



UNIVERSITÉ CATHOLIQUE DE LOUVAIN
ÉCOLE POLYTECHNIQUE DE LOUVAIN
INSTITUT DE MÉCANIQUE, MATÉRIAUX ET GÉNIE CIVIL

FINITE ELEMENT METHODS FOR SEA ICE MODELING

DOCTORAL DISSERTATION PRESENTED BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR IN APPLIED SCIENCES

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LOUVAIN-LA-NEUVE
OCTOBER 2011

Une fin de thèse, c'est avant tout une partie de vie qui s'achève, et non pas une page, mais bien des centaines de pages qui se tournent. Et pendant ces 6 années de recherche et d'encadrement didactique, j'ai eu la chance d'être entouré de nombreuses personnes qui ont permis la réalisation de ma thèse. Je veux les remercier ici.

Tout d'abord, mes promoteurs: Vincent Legat et Thierry Fichet. Je souhaite leur exprimer toute ma gratitude car c'est grâce à leur confiance et à leur soutien que j'ai pu entamer et mener à bien mon doctorat. Merci à Vincent pour son engouement pour les jolies méthodes numériques, ses précieux conseils et son enthousiasme. Ta rigueur scientifique et ton expérience en matière de communication scientifique n'auront pas manqué d'enrichir ma formation. Merci aussi d'avoir si souvent encouragé et participé aux activités eulériennes, cela a été un réel plaisir de travailler dans une aussi bonne ambiance pendant ma thèse. Merci à Thierry de m'avoir aiguillé, lors d'une journée portes ouvertes pour les rhétos au cours de ma deuxième candidature à l'ULB, sur les possibilités de suivre des cours d'océanographie en ASTR tout en continuant mon cursus d'ingénieur. Et par la suite de m'avoir suivi au fil de nos réunions, pour ta super-vision qui exige toujours de voir à long terme et concrètement la suite des développements. Merci pour ton engouement pour ce magnifique sujet qu'est la glace de mer et pour l'énergie que tu mets dans la gestion d'un modèle reconnu à travers le monde. Et si ma prose en anglais a tant progressé, c'est probablement grâce à toi. Merci enfin à vous deux de m'avoir permis de participer à de nombreuses conférences, souvent en votre compagnie.

Je souhaite également remercier les membres de mon jury (Hugues Goosse, Miguel Angel Morales Maqueda, Jérôme Weiss et Thomas Pardoën) pour leur lecture attentive et constructive de mon manuscrit, ainsi que pour les discussions qui en ont suivi. Merci aussi à Laurent Delannay de m'avoir suivi pendant ma recherche et pour ses conseils avisés. Je tenais également à remercier Eric Deleersnijder de m'avoir poussé à investiguer la thématique de l'âge de la glace. Enfin, un énorme merci à toi Martin, pour ces discussions passionnantes et enrichissantes ("Ha oui, je vois, ya un papier... Untel ... JGR ... 2003 je pense"), au 5^e étage ou autour d'un verre, ou par tes conseils pour orienter ma recherche.

Je ne pourrais pas ne pas mentionner tous mes collègues qui, de près ou de loin, m'ont donné des coups de main dans la réalisation de mes travaux. Merci à Laurent White de m'avoir guidé dans mes débuts, parfois remis en question, cela a été un plaisir d'être ton collègue de bureau et de partager mes premières séances de TP avec toi! Olivier, Sylvain, Richard: vous avez été des collègues de bureau extras! Jon, un tout grand merci pour tes nombreux dépannages informatiques! Seb, Christelle, Mel, Thom et Greg: j'ai passé 3 terribles années en votre compagnie rue de la Ramée! Merci enfin à tous les collègues du bâtiment Euler, en particulier à ceux qui ont cédé

à mes propositions d'activités (à commencer par le foot évidemment!). OliGou, une palme pour toi et ton éternelle motivation à te jeter la tête la première dans nos sorties ;-) Merci à ceux qui étaient présents à mon EVG, ça m'a vraiment touché (Max, Oli, Bruno, Seb). Bien plus que des collègues, vous êtes tous devenus des amis... Merci à Michèle, Astrid et à Gaëtan D pour leurs coups de main divers et toujours appréciés :-)

Je tenais également à remercier mes mémorants Pierre-Nicolas et Martin pour l'excellent travail qu'ils ont effectué sur le modèle Lagrangien avec Gaëtan.

Enfin, je ne terminerai pas sans faire un petit clin d'oeil à Y. Daels, Professeur au Collège-Saint-Pierre à Uccle, sous la direction de qui j'avais effectué un travail de fin d'études intitulé: "Mouvements de régression et de transgression du volume des glaciers". Hé oui, des glaçons, déjà...

Une thèse, c'est avant tout un projet personnel, et si j'étais parfois plongé dans ma caverne, je tiens à remercier tous ceux qui ont su m'en sortir et me changer les idées quand j'en avais besoin. Merci aux fofes de la W4 (Cara, Campa, Cabe, Luss, Buffle, Fab, Logan, VD, Spat, Pierre, Zibe, Tim) pour tous ces matchs de foot entre potes, ces s'tages à n'en plus finir, ces soirées passées ensemble. Merci en particulier à mes 3 témoins pour l'intérêt qu'ils ont toujours porté à ma thèse ("Alors, ils fondent tes glaçons?"). Merci aux 3 mousquetaires (Fabrice, Nico, Pierre) et Mathieu pour toutes nos retrouvailles, WE's goudés, WE's mariage et autres sorties. Restez comme vous êtes! Merci à tous mes autres potes pour leur intérêt et leur compréhension quand j'étais fort occupé (KDF, MAP, Linkebeek, ...) et merci à tous les potes de Nath pour leur soutien.

J'ai la chance d'avoir une famille extraordinaire (et à présent une belle-famille ;-) et en particulier des parents en or: merci de vous être tellement investi dans notre éducation, de vous être toujours coupé en 4 pour nous, de vous impliquer encore et toujours avec la même ferveur. Cette thèse se doit aussi d'être votre réussite! Merci à Caro, Oli, Jean-Phi et Val et aux couz.

On garde toujours le meilleur pour la fin, pas vrai? Nath, merci pour ta patience, ton écoute, ta compréhension. Merci de t'être intéressée à cette "bitch" ;-) de m'avoir conseillé et surtout, de m'avoir redonné de l'espoir et de la motivation quand j'étais à la limite de l'épuisement et du ras-le-bol. Obrigado!

*A mes grands-pères,
pour l'intérêt qu'ils ont toujours porté à mes études.*

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CHAPTER 1

INTRODUCTION

*"White men think of ice as frozen water, but Inuit think of water as melted ice.
To us, ice is the natural state"*

–Nukapinguaq, Inuit hunter (Kobalenko, 2002).

Water has several properties that make it a unique and exceptional material in many regards. For example, all three states of matter can be found in natural conditions. Due to the presence of hydrogen bonds, water has one of the highest heat capacity of all liquids and very high latent heat of evaporation and fusion. At atmospheric pressure, water density increases with decreasing temperature until 4 °C, like any other liquid. But below this threshold temperature, the water molecules begin to form hexagonal crystals, resulting in a decrease of the density which further undergoes a strong shift when ice forms. In consequence, its solid phase is less dense than its liquid phase and ice floats on water. Without this fundamental characteristic, one would not know sea ice. According to the World Meteorologic Organization, sea ice is *any form of ice found at sea which has originated from the freezing of sea water*.

During the sea ice formation, salt and impurities can hardly be incorporated into the crystal lattice and are dissolved in liquid inclusions. Sea ice is thus a two-

phase, four-components material: it is made of pure freshwater ice with brine inclusions, air pockets, and solid salt deposits (e.g., Weeks and Ackley, 1986). Sea ice constitutes an unexpected habitat of life: algae, bacteria, viruses and other microorganisms survive at very low temperatures in the complex networks of brine channels and at the ice surface and bottom (e.g., Palmisano and Garrison, 1993; Staley and Gosnik, 1999; Lizotte, 2003). To protect them from cold winters, bacteria have developed the aptitude to synthesize fatty materials (EPS: extracellular polymeric substance, Krembs et al. (2002)).

As an echo to the Inuit point of view, most of the water in the Solar System is in solid phase (Deming and Eicken, 2007). Like sea ice, ice is rarely pure and impurities trigger the presence of liquid inclusions that could potentially host microorganisms, a domain of research actively investigated by astrobiology. Another astrobiological field of inquiry concerns viruses. These bacteriophages are the vector of “horizontal” transfer of genes (in contrast to vertical –parent-to-child– transfer), raising the question of their role in the evolution of life (Deming and Eicken, 2007). Understanding the physics and biology of sea ice is hence mandatory to investigate possible extraterrestrial life in frozen environments (e.g., Europa, Mars, Titan) and constitutes one of the many motivations to study sea ice.

Sea ice in the climate system

Volumetrically, sea ice is insignificant compared to the total volume of water in the World Ocean (roughly 0.004%, Weeks (2010)). In extent though, sea ice covers about 4.5% of the Earth surface and 7% of the ocean surface, on annual average. It is the natural barrier that sea ice constitutes between the atmosphere and ocean that makes it a key component of the high-latitude climates (e.g., Serreze et al., 2007). In regions where the incoming solar radiation at the top of the atmosphere is higher during local summer than elsewhere on Earth (e.g., Wallace and Hobbs, 1977), most of the downwelling shortwave radiation is reflected due to the high snow and ice albedos (e.g., Perovich et al., 2002). Due to its low thermal conductivity, sea ice further insulates the relatively warm ocean waters from the cold atmosphere. The ice cover hence drastically limits heat, momentum and water mass exchanges between the low atmosphere and the upper ocean. Moreover, the salt rejected into the underlying water during sea ice formation takes an important part in high-latitude oceanic convection and thus in the global thermohaline circulation (e.g., Aagaard and Carmack, 1989). All these interactions illustrate the complex role played by sea ice in the climate system. In particular, the ice-albedo feedback (Ebert and Curry, 1993) is considered to be largely responsible for the high sea ice sensitivity to climate change, as highlighted by simulations with climate general circulation models (CGCMs) on the one hand (e.g., Manabe and Stouffer, 1980; Holland and Bitz, 2003), and by the acceleration of the recent decline of the Arctic sea ice cover on the

other hand (Stroeve et al., 2011). Nevertheless, recent comparisons between CGCM simulations (e.g., Holland and Bitz, 2003; Arzel et al., 2006; Zhang and Walsh, 2006; Lefebvre and Goosse, 2008) plead for the need to improve our understanding of sea ice and its representation in climate models.

Sea ice has been given growing media coverage over the last decade due to the repetition of Arctic sea ice extent minima (Stroeve et al., 2005; Comiso et al., 2008; Stroeve et al., 2008; Kwok et al., 2009), initiating a transition phase to a summer ice-free Arctic Ocean. More importantly, this decrease in extent has been coupled with a decrease in the ice volume and ice age. Observations based on upward-looking sonar onboard submarines (ULS, Rothrock et al. (1999); Wadhams and Davis (2000)) have shown that the ice draft (the component of the ice thickness that is submerged under the sea) has lost about 40% between the mid-seventies and the mid-nineties. Recently, helicopter-borne electromagnetic thickness measurements indicate a loss of up to 53% between 2001 and 2007 (Haas et al., 2008). Regarding the ice age, within the central Arctic Basin for example, 57% of the ice was 5 or more years old in 1987 (25% at least 9 years old) while, by 2007, the fraction of 5 or more years old shrank to 7% and no very old ice survived (Maslanik et al., 2007). Nowadays, an increasing fraction of the Arctic Ocean is thus covered by thin, first-year ice that is more sensible to summer melt and is associated to another facet of the ice-albedo feedback: more open water leads to more solar heat absorbed, which results in more melting and more open water (Perovich et al., 2007, 2008).

The need for unstructured grids

In order to study and understand the behavior of sea ice, numerical sea ice models have been developed since the early seventies. The first detailed sea ice model is the one presented by Maykut and Untersteiner (1971) and deals with vertical heat transfers –the so-called thermodynamic component. In the mid-seventies, intense efforts have been dedicated to the modeling of dynamical processes (e.g., Coon et al., 1974; Pritchard et al., 1976; Hibler, 1977). Since then, numerous improvements have been achieved in sea ice modeling, regarding thermodynamics (e.g., Bitz and Lipscomb, 1999; Notz and Worster, 2006; Flocco and Feltham, 2007; Vancoppenolle et al., 2007), dynamics (e.g., Schreyer et al., 2006; Wilchinsky and Feltham, 2006; Coon et al., 2007; Sulsky et al., 2007; Weiss et al., 2007; Girard et al., 2011) or numerics (e.g., Hunke and Dukowicz, 1997; Lipscomb and Hunke, 2004; Lemieux et al., 2010). Sea ice models are further included in climate models to investigate the interactions between sea ice and the other components of the climate system, understand the past variations of the sea ice cover and project its future evolution in the framework of IPCC¹ assessments.

¹IPCC stands for Intergovernmental Panel on Climate Change.

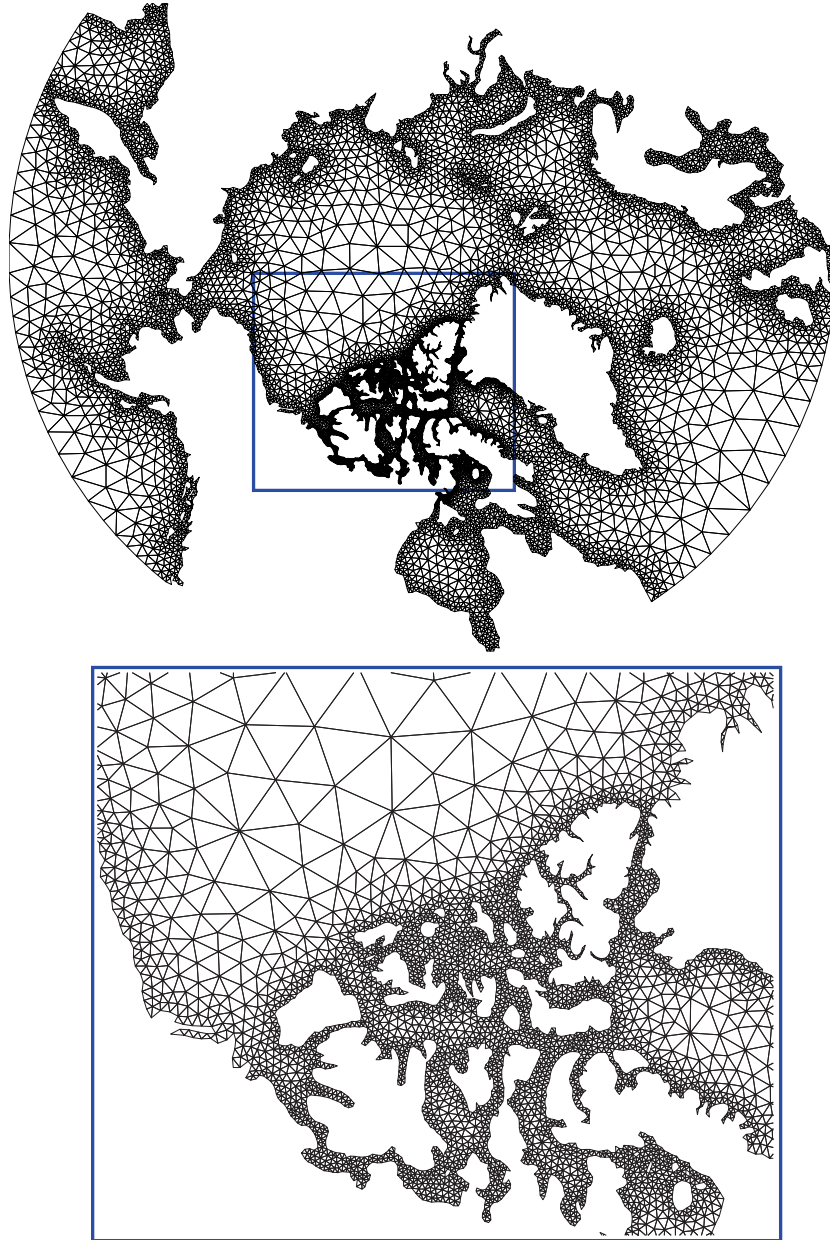


Figure 1.1: Finite element mesh of the Arctic with a close-up of the Canadian Arctic Archipelago. The domain extends north of the parallel 50 degrees North. The mesh contains 17053 triangles and 10505 nodes and its resolution varies from 10 to 400 km.

Sea ice models have traditionally been based on structured grids and finite difference schemes due to their ease of implementation and widespread use. These algorithms have also the advantage of being very fast. However, structured grids lack geometrical flexibility because the grid is an image of the coordinate system. There exists alternative numerical methods to solve differential problems. In this thesis, we will focus on the finite element method (FEM). The FEM has in common with finite differences (FD) the computation of approximate values of the solution at some particular points (nodes) of the computational domain. But where FD discretize the differential operators, the FEM is based on the discretization of the fields, on which the differential operators are applied. The discretization relies on the choice of interpolation functions, allowing to derive the solution at any position from the nodal values. Finally, the mesh divides the domain of the problem in small entities (Fig. 1.1).

Finite element methods have been proposed at the onset of sea ice modeling. In the framework of the AIDJEX project (Arctic Ice Dynamics Joint Experiment), Mukherji (1973) was the first to make use of a finite element method code for its aptitude to simulate the crack propagation in sea ice. A few years later, Becker (1976) introduced the guiding lines of the method and asserted: *“Because of their generality and widespread use, finite element techniques seem a worthwhile alternative to the difference scheme. [...] The ease with which finite element techniques can be used to model complicated shapes with arbitrary variation in the mesh spacing is likely to be the motivating factor leading to any such use.”*. In the early eighties, Sodhi and Hibler (1980) used in a pioneer study an unstructured grid along with a finite element method in order to resolve the ice drift in the complex region of Strait of Belle Isle. Another work investigating the finite element method in sea ice modeling was held by Thomson and McKenna (1988). They performed a comprehensive comparison between different constitutive laws for Eulerian and Lagrangian descriptions in order to model the short-term ice motion in the Beaufort Sea. Over the last decade, many efforts have been directed towards the implementation of unstructured meshes in sea ice modeling (Schulkes et al., 1998; Yakovlev, 2003; Wang and Ikeda, 2004; Sulsky et al., 2007). Recently, Timmermann et al. (2009) successfully performed a run on a global configuration with a coupled ice-ocean model.

Unstructured meshes have among others the advantage to avoid the pole singularity and related stability problems (Williamson, 1979) and to represent in a more accurate fashion coastlines (e.g., Adcroft and Marshall, 1998; Legrand et al., 2006). But with their natural ability to locally increase the mesh resolution, unstructured meshes propose above all an alternative framework to capture the complex oceanic areas formed by coasts and islands. They are also particularly well designed for multi-scale problems: the mesh resolution can be increased where and when needed. The mesh design can also be based upon the bathymetry (Gorman et al., 2006; Legrand et al., 2007), or any given field (Legrand et al., 2000).

Motivation for this thesis

This thesis falls within the framework of the SLIM² project. This project aims at developing a coupled sea ice-ocean model based on the finite element method and unstructured grids. The ocean model solves the shallow water equations by means of the discontinuous Galerkin finite element method. It features a 1D river model, 2D depth averaged model and a full 3D model (de Brye et al., 2010; Blaise et al., 2010; Comblen et al., 2010a). The model is the result of the collaboration between the entities of Applied Mechanics (MEMA) and the G. Lemaître Institute of Astronomy and Geophysics (ASTR) at the Université catholique de Louvain (Louvain-la-Neuve, Belgium).

For more than a decade, ASTR has been at the leading edge of sea ice modeling. LIM³ was originally a large-scale dynamic-thermodynamic sea ice model developed by Fichefet and Morales Maqueda (1997). The first coupling effort was due to Goosse and Fichefet (1999) before LIM was coupled to the French ocean model OPA⁴ (Timmermann et al., 2005). Recently, a new version of the sea ice model has been released (LIM3, Vancoppenolle et al. (2009)) with major improvements. In order to account for unresolved variations in ice thickness, several thickness categories have been included in the model. A new multi-layer halo-thermodynamic component incorporates an explicit representation of brine entrapment and drainage, as well as the impact of brine on sea ice growth and decay. LIM3 is part of the NEMO⁵ ocean modeling system and is used in numerous laboratories involved in climate studies and operational oceanography.

This thesis is dedicated to the development of the sea ice component of the SLIM model, and as such lies between the worlds of sea ice modeling and applied mechanics. The key developments that constitute this work are associated to realistic applications. They can be summarized as follows:

- **What are the advantages of using finite element methods for sea ice modeling?** Associated to unstructured meshes, the finite element method undoubtedly yields large geometrical flexibility to represent coastlines and capture narrow straits (Fig. 1.2). Contrary to finite differences, finite element methods do not need a global coordinate system to define the discrete differential operators. Instead, the combined use of a local coordinate system on each element and suitable changes of variable enable to express partial differential equations on curved surfaces (Comblen et al., 2009). In particular,

²Second-generation Louvain-la-Neuve Ice-ocean Model, <http://www.climate.be/SLIM>.

³Louvain-la-Neuve sea Ice Model, <http://www.climate.be/LIM>.

⁴OPA (Océan Parallélisé) was developed at LOCEAN, Université de Paris VI, Paris, France.

⁵NEMO stands for Nucleus for European Modelling of the Ocean.

this method circumvents the pole singularity issue arising when the numerical grid is based on a geographical coordinate system.

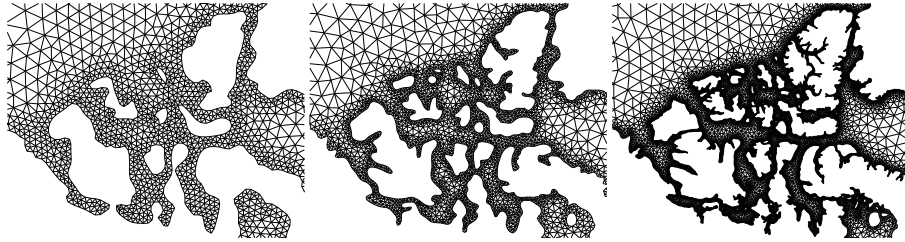


Figure 1.2: Finite element mesh of the Canadian Arctic Archipelago. From left to right, the coast resolution is increasing (75 km, 40 km and 10 km, respectively). The mesh resolution is also increasing: 40 km, 20 km and 10 km.

- **What is the impact of resolving the narrow straits of the Canadian Arctic Archipelago for sea ice modeling?** The complex area formed by the numerous islands and coastlines in the Canadian Arctic Archipelago (CAA, see Fig. 1.1) presents a natural application for unstructured meshes. The straits constitute an important pathway for the cold waters from the Arctic Ocean to the Atlantic. This area is also currently being of strong economical importance and political interest in the context of the possible opening of the Northwest Passage during summer months in the coming decades. Clearly, modeling sea ice in the CAA is challenging and involves many scales: its ice cover characteristics are influenced by the proximity of the Arctic Basin, which is itself dependent on the World Ocean circulation; on the other hand, small scales involve static arches formation and landfast ice, which play a key role in the ice circulation of the area. In a first study of the CAA (Lietaer et al., 2008), we have shown that, on average, the sea ice volume in the CAA represents a non-negligible 10% of the total sea ice volume in our model (Fig. 1.3). The CAA does thus matter in terms of the Arctic sea ice mass balance.
- **What are the interest and potential applications of developing a Lagrangian sea ice model?** The Lagrangian description offers several attractive advantages such as the easy tracking of free surfaces or the direct access to the material deformation history. At least two natural applications arise from a Lagrangian sea ice model: first, this approach can be particularly efficient to track the ice edge, which can be especially useful to operational oceanography. Second, it reveals an appropriate tool to develop new sea ice rheologies since the non linear advection term is naturally handled and the computation of the rheological contribution to the momentum equation is simplified as we follow material points.

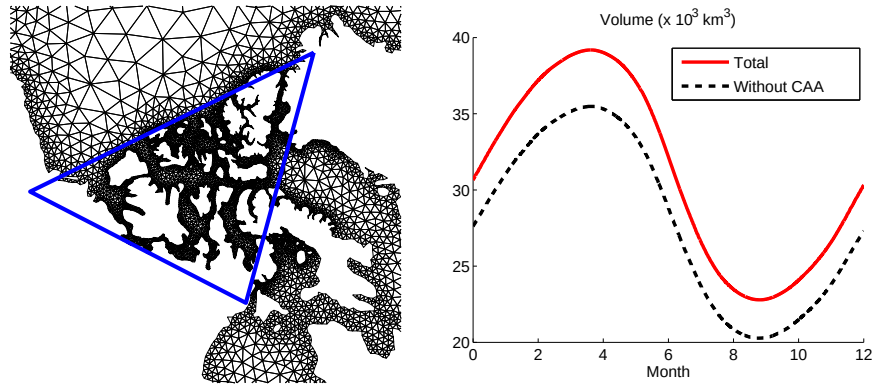


Figure 1.3: Left: close-up view of the mesh in the CAA. Right: seasonal cycles of the Arctic sea ice volume with (solid) and without (dashed) the contribution of the CAA. All the elements included in the blue triangle in the left figure are considered being part of the CAA.

- **What can be gained from mesh adaptivity?** Dynamic mesh adaptivity delivers a mean to increase the mesh resolution where and when it is needed in the domain. Is it particularly powerful for very transient processes with strong gradients. In practice, mesh adaptivity follows a criteria that can take several forms: error estimator, field isovalue, predefined mesh size field distribution, ... As far as sea ice is concerned, this high mesh flexibility could be used to better resolve the ice edge or regions of strong deformation.
- **How sea ice age can be modeled and what is the influence of the ice age definition on the results?** The sea ice age is an interesting diagnostic tool because it may provide a proxy for the sea ice thickness and is easier to infer from observations than the sea ice thickness. Remote sensing algorithms and modeling approaches proposed in the literature indicate significant methodological uncertainties, leading to different ice age values and physical interpretations. The Lagrangian sea ice model is a nice modeling tool to compare in the same framework different sea ice age definitions and analyze their impact on the age values.

However, the use of unstructured grids for sea ice modeling still faces several constraints that currently limit their widespread use. The main limitation comes from the higher computational time compared to structured grids. Indeed, the use of unstructured grids is associated to more complex (thus more expensive) numerical methods and suffers from the Courant-Friedrichs-Lewy (CFL) condition: in explicit methods, the time step must be smaller than the travel time of the fastest physical process over the smallest space increment. In other words, when the mesh is refined, the time step has to be decreased in accordance with the CFL condition.

On the other hand, as the mesh is refined, some physical parameterizations need to be updated accordingly. Indeed, model subgrid-scale parameterizations take into account unresolved processes at a given spatial resolution. With the resolution of smaller scales comes the need to update these parameterizations accordingly. In a certain sense, the Lagrangian model developed during this thesis constitutes a nice *numerical laboratory* to study new rheologies or new physical parameterizations. This is certainly one of the most attractive applications of the model. The different limitations mentioned above explain why unstructured grids are not yet generalized in current ocean and sea ice models and demonstrate that there is still large efforts and research to be done before it can be achieved successfully.

Structure of the thesis

In **Chapter 2**, we focus on vertical heat transfer in sea ice and the common hypotheses and general principles of a thermodynamic model are delivered. In order to investigate the vertical age profile in sea ice, we propose a vertically-variable sea ice age definition. An analytical solution is derived in a simplified setup and two numerical methods to solve the age equation are proposed. Both methods are validated against the analytical solution and finally applied to a stand-alone thermodynamic sea ice model of the Arctic.

In order to resolve the ice circulation in the Arctic, the equations for ice dynamics are derived in **Chapter 3** and the principles of the finite element method are introduced. The discretization of the sea ice momentum equations leading to the algebraic system is described. The sea ice model is validated against satellite and buoys measurements and the influence of closing the narrow straits of the Canadian Arctic Archipelago in the numerical simulation is investigated. In particular, we show that the annual mean ice export towards Baffin Bay is not negligible and may play an important role on the convective overturning in the Labrador Sea.

In **Chapter 4**, we develop a Lagrangian, adaptive sea ice model allowing the computational grid to move with the ice drift. In order to maintain a good quality of the mesh, the mesh has to be adapted during the simulation, involving particular mesh adaptation techniques. When the mesh is adapted, the fields have to be updated accordingly. Based on a discontinuous representation of the fields, we use a local Galerkin projection based on the geometrical mesh intersection of the source and target mesh elements. The mesh adaptation and field update methods are validated on standard benchmarks. Finally, the Lagrangian model is run on the Arctic Basin and different ice age definitions are implemented in the model and compared.

Conclusions and perspectives are eventually proposed in **Chapter 5**.

CHAPTER 2

THERMODYNAMIC PROCESSES: THE VERTICAL AGE PROFILE IN SEA ICE

This Chapter reproduces an updated version of the following paper:

O. Lietaer, E. Deleersnijder, T. Fichefet, M. Vancoppenolle, R. Comblen, S. Bouillon, and V. Legat. The vertical age profile in sea ice: theory and numerical results. *Ocean Model.*, 2011 (in press).
doi: 10.1016/j.ocemod.2011.09.002.

Summary

Changes in the surface energy budget shape the seasonal cycle of ice thickness. The sea ice age is often considered as a proxy for the sea ice thickness. However, significant methodological uncertainties appear in sea ice age studies. In this chapter, we focus on the thermodynamic evolution of sea ice and propose a vertically-variable sea ice age definition. An analytical solution of an idealized sea ice age problem is derived and allows for assessing two numerical methods. Both methods are applied to a thermodynamic sea ice model of the Arctic and the age-thickness relationship is analyzed.

In this section, we focus on vertical heat transfer in sea ice and the common hypotheses and general principles of a thermodynamic model are delivered. We first present a one-dimensional analytical model of sea ice growth and melt forced by changes in air temperature. In order to validate sea ice models, modelers use available sets of observational data, including sea ice thickness. Sea ice thickness, however, is difficult to infer from satellites, so that sea ice age has been given growing attention recently. Indeed, it may provide a proxy for the sea ice thickness (“the older the ice, the thicker the ice”) and is easier to infer from observations than the sea ice thickness. Since satellites do not see subsurface processes, remote sensing studies track the surface ice age. Modelers tend to mimic the sea ice age derived from satellite observations and new ice formed at the base or at the surface is assumed to have the same age as the rest of the column. In contrast to previous modeling studies, we propose a vertically-variable sea ice age definition in order to investigate the vertical age profile in sea ice.

2.1 A one-dimensional model of sea ice accretion and melting

Changes in the surface energy budget shape the seasonal cycle of ice thickness. The first sea ice model that has been established is probably Stefan’s one (1891), which describes the evolution of ice growth. In this section, a one-dimensional analytical model of sea ice growth and melt forced by changes in air temperature is introduced. The model combines the classical model of Stefan (1891) for sea ice growth and a linear parameterization of melt. This model is able to reproduce the salient features of sea ice growth and melt. However, being highly idealized, it is not meant to provide accurate simulations of ice thickness.

2.1.1 Stefan’s law

Assume that the ice layer under study is horizontally homogeneous and infinite, implying that only vertical heat fluxes are to be taken into account. During the accretion period, we assume that only ice accretion occurs at the bottom of the ice. Let t denote time ($t = 0$ at the beginning of the accretion period). At the lower boundary of the ice, the ice temperature is that of the melting point. At the upper boundary, the air temperature largely influences the ice temperature. The difference, $\Delta\theta$, between the temperatures of the ice bottom and the ice surface is positive during the accretion season, and is considered constant in the present approach. By neglecting thermal inertia and internal heat sources, the temperature difference $\Delta\theta$ is associated with a heat flux F_C , which, in Stefan’s model, is assumed to be constant over the height of the ice (the fluxes directed to the ice surface are taken to be positive). This simplification holds for thin ice, but the model validity is questionable for ice thicker than about 1 m. If we neglect the oceanic heat flux, the growth

rate of $H(t)$, the ice thickness, satisfies the following expression:

$$L \frac{dH}{dt} = F_C, \quad (2.1)$$

where L represents the volumetric latent heat of fusion of ice (see Table 2.1).

Parameter	Symbol	Value	Unit
Volumetric latent heat of fusion	L	3×10^8	J m^{-3}
Thermal conductivity	k	2.1	$\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$
1 year	T	365×86400	s
Duration of accretion season	T_a	200×86400	s
Duration of melting season	T_m	165×86400	s
Bottom temp. - top temp.	$\Delta\theta$	20	$^\circ\text{C}$
Conductive heat flux	F_C	parameterized	W m^{-2}

Table 2.1: Table with principal parameters and values used for the simple model of ice accretion and melting.

