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Long-term care with myopic carers*

Justina Klimaviciute†, Pierre Pestieau‡ and Jerome Schoenmaeckers§

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Abstract

This paper analyzes the design of a social policy scheme for long-term care in a setting where disabled elderly individuals receive care both informally, from family members, and formally, from professional providers. We introduce two key features: informal caregivers bear substantial physiological and psychological costs, and they tend to be myopic with respect to these health consequences. Under perfect observability, an optimal policy can induce caregivers to exercise caution and provide an appropriate level of informal care. However, when individual characteristics are not common knowledge, public policy must resort to distortions and indirect instruments to guide agents toward optimal behavior - a classic second-best problem.

Keywords: Long-term care, Myopia, Caregiving burden, Social insurance

JEL codes: H21, J14, D91

1 Introduction

Long-term care (LTC) in old age, which can be defined as help with basic daily and household activities, constitutes a major challenge in today's aging world. There are many issues associated with LTC which relate to three main institutions providing and financing this care, namely, the family, the market and the state. These issues range from family care arrangements and private

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insurance purchases to social policy design and involve different institutional as well as behavioral aspects.[1]

In this paper, we aim to analyze a twofold problem concerning the informal caregivers of the elderly. The first dimension of this problem is related to the physiological and psychological costs borne by informal caregivers, usually family members or close relatives, who provide unpaid assistance to dependent elderly persons. This burden is well-documented in the literature and involves elevated risks of depression, anxiety, physical illness and diminished quality of life. For instance, Pinquart and Sörensen (2003), in a meta-analysis of 84 studies, report that caregivers had significantly higher levels of perceived stress and depression than non-caregivers. Physical health differences were smaller but also statistically significant and larger between dementia caregivers and non-caregivers. Other studies, such as Schulz and Sherwood (2008), Coe and Van Houtven (2009) or Bom et al. (2019), also document both physical and mental health effects of caregiving. Stöckel and Bom (2022) analyze longer-term and dynamic health effects of caregiving in the United Kingdom and find important negative effects on mental health, especially borne by caregivers providing more than 20 hours of weekly care.

The second dimension, which even worsens the problem, is that caregivers themselves may act in a somewhat myopic way when it comes to their health. Indeed, evidence shows that caregivers may sacrifice their own health when taking care of others (see, for instance, The Associated Press and NORC (2018), Martinez-Marcos and De la Cuesta-Benjumea (2014), Stein et al. (2000), Vanderwerker et al. (2005)). This suggests that the negative consequences of caregiving might be underestimated when caregivers take their caring decisions, and these decisions might therefore be suboptimal. On top of that, a recent study by Amar et al. (2022) directly measured the awareness of elderly caregivers regarding the health risks associated with caregiving and found a low-moderate awareness level.

The present paper examines the policy implications that arise from the conjunction of these two observations. In particular, we consider a setting where caregiving has negative effects on the caregivers' health and where certain individuals are myopic about these negative consequences. Individuals also differ in their altruism towards their dependent parents who require LTC. We first study the *laissez-faire* situation and then develop a normative framework for the design of optimal LTC policy in this context.

In that respect, our paper belongs to the theoretical literature studying optimal LTC policy design in the presence of myopia. However, most of this literature is concerned with a different type

[1] For an overview of the economics of LTC, see Klimaviciute and Pestieau (2023) or Pestieau (2026).

of myopia, namely, myopia about the probability of becoming dependent (Cremer and Roeder, 2013, 2017; Jasaityte and Klimaviciute, 2024).[2] Myopia about the negative health effects of caregiving is analyzed by Klimaviciute (2020), but that paper focuses on spousal caregiving and studies a public policy targeted at private LTC insurance.

The rest of the paper is organized as follows. In Section 2, we present the model and describe the *laissez-faire* situation. In Section 3, the first-best optimum is derived. Sections 4 and 5 are devoted to the second-best optimum under different informational assumptions. Finally, Section 6 provides some concluding remarks, while the Appendixes include some more technical details and additional analysis.

2 The model

Consider three types of individuals: myopic altruists, denoted by MA, rational altruists, denoted by RA, and non-altruists, denoted by NA. They all have a dependent parent that requires care. We assume at the outset that devoting time in care has an effect on the carer's health. We thus expect that the NA provide no care, the MA provide care in time without correctly considering the collateral effect on their own health, whereas the RA provide less care in time and support formal care. As a consequence, one expects the MA to be in bad health when their dependent parents pass away relative to the two other types of individuals. Further we assume that each parent has an amount of resources equal to R. In an altruistic family, if R is big enough it can be used to finance formal LTC and some bequest. If R is small, there will be no bequest and the child may have to finance part of the formal LTC.

We will use the subscript 1 for the MA, the subscript 2 for the RA and the subscript 3 for the NA. We denote by w_i and l_i (for $i = 1, 2, 3$) the children's wage rates and labor supply respectively and define the children's gross income as $y_i = w_i l_i$. Moreover, we denote by a_1 and a_2 the time that the MA and the RA children respectively devote to providing care to their parents. Finally, we define the leisure time z_i of the three types as $z_1 = 1 - a_1 - l_1$, $z_2 = 1 - a_2 - l_2$ and $z_3 = 1 - l_3$.

In the *laissez-faire*, the problem for the MA and the RA child is to maximize the following utility (for $i = 1, 2$):

$$U_i = u(y_i + R - s_i) + v(1 - a_i - \frac{y_i}{w_i}) + H(\beta a_i + s_i) - \gamma_i \varphi(a_i)$$

where $u(c_i)$ is the child's utility of consumption c_i (with $u'(c_i) > 0$ and $u''(c_i) < 0$), $v(z_i)$

[2] De Donder and Leroux (2013) study this type of myopia in a political economy setting.

is the child's utility of leisure (with $v'(z_i) > 0$ and $v''(z_i) < 0$), $H(m_i)$ is the parent's utility of LTC m_i (with $H'(m_i) > 0$ and $H''(m_i) < 0$), s_i denotes the cost of formal care and $R - s_i$ is the bequest remaining to the child. The parameter $\beta > 1$ reflects the parent's preference for personal care, $\gamma_i \in [0, 1]$ is a parameter of myopia and $\varphi(a_i)$ is the health cost to the child resulting from the personal care provision (with $\varphi'(a_i) > 0$ and $\varphi''(a_i) > 0$). We assume that $\gamma_2 = 1$, whereas $\gamma_1 = \gamma < 1$, meaning that the RA child fully understands and takes into account this health cost, while the MA child underestimates it, i.e. is myopic. Finally, we assume that $\beta > w_i$.

