

Experimental evaluation of the friction slope in unsteady flows

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ABSTRACT: Experiments of moderate unsteady flows over mobile beds are used to determine the friction slope from the measured flow conditions. The experiments concern the progressive bed evolution in a super-critical flow over steep slopes made of coarse sand and a dike-overtopping case, where the dike is made of fine sediment. Bed and water elevation were measured by digital imaging techniques. Using these measurements, the friction slope S_f is derived from three shallow-water models used for the description of such flows. The resulting S_f by these three models are compared in order to assess the validity of the proposed approaches. Then, the measured friction slopes are used to calculate an equivalent Manning friction coefficient for these flows. The preliminary results tend to indicate that some unsteady flow effects can be identified through the comparison between the Manning coefficients obtained from the measurements and the values obtained using empirical formulas based on grain diameters.

1 INTRODUCTION

The flow in open channels is highly dependent on the shear stresses acting along the boundaries. These shear stresses are mainly controlled by the roughness of these boundaries, and induce erosion of sediment from a mobile bed. In open-channel flows, these shear stresses are usually related to the friction slope S_f . For steady uniform flows, many well validated formulations of S_f exist (e.g. Manning, Darcy-Weisbach, Chezy) that have been developed under steady-flow conditions thanks to a large number of experiments realized in the laboratory or carried out on the field (Brownlie 1983, Strickler 1923). The same formulations are commonly used to represent the friction losses in non-uniform and unsteady flows, but in these cases the uncertainty in the estimation of S_f might become significant.

One most common example of this kind of formulation is given by the Manning formula. S_f is computed as a function of the depth-averaged velocity u , the hydraulic radius R_H , and a friction coefficient n that depends on the diameter of sediments on the bed.

$$S_f = n^2 \frac{u^2}{R_H^{4/3}} \quad (1)$$

where the Manning friction coefficient n can be estimated using empirical formulations of Strickler (1923), Meyer-Peter and Müller (1948), or Bray (1979), as follows

$$n_{Strickler} = d_{50}^{1/6} / 21 \quad (2a)$$

$$n_{MPM} = d_{90}^{1/6} / 26 \quad (2b)$$

$$n_{Bray} = 0.0593 d_{50}^{1/6} \quad (2c)$$

The friction term S_f is generally used to represent the entire flow resistance, *i.e.* not only the shear stresses due to the boundary roughness, but also other effects such changes in the channel geometry or the flow unsteadiness (Yen 2002).

Some authors studied the influence of unsteady flow characteristics to determine more realistic values for the friction term S_f in unsteady flows. For example Becker and Yeh (1972) used measured velocities from field observations, for unsteady open channel flows, in order to evaluate the friction term. A study on the friction effects in unsteady open-channel flows over gravel beds is presented by Haizhou and Graf (1993), where the authors show the evolution of the friction parameter during the passage of a hydrograph. They concluded that for accelerating flows the friction parameter was usually larger than in decelerating flows, meaning that the

flow unsteadiness influences the effective values of S_f , as also shown by Graf and Song (1995).

Another important contribution in this field is given by Recking et al. (2008), who consider the influence of erosion and deposition on the bed shear stress. Their experiments were conducted over uniform gravel beds to obtain friction factor values under bedload equilibrium flow conditions. Analysis of these data showed that bedload transport produces a significant additional resistance.

Focusing in particular on the observations by Recking et al. (2008), the main objective of this paper is the study of the friction slope, with a special attention on the evaluation of this term in the case of unsteady flows over erodible bed.

Based on experimental measurements described in Section 2, the friction slope is evaluated using the values of the flow variables extracted from the experimental data considering different models for the representation of the flow. The first experiment shows the progressive evolution of an erodible bed featuring an initial knickpoint (Juez et al, submitted). The second experiment investigates the erosion of a non-cohesive sand embankment by overtopping (Van Emelen et al, 2015).

The models used to evaluate the friction slope are described in Section 3. They consist of depth-averaged representations of the flow, with different approaches for the morphological evolution of the bed, all based on the Saint-Venant and Exner equations.

Section 4 presents the results of the friction slope evaluation and finally conclusions are provided.