According to Fourier's law, the upward conductive heat flux F_C taking place in the ice may be parameterized as $k\Delta\theta/H$, where k is the thermal conductivity of ice (Table 2.1). The differential equation that the ice thickness must satisfy during the accretion period is:

$$H \frac{dH}{dt} = k \frac{\Delta\theta}{L}. \quad (2.2)$$

Without any loss of generality, the ice accretion period may be assumed to start at time $t = 0$. If $H_0 = H(0)$ and if T_a is the duration of the accretion period, then the ice thickness grows as

$$H(t) = \sqrt{H_0^2 + \alpha t}, \quad 0 \leq t \leq T_a, \quad (2.3)$$

where α depends on various physical parameters (see Table 2.1):

$$\alpha = \frac{2k\Delta\theta}{L}. \quad (2.4)$$

Then, the ice thickness at the end of the cold season is:

$$H_1 = \sqrt{H_0^2 + \alpha T_a}. \quad (2.5)$$

2.1.2 The melting period and the periodic regime

During the warm season, we assume that only the upper part of the ice melts, and, for simplicity, that the melting rate is constant. It is now further assumed that

the ice layer evolution exhibits a periodic regime (Fig. 2.1.) with a period of one year, implying that the ice thickness at the beginning of the accretion period (H_0) must be equal to that attained at the end of the warm season (H_2). The periodicity of the ice thickness seasonal cycle allows us to compute the ice height ΔH frozen and melted each year:

$$\Delta H = H(T_a) - H_0 = \sqrt{H_0^2 + \alpha T_a} - H_0. \quad (2.6)$$

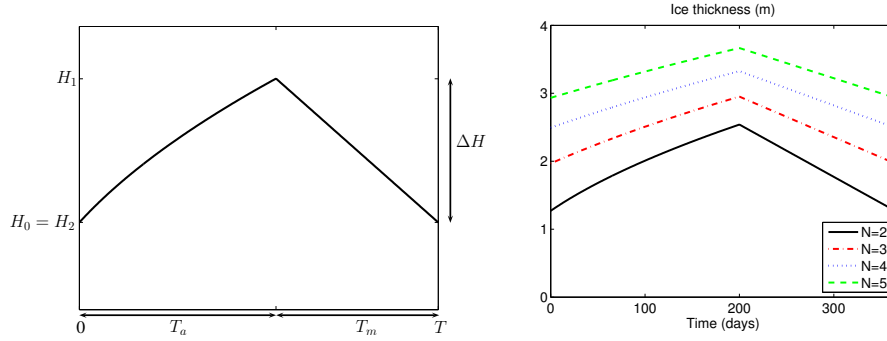


Figure 2.1: Left: periodic regime of the ice thickness ($H_2 = H_0$); ΔH denotes the ice height frozen and melted each year. Right: ice thickness seasonal cycles obtained for the parameters described in Table 2.1 and for various values of N (integer relating the maximum ice thickness to ΔH).

Taking the hypothesis of the constant melting rate into account, the ice thickness evolves according to:

$$H(t) = \begin{cases} \sqrt{H_0^2 + \alpha t}, & 0 \leq t \leq T_a \\ \sqrt{H_0^2 + \alpha T_a} - \frac{t - T_a}{T_m} \Delta H, & T_a \leq t \leq T_a + T_m \end{cases} \quad (2.7)$$

where T_m is the duration of the melting period and ΔH is given by equation (2.6). In order to further simplify the problem, we consider that the ice layer is made of N annual layers of ice height ΔH :

$$H_0 + \Delta H = N \Delta H. \quad (2.8)$$

We can derive an analytical expression for H_0 :

$$H_0 = \sqrt{\frac{(N-1)^2 \alpha T_a}{2N-1}}. \quad (2.9)$$

As illustrated in Fig. 2.1, the parameter N gives rise to different ice regimes. To further understand the thermodynamic evolution of the ice layer, we will refer to the concept of sea ice age.

2.2 Introduction to the sea ice age

Sea ice age patterns and how they change in time provide an integrated view of the recent evolution of sea ice growth, melt and circulation, and therefore give some information on the recent thinning of Arctic sea ice. The sea ice age may also provide a means to retrieve the ice thickness, as older ice is typically thicker. For these reasons, the age of Arctic sea ice has recently been the subject of several remote sensing and modeling studies (Walsh and Zwally, 1990; Nghiem et al., 2007; Maslanik et al., 2007; Vancoppenolle et al., 2009; Hunke and Bitz, 2009).

Sea ice age inferred from satellite ice concentration and motion data has been used as a proxy for ice thickness because of the difficulties to observe ice thickness from space. Typically, the satellite sea ice age products (Rigor and Wallace, 2004; Fowler et al., 2004; Belchansky et al., 2005) are based on (i) the creation of sea ice with age zero in open water, (ii) horizontal advection of the age using gridded fields of ice motion, (iii) ageing of sea ice with time and (iv) disappearance of ice of any age lying outside the limits of the ice edge. The age evaluated in this manner corresponds to the time elapsed since the creation of an ice parcel and is available at large scales. However, there are significant differences in sea ice age among the different algorithms used. In addition, those algorithms neglect several processes, including the vertical processes of sea ice growth and melt, which could have a significant role.

In order to understand how ice thickness can be inferred from sea ice age, the latter has been introduced in large-scale sea ice models (Vancoppenolle et al., 2009; Hunke and Bitz, 2009). Sea ice age is considered in those models as a two-dimensional tracer, following ice area (Vancoppenolle et al., 2009) or ice volume (Hunke and Bitz, 2009). This implies horizontal transport and redistribution in thickness space due to thermodynamic and mechanical processes. Besides, new ice formed at the base or at the surface is assumed to have the same age as the rest of the column. Thereby, modelers intend to mimic the sea ice age derived from satellites which observe only the surface age and do not consider the role of vertical processes. Using such definitions for sea ice age, model studies found some difficulties to derive ice thickness from ice age. Besides, those studies indicate large methodological uncertainties. For instance, Harder (1997) proposes another approach where new bottom ice is considered as a sink of ice age, and Hunke and Bitz (2009, Fig. 9) show that integrating bottom growth in the ice age induces a decrease in the simulated sea ice age by more than a factor of 2. This illustrates how the choice of the age def-

initiation leads to different values and physical interpretations, which is well known in marine modeling (see Deleersnijder et al., 2001).

The ice age vertical profile accounts for subsurface processes that satellites cannot see. Hence, understanding the sea ice age vertical profile provides a theoretical framework for understanding satellite retrievals. Of particular interest is bottom ice growth that reduces the average ice age. The sea ice age vertical profile also tells us about the history of the ice formation, and therefore about recent changes in the thermodynamic evolution of the ice pack in the context of climate change. Furthermore, sea ice age is an interesting diagnostic that is related to several sea ice physical properties (ice strength, salinity, surface topography).

There are two fundamental questions that are addressed here. First, what are the ranges for sea ice age associated with vertical thermodynamic processes? Second, what is the nature of the relation between ice age and thickness, considering the vertical thermodynamics of sea ice? To address these questions, we consider a vertically-variable sea ice age, based on the age theory developed for marine modeling by Delhez et al. (1999) and Deleersnijder et al. (2001)¹. In contrast to previous modeling studies, we define the sea ice age as the time elapsed since the ice particle under consideration has been formed at the ice base and explicitly compute the vertical age profile. Horizontal advection and redistribution in thickness space are not taken into account.

2.3 The ice age: concepts and analytical solution

The concepts of the ice age are presented and a general equation of evolution of the ice age is derived in a three-dimensional framework. We will then deduce the ice age equation for different vertical coordinates. Finally, an analytical solution of the vertical age profile in sea ice is deduced.

2.3.1 The theory of the age applied to sea ice

Let us first derive the evolution equation for age in sea ice. From a general point of view, the sea ice age can be considered as a three-dimensional tracer. If (x, y, z) denote the cartesian coordinates, we attach the coordinate system to a given material surface of the ice at time $t = 0$, defining the level of reference $z = 0$ (Fig. 2.2). The z axis points upwards and the x and y axes are horizontal and define an orthogonal basis. The ice thickness can be defined as:

$$H(x, y, t) = z_s(x, y, t) - z_b(x, y, t),$$

¹This theory is a component of CART (Constituent-oriented Age and Residence time Theory, <http://www.climate.be/CART>).

where $z_b(x, y, t)$ and $z_s(x, y, t)$ denote the height (positive upwards) of the bottom and surface of the ice column. These interfaces are time-dependent and define a domain with moving boundaries. The boundaries evolve according to the rate of change in ice thickness (q) which can be decomposed in surface (q_s) and basal (q_b) components:

$$q(x, y, t) = q_s(x, y, t) + q_b(x, y, t). \quad (2.10)$$

We further assume that the ice is moving in a “solid block” way -that is, the ice velocity does not depend upon the vertical coordinate. Accordingly, in this reference frame, ice particles are created or removed at the two free surfaces and keep the same height: the vertical velocity is zero (the trajectories of several ice particles are illustrated in Fig. 2.3). In particular, the Eulerian and Lagrangian kinematical descriptions are thus equivalent in the vertical coordinate. Obviously, this referential is not the physical frame of reference, as the sea ice vertical velocity is not vanishing in the general case. Moreover, Archimedes’ force acting differently at each location would induce an additional motion that could create internal stress and deformation in the ice layer.

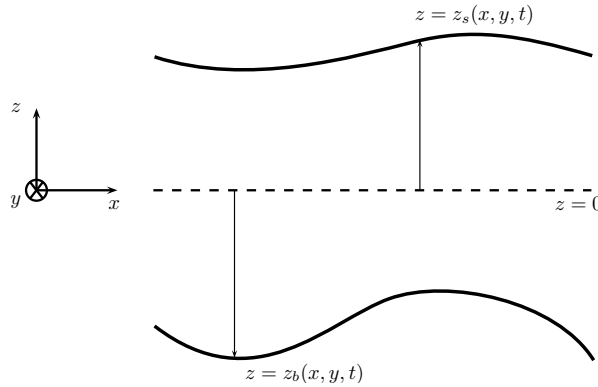


Figure 2.2: Sketch of the domain $\Omega(t)$. We define the coordinate system as attached to a given material surface of the ice at time $t = 0$. The ice thickness is given by $H(x, y, t) = z_s(x, y, t) - z_b(x, y, t)$.

Following Delhez et al. (1999), the age of a particle of a given continuum is defined as “the time elapsed since the particle under consideration left the region in which its age is prescribed to be zero”. From a Lagrangian point of view, an ice particle “carries along” its age which increases at the same rate as time passes. In other words, the age of an ice particle is the time elapsed since it was “created”. In mathematical terms, the equation for the three-dimensional ice age states that the ma-

terial derivative of the ice age $a(x, y, z, t)$ must be equal to unity:

$$\frac{Da}{Dt}(x, y, z, t) = 1. \quad (2.11)$$

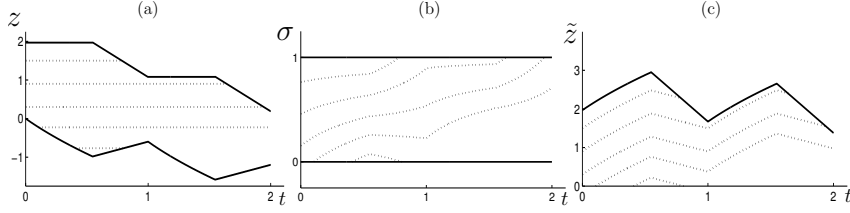


Figure 2.3: Evolution of the ice thickness as a function of time (years) illustrated in the three coordinate systems. In this sketch, ice accretion occurs at the ice bottom and ice ablation at both interfaces. The same particles trajectories in the vertical direction (dotted lines) are illustrated in each domain. (a) Original domain. The level of reference is 0. In this referential, particles have a zero vertical velocity and thus keep the same height. (b) Same, but using the σ coordinates. Using this ALE description, particles are now moving according to the accretion/melting rates at both interfaces, which is expressed by the velocity ω of the particles in the referential domain (see equation (2.20)). (c) Same, but using the distance to the bottom coordinates ($\tilde{z}$). Particles are moving according to the accretion/melting rates at the ice bottom only, which is expressed by the velocity $\tilde{\omega}$ of the particles in this domain (see equation (2.26)).

The general boundary value problem for the ice age $a(x, y, z, t)$ consists in solving equation (2.11)

$$\frac{\partial a}{\partial t}(x, y, z, t) + u(x, y, t) \frac{\partial a}{\partial x}(x, y, z, t) + v(x, y, t) \frac{\partial a}{\partial y}(x, y, z, t) = 1, \quad (2.12)$$

on the unknown moving domain

$$\Omega(t) = \{(x, y, z) \in R^3 \text{ such that } z_b(x, y, t) \leq z \leq z_s(x, y, t)\}. \quad (2.13)$$

We denote by (u, v) the horizontal components of the ice velocity. In terms of boundary conditions, we impose that the age of newly frozen ice is zero. We need to define an initial age field, but its importance is weak: as soon as all the initial ice has melted, the ice age will be correctly defined in the whole ice layer. Moreover, in usual hindcast simulations, a spin-up is generally performed in order to initialize correctly all physical variables. Equation (2.12) is hyperbolic, and the ice age must be prescribed where ice is formed:

$$a(x, y, z, 0) = a_0(x, y, z), \quad (2.14)$$

$$a(x, y, z_b, t) = 0, \quad \text{if } q_b(x, y, t) > 0, \quad (2.15)$$

$$a(x, y, z_s, t) = 0, \quad \text{if } q_s(x, y, t) > 0. \quad (2.16)$$

The motion of the ice bottom and surface are prescribed by the thermodynamic rates of change in ice thickness:

$$\frac{\partial z_b}{\partial t}(x, y, t) + u(x, y, t) \frac{\partial z_b}{\partial x}(x, y, t) + v(x, y, t) \frac{\partial z_b}{\partial y}(x, y, t) = -q_b(x, y, t), \quad (2.17)$$

$$\frac{\partial z_s}{\partial t}(x, y, t) + u(x, y, t) \frac{\partial z_s}{\partial x}(x, y, t) + v(x, y, t) \frac{\partial z_s}{\partial y}(x, y, t) = q_s(x, y, t). \quad (2.18)$$

Note that the boundary condition (2.16) covers the case where ice is formed at the surface, so that the boundary value problem (2.12)-(2.18) is general.

In order to transform the problem from a moving domain to a fixed reference domain, we introduce the following change of variable for the vertical coordinate:

$$\sigma(x, y, z, t) = \frac{z - z_b(x, y, t)}{z_s(x, y, t) - z_b(x, y, t)},$$

defining a new reference frame which has the advantage of keeping the height of the domain constant. Actually, these coordinates define an Arbitrary Lagrangian-Eulerian (ALE) formalism (e.g., Donea et al., 2004). In the ALE description, the computational grid is neither fixed in space, nor necessarily associated with material points. In this formalism, the ice bottom is mapped onto $\sigma = 0$ and the surface onto $\sigma = 1$. Note that this coordinate is similar to the sigma coordinate defined in ocean modeling (Phillips, 1957). In the equation for the age, a vertical velocity ω has to be introduced in order to take into account the motion in the domain:

$$\begin{aligned} \frac{\partial a}{\partial t}(x, y, \sigma, t) + u(x, y, t) \frac{\partial a}{\partial x}(x, y, \sigma, t) + v(x, y, t) \frac{\partial a}{\partial y}(x, y, \sigma, t) \\ + \omega(x, y, \sigma, t) \frac{\partial a}{\partial \sigma}(x, y, \sigma, t) = 1. \end{aligned} \quad (2.19)$$

The particle velocity ω can be directly obtained as a linear combination of the rates of change in ice thickness at the ice bottom and surface (Fig. 2.3b):

$$\omega(x, y, \sigma, t) = \frac{(1 - \sigma) q_b(x, y, t) - \sigma q_s(x, y, t)}{z_s(x, y, t) - z_b(x, y, t)}. \quad (2.20)$$

The initial and boundary conditions associated to equation (2.19) are:

$$a(x, y, \sigma, 0) = a_0(x, y, \sigma), \quad (2.21)$$

$$a(x, y, 0, t) = 0, \quad \text{if } q_b(x, y, t) > 0, \quad (2.22)$$

$$a(x, y, 1, t) = 0, \quad \text{if } q_s(x, y, t) > 0. \quad (2.23)$$

For notational convenience, we also define a local vertical coordinate by:

$$\tilde{z}(x, y, z, t) = z - z_b(x, y, t), \quad (2.24)$$

which refers to the distance to the ice bottom (Fig. 2.3c). This transformation belongs again to an ALE formalism and equation (2.11) becomes:

$$\frac{\partial a}{\partial t}(x, y, \tilde{z}, t) + u(x, y, t) \frac{\partial a}{\partial x}(x, y, \tilde{z}, t) + v(x, y, t) \frac{\partial a}{\partial y}(x, y, \tilde{z}, t) + \tilde{w}(x, y, t) \frac{\partial a}{\partial \tilde{z}}(x, y, \tilde{z}, t) = 1, \quad (2.25)$$

where

$$\tilde{w}(x, y, t) = q_b(x, y, t), \quad (2.26)$$

corresponding to the intuition that the distance separating a material point from the bottom of the ice is only dictated by the ice growth/melt at the ice bottom. Accordingly, in absence of ice accretion or melting at the bottom, an ice particle keeps the same relative height in the ice. The converted initial and boundary conditions to close the problem are:

$$a(x, y, \tilde{z}, 0) = a_0(x, y, \tilde{z}), \quad (2.27)$$

$$a(x, y, 0, t) = 0, \quad \text{if } q_b(x, y, t) > 0, \quad (2.28)$$

$$a(x, y, z_s - z_b, t) = 0, \quad \text{if } q_s(x, y, t) > 0. \quad (2.29)$$

In the remainder of this chapter, we will focus on the vertical profile of the ice age, neglecting the horizontal advection.

2.3.2 An analytical solution of the ice age for the periodic regime

In order to determine an analytical solution for the vertical profile of the sea ice age, we will consider the model developed in Section 2.1 and based on the following hypotheses: (i) the ice layer is horizontally homogeneous, (ii) horizontal advection is neglected, (iii) the ice thickness seasonal cycle is split into two non overlapping periods (growth and melt), (iv) the simple thermodynamic model of Stefan (1891) is used to compute the ice growth, (v) the ice thickness decreases linearly during the melting period, (vi) a periodic regime of the ice thickness seasonal cycle is considered and (vii) the maximum ice thickness is an integer multiple (N) of the ice height ΔH frozen and melted each year.

The first step of the derivation of the analytical solution focusses on the newly formed ice layer (see Fig. 2.4). We first consider the ice accretion period. Let $\Psi(t)$ denote the total ice growth since the beginning of the growth season ($t = 0$):

$$\Psi(t) = H(t) - H_0 = \sqrt{H_0^2 + \alpha t} - H_0. \quad (2.30)$$

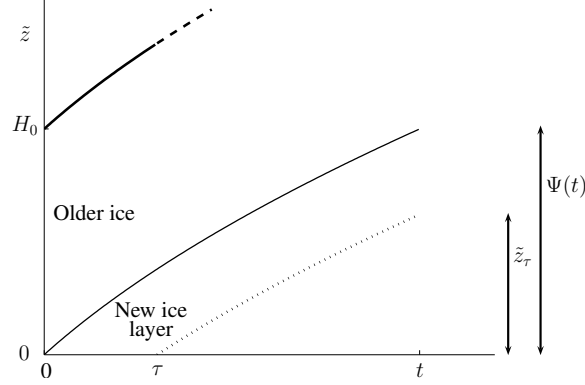


Figure 2.4: Sketch of the evolution of the newly formed ice layer. H_0 denotes the ice thickness at the beginning of the accretion period ($t = 0$). $\Psi(t)$ represents the ice thickness accumulated since the beginning of the ice accretion period. $\tilde{z}_\tau(t)$ is the distance to the ice bottom at time t of the particle formed at time τ .

We naturally have the following requirements for the ice age of the newly formed ice:

$$0 \leq t \leq T_a, \quad (2.31)$$

$$0 \leq \tilde{z} \leq \Psi(t), \quad (2.32)$$

$$a(0, t) = 0, \quad (2.33)$$

$$a(\Psi(t), t) = t, \quad (2.34)$$

$$0 \leq a(\tilde{z}, t) \leq t, \quad (2.35)$$

which state that the ice age is greater than zero (for newly formed ice) and smaller than t (for ice frozen at the beginning of the accretion period). In one dimension, equation (2.25) is:

$$\frac{\partial a}{\partial t}(\tilde{z}, t) + q_b(t) \frac{\partial a}{\partial \tilde{z}}(\tilde{z}, t) = 1. \quad (2.36)$$

An elegant way of solving the ice age equation (2.36) is to consider the growth of the ice layer as a streakline, i.e. a geometrical curve generated by the continuous emission of particles from a given position in space (here $\tilde{z} = 0$). Note that this method is equivalent to the method of characteristics. The streakline is described at the time t as the ensemble of particles of Lagrangian coordinate z such that at a given time τ we have $\tilde{z}(z, \tau) = 0$. Its parametric equation is given by:

$$\tilde{z}_\tau = \tilde{z}(z(0, \tau), t), \quad 0 \leq \tau \leq t \quad (2.37)$$

where $z = z(\tilde{z}, t)$ is the inverse one-to-one mapping from the referential to the original domain. Following equation (2.26), the parametric equation of the streakline

is the solution of the following differential expression:

$$\frac{\partial \tilde{z}_\tau}{\partial t} = q_b(t), \quad (2.38)$$

and, according to hypothesis (iii), $q_b(t) = \frac{dH(t)}{dt}$ here. Equation (2.38) gives then by integration between τ and t :

$$\tilde{z}_\tau = \sqrt{H_0^2 + \alpha t} - \sqrt{H_0^2 + \alpha \tau}, \quad (2.39)$$

thanks to equation (2.3). In a Lagrangian framework, the age of this particle must satisfy equation (2.12):

$$\frac{\partial a}{\partial t}(z(0, \tau), t) = 1, \quad (2.40)$$

with $a(z(0, \tau), \tau) = 0$, yielding by integration again:

$$a(z(0, \tau), t) = t - \tau = a(\tilde{z}_\tau, t). \quad (2.41)$$

Equation (2.39) can be rearranged as (dropping the subscript τ):

$$\tau = t - \frac{2\tilde{z}}{\alpha} \sqrt{H_0^2 + \alpha t} + \frac{\tilde{z}^2}{\alpha}, \quad 0 \leq \tilde{z} \leq \Psi(t). \quad (2.42)$$

Substituting equation (3.2) in equation (2.41), the age of the newly formed ice is finally given by:

$$a(\tilde{z}, t) = \frac{2\tilde{z}}{\alpha} \sqrt{H_0^2 + \alpha t} - \frac{\tilde{z}^2}{\alpha}, \quad 0 \leq \tilde{z} \leq \Psi(t). \quad (2.43)$$

Using equation (2.43), we now can get the age in the newly formed ice layer during the whole year:

$$a^{new}(\tilde{z}, t) = \begin{cases} \frac{2\tilde{z}}{\alpha} \sqrt{H_0^2 + \alpha t} - \frac{\tilde{z}^2}{\alpha}, & 0 \leq t \leq T_a \\ \frac{2\tilde{z}}{\alpha} \sqrt{H_0^2 + \alpha T_a} - \frac{\tilde{z}^2}{\alpha} + t - T_a, & T_a \leq t \leq T_a + T_m \end{cases}$$

Finally, for the remainder of the ice column, we introduce the integer i to determine in which annual layer we are:

$$i = \left\lfloor \frac{\tilde{z} - \Psi(t)}{\Delta H} \right\rfloor \quad (2.44)$$

where $\lfloor x \rfloor$ is the largest integer smaller than or equal to x . The first annual layer is characterized by $i = 0$, and its age is comprised between t and $t + 1$ year, etc. We can now finally generalize the ice age for the whole column:

$$a(\tilde{z}, t) = \begin{cases} a^{new}(\tilde{z}, t), & 0 \leq \tilde{z} \leq \Psi(t) \\ a^{new}(\tilde{z} - \Psi(t) - i\Delta H, T) + t + iT, & \Psi(t) \leq \tilde{z} \leq H(t), \end{cases}$$

where $T = 1$ year.

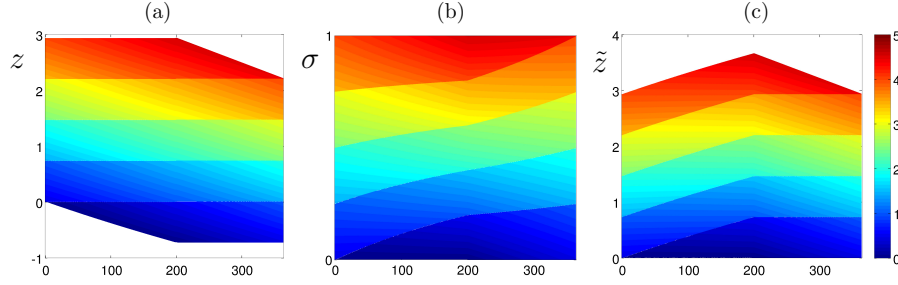


Figure 2.5: (a) Time series (in days) of the analytical sea ice age ($N = 5$, 0-5 years) contoured in the sea ice domain. (b) Same, but in the σ domain. (c) Same, but in the $\tilde{z}$ domain. As expected, the age gets older with height. The sharp variations in colour between two growth seasons illustrate the annual stratification of the ice column and the discontinuities in the age profile.

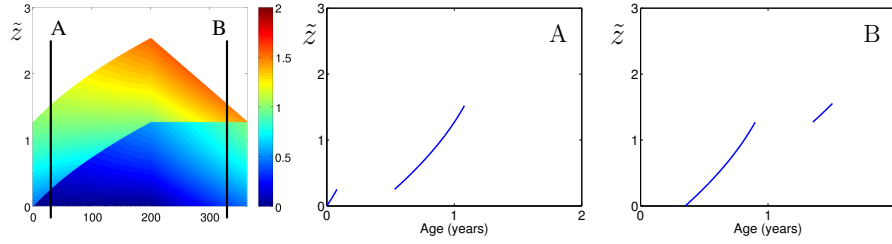


Figure 2.6: Left: time series (in days) of the analytical sea ice age ($N = 2$, 0-2 years) contoured in the $\tilde{z}$ domain. Centre: profile A (snapshot after 1 month) of sea ice age. Right: profile B (after 11 months). The discontinuity in ice age corresponds to the duration of the melt season.

The analytical solution is illustrated in figures 2.5 and 2.6. Fig. 2.5 shows the time series (in days) of the analytical sea ice age contoured in the sea ice domain for the three different coordinates (case $N = 5$). The periodicity of the ice regime is obvious. In the z domain (Fig. 2.5a), the reference level is taken to be the ice bottom at the beginning of the accretion period. In this simple model, ice particles are formed at the ice bottom during the accretion period and keep the same height until they melt at the ice surface during the melting period. The age thus gets older with height and is comprised between 0 and 5 years. The sharp variations in colour between two growth seasons illustrate the annual stratification of the ice column and the discontinuities in the age profile. Note that the number of annual growth layers corresponds to N and the number of age discontinuities to $N - 1$. When

the ice column is described as a function of the distance to the bottom (panel (c)), accretion at the ice bottom generates a vertical velocity (see equation (2.26)), while during summer, we have the same age contour as in the previous reference frame due to the absence of basal melting. Panel (b) illustrates the same configuration, but using σ coordinates. The fact that the domain is fixed is counterbalanced by a much more complex particles movement depending on the accretion/melting rates at both interfaces (see equation (2.20)). Fig. 2.6 illustrates two ice age profiles in the $\tilde{z}$ domain. Profile A corresponds to the growth period. New particles of age 0 are accreted at the ice bottom and the discontinuity in ice age corresponds to the duration of the melt season. Profile B illustrates the melt season. Particles at the ice surface are melted, while the rest of the particles see their age increasing at the same rate as time passes.

2.4 Numerical methods

In this section, we present two numerical methods for solving the sea ice age equation. The first method is designed to capture the vertical profile of sea ice age and its discontinuities. This method can be viewed as a Lagrangian method because the vertical velocity vanishes in our model. Therefore, the Eulerian grid nodes correspond to Lagrangian particles. The second one is based on a finite element method, is computationally cheaper and aims at simulating the mean and standard deviation of the age only. To illustrate both methods' respective strengths and weaknesses, we compare them to the analytical solution.

2.4.1 Spatial adaptive discretization

We present a simple adaptive approach to compute the vertical profile of the ice age. It consists in updating the age of a large number of ice particles. The vertical resolution can be chosen arbitrarily, the point here being not to enhance the precision but merely the "picture" of the vertical distribution of the ice age that we get. Actually, such a method consists in integrating a partial differential equation along the characteristic and will provide an exact solution if the temporal discretization is performed exactly. This is particularly helpful in a realistic simulation where no analytical solution exists. Since particles only move vertically in this work, the variables in this method are the number n of particles and the distance of the first particle to the bottom (z_0). Also of use are the distance of the last particle to the surface (z_n), the ice thickness (H) and the growth/melt rates. The method is illustrated in Fig. 2.7.

The particles are updated at each time step in function of the basal and surface growth/melt rates. For the sake of generality, in the sketch of the method (Fig. 2.7), summer melt has bottom and surface contributions, whereas the analytical solution involves surface melt only. If the distance z_0 (z_n) is larger than the prescribed

vertical resolution Δz , we add a particle at the bottom (surface) of the ice layer. If the distance to the bottom (surface) is negative, we remove the first (last) particle. In practice, we store the accretion time of the particles instead of their age, avoiding any further time update. The computational overhead to handle the Lagrangian particles is almost negligible, unlike the memory footprint of this method.

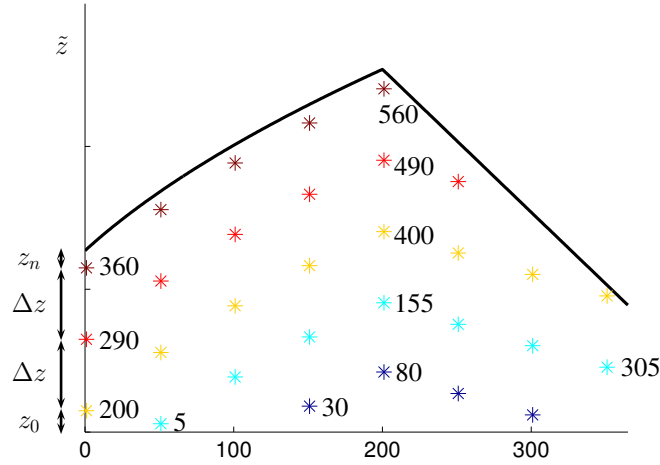


Figure 2.7: Ice thickness seasonal cycle illustrating the Lagrangian particles method used in this work. z_0 and z_n are the distance of the first particle to the bottom and the distance of the last particle to the surface, respectively, and Δz denotes the vertical resolution. The age of some particles is noted (in days). In this illustration, summer melt has bottom and surface contributions. Bottom melt is responsible for downward motion of the particles in this referential.

2.4.2 Spatial ALE discretization

The aim of this method is to reproduce the mean and standard deviation of the vertical distribution of the ice age using as few degrees of freedom as possible. The problem is easier to solve numerically on a fixed domain, using the σ coordinates. The boundary value problem (see equations (2.19)-(2.23)) that needs to be solved for the vertical ice age profile $a(\sigma, t)$ is summarized as follows:

$$\frac{\partial a}{\partial t}(\sigma, t) + \frac{(1 - \sigma)q_b(t) - \sigma q_s(t)}{z_s(t) - z_b(t)} \frac{\partial a}{\partial \sigma}(\sigma, t) = 1, \quad (2.45)$$

with the following boundary conditions:

$$a(\sigma, 0) = a_0(\sigma), \quad (2.46)$$

$$a(0, t) = 0, \quad \text{if } q_b(t) > 0, \quad (2.47)$$

$$a(1, t) = 0, \quad \text{if } q_s(t) > 0, \quad (2.48)$$

where $q_b(t)$ and $q_s(t)$ are prescribed by the ice thermodynamics. In order to compute the age profile by the finite element method, a variational formulation of this problem must be obtained. To this end, we first multiply equation (2.45) by a test function $\hat{a}$ and we integrate the partial differential equation over the height of the domain:

$$\frac{d}{dt} \int_0^1 a \hat{a} d\sigma = -[\omega a \hat{a}]_0^1 + \int_0^1 \left(\frac{\partial(\omega \hat{a})}{\partial \sigma} a + \hat{a} \right) d\sigma, \quad (2.49)$$

where the second term of equation (2.45) has been integrated by parts. Furthermore, the unknown field $a(\sigma, t)$ is approximated by piecewise polynomial functions defined as follows:

$$a(\sigma, t) \approx a^h(\sigma, t) = \sum_{j=0}^p a_j(t) \phi_j(\sigma) \quad (2.50)$$

where $\phi_j(\sigma)$ ($j = 0, 1, \dots, p$) are the polynomial shape functions and $a_j(t)$ the unknown degrees of freedom. In a usual Galerkin formulation, the shape functions are used as test functions $\hat{a}$. Taking the boundary condition (2.47)-(2.48) and the expression for ω into account, the first term of second member of equation (2.49) now reads:

$$-[\omega a \phi_i]_0^1 = \frac{q_b^- a(0, t) \phi_i(0) + q_s^- a(1, t) \phi_i(1)}{H}, \quad i = 0, 1, \dots, p \quad (2.51)$$

where the $-$ superscripts denote the melt rates. Substituting equations (2.50) and (2.51) in (2.49) gives rise to a system of $p+1$ ordinary differential equations and $p+1$ unknowns $a_j(t)$:

$$\sum_{j=0}^p \int_0^1 \phi_i(\sigma) \phi_j(\sigma) d\sigma \frac{da_j}{dt} = \sum_{j=0}^p A_{ij} a_j + \int_0^1 \phi_i(\sigma) d\sigma, \quad i = 0, 1, \dots, p \quad (2.52)$$

where $A_{ij} = \frac{q_b^- \phi_i(0) \phi_j(0) + q_s^- \phi_i(1) \phi_j(1)}{H} + \int_0^1 \left(\frac{\partial(\omega \phi_i)}{\partial \sigma} \phi_j \right) d\sigma$. The system of equations gets even simpler if one decides to opt for orthonormal shape functions, i.e.

$$\int_0^1 \phi_i(\sigma) \phi_j(\sigma) d\sigma = \delta_{ij}, \quad (2.53)$$

where δ_{ij} denotes Kronecker's delta, diagonalizing the mass matrix. Such an orthogonal property is provided by the Legendre polynomials P_j , from which we derive the shape functions:

$$\phi_j(\sigma) = \sqrt{2j+1} P_j(2\sigma-1). \quad (2.54)$$

These polynomials are illustrated in Fig. 2.8. The system now takes the following form:

$$\frac{da_i}{dt} = \sum_{j=0}^p A_{ij} a_j + \delta_{0i}, \quad i = 0, 1, \dots, p \quad (2.55)$$

and is easily discretized in time, using in this work the forward Euler scheme for simplicity. The coefficients of the matrix A are given in the appendix A.1. The question as to whether it is better to use several piecewise polynomials of low order or one polynomial of higher order resorts to the interpolation problematic and is treated in the appendix A.2. We find that increasing the number of elements does not enhance significantly the approximation and, in the remainder of the study, we will restrict ourselves to polynomials of degree up to three on one element.

