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Optimal Compression and Transmission Policies for Energy Harvesting Nodes

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***Abstract*—We consider an energy harvesting transmitter which may need to compress received packets before forwarding them over a flat fading channel. Data compression is required to meet the bandwidth or energy constraint at the cost of data distortion. The objective is to design optimal compression and transmission policies, namely optimal transmission and compression powers, transmission and compression rates and transmission and compression times, such that the total distortion is minimized. In this paper, we consider a time slotted system where new data and energy packets arrive at the beginning of each time slot (TS) and channel gains are assumed to remain constant during each TS. Under the assumption that the energy and data arrivals and channel gains are known non-causally which corresponds to off-line optimization, we formulate the compression and transmission scheduling optimization as a convex optimization problem and characterize the properties of optimal scheduling. For the strict delay case where the transmission and compression of each packet must be executed within the corresponding TS, we provide an iterative algorithm which mimics the iterative directional water-filling (IDWF) algorithm. Numerical results are provided to illustrate our results and the properties of optimal scheduling.**

I. Introduction

In any large scale wireless network which accommodates a high traffic volume of data, data compression is inevitable to meet the bandwidth and energy constraints leading thereby to data distortion. Although data compression can reduce the transmission energy, it adds computational burdens on nodes, which makes the management of extremely limited energy resources very important in order to ensure minimum distortion.

Energy harvesting (EH) alleviates the energy limitations of the wireless devices by harvesting the energy they need from the environment. Due to the time-varying nature of harvested energy, energy management of an EH node is more complex than that of battery-operated node because it requires to jointly consider energy harvesting, channel and data arrival processes in order to minimize the total distortion. Recently, the problem of distortion minimization of an EH node has received considerable attention (see [2]–[4], [7]–[9], [11], and references therein). Most of the literature focuses on the derivation of optimal transmission strategies without taking into account the time and energy allocated for the compression process. Among these works, the work in [2] is of particular interest and is closely related to the current work. In [2], the authors have considered the compression time and the compression energy as the scheduling parameters under the assumption

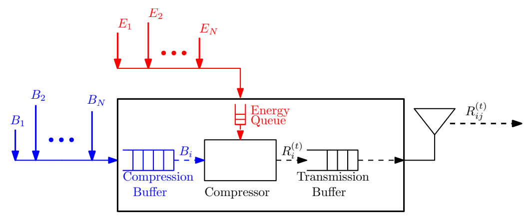

Fig. 1. A EH transmitter where the received packets can be compressed before transmission.

that the compression power is chosen at the beginning of the system operation and remains constant across the whole duration of transmission. However, both compression powers and compression duration time slots need to be judiciously optimized to ensure the minimum total distortion. In this work, we model the amount of time and energy consumption for compression as functions of the frequency of CPU scheduled for the compression process, which can be adjusted based on the current status of the compression and transmission data buffers, the battery, the harvested energy and the data arrivals to minimize the distortion. To the best of our knowledge, this study is amongst the first works to simultaneously optimize the compression energy and time, modeled as functions of CPU frequency. Depending on the deadline on the packet transmission, we consider two different cases: the first case corresponds to no delay constraint, and the second case corresponds to the strict delay constraint when a packet needs to be transmitted within the following TS. For the first case, we derive some properties of optimal policy which mimics the classical water-filling interpretations. For the second case, similarly to [2], we crave out an IDWF algorithm to calculate the optimal compression and transmission rate, and the compression and transmission time.

II. System Model

We consider an energy harvesting (EH) transmitter node, depicted in Fig. 1, which receives packets from different source nodes, compresses them if needed and forwards them to the destination by a deadline T. We assume that time is slotted, and we denote the time slot lengths by $\{\ell_i\}$. At the beginning of slot $i \in \{1, \cdots, N\}$, the node receives a data packet of size $\{B_i\}$ bits, and harvests an energy packet of size $\{E_i\}$. Due to the limited availability of energy, the node may not be able to transmit all packets, and thus has to compress them prior to transmission. Upon reception of a new packet, the node first stores it in the compression data

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buffer and then passes it to the compression algorithm. The compressed packet is then stored in the transmission buffer, waiting to be transmitted. Stored in a battery, a received energy packet is used for compression and transmission of packets. In this paper, the sizes of data buffers and battery are assumed to be infinite, but the analysis can easily be extended to the case of finite sizes. The node compresses and transmits the packets according to a schedule that determines how the available time and energy should be shared between compression and transmission.

