Fertility Differentials and Uncertainty: An Economic Theory of the Minority Status Hypothesis

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Fertility Differentials and Uncertainty: An Economic Theory of the Minority Status Hypothesis

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Abstract

We revisit the question of why fertility behaviors and educational decisions appear to vary systematically across ethnic groups. We assess the possibility that fertility differentials across groups remain even though their socio-economic characteristics are similar. More specifically, we consider that parents' fertility decisions are affected by the uncertainty concerning the future economic status of their off-spring. We assume that this uncertainty varies across groups and is linked to the size of the group one belongs to. The transition path gives support to the minority status hypothesis according to which minority members usually have a higher fertility facing low potential for social mobility but may in some instances strategically decrease their fertility.

JEL Classification: D81, J13, J15

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1 Introduction

Differences in fertility behavior and educational attainment between minority and majority populations around the world are persistent and have existed during most of history. Ethnic minorities are often found to exhibit higher fertility levels and lower educational investments than the majority counterpart, it is however the contrary in some cases. For instance, in the U.S. in 2006, the Black and Hispanic minorities had a total fertility rate[1] of respectively 2,155 and 2,960 children, while the White had 2,101 and the Asian 2,043. As already ascertained in the quality/quantity trade-off literature, fertility is inversely related to educational investment in children. Data for 2006[2] gives that 81,7% of the Black and 59,3% of the Hispanic were high school graduates or more, while 86,1% of the White and 87,4% of the Asian[3].

Existing models of endogenous fertility give theoretical support to the evidence that choices of fertility and education are interdependent. Parents face a tradeoff between the number of children they have and the amount of resources they devote to education of each child. Socio-economic characteristics of the parents determine the outcome of this trade-off. In other words, if characteristics of parents converge then so do the fertility rates. For instance, in de la Croix and Doepke (2003, 2004), fertility differentials collapse as inequality across households disappears. Persistent differentials in fertility and educational investment therefore only arise if socio-economic characteristics do not converge across groups. For example, Chimswhick (1988) suggests that different rates of return from investment in schooling are responsible for persistent gaps in educational investment across ethnic groups.

More recently, an emerging strand of the literature has underlined the important role of culture in fertility choices. For instance, Fernandez and Fogli (2005, 2006) find that cultural proxies are significant determinants of fertility controlling for socio-economic characteristics. For instance, fertility and labor force participation rates in

[1]The total fertility rate is the number of births that 1000 women would have in their lifetime if, at each year of age, they experienced the birth rates occurring in the specified year. Source: U.S. National Center for Health Statistics

[2]Source: U.S. Census Bureau

[3]These differentials become even bigger when we look at the proportion of college graduates or more.

the origin country of migrants or number of siblings of parents have been found to be significantly correlated to fertility decisions of migrant women in the US. In this view, fertility differentials may persist as beliefs and norms of different groups evolve differently than socio-economic characteristics.

A third factor considered in the literature to be an important determinant of fertility choices is uncertainty. Kalemli-Ozcan (2003) builds a stochastic model of endogenous fertility where the number of surviving children is a random variable. She finds that the risk stemming from the loss of a child leads to a precautionary demand for children that leads parents to overshoot their desired fertility. A key result is that an exogenous decrease in the infant mortality rate reduces the uncertainty faced by parents and thus decreases the precautionary demand in favor of educational investment. On the other hand, Hondroyiannis (2010) argues that uncertainty about macro-economic variables may be detrimental to child-bearing as responsible parents would not decide to have one more child if they were to face a high risk of unemployment or low income in the future. He uses a panel data of European countries to show that measures of economic uncertainty such as output volatility and the unemployment rate are negatively related to fertility rates. In such frameworks, the fact that groups face different levels of uncertainty would explain persistent fertility gaps.

The sociological literature has viewed the behavior of minorities in terms of fertility and educational decisions from two complementary perspectives. The first, know as *assimilation* or *characteristic hypothesis* asserts that differences in fertility arise from differences in socio-economic achievement. According to this view, as a minority group reaches the socio-economic characteristics of the majority group, fertility differences disappear[4]. A second approach, namely the *minority status hypothesis*, argues that minority group membership can have an independent effect on fertility. This theory, as formulated in the seminal work of Goldscheider and Uhlenberg (1969) admits the relevance of socio-economic factors in explaining differences in fertility behaviors between majority and minority groups, but highlights the importance of several other factors: desire for social and economic mobility, psychological insecurity, segregation,

[4]See for instance Gordon (1964, 1978), Ryder (1973), Bean and Marcum (1978), Bean and Swicegood (1985), Trovato (1987) for an exhaustive discussion on the social characteristics hypothesis in sociology.

intragroup and intergroup socio-economic relationships, that may induce fertility differences to persist even though socio-economic characteristics of the minority converge towards those of the majority.

In this view, it may be that minority membership has per se an effect on fertility. One possible reason might be the presence of barriers to minorities for upward economic mobility. These obstacles may lead minority members to modify their fertility and educational investment in order to compensate for potential limitations to economic achievement even when social, economic, and demographic attributes for majority and minority populations are very similar. On the one hand, according to Goldscheider and Uhlenberg (1969) *'the residual lower fertility of minority groups may result from the insecurity associated with minority group status'.* This mechanism of fertility limitation among minority members operates only when it is thought as a means to improve socio-economic perspectives[5]. On the other hand, when the obstacles to upward mobility are substantial enough, the marginality and the insecurity associated with the minority group status may induce minority members to choose to increase their fertility with respect to majority members with similar characteristics as a way to self-insure against potential discriminations.

We develop a model of endogenous fertility that gives theoretical support to the minority status hypothesis. This theory has been tested in several empirical studies but, to our knowledge, it has never been investigated in a theoretical framework[6]. We revisit the question of why fertility behaviors and educational decisions appear to vary systematically across ethnic groups. We assess the possibility that fertility differentials across groups remain even though their socio-economic characteristics are similar. In this work we claim that, to this purpose, it is crucial to take into account uncertainty about future earnings or expected socio-economic conditions.

The data on completed fertility by race and hispanic origin and educational attainment shown in Table 1 seem to support this hypothesis. Indeed, on the one hand, for

[5]In a study about Chinese American women, Espenshade and Ye (1994) explain the lower fertility of this ethnic group with the sacrificial effort in their pursuit of social and economic equality.

[6]See Sly (1970), Roberts and Lee (1974), Jiobu and Marshall (1975), Johnson and Nishida (1980), Boyd (1994).

Table 1: Children ever born per 1000 women aged 35-44 by race and hispanic origin and educational attainment in the U.S. 2006.

Education	White	Asian	Hispanic	Black
...Total	1.764	1.765	2.286	1.978
...Not high school graduated	2.197	2.689	2.758	2.735
...High school (4 years)	1.860	1.905	2.244	2.105
...Some college (no degree)	1.829	1.848	1.970	1.827
...Bachelor degree	1.626	1.596	1.834	1.648

Source: US Census Bureau, Current Population Survey, 2006.

low levels of education, all the minority groups exhibit a higher fertility than the White. On the other hand, for higher levels of education, some groups still exhibit a higher fertility (the Hispanic) while others adopted a fertility rate that is similar to that of the majority or even lower (the Black and the Asian respectively). Notice that the data also clearly shows the interdependency between fertility and education: the more educated women are, the lower the fertility rate.