As to the NA child, his utility is given by:

$$U_3 = u(y_3) + v(1 - \frac{y_3}{w_3})$$

where this child's consumption c_3 is simply equal to y_3, and the utility functions $u(c_3)$ and $v(z_3)$ have the same properties as in the case of the MA and the RA child. The dependent parent in the NA family has a utility given by $H(m_3) = H(R)$. In other words, the parent and the child in such a family simply act as independent agents.

A word on the income of the parent is in order. We have two extreme cases in mind. In one case, $R = 0$, which implies that in the *laissez-faire* the parent of the NA has a level of LTC $m_3 = 0$. In the other case, R is quite large, which means that the MA and the RA can benefit from some bequests equal to $R - s_i$ (for $i = 1, 2$) and that the parents of the NA can have a level of care above the consumption of their children: $m_3 = R > y_3 = c_3$. In the next sections we assume that $R = 0$, thus excluding the possibility of bequests and implying that the MA and the RA children finance the formal LTC of their parents.

The first-order conditions (FOCs) for the MA and the RA child can be written as follows (for $i = 1, 2$):

$$\frac{\partial U_i}{\partial y_i} = u'(c_i) - v'(z_i)/w_i = 0 \tag{1}$$

$$\frac{\partial U_i}{\partial s_i} = -u'(c_i) + H'(m_i) \leq 0 \tag{2}$$

$$\frac{\partial U_i}{\partial a_i} = -v'(z_i) + \beta H'(m_i) - \gamma_i \varphi'(a_i) \leq 0 \tag{3}$$

The child's choice of y_i (or, equivalently, labor supply) involves a trade-off between increasing his consumption and decreasing his leisure time. The choice of formal care to his parent takes into account the reduction in his own consumption but also the increase in the parent's LTC level. Finally, the choice of personal care considers three factors: the decrease in the child's leisure time, the increase in the parent's LTC level (taking into account that personal care is preferred to formal care by the parent) and the fact that personal care provision has a negative effect on the child's health. However, the negative health effect is underestimated by the MA child.

Assuming interior solutions and combining those FOCs yields:

$$(\beta - w_i)\, H'(m_i) = \gamma_i \varphi'(a_i)$$

Note that our assumption of $\beta > w_i$ is needed to get an interior solution for a_i.

In Appendix A, we show that $\frac{\partial a_i}{\partial \gamma_i} < 0$ and $\frac{\partial s_i}{\partial \gamma_i} > 0$, which means that less myopic children provide less personal care and privilege more formal care to their parents. This implies that, all other things being equal, the RA children will provide less personal care than the MA ones and, consequently, will end up having a better health. We also show in Appendix A that $\frac{\partial y_i}{\partial \gamma_i} > 0$, meaning that less myopic children work more in the labor market, which is quite intuitive given that they spend less time for personal care and need more resources to finance their parents' formal care.

Finally, the NA child chooses only his labor supply and his FOC is given by

$$\frac{\partial U_3}{\partial y_3} = u'(c_3) - v'(z_3)/w_3 = 0$$

3 First-best optimum

We now assume the presence of a utilitarian government which has full information about the economy. There are several ways of expressing the first-best optimum. Here we choose the problem expressed by the following Lagrangian:

$$\mathcal{L} = n_1 \left[u(c_1) + v(1 - a_1 - \frac{y_1}{w_1}) + H(\beta a_1 + s_1) - \varphi(a_1) \right] +$$

$$+ n_2 \left[u(c_2) + v(1 - a_2 - \frac{y_2}{w_2}) + H(\beta a_2 + s_2) - \varphi(a_2) \right] +$$

$$+ n_3 \left[u(c_3) + v(1 - \frac{y_3}{w_3}) + H(m_3) \right] - \mu \left(n_1 \left(c_1 + s_1 - y_1 \right) + n_2 \left(c_2 + s_2 - y_2 \right) + n_3 \left(c_3 + m_3 - y_3 \right) \right)$$

where n_1, n_2 and n_3 are the population shares of respectively the MA, the RA and the NA families and μ is the Lagrange multiplier associated with the resource constraint. Note that in this specification, we assume that the government takes full account of the health cost of a_1 and acknowledges that the NA children cannot be forced to provide any assistance to their parents. Moreover, the utility of the MA and RA children is considered without its altruistic component to avoid the double counting of their parents' utility.

The FOCs for interior solutions imply:

$$\mu = u'(c_i) = H'(m_i) \tag{4}$$

$$v'(z_i)/w_i = \mu \tag{5}$$

$$v'(z_1) = \beta H'(m_1) - \varphi'(a_1) = w_1 H'(m_1) \tag{6}$$

$$v'(z_2) = \beta H'(m_2) - \varphi'(a_2) = w_2 H'(m_2) \tag{7}$$

where $i = 1, 2, 3$.

This means that $c_1^* = c_2^* = c_3^* = c^*$ and $m_1^* = m_2^* = m_3^* = m^*$, whereas a_1^* and a_2^* are respectively given by:

$$(\beta - w_1)H'(m_1) = \varphi'(a_1)$$

and

$$(\beta - w_2)H'(m_2) = \varphi'(a_2)$$

These equations imply that $a_1^* \lesseqgtr a_2^*$ if $w_1 \gtreqless w_2$. Moreover, if $w_1 = w_2 = w_3$, we have $z_1^* = z_2^* = z_3^* = z^*$ and thus $l_3^* > l_1^* = l_2^*$. If the wage rates of the three types are not equal, we may have different cases of comparison between the levels of labor supply.