2 EXPERIMENTAL SET-UP

The experiments were conducted in the Hydraulics Laboratory of the Institute of Mechanics, Materials and Civil Engineering at the Université catholique de Louvain, Belgium. Facilities used in the laboratory to record the evolution of the phenomena consisted of a camera taking images of the flow at a rate of 12 fps; the flow being illuminated by a laser sheet allowing to isolate a given vertical plane on which the free surface and the bed elevation can be identified. Water-level probes were also used to measure the water level in areas not covered by the laser sheet.

2.1 Supercritical flow over steep slopes

The first experiment (Juez et al., submitted) focuses on the evolution of an erodible bed, composed by non-cohesive sand. The initial bed profile is illustrated in Figure 1: the upstream bed slope is 5.05 % while the downstream slope is 17.6 %. A constant discharge is supplied at the inlet and it is responsible of the increased erosion of the bed which progressively reaches a constant uniform slope.

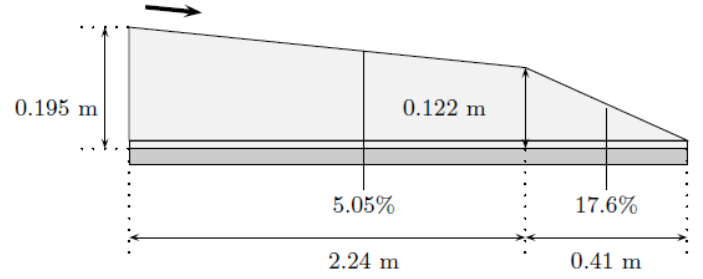


Figure 1: Initial conditions for the experiment of supercritical flow over steep slopes (Juez et al., submitted).

The channel cross section is rectangular with a width of 0.2 m. The erodible bed is made of sediment having grain-size distribution that can be considered as quasi-uniform, with the following characteristics: the mean diameter $d_{50} = 1.82$ mm, the friction angle $\phi = 30^\circ$ and the bed porosity $\epsilon_0 = 0.5$.

This experiment is conducted as follows. The bed is initially dry. The water discharge is progressively increased from 0 to $0.001 \text{ m}^3/\text{s}$ in 10 s and then remains constant during the entire experiment. No sediment supply is provided at the upstream boundary. The time evolution of the bed profile is illustrated in Figure 2. The wetting front progressively inundates the flume (Figure 2a) and a supercritical flow establishes. This flow starts eroding the knickpoint (Figure 2b) until the complete softening of the initial profile (Figure 2c).

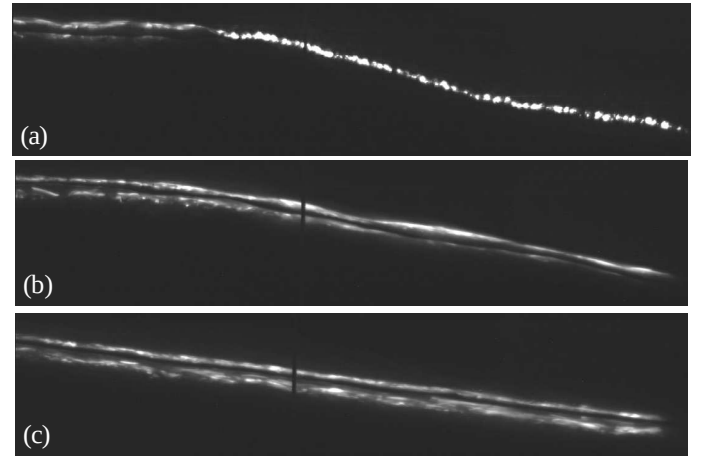


Figure 2: Progressive erosion of the initial knickpoint: (a) $t = 1.26$ s, (b) $t = 3.12$ s, (c) $t = 16.53$ s

2.2 Dike overtopping

The second experiment consists of the erosion of a non-cohesive sand embankment due to overtopping (Van Emelen et al. 2014, Van Emelen 2014). The experimental setup is sketched in Figure 3.

