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Electromechanical model for the dynamical behavior of magnetic bearings subjected to induced currents

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28 September 2011
Doctoral dissertation

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Acknowledgment

La rédaction de cette thèse est l'aboutissement d'un travail de recherche de longue haleine, qui n'aurait pas été possible sans certaines personnes qui m'entourent.

Je suis contente de pouvoir exprimer ma gratitude envers toutes ces personnes qui d'une manière ou d'une autre, directement ou indirectement m'ont aidé à réaliser ce travail.

Tout d'abord, un merci particulier à Bruno de m'avoir encadré tout au long de cette thèse. Je tiens à le remercier pour ses qualités tant scientifiques que humaines. Je voudrais exprimer ma reconnaissance pour tout le temps consacré à cet encadrement, pour la pertinence de ses conseils, que ce soit concernant de nouvelles idées, concernant le modèle développé et son formalisme, les éléments finis, la structure du travail, le banc d'essai, ou concernant la rédaction, et enfin pour son optimisme.

Je voudrais également remercier Damien pour avoir proposé la problématique initiale sur les paliers magnétiques, et pour son accueil lors de mes visites à SATIE. Je tiens aussi à remercier Francis pour ses analyses scientifiques pertinentes, sa capacité à exprimer et expliquer les phénomènes physiques simplement ainsi que pour son soutien lors de moments plus difficiles. Je voudrais également remercier Paul de m'avoir accueilli au sein de l'équipe du CEREM et d'avoir effectué une relecture minutieuse de ma thèse. Je remercie enfin Ernest pour sa disponibilité lorsque je me suis retrouvée noyée dans les équations. Je remercie Hamid pour toutes les remises en question qu'il a pu apporter vis-à-vis du travail effectué, et pour son accueil au sein de SATIE. Je remercie tous les membres du jury pour leurs conseils et

leurs remarques constructives.

Enfin je tiens à remercier tous mes collègues pour leur disponibilité et leur bonne humeur. En particulier, merci à Thierry, André, Jovino, Axel, Alban et Jean pour leur aide sur les bancs d'essai. Merci également à Paul pour son aide concernant mon ordinateur. Je tiens aussi à remercier Julien pour son aide concernant LaTeX et ses conseils sur la rédaction. Merci à tous ceux qui m'ont supporté à leurs côtés: Olivier, Sophie et Xavier. Je remercie Ghislaine et Marie-Christine pour leur aide sur les aspects administratifs. Je remercie tous les membres du CEREM pour les conversations intéressantes ou divertissantes que nous avons pu partager, que ce soit à propos de notre travail de recherche, à propos des rénovations de maison, à propos des enfants ou encore d'autres choses.

En rédigeant ces remerciements, je pense aussi aux étudiants pour leur travail de fin d'étude concernant de près ou de loin ma thèse.

Pour terminer, je voudrais remercier certaines personnes ne faisant pas partie de mon environnement professionnel, mais si importantes dans ma vie. Merci à Gil pour son soutien moral durant toutes ces années, son esprit critique et le regard extérieur qu'il a apporté sur mon travail, et merci pour tout le reste.

Merci à Olivier, Eloïse et Aurélie d'être mes petits soleils.

Merci également à mon frère, ma soeur, mes belles-soeurs et beaux-frères, et à Brigitte.

Je remercie aussi Agnès de s'occuper si bien de ses petits- et arrière-petits-enfants.

Je terminerai en remerciant mes parents de m'avoir appris la persévérance, d'essayer de faire de son mieux, deux qualités importantes lors d'une thèse ainsi que de m'avoir soutenue moralement en s'occupant si bien de leurs petits-enfants.

Abstract

Magnetic bearings can prove to be useful in various situations, like high-speed or vacuum applications where classical roller bearings reach their limits. These bearings can be classified into different categories: active, semi-active, passive or semi-passive.

Advantages of passive and semi-passive suspensions with electrodynamic bearings are to be cheap, to be completely stable at ambient temperature on certain conditions, but they are very difficult to design. And a bad design of these latter may lead to important losses, or even to be unstable for rotation speeds where it is supposed to behave stable.

In some applications, a good compromise between low cost and high performance can be achieved via semi-active magnetic suspensions, some degrees of freedom being controlled, others not. However, a study on this kind of magnetic bearing has shown the onset of unexpected vibrations [Faure, 2003]. These are unexpected because they take place after the critical speeds of the system have been passed, and they are not due to gyroscopic effects.

In the present dissertation, we develop an electromechanical model able to predict the dynamic behavior of magnetic bearings. This model is based on a general perspective for the electromechanical systems like magnetic bearings. It is built on the idea that the forces in magnetic bearings can be modeled by mechanical components. In particular, the reluctance forces and the forces between permanent magnets can be modeled by stiffness, and the forces due to the interaction between the eddy currents and the magnetic fields can be modeled by intro-

ducing damping into the mechanical equations governing the magnetic bearing motion. Let's put the emphasis on the fact that a macroscopic point of view is chosen to develop the electromechanical model. This point of view has been chosen to keep a model that is general and can be applied to all kind of magnetic bearings subject to eddy currents.

The first objective of the electromechanical model presented in this dissertation is to fully understand and explain the physical phenomena present in magnetic bearings, and in particular the consequences of forces due to eddy currents which may lead to stable and unstable speed ranges.

The second objective is to have a tool which can be used to predict the dynamic behavior of passive magnetic bearings. This is necessary to design new stable magnetic bearings, or possibly to stabilize existing magnetic bearings for a desired speed range. The parameters of the model have to be identified for a particular magnetic bearing configuration and this can be done on the basis of a few experimental measurements, or on the basis of a quasi-static finite element model of the bearing. Once its parameters identified, the presented electromechanical model enables to predict the forces evolution with the spin speed, for any position of the magnetic bearing. This allows to study the dynamic behavior of these bearings as well as independent systems as integrated in a more complex mechanical system.

The validation of the parameterized electromechanical model we developed showed on different cases that the forces prediction with the spin speed fit very well on finite elements model force calculations. The validation also showed that two phenomenological models to take the skin effect into account improve the electromechanical model predictions.

Stability issues for a simple Jeffcott rotor supported by electrodynamic magnetic bearings are studied based on this electromechanical model, and different parameters influencing this stability are examined. It is shown that the resistive behavior is damageable for the stability spin speeds and for the induced stiffness of an electrodynamic bearing. A contrario high inductive behavior is beneficial for the stability but it can also be to the detriment of the induced stiffness. The

damping in the stator is, on the other hand, beneficial for the stability without changing the induced stiffness.

Finally the electromechanical model is used to study more complex dynamical systems than a simple Jeffcott rotor and the consequences of a static unbalance are examined. A Jeffcott rotor supported by electrodynamic magnetic bearings is also investigated, and it is shown that an eccentricity excites the system, which then becomes unstable, even with no exterior perturbation, when the spin speed is situated within an unstable speed range.

A system with the stator connected to the ground by a spring-damper in parallel is also investigated and it is concluded that when the damping is chosen appropriately, the speed range where the rotor behaves unstable can be reduced.

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Introduction

1 Literature review

1.1 Magnetic bearing applications

Magnetic bearings are a very appealing technology for high-speed drives and/or systems in a vacuum environment due to the non-existent contact between the moving and the fixed parts. First in research labs, these contact free bearings showed their abilities:

- to work in vacuum with no lubrication and no contamination,
- to run at high speed,
- to shape novel rotor dynamics,
- to function without friction losses or wear due to contact.

Nowadays, magnetic bearings have been introduced into the industrial world for applications domains like:

- high-speed drives: the lack of contact between the moving and the fixed parts allows higher speeds. In rotation drives, the rotation speed is then limited by the risk of mechanical burst of the rotor due to centrifugal forces,
- vacuum: in this case lubrication is excluded,
- low loss: magnetic bearings do not present losses due to friction, although other losses may be present,
- low wear: because there is no contact between the different parts, there is also no wear,

- high precision: an active magnetic bearing enables a very precise positioning of pieces against each other, depending mostly of sensor's quality,
- high temperature range: when appropriate materials are chosen for the design of the magnetic bearing, the bearing will support very high or very low temperatures.

However, magnetic bearings do not support overload even for a short time, when classical roller bearings do. Magnetic bearings are also more expensive than roller bearings. Space occupied by magnetic bearings is relatively large.

Examples of industrial applications can be found in [Spirig et al., 2002], [Mitsuda et al., 2009] and [Lantto and Tommila, 2009]. Let's mention a few examples:

- flywheel energy storage,
- milling machines where the spindle equipped with magnetic bearings rotates fast and with a high precision,
- flywheel in satellites for energy storage or satellite stabilization,
- turbocompressors and turboexpanders used in the oil and gas industry,
- turbomolecular pumps which can reach very high vacuum performances,
- magnetic levitation trains (linear magnetic bearing).

1.2 Magnetic bearing classification

Physical principles

With regard to the physical principles involved, electromagnetic levitation can be done by using repulsive or attractive electromagnetic forces.

A classification of the magnetic bearings can be done based on these physical principles ([Bleuler et al., 2009]). This classification distinguishes two groups.

In the first group, the magnetic force is based on the Lorentz Force. This force is perpendicular to the flux lines and to the current direction. It does not depend on an airgap, but depends on the current. In the second group, the magnetic force is a reluctance force, and it appears at the border between two materials of different permeabilities. The force direction is perpendicular to the materials border, and tends to minimize the magnetic circuit reluctance. The attractive or repulsive magnetic forces between two permanent magnets in a system are also considered as reluctant forces.

Magnetic bearings static stability

In [Earnshaw, 1842], Earnshaw states that a group of charged particles can not be maintained in a stable equilibrium by only electrostatic interactions. The theorem is based on the consequence that the inverse-square law forces derive from a potential U .

If

$$F = -\nabla U \tag{1}$$

then

$$\nabla F = \nabla(-\nabla U) \tag{2}$$

$$= -\nabla^2 U \tag{3}$$

$$= 0 \tag{4}$$

which is the Laplace differential equation. Since the solution of this equation does not have any local maxima nor minima but only saddle-type equilibrium points exist, there can be no equilibrium.

This theorem was later expanded in [Braunbek, 1939b] and [Braunbek, 1939a] to include materials with magnetic permeability and electrical permittivity greater than one. Both results are collectively referred to as Earnshaw theorem.

In particular, for bearings based on forces between permanent magnets or between permanent magnets and soft magnetic elements, Earnshaw theorem implies that the sum of all the stiffnesses of the suspension is zero or less than zero:

$$k_x + k_y + k_z \leq 0 \quad (5)$$

And for an axially symmetric bearing, this equation reduces to

$$k_{axial} + 2k_{radial} \leq 0 \quad (6)$$

where k_{axial} is the axial stiffness, and k_{radial} is the radial stiffness, as shown in [Yonnet, 1981]

For example, let us consider a simple radial bearing formed by two axially magnetized permanent magnets, as illustrated in Fig. 1. When one of the permanent magnets is off-centered, a restoring force re-centers the permanent magnets. The stiffness in the radial direction is positive. However, this system is unstable, as in the axial direction, the permanent magnets attract each other, and if left free they will stick together. The stiffness in the axial direction is negative, with respect to eq. 6

In short, the Earnshaw theorem implies that a static body in a magnetostatic field can not levitate in a stable way if the relative permeability of the whole system is greater than one. In other words,

- a magnet in a magnetostatic field can not be in a stable equilibrium,
- a body with positive magnetic susceptibility can not be in a stable equilibrium in a magnetostatic field,
- a body with negative magnetic susceptibility can be in a stable equilibrium in a magnetostatic field

Another way to categorize magnetic bearings is to look at how they achieve stability. This may lead the usual and simple way to sort magnetic bearings by type, active or passive magnetic bearings ([Delamare, 1994]).

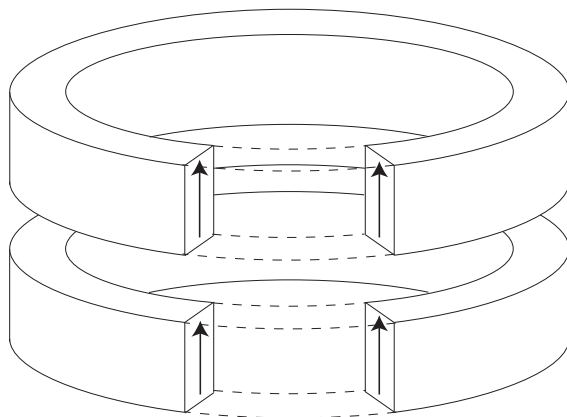


Figure 1: Radial bearing with axially magnetized permanent magnets

We will go a bit further by adding three other types of bearings: the semi-active magnetic bearings, the hybrid magnetic bearings and the semi-passive magnetic bearings.

A. Active magnetic suspensions

An active magnetic suspension employs control systems to vary the strength of the forces in function of the rotor displacement. This means that this type of suspension needs an electrical energy contribution in order to work. Most common active magnetic bearings are based on reluctant forces, with an electromagnet. The forces are then attractive, and the magnetic bearing can not be stable without a control on these latter. This can be done by controlling the electromagnet current. Active magnetic bearings allow high stiffnesses, high precision positioning, and have a greater industrial maturation. But they suffer from power consumption, and they need material like power supply and sensors. As a consequence, they are expensive and limitations as time response due to sensors, saturation, and to control dynamics exist ([Pichot and Driga, 2005]).

B. Passive magnetic suspensions

In opposition to an active magnetic suspension, a passive magnetic suspension does not employ control systems.

This means that passive magnetic bearings have a lower power consumption, are simplified and cost effective because they don't need electronics or control. This also means a greater reliability. Nevertheless, passive magnetic bearings submitted to Earnshaw theorem suffer from stability issues. However, complete passive magnetic suspensions exist without being in contradiction with this theorem. They lay outside the scope of Earnshaw theorem.

Diamagnetic bearings Magnetic bearings constructed with diamagnetic materials, in other words with materials having a relative permeability less than one, are therefore appealing because they are not submitted to Earnshaw theorem. This means that completely stable passive suspension can be conceived. Examples of diamagnetic materials are graphite, bismuth, . . .

Unfortunately, the forces and their stiffnesses developed by such systems remain small. For instance, in [Moser et al., 2001], five degrees of freedom are stabilized using diamagnetic material, and the weight of the rotor is compensated with permanent magnets. The experimental result shows a motor of about 1 cm^3 rotating stably up to about 800 rpm, see Fig. 2.

Superconducting bearings Superconductors may be viewed as perfect diamagnetic materials since they expel magnetic field from their interior.

But unlike diamagnetics cited in previous section, forces developed by superconductors may be significant. However, the requirement of cooling the superconductors to cryogenic temperatures is a big drawback for this technology.

Bearings stabilized by a gyroscopic torque A stable passive suspension may be realized using only permanent magnets, when one of the magnet is rotating, thanks to the dynamic effects of rotation. This means that this suspension only works when the rotor is already in

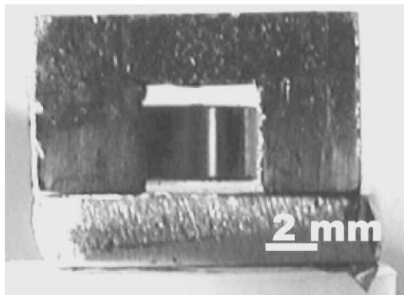


Figure 2: Picture of a diamagnetic suspension system, without the weight compensating permanent magnet [Moser et al., 2001]

motion. A famous illustration of this kind of suspension is the Levitron, see Fig. 3.

Unfortunately, this kind of suspension is only stable in a narrow speed range, has a low stiffness, and is very sensitive.

Electrodynamic bearings Another way to design passive suspensions is to use a non static magnetic field. In this case, the magnetic field can induce currents in conductors in order to generate restoring forces by the interaction of these currents and the magnetic field. This kind of bearing, particularly concerned by our model, is examined in detail in the section below (page 14).

C. Semi-active magnetic suspensions

Another way to achieve passive magnetic bearings is to use either the interaction between permanent magnets on the fixed and on the moving parts, either the reluctant forces on ferromagnetic material. This interaction has been studied and used by many. Already in 1978, Yonnet showed that for identically shaped bearings with permanent magnets, strength and stiffnesses are the same whether the magnetization is axial or radial [Yonnet, 1978]. In 1981, he studies the coupling between magnets for different basic configurations for magnetic bearings [Yonnet, 1981]. This article shows a method for prediction of all



Figure 3: Levitron: magnet rotating above another magnet

different basic arrangements of the direction of magnetization and the calculation of force and stiffness in simple cases.

But according to Earnshaw theorem, magnetic bearings only based on these kind of interactions are not stable. This is why magnetic bearings based on this kind of interactions for some degrees of freedom need to be combined with another type of magnetic bearing for the remaining degrees of freedom, in order to stabilize the whole system.

For a semi-active suspension, some degrees of freedom are maintained by a passive magnetic bearing, and the other degrees of freedom are controlled by active means. This kind of suspension allows to gather the advantages of active and passive magnetic bearings. They are simple, cheap, and controlled for some degrees of freedom.

More generally, inserting permanent magnets in an active magnetic bearing is advantageous for the power consumption.

[Yanliang et al., 2006] shows the advantage in power consumption by using a permanent magnet to generate the principal magnetic flux instead of an electromagnet. A prototype of a new radial bearing with permanent magnets in the rotor and an active magnetic bearing with the same force capacity are built, implemented on the same flywheels

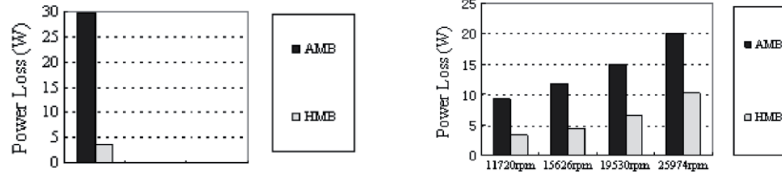


Figure 4: Comparison of the power losses in an active magnetic bearing and permanent magnet biased magnetic bearing [Yanliang et al., 2006]

and compared, as can be seen on Fig. 4. The first figure shows the needed input power of the magnetic bearing amplifiers (sum of the coil copper and the power amplifier losses), when the flywheel is suspended statically. The second figure shows the iron losses at different speeds.

An example of a semi-active magnetic suspension is described briefly below. In 2003, F. Faure [Faure, 2003] made a feasibility study of a cheap magnetic suspension for a flywheel energy storage of 3 kWh. The flywheel is a 150 kg plain steel cylinder of 30cm diameter and 28cm height. As can be seen on a detail view of the magnetic suspension in Fig. 5, the rotation axis of the system is vertical, and radial centering is ensured passively by reluctance teeth magnetized by the permanent magnet flux. Those teeth are located face to face on the rotor and the stator at the outer diameter of the bearing. The radial axis is centered by passive means.

The axial axis is controlled actively. These bearings are used by pairs: the upper bearing pulls the flywheel up, acting against gravity, and the lower bearing acts against the first one. The principal magnetic flux is generated by permanent magnets but can be modulated by the electromagnets. The coils allow to control the magnetic flux circulating inside the bearing, in the aim to stabilize the system.

At equilibrium, when the system is vertical, the mean current in the bearings can be canceled if the position of the rotor is slightly closer to the upper bearing. This is due to the fact that in this position the upper bearing attractive force becomes bigger than the lower bearing attractive force in order to counterbalance the rotor weight.

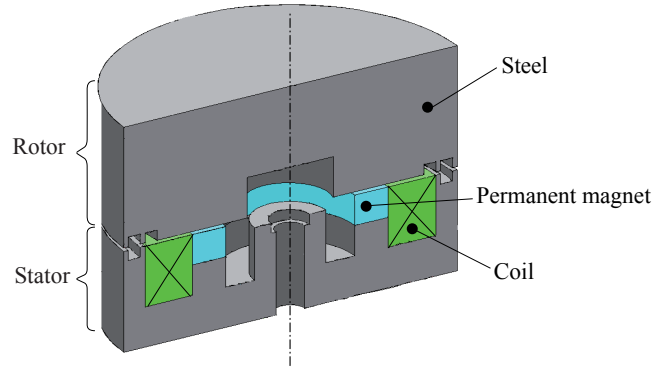


Figure 5: Semi-passive suspension for flywheel energy storage [Faure, 2003]

D. Hybrid magnetic suspensions

Hybrid suspensions combine a passive magnetic bearing and another type of bearing, like a mechanical roller bearing.

An example of hybrid magnetic bearing is the system developed by the NASA in [Siebert et al., 2002]. This system, see Fig. 6, has a horizontal rotation axis and has been tested at speeds up to 5500 rpm, the radial bearings are permanent magnets in repulsive mode, and the thrust bearing is a jewel bearing. The radial and the axial stiffnesses of the magnetic bearing are measured, and it can be noted that the axial stiffness is equal to twice the radial stiffness, which is consistent with Earnshaw theorem (eq. 6). The radial stiffness achieved by this suspension is worth $1,78 \cdot 10^5 \text{N/m}$.

In 2006, [Moser et al., 2006] showed, with a radial passive magnetic bearing and an axial mechanical single-point bearing, that for a given cylindrical volume, an optimized repulsive bearing design can be found. This optimum depends on the air gap width.

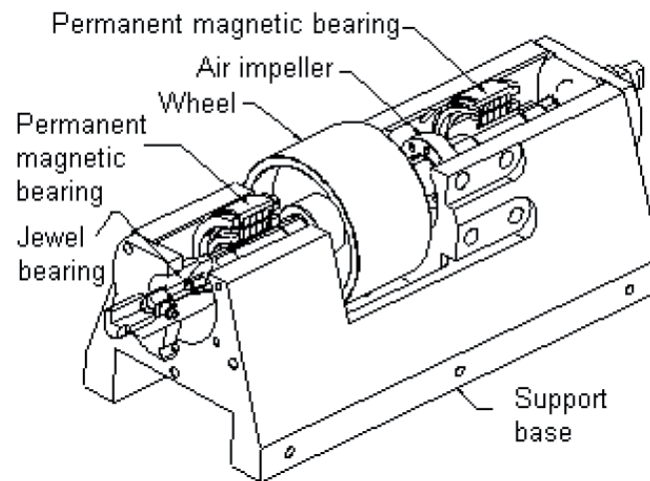


Figure 6: Schematic view of an hybrid magnetic suspension combining a magnetic bearing and a jewel bearing [Siebert et al., 2002]

E. Semi-passive magnetic bearings

Semi-passive magnetic suspensions combine an unstable passive magnetic bearing with a stable passive magnetic bearing, like a diamagnetic or an electrodynamic bearing in order to stabilize the system. When combined with an electrodynamic bearing, they only work for certain values of spin speed, usually they have a lower limit spin speed. When spinning more slowly than this spin speed they are supported by backup bearings. In this case, stabilization only occurs when the rotor spins. This means that some mechanical energy needs to be injected in the system, to make the rotor spin.

1.3 Eddy currents and magnetic bearings

Eddy currents play a significant role in magnetic bearings, whether they are active or passive.

A theoretical description and quantification of the eddy current losses in passive magnetic bearings by means of Faraday's law is made in [Bouty, 2002]. It is shown that the losses are proportional to the square of the rotation speed and to a function of the bearing geometry, under the assumption the magnetic field due to the eddy currents remains smaller than the excitation magnetic field.

Unvaluable eddy currents

When talking about eddy currents and magnetic bearings, one may first think of electrodynamic bearings, where eddy currents are deliberately induced in order to create desired forces. But unintentional eddy currents may also play a significant role in other kinds of magnetic bearings. This is also true for any electrical machine in which eddy currents may arise.

For instance in [Meeker et al., 1996], an eddy current correction is made for an active radial magnetic bearing. This correction is based on a one dimension eddy current model often employed for losses in transformer cores. A comparison is made of the electrical impedance

prediction with this correction to experimental results, and an amelioration is noticed.

In [Murakami et al., 2008], the vibration characteristic of a rotor supported by a PID controlled magnetic bearing is investigated. The differential equation of motion is derived with non linearities of the magnetic force for the air gap, the current, and the characteristics of the current amplifier together with a decrement of the magnetic force, considered due to eddy currents caused by the rotation of the rotor. The influence of the rotational speed on the magnetic force is approximated by a fractional function and the coefficients are determined experimentally. Finally the resonant curves for the major critical speed show a good agreement with the experimental results.

The effect of eddy current is also very important in self-sensing technology. In [Mizuno et al., 1996], it is shown that eddy currents in self-sensing magnetic bearings with solid iron cores may destabilize the controller.

The model of the impedance of the magnetic bearing has to be as close as possible to the reality. Indeed, when using self-sensing technology, the information in the coil of the electromagnet of the active magnetic bearing is used to deduce rotor position and/or speed. The model of the system is of great importance in order to have a good estimation of these informations about the rotor. In [Herzog et al., 2009] eddy currents are modeled by a linear lumped RL ladder network.

In [Faure, 2003], the dynamic effects of eddy currents was also experienced. The semi-active magnetic bearing developed in [Faure, 2003] was designed for a flywheel energy storage system, which was intended to rotate at 30000 to 40000 rpm, but unexpected vibrations appeared around 2800 rpm. The suspected reason to these vibrations is parasitic forces due to currents induced in the rotor. Indeed, when the rotor is off-centered, the magnetic axis does not coincide with the rotation axis anymore, and the rotor experiences a flux variation along its rotation.

More generally, in electrical machines, when the rotor is off-centered, the generated eddy currents create a force called in the literature un-

balanced magnetic pull. An interesting finding is that the unbalanced magnetic pull is not aligned with the eccentricity, but has a parallel and a perpendicular component.

In [Belmans et al., 1987], the relationship between the unbalanced magnetic pull and the homopolar flux is analyzed, and equations of motion are derived.

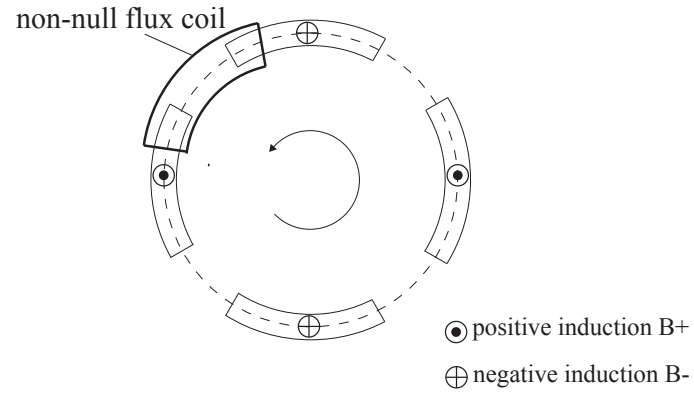
Valuable eddy currents: electrodynamic bearings

As it has been mentioned previously (section 1.2), another way to get around the Earnshaw stability theorem, is to be in a non-static situation: either the body is moving and sees a varying magnetic field, either the magnetic field is time-dependent. Electrodynamic bearings operate on this principle, and as predicted by Earnshaw, research on this kind of bearings show that they can be fully stable and passive. However, electrodynamic bearings can generate non negligible losses, and their equilibrium is not always easy to predict.

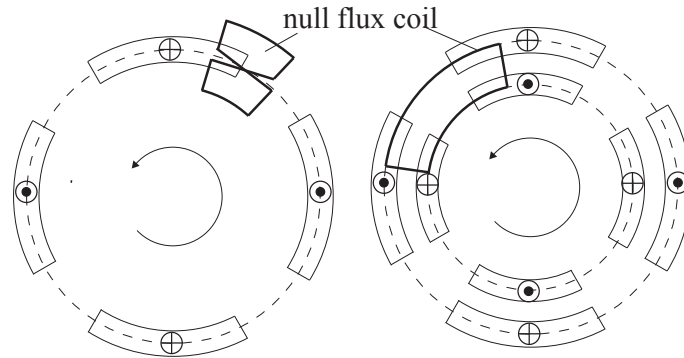
In 1992, [Ting and Tichy, 1992] already raised the difficulty of designing such bearings. With the perturbation method and the help of a symbolic computation software, they solved the Maxwell equations to have a theoretical prediction of the forces, the stiffnesses and the damping of such a bearing. Their conclusions were that the stiffnesses were low, the damping negative, and that cross-coupled stiffness coefficients generate rotor-dynamic problems.

However, in 1994 a patent [Pinkerton, 1994] based on the electrodynamic principle for a magnetic bearing was deposited. The null-flux principle is employed in this bearing. Null flux coils are coils where the flux variation within each loop is zero when the rotor is centered. But when the rotor is off-centered, this flux variation is not zero anymore, which results in eddy currents circulating in the coils. An example of rotating null flux coils is shown in Fig. 7 (b). They have the advantage that no losses are present at the equilibrium position, contrary to Fig. 7 (a).

In 2000, [Bender and Fost, 2000] designed a magnetic bearing for flywheel energy storage. It is composed of different elements as illus-



(a)



(b)

Figure 7: The operational principle of a rotational null-flux radial bearing: (a) Non-null flux coil rotating above induction and (b) Null flux coil rotating above induction: two basic geometries

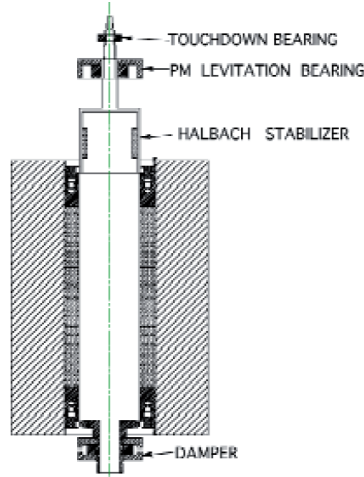


Figure 8: Section drawing of passive magnetic suspension designed for flywheel energy storage by [Bender and Fost, 2000]

trated in Fig. 8: axially symmetric levitation elements, electrodynamic stabilizers and eddy currents type vibration dampers. The electrodynamic stabilizers employ Hallbach arrays on the rotating system which interact inductively with specially wound electrically shorted stator circuits to create centering forces. The eddy-current dampers employ axially symmetric rotating pole assemblies interacting with stationary metallic discs. This article outlines the difficulty to find the specific quantitative criteria insuring that when rotating above a low critical speed, the bearing is dynamically stable. It also outlines the design problem to measure or calculate the relevant stiffnesses and drag coefficients. Note that the damping coefficients of eddy currents were estimated under the assumption that no skin effect is present, but at higher rotating speed, this assumption was not verified, and the damping coefficient measured on a test rig decreased.

Also in 2000, [Murakami and Satoh, 2000] show experimental results up to 5000 rpm of an electrodynamic bearing constituted of axially asymmetric annular magnetic gap, and a copper disc inserted in this

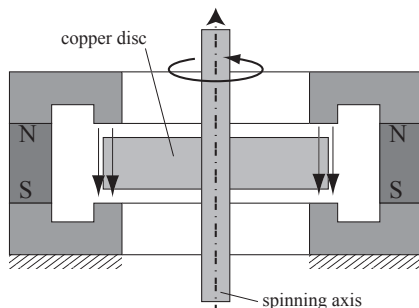


Figure 9: structure of the tested radial electrodynamic bearing in [Murakami and Satoh, 2000]

gap (see Fig. 9). This means that as long as the rotor is centered, no losses and no forces are produced. The inserted part of the conducting disc is sandwiched by two iron pole pieces, which ensures a strongly inductive character. The bearing has been studied by approximating the eddy current distribution, using a distributed parameter model, and the forces are then calculated as being the vectorial product of these currents with the magnetic induction.

Other experimental results of the electrodynamic principle are shown in [Bermudez et al., 2000]. An outer rotor made of a copper tube rotates outside cylindrical axially magnetized NeFeB magnets. The arrangement is such that no eddy current losses occur in centered position. The measured forces (perpendicular and parallel to eccentricity) show that the crossing and the restoring stiffnesses grow with the rotation speed. The resultant force-displacement angle also grows with the rotation speed.

In [Filatov, 2002] A. Filatov proposes a structure of a semi-passive magnetic suspension which is able to provide noncontact stable suspension of a rotor above a certain critical speed. This suspension is based on the combination of the static interaction between permanent magnets, to achieve axial suspension, and the dynamic interaction between conductors and permanent magnets, to achieve radial stabilization. The used electrodynamic principle is a homopolar null flux

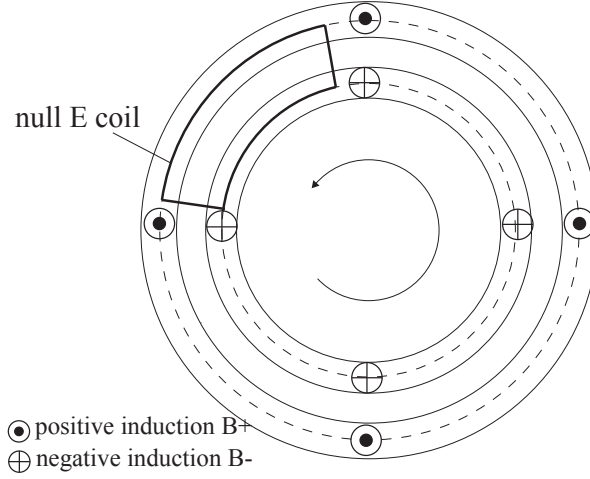


Figure 10: The operational principle of a homopolar null flux coil

coil, and is called by the author “null E coils”, as shown in Fig. 10. A theoretical analysis of the proposed principle is made. First the interaction between a rotating conductor and static permanent magnets is analyzed. It is shown that in this case, the behavior is always unstable. Second, the interaction between a static conductor, neglecting its inductive behavior, and rotating permanent magnets is analyzed. Making the approximation that the damping coefficient introduced by the system composed of the static conductor and rotating permanent magnet is much higher than the damping coefficient developed by the system composed of the rotating conductor and static permanent magnet, a critical damping factor is derived beyond which the system has a stable equilibrium. Finally an analysis is made including the axial suspension, and a modified stability condition (under the same assumptions) is derived, including the inductive behavior of the static conductor. It is also shown that in the developed suspension principle, conducting loops are more advantageous than a bulk conductor. The major theoretical results are verified with a prototype in which a 3.2 kg rotor is suspended stably when it rotates above 18 Hz.

The design of homopolar null flux coils for rotating systems is also

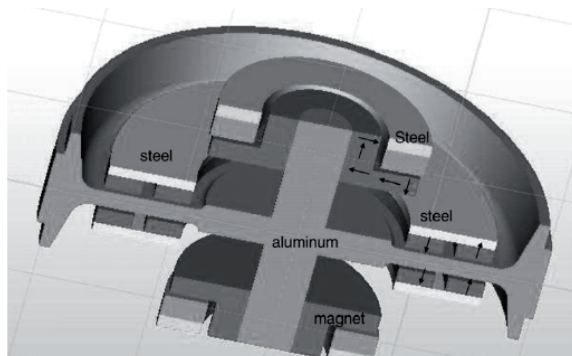


Figure 11: Complete suspension developed in [Davey et al., 2005]

analyzed in [Davey et al., 2005]. Homopolar null flux coils are used for the radial bearing, they consist of a bulk conductor and of axially symmetric magnets on the stationary part. The completely stable passive suspension, represented in Fig. 11 is conceived in this paper following this design sequence: first the restoring forces (radial and azimuthal) of the homopolar null flux bearing are computed using a 2D-FEM analysis, assigning a lateral velocity to the conductor. Next, the necessary damping requirements are determined for a stable behavior: the equations of motion, taking into account the stiffness of the bearing are solved. The stiffness is deducted from a table where it is estimated for different rotation speed and off-centering distances. This allows to figure out how much damping is necessary for a given rotation speed, and the required damping must grow as $\sqrt{\omega}$. Afterward, the damper, and the axial support bearing are designed. By finite element simulations different damper designs are tried in order to satisfy the damping required by previous analysis. Finally, the radial and axial stability are verified.

The electrodynamic bearing developed in [Lembke, 2005] is based on a null flux scheme: no currents are induced until needed, thanks to the homopolar design, see Fig. 12. The conductor is a bulk cylinder, which is better suited for high speed applications. The forces are first derived analytically by looking at the magnetic induction variation in a frame fixed to the rotor, when the rotor is off-centered. The induced

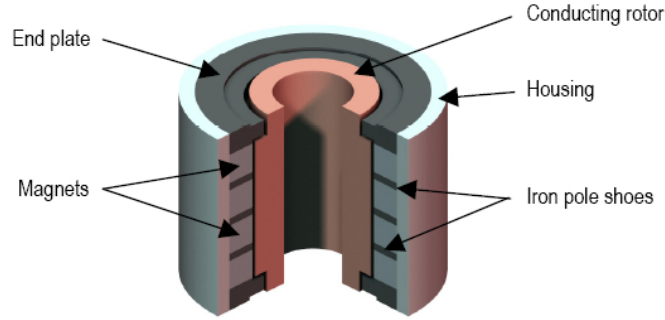


Figure 12: Homopolar radial electrodynamic bearing in [Lembke, 2005]

voltage is derived around an arbitrary eddy current path area. The resistance, taking into account the skin effect, and the inductance of this current path are calculated, and the eddy currents are deduced. The forces are then the vectorial product of these currents with the magnetic induction.

A finite element analysis of the proposed bearing is also made, and the difficulties of modeling correctly these currents with the finite elements is shown. Finally, an experimental validation of the bearing is made.

[Sandtner and Bleuler, 2004] and [Sandtner and Bleuler, 2008] present a thrust electrodynamic bearing. The rotor weight is magnetically compensated: the gravity forces are canceled by permanent magnet forces. The external rotor has permanent magnets and the stator has interconnected copper wire segments, similar to a squirrel cage. Above a given moderate rotational speed, complete thrust stabilization is achieved: the restoring force depends on the rotational speed, and increases with higher speed. In centered position, half of the magnetic flux penetrates inwards and the other half outwards the structure, therefore no currents are induced in the loops. But when it is off-centered, a voltage is induced, currents flow in the copper structure, and rotate synchronously with the rotor. These currents

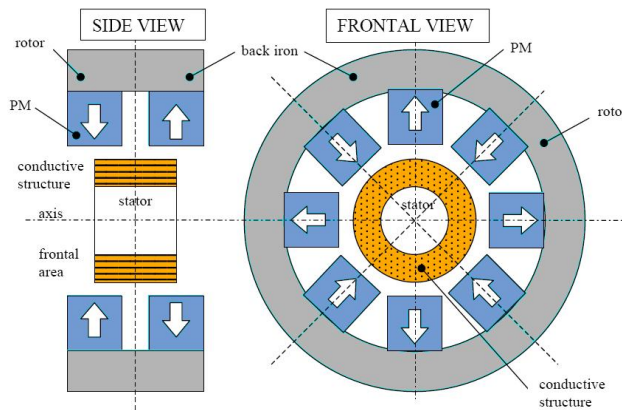


Figure 13: Principle of thrust electrodynamic bearing by [Sandtner and Bleuler, 2008]

arise the axial restoring force. The advantages of the proposed design is to generate no losses at nominal position, no braking torque at low speeds, and no force ripples. It is also mentioned that permanent magnets are preferably placed on the rotor for stability reasons. A prototype has been presented at the ISMB10 conference in 2006.

Finally, let us mention that deliberate eddy currents are also used in electrodynamic dampers. Their principle is similar to electrodynamic bearings, only the finality changes.

2 Objectives and Motivations

The literature review has shown that because of the losses and because of the mechanical parasitic forces the eddy currents develop, these currents may be undergone negatively, for instance in active magnetic bearings or in semi-passive suspensions.

On the other hand, the eddy currents may also be deliberately generated in order to create useful forces, like in electrodynamic bearings. In both cases, modeling the forces due to eddy currents adequately in dynamic conditions is important in order to be able to simulate the

dynamical behavior of the bearing and to predict its performances. But this study is not easy because two aspects have to be taken into account: the electromagnetic nature of the forces present in the bearing and the rotational machinery aspects. Indeed, magnetic bearings are devices where a very strong interaction between electromagnetic and mechanical science arise.

Modeling the dynamical behavior of a magnetic bearing with finite elements is not possible. These models have to include the mechanic and electric dynamics of the bearing, which would be far too heavy to simulate, and is not possible to realize reasonably nowadays. Furthermore, finite elements model do not offer any insight on the physics involved nor on the parameters influencing the bearing behavior. Analytical models have already been developed for particular passive electrodynamic bearings ([Filatov, 2002],[Lembke, 2005]). In each case, the particular model of the proposed magnetic bearing structure is then used to determine the necessary damping and to predict the dynamical behavior of the bearing. These models were developed by solving the Maxwell's equations, and when the topology of the bearing is changed, the whole process needs to be done again. Furthermore, in both models restrictive hypothesis are made in order to be able to solve the Maxwell's equations, or even in order to be able to study the dynamic stability. Besides, such a model does not allow a comprehensive understanding of all the physical principles involved and interactions, nor an easy analyze of the parameters influence, nor to be used to draw general conclusions on magnetic bearings development. This shows the need for a general comprehensive electromechanical model able to predict the dynamical behavior of such bearings. In [Amati et al., 2008], a general parameterized electromechanical model is described. The model is based on the analogy between the resistive-inductive dynamics of eddy currents and the spring-damper in series dynamics. The model focuses on the dynamical behavior of purely electrodynamic magnetic radial and thrust bearings. Stability is examined for a Jeffcott rotor on these kind of bearings. It is shown, based on a root loci study, that the rotor is wholeheartedly unstable when used alone: external non rotating damping is necessary to stabilize the system. Solutions are proposed in [Tonoli et al., 2011].

The objective of this work is to develop a model able to:

- predict and explain the dynamical behavior and performances of any magnetic bearing subject to eddy currents or of a system integrating such magnetic bearings,
- highlight the parameters influencing the dynamical behavior and the performances of any magnetic bearing subject to eddy currents or of a system integrating such magnetic bearings.
- explain how the parameters influence the dynamical behavior and the performances of any magnetic bearing subject to eddy currents or of a system integrating such magnetic bearings.

The purpose of this model is to

- develop a comprehensive understanding of the phenomena and the parameters determining the performances and the dynamical behavior of any magnetic bearing subject to eddy currents or of a system integrating such magnetic bearings,
- deduce from that some design rules to be applied to these magnetic bearing subject to eddy currents,
- predict and simulate the dynamical behavior of any magnetic bearing subject to eddy currents and predict its performances.