Comparing equations (4)-(7) to the individual FOCs in the *laissez-faire*, we see that all the trade-offs are optimal for the RA children. On the other hand, the MA children face optimal trade-offs for labor supply and formal care choices, but their choice of personal care is different from the optimal one because $\gamma_1 = \gamma < 1$. Since the MA individuals do not fully take into account the negative effect of caregiving on their health, their *laissez-faire* amount of personal care is too large compared to the optimum. Thus, to decentralize the first-best, their personal care provision has to be discouraged through a linear tax on a_1. We show in Appendix B that the rate of this tax is given by

$$t_1^{FB} = \frac{(1-\gamma)\,\varphi'(a_1^*)}{u'(c_1^*)} \geq 0$$

The NA children also face an optimal trade-off for labor supply in the *laissez-faire*; however, they do not provide any type of care to their parents. The optimal amount of LTC for their parents can only be obtained in the form of formal care, by providing or financing this care publicly. In particular, the parents of the NA children should receive a public LTC benefit $g = m^*$.

In addition to these instruments, we also need lump-sum taxes/transfers on the three types of individuals in order to compensate for the linear tax, finance g and ensure the first-best equalities in terms of consumption and LTC of the three individual types.

The decentralization of the first-best optimum is summarized in Proposition 1.

Proposition 1. *The first-best optimum can be decentralized by:*

i) introducing a linear tax of rate $t_1^{FB} = \frac{(1-\gamma)\varphi'(a_1^*)}{u'(c_1^*)} \geq 0$ *on the personal care provided by the MA individuals;*

ii) publicly providing or financing the amount $g = m^*$ *of formal care to the parents of the NA individuals;*

iii) introducing lump-sum taxes/transfers on the MA, RA and NA individuals so as to compensate for the linear tax, finance g *and equate consumptions and LTC of the three types.*

4 Second-best optimum

To simplify the analysis, we now assume that the society consists of only two types of individuals: altruists (type 1) and non-altruists (type 2), where all the altruistic children are either myopic or rational, i.e. they all have the same myopia parameter γ which can be either smaller or equal to 1.

Moreover, we assume that the government cannot observe the individual types. In particular, it cannot observe whether the individuals are altruistic or not and also their wage rates w_i, their labor supplies l_i and their leisure time z_i (for i now being equal to 1 or 2). All the other variables, i.e. gross income as well as formal and personal care provision, are observable.[3]

In that case, it can be tempting for the altruistic children to mimic the NA ones so that the LTC of their parents is ensured by the government. That way they can consume more and avoid the health problems related to personal care. We therefore assume that the relevant self-selection

[3]In the next section, we will drop the assumption of personal care being observable.

constraint is that of the altruistic children. This is quite obvious if $w_1 \geq w_2$, but even if the altruistic child has a lower wage, he could still be tempted to pass for a NA. This depends on the difference between the two wages but also on the levels of γ and β.[4]

4.1 Optimal allocations

The problem of the government can be expressed by the following Lagrangian expression:

$$\mathcal{L} = n_1 \left[u(c_1) + v(1 - a_1 - \frac{y_1}{w_1}) + H(\beta a_1 + s_1) - \varphi(a_1) \right] + n_2 \left[u(c_2) + v(1 - \frac{y_2}{w_2}) + H(m_2) \right] -$$

$$-\mu \left(n_1 \left(c_1 + s_1 - y_1 \right) + n_2 \left(c_2 + m_2 - y_2 \right) \right) +$$

$$+\lambda \left[u(c_1) + v(1 - a_1 - \frac{y_1}{w_1}) + H(\beta a_1 + s_1) - \gamma\varphi(a_1) - u(c_2) - v(1 - \frac{y_2}{w_1}) - H(m_2) \right]$$

where λ is the multiplier associated with the self-selection constraint of the altruistic children. Note that in the self-selection constraint we introduce the myopia parameter that does not appear in the welfare function.

We now have the following FOCs:

$$u'(c_1)(n_1 + \lambda) = \mu n_1 \tag{8}$$

$$u'(c_2)(n_2 - \lambda) = \mu n_2 \tag{9}$$

$$H'(m_1)(n_1 + \lambda) = \mu n_1 \tag{10}$$

$$H'(m_2)(n_2 - \lambda) = \mu n_2 \tag{11}$$

$$\left[\beta H'(m_1) - \varphi'(a_1) \right] (n_1 + \lambda) = \mu n_1 w_1 - \lambda \left(1 - \gamma \right) \varphi'(a_1) \tag{12}$$

$$\left(v'(z_1)/w_1 \right) (n_1 + \lambda) = \mu n_1 \tag{13}$$

[4]The direction of the self-selection constraints is less obvious with three individual types. Nevertheless, in Appendix C we come back to our initial setting with three types and provide an analysis of the second-best assuming that the MA may be tempted to mimic the RA, while the RA may be tempted to mimic the NA.

$$\left(v'(z_2)/w_2\right) n_2 = \mu n_2 + \lambda v'(1 - \frac{y_2}{w_1})/w_1 \tag{14}$$

One sees right away that in case of perfect observability ($\lambda = 0$), we would have the first-best solutions of the previous section.

Rearranging equations (8)-(11), we get:

$$u'(c_1) = H'(m_1) < u'(c_2) = H'(m_2)$$

and thus

$$c_1 > c_2; m_1 > m_2$$

To prevent the altruistic children from mimicking the NA ones, the government needs to make sure that the consumption and LTC levels in the altruistic families are higher than in the NA ones.