Let $R_i^{(t)}$ denote the size of the i-th compressed packet, and let $R_i^{(c)} \overset{\text{def}}{=} B_i - R_i^{(t)}$ denote the number of bits removed during the compression, respectively. Assuming that the node can not simultaneously transmit and compress a packet, we denote the transmission duration and compression duration within the i-th TS by $t_i^{(t)}$ and $t_i^{(c)}$. Similar to [7], the exponential function is used in this paper to measure the amount of distortion $\mathcal{D}_i$ for packet i: $\mathcal{D}_i = 2^{B_i - R_i^{(t)}}$, which is a decreasing and convex function of the size of i-th compressed packet. The total distortion is thus given by $\mathcal{D} \overset{\text{def}}{=} \sum_{i=1}^N \mathcal{D}_i$.

A. Packet Compression

Upon reception of a packet, the node employs the compression algorithm to reduce the number of bits to be transmitted. For the sake of tractability, we assume that the packet must be compressed before the arrival of the next packet. Let w denote the number of CPU cycles required for compressing one bit, and let $r_i^{(c)}$ denote the compression ratio, defined by $r_i^{(c)} \overset{\text{def}}{=} R_i^{(c)}/B_i$. As shown in [10, Figure 3.b], for lossy compression algorithms such as Polynomial Regression or Auto-Regressive Moving Average methods, the number of clock cycles per bit to compress the received packet can be expressed as $w(r_i^{(c)}) = \eta_1 r_i^{(c)} + \eta_2$, where η_1, η_2 are two constants depending on the compression algorithm. Since w is an increasing function of $r_i^{(c)}$, higher compression rates require more compression energy per bit. Thus, there is a trade-off between transmission and compression energy costs: although the packet compression reduces the transmission energy, much packet compression may consume more energy than transmitting raw data. For the sake of simplicity, we here neglect the constant term η_2[1]. Consequently, the number of cycles required to compress a packet of size B_i to that of size $R_i^{(t)}$, denoted by $W(R_i^{(c)}, B_i)$, is given by: $W(R_i^{(c)}, B_i) = B_i w(r_i^{(c)})$. Denoting by f_i the frequency scheduled for the compression of i-th packet, the execution time $t_i^{(c)}$ and the energy consumption $E_i^{(c)}$ for the i-th packet compression can be expressed as [6]:

$$E_i^{(c)} = \eta_h W(R_i^{(c)}, B_i) f_i^2 \,, \;\; t_i^{(c)} = \frac{W(R_i^{(c)}, B_i)}{f_i} \,, \tag{1}$$

respectively, where η_h is a constant related to the hardware architecture.

[1]The general case may easily be derived by a change of variable: $\tilde{r}_i^{(c)} = r_i^{(c)} + \frac{\eta_2}{\eta_1}$

We define the compression capacity $c_i^{(c)}$ as the maximum number of bits per second that can be compressed for a given compression power $p_i^{(c)} \geq 0$, which according to eq. (1) can be expressed as $c_i^{(c)} = \frac{(p_i^{(c)})^{1/3}}{\eta}$, where $\eta \overset{\text{def}}{=} \sqrt[3]{\eta_h}\eta_1$. Hence, the number of compressed bits within the i-th TS must satisfy:

$$R_i^{(c)} \leq C_i^{(c)} \,, \;\; \forall i \in \{1, \cdots, N\} \tag{2}$$

where $C_i^{(c)} \overset{\text{def}}{=} t_i^{(c)} c_i^{(c)}$. We note that at the optimal solution, these inequalities must be satisfied with equality.

B. Packet Transmission

We denote by $R_{ij}^{(t)}$ the portion of i-th compressed packet transmitted in the j-th TS. Obviously, we have $R_i^{(t)} = \sum_{j=i}^N R_{ij}^{(t)}$. The instantaneous rate during the epoch i is given by $c_i^{(t)} \overset{\text{def}}{=} \frac{1}{2}\log_2(1 + \frac{h_i p_i^{(t)}}{n_i})$, where h_i, n_i and $p_i^{(t)}$ are the channel gain, the noise power and the transmission power during epoch i, respectively. Denoting the amount of transmitted data within the i-th TS as $C_i^{(t)} = t_i^{(t)} c_i^{(t)}$, where $t_i^{(t)}$ is the transmission slot time during the i-th time slot, we have:

$$\sum_{i=1}^{j} R_{ij}^{(\mathrm{t})} \leq C_j^{(t)} \,, \quad \forall j \in \{1, \cdots, N\}. \tag{3}$$

As shown in [7, Appendix], the Fourier-Motzkin elimination technique can be applied to show that the above constraints in eq. (3) are equivalent to the following set of inequalities:

$$\sum_{j=i}^{N} R_j^{(t)} \leq \sum_{j=i}^{N} C_j^{(t)} \,, \quad \forall i \in \{1, \cdots, N\} \tag{4}$$