In this dissertation, we present the general parameterized electromechanical model we developed leading to the study of the radial dynamical behavior of magnetic bearings subject to eddy currents. A macroscopic point of view is chosen as to the electromagnetic phenomena involved in the system. This allows us to generalize the model to all kind of radial magnetic bearing subject to eddy currents, as presented in [Kluysskens et al., 2007], [Kluysskens and Dehez, 2008] and in [Kluysskens and Dehez, 2009]: electrodynamic bearings, permanent magnet bearings... These bearings are modeled by mechanical components, like springs, for reluctant forces, and dampers, for eddy current forces. The choice of the way to introduce these mechanical components in the electromechanical model is the result of a rigorous approach based on the electrical and the mechanical power present in the system. With our parameterized model, there is no

need to develop an often complex analytical solution for the electric and the magnetic fields. The resistive and the inductive behavior of the eddy currents is also taken into account, and an analysis is made of the skin effect on this behavior at high rotating spin speed ([Kluyssens and Dehez, 2011]).

The parameters of the electromechanical system can be identified by forces measurements at different spin speed either through a finite element simulations or through experiments.

The developed electromechanical system shows the existence of an unstable spin speed range, or in other words the existence of two limit spin speeds between which the system behaves unstable in the radial plane. The lower limit spin speed is confirmed by experiences, which also show the dependency of this unstable speed range with the damping present in the stator. The existence of the upper limit spin speed is confirmed by literature. Above this second limit spin speed, the system is always stable. Based on the presented electromechanical model, the stability of a simple Jeffcott rotor supported by magnetic bearings is analyzed. The influence of the different parameters of the electromechanical model on the unstable speed range and on the stiffness is examined, in order to express some design rules.

The electromechanical system also allows to predict the value of the stiffness achievable by the induced current forces, so that it can be verified that a specific magnetic suspension corresponds to desired specifications.

In this work, simulations of the behavior of two simple examples, based on the exploitation of the electromechanical model are shown.

The structure of this work is the following. In the first part of the dissertation, chapter 1 reminds some mechanical principles of rotating machinery: when damping is associated to the rotor spinning, it may destabilize the system.

Next, chapter 2 develops the macroscopic parameterized model for the dynamical behavior of magnetic bearings. The starting point is the mechanical model presented in previous chapter, but the differences with it are outlined. Then the electromechanical model is obtained by a comparison between the electrical and the mechanical power present in the system.

Finally, to be able to take the skin effect in the conductors into ac-

count, models for the impedance dependency with the frequency are presented in chapter 3.

In the second part of the dissertation, a qualitative validation is carried out with two basic experiments in chapter 4. A quantitative validation of the model on various cases of radial electrodynamic systems is done in chapter 5. A few words are also written on the identification method in this chapter. It is shown that a quasi-static finite element simulation or experiment is sufficient to identify the parameters of the model on a the least square criterion. The importance of the choice of the skin effect model is also shown. In parallel, the finite element simulation results have been validated by experimental measurements.

In the third part of the dissertation, chapter 6 analyzes stability issues thanks to the developed electromechanical model. The influence of the different parameters is examined, from the specific to the electromechanical model parameters point of view and from the physical parameters point of view. Finally some examples of the exploitation of the model are shown in chapter 7.

Finally, the dissertation concludes with a summary of the results and some suggestions for future research.

Part I

Modeling

Chapter 1

Mechanical model

The content of this chapter is a presentation of some useful rotor dynamics basic knowledge. The aim is to study and understand the role of damping in rotor dynamics through a purely mechanical approach. As it will be explained, depending on the origin of the damping, linked to the rotor spinning or whirling, this damping may have a stabilizing or a destabilizing role.

1.1 Equations of motion

Let us write the equations of motion of a simplified linear Jeffcott rotor. The Jeffcott rotor, also called the Laval rotor, is a single disk centrally mounted on a uniform flexible massless shaft and supported by two identically rigid bearings, as illustrated in Fig. 1.1(a). This rotor model is widely used to study and understand rotor dynamics phenomena. We will consider a simplified Jeffcott rotor, by supposing the disk to be a point mass, with mass m , Fig. 1.1(b). The shaft produces an elastic restoring force with stiffness k on the rotor. The spinning speed is ω . The damping in the system can be split into two components:

- on the one hand, the damping linked to the rotor whirling motion, ie the motion of the rotor in the radial plane, called "non-rotating damping", and noted c_{nr} in the following sections,

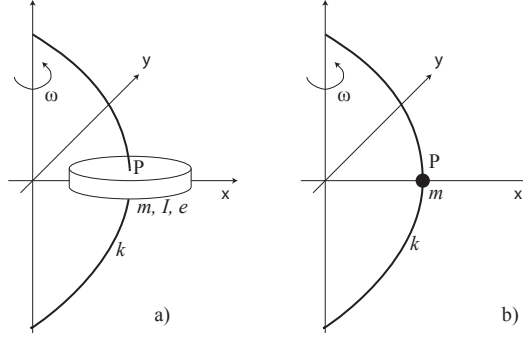


Figure 1.1: (a) Sketch of a Jeffcott rotor, in its deformed configuration (b) simplified Jeffcott rotor, as presented in [Genta, 1999]

- on the other hand, the damping linked to the rotor spinning motion, ie the rotor rotation around the axial axis, called "rotating damping", and noted c_r in the following sections.

The whirl and spinning motions of the rotor are illustrated in Fig 1.2. In mechanics, damping associated to the rotor spinning motion may come from all mechanisms creating energy dissipation within the rotor, such as material damping, friction in connections in built-up rotors, intershaft dampers in multishaft machines, and so on.

We shall now develop in detail the equations of motion of a simplified Jeffcott rotor, as illustrated in Fig. 1.1 (b).

An illustration of rotating and non rotating damping can be made by studying the behavior of a rigid shaft rotating at constant speed ω whose bearing is composed of two elastomers in parallel. The first elastomer is fixed to the stator: it is not rotating. It will only produce forces when the rotor is whirling. The second elastomer is fixed to the rotor: it is rotating with the rotor, and will be excited by the rotor spinning motion. The non-rotating elastomer has stiffness k_{nr} and damping factor c_{nr} . The rotating elastomer has stiffness k_r and damping factor c_r (see Figs. 1.3 and 1.4). The elastomer properties are assumed to be frequency independent, which allows to remain in a situation similar to the "Jeffcott rotor".

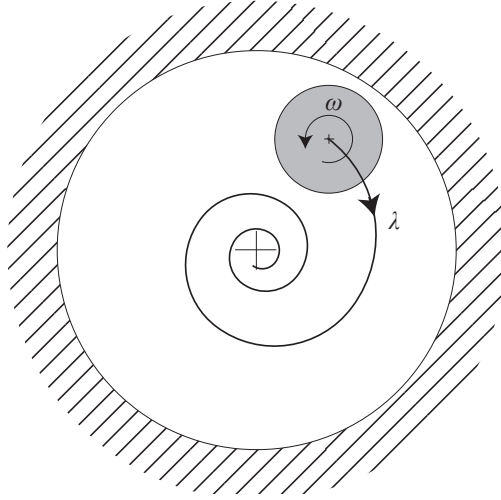


Figure 1.2: Schematic representation of the rotor spinning at speed ω and whirling at speed λ

When looking on a cut into the radial plane, two frames can be associated with this system (Fig. 1.4). The first one has its origin at the center of the stator and is not rotating. It is the inertial frame $\{\hat{I}\}$. The second frame $\{\hat{X}\}$, has the same origin but is rotating at the spin speed ω . The rotor position coordinates in the plane perpendicular to the rotation axis are:

- in the inertial frame $\{\hat{I}\}$: $(x_I(t), y_I(t))$,
- in the rotating frame $\{\hat{X}\}$: $(x_X(t), y_X(t))$.

The forces acting on the rotor are:

$$\begin{aligned}
 & -k_r x_X \hat{X}_1 - k_r y_X \hat{X}_2 - c_r \dot{x}_X \hat{X}_1 - c_r \dot{y}_X \hat{X}_2 - \\
 & k_{nr} x_I \hat{I}_1 - k_{nr} y_I \hat{I}_2 - c_{nr} \dot{x}_I \hat{I}_1 - c_{nr} \dot{y}_I \hat{I}_2.
 \end{aligned} \tag{1.1}$$

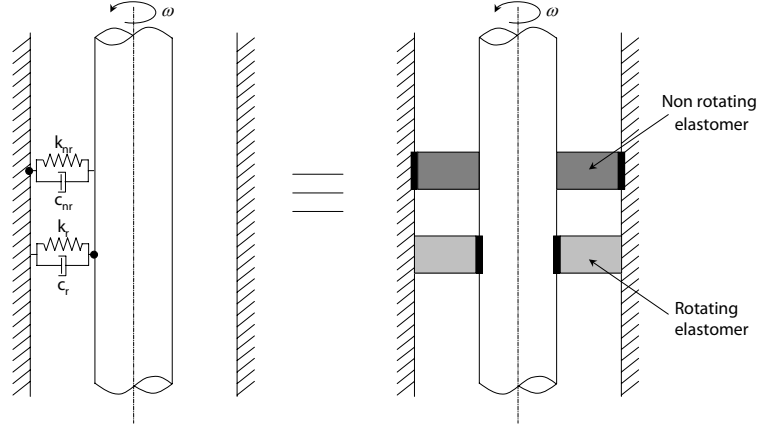


Figure 1.3: Shaft with rotating and non rotating elastomers - side view

By carrying out the appropriate coordinate and frame changes, the following equations of motion for the rotor are obtained:

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} \ddot{x}_I \\ \ddot{y}_I \end{pmatrix} + \begin{pmatrix} c_{nr} + c_r & 0 \\ 0 & c_{nr} + c_r \end{pmatrix} \begin{pmatrix} \dot{x}_I \\ \dot{y}_I \end{pmatrix} + \begin{pmatrix} k_{nr} + k_r & c_r \omega \\ -c_r \omega & k_{nr} + k_r \end{pmatrix} \begin{pmatrix} x_I \\ y_I \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (1.2)$$

As observed, the rotating damping introduces off-diagonal terms in the stiffness matrix. They are proportional to ω .

The position (x, y) of the rotor in the plane perpendicular to the rotation can also be expressed by a complex vector $\bar{z} = x + jy$. We shall suppose that the spinning speed ω is constant and that there is no unbalance. The homogeneous equation of motion then becomes, as explained in [Genta, 1999]:

$$m\ddot{\bar{z}} + (c_{nr} + c_r)\dot{\bar{z}} + (k - jc_r\omega)\bar{z} = 0. \quad (1.3)$$

which is the same as eq. 1.2 with $k = k_r + k_{nr}$.

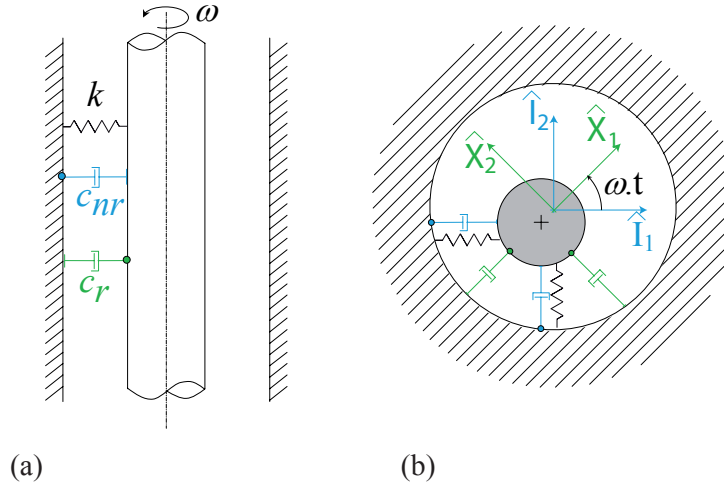


Figure 1.4: Shaft with rotating and non rotating elastomers (a) side schematic view (b) upper schematic view

1.2 Limit spin speed

The solution of eq. 1.3 may be expressed as $\bar{z} = z_0 \exp(\bar{\lambda}t)$, with $\bar{\lambda} = \lambda_R + \lambda_I j$ a complex number, which corresponds to:

$$\begin{cases} x = z_0 \exp(\lambda_R t) \cos(\lambda_I t) \\ y = z_0 \exp(\lambda_R t) \sin(\lambda_I t) \end{cases}, \quad (1.4)$$

where λ_I is the whirl motion frequency and λ_R is the whirl motion damping rate. z_0 depends on the initial conditions. By replacing $\bar{z}$ by its solution in the equation 1.3, this yields for $\bar{\lambda}$:

$$\bar{\lambda} = - \left(\frac{c_{nr} + c_r}{2m} \pm \frac{1}{\sqrt{2}} \sqrt{-\Gamma + \sqrt{\Gamma^2 + \left(\frac{\omega c_r}{m}\right)^2}} \right) \mp j \frac{1}{\sqrt{2}} \sqrt{\Gamma + \sqrt{\Gamma^2 + \left(\frac{\omega c_r}{m}\right)^2}}, \quad (1.5)$$

with:

$$\Gamma = \frac{k}{m} - \frac{(c_{nr} + c_r)^2}{4m^2}. \quad (1.6)$$

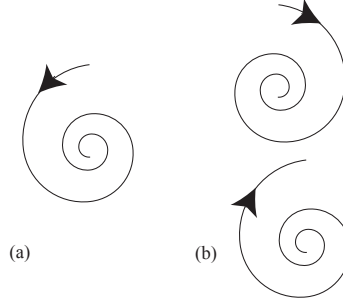


Figure 1.5: Schematic representation for a rotor spinning in clockwise direction, of (a) an always damped backward whirl motion (b) a damped and an undamped forward whirl motion

The first solution, corresponding to the negative imaginary part of $\bar{\lambda}$, has always a negative real part. It represents an always-damped backward whirl motion, see Fig. 1.5 (a).

The second solution, corresponding to the positive imaginary part of $\bar{\lambda}$, represents a forward whirl motion, see Fig. 1.5 (b). It is only damped when λ_R (eq. 1.5) is negative, that is to say for an angular velocity ω smaller than the limit angular velocity ω_{lim} given by:

$$\omega_{lim} = \omega_{cr} \left(1 + \frac{c_{nr}}{c_r} \right), \quad (1.7)$$

with:

$$\omega_{cr} = \sqrt{\frac{k}{m}}, \quad (1.8)$$

the critical speed of the system. If the angular velocity is higher than the limit speed, the real part of λ is positive and the forward whirl motion is permanently amplified.

The system stability may also be studied by working in the Laplace domain. The denominator of the system transfer function is then:

$$\begin{aligned} m^2 s^4 + 2m(c_{nr} + c_r)s^3 + (2mk + (c_{nr} + c_r)^2)s^2 \\ + 2(c_{nr} + c_r)ks + k^2 + c_r^2\omega^2. \end{aligned} \quad (1.9)$$

Applying the Routh-Hurwitz criterion to this denominator, we get the coefficient table shown in Table 1.1. A necessary and sufficient condition for the system to be stable is for all the first column coefficients to have the same sign. Here this implies that:

$$\omega < \sqrt{\frac{k}{m}} \left(1 + \frac{c_{nr}}{c_r} \right). \quad (1.10)$$

When this condition is met, the system is asymptotically stable.

Table 1.1: Routh-Hurwitz coefficient table

m^2	$2mk$ $+(c_{nr} + c_r)^2$	k^2 $+c_r^2\omega^2$
$2m(c_{nr} + c_r)$	$2(c_{nr} + c_r)k$	
$mk + (c_{nr} + c_r)^2$	$k^2 + c_r^2\omega^2$	
$\frac{2(c_{nr}+c_r)}{mk+(c_{nr}+c_r)^2}$ $((c_{nr} + c_r)^2 k - mc_r^2 \omega^2)$		

When c_r is nonexistent, the limit angular velocity is infinite and there is no problem. But when c_r is not zero, the behavior of the rotor is unstable beyond a finite limit angular velocity ω_{lim} . There is no stable behavior possible beyond this limit speed anymore. As illustrated in Fig. 1.6, this limit angular velocity decreases when c_r increases. When c_{nr} is nonexistent or negligible compared to c_r , the limit angular velocity is minimum and is equal to ω_{cr} , the critical speed. In opposition to the usual stabilizing role of damping, it can be observed here that the rotating damping has a destabilizing role. Thus, when damping is associated to rotating parts, it may induce an unstable behavior, where the amplitude of the motion increases instead of decreasing like it could be expected when introducing damping into the system. This means that attention has to be paid to the way damping is introduced in rotating systems.

1.3 Numerical case

To illustrate the equations obtained in previous section, let us look at the results of simulations for a numerical case.

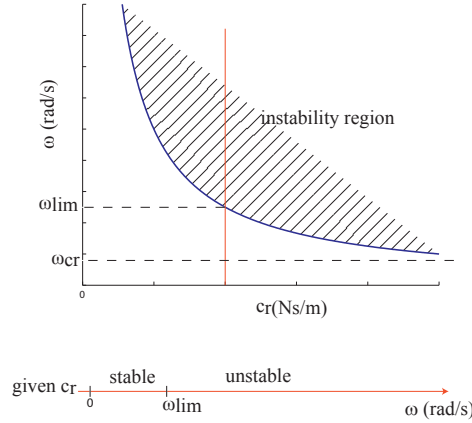


Figure 1.6: Limit spin speed for a given set of parameters as a function of the rotating damping (c_r)

A system where:

- the rotor weighs 0.02 kg,
- the elastomers have stiffnesses of 5 N/m each,
- the non rotating damping is worth 0.1 Ns/m,
- and the rotating damping is worth 1 Ns/m.

From eqs. 1.7 and 1.8, the critical speed of the system equals 22.36 rad/s and the limit angular velocity equals 24.6 rad/s.

Via numerical simulations, it can be observed (Figs. 1.7 and 1.8) that as long as the rotation speed is smaller than the limit speed, the system is stable. On the other hand, for $\omega = 30\text{rad/s}$, the system is unstable.

1.4 Intuitive explanation

This instability can also be explained in a more intuitive manner on the system depicted by Figs. 1.3 and 1.4, by using the superposition

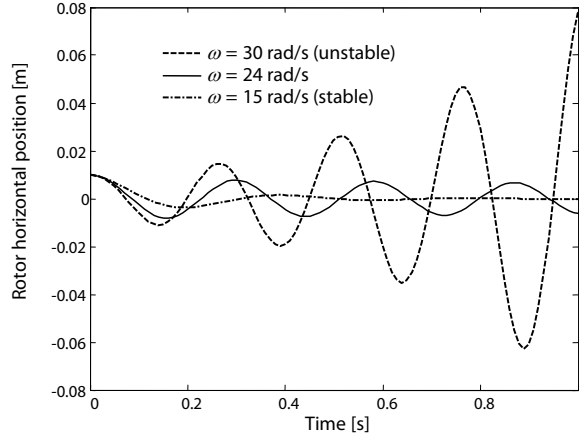


Figure 1.7: Rotor motion as a function of time

principle and examining the forces acting on the rotor. In the following analysis, let us only consider the forces due to the damping, not the restoring force due to the stiffness.

Consider first the rotor spinning in a fixed off-centered position. The elastomer rotating with the rotor will in turn be compressed and stretched. This results in a damping force induced by the radial velocity in the moving frame which is ωd , and directed in the perpendicular direction (Fig. 1.9 (a)). The elastomer fixed to the stator, does not move, and does not produce any damping force. In other words, the non-rotating damping does not produce any force because the center shift is fixed. But the rotating damping produces a force perpendicular to the center shift, and proportional to ωd :

$$F_{\perp 1} = c_r \omega d. \quad (1.11)$$

Second, let us look at the forces acting on the rotor in the case where the rotor does not spin (Fig. 1.9 (b)), but its center turns around

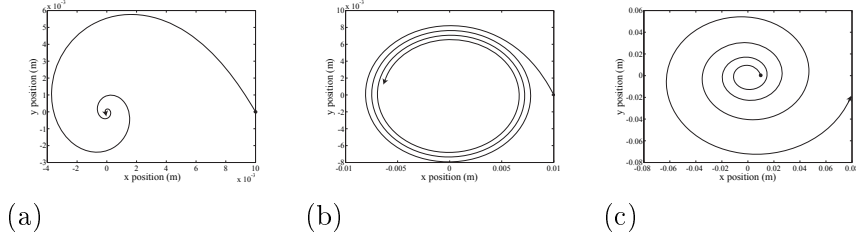


Figure 1.8: Radial rotor motion for $\omega =$ (a) 15 rad/s, (b) 24 rad/s and (c) 30 rad/s

the center of the stator at the critical speed $\omega_{cr} = \sqrt{\frac{k}{m}}$. Both elastomers, the one linked to the rotor, and the one linked to the stator, produce a force. This force is proportional to the tangential speed of the rotor:

$$F_{\perp 2} = (c_{nr} + c_r) \sqrt{\frac{k}{m}} d, \quad (1.12)$$

and directed in the opposite direction to $F_{\perp 1}$.

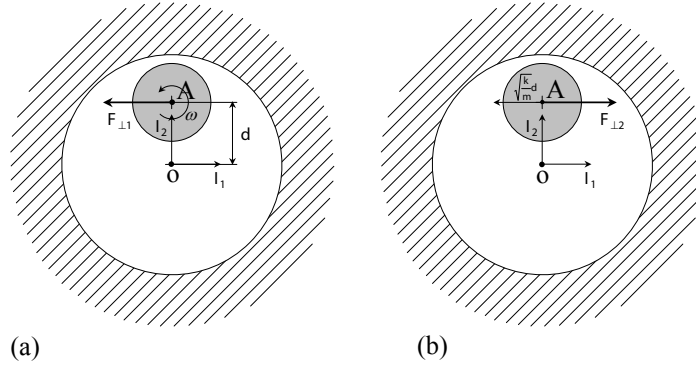


Figure 1.9: Forces acting on the rotor when (a) rotating around a fixed off-centered point A, (b) A is moving circularly around O

The sum of the forces due to the dampers, $F_{\perp 1}$ and $F_{\perp 2}$, act in the direction perpendicular to the center shift. As long as this sum is negative, its action is opposed to the tangential speed, and the system follows a shrinking elliptic motion. But when this sum is positive, the system undergoes an expanding elliptic trajectory, and is unstable.

The limit angular velocity is defined by the relation:

$$(c_r + c_{nr})\sqrt{\frac{k}{m}} = c_r\omega_{lim}, \quad (1.13)$$

leading to the same result as predicted by the mechanical model in Eq. (1.10).

1.5 Synthesis

In this section, it has been explained that for a point rotor with mass m with no eccentricity, submitted to spring forces, to rotating and non-rotating damping forces, the equations of motion can be written as:

$$m\ddot{\bar{z}} + (c_{nr} + c_r)\dot{\bar{z}} + (k - jc_r\omega)\bar{z} = 0. \quad (1.14)$$

The analysis of this equation has shown the existence of a rotor limit spin speed beyond which no stable behavior is possible:

$$\omega_{lim} = \sqrt{\frac{k}{m}} \left(1 + \frac{c_{nr}}{c_r} \right). \quad (1.15)$$

Chapter 2

Electromechanical model

The electromechanical model we developed is presented in this section. It is based on the idea that the electromagnetic forces acting within a magnetic bearing can be modeled using mechanical components like springs and dampers. Indeed, on the one hand, the reluctance forces and the forces between permanent magnets can be modeled by stiffnesses, as they are proportional to the relative displacement, when these displacements remain small enough to linearize the forces. On the other hand, the Lorentz forces due to the interaction between induced currents, due to relative speed, and magnetic fields, can be modeled by introducing damping into the equations. Depending on following which kind of motion the eddy currents result from, they are modeled by rotating damping or by non-rotating damping.

However, these Lorentz forces can not be modeled by simply replacing the rotating damping coefficient by an appropriate constant in the previous mechanical model (Eq. 1.3).

Actually, when an electromotive force is induced on a conducting piece, the generated currents are subjected to inductive and resistive effects. Since the amplitude and the direction of the Lorentz forces depend on these induced currents, these forces are also subjected to these resistive and inductive effects.

These effects will be felt on the orientation of the force. The induced currents undergo a phase shift relative to the electromotive force function of the inductive and resistive effects. It will be shown in this

chapter that this phase shift results in an angle shift between the displacement and the forces.

The resistive and inductive effects also have consequences on the norm of the force. Indeed, the norm of these induced currents depends on the norm of the electromotive force through the impedance, and then, through the resistive and inductive effects.

2.1 Assumptions

In order to formalize these effects, we will consider that:

- the spinning speed ω is constant;
- the rotor is a rigid point mass;
- there is no unbalance, i.e. the point mass is situated on the rotation axis;
- the relative displacements remain small;
- the electromagnetic variables vary in a way that may be approximated by a sinusoidal behavior;
- there are only eddy currents due to the spinning motion of the rotor;
- the material are linear and do not have any magnetic saturation;
- the thermal regime is a steady-state regime;
- the electrical behavior of the conductor can be modeled by a circuit with a global resistance R_r and a global inductance L_r .

2.2 Consequence of an angular shift on the equations of motion

Given that the resistive and inductive effects of the currents generate a phase shift between these currents and the electromotive force appearing when the rotor moves, we can suppose that the induced current

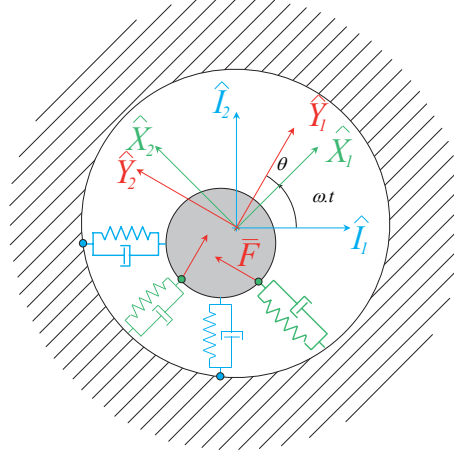


Figure 2.1: Shaft with rotating and non rotating elastomers and angular shift - upper schematic view

forces will experience an angular shift with respect to the rotor speed $\dot{z}$. We do not know how this angular and phase shift are linked yet, but we can already analyze the mechanical implications of an angular shift of the forces, called θ , on the equations of motion.

To do this, we shall now study the behavior of a shaft rotating at constant speed ω , similar to the practical case of the linear simplified Jeffcott rotor studied in the section 1.3. For recall, the bearing of the shaft could be assimilated to two elastomers in parallel. The first elastomer being fixed to the stator, it is not rotating. The second elastomer is fixed to the rotor: it is rotating. Let us now imagine that the forces due to the damping of the rotating elastomer undergo an angular shift θ .

Three frames can be associated with this system (Fig. 2.1). The first one has its origin at the center of the stator and is not rotating. It is the inertial frame $\{\hat{I}\}$. The second frame $\{\hat{X}\}$, has the same origin but is rotating at speed ω . The third frame $\{\hat{Y}\}$, has also the same

origin, is also rotating at speed ω , but with an angular shift θ , such that

$$\{\hat{Y}\} = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \{\hat{X}\}.$$

The coordinates of the rotor center position in the plane perpendicular to the rotation axis are:

- in the inertial frame $\{\hat{I}\}$: $(x_I(t), y_I(t))$,
- in the rotating frame $\{\hat{X}\}$: $(x_X(t), y_X(t))$.

The forces acting on the rotor are:

$$-c_r \dot{x}_X \hat{Y}_1 - c_r \dot{y}_X \hat{Y}_2 - k x_I \hat{I}_1 - k y_I \hat{I}_2 - c_{nr} \dot{x}_I \hat{I}_1 - c_{nr} \dot{y}_I \hat{I}_2. \quad (2.1)$$

By carrying out the appropriate coordinate and frame changes, the following equations of motion for the rotor are obtained:

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} \ddot{x}_I \\ \ddot{y}_I \end{pmatrix} + \begin{pmatrix} c_{nr} + c_r \cos \theta & -c_r \sin \theta \\ c_r \sin \theta & c_{nr} + c_r \cos \theta \end{pmatrix} \begin{pmatrix} \dot{x}_I \\ \dot{y}_I \end{pmatrix} + \begin{pmatrix} k + c_r \omega \sin \theta & c_r \omega \cos \theta \\ -c_r \omega \cos \theta & k + c_r \omega \sin \theta \end{pmatrix} \begin{pmatrix} x_I \\ y_I \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (2.2)$$

It can be observed that in this case, there are non-diagonal terms not only in the stiffness matrix, but also in the damping matrix. Furthermore, in the diagonal terms of the stiffness matrix, we observe that this angular shift creates an stiffness, which is worth $c_r \omega \sin \theta$. Obviously, with no angular shift ($\theta = 0$) we find again the same equations as the equations of motion without angular shift described in the equation system (1.2).

The same result could be reached by introducing a complex damping coefficient $\bar{c}_r = c_r \cos \theta + j c_r \sin \theta$ into Eq. (1.3). By substituting the complex damping coefficient $\bar{c}_r$ into Eq.(1.3), we obtain:

$$m \ddot{\bar{z}} + (c_{nr} + c_r \cos \theta + j c_r \sin \theta) \dot{\bar{z}} + (k + c_r \omega \sin \theta - j c_r \omega \cos \theta) \bar{z} = 0, \quad (2.3)$$

which is equivalent to Eq.(2.2), with $\bar{z} = x + jy$.

2.3 Power considerations

The objective in this section is to put into relation the consequences of the resistive and inductive effects on the norm and the phase shift of the induced currents with the angle shift and the norm of the forces due to the rotating damping.

To do this, let's have a closer look at the different powers present in an electromechanical system.

2.3.1 Mechanical power

To examine the mechanical power dissipated by the rotor spin motion, we place ourselves on the rotor, rotating at spin speed ω with the rotating frame $\{\hat{X}\}$ (see Fig. 2.1). Because we are only interested in the power dissipated by the rotor spin motion, we will only look at the forces created by the rotating damping.

In this case, the forces acting on the rotor, and expressed in the rotating frame, predicted by the electromechanical model are equal to

$$\bar{F}_X = -(c_r \cos \theta + j c_r \sin(\theta)) \dot{\bar{z}}_X, \quad (2.4)$$

where $\bar{z}_X$ is a complex number expressing the rotor coordinates in the rotating frame: $\bar{z}_X = x_X + jy_X$. As explained in the previous section, the appropriate frame changes allow to express the rotor coordinates in the inertial frame ($\bar{z}_I$) or in the rotating frame: $\bar{z}_X = \bar{z}_I \exp(-j\omega t)$. This means that the rotor speed is:

$$\dot{\bar{z}}_X = \dot{\bar{z}}_I \exp(-j\omega t) - \bar{z}_I j\omega \exp(-j\omega t) \quad (2.5)$$

The first term of this equation is the relative derivative in the rotating frame, and the second term arises because we are in a frame where the orientation is not constant with respect to the inertial frame.

We are interested in the mechanical power dissipated by the rotating damping coefficient, given that this is the mechanical component we want to put into relation with the induced currents into the rotor.

The complex mechanical power dissipated by the rotating damping can now be expressed as:

$$\begin{aligned}\bar{S}_{MECA} &= \bar{F}_X \dot{z}_X^* \\ &= -(c_r \cos \theta + j c_r \sin \theta) \dot{z}_X \dot{z}_X^*\end{aligned}\quad (2.6)$$

From Eqs. (2.5) and (2.6), it can be seen the mechanical power is worth:

$$\begin{aligned}\bar{S}_{MECA} &= -(c_r \cos \theta + j c_r \sin \theta) \\ &\quad \left(\dot{Z}_I^2 + j \omega (\dot{z}_I \bar{z}_I^* - \dot{z}_I^* \bar{z}_I) + \omega^2 Z_I^2 \right)\end{aligned}\quad (2.7)$$

As the motion of the rotor can be described by a whirl motion of frequency λ_I , and thus can be expressed by:

$$\bar{z}_I(t) = Z_I(t) \exp j \lambda_I t, \quad (2.8)$$

The mechanical power, with Eqs. 2.8 and 2.7, can finally be expressed by:

$$\begin{aligned}\bar{S}_{MECA} &= -(c_r \cos \theta + j c_r \sin \theta) (\lambda_I^2 Z_I^2 + j \omega (2j \lambda_I Z_I^2) + \omega^2 Z_I^2) \\ &= -(c_r \cos \theta + j c_r \sin \theta) (\omega + \lambda_I)^2 Z_I^2\end{aligned}\quad (2.9)$$

From the expression of the mechanical power in Eq. 2.9, on one side the active part, which is the real part and on the other side the reactive part, which is the imaginary part may be dissociated:

$$Re(\bar{S}_{MECA}) = -c_r (\omega + \lambda_I)^2 Z_I^2 \cos \theta \quad (2.10)$$

$$Im(\bar{S}_{MECA}) = -c_r (\omega + \lambda_I)^2 Z_I^2 \sin \theta \quad (2.11)$$

Let us point out that from a mechanical point of view, with regard to Eq. 2.10 and Eq. 2.11, the real part of $\bar{c}_r$, this is to say $c_r \cos \theta$, corresponds to the damping and is the part which dissipates, whereas the imaginary part, this is to say $c_r \sin \theta$, corresponds to a stiffness in the sense that it does not imply any energy dissipation.

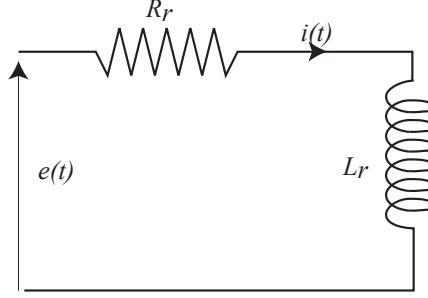


Figure 2.2: Equivalent electrical circuit for the induced currents

2.3.2 Electrical power

Considering that the conductor experiencing a flux variation due to the rotor spin can be brought to an equivalent circuit in which the eddy currents, called $i(t)$ flow, as shown in Fig. 2.2, the electrical equations of what is happening can be written as:

$$0 = R_r i(t) + \frac{d\psi_{tot}}{dt},$$

where $\psi_{tot} = \psi_0 + L_r i(t)$. ψ_0 is the magnetic flux generated by the permanent magnets. The resistance R_r represents the resistive effects, and the inductance L_r represents the inductive effects on the induced currents in this electrical circuit.

Let's define $e(t) = -\frac{d\psi_0}{dt}$ the electromotive force due to the variations of magnetic flux seen by the conductor. This electromotive force $e(t)$ is the excitation of the system, and the current $i(t)$ flowing in the circuit is the response. The last equation can be rewritten:

$$e(t) = R_r i(t) + L_r \frac{di(t)}{dt} \quad (2.12)$$

As stated in the assumptions, the excitation can be approximated by the first harmonic, with a frequency ω_{elec} . This frequency is di-

rectly linked to the rotor motion and is the sum of the rotor spinning frequency ω and the rotor whirling frequency λ_I .

The first harmonic assumption may at first sight seem strong or restrictive. But as it will be illustrated in the validation part in chapter 5, this hypothesis is justified for magnetic bearings. Indeed when developing a magnetic bearing, to limit the losses, it will always be tried to avoid induced currents in the equilibrium position. This means that the magnetic field has to be homopolar and axisymmetric.

In this case, when the bearing is out-centered, the approximation of a sinusoidal excitation $e(t)$ is not restrictive, and the resulting electromagnetic quantities vary in a sinusoidal manner.

Manipulating sinusoidal quantities, we can use the phasor notations:

$$\begin{aligned}\bar{E} &= E \\ \bar{I} &= I \exp(j\varphi)\end{aligned}$$

linked to the time notations by

$$\begin{aligned}e(t) &= \operatorname{Re}(\sqrt{2}E \exp(j\omega_{elec}t)) \\ i(t) &= \operatorname{Re}(\sqrt{2}I \exp(j(\omega_{elec}t + \varphi)))\end{aligned}$$

The complex electric power dissipated in the rotor can then be expressed by

$$\begin{aligned}\bar{S}_{ELEC} &= \bar{E}\bar{I}^* \\ &= EI \cos \varphi + jEI \sin \varphi\end{aligned}\tag{2.13}$$

Considering the global impedance of the rotor, the phase shift between the electromotive force $\bar{E}$ and the induced current $\bar{I}$, called φ is given by:

$$\tan \varphi = \frac{\omega_{elec}L_r}{R_r}\tag{2.14}$$

The relation between the norm of the induced current I and the norm of the electromotive force E is:

$$I = \frac{E}{\sqrt{R_r^2 + (\omega_{elec}L_r)^2}}\tag{2.15}$$

Finally, the electric power dissipated in the rotor can be developed into the following expression:

$$\bar{S}_{ELEC} = \frac{E^2}{\sqrt{R_r^2 + (\omega_{elec}L_r)^2}} (\cos \varphi + j \sin \varphi) \quad (2.16)$$

and dissociated into the real or active and the imaginary or reactive parts:

$$Re(\bar{S}_{ELEC}) = \frac{E^2}{\sqrt{R_r^2 + (\omega_{elec}L_r)^2}} \cos \varphi \quad (2.17)$$

$$Im(\bar{S}_{ELEC}) = \frac{E^2}{\sqrt{R_r^2 + (\omega_{elec}L_r)^2}} \sin \varphi \quad (2.18)$$

2.3.3 Power balance

If we consider that the other mechanical losses and the magnetic hysteresis losses in the system are negligible, there is no other power exchange in the system. Indeed, no power is transmitted between the stator and the rotor. As a conclusion the mechanical power injected into the rotor has to be equal to the electric power dissipated in this rotor.

This means that the eqs. (2.16) and (2.9) have to be equal. This leads to two equations, one for the real part and one for the imaginary part:

$$c_r(\omega + \lambda_I)^2 Z_I^2 \cos \theta = \frac{E^2}{\sqrt{R_r^2 + (\omega_{elec}L_r)^2}} \cos \varphi \quad (2.19)$$

$$c_r(\omega + \lambda_I)^2 Z_I^2 \sin \theta = \frac{E^2}{\sqrt{R_r^2 + (\omega_{elec}L_r)^2}} \sin \varphi \quad (2.20)$$

The ratio between the two Eqs 2.19 and 2.20 leads to the conclusion that:

$$\theta = \varphi = \arctan \frac{\omega_{elec}L_r}{R_r} \quad (2.21)$$

The phase shift between the electromotive force, due to the variation of magnetic flux, and the induced currents therefore result in an angular shift on the rotating damping coefficient. This is coherent with the physics: the orientation of the force due to the induced currents is determined by the angular shift between the induced currents and the magnetic field inducing them, and thus by the electrical phase shift between the electromotive force and the induced currents.

This result, eq. (2.21), leads eq. (2.19) and eq. (2.20) to:

$$\begin{aligned} c_r(\omega + \lambda_I)^2 Z_I^2 &= \frac{E^2}{\sqrt{R_r^2 + (\omega_{elec} L_r)^2}} \\ c_r &= \frac{E^2}{(\omega + \lambda_I)^2 Z_I^2 \sqrt{R_r^2 + (\omega_{elec} L_r)^2}} \end{aligned} \quad (2.22)$$

As the electromotive force is proportional to the electrical frequency $\omega_{elec} = \omega + \lambda_I$ exciting the system, since:

$$e = -\frac{d\psi}{dt} = -\frac{d\psi}{d\theta_{elec}} \frac{d\theta_{elec}}{dt} = -\frac{d\psi}{d\theta_{elec}} \omega_{elec} \quad (2.23)$$

Eq. (2.22) reduces to:

$$c_r = \frac{c_m^2}{Z_I^2 \sqrt{R_r^2 + ((\omega + \lambda_I) L_r)^2}} \quad (2.24)$$

calling c_m the proportional constant between the electromotive force amplitude e and the electrical frequency ω_{elec} . It appears that the amplitude of the rotating damping is inversely proportional to the global impedance of the rotor. This also is coherent since the force due to the induced currents is proportional to these currents, and the norm of these currents is inversely proportional to the global impedance of the conductor where they take place.

It can also be noticed that rotating damping is inversely proportional to the square of the amplitude of the motion of the rotor.

The global resistance R_r , the global inductance L_r and the proportionality constant c_m are parameters of the model specific for each magnetic bearing.

2.4 Limit spin speed

Like for the mechanical model, the system stability can be studied by working in the Laplace domain. Based on eq. (2.3), the denominator of the system transfer function is:

$$\begin{aligned}
 m^2 s^4 + 2m(c_{nr} + c_r \cos \theta) s^3 + \\
 (2m(k + c_r \omega \sin \theta) + c_{nr}^2 + c_r^2 + 2c_{nr}c_r \cos \theta) s^2 \\
 + 2(c_{nr}k + c_r k \cos \theta + c_{nr}c_r \omega \sin \theta) s + k^2 + c_r^2 \omega^2 + 2c_r k \omega \sin \theta.
 \end{aligned} \tag{2.25}$$

Applying the Routh-Hurwitz criterion to this denominator, we get the coefficient table shown in Table 2.1.

Table 2.1: Routh-Hurwitz coefficient table

m^2	$2m(k + c_r \omega \sin \theta) + c_{nr}^2 + c_r^2 + 2c_{nr}c_r \cos \theta$	$k^2 + c_r^2 \omega^2 + 2c_r k \omega \sin \theta$
$2m(c_{nr} + c_r \cos \theta)$	$2(c_{nr}k + c_r k \cos \theta + c_{nr}c_r \omega \sin \theta)$	
c_1 (see eq. (2.26))	$k^2 + c_r^2 \omega^2 + 2c_r k \omega \sin \theta$	
d_1 (see eq. (2.27))		
$k^2 + c_r^2 \omega^2 + 2c_r k \omega \sin \theta$		

In Table 2.1, the coefficient c_1 is worth

$$\begin{aligned}
 c_1 = & \frac{1}{c_{nr} + c_r \cos \theta} (c_{nr}^3 + 2c_{nr}c_r^2 + c_{nr}km + c_r \cos \theta (3c_{nr}^2 + c_r^2 \\
 & + km) + c_{nr}c_r^2 \cos(2\theta) + c_{nr}c_r m \omega \sin \theta + c_r^2 m \omega \sin 2\theta) \tag{2.26}
 \end{aligned}$$

This coefficient c_1 is always positive because $c_r(\omega)$, defined by eq. (2.24) is always positive and θ defined by eq. 2.21 is always included between 0 and $\pi/2$, since the inductive character of the currents induced into the rotor.