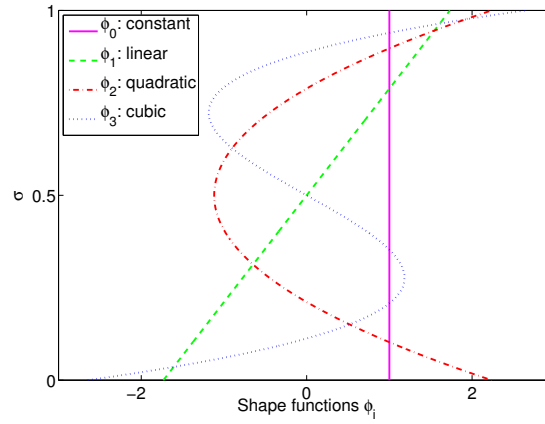


Figure 2.8: Orthogonal Legendre shape functions used to solve numerically the age equation in function of the normalized vertical coordinate σ .

2.4.3 Validation with the analytical solution

Numerical results of the Lagrangian particles and the finite element method are compared against the analytical solution derived in section 2.3. The solutions are shown after a spin-up of 25 years of integration. The vertical resolution of the Lagrangian particles is taken to be $\Delta z = 0.05$ m. For the finite element solutions, we use polynomials of degree up to three on one element.

There is a very good agreement between the analytical solution and the Lagrangian particles (Fig. 2.9) which are clearly able to reproduce the ice age discontinuities across melting periods. As one could have expected, the constant finite element approximation clearly is to be rejected. To explain this, we show that the

constant approximation corresponds to the mean value $\mu(t)$ of the age distribution:

$$\mu(t) = \int_0^1 a^h(\sigma, t) d\sigma = \sum_{j=0}^p a_j(t) \underbrace{\int_0^1 \phi_j(\sigma) d\sigma}_{=\delta_{0j}} = a_0(t).$$

In the particular framework of the analytical solution, equation (2.55) gives the following relationship for the coefficient $a_0(t)$:

$$\frac{da_0(t)}{dt} = \begin{cases} -\frac{q_b(t)}{H(t)} a_0(t) + 1, & 0 \leq t \leq T_a \\ 1, & T_a \leq t \leq T_a + T_m. \end{cases} \quad (2.56)$$

During the accretion period, as the newly frozen ice has an age of zero, the ice growth tends to decrease the mean age ($-\frac{q_b(t)}{H(t)} a_0(t)$ term), counterbalancing the ice ageing process (term equal to 1). During the melting period, the oldest ice is supposed to “melt and carry along its age”, decreasing the mean value of the age. However, this effect is not taken into account with a constant age approximation, explaining the (inconsistent) large values of the computed mean age (Fig. 2.9). In consequence, we will omit the constant approximation for the remainder of this study.

For young ice ($N = 2$, panels a-b), differences between the finite element solutions are clear. In the early accretion period, the cubic approximation gives a negative ice age which has to be rejected. For multiyear ice ($N = 5$, panels c-d), all finite element solutions (except the constant approximation) converge towards a linear evolution of the ice age inside the ice, since no discontinuity is introduced in the solution. This result is similar to the conclusion drawn in appendix A.2.

In order to determine the accuracy of each method, we compute the root mean square (RMS) error between the numerical approximations and the analytical solution (Table 2.2). As expected, the Lagrangian particles method shows excellent accuracy to simulate sea ice age and the error is about two orders of magnitude smaller than the error produced by the finite element approximations. As mentioned before, the error is not affected by the number of particles. On the other hand, due to the forward Euler scheme, the error is $O(\Delta t)$ so that, asymptotically, decreasing the time step by two yields an error divided by two. The RMS error of the finite element solutions is approximately the same for any value p . Thus increasing the order of the approximation does not enhance the solution in the presence of discontinuities.

The main goal of the finite element method is to approximate the mean value and standard deviation of the ice age distribution. To evaluate this ability, we compare the mean and standard deviation of the analytical solution age distribution with the

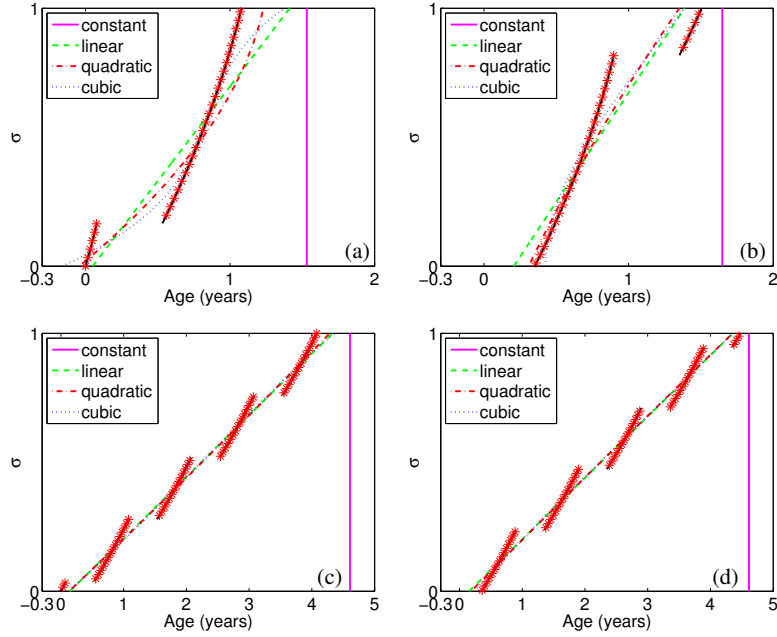


Figure 2.9: Analytical (solid black line) and numerical solutions of the ice age as a function of the normalized vertical coordinate. The red stars represent the Lagrangian particles. The degree of the Legendre polynomials is specified in the legend. (a)-(b) Snapshot of the case $N = 2$ after month 1 (a) and 11 (b). (c)-(d) Same for $N = 5$. Note that each value of the integer N corresponds to an equilibrium state that is reached by the numerical solution after 25 years of integration.

N	2	3	4	5	6	7
Lagr.	0.0016	0.0015	0.0017	0.0015	0.0015	0.0015
P_1	0.1326	0.1314	0.1308	0.1305	0.1304	0.1304
P_2	0.1198	0.1253	0.1276	0.1286	0.1291	0.1294
P_3	0.1034	0.1245	0.1281	0.1292	0.1296	0.1298

Table 2.2: RMS error (in years): Lagrangian particles method (Lagr.) and finite element method: linear (P_1), quadratic (P_2) and cubic (P_3) approximations.

numerical solution. Based on the finite element method, the standard deviation of the age distribution is computed as:

$$\zeta(t) = \sqrt{\int_0^1 [a^h(\sigma, t)]^2 d\sigma - \left[\int_0^1 a^h(\sigma, t) d\sigma \right]^2} = \sqrt{\int_0^1 \left[\sum_{j=0}^p a_j(t) \phi_j(\sigma) \right]^2 d\sigma - [a_0(t)]^2}.$$

Thanks to the orthogonality of the shape functions, the standard deviation reads finally:

$$\zeta(t) = \sqrt{\sum_{j=1}^p [a_j(t)]^2}.$$

Tables 2.3 and 2.4 give the mean and standard deviation of the ice age for the finite element solutions. The linear approximation (P_1) yields satisfactory results which improve for increasing N . Actually, the analytical solution is composed of a linear part with a deviation of mode $N-1$ and of amplitude of the order of half a year. The larger N , the better the linear approximation since the relative importance of the deviation decreases. Again, increasing the order of the finite element approximation does not remarkably enhance the solution.

N	2	3	4	5	6	7
P_1	0.0174	0.0106	0.0071	0.0053	0.0041	0.0032
P_2	0.0229	0.0118	0.0075	0.0055	0.0043	0.0035
P_3	0.0166	0.0095	0.0066	0.0051	0.0041	0.0033
$\bar{\mu}$	0.7634	1.2563	1.7532	2.2512	2.7497	3.2485

Table 2.3: RMS error of the finite element method on the mean value $\mu(t)$ of the ice age (in years): linear (P_1), quadratic (P_2) and cubic (P_3) approximations. The annual mean of $\mu(t)$ ($\bar{\mu}$) is also shown.

N	2	3	4	5	6	7
P_1	0.0428	0.0246	0.0166	0.0123	0.0094	0.0078
P_2	0.0348	0.0171	0.0111	0.0084	0.0068	0.0058
P_3	0.0246	0.0136	0.0098	0.0078	0.0065	0.0056
$\bar{\zeta}$	0.4247	0.7165	1.0063	1.2956	1.5846	1.8735

Table 2.4: RMS error of the finite element method on the standard deviation $\zeta(t)$ of the ice age (in years): linear (P_1), quadratic (P_2) and cubic (P_3) approximations. The annual mean of $\zeta(t)$ ($\bar{\zeta}$) is also shown.

2.5 Application to the large-scale distribution of sea ice age

In this preliminary study, both numerical methods are applied to a stand-alone thermodynamic model of the Arctic sea ice to evaluate the vertical profile of the sea ice age, which, in the case of undeformed ice, is essentially thermodynamically driven. The model used in this study corresponds to the thermodynamic component of the model presented in Lietaer et al. (2008), based on Semtner (1976)'s model. In the absence of ice transport, the modeled age field gives an approximation of the time needed to thermodynamically grow an ice layer as a function of local atmospheric conditions.

2.5.1 A stand-alone thermodynamic sea ice model of the Arctic

The computation of heat diffusion within the thick ice is based on Semtner (1976)'s zero-layer model. This model neglects the storage of sensible and latent heat, resulting in a linear temperature profile in the ice. Following Fichefet et al. (1998), two boundary conditions are needed, expressed as a heat budget at the surface and at the bottom of the ice. At the upper surface, an equilibrium surface temperature T_s is computed from the heat balance:

$$\underbrace{(1 - \alpha)F_r + F_L + F_s + F_l + F_C}_{F_A} = 0, \quad (2.57)$$

where α is the surface albedo, which in our model is prescribed for each month and takes the presence of snow into account. F_A is the net atmospheric flux to the upper ice surface and F_C is the conductive flux through ice. Fluxes directed to the ice surface are taken to be positive. In order to solve the heat balance equation, the model includes a parameterization of the solar radiation F_r (Zillmann, 1972), the net longwave radiation F_L (Berliand and Berliand, 1952) and turbulent fluxes of sensible (F_s) and latent (F_l) heat (bulk aerodynamic formulas, see Goosse (1997)). If the predicted T_s is above the melting point T_f , its value is fixed to T_f and the excess of energy is used to melt ice:

$$q_s = -\frac{F_A + F_C}{L}, \quad (2.58)$$

where L represents the volumetric latent heat of fusion of ice.

At the ice base, temperature is kept at the freezing point of seawater. The melt or growth rate of the ice depends only on the imbalance between the conductive heat flux F_C and the oceanic heat flux F_b :

$$q_b = \frac{F_C - F_b}{L}. \quad (2.59)$$

We use a simple slab ocean model and assume that the ocean has a constant mixed layer depth of 30 m. This ocean layer is characterized by a unique temperature T_w that is equal to the freezing point of seawater if there is ice on the area element. When the area element is ice-free, the ocean temperature is computed thanks to the following prognostic equation:

$$\rho_w C_{pw} h_{mixl} \frac{\partial T_w}{\partial t} = B_l. \quad (2.60)$$

B_l is the heat budget of the oceanic area:

$$B_l = (1 - \alpha_w) F_r + F_L + F_s + F_l + F_b, \quad (2.61)$$

where α_w stands for the ocean surface albedo. The oceanic flux consists of a restoring term towards a climatological temperature of the mixed layer (T_{mixl}) taken from the Polar science center Hydrographic Climatology (PHC 3.0, updated from: Steele et al. (2001)):

$$F_b = \rho_w C_{pw} h_{mixl} \gamma_t (T_{mixl} - T_w), \quad (2.62)$$

where C_{pw} is the specific heat of seawater and γ_t a relaxation constant ($6 \times 10^{-8} \text{ s}^{-1}$).

Finally, the atmospheric forcing data sets used to run the model are daily NCEP/NCAR reanalysis data for the air temperature and the wind velocity, and monthly climatologies for the relative humidity (Trenberth et al., 1989) and the total cloudiness (Berliand and Strokina, 1980). No snow precipitation is included in the model.

The model here is purely thermodynamic, neglects new ice growth in leads as well as ice transport and deformation. Consequently, ice production and piling are underestimated and the simulated ice thickness gradient between Siberia and Canada is too weak (Fig. 2.10). However, despite the absence of ice dynamics, ice thickness of up to 6 m is produced along the Canadian Arctic Archipelago and Greenland, which is in agreement with observations (e.g., Bourke and Garrett, 1987; Haas et al., 2010). The presence of very thick ice is the signature of the (low) observed air temperatures used to force the model there. The reanalyses of air temperature are naturally influenced by the cold winds around Greenland and other glaciers and by the very thick ice present along the Canadian Arctic Archipelago. In consequence, the thermodynamic sea ice model reproduces very thick ice in turn. Despite being simple, the model does reasonably simulate the features of Arctic sea ice thickness that are required to understand the thermodynamic controls on the vertical profile of sea ice age.

2.5.2 Results of the large-scale distribution of Arctic sea ice age

The thermodynamic model is spun up for 12 years (so that the initial state of the ice age has vanished) and is then integrated with daily wind and air temperature

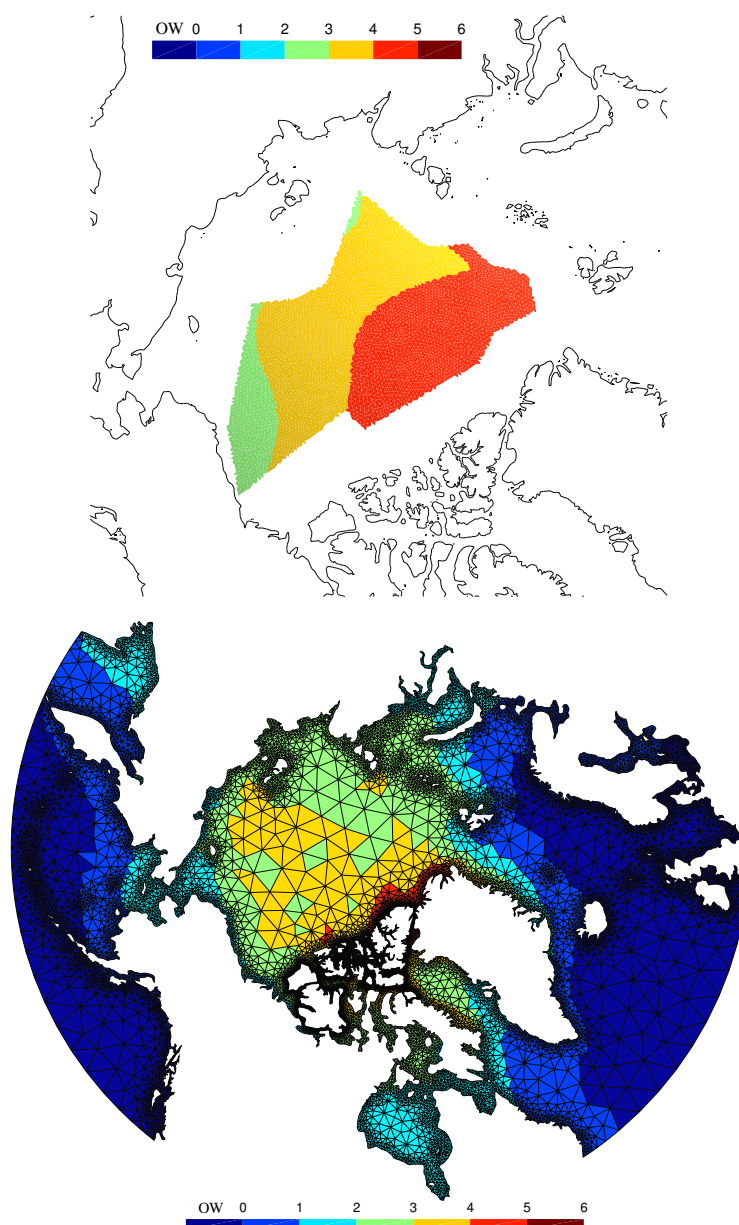


Figure 2.10: Top: ice thickness climatology from Rothrock et al. (2008) converted from ice draft data collected by naval submarines (April 1975-2000, 0-6 m). Bottom: global sea ice thickness pattern as computed by the thermodynamic model (April 1979-2000, 0-6 m). OW stands for “open water”.

forcings between 1979 and 2009. The computational domain is situated north of the parallel 50 degrees North and englobes most of the Arctic sea ice. It is composed of triangular elements at the barycenter of which we compute the ice age, following the two methods exposed before. In absence of ice dynamics, everything happens as if each element corresponds to an ice column. In this section, the vertical resolution of the Lagrangian particles is set to $\Delta z = 0.1$ m.

First, a general overview of the maximum age of the ice columns is presented in Fig. 2.11, based on the Lagrangian particles method. We compute the 1979-2009 March mean age of the oldest Lagrangian particle of each ice column. Within our general framework and hypotheses, in this numerical simulation, most of the sea ice of the Arctic Basin is second- or third-year ice. Due to the influence of Greenland on the air temperature reanalyses, the ice along the coast of Greenland and the Eastern Canadian Arctic Archipelago is thick and old (older than 6 years). In absence of dynamics, the first-year ice (comprised between 0 and 1 year) matches, by definition, the seasonal ice zone.

The bottom panel of Fig. 2.11 corresponds to the same result as the top panel, except that the average age of all Lagrangian particles on each vertical profile is shown instead of the age of the oldest particle. This illustrates the two different approaches discussed in the introduction: tracking the surface ice age or allowing bottom ice growth to decrease the ice age, i.e. a volume-averaged ice age. Compared to the top panel, the spatial pattern of sea ice age is very similar, except that the color scale has been divided by two. This factor of approximately two confirms the discrepancy between the two approaches noted by Hunke and Bitz (2009).

At first glance, the correlation between ice thickness (Fig. 2.10) and the age of the oldest ice particle (Fig. 2.11, top) seems evident (e.g., most of the first-year ice is smaller than 2 m). Indeed, the overall correlation amounts to about 0.84 for the average month of March 1979-2009. However, this correlation is not uniform over the Arctic Basin as shown in Fig. 2.12. In particular, the correlation is highest in the Beaufort sector (0.97) and lowest in the Kara sector (0.71). It should be noted that, judged from the upper and lower bounds of each year distribution, the scatter follows the typical curve of purely thermodynamically growing ice, i.e. decreasing growth rates with increasing ice thickness and growth towards a cyclostationary state. Therefore, a linear regression fit should not match perfectly by definition. We further compute the mean ice thickness for each age class for the same period as Maslanik et al. (2007), i.e. March 2003-2006 (Fig. 2.13). Our model tends to overestimate the ice thickness, except for first-year ice, and the ranges of the ice thicknesses within each ice class are larger than those obtained by Maslanik et al. (2007). We obtain a strong relationship between mean ice age and mean ice thickness (0.87 for all classes and 0.95 for 1-8 years). However, only the first ice class preserves a high

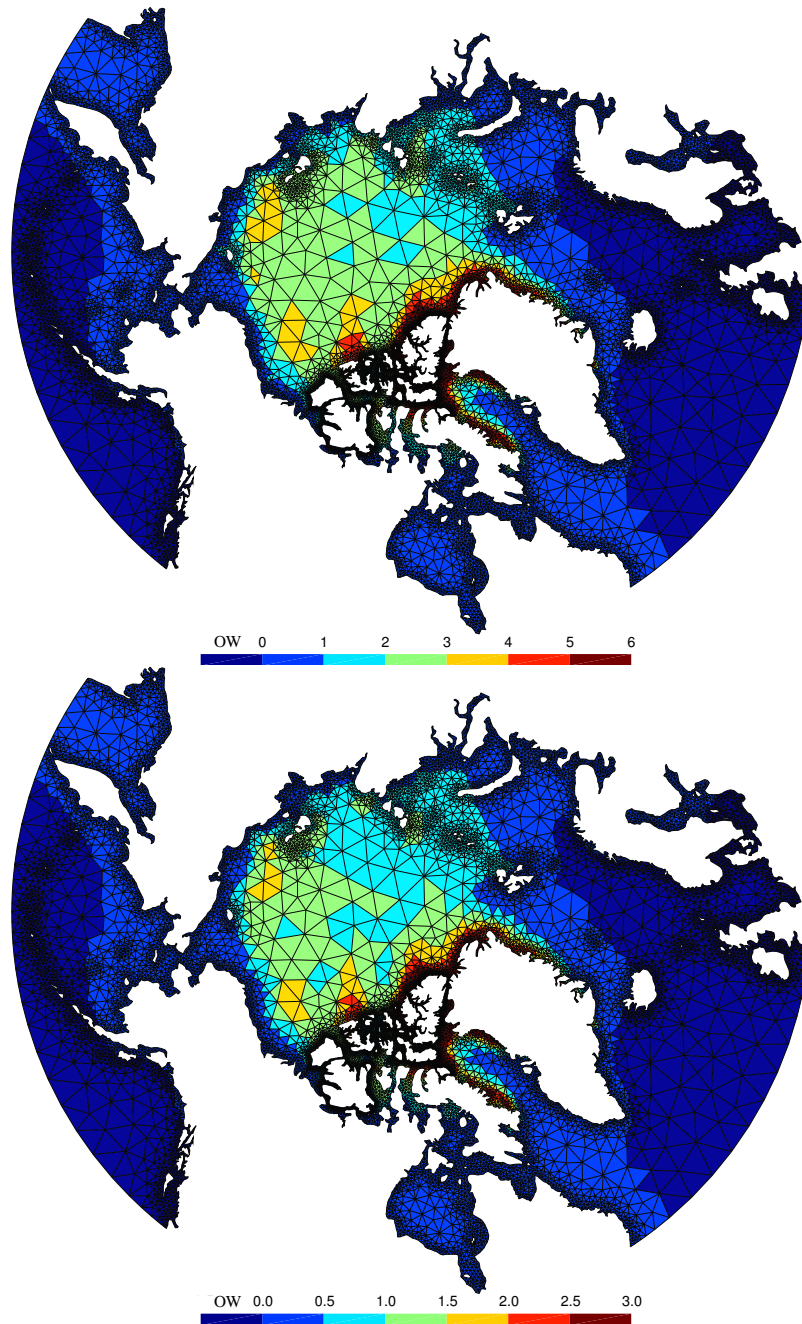


Figure 2.11: Ice age (March 1979-2009) computed north of the parallel 50 degrees North. Top panel: for each triangular element of this numerical simulation, the long-term mean of the age of the oldest Lagrangian particle is represented. Bottom: same, but the average age of all Lagrangian particles on each vertical profile is shown instead of the age of the oldest particle. Note the different scale. OW stands for “open water”.

correlation (0.77) when computing the correlation within a single ice class. In our model, the relationship is the image of the growth/melt rates and is thereby subject to the atmospheric variability. Our results are in agreement with Hunke and Bitz (2009): outputs from numerical models show that sea ice age is a good proxy for sea ice thickness when averaged over large spatial and time scales, while the correlation breaks down at smaller scales. Presumably, improvements could be achieved by using information on, for example, the history of atmospheric forcing fields.

An overview of the evolution of sea ice age is shown in Fig. 2.14. We compare two different years to illustrate the thermodynamical evolution of the ice near the Canadian Archipelago in our simulation. In this case, we observe ice thinning in the model between 1979-1980 and 2008-2009. This decrease of about 1.5 m corresponds to the signature of the global warming in the air temperature data used to force the model. As can be seen in Fig. 2.14, the ice column essentially undergoes surface melting, so the ice thinning is coupled to a decrease in sea ice age of about 2 years.

Finally, a couple of snapshots of Arctic ice age profiles are provided in Fig. 2.15. The right figure illustrates the vertical age profile for the same place as in Fig. 2.14 for March 2009. The finite element solutions converge towards a linear age profile, missing the age discontinuities between annual layers revealed by the Lagrangian particles method. The ice column surface is almost 7 years old. The centre panel shows a snapshot of second-year ice in Beaufort Sea. Differences between the finite element solutions are clearly marked. The quadratic solution inconsistently predicts a younger ice inside the ice column than at the bottom of it. However, it must be stressed that this method has been designed to compute the mean and standard deviation of the age distribution. For both snapshots, the error in mean age computed by the finite element solutions compared to the particles method is smaller than 10 days and the error in standard deviation does not exceed 13 days. Moreover, the finite element solutions give an approximation of the maximum age of the column (that is, the age at the surface in the absence of ice formation at the surface) that is reasonable, considering that sea ice is traditionally classified in year classes (first-year, second-year, ...) for climatological analyses. On the left, a first-year ice vertical profile in the Laptev Sea is shown. In the absence of age discontinuities, quadratic and cubic approximations are in very good agreement with the particles method. Since the Arctic Ocean is the subject of a shift from perennial to seasonal ice (e.g., Maslanik et al., 2007; Nghiem et al., 2007), this is an interesting feature of the method.

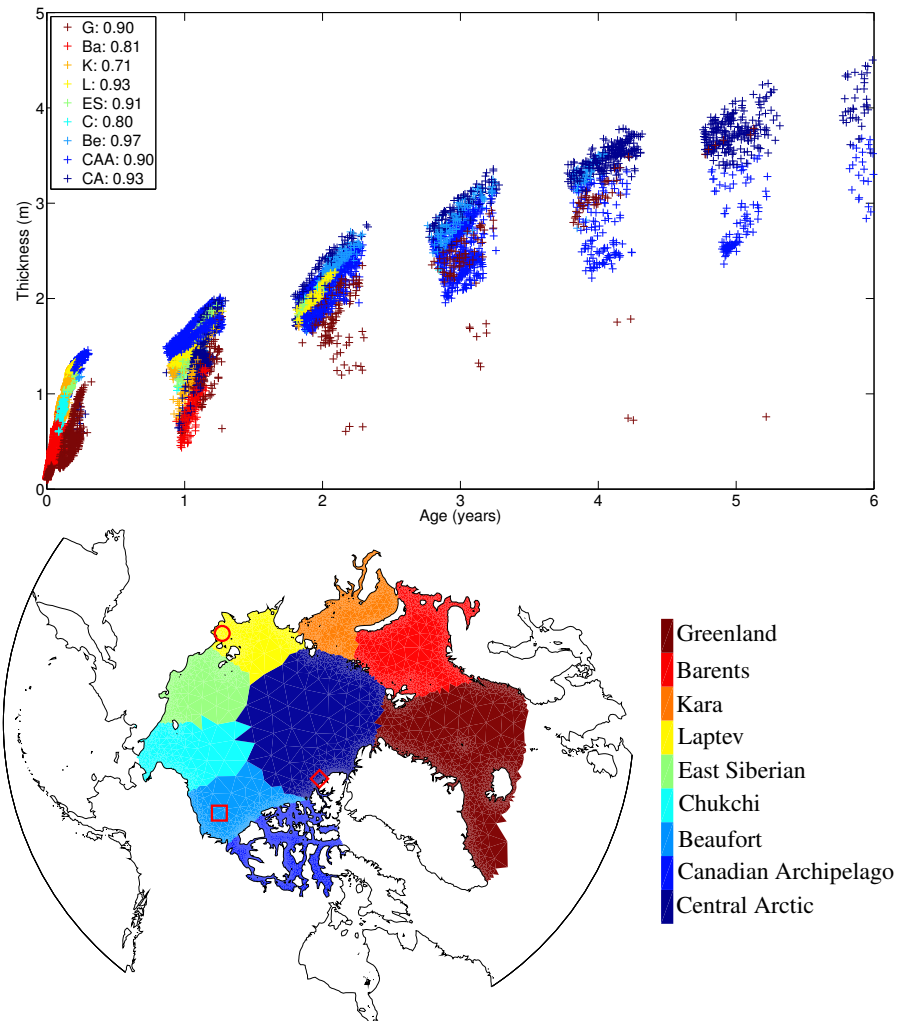


Figure 2.12: Ice age of the oldest particle at the end of the simulation for each column belonging to one of the sectors described in the bottom figure. The correlation of each sector between the ice age and the ice thickness is mentioned in the legend of the top figure. The melting period induces gaps between crosses clusters that clearly identify ice types (first-year, second-year, ...). Note that for the sake of readability, the top figure has been cropped to ice particles of maximum 6 years.

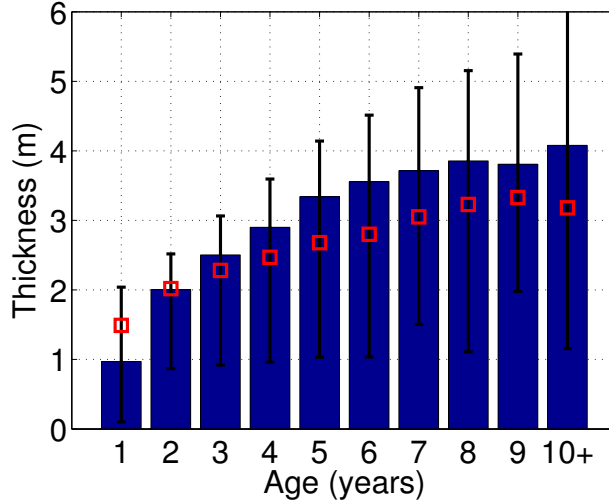


Figure 2.13: Average March 2003-2006 ice thickness as a function of the ice age. 10+ corresponds to ice that is at least 10-years old. The ranges of mean thicknesses over the period are indicated. Also shown is the proxy ice thickness data from Maslanik et al. (2007) for the same period (red squares).

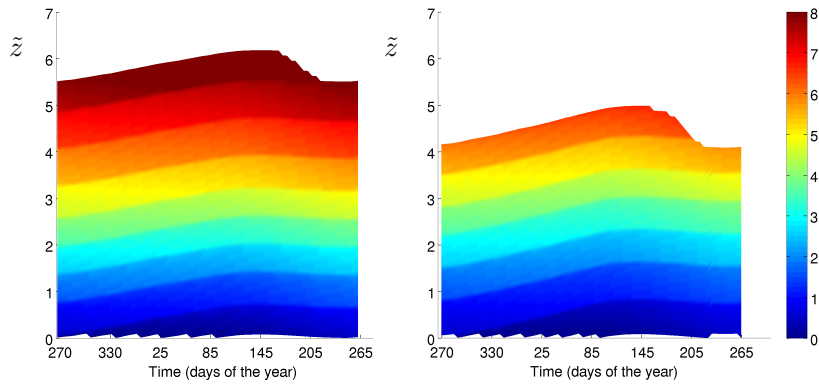


Figure 2.14: Left: time series of sea ice age (0-8 years) contoured in the $\tilde{z}$ domain. The ice column is situated along Ellesmere Island (near the Canadian Arctic Archipelago, see the red diamond in Fig. 2.12) for the years 1979-1980, starting at the beginning of the accretion period (days of the year). The sharp variations in colour between two growth seasons are clearly marked, illustrating again the annual layering of the ice column. Right: same, but for the years 2008-2009.

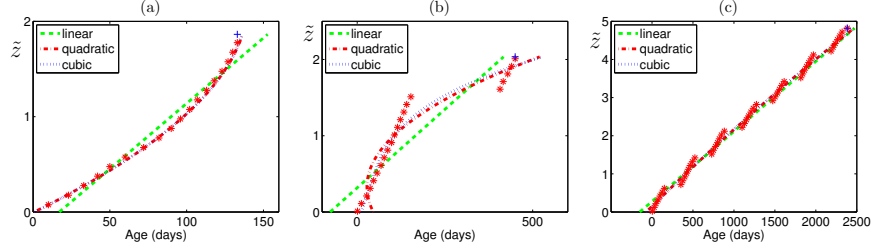


Figure 2.15: Comparison of vertical age profiles (March 2009) as computed by the Lagrangian particles method (red stars) and the finite element method. The blue cross indicates the ice surface. (a) First-year ice (Laptev Sea, see the red circle in Fig. 2.12). In absence of age discontinuities, quadratic and cubic approximations are in very good agreement with the particles method. The linear approximation gives a measure of the mean and standard deviation of the age distribution that is as accurate as higher order polynomials. (b) Second-year ice (Beaufort Sea, see the red square in Fig. 2.12). Differences between the finite element solutions are clearly marked. (c) Multiyear ice (along Ellesmere Island, see the red diamond in Fig. 2.12).