III. Optimal Compression and Transmission Schedule

Our goal is to find an optimal scheduling for the compression $\{t_i^{(c)}, R_i^{(c)}, p_i^{(c)}\}$ and transmission $\{t_i^{(t)}, R_i^{(t)}, p_i^{(t)}\}$ policies that minimizes the total distortion $\mathcal{D}$, which can be formulated as:

$$\min_{t_i^{(t)}, R_i^{(t)}, C_i^{(t)}, t_i^{(c)}, R_i^{(c)}, C_i^{(c)}} \sum_{i=1}^{N} 2^{B_i - R_i^{(t)}}$$

$$\text{s.t.} \; \sum_{j=1}^{i} \left[t_j^{(t)} \left(\frac{2^{C_j^{(t)}/t_j^{(t)}} - 1}{h_j} \right) + t_j^{(c)} \left(\frac{\eta C_j^{(c)}}{t_j^{(c)}} \right)^3 \right] \leq \sum_{j=1}^{i} E_j \,, \tag{5a}$$

$$\sum_{j=i}^{N} R_j^{(t)} \leq \sum_{j=i}^{N} C_j^{(t)} \,, \tag{5b}$$

$$R_j^{(c)} \leq C_j^{(c)} \,, \tag{5c}$$

$$R_i^{(t)} + R_i^{(c)} \geq B_i \,, \tag{5d}$$

$$R_i^{(t)} \leq B_i, \;\; R_i^{(c)} \geq 0 \,, \tag{5e}$$

$$t_i^{(t)} + t_i^{(c)} \leq \ell_i \,, \tag{5f}$$

$$t_i^{(t)} \geq 0, \;\; t_i^{(c)} \geq 0 \,, \tag{5g}$$

where (5a) represents the energy causality constraint, which implies that the energy consumed for transmission and compression should not exceed the total harvested energy. The constraints in (5d) and (5e) ensure that the number of compressed and transmitted bits of each packet is equal to the size of packet. We note that at the optimal solution, the constraint in (5d) must be satisfied with equality for any i, otherwise, we can increase the number of transmitted bits by reducing the compression energy which decreases the total distortion. The constraints in (5f) and (5g) guarantee that the sum of compression and transmission durations is less than the length of their corresponding time slot. Due to the convexity of transmission and computation power-rate functions, the constraint in (5f) must also be satisfied with equality.

It can easily be verified that the above optimization problem is convex since the objective function is convex in $\{t_i^{(t)}, R_i^{(t)}, C_i^{(t)}, t_i^{(c)}, R_i^{(c)}, C_i^{(c)}\}$ and the constraints are either affine or convex functions in the optimization parameters. Hence the KKT conditions can be used to derive properties of the optimal solution. The Lagrangian of (5) is defined as follows:

$$\begin{aligned}\mathcal{L} = &\sum_{i=1}^{N} 2^{B_i - R_i^{(t)}} \qquad (6)\\ &+ \sum_{i=1}^{N} \lambda_i \left(\sum_{j=1}^{i} E_j^{(t)}(t_j^{(t)}, C_j^{(t)}) + E_j^{(c)}(t_j^{(c)}, C_j^{(c)}) - E_j \right)\\ &+ \sum_{i=1}^{N} \gamma_i^{(t)} \left(\sum_{j=i}^{N} R_j^{(t)} - C_j^{(t)} \right) + \sum_{i=1}^{N} \gamma_i^{(c)} (R_i^{(c)} - C_i^{(c)})\\ &+ \sum_{i=1}^{N} \beta_i^{(t)} (R_i^{(t)} - B_i) - \sum_{i=1}^{N} \beta_i^{(c)} R_i^{(c)}\\ &+ \sum_{i=1}^{N} \kappa_i (B_i - R_i^{(t)} - R_i^{(c)}) - \sum_{i=1}^{N} \xi_i^{(t)} C_i^{(t)} - \sum_{i=1}^{N} \xi_i^{(c)} C_i^{(c)}\\ &- \sum_{i=1}^{N} \delta_i^{(t)} t_i^{(t)} - \sum_{i=1}^{N} \delta_i^{(c)} t_i^{(c)} + \sum_{i=1}^{N} \psi_i (t_i^{(t)} + t_i^{(c)} - \ell_i) .\end{aligned}$$

where $\{\lambda_i, \gamma_i^{(t)}, \gamma_i^{(c)}, \beta_i^{(t)}, \beta_i^{(c)}, \kappa_i, \xi_i^{(t)}, \xi_i^{(c)}, \delta_i^{(t)}, \delta_i^{(c)}, \psi_i\}$ are the non-negative Lagrangian multipliers corresponding to (5a)-(5g), respectively.

