N \log N)$ matrix-vector multiplication complexity is established.

III. Numerical results

Let us consider the ground plane depicted in Fig. 2. The ground plane lies on the interface between free-space and a semi-infinite layer representing the soil. In this case a soil with relative permittivity $\epsilon_r = 3.66$ is selected. The influence of the layered media is accounted for by using the appropriate Green's functions [14]. The ground plane is described with 9640 rooftop basis functions of size $\lambda/10 \times \lambda/20$, with λ the free-space wavelength. A broadside incident plane wave is considered as excitation. The GMRES iterative solver is used to solve the MoM both with the technique presented in Section II and without preconditioning. The convergence of the initial MoM system is compared to the one of (1) in Fig. 3. The threshold for convergence is set at a relative residual error of 10^{-6}. The GMRES solver is able to converge in 55 iterations using the Calderón preconditioner while it needs 539 iterations with the initial MoM system of equations.

IV. Conclusion

An efficient iterative scheme for solving the MoM with a Calderón preconditioner is presented. Thanks to the use of

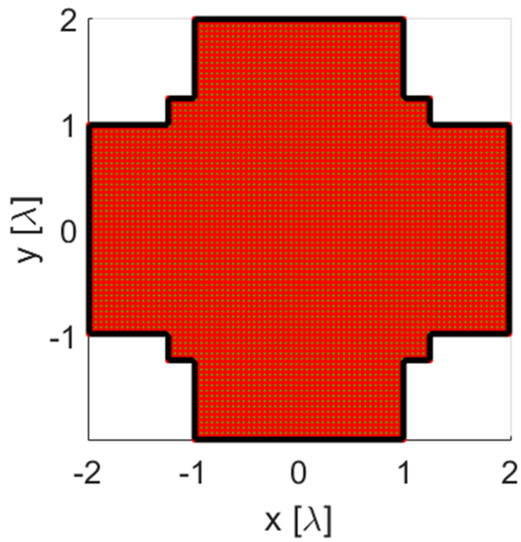

Fig. 2. Ground plane lying on a semi-infinite soil, described with rooftop basis functions.

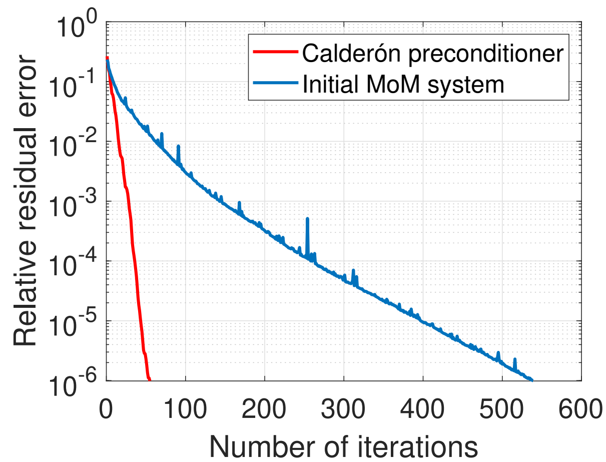

Fig. 3. Convergence of GMRES both with and without preconditioning.

regularly disposed rooftop basis functions both in the primary and the refined meshes, we obtain an algorithm with $N \log N$ complexity. At each iteration, the basis functions outside of the domain of interest are canceled thanks to multiplication by a simple diagonal matrix. Thus, the technique appears ideal to simulate large planar structures with arbitrary shapes such as large ground planes.

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