2.6 Discussion and conclusions

In this chapter, we have introduced a vertically-variable sea ice age, based on the age theory in marine modeling (see, e.g., Delhez et al. (1999) and Deleersnijder et al. (2001)). The age of a sea ice particle is a function of the three spatial coordinates and is interpreted as the time elapsed since its creation. The age is defined here in a geological sense and differs from the ice age derived from satellite data measuring the time elapsed since a parcel of ice has been created. Our developments include the particular case of ice formation at the surface (see the boundary condition (2.16)).

We have first resolved the sea ice age equation in the framework of an analytical model of sea ice, combining the classical formulation of ice growth by Stefan (1891) and a linear parameterization of melt. An analytical solution of this model has been derived. In absence of ice formation at the surface, ice age typically increases upwards and features discontinuities corresponding to the length of the melting periods. Based on the age, the ice column can thus be decomposed in annual growth layers. This annual layering has been reported in past studies (e.g., Cherepanov, 1957; Schwarzacher, 1959). However, these observations of ideal, undeformed multiyear ice are sparse and the stratification is generally less defined in the upper part of the ice experiencing melting and refreezing (Weeks, 2010).

Two numerical methods have been further introduced. The first method involves Lagrangian particles and thereby follows an approach similar to what has been done in, e.g., ice sheet modeling (see Mugge et al. (1999); Rybak and Huybrechts (2003)). This method is adaptive and designed to capture the vertical profile of sea ice age and its discontinuities. Based on the characteristic method, it provides excellent agreement with the analytical solution but is expensive in terms of memory usage (e.g., adding 10 particles to each element is equivalent to adding 10 P_0 tracers on the domain). Increasing the number of particles does not enhance the precision, but merely the “picture” of the vertical age profile that we get. The aim of the second method is to reproduce the mean and standard deviation of the vertical age profile using as few degrees of freedom as possible. The method uses an ALE spatial discretization and is based on the finite element method. Due to the age discontinuities, the approximations of degree up to three do not accurately represent the vertical distribution in sea ice age, but a linear approximation is sufficient to simulate the mean and standard deviation of the age in very good agreement with the analytical solution.

Both methods have been used in a large-scale simulation of the Arctic by accounting for thermodynamic processes only. In this framework, the correlation between ice thickness and ice age as computed by the Lagrangian method is typically high. However, the scatter is significant, and it does not seem feasible to find a unique relationship between ice age and thickness, as pointed out earlier by Hunke and Bitz (2009) and Vancoppenolle et al. (2009) using a different definition. Furthermore, the growth of new ice in leads, transport and piling of ice by deformation may further reduce the age-thickness relationship. In conclusion, our study suggests that the ice age is a good proxy for ice thickness where ice deformation is small or absent. But even in this simplified framework, it is not sufficient to reconstruct ice thickness. The finite element method is able to compute the mean and standard deviation of the age distribution in good agreement with the particles method, and, in the particular case of first-year ice, even its profile is well reproduced. Moreover, the method gives an order of magnitude of the maximum age of the column that is reasonable, considering that sea ice age is typically presented in year classes for climate studies.

Though the principal contribution to compute the ice age comes from the ice growth/melt rates, the role of dynamics is important in redistributing the ice age profile. The ice transport may change dramatically the atmospheric and oceanic conditions (e.g., multiyear ice exported through Fram Strait) experienced by an ice particle. Piling of ice in regions of strong deformation redistributes the age profile. Taking into account new ice formation in leads should largely increase the ice production and consequently provide a greater volume of young ice. It is therefore expected that the age distribution should significantly deviate from a quasi linear profile, which should reduce the accuracy of the finite element method based

on a linear polynomial and should increase the interest of higher degree polynomials. The results should also clearly benefit from an oceanic feedback and the oceanic heat advection missing in this model. This should substantially reduce the ice thickness in closer agreement to observations during the last decade.

A perspective of this work is to take the ice dynamics into account and transport the ice age distribution. On the other hand, in future developments, the age must be integrated in the framework of the subgrid-scale ice thickness distribution (ITD, Thorndike et al. (1975)). In both cases, an open issue concerns the “mixing” of age profiles from different categories (ITD) or from different ice columns (transport). Though it should in principle be easy to get a vertically average age content, it sounds much more complex to reconstruct a consistent age profile. However, this is true for other properties that vary in the vertical (temperature, salinity, ...) and a solution for the finite element method can be devised. On the contrary, considering that the Lagrangian particles method is exact if the temporal integration is performed exactly, averaging different profiles would induce a significant loss of information (in particular, the position of the discontinuities). Lagrangian particles should thus be considered as anchored in a parcel of undeformed ice (i.e., neglecting the ice deformation) to measure the thermodynamical controls on the ice age. In this respect, Lagrangian particles might constitute an interesting method to study the evolution of passive tracers in the ice (e.g., in the biogeochemistry field). Other passive tracers of interest in the ice are $\delta^{18}\text{O}$ profiles (the ratio between stable oxygen isotopes), which could be easily computed thanks to a Lagrangian sea ice model. Following Pfirman et al. (2004), this would constitute an effective tool to study the evolution of Arctic surface water masses. By computing the sea ice age, the comparison between modeled isotopes profiles and sea ice cores would be easier for the samples presenting an annual stratification. Finally, modeled vertical profiles of sea ice age could be evaluated thanks to ice mass balance buoy data that measure ice growth and ablation.

CHAPTER 3

SEA ICE DYNAMICS: THE EFFECTS OF RESOLVING THE CANADIAN ARCTIC ARCHIPELAGO IN A FINITE ELEMENT SEA ICE MODEL

This Chapter reproduces an updated version of the following paper:

O. Lietaer, T. Fichefet, and V. Legat. The effects of resolving the Canadian Arctic Archipelago in a finite element sea ice model. *Ocean Modelling*, 24:140–152, 2008. doi: 10.1016/j.ocemod.2008.06.002.

Summary

Sea ice dynamics are accounted for in a two-level dynamic-thermodynamic sea ice model, including a viscous-plastic rheology. A finite element method is proposed to solve the sea ice dynamics equations. The model is run over 1979–2005, forced by daily NCEP/NCAR reanalysis data, and is validated against different sets of observational data. The sensitivity of the Arctic sea ice cover to the resolution of the Canadian Arctic Archipelago (CAA) is investigated. The importance of the CAA in terms of mass balance is highlighted.

3.1 Introduction

Sea ice is drifting at the surface of the ocean, mainly driven by the wind and the oceanic currents. The large-scale Arctic ice circulation is characterized by two main features. In the western part of the Arctic, the ice describes a large anticyclonic gyre named the Beaufort Gyre where the ice can be trapped for up to 10 years (Hibler, 1980; Rigor and Wallace, 2004), with typical ice velocities of 2 cm s^{-1} . In the east, the ice is transported by the Transpolar Drift from the north of Siberia to the North Atlantic through Fram Strait. This strait is approximately 435 km wide and constitutes the main exit for the Arctic sea ice, the ice volume export reaching about $2200 \text{ km}^3 \text{ yr}^{-1}$ ($\approx 0.07 \text{ Sv}$) (Kwok et al., 2004). Narrow straits, on their part, may have a strong impact on the geophysical flows in general (e.g. water mass exchanges between different oceanic basins, complex flow in localized areas, etc.) and, in the polar regions, they can be responsible for a complex dynamical behavior of sea ice: due to the high ice concentration and low ice temperature, the formation of ice bridges in channels like Nares Strait (Samelson et al., 2006) may prevent any ice flow for several months; later, the ice is suddenly released when the plugs collapse. Moreover, the ice characteristics in these channels have a strong interannual variability (Melling, 2002; Kwok, 2006).

Of particular concern is the complex area formed by the numerous islands and coastlines in the Canadian Arctic Archipelago (CAA, see Fig. 3.1). The straits constitute an important pathway for the cold waters from the Arctic Ocean to the Atlantic. Even if many recent studies have explored the freshwater cycle of the Arctic (e.g., Stigebrandt, 2000; Serreze et al., 2006), large uncertainties remain, especially concerning the sea ice contribution. Assembling different sources, we get the following picture: estimates of the ice freshwater outflow through the southern part of the CAA based on observations in Lancaster Sound (Prinsenberg and Hamilton, 2005) roughly amount to $160 \text{ km}^3 \text{ yr}^{-1}$ ($\approx 0.005 \text{ Sv}$) (Serreze et al., 2006), while, in the north, the Nares Strait sea ice flux (Kwok et al., 2004) accounts for $\sim 100 \text{ km}^3 \text{ yr}^{-1}$ ($\approx 0.003 \text{ Sv}$) of freshwater (assuming a sea ice salinity of 4 psu, for a reference seawater salinity of 34.8 psu, and a density of 900 kg m^{-3} , see Serreze et al. (2006)). In comparison, the oceanic freshwater flux through the CAA is evaluated by Prinsenberg and Hamilton (2005) at $3200 \text{ km}^3 \text{ yr}^{-1}$ ($\approx 0.1 \text{ Sv}$). The rest of the northern gates of the CAA contributes to a net inflow into the Arctic (Kwok, 2006), suggesting, as stated in the latter study, that most of the ice exported into Baffin Bay (except from Nares Strait) is produced in the CAA rather than exported from the Arctic. Because sea ice is landfast during a large part of the year in the northern CAA (Melling, 2002), the different straits act as a “buffer” on sea ice between the Arctic Ocean and Labrador Sea, preventing Arctic ice outflow in the north and providing ice generation in the south (Sou, 2007).

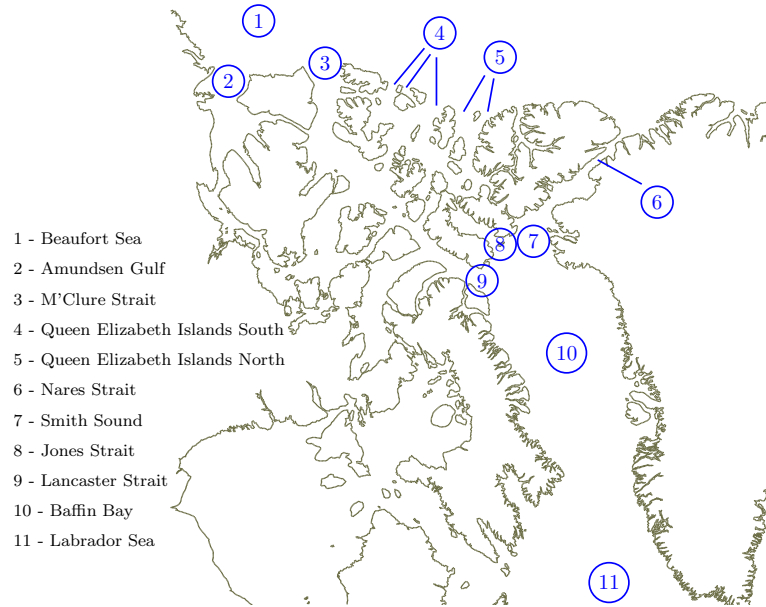


Figure 3.1: The Canadian Arctic Archipelago is composed of numerous islands and coastlines. All the geographical places of interest for this study are clearly labeled on the map. Most of them correspond to the gates for the computation of ice area/volume fluxes.

The role of the freshwater flux through the CAA is thought to be important regarding the freshwater budget of the region (Stigebrandt, 2000; Dickson et al., 2007), and may play an important role in the Labrador Sea convective overturning (Goosse et al., 1997). Less known is the influence of the ice contribution to this freshwater export, on the one hand, and the impact of the ice flux on the water stratification in Baffin Bay, on the other hand. Together with the North Water Polynya formation (northern Baffin Bay, see Barber et al. (2001)), the latter impact might constitute a part of the explanation why most of the models miss the decay of the summer sea ice cover in Baffin Bay (e.g., Timmermann et al., 2005; Johnson and Gerdes, 2007; Vancoppenolle et al., 2009). Indeed, observations show that, in northern Baffin Bay, the North Water Polynya existence is partly due to the formation of an ice bridge that prevents any ice flow. The polynya further extends southwardly during summer and promotes the ice retreat in Baffin Bay. Enhancing the modeled ice flux in the CAA, and in particular in Nares Strait, should thus help better representing the ice decay. Finally, data in this harsh region of the world are scarce (e.g., Melling, 2002), especially concerning the ice thickness, ranging between 2 and 6 m in the CAA (Bourke and Garrett, 1987; Melling, 2002; Kwok, 2005), underlining the need for numerical experiments. Furthermore, this area is currently being of strong

economical importance and political interest in the context of the opening of the Northwest Passage.

Very few modeling research has been performed on the CAA. Three studies are based on regional models: Kliem and Greenberg (2003) constructed gridded fields of potential temperature and salinity, used in a second step to simulate the mean circulation in the area on an unstructured grid, while Sou (2007) studied the impact of future climate scenarios on the CAA. Recently, Rasmussen et al. (2010) have modeled the sea ice in Nares Strait in a regional setup. The present work in turn presents a large-scale sea ice model operational for climate studies which, to our knowledge, is the first to investigate the effects of resolving the CAA on the ice cover features. This work has served as a basis for a recent study (Terwisscha van Scheltinga et al., 2010).

The model presented is aimed at being coupled to the finite element ocean model SLIM (White et al. (2008); Blaise et al. (2010); Comblen et al. (2010a)). As both models are still under development, and because a large part of the variability of the ice circulation can be explained by the winds (Thorndike and Colony, 1982; Serreze et al., 1989), the study presented here is conducted with an uncoupled sea ice model with no ocean dynamics. Furthermore, the ocean stress is not significant in most of the northern CAA due to the weak ocean currents (Stronach et al., 1987). Though tides are strong in other parts of the CAA (e.g., Hell Gate, Nares strait, Gulf of Boothia), they are not represented in this model. Tidal effects on sea ice may include a stronger mobilizing and fracturing of the ice (Holloway and Proshutinsky, 2007). Tides are also responsible for an enhanced ocean heat mixing to the surface that could result in ice thinning. These effects are clearly beyond the scope of the present study. Since most of the results are considered on a climatological time scale, the effects of tides on the ice dynamics should not alter significantly the results. Other physical processes missing or crudely represented in the present study include katabatic winds along the glaciers and landfast ice. Landfast ice stabilization is associated to the presence of grounded ridges (Mahoney et al., 2007). Any attempt to model landfast ice should thus necessitate information extracted from bathymetry data. The sea ice model includes a viscous-plastic rheology and the Semtner (1976) zero-layer thermodynamic scheme presented in Chapter 2. It is evident that some caution must be taken while analyzing the model results because of the simple thermodynamics used, the absence of ocean currents and other limitations of the model. These limitations prevent us to compare model outputs to detailed in situ data, but the model complexity is nevertheless found sufficient, for the purpose of this chapter is to focus (i) on the feasibility and advantages of unstructured grids in large-scale sea ice modeling, and (ii) on the overall influence of the CAA on the Arctic ice cover and freshwater budget.

3.2 Model description

3.2.1 The sea ice dynamics

Following Hibler (1979), sea ice is assumed to behave dynamically as a two-dimensional viscous-plastic fluid with nonlinear viscosities. The model includes two thickness categories (see Fig. 3.2): thick ice with an actual thickness H and thin ice corresponding to a thickness lower than a cutoff constant h_0 . In this so-called two-level model, we introduce two variables to describe sea ice on a given oceanic area: the ice volume v^i and the ice concentration A , which is defined as the proportion of thick ice covering the oceanic area. The relation between those variables is illustrated in Fig. 3.2. In this approach, the ice thickness H corresponds to an equivalent (intensive) variable that is only used in thermodynamic computations, while the ice volume v^i and the ice concentration A are the state variables.

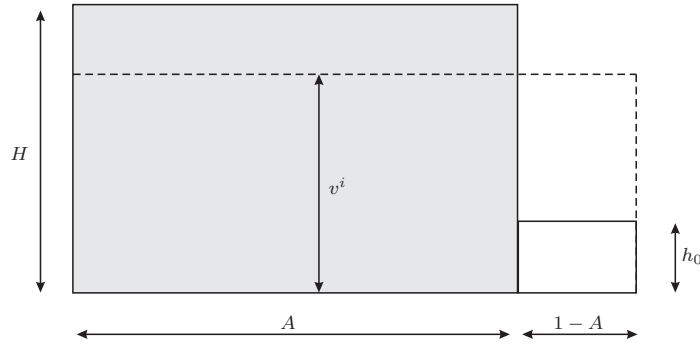


Figure 3.2: Two-level model: neglecting h_0 leads to a simplified relation between ice volume (v^i) and ice thickness (H): $v^i = AH$.

According to the aforementioned definitions, the equation of momentum conservation reads:

$$\rho v^i \frac{D\mathbf{u}}{Dt} = -\rho v^i f \mathbf{k} \times \mathbf{u} + A(\boldsymbol{\tau}_a + \boldsymbol{\tau}_w) - \rho v^i g \nabla \xi + \nabla \cdot \boldsymbol{\sigma} \quad (3.1)$$

where ρv^i is the ice mass per unit area, $\mathbf{u}$ is the ice velocity, $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$ is the material derivative, f is the Coriolis parameter (taken to be constant for the high latitudes), $\mathbf{k}$ is a unit vector normal to the surface, $\boldsymbol{\tau}_a$ et $\boldsymbol{\tau}_w$ are the air and ocean surface stresses on the ice weighted by the ice concentration, respectively (Connolley et al., 2004), g is the acceleration due to gravity, ξ is the sea surface dynamic height and $\boldsymbol{\sigma}$ is the ice internal stress tensor. Both the air and water stresses are parameterized by quadratic drag laws, the water stress depending on the ice velocity:

$$\boldsymbol{\tau}_w = \rho_w C_w |\mathbf{U}_w - \mathbf{u}| [(\mathbf{U}_w - \mathbf{u}) \cos \theta_w + \mathbf{k} \times (\mathbf{U}_w - \mathbf{u}) \sin \theta_w] \quad (3.2)$$

where C_w is the water drag coefficient, ρ_w the water density and θ_w the water turning angle. The force due to the ocean tilt is commonly computed by establishing the geostrophic balance with the ocean current:

$$-\rho v^i g \nabla \xi = \rho v^i f \mathbf{k} \times \mathbf{U}_w \quad (3.3)$$

In our model, the ocean velocity $\mathbf{U}_w$ is uniformly equal to zero, hence reducing the water surface stress to a simple drag of the ocean on the ice. Additionally, the contribution of the ocean tilt vanishes. We shall nevertheless take these terms into account in the rest of this chapter for the sake of generality.

The viscous-plastic rheology assumes that sea ice is behaving as a nonlinear compressible viscous fluid with the general form:

$$\boldsymbol{\sigma} = 2\eta(\dot{\boldsymbol{\epsilon}}, P) \dot{\boldsymbol{\epsilon}} + \left(\zeta(\dot{\boldsymbol{\epsilon}}, P) - \eta(\dot{\boldsymbol{\epsilon}}, P) \right) \text{tr}(\dot{\boldsymbol{\epsilon}}) \mathbf{I} - \frac{P}{2} \mathbf{I} \quad (3.4)$$

where $\dot{\boldsymbol{\epsilon}} = \frac{1}{2} (\nabla \mathbf{u} + \nabla \mathbf{u}^T)$ is the strain rate tensor, $\mathbf{I}$ is the identity tensor, η and ζ are the shear and bulk viscosities, respectively, and P is the ice strength. For typical strain rates, ice is assumed to behave plastically, while for small strain rates, ice is supposed to flow very slowly in a viscous way. In the one-dimensional case, a fixed strain rate $\dot{\epsilon}_0$ defines the boundary between the plastic and the viscous regimes. In two dimensions, this threshold concept is generalized through a yield curve in the principal stress plane. If the stress state lies inside this curve, ice behaves viscously; otherwise, ice is constrained to plasticity and the state lies on the curve. Hibler (1979)'s original elliptical yield curve has been widely used, and, combined with the normal flow rule, it yields the following viscosities:

$$\zeta = \min \left(\frac{P}{2\gamma}, 2.5 \times 10^8 P \right) \quad \text{and} \quad \eta = \zeta / e^2 \quad (3.5)$$

where e is the ellipse eccentricity (typically 2), P determines the ellipse size and γ is given by :

$$\gamma = [(\dot{\epsilon}_{xx}^2 + \dot{\epsilon}_{yy}^2)(1 + 1/e^2) + 4\dot{\epsilon}_{xy}^2/e^2 + 2\dot{\epsilon}_{xx}\dot{\epsilon}_{yy}(1 - 1/e^2)]^{1/2} \quad (3.6)$$

and can be interpreted as a measure of the strain rate determining whether the material undergoes plastic deformation or not. In the principal stress plane, all the stress states characterized by the same γ value lie on an ellipse centered on $(-1/2, -1/2)$ whose size normalized by P is proportional to γ (Fig. 3.3). A one-dimensional testcase that illustrates the ice rheology can be found in the appendix B.1. Finally, we need an expression of the pressure term to close the system. Following Hibler (1979), P is computed as:

$$P = p^* v^i e^{-C(1-A)} \quad (3.7)$$

with two empirical parameters p^* and C .

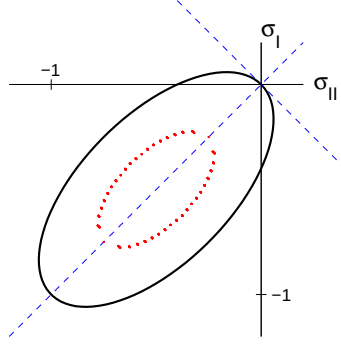


Figure 3.3: Principal stress diagram of the constitutive relationship. Inside the ellipse, the material is considered as viscous. On the ellipse itself, the material undergoes plastic deformation, keeping a constant stress even if the deformation rate is higher than the limit defined by the geometrical locus ($\gamma = 2 \times 10^{-9} \text{ s}^{-1}$). Material points yielding a $\gamma = 10^{-9} \text{ s}^{-1}$ are also plotted in red.

Mass conservation is ensured by an advection equation for the mean ice thickness, while another continuity equation is required for the ice concentration:

$$\frac{\partial v^i}{\partial t} + \nabla \cdot (\mathbf{u} v^i) = q_v \quad (3.8)$$

$$\frac{\partial A}{\partial t} + \nabla \cdot (\mathbf{u} A) = q_A \quad (3.9)$$

where q_v and q_A are source terms accounting for sea ice ablation or accretion. These terms are computed thanks to three heat budgets: two at the top and the bottom of the thick ice and one over the thin ice/open water area. This explains why the actual ice thickness H will be used as variable when computing the thermodynamical processes. In addition, the ice concentration is constrained ($A \leq 1$) to roughly account for the mechanical redistribution of sea ice.

In the two-level approach, compared to the thermodynamic model presented in Chapter 2, we need to account for the lateral growth and decay of sea ice (q_A). In addition of heat balances (2.58) and (2.59), the heat budget B_l of the lead area is computed in the same fashion as equation (2.61). If it is negative, the lead is partially filled with ice of thickness h_0 (Fichefet et al., 1998):

$$q_A^+ = -\frac{(1-A)B_l}{Lh_0}, \quad (3.10)$$

where the + superscript denotes the accretion rate. In the case of open water, the ocean temperature has first to be decreased (equation (2.60)) to the freezing point

T_f before a new ice layer can be formed. When the heat balance is positive, the lateral decay depends on the melt rate of thick ice:

$$q_A^- = -\frac{A}{2H}[-(q^s + q^b)]^+. \quad (3.11)$$

The source term q_v is hence made of different contributions:

$$q_v = A(q^s + q^b) + q_A^+ h_0 + q_A^- H, \quad (3.12)$$

taking into account the surface melt, the basal accretion or melt and the lateral growth and decay.

3.3 Numerical method

In this section, the equations and the initial and boundary conditions of the sea ice model are presented. The finite element spatial discretization of this boundary value problem is derived and the system of equations obtained is integrated with respect to time.

The sea ice model consists in finding the ice velocity $\mathbf{u}(x, y, t)$, the ice volume $v^i(x, y, t)$ and the ice concentration $A(x, y, t)$ satisfying the following partial differential equations:

$$\rho v^i \frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \boldsymbol{\sigma}(\mathbf{u}, v^i, A) + \mathbf{q}_u(\mathbf{u}, v^i, A) \quad (3.13)$$

$$\frac{\partial v^i}{\partial t} + \nabla \cdot (\mathbf{u} v^i) = q_v(v^i, A) \quad (3.14)$$

$$\frac{\partial A}{\partial t} + \nabla \cdot (\mathbf{u} A) = q_A(v^i, A) \quad (3.15)$$

on the two-dimensional domain Ω encompassing the whole Arctic sea ice north of the parallel 50 degrees North. In the momentum equation (3.13), the term $\mathbf{q}_u$ takes into account the wind and ocean forcings, the Coriolis force and the force due to the tilt of the ocean, and the advection has been neglected (Fichefet et al., 1998) in the momentum balance (equation (3.1)). All the calculations are performed from a given initial situation where all fields are specified. The kind of boundary conditions is twofold: on the one hand, a no-slip condition is prescribed along the coastline and islands. On the other hand, along the open sea boundary, a stress-free boundary condition is applied, while the inward ice thickness and concentration fluxes are prescribed.

The finite element method consists of three major steps.

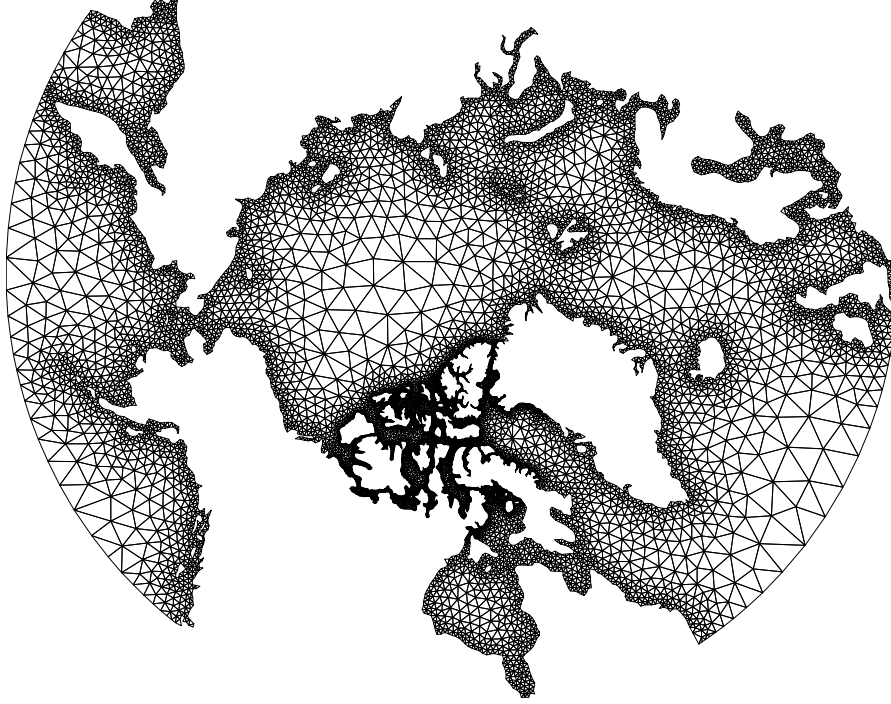


Figure 3.4: Finite element mesh of the Arctic. The domain extends north of the parallel 50 degrees North. The mesh contains 17053 triangles and 10505 nodes and its resolution varies from 10 to 400 km.

- We divide the domain Ω into a triangulation of non overlapping elements Ω_e as shown in Fig. 3.4. Unstructured grids enable a flexible distribution of the degrees of freedom in the domain according to the areas of interest. As far as sea ice is concerned, the mesh resolution can be coarse where the spatial variability is mostly on large scale and it should be fine where smaller scales are relevant. Hence, in this study focusing on the influence of the CAA on the Arctic ice cover, large elements are used in Central Arctic and in areas of low latitudes, while the mesh is refined along coastlines, especially in the CAA.

- We replace the three unknown fields by piecewise polynomial approximations defined as follows:

$$\mathbf{u}(x, y, t) \approx \mathbf{u}^h(x, y, t) = \sum_{j=1}^p \mathbf{U}_j(t) \phi_j(x, y) \quad (3.16)$$

$$v^i(x, y, t) \approx v^{ih}(x, y, t) = \sum_{j=1}^m V_j^i(t) \psi_j(x, y) \quad (3.17)$$

$$A(x, y, t) \approx A^h(x, y, t) = \sum_{j=1}^m A_j(t) \psi_j(x, y) \quad (3.18)$$

where the shape functions ϕ and ψ are defined a priori, and $\mathbf{U}_j(t)$, $V_j^i(t)$ and $A_j(t)$ are the unknown degrees of freedom. Generally, they correspond to the nodal values in a usual continuous finite element discretization. The local support of those polynomial shape functions is the key ingredient of the finite element method allowing us to derive a discrete algebraic system enjoying nice sparsity properties.

- Finally, we define a discrete Galerkin formulation to obtain numerical values for the unknown degrees of freedom. Such a procedure imposes that the weighted residuals of the partial differential equations have to vanish. Typically, we integrate over the whole domain the partial differential equations weighted by the shape functions. However, even if those residuals vanish, it does not ensure that the differential equations will be satisfied everywhere by $\mathbf{u}^h$, v^{ih} and A^h . Moreover, some integrals by parts are performed in order to reduce the degree of differentiability required by the shape functions. One clear advantage of the finite element method for sea ice modeling is its facility to handle Neumann or so-called natural boundary conditions (Thomson and McKenna, 1988). Taking advantage of the homogeneous stress boundary condition, the spatial discretization reads:

Find $\mathbf{u}^h$, v^{ih} and A^h such that:

$$\begin{aligned} \langle \phi_j \rho v^{ih} \frac{\partial \mathbf{u}^h}{\partial t} \rangle &= - \langle \nabla \phi_j \cdot \boldsymbol{\sigma}^h \rangle + \langle \phi_j A^h \mathbf{q}_u(\mathbf{u}^h, v^{ih}, A^h) \rangle & j = 1, \dots, p \\ \langle \psi_j \frac{\partial v^{ih}}{\partial t} \rangle &= \langle \nabla \psi_j \cdot (\mathbf{u}^h v^{ih}) \rangle - \langle \psi_j v^{ih} (\mathbf{u}^h \cdot \mathbf{n}) \rangle + \langle \psi_j q_v(v^{ih}, A^h) \rangle & j = 1, \dots, m \\ \langle \psi_j \frac{\partial A^h}{\partial t} \rangle &= \langle \nabla \psi_j \cdot (\mathbf{u}^h A^h) \rangle - \langle \psi_j A^h (\mathbf{u}^h \cdot \mathbf{n}) \rangle + \langle \psi_j q_A(v^{ih}, A^h) \rangle & j = 1, \dots, m \end{aligned} \quad (3.19)$$

where $\mathbf{n}$ denotes the outward normal, the notation $\langle . \rangle$ holds for $\int_{\Omega} . \, d\Omega$ and $\ll . \gg$ for $\int_{\partial\Omega} . \, d\Gamma$. p and m are the number of nodes corresponding to the degrees of freedom for the velocity field and the ice volume and concentration fields, respectively.

This spatial discretization leads to a nonlinear system at each time step whose unknowns are the discrete degrees of freedom. A nonconforming linear approximation (Hua and Thomasset, 1984) is chosen for the velocity field, while the ice volume and concentration are approximated by constant shape functions (Fig. 3.5). To illustrate this spatial discretization, the degrees of freedom for velocities are lo-

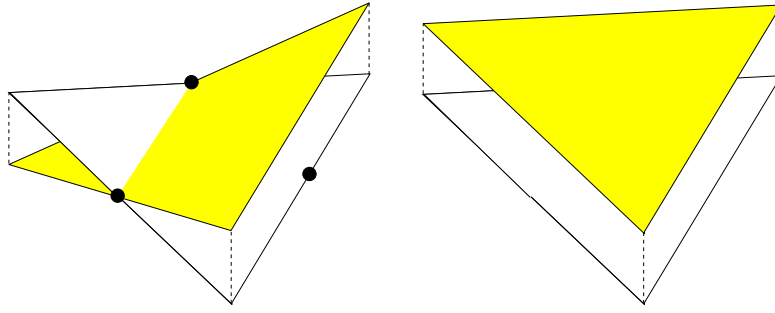


Figure 3.5: Linear nonconforming (left) and constant (right) shape functions.

cated on the edges of the triangles, while the degrees of freedom for scalar properties A and v^i are located in the center of the element and represent a value that is constant over the element - similar to an Arakawa C grid. The two-level model proposed by Hibler (1979) suggests indeed a constant ice volume and concentration in each cell, even if not excluding a higher order representation of both variables. Advantages of the linear nonconforming elements include an easier computation of the fluxes through the edges and the orthogonality of the shape functions. In the set of discrete equations, it is important to emphasize that the integral over the domain has to be splitted into the sum of the integrals over each element because the derivative no longer exists on the edge. This is so because nonconforming shape functions are discontinuous across elements edges. In the same way, the boundary integral is now the sum of the integrals along all internal and external edges. For more details, we refer the reader to Hanert et al. (2004). Finally, we perform a hybrid time stepping scheme where Coriolis force and external forcings due to the ocean are implicitly integrated. To take into account nonlinear terms, a standard linearization based on a Newton-Raphson scheme is introduced, and the resulting sparse matrix is solved with a “skyline” linear solver. Typically, for a simple Euler scheme, the discrete equations read:

$$\begin{aligned}
\langle \phi_j \rho v_n^{i,h} (\mathbf{u}_{n+1}^h - \mathbf{u}_n^h) \rangle &= \Delta t \left(- \langle \nabla \phi_j \cdot \boldsymbol{\sigma}_n^h \rangle + \langle \phi_j \mathbf{q}_u^h(\mathbf{u}_{n+1}^h, v_n^{i,h}, A_n^h) \rangle \right) \\
\langle \psi_j (v_{n+1}^{i,h} - v_n^{i,h}) \rangle &= \Delta t \left(- \ll \psi_j v_n^{i,h} (\mathbf{u}_{n+1}^h \cdot \mathbf{n}) \gg + \langle \psi_j q_v(v_n^{i,h}, A_n^h) \rangle \right) \\
\langle \psi_j (A_{n+1}^h - A_n^h) \rangle &= \Delta t \left(- \ll \psi_j A_n^h (\mathbf{u}_{n+1}^h \cdot \mathbf{n}) \gg + \langle \psi_j q_A(v_n^{i,h}, A_n^h) \rangle \right)
\end{aligned}$$

where the subscript denotes the time evaluation.