The coefficient d_1 is the only coefficient likely to be negative, and is

worth

$$d_1 c_1 = -\frac{2c_r^2 m \cos^2 \theta (c_{nr}^2 + c_r^2 + 2c_{nr}c_r \cos \theta)}{c_{nr} + c_r \cos \theta} \omega^2 + 2c_{nr}c_r \sin \theta (c_{nr}^2 + c_r^2 + 2c_{nr}c_r \cos \theta) \omega + 2k(c_{nr} + c_r \cos \theta)(c_{nr}^2 + c_r^2 + 2c_{nr}c_r \cos \theta). \quad (2.27)$$

The system is unstable when eq. (2.27) is negative. Equation (2.27) can be solved as a quadratic equation whose roots are then:

$$\omega_{lim,1} = (1 + \frac{c_{nr}}{c_r \cos \theta})(\omega_{dyn} - \sqrt{\omega_{dyn}^2 + \omega_{cr}^2}), \quad (2.28)$$

$$\omega_{lim,2} = (1 + \frac{c_{nr}}{c_r \cos \theta})(\omega_{dyn} + \sqrt{\omega_{dyn}^2 + \omega_{cr}^2}), \quad (2.29)$$

with

$$\omega_{cr} = \sqrt{\frac{k}{m}},$$

and

$$\omega_{dyn} = \frac{c_{nr}}{2m} \tan \theta.$$

It has to be noticed that eqs. (2.28) and (2.29) are non-linear implicit equations since θ and c_r depend on ω in a non-linear way (see eqs. (2.31) and (2.32)), and since θ and c_r also depend non-linearly on λ_I , which is the rotor whirl frequency, thus the solution of the equation of motion.

The roots of this system may give a limited speed range where the behavior of the system is unstable. Indeed, the spin speed can pass through a region where the system is unstable, and afterward reach a region where the behavior is stable.

When there is no phase shift ($\theta = 0$) we find again the previous stability condition of the purely mechanical system.

For instance, we can look at the instability region in figure 2.3. It is clear on this figure that the instability region has a finite upper and lower limit.

From a practical point of view, to pass through a region where the spin speed leads to an unstable behavior, the system will need backup bearings.

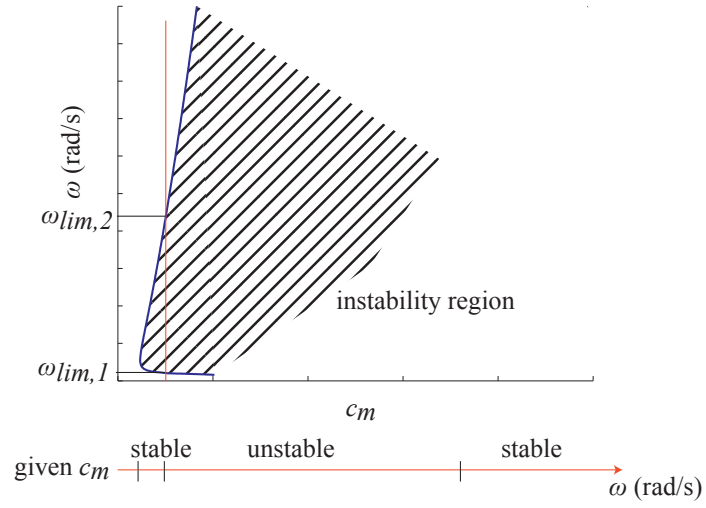


Figure 2.3: Unstable speed range for a given set of parameters as a function of the proportionality constant of rotating damping (c_m) for the electromechanical model

2.5 Numerical case

Let us illustrate the obtained equations for the electromechanical model for a system where:

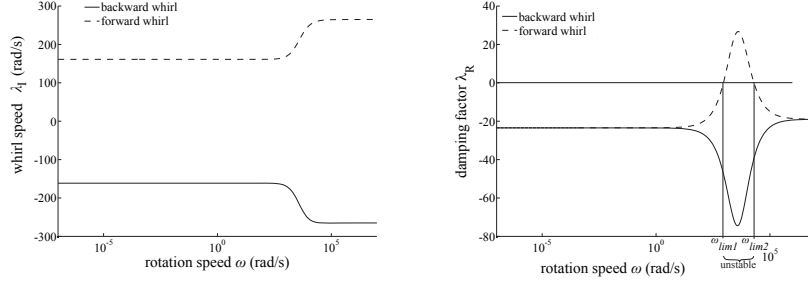
- the rotor weight is $m = 0.2108$ kg,
- the stiffness is $k = 5600$ N/m,
- the non rotating damping is $c_{nr} = 8$ Ns/m,
- the global resistance is $R_r = 0.5286\Omega$,
- the global inductance is $L_r = 1.0777 \cdot 10^{-4}$ H.

These numerical values correspond to the experimental case examined in section 4.2.

When solving the equations of motion eq. 2.30 for the forward and backward whirl frequencies, we get the whirl frequencies and their respective damping coefficients illustrated in figure 2.4 as a function of ω . In this figure, we can first notice that the damping coefficient λ_R associated to the backward whirl motion is negative for all the values of spin speed: this motion is always damped. However, it can also be seen that the damping coefficient λ_R associated to the forward whirl is only negative until a spin speed ω worth 900 rad/s. It is then positive until ω is worth 20000 rad/s; where it becomes negative again.

This means that any perturbation on the rotating conducting disk is damped until 900 rad/s, and after 20000 rad/s. Between these two values of spin speed, any perturbation on the rotor is amplified.

Let us simulate the rotor motion when it is initially out-centered in the x-axis, and has no lateral speed. Figure 2.5 describes the rotor motion in the radial plane. It is clear in Fig 2.5 (a) that there is no stability problem, the rotor goes back to its centered position. But between 900 and 20000 rad/s the rotor is unstable, it describes an out-going spiral (figure 2.5 (b) and (c)). And for higher rotating speed, in Fig 2.5 (d), the rotor will go back to its centered position again, when waiting long enough.



(a)

(b)

Figure 2.4: Whirl motion as a function of the spin speed ω (a) whirl frequency λ_I and (b) damping λ_R

2.6 Synthesis

An electromechanical model has been presented, taking into account the electromagnetic nature of the forces present in magnetic bearings and the mechanical aspects related to rotor dynamics also present in magnetic bearings.

Under the following assumptions:

- the spinning speed ω is constant;
- the rotor is a rigid point mass;
- there is no unbalance, i.e. the point mass is situated on the rotation axis;
- the relative displacements remain small;
- the electromagnetic variables vary in a way that may be approximated by a sinusoidal behavior;
- there are only eddy currents due to the spinning motion of the rotor;
- the material are linear and do not have any magnetic saturation;

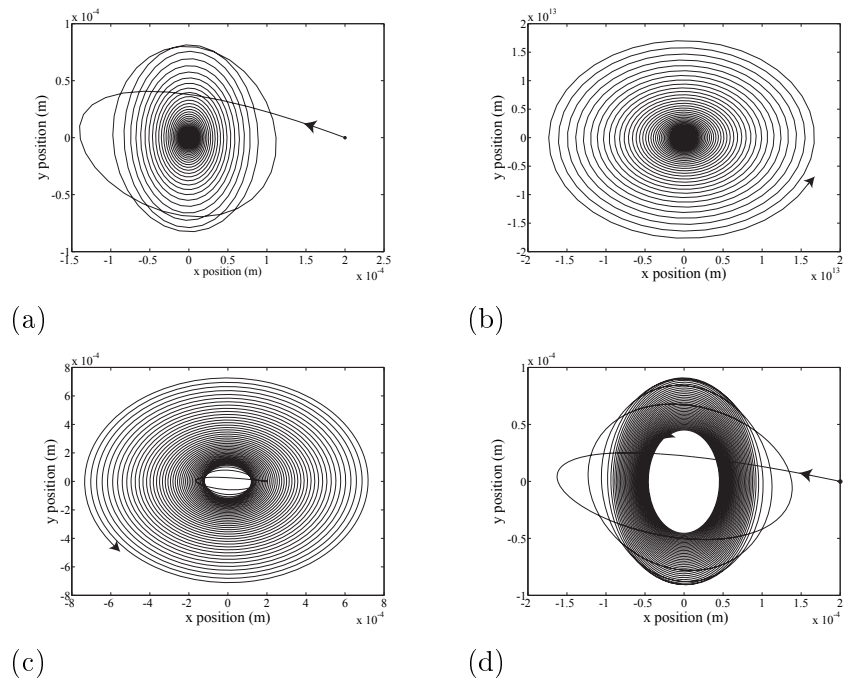


Figure 2.5: Rotor motion in the xy radial plane for a spin speed ω (a) of 800 rad/s: the rotor is stable, (b) of 1000 rad/s: the rotor is unstable, (c) of 18000 rad/s: the rotor is unstable, (d) of 21000 rad/s: the rotor is stable

- the thermal regime is a steady-state regime;
- the electrical behavior of the conductor can be modeled by a circuit with a global resistance R_r and a global inductance L_r .

we obtained the equations of motion described by

$$m\ddot{z} + (c_{nr} + c_r \cos \theta + j c_r \sin \theta)\dot{z} + (k + c_r \omega \sin \theta - j c_r \omega \cos \theta)z = 0, \quad (2.30)$$

where θ the angular shift is defined by:

$$\tan \theta = \frac{(\omega + \lambda_I)L_r}{R_r}, \quad (2.31)$$

and the rotating damping c_r is worth:

$$c_r = \frac{c_m^2}{Z_I^2} \frac{1}{\sqrt{((\omega + \lambda_I)L_r)^2 + R_r^2}}, \quad (2.32)$$

with $c_m = \frac{d\psi}{d\theta_{elec}}$ the proportionality constant, ω the rotor spin speed, λ_I the whirl speed, L_r and R_r the global conductor inductance and resistance, c_{nr} the non-rotating damping and k the stiffness of the system.

The stability of this system is conditioned by two implicit equations:

$$\omega_{lim,1} = \left(1 + \frac{c_{nr}}{c_r \cos \theta}\right)(\omega_{dyn} - \sqrt{\omega_{dyn}^2 + \omega_{cr}^2}), \quad (2.33)$$

$$\omega_{lim,2} = \left(1 + \frac{c_{nr}}{c_r \cos \theta}\right)(\omega_{dyn} + \sqrt{\omega_{dyn}^2 + \omega_{cr}^2}), \quad (2.34)$$

with:

$$\omega_{cr} = \sqrt{\frac{k}{m}},$$

and:

$$\omega_{dyn} = \frac{c_{nr}}{2m} \tan \theta.$$

The roots of the system may give a limited speed range where the behavior of the system is unstable

Chapter 3

Advanced electromechanical model: evolution of the impedances with the frequency

The skin effect is a phenomenon which tends to concentrate currents on the surfaces of conductors nearest to the field producing them. This redistribution of the current produces an evolution of the apparent resistance and inductance, i.e. the real and the imaginary part of the impedance of the conductor.

The skin depth is defined by

$$\delta = \frac{1}{\sqrt{\frac{\omega\mu\sigma}{2}}} \quad (3.1)$$

This length is thus characterized by the material properties and the excitation frequency.

Magnetic bearings are susceptible to work with a broad frequency spectrum and pass gradually from a situation where the skin effect is negligible to a situation where it becomes predominant. It is thus important to have at our disposal a model of the impedance dependence

with respect to skin depth.

In this context, the aim of this section is to present and compare different models predicting the evolution of the total impedance with the skin depth, these models have been found in the literature, or have been adapted from the literature.

These models are especially compared for the intermediate frequency range where it is impossible either to neglect the skin effect, either to suppose the skin effect to be dominant.

These results were presented in [Kluyskens and Dehez, 2011].

First, various models predicting the evolution law of the internal impedance or the global impedance are presented (section 3.1). Then, in section 3.2, the models are compared to FEM results for different geometries from a simple round wire to a complex shaped conductor with a varying cross-section. Results are analyzed, and it is shown that two models based on a phenomenological approach give good prediction of the frequency evolution of the impedance.

3.1 Evolution laws

The considered evolution laws are developed on an approach based on the diffusion equation or on a phenomenological approach.

3.1.1 Approach based on the diffusion equation

The two first models considered are based on the models described by [Kerst and Sprott, 1986]. In their paper, the frequency-dependent impedances are calculated by the following approach. The time-dependent diffusion equation is applied to different geometries and is solved for the soak-in of the magnetic field for the special case of a constant magnetic field at the surface of the conductor. This equation is solved using the appropriate coordinates in function of the geometry involved. For example, for a cylindrical rod with axial current j_z , resistivity ρ , and permeability μ , as illustrated in Fig. 3.1 the diffusion equation can be written as:

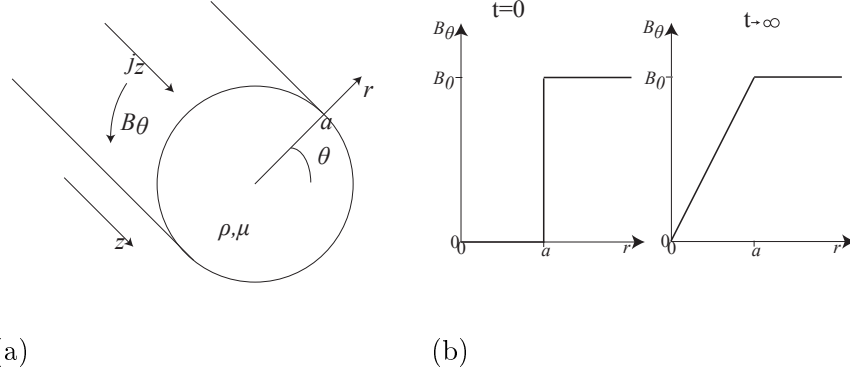


Figure 3.1: Cylindrical rod with axial current (a) for which the diffusion equation is solved (b) for $t = 0$ and $t \rightarrow \infty$ in [Kerst and Sprott, 1986]

$$\frac{\partial B_\theta}{\partial t} = \frac{\rho}{\mu} \left(\frac{\partial^2 B_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial B_\theta}{\partial r} - \frac{B_\theta}{r^2} \right) \quad (3.2)$$

which gives the evolution of the azimuthal component of the induction B with the time t and the appropriate coordinates (here, the radius r). These results are converted to a time-dependent impedance using the fact that the voltage is the rate at which the magnetic flux penetrates the conductor.

$$Z(t) = \frac{V(t)}{I} = \frac{\int \rho j_0 dl}{I} \quad (3.3)$$

where $V(t)$ is the voltage drop across some length at the surface of the conductor, j_0 is the current density at the surface, and I the total current. The obtained time-dependant impedance is an infinite sum of exponential terms with a time-dependant amplitude and progressively shorter time constants. This can be represented by a linear electric circuit as in Fig. 3.2.

From there on, the time-dependent impedance is easily converted to the frequency domain into a frequency-dependent impedance. For all the considered cases [Kerst and Sprott, 1986], the frequency-dependent impedance has the following form:

$$\bar{Z}(\omega) = R(\omega) + j\omega L(\omega), \quad (3.4)$$

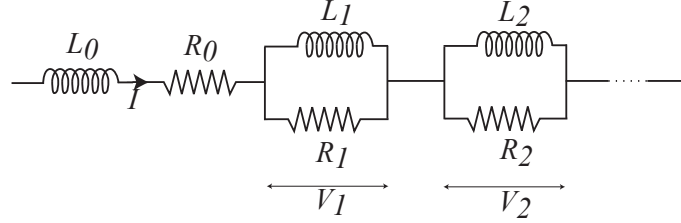


Figure 3.2: Electrical equivalent circuit representation for the impedances calculated in [Kerst and Sprott, 1986]

with

$$R(\omega) = R_0 + R_1 \sum_{n=1}^{\infty} \frac{\omega^2 \tau_n^2}{1 + \omega^2 \tau_n^2}, \quad (3.5)$$

$$L(\omega) = L_0 + \sum_{n=1}^{\infty} \frac{L_n}{1 + \omega^2 \tau_n^2}, \quad (3.6)$$

$$\tau_n = \frac{L_n}{R_n}. \quad (3.7)$$

Given that the impedance has the same form for all the calculated cases [Kerst and Sprott, 1986], we generalize it and used it for any case, but with the parameters to be identified properly.

So, the first model we consider are the expressions (3.5) and (3.6), truncated after 5 terms ($n = 4$). It thus comprises 10 parameters: $R_0, R_1, R_2, R_3, R_4, L_0, L_1, L_2, L_3$ and L_4 .

The problem of the first model is that we truncate the series, without introducing any compensation for the remaining terms.

The second model, also developed in [Kerst and Sprott, 1986], is an analytical approximation of Eqs. (3.5) and (3.6) that does not involve infinite series:

$$R(\omega) = R_0 + R_1 \left[(K/\omega\tau_1)^{2k} + (2/\pi A\omega\tau_1)^{k/2} \right]^{-1/k}, \quad (3.8)$$

$$L(\omega) = L_0 + L_1 \left[M^6 + (2\omega\tau_1/\pi A)^3 \right]^{-1/6}. \quad (3.9)$$

Subsequently, this second model includes 8 parameters: R_0 , R_1 , L_0 , L_1 , K , k , A and M .

3.1.2 Phenomenological approaches

In this second approach, [Engin et al., 2005] propose a phenomenological approach based on the evolution law of the impedance at high frequency:

$$\bar{Z} = B\sqrt{j\omega}.$$

Based on this, they propose three different models where the parameters can be identified via three important variables, the DC resistance, the DC internal inductance, and the skin effect resistance at a sufficiently high frequency. The external inductance however, is not taken into account directly in the models. Another model based on a weight function between the low- and the high-frequency impedance is also proposed.

Three different approaches are considered in [Engin et al., 2005] : a rational polynomial approximation, a continued fraction expansion and finally a non uniformly discretized pi-model. First, they write the internal impedance with a rational polynomial approximation as:

$$\bar{Z}(\omega) = A + B_1 \frac{\sum_{r=0}^p \binom{n+1}{2r} (B_2 j\omega)^r}{\sum_{r=0}^q \binom{n+1}{2r+1} (B_2 j\omega)^r}. \quad (3.10)$$

where $p = q = n/2$ for n even and $p = (n+1)/2$, $q = (n-1)/2$ for n odd. The parameters can be found from equations:

$$R_{ac} = B_1 \sqrt{B_2 \omega_{ac} / 2} \quad (3.11)$$

$$L_{dc} = B_1 B_2 \frac{n^2 + 2n}{3n + 3} \quad (3.12)$$

$$R_{dc} = A + \frac{B_1}{n+1}, \quad (3.13)$$

where R_{dc} is the DC resistance, L_{dc} is the DC internal inductance and R_{ac} the skin effect resistance at a sufficiently high frequency ω_{ac} .

Second [Engin et al., 2005], express the internal impedance of a half infinite plane, solving Maxwell's equations, with a coth function:

$$Z = \sqrt{\frac{sL}{G}} \coth(\sqrt{sLG}).$$

Using the continued fraction expansion of the tanh function, the impedance can be finally written as:

$$\bar{Z}(\omega) = R_{dc} + \frac{1}{\frac{3}{j\omega L} + \frac{1}{\frac{5}{G} + \frac{1}{\frac{7}{j\omega L} + \dots}}}. \quad (3.14)$$

The parameters can be found from equations:

$$L = 3L_{dc} \quad (3.15)$$

$$G = \frac{\omega_{ac}L}{2R_{ac}^2} \quad (3.16)$$

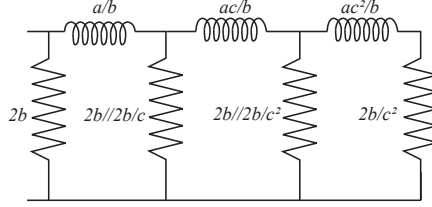
Finally, the impedance can also be modeled using iterated lumped-parameter networks, like cascaded π models. [Engin et al., 2005] has observed that a non-uniform discretization based on a constant resistance and inductance ratio between subsequent π models improves stability. Fig. 3.3 shows a sample ladder network, using three cascaded π models. The parameters on this figure can be obtained by:

$$a = \frac{2R_{ac}^2}{\omega_{ac}} \quad (3.17)$$

$$b = \frac{\sqrt{\omega_{max}/2a}}{2} \quad (3.18)$$

$$c = \frac{-1 + \sqrt{1 - 4(1 - (b/R_{dc}))}}{2} \quad (3.19)$$

In these three latter models, the internal impedance model is based on parameters that can always be found based on three values: the

Figure 3.3: Non-uniformly discretized π model for the skin effect

DC resistance R_{dc} , the DC internal inductance L_{dc} and the skin effect resistance R_{ac} at a sufficiently high frequency ω_{ac} . But these models do not consider the external inductance. To take the external inductance L_0 into account, we can introduce a slight modification of previous equations: the term $j\omega L_0$ has to be added to expressions of the impedance in Eqs (3.10) and (3.14), and in Fig.3.3. For the rational polynomial model, the identification of the parameters has also to be changed, and Eq. (3.12) becomes:

$$L_0 + L_{dc} = B_1 B_2 \frac{n^2 + 2n}{3n + 3}. \quad (3.20)$$

Finally, four values are needed to determine the global impedance: L_0 , L_{dc} , R_{dc} and R_{ac} .

The last model considered ([Kluyskens and Dehez, 2011]) is based on the principle that when the frequency is low, the impedance has a constant value and that at high frequencies, the internal impedance has a known evolution law with the frequency [Ramo et al., 1965]. At intermediate frequencies, we suppose that the impedance is a combination of the low and high frequencies impedances. The weight of one compared to the other one depends directly on the value of the frequency through a weighing function.

The low frequency impedance (when the skin effect is negligible)

is:

$$\bar{Z}_{lf} = R_{dc} + j\omega(L_0 + L_{int}) \quad (3.21)$$

where R_{dc} represents the DC resistance of the conductor, L_0 represents the external inductance of the conductor, and L_{int} the internal inductance of the conductor when the current is uniformly distributed inside the conductor.

Let us now look at what happens when the skin effect is dominant. Considering a semi-infinite conducting environment ($z > 0$) separated of the vacuum by an interface vacuum-conductor in $z = 0$ and supposing that the field source placed in the vacuum is uniform in x and y and produces only a field E_x , the solution for the electrical field in the conductor is:

$$E_x = E_0 \exp(-(1+j)(z/\delta)) \quad (3.22)$$

where $\delta = 1/\sqrt{\omega\mu\sigma/2}$ is the skin depth. The current density by width unit is:

$$I_x = \int_0^\infty J_x dz = J_0 \delta / (1+j) \quad (3.23)$$

with $J_0 = \sigma E_0$. The internal impedance can then be calculated by the relation $Z_{int} = E_0/I_x = \sqrt{j\omega\mu/\sigma}$. The evolution law of the impedance with the frequency is therefore:

$$\bar{Z}_{int} = R_{hf} + j\omega L_{hf} = A\sqrt{\omega}(1+j), \quad (3.24)$$

which implies:

$$R_{hf} = A\sqrt{\omega} \quad \text{and} \quad L_{hf} = A/\sqrt{\omega}. \quad (3.25)$$

with a constant A proportional to the square root of the material permeability and conductivity.

Finally, the high frequency impedance is:

$$\bar{Z}_{hf} = R_{hf} + j\omega(L_0 + L_{hf}). \quad (3.26)$$

If the skin depth is thin enough compared to characteristic dimensions of the conductor, then the frequency evolution at high frequencies can

be compared to the frequency evolution described in Eq. (3.26).

For intermediate frequencies, the impedance is a combination of the low frequency impedance, Eq. (3.21), and the high frequency impedance, Eq. (3.26), according to the following expression:

$$R(\omega) = R_{dc} + R_{hf}g(\omega), \quad (3.27)$$

$$L(\omega) = L_0 + L_{hf}g(\omega) + (1 - g(\omega))L_{int}. \quad (3.28)$$

For the weight function, a normalized function based on arctangent evolution has been chosen:

$$g(\omega) = 1/2 + 1/\pi \arctan(\alpha\omega + \beta) \quad (3.29)$$

We noticed better results of this model when the R_{dc} is not multiplied by $1 - g(\omega)$, as expressed in eq. 3.27. This is probably because the weight function is an arctangent, its value is not zero for $\omega = 0$, although it tends to zero.

There are 6 parameters for this third model: A , R_{dc} , L_0 , L_{int} , α , β .

Let us note, that, as said before, the parameter A is proportional to the square root of the material permeability and conductivity. However, it is multiplied by the weight function $g(\omega)$, which also depends on the material permeability and conductivity, so A can not be calculated directly.

3.2 Model comparison

These models can be applied to three different cases in order to compare their prediction of the evolution of the impedances with the frequency. The cases presented in this section vary from a simple extruded conductors with simple-shaped section (i.e. circular or square) to an extruded conductor with a complex-shaped section, and even to more complex conductors with variable sections. This allows to compare the frequency evolution of the impedance in complicated conductors, which are more similar to bulk conductors, as in some magnetic

bearings.

In these three cases, the conductors are submitted to an oscillating voltage at their terminals.

The parameters can be found by identification with a finite elements model of the conductor. The parameters of the models constructed on an approach based on the diffusion equation, Eqs. (3.5) and (3.6), (3.8) and (3.9), (3.27) and (3.28) are found by minimizing the summation of the square of the errors between the prediction law and the finite element model, whereas the parameters of the model based on a rational polynomial approximation, Eqs. (3.10), of the continued fraction expansion, (3.14) and the cascaded π model (3.17) are found based on a single value for R_{dc}, L_{dc}, L_0 (at $f = 0$) and R_{ac} (at $f = f_{ac}$) in the FEM model.

For a conductor with a constant simple-shaped section, the parameters of each model can also be found analytically.

The results of the internal impedance for a round wire are shown in Fig. 3.4. It can also be computed analytically as [Ramo et al., 1965]:

$$Z = \frac{\sqrt{j\omega\mu/\sigma}}{2\pi r} \frac{I_0(r\sqrt{j\omega\mu/\sigma})}{I_1(r\sqrt{j\omega\mu/\sigma})} \quad (3.30)$$

where I_0 and I_1 represent the modified Bessel functions of order zero and one, and r is the radius of the wire. It can be seen in Fig. 3.4 that the approach based on a rational approximation Eq. 3.10 does not give very good results. Except for that model, the approaches based on a phenomenological approach give better results, particularly the continued fraction expansion model.

In the case of a constant complex-shaped conductor, the current density predicted by the finite element model is shown in Fig. 3.5. This conductor is 1 m long, is made of copper, and submitted to an oscillating voltage of 1V amplitude, as shown in Fig. 3.6. The finite element model is a 2D cross-section plane inside a circular air domain of 1m diameter, as shown in Fig. 3.7. The boundary conditions outside the air domain are magnetic insulation: the magnetic field is supposed to be tangential to this boundary.

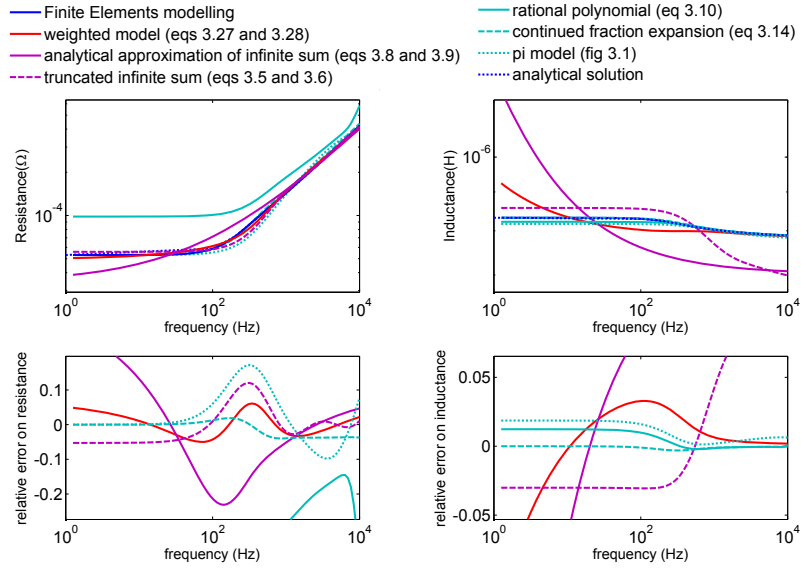


Figure 3.4: Comparison of the frequency evolution of the internal impedance predicted by the different models for a round wire: the resistance, the inductance, the relative error of the resistance compared to FEM, the relative error of the inductance compared to the FEM

Table 3.1: Value of the parameters for a simple conductor

truncated infinite sum (Eqs. 3.5 and 3.6)	$R_0 = 5.59 \cdot 10^{-5} \Omega$, $R_1 = 1.3 \cdot 10^{-4} \Omega$, $R_2 = 0.0013 \Omega$, $R_3 = 2.68 \cdot 10^{-5} \Omega$, $R_4 = 6.53 \cdot 10^{-4} \Omega$, $L_1 = 1.47 \cdot 10^{-9} \text{H}$, $L_2 = 3 \cdot 10^{-8} \text{H}$, $L_3 = 8.08 \cdot 10^{-10} \text{H}$, $L_4 = 1.38 \cdot 10^{-7} \text{H}$
analytical approximation of infinite sum (Eqs. 3.8 and 3.9)	$R_0 = 3.45 \cdot 10^{-5} \Omega$, $R_1 = 3.35 \cdot 10^{-9} \Omega$, $k = 3.08$, $A = 0.24$, $K = 7.3$, $L_1 = 0.002 \text{H}$, $M = 9.68 \cdot 10^{-4}$,
weighted model (Eqs. 3.27 and 3.28)	$R_{dc} = 4.94 \cdot 10^{-5} \Omega$, $L_{int} = 3.03 \cdot 10^{-9} \text{H}$, $A = 1.47 \cdot 10^{-6}$, $\alpha = 4.3 \cdot 10^{-4}$, $\beta = -0.89$
other models (Eqs. 3.10, 3.14 and Fig.1)	$R_{dc} = 5.3 \cdot 10^{-5} \Omega$, $L_{dc} = 5 \cdot 10^{-8} \text{H}$, $R_{ac} = 4.22 \cdot 10^{-4} \Omega$ ($\omega_{ac} = 2\pi 10^4 \text{ Hz}$)

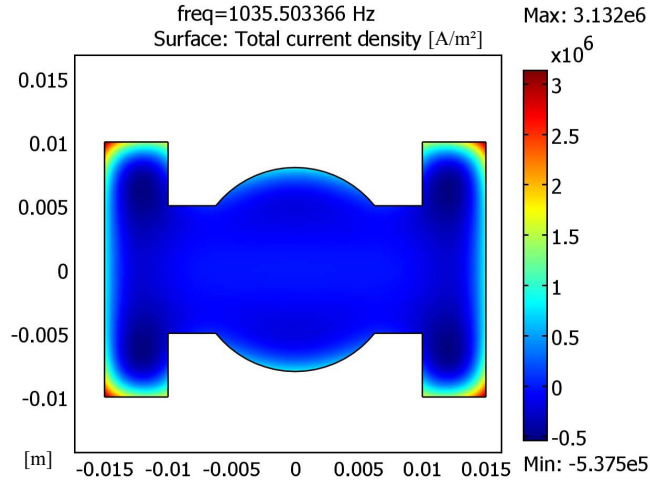


Figure 3.5: Current density in the cross-section of a complex-shaped conductor for a 1035 Hz frequency

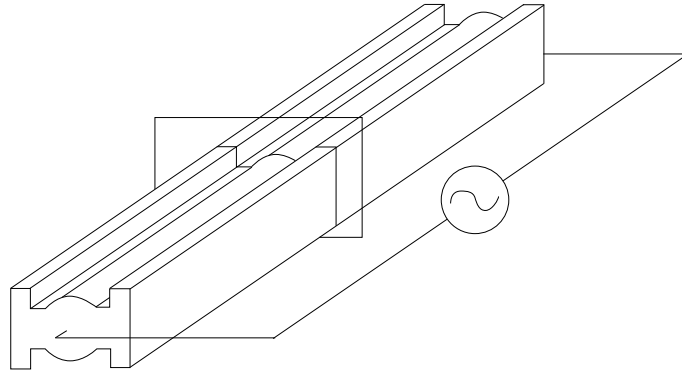


Figure 3.6: Schematic representation of the complex-shaped extruded conductor, submitted to an oscillating voltage

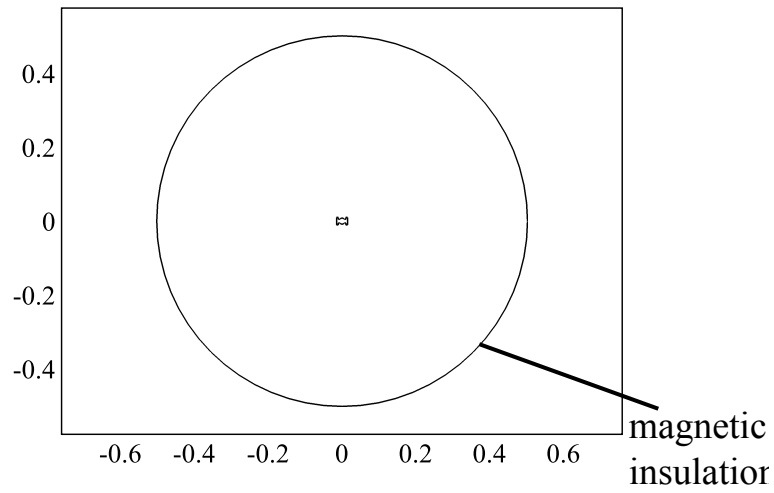


Figure 3.7: cross-section of conductor inside an air domain for the finite element model

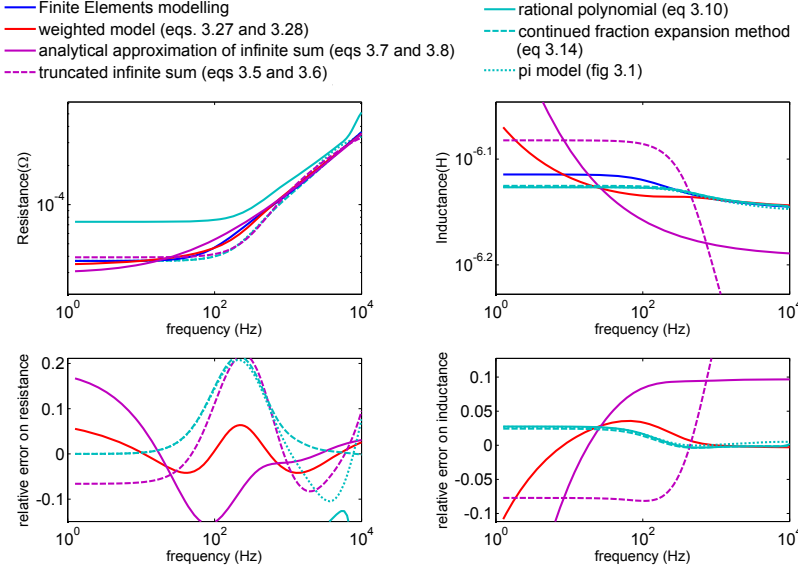


Figure 3.8: Comparison of the frequency evolution of the impedance predicted by the different models for a conductor with a constant complex cross-section: the resistance, the inductance, the relative error of the resistance compared to FEM, the relative error of the inductance compared to the FEM

The results of the resistance and the internal inductance of this complex-shaped extruded conductor are shown in Fig. 3.8.

The value of the parameters are shown in Table 3.2.

Once more, it can be seen that the continued fraction expansion method gives good results, as well as the weighted model. For the weighted model, the importance of the zero-frequency impedance versus the high-frequency impedance can be analyzed: the function $g(\omega)$ of this model gives the weight of the high-frequency impedance, while the function $(1 - g(\omega))$ gives the weight of the zero-frequency impedance. In the case of the extruded conductor, the weight of the high-frequency impedance becomes quickly predominant (Fig. 3.9).

Table 3.2: Value of the parameters for a complex cross-section extruded conductor

truncated infinite sum (Eqs. 3.5 and 3.6)	$R_0 = 3.93 \cdot 10^{-5}\Omega$, $R_1 = 8.55 \cdot 10^{-5}\Omega$, $R_2 = 0.0011\Omega$, $R_3 = 0.0014\Omega$, $R_4 = 1.44 \cdot 10^{-7}\Omega$, $L_0 = 4.4 \cdot 10^7\text{H}$, $L_1 = 2.05 \cdot 10^{-9}\text{H}$, $L_2 = 2.62 \cdot 10^{-7}\text{H}$, $L_3 = 1.25 \cdot 10^{-7}\text{H}$, $L_4 = 3.6 \cdot 10^{-12}\text{H}$
analytical approximation of infinite sum (Eqs. 3.8 and 3.9)	$R_0 = 3.002 \cdot 10^{-5}\Omega$, $R_1 = 1.12 \cdot 10^{-7}\Omega$, $k = 0.27$, $A = 7.31$, $L_0 = 6.41 \cdot 10^{-7}\text{H}$, $L_1 = 1.41 \cdot 10^{-6}\text{H}$, $M = 0.0027$, $K = 6.42$
weighted model (Eqs. 3.27 and 3.28)	$R_{dc} = 3.37 \cdot 10^{-5}\Omega$, $L_{int} = 1.55 \cdot 10^{-10}\text{H}$, $L_0 = 7.13 \cdot 10^{-7}\text{H}$, $A = 1.28 \cdot 10^{-6}$, $\alpha = 6.13 \cdot 10^{-4}$, $\beta = -0.72$
other models (Eqs. 3.10, 3.14 and Fig.1)	$R_{dc} = 3.69 \cdot 10^{-5}\Omega$, $L_0 = 7.11 \cdot 10^{-7}\text{H}$, $L_{dc} = 3.82 \cdot 10^{-8}\text{H}$ $R_{ac} = 3.62 \cdot 10^{-4}\Omega$ ($\omega_{ac} = 2\pi 10^4 \text{ Hz}$)

For an axisymmetric conductor, with a variable section, in aluminum, submitted to an oscillating voltage of 10V, represented in Fig. 3.10 the current distribution inside the axisymmetric conductor is shown in Fig. 3.11.

In this finite element model, the conductor is axisymmetric, and the magnetic boundary conditions on the external boundaries of the conductor are magnetic insulation. The electric boundary conditions are a ground potential at one extreme, and a fixed potential at the other extreme, and the external radial boundary has a normal current density which is zero.

The comparison between the different evolution laws of the resistance and the inductance of this conductor are shown in Fig. 3.12. The value of the parameters are listed in Table 3.3.

Here again, when we look at the entire frequency range, from low to high frequencies, the continued fraction expansion and the weighted model give better results.

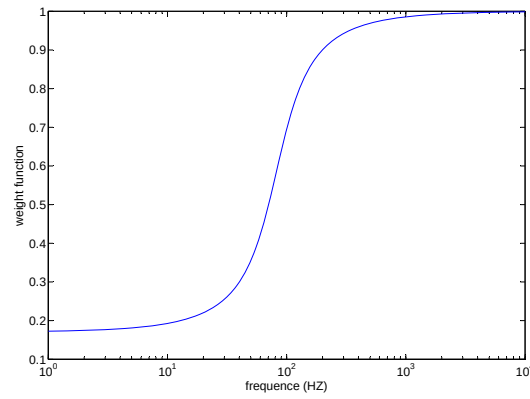


Figure 3.9: Weight of the high-frequency impedance in function of the frequency for a conductor with constant cross-section

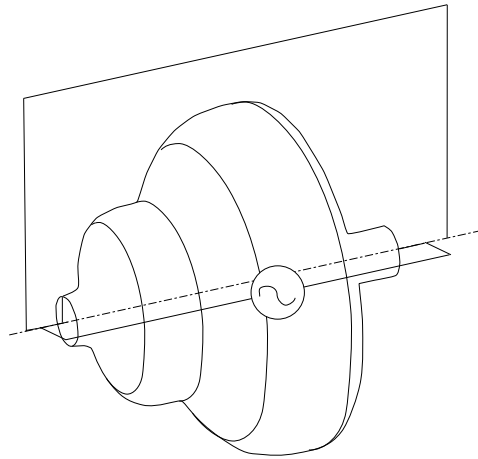


Figure 3.10: Schematic representation of an axisymmetric conductor, submitted to an oscillating voltage

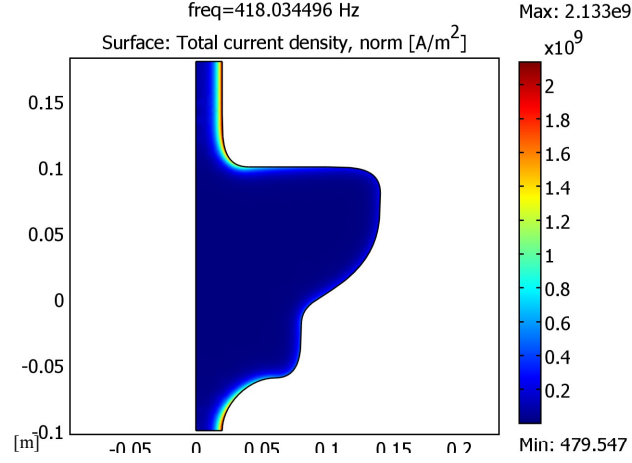


Figure 3.11: Current density in the axial section of a complex-shaped conductor with variable section for a 420 Hz frequency

Table 3.3: Value of the parameters for a conductor with variable cross-section

truncated infinite sum (Eqs. 3.5 and 3.6)	$R_0 = 3.16 \cdot 10^{-6} \Omega$, $R_1 = 1.3 \cdot 10^{-5} \Omega$, $R_2 = 3.45 \cdot 10^{-5} \Omega$, $R_3 = 1.69 \cdot 10^{-5} \Omega$, $R_4 = 6 \cdot 10^{-5} \Omega$, $L_0 = 2.43 \cdot 10^{-9} \text{H}$, $L_1 = 3.13 \cdot 10^{-10} \text{H}$, $L_2 = 1.6 \cdot 10^{-9} \text{H}$, $L_3 = 7.97 \cdot 10^{-9} \text{H}$, $L_4 = 1.7 \cdot 10^{-9} \text{H}$
analytical approximation of infinite sum (Eqs. 3.8 and 3.9)	$R_0 = 1.41 \cdot 10^{-6} \Omega$, $R_1 = 1.43 \cdot 10^{-12} \Omega$, $k = 0.16$, $A = 7$, $L_0 = 2.3 \cdot 10^{-18} \text{H}$, $L_1 = 0.0022 \text{H}$, $M = 0.0017$, $K = 0.37$,
weighted model (Eqs. 3.27 and 3.28)	$R_{dc} = 2.2 \cdot 10^{-6} \Omega$, $L_{int} = 1.7 \cdot 10^{-9} \text{H}$, $L_0 = 1.17 \cdot 10^{-9} \text{H}$, $A = 1.84 \cdot 10^{-7}$, $\alpha = 0.0011$, $\beta = -0.12$
other models (Eqs. 3.10, 3.14 and Fig.1)	$R_{dc} = 2.5 \cdot 10^{-6} \Omega$, $L_0 = 0 \text{H}$, $L_{dc} = 2.37 \cdot 10^{-8} \text{H}$, $R_{ac} = 4.8 \cdot 10^{-5} \Omega$ ($\omega_{ac} = 2\pi \cdot 10^4 \text{ Hz}$)

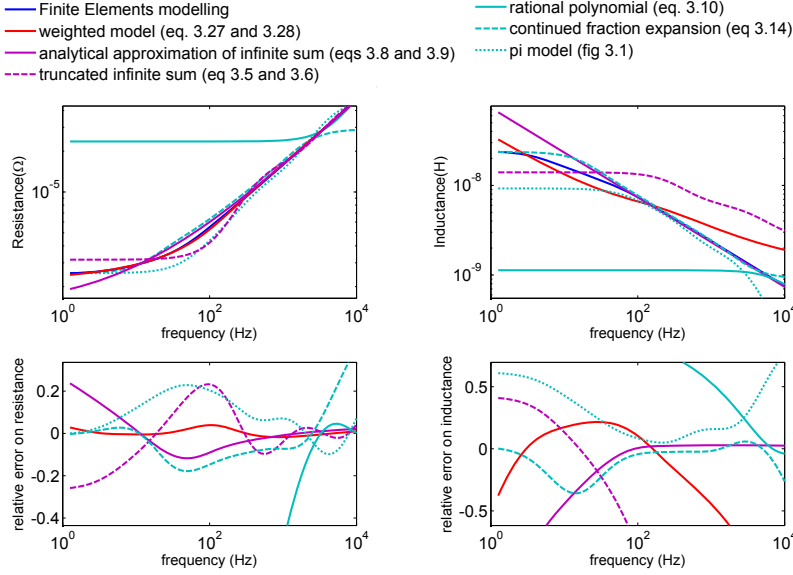


Figure 3.12: Comparison of the frequency evolution of the internal impedance predicted by the different models for a conductor with variable cross-section: the resistance, the inductance, the relative error of the resistance compared to FEM, the relative error of the inductance compared to the FEM

3.3 Synthesis

Different models predicting the evolution of the total impedance of a conductor with the frequency have been presented and compared in preceding section. These models have only been validated on conductors submitted to an oscillating voltage, with as a consequence, a variation of the skin depth.