In our economy, parents decide on their number of children and on the education level of each child. We take a standard model of endogenous fertility, but accounting for the fact that parents care about the future earnings of their children. Unlike the existing literature, this paper stresses the role of uncertainty and in particular of the insecurity about the expected economic status of children[7]. In our opinion, uncertainty is crucial to explain fertility differentials across groups because minority membership might bring on an independent effect on parents' optimal behavior. Fewer obstacles to upward socio-economic mobility reduces the psychological insecurity and the marginality of minority members, which might stimulate the investment in the human capital of children instead of fertility. We use a restricted pool matching to model the uncertainty parents face with respect to job market outcomes of their children. It provides first a convenient link between the level of uncertainty faced by parents and the size of the group they

[7]Some papers already introduced uncertainty in endogenous fertility models, but, to our knowledge, the uncertainty focuses on the survival rate, never on the expected economic status of children.

belong to, which is crucial to understand the interplay between fertility and the group size dynamics. Furthermore it allows us to remain very general in our conclusions as random matching is nothing but a particular case of the restricted pool matching.

As a result, this paper formulates a contribution to the theoretical literature on endogenous fertility: we argue that non-convergence in the socio-economic characteristics might not be the only determinant of persistent fertility gaps. We claim that minority membership may per se influence fertility and educational investment behaviors in various directions, which constitutes a theoretical support of the minority status hypothesis. We link group size and uncertainty about future economic status of the children and introduce the fact that this uncertainty may influence fertility decisions. We then investigate the implications on population dynamics. Section 2 introduces the static model, while section 3 presents the matching process. Section 4 solves the dynamic system and section 5 shows a computational experiment. Section 6 concludes.

2 The static model

Let us take a simple OLG model of endogenous fertility with uncertainty, as given for instance in Kalemli-Ozcan (2003), where, instead of the survival rate, we consider the future wage rate of children as stochastic. Parents maximize their lifetime utility choosing their own consumption c_t, the number of offsprings n_t and the schooling time per child r_t. Let us assume a production function that is linear in human capital, as it is often assumed in this literature. The labor market is competitive, so that agents act as price takers with respect to the wage rate per unit of human capital. Future earnings of the children are therefore well captured by the expression $h_{t+1}w^e_{t+1}$, where h_{t+1} is the human capital of children and w^e_{t+1} their expected wage rate per unit of human capital:

$$\max_{c_t,n_t,r_t} (1-\gamma)\ln(c_t) + \gamma\ln(n_t h_{t+1} w^e_{t+1}) \tag{1}$$

s.t.

$$c_t = (1 - n_t(\phi + r_t))\, h_t w_t \tag{2}$$

$$h_{t+1} = \eta r_t^{\tau} h_t^{\psi} \tag{3}$$

(2) is the budget constraint and (3) the human capital production function. γ is the altruism parameter, ϕ represents the fixed time cost of bearing a child, it is thus taken between 0 and 1. ψ and τ are strictly positive parameters whose sum is less or equal than 1 so that returns to parental human capital and education are decreasing; η is a strictly positive multiplier.

Taking the first order conditions and checking the second order conditions are verified, we obtain the following optimal decisions:

$$r_t^* = \frac{\tau\phi}{1-\tau} \tag{4}$$

$$n_t^* = \frac{\gamma(1-\tau)}{\phi} \tag{5}$$

Notice that optimal decisions do not depend on parental human capital nor expected wage rate. Moreover, we have that decisions are left unaffected by any variation in the expected wage rate or parental human capital. The intuition is straightforward (and was already given by Kalemli-Ozcan (2003) about the survival probability) in the sense that any exogenous change in w_{t+1}^e represents a change in the price of having a well-off children and this price change has two contradicting effects: income and substitution, which exactly cancel out with logarithmic utility functions. This means that we do not observe a quality/quantity trade-off in the sense that more educated parents will not prefer having less children in order to educate them more, at least as long as w_{t+1}^e is certain[8]. Studying the logarithmic utility case cancels out the effect of parental human capital on optimal decisions and allows us to focus essentially on the role of uncertainty on differential fertility.

Let us now introduce uncertainty assuming w_{t+1}^e is a random variable with expected value μ and variance σ, reflecting the fact that job market opportunities are expected to be uncertain. Let us define $W_{t+1} = \sum_{\nu=0}^{n_t} h_{t+1} w_{\nu,t+1}^e$ as the total income of a cohort of

[8]The logarithmic formulation implies that a fixed fraction of parental time (and thus income) is devoted to children, so that consumption is never affected. Furthermore, if the price of a well-off children falls (through an increase in the expected wage rate), the relative price of education with respect to child bearing has not changed, which is why the trade-off between quality and quantity of children is not affected by the expected outcome on the labour market and that optimal decisions remain the same.

children. It is a random variable with following expected value and variance:

$$
\begin{aligned}
E[W_{t+1}] &= n_t h_{t+1} \mu \\
V[W_{t+1}] &= n_t (h_{t+1})^2 \sigma
\end{aligned}
$$

Let us write here the expected utility of parents:

$$
E_t U_t = \int_0^{n_t} \{(1-\gamma)\ln(c_t) + \gamma \ln(W_{t+1})\} f(W_{t+1}) dW_{t+1} \tag{6}
$$

with $f(W_{t+1})$ the density function of W_{t+1}. Note that fertility n_t is the upper bound of the integral, meaning that increasing fertility increases the range over which one derives utility.

In order to solve this program, we use the Delta method, that is we take a Taylor approximation of this expected utility around its mean. First taking the Taylor approximation around the mean and dropping the time subscripts, we get:

$$
\begin{aligned}
U(W) &= U(nh\mu) + (W - nh\mu) U_W(nh\mu) \\
&\quad + \frac{(W-nh\mu)^2}{2} U_{WW}(nh\mu) + o(W^2)
\end{aligned}
$$

Now, taking the expected value of this, we obtain:

$$
\begin{aligned}
EU(W) &= U(nh\mu) + \overbrace{E[W - nh\mu]}^{=0} U_W(nh\mu) \\
&\quad - \frac{\overbrace{n(h)^2\sigma}^{=V(W)}}{2!} \frac{\gamma}{(nh\mu)^2} \\
&= U(nh\mu) - \frac{\sigma\gamma}{2n\mu^2}
\end{aligned} \tag{7}
$$

The expected value of the difference between a random variable and its mean is zero, that of the square of this difference is its variance. The second term consequently cancels out, while the third remains, representing the risk effect.

Let us take the first order conditions of the maximization of the expected utility given in expression (7) subject to the constraints (2) and (3). We obtain:

$$\frac{\gamma}{n_t} - \frac{(1-\gamma)(\phi + r_t)}{n_t(\phi + r_t) - 1} + \frac{\gamma\sigma}{2n_t^2\mu^2} = 0 \tag{8}$$

$$\frac{\gamma\tau}{r_t} - \frac{(1-\gamma)n_t}{n_t(r_t + \phi) - 1} = 0 \tag{9}$$

The optimal choices n^{**} and r^{**} are given by the following expressions:

$$\begin{aligned} n^{**} &= \frac{1}{4\mu^2\phi}(\sqrt{\gamma(8\sigma\mu^2\phi + \gamma A^2)} - \gamma A) \\ r^{**} &= \frac{4\mu^2\phi\gamma\tau}{\sqrt{\gamma(8\sigma\mu^2\phi + \gamma A^2)} - \gamma A} \end{aligned}$$

with

$$A = 2\mu^2(\tau - 1) + \sigma\phi$$

Proposition 1 *Under uncertainty, parents engage in a self-insurance mechanism against risk overshooting fertility with respect to the certainty case. Fertility increases with the risk term, that is when the variance increases and/or when the expected value decreases.*

$$\begin{aligned} n^{**} &\geq n^* \\ \frac{\partial n^{**}}{\partial \sigma} &> 0 \\ \frac{\partial n^{**}}{\partial \mu} &< 0 \end{aligned}$$

All these relations apply to r^{**} with opposite signs.