Furthermore, using equation (12), we can see that the second-best optimal level of personal care is given by:

$$(\beta - w_1)\, H'(m_1) = \varphi'(a_1) \left[1 - \frac{\lambda\,(1-\gamma)}{(n_1 + \lambda)}\right] \leq \varphi'(a_1) \tag{15}$$

This condition implies that the second-best level tends to be higher than the first-best one as long as $\gamma < 1$. This is because in the second-best the government has to respect the self-selection constraint of the altruistic children, and if $\gamma < 1$, these children perceive personal care more positively than the government (i.e. they underestimate its negative effect on their health). Note that if there is no myopia, i.e. $\gamma = 1$, the second-best condition becomes the same as the first-best one.

As far as the labor supply is concerned, for the altruistic children we have

$$\frac{v'(z_1)}{u'(c_1)} = w_1, \tag{16}$$

which is the same trade-off as in the first-best, meaning that there is no need to distort the labor supply of these children.

On the other hand, for the NA children, using equation (14) we can obtain

$$\frac{v'(z_2)}{u'(c_2)} = \frac{w_2 \left(1 - \frac{\lambda}{n_2}\right)}{\left(1 - \frac{\lambda w_2 v'(1 - \frac{y_2}{w_1})}{n_2 w_1 v'(1 - \frac{y_2}{w_2})}\right)} \lesseqgtr w_2 \tag{17}$$

If $w_1 = w_2$, it can be easily seen that $\frac{v'(z_2)}{u'(c_2)} = w_2$, implying that the labor supply of the NA children is not distorted. Indeed, in that case, the mimicker (altruistic child) and the mimicked (NA child) value leisure in the same way, so distorting the labor supply of the NA cannot improve social welfare.

If $w_1 > w_2$, one can verify that $\frac{v'(z_2)}{u'(c_2)} < w_2$, which implies a downward distortion of the NA children's labor supply. In that case, the mimicker (altruistic child) is willing to work more than the mimicked (NA child), so distorting the labor supply of the NA downwards hurts the mimicker more than the mimicked and can thus improve social welfare.

Finally, if $w_1 < w_2$, we have $\frac{v'(z_2)}{u'(c_2)} > w_2$, implying that the labor supply of the NA children is distorted upwards. This is an opposite situation to the previous one: the mimicker (altruistic child) is now willing to work less than the mimicked (NA child), so an upward distortion of l_2 hurts the mimicker more.

4.2 Decentralization

To decentralize the second-best optimum, we consider a policy consisting of a non-linear income tax $T(y_i)$, a linear tax of rate t on personal care and a public LTC benefit g provided to the parents of the NA children.

Given the policy, the problem of an altruistic child is to maximize the following utility:

$$U_1 = u(y_1 - s_1 - ta_1 - T(y_1)) + v(1 - a_1 - \frac{y_1}{w_1}) + H(\beta a_1 + s_1) - \gamma\varphi(a_1)$$

This gives the following FOCs:

$$\frac{\partial U_1}{\partial y_1} = u'(c_1)(1 - T'(y_1)) - v'(z_1)/w_1 = 0 \tag{18}$$

$$\frac{\partial U_1}{\partial s_1} = -u'(c_1) + H'(m_1) \leq 0 \tag{19}$$

$$\frac{\partial U_1}{\partial a_1} = -u'(c_1)t - v'(z_1) + \beta H'(m_1) - \gamma\varphi'(a_1) \leq 0 \tag{20}$$

Combining (18) with (16), we can easily see that $T'(y_1) = 0$, i.e. the altruistic children face a zero marginal tax rate on their income. On the other hand, combining (20) with (18), (19) and then with (15), we get

$$t^{SB} = \frac{n_1}{(n_1+\lambda)} \frac{(1-\gamma)\,\varphi'(a_1)}{u'(c_1)} \geq 0$$

We can see that this tax rate is strictly positive as long as the altruistic children are myopic, i.e. $\gamma < 1$. Moreover, it is scaled down compared to the one needed for the MA children in the first-best, namely t_1^{FB}. Indeed, as we saw above, the second-best optimal level of personal care tends to be higher than the first-best one under $\gamma < 1$, so the tax aiming to discourage care provision has to be smaller. In other words, in the second-best the government does not fully correct for the myopia of the altruistic children.

Turning to the NA, a child of this type maximizes the following utility:

$$U_2 = u(y_2 - T(y_2)) + v(1 - \frac{y_2}{w_2})$$

The FOC writes:

$$\frac{\partial U_2}{\partial y_2} = u'(c_2)(1 - T'(y_2)) - v'(z_2)/w_2 = 0$$

Combining this FOC with equation (17), we obtain

$$T'(y_2) = 1 - \frac{\left(1 - \frac{\lambda}{n_2}\right)}{\left(1 - \frac{\lambda w_2 v'(1-\frac{y_2}{w_1})}{n_2 w_1 v'(1-\frac{y_2}{w_2})}\right)} \lesseqqgtr 0 \tag{21}$$

It can be verified that $T'(y_2) = 0$ if $w_1 = w_2$, $T'(y_2) > 0$ if $w_1 > w_2$ and $T'(y_2) < 0$ if $w_1 < w_2$. Therefore, the NA children face a zero marginal tax rate if their wage rate is the same as that of the altruistic ones, while they face a marginal tax (resp. a marginal subsidy) on their labor income if their wage rate is lower (resp. higher) than that of the altruists. This is directly implied by the analysis of the second-best optimum above.

Moreover, the parents of the NA children should receive a public LTC benefit g equal to the second-best optimal level of m_2, which, as we have seen above, is lower than the LTC level received by the parents of the altruistic children. Finally, the non-linear income tax $T(y_i)$ should finance g and ensure that all the consumption levels are at their second-best optimal values.

Proposition 2 provides a summary of the decentralization of the second-best optimum.