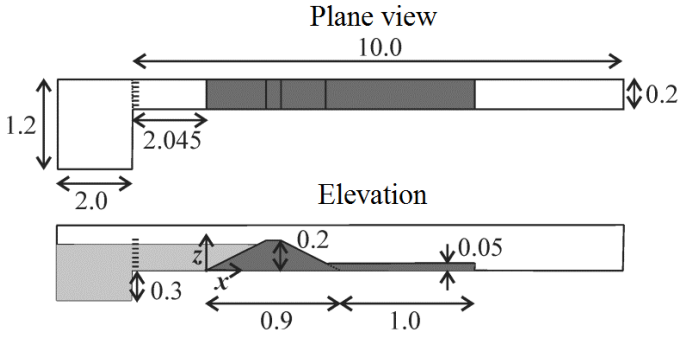


Figure 3: Flume and dike geometry (Van Emelen 2014).

The flume is 0.2 m wide, the initial dike has upstream and downstream slopes of 2:1, and the crest is 0.10 m long, at a level of 0.2 m above the bed of the flume. Downstream of the dike, a 0.05 m layer of sediment extends over 1.0 m. The sediment used to create the dike and the downstream layer, has a uniform size distribution with a median diameter $d_{50} = 0.61$ mm.

The upstream reservoir is filled continuously during the experiment with a constant inflow of $0.005 \text{ m}^3/\text{s}$. The experiment is considered to start when the water level reaches the dike crest and the overtopping starts. Downstream of the flume, water and sediment flow freely into a lower reservoir, preventing any influence on the upstream flow.

Different stages of the dike evolution are illustrated in Figure 4. The water level in the reservoir upstream the dike increase due to the constant discharge supplied at the channel inlet. Once the water level reaches the top of the dike, the water overtops the crest (Figure 4a). The water flows along the downstream slope, causing gradual erosion and rounding of the top of the dike, and causing deposition at the toe of the dike (Figure 4b). The downstream dike slope flattens until reaching an equilibrium shape (Figure 4c). The final state consists of a nearly horizontal sand layer.

The breaching hydrograph, measured from the time evolution of the water level in the upstream reservoir, is illustrated in Figure 5. During the first stage of fast erosion of the dike, the discharge increases until reaching a peak discharge of $0.009 \text{ m}^3/\text{s}$, and then decreases towards the constant inflow discharge.

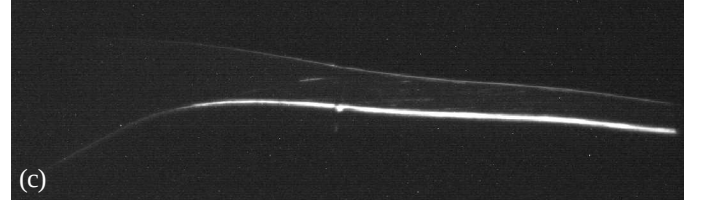
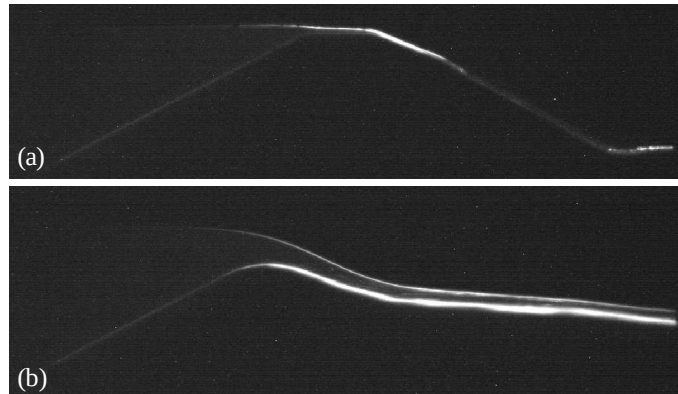


Figure 4: Dike evolution during breaching experiment: (a) $t = 2.66$ s, (b) $t = 10.06$ s, (c) $t = 26.56$ s