Proof First, notice that when $\sigma = 0$, the FOCs of the problem under uncertainty amount to those under certainty. It therefore follows that optimal decisions are the same in this case.

From equation (9), we obtain an expression of r_t that we substitute in equation (8). We obtain:

$$\begin{aligned} r_t &= \frac{\gamma\tau(1 - n_t\phi)}{n_t(1 - \gamma + \gamma\tau)} \\ G(n_t, \sigma, \mu) &= 0 \end{aligned}$$

with

$$G(n_t, \sigma, \mu) = -\frac{2\mu^2 n_t^2 \phi - \gamma(1 - n_t\phi)\sigma + 2\mu^2 n_t(1-\tau)}{2\mu^2 n_t^2(1 - n_t\phi)}$$

We suppress the time subscript and apply the implicit function theorem on G, we obtain the following expression:

$$\begin{aligned}
\frac{\partial n}{\partial \sigma} &= -\left(\frac{\partial G}{\partial n}\right)^{-1} \frac{\partial G}{\partial \sigma} \\
&= -\left(-\frac{\mu^2 n^3 (1-n\phi)^2}{(\mu^2 n^3 \phi^2 + \gamma((1-n\phi)^2\sigma + \mu^2 n(1-2n\phi)(1-\tau))}\right)\frac{\gamma}{2\mu^2 n^2} \\
&> 0 \quad \text{iff} \quad n < \frac{1}{2\phi}
\end{aligned}$$

Notice that the budget constraint (2) gives an implicit upper bound on fertility that is: $n < \frac{1}{\phi}$. It is therefore straightforward to observe that this derivative is always positive. We therefore get that n^{**} is increasing in σ and always greater or equal than n^*.

We now apply again the implicit function theorem to $G(n, \sigma, \mu) = 0$:

$$\begin{aligned}
\frac{\partial n}{\partial \mu} &= -\left(\frac{\partial G}{\partial n}\right)^{-1} \frac{\partial G}{\partial \mu} \\
&= -\left(-\frac{\mu^2 n^3 (1-n\phi)^2}{(\mu^2 n^3 \phi^2 + \gamma((1-n\phi)^2\sigma + \mu^2 n(1-2n\phi)(1-\tau))}\right)\frac{-\gamma\sigma}{\mu^3 n^2} \\
&< 0 \quad \text{iff} \quad n < \frac{1}{2\phi}
\end{aligned}$$

From which it follows that n^{**} is decreasing in μ.

We can write r^{**} as follows:

$$r^{**}(\sigma, \mu) = \frac{\gamma\tau(1 - n^{**}(\sigma,\mu)\phi)}{n^{**}(\sigma,\mu)(1-\gamma+\gamma\tau))}$$

Taking the derivative with respect to σ and μ, we get:

$$\begin{aligned}
\frac{\partial r^{**}(\sigma,\mu)}{\partial \sigma} &= -\frac{\gamma\tau\frac{\partial n^{**}(\sigma,\mu)}{\partial\sigma}}{(1-\gamma+\gamma\tau)n^{**}(\sigma,\mu)^2} < 0 \\
\frac{\partial r^{**}(\sigma,\mu)}{\partial \mu} &= -\frac{\gamma\tau\frac{\partial n^{**}(\sigma,\mu)}{\partial\mu}}{(1-\gamma+\gamma\tau)n^{**}(\sigma,\mu)^2} > 0
\end{aligned}$$

r^{**} is therefore increasing in μ and decreasing in σ. ∎

Under uncertainty, only a risk term is added to the utility function with respect to the certainty case. This risk term is monotonically increasing in σ and decreasing in μ

and n_t. So the greater is the variance of the random variable and the lower its expected value, the higher is the perceived risk with respect to the future income of the cohort of children. Thus increasing one's number of children reduces the risk. In other words, parents use fertility as a self-insurance mechanism[9].

3 Matching

Let us now consider a specification for the random variable giving the expected wage rate per unit of human capital. We consider that everyone needs to match someone in the society in order to produce output and get paid, which represents a reasonable assumption about the job market. The production function is assumed to be linear in own human capital and to depend on the quality of the match. This match quality follows a random variable whose expected value and variance depend on whether you match someone from your own group or not. Everyone matches somebody with probability one, but we assume individuals may be biased towards matching someone belonging to their own group. Indeed, it is now established that networks play an important role in the job seeking process[10]. These networks usually appear within a given community. This is why, as in intermarriage models[11], agents will first try to match someone from their group and, only in a second attempt, they will match anybody in the society. More specifically, there is an exogenous probability π that an individual matches in the restricted pool of coworkers from the same group, in which case the outcome follows a random variable W_s with mean μ_s and variance σ_s. With probability $1-\pi$ instead, an agent of group i matches anybody in the society, that is someone from the same group with probability x_i, where x_i represents the proportion of group i agents in the society, and someone from another group with probability $1-x_i$, in which case the wage rate follows a random variable W_d, with mean μ_d and variance $\sigma_d \geq \sigma_s$. This assumption allows us to capture the fact that belonging to a group implies lower insecurity about the quality of the match inside this group. To support this assumption, Simon and

[9]The same feature appears in Kalemli-Ozcan (2003) about the infant mortality rate.
[10]See Simon and Warner, (1992).
[11]See Bisin and Verdier (2000).

Warner (1972) find empirical evidence that hiring through the old boy network reduces employer's uncertainty about worker productivity. The job market outcome for a given worker thus follows the following random variable:

$$W_i = \pi W_s + (1-\pi)(x_i W_s + (1-x_i) W_d)$$

where π proxies the bias towards own group and x_i is the proportion of group i agents in the society. We interpret a high value of π as a situation in which there are less opportunities and/or more costs for two agents from different groups to match, due to physical or cultural distance. For instance, it is often considered in labor economics that language creates an extra cost to match outside one's own group. Lang (1986) shows that a competitive labor market tends to minimize communication costs through segregation. The expected value of a match is given by: $x_i\mu_s + (1-x_i)\mu_d$ and the variance $x_i^2\sigma_s + (1-x_i)^2\sigma_d$. We therefore have that W_i follows a random variable with:

$$\begin{aligned} E(W_i) &= \pi\mu_s + (1-\pi)(x_i\mu_s + (1-x_i)\mu_d) && (10)\\ V(W_i) &= \pi^2\sigma_s + (1-\pi)^2(x_i^2\sigma_s + (1-x_i)^2\sigma_d) && (11) \end{aligned}$$

We can now rewrite the maximization program as follows:

$$\max_{c_{i,t}, n_{i,t}, r_{i,t}} (1-\gamma)\ln(c_{i,t}) + \gamma \ln(n_{i,t} h^e_{i,t+1} E(W_{i,t+1})) - \frac{\gamma V(W_{i,t+1})}{2n_{i,t}(E(W_{i,t+1}))^2} \qquad (12)$$

s.t.

$$\begin{aligned} c_{i,t} &= (1 - n_{i,t}(\phi + r_{i,t}))h_{i,t}w_{i,t}\\ h^e_{i,t+1} &= \eta r^\tau_{i,t} h^\psi_{i,t} \end{aligned}$$