As a conclusion to this validation, two models are adopted: the continued fraction expansion model and the weighted model.

The model describing the frequency evolution of the impedance on the basis of a continued fraction expansion gives very good results. It only needs four measurements to represent the global impedance for

every frequency.

This model is summarized by the following equations:

$$\bar{Z}(\omega) = R_{dc} + j\omega L_0 + \frac{1}{\frac{3}{j\omega L} + \frac{1}{\frac{5}{G} + \frac{1}{\frac{7}{j\omega L} + \dots}}}, \quad (3.31)$$

and the parameters can be found from equations:

$$L = 3L_{dc} \quad (3.32)$$

$$G = \frac{\omega_{ac} L}{2R_{ac}^2} \quad (3.33)$$

There are 4 parameters for this model: L_{dc} , R_{dc} , R_{ac} and L_0 .

The model weighting the zero-frequency impedance versus the high frequency impedance has six parameters to be identified by an appropriate identification method, like the least square criterion. It also gives good results.

This model is summarized by the following equations:

$$R(\omega) = R_{dc} + A\sqrt{\omega}g(\omega), \quad (3.34)$$

$$L(\omega) = L_0 + A/\sqrt{\omega}g(\omega) + (1 - g(\omega))L_{int}. \quad (3.35)$$

For the weighting function, a normalized function based on arctangent evolution has been chosen:

$$g(\omega) = 1/2 + 1/\pi \arctan(\alpha\omega + \beta) \quad (3.36)$$

There are 6 parameters for this model: A , R_{dc} , L_0 , L_{int} , α , β .

Emphasis should be put on the fact that this phenomenological model is a choice for the dependency of the impedance with the frequency, and that this choice is independent of the electromechanical model developed in preceding chapter 2.

In the next part of this work, the validation part, both models, continued fraction expansion and weighted model, are used to judge of the qualities of each one.

Part II

Validation

Chapter 4

Experimental validation

The electromechanical model developed in previous part predicts the forces acting on an electromechanical system when induced currents are generated due to the spinning motion of the rotor. In relation with the forces, it was shown that stable or unstable behavior can occur, depending on the rotation speed ω of the rotor.

The aim of this chapter is to show qualitatively through experimental observations that rotating damping introduces instability in a system. A second experiment will also show, as expected from the model, that the stability threshold depends on the non-rotating damping and on the stiffness present in the system.

4.1 Elementary experiment

4.1.1 Setup description

The first experience we carry out is very simple and classic. A body is suspended by a cable, and is put into rotation by a motor. The driving torque is transmitted through this cable. This latter leaves the body free to behave as a pendulum.

In a first configuration, the suspended rotating body is a magnet. Below this magnet is a static conductor, as shown in Fig. 4.1.



Figure 4.1: Picture of a suspended rotating magnet above a static conductor

In this configuration, the currents can only be induced in the conductor when the rotor whirls, and therefore they generate non-rotating damping.

The second studied configuration is the inverse configuration: the suspended rotating body is a conductor facing a static permanent magnet, see Fig. 4.2).

In this system, currents are induced in the conductor as soon as the rotor spins in an off-centered position. The damping is thus associated to the spinning motion and is consequently rotating damping.

4.1.2 Observations

In the first configuration, when giving an external perturbation to the rotating magnet, we can observe that this pendulum motion is rapidly damped.

Indeed, in this system, the induced currents generate non-rotating damping. As we saw in previous chapters, non-rotating damping acts as usual damping, and damps the radial motion of the rotating part. Besides this non-rotating damping, this setup does not undergo any



Figure 4.2: Picture of a rotating conductor above a static magnet

rotating damping and its stiffness is very low. This means, referring to Eq. 1.15, that the limit spin speed is almost infinite.

In the second configuration, when the spinning rotor is off-centered, currents are induced in the conducting rotor, creating rotating damping. The non-rotating damping is negligible, and again the stiffness is very low. The limit spin speed is very small. According to the model, the rotating damping enhances the radial forward whirl of the rotor, instead of damping it, when the spin speed is higher than the limit spin speed. On the contrary, the backward whirl motion is always damped.

Indeed, this is what we observed. When giving an external perturbation in the forward whirl direction, the rotor describes a pendulum motion with a bigger and bigger amplitude. Actually, even without any external perturbation (only the inherent perturbations), this outwards spiral was observed. And when giving an external perturbation in the backward whirl direction, we could observe that this motion was first damped, and then the outwards forward whirl spiral appeared.

4.2 Basic experiment

The second experience allows to study a rotating conductor above a permanent magnet when external stiffness, rotating and non-rotating damping are present.

In theory, the fact that an external stiffness is present in the system should allow us to observe two limit spin speeds, as shown in Fig. 2.3. But as it will be explained further, the second limit spin speed has a too high value and is not observable.

However, two configurations of the test bench are experimented: in the first configuration, almost no non-rotating damping is present, and in the second configuration, non-rotating damping is added. This will affect the value of the observed limit spin speeds.

4.2.1 Setup description

The system consists of a conductor rotating above a static axially magnetized ring magnet ((4) and (5) in Figs. 4.3 and 4.4). As long as the rotating axis of the conducting disk coincides with the magnetic axis of the permanent magnet, no currents are induced. When there is a center shift between these two axis, a point of the conductor sees a magnetic induction variation when it rotates, currents are induced which results in electromagnetic forces.

The principle of the test bench is to allow the rotating disk ((4) in Figs. 4.3 and 4.4) to move freely in the radial directions above the permanent magnet ((7) in Fig. 4.4) while keeping a constant air gap. This part of the test bench spin and whirls. The rotating torque is transmitted to this conductor so as to make it spin at a given constant rotation speed, and the radial whirl behavior is observed. Spin speed is measured with an optical tachometer.

The axial positioning of the conductor above the permanent magnet is obtained thanks to an aerostatic bearing ((3) in Figs. 4.3 and 4.4) levitating the conductor above the permanent magnet. This aerostatic bearing consists in blowing pressurized air all around the external diameter of the conductor.

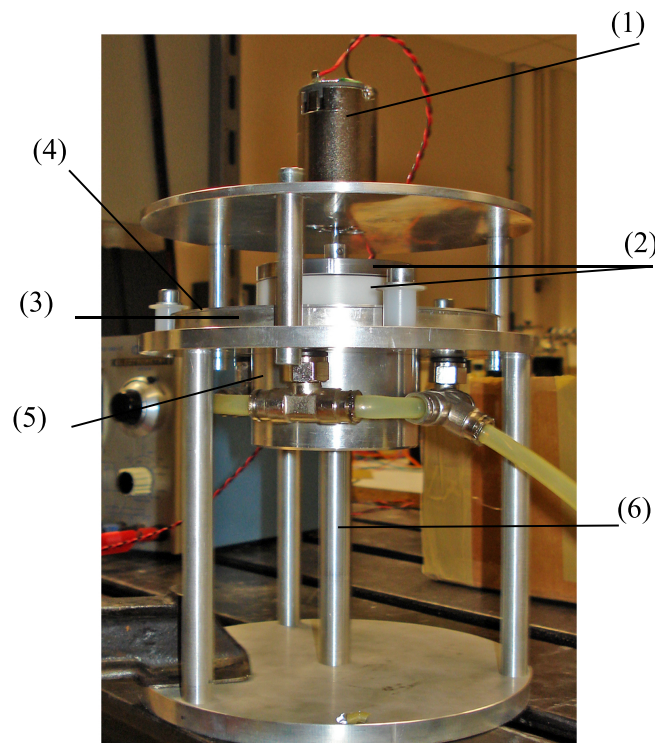


Figure 4.3: Picture of the test bench, where (1) is the motor, (2) a magnetic coupling, (3) an aerostatic bearing, (4) the conducting disk levitated by the aerostatic bearing, (5) support for the permanent magnet, (6) axis for an eventual force sensor

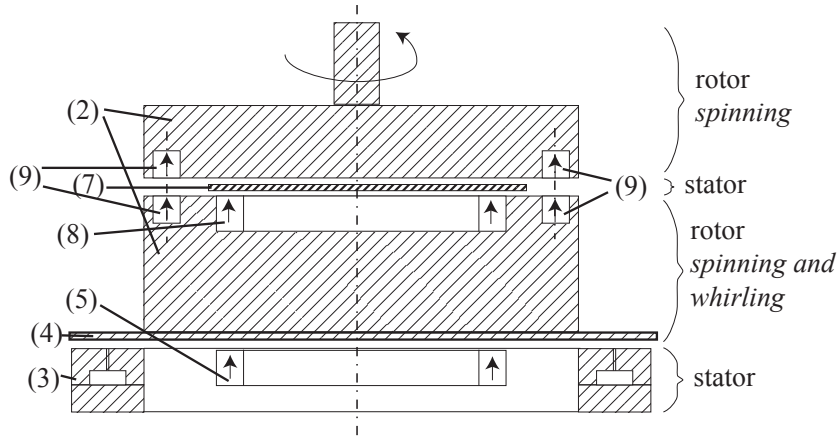


Figure 4.4: Schematized section view of the test bench, where (2) is the magnetic coupling, (3) the aerostatic bearing, (4) the rotating conducting disk levitated by the aerostatic bearing, (5) the static annular permanent magnet inducing rotating damping, (7) a static conducting disk to create non rotating damping (8) annular permanent magnet inducing non rotating damping, (9) cylindrical permanent magnets pairs for the magnetic coupling

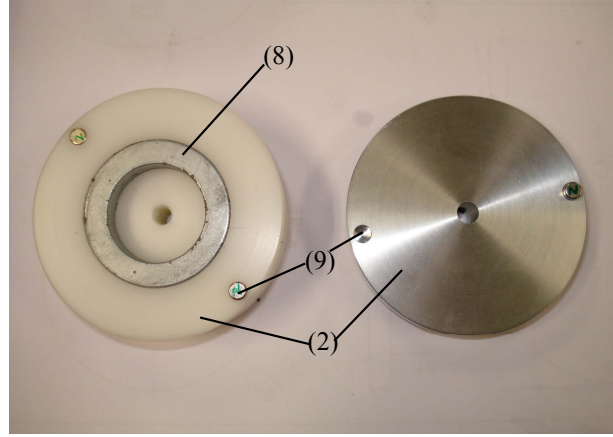


Figure 4.5: Magnetic coupling

The coupling between the motor and the conductor is done by two permanent magnet pairs ((2) in Figs. 4.3 and 4.4). A part of the coupling (Fig. 4.5), is fixed to the motor, and the second part, in non-conducting plastic material, is fixed to the conducting disk by a central plastic screw. On each part of the coupling two permanent magnets are mounted in holes, (9) in Figs. 4.4 and 4.5. Each permanent magnets pair (consisting in one magnet on each part of the coupling) is mounted in such a way that each magnet of the pair has its opposed poles facing each other. A magnet on the part of the coupling attached to the motor attracts respectively one magnet of the other part of the coupling, its magnetic flux path traveling into the air and as a result, the motor torque is transmitted. This coupling introduces an external radial stiffness into the system between the permanent magnets.

The stator of the system, which consists of a permanent magnet, is mounted on an axis leaving the possibility to glue strain gauges on it ((6) in Fig. 4.3). This would allow to measure the forces exerted on the static permanent magnet, if desired.

An additional annular permanent magnet is associated to the whirling and rotating motion of the conducting disk ((8) in Fig. 4.4). This annular permanent magnet has two functions: a non-rotating damper function as explained below, and an opposing function to the attraction between the permanent magnets of the magnetic coupling, in order to avoid for the two parts of this coupling to stick together. The annular permanent magnet of the non-rotating damper, although situated at a certain distance from the stator's annular permanent magnet of the magnetic bearing studied interacts with it and introduces another external radial stiffness.

This radial stiffness can be calculated thanks to an evaluation of the axial stiffness, done with a simple 2D FEM. Earnshaw theorem says that for permanent magnets the double of the radial stiffness is equal to the opposite of the axial stiffness, as stated in eq. 6:

$$2k_{radial} = -k_{axial}.$$

So the radial stiffness is worth about 1420 N/m.

In a first configuration, the measures are performed according to the following procedure.

The conducting disk is accelerated by supplying a torque through the motor and the magnetic coupling. Once the conducting disk spins fast enough, the motor and the connected part of the magnetic coupling are removed, so that they do not influence the observations. The only external stiffness still present is due to the annular permanent magnets. The only non rotating damping present in the system is due to the aerostatic bearing and is very low. This means that when removing the motor and its associated part of the magnetic coupling, we lessen the non-rotating damping and the external stiffness, which means that we also lessen the first limit spin speed. Without the driving magnetic coupling, the conducting disks whirls in an unstable manner, and when decelerating, it crosses this first limit spin speed and goes back to stability. An optical tachometer gives the rotation speed. We can then measure at which spin speed the rotor goes back to stability, this measure is taken visually.

In a second configuration, some non-rotating damping is added. This additional damping may have no contact with the levitating ro-

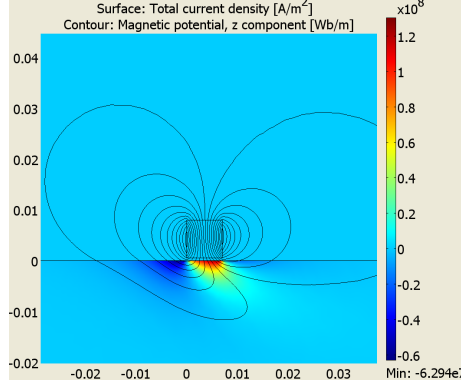


Figure 4.6: 2D-FEM study of the non rotating damping

tor, so we choose to introduce this non-rotating damping by an electromechanical system, composed of a rotating permanent magnet and a static conductor. The rotating permanent magnet is the annular permanent magnet associated to the magnetic coupling ((8) in Fig. 4.4). It faces a fixed conducting disk ((2) in Fig. 4.4). This is non-rotating damping.

A basic 2D-FEM study of this principle is made, in order to have an evaluation of this non-rotating damping, see Fig. 4.6. The non rotating damping as a function of speed is shown in Fig. 4.7.

Because we have an annular permanent magnet, the total non-rotating damping is estimated to be:

$$\begin{aligned} c_{nr} &\approx 0.075L(mm) \\ &\approx 8N/m/s \end{aligned} \quad (4.1)$$

for an air gap of 0.6 mm inside the non-rotating damper.

With this additional non-rotating damping, the first limit spin speed should be higher than the one observed in the first configuration, as predicted by Fig. 4.8.

Based on the different characteristics of the system, and thanks to

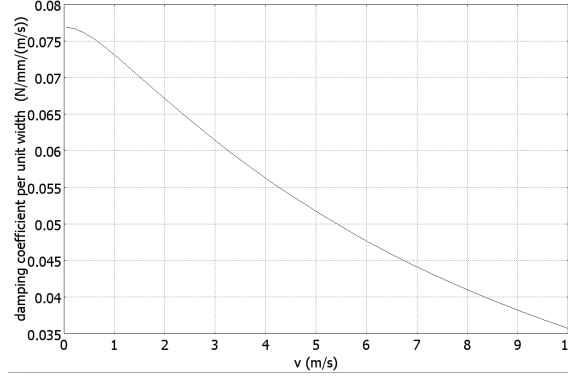


Figure 4.7: Damping coefficient per unit width computed by the FEM, for a 0.6 mm air gap inside the non-rotating damper

the model, an evaluation has been made of its limit spin speeds. It is supposed to behave stably until a first limit spin speed, then become unstable, and finally go back into stability for a second limit spin speed, as illustrated in Fig. 4.8.

Unfortunately, the evaluations for this second limit spin speed predict to go back to a stable behavior at a limit spin speed that can not be reached by the test bench (around 30.000 rad/s for a non rotating damping worth 8 Ns/m). This second limit spin speed could be reduced when increasing the non rotating damping. But this increase has to be important, and introducing an important amount of non rotating damping without contact between the stator and the rotor is not easy. This means that only the first threshold between stability and instability can be observed, but not the fact that it goes back to stability at higher rotation speeds.

4.2.2 Observations

In the first configuration, we observe that as soon as the motor is removed, the rotor starts to whirl in an unstable way. The rotor then

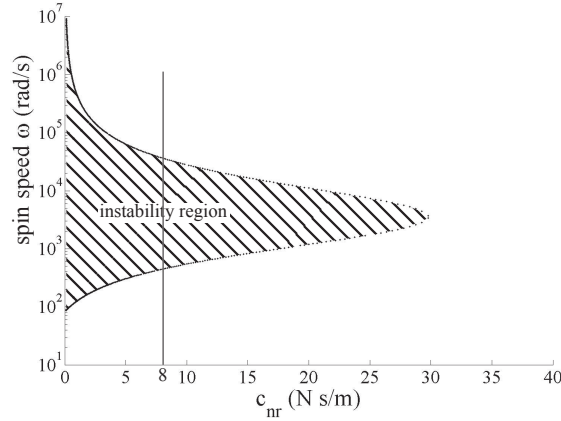


Figure 4.8: Predicted limit spin speed for the test bench as a function of the non-rotating damping c_{nr}

decelerates until a certain spin speed where it does not whirl anymore.

In these conditions, the model predicts a first limit spin speed very close to the critical speed of the system. The critical speed, $\sqrt{k/m}$ is worth 99,51 rad/s, and the first limit spin speed is 100 rad/s. So experimentally it is not easy to dissociate one from the other. But we observed that above a value around 105 rad/s the conducting disk is unstable.

For the second configuration, the observed limit spin speed is situated around 170 rad/s, and is clearly dissociated from the critical spin speed which was then observed to be around 115 rad/s.

As expected, on this test bench with rotating damping and stiffness, we observe that under a certain value of spin speed, the conducting disk behaves stable, and above this limit spin speed, the behavior is unstable.

We could also observe that the higher the amount of non rotating damping, the further this limit spin speed is situated from the critical

speed, which confirms the electromechanical model predictions.

4.3 Synthesis

The first basic experiment showed that when currents are induced due to the spinning motion of the rotor, the forward whirl motion was amplified and the backward whirl motion was damped. A contrario, it also showed that when currents are induced due to the whirling motion of the rotor, any whirl motion was damped.

The second experiment showed, when currents are induced in the rotor of a system presenting some stiffness, the existence of a limit spin speed beyond which the rotor behaves unstable. Below this limit spin speed the rotor behaves stable. We also observed that when increasing the amount of non rotating damping present in the system, this first limit spin speed also increases, and moves away from the critical speed of the system.

We were not able to observe the theoretical second limit spin speed above which the rotor goes back to stability. However, the existence of this limit spin speed is well known, and is the minimum spin speed above which the existing electrodynamic magnetic bearing must spin in order to function, see [Filatov, 2002], [Lembke, 2005].

Chapter 5

Numerical validation

In previous chapter, it was shown qualitatively that unstable speed ranges exist. The aim of this chapter is to validate the electromechanical model in itself.

When quantitative predictions are desired for a particular magnetic bearing, the parameters of the model described in Chapter 2 need to be identified. In this chapter, first a procedure is presented to identify the parameters of the electromechanical model, on the basis of experimental or finite elements results. The numerical methods used for the optimization of the identification are briefly presented.

Second, four practical cases will be examined, to confirm that the finite element forces evaluation and the experimental forces measures fit well on the electromechanical model. For each case, the numerical validation is done on forces evaluated in quasi-static conditions: the rotor spins in an off-centered direction. Indeed, simulating these cases in 3D time-dependent FEM models is too heavy.

The two first cases simply consists in a rotating conductor in an homopolar magnetic field. These cases are not designed to be magnetic bearings, but their principle is the same as in an electrodynamic magnetic bearing. Their advantage is to have a very simple geometry. The third case examined, is the electrodynamic bearing designed in [Lembke, 2005].

The last case will examine how the electromechanical model behaves

when located outside the domain validity of the model: a conducting rotor is submitted to a non-sinusoidal excitation.

5.1 Model parameter identification

In the electromechanical model described by the equations of motion 2.30, the parameters are to be determined for each different magnetic system.

The different parameters are

- the stiffness k ;
- the non rotating damping coefficient c_{nr} ,
- the proportionality constant c_m for the rotating damping coefficient,
- the global inductance L_r , and the global resistance R_r , or eventually the parameters of the model chosen to describe the skin effect of these impedances.

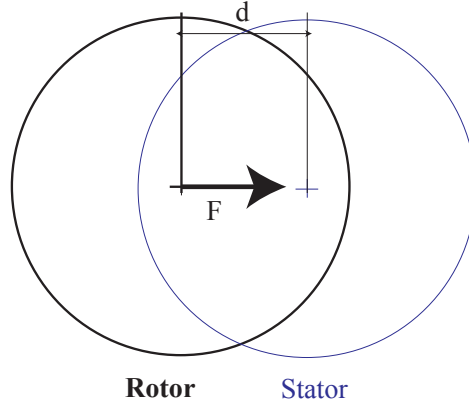
Each of these parameters can be identified by a force measurement. Forces measurements present the advantage to be a physical element easy to calculate within a finite element model or to measure on a test bench.

5.1.1 Stiffness

Concerning the stiffness parameter k , an identification can simply be done on basis of the forces values when the rotor is off-centered and is not moving ($\dot{z} = 0$) nor spinning ($\omega = 0$), like illustrated in Fig. 5.1. The forces predicted by the model, in Eq. 2.30 then reduce to:

$$\bar{F} = -k\bar{z}. \quad (5.1)$$

If no stiffness forces are present in the system when the rotor is not rotating, or in other words when there are no reluctant forces, it can

Figure 5.1: Configuration to identify the stiffness k

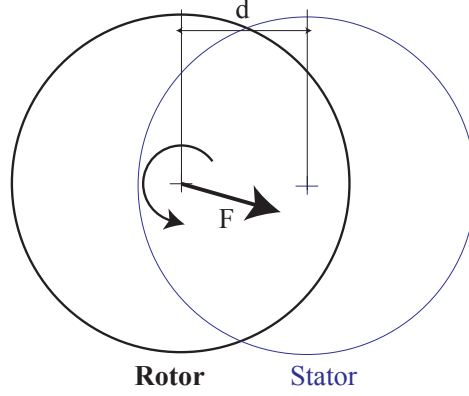
be stated that $k = 0$.

5.1.2 Rotating damping parameters

The parameters characterizing the rotating damping: c_m , L_r and R_r , need a more advanced analysis.

In practice, we have a value for the forces acting on the system. This means that those parameters c_m , L_r and R_r can be identified by fitting the forces evolution predicted by the electromechanical model on known values for the forces exerted on the magnetic bearing for different rotation speeds.

To identify these latter, we can fix the spinning rotor in an out-centered position, and write the values of the forces for different rotation speeds (see Fig. 5.2). In this particular configuration, we know that $\dot{z} = 0$ and $\lambda_I = 0$. The stiffness k has previously been identified, and can thus be eliminated from the equations.

Figure 5.2: Configuration to identify the rotating damping $c_r(\omega)$

The forces predicted by the model then simplify to:

$$\bar{F} = -(c_r \omega \sin \theta - j c_r \omega \cos \theta) \bar{z} \quad (5.2)$$

When expanding the Eq. 5.2 by replacing the rotating damping and the phase shift by their expressions, we obtain:

$$\bar{F} = -\left(\frac{c_m^2 \omega^2 L_r}{|z|^2 (R_r^2 + (\omega L_r)^2)} - j \frac{c_m^2 \omega R_r}{|z|^2 (R_r^2 + (\omega L_r)^2)} \right) \bar{z} \quad (5.3)$$

This expression may be split between its real and its imaginary part. The real part is the restoring force in the direction of the center shift, and is called centering/parallel force and is written F_{para} . The imaginary part is the restoring force in the direction perpendicular to the center shift, and is called perpendicular force and is written F_{perp} . If

d is the amplitude of the center shift, then:

$$\begin{aligned} F_{\text{paral}} &= \frac{c_m^2 \omega^2 L_r}{d^2 (R_r^2 + (\omega L_r)^2)} d \\ &= \frac{\omega^2 (L_r/c_m^2) d^2}{((R_r/c_m^2) d^2)^2 + (\omega (L_r/c_m^2) d^2)^2} d \end{aligned} \quad (5.4)$$

$$\begin{aligned} F_{\text{perp}} &= \frac{c_m^2 \omega R_r}{d^2 (R_r^2 + (\omega L_r)^2)} d \\ &= \frac{\omega (R_r/c_m^2) d^2}{d^2 ((R_r/c_m^2) d^2)^2 + (\omega (L_r/c_m^2) d^2)^2} d \end{aligned} \quad (5.5)$$

It can thus be noticed that we have two equations (Eqs. (5.4) and (5.5)) for three parameters. The parameters are linked to each other by a scale effect: for instance when multiplying c_m^2 , R_r and L_r by a constant factor the expressions remain identical. Only the ratios R_r/c_m^2 and L_r/c_m^2 can therefore be identified.

In addition, we can a priori suppose c_m proportional to the center shift d , while the inductance L_r and the resistance R_r are independent of d .

This makes physically sense: c_m represents the amplitude of the flux variation with the angular position (Eq. 2.23), and it seems coherent that this amplitude depends linearly on the center shift d . The rotor impedance characteristics are independent of the center shift, which is also coherent because the conductivity and the permeability of the rotor have not changed.

As a consequence, the forces are proportional to d ,

$$cst_{\text{paral}} = \frac{\omega^2 (L_r d^2 / c_m^2)}{((R_r d^2 / c_m^2)^2 + (\omega (L_r d^2 / c_m^2))^2)} \quad (5.6)$$

and

$$cst_{\text{perp}} = \frac{\omega (R_r d^2 / c_m^2)}{d^2 ((R_r d^2 / c_m^2)^2 + (\omega (L_r d^2 / c_m^2))^2)} \quad (5.7)$$

such as, according to Eq. 5.4

$$F_{\text{paral}} = cst_{\text{paral}} d$$

and, according to Eq. 5.5

$$F_{perp} = cst_{perp}d$$

are only dependent on d through the ratio c_m/d supposed constant. This assumption will be verified in the next sections, for center shifts d small enough.

Finally, we normalize the excitation by imposing the ratio c_m/d to 1, and this reduces the remaining degree of freedom.

The fact that the forces are proportional to d implies that the forces only need to be measured or simulated for one center shift in order to identify the parameters properly. This also implies that the rotating speeds for which the rotor behaves in a stable way does not depend on the center shift.

Of course, when the center shift is too big, the field distribution is changed, the currents paths are changed. The forces are therefore not proportional to d anymore, and as well the inductance L_r as the resistance R_r depend on the center shift. The limit stability spin speed then would depend on the center shift.

5.1.3 Non rotating damping

Finally, let us look at the last parameter, the non-rotating damping coefficient c_{nr} . Either, this parameter is known because the characteristics of an external damper are known, either this last parameter can be identified by giving a whirl constant velocity to the off-centered rotor, which is spinning at the same frequency, and by noting the values of the forces for different velocities, as in Fig. 5.3. In this configuration, the whirl frequency is worth $\lambda_I = \omega$, the position of the rotor is $\bar{z} = d \exp(j\lambda_I t)$ and its speed is $\dot{\bar{z}} = jd\lambda_I \exp(j\lambda_I t)$. Again, the stiffness k has previously been identified, and its action can therefore be eliminated from the equation. The expression of the forces reduces to:

$$\bar{F} = -jc_{nr}\lambda_I\bar{z} \quad (5.8)$$

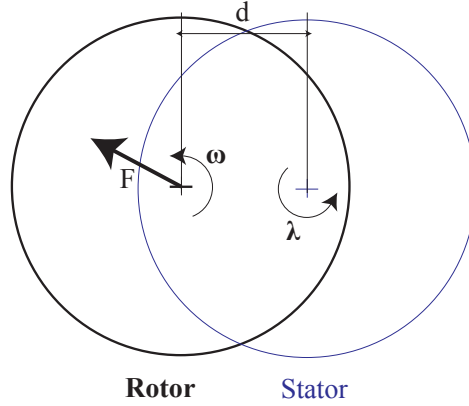


Figure 5.3: Configuration to identify the rotating damping c_{nr}

Only the non-rotating damping is excited: the rotating damping does not act. The amplitude of the force in Eq. 5.8 gives the value of the non-rotating damping.

The non-rotating damping can also be identified by fixing the spinning rotor in an off-centered position, and making the stator spin at the frequency ω (Fig. 5.4). This will yield the same equation as eq. 5.8. This second configuration for the identification of the non-rotating damping may be easier to implement in a finite element model for instance, whereas, the first configuration may be easier to implement on an experimental test bench where it is impossible to move the stator. These two possible configurations highlight the reciprocity of the model.

5.1.4 Numerical identification

The model parameters for the rotating damping are identified by minimizing the quadratic error between the forces predicted by the model and those measured or obtained by FEM: the least square criterion.

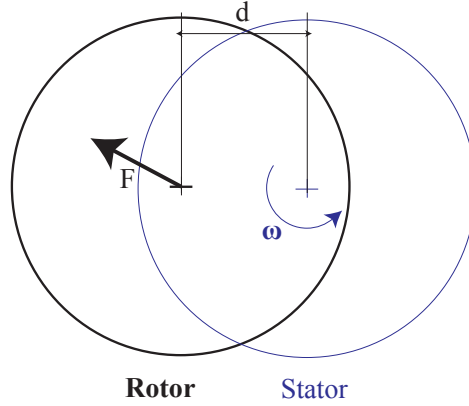


Figure 5.4: Second configuration to identify the rotating damping c_{nr}

The least-squares criterion is a specific case of the prediction-error approach, whose principle is to minimize the prediction error. This prediction error can be filtered, then a norm is applied to this filtered error. The advantage of a prediction-error approach is to be applicable to all model structures ([Ljung, 1999]), as well linear as non-linear, which is the case of our model.

In the least-square criterion, the filter is equal to 1, and the norm is the quadratic norm. The benefits of this criterion are to be simple to use and to understand. But if there is noise on the data, and that this noise is not white noise, the least square estimate will not converge to the true value of the parameters.

Concerning the minimization of the error, we noticed the existence of a number of local minima in this optimization function. It has been shown that when we restrict ourself to a local minimum, the prediction of the stability behavior of the rotor is affected. For instance, in ([Lafont, 2010]) several optimizations gave the results in Table 5.1.

A procedure has therefore been developed to raise the chances to find

Table 5.1: Value of the parameters of the electromechanical model for several optimizations in [Lafont, 2010]

optimization function value [$\ast 10^{-10}$]	R_{dc} [$\ast 10^{-2}$]	L_0 [$\ast 10^{-5}$]	L_{int} [$\ast 10^{-7}$]	A [$\ast 10^{-6}$]	α	β
8.41	2.9	1.33	1.97	4.96	0.0062	-1.88
9.74	2.9	1.33	1.98	5.18	0.0058	-1.77
10	2.9	1.33	1.98	5.2	0.0057	-1.76
10.6	2.9	1.33	1.98	5.26	0.0056	-1.73
3580	2.9	$4.33 \cdot 10^{-3}$	267	7.49	$3.64 \cdot 10^{-14}$	$-4.84 \cdot 10^{-5}$

the global minimum.

When using only the method of the preconditioned conjugate gradients, the chances exist to be stuck in a local minimum. This is why a stochastic approach is chosen. The stochastic approach reduces the chances to be stuck in a local minimum because the parameters space is traveled all over more efficiently.

The developed procedure consists in first applying a genetic algorithm on an initial population of parameters. After a number of generations, the procedure keeps the best elements of the population, on which a method of preconditioned conjugate gradients is applied. This hybrid approach accelerates the research of the global minimum, and is more robust.

A last word can be said about which least square error has to be minimized. The measure we have at our disposal are the forces values. This is our starting point, from there on, two approaches are possible. The first approach consists in identifying R_r/c_m^2 and L_r/c_m^2 for each spin speed without pre-supposing a model for the impedance frequency dependence. Then the second step of this approach consists in choosing an evolution law of the impedance with the frequency, and to identify the parameters of this evolution law on the basis of the obtained values for $R_r(\omega)/c_m^2$ and $L_r(\omega)/c_m^2$. If the subscript *mes* refers to the

data coming from experimental measures or finite element model simulations, depending on which evolution law is chosen, the optimization function can be written:

$$\min_{\substack{p=\{R_{dc}, A, L_0, \\ L_{int}, \alpha, \beta\}}} \sum_{i=1}^n (R_{mes}(\omega_i) - R(p, \omega_i))^2 + (L_{mes}(\omega_i) - L(p, \omega_i))^2 \quad (5.9)$$

when the weighting function model is chosen or

$$\min_{\substack{p=\{R_{dc}, c_{fe}, G_{cfe}, \\ L_0, c_{fe}, L_{cfe}\}}} \sum_{i=1}^n (R_{mes}(\omega_i) - R(p, \omega_i))^2 + (L_{mes}(\omega_i) - L(p, \omega_i))^2 \quad (5.10)$$

when the continued fraction expansion model is chosen.

The second approach consists in choosing a priori an evolution law of the impedance with the frequency and to identify directly all the parameters on the forces evolution with ω . The optimization function is then:

$$\min_{\substack{p=\{R_{dc}, A, L_0, \\ L_{int}, \alpha, \beta\}}} \sum_{i=1}^n (F_{paral,mes}(\omega_i) - F_{paral}(p, \omega_i))^2 + (F_{perp,mes}(\omega_i) - F_{perp}(p, \omega_i))^2 \quad (5.11)$$

when the weighting function model is chosen or

$$\min_{\substack{p=\{R_{dc}, c_{fe}, G_{cfe}, \\ L_0, c_{fe}, L_{cfe}\}}} \sum_{i=1}^n (F_{paral,mes}(\omega_i) - F_{paral}(p, \omega_i))^2 + (F_{perp,mes}(\omega_i) - F_{perp}(p, \omega_i))^2 \quad (5.12)$$

when the continued fraction expansion model is chosen.

The advantage of the first approach is that no pre-supposition of the impedance evolution with the frequency needs to be done, but its inconvenient is that it needs two steps.

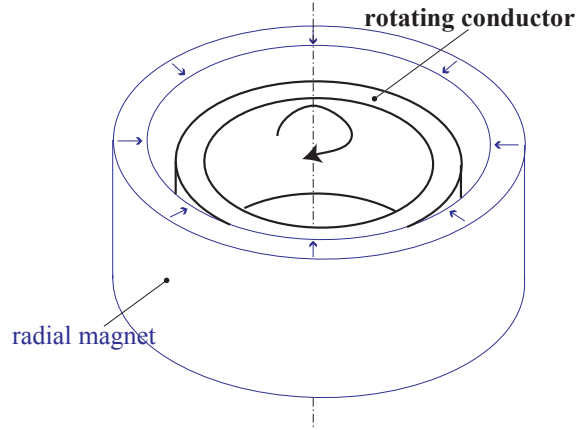


Figure 5.5: Schematic representation of a simple case for the validation of the electromechanical model

5.2 Simple case with radial flux

5.2.1 Geometrical and physical characteristics

To validate the electromechanical model presented in chapter 2 and 3, the model is confronted to a case study modeled by a finite element modeling software. The aim of this simple electromechanical system is not to develop a magnetic bearing but to study the forces due to induced currents within a simple configuration.

The simple case chosen for this comparison is a cylindrical system, where an annular conductor rotates inside a magnetic field generated by a tubular permanent magnet with a radial magnetization, as illustrated in Fig. 5.5. The geometric and physical properties of the system are summarized in Table 5.2.

As long as the rotor is centered, no currents are induced, and no forces are generated on this rotor. When the rotating conductor is out-centered, the resulting induced currents interact with the magnetic field to generate forces. One force component tends to re-center the rotor, the other one is tangential to the center shift and leads to an inwards or outwards whirl motion. This is illustrated on the schematic

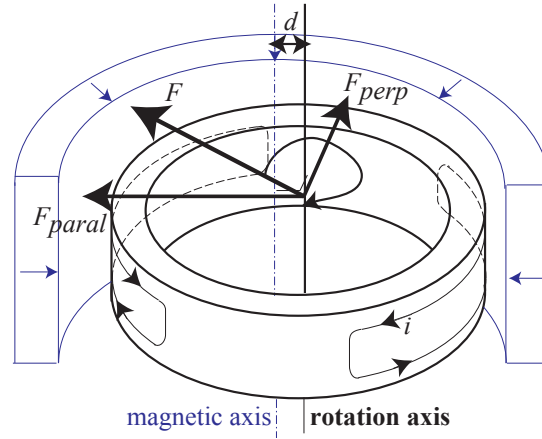


Figure 5.6: Schematic representation of the principle of the generation of forces in this simple case

Table 5.2: Geometric and physical data for a simple case with radial flux

ring magnet	outer diameter 45 mm inner diameter 41 mm height 20 mm	$B_r = 1.17 \text{ T}$
conducting ring	outer diameter 40 mm inner diameter 36 mm height 16 mm	$\sigma = 60 \cdot 10^6 \text{ S/m}$
nominal air gap	0.5 mm	

drawing, in Fig. 5.6.

5.2.2 Finite Element Results

The forces generated on the rotor are calculated using a FEM where the rotor is fixed in an off-centered position. A plane of symmetry at the half of the height of the system is used, to simplify the FEM.

The used FEM software is Comsol multiphysics 3.5. It solves the electric and the magnetic potential for a static configuration, taking into account the induced currents. More details about the physical equations, the mesh and the numerical solver used are given in Appendix A and in [Kluyskens and Dehez, 2007].

Once the FEM solved, the forces due to the induced currents are calculated by integrating the expression

$$d\vec{F} = \vec{J} \times \vec{B} \quad (5.13)$$

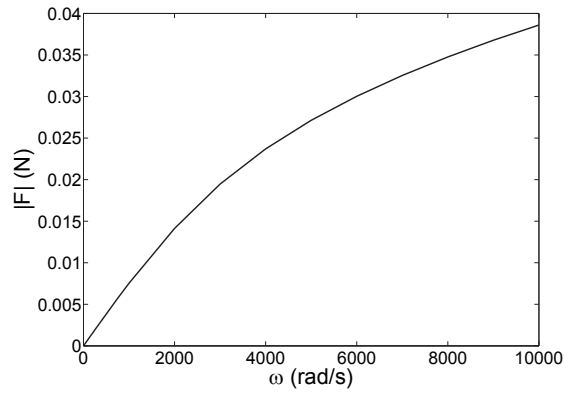
on the volume where the induced currents take place.

The evolution of the force with the rotation speed is illustrated in Fig. 5.7. As expected, the norm and the angular shift of the force due to the induced currents in the rotor increase with the rotation speed. This is coherent with the evolution of the norm of the induced currents, and their angular shift with the rotation speed, observed in Fig. 5.8.

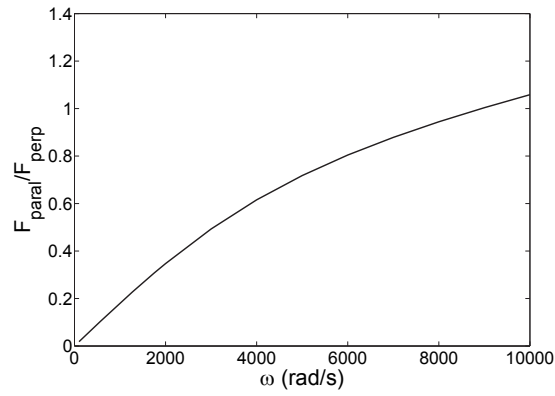
The forces evolution for various center shift d are represented in Fig. 5.9. It can be observed on this figure that the forces vary in a linear way with the center shift. This confirms the assumption made in section 5.1.2, and the possibility to identify the parameters of the electromechanical model on the basis of forces for only one center shift.