Advection is computed via a first-order, finite volume upwind-weighted scheme. Though numerically diffusive, it is monotonic and conservative, two essential ingredients to ensure the positive-definiteness of the transported variables v^i and A .

3.4 Results and discussion

The model has been run over the period 1979-2005. We first present results from the control simulation, essentially focusing on the patterns of ice concentration, volume and drift. A numerical experiment is further presented where we investigate the influence of the ice flow through the narrow straits of the CAA on the Arctic ice thickness distribution and freshwater budget.

3.4.1 Control run

The model is spun up for only 6 years with annual mean forcings and then integrated between 1979 and 2005. The atmospheric forcing data sets used to run the model are daily NCEP/NCAR reanalysis data for the air temperature and the wind velocity, and monthly climatologies for the relative humidity (Trenberth et al., 1989) and the total cloudiness (Berliand and Strokina, 1980). The mesh used for this simulation has a varying resolution between 10 km in the CAA and 400 km (Fig. 3.4). Since the resolution in Central Arctic is low, some small-scale and short-term processes might be neglected. However, on the one hand, the results should not be significantly altered, especially as most attention is being paid to the climatological results of the model. On the other hand, the aim of this study being to focus on the influence of the CAA on the Arctic ice cover, this resolution was found sufficient in Central Arctic. The integration of one simulation year on this mesh takes about 2 hours on an ordinary computer. Finally, the oceanic heat flux is computed exactly as in Chapter 2 (see equation (2.62)): it consists of a restoring term towards a climatological temperature of the mixed layer.

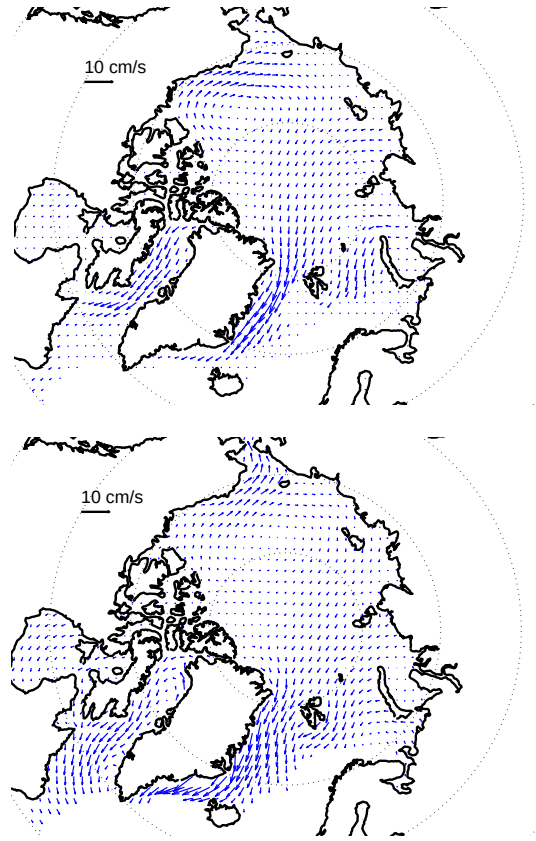


Figure 3.6: Mean annual climatological ice drifts (1979-2005). Top: observations; bottom: the velocity field as computed by our model. Both fields have been interpolated on a regular grid for visual convenience.

We first compare the annual mean sea ice drifts simulated during the whole 1979-2005 period to the available data (Fig. 3.6). The daily observed ice motion vectors for the 1979-2005 period are derived from data of Advanced Very High Resolution Radiometer (AVHRR), Scanning Multichannel Microwave Radiometer (SMMR), Special Sensor Microwave/Imager (SSM/I), and International Arctic Buoy Program (IABP) (<http://nsidc.org>, Fowler (2003)). The simulated ice drift vectors are only drawn on grid points where the annual mean ice concentration is higher than 15%, a value that will be considered here as delimiting the ice edge.

Both the Beaufort Gyre and the Transpolar Drift Stream are recognizable in the observations and the simulation, attesting that the model captures well the essen-

tials of the ice circulation in the Arctic. However, the simulated Beaufort Gyre appears too weak and the model exhibits a clear ice outflow from the Arctic in the region of Bering Strait. But on a climatological time scale, observations suggest that the ice is almost static there. This difference is attributed to the effects of the oceanic surface currents flowing from the North Pacific to the Arctic Basin through Bering Strait which are missing in our model and tend to counterbalance the wind stress. Plug flow might be also a part of the explanation. The simulated ice dynamics indicates also a higher ice drift along the east coast of Greenland, while a parabolic-like velocity profile across Fram Strait (e.g., Kwok et al., 2004) is expected. It is worth noting that the model does not include any parameterization of landfast sea ice. We suppose that the fine mesh resolution along the coastline allows the influence of the no-slip boundary condition to vanish rapidly, resulting in a steep variation of the velocity profile. We also note in the simulation the too large ice tongue south of Fram Strait.

Fig. 3.7 compares the mean ice concentrations from our simulation to those derived from SMMR and SSM/I data sets (<http://nsidc.org>, Comiso (1999, updated 2005)) and interpolated on the finite element mesh. As can be seen from the observations, in winter, the ice is very concentrated in the Arctic Basin ($> 90\%$). The simulated ice pack represents this feature, but the ice is too extensive in the southernmost part of the domain. The missing oceanic heat advection is clearly responsible for this overestimation. In the North Atlantic, a huge amount of heat is brought into the Nordic Seas by the North Atlantic Drift Current that originates from the Gulf Stream. This feature is not present in our simulation and mostly explains the too large presence of sea ice in the Norwegian and Labrador Seas. The simple parameterization of the oceanic heat flux, consisting of a restoring term towards a climatological mixed layer temperature, may also be partly responsible for this discrepancy. In summer, only the central Arctic Basin remains ice-covered. The simulated ice concentration better matches the observations, even if the model still overestimates the ice cover, especially in Baffin Bay and the Kara Sea.

A question regards whether such a simple model is able to capture the interannual variability of the sea ice cover. For this purpose, Fig. 3.8 shows the time series of the monthly mean anomalies of the ice extent as simulated by the model and in observations for the 1979-2005 period. Even if the mean climatological ice extent is overestimated in our model, the interannual variability is in reasonable agreement with the observations. Surprisingly, the simulated ice extent is clearly underestimated during the first three years of the simulation. This may be attributed to two factors: the necessary time adaptation after the model spin-up and the too high NCEP/NCAR reanalyzed winter temperatures in the Northern Siberian Coast in 1980 and 1981 (Vancoppenolle et al., 2009). Moreover, the extent minima are not accurately represented and their phase is somewhat delayed.

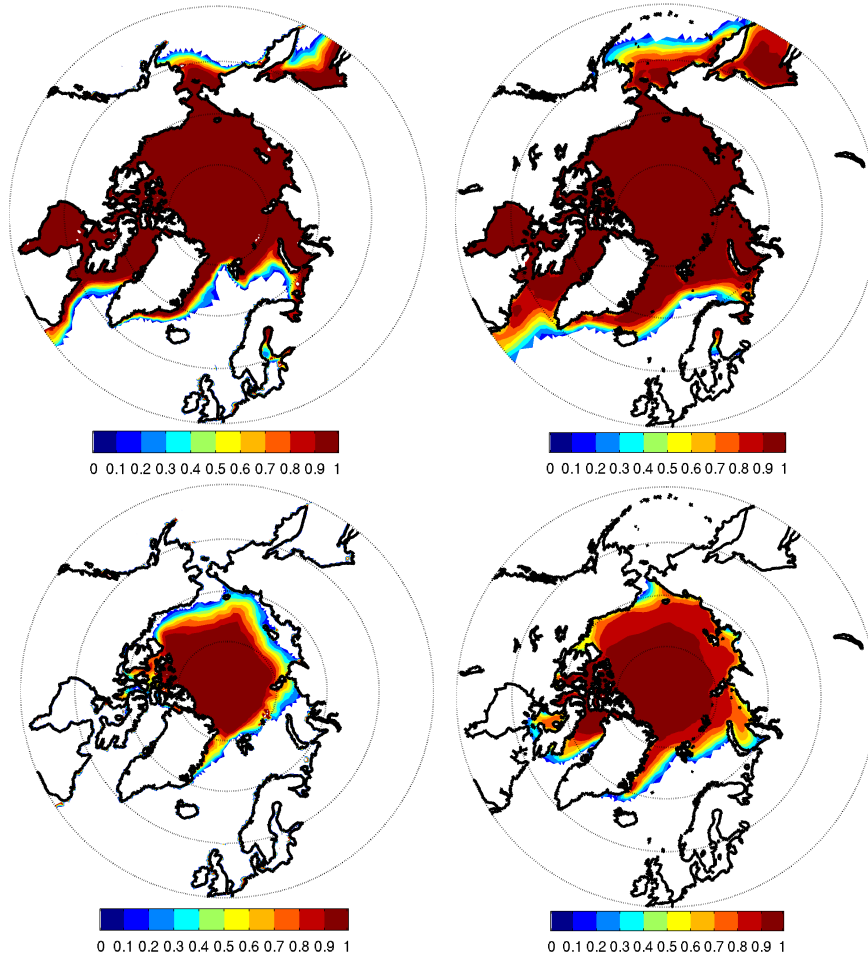


Figure 3.7: Mean ice concentrations in March (top) and September (bottom). For both months, we compare the data sets (left, 1979-2005) derived from SMMR and SSM/I and interpolated on the finite element grid with the simulated ice concentrations (right, 1979-2005).

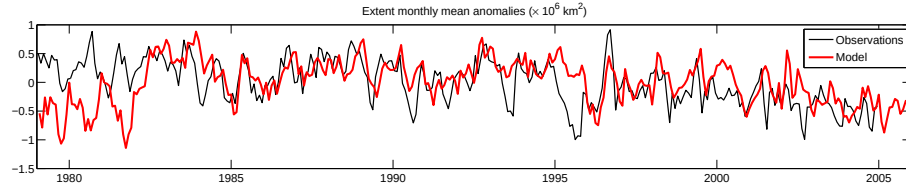


Figure 3.8: Time series of monthly mean anomalies of sea ice extent for the Arctic as simulated by the model (red line) and in observations (black line) derived from satellite microwave recordings (Comiso, 1999, updated 2005). The anomalies were obtained by taking the monthly ice extent for each individual month and by subtracting the mean ice extent for that month over the 1979-2005 period.

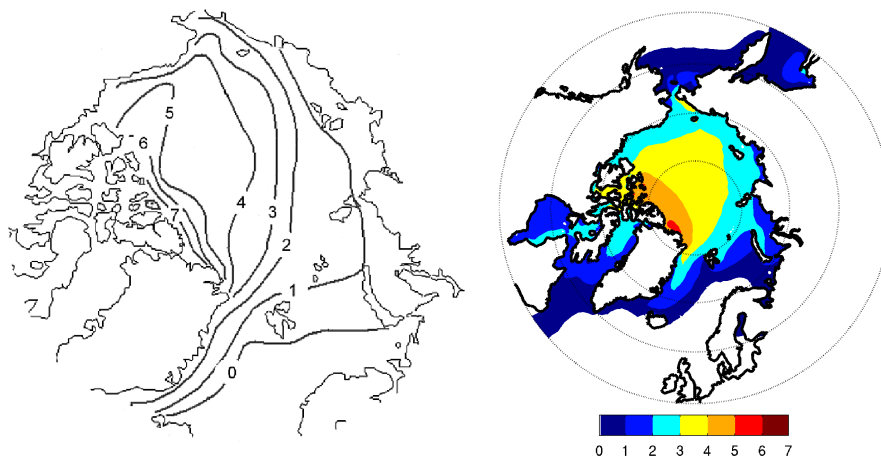


Figure 3.9: Global sea ice thickness pattern (in m) as computed by the model (March 1979-2005, right) and as observed (January-April 1960-1982, left, Bourke and Garrett (1987)).

Finally, there is a general agreement between the simulated ice thicknesses and observations (Fig. 3.9). In the model Central Arctic, ice thickness lies between 3 and 4 m. Thickness progressively increases from Siberia to the CAA “*providing natural barriers to the movement of the ice*” (Bourke and Garrett, 1987), where thickness reaches up to 6 m. The essential of the ice ridging occurs indeed in front of the CAA. Inside the CAA, some spots attain about 7 m (Fig. 3.11). The Beaufort maximum visible in the observations is not captured by the model. This feature is however rarely represented in sea ice models. The influence of resolving the narrow straits constituting the CAA will be investigated in a separate subsection.

3.4.2 Sensitivity experiment

In order to explore the sensitivity of the sea ice cover to the resolution of the narrow straits of the CAA, two numerical experiments are conducted. The first one (OP) corresponds to the control simulation with the CAA open. In the second ex-

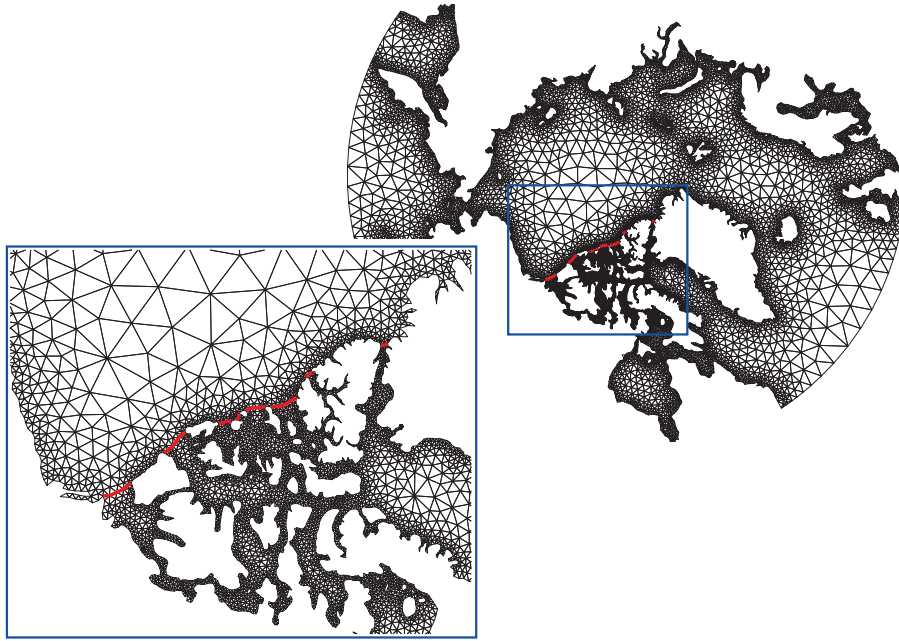


Figure 3.10: Zoom on the CAA. The edges in red are considered as land in the CL experiment, thereby closing artificially the Arctic Basin in the Canadian area.

periment (CL), the straits connecting the Arctic Basin to the CAA are closed. For this purpose, the edges of the mesh crossing those straits are converted to land boundaries, thereby disconnecting the Arctic Basin from the CAA. This finally reduces to imposing a zero velocity for the velocity degrees of freedom situated on the edges in red (see Fig. 3.10). This procedure is meant to mimic the situation encountered in coarse resolution sea ice models that are not able to represent the exchanges of sea ice through the CAA.

Fig. 3.11 illustrates the climatological ice thicknesses in March for OP and CL minus OP. As mentioned earlier, the mean ice thickness reaches about 6 m along the CAA. Inside the CAA, thickness progressively decreases southward in the direction of Baffin Bay and the Labrador Sea. What strikes first when analyzing the difference

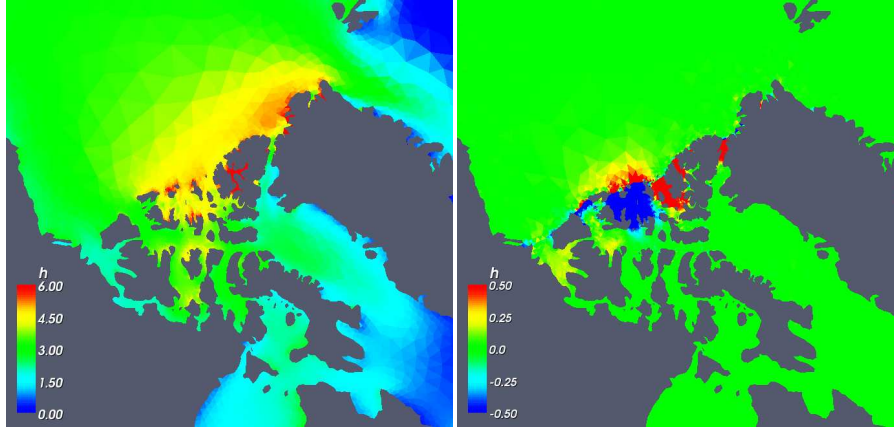


Figure 3.11: March ice thicknesses in the CAA for the 1979-2005 period. On the left: control simulation OP, the scale ranges from 0 to 6 m; on the right: difference between CL and OP, the scale ranges from -0.5 m to 0.5 m.

between CL and OP is the high similarity between results of both experiments. Outside a restricted area surrounding the CAA entrance, there is no clear difference between OP and CL. Henceforth, on a climatological time scale, this experiment tends to attest that closing the CAA does not significantly alter the general ice thickness pattern. Nevertheless, the local thickness pattern shows a clear sensitivity to the resolution of the narrow straits north of Canada. In particular, the ice thickness difference between OP and CL is up to 2 m (not shown) in the southern gates of the Queen Elizabeth Islands (QEI-South, see Fig. 3.1), because of the stopped ice flux from the Arctic to the CAA.

On a daily basis, the comparison between OP and CL reveals stronger differences. The mean ice thickness patterns are compared for late October 2005 in Fig. 3.12. During that period, the wind is blowing offshore the CAA. Two points are to be outlined here. First, along the Queen Elizabeth Islands, the ice divergence caused by the wind is much more pronounced in CL because the closed straits prevent any ice thickness outflow from the CAA. The influence of the domain boundary and the related boundary condition diffuses inside the domain. Moreover, even a very thick ice cover, which is supposed to have a high ice strength, is very sensitive to the wind direction. In a few days, the mean ice thickness along the CAA has decreased by more than 2 m. It is worth mentioning that, in Hibler (1979)'s model, the tensile strength applied to the ice is weak and that no landfast parameterization is included. The ice accumulated against the CAA is hence more sensitive to the wind direction in the CL experiment. Second, for the same reason, closing Amundsen Gulf (southern strait in Fig. 3.10) in CL leads to an ice accumulation inside the CAA,

while in Amundsen Gulf, it generates a large open water area, with all the physical processes involved. The ice fluxes between both regions are thus important to accurately represent the ice flow in northern Canada.

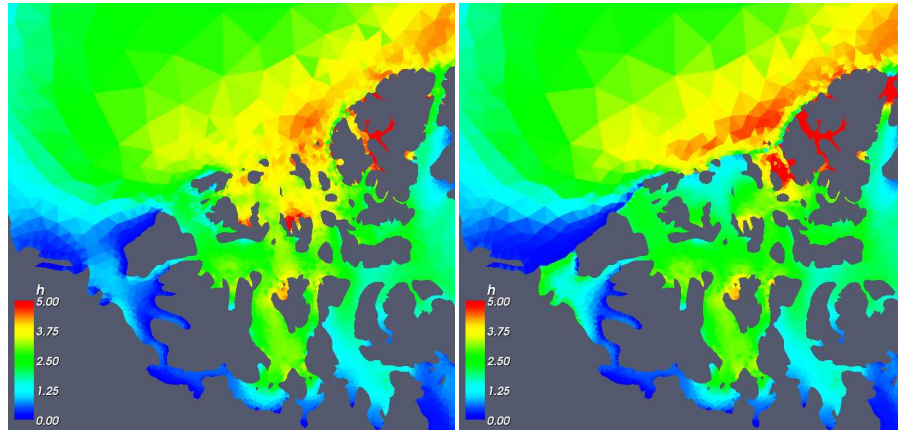


Figure 3.12: Ice thicknesses (from 0 to 5 m) in the CAA simulated by the model for late October 2005. On the left: control simulation OP; on the right: CL. Note the formation of open water in the Beaufort Sea for simulation CL and the difference in thickness between OP and CL along the Queen Elizabeth Islands.

Exchanges of sea ice between the CAA and the Arctic are extensively described and quantified by Kwok (2006). This study based on RADARSAT ice motion provides a 5-year complete record of ice area fluxes in the northern CAA. The mean annual observed ice fluxes for the 1998-2002 period (Table 1) indicate a northward ice movement in Amundsen Gulf and M'Clure Strait and an Arctic outflow in the Queen Elizabeth Islands. Overall, ice area is exported from the CAA to the Arctic, but the seasonal and the interannual variability of these ice fluxes are very high. Ice volume fluxes are estimated on the basis of the ice area fluxes and representative ice thicknesses.

Sou (2007)'s study uses a finite difference regional ice-ocean model forced by NCEP/NCAR reanalyzes data with two ice thickness categories and viscous-plastic rheology. The model is run on a nested grid with a 0.2° horizontal resolution which covers the CAA region. The ice area fluxes computed by this model (Table 1) compare reasonably well with the observations, in particular in the QEI region. On the low side, the ice outflow in Amundsen Gulf is too large and the direction of the flux in M'Clure Strait is opposite to what is observed. Sou (2007) has reservations about the accuracy of the use of representative ice thicknesses to estimate the "observed"

	Kwok (2006)			Sou (2007)			Model		
	Area	Vol.	Thick.	Area	Vol.	Thick.	Area	Vol.	Thick.
A G	-85	-85	1.0	-130	-97	1.1	-70	-87	1.5
MS	-20	-80	4.0	2	54	2.0	-19	-32	2.6
QEI	8	27	3.4	11	72	3.6	33	131	4.2
Net	-97	-138		-117	29		-56	12	

Table 3.1: Mean annual ice area ($\times 10^3 \text{ km}^2 \text{ yr}^{-1}$) and volume ($\text{km}^3 \text{ yr}^{-1}$) fluxes for the 1998-2002 period through Amundsen Gulf (AG), M’Clure Strait (MS) and Queen Elizabeth Islands (QEI) in observations (Kwok, 2006) and as computed by Sou (2007)’s model and by our model. Positive sign indicates Arctic outflow. The ice volume estimated by Kwok (2006) is based on a representative ice thickness (m). For comparison, a representative ice thickness is also provided for both models, but not used in the volume fluxes computation.

ice volume fluxes in regions of high thickness or ice direction variability. For example, in the case of an oscillating ice flux through a gate whose ice thickness is higher at one of its sides, the net ice area flux would be close to zero, while the ice volume flux will not. The weighting played by the non uniform ice thickness appears to be the leading factor to explain large differences that one can get between the area and the volume fluxes.

Our model shows a good agreement with the observed ice area fluxes, with typically the same direction and order of magnitude. The model does particularly well in M’Clure Strait. Discrepancies of the model include too large an ice flux through the Queen Elizabeth Islands, especially in winter when the model is missing the landfast sea ice. The net exchange in ice area between the Arctic and the CAA is hence smaller than in observations. Due to the method used, the agreement between modeled ice volume flux to the “observed” one depends highly on the variability of the region (Sou, 2007). In both models for example, the estimated ice volume flux based on the representative ice thickness overestimates the modeled ice volume flux in M’Clure Strait (+54% in our simulation). However, in the QEI, the estimated ice volume flux ($33 \times 10^3 \text{ km}^2 \text{ yr}^{-1} \times 4.2 \text{ m} \approx 139 \text{ km}^3 \text{ yr}^{-1}$) is very close to the one computed by our model ($131 \text{ km}^3 \text{ yr}^{-1}$). Both models exhibit the same discrepancy between the total net ice area (Arctic inflow, consistently with observations) and volume (Arctic outflow) fluxes. The opposite signs between net area and volume fluxes is striking, but can be explained by the arithmetic weighting of the ice area fluxes by the ice thicknesses.

As expected, North Nares Strait monthly ice area fluxes are weak on average ($< 200 \text{ km}^2$, not shown here), but with large northward exports for a couple of months, yielding a net Arctic inflow ($-3 \times 10^3 \text{ km}^2 \text{ yr}^{-1}$, 1998-2002). Note that the Arctic inflow can also be deduced from Fig. 3.11: more ice is present in Nares Strait when it is closed. This discrepancy with the observations is also present in Sou (2007)’s study and is mainly attributed to the coarse wind forcing missing the effects of the

topography on the local wind pattern (Sou, 2007). The absence of oceanic currents and the lack of mesh resolution in North Nares Strait can also be adduced.

A recent paper by Terwisscha van Scheltinga et al. (2010) mimicked the present study and gives another point of comparison. Their model is based on FESOM (Timmermann et al., 2009) and is similar to the one presented in this chapter in all points but the forcing data sets and the finite element discretization ($P_1 - P_1$ formulation). As expected, their results compare well with the ones presented in this chapter. However, their model has better representation of the ice fluxes in the QEI compared to the observations (Kwok, 2006). Though their mesh resolution is clearly higher in the narrow straits than in our simulations, some numerical tests have shown that the ice fluxes are not very sensitive to the resolution. The difference in the ice fluxes is thus probably linked to the different data set that was used to force the model. In the next subsection, we investigate the influence of those ice fluxes on the freshwater budget of the Arctic.

3.4.3 Importance of the CAA for the mass balance of Arctic sea ice

The resolution of the CAA still constitutes in the large majority of sea ice models a missing piece of the Jigsaw puzzle. We have already highlighted the importance of the CAA for the mass balance of Arctic sea ice in the introduction (Fig. 1.3). According to our model, the mean CAA ice mass as computed in the control run OP represents a rough 10 % of the total simulated sea ice volume, which is far from being negligible.

Furthermore, we perform an additional analysis: the ice volume fluxes towards Baffin Bay are evaluated at three gates described in Fig. 3.13 and situated in Lancaster Sound, Jones Sound and Smith Sound. The fluxes experience a strong interannual variability and, except for three years in Lancaster Sound, contribute to a positive ice flux to Baffin Bay. As expected, Smith Sound yields the major contribution with roughly 70 % of the mean total sea ice export, while Lancaster and Jones Sounds contribute to 20 % and 10 %, respectively. Converted into mean annual freshwater fluxes, we find a contribution of $\sim 20 \text{ km}^3 \text{ yr}^{-1}$ for Lancaster Sound, $\sim 15 \text{ km}^3 \text{ yr}^{-1}$ for Jones Sound and $\sim 90 \text{ km}^3 \text{ yr}^{-1}$ for Smith Sound. The mean total sea ice freshwater export towards Baffin Bay is hence of $125 \text{ km}^3 \text{ yr}^{-1}$, which is close to the $160 \text{ km}^3 \text{ yr}^{-1}$ estimated by Serreze et al. (2006), on the basis of Prinsenberg and Hamilton (2005)'s study, and computed by Sou (2007) on a long period (1950-2004). As mentioned by Sou (2007), the ice volume transport is probably underestimated in models in narrow straits like Jones and Lancaster Sound due to the coarse wind forcing and the low mesh resolution. In order to evaluate the relative importance of the ice export through the CAA, this value is to be compared to the mean freshwater export through Fram Strait. According to recent observations, our estimate represents only 7% of the freshwater equivalent of the ice export through

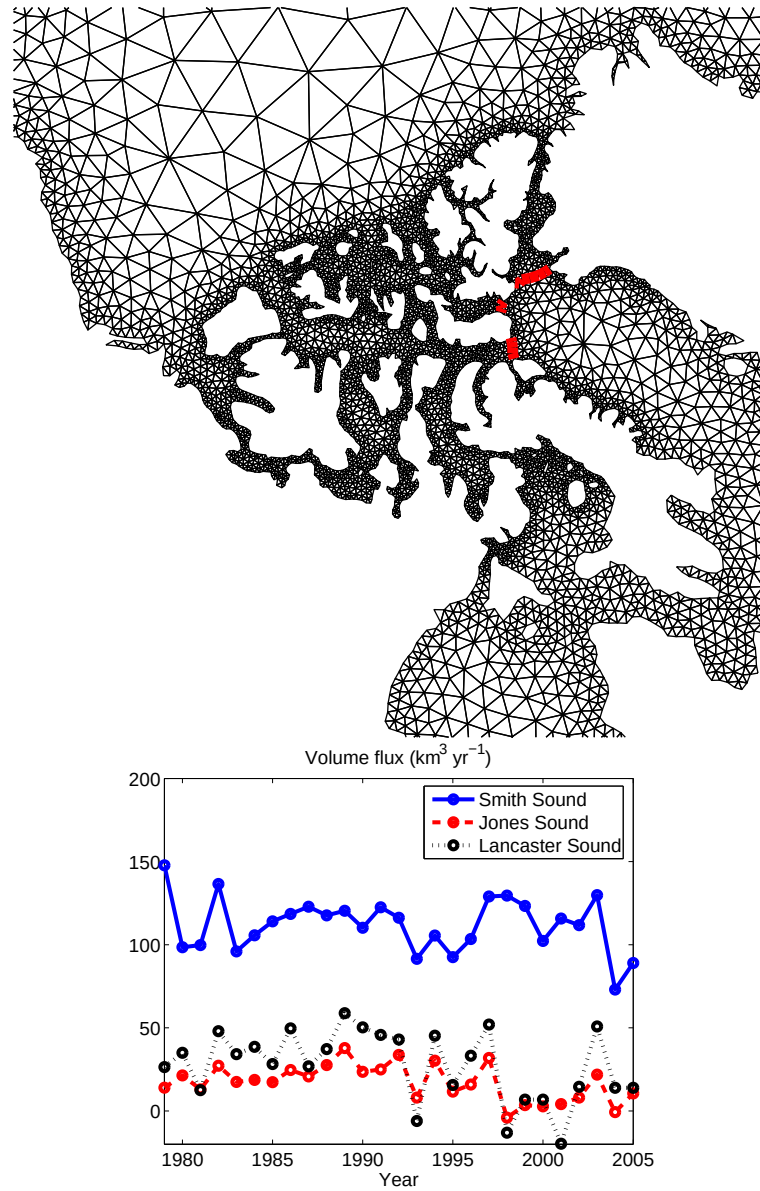


Figure 3.13: Top: close-up view of the mesh in the CAA. Bottom: annual ice volume fluxes (positive if directed towards Baffin Bay) through the three gates represented in the upper figure (bottom-up: Lancaster Sound, Jones Sound and Smith Sound). See also Fig. 3.1 for the geographical locations.

Fram Strait ($1800 \text{ km}^3 \text{ yr}^{-1}$, estimation from Kwok et al. (2004) converted into freshwater equivalent) and 5% of Fram Strait's liquid freshwater outflow ($2400 \text{ km}^3 \text{ yr}^{-1}$, Serreze et al. (2006)). Though relatively small, the impact of this ice export on the water stratification in Baffin Bay and on the convective overturning downstream in Labrador Sea has to be investigated.

3.5 Conclusions

In this chapter, we have presented a finite element, unstructured grid sea ice model. The model includes a viscous-plastic rheology along with a complete parameterization of the atmospheric fluxes and is driven by daily NCEP/NCAR reanalysis data. The climatological sea ice drifts, thicknesses and concentrations computed by the model compare qualitatively well with the observations. The finite element method presented in this chapter reveals effectiveness for sea ice modeling and enables the use of unstructured grids with all their potential and flexibility.

A clear advantage of unstructured grids is to enable the resolution of the narrow straits of the Canadian Arctic Archipelago (CAA). A numerical experiment has hence been performed to investigate the influence of the ice flow through the CAA. Focusing on the large-scale sea ice thickness pattern, within our general framework and hypotheses, we have shown that the inclusion of those straits is not essential, the impact being merely local. This tends to validate the idea of the role of the CAA as a "buffer" on sea ice between the Arctic Basin and Baffin Bay. However, we have shown that the local and short-term influences of the ice exchanges are nonnegligible. In particular, depending on whether the straits are open or closed in the numerical experiment, the domain boundary and the associated boundary condition influence directly the numerical solution in the proximity of those straits. Moreover, on average, the sea ice volume in the CAA represents a nonnegligible 10 % of the total sea ice volume in our model. Finally, the annual sea ice volume flux towards Baffin Bay has been evaluated in order to assess its importance in the freshwater balance of the Arctic and yields a mean freshwater outflow of $125 \text{ km}^3 \text{ yr}^{-1}$, which is in good agreement with the $160 \text{ km}^3 \text{ yr}^{-1}$ suggested by Serreze et al. (2006), on the basis of Prinsenberg and Hamilton (2005)'s study. Though relatively small compared to the ice contribution of the freshwater outflow through Fram Strait (7%), this ice export, together with the North Water Polynya formation, might be important regarding the water stratification in Baffin Bay and the decay of the summer sea ice cover in Baffin Bay. All these diagnoses show how important the CAA is on the mass balance of the Arctic sea ice and plead for the need to better understand the complex processes and interactions being held in this region.