The first order conditions with respect to $n_{i,t}$ and $r_{i,t}$ respectively are:

$$\begin{aligned} \frac{\gamma\left(\pi^2\sigma_s + (1-\pi)^2(x_i^2\sigma_s + (1-x_i)^2\sigma_d)\right)}{2n_{i,t}^2(\pi\mu_s + (1-\pi)(x_i\mu_s + (1-x_i)\mu_d))^2} - \frac{(\gamma-1)(r_{i,t}+\phi)}{n_{i,t}(r_{i,t}+\phi)-1} + \frac{\gamma}{n_{i,t}} &= 0 && (13)\\ \frac{n_{i,t}(1-\gamma)}{n_{i,t}(r_{i,t}+\phi)-1} + \frac{\gamma\tau}{r_{i,t}} &= 0 && (14) \end{aligned}$$

Observe that the first order condition with respect to $r_{i,t}$ (14) remains the same whatever the group affiliation, while, in the one with respect to $n_{i,t}$ (13), only the risk term (the second element) varies across groups. More specifically, agents from different groups take the same decisions if their risk term is equal. Otherwise, those with a larger risk term prefer to have a higher fertility and a lower investment in human capital. We derive the optimal fertility and education choices conditional on group affiliation:

$$n_i^{***} = f(x_i, \Lambda) \tag{15}$$

$$r_i^{***} = g(x_i, \Lambda) \tag{16}$$

where Λ represents all the parameters of the model. It is important to observe that the optimal decisions are only a function of group size and not of parental human capital. It allows us to obtain fertility differentials even though socio-economic characteristics of the parents are the same. Unlike existing literature, non convergence in socio-economic characteristics is not a necessary condition for fertility differentials.

4 Dynamics

Given initial conditions on human capital, $h_{i,0}$, and on population share $x_{i,0}$, an intertemporal equilibrium is a sequence of temporary equilibria of the static problem defined by equations (15) and (16) that satisfies for all $t \geq 0$ the following laws of motion:

$$h_{i,t+1} \equiv \xi(x_{i,t}, h_{i,t}) = \eta r_{i,t}^{\tau} h_{i,t}^{\psi} \tag{17}$$

$$x_{i,t+1} \equiv \chi(x_{i,t}) = \frac{x_{i,t} n_{i,t}}{x_{i,t} n_{i,t} + (1 - x_{i,t}) \overline{n}_{i,t}} \tag{18}$$

where $\overline{n}_{i,t}$ represents the average fertility of the other groups in the society. Notice that $x_{i,t+1}$ is a function of $x_{i,t}$, $\overline{n}_{i,t}$ and the parameters while h_{t+1} depends on $x_{i,t}$, h_t and the parameters. As a first step, we will consider the two-group case that simplifies $x_{i,t+1}$ down to a function of $x_{i,t}$ only. Indeed, in the two-group case, $\overline{n}_{i,t}$ depends only on $x_{i,t}$ while, in a N group generalization, it would depend also on the relative sizes of the other groups.

4.1 The two-group case

Let us now turn to the simple two-group case in order for us to derive analytically the steady states and their stability properties. From equation (18), one may see that for x_i to be at a steady state, one needs $x_i = 0$ or $x_i = 1$ or $n_i = \overline{n}_i$. For this last equality to occur in the two-group case, it is necessary and sufficient that the risk term be the same for the two groups. Indeed, agents from the two groups then solve the same problem and necessarily have the same fertility. We therefore have that $\overline{x}_i$ is a steady state if and only if $\overline{x}_i = 0$, $\overline{x}_i = 1$ or $\overline{x}_i$ is such that:

$$\frac{\gamma\left(\pi^2\sigma_s + (1-\pi)^2(\overline{x}_i^2\sigma_s + (1-\overline{x}_i)^2\sigma_d)\right)}{2(\pi\mu_s + (1-\pi)(\overline{x}_i\mu_s + (1-\overline{x}_i)\mu_d))^2} = \frac{\gamma\left(\pi^2\sigma_s + (1-\pi)^2(\overline{x}_i^2\sigma_d + (1-\overline{x}_i)^2\sigma_s)\right)}{2(\pi\mu_s + (1-\pi)(\overline{x}_i\mu_d + (1-\overline{x}_i)\mu_s))^2} \tag{19}$$

Equation (19) stems from the comparison between the risk term for an agent in group i and an agent in the other group. It may be simplified into a 3rd degree polynomial as follows [12]:

$$2\gamma(1-\pi)(2\overline{x}_i - 1)\left(a\overline{x}_i^2 + b\overline{x}_i + c\right) = 0 \tag{20}$$

with

$$\begin{aligned}
a &= 2(1-\pi)^2(\mu_s - \mu_d)\left((1-\pi)\mu_d\sigma_s + \mu_s(\sigma_d + \pi\sigma_s)\right) \\
b &= -a \\
c &= \mu_s^2\sigma_d(1-\pi) + 2\pi^3\mu_s^2\sigma_s + \mu_d^2\sigma_s(2\pi^3 - 4\pi^2 + 3\pi - 1) - 2\mu_d\mu_s\sigma_s(2\pi^3 - 2\pi^2 + \pi)
\end{aligned}$$

It can be inferred that $\overline{x}_i = 1/2$ is always a steady state and that there are at most two others. Indeed, observe that $\overline{n}_i = f(1 - x_i, \Lambda)$. Then the fact that 1/2 is a steady state is actually very intuitive in the sense that if the total population is divided into two halves, the risk terms are equal for both group, they necessarily have the same fertility and the shares remain constant at 1/2.

Now let us study the remaining 2nd order polynomial. The potential real solutions are of the following form:

$$\overline{x}_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1}{2} \pm \frac{\sqrt{\Delta}}{2a}$$

[12] See Appendix A1

Recalling $b = -a$, we obtain $\Delta = a^2 - 4ac$ and

$$\overline{x} = \frac{1}{2}\left(1 \pm \frac{\sqrt{a(a-4c)}}{a}\right)$$

A first condition for the existence of a real solution is the non negativity of the discriminant Δ. We therefore need $a(a-4c) \geq 0$ which is equivalent to:

$$\frac{c}{a} \leq 1/4 \tag{21}$$

A second condition in order to have an admissible steady state is that $\overline{x} \in [0;1]$. It can be shown that roots are admissible if and only if a and c are of the same sign and that, in this case, both roots are admissible[13].

$$ac > 0 \iff \overline{x}_1, \overline{x}_2 \in [0;1] \tag{22}$$

In order to discuss whether conditions (21) and (22) are fulfilled, we need to list different possible scenarii:

- $\mu_s > \mu_d$ implies $a > 0$

  We obtain in this case that condition (18) is never verified and therefore that there are no other admissible steady state than 0, 1 and 1/2.

- $\mu_s < \mu_d$ implies $a < 0$

  In this case, condition (21) and (22) are verified if and only if, respectively, (23) and (24) hold:

$$\underline{\sigma} \leq \sigma_d \tag{23}$$

$$\sigma_d \leq \overline{\sigma} \tag{24}$$

  with

$$\begin{aligned}
\underline{\sigma} &= \frac{\left((3\pi^2 - 2\pi + 1)\mu_d + (1-3\pi)\pi\mu_s\right)\sigma_s}{(1-\pi)\mu_s} > 0 \\
\overline{\sigma} &= \frac{1}{(1-\pi)\mu_s^2}\left(-(2\pi^3 - 4\pi^2 + 3\pi - 1)\mu_d^2 + 2\pi(2\pi^2 - 2\pi + 1)\mu_d\mu_s - 2\pi^3\mu_s^2\right)\sigma_s > \underline{\sigma}
\end{aligned}$$

[13]See Appendix A2

We therefore have 5 admissible steady states if and only if $\sigma_d \in]\underline{\sigma};\overline{\sigma}[$ and only 3 otherwise. Note that there always exists a parametrical space for which there are 5 steady states as $\overline{\sigma} > \underline{\sigma} > 0$ [14].