A. Optimal Distortion Allocation

Now, we apply the KKT optimality conditions to characterize the properties of the optimal compression and transmission policy. Taking the derivative of (6) w.r.t $R_i^{(t)}$ and $R_i^{(c)}$ and imposing the relevant complementary slackness conditions, we can obtain the optimal i-th packet distortion $\mathcal{D}_i^* = 2^{B_i - R_i^{(t)}}$ as:

$$\mathcal{D}_i^* = \begin{cases} 1 & \text{if } \zeta_i < 1 \\ \zeta_i & \text{if } 1 \le \zeta_i \le 2^{B_i} \\ 2^{B_i} & \text{if } \zeta_i > 2^{B_i} , \end{cases} \qquad (7)$$

where $\zeta_i \overset{\text{def}}{=} \frac{\sum_{j=1}^{i} \gamma_j^{(t)} - \gamma_i^{(c)}}{\ln 2}$. Note that ζ_i mimics the reverse water level in the optimal solution for distortion minimization of parallel Gaussian channels [5, Chapter 5], with this difference, that here the reverse water level ζ_i is not independent of i and is a function of the Lagrangian multipliers associated with the constraints on compression and transmission buffers corresponding to the previous time slots.

B. Optimal Power Allocation

Differentiating the Lagrangian function w.r.t $C_i^{(t)}$ and $C_i^{(c)}$ and considering the relevant complementary slackness conditions, we get

$$p_i^{(t)} = \left[\pi_i^{(t)} - \frac{n_i}{h_i} \right]^+ , \qquad (8)$$

and

$$p_i^{(c)} = \sqrt{\pi_i^{(c)}} , \qquad (9)$$

where $\pi_i^{(t)}$ and $\pi_i^{(c)}$ are defined by

$$\pi_i^{(t)} \overset{\text{def}}{=} \frac{\sum_{j=1}^{i} \gamma_j^{(t)}}{\ln(2) \sum_{j=i}^{N} \lambda_j} , \; \pi_i^{(c)} \overset{\text{def}}{=} \frac{\gamma_i^{(c)}}{3\eta^3 \sum_{j=i}^{N} \lambda_j} ,$$

respectively. Note that $\pi_i^{(t)}$ and $\pi_i^{(c)}$ can be interpreted similarly to the water levels of directional water-filling [7], where both water levels depend on the time slot index i and the interdependency of these two water levels is mediated through the Lagrangian multipliers $\{\lambda_j\}_{j=i}^{N}$ associated with the energy causality constraints corresponding to the next time slots.

Lemma 1. *The water levels $\{\pi_i^{(t)}\}$ constitute a non-decreasing sequence. Moreover, whenever $\pi_i^{(t)}$ increases from TS i to TS $i+1$, either transmission buffer or the battery depletes at the end of TS i.*

Now, we consider the timing parameters, i.e., $t_i^{(t)}$ and $t_i^{(c)}$. Taking the derivative of the Lagrangian w.r.t $t_i^{(t)}$ and $t_i^{(c)}$ and imposing the complementary slackness conditions, we can obtain the following lemma, which provides a one-to-one map between the optimal compression capacity and the optimal transmission capacity.

Lemma 2. *For a given transmission capacity $c_i^{(t)}$, the compression capacity at slot i is given by:*

$$c_i^{(c)} = \frac{1}{\eta \sqrt[3]{2h_i/n_i}} \sqrt[3]{W(c_i^{(t)})} , \qquad (10)$$

where $W(r) \overset{def}{=} (\ln 2 \times r - 1)2^r + 1$.

C. Optimal Solution

As discussed in the previous section, the optimal size of compressed packet (7), the optimal transmission and computational powers, (8) and (9), are coupled by the set of Lagrangian variables $\{\boldsymbol{\lambda}, \boldsymbol{\gamma}^{(\mathrm{t})}, \boldsymbol{\gamma}^{(\mathrm{c})}\}$. Since problem (5) satisfies the strong duality [1, Sec. 6.1], the projected sub-gradient method can be used to solve the corresponding dual problem in order to find the optimal compression and transmission scheduling, as follows. At the $n+1$-th iteration and for given values of $\{\boldsymbol{\lambda}, \boldsymbol{\gamma}^{(\mathrm{t})}, \boldsymbol{\gamma}^{(\mathrm{c})}\}(n)$, the primal variables $R_i^{(t)}(n+1)$, $p_i^{(t)}(n+1)$ and $p_i^{(c)}(n+1)$ are computed as in (7), (8), (9), respectively, and the remaining optimization variables are calculated according to (5d), (5f) and Lemma 2. The dual variables are then updated as follows:

$$\lambda_i(n+1) = \left[\lambda_i(n) + \alpha_1(n)\left(\sum_{j=1}^{i} E_j^{(t)}(n) + E_j^{(c)}(n) - E_j\right)\right]^+$$

$$\gamma_i^{(t)}(n+1) = \left[\gamma_i^{(t)}(n) + \alpha_2(n)\left(\sum_{j=1}^{i} R_j^{(t)}(n) - C_j^{(t)}(n)\right)\right]^+$$

$$\gamma_i^{(c)}(n+1) = \left[\gamma_i^{(c)}(n) + \alpha_3(n)(R_i^{(c)}(n) - C_i^{(c)}(n))\right]^+,$$

where $\{\alpha_i(n)\}$ denotes the corresponding step size.

IV. Compression and Transmission in a Single Epoch

In this section, we assume that a data packet arriving at the beginning of a time slot must be transmitted within that time slot. This scenario corresponds to the case of strictly delay-constrained streaming or the case of very small data buffer size. In this case we have

$$R_i^{(t)} = C_i^{(t)}, \quad R_i^{(c)} = C_i^{(c)}. \tag{11}$$

The Lagrangian function corresponding to this case can be obtained by replacing (11) in (6). By using the Lagrangian multiplier method, we can characterize the optimal compression and transmission as follows. Assume that at the i-th time slot, $R_i^{(t)} \neq B_i$. For this time slot, we can show that the optimal transmission time and size of compressed packet must satisfy the following condition:

$$\Omega_i(R_i^{(t)}, t_i^{(t)}) \overset{\text{def}}{=} 2^{R_i^{(t)} - B_i} \times \tag{12}$$

$$\left[\frac{\ln 2}{h_i/n_i} 2^{R_i^{(t)}/t_i^{(t)}} - 3\eta\left(\sqrt[3]{\frac{W(R_i^{(t)}/t_i^{(t)})}{2h_i/n_i}}\right)^2\right]$$

$$= \frac{\ln 2}{\sum_{j=i}^{N} \lambda_j}, \tag{13}$$

where $\{\lambda_j\} \geq 0$ are the Lagrange multipliers corresponding to (5a). From (12), it is clear that the $\{\Omega_i \overset{\text{def}}{=} \Omega_i(R_i^{(t)}, t_i^{(t)})\}_{i=1}^N$ form a well-defined and non-decreasing sequence since $\lambda_i \geq 0$, and $\lambda_N > 0$. Moreover, form (12), we can easily conclude that whenever Ω_i increases from TS i to TS $i+1$, the battery is depleted at the end of TS i, that is, $\lambda_i > 0$. Based on this observation, we can interpret $\{\Omega_i\}$ as *compression-transmission water levels* by noting that, similarly to [7], these water levels have to be equalized as much as possible among time slots. However, due to the non-linear relationship between Ω_i and the total amount of consumed energy at TS i, denoted by $E_i^{(\text{tot})} \overset{\text{def}}{=} E_i^{(t)} + E_i^{(c)}$, we need to handle the energy redistribution among the slots by an iterative method described as follows.

Iterative Directional Water-Filling

Similar to [2], we devise an iterative directional water-filling algorithm based on the following principles:

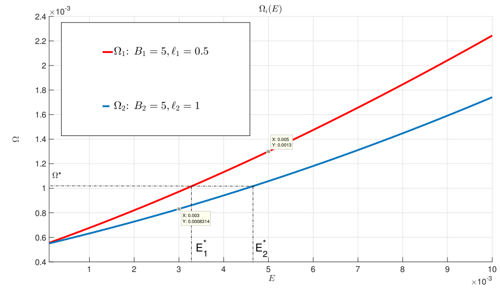

Fig. 2. The optimal energy redistribution for 2 adjacent slots.

- The harvested energy must be exhausted at the end of last time slot.
- The optimal water levels $\{\Omega_i\}$ are non-decreasing, and $\Omega_i = \Omega_{i+1}$ if the battery is not exhausted at the end of slot i.