Future work on the sea ice model will include a better representation of the sea ice physics (halo-thermodynamic model (Vancoppenolle et al., 2007), ice-thickness

distribution (Thorndike et al., 1975), etc.). An intermediate step towards the development of a coupled ice-ocean model might consist of including climatological modeled currents (e.g., Maslowski et al., 2004; Vancoppenolle et al., 2009) in order to improve the ice circulation in areas where these currents play an important role (Bering Strait, Nares Strait, etc.). Some additional investigation based on a coupled sea ice-ocean model should be carried out to assess the actual role of sea ice passing through the CAA on the global ocean circulation, by way of the interactions between the associated freshwater flux and the water mass properties of the adjacent seas. Recently, this issue has been readdressed by Kwok (2005) concerning the impact of the ice export through Nares Strait and more generally from the CAA on the convective overturning downstream in the Labrador Sea. A coupled model might also enable to assess the role of static arches on the ice circulation and the interactions with the ocean, or propose lacking data (such as thickness order of magnitude for estimations of mass budgets). As outlined by Dickson et al. (2007), numerical models have indeed a role to play to make up for the inherent difficulties and financial limitations to collect data, and in particular ocean fluxes.

The results presented in this study must be considered with some care though since no ocean dynamics nor oceanic feedback are included in this uncoupled version of the model. The deficiencies of the model, including the Bering outflow, are to be attributed to these factors. In particular, the absence of oceanic heat advection is clearly responsible for the overestimation of the simulated sea ice extent. The simple parameterization of the oceanic heat flux is also a weak means to compensate for the lack of ocean interactions. An important issue is to know whether or not the coarse resolution atmospheric forcings typically used in the models is sufficiently accurate to enable realistic simulations in the CAA. The resolution of the air temperature and wind velocity data sets used in this study amounts to about 200 km, while the mesh resolution reaches 10 km in some parts of the domain. Additionally, coarse wind forcing do not resolve the effects of the topography on the local wind pattern (channeled wind, katabatic winds, ...). Sou (2007) showed that the average wind stress (1950-2004) from NCEP/NCAR indicates a northward direction in Nares Strait, while a southward ice flow is observed. As stated by Wang et al. (2003), high resolution atmospheric forcing fields might help to solve these issues.

The mesh resolution must be ideally sufficiently fine to resolve the ice circulation everywhere, in particular where smaller scales are relevant. In a series of numerical simulations that we performed on different meshes, we found little dependence of the sea ice fluxes and the ice features to the mesh resolution. We can thus consider that most of the ice flow is resolved. One exception is probably Nares Strait, where we have shown that the ice circulation is poorly resolved. However, this particular place seems to collect the modeling challenges: the wind is subject to the particular topography and is channeled, ice arches form during winter in the northern (Lincoln Sea) and southern (North Water Polynya) parts and some portions of the

strait are occupied by landfast ice. We have shown that, with our model, we are not able to represent static landfast sea ice in the North Canadian Archipelago. This issue might be resolved by including some parameterization of the landfast sea ice in the model. However, detailed bathymetry and well-resolved coastal currents are needed for an appropriate simulation of landfast sea ice (Wang et al., 2003). The formation of static arches should be investigated in a first step thanks to some test problems (see Dumont et al. (e.g., 2009)). In light of these model deficiencies, it appears that, instead of adding new features and parameterizations to the actual model, significant improvements might be gained by investigating a new rheology integrating recent laboratory research work on the mechanical properties of the ice and observations based on buoys tracking (e.g., Schreyer et al., 2006; Coon et al., 2007; Weiss et al., 2007; Girard et al., 2011). It is our believe that the developments presented in the following chapter towards a Lagrangian model will greatly help investigate these issues and better understand the mechanical behavior of sea ice. As such, Nares Strait constitutes a nice testcase for new model developments.

CHAPTER 4

A LAGRANGIAN, ADAPTIVE, FINITE ELEMENT SEA ICE MODEL

Summary

We present a Lagrangian, adaptive, finite element sea ice model allowing the computational grid to move with the ice. The Lagrangian description offers several attractive advantages such as the easy tracking of free surfaces or the direct access to the material deformation history. We take advantage of the locality of the mesh adaptation procedure to update the discontinuous fields thanks to a local Galerkin projection. The mesh adaptation procedure is first validated. Then the conservation and convergence properties of the field projection are demonstrated through different benchmarks and the transport algorithm is evaluated by a classical advection test case. Finally, the model is run on the Arctic Basin and different ice age definitions are implemented and compared.

4.1 Introduction

Eulerian models associated to the finite difference method have constituted the larger part of sea ice models in the past. The Lagrangian description is an attractive alternative as it facilitates the tracking of moving boundaries and free surfaces. The Lagrangian approach is also mathematically well-suited for the description of conservation equations. In particular, ongoing research is dedicated to the develop-

ment of new sea ice rheologies (e.g., Schreyer et al., 2006; Wilchinsky and Feltham, 2006; Coon et al., 2007; Sulsky et al., 2007; Weiss et al., 2007; Girard et al., 2011). The inclusion of material properties and the access to the history of deformation are key elements to improve sea ice dynamics representation. The implementation of these features is straightforward in a Lagrangian framework. Finally, the Lagrangian description clearly makes easier the assimilation of sea ice trajectories from buoy motion or satellite tracking (Meier and Maslanik, 2001; Lindsay and Stern, 2004).

A series of studies focusing on Lagrangian or Eulerian-Lagrangian descriptions for sea ice modeling have been reported. Our objective is not to conduct an extensive survey (we refer the reader to Lindsay and Stern (2004) and Sulsky et al. (2007)), but to report on different approaches developed so far. Pritchard et al. (1990) utilized an adaptive Arbitrary Eulerian-Lagrangian model to forecast the ice edge position. Another approach has been investigated by Hopkins and Hibler (1991) in which sea ice behaviour is dictated by interactions between discrete particles. A couple of studies had recourse to the Particle In Cell (PIC) method that mixes Eulerian and Lagrangian descriptions (Flato, 1993; Huang and Savage, 1998; Ligett, 2000). A fixed grid is used to solve the ice momentum equation, while Lagrangian particles are transported by the ice flow. The Material Point Method (MPM) can be viewed as an alternative of the PIC method applied to solids (Sulsky et al., 2007). Finally, the Smoothed Particule Hydrodynamics (SPH) is a Lagrangian method based on a large number of particles without any grid (Gutfraind and Savage, 1997; Lindsay and Stern, 2004). Because unstructured meshes enable the use of mesh adaptivity, we had the opportunity to enjoy the advantages of a purely Lagrangian approach and thereby followed the preliminary work of Wang and Ikeda (2004) consisting in a Lagrangian, finite element, adaptive model.

Lagrangian methods involve frequent remeshing operations to maintain a good quality of the mesh, thus necessitating the use of interpolation techniques to transfer the fields from the source mesh to the target mesh. This naturally yields a loss of accuracy which can be viewed as the counterpart of diffusion in the Eulerian description. In this chapter, we investigate a local and conservative projection to handle fields updates. Our model has been tested on two specific applications. The first one deals with the ability of the model to track the ice edge. The second application concerns the ice age. In Chapter 2, we presented a vertically-varying sea ice age. In this chapter, we take advantage of the Lagrangian description to take ice dynamics into account and compare the results with other ice age definitions.

4.2 Mesh adaptation

The mesh design is supported by a prescribed mesh size field that describes the desired element size and shape distribution. Depending on the modeler's objec-

tives, it depends generally upon space, but possibly upon any other variables, including time, direction (anisotropic mesh) or any field of the problem studied. The latter possibility of refining the mesh in areas of physical interest renders adaptive mesh simulations particularly attractive. The mesh adaptation procedure used in this study modifies the given mesh thanks to local operations in order to satisfy at best the desired element size and shape distribution. These operations are handled by a mesh adaptivity library (MAdLib, <http://www.madlib.be>). The definition of the mesh size field and the choice of the local operations constitute the core of the mesh adaptation procedure described below and have been largely discussed and validated (Li, 2003; Remacle et al., 2005; Shephard et al., 2005; Compère et al., 2008).

4.2.1 The mesh size field

In three dimensions, the mesh size field can be represented at any point by a mesh metric tensor describing an ellipsoidal surface whose points yield the desired mesh edge length for any direction. An appropriate space transformation mapping the ellipsoidal surface on a unit sphere allows us to deduce naturally a mesh conformity criteria. If an entity of the given mesh does not satisfy this criteria, the adaptation procedure is activated, using the information contained in the size field to correct the mesh.

The mesh metric field at a point P is defined as “a 3×3 symmetric positive definite mesh metric tensor $\mathbf{M}(P)$ such that the desired directional mesh edge length distribution at this point follows an ellipsoidal surface

$$\mathbf{x}\mathbf{M}(P)\mathbf{x}^T = 1 \quad (4.1)$$

where $\mathbf{x} = \{x_1, x_2, x_3\}$ is a coordinate vector” (Li, 2003). In two dimensions, the metric tensor is of order two and the ellipsoidal surface reduces to an elliptical surface (Fig. 4.1). Let $h(P, \mathbf{e})$ denote the desired mesh edge length at the point P along the direction given by the unit vector $\mathbf{e}$. By construction of the metric tensor, the vector $h(P, \mathbf{e})\mathbf{e}$ starting from P points at the ellipsoidal surface, hence satisfying equation (4.1):

$$h(P, \mathbf{e})\mathbf{e}\mathbf{M}(P)h(P, \mathbf{e})\mathbf{e}^T = 1,$$

from which we can derive an expression for $h(P, \mathbf{e})$:

$$h(P, \mathbf{e}) = \frac{1}{\sqrt{\mathbf{e}\mathbf{M}(P)\mathbf{e}^T}}. \quad (4.2)$$

Due to its symmetry, the tensor $\mathbf{M}(P)$ can be classically diagonalized as follows (e.g., Lancaster and Tismenetsky, 1985):

$$\mathbf{M}(P) = \mathbf{R}(P)\mathbf{\Lambda}(P)\mathbf{R}(P)^T$$

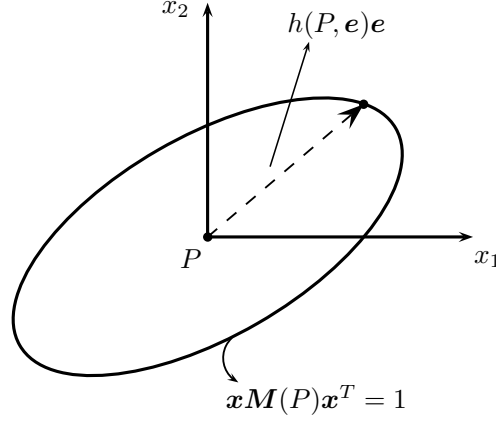


Figure 4.1: In two dimensions, the mesh metric field can be represented at each point P by a symmetric positive definite tensor $\mathbf{M}(P)$ describing the desired mesh edge length $h(P, \mathbf{e})$ at the point P along any direction $\mathbf{e}$.

where $\mathbf{R}(P)$ is an orthogonal matrix containing the eigenvectors of $\mathbf{M}(P)$ and $\mathbf{\Lambda}(P) = \text{diag}\{\lambda_1, \dots, \lambda_n\}$ is the diagonal matrix composed of the eigenvalues of $\mathbf{M}(P)$. The tensor being positive definite, the eigenvalues are all positive and the tensor can be finally decomposed as:

$$\mathbf{M}(P) = (\mathbf{R}(P)\mathbf{D}(P))(\mathbf{R}(P)\mathbf{D}(P))^T = \mathbf{Q}(P)\mathbf{Q}(P)^T$$

with $\mathbf{D}(P) = \text{diag}\{\sqrt{\lambda_1}, \dots, \sqrt{\lambda_n}\}$. The tensor $\mathbf{Q}(P)$ can be geometrically interpreted as the combination of a rotation ($\mathbf{R}$) and a distortion ($\mathbf{D}$), thereby defining a space transformation $\mathbf{x}' = \mathbf{x}\mathbf{Q}(P)$. The latter has the particular property of mapping any edge from the physical space into an isotropic and normalized transformed space (see Fig. 4.2):

$$\mathbf{x}'\mathbf{x}'^T = \mathbf{x}\mathbf{Q}(P)\mathbf{Q}(P)^T\mathbf{x}^T = \mathbf{x}\mathbf{M}(P)\mathbf{x}^T = 1.$$

Ideally, all mesh edges should have a unit length in the transformed space, providing a criteria to measure with what accuracy the mesh size field is satisfied. Note that the desired edge length depends on the metric of both vertices of the edge. A piecewise interpolation between these vertices is hence applied to yield the final edge length.

In practice, it is often impossible to satisfy exactly the mesh size field for each mesh entity. The ideal criteria is hence relaxed towards an interval $[L_{min}, L_{max}]$ defined in the transformed space. In order to avoid infinite loops between refining and coarsening operations, this interval must be chosen such as $L_{min} \leq 0.5 L_{max}$. This condition ensures that whenever an edge larger than L_{max} is split in two (*splitting* operation), it then belongs to the interval. Finally, Fig. 4.3 illustrates a mesh

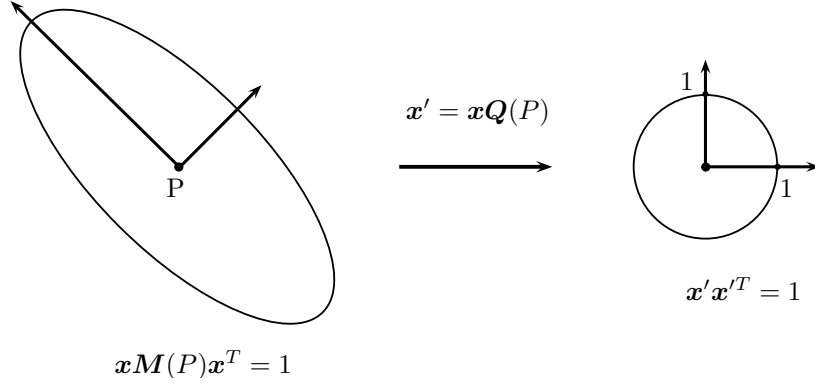


Figure 4.2: Linear transformation from the physical space to the transformed space associated to the tensor $Q(P)$. In the latter space, ideal mesh edge lengths are normalized.

size field used to generate a typical mesh of the Arctic. The mesh resolution varies between 10 and 200 km.

4.2.2 Mesh adaptation operations

All mesh modification operations described and used in this work are local, thereby meaning that only a small set of mesh triangles, called *cavity* and noted $\mathcal{C}$, are involved by the operation and replaced by another cavity $\mathcal{C}'$ with the same closure. An advantage of this approach is that only a small part of the mesh is modified, allowing frequent mesh adaptations and local conservative field projections (see Section 4.3).

Among these operations, two are aimed at better satisfying the mesh metric field. If the length of an edge in the transformed space is longer than L_{max} , the *splitting* operation allows us to refine the mesh. The edge is divided into two equal parts and a new vertex is inserted, while the cavity goes from two to four triangles (see Fig. 4.4). If the length of an edge is smaller than L_{min} , the *collapsing* operation removes the shortest edge and collapses its vertices (Fig. 4.5). The number of elements in the cavity decreases by two.

The last operation regards the quality of the triangles. The shape measure of an element e is evaluated according to the square *mean ratio* (Liu and Joe, 1994) expressed as:

$$\eta_e^2 = 48 \frac{A_e'^2}{\sum_{i=1}^3 l_i'^2}, \quad (4.3)$$

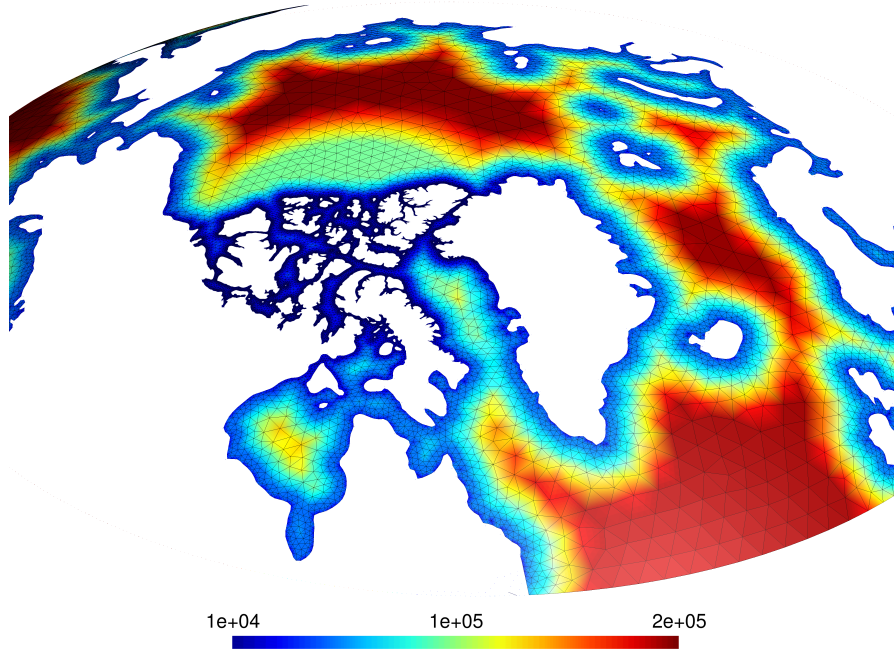


Figure 4.3: Mesh size field used to generate a typical mesh of the Arctic and surrounding seas. The mesh resolution varies between 10 and 200 km.

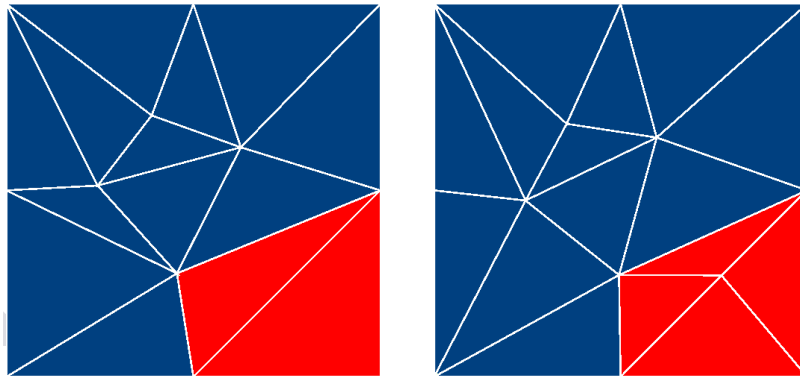


Figure 4.4: Splitting operation: if an edge is too long, it is split in its middle by inserting a new vertex.

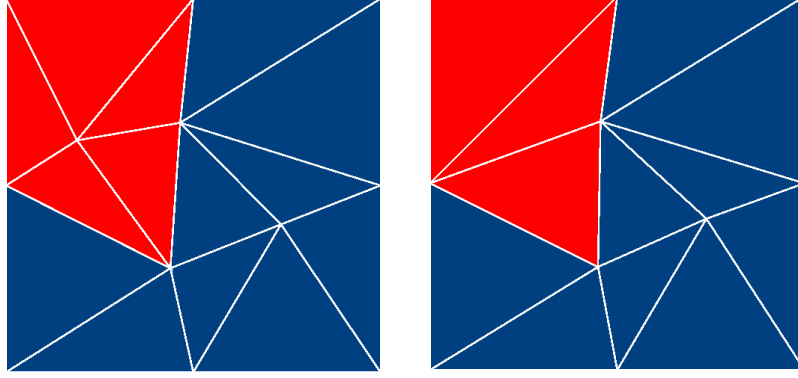


Figure 4.5: Collapsing operation: if an edge is too short, it is removed and its vertices are collapsed.

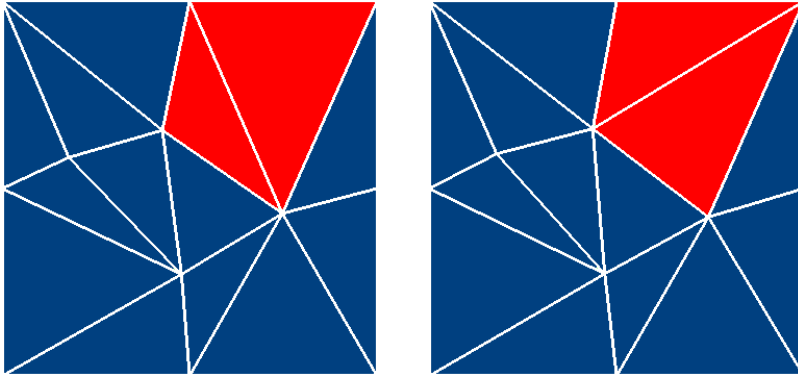


Figure 4.6: Swapping operation: if a triangle is too flat, the cavity formed with a neighboring triangle is modified in order to create two new triangles.

where A'_e and l'_i are the area of the element e and the length of the i th element edge, respectively, both measured in the transformed space. The value of the criterion is 1 for an equilateral triangle and 0 for a flat element. According to this criterion, if a triangle is detected as badly shaped, a *swapping* operation is attempted and retained only if the overall quality of the considered triangles is enhanced by the operation. This operation simply consists in modifying the mesh topology in the cavity so that the two existing triangles are replaced by two others (see Fig. 4.6) by swapping the edge. Finally, according to the triangle shape measure, flat elements (*slivers*) are eliminated. This step is very important for the robustness of the algorithm since badly shaped elements are particularly harmful for the numerical solution. When such elements are detected, the above three basic operations are successively ap-

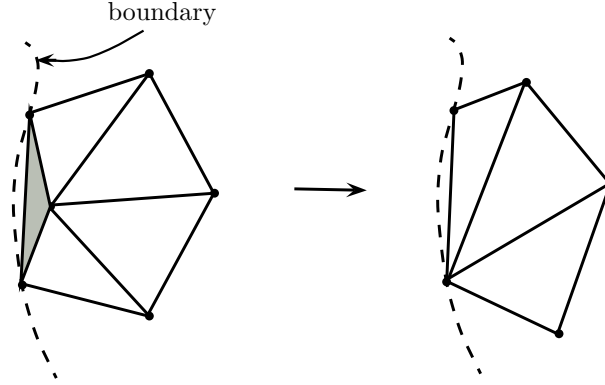


Figure 4.7: Sliver removal procedure on the boundary: typically, a collapse operation is applied to remove the sliver (greyed element).

plied to try and remove them. When a sliver is detected on the boundary (Fig. 4.7), collapse and swap are temporarily enabled on the boundary.

4.2.3 Mesh adaptation procedure

Once the mesh adaptation operations have been described, one has to make up his mind on the exact order of succession of these operations. Each sequence is operated in a row, and the order of succession of these operations constitutes a degree of freedom of the mesh adaptation procedure. This one being rather empirical, the influence of applying different operation sequences on a test problem has been investigated. We consider four adaptation sequences inspired by the literature (Remacle et al., 2002; Li, 2003; Shephard et al., 2005) (see Fig. 4.8). Each sequence has been studied twice: by applying the sequence once, and by integrating the sequence into a minimum quality objective algorithm: the sequence is repeated until a minimum quality threshold (here fixed at 0.14) is satisfied, meaning that the mesh is free of sliver triangles. In order to avoid infinite loops, a maximum number of iterations (here 10) is imposed. To validate the mesh adaptation procedure, the following test case is studied. The domain of study corresponds to a square representing a parcel of ocean in which an idealized island is placed (Fig. 4.9). The velocity field used to challenge the adaptation procedure needs to cancel out on the boundaries. Therefore, a non-uniform velocity field $\mathbf{u}(\mathbf{x}(t))$ is computed by multiplying a rotational velocity field by the solution of the Poisson equation ($\nabla\phi = -1$). The resulting field is illustrated in Fig. 4.9. The displacement of the nodes is computed by integrating the velocity field:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{u}(\mathbf{x}(t)), \quad (4.4)$$

Sequence 1	Sequence 2	Sequence 3	Sequence 4
RemoveSlivers Collapse Split Swap	RemoveSlivers Collapse Swap Split Swap	Split RemoveSlivers Collapse Swap	Split Swap RemoveSlivers Collapse Swap

Figure 4.8: Mesh adaptation sequences considered. Each component of a sequence corresponds to a loop on the concerned mesh entities. The operation `RemoveSlivers` refers to the sliver removing procedure.

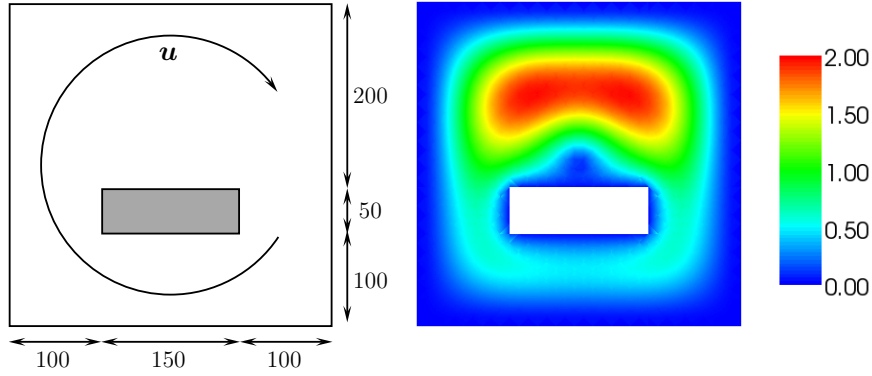


Figure 4.9: Left: configuration of the mesh adaptation test case. The greyed rectangle corresponds to an island. Right: norm of the velocity field used to displace the mesh.

thanks to a simple Euler forward scheme. The first five displacements of the mesh are shown in Fig. 4.10.

The mean and standard deviation of the element shape measure are computed for every adaptation sequence listed in Fig. 4.11 during 400 time steps. Also shown are the minimum and maximum values of the shape measure. As can be seen, the mean and standard deviation are very similar for all sequences. It must be stressed that the most critical operation is the sliver removal. In this regard, the minimum value is particularly important since we want to avoid slivers. As expected, the sequences integrated into a minimum quality objective algorithm provide better results than a single iteration of each sequence. Two sequences yield better results in terms of minimum quality measure (2 and 4). Sequence 2 has the additional advantage to begin with a collapse loop, which tends to decrease the memory peak at the

begin of the procedure as collapsing operations decrease the number of elements. We therefore chose this sequence for all numerical simulations in this work. To save cpu time, the sequence is not integrated in a loop depending on a quality threshold, but is repeated a maximum number of times as long as some mesh adaptation operations are applied. This sequence has proven to be well suited to remove the slivers.

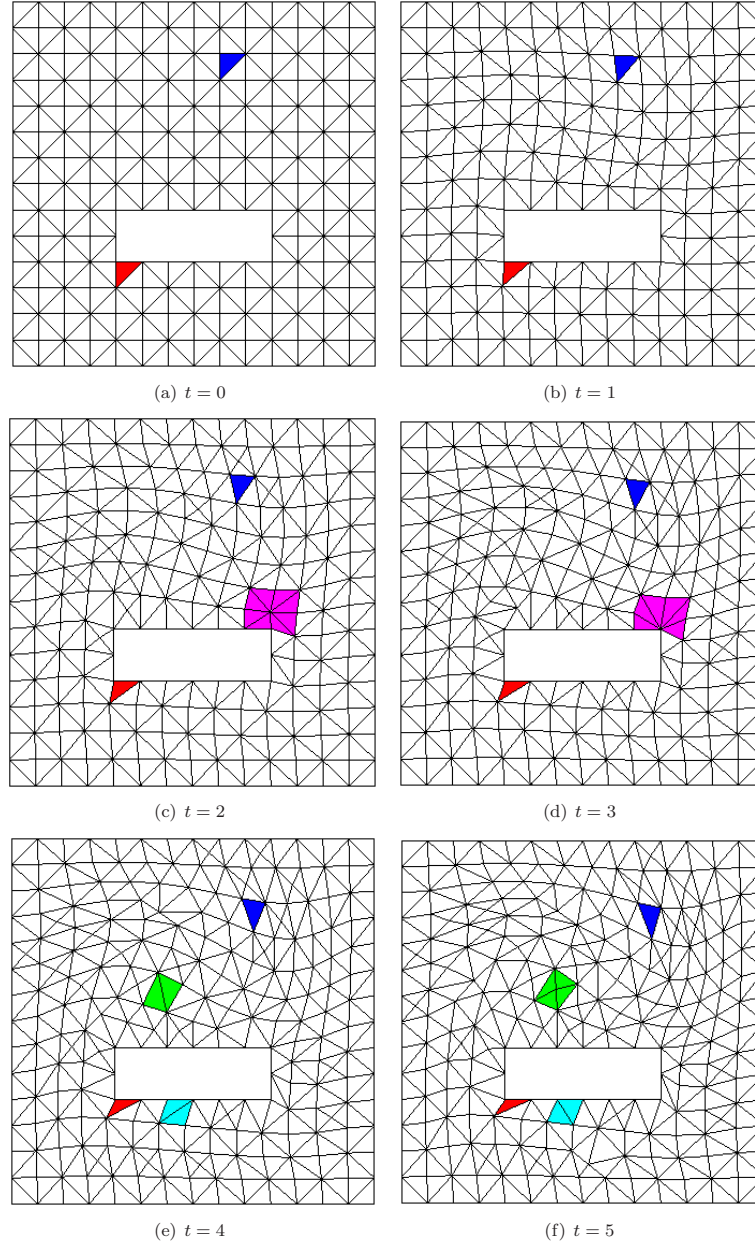


Figure 4.10: Mesh position of the initial and five first time steps of the mesh adaptation test case. Blue and red triangles help guiding the eyes. Examples of the three basic mesh operations are shown (*split* in green, *swap* in light blue and *collapse* in mauve).

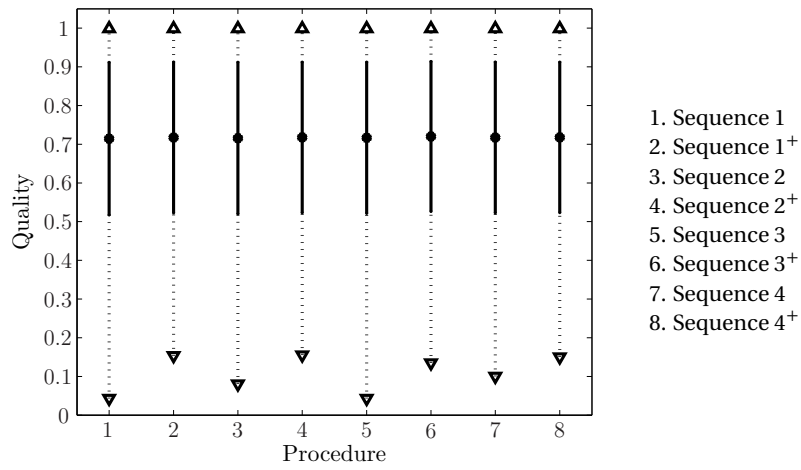


Figure 4.11: Comparison of the mean mesh quality between different sequences of operations. The symbol ⁺ refers to repeated sequences integrated in the algorithm with minimal quality objective. The mean values (μ) are represented by • and the 68% confidence intervals (corresponding to one standard deviation) by thick lines. Maximum and minimum quality values are indicated by Δ and ∇ , respectively.

4.3 The field update procedure

In the previous section, we have described a mesh adaptation procedure in which source and target meshes only differ in a finite number of element cavities. In practice, the two meshes are hence very similar and, thanks to the discontinuous representation of the fields, the latters must only be updated on the adapted cavities. In this section, we describe a local and conservative field projection. The locality of the method makes its efficiency and its originality.

4.3.1 The need for conservative projection

The problem of field transfer between meshes arises in many applications of numerical modeling: h -adaptivity, multiphysics simulations (fluid-structure interactions, problem with interfaces), multigrid methods, etc. Let us formulate the problem: let C^S be a field given on the source mesh that we want to project on the target mesh to obtain the field C^T . The two most common properties that the field transfer must satisfy are that it must be as accurate as possible (we want to minimize the L_2 norm of the residual $\|C^S - C^T\|_2 = \sqrt{\int_{\Omega} (C^S - C^T)^2 dx}$ and be conservative: $\int_{\Omega} C^S dx = \int_{\Omega} C^T dx$. Both fields are classically discretized by the finite element method as $C^S = \sum_{i=1}^{n_S} \phi_i^S C_i^S$ and $C^T = \sum_{i=1}^{n_T} \phi_i^T C_i^T$, where ϕ_i^S (ϕ_i^T) correspond to the n_S (n_T) shape functions of the source (target) mesh and C_i^S (C_i^T) are the associated nodal values.