- $\mu_s = \mu_d$ implies $a = b = 0$

  and there are no other steady state than $0, 1/2$ and 1.

We therefore derive the following proposition:

Proposition 2 *$\overline{x} = 1/2$ is always a stable steady state unless $\mu_s < \mu_d$ and $\sigma_d < \underline{\sigma}$. It is moreover the unique stable steady state, unless $\mu_s < \mu_d$ and $\sigma_d \in [\underline{\sigma}, \overline{\sigma}]$, in which case 0 and 1 are also stable.*

Proof From the dynamic equation (18), we observe that 0 is a stable steady state if and only if the fertility of group i is strictly smaller than the fertility of the other group when x_i goes to 0. Likewise, 1 is a stable steady state if and only if the fertility of group i is strictly bigger than the fertility of the other group when x_i goes to 1. This amounts respectively to the following conditions: $\lim_{x_i \to 0} f(x_i, \Lambda) < \lim_{x_i \to 0} f(1 - x_i, \Lambda)$ and $\lim_{x_i \to 1} f(x_i, \Lambda) > \lim_{x_i \to 1} f(1 - x_i, \Lambda)$. In order to check under which conditions these inequalities are verified, we proceed to the comparison of the risk terms of the two groups in these two cases. The trivial steady state 0 (respectively 1), is therefore stable if and only if the risk term of group i is smaller (respectively bigger) than that of the other group when x_i goes to 0 (respectively 1). These conditions can be written respectively as follows:

$$\lim_{x_i \to 0} \frac{\gamma\left(\pi^2\sigma_s + (1-\pi)^2(x^2\sigma_s + (1-x)^2\sigma_d)\right)}{2\left(\pi\mu_s + (1-\pi)(x\mu_s + (1-x)\mu_d)\right)^2} < \lim_{x_i \to 0} \frac{\gamma\left(\pi^2\sigma_s + (1-\pi)^2(x^2\sigma_d + (1-x)^2\sigma_s)\right)}{2\left(\pi\mu_s + (1-\pi)(x\mu_d + (1-x)\mu_s)\right)^2} \tag{25}$$

$$\lim_{x_i \to 1} \frac{\gamma\left(\pi^2\sigma_s + (1-\pi)^2(x^2\sigma_s + (1-x)^2\sigma_d)\right)}{2\left(\pi\mu_s + (1-\pi)(x\mu_s + (1-x)\mu_d)\right)^2} > \lim_{x_i \to 1} \frac{\gamma\left(\pi^2\sigma_s + (1-\pi)^2(x^2\sigma_d + (1-x)^2\sigma_s)\right)}{2\left(\pi\mu_s + (1-\pi)(x\mu_d + (1-x)\mu_s)\right)^2} \tag{26}$$

Equations (25) and (26) turn out to be the same condition that we rewrite as follows:

$$\frac{\pi^2\sigma_s + (1-\pi)^2\sigma_d}{(\pi\mu_s + (1-\pi)\mu_d)^2} \leq \frac{(\pi^2 + (1-\pi)^2)\sigma_s}{\mu_s^2} \tag{27}$$

[14] See appendix A3

Table 2: Stability of the steady states

Scenarios	$\sigma_d \leq \underline{\sigma}$	$\underline{\sigma} < \sigma_d < \overline{\sigma}$	$\sigma_d \geq \overline{\sigma}$
$\mu_s < \mu_d$	0,1 locally stable $\frac{1}{2}$ unstable	0, 1, $\frac{1}{2}$ locally stable $\overline{x}_1$, $\overline{x}_2$ unstable	$\frac{1}{2}$ globally stable 0,1 unstable
$\mu_s \geq \mu_d$	$\frac{1}{2}$ globally stable; 0,1 unstable		

It is noticeable that equation (27) is never verified in the case $\mu_s \geq \mu_d$, so that 0 and 1 are always unstable. As χ, in the dynamic equation (18), is continuous over $[0,1]$, we infer that 1/2 is therefore necessarily stable. Alternatively, when $\mu_s < \mu_d$, (27) may be rearranged as follows:

$$\sigma_d < \overline{\sigma} \tag{28}$$

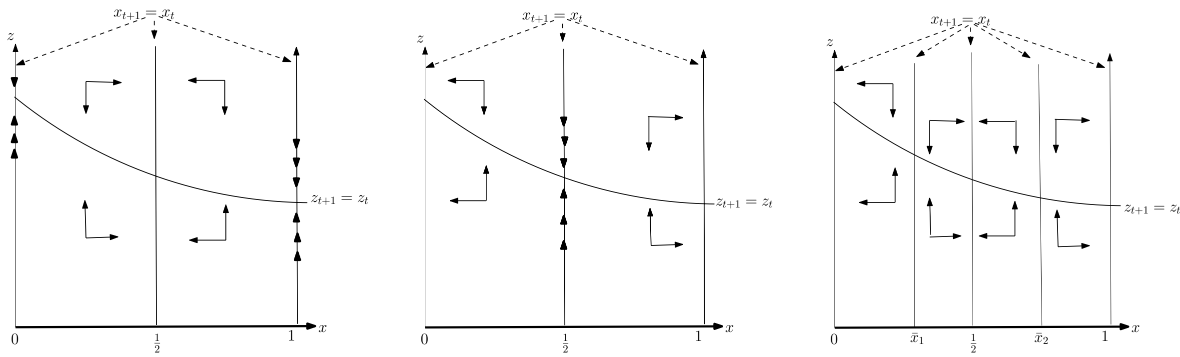
When $\sigma_d \in]\underline{\sigma}; \overline{\sigma}[$, then the two non trivial steady states $\bar{x}_{1,2}$ exist and are admissible but are always unstable. Indeed, recalling that $\overline{x}_{1,2} = \frac{1}{2} \pm \frac{\sqrt{\Delta}}{2a}$, that is $1/2 \in [\bar{x}_1; \bar{x}_2]$. By continuity of χ, we obtain that in this case 1/2 is stable. Finally, if $\sigma_d \leq \underline{\sigma}$, then 0 and 1 are still stable, but $\bar{x}_{1,2}$ are complex, so, by continuity of χ, 1/2 is necessarily unstable. ∎

We sum up the stability properties in table 2.

In the case $\mu_s < \mu_d$ and $\sigma_d \leq \underline{\sigma}$, the group that is initially in minority decreases in proportion towards 0 and tends to disappear. In the case $\mu_s < \mu_d$ and $\sigma_d \in]\underline{\sigma}, \overline{\sigma}[$, only if the minority is initially big enough ($> \min(\overline{x}_1, \overline{x}_2)$), does the dynamics converge to 1/2, otherwise the share of the minority in the population will tend to decrease towards 0.