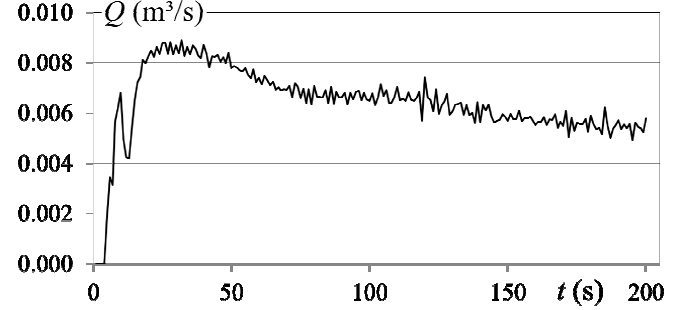


Figure 5: Measured discharge over the dike (Van Emelen 2014)

3 EVALUATION OF THE FRICTION TERM

Based on the shallow-water equations, different mathematical models exist for the representation of flow involving sediment transport and morphological evolution of the bed (e.g. Soares-Frazão et al., 2012 for an overview of different modelling approaches). For slow transient flows where inertia effects due to the entrainment of large amounts of sediments in the flowing layer can be neglected, the bed evolution is accounted for by an Exner-type equation for the continuity of the sediment. The resulting system of equations reads (Cunge and Perdreau, 1973)

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (3a)$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{q^2}{h} + g \frac{h^2}{2} \right) = g h (S_0 - S_f) \quad (3b)$$

$$\frac{\partial z_b}{\partial t} + \frac{1}{1 - \epsilon_0} \frac{\partial q_s}{\partial x} = 0 \quad (3c)$$

where h is the water depth, q is the unit discharge passing through the section, g is the gravitational acceleration, $S_0 = -\partial z_b / \partial x$ is the bed slope along x , the main direction of the flow, S_f is the friction slope, z_b is the bed elevation, ϵ_0 is the bed porosity, and q_s is the sediment discharge. The bed shear stress is usually expressed as $\tau = \rho g h S_f$ where ρ is the density of the fluid.

Starting from system (3), it is possible to obtain an expression for the friction slope S_f that depends on all the other variables of the flow, including their time and space evolution. However, the parameters for the calculation of S_f from system (3) are not all

directly measurable and have thus to be deduced from the experimental data using appropriate assumptions (Becker and Yeh, 1972). In this paper, three methods based on equations (3) are proposed, and their results will be compared.

3.1 Direct evaluation from the momentum equation

The first method applies directly the momentum equation (3b), where the unit discharge q was first developed as $q = uh$. The friction term is then explicitly written as a function of the time variation of the water discharge, and of the spatial variation of the water depth, velocity and water level, considering that $z_w = z_b + h$.

$$S_f = -\frac{1}{gh} \frac{\partial q}{\partial t} - \frac{u^2}{gh} \frac{\partial h}{\partial x} - 2 \frac{u}{g} \frac{\partial u}{\partial x} - \frac{\partial z_w}{\partial x} \quad (4)$$

In order to evaluate the spatial derivatives of h and z_w on the right-hand side of (4), the measured water and bed profiles are used. Indeed, extracting these values from the filmed images (Figures 2 and 4) a sufficient spatial discretisation can be obtained to evaluate these derivatives accurately. The time evolution of the discharge is directly obtained from the known flow hydrograph in the considered experimental cases. The velocity is calculated as $u = q/h$, using the direct measurements of h .

3.2 Method of Song and Graf (1996)

Assuming a power law for the velocity profile in open-channel flows, Song and Graf (1996) derived an expression for the direct evaluation of the friction term S_f considering the Boussinesq coefficient β that accounts for the non-uniformity of the velocity profile. In the present case, the vertical distribution of the velocity is assumed to be constant, and thus $\beta = 1$. As a result, the model proposed by Song and Graf (1996) reduces to

$$S_f = -\frac{1}{gh} \frac{\partial q}{\partial t} + 2 \frac{u}{gh} \frac{\partial h}{\partial t} + \frac{u^2}{gh} \frac{\partial h}{\partial x} - \frac{\partial z_w}{\partial x} \quad (5)$$

where now the time derivative of the water depth h appears instead of the spatial derivatives of the velocity u . With a sufficient density of filmed images, it is possible to extract also time derivatives from the measurements. It must be noted that (4) and (5) are equivalent, and that (5) can be obtained also from (4) after some mathematical developments using the continuity equation (3a).