Proposition 2. *Assume that there are two types of individuals (altruists (type 1) and non-altruists (type 2)) and that the government cannot observe the individuals' altruism, their wage* *rates w_i, their labor supplies l_i and their leisure time z_i. The second-best optimum in this setting can be decentralized by:*

i) introducing a linear tax of rate $t^{SB} = \frac{n_1}{(n_1+\lambda)} \frac{(1-\gamma)\varphi'(a_1)}{u'(c_1)} \geq 0$ on personal care provision;

ii) providing a public LTC benefit $g = m_2 < m_1$ to the parents of the non-altruistic individuals;

iii) introducing a non-linear income tax $T(y_i)$ which finances g, ensures that all the consumption levels are at their second-best optimal values and has the following properties:

a) $T'(y_1) = 0$;

b) $T'(y_2) = 0$ if $w_1 = w_2$, $T'(y_2) > 0$ if $w_1 > w_2$ and $T'(y_2) < 0$ if $w_1 < w_2$.

5 Second-best optimum with unobservable personal care

In the previous section, we assumed that the government could observe the personal care provided by children to their parents. Since this may be a strong assumption, we now study another version of the second-best in which personal care is not observable to the government. All the other assumptions remain the same as in the previous section.

5.1 Policy instruments and individual choices

Since personal care is not observable, it can no longer be taxed directly. Therefore, the policy that we now consider consists of a non-linear income tax $T(y_i)$, a linear subsidy of rate σ on formal care and a public LTC benefit g provided to the parents of the NA children.

Given this policy, the problem of an altruistic child is to maximize the following utility:

$$U_1 = u(y_1 - s_1 + \sigma s_1 - T(y_1)) + v(1 - a_1 - \frac{y_1}{w_1}) + H(\beta a_1 + s_1) - \gamma\varphi(a_1)$$

This yields the following FOCs:

$$\frac{\partial U_1}{\partial y_1} = u'(c_1)(1 - T'(y_1)) - v'(z_1)/w_1 = 0 \tag{22}$$

$$\frac{\partial U_1}{\partial s_1} = u'(c_1)(\sigma - 1) + H'(m_1) \leq 0 \tag{23}$$

$$\frac{\partial U_1}{\partial a_1} = -v'(z_1) + \beta H'(m_1) - \gamma\varphi'(a_1) \leq 0 \tag{24}$$

The problem and the FOC of a type NA child writes in the same way as in the previous section.

5.2 Optimal allocations and decentralization

For simplicity, we assume that g, the public LTC provided to the parents of the NA, takes the form of a full-time residence in a public nursing home, implying that there is no possibility of adding some personal care on top of that if the altruistic children mimic the NA ones (even though personal care is unobservable). In this case, the level of g can be seen as the quality of the nursing home care. The Lagrangian of the government can thus be written as

$$\mathcal{L} = n_1 \left[u(c_1) + v(1 - a_1 - \frac{y_1}{w_1}) + H(\beta a_1 + s_1) - \varphi(a_1) \right] + n_2 \left[u(c_2) + v(1 - \frac{y_2}{w_2}) + H(g) \right] -$$

$$-\mu \left(n_1 \left(c_1 + s_1 - y_1 \right) + n_2 \left(c_2 + g - y_2 \right) \right) +$$

$$+\lambda \left[u(c_1) + v(1 - a_1 - \frac{y_1}{w_1}) + H(\beta a_1 + s_1) - \gamma\varphi(a_1) - u(c_2) - v(1 - \frac{y_2}{w_1}) - H(g) \right]$$

Note that the government can no longer control a_1 directly. Instead, it has to take into account the effect that other variables (namely, s_1 and y_1) have on the altruistic child's choice of personal care. Therefore, the FOCs now write as follows:

$$u'(c_1)(n_1 + \lambda) = \mu n_1 \tag{25}$$

$$u'(c_2)(n_2 - \lambda) = \mu n_2 \tag{26}$$

$$H'(m_1)(n_1 + \lambda) = \mu n_1 + \frac{\partial a_1}{\partial s_1} n_1 \left(1 - \gamma \right) \varphi'(a_1) \tag{27}$$

$$H'(g)(n_2 - \lambda) = \mu n_2 \tag{28}$$

$$\left(v'(z_1)/w_1 \right) \left(n_1 + \lambda \right) = \mu n_1 - \frac{\partial a_1}{\partial y_1} n_1 \left(1 - \gamma \right) \varphi'(a_1) \tag{29}$$

$$\left(v'(z_2)/w_2 \right) n_2 = \mu n_2 + \lambda v'(1 - \frac{y_2}{w_1})/w_1 \tag{30}$$

where in (27) and (29) we have used the altruistic child's FOC for a_1 (equation (24)).

Combining (27) with (25), we have

$$H'(m_1) = u'(c_1) + \frac{\frac{\partial a_1}{\partial s_1} n_1 (1-\gamma) \varphi'(a_1)}{(n_1+\lambda)} \leq u'(c_1) \tag{31}$$

The inequality follows from the fact that $\frac{\partial a_1}{\partial s_1} < 0$ (we show this in Appendix D). Indeed, formal and personal care are substitutes in our setting; thus, the higher the level of formal care, the lower amount of personal care is needed. Equation (31) implies that the optimal level of s_1 in this second-best scenario tends to be higher than in the scenario with observable personal care and in the first-best as long as $\gamma < 1$. In other words, formal care in the altruistic families is distorted upwards. Since the government needs to correct for the myopia regarding personal care provision but can influence a_1 only indirectly, it is now optimal to impose more formal care for the altruistic families as this will decrease the level of a_1 chosen by the altruistic children. Note that this holds only when $\gamma < 1$. If $\gamma = 1$, there is no need to correct the choice of a_1, and equation (31) then simply yields $H'(m_1) = u'(c_1)$.

Overall, equations (25)-(28) imply

$$H'(m_1) < u'(c_1) < u'(c_2) = H'(g)$$

and thus we have $c_1 > c_2$ and $m_1 > g$, meaning that, as in the previous scenario, the consumption and LTC levels in the altruistic families are higher than in the NA ones.