First, we construct an initial feasible solution for the problem (5) with additional constraints (11) by decoupling the problem into N single-slot optimization problems where the i-th single-slot optimization problem is equivalent to the i-th packet distortion minimization where the harvested energy E_i is exhausted within that slot. For any given positive values of $\{\ell_i, B_i, E_i, h_i, n_i\}$, we define the following two functions:

$$G_i^{(\text{b})}(t, r) \overset{\text{def}}{=} tr + (\ell_i - t)\frac{1}{\eta\sqrt[3]{2h_i/n_i}}\sqrt[3]{W(r)} - B_i, \tag{14}$$

$$G_i^{(\text{e})}(t, r) \overset{\text{def}}{=} \frac{t2^{r+1} + (\ell_i - t)W(r) - 2}{2h_i/n_i} - E_i, \tag{15}$$

for any $t \in (0, \ell_i]$ and $r \in (0, B/\ell_i]$.

Lemma 3. *Within a time slot of duration ℓ_i, the optimal $\{t_i^{(t)}, R_i^{(t)}\}$ for the distortion minimization of a packet of size B_i by using E_i amount of energy for compression and transmission over a fading channel with channel gain h_i and noise power n_i is given by*

$$(t_i^{(t)}, R_i^{(t)}) = \begin{cases} (\ell_i, B_i/\ell_i) & \text{if } B_i \leq \ell_i \log\left(1 + \frac{h_i E_i}{n_i \ell_i}\right) \\ (t_i^*, t_i^* \times r_i^*) & \text{otherwise}, \end{cases}$$

where (t_i^*, r_i^*) is the unique solution of this pair of equations:

$$G_i^{(b)}(t, r) = 0, \quad G_i^{(e)}(t, r) = 0.$$

The optimal compression and transmission algorithm, given in Algorithm 1, mimics the iterative directional water-filling algorithm, where the time slots are separated by walls with right permeable water tap in each wall which allows energy to flow only to the right. This algorithm starts with the set of initial points $(t_i^{(t)}, R_i^{(t)})$ computed according to Lemma 3, and the corresponding Ω_i for all $i \in \{0, \cdots, N\}$. Then, we check if there exists any set of slots $\{i, \cdots, j+k+1\}$ such that $\Omega_i = \cdots = \Omega_{i+j} > \Omega_{i+j+1} = \cdots = \Omega_{i+j+k}$. According to eq. (12), the energy among these slots should be optimally redistributed to have $\Omega_i = \cdots = \Omega_{i+j+k} = \Omega^*$, for a positive Ω^*. Contrary to the water levels adjustment in [7], we can not simply set $\Omega^* = \frac{\sum_{m=i}^{i+j+k} \Omega_m}{i+j+k}$, since the corresponding energy

redistribution may violate the energy causality constraint due to the non-linearity of $E_i^{(\text{tot})}(\Omega)$, where $E_i^{(\text{tot})}$ denotes the total amount of consumed energy in the i-th slot for a given set of $\{B_i, \ell_i, h_i\}$. To this end, we need first to establish a one-to-one map between Ω and $E_i^{(\text{tot})}$. The existence of one-to-one map between $E_i^{(\text{tot})}$ and Ω is guaranteed by the fact that for any given energy budget $E_i^{(\text{tot})}$, the corresponding optimal pair of $\{t_i^{(t)}, R_i^{(t)}\}$ is unique and can be computed according to Lemma 3. The redistribution of energy between time slots can be done as follows. Given the set of functions $E_m^{(\text{tot})}(\Omega)$ for all $m \in \{i, \cdots, i+j+k\}$, we search for the unique Ω^* such that $\sum_{m=i}^{i+j+k} E_m^{(\text{tot})}(\Omega^*) = \sum_{m=i}^{i+j+k} E_m^{(\text{tot})}(\Omega_m)$. This search is accomplished by moving along Ω-axis as shown in Fig. 2. Then, we set $E_m^{(\text{tot})} = E_m^{(\text{tot})}(\Omega^*)$ for $m \in \{j, \cdots, i+k\}$, and we use Lemma 3 to adjust the optimization variables $\{t_m^{(\text{t})}, R_m^{(\text{t})}\}$. We repeat this process until we obtain a non-decreasing sequence of $\{\Omega_i\}$.
A more tractable result can be derived for the special case when $B_1 = \cdots = B_N$, $\ell_1 = \cdots = \ell_N$, $h_1 = \cdots = h_N$ and $n_1 = \cdots = n_N$, as follows.