Several methods have been proposed to solve this issue. The first solution found in the literature is the consistent interpolation that consists in determining the nodal values C_i^T pointwise by evaluating the source interpolants at the geometric locations of the target nodes. As outlined by Farrell and Maddison (2010), this method suffers from several drawbacks: it is in general not conservative, it tends to diffuse the extrema of the source function and it is not well defined for (partly) discontinuous shape functions. Cubic splines have proven to be very accurate to transfer information between meshes (Jiao and Heath, 2004), but conservativity is not achieved. The L_2 minimization consists in minimizing the L_2 norm of the residual by requiring that the derivative of the norm vanishes:

$$\frac{\partial \int_{\Omega} (C^T - C^S)^2 dx}{\partial C_i^T} = \int_{\Omega} 2\phi_i^T (C^T - C^S) dx = 2 \sum_{j=1}^{n_T} \int_{\Omega} \phi_i^T \phi_j^T dx - 2 \int_{\Omega} \phi_i^T C^S dx = 0, \quad i = 1, \dots, n_T, \quad (4.5)$$

leading to the system of equations $\mathbf{M}^{TT} \mathbf{C}^T = \mathbf{M}^{TS} \mathbf{C}^S$ where $M_{ij}^{TT} = \int_{\Omega} \phi_i^T \phi_j^T dx$ denotes the consistent mass matrix based on the target shape functions, $M_{ij}^{TS} = \int_{\Omega} \phi_i^T \phi_j^S dx$ denotes the mixed mass matrix based on both shape functions and

C represents the nodal values on each mesh. Because the same system of equations can be seen as an orthogonal projection of the residual on the function space spanned by the target shape functions, this method is generally referred to as *Galerkin projection*. It is conservative, suitable for discontinuous Galerkin discretization and optimal for the L_2 norm in the above-mentioned space.

4.3.2 Local projection on the mesh cavities

Regarding the projection of the solution between the source and target meshes, the mixed mass matrix M^{TS} needs a particular treatment: the shape functions of the source mesh are piecewise polynomials over the elements of the target mesh. Integrating exactly functions that are piecewise is not an easy task as the classical quadrature rules are generated for smooth functions. Increasing the order of integration does not achieve to compute exactly the integrals, not even to a prescribed precision (Farrell and Maddison, 2010). The usual way to bypass this difficulty is to integrate (exactly) the fields on each intersection between the source and the target elements (George and Borouchaki, 1998; Jiao and Heath, 2004; Alauzet and Mehrenberger, 2009; Farrell and Maddison, 2010). On the other hand, the use of a discontinuous representation of the fields introduces one great simplification: fields only need to be projected locally, that is, on the mesh cavities altered by the mesh adaptation procedure (see Fig. 4.12). Indeed, these cavities contain a limited number of elements and, on each element, the field relies on discontinuous shape functions (that are thus independent of the neighboring elements). In this work, we have implemented a local Galerkin projection based on the element cavities and generalized for any order of shape functions.

We first need to construct the intersection mesh of the cavity. The mesh adaptation algorithm returns the source and target elements constituting the mesh cavities. Hence no global search algorithm is needed in this approach and, due to the small number of elements involved in a cavity, the intersection of the source and target cavities is computed thanks to a brute-force segment-by-segment intersection. The locality of the method constitutes an important simplification and advantage compared to methods dealing with constructing a complete intersection mesh (e.g., Farrell et al., 2009). Degenerated cases such as aligned or coincident edges, or element included in another, are first detected.

The mixed mass matrix is integrated on the mesh intersection and assembled. A noteworthy feature of the method is that all the computations needed to assemble both mass matrices are essentially based on geometrical considerations and must be done only once for different fields of the same order. Finally, once the contributions of all the intersections on a target element have been assembled in the mixed mass matrix, the fields can be projected. The consistent mass matrix must be inverted only once for all fields of the same order, leaving only matrix-vector products

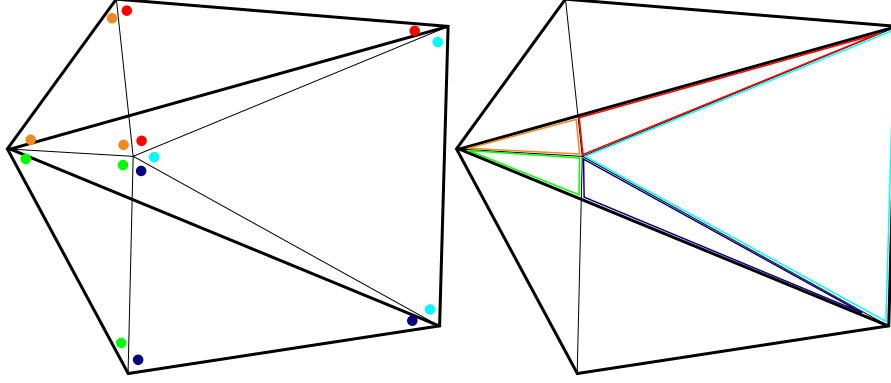


Figure 4.12: Projection of a linear discontinuous field on a mesh cavity (example of a collapse). The source cavity is in thin lines and the target cavity in thick lines. Left: the shape functions associated to the source cavity are denoted by their degrees of freedom (color points) and are discontinuous across the elements of the target cavity. In this case, the projection of the field on the target central element (right) involves integration of the field on all the elements of the source cavity.

to derive the updated nodal values for each field. In addition of the locality of the method, increasing the number of fields of the same order is thus almost negligible in terms of cpu time and the projection proposed here is conservative, trivially parallelizable and efficient.

The locality further yields a nice simplification to limit fields based on linear shape functions (P_1^{DG}): by inspecting the source cavity, we have directly access to the lower and upper bounds of the field that the projected field must satisfy on the target cavity in order to avoid new extrema, whereas classical formulations have recourse to an extrapolation of the field on the neighboring elements (Cockburn and Shu, 1998). The following slope limiting procedure is therefore adapted from Cockburn and Shu (1998). Let us denote by U_i ($i = 1, 2, 3$) the nodal values of the field to be limited after the projection and by $\bar{U}$ the mean of these nodal values. We first compute the maximum (Δ_{max}) and minimum (Δ_{min}) deviations of the field on the source cavity compared to the mean $\bar{U}$. The largest (in absolute value) deviation Δ_i allowed on the target cavity in order to avoid new over/undershoots is given by

$$\Delta_i = \begin{cases} \min(U_i - \bar{U}, \Delta_{max}), & \text{if } U_i - \bar{U} > 0, \\ \max(U_i - \bar{U}, -\Delta_{min}), & \text{otherwise.} \end{cases} \quad (4.6)$$

Then, if $\sum_{i=1}^3 \Delta_i = 0$, the mean value is kept constant after the limiting stage and the deviations can be applied directly to determine the limited nodal values U_i^* :

$$U_i^* = \bar{U} + \Delta_i, \quad i = 1, 2, 3. \quad (4.7)$$

In the negative, one has to define the maximum proportion of the positive and negative deviations that are allowed:

$$\text{pos} = \sum_{i=1}^3 \max(0, \Delta_i), \quad (4.8)$$

$$\text{neg} = \sum_{i=1}^3 \max(0, -\Delta_i), \quad (4.9)$$

and the proportions are set to:

$$\theta^+ = \min\left(1, \frac{\text{neg}}{\text{pos}}\right), \quad (4.10)$$

$$\theta^- = \min\left(1, \frac{\text{pos}}{\text{neg}}\right). \quad (4.11)$$

The conservative deviations $\hat{\Delta}_i$ read:

$$\hat{\Delta}_i = \theta^+ \max(0, \Delta_i) - \theta^- \max(0, -\Delta_i), \quad i = 1, 2, 3, \quad (4.12)$$

and are applied in the same manner as equation (4.7).

4.3.3 Field update benchmarks

To validate the field projection procedure, we perform a convergence analysis of the L_2 error between an analytical field and its local projection. In order to involve the maximum element cavities possible in the projection procedure, we first force a strong mesh distortion by imposing a (weak) average mesh element shape quality. This step is realized thanks to an edge collapse loop. Second, the mesh is adapted following the initial characteristic mesh size h to restore a good average mesh element quality. This artificial benchmark is intended to challenge the projection procedure whereas, in usual simulations, only a few element cavities are adapted at each time step. An example of source and target meshes is provided in Fig. 4.13. The analytical field is taken from Farrell and Maddison (2010) and given by:

$$C(x, y) = \exp(x^2) + 2y. \quad (4.13)$$

The results of the convergence analysis is shown in Fig. 4.13. As expected, the theoretical rates of convergence are reached for constant (P_0), linear (P_1^{DG}) and quadratic (P_2^{DG}) shape functions.

Finally, an analytical field is projected repeatedly 100 times to check that the integral of the fields are conserved up to machine precision and to analyze the evolution of the extrema of the fields. To this end, we use the peaks function proposed

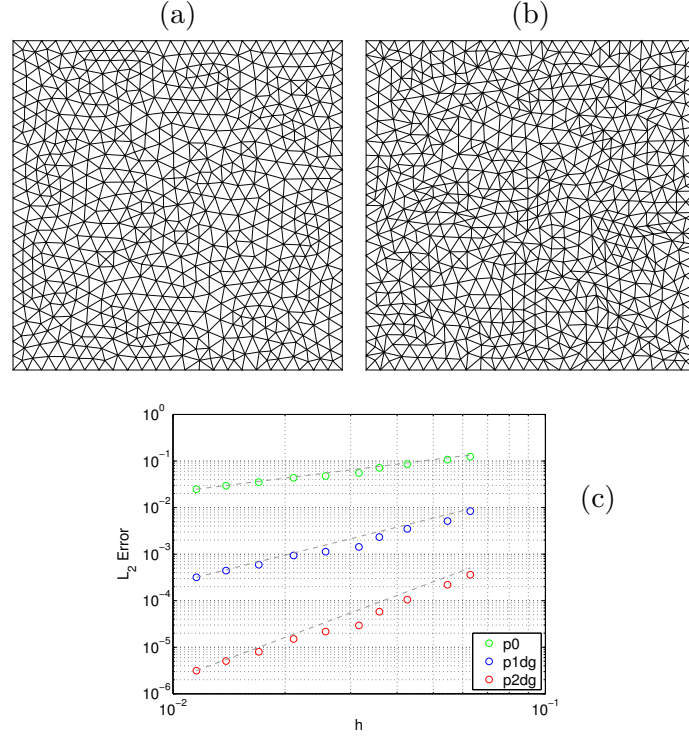


Figure 4.13: Source ((a), 1216 elements) and target ((b), 1380 elements) illustrative meshes used to perform the projection benchmarks. (c) Convergence of the L_2 error of the projection of the field C between source and target meshes with the element size h . Theoretical convergence rates (dashed lines) are reached.

by Jiao and Heath (2004):

$$\begin{aligned} \text{peaks}(x, y) = & 3(1-x)^2 \exp(-x^2 - (1+y)^2) - 10\left(\frac{x}{5} - x^3 - y^5\right) \exp(-x^2 - y^2) \\ & - \frac{1}{3} \exp(-(1+x)^2 - y^2). \end{aligned} \quad (4.14)$$

The contours of the P_2^{DG} field after 100 iterations are in very good agreement with the initial field (Fig. 4.14). In the absence of discontinuity in the analytical function, the repeated projection of the P_2^{DG} field is indeed remarkably low diffusive (Fig. 4.15). The P_1^{DG} field is slightly more diffusive. However, both fields are subject to strong over- and undershoots when the initial function features discontinuities (e.g., a cylinder of unit height, not shown here). The effect of the slope limiter on the P_1^{DG} field is also illustrated. As expected, the initial bounds are respected strictly at

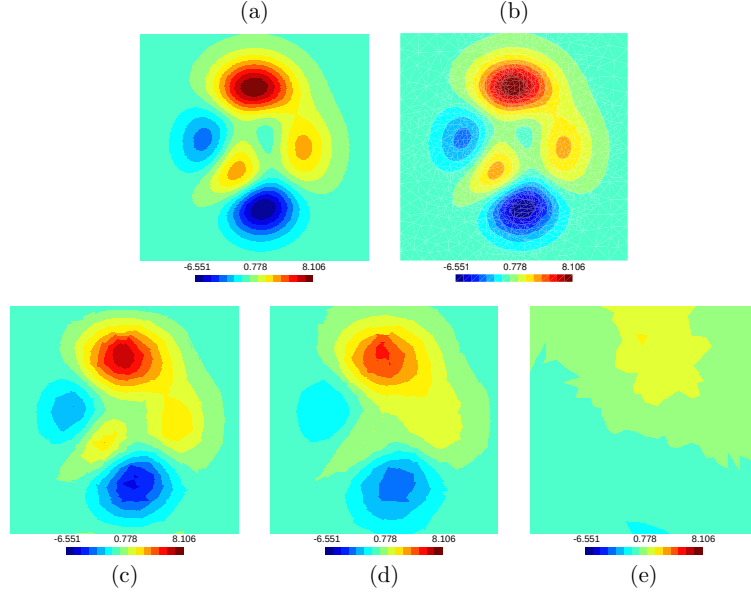


Figure 4.14: The peaks function after 100 repeated projections: (a) analytical solution, (b) P_2^{DG} , (c) P_1^{DG} , (d) P_1^{DG} with limiter and (e) P_0 .

the cost of diffusing these extrema. The P_0 field is by far the most diffusive. The P_1^{DG} with limiter seems to constitute a good compromise between accuracy and extrema-preserving, but, again, one must keep in mind that repeating intensively the projections is not likely to occur in realistic simulations. It is only intended to magnify the errors generated by the field transfer method.

4.3.4 Lagrangian formulation

In this section, we formulate the Lagrangian approach and analyze the ability of the Lagrangian model to transport tracers. The Lagrangian description is based on a reference fixed configuration $\Omega_0(\mathbf{X})$ (e.g., the domain configuration at $t = t_0$), where $\mathbf{X}$ denote the Lagrangian coordinates. We define the mapping between the reference and the spatial domains as:

$$\phi : \Omega_0 \longrightarrow \Omega : \mathbf{X} \longrightarrow \phi(\mathbf{X}, t) = \mathbf{x}, \quad (4.15)$$

and require that this transformation be invertible:

$$J = \det \left(\frac{\partial \mathbf{x}}{\partial \mathbf{X}} \right) > 0, \quad (4.16)$$

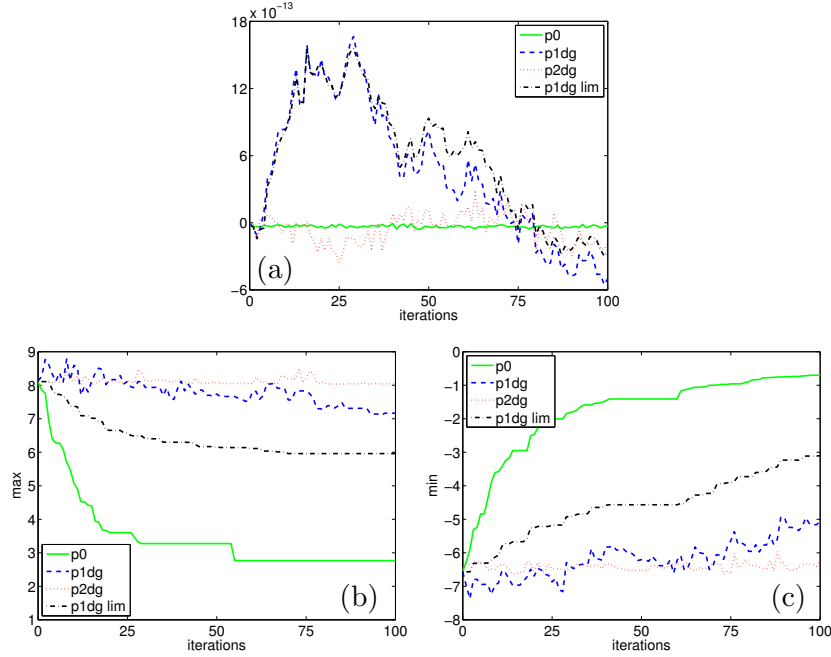


Figure 4.15: (a) Difference between the integral of the fields during 100 repeated projections of the peaks function and the integral of the initial field. Conservation between two iterations is achieved up to machine precision. Evolution of the maximum (b) and minimum (c) of the peaks function. Projection of P_0 fields is very diffusive, while P_1^{DG} and P_2^{DG} are subjects to some over- and undershoots. Adding a limiter allows to respect strictly the initial bounds of the field at the cost of diffusing these extrema.

where J is the Jacobian determinant of the transformation and represents the ratio between the elementary volumes in the spatial and in the reference configuration.

Since the mesh moves with the flow, we need to recast the tracer equation in the Lagrangian framework. The equation of tracer conservation reads:

$$\frac{\partial C}{\partial t} + \nabla \cdot (C \mathbf{u}) = 0, \quad (4.17)$$

where $C(\mathbf{x}, t)$ is the tracer concentration and $\mathbf{u}(\mathbf{x}, t)$ is the (mesh) velocity field. We first rewrite this equation in non-conservative form:

$$\frac{DC}{Dt} + C (\nabla \cdot \mathbf{u}) = 0, \quad (4.18)$$

that makes the material derivative $\frac{D}{Dt}$ appear. In order to conserve the tracer, we thus need to take into account the contribution of the divergence term. From continuum mechanics, we know that the divergence of the mesh velocity is related to the Jacobian as follows:

$$\nabla \cdot \mathbf{u} = \frac{1}{J} \frac{DJ}{Dt}. \quad (4.19)$$

Equation (4.18) can be rewritten as:

$$J \frac{DC}{Dt} + C \frac{DJ}{Dt} = \frac{D(CJ)}{Dt} = 0. \quad (4.20)$$

Intuitively, when the mesh is displaced, the product of the Jacobian and the tracer is conserved. In a finite element framework, the tracer equation is thus satisfied if the nodal values of the field are updated as a function of the ratio of the elements before and after the displacement.

4.3.5 The frontogenesis advection test case

The frontogenesis problem (Doswell, 1984) consists of a divergence free rotational velocity field in an initially strait front. The domain covers the square $[-4; 4] \times [-4; 4]$. The analytical velocity field is given by the following expression:

$$\mathbf{u}(r) = \frac{1}{u_0} \tanh(r) / \cosh(r)^2 \quad (4.21)$$

where $r = \sqrt{x^2 + y^2}$ and $u_0 = 0.385$. The initial tracer concentration reads:

$$C(x, y, 0) = -\tanh(y/2). \quad (4.22)$$

These fields are illustrated in Fig. 4.16.

We performed a convergence analysis of the error of the tracer field compared to the analytical solution for constant and linear shape functions (Fig. 4.17). The P_0 tracer yields a linear rate of convergence and the P_1^{DG} field a quadratic rate. The limiting procedure described by equations (4.6)-(4.12) decreases slightly the accuracy of the original P_1^{DG} field but gives the same order of convergence. Note that in this test case, no update of the field as a function of the element area change has been applied since the analytical velocity field is divergence-free. However, to be consistent, one should ensure that the velocity field is discretely divergence-free. The latter work is beyond the scope of this chapter.

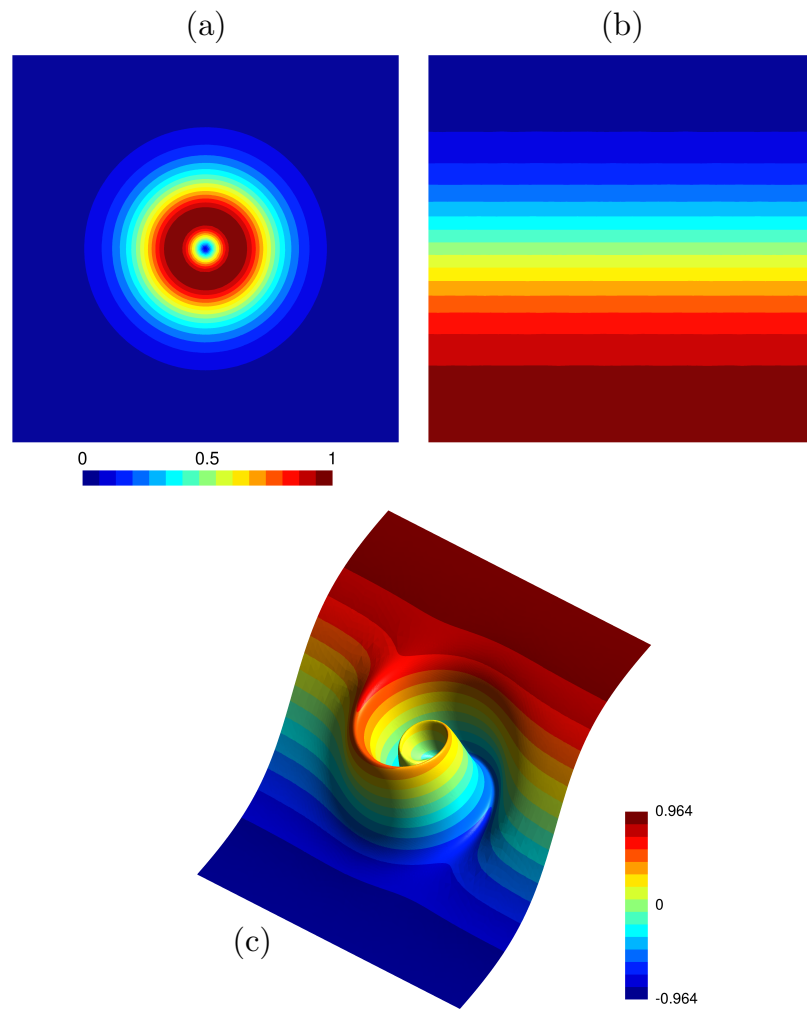


Figure 4.16: The frontogenesis problem (Doswell, 1984): (a) norm of the velocity field, (b) initial tracer field and (c) analytical solution.

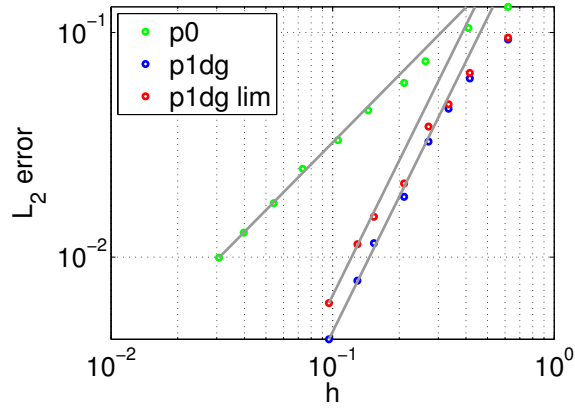


Figure 4.17: Convergence of the L_2 error of the tracer with the grid size h for constant (P_0) and linear (P_1^{DG}) shape functions. Also shown is the P_1^{DG} tracer with the limiting procedure described by equations (4.6)-(4.12). Linear and quadratic rates of convergence are depicted by the grey lines.

4.4 Numerical simulations of the Arctic sea ice cover

The Lagrangian sea ice model is described and the model is run on the Arctic. Sea ice thermodynamics and dynamics are essentially similar to the previous chapters, except that the model equations are solved on spherical geometry. Two specific applications highlighting the benefits of the Lagrangian approach are presented.

4.4.1 Configuration of the Lagrangian sea ice model

The equations are solved on spherical geometry following the approach described by Comblen et al. (2009). This approach involves two coordinate systems with appropriate changes of variable, one defined locally on each triangle and the other associated to the vectorial degrees of freedom. As a result, it enables the resolution of partial differential equations on curved surfaces and avoids the pole singularity issue arising when the numerical grid is based on a geographical coordinate system. The spherical geometry introduces a couple of difficulties regarding the mesh displacement. As the velocities are defined on a plane tangent to the sphere, the vertices will thus no longer belong to the spherical surface and need to be projected back on the surface. The error generated by this stage is however small since typical element sizes and mesh displacements are small compared to the Earth radius. Another consequence of the spherical geometry is that we need to project the mesh cavities to apply the local projection described in Section 4.3. To this end, an equal-area Lambert projection is applied on the vertices of each cavity. In this manner, the Galerkin projection can be applied and conservativity is achieved. In our numerical simulations, the mesh adaptation algorithm is run at each time step. Though the small mesh displacements would not require a systematic mesh adaptation, in practice the procedure is more efficient when applied often on a small number of mesh cavities than more sporadically with numerous rearrangements.

The sea ice momentum equations solved for the numerical simulations of this chapter are the same as equations (3.1), except the acceleration term that has been disregarded. Scale analysis has shown that the order of magnitude of this term is very small compared to the other forces in presence (Rothrock, 1970; Thorndike, 1986). This is particularly true for time averages above half an hour, whereas our time step typically amounts several hours. This simplification allows us to focus on the resolution of the non-linear system obtained from the discretization of the momentum equations. To this end, a subiteration stage ensures the convergence of the solution and a better resolution of the ice rheology. In consequence, sea ice behaves more plastically and responds faster to changing forcings. In particular, we find a decrease in ice mobility along the Canadian Arctic Archipelago. To increase the ice mobility, the empirical constant p^* (Eq. (3.7)) is set to 10000 N m^{-2} instead of 20000 N m^{-2} . Finally, a mask is applied on the velocity nodes lying in open water, saving cpu time during the system resolution.

Ice thermodynamics and forcing data sets are identical to chapter 3. We only changed the lateral melt contribution to the change in ice volume (Eq. (3.12)). In the slab ocean used in this thesis, the ocean temperature T_w is kept at the freezing point as long as there is ice on the element. The lateral melt parameterization (same as Eq. (3.11)) prescribes that the loss in ice area is equal to the ice area recovered by ice thinner than the ice quantity melted, assuming that the ice thickness is distributed between 0 and $2H$:

$$q_A^- = -\frac{A}{2H} [-(q^s + q^b)]^+, \quad (4.23)$$

where H is the ice thickness, A is the ice concentration and q^s and q^b are the surface and basal rates of change in ice thickness, respectively. Thus the heat gained in the leads is not explicitly used to melt ice (nor to heat the water) and the lateral melt depends only on the vertical ice ablation. In consequence, the increase in open water and the associated increase in heat absorbed in the leads do not enhance the ice melting following a classical ice-albedo retroaction. In this chapter, we keep the assumption that the decrease in ice concentration is, for the most part, due to the vertical melting of thin ice. Equation (4.23) is thus the same as in chapter 3, and we considered that the ice volume loss associated to the lateral melt was equal to the change in area multiplied by the ice thickness ($q_A^- H$, see Eq. (3.12)). Here, to integrate the effects of the increase in heat absorbed in the leads, the lateral melting is associated to a decrease in ice volume depending on the heat budget of the open water area. The last term of equation (3.12) is thus changed and the equation now reads:

$$q_v = A (q^s + q^b) + q_A^+ h_0 + \frac{(1-A)B_l}{L}, \quad (4.24)$$

where q_A^+ and h_0 denote the lateral growth and the threshold ice thickness created in the leads, respectively, while B_l and L correspond to the heat budget of the lead area and the volumetric latent heat of fusion of ice, respectively.

The velocity field is discretized thanks to linear continuous shape functions and the other fields by constant P_0 shape functions. While very attractive, the use of linear discontinuous interpolants for the ice volume or concentration still poses problem for sea ice applications since the slope of the ice volume on a given finite element does not affect the ice momentum equations. Indeed, in the finite element formulation (3.19), the term related to the ice strength (Eq. (3.7)) is integrated by parts: $-\langle \nabla \phi_j \cdot \sigma^h \rangle$. Thus the ice volume (see the ice strength term) is integrated and the slope of the ice volume has no influence on the momentum equation.

Since we impose that the domain boundaries do not move, no flux boundary conditions are naturally satisfied. The typical meshes of the Arctic Ocean and surrounding seas used in our studies yield two open boundaries along the parallel 50

degrees North in the Atlantic and Pacific Oceans. To satisfy open boundary conditions, one should compute a flux in the ALE fashion: the mesh velocity $\mathbf{u}_m$ of interior nodes is equal to the velocity $\mathbf{u}$ computed by the model as we are in a Lagrangian framework, while the mesh velocity of boundary nodes is equal to zero. In the ALE formalism, a flux appears in equation (4.20):

$$\frac{D(CJ)}{Dt} + J \nabla \cdot ((\mathbf{u} - \mathbf{u}_m) C) = 0, \quad (4.25)$$

that cancels out for interior edges. We opted for a simple solution since we deal with P_O fields: for any element in contact with the open boundary, we do not update the field as a function of the element change in area (see Eq. (4.20)). In this manner, the integral of the field is increased/decreased by a quantity equal to the difference in area multiplied by its value and we only neglect the contribution due to the divergence of the velocity field. Since the open boundaries are far from the domain of interest and correspond to areas where the presence of sea ice is rare, it will not be harmful for the numerical simulation.

4.4.2 Model validation

The model is spun up for 12 years and then integrated over the period 1979-2009. Several numerical simulations have been performed on different meshes. Unless specified, the mesh adaptation criterion used to control the distribution of the edges length preserves the initial size field. To this end, the latter is exported on a structured grid at the begin of the simulation. During the adaptation procedure, the initial field is loaded and the mesh size is interpolated at the mesh vertices.

Fig. 4.18 compares the mean modeled ice concentrations over the period 1979-2005 (the same as in Chapter 3) with the satellite observations (SMMR and SSM/I data sets, <http://nsidc.org>, Comiso (1999, updated 2005)). To compute mean field values on a moving mesh, the model outputs are generated on a fine structured grid on which all mean computations are done. The months of the climatological annual minimum (September) and maximum (March) in sea ice extent are illustrated. The March distribution of ice concentration is very similar to the one computed with the Eulerian model and has the same discrepancy: the ice is too extensive in the southernmost parts of the domain. In the Labrador Sea and the Barents Sea, the model clearly misses the ocean heat advection. A higher mesh resolution might contribute to better match the observations in the Greenland Sea. However, in summer, the correction brought to the lateral melt contribution is successful in better representing the ice extent. The model now presents open water in the Laptev Sea, Chukshi Sea and Beaufort Sea. The ice is also less extensive in the Kara Sea, Greenland Sea and Baffin Bay. Also shown is the state of the Arctic on September 2007, the year of the record minimum in ice coverage. In accordance with the observations, the model successfully reproduces an important ice loss in

the western part of the Arctic. Meanwhile, the Central Arctic has lost a few percents of ice concentration in summer (panel (b)), suggesting that the new lateral melt contribution tends to overestimate the lateral decay.

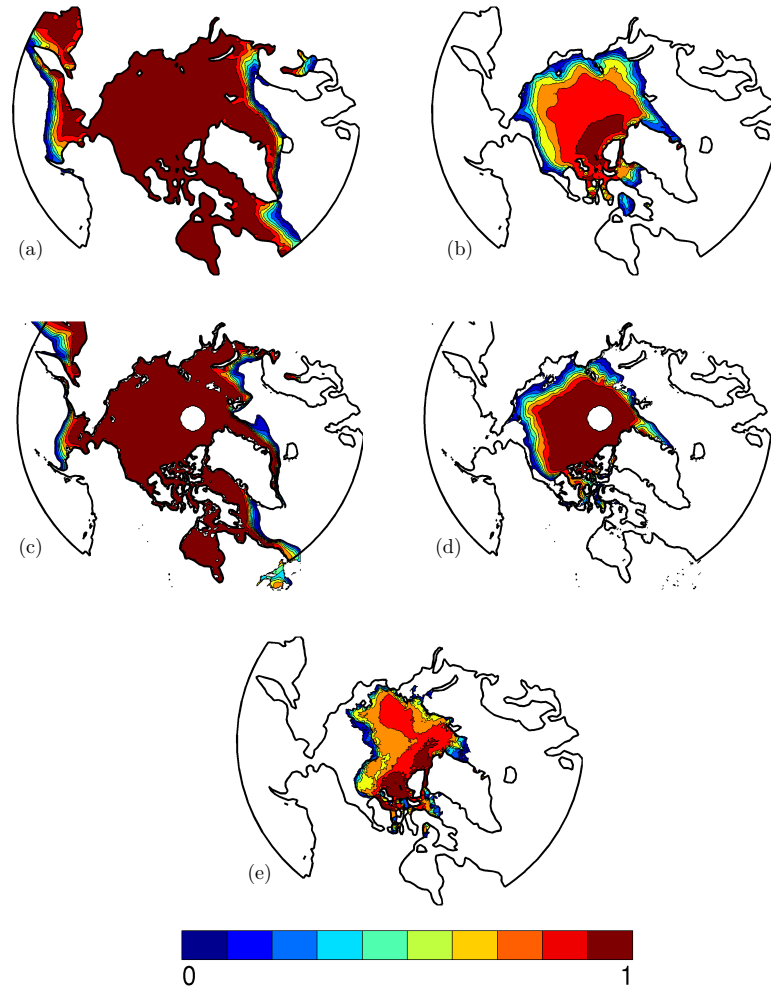


Figure 4.18: Simulated March (a) and September (b) mean ice concentrations (1979-2005). The corresponding observed mean ice concentrations are shown in panels (c) and (d) for the same period. (e) Mean ice concentrations in September 2007 as computed by the model. All fields have been interpolated on the same structured grid.

Fig. 4.19 compares the mean sea ice thicknesses in April over the period 1979-2000 as simulated by the model with the ice thickness climatology from Rothrock et al. (2008) converted from ice draft data collected by naval submarines (April 1975-2000). Compared to the climatology, the modeled ice thickness is about 1m lower over the Central Arctic and the thickness gradient between North Siberia and the Canadian Arctic Archipelago is also underestimated. This discrepancy of the model is attributed to the larger lateral melt contribution and to a strong Beaufort Gyre in the model. In consequence, the influence of the Transpolar Drift is weakened as ice that should be pushed against the Canadian Arctic Archipelago or exported through Fram Strait recirculates back in the Beaufort Gyre.