3.3 Mixture-layer model

The mixture-layer model (Cao et al. 2004) for flows involving sediment transport and morphological evolution of the bed is written as

$$\frac{\partial z_w}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (6a)$$

$$\begin{aligned} \frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{q^2}{h} + g \frac{h^2}{2} \right) &= g h (S_0 - S_f) \\ -\frac{1}{2} \frac{\rho_s - \rho_w}{\rho} g h^2 \frac{\partial C}{\partial x} + \left(\frac{\rho_0}{\rho} - 1 \right) u \frac{\partial z_b}{\partial t} \end{aligned} \quad (6b)$$

$$\frac{\partial z_b}{\partial t} = -\frac{E - D}{1 - \epsilon_0} \quad (6c)$$

where ρ_s and ρ_w are respectively the solid and water densities, C is the sediment concentration within the fluid, ρ_0 is the density of the bed and ρ is the depth averaged density of the water-sediment flow mixture, E and D are respectively the erosion and deposition rate of sediments.

These equations differ from system (3) by several aspects. First, the water level and not only the water depth is considered in the continuity equation (6a), which might be more appropriate in cases of significant time evolution of the bed. Secondly, the last two terms in the right hand side of the momentum equation (6b) account for the presence of sediment in the flow and play a predominant role in case of rapid varying flows with significant sediment entrainment, such as dam-break flows, for which the sediment concentration can be very important (Cao et al. 2004). However, following the experience of numerical simulations by Brufau et al. (2000) with a similar model, it was shown that the density variation had only a very limited influence on the results in a wide range of examples of practical interest. Finally, the bed evolution in (6c) is calculated considering non-equilibrium between erosion and deposition, instead of a sediment flux q_s as in (3c).

In the case of the experiments considered here, the density variation in the flow can be assumed to be negligible, because sediment transport occurs only by bed load. So, neglecting the two last terms of (6b), the following formulation for the friction term is obtained:

$$S_f = -\frac{1}{gh} \frac{\partial q}{\partial t} + 2 \frac{u}{gh} \frac{\partial z_w}{\partial t} + \frac{u^2}{gh} \frac{\partial h}{\partial x} - \frac{\partial z_w}{\partial x} \quad (7)$$

Expression (7) only differs from (5) by the fact that the spatial derivative of the water level is considered in the second term on the right-hand side instead of the water depth.

4 RESULTS

To obtain the values of S_f , all derivatives in Eqs. (4), (5) and (7) need to be calculated from the pictures of

the experiments, for all variables of interest (h , z_b , z_w , u , q).

Using the pictures of the experiment, an automatic detection of the free-surface and of the bed elevation (Soares-Frazão et al., 2007) is first applied to extract the values of z_b and z_w . This allows to obtain the water depth $h = z_w - z_b$. Knowing the unit discharge q , the flow velocity is determined as $u = q / h$.

In the case of the knickpoint experiment, the water discharge q was constant, and this assumption yields a good estimate of the depth-averaged velocity. In the case of the dike overtopping, it was assumed that the instantaneous discharge over the dike was representative of the discharge throughout the flow at the considered time. The validity of this assumption was checked as described below.

First, as the considered transient flows were quite slow, the images were acquired at a frequency of about 5 Hz. This allows calculating time derivatives of the variables, for example the water depth h , as

$$\frac{\partial h}{\partial t} = \frac{h^{(t)} - h^{(t-\Delta t)}}{\Delta t} \quad (8)$$

with $\Delta t \cong 0.2$ s between two successive images. The spatial derivatives are calculated in centered way, on a single image, using $\Delta x = 0.01$ m. Before calculating the spatial derivatives, a simple moving average filter is applied to the measured water and bed levels to eliminate spurious peaks in the extracted values. No time averaging is applied in the calculation of the spatial derivatives.

The assumption made for the discharge implies that throughout the flow

$$\frac{\partial q}{\partial x} = \frac{\partial(uh)}{\partial x} = h \frac{\partial u}{\partial x} + u \frac{\partial h}{\partial x} = 0 \quad (9)$$

or that

$$h^{(x)} \frac{u^{(x+\Delta x)} - u^{(x-\Delta x)}}{2\Delta x} + u^{(x)} \frac{h^{(x+\Delta x)} - h^{(x-\Delta x)}}{2\Delta x} = 0 \quad (10)$$

For the knickpoint experiment, where the inflow discharge is constant, equation (10) estimated using the measured values is well verified. For the dike overtopping case, slight discrepancies are observed, showing that the assumption of considering $q(x) = cst$ for a fixed t is not completely valid. However, as no other measurements are available, the assumption will be made for the present analysis.