Before the identification of the parameters of the electromechanical model, two considerations are worth noticing:

- as said before, the identification of these parameters can be done for only one center shift, because their value is independent from the center shift. Here, the identification is carried on the forces for a center shift $d = 0.2$ mm, the nominal air gap being 0.5 mm

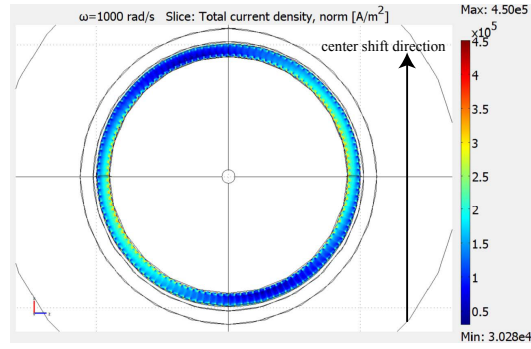


(a)

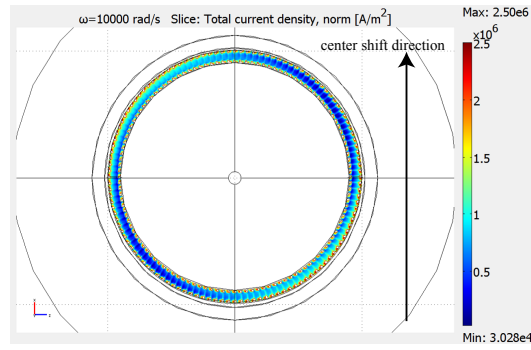


(b)

Figure 5.7: Norm and angular shift of the force due to the induced currents in the rotor for a center shift $d = 0,2\text{mm}$ (40% of nominal air gap)

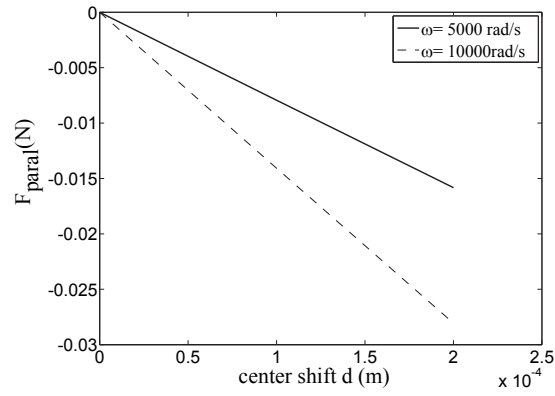


(a)

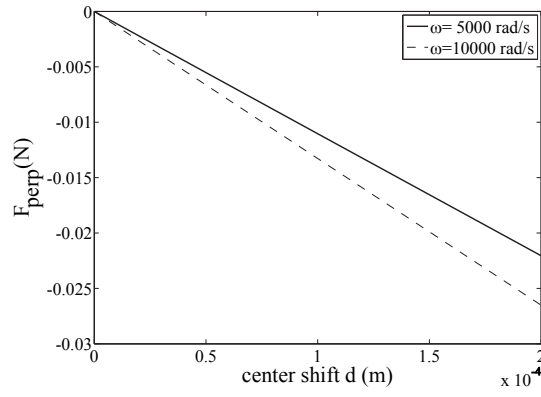


(b)

Figure 5.8: Radial slice in the conducting rotor representing the norm of the induced currents, when the rotor has a center shift $d = 0.2$ mm, for a rotation speed (a) $\omega = 1000$ rad/s, (b) $\omega = 10000$ rad/s



(a)



(b)

Figure 5.9: (a) Parallel and (b) perpendicular component of the force due to induced currents in the rotor as a function of the center shift

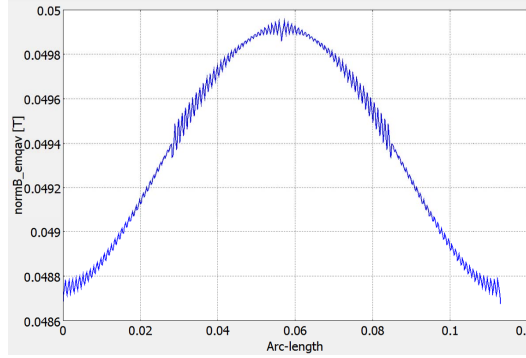


Figure 5.10: Norm of the magnetic induction along the rotor periphery, for $d = 0.2$ mm

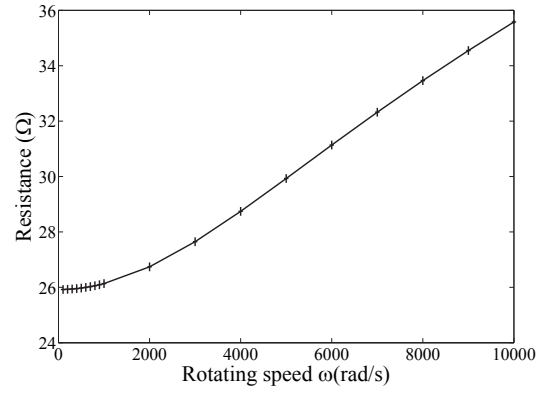
- one of the restrictions on which the electromechanical model is based is that, when out-centered, the evolution of the magnetic induction can be approximated by the first harmonic. When plotting the norm of the magnetic induction along the rotor periphery for a center shift $d = 0.2$ mm, we can see (Fig. 5.10) that this case respects this restriction. The small variations around the first harmonic behavior are due to the mesh.

5.2.3 Electromechanical model validation

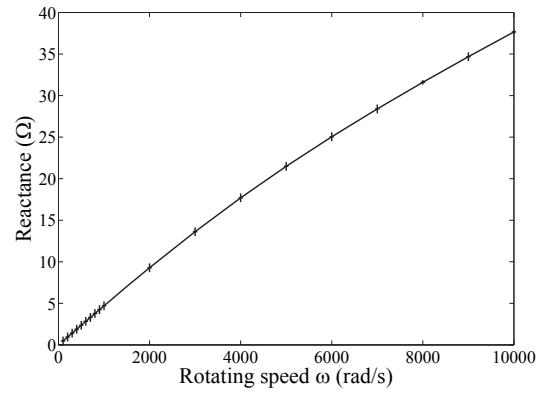
The FEM gives the parallel and the perpendicular forces value for different spin speed when the rotor spins in an out-centered position. In this case the forces are then worth Eq. 5.4 and 5.5, since stiffness k is equal to 0.

This means that for each spin speed, an identification can be done on these forces to know the value of the global resistance R_r and global reactance ωL_r . The variation of these latter with the spin speed is shown in Fig. 5.11.

According to the skin effect, it seems logical that the resistance increases with spin speed and that the reactance grows slower than the tangent at the origin with the spin speed, but so far no model for the evolution of the impedance with the spin speed has been chosen.



(a)



(b)

Figure 5.11: Evolution of the global resistance R_r and the global reactance ωL_r with the spin speed

Table 5.3: Value of the parameters of the electromechanical model for a simple case with radial flux, skin effect described by a continued fraction expansion model for the impedance

c_m/d	1
R_{dc}	25,94 Ω
G	1/7, 1 Ω^{-1}
L_0	2,02 10^{-3}H
L	8,13 10^{-3}H

In chapter 3, we have seen that two models based on a phenomenological approach give very good results in describing the skin effect on the impedances. The first model is based on a continued fraction expansion, and the second model is based on a weight function between the low-frequency impedance and the high-frequency impedance.

First, we choose to describe the impedance evolution with the spin speed by the continued fraction expansion, see eq. (3.31). This means that the parameters L , R_{dc} , G and L_0 have to be identified. This can be done on the basis of the resistance and inductance values illustrated in Fig. 5.11 (a) and (b).

Let us note that all the identifications are carried out as described in section 5.1.4.

The results of this parameter identification are listed in Table 5.3.

Fig. 5.12 shows the evolution of the global resistance and reactance predicted by the continued fraction expansion skin effect model, and the resistance and reactance values identified on the basis of FEM simulations. It can be concluded that this skin effect model fits the actual evolution of the impedances very well.

The result on the forces evolution with the spin speed and the comparison with the FEM calculated forces is shown in Fig. 5.13. The relative residue on the forces has a value of order 10^{-6} . Indeed the forces evolution predicted by the electromechanical model and the forces simulated by FEM match very well.

Second, we choose to describe the impedance evolution with the

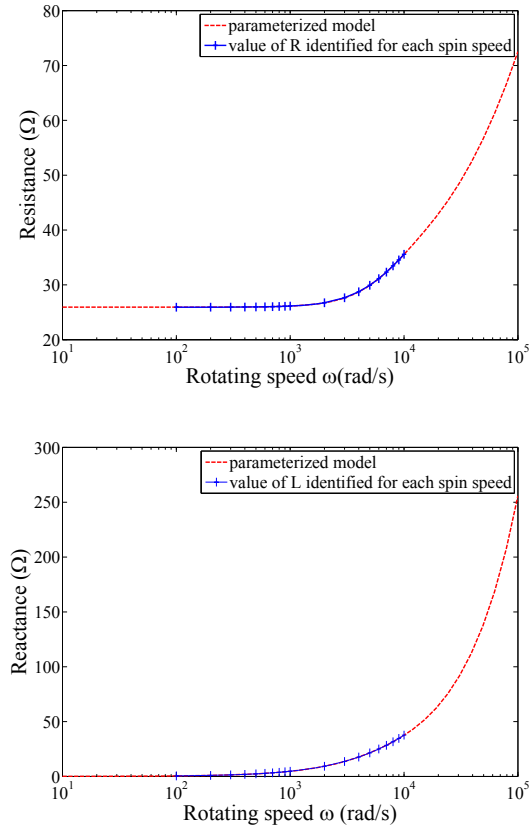


Figure 5.12: Comparison of the global rotor impedance evolution after identification of the continued fraction expansion model parameters, logarithmic scale for the spin speed

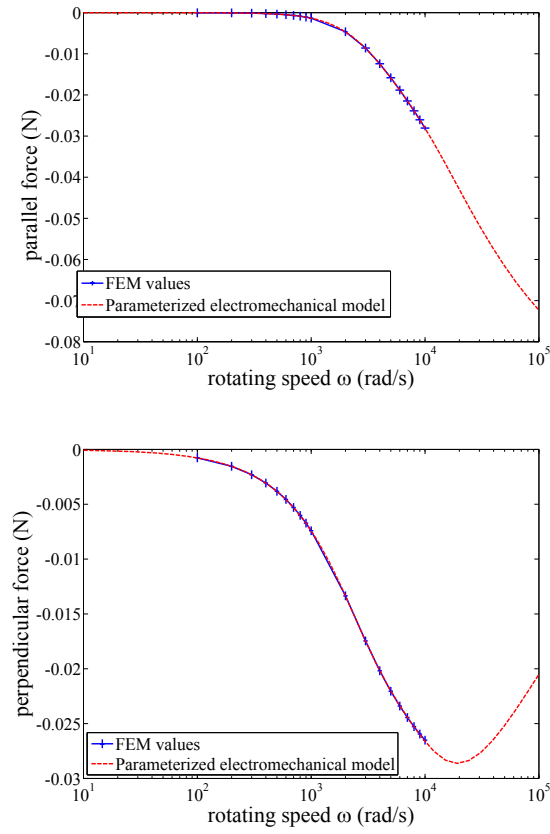


Figure 5.13: Comparison of the forces evolution with the parameters of the continued fraction expansion skin effect model identified, for a center shift $d=0,2\text{mm}$, logarithmic scale for the spin speed

spin speed by the weighted model described by eqs.(3.34), (3.35) and (3.36). This means that now the parameters R_{dc} , L_0 , L_{int} , A , α and β can be identified on basis of the resistance and inductance values illustrated in Fig. 5.11 (a) and (b) as previously, or also directly on the basis of the forces value, which are illustrated in Fig. 5.7 (a) and (b). The results of this identification are listed in Table 5.4.

Starting from the forces values (1 step) or from the impedances value (2 step) give the same relative residue for the error on the forces of order 10^{-5} . In this case, the 1 step method is a bit more faster than the 2 step method (447 sec vs 1250 sec).

Fig. 5.14 shows the evolution of the global resistance and reactance and the evolution predicted by the weighted skin effect model. Here again, it can be concluded that this skin effect model fits the actual evolution of the impedances well. However, for the resistance, it can be seen that around $\omega = 4000$ rad/s, the model resistance evolution has moves slightly away from the finite element results and then comes back to them. The look of this behavior reminds of the form of an arctangent, which is the weight function.

The result on the forces evolution with the spin speed and the comparison with the FEM calculated forces is shown in Fig. 5.15. It can also be observed on these figures that the whole model, electromechanical model with a model for the skin effect in the impedance fits the forces evolution with the spin speed well, but a little less good than when using the continued fraction expansion model.

Whatever choice is made for the skin effect, we observe that when extrapolating the forces evolution with the spin speed, we can see that the perpendicular component of the force decreases, where the parallel component continues to increase. This actually corresponds to the observed force evolution, as shown in a next case (see section 5.4).

Both skin effect models fit the resistance and the reactance evolution with the spin speed very well, although the continued fraction expansion gives slightly better results.

Once the identification is done, it may be interesting to look at the evolution with the spin speed of the rotating damping coefficient c_r , the phase shift θ , or the induced stiffness, $c_r \omega \sin \theta$. Examining these results using the identified weighted model, we ob-

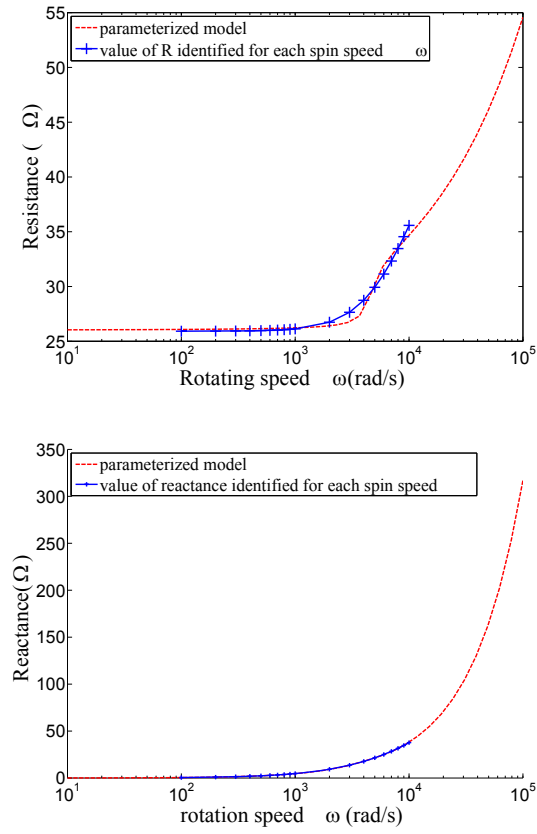


Figure 5.14: Comparison of the global rotor impedance evolution after identification of the weighted model parameters, logarithmic scale for the spin speed

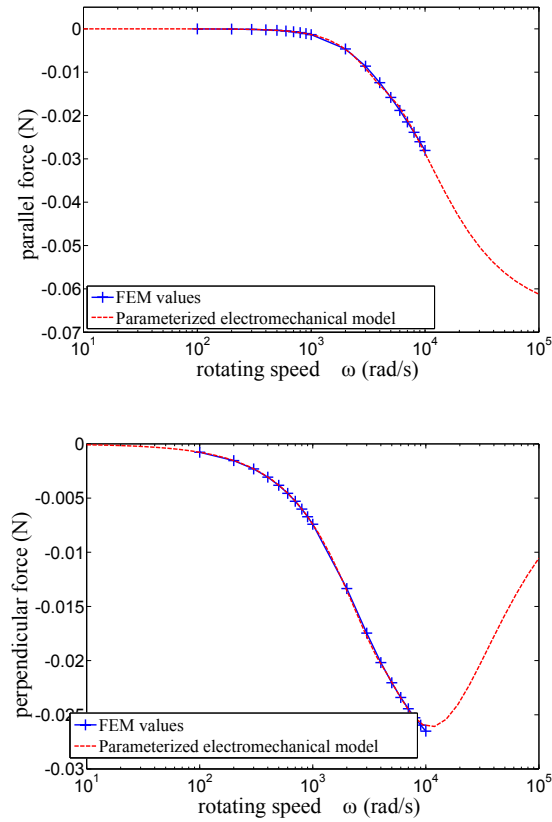


Figure 5.15: Comparison of the forces evolution after identification of the weighted model parameters for the parallel and the perpendicular component, for a center shift $d=0,2\text{mm}$, logarithmic scale for the spin speed

Table 5.4: Value of the parameters of the electromechanical model for a simple case with radial flux, skin effect described by weighted model for the impedance

c_m/d	1
R_{dc}	26,03 Ω
A	0,091
L_0	2,88 10^{-3} H
L_{int}	1,67 10^{-3} H
α	1,29 10^{-3}
β	-5,85

serve the phase shift gradually increases from 0 to $\pi/2$ (Fig. 5.18): when the spin speed increases the inductive part of the impedance becomes more and more important.

In parallel, we can see (Fig. 5.16) that the rotating damping coefficient decreases with the spin speed, in opposition to the induced stiffness (Fig. 5.17) which increases with the spin speed. However, the stiffness due to the induced currents stays very low until very high rotation speeds are reached.

The evolution of the induced stiffness with the spin speed in Fig. 5.17 can be describes as follows, first it increases with the spin speed, and for very high values of spin speed it tends to a constant. This behavior can be confirmed when looking at the expression of the induced stiffness in eq. 5.14:

$$c_r \omega \sin \theta = \frac{c_m^2}{Z_I^2 \sqrt{R_r^2 + (\omega L_r)^2}} \omega \frac{\omega L_r}{\sqrt{R_r^2 + (\omega L_r)^2}}. \quad (5.14)$$

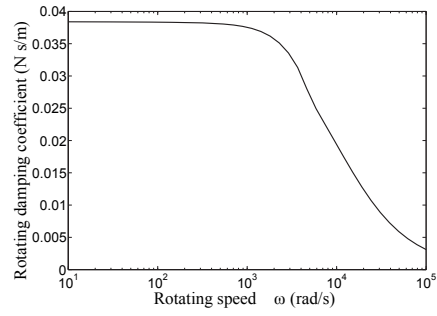
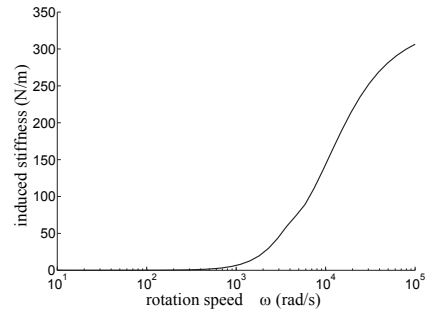
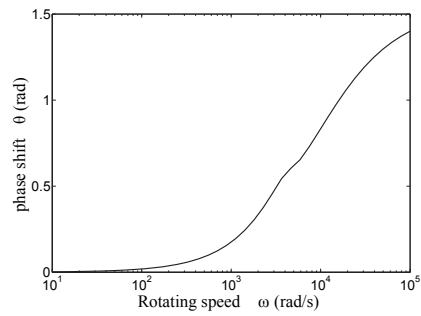
When $\omega L_r \ll R_r$, examining the three terms of eq. 5.14, the induced stiffness increases with the spin speed ω :

- the amplitude of the rotating damping $c_r \rightarrow \frac{c_m^2}{Z_I^2 R_r}$ stays constant,
- ω increases,
- the orientation of the force expressed by $\sin \theta \rightarrow \frac{1}{\sqrt{R_r^2 + 1}}$ stays constant.

When $\omega L_r \gg R_r$, in eq. 5.14, the induced stiffness is constant:

- the amplitude of the rotating damping $c_r \rightarrow \frac{c_m^2}{Z_I^2 \omega L_r}$ decreases in ω ,
- ω increases,
- the orientation of the force $\sin \theta \rightarrow \frac{\omega L_r}{\omega L_r}$ stays constant.

Another interesting comparison would have been the comparison between the losses (the active power) predicted by the electromechanical model, with its parameters identified, and the losses calculated in the FEM. Unfortunately, the Joule losses in the FEM are over-evaluated because of false eddy currents. These false eddy currents do not influence the calculations of the forces, but they inflate the Joule losses in the conductor. This comparison is thus not representative. More details about the finite element modeling and these false eddy currents can be found in Appendix A.

Figure 5.16: Rotating damping coefficient c_r Figure 5.17: Induced stiffness $c_r \omega \sin \theta$, logarithmic scale for the rotation speedFigure 5.18: Phase shift θ , logarithmic scale for the rotation speed

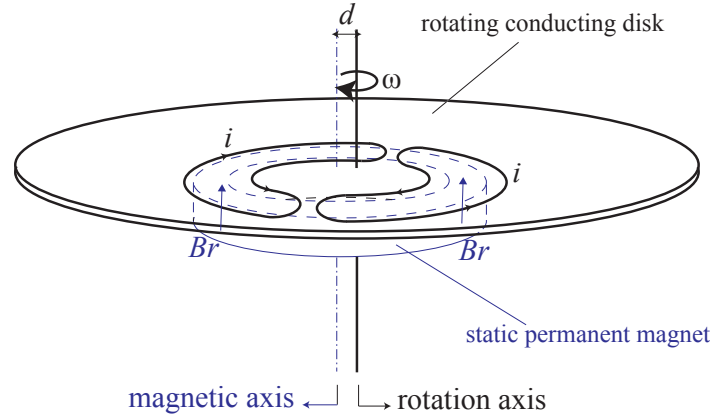


Figure 5.19: Principle of induced currents due to an out-centered position for a system with an axial flux

5.3 Simple case with axial flux

5.3.1 Geometrical and physical characteristics

The electromechanical model is now confronted to another simple study case, but with axial flux. This case is the same as the one examined in the qualitative validation (Fig. 5.19). The geometric and physical properties of this study case are listed in Table 5.5. The principle is similar to the previous study case: when the rotating axis of the conducting disk is aligned with the magnetic axis of the permanent magnet, no currents are induced. When there is a center shift between these two axis, a point of the conductor sees a magnetic induction variation when it rotates, and currents are induced which results in electromagnetic forces.

5.3.2 Experimental and finite element results

The forces on the conducting disk are calculated as a function of the rotation speed by a FEM (see Fig. 5.20). The FEM software and the FE modelling procedure are the same as in previous case. The

Table 5.5: Geometric and physical data for a simple case with axial flux

magnet ring	outer diameter 41 mm inner diameter 27 mm height 7.5 mm	$B_r = 1.17\text{T}$
conducting disk	diameter 145 mm thickness 2 mm	$\sigma = 37.7 \cdot 10^6 \text{ S/m}$
air gap	1 mm	

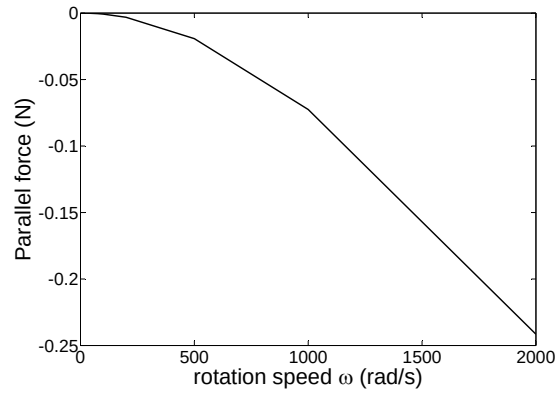
induced currents developed in the conducting disk are represented on Fig. 5.21.

It can be underlined, that in this case also, the forces depend in a linear way on the center shift. This is clearly shown in Fig. 5.22.

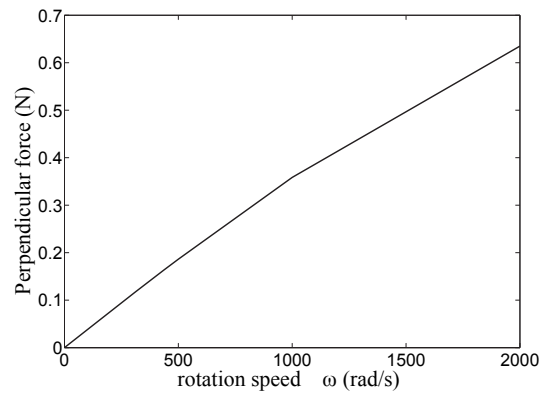
An experimental test bench has also been developed to measure the force exerted on the rotating conducting disk spinning in a fixed off-centered position (Fig. 5.23). The aluminum disk is attached to a plastic axis which is driven by a milling machine at constant rotation speeds up to 660 rad/s. The permanent magnet (stator) is fixed to a force sensor. This sensor measures the forces acting on the magnet, which, in pursuance of the action-reaction principle, are equal to the opposite of the forces acting on the rotor. The force sensor has a sensing range of 65N and a resolution of 0.05 *N*.

The milling machine allows a precise positioning of the rotor with respect to the stator, and a constant spinning speed. To be able to measure the forces with enough precision, the center shift for the measurements is fixed to 0.8 mm.

Thanks to these experimental measurements, we validate the FEM simulations results. But the FE method does not allow too high center shift, and only simulations of center shift up to 0.2 mm have been carried out: with a too high center shift, the deformed mesh is of a too poor quality. Although, because the forces are linear with the center shift, it is sufficient to apply a factor 4 on these forces to have



(a)



(b)

Figure 5.20: Parallel and perpendicular component of the forces due to the induced currents in an aluminum rotor, for a center shift of 0.2 mm

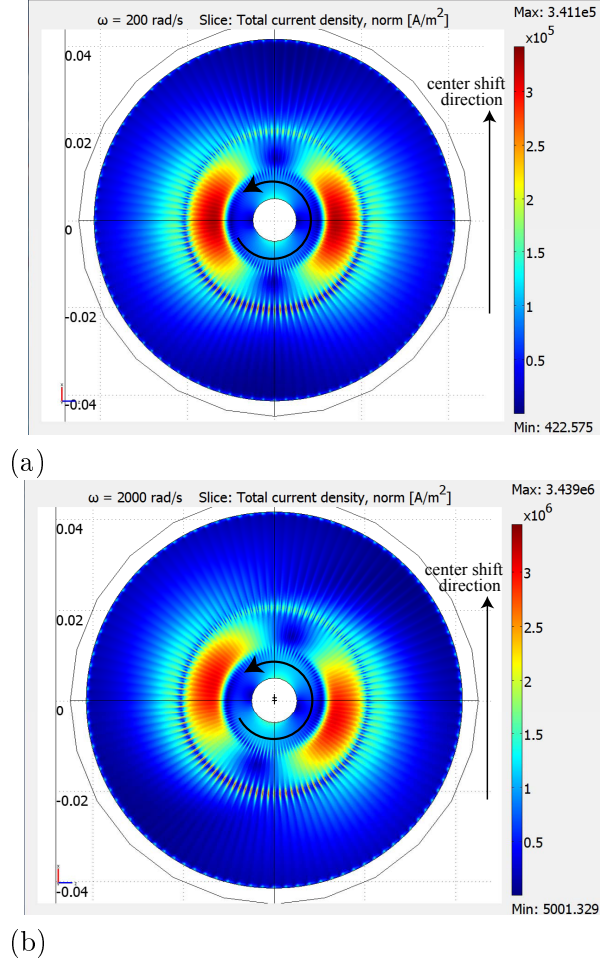
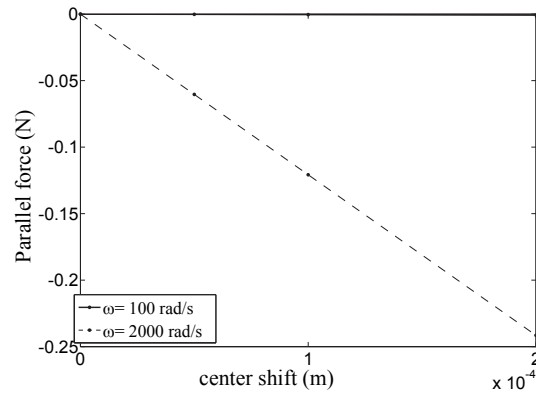
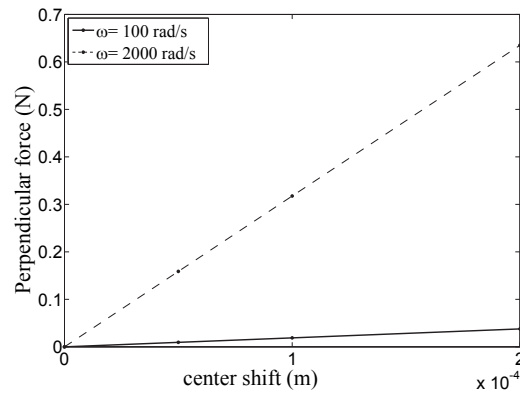


Figure 5.21: Norm of the induced currents in a aluminum disk rotating above a permanent magnet of induction 1.2T, with an air gap of 1 mm, (a) for a rotation speed of 200 rad/s, and (b) for a rotation speed of 2000 rad/s



(a)



(b)

Figure 5.22: (a) Parallel and (b) perpendicular forces in function of the center shift for a 2mm aluminum disk

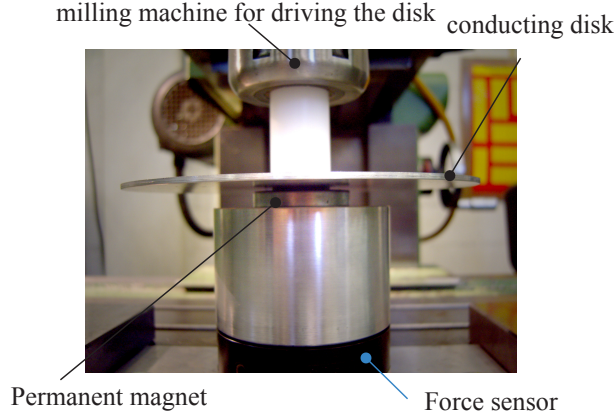


Figure 5.23: Experimental test bench to measure the forces on the rotor spinning in an off-centered position

the values of the forces estimated by the FEM with a center shift of 0.8 mm. In Fig. 5.24 and 5.25, the comparisons of the experimental measurements and of the FEM calculations are shown. It can be seen, that in both cases, the values correspond. There is a higher variation for the centering (parallel) component of the forces, this might be due to the fact that these forces are smaller, and that the measures are less precise, or to the fact that the material properties do not correspond exactly to the FEM material properties.

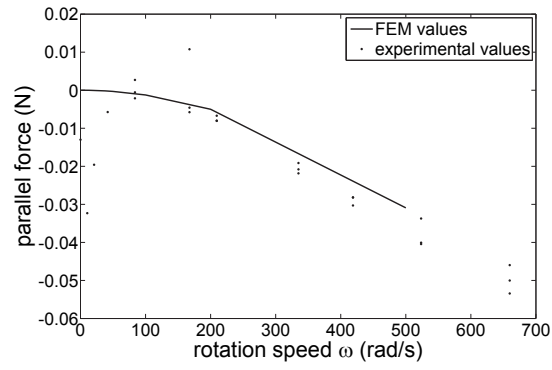
5.3.3 Electromechanical model validation

The identification is carried out on the FEM calculated values for the forces.

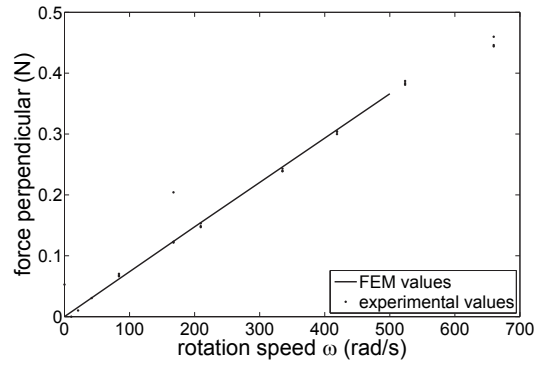
This identification is done in one step, supposing that the impedances vary with the spin speed as the weighted model described by eqs. (3.27), (3.28) and (3.29).

After identification, the relative residue on the error for the forces is of order 10^{-8} . The value of the different parameters is shown in table 5.6.

In Fig. 5.26, we see the evolution of the global resistance and

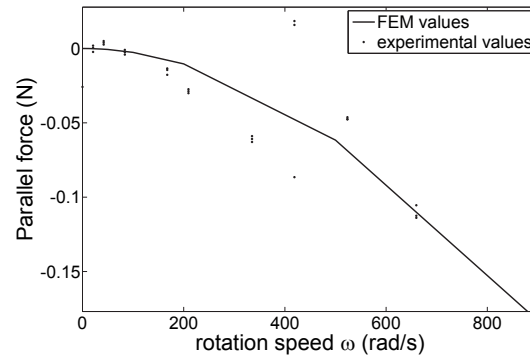


(a)

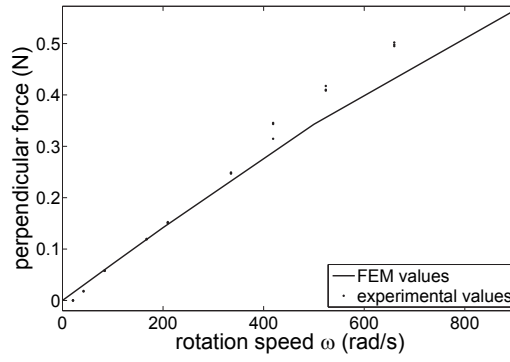


(b)

Figure 5.24: Comparison between the experimental measurements and the FEM calculations of the forces evolution with the spin speed, for an 1mm aluminum disk



(a)



(b)

Figure 5.25: Comparison between the experimental measurements and the FEM calculations of the forces evolution with the spin speed, for a 2mm aluminum disk

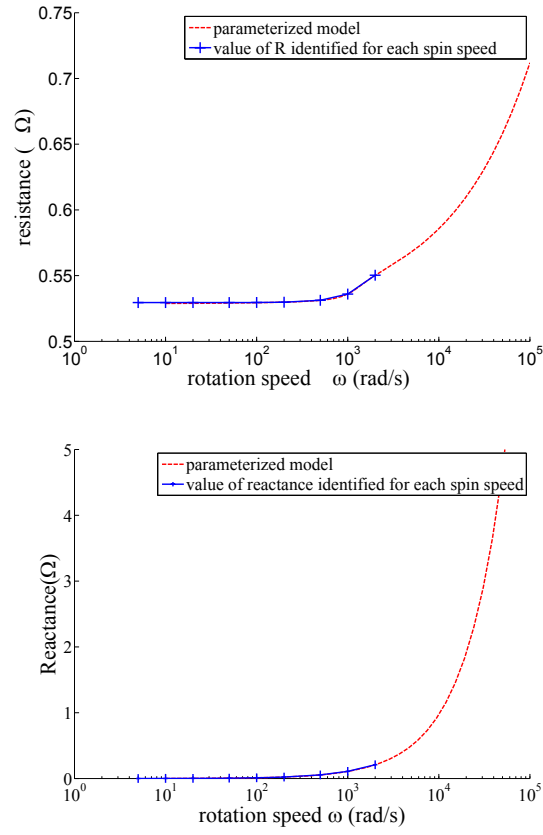


Figure 5.26: Comparison of the global rotor impedance evolution after identification of the parameters

Table 5.6: Value of the parameters of the electromechanical model for a simple case with axial flux

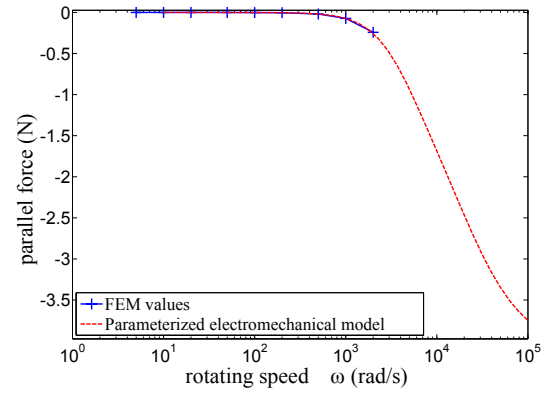
c_m/d	1
R_{dc}	0.53Ω
A	$5.81 \cdot 10^{-4}$
L_0	$9.1 \cdot 10^{-5}\text{H}$
L_{int}	$1.67 \cdot 10^{-5} \text{ H}$
α	$2.07 \cdot 10^{-3}$
β	-2.4

inductance with the spin speed. The model fits very well for the resistance and for the reactance, especially at high frequencies. It also fits well for the forces evolution with the spin speed, see Fig. 5.27.

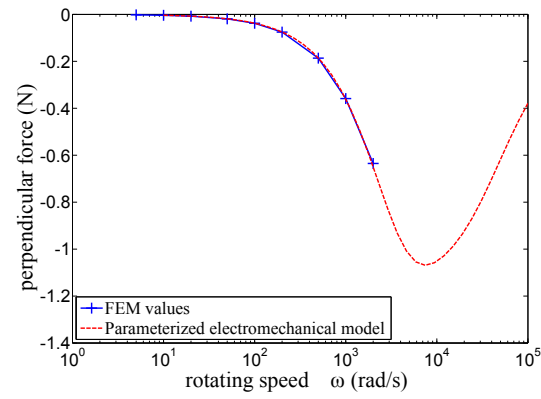
Let us now look at the evolution with the spin speed of the rotating damping coefficient c_r , the induced stiffness $c_r\omega \sin \theta$, the phase shift θ and the weight function of the low vs high frequency impedance. Here again, the phase shift gradually increases from 0 to $\pi/2$ (Fig. 5.30). In parallel, the damping decreases (Fig. 5.28) and the induced stiffness increases (Fig. 5.29). In this case also, the stiffness produced by the eddy current forces is low until high rotation speeds are reached.

Let us now look at posteriori at the differences when the skin effect is considered, and when the impedances are considered as constants to visualize the impact of the skin effect model. Fig. 5.31 shows the difference on the force identification when the skin effect is considered, and when it is not. Clearly, the model fits better when taking the skin effect into account (zoom in Fig. 5.31). And this has an impact on the extrapolation of the forces at higher spin speeds, where the skin effect is more important.

Let's now look at Fig. 5.32 at the difference on the limit spin speeds predicted by the electromechanical model for different external stiffness values, when the skin effect is taken into account and when it is not. The difference on the limit spin speed is more obvious for higher



(a)



(b)

Figure 5.27: Comparison of the forces evolution after identification of the parameters for the parallel and the perpendicular component, for a center shift $d=0,2\text{mm}$

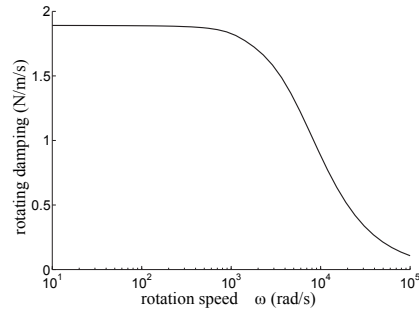


Figure 5.28: Rotating damping coefficient c_r , logarithmic scale for the spin speed

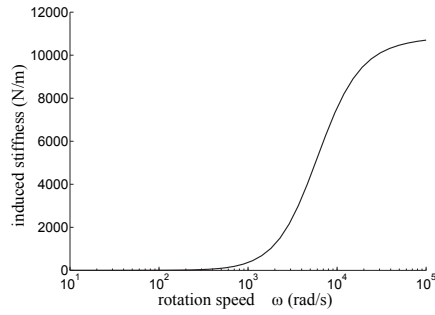


Figure 5.29: Induced stiffness $c_r \omega \sin \theta$, logarithmic scale for the spin speed

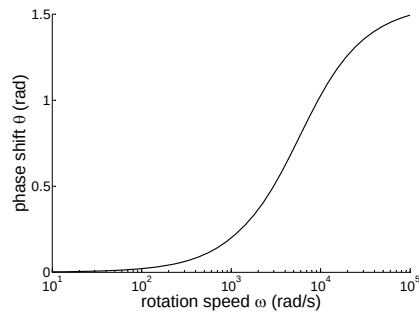
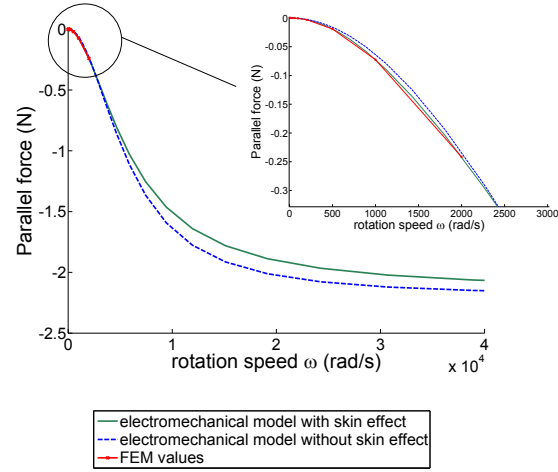
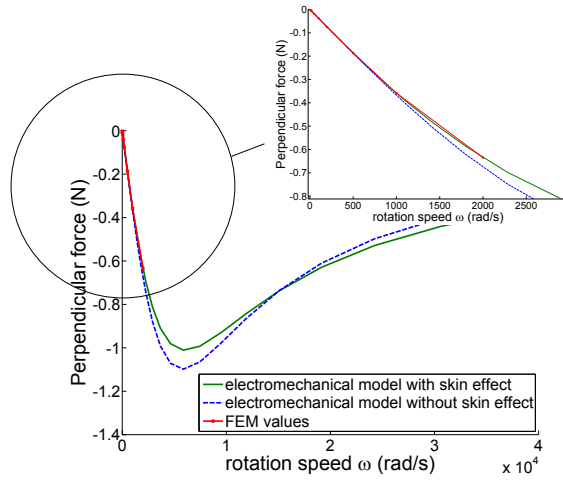


Figure 5.30: Phase shift θ , logarithmic scale for the spin speed



(a)



(b)

Figure 5.31: Comparison between the forces identified for an electromechanical model considering the resistance and the inductance as constant, or considering them dependent of the frequency (a) Parallel force, (b) Perpendicular force

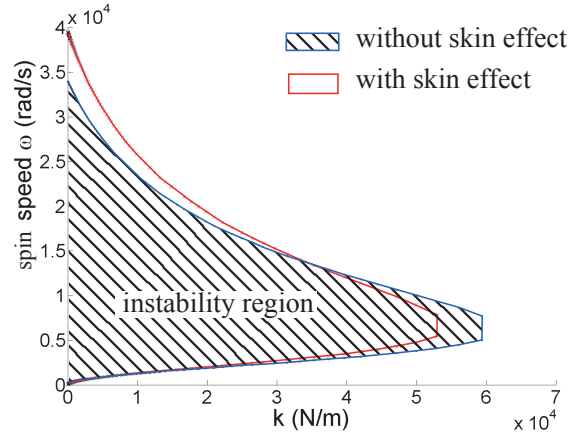


Figure 5.32: Evolution of the limit spin speed with the external stiffness, when considering the skin effect on the rotor impedance or not

spin speed, but also for high values of external stiffnesses. Although these differences remain relatively small (less than 20 %). The value of the first limit spin speed for small external stiffness is not significant. But the value of the limit spin speed prediction where the system goes back into stability are influenced by the choice of the skin effect model, because of the high values for ω .