Lemma 4. *When $B_1 = \cdots = B_N$, $h_1 = \cdots = h_N$, $n_1 = \cdots = n_N$, and $\ell_1 = \cdots = \ell_N$, the optimal size of compressed packets, $\{R_i^{(t)}\}_{i=1}^N$ constitutes a non-decreasing sequence.*

Proof of Lemma 4: For this case, the function $E^{(\text{tot})}(\Omega)$ is the same for all slots, and thus the monotonicity of $\{\Omega_i\}$ implies the monotonicity of $\{R_i^{(t)}\}$. ■

Algorithm 1 Off-line optimal scheduling for the delay-constrained case

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Procedure: OFF-Line Scheduling

Initialisation :

1: Set $E_i^{(\text{tot})} = E_i$.
2: Compute $\{\boldsymbol{t}^{\text{t}}, \boldsymbol{r}^{\text{t}}\}$ according to Lemma 3, and the corresponding $\{\Omega_i\}_{i=1}^N$.
3: Search for slots $[l, \cdots, l+m+n]$ such that $\Omega_l = \cdots = \Omega_{l+m} > \Omega_{l+m+1} = \cdots = \Omega_{l+m+n}$.
4: Redistribute the total consumed energy within $[l, \cdots, l+m+n]$ as described in 6-8.
5: Repeat steps 2-4 until we obtain $\Omega_1 \leq \cdots \leq \Omega_N$.
6: Based on $E_i^{(\text{tot})}(\Omega_i)$ functions, search for Ω^* such that $\sum_{i=l}^{l+m+n} E_i^{(\text{tot})}(\Omega^*) = \sum_{i=l}^{l+m+n} E_i^{(\text{tot})}(\Omega)$.
7: Set $E_i^{(\text{tot})} = E_i^{(\text{tot})}(\Omega^*)$, $\forall i \in [l, \cdots, l+m+n]$.
8: Compute $\{t_i^{(t)}, R_i^{(t)}\}_{i=l}^{l+m+n}$ according to Lemma 3.

V. Numerical Results

In this section, we provide numerical results to show how our IDWF algorithm allocates time and energy for compression and transmission and compare its performance with the greedy algorithm where the energy harvested at the beginning of the slot is exhausted within that slot, which is equivalent to the initial feasible solution of Algorithm 1. We set $N = 10$, $h_i = 1$, $\ell_i = 1$s, $B_i = 10$ bits, $\eta = 10^{-4}$ and noise power of channel 5mW. Note that we have here normalized the actual value of η w.r.t the bandwidth: for $\eta_1 = 100$, $\eta_h = 10^{-36}$ and

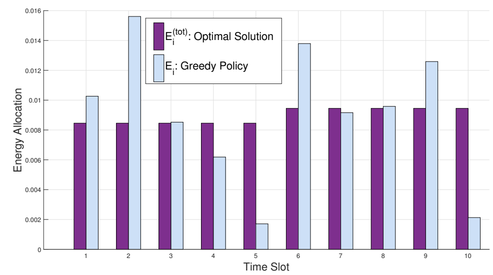

Fig. 3. The redistributed energy with the optimal-offline (left blocks) and the greedy (right blocks) policies.

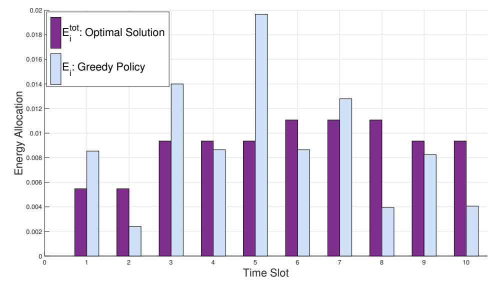

Fig. 4. The redistributed energy with the optimal-offline (left blocks) and the greedy (right blocks) policies for $\boldsymbol{\ell} = [1, 1, 1, 1, 1, 2, 2, 2, 1, 1]$. The optimal value of $\boldsymbol{\Omega}$ satisfies: $\Omega_1 = \Omega_2 < \Omega_3 = \cdots = \Omega_{10}$.

the bandwidth $W = 10^6$, from eq. (1), the normalized η is given by $\eta = \eta_1 \sqrt[3]{\eta_h} W = 10^{-4}$.
The IDWF solution is shown in Fig. 3. As noted in Lemma 4, we observe that the amount of energy consumption, and therefore the size of compressed packets form a non-decreasing sequence. Comparing to the greedy solution, we see also that in the optimal solution, the energy is allocated equally among the slots to the extent possible. Also, we note that to achieve this fair energy budget distribution, some amount of energy has been transferred from slots with high amount of harvested energy (e.g. the first two slots) to the slots with less amount of harvested energy (the fourth and fifth slots). Now, we let the durations of slots vary to see how the variation of slot durations affects the optimal policy. We assume that the slot duration can be either 1s or 2s. In Fig. 4, the energy distribution of the optimal policy is shown. Firstly, we observe that in this scenario, although the energy consumption $\boldsymbol{E}^{(\text{tot})}$ does not form a non-decreasing sequence over all slots, in the adjacent slots with the same duration, the sub-sequence of their corresponding energy consumption is a non-decreasing sequence (e.g. the first fifth slots). Secondly, we note that among the slots with the same value of Ω (e.g. the fifth and sixth slots), the slot with longer duration consumes more energy which is due to the fact that in the optimal solution, within two TSs with the same value of water level Ω, the higher the ℓ_i, the higher is the number of transmitted bits, which in turn implies that more energy is allocated to that time slot, i.e., $E_i^{(\text{tot})}(\Omega) \leq E_j^{(\text{tot})}(\Omega)$ if $\ell_i \leq \ell_j$.