Finally, from the experimental values of S_f , the value of the Manning coefficient will be extracted using (1) and compared to values obtained using (2).

4.1 Knickpoint experiment

The first experiment (Juez et al., submitted) is characterized by a wetting front that progressively inundates the flume, establishing a supercritical

flow, and in the meanwhile erodes the bottom until reaching a final equilibrium slope of 7.8 %. The analysis of S_f focuses on two crucial moment of the experiment: the first one at the beginning of the erosion of the bed by the flow (Figure 2b) and the second one when a constant slope is established (Figure 2c).

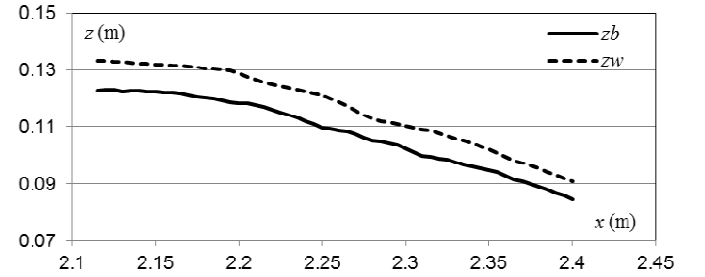


Figure 6: Measured bed and water profiles at $t = 3.122$ s

At time $t = 3.122$ s, as illustrated in Figure 6, the initial knickpoint can still be identified. The friction slope is then calculated using the models of (4), (5) and (7), yielding the results of Figure 7. All methods give very close results, which proves that the time and space derivatives calculated from the experimental profiles are consistent. These estimated values of S_f however present quite an important scatter. This might be due to the initial scatter in the experimental values of the variables extracted from the images and these irregularities are thus even amplified when calculating the derivatives required to apply the methods (4), (5) and (7). It must be noted that therefore, in some cases, negative values of S_f may arise from the calculations. These were discarded from the analysis.

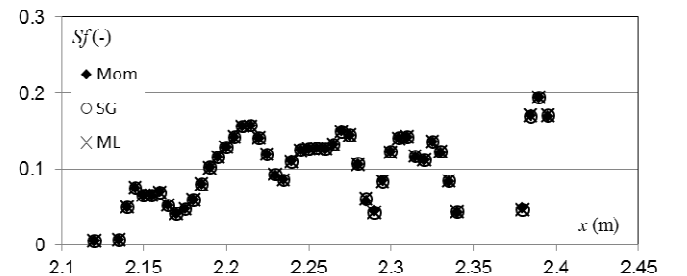


Figure 7: Friction slope calculated for $t = 3.122$ s

Using the results of Figure 7, it is now possible to obtain an estimate of the corresponding Manning friction coefficient. This is illustrated in Figure 8, where it can be observed that its average value is 0.025 with a standard deviation of 0.007. Using the formulations (2) to determine n only from the grain size yields $n = 0.017$ for Strickler (2a), $n = 0.013$ for Meyer-Peter and Müller (2b) and $n = 0.021$ for Bray (2c). These values are lower than the experimental one, which could indicate that the grain friction is not the only effect in the global losses for this flow.

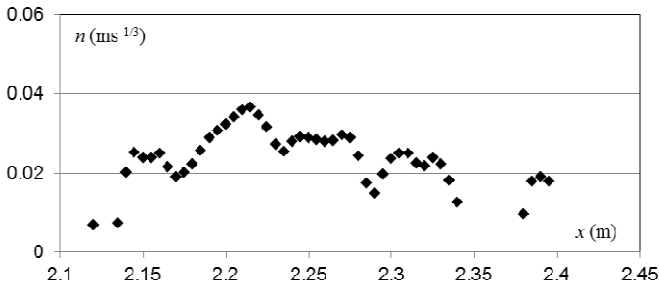


Figure 8: Values of the Manning coefficient extracted from S_f at time $t = 3.122$ s