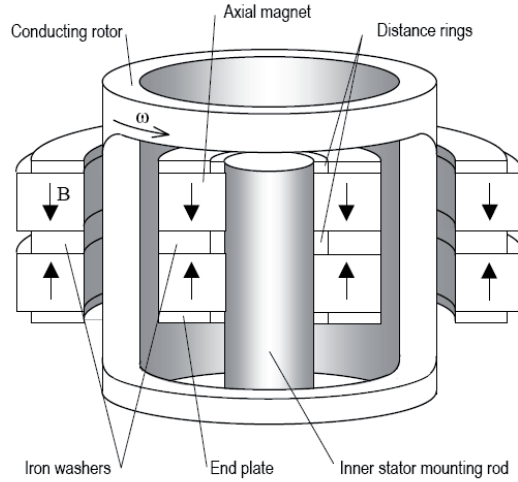


Figure 5.33: Homopolar radial electrodynamic bearing in [Lembke, 2005]

5.4 Electrodynamic magnetic bearing

5.4.1 Geometrical and physical characteristics

Let us now confront the electromechanical model to an existing electrodynamic magnetic bearing: the electrodynamic bearing developed in [Lembke, 2005]. This electrodynamic bearing is based on a null flux scheme: no currents are induced until needed, thanks to the homopolar design, see Fig. 5.33. In other words, currents are only generated when the rotor is out-centered, just as the two previous studied cases, the principle is illustrated in Fig. 5.34.

The rotating conductor is a bulk tubular cylinder, the inner and outer magnets are axially magnetized. The magnetic field generated is homopolar and axi-symmetric.

Let us look at the radial magnetic induction seen by one point on the conductor when out-centered and rotating of 360° . This is

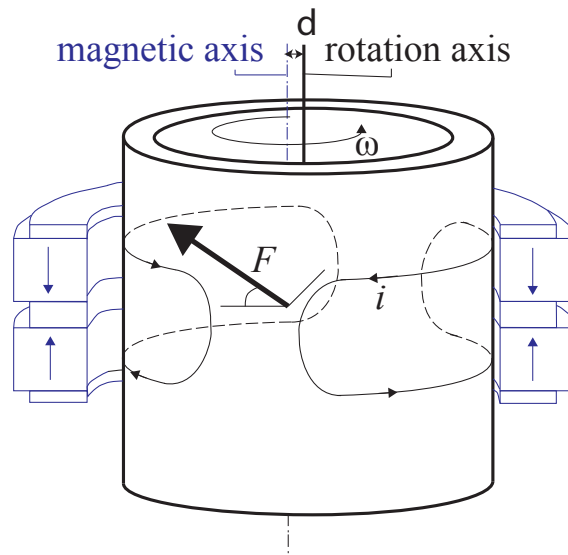


Figure 5.34: Principle of the electrodynamic bearing: resulting force between induced currents and magnetic field

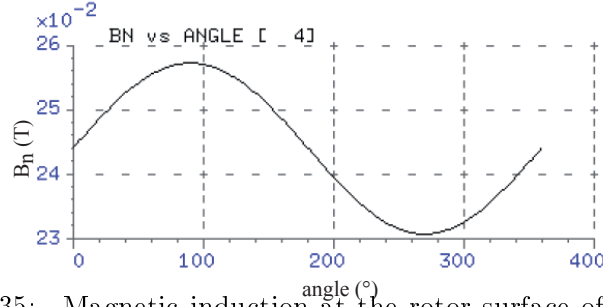


Figure 5.35: Magnetic induction at the rotor surface of the electrodynamic bearing in [Lembke, 2005]

illustrated in Fig. 5.35. It can be seen that it varies as a sinus like it was supposed in the assumptions.

For a bearing whose dimensions are recapitulated in Table 5.7, and an eccentricity of $50 \mu m$, that is 10% of the air gap, the forces evolution with the rotating speed is illustrated in Fig. 5.36. These forces were obtained by FEM simulations in [Lembke, 2005].

5.4.2 Electromechanical model validation

The FEM developed in [Lembke, 2005] gives the tangential and the perpendicular forces value for different spin speed when the rotor spins in an out-centered position. In this case again, the forces are then worth Eq. 5.4 and 5.5.

For each spin speed, according to section 5.1.4, an identification can be done to determine the value of the global resistance R_r and global reactance ωL_r : Fig. 5.37.

The model chosen to describe the impedance evolution with the spin speed is the weighted model described by eqs. (3.27), (3.28) and (3.29), and the parameters R_{dc} , L_0 , L_{int} , A , α and β have to be identified. The results of this identification, done on the basis of the forces predicted by a FEM, are listed in Table 5.8.

Table 5.7: Geometrical data of the electrodynamic bearing in [Lembke, 2005]

outer magnet rings	outer diameter 38 mm inner diameter 30 mm height 4 mm	$B_r = 1 \text{ T}$
inner magnet rings	outer diameter 24 mm inner diameter 16 mm height 4 mm	$B_r = 1 \text{ T}$
outer intermediate washer	outer diameter 38 mm inner diameter 30 mm height 2 mm	
outer end plates	outer diameter 38 mm inner diameter 30 mm height 1 mm	
inner intermediate washer	outer diameter 24 mm inner diameter 16 mm height 2 mm	
inner end plates	outer diameter 24 mm inner diameter 16 mm height 1 mm	
Rotating Copper cylinder	outer diameter 29 mm inner diameter 25 mm height 9 mm	$\sigma = 60 \cdot 10^6 (\Omega m)^{-1}$

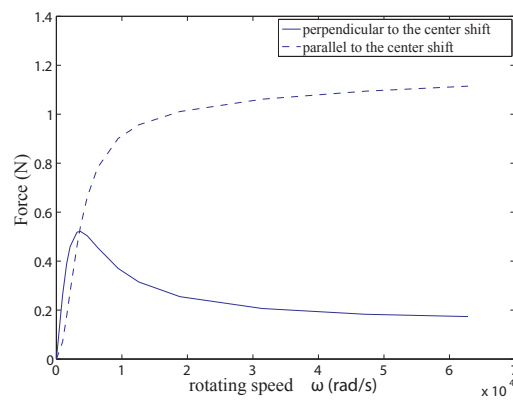
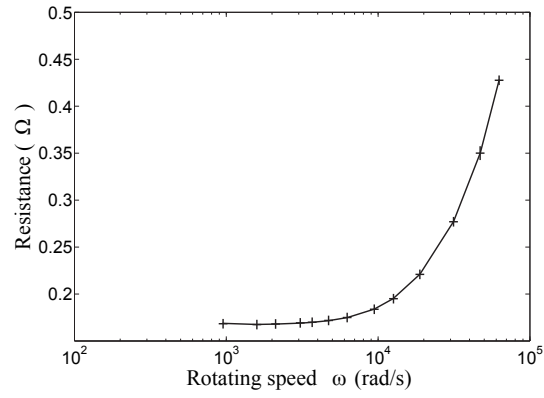
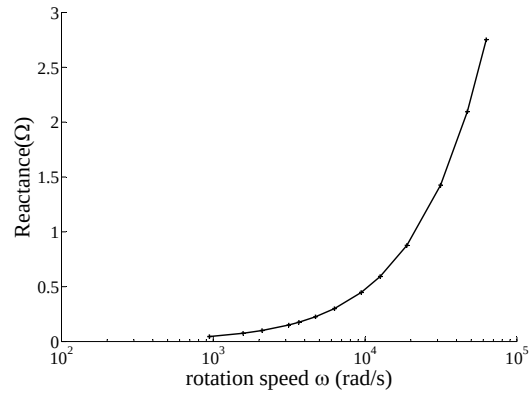


Figure 5.36: Magnetic bearing force evolution with the spin speed, obtained by FEM



(a)



(b)

Figure 5.37: Evolution of the global resistance R_r and the global reactance ωL_r with the spin speed

Table 5.8: Value of the parameters of the electromechanical model for an electrodynamic homopolar magnetic bearing with radial flux

Parameters with skin effect	
c_m/d	1
R_{dc}	0,16 Ω
A	1,27 10^{-3}
L_0	3,89 10^{-5} H
L_{int}	5,58 10^{-6} H
α	5,35 10^{-5}
β	-1,62

Fig. 5.38 shows the evolution of the global resistance and reactance and the evolution of the global resistance and reactance predicted by the skin effect model after identification. It can be concluded that this skin effect model fits the actual evolution of the impedances very well. The result on the forces evolution with the spin speed and the comparison with the FEM calculated forces is shown in Fig. 5.39. The relative residue on the error for the forces is of order 10^{-2} . Again, it can be said that the model predicts the forces evolution very good. In this case, we can clearly see that the evolution of the perpendicular component of the force with the spin speed is to first increase until a maximum value, and then to decrease. And for the parallel component of the force, it continues to increase with the spin speed. The force evolution predicted by the model shows this behavior.

Let us now observe the rotating damping coefficient c_r and the induced stiffness $c_r \omega \sin \theta$ in figs. 5.40 and 5.41. The induced stiffness is much higher than in the first case, but at low rotation speed, it remains small. This means that until a certain rotation speed, the bearing will have to be helped by other bearings to launch the rotor.

In order to have the more accurate model possible for a bearing, the parameter identification has to be carried out carefully. This iden-

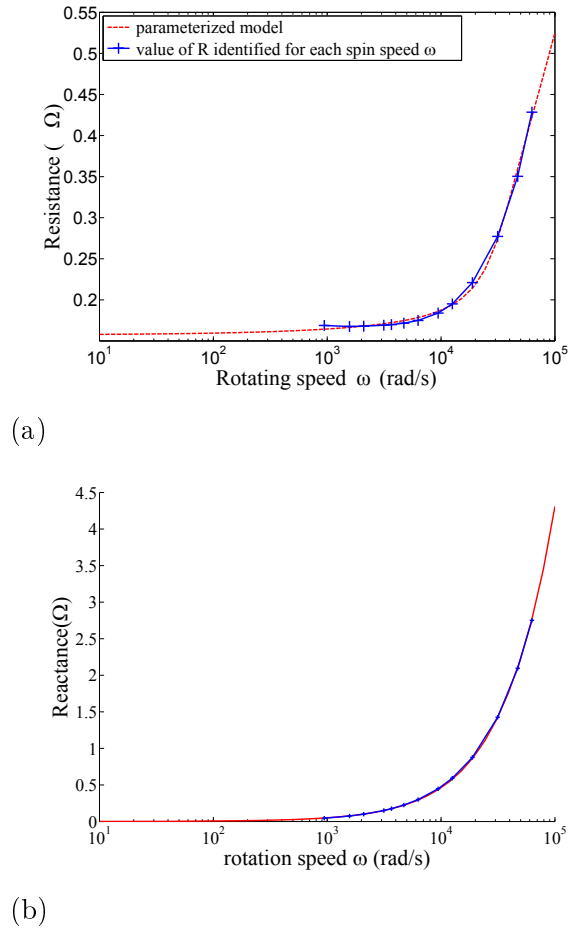
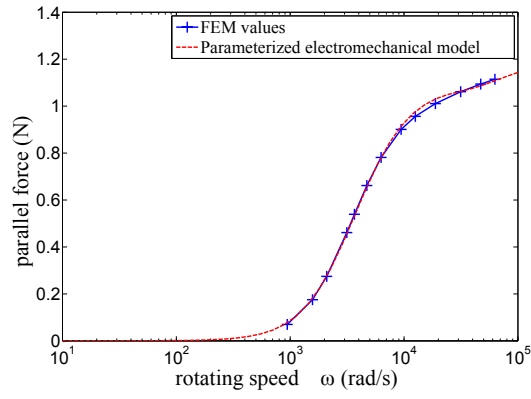
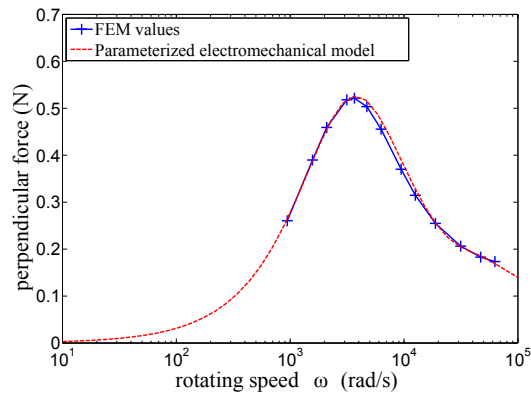


Figure 5.38: Comparison of the global rotor impedance evolution after identification of the parameters



(a)



(b)

Figure 5.39: Comparison of the forces evolution after identification of the parameters for (a) the parallel and (b) the perpendicular component, for a center shift $d=0.05$ mm

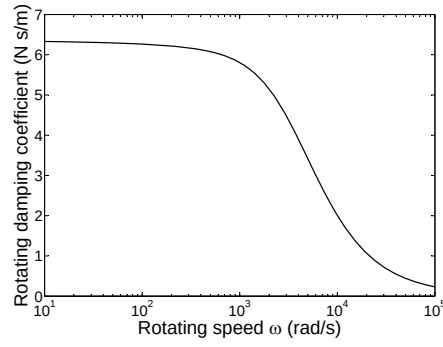


Figure 5.40: Rotating damping coefficient c_r , logarithmic scale for the spin speed

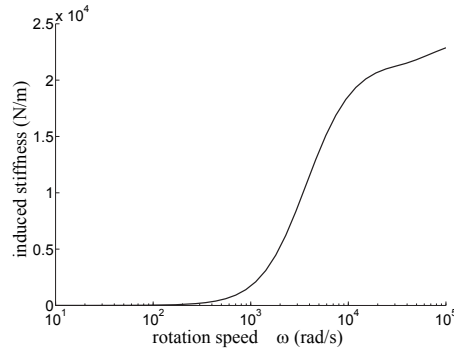


Figure 5.41: Induced stiffness $c_r \omega \sin \theta$, logarithmic scale for the spin speed

Table 5.9: Value of the parameters for an electrodynamic homopolar magnetic bearing with radial flux, identification done on forces for spin speed smaller than 4000 rad/s

Parameters with skin effect	
c_m/d	1
R_{dc}	0,17 Ω
A	7,35 10^{-5}
L_0	4,68 10^{-5} H
L_{int}	1,07 10^{-6} H
α	1,17 10^{-3}
β	-4,25

tification will be more precise when having force measurements for a wide range of spin speeds. However, sometimes it is not possible to have all these forces measurements at our disposal. Therefore, we will now examine the results of an identification on this case, but using only the forces values until 4000 rad/s. We will then compare the results of the forces model extrapolation with the other FEM forces values. The results of this identification on the parameters are gathered in Table 5.9. Fig. 5.42 shows the comparison for both forces components of the model extrapolation and the FEM values. We see that the model prediction is less good than in Fig. 5.39, but it is still correct.

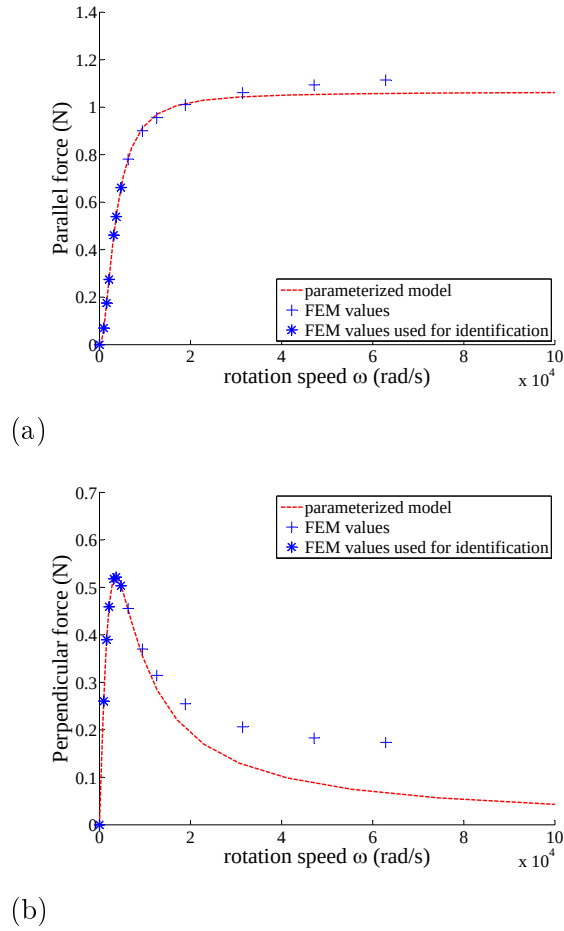


Figure 5.42: Comparison of the forces evolution after identification of the parameters for (a) the parallel and (b) the perpendicular component, for a center shift $d=0.05$ mm

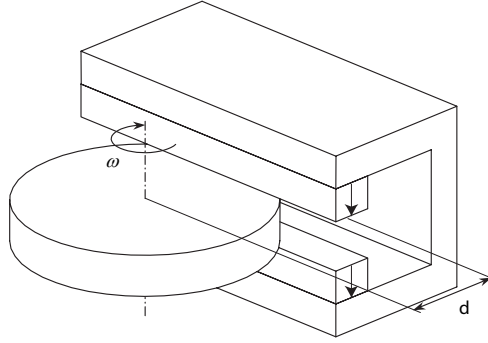


Figure 5.43: Schematic representation of a study case with a non sinusoidal flux variation

5.5 Non sinusoidal case

5.5.1 Geometrical and physical characteristics

The electromechanical model has been confronted to another simple study case where forces due to induced currents in the rotor are generated ([Kluyskens and Dehez, 2006]). However, in this case, the first harmonic approximation is not valid.

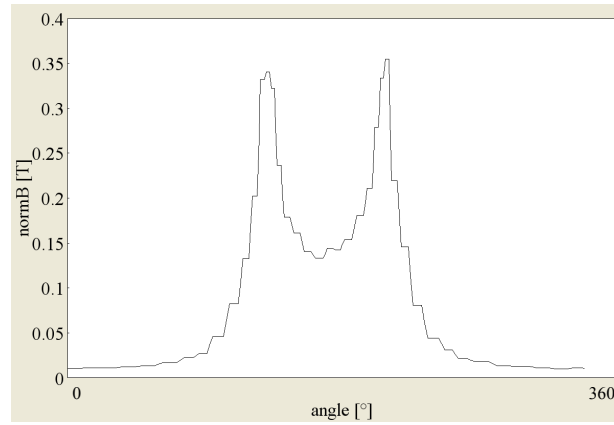
The magnetic field is generated by two permanent magnets in a magnetic circuit, as represented in Fig. 5.43, and the conducting disk is rotating in the middle of the air gap. A point of this conductor sees a varying magnetic induction, but this variation can not be approximated by the fundamental, as seen in Fig. 5.44. This means that in this system, one of the basis hypothesis on which the electromechanical model relies is not respected.

5.5.2 Experimental and finite element results

A FEM model of this case has been developed, more details are given in Appendix A. The resulting induced currents are shown in Fig. 5.45.

Table 5.10: Geometrical data of the non sinusoidal study case

2 permanent magnets	height 5 mm width 10 mm length 40 mm	$B_r = 1.21 \text{ T}$
1 magnetic circuit	outer dimensions: height 40mm width 40 mm length 40 mm thickness 10 mm	steel $\mu_r = 600$
1 rotating disk	diameter 60 mm height 12 mm or height 4 mm	aluminum $\sigma = 3.774 \cdot 10^7 (\Omega m)^{-1}$

Figure 5.44: Variation of the norm of the induction seen by a point situated on the external border of the conducting disk, when rotating of 360°

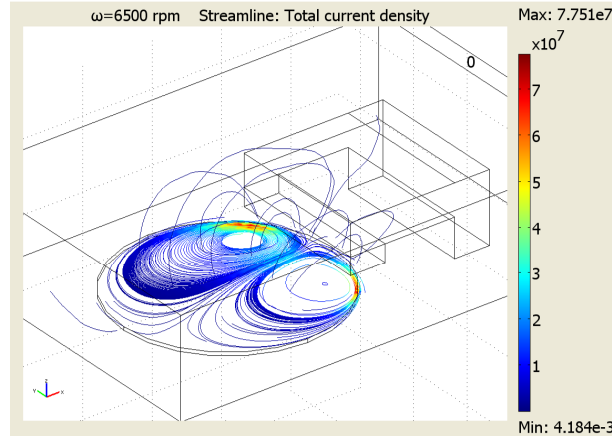


Figure 5.45: Induced currents in the conducting disk

We have completed our study of this case by an experimental validation.

The experimental set up of the system is similar as the setup in case 5.3 (see Fig. 5.46): the aluminum disk is driven by a milling machine allowing rotation speeds up to 660 rad/s. Through the action-reaction principle, we measure the forces by a force/torque sensor attached to the magnetic circuit. The results are shown in Fig. 5.47.

The experimental measurements and the predicted forces by the 3D FE model correspond to each other. The small differences observed are probably due to the heating of the aluminum disk via the induced currents, influencing its resistivity, to measurement uncertainties, and to FE calculation inaccuracies.

About the linearity of the forces with the center shift, this affirmation can not be maintained in this case. When positioning the conducting disk at different center shifts with respect to the permanent magnet, it can be seen in Fig. 5.48 that in this case, the forces are not linear with respect to the distance. This means that when carrying out an identification on the basis of forces for one given center shift, the obtained values of the parameters are only valid for this

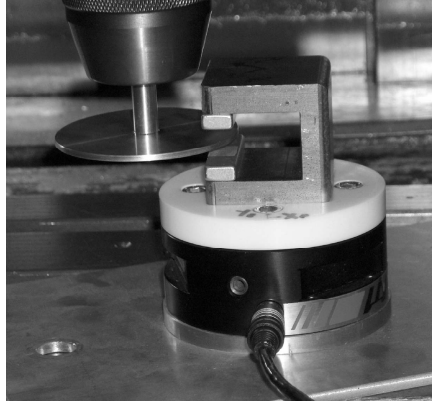


Figure 5.46: Experimental setup to measure the forces on the rotating conducting disk

particular center shift.

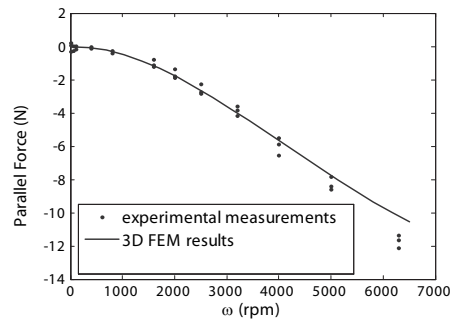
5.5.3 Electromechanical model

The identification of the parameters has been done with the forces for a distance $d = 0.02$ m. This identification has been done in two steps, in order to be able to observe the resistance and the inductance dependency with the rotation speed in this specific case.

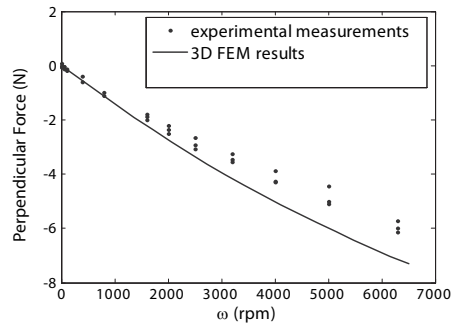
Oddly, the resistance decreases with the rotation speed (Fig. 5.50 (a)), and the reactance starts by increasing faster than the tangent at the origin, and then slower (Fig. 5.50 (b)). This is in opposition with our expectations. As a consequence, our model for the skin effect does not fit this evolution very well for this case.

But the question may be asked if the effect of the frequency is limited to the depth direction, as in previous cases. For instance, the current distribution with the frequency is shown in Fig. 5.51.

In this case, the current paths are free in all the directions and may change. It is more difficult for this to have an appropriate model predicting the variation of the impedance with the frequency. Even if a tendency to keep the same form for the current paths can be noticed in Fig. 5.51, no conclusion can be drawn on the impedance evolution

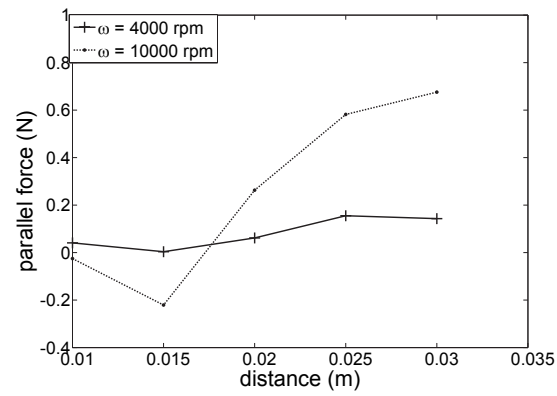


(a)

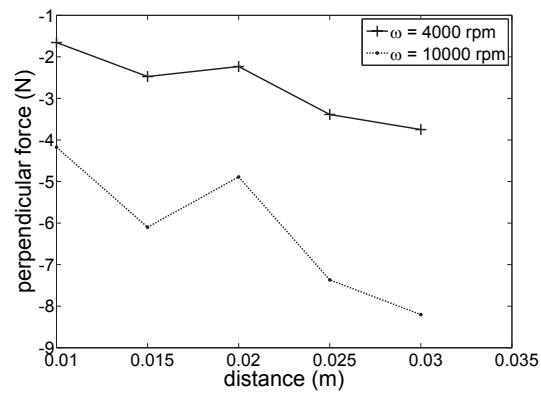


(b)

Figure 5.47: Comparison between the experimental measurements and the FEM calculations of the forces evolution with the spin speed, for a 10mm aluminum disk

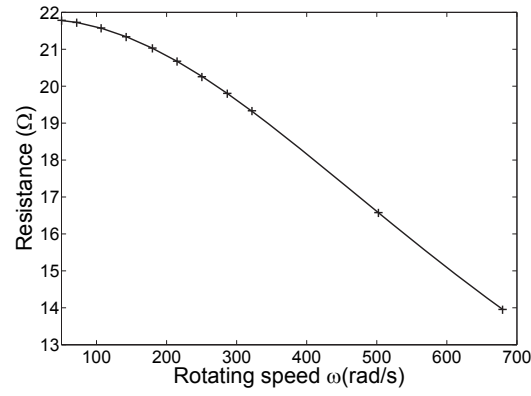


(a)

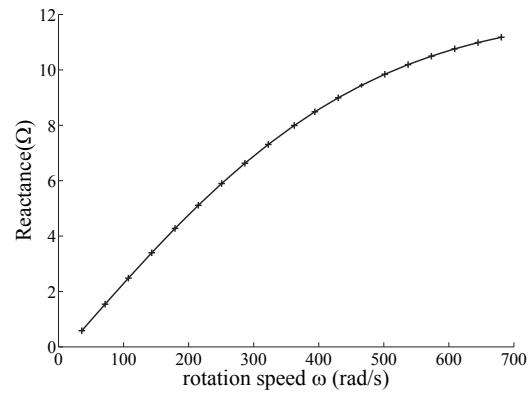


(b)

Figure 5.48: Forces evolution with the center shift, for a 10mm aluminum disk

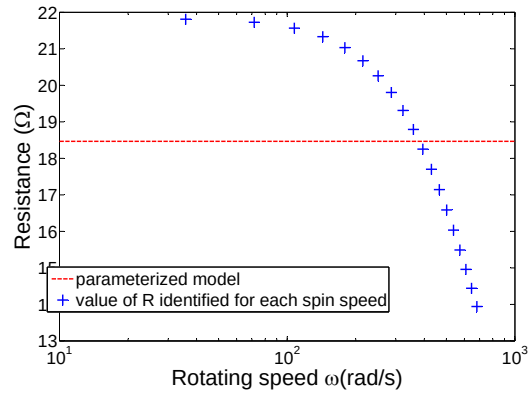


(a)

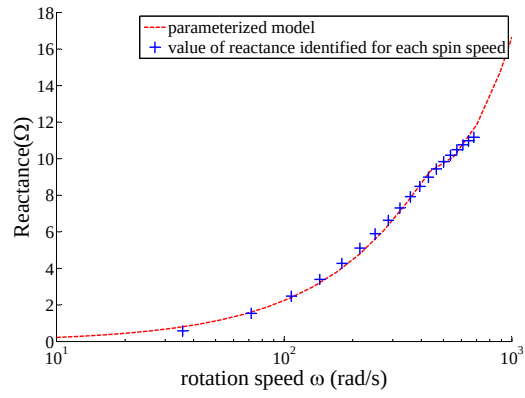


(b)

Figure 5.49: Evolution of the global resistance R_r and the global reactance ωL_r with the spin speed



(a)



(b)

Figure 5.50: Comparison of the global rotor impedance evolution after identification of the parameters

Table 5.11: Value of the parameters of the electromechanical model for a simple case with a non sinusoidal flux variation

c_m/d	1
R_{dc}	18.47Ω
A	$2.22 \cdot 10^{-14}$
L_0	0.016H
L_{int}	0.0064H
α	0.016
β	-8.28

with the frequency regarding this phenomenon.

The predicted force evolution by the electromechanical model is not bad for the spin speed range on which the identification was carried out, as it can be seen in Fig. 5.52. However, it can be seen that for the high spin speed, the model tends to diverge, and it is not sure that the force extrapolation remains correct.

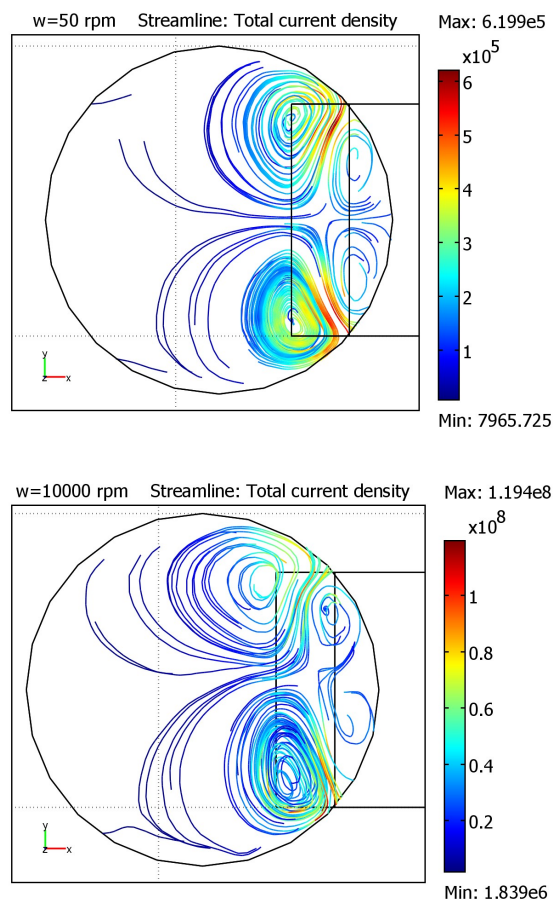
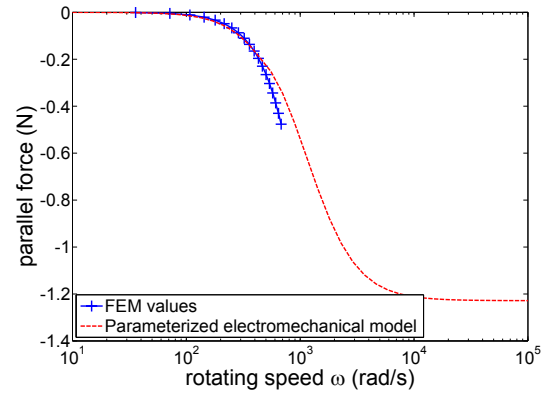
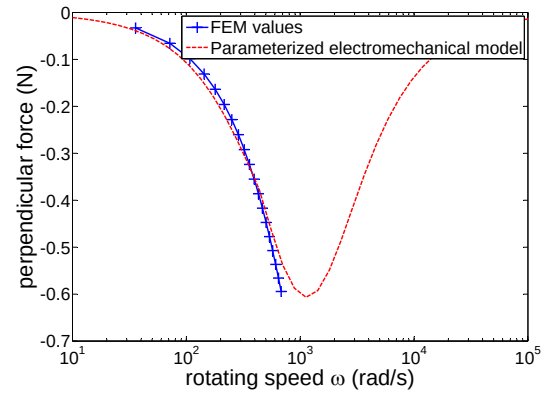


Figure 5.51: Current distribution, solved by FEM, in plain conducting disk excited by a non uniform magnetic field voltage, rotating at 50 rpm versus rotating at 10000 rpm



(a)



(b)

Figure 5.52: Comparison of the forces evolution after identification of the parameters for the parallel and the perpendicular component

5.6 Synthesis

In this chapter we have first proposed a systematic identification method for the electromechanical model parameters. Second, we have validated the parameterized electromechanical model by comparing it to finite elements results, inside the validity domain posed by the model assumptions. The used finite element models have also been validated by experimental measurements.

When not respecting the first harmonic hypothesis, we have seen that the parameterized electromechanical model fits not too bad, but nothing can be said about the forces extrapolation.

We have also seen that the rotor impedance varies with the frequency, and that the two models which gave good results in chapter 3, the continued fraction expansion and the weighted model are appropriate to model this frequency dependency, and the continued fraction expansion models gives better results. We saw that neglecting this skin effect, and considering the rotor impedance constant introduce small errors in the forces prediction and in the limit spin speed prediction. This error grows with the spin speed.

Part III

Exploitation

Chapter 6

Parameter analysis

Thanks to the previously presented electromechanical model, for a system whose parameters have been identified, we can now predict for which spin speeds it behaves stable or not, and how much induced stiffness it will produce.

In this section, we will analyze

- the influence of the model parameters, like the rotor weight m , the non-rotating damping c_{nr} , the external stiffness k , the rotating damping characterized by the global resistance R_r and inductance L_r ,
- and the influence of the physical parameters defining the magnetic bearing like the dimensions, the material,

on the stability and on the induced stiffness.

6.1 Model parameters

6.1.1 Global analysis

It might be instructive to look at the evolution of the limit spin speeds and of the induced stiffness when the different parameters characterizing the system change.

To do so, we will focus on the simple case with axial flux, for a 2 mm

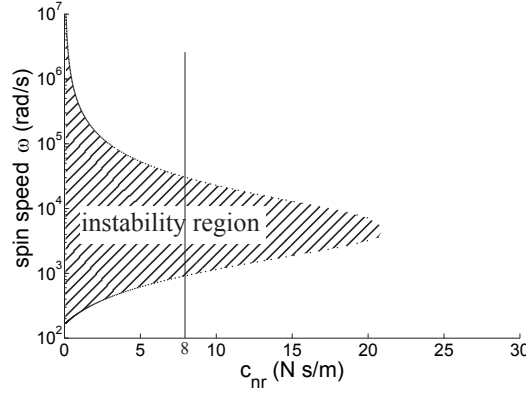


Figure 6.1: Evolution of the limit spin speed with the non rotating damping coefficient

aluminum conducting disk, presented in the validation part, in section 5.3.

The different rotating damping parameters of this simple case are summarized in Table 5.6. The non rotating damping c_{nr} is worth 8 Ns/m. The mass of the conducting disk (thickness of 2mm), augmented by the magnetic coupling attached to it is worth 0.2108 kg. The magnetic coupling and the non rotating damper have a radial stiffness worth 5600 N/m.

Non rotating damping

When the non rotating damping increases, the range of spin speed where the rotor is unstable decreases. This is illustrated in Fig. 6.1. For instance, when c_{nr} is worth 8 Ns/m (Fig. 6.1), we can see that we cross the stability limit at a spin speed ω worth 920 rad/s and we go back to stability for a spin speed worth 29800 rad/s. For a higher value of non-rotating damping coefficient, $c_{nr} = 20$ Ns/m for instance, the unstable speed range is situated between 3184 rad/s and 7197 rad/s.

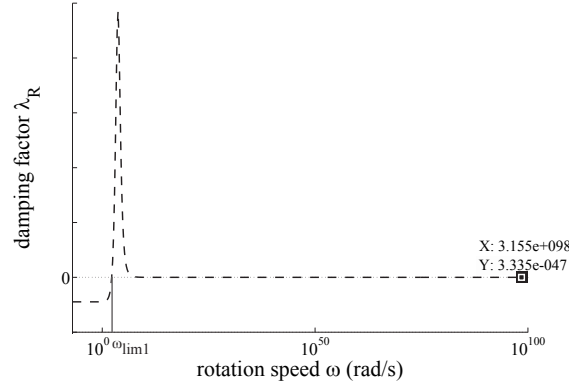


Figure 6.2: Evolution of the damping factor λ_R of the forward whirl with the spin speed for a zero non rotating damping coefficient

Beyond a certain amount of non rotating damping in the system, the behavior is never unstable.

On the other extreme, when the non-rotating damping is zero, there is only one stability threshold, and once this limit is passed through, the system is always unstable. This is confirmed by looking at Fig. 6.2 illustrating the value of the damping coefficient associated to the whirl motion λ_R as a function of ω . When λ_R is negative, the whirl motion is damped, but when λ_R is positive, the whirl motion is amplified. We can see that once the first limit spin speed has been reached, the damping ratio λ_R remains always positive.

In practice, introducing non rotating damping in the system is not easy to realize. This damping has to be provided without contact between the stator and the rotor. One practical solution consist in inducing currents when the rotor whirls, but not when it spins, by mounting an homopolar permanent magnet on the rotor in front of a conductive part on the stator.

Introducing non-rotating damping may also be beneficial for the

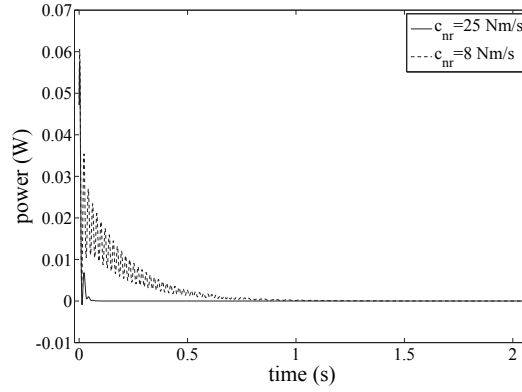


Figure 6.3: Evolution of the power consumption of a perfectly balanced system, rotating at spin speed $\omega = 800$ rad/s, when initially out-centered

transient power consumption, see Fig. 6.3. For a higher non-rotating damping coefficient, the motion of a perfectly balanced and initially out-centered rotor is damped faster, which means that the induced currents are present for a shorter time in the conductor, where they dissipate in the resistance.

However, when the system is unbalanced, it is permanently excited, and it will continually dissipate, as shown in Fig. 6.4. We observe in this figure that when the non-rotating damping coefficient is higher (Fig. 6.4 (b)), the amplitude of the power dissipation is also higher.

At the same time, the non-rotating damping has no influence on the induced stiffness value.

This decoupling may be positive because it means that we can act on the stability thanks to this parameter without influencing the induced stiffness.

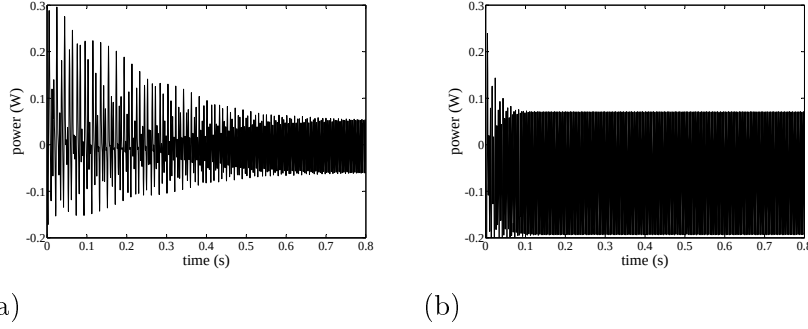


Figure 6.4: Evolution of the power consumption of an unbalanced system, rotating at spin speed $\omega = 800$ rad/s for a non-rotating damping coefficient of (a) 8 Ns/m and (b) 25 Ns/m

Mass

The instability speed range as a function of the system mass m is represented in Fig. 6.5. We can see that the smaller the mass is, the smaller the unstable speed range is. However, the conductor mass has to have a minimal value.

The system mass has no influence on the induced stiffness value.

Stiffness

Regarding the stiffness k of the system, when this latter increases, the critical speed of the system increases accordingly, and the spin speed range where the rotor is unstable decreases. This is shown in Fig. 6.6, where we see that how bigger this stiffness is, how smaller the instability speed range is. Beyond a certain amount of stiffness in the system, the behavior is never unstable.

When the wish is to design a purely electrodynamic magnetic bearing, the stiffness, named induced stiffness, is produced by the rotating damping, and there is no other stiffness. This particular case is discussed in the next section 6.1.2.

However, electrodynamic bearings are usually used in addition to a

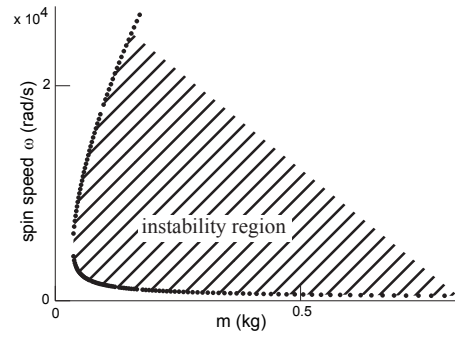


Figure 6.5: Evolution of the limit spin speed with the rotor weight

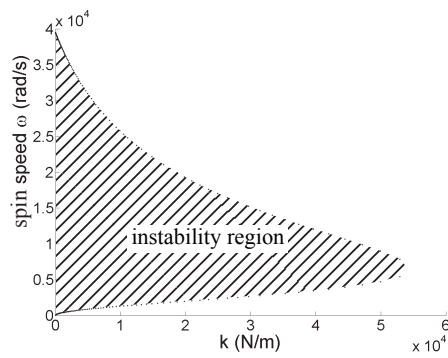


Figure 6.6: Evolution of the limit spin speed with the external stiffness

passive magnetic bearing with permanent magnets, which then creates an external radial and axial stiffness, one being negative. When the radial stiffness is negative, this has of course a negative impact on stability. When plotting the limit spin speed as a function of radial stiffness, for negative radial stiffnesses, the instability speed range increases. The upper limit spin speed, which allows a come back to stability, grows fast.