Turning to the labor supply, combining (29) with (25), we obtain

$$\frac{v'(z_1)}{u'(c_1)} = w_1 - \frac{\frac{\partial a_1}{\partial y_1} n_1 (1-\gamma) \varphi'(a_1) w_1}{u'(c_1)(n_1+\lambda)} \geq w_1 \tag{32}$$

The inequality is implied by the fact that $\frac{\partial a_1}{\partial y_1} < 0$ (also shown in Appendix D). As the altruistic children increase their labor supply (or, equivalently, y_1), their leisure time decreases, so providing care becomes more "costly" because it reduces the already scarce leisure time even more. Equation (32) implies that, as long as $\gamma < 1$, the labor supply of the altruistic children is distorted upwards compared to the first-best and the second-best scenario with observable personal care. Again, this is a way for the government to influence the level of a_1: since personal care decreases with y_1, it is optimal to have a higher level of labor supply than in the case where personal care can be controlled directly. Just like for s_1, this holds only as long as the altruistic children are myopic, i.e. $\gamma < 1$.

As far as the NA children are concerned, the analysis of their labor supply is the same as in the second-best scenario with observable personal care. Depending on the comparison between w_1 and w_2, they may face a downward, an upward or no distortion of their labor supply.

Turning to the decentralization of this second-best optimum, combining equation (31) with the altruistic child's FOC for s_1 (equation (23)), we get

$$\sigma^{SB} = \frac{-\frac{\partial a_1}{\partial s_1} n_1 (1-\gamma)\varphi'(a_1)}{(n_1+\lambda)u'(c_1)} \geq 0$$

Therefore, in this scenario we need a subsidy on formal care as long as $\gamma < 1$. This follows directly from the need to encourage more formal care in the altruistic families as discussed above.

Furthermore, combining (32) with the altruistic child's FOC for y_1 (equation (22)), we obtain

$$T'(y_1) = \frac{\frac{\partial a_1}{\partial y_1} n_1 (1-\gamma)\varphi'(a_1)}{u'(c_1)(n_1+\lambda)} \leq 0$$

This means that, as long as $\gamma < 1$, the altruistic children face a negative marginal tax, i.e. a marginal subsidy, on their income. This is again a direct consequence of the above discussed need to distort upwards the labor supply of these children. Together with the subsidy on formal care, the subsidy on labor supply replaces the tax on personal care provision in this setting where such a tax is impossible due to personal care being unobservable.

Finally, the results regarding the marginal tax for the NA children are the same as in the previous section.

The decentralization of the second-best optimum in the scenario with unobservable personal care is summarized in Proposition 3.

Proposition 3. *Assume that there are two types of individuals (altruists (type 1) and non-altruists (type 2)) and that the government cannot observe the individuals' altruism, their wage rates w_i, their labor supplies l_i, their leisure time z_i and their personal care provision a_i. The second-best optimum in this setting can be decentralized by:*

i) introducing a linear subsidy of rate $\sigma^{SB} = \frac{-\frac{\partial a_1}{\partial s_1} n_1(1-\gamma)\varphi'(a_1)}{(n_1+\lambda)u'(c_1)} \geq 0$ on formal care provision;

ii) providing a public nursing home LTC $g < m_1$ to the parents of the non-altruistic individuals;

iii) introducing a non-linear income tax $T(y_i)$ which finances the formal care subsidy and g, ensures that all the consumption levels are at their second-best optimal values and has the following properties:

a) $T'(y_1) \leq 0$;

b) $T'(y_2) = 0$ *if* $w_1 = w_2$, $T'(y_2) > 0$ *if* $w_1 > w_2$ *and* $T'(y_2) < 0$ *if* $w_1 < w_2$.

6 Conclusion

This paper has examined the optimal design of social policy for LTC in a setting where informal caregivers (children who have dependent parents) face significant physiological and psychological burden and may act myopically, systematically underestimating the health costs of their caregiving effort. The analysis distinguishes among myopic altruists, rational altruists, and non-altruists and characterizes both first-best and second-best optimal policies under varying informational constraints. A central finding is that myopic caregivers tend to supply excessive informal care, neglecting their own health in the process - an outcome that proves harmful both to caregivers themselves and to the overall efficiency of LTC provision.

Under full observability of individual characteristics, the optimal policy combines a linear tax on personal care provided by myopic caregivers, public provision of formal care for non-altruistic households and lump-sum transfers to ensure equity in consumption and care levels. Together, these instruments correct for myopia and align private incentives with social welfare.

When the government can observe personal care but not individual types, a tax on informal care provided by myopic agents remains optimal, though the corrective rate is lower than in the first-best. Non-linear income taxes and targeted LTC benefits are employed alongside it to preserve incentive compatibility and finance the system. If personal care is unobservable, direct taxation becomes infeasible. Policy must then rely on subsidies for formal care and labor income, supplemented by public LTC benefits for non-altruists - instruments that indirectly curb excessive informal caregiving and compensate for myopia through second-best means.

Taken together, these results show that caregiver myopia and informational constraints call for indirect policy instruments and careful incentive design. The optimal mix of taxes, subsidies and public benefits depends critically on which aspects of caregiving behavior and individual heterogeneity are observable to policymakers. By engaging both the behavioral biases of informal caregivers and the practical limits of policy implementation, the paper offers a unified framework for improving the efficiency and equity of LTC systems.