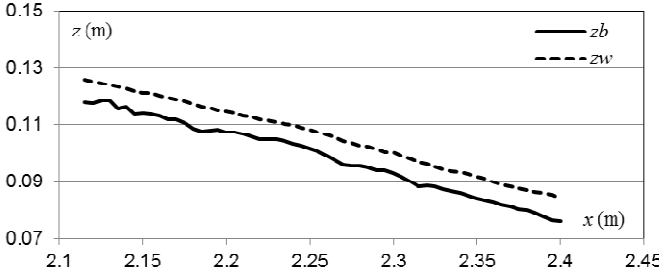


Figure 9: Measured bed and water profiles at $t = 16.532$ s

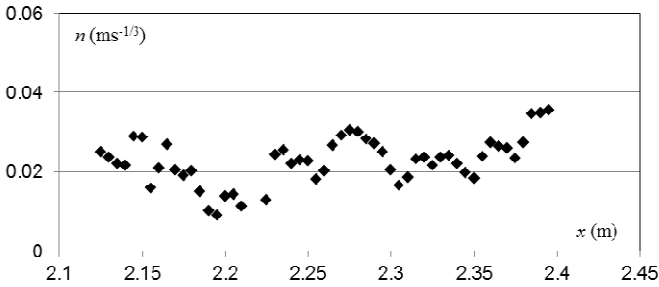


Figure 10: Values of the Manning coefficient extracted from S_f at time $t = 16.532$ s

The same analysis is performed at $t = 16.532$ s, when the knickpoint has completely vanished. Results are provided in Figures 9 and 10 for the flow profile and the corresponding Manning coefficient, respectively. Here, the average value of n obtained from the analysis is 0.022 with a standard deviation of 0.006. This value is lower than the previous one and thus closer to the values obtained using (2), which would indicate that the flow becomes closer to a uniform flow, corresponding to the conditions for which the Manning equation was initially developed. However, such preliminary conclusion should be strengthened with further experiments.

4.2 Dike-overtopping experiment

The results of the second experiment focus on three stage of the dike overtopping end erosion. The first moment is taken 9.108 s after the beginning of dike overtopping, the second after 40.034 s and the last at the final part of the experiment, after 158.050 s. The measured bed and water levels are illustrated in Figure 11 for these three times: the bed slope progressively reduces and the water depth increases, while bed material is transported, resulting in a lowering of the bed elevation.

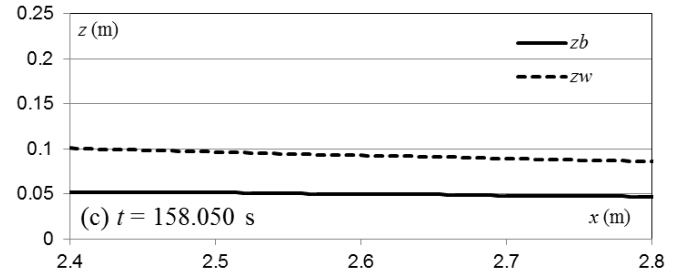
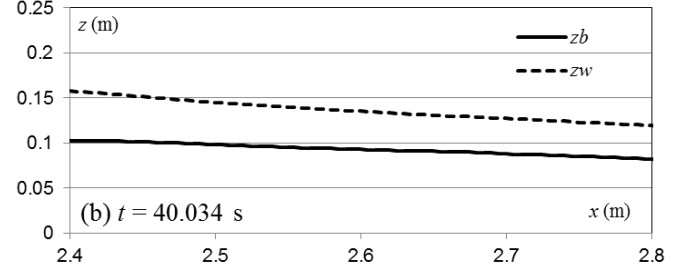
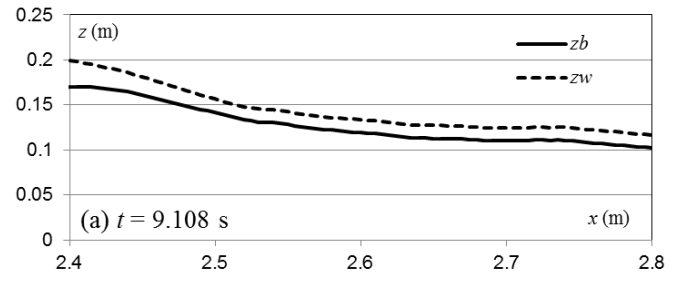