Once we have solved for the population share, the remaining problem concerning human capital is straightforward. Indeed, the long-run value for human capital depends

Table 3: Phase diagrams



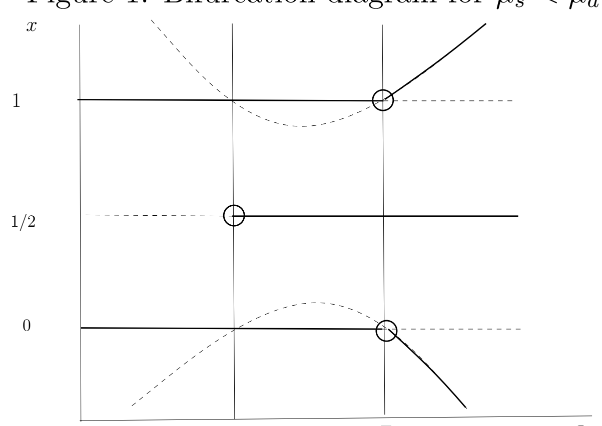
$\mu_s \geq \mu_d$ or $\mu_s < \mu_d$ and $\sigma_d \geq \overline{\sigma}$

$\mu_s < \mu_d$ and $\sigma_d \leq \underline{\sigma}$

$\mu_s < \mu_d$ and $\underline{\sigma} < \sigma_d < \overline{\sigma}$

only on the population share and the parameters. From proposition 1, we know that fertility and education vary in opposite direction following a shock on the parameters. We thus observe a quality-quantity trade-off as the group shares evolve towards the steady state. Table 3 gives the phase diagrams for the different scenarii, where z denotes the relative human capital of group i with respect to the other group, and figure 1 displays the bifurcation in the case when $\mu_s < \mu_d$. The fact that the $z_{t+1} = z_t$ schedule is a decreasing curve illustrates the quality-quantity trade-off. The $x_{t+1} = x_t$ schedules are vertical lines because the steady states in x do not depend on z.

Figure 1: Bifurcation diagram for $\mu_s < \mu_d$



Risk aversion drives the fertility choices of parents. Whenever $\mu_s \geq \mu_d$, meaning there is a higher surplus for an inside group match, and irrespective of the values of σ_s, σ_d and π, we observe convergence towards groups of equal size and convergence in human capital. 1/2 is a globally stable steady state, as depicted in the left panel of table 3. This is due to the fact that, in this case, the risk term for the minority is always greater than that of the majority. Minorities therefore exhibit a higher fertility than the majority up to the point when it ceases to be a minority and socio-economic characteristics become equal. This result illustrates the fact that, when insecurity is perceived as an obstacle to social mobility, minorities react by increasing fertility above majority levels in such a way that they catch up in size in the long-run.

On the other hand, when $\mu_d > \mu_s$, we may observe situations that support the minority status hypothesis. This can be interpreted as a case when intergroup matching leads to a higher total surplus. One reason for this may be the complementarity between different labor types. Think for instance of a situation in which the minority members are skilled and own an unskilled labor intensive technology and the majority is composed of unskilled workers. They both would gain more from intergroup than intragroup matching. For instance, Meng and Gregory (2005) show that intermarried immigrants in Australia have on average higher earnings than those married within their own ethnic groups[15]. As a matter of fact, insecurity about future labor market outcomes may in some instances lead minority members to have a lower fertility than their majority counterparts. This happens in two cases: it is always the case that minority members have a lower fertility and therefore a decreasing share in the population, as well as a higher human capital, when σ_d is low enough (smaller than $\underline{\sigma}$). The intuition behind this result is that, if minority parents expect the interaction with an alien to be positive and that the variance associated to this this match is low enough, then they become less risk averse than their majority counterpart and feel less need to increase fertility as an insurance mechanism. We interpret this result as the situation when fewer obstacles to social mobility exist. In this case, the reaction of minorities to insecurity is to decrease fertility lower than their majority counterparts, which implies a decreasing share in the

[15]We take the same μ_s and μ_d for both groups, that is we assume equal splitting of the surplus, for analytical reasons.

population and a higher human capital.

Notice the important role played by σ_d when $\mu_d > \mu_s$. Indeed, when σ_d is big enough (bigger than $\overline{\sigma}$), we are back into the equal splitting case. This means that the variance is big enough to compensate the gain of the intergroup match in the risk term, making the minority again more risk averse than the majority. If, instead, σ_d is between $\underline{\sigma}$ and $\overline{\sigma}$, both scenarii are possible. In this case, the initial share of the minority compared to $\min(\bar{x}_{1,2})$ determines whether we converge towards the splitting case or towards an ever declining minority in share. The factor of bias towards own group π plays here a role in determining the level of $\min(\bar{x}_{1,2})$.

4.2 The case with more than two groups

It is noticeable from equation (18) that the only way to have a society divided into steady shares is that either one group tends to 1 and all the others to 0 or fertility of the various groups should be equal. In this latter case, the risk term should be the same for all groups and thus they should all be of equal size. We are thus able to say that a one-group society or a society divided among groups of equal size are potential steady states. It is however very difficult to derive more analytical results.

5 Computational experiment

The theoretical results in the previous sections show that persistent fertility differentials may be explained through other channels than differences in socio-economic characteristics. Indeed, in our model, fertility relies on how parents perceive the uncertainty about the job market opportunities their children will face. In this section, we construct two numerical examples that aim at replicating the fertility behaviors and group shares of the US society. We first calibrate the "traditional" parameters and then find values for the remaining parameters that allow to fit the data. This numerical exercise shows that observed fertility differentials may be replicated by our model and allows to draw conclusions about the expected population dynamics that are in line with the projections of the US Census Bureau (zero net migration scenario).

5.1 Calibration

We extend the model to four groups (White, Black, Hispanic and Asian) and we let the parameters μ vary across groups. This illustrates the fact that different groups may own a different technology and/or exhibit different network externalities. Our model is calibrated so that a period has a length of 30 years. The time cost parameter ϕ for bearing a child is set to $0,075$ based on the evidence in Haveman and Wolfe (1995) according to which the time cost of having a child is approximatively 15% of a parent's time endowment. If we assume that a child lives 15 years with the parents, this opportunity cost amounts to 15% of 15 years, which represent 7,5% of a parent's total time endowment. For the parameters τ and ψ, respectively the return to parental time and to parental human capital in education, we use the calibration in de la Croix and Doepke (2003). They set $\tau = 0.635$ and let ψ vary. We take $\psi = 0.2$ as a benchmark case. Notice that it does not impact optimal decisions anyway. In our model, we know that agents will spend optimally γ percent of their income on children and the rest on consumption. We therefore choose γ to match the expenditures on education over expenditures on private consumption ratio[16]. We obtain $\gamma = 0.09$.

We then follow two strategies to calibrate the remaining parameters.

5.2 Strategy 1

In this strategy, we focus on the calibration of π, the bias towards own group. To this purpose, we exploit the evidence in Kao and Joyner (2004) about homophily in teenagers' friendship relationships. Kao and Joyner extract from the National Longitudinal Study of Adolescent Health (or Add Health) survey the percentage of teenagers that declare to have a same-race best friend. We extrapolate this data in order to proxy the bias agents exhibit towards intra-group matches. We use these values and the observed group shares in order to compute for each group a π, that is the proxy for homophily (bias towards own group), that allows the percentage of same-group matches on the labor market predicted by our model to fit the observed figures of same-group

[16]According to the Bureau of Economic analysis, expenditures on personal consumption were of 9,268.9 billion dollars whereas OECD data gives 888.033 billion dollars spent on education.

friendships according to the following formula:

$$\text{percentage of same-group matches}_i = \pi_i + (1 - \pi_i)x_i$$

The values we obtain are shown in table 4 below.

Table 4: Calibration of π

	white	black	hispanic	asian
group share	0.687	0.128	0.143	0.041
percentage of same-race best friend	0.918	0.850	0.611	0.649
Implied value of π	0.74	0.82	0.55	0.63

Table 5 shows data about mean wage and mean number of years of schooling (that we take as a proxy for human capital) per group in 2006 from the Census Bureau. From these we compute the wage per unit of human capital for each group and normalize that of the white to 1 for simplicity. We then set the expected outcome of a cross-group match, μ_d, equal to 1 for all groups so that differentials are only driven by μ_s, the expected outcome of an intra-group match[17]. We then infer the implied values of μ_s for each group according to the equation for the expected wage per unit of human capital:

$$E[w_i] = \pi_i \mu_{s,i} + (1 - \pi_i)(x_i \mu_{s,i} + (1 - x_i)\mu_d)$$

Table 6 shows the calibrated values for the μ.