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Appendix A: comparative statics with respect to γ_i

Assuming interior solutions, fully differentiating equations (1)-(3) with respect to γ_i and combining, we obtain

$$\frac{\partial a_i}{\partial \gamma_i} = \frac{\varphi'(a_i)\left[u''(c_i)\frac{v''(z_i)}{w_i^2} + u''(c_i)H''(m_i) + H''(m_i)\frac{v''(z_i)}{w_i^2}\right]}{\left(1-\frac{\beta}{w_i}\right)^2 u''(c_i)H''(m_i)v''(z_i) - \gamma_i\varphi''(a_i)\left[u''(c_i)\frac{v''(z_i)}{w_i^2} + u''(c_i)H''(m_i) + H''(m_i)\frac{v''(z_i)}{w_i^2}\right]} < 0,$$

$$\frac{\partial s_i}{\partial \gamma_i} = -\frac{\partial a_i}{\partial \gamma_i}\left[\frac{u''(c_i)\frac{v''(z_i)}{w_i} + \beta u''(c_i)H''(m_i) + \beta H''(m_i)\frac{v''(z_i)}{w_i^2}}{u''(c_i)\frac{v''(z_i)}{w_i^2} + u''(c_i)H''(m_i) + H''(m_i)\frac{v''(z_i)}{w_i^2}}\right] > 0,$$

$$\frac{\partial y_i}{\partial \gamma_i} = \frac{u''(c_i)\frac{\partial s_i}{\partial \gamma_i} - \frac{v''(z_i)}{w_i}\frac{\partial a_i}{\partial \gamma_i}}{u''(c_i) + \frac{v''(z_i)}{w_i^2}} > 0.$$

Appendix B: first-best tax rate for the MA children

Let us consider a policy targeted at the MA children and consisting of a linear tax of rate t_1 on their personal care provision and a lump-sum tax/transfer T_1. The problem of a child of type MA in this setting is to maximize the following utility:

$$U_1 = u(y_1 - s_1 - t_1 a_1 - T_1) + v(1 - a_1 - \frac{y_1}{w_1}) + H(\beta a_1 + s_1) - \gamma\varphi(a_1)$$

This gives the following FOCs:

$$\frac{\partial U_1}{\partial y_1} = u'(c_1) - v'(z_1)/w_1 = 0$$

$$\frac{\partial U_1}{\partial s_1} = -u'(c_1) + H'(m_1) \leq 0$$

$$\frac{\partial U_1}{\partial a_1} = -u'(c_1)t_1 - v'(z_1) + \beta H'(m_1) - \gamma\varphi'(a_1) \leq 0 \tag{33}$$

Combining (33) with the first-best optimality condition (6), we get

$$t_1^{FB} = \frac{(1-\gamma)\,\varphi'(a_1^*)}{u'(c_1^*)} \geq 0$$

Appendix C: second-best optimum with three types

In this appendix, we come back to our initial setting with three individual types: MA (type 1), RA (type 2) and NA (type 3). We assume that the government cannot observe the type of an individual and keep the same informational assumptions as in Section 4. The direction of the self-selection constraints is less obvious in this case, but we propose an analysis of the second-best under the assumption that the MA may be tempted to mimic the RA, while the RA may be tempted to mimic the NA.

The problem of the government in this case can be expressed with the following Lagrangian:

$$\mathcal{L} = \sum_{1,2} n_i \left[u(c_i) + v(1 - a_i - \frac{y_i}{w_i}) + H(\beta a_i + s_i) - \varphi(a_i) \right] +$$

$$+ n_3 \left[u(c_3) + v(1 - \frac{y_3}{w_3}) + H(m_3) \right] - \mu \left(\sum_{1,2} n_i \left(c_i + s_i - y_i \right) + n_3 \left(c_3 + m_3 - y_3 \right) \right) +$$

$$+\lambda_1\left[u(c_1)+v(1-a_1-\frac{y_1}{w_1})+H(\beta a_1+s_1)-\gamma\varphi(a_1)-u(c_2)-v(1-a_2-\frac{y_2}{w_1})-H(\beta a_2+s_2)+\gamma\varphi(a_2)\right]+$$

$$+\lambda_2\left[u(c_2)+v(1-a_2-\frac{y_2}{w_2})+H(\beta a_2+s_2)-\varphi(a_2)-u(c_3)-v(1-\frac{y_3}{w_2})-H(m_3)\right]$$

where λ_1 and λ_2 are the multipliers associated with the self-selection constraints of the MA and the RA children respectively.

The FOCs are:

$$u'(c_1)(n_1+\lambda_1)=\mu n_1 \tag{34}$$

$$H'(m_1)(n_1+\lambda_1)=\mu n_1 \tag{35}$$

$$\left[\beta H'(m_1)-\varphi'(a_1)\right](n_1+\lambda_1)=\mu n_1 w_1-\lambda_1(1-\gamma)\varphi'(a_1) \tag{36}$$

$$\left(v'(z_1)/w_1\right)(n_1+\lambda_1)=\mu n_1 \tag{37}$$

$$u'(c_2)(n_2-\lambda_1+\lambda_2)=\mu n_2 \tag{38}$$

$$H'(m_2)(n_2-\lambda_1+\lambda_2)=\mu n_2 \tag{39}$$

$$\left[\beta H'(m_2)-\varphi'(a_2)-v'(z_2)\right](n_2+\lambda_2)-\lambda_1\left[\beta H'(m_2)-\gamma\varphi'(a_2)-v'(1-a_2-\frac{y_2}{w_1})\right]=0 \tag{40}$$

$$\left(v'(z_2)/w_2\right)(n_2+\lambda_2)=\mu n_2+\lambda_1 v'(1-a_2-\frac{y_2}{w_1})/w_1 \tag{41}$$

$$u'(c_3)(n_3-\lambda_2)=\mu n_3 \tag{42}$$

$$H'(m_3)(n_3 - \lambda_2) = \mu n_3 \tag{43}$$

$$\left(v'(z_3)/w_3\right) n_3 = \mu n_3 + \lambda_2 v'(1 - \frac{y_3}{w_2})/w_2 \tag{44}$$

We can see that the optimum implies

$$u'(c_1) = H'(m_1) < u'(c_3) = H'(m_3)$$

and thus

$$c_1 > c_3; m_1 > m_3$$

For type 2, we also have $u'(c_2) = H'(m_2)$, but the comparison of c_2 and m_2 with the respective variables of the other types is ambiguous.

As far as the labor supply is concerned, for the MA children we have the first-best trade-off

$$\frac{v'(z_1)}{u'(c_1)} = w_1,$$

meaning no need to distort the labor supply of these children.