Besides, the external stiffness has no influence on the induced stiffness value.

Rotating damping

To complete this study, let us make some considerations on the link between instability region and the rotating damping, which is characterized by the global inductance and resistance. Only the global inductance and resistance will be studied here, not the parameters of the model describing the skin effect on the impedance. Indeed, the skin effect can be modeled by different laws as seen previously and results on skin effect model parameters would be more difficult to interpret, without bringing much more information.

Inductance In Fig. 6.7, it is clear that when the inductance is multiplied by a constant factor, the instability region shrinks. This may be said in a more general manner: the inductive behavior has a positive impact on the stability of the system. Indeed, when increasing the inductance of the conductor, the phase shift and thus the orientation of the forces θ increases towards $\pi/2$.

This observed behavior of the limit spin speed is confirmed by analyzing the equations of the limit spin speed eqs. 2.28 and 2.29.

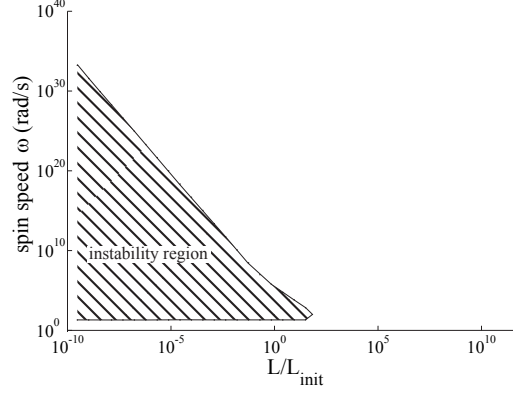


Figure 6.7: Comparison between the instability region when the rotor inductance L_r increases

The limit spin speed can be written:

$$\omega_{lim,1,2} = \left(1 + \frac{c_{nr} Z_I^2}{c_m^2} \frac{(\omega_{lim} L_r)^2 + R_r^2}{R_r}\right) \left(\frac{c_{nr}}{2m} \frac{\omega_{lim} L_r}{R_r} \pm \sqrt{\left(\frac{c_{nr}}{2m}\right)^2 \left(\frac{\omega_{lim} L_r}{R_r}\right)^2 + \frac{k}{m}}\right), \quad (6.1)$$

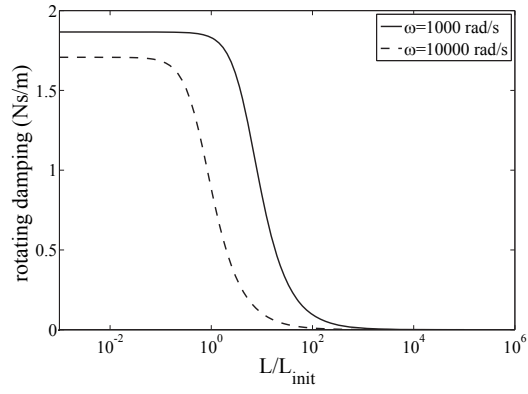
For L_r high enough, one limit spin speed tends to infinity, and the other one tends to zero: the system becomes unconditionally stable.

Regarding the induced stiffness $c_r \omega \sin \theta$, it can be written:

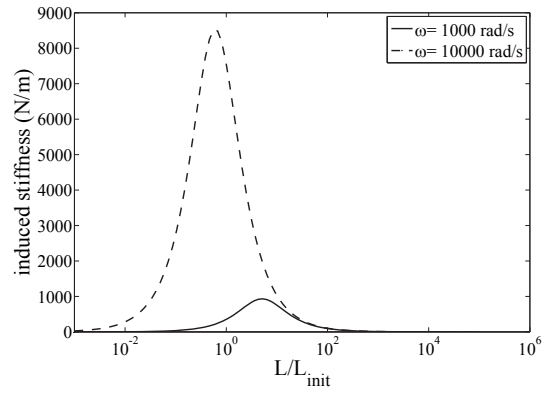
$$c_r \omega \sin \theta = \frac{c_m^2}{Z_I^2 \sqrt{R_r^2 + (\omega L_r)^2}} \omega \frac{\omega L_r}{\sqrt{R_r^2 + (\omega L_r)^2}}. \quad (6.2)$$

Eq. 6.2 reaches a maximum when $R_r = \omega L_r$, when the resistance and reactance are equal. The inductance L_r can then be chosen to reach the maximal induced stiffness at the nominal spin speed ω in function of the resistance R_r .

The induced stiffness is maximum when the phase shift of the induced currents θ is worth $\pi/4$. Indeed, a first effect predominates when the reactance is smaller than the resistance, the induced stiffness increases



(a)



(b)

Figure 6.8: Comparison when the rotor inductance increases of (a) the rotating damping and (b) the induced stiffness

with the spin speed because the reactive part of the impedance becomes more important which is favorable to the induced stiffness. But when the reactance is higher than the resistance, this effect is counteracted by the fact that the norm of the induced currents decreases with the spin speed, which is unfavorable to the induced stiffness. When $\omega L_r \gg R_r$, in eq. 6.2, the induced stiffness decreases with the spin speed ω : The impact of the inductance on the induced stiffness of the system, is shown in Fig. 6.8 (b)).

Resistance On the other hand, in Fig. 6.9, it is clear that when the resistance increases the instability region grows (logarithmic scale for the y-axis representing ω in Fig. 6.9). This may be said in a more general manner: the resistive behavior has a negative impact on the stability of the system.

The increase of the resistance also has a negative impact on the induced stiffness, see Fig. 6.10(b). The expression for the induced stiffness

$$c_r \omega \sin \theta = \frac{c_m^2 \omega^2 L_r}{Z_I^2 (R_r^2 + (\omega L_r)^2)}$$

means that for a given spin speed, when the resistance increases, the induced stiffness decreases. The resistance has a double effect on the induced stiffness: when it increases, the resistive part of the impedance becomes more important and the phase shift θ approaches zero, and the norm of the induced currents decreases when the impedance increases.

6.1.2 Particular case

Finally, let us look at the stability conditions described by the Eq. 2.28 and 2.29 when there is no external stiffness ($k = 0$). This would be the case for a purely electro-dynamical bearing. This means that there are only forces due to induced currents.

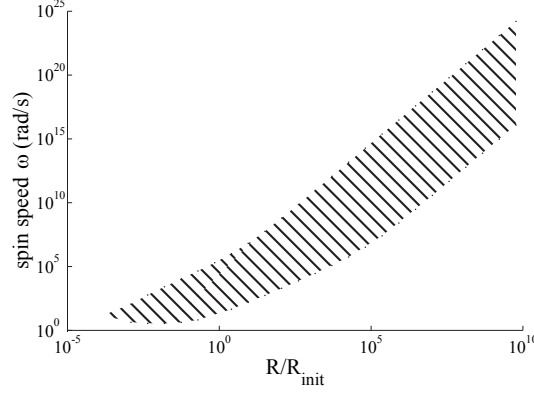


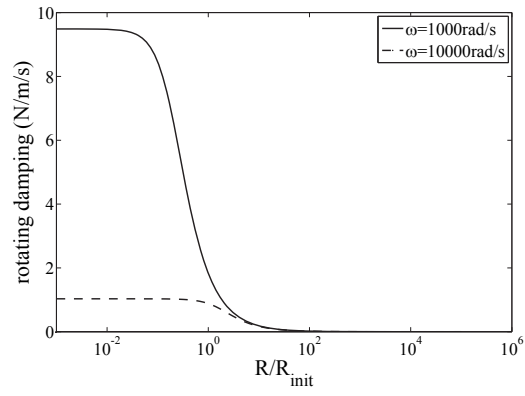
Figure 6.9: Comparison between the instability region when the rotor resistance R_r increases

These equations become:

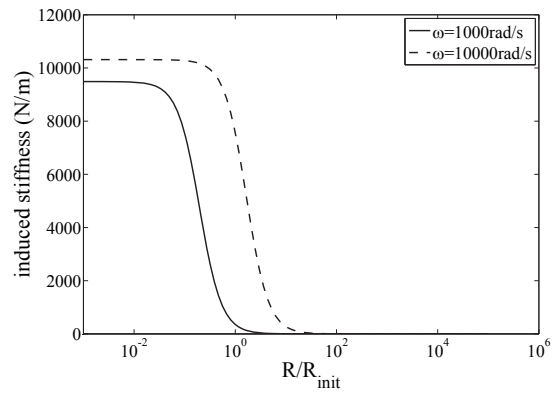
$$\begin{aligned}
 \omega_{lim,1} &= 0 \\
 \omega_{lim,2} &= \left(1 + \frac{c_{nr}}{c_r \cos \theta}\right) \frac{c_{nr}}{m} \tan \theta \\
 &= \left(1 + \frac{c_{nr} Z_I^2}{c_m^2} \frac{(\omega L_r)^2 + R_r^2}{R_r}\right) \frac{c_{nr}}{m} \frac{\omega L_r}{R_r}, \quad (6.3)
 \end{aligned}$$

Non rotating damping

When there is no non rotating damping in this case, the limit spin speed is zero and the system is always unstable. When the non-rotating damping coefficient grows, we can see that eq. 6.3 is a quadratic equation in c_{nr} , which then gives two roots delimitating an unstable speed range, as illustrated in Fig. 6.11. However, the first limit spin speed is so small that in practice, only the second threshold from instability to stability will be noticed.



(a)



(b)

Figure 6.10: Comparison when the rotor resistance increases of (a) the rotating damping and (b) the induced stiffness

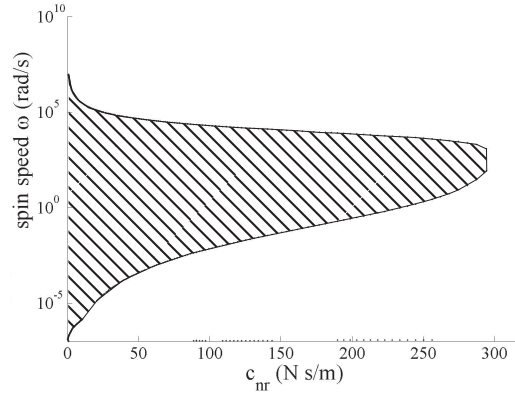


Figure 6.11: Evolution of the limit spin speed with the non rotating damping coefficient, for a system with no external stiffness

Mass

When the mass of the system tends to zero, the limit spin speed tends to infinity, and the system remains always stable. But when the mass grows, the limit spin speed decreases, and the instability region grows, see Fig. 6.12.

Rotating damping

Inductance The limit spin speed (eq. 6.3) shows that when the inductance becomes high enough, the system becomes unconditionally stable, as illustrated in Fig. 6.13.

Resistance In this case, the limit spin speed described by eq. (6.3) becomes, for R_r tending to infinity,

$$\omega_{lim}(R_r \rightarrow \infty) \rightarrow \frac{c_{nr}\omega L_r Z_I^2}{mc_m^2}.$$

When the inductance is constant, the limit spin speed tends to be proportional to ω . This is illustrated in Fig. 6.14

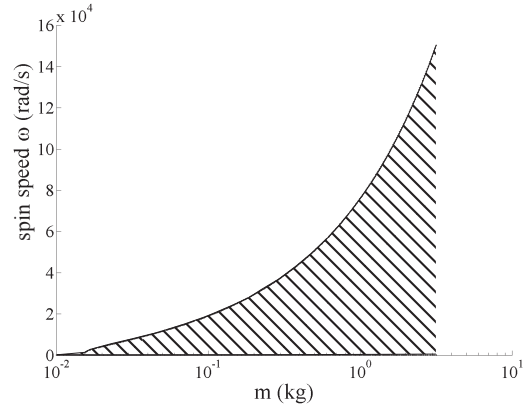


Figure 6.12: Evolution of the limit spin speed with the rotor weight, when $k = 0$

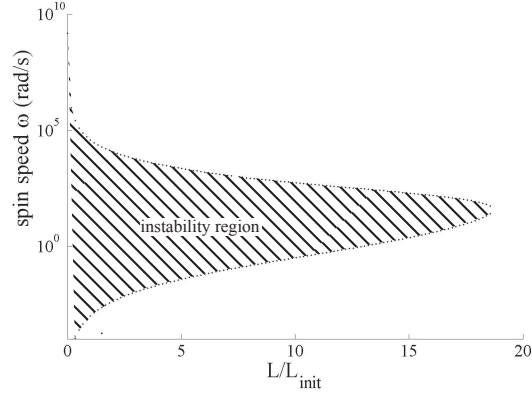


Figure 6.13: Evolution of the limit spin speed with the inductance L_r , when $k = 0$

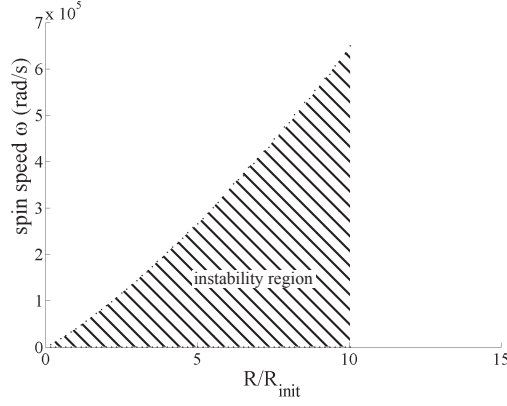


Figure 6.14: Evolution of the limit spin speed with the resistance R_r , when $k = 0$

6.2 Physical parameters

In the previous section, we have analyzed the impact of each of the model parameters on the system stability, and on the induced stiffness when they impact this latter. But when working on a magnetic bearing, to modify these parameter, we will work on physical parameters of the system, like its material, its topology. When changing the material for instance, this will affect various model parameters simultaneously, like the mass, the resistance and the inductance.

So in this section, we will analyze the effect of two changes: the material, and the scale of the whole system.

6.2.1 Material

The simple case with axial flux presented in section 5.3 has been studied for a conducting disk in two different materials, one in aluminum, and one in copper. These two materials are considered as good conductors.

The influence of the material on the forces when the rotor is spinning in an off-centered position is shown on the forces calculated by FEM, and illustrated in Fig. 6.15.

As well for the parallel component of the force as for the perpendicular component of the force, their value is higher for the copper, which is a better conductor.

For each case, considering a skin effect modeled by a weighted model, the parameters of the system have been identified, and their values are summarized in Table 6.1.

About the value of the identified parameters, a first statement can be done concerning the value of the low-frequency resistance parameter: R_{dc} . The ratio between the two parameters R_{dc} , representing the DC resistance of the equivalent circuit in the conductor is equal to the ratio between the resistivity of the two materials:

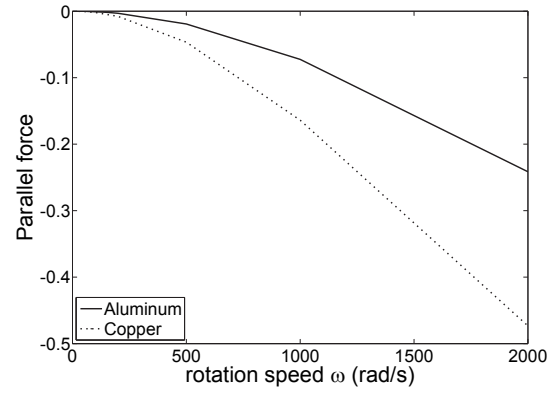
$$\frac{1/\sigma_{alu}}{1/\sigma_{copper}} = \frac{R_{dc,alu}}{R_{dc,copper}} = 1.58$$

About the inductances, we observe that the value of the parameters L_0 and L_{int} vary also, although the permeability has not changed. This means that when the conductivity of the material changes, the currents path change, and so does the inductance. We could have expected that the ratio of A_{cu}/A_{al} was worth $\sqrt{\sigma_{al}/\sigma_{cu}}$, but it is not the case; this is because of the intervention of the weight function in the inductance expression. The skin effect not only appears in the factor A for the high-frequency inductance, but also in the parameters α and β characterizing the transition between the low- and the high-frequency inductance.

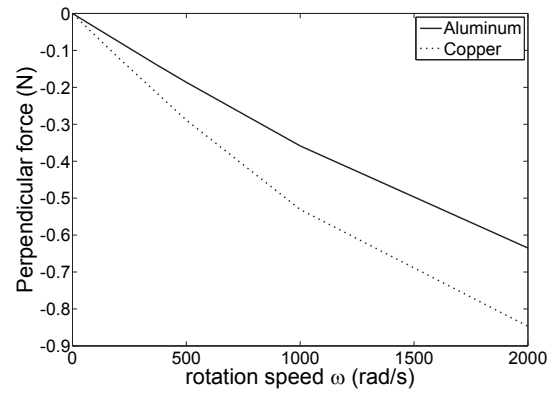
Considering now a skin effect modeled by the continued fraction expansion, the parameters of the system have also been identified. Their values are gathered in Table 6.2.

We can notice that in this case, the value of the inductances are almost the same for both materials. The ratio between the DC resistances is also equal to the ratio between the resistivities:

$$\frac{1/\sigma_{alu}}{1/\sigma_{copper}} = \frac{R_{dc,alu}}{R_{dc,copper}} = 1.58.$$



(a)



(b)

Figure 6.15: (a) Parallel and (b) perpendicular force for a copper and an aluminum conducting disc as a function of the spin speed

Table 6.1: Value of the electromechanical model parameters for a simple case with axial flux, comparison between copper and aluminum

material	aluminum	copper
conductivity	$37.7 \cdot 10^6 \text{ S/m}$	$59.6 \cdot 10^6 \text{ S/m}$
density	2698 kg/m^3	8960 kg/m^3
permeability	$\mu_r = 1$	$\mu_r = 1$
model parameters		
c_m/d	1	1
R_{dc}	0.5286Ω	0.3336Ω
A	$5.807 \cdot 10^{-4}$	$7.1 \cdot 10^{-4}$
L_0	$9.105 \cdot 10^{-5} \text{ H}$	$8.34 \cdot 10^{-5} \text{ H}$
L_{int}	$1.672 \cdot 10^{-5} \text{ H}$	$2.335 \cdot 10^{-5} \text{ H}$
α	$2.074 \cdot 10^{-3}$	$1.758 \cdot 10^{-3}$
β	-2.402	-1.875

The ratio between the parameters $G = \frac{\omega_{ac}L}{2R_{ac}^2}$ is worth 1.4 and moves away from the ratio between the conductivities, even if it is still close. Concerning the inductances, they stay almost equal. The continued fraction expansion model gives us a better insight on its parameters than above analyzed weighted model.

In Fig. 6.16, it can be seen that the induced stiffness obtained with a copper disc is slightly more important than with an aluminum disc; In Fig. 6.17, it can be seen that the value of the rotating damping coefficient obtained with a copper disc is also slightly more important than with an aluminum disc.

We can compare the effects of a change of material on the stability behavior of the magnetic bearing in Fig. 6.18. The copper, which had higher value of forces and induced stiffness, needs a higher non rotating damping coefficient to be unconditionally stable. For the system with a copper disc, the instability region is bigger. Indeed, the copper is a better conductor, but its density is also more important. This means that copper will have a lower resistance, but on another hand weights more. Both these parameters influence stability. In Fig. 6.17, we

Table 6.2: Value of the electromechanical model parameters for a simple case with axial flux, comparison between copper and aluminum

material	aluminum	copper
conductivity	$37.7 \cdot 10^6 \text{ S/m}$	$59.6 \cdot 10^6 \text{ S/m}$
density	2698 kg/m^3	8960 kg/m^3
permeability	$\mu_r = 1$	$\mu_r = 1$
model parameters		
c_m/d	1	1
R_{dc}	0.5297Ω	0.3354Ω
L_0	$7.4712 \cdot 10^{-5}\text{H}$	$7.35 \cdot 10^{-5}\text{H}$
L	$1.0625 \cdot 10^{-5} \text{ H}$	$1.085 \cdot 10^{-5} \text{ H}$
G	26.7073	37.2995

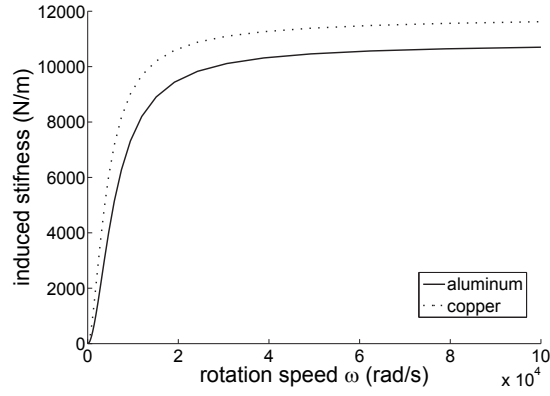


Figure 6.16: Induced stiffness $c_r \omega \sin \theta$ due the induced currents in a copper and an aluminum conducting disc as a function of the spin speed

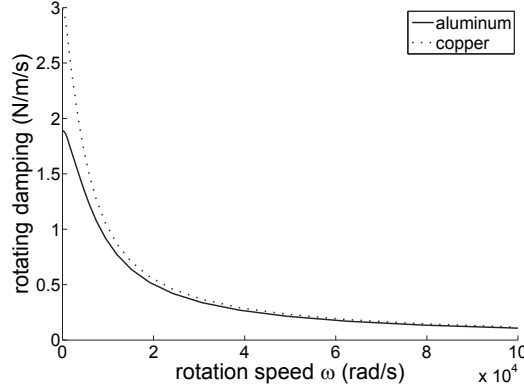


Figure 6.17: Rotating damping coefficient for a copper and an aluminum conducting disc as a function of the spin speed

observe that the copper rotating damping coefficient is higher. This means that for a given excitation, the power dissipated in the copper conducting disc will be higher than for the aluminum conducting disc.

6.2.2 Scale effect

The simple case with radial flux presented in section 5.2 is studied with two sets of dimensions.

The first set is the dimensions presented in section 5.2, and the second set has the same dimensions multiplied by two. The center shift is also multiplied by two, to keep the same relative center shift in the two studied cases: 20% of the air gap.

The resulting forces are compared in Fig. 6.19. Obviously, the forces developed in the larger system are more important.

The comparison of the parameters of the identified electromechan-

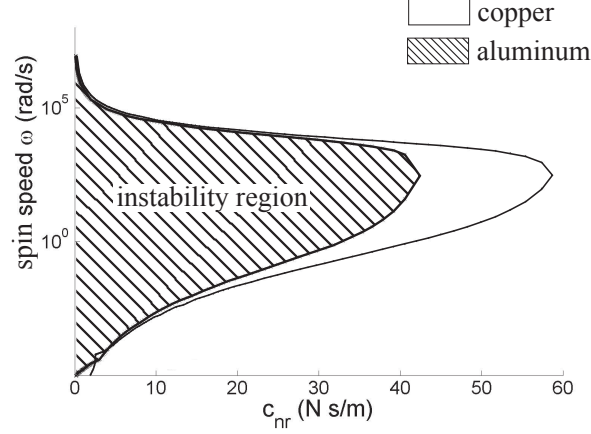


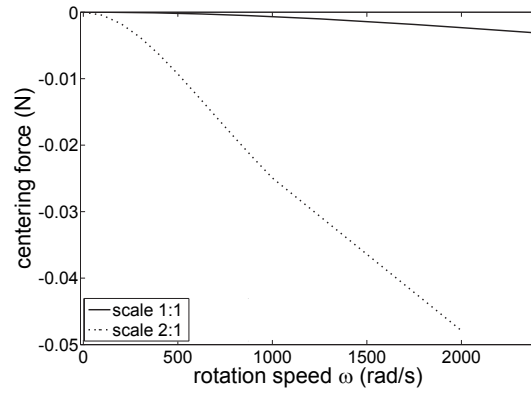
Figure 6.18: Evolution of the limit spin speed with the non rotating damping coefficient

ical model can be done in Table 6.3.

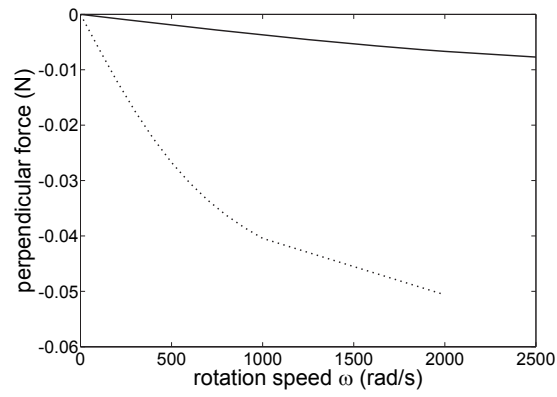
Prior to the analysis of the value of these parameters, a remark needs to be done. The parameters are identified by normalizing the parameter c_m/d . In the first case, this parameter is normalized to one, so to make a valid comparison, this parameter has to be normalized to two in the second case. The proportionality constant c_m represents $d\psi/d\theta$, and the flux in the second case is multiplied by 4, as the surface is 4 times larger and the flux density is constant. The distance d is 2 times bigger, so the ratio c_m/d has to be normalized to 2 in the second case.

Doing so, we can notice that:

- $R_{dc,1}/R_{dc,2} = 2$
- $L_{0,1}/L_{0,2} = 1/2$
- $L_{int,1}/L_{int,2} = 1/2$
- $A_1/A_2 = 1$



(a)



(b)

Figure 6.19: (a) Parallel and (b) perpendicular force of two identical electromechanical systems, but with a ratio 2 for the geometric dimensions

Table 6.3: Value of the parameters of the electromechanical model for a simple case with radial flux, for two sets of geometrical dimensions

scale	1:1	2:1
c_m/d	1	2
R_{dc}	26,03 Ω	12,98 Ω
A	0,0905	0,0892
L_0	2,88 10^{-3} H	5,788 10^{-3} H
L_{int}	1,67 10^{-3} H	33,26 10^{-4} H
α	1,29 10^{-3}	3,77 10^{-3}
β	-5,847	-3,66

The ratio between the resistances is worth 2, this is the exact scale factor between the two cases. This is coherent with the general expression of a resistance in a wire $R = \rho l/S$. When multiplying all dimensions by two, the length is multiplied by two and the surface by four. The ratio between the resistance of the smaller geometry and the bigger geometry is two.

The ratio between the inductances is worth 1/2. This is also coherent with the general expression of an inductance in a solenoïd $L = \mu N^2 S/l$. When multiplying all dimensions by two, the ratio between the inductance of the smaller geometry and the bigger geometry is one half.

The parameters A , representing the impedance dependence with the frequency at high frequency, are almost equal. At high frequency not all the volume of the conductor is used because of the skin effect, and because the material are identical in both cases, they both depend in the same way on the frequency.

When comparing the induced stiffness (Fig. 6.20) for both cases, it can be noticed that the induced stiffness is higher when the dimensions are bigger, which is positive. However, for a ratio two between the dimensions, this is to say a ratio eight between the volume occupied by each bearing, the induced stiffness does not increase accordingly. For the rotating damping coefficient value (Fig. 6.21), it is also more important in the larger system.

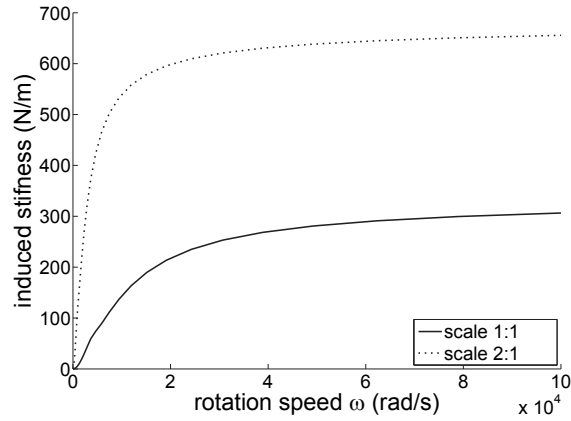


Figure 6.20: Induced stiffness $c_r \omega \sin \theta$ due the induced currents for two identical electromechanical systems, but with a ratio 2 for the geometric dimensions

The effect on the stability may be studied in Fig. 6.22. Of course, the system with bigger dimensions needs a bigger non rotating damping coefficient value to be unconditionally stable. For a given non rotating damping coefficient, the larger system has also a larger unstable speed range.

Constructing a larger electrodynamic magnetic bearing allows higher induced stiffness, but it also means that more non rotating damping coefficient is needed to keep the same useful speed range. Another problem with scaling is the higher risk of mechanical failure due to centrifugal effects, and the larger space needed on the axis.

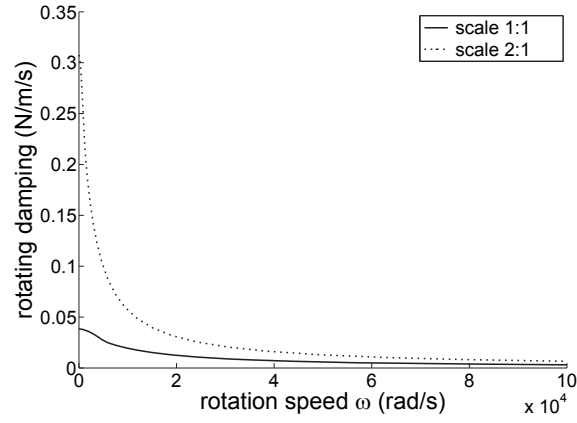


Figure 6.21: Rotating damping coefficient for two identical electromechanical systems, but with a ratio 2 for the geometric dimensions

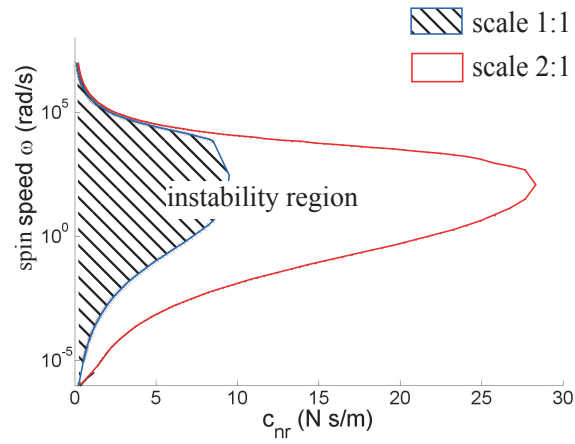


Figure 6.22: Evolution of the limit spin speed with the non rotating damping coefficient

6.3 Synthesis

Previous considerations have shown that when designing electrodynamic bearing attention has to be paid to an optimal inductance/resistance ratio, in function of the working spin speed. A high inductance, is profitable for the stability, but it may be prejudicial for the induced stiffness, if the reactance is higher than the resistance. But a high resistance is not only disadvantageous for the stability, but also for the induced stiffness. This ratio inductance/resistance can be realized by an appropriate choice of material, by inducing the currents not in a bulk conductor, but in coils, or even by adding an external inductance to these coils. But this last proposition is not easy to realize when the coils are on the rotor.

Another way to increase the induced stiffness is to increase the size of the bearing, but keeping in mind that this influences the stability of the system, which means that more non rotating damping coefficient is needed to keep the same useful speed range. Furthermore, the rotor has to mechanically resist to the maximum spin speed.

Introducing non-rotating damping into the system is a solution to have acceptable unstable spin speeds without damaging the induced stiffness. But realizing non rotating damping without contact between the rotor and the stator is not easy: a way to realize this is to induce currents when the rotor whirls, by inducing currents in a statoric conductor with an homopolar permanent magnet on the rotor for instance. This means mounting permanent magnets on the rotor, which spins very fast!

Another solution could be to act externally on the stator which is not spinning, by placing a damper-spring between the stator and the ground for instance. This solution will be examined in the next chapter (section 7.2).

We have also seen that the stiffness k is beneficial for the stability. But inside a magnetic suspension, this stiffness is often realized by a passive magnetic bearing, which means that it will also introduce a negative stiffness, which is then harmful.

Chapter 7

Rotor dynamic simulations

Previous developed parameterized electromechanical model considered that the rotor was a simple Jeffcott rotor with a constant spin speed, which is an oversimplification of the real world. In reality, the center of mass will not coincide with geometric center, the rotor will not be a point mass, but will have inertia, when rotating at very high spin speed, the flexural modes of the rotor might have to be taken into account, and so on.

The Jeffcott rotor has allowed us to focus on the typical phenomena encountered when working with forces generated by induced currents. And even when in presence of more complex systems, the electromechanical model still gives the form of the forces developed by the induced currents on the bearing. This forces can then be introduced into the mechanical equation governing the system, according to the hypothesis that are then considered: point mass rotor or not, perfectly balanced or not, small displacements or not,...

Two examples of more complex systems are shown below: first the rotor is still considered as a point mass, but with a static unbalance, and second we consider a system where damping and stiffness are introduced between the stator and the ground.

7.1 Unbalance on the rotor

In previous sections, the equations of motions were written supposing that the rotor was a simple point mass perfectly balanced. The Jeffcott rotor is an oversimplification of the real world: in the real world, the center of gravity of mass will never exactly coincide with the geometric center of the cross section of the shaft. The presence of this eccentricity ϵ will cause a static unbalance $m\epsilon$. Supposing that the spin speed ω is constant, this unbalance generates a synchronous excitation $m\epsilon\omega^2 \exp(j\omega t)$, that can be introduced in eq. 2.30:

$$m\ddot{\bar{z}} + (c_{nr} + c_r \cos \theta + jc_r \sin \theta)\dot{\bar{z}} + (k + c_r\omega \sin \theta - jc_r\omega \cos \theta)\bar{z} = m\epsilon(\omega^2 - j\dot{\omega}) \exp j\omega t. \quad (7.1)$$

Then the motion of the geometric center will consist on the superimposition of the free motion, as examined previously, solution of the homogeneous differential equation 2.30, and a circular motion with an angular speed ω , particular solution of the differential equation. This means that the stability of the unbalanced system depends on the stability of the solution for free whirling. When this latter leads to a stable behavior, the unbalanced system is stable, but when it leads to an unstable behavior, this behavior will occur even if perfectly centered and without exterior disturbance because the unbalance will create the excitation.

Let us examine the same case as the simple case with axial flux, with $c_{nr} = 8 \text{ Ns/m}$, but with its center of mass situated at a distance of 0.1 mm of the geometric center of the rotor. As stated in previous section, the stable speed range is situated before 920 rad/s and after 29800 rad/s. This does not change, but the unbalance will give an excitation to the rotor, even when initially centered. In figure 7.1, we observe that the rotor motion is the superimposition of motion at spin frequency ω , which is the particular solution to the differential equation of motion, and at whirl frequency λ_I , which is the solution to the homogeneous differential equation of motion.

Furthermore, we observe that when spinning at spin speed leading to a stable behavior, in Fig. 7.1 (a) and (d), the equilibrium is situated

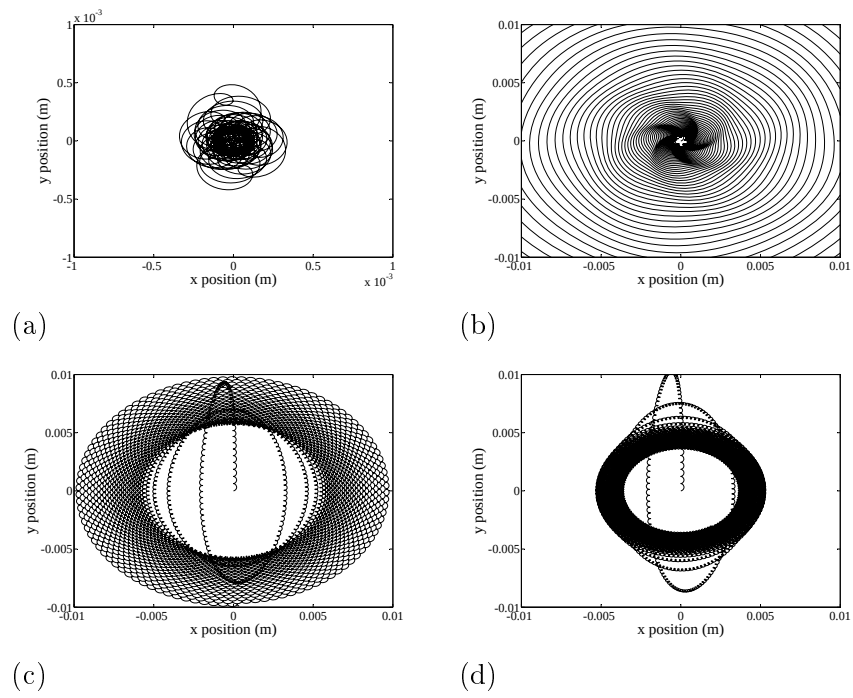


Figure 7.1: Rotor motion in the xy radial plane for a (a) spin speed ω of 800 rad/s: the rotor is stable, (b) spin speed ω of 1000 rad/s: the rotor is unstable, (c) spin speed ω of 28000 rad/s: the rotor is unstable, (d) spin speed ω of 31000 rad/s: the rotor is stable

around a radius of 0.1 mm, which is equal to the unbalance ϵ . This is because when rotating in supercritical conditions, the rotor tends to selfcenter: it rotates around its center of mass instead of around its geometric center.

7.2 Damped stator

The dynamic model also allows to simulate the behavior of an electrodynamic bearing included in a more complex system. For instance, the previously solution to stabilize the system was to introduce non-rotating damping into the system. Another solution would be to introduce stiffness and damping between the stator and the ground. This other strategy was already proposed in other works, like in [Dai et al., 2001] and in [Impinna, 2010].

This damping between the stator and the ground can be realized in the form of elastomere rings, or a squeeze-film. The advantage of this system is to be easier to implement than creating directly non-rotating damping into the stator. However, the effects of this damping are not as easy to predict.

In such a system, sketched in Fig. 7.2, the forces acting on the rotor still have the same form as the ones predicted by the parameterized electromechanical model. These forces also act on the stator, according to the action-reaction principle. Furthermore, the stator is submitted to the forces due to damper-spring between the stator and the ground.

The new parameters influencing on the overall system behavior are:

- the stator mass M_{stator} ,
- the spring stiffness K_{stator} ,
- the damping value of the damper C_{stator} .

The equations of motion for the two bodies (stator and rotor)

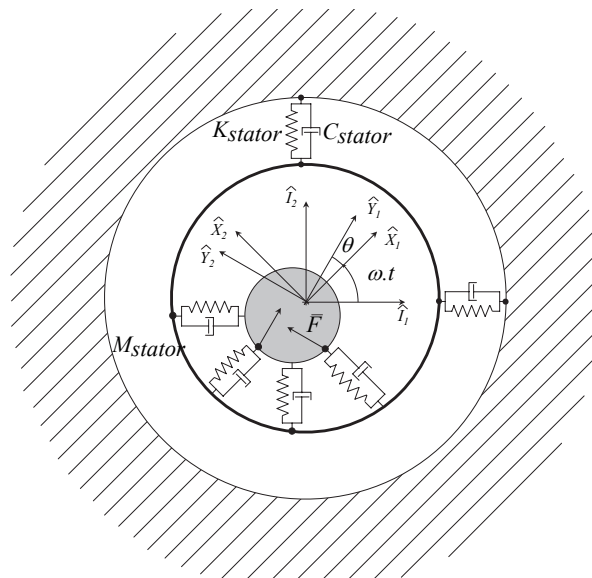


Figure 7.2: Upper schematic view of a system submitted to rotating damping with a damped stator

present in the system are then:

$$\begin{aligned}
 & \begin{pmatrix} m & 0 & m & 0 \\ 0 & m & 0 & m \\ 0 & 0 & M_{stator} & 0 \\ 0 & 0 & 0 & M_{stator} \end{pmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{x}_{stator} \\ \ddot{y}_{stator} \end{pmatrix} + \\
 & \begin{pmatrix} c_{nr} + c_r \cos \theta & -c_r \sin \theta & 0 & 0 \\ c_r \sin \theta & c_{nr} + c_r \cos \theta & 0 & 0 \\ -c_{nr} - c_r \cos \theta & c_r \sin \theta & C_{stator} & 0 \\ -c_r \sin \theta & -c_{nr} - c_r \cos \theta & 0 & C_{stator} \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{x}_{stator} \\ \dot{y}_{stator} \end{pmatrix} + \\
 & \begin{pmatrix} k + c_r \omega \sin \theta & c_r \omega \cos \theta & 0 & 0 \\ -c_r \omega \cos \theta & k + c_r \omega \sin \theta & 0 & 0 \\ -k - c_r \omega \sin \theta & -c_r \omega \cos \theta & K_{stator} & 0 \\ c_r \omega \cos \theta & -k - c_r \omega \sin \theta & 0 & K_{stator} \end{pmatrix} \begin{pmatrix} x \\ y \\ x_{stator} \\ y_{stator} \end{pmatrix} \\
 & = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{7.2}
 \end{aligned}$$

where

$$\tan \theta = \frac{(\omega + \lambda_I)L_r}{R_r}, \tag{7.3}$$

and

$$c_r = \frac{c_m^2}{Z_I^2} \frac{1}{\sqrt{((\omega + \lambda_I)L_r)^2 + R_r^2}}, \tag{7.4}$$

with c_m the proportionality constant, ω the rotor spin speed, and λ_I the rotor whirl speed.

These equations can be written in a state-space form, and the eigen-values of the matrix of the system can be found numerically. There are four complex eigen-values, which relate to the four eigen modes of the system: rotor forward whirl motion, rotor backward whirl motion, stator forward whirl motion, and stator backward whirl motion.

It can then be examined if their real part is positive, which means that the system is unstable or negative, which means that the motion

is damped. The limit spin speed is the value of the spin speed where the real part of the eigen-value crosses the zero.

Let us first examine what happens when the stator stiffness K_{stator} is zero, and the stator mass M_{stator} is fixed to 1 kg. The rotor system is still the same as in section 7.1, but with no eccentricity. For recall, the system was unstable between ω 920 and 29800 rad/s.

The numerically found limit spin speed are illustrated in Fig. 7.3. It can be observed that as well for the first limit spin speed (end of stable behavior), as for the second limit spin speed (return to stable behavior) there is an optimal value for the damper between the stator and the ground. This optimal value is situated around $C_{stator} = 350$ Ns/m, in this case.