The only remaining unknown parameters that matter for optimal decisions are σ_s and σ_d. We choose them in order to fit the observed fertility for each group. We therefore solve the following constrained minimization program:

$$\min_{\sigma_s, \sigma_d} \sum_i [n_i(\sigma_s, \sigma_d) - \tilde{n}_i]^2 \text{ s.t. } \sigma_d \geq \sigma_s \geq 0$$

Where $\tilde{n}_i$ are the observed fertilities. The minimization gives us $\sigma_d = 3.66$ and $\sigma_s = 0.81$. Then we compute the fertilities predicted by the model given these parameters and compare them to the observed ones in Table 7.

[17]The results are not sensitive to the normalization of μ_s to 1, instead of μ_d

Table 5: Mean yearly wage, years of schooling and wage rate per unit of human capital per group

	white	black	hispanic	asian
mean wage in $	43899	31837	28815	49690
mean human capital in years	14.003	13.346	12.399	15.207
wage rate per unit of human capital	1	0,76	0,74	1,04

Table 6: Calibration of μ

μ_d	$\mu_{s,w}$	$\mu_{s,b}$	$\mu_{s,h}$	$\mu_{s,a}$
1	1	0.717	0.582	1.065

Interestingly enough, calibrating our parameters to fit the wage differentials across groups as well as the bias towards own group and the altruism parameter, it is possible to find risk parameters σ_s and σ_d common to all groups in order to obtain fertility differentials that are quite similar to the actual ones. The only outlier is the fertility of asians that is found to be higher than that of the white, which is counterfactual. It may though remain low enough to be lower than the average fertility in the US. This would imply a decreasing share of asians in the American society, which is in line with the predictions of the Census Bureau at the horizon 2030 (zero net migration scenario). However to advocate the case for our model, it needs to be said that our model do not feature any correlation between the fertility and educational investment decision, for the sake of tractability. It remains to be investigated whether our model could explain the lower fertility of the asians in the case where investment in education would allow to face a job market with different expected wage and variance. We are quite confident in the possibility that such a model would replicate more closely actual data, at the expense though of analytical tractability.

Table 7: Predicted and observed fertility

	white	black	hispanic	asian
predicted	0.793	1.031	1.132	0.889
observed	0.88	1.00	1.15	0.84

5.3 Strategy 2

In this subsection, we do not calibrate different values of bias towards own group for every group. As in the original model, we assume that this bias is equal across groups. We take different values for π (0.4, 0.6 and 0.8) that are in line with the calibrations of the previous subsection. Then as in the first strategy, we calibrate the μ in order to fit observed wage differentials. We set μ_d to 1 for all groups and recover the implied μ_s for each group. Table 8 shows the results.

Table 8: Calibration of μ_s

	white	black	hispanic	asian
$\pi = 0.4$	1	1.031	1.132	0.889
$\pi = 0.6$	1	1.00	1.15	0.84
$\pi = 0.8$	1	1.00	1.15	0.84

For every value of π, we then choose the σ in order to fit the observed fertility for each group. We therefore solve the following constrained minimization program:

$$\min_{\sigma_s,\sigma_d} \sum_i \left[n_i(\sigma_s,\sigma_d) - \tilde{n}_i\right]^2 \text{ s.t. } \sigma_d \geq \sigma_s \geq 0$$

Where $\tilde{n}_i$ are the observed fertilities. The minimization gives us respectively for $\pi = 0.4$, $\sigma_d = 1.608$ and $\sigma_s = 1.608$, for $\pi = 0.6$, $\sigma_d = 1.454$ and $\sigma_s = 1.454$, for $\pi = 0.8$, $\sigma_d = 1.046$ and $\sigma_s = 1.045$. Then we compute the fertilities predicted by the model given these parameters and compare them to the observed ones.

We find that, calibrating our parameters to fit the wage differentials across groups

Table 9: Predicted and observed fertility

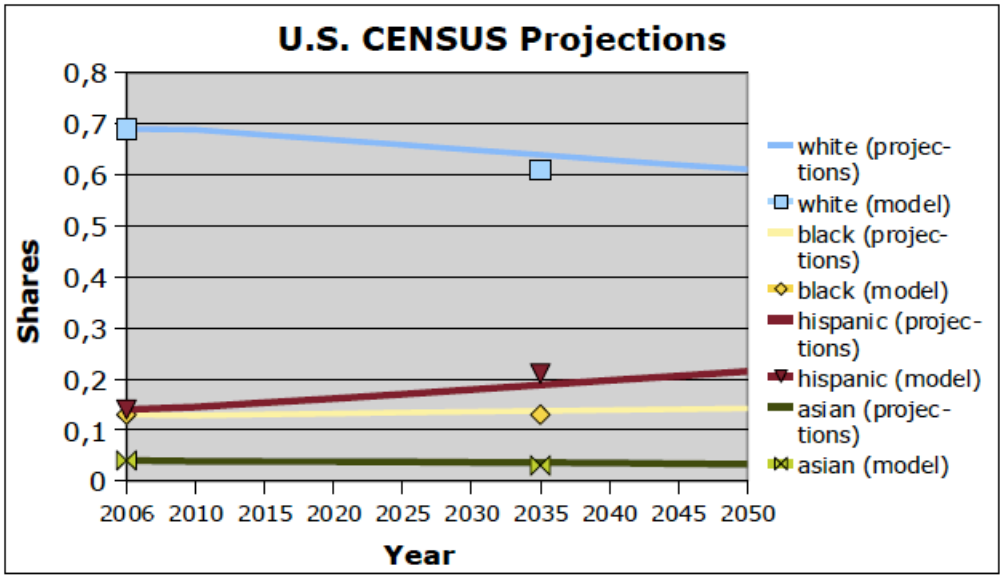
	π	white	black	hispanic	asian
predicted	0.4	0.834	1.069	1.082	0.894
	0.6	0.864	1.067	1.085	0.874
	0.8	0.880	1.066	1.085	0.862
observed		0.88	1.00	1.15	0.84

as well as the altruism parameter and for different values of the bias towards own group π, it is possible to find risk parameters σ_s and σ_d common to all groups in order to obtain fertility differentials that are close to the actual ones. It is also noticeable that we obtain the correct ranking of fertilities if π is high enough. Other values of π are problematic as they imply that the fertility of asians is found to be higher than that of the white, which is counterfactual. Nevertheless, in all our three cases, the fertility of the asian remains lower than the average fertility, which implies a decreasing share of this group in the American society. This result is in line with the predictions of the Census Bureau at the 2030 horizon (zero net migration scenario), as illustrated in the following figure.

6 Conclusion

The aim of this paper was to study the fertility and educational investment differentials between racial and ethnic minorities within an endogenous fertility framework that explicitly accounts for uncertainty about future earnings or expected socio-economic conditions. The analysis led to three main conclusions. First, our model may be interpreted as a theoretical support of the minority status hypothesis according to which minority members usually have a higher fertility facing low potential for social mobility but may in some instances strategically decrease their fertility. This mechanism arises from the fact that the marginality and the insecurity associated with the minority status may influence individuals expectations about potential social mobility. Second, with respect

Figure 2: US Census Projections and simulations with $\pi = 0.8$



to the existing literature in which differential in fertility and educational investments are persistent only if socio-economic characteristic do not converge across groups, we observed the possibility of persistent differentials even though socio-economic characteristics are equivalent. Third, a calibration exercise for the model with more than two groups shows that our model may roughly reproduce observed fertility differentials, although the effect of socio-economic characteristics on fertility is not modeled. Including an interaction between fertility and educational investment decisions may help to reproduce more closely the data. Finally, a natural extension of this article would be to incorporate the effect of parental human capital on fertility in order to quantitatively assess the respective importance of socio-economic characteristics versus minority status as determinants of fertility differentials.