For the RA children, using equation (41) we can obtain

$$\frac{v'(z_2)}{u'(c_2)} = \frac{w_2\left(1 - \frac{\lambda_1}{(n_2+\lambda_2)}\right)}{\left(1 - \frac{\lambda_1 w_2 v'(1-a_2-\frac{y_2}{w_1})}{(n_2+\lambda_2) w_1 v'(1-a_2-\frac{y_2}{w_2})}\right)} \lesseqqgtr w_2 \tag{45}$$

If $w_1 = w_2$, it can be easily seen that $\frac{v'(z_2)}{u'(c_2)} = w_2$, implying that the labor supply of the RA children is not distorted. On the other hand, if $w_1 > (<) w_2$, one can verify that $\frac{v'(z_2)}{u'(c_2)} < (>) w_2$, which implies a downward (upward) distortion of the RA children's labor supply. The intuition of these results is the same as in the discussion of altruistic and non-altruistic children in Section 4, with the difference that here we have the MA children potentially mimicking the RA ones.

For the NA children, using equation (44) we get

$$\frac{v'(z_3)}{u'(c_3)} = \frac{w_3\left(1 - \frac{\lambda_2}{n_3}\right)}{\left(1 - \frac{\lambda_2 w_3 v'(1-\frac{y_3}{w_2})}{n_3 w_2 v'(1-\frac{y_3}{w_3})}\right)} \lesseqqgtr w_3 \tag{46}$$

The NA children are potentially mimicked by the RA ones; thus, the results depend on the comparison between w_3 and w_2. If $w_2 > (<) w_3$, we have $\frac{v'(z_3)}{u'(c_3)} < (>) w_3$, implying a downward (upward) distortion of the NA children's labor supply. If $w_2 = w_3$, the NA children's labor supply is not distorted. Again, the same intuition as in Section 4 applies.

Turning to personal care, using equation (36) we can see that the second-best optimal level of personal care provided by the MA children is given by:

$$(\beta - w_1)\, H'(m_1) = \varphi'(a_1) \left[1 - \frac{\lambda_1 (1-\gamma)}{(n_1 + \lambda_1)}\right] \leq \varphi'(a_1) \tag{47}$$

This condition is the same as the one for altruistic children in Section 4 and implies that the second-best level of the MA children's personal care tends to be higher than the first-best one. This is because in the second-best the government has to prevent the MA children from mimicking the RA ones, and since the MA children perceive personal care more positively than the government, allowing a higher level of personal care makes the allocation of the MA children more attractive to them.

The most interesting result compared to Section 4 concerns the personal care provided by the RA children. Using equation (40), we can get the following condition for the second-best optimal level of this care:

$$(\beta - w_2)\, H'(m_2) = \varphi'(a_2) \frac{[n_2 + \lambda_2 - \lambda_1 \gamma]}{[n_2 + \lambda_2 - \lambda_1]} - \frac{\lambda_1}{[n_2 + \lambda_2 - \lambda_1]} v'(1 - a_2 - \frac{y_2}{w_1}) \left(\frac{w_1 - w_2}{w_1}\right) \gtreqless \varphi'(a_2) \tag{48}$$

The right-hand side of this equation consists of two terms. The first term, $\varphi'(a_2)\frac{[n_2+\lambda_2-\lambda_1\gamma]}{[n_2+\lambda_2-\lambda_1]}$, is greater than $\varphi'(a_2)$ and pushes for a downward distortion of the personal care provided by the RA. This is needed to prevent the MA from mimicking the RA. Indeed, the mimicker (MA child) perceives personal care more positively than the mimicked (RA child), so distorting the personal care of the RA downwards hurts the mimicker more than the mimicked and can thus improve social welfare. The second term of the equation depends on the comparison between the wage rates of the MA and the RA. If $w_1 > w_2$, this term is negative and pushes for a higher level of personal care. In that case, the mimicker (MA child) has a higher opportunity cost of providing personal care and is thus willing to provide less care than the mimicked (RA child), so distorting the personal care of the RA upwards hurts the mimicker more than the mimicked. Therefore, if $w_1 > w_2$, the two terms of the right-hand side of the equation push to opposite directions and it remains unclear whether the personal care of the RA is distorted downwards or upwards.

If $w_1 < w_2$, the second term is positive and pushes for a lower level of personal care. In that case, the mimicker (MA child) has a lower opportunity cost and is willing to provide more care than the mimicked (RA child), so distorting the personal care of the RA downwards hurts the mimicker more. Thus, the two terms push to the same direction in that case, which implies a clear-cut downward distortion of the personal care provided by the RA.

Finally, if $w_1 = w_2$, the second term disappears since both the mimicker and the mimicked have the same opportunity cost of providing personal care, so, in that respect, a distortion cannot improve social welfare. However, the first term, which appears due to the myopia of the MA, still implies a downward distortion in that case.

In a similar way as in Section 4, one can then verify that this second-best optimum can be decentralized by the following instruments:

i) a marginal tax on the personal care provision of the MA children (but at a lower rate than in the first-best);

ii) a marginal tax on the personal care provision of the RA children if $w_1 \leq w_2$ and a marginal tax or a marginal subsidy if $w_1 > w_2$;

iii) a public LTC benefit to the parents of the NA children;

iv) a non-linear income tax which finances public LTC, ensures that all the consumption levels are at their second-best optimal values and has the following properties:

a) zero marginal tax for the MA children;

b) zero (positive, negative) marginal tax for the RA children if $w_1 = (>, <) w_2$;

c) zero (positive, negative) marginal tax for the NA children if $w_2 = (>, <) w_3$.

Appendix D: $\partial a_1/\partial s_1$ and $\partial a_1/\partial y_1$ with unobservable personal care

Using equation (24), we can obtain

$$\frac{\partial a_1}{\partial s_1} = -\frac{\beta H''(m_1)}{v''(z_1) + \beta^2 H''(m_1) - \gamma \varphi''(a_1)} < 0$$

and

$$\frac{\partial a_1}{\partial y_1} = -\frac{v''(z_1)/w_1}{v''(z_1) + \beta^2 H''(m_1) - \gamma \varphi''(a_1)} < 0$$