When comparing the limit spin speed obtained with a damped stator to the system where the stator is directly linked to the ground, the unstable speed range is smaller, when the damper between the stator and the ground is chosen appropriately.

When the damping value is too high, as well for the first limit spin speed (Fig. 7.4) as for the second limit spin speed, the unstable speed range increases.

When increasing the value of the stator mass, the gain on the width of the unstable speed region decreases, see Fig. 7.5.

When the stator weights 10 kg, it is not so obvious anymore that there is an optimal damping value.

From this example, we can see that stability is influenced by the mass of the stator, by the value of the stiffness and the damping between the stator and the ground. However, the impact of their presence can be positive or negative on the unstable speed range, and a careful choice of their values may result in an improvement on stability.

Thanks to the electromechanical model, we can also simulate the rotor and the stator behavior for different spin speeds, see figs. 7.6, 7.7 and 7.8. In the case shown on these figures, the rotor is perfectly balanced and is initially off-centered, the stator mass is $M_{stator} = 1$ kg, there is no stiffness, $K_{stator} = 0$ N/m, and the damper has a value of $C_{stator} = 400$ Ns/m.

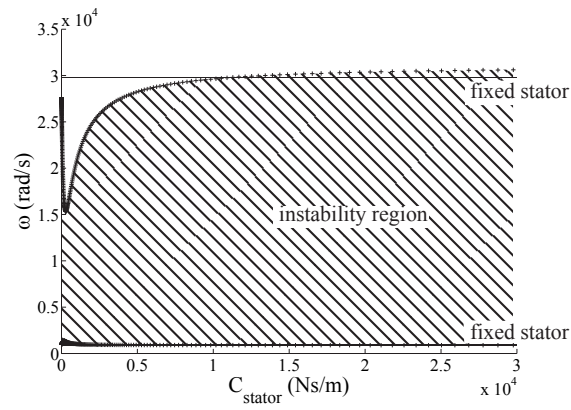


Figure 7.3: Limit spin speed for a system with a damped stator, weighing 1kg, for various values of damping

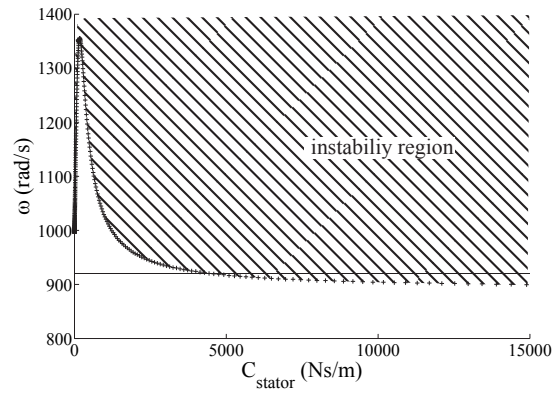


Figure 7.4: Zoom on the first limit spin speed for a system with a damped stator, weighing 1kg, for various values of damping

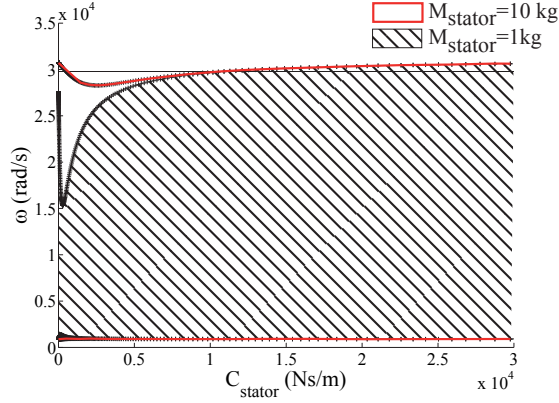


Figure 7.5: Limit spin speed for a system with a damped stator, for various values of damping, the stator weight is 1kg and 10 kg

Concerning the power consumption of this system, we compare it to the power consumption of the same system but with a fixed stator, in the case of an unbalanced rotor spinning at $\omega = 800$ rad/s. For the damped stator, the values are $M_{stator} = 1$ kg, $K_{stator} = 5000$ N/m, and the damper has a value of $C_{stator} = 400$ Ns/m. We notice that the power consumption is higher in transient state but lower in steady state, as shown in Fig. 7.9.

7.3 Synthesis

This chapter has shown that the parameterized electromechanical model developed in this work can be integrated in the equations of motion of more complex system, in order to simulate or to predict the dynamical behavior of this system.

Here, we applied this on a simple Jeffcott rotor with a static unbalance. This showed that the static unbalance excites the system, and

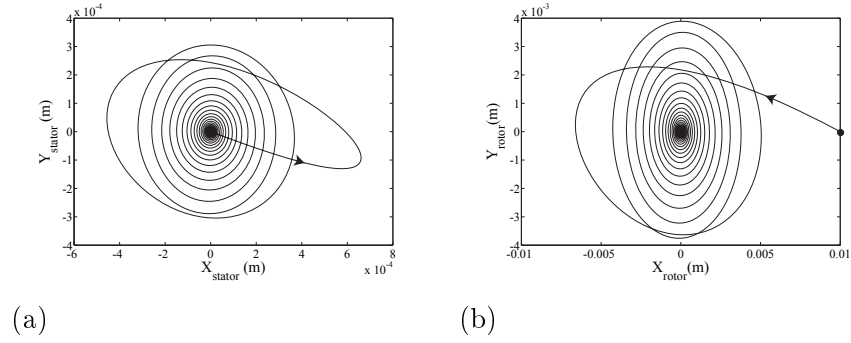


Figure 7.6: (a) Stator and (b) Rotor motion in the xy radial plane for a spin speed ω of 1000 rad/s: the rotor is stable

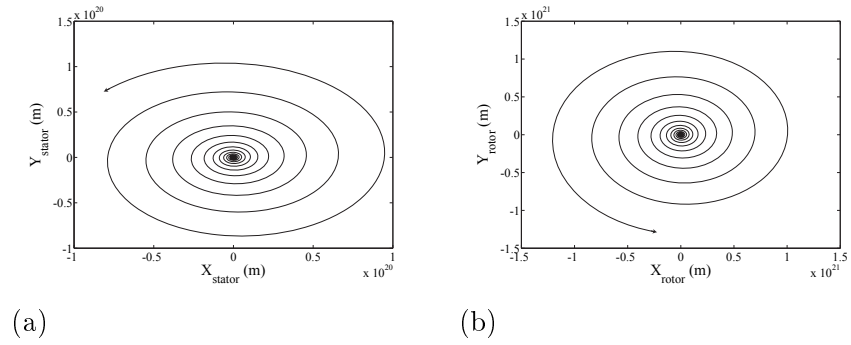


Figure 7.7: (a) Stator and (b) rotor motion in the xy radial plane for a spin speed ω of 2000 rad/s: the rotor is unstable

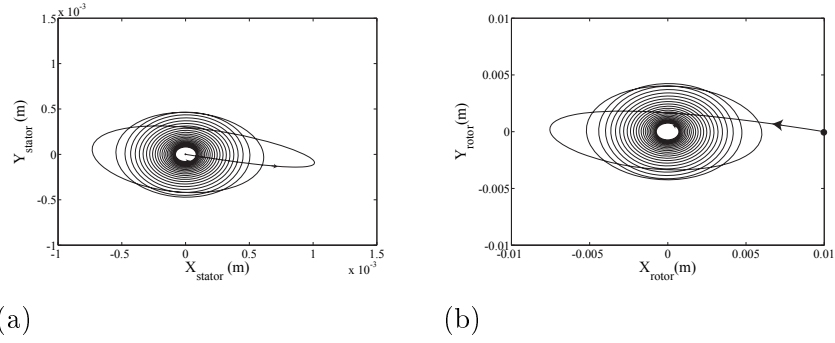


Figure 7.8: (a) Stator and (b) rotor motion in the xy radial plane for a spin speed ω of 18000 rad/s: the rotor is stable

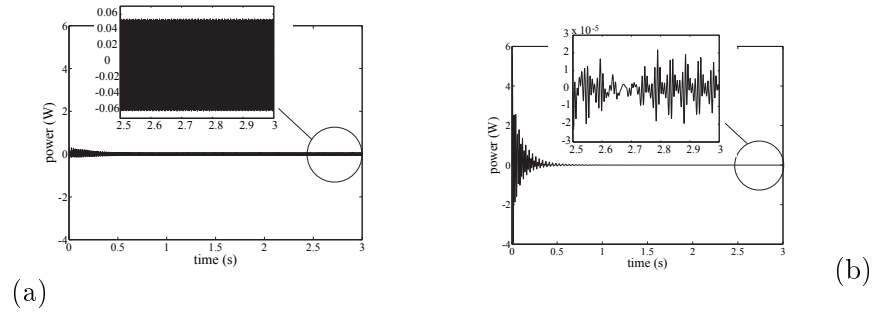


Figure 7.9: Power consumption of a system with (a) a fixed stator and (b) a damped stator when the unbalanced rotor spins at $\omega = 800$ rad/s

that even when initially centered, and when rotating at an unstable spin speed, the rotor start an outwards spiral whirl motion.

We also applied the electromechanical model to a rotor whirling inside a stator which is attached to the ground by a spring-damper in parallel. We saw that benefits on the unstable spin speed range can be achieved.

Part IV

Conclusion and perspectives

Conclusion and perspectives

The work presented in this dissertation develops a parameterized electromechanical model which allows to predict and explain the dynamical radial behavior of rotating electromagnetic systems.

This electromechanical model takes both mechanical and electromagnetic aspects into account for systems where as well the electromagnetic nature of the forces as the rotor dynamics are important. A typical example of such a system is a radial magnetic bearing, submitted to forces due to induced currents.

More precisely, the electromechanical forces are represented by mechanical components. Reluctant forces are represented by springs, and forces due to eddy currents are represented by dampers, but are submitted to resistive and inductive effects, which have consequences on the force orientation and norm.

The influence of the inductive and the resistive effects on the forces and the representation of these effects by mechanical components is obtained by a rigorous approach based on the power balance in the system.

In addition, the impact of the skin effect on the resistive and the inductive behavior of the forces have been taken into account thanks to phenomenological models. These skin effect models have been compared to each other first on conductors submitted to oscillating voltages and second on the forces exerted by magnetic bearings.

A method for the identification of the model parameters and eventually the skin effect parameters has been presented. Based on this identification, the validation of the electromechanical model showed on different cases that the forces prediction with the spin speed, as

well for the centering force, parallel to the center shift, as for the perpendicular force, perpendicular to the center shift was good.

We saw that the forces prediction with the spin speed was better when taking the skin effect into account. Of course the amelioration was more significant for high spin speed, which is important because systems supported by magnetic bearings often are dedicated to high spin speed.

The electromechanical model we developed allows to fully understand the phenomena present in rotating electromagnetic systems when submitted to eddy current forces. We have seen that the rotating damping introduced by the eddy current forces predicts a first limit spin speed above which the rotor behaves unstable. We have also seen that the resistive and inductive effects of the conductor leads to a second limit spin speed above which the rotor behavior is stable again.

The first limit spin speed has been confirmed by experiments, which showed a transition from unstable behavior to stable behavior when the spin speed decreases. These experiments also showed that this transition depends on the amount of damping present when the rotor whirls.

In addition, the model allows to predict on one hand the amount of induced stiffness resulting from the spinning motion of the rotor and on another hand the value of the limit spin speed for which the rotor behaves unstable. In this work, we used our model to analyze the link between stability, induced stiffness and the different model parameters. Some modifications of physical parameters, such as the rotor material and the system size are also examined. To reach acceptable values for the unstable spin speed range, a first solution consists in adding non-rotating damping to the system. However, introducing non-rotating damping in the system is not always easy to realize: this damping has to be contact-free. The stability thresholds are also affected by the global inductance and resistance, which depend on the conductor material choice and the bearing topology. But one has to keep in mind that these parameters also influence the induced stiffness.

Therefore, when designing or when working with electrodynamic bearings, attention has to be paid to those both aspects: instability and

stiffness.

Therefore, the electromechanical model then can be a tool to predict and analyze the dynamical behavior of a rotor supported by magnetic bearings and the stiffness produced by these magnetic bearings. On the contrary to finite elements models, which do not allow to simulate the dynamical behavior of magnetic bearings.

But also, the electromechanical model is useful to study the dynamical behavior of more complex systems integrating magnetic bearings. In this dissertation, we have seen that when an eccentricity is present, this generates a synchronous excitation of the system, which then behaves unstable between two limit spin speed even when no exterior perturbation is present.

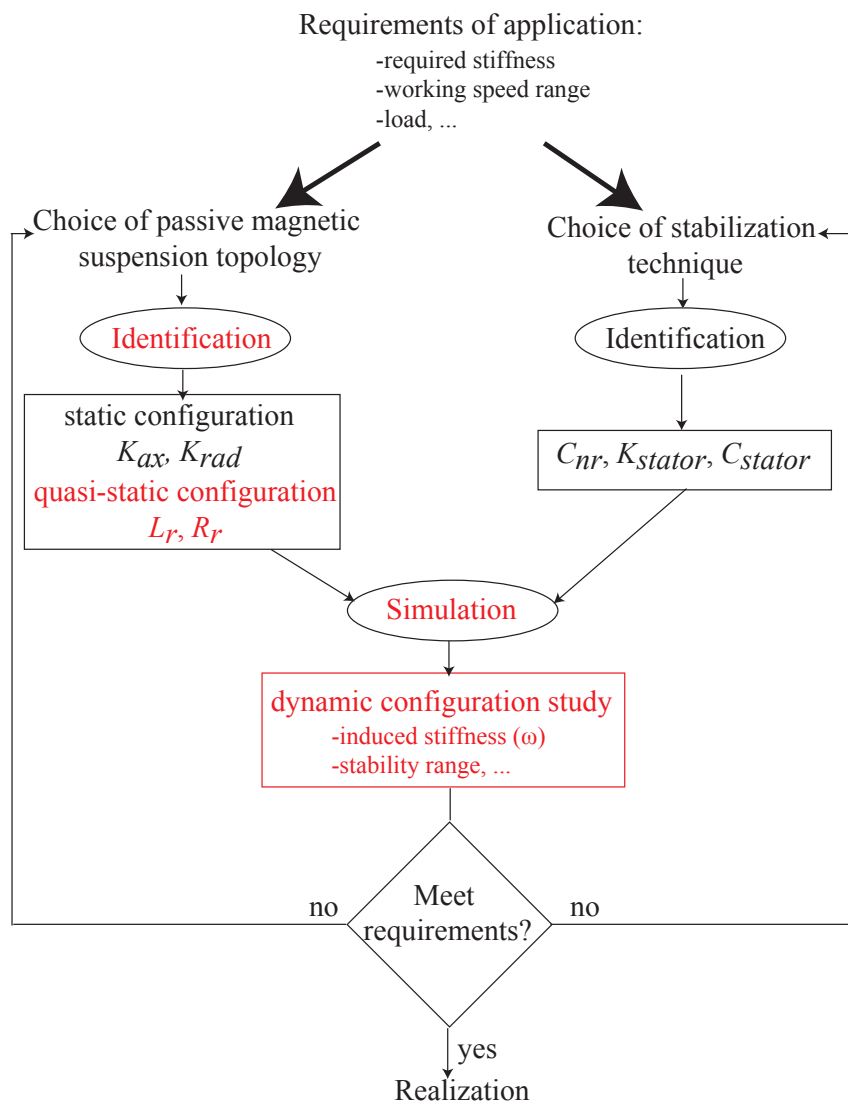
The electromechanical model was also used to examine a system where a parallel damper-spring mechanism is placed between the stator and the ground. We have seen that when the damping value of the damper was chosen appropriately, the unstable speed range of the system can be reduced. This is thus an alternative to the solution of introducing non-rotating damping to the system.

Another use of the electromechanical model can be a prediction for the power consumption of magnetic bearings. This may be an important factor in applications like flywheel energy storage.

In conclusion, the model can be used to support a general flow chart for designing a passive magnetic suspension, as illustrated further. The model can also be used to ameliorate existing magnetic bearings.

Of course this work raises new questions which may be interesting to look at in the future.

The experimental validation done in this work only showed the first transition from stable to unstable behavior when the spin speed increases. The second transition is known and has been observed on purely electrodynamic radial bearings. An interesting validation would be to develop a test bench where these both limits are present and observable. A solution to realize this would be to use a real electrodynamic bearing, and eventually do some reverse engineering to have the two limit spin speed present in a reasonable range.



Flowchart for passive magnetic suspension design thanks to electromechanical model

As for the model, it would be interesting and not too heavy work to complete it by introducing forces due to eddy currents generated by the whirling rotor motion. These forces could be modeled by non-rotating damping, but the resistive and inductive effects of the conductor have also to be taken into account.

Another perspective of the work presented in this dissertation is the use of the model to develop new electrodynamic magnetic bearings, following the presented flow chart. And given that the power consumption of the bearings can be estimated thanks to our model, it would be interesting for this new design to approach the magnetic bearing suspension design from energetics point of view.

Finally in this work, we focused the model on one type of applications: electrodynamic bearings, but it could also be used to predict parasitic induced current forces inside a motor or to improve active magnetic bearings control strategies when the induced currents forces become significant.

Conclusion and perspectives

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Nomenclature

Glossary

Definition of the main mathematical notations used

Notation	Description	Page n ^o
A	constant describing the evolution of the impedance of a conductor at very high frequency: $\bar{Z}_{int} = A\sqrt{\omega}(1 + j)$	66
α	constant describing the slope of the weighting function $g(\omega) = 1/\pi (\pi/2 + \arctan(\alpha\omega + \beta))$ taking into account the impedance variation with the frequency	67
β	constant describing the offset of the weighting function $g(\omega) = 1/\pi (\pi/2 + \arctan(\alpha\omega + \beta))$ taking into account the impedance variation with the frequency	67
c_m	proportional constant between the electromotive force e and the electrical frequency ω_{elec}	50
c_{nr}	non-rotating damping coefficient (N s/m)	29
c_r	rotating damping coefficient (N s/m)	29
$c_r\omega \sin \theta$	induced stiffness	44
C_{stator}	damping value of a damper acting between the stator and the ground (Ns/m)	191
d	center shift between the magnetic axis of the stator and the rotation axis of the rotor (m)	96

Notation	Description	Page n ^o
$e(t)$	electromotive force on the electrical circuit due to the variation of magnetic flux seen by this circuit $e(t) = -\frac{d\psi_0}{dt}$	47
$\bar{E}$	phasor representing the electromotive force on the electrical circuit due to the variation of magnetic flux seen by this circuit	48
E	amplitude of the phasor $\bar{e}$ representing the electromotive force on the electrical circuit due to the variation of magnetic flux seen by this circuit	48
ϵ	eccentricity (m)	187
$\bar{F}$	phasor expressing the forces acting on the rotor	45
$\bar{F}_I$	phasor expressing the forces acting on the rotor in the inertial frame $\{\hat{I}\}$	45
$\bar{F}_X$	phasor expressing the forces acting on the rotor in the rotating frame $\{\hat{X}\}$	45
F_{paral}	force in the direction of the center shift (N)	96
F_{perp}	force perpendicular to the center shift (N)	96
φ	phase shift between the phasor induced currents $\bar{i}$ and the phasor electromotive force $\bar{e}$	48
$g(\omega)$	weighting function of the low frequency impedance versus the high frequency impedance	67
$\{\hat{I}\}$	inertial frame	31
$i(t)$	induced currents flowing in the electrical circuit as a result of the electromotive force applied on it	47
$\bar{I}$	phasor representing the induced currents flowing in the electrical circuit as a result of the electromotive force applied on it	48

Notation	Description	Page n ^o
I	amplitude of the phasor $\bar{i}$ representing the induced currents flowing in the electrical circuit as a result of the electromotive force applied on it	48
k	external stiffness (N/m)	29
K_{stator}	stiffness of a spring acting between the stator and the ground (N/m)	191
L_0	external inductance of a conductor (H)	66
L_{int}	internal inductance of a conductor when the current is uniformly distributed inside this conductor (H)	66
L_r	global inductance of the conductor, representing the inductive effects of the conductor in an electrical circuit (H)	47
$\bar{\lambda}$	whirl motion phasor $\bar{\lambda} = \lambda_R + \lambda_I j$	32
λ_I	whirl motion frequency (rad/s)	32
λ_R	whirl motion damping rate	32
m	rotor weight (kg)	29
M_{stator}	stator weight (kg)	191
μ	magnetic permeability (H/m)	59
μ_r	relative magnetic permeability	146
P_{ELEC}	electrical active power in the rotor $P_{ELEC} = Re(\bar{S}_{ELEC})$	49
P_{MECA}	mechanical active power in the rotor $P_{MECA} = Re(\bar{S}_{MECA})$	46
Q_{ELEC}	electrical reactive power in the rotor $Q_{ELEC} = Im(\bar{S}_{ELEC})$	49
Q_{MECA}	mechanical reactive power in the rotor $Q_{MECA} = Im(\bar{S}_{MECA})$	46
R_{dc}	DC resistance of a conductor (Ω)	66

Notation	Description	Page n ^o
R_r	global resistance of the conductor, representing the resistive effects of the conductor in an electrical circuit (Ω)	47
$\bar{S}_{ELEC}$	phasor expressing the electrical power in the rotor $\bar{S}_{ELEC} = P_{ELEC} + jQ_{ELEC}$	49
σ	electric conductivity ($(\Omega m)^{-1}$)	59
$\bar{S}_{MECA}$	phasor expressing the mechanical power in the rotor $\bar{S}_{MECA} = P_{MECA} + jQ_{MECA}$	46
θ	angular shift between the position $\vec{z}$ and the forces $\vec{F}$	42
ω	rotor spinning speed (rad/s)	29
ω_{elec}	frequency of excitation of the electrical circuit (rad/s)	47
ω_{lim}	limit rotor spin speed for which the rotor behavior changes from a stable to an unstable behavior or vice versa	34
$\{\hat{X}\}$	rotating frame at rotor spin speed ω	31
$\{\hat{Y}\}$	rotating frame at rotor spin speed ω , with an angular shift θ	44
$\bar{z}$	phasor expressing the radial position of the rotor $\bar{z} = x + jy$ (m)	32
Z_I	amplitude of phasor $\bar{z}_I$ for the radial position of the rotor in the inertial frame $\{\hat{I}\}$ (m)	46
$\bar{z}_I$	phasor expressing the radial position of the rotor in the inertial frame $\{\hat{I}\}$ (m)	45
Z_X	amplitude of phasor $\bar{z}_X$ for the radial position of the rotor in the rotating frame $\{\hat{X}\}$ (m)	46
$\bar{z}_X$	phasor expressing the radial position of the rotor in the rotating frame $\{\hat{X}\}$ (m)	45
$\bar{Z}(\omega)$	phasor representing the impedance of an electrical circuit $\bar{Z}(\omega) = R(\omega) + j\omega L(\omega)$ (Ω)	61

Glossary

Notation	Description	Page n ^o
$\bar{Z}_{hf}$	phasor representing the impedance of an electrical circuit at high frequency (Ω)	66
$\bar{Z}_{int}$	phasor representing the internal impedance of an electrical circuit at high frequency (Ω)	66
$\bar{Z}_{lf}$	phasor representing the impedance of an electrical circuit at low frequency (Ω)	66

Glossary

Appendix

Appendix A

Finite element modeling

The finite element models presented in this thesis are all magneto-static models. A conductor rotates in a fixed off-centered position, sees a varying induction field, and thus currents are induced in this conductor.

Since the studied problems are true 3D problems, in which in the best case only one plane of symmetry exists, 3D-FEM calculations were needed. These calculations are very time consuming, and this appendix describes the choices made in order to have the more exact calculations.

The software which has been used for these finite element models is called Comsol multiphysics 3.5a [Com, 2008]. Although Comsol multiphysics solves magnetostatic problems, it is not specialized in electric or magnetic problems. It's force is to be able to take multiple physics into account.

The less memory consuming problems were run on a computer with Windows 32 bit, with 2 Giga RAM, and a dual processor. The more complicated problems were run on a computer with Windows 64 bit, with 24 Giga RAM and 8 processors. Even with that computer some problems took more than 24 hours to be solved.

1 Physical equations

The FEM problems we are interested in solving, are magnetostatic problems. Because induced currents are present, the magnetic potential but also the electric potential have to be solved.

The actual equations solved by Comsol multiphysics derive from Maxwell equations in the following way.

The Maxwell's equations can be written their differential form as

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \quad (\text{A.1})$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad (\text{A.2})$$

$$\nabla \bar{D} = \rho \quad (\text{A.3})$$

$$\nabla \bar{B} = 0 \quad (\text{A.4})$$

where $\bar{E}$ is the electric field intensity, $\bar{D}$ is the electric displacement, $\bar{H}$ is the magnetic field intensity, $\bar{B}$ is the magnetic flux density, $\bar{J}$ is the current density, and ρ is the electric charge density.

Another important equation is

$$\nabla \bar{J} = -\frac{\partial \rho}{\partial t} \quad (\text{A.5})$$

The constitutive equations are:

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P} \quad (\text{A.6})$$

$$\bar{B} = \mu_0 \bar{H} + \bar{B}_r \quad (\text{A.7})$$

$$\bar{J} = \sigma \bar{E} + \bar{J}_e \quad (\text{A.8})$$

with ϵ_0 the vacuum permittivity, $\bar{P}$ the electric polarization vector, μ_0 the vacuum permeability, σ the electric conductivity and $\bar{J}_e$ the external current density.

The equations in the FEM software are expressed in terms of the electric potential V and the magnetic potential $\bar{A}$, which are defined by:

$$\bar{B} = \nabla \times \bar{A} \quad (\text{A.9})$$

$$\bar{E} = -\nabla V - \frac{\partial \bar{A}}{\partial t} \quad (\text{A.10})$$

In the problems we solved, we can make the quasi-static approximation, this is to say that we consider stationary currents at every instant.

The eqs. A.1 to A.5 reduce to:

$$\begin{aligned}\nabla \times \bar{H} &= \bar{J} \\ &= \sigma(\bar{E} + \bar{v} \times \bar{B}) + \bar{J}_e\end{aligned}\quad (\text{A.11})$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad (\text{A.12})$$

$$\nabla \bar{D} = \rho \quad (\text{A.13})$$

$$\nabla \bar{B} = 0 \quad (\text{A.14})$$

$$\nabla \bar{J} = 0 \quad (\text{A.15})$$

Remembering the definition of the potentials A.9 and A.10, and the constitutive eq. A.7, the Ampere's law A.11 can be written:

$$\begin{aligned}\sigma \frac{\partial \bar{A}}{\partial t} + \nabla \times \left(\frac{1}{\mu_0} \nabla \times \bar{A} - \bar{M} \right) - \\ \sigma \bar{v} \times (\nabla \times \bar{A}) + \sigma \nabla V = \bar{J}_e\end{aligned}\quad (\text{A.16})$$

The eq. A.15 can be written in terms of the potentials as:

$$-\nabla \left(\sigma \frac{\partial \bar{A}}{\partial t} - \sigma \bar{v} \times (\nabla \times \bar{A}) + \sigma \nabla V - \bar{J}_e \right) = 0 \quad (\text{A.17})$$

Given that our problems are magnetostatic, with induction currents, eq. A.16 and A.17 can even be simplified to:

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \bar{A} - \bar{M} \right) - \sigma \bar{v} \times (\nabla \times \bar{A}) + \sigma \nabla V = \bar{J}_e \quad (\text{A.18})$$

$$-\nabla (-\sigma \bar{v} \times (\nabla \times \bar{A}) + \sigma \nabla V - \bar{J}_e) = 0 \quad (\text{A.19})$$

Eq. A.18 and A.19 are the differential equations solved by comsol multiphysics, and the variables are the electric and the magnetic potential, V and $\bar{A}$.

When examining Eq. A.18 and A.19, we can see that the electrical conductivity σ can never be equal to zero, because the electric potential V then vanishes from the equations and it can not be solved.

This is why in the problems we solved; the air for instance has a conductivity of 1.

Concerning the differential equations, it has also to be noted that the variables are not uniquely defined. Introducing a gauge transformation like:

$$\bar{A}' = \bar{A} + \nabla\psi \quad (\text{A.20})$$

$$V' = V - \frac{\partial\psi}{\partial t} \quad (\text{A.21})$$

The new electric potential and magnetic vector potential V' and $\bar{A}'$ give the same electric and magnetic field as V and $\bar{A}$. This is why constraints have to be put on the gauge to have a unique solution. The constraint put on the gauge for magnetostatic problems by comsol multiphysics is the Coulomb gauge:

$$\nabla\bar{A} = 0 \quad (\text{A.22})$$

Large 3D electromagnetic problems often need an iterative solver because a direct solver requires too much memory.

Gauge fixed magnetostatic problems can be solved using a GMG iterative linear solver, and the Vanka preconditioner as presmoothing and postsmoothing.

An alternative to explicit gauge fixing exists. This alternative consists in fixing the gauge numerically with one of the SOR gauge smoothers. For more technical details, you can refer to comsol multiphysics help.

2 Mesh

We paid great attention to the mesh of the FE model. An inaccurate mesh leads to results in which no confidence can be made.

For the non sinusoidal case presented in section 5.5, the results are satisfactory by applying a free mesh not too fine, and by solving the system with a direct solver.

For this 3D model, it can be noted that the system includes a plane of

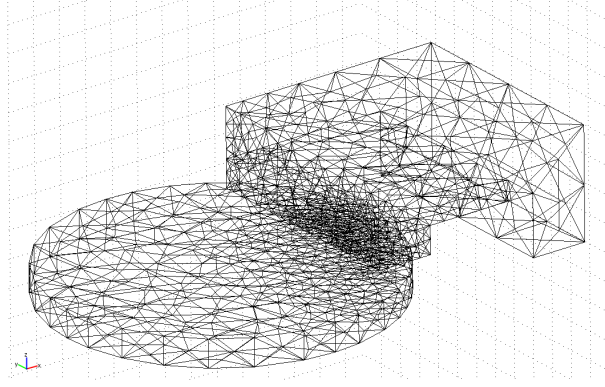


Figure A.1: Mesh of the 3D FEM model, with a plane of symmetry

symmetry, so only half of the model has to be solved (see Fig. A.1).

But except for this case, we rapidly found out that we could not content ourselves of a triangular mesh, automatically generated, even if it is very fine and of good quality. Indeed, when doing so, we established that non negligible eddy currents were generated, even when the rotation axis of the rotor coincides exactly with the magnetic axis of the stator, see fig.A.2. These eddy currents are only due to the mesh. The same facts were reported in the thesis [Lembke, 2005], although he used another finite element software.

These false induced currents, due to the mesh, resulted non negligible forces when the rotor was centered instead of being zero. When opting for an axi-symmetric mesh, the forces with a rotor spinning in a centered position are much smaller, see fig. A.3.

Some false eddy current remain with an axi-symmetric mesh (see fig. A.4, but they are less important, and because the error due to these false eddy current are distributed axi-symmetrically, their influence is much smaller.

When opting for an axi-symmetric mesh, the elements are not

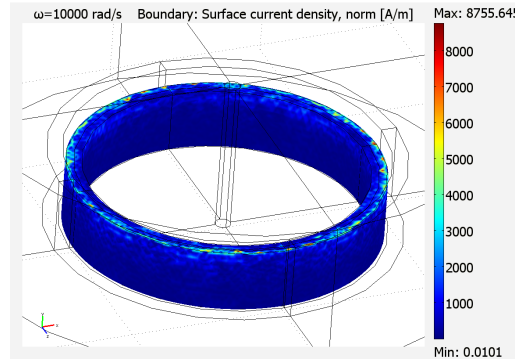


Figure A.2: Induced currents in the simple case with a radial flux, when the rotor is spinning in a centered position

tetrahedral anymore, but prisms, as illustrated on fig. A.5. This mesh is first generated in a 2D cut, and then revolved around the symmetry axis. The mesh elements can not touch the symmetry axis, this is why, there is always a hole in the center of these models. Careful attention has to be paid to the number of mesh elements in the revolution of 360 degrees. If a too small number of elements is chosen, the geometric circles risk not to be a circle anymore, which can also generate false eddy currents, when rotating. However, a too important number of elements can generate a model which is too much memory consuming. Furthermore, a too important number of elements in the 360 degrees revolution with respect to the 2D mesh in the cut may lead to inverted mesh elements, which can lead to convergence problems when solving the problem with an iterative solver.

The problem is that an axi-symmetric mesh needs an axi-symmetric geometry, which is not the case anymore when the rotor is out-centered. When the rotor is out-centered, we started from the axi-symmetric mesh of the centered case, that we deform in order to move the rotor from the prescribed center shift. The magnetostatic problem is then solved in this new configuration, with an out-centered rotor and a de-

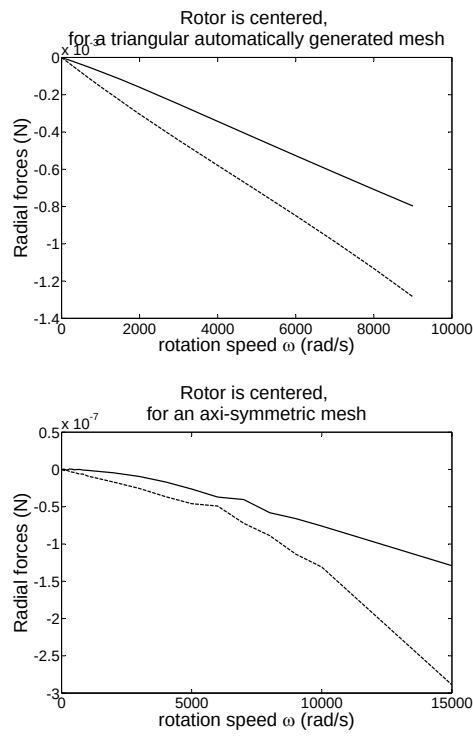


Figure A.3: Computed forces, due to numerical eddy currents, on the rotor of a simple case with radial flux, when spinning in a centered position: for a free tetrahedral mesh, and for an axisymmetric mesh

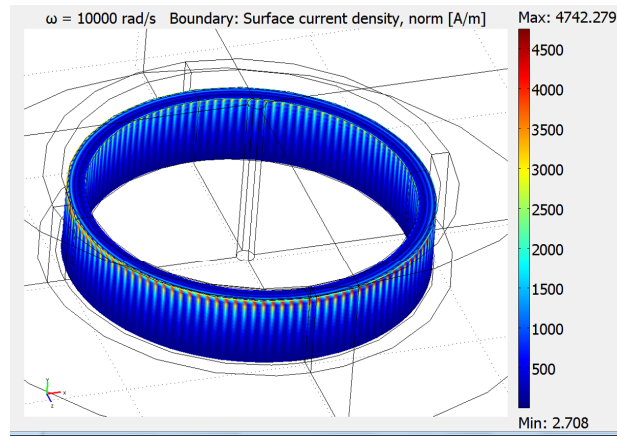


Figure A.4: Induced currents in the simple case with a radial flux, when the rotor is spinning in a centered position, with an axisymmetric mesh

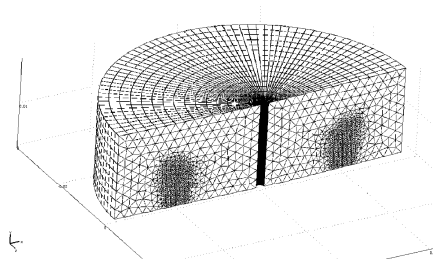


Figure A.5: Axi-symmetric mesh of a simple case with radial flux

formed mesh.

Technically, the followed procedure is:

- create the 2D cut geometry of the problem.
- mesh this 2D geometry automatically with prisms elements, or refining the automatic mesh with the necessary parameters.
- revolve the 2D mesh to a new 3D geometry. As said before, attention has to be given to the number of mesh elements in 360 degrees.
- move the rotor of the prescribed center shift by deforming the mesh. This is done with the physics mode called ALE in comsol mulipysics. All the subdomains are set to be fixed (the stator), and the rotor and the air surrounding the rotor is set to be free. This means that they are fixed by their boundaries displacement. The rotor boundaries undergo a prescribed displacement corresponding to the center shift.
- the mesh deformation is solved by selecting the ALE application mode in the solver manager, with an appropriate solver.
- this deformed mesh is stored by storing the solution of the ALE application mode in the solver manager.
- a new 3D geometry is created from the deformed mesh. In this new geometry the magnetostatic problem will be solved, with application mode called EMQAV in comsol multiphysics. The magnetic and electric properties have to be given to the rotor, the air and the stator. Boundary conditions have also to be set.
- Choose an appropriate parametric solver (probably iterative), to solve for different rotation speed. In the solver manager, on the initial value tab, under values of variables not solved for, select stored solution. In the solve for tab, select the magnetostatic application mode, and solve the problem

To be sure the mesh is fine enough, a finer mesh can be generated, and the forces can be compared in both cases. This is illustrated in

fig.A.6. it can be seen that the first mesh case, which is coarser seems fine enough, because, the forces calculations with a finer mesh give the same results, consuming more memory, more time, and having a harder time to converge.

3 Force calculation

To compute the forces acting on the conductor, three methods are possible. The three were tested, when the three methods gave the same force variations with the spin speed, there was an offset between the three results.

3.1 Maxwell stress tensor

The first way to calculate the force on a body surrounded by air is to integrate the Maxwell stress tensor on its surface.

The Maxwell stress tensor is equal to:

$$T_{ij} = \epsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2} (\epsilon_0 E^2 + \frac{1}{\mu_0} B^2) \delta_{ij} \quad (\text{A.23})$$

The force applied on the body is then obtained by integrating the tensor on the surface of the body:

$$\bar{F} = \int \bar{T} \bar{n} dS \quad (\text{A.24})$$

where $\bar{n}$ is the outward normal of the body.

So, by integrating the Maxwell stress tensor on the surface of the rotor, we obtain a value of the force acting on this rotor.

The Maxwell stress tensor can also be applied by integrating on the surface of the stator, to have a value of the force acting on the stator. By action-reaction, this force is equal to the opposite of the force acting on the rotor.

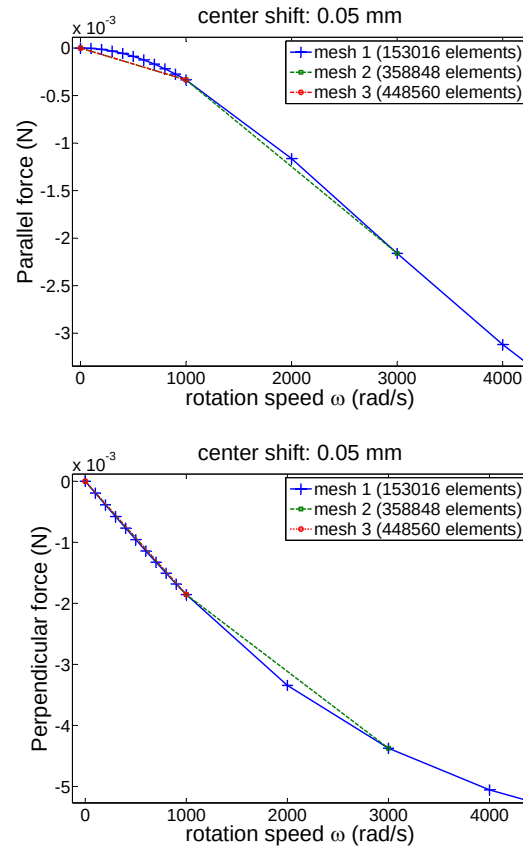


Figure A.6: Computed forces on the out-centered rotor of a simple case with radial flux, for different axisymmetric mesh cases

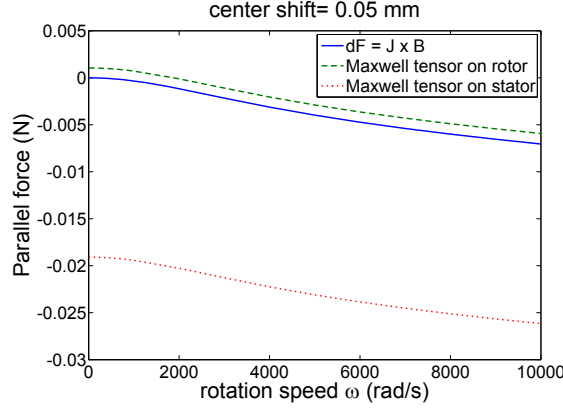


Figure A.7: Computation of the parallel forces acting on the simple case (radial flux) with three different methods

3.2 Lorentz force

Another way to compute the force acting on a conductor in this case is to use the expression for the Lorentz Force:

$$\bar{F} = \int \bar{J} \times \bar{B} dV \quad (\text{A.25})$$

integrated on the volume of the conductor.

3.3 Comparison

For the perpendicular forces (fig.A.8), the three methods give the same results. But for the parallel forces (fig. A.7), an important offset exists between the three forces computed.

This offset, present when using the Maxwell stress tensor, is not correct: there are no forces present when the rotor is not spinning. This error is probably due to the fact that the tensor was calculated at the border between the rotor/stator and the air. This could have been avoided by taking a closed surface in the air close to the stator/rotor boundaries.

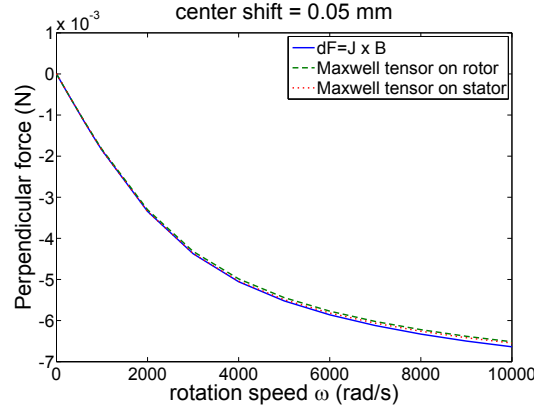


Figure A.8: Computation of the perpendicular forces acting on the simple case (radial flux) with three different methods

4 Some additional remarks

The convergence of the problem is improved with a greater air conductivity, especially when the computational domain is small.

Induced currents are mathematically a convective problem, so when great conductivities and great permeabilities are present in the problem, convergence is very difficult, and in some cases impossible to reach.

This happens when the conductor where the induced currents are present is part of the magnetic circuit, and is iron, for instance.