Appendices

A Fixed points of $x_{t+1} = f(x_t)$

A.1 3rd degree polynomial

Notice that substituting $\overline{x}$ by $1/2$, we have that (18) is verified. Rearranging to get rid of the quotient and keeping everything on the left hand side, we should also be able to factor by $2\overline{x}-1$. Then we collect the terms in the remaining bracket to get a 2nd order polynomial.

$$
\begin{aligned}
& \gamma\left(\pi^2\sigma_s + (1-\pi)^2(\overline{x}^2\sigma_s + (1-\overline{x})^2\sigma_d)\right) 2(\pi\mu_s + (1-\pi)(\overline{x}\mu_d + (1-\overline{x})\mu_s))^2 \\
& -\gamma\left(\pi^2\sigma_s + (1-\pi)^2(\overline{x}^2\sigma_d + (1-\overline{x})^2\sigma_s)\right) 2(\pi\mu_s + (1-\pi)(\overline{x}\mu_s + (1-\overline{x})\mu_d))^2 && = 0 \\
\iff\quad & 2\gamma(2\overline{x}-1)\left[\pi^2(1-\pi)^2\mu_s^2(\sigma_s-\sigma_d) + (1-\pi)^2\pi^2(\mu_d-\mu_s)(\mu_d+\mu_s)\sigma_s\right. \\
& +(1-\pi)^4\left((2\overline{x}^2-2\overline{x}+1)(\mu_d^2\sigma_s-\mu_s^2\sigma_d) + (2\overline{x}-2\overline{x}^2)\mu_s\mu_d(\sigma_s-\sigma_d)\right) \\
& \left.+2\pi^3(1-\pi)\mu_s(\mu_d-\mu_s)\sigma_s + 2\pi(1-\pi)^3\mu_s((\overline{x}^2-\overline{x}+1)(\mu_d\sigma_s-\mu_s\sigma_d) + (\overline{x}^2-\overline{x})(\mu_d\sigma_d-\mu_s\sigma_s))\right] && = 0 \\
\iff\quad & 2\gamma(1-\pi)(2\overline{x}-1)\left[2(1-\pi)^2(\mu_s-\mu_d)\left((1-\pi)\mu_d\sigma_s + \mu_s(\sigma_d+\pi\sigma_s)\right)\overline{x}^2\right. \\
& -2(1-\pi)^2(\mu_s-\mu_d)\left((1-\pi)\mu_d\sigma_s + \mu_s(\sigma_d+\pi\sigma_s)\right)\overline{x} \\
& \left.+\mu_s^2\sigma_d(1-\pi) + 2\pi^3\mu_s^2\sigma_s + \mu_d^2\sigma_s(2\pi^3-4\pi^2+3\pi-1) - 2\mu_d\mu_s\sigma_s(2\pi^3-2\pi^2+\pi)\right] && = 0
\end{aligned}
$$

A.2 Admissible roots

Recall we have the following:

$$
\begin{aligned}
\overline{x}_1 &= \frac{1}{2}\left(1 - \frac{\sqrt{a^2-4ac}}{a}\right) \\
\overline{x}_2 &= \frac{1}{2}\left(1 + \frac{\sqrt{a^2-4ac}}{a}\right)
\end{aligned}
$$

Assume the condition for existence is fulfilled, that is $a^2 - 4ac \geq 0$. For $\overline{x}_1, \overline{x}_2 \in [0,1]$, it is necessary and sufficient to have $\left|\frac{\sqrt{a^2-4ac}}{a}\right| \leq 1$. This is the case if and only if $\sqrt{a^2-4ac} \leq |a|$, which is equivalent to $a^2 - 4ac \leq a^2$, i.e. $ac \geq 0$. ∎

A.3 Thresholds $\underline{\sigma}$ and $\overline{\sigma}$

Recall that

$$\underline{\sigma} = \frac{\left((3\pi^2 - 2\pi + 1)\mu_d + (1 - 3\pi)\pi\mu_s\right)\sigma_s}{(1-\pi)\mu_s}$$

Dividing by $\mu_d > 0$ and defining $\alpha = \frac{\mu_s}{\mu_d}$ [18], we have: $\underline{\sigma} > 0$ iff

$$3\pi^2 - 2\pi + 1 + (1 - 3\pi)\pi\alpha > 0$$

Let us prove that a minimum of this expression, say A, over $\pi, \alpha \in [0,1] \times]0,1[$ is always strictly positive. The FOCs with respect to π and α are respectively:

$$\begin{aligned} \frac{\partial A}{\partial \pi} &= 6\pi(1-\alpha) + (\alpha - 2) \\ \frac{\partial A}{\partial \alpha} &= \pi(1 - 3\pi) \end{aligned}$$

A increases in α when $\pi \in [0, 1/3]$, so that a minimum is necessarily reached for $\alpha \to 0$ and that A decreases in α when $\pi \in [1/3, 1]$, so that a minimum is necessarily reached for $\alpha \to 1$. Substituting these values into the derivative with respect to π, we obtain:

- for $\alpha = 0$, $\frac{\partial A}{\partial \pi} = 6\pi - 2$, so that a minimum is reached for $\pi = 1/3$ and equals $2/3$.
- for $\alpha = 1$, $\frac{\partial A}{\partial \pi} = -1$, so that a minimum is reached for $\pi = 1$ and equals 0. However, as α never reaches the value 1, A always remains above 0. ∎

Let us now prove that $\underline{\sigma} < \overline{\sigma}$. It is the case iff:

$$\frac{\sigma_s}{(1-\pi)\mu_s^2}\left(\left(-(2\pi^3 - 4\pi^2 + 3\pi - 1)\mu_d^2 + 2\pi(2\pi^2 - 2\pi + 1)\mu_d\mu_s - 2\pi^3\mu_s^2\right) - \left((3\pi^2 - 2\pi + 1)\mu_d\mu_s + (1 - 3\pi)\pi\right.\right.$$

Rearranging and noticing we can factorize by $1 - \pi$, we obtain:

$$\frac{\sigma_s}{\mu_s^2}\left((2\pi^2 - 2\pi + 1)\mu_d^2 - (4\pi^2 - 3\pi + 1)\mu_d\mu_s - \pi(2\pi - 1)\mu_s^2\right) > 0$$

Noticing now that putting $\mu_s = \mu_d$ we obtain 0, we factorize by $\mu_d - \mu_s$:

$$\frac{\sigma_s\overbrace{(\mu_d - \mu_s)}^{>0}}{\mu_s^2}\overbrace{\left((2\pi^2 - 2\pi + 1)\mu_d + \pi(1 - 2\pi)\mu_s^2\right)}^{B} > 0$$

As previously done, let us notice that $\frac{\partial B}{\partial \mu_s}$ is positive for $\pi \in [0, 1/2]$ and negative for $\pi \in [1/2, 1]$, so that a minimum is reached respectively for $\mu_s \to 0$ and $\mu_s \to \mu_d$. In

[18] Note that $\alpha \in]0,1[$

the first case, one may check that a minimum is reached for $\pi = 1/2$ and B equals $1/2$. In the second case, B is minimal for $\pi = 1$ and tends to 0, though this value is never reached because we are in the case $\mu_d > \mu_s$. B is therefore always strictly positive. ∎

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