In Fig. 5, we illustrate the variation of the total distortion

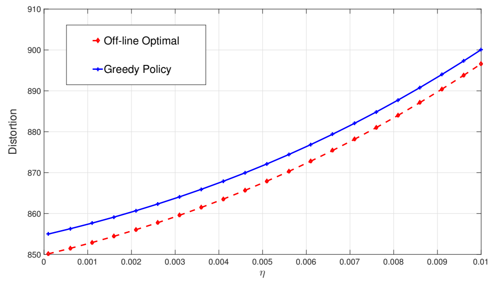

Fig. 5. Distortion versus η for $N = 5$, $h_i = 1$, noise power 5mW and $\boldsymbol{E} = \{0.0153, 0.0053, 0.0022, 0.0156, 0.0138\}$.

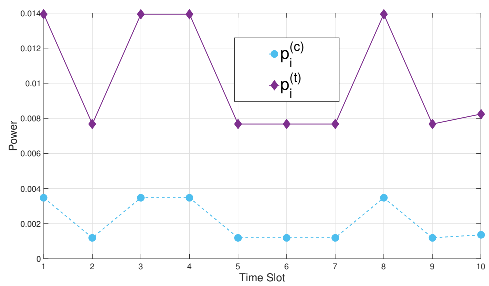

Fig. 6. Optimal compression and transmission powers $\{\boldsymbol{p}^{(c)}, \boldsymbol{p}^{(t)}\}$ for $N = 10$, $h_i = 1$, $B_i = 10$ noise power 5mW, $\boldsymbol{E} = \{1.69, 1.26, 1.19, 0.72, 0.97, 1.46, 1.78, 1.47, 0.14, 1.38\} \times 10^{-2}$ and $\eta = 0.005$.

with η. As we can see, the average distortion increases with η, which is due to the fact that as η increases, for a given compression rate and given amount of time and energy, we need to increase the compression power, and therefore, decrease the transmission power which results in a lower number of transmitted bits, and hence, more data distortion. Finally, we present in Fig. 6 and in Fig. 7, the optimal compression power and the optimal compression and transmission durations for $\eta = 5 \times 10^{-3}$, respectively. As we can see in Fig. 6, the optimal compression power varies from slot to slot which illustrates that the fixed compression power assumption in [2] is not optimal for the distortion minimization problem. Also, we can observe from Fig. 7 that for the longer slots with higher amounts of allocated energy, compression time durations increase in order to decrease the compression energy which renders the saved energy available for more transmission.

VI. Conclusion

The offline-packet distortion minimization problem for an EH node has been investigated for a time-slotted system under two different scenarios. For the non-delay scenario, we have obtained structural properties of optimization variables, i.e. compression time/rate and transmission time/rate, reminiscent of the properties obtained in the literature of EH node optimal scheduling. For the second scenario, i.e. strict delay constraint, we have craved out an IDWF algorithm where the energy distribution among the slots is meticulously handled in order to satisfy the causality constraint. Analytical and numerical

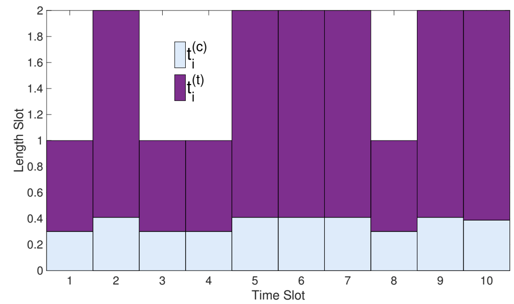

Fig. 7. Optimal compression and transmission times $\{\boldsymbol{t}^{(c)}, \boldsymbol{t}^{(t)}\}$ for $N = 10$, $h_i = 1$, $B_i = 10$, noise power 5mW, $\boldsymbol{E} = \{1.69, 1.26, 1.19, 0.72, 0.97, 1.46, 1.78, 1.47, 0.14, 1.38\} \times 10^{-2}$ and $\eta = 0.005$.

results show that when all other parameters are equal except the amount of harvested energy at the beginning of slots, the optimal policy tends to increase the allocated energy and thus, the size of compressed packets as time increases.

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