Figure 11: Measured water and bed elevation during the dike-overtopping experiment

Applying the procedure described before to calculate S_f using (4), (5) and (7) yields the results of Figures 12 and 13 for $t = 158.050$ s. At that time, the discharge is estimated as being $Q = 0.00534$ m³/s (Figure 5). Fluctuations of S_f can be observed, that could indicate the progressive formation of bed-forms, which were indeed observed at later times and further downstream in the experiments. From these values of S_f , using again the results from (5), an average value of the Manning coefficient is obtained as $n = 0.021$ with a standard deviation of 0.006 (Figure 13). The results for the three considered times of the experiments are summarized in Table 1, while the values of n obtained from (2) are 0.014, 0.011 and 0.017, again mostly below the values deduced from the experiments. It must be noted however that at time $t = 9.108$ s, the measured Manning coefficient is lower, which might indicate that at that time, as the flow unsteadiness is more significant, the assumption of a constant discharge all over the considered flow region may not hold.

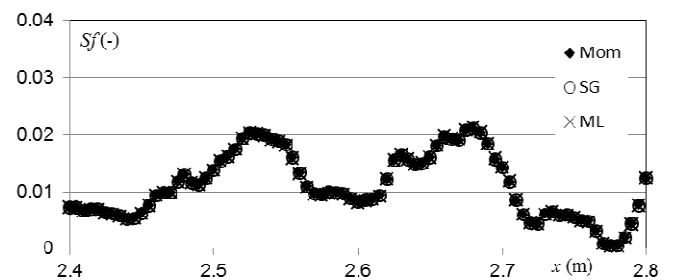


Figure 12: Friction slope calculated for $t = 158.050$ s

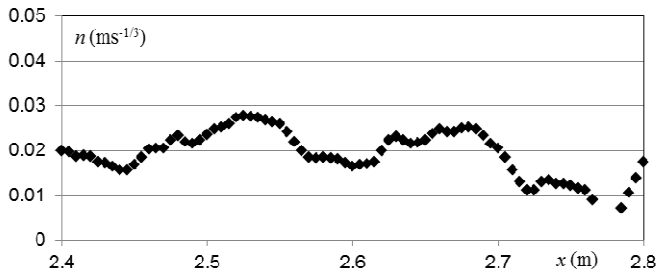


Figure 13: Values of the Manning coefficient extracted from S_f at time $t = 158.050$ s

Table 1: Manning coefficient evaluated from the dike-overtopping experiment

t (s)	n ($\text{ms}^{-1/3}$)	STD ($\text{ms}^{-1/3}$)
9.108	0.010	0.003
40.034	0.019	0.006
158.050	0.021	0.006

5 CONCLUSIONS AND FUTURE WORK

Three different shallow-water models were used to obtain estimates of the friction slope S_f from experimental measurements of flow discharge, water level and bed elevation in two moderate transient flow cases involving sediment transport. Among the three shallow-water models considered, one only involves spatial derivatives; the two other involve also temporal derivatives of the flow variables. These values were extracted from the measurements obtained using digital-imaging techniques. The three methods provide consistent results for the calculation of the friction slope.

Considering the Manning formula, estimates of the Manning friction coefficient n was derived from the measured friction slopes and compared to the values obtained using empirical relations based only on the grain diameter. In both cases, the values of n obtained from the experimental calculation of S_f from the shallow-water models are higher than the values obtained using the empirical relations based on the grain diameter. Also, the values extracted from the experiments tend to evolve in time, which would indicate that unsteady flow effects are present and influence the friction slope.

However, quite a significant scatter was observed in the values of the flow variables and estimates of S_f from the measurements. Therefore, further work will concentrate first on improve data treatment to remove the effects of noise in the measurements. This however needs to be done carefully in order to avoid spurious effects due to excessive smoothing or interpolation of the data. The analysis will then also be extended to consider all the time steps in the experiments with the aim of finding some trends in the evolution of S_f and n , if any.

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