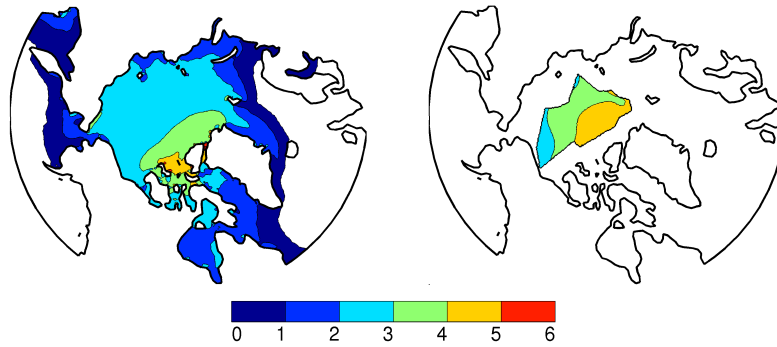


Figure 4.19: Simulated April mean ice thicknesses (left, 1979-2000). Right: ice thickness climatology from Rothrock et al. (2008) converted from ice draft data collected by naval submarines (April 1975-2000, 0-6 m). All fields have been interpolated on the same structured grid.

4.4.3 Mesh adaptivity and the ice edge tracking

We illustrate here the ability of the model to refine the mesh in areas of interest. The marginal ice zone is defined as the area where the ice cover is significantly altered by ocean processes, including the ocean swell. It is a narrow but crucial region for physical processes (heat exchanges) and biogeochemistry (algae, plankton, gas exchanges). Moreover, a particular attention has been brought recently to the exact location of the ice edge in the context of the possible opening of the Northwest Passage during summer months. On the other hand, forecast simulations (i.e., short-term prediction of the ice conditions) are bound to become an increasingly asked discipline. Adaptive, finite element methods render local mesh refinement possible following the modeler's objectives, hence yielding an attractive tool to track the ice edge.

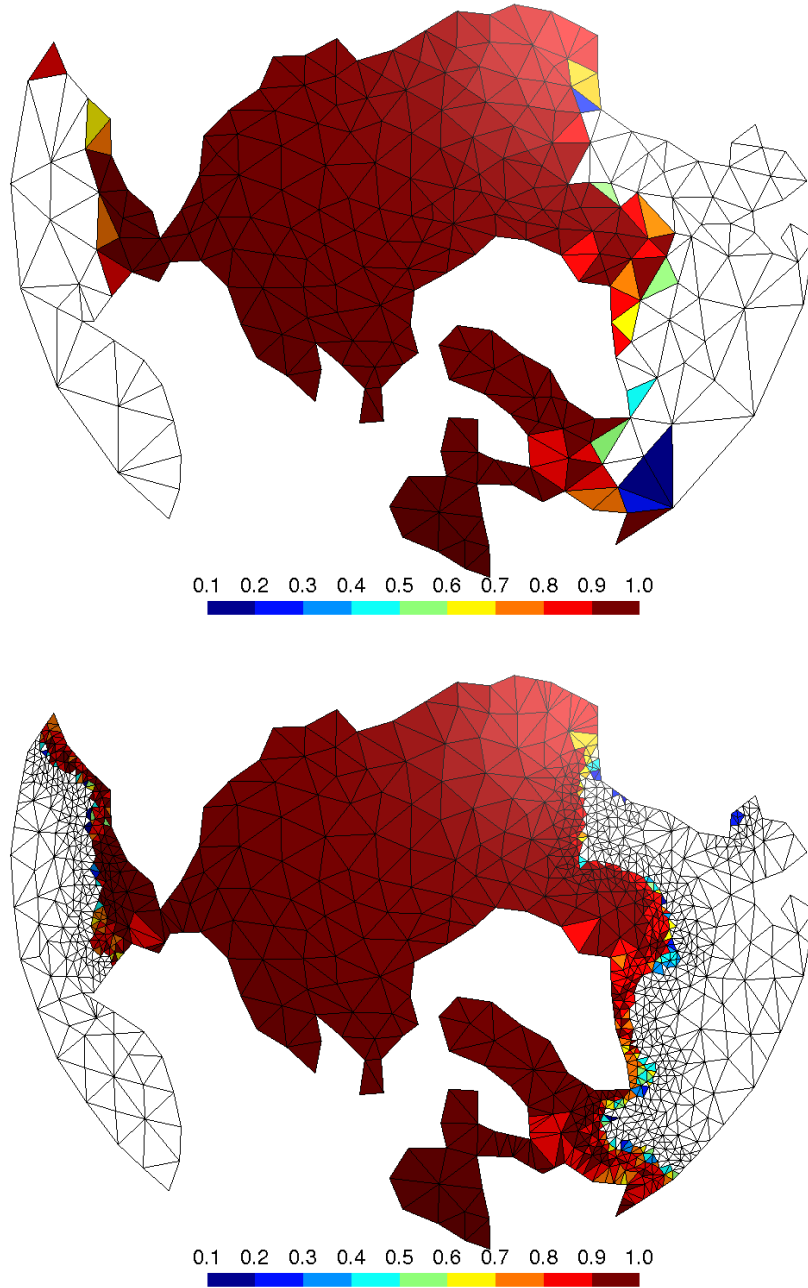


Figure 4.20: Example of mesh refinement along the ice edge for P_0 shape functions. The finer mesh resolution is about 100 km and the typical edge length far from the ice edge amounts to 400 km.

In order to adapt dynamically the mesh size field in function of the distance to the ice edge, we first need to compute an ensemble of points corresponding to the ice edge position. Then these points are stored in a particular data structure (a tree) to search efficiently the nearest points (Approximate Nearest Neighbor Searching, ANN library). Based on the distance information, the mesh size field is prescribed in the same fashion as for boundaries: we define a fine mesh resolution near the ice edge and a coarse resolution far from it, and we adjust a transition zone between these two extremes. For P_0 approximations, the ice edge is necessarily situated along an edge shared by an element with a higher concentration than the threshold and an other one whose ice concentration is under the threshold. When linear shape functions are used for the ice concentration, the exact location of the ice edge can be derived on the element edges that are crossed by the ice edge (except in the presence of jumps in the solution). For higher order interpolations (at least quadratic), an efficient algorithm computes the ice edge location that relies on a recursive division of the elements into sub-elements (Marchandise et al., 2007). The field (here, the ice concentration) is approximated by a linear function on the sub-element edges, allowing to compute the iso-zero easily. An example of ice edge refinement is shown in Fig. 4.20. Note that the mesh refinement along the ice edge can be straightforwardly applied to Eulerian simulations.

4.4.4 Buoy data assimilation

The sea ice mean speed is the subject of a positive trend and could play an important role in the ice-albedo feedback: more deformation yields more fracturing, hence more ice opening that leads to a decrease in the albedo (Rampal et al., 2009). On the other hand, IPCC climate models fail in capturing the observed ice motion acceleration (Rampal et al., 2011). The trajectory and velocity of ice particles give an immediate point of comparison with drifting buoys, using for example the IABP (International Arctic Buoy Program) buoy data sets, and could serve as a basis for drift analysis (Rampal et al., 2009) or data assimilation (e.g., Meier and Maslanik, 2001; Dulière and Fichefet, 2007). As an illustration, the trajectory of several ice particles are shown in Fig. 4.21.

4.5 Comparison of different sea ice age definitions

4.5.1 Introduction

Typically, satellite sea ice age retrievals (Rigor and Wallace, 2004; Fowler et al., 2004; Belchansky et al., 2005) are obtained by treating each sea ice covered grid cell as a Lagrangian particle that is advected by means of gridded fields of ice motion. New particles are created in open water with age zero and aged until they cross the limits of the ice edge. If a grid cell contains particles of different ages, the age of the oldest particle is retained (Maslanik et al., 2011). Uncertainties on this generic



Figure 4.21: The trajectory of several ice particles is shown. The time integration corresponds to the months of January and February 1981 with a time step of 6 hours.

method are primarily attributed to the errors in the underlying data sets (ice motion and ice concentration) and, in particular, to their low spatial and temporal resolution. Results are also dependent on the ice concentration threshold used for delimiting the ice edge: a pixel containing as little as 15% ice cover is retained with equivalent status as one with 100% cover (Belchansky et al., 2005). Thus the higher the threshold concentration, the more restrictive the algorithm regarding old ice survival.

Another satellite product related to sea ice age estimations is the multiyear coverage. It has been first derived from passive microwave brightness temperature (e.g., Comiso, 1990; Belchansky et al., 2004). However, this method has been shown to be unreliable because of spatial and temporal variations in the passive microwave signature (Grenfell, 1992; Kwok et al., 1996). Later, global backscatter data providing an enhanced confidence in the ice types delineation have been used (Kwok et al.,

1999; Kwok, 2004). Note that this data set also shows significant variability in the signal at the beginning of freeze up due to moisture on the surface (Kwok, 2004). While the multiyear ice distribution data have a higher spatial resolution, age estimates derived from advection of sea ice parcels provide a complete classification between ice age classes (first-year, second-year, ...). Nghiem et al. (2007) give a detailed comparison between these two types of age estimates. Finally, passive microwave satellite observations have been recently blended with buoy motions to improve the temporal resolution of the ice motion data set, generating a daily product (Tschudi et al., 2010).

In Chapter 2, we demonstrated the differences in age values obtained with different sea ice age definitions. We also showed that the correlation between ice thickness and ice age is high when averaged over large spatial and time scales, but the scatter is significant at smaller scales. In this section, to further investigate the influence of the sea ice age definition on the age values, we take advantage of the Lagrangian model to compute in the same formalism different sea ice ages and compare the results. The Lagrangian model enables us to compute sea ice age estimates in two distinct fashions: by having recourse to Lagrangian particles or by finite element fields transported by the model.

4.5.2 Sea ice age definitions used in this study

First, as the mesh moves with the ice, Lagrangian particles are naturally following the ice drift. Compared to Chapter 2, we are thus able to take sea ice dynamics into account and to transport the whole sea ice column since we assume that the ice velocity does not depend upon the vertical coordinate. Therefore, sea ice particles determining the vertical sea ice age profile will stay vertically-aligned throughout the numerical simulation. We also consider that the *vertical sea ice age* is not affected by ice deformation. There are two reasons: first, the vertical distribution measures the thermodynamical controls on the ice age; second, it is complex to reconstruct a consistent age distribution from several mixed profiles. In other words, this sea ice age is considered as anchored in a parcel of undeformed ice. This is an important simplification since sea ice dynamics affect sea ice thickness, which in turn influences sea ice thermodynamics. To evaluate the previous hypothesis, we illustrate in Fig. 4.22 the difference between the ice thicknesses computed by the Lagrangian model (with deformation) and by the Lagrangian particles (no deformation), evaluated at the location of the particles. The majority of the particles show a very small (< 0.35 m in absolute value) difference in ice thickness. In these locations, the ice deformation plays a limited role, as most of the first-year ice. The ice thickness difference is large (> 1.05 m in absolute value) in a few points, meaning that the ice deformation has piled the ice (positive difference, typically along coastlines or in the Canadian Archipelago) or that the ice undergoes strong diver-

gence (negative difference). Note that the numerical diffusion associated with the Lagrangian model can explain negative differences also.

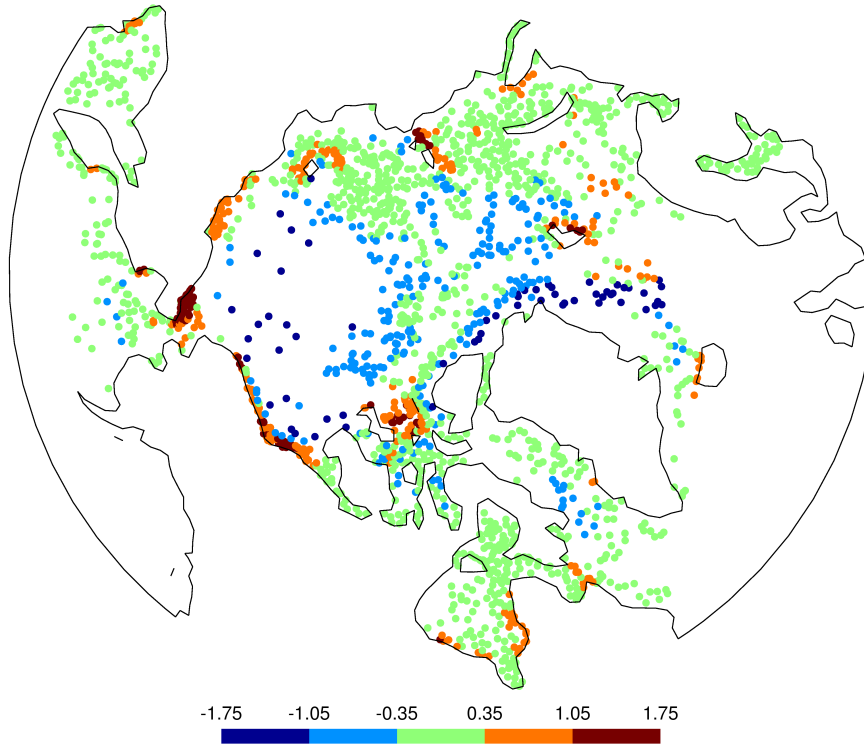


Figure 4.22: Difference between the ice thicknesses computed by the Lagrangian model (with deformation) and by the Lagrangian particles (no deformation). The scale ranges between -1.75 m and 1.75 m.

The second generic age type that we will consider measures the length of time that an ice parcel has spent in the Arctic. Different approaches are possible to compute this kind of age. The first one relies on horizontal Lagrangian particles in the fashion of the ice age derived from satellite observations. If the ice particle lies outside the ice edge, its age is set to zero. In practice, Lagrangian ice “columns” are released in newly frozen ice to maintain a sufficient number of particles from which both the *vertical* and the *satellite* sea ice ages are computed.

The second approach is similar to the *satellite* sea ice age and aimed to mimic the satellite retrievals but has been adapted for finite element fields. Exploiting the

Lagrangian model, we consider each finite element as an independent Lagrangian particle. The advantage of the method is to follow even more closely the satellite algorithm with the presence of a grid. The disadvantage compared to Lagrangian particles is that a rule must be defined to compute the age on the adapted mesh cavities. The sea ice age is dependent on the way different ages are mixed and the Lagrangian model is particularly helpful to investigate this impact. We will consider two different sea ice ages: the *maximum area sea ice age* assigns the age of the largest intersection with the source elements, while the *mean area sea ice age* computes an area weighted age. We could also decide to assign the maximum age from the source elements. Due to the numerical diffusion associated with the mesh adaptation, the Arctic would rapidly be characterized by a uniform (very high) age. This alternative has thus been rejected. This type of age is not conservative and not affected by dynamics. The more refined the grid, the closer we get to Lagrangian particles.

Finally, we followed the approach used by Vancoppenolle et al. (2009) in LIM3 that we will refer to as the *modeled sea ice age*. This ice age $a_{mod}(\mathbf{x}, t)$ corresponds to an usual horizontal tracer and is affected by ageing, thermodynamics and dynamics. Contrary to the first ice age, new ice formed at the ice base or surface is assumed to have the same age as the rest of the column. The age is thus independent of the vertical coordinate and newly frozen ice in open water areas is given an age zero. The age content is defined as the product of the age and the ice concentration $A(\mathbf{x}, t)$. Note that in another study by Hunke and Bitz (2009), the age is weighed by the ice volume. The idea here is to follow as closely as possible satellite retrievals of sea ice age that do not consider subsurface processes. The equation of evolution of the age content thus reads:

$$\frac{D(A(\mathbf{x}, t) a_{mod}(\mathbf{x}, t))}{Dt} + A(\mathbf{x}, t) a_{mod}(\mathbf{x}, t) (\nabla \cdot \mathbf{u}(\mathbf{x}, t)) = A(\mathbf{x}, t) + q_A^-(\mathbf{x}, t) a_{mod}(\mathbf{x}, t), \quad (4.26)$$

where the first term on the right hand side corresponds to the ageing process and the second term to the age content loss associated with lateral melt. Lateral accretion of ice in open water areas has a zero contribution. Combining equation (4.26) to equation (3.9), we get the following equation:

$$\frac{Da_{mod}(\mathbf{x}, t)}{Dt} = 1 - \frac{q_A^+(\mathbf{x}, t)}{A(\mathbf{x}, t)} a_{mod}(\mathbf{x}, t). \quad (4.27)$$

Compared to equation (2.11) derived for the vertically-variable sea ice age, we have an additional term in the right hand side accounting for a decrease in modeled ice age with lateral growth: the mean age of the ice parcel is decreased by the incorporation of age zero.

4.5.3 Results and analyses

Fig. 4.23 compares the different sea ice age types at the end of January 2008. The *satellite sea ice age*, based on Lagrangian particles, gives us the most reliable comparison with what is inferred from satellite data. First, ice parcels that are as old as the simulation duration are found in the Canadian Arctic Archipelago (CAA) and Nares Strait because of the lack of ice motion. This feature is also present in satellite retrievals where ice motion cannot be resolved, these areas are thus generally excluded from the analyses (e.g., Maslanik et al., 2011). Due to the large summer 2007 ice loss, much of the Arctic sea ice is first-year ice. Since open water is rarely found at the end of the summer along the CAA, very few ice particles are added during the numerical simulation in this region. This explains the two large areas depleted of particles. According to the algorithm used for satellite retrievals, the ice parcels are aged as long as they lie inside the ice concentration threshold. Since the depleted regions correspond to ice export from the CAA, we consider that they correspond to old ice. One of them is entrapped in the Beaufort Gyre along the Alaskan coast. The other important old ice pattern is found in the Central and Eastern Arctic, where the ice will eventually be exported through Fram Strait and melt out in the Greenland Sea.

The *vertical sea ice age* is illustrated on panel (b) and gives the time needed to growth thermodynamically the ice column. As can be seen, the age values rarely exceed 3 years old. One can also see the similarity between first-year ice patterns as computed by the satellite approach and the vertical ice age profile. In fact, until sea ice undergoes surface melt (where the oldest vertical ice particle lies), the values of the vertical and satellite approaches are equal. After the first summer, age values differ and the vertical age increases as the ice thickens because more annual layers develop.

Panel (c) shows the *maximum area sea ice age*. The sea ice age distribution matches surprisingly well the satellite approach. Clearly, some details of the *satellite* sea ice age patterns are diffused, especially when the mesh is coarsened (e.g., in the Greenland Sea), but the essential of the sea ice age distribution is well represented. In this regard, the choice of the maximum area age as a rule to assign an ice age to target elements seems to somehow counteract the diffusion associated with mesh adaptation. In particular, the transition between different ice age categories is sharp and agrees well with the ice particles. We now clearly see that the areas depleted of ice particles correspond to old ice. Contrary to the *maximum area sea ice age*, the *mean area sea ice age* (panel (d)) presents additional diffusion compared to the maximum area age. Lateral growth in the leads decreases the *modeled sea ice age* by incorporating new ice of age zero, while the other age approaches consider the ice parcel with the same status as long as it does not fall below the ice concentration threshold. However, the lateral growth contribution (term q_A^+) in the Arctic

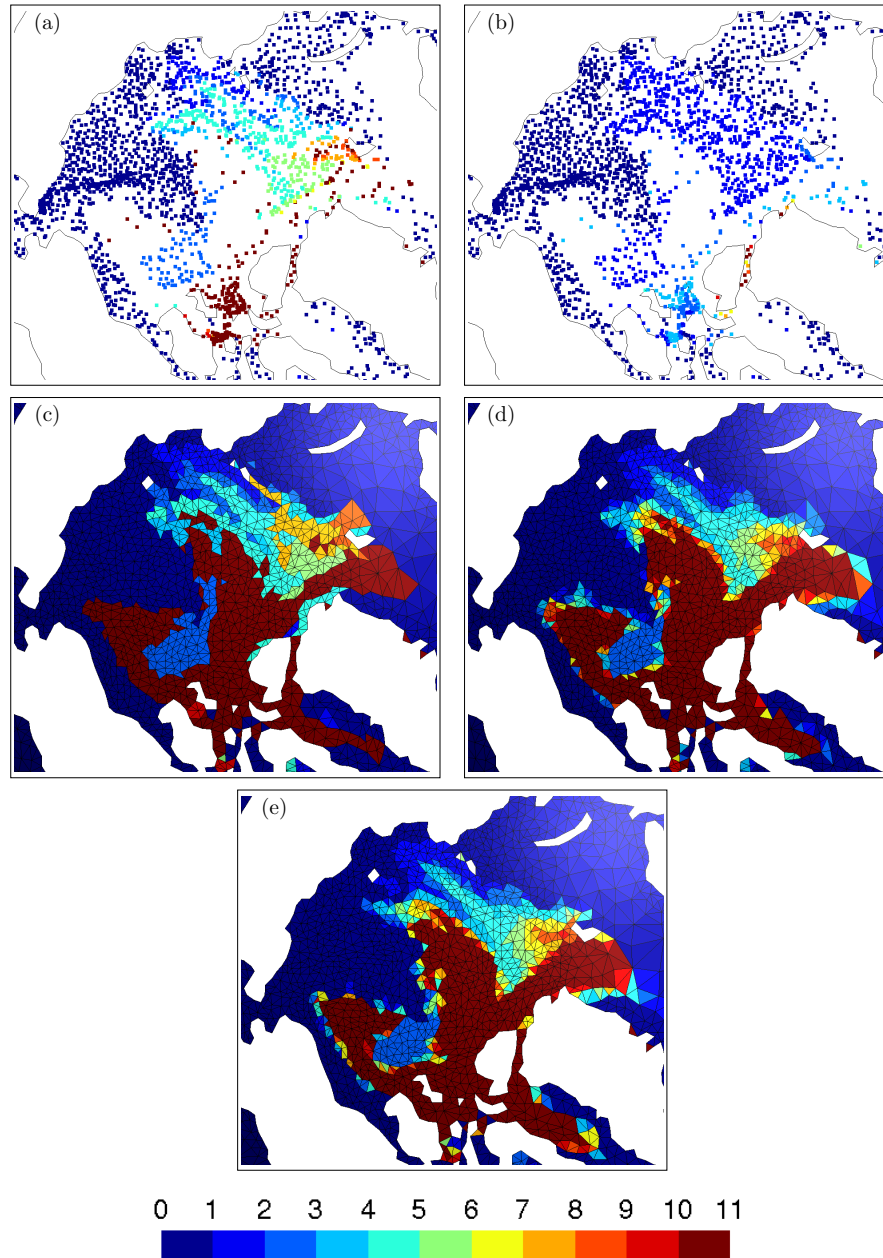


Figure 4.23: Simulated ice age at the end of January 2008 (0-10+ years). The last ice age class regroups ice age larger than 10 years. (a) Satellite ice age: Lagrangian particles are transported by the model. (b) Vertical ice age displaying the oldest particle on the ice column. (c) Maximum area ice age: the age of the finite element corresponds to the age of the largest intersection with the source elements. (d) Mean area ice age: the age is an area weighted age. (e) Modeled ice age: the age is a conservative tracer influenced by ice dynamics.

Basin is small because the ice concentration is high in most of the domain. The modeled age fields are thus very similar to the previous ice ages.

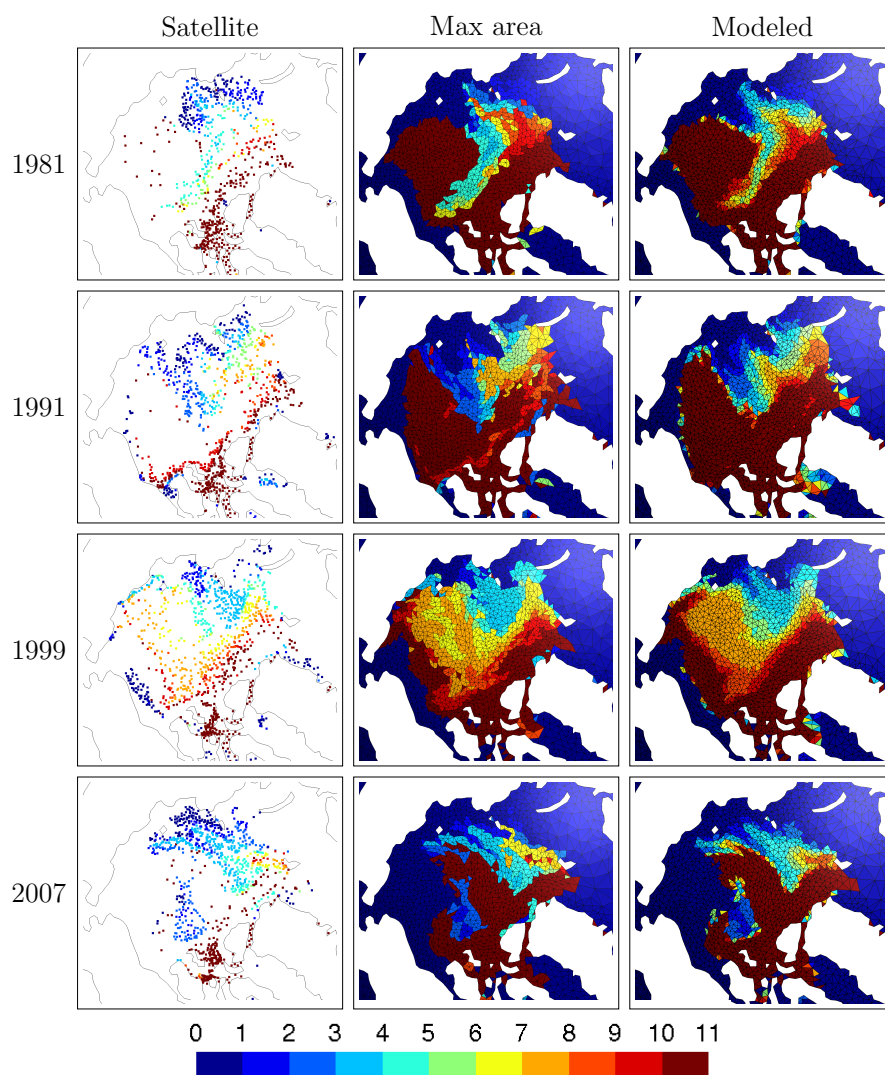


Figure 4.24: Changes in the simulated ice age in the Arctic Ocean (0-10+ years). In September, the first age class essentially corresponds to open water areas. The last ice age class regroups ice age larger than 10 years. Satellite (simulated by the model), maximum area and modeled ice ages are represented for September 1981, 1991, 1999 and 2007.

Sea ice age distributions and their temporal evolution deliver us an integrated view of the ice circulation, growth and melt. An overview of the changes in ice age in the Arctic Ocean is provided in Fig. 4.24. Some key observations can be made that are consistent with satellite observations: the Beaufort Gyre entraps the ice during several years, producing a large area of very old ice in the Western Arctic; the Transpolar Drift pushes large amounts of this old ice through Fram Strait where sea ice eventually melts out; the first-year ice survival, that is the amount of ice that passes the first melt season, is highly variable as outlined by Belchansky et al. (2005).

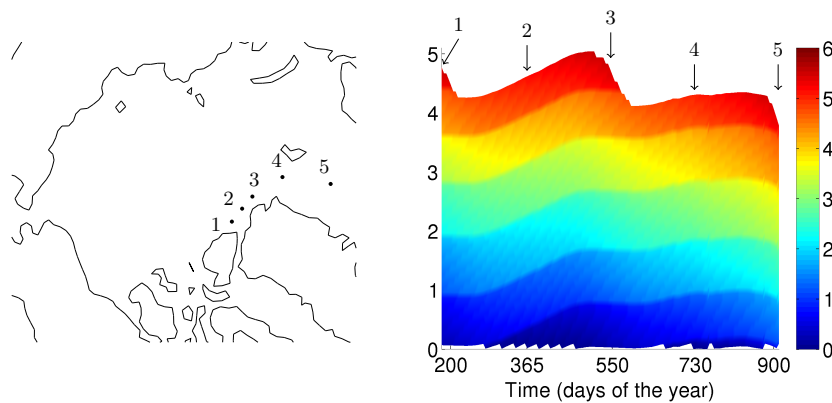


Figure 4.25: Left: the trajectory of an ice column initially situated north of Ellesmere Island is depicted by dots every 6 months, starting the 1st of July 2005 (location 1), 1st January 2006 (location 2), 1st July 2006 (location 3), 1st January 2007 (location 4) before it disappears in the Greenland Sea (1st July 2007, location 5). Right: time series of the vertical sea ice age (0-6 years) of the ice column.

Finally, we illustrate the effects of changing atmospheric and oceanic conditions on the vertical age profile. To this end, we follow the trajectory of an ice column initially situated north of Ellesmere Island (Fig. 4.25). On the 1st January 2005, the ice is about 5 m thick and more than 5 years old. During the first year, the time series of the ice age yields a classical thermodynamic growth and the ice thickness exceeds 5 m. During the 2006 summer, about 1 m ice has melted at the ice surface. The following winter, as the ice column passes Fram Strait, very few bottom ice is produced because the temperature of the mixed layer is higher than in the Central Arctic. By the first days of July 2007, the ice parcel reaches Greenland Sea where it eventually melts out in a couple of days.

4.6 Discussion and conclusions

In this chapter, we have developed and validated a Lagrangian, adaptive, finite element sea ice model. Mesh adaptation is performed on small ensembles of elements called *cavities*. Combined to a discontinuous representation of the fields, the field projection is limited to a small ensemble of cavities and consists in a local Galerkin projection, whereas complete remeshing requires systematic mesh-to-mesh projections. The locality of the method makes its efficiency and its originality. To ensure the conservativity of the projection, the discontinuous fields are integrated exactly on each intersection between the source and the target cavities. Moreover, the method is generic as it can handle fields of any degree. Based on a geometrical construction and limited to the element cavities, the method proves to be particularly efficient for transporting a large number of tracers: each additional tracer only requires small matrix-vector products. As modern sea ice models include a large number of tracers to transport (e.g., snow and ice volumes, ice area and enthalpy for each ice thickness category), this is promising. In order to respect strictly the bounds of the fields, a classical limiter is implemented for linear shape functions. The theoretical orders of convergence of the projection have been obtained and the impact of repeated projections has been investigated on analytical fields. Finally, the Lagrangian algorithm has been validated on a frontogenesis problem (Doswell, 1984). First order and second order of convergence of the numerical solution are obtained for constant and linear approximations, respectively. To better evaluate the Lagrangian algorithm, one should evaluate its numerical diffusion. A couple of methods exist to measure the numerical diffusion of an advection scheme (e.g., Morales Maqueda and Holloway, 2006; Burchard and Rennau, 2008) and could serve as a basis for such investigations.

The Lagrangian model has been run on the Arctic with a configuration very close to the one used in chapter 3. There are three main differences: following Comblen et al. (2010b), the ice momentum equations are solved on spherical geometry and a sub-cycling procedure has been dedicated to their resolution. In addition, the contribution of lateral melt to the change in ice volume has been corrected to better account for the ice-albedo feedback. Indeed, the increase in open water area is thought to be largely responsible for the recent ice cover decline in the Arctic as it is associated to an increased absorption of solar radiation (Perovich et al., 2007, 2008). In consequence, the representation of the ice extent in summer has been considerably improved in the model and the September 2007 simulated sea ice cover successfully marks a severe decline, though not as strong as it has been observed. However, the resulting ice thicknesses and the ice thickness gradient between northern Siberia and the Canadian Arctic Archipelago are underestimated. This change in the lateral melt parameterization is questionable. Clearly, the ice-albedo feedback was underestimated and the simulated ice extent in summer has been considerably improved. But the decrease in ice volume points out the real

caveat: the missing subgrid-scale ice thickness distribution that is known to better represent the ice-albedo feedback. In winter, the same observation as in chapter 3 is done: the model misses the oceanic heat advection that should substantially reduce the ice extent. Model improvements also include the development of a more sophisticated slab ocean model, or the use of observed sea surface temperatures to calculate the oceanic heat flux into the ice.

We have taken advantage of the Lagrangian model to study the impact of different sea ice age definitions on the age values. It must be stressed that the aim is not to proceed to a quantitative comparison, but merely to a qualitative analysis. The satellite sea ice age gives us the best estimate to what is inferred from satellite data. The Lagrangian particles are not subject to mixing or diffusion, and basically follow the satellite algorithm: particles are added where new ice is formed and aged until they cross the ice edge. However, the sea ice ages based on finite element fields allow us to perform some investigations on the impact of the rule applied to assign an age value to the target elements. When the maximum value of the source elements is considered, the age distribution tends to a uniform value. This option has thus been disregarded. Computing the mean area-weighted age smoothes out the details of the age distribution. The maximum area sea ice age gives us satisfactory results that are in close agreement with the ice age particles. The modeled sea ice age is only weakly affected by the incorporation of age zero as lateral growth occurs. As a result, the age values and distribution are very similar to the previous approaches. In other words, any two-level sea ice model is able to compute sea ice age estimates that are in good agreement with the satellite approach. However, it only highlights the over-simplicity of the two-level model: with a subgrid-scale ice thickness distribution, the ice age would be averaged over all thickness categories and would differ significantly from ice age values derived from satellite data (Van-coppenolle et al., 2009).

Sea ice age distributions and their temporal evolution deliver us an integrated view of the ice circulation, growth and melt. Several characteristics consistent with satellite observations can be deduced from our numerical simulations: the Beaufort Gyre entraps the ice during several years, producing a large area of very old ice in the Western Arctic; the Transpolar Drift pushes large amounts of this old ice through Fram Strait where sea ice eventually melts out; the first-year ice survival, that is the amount of ice that passes the first melt season, is highly variable as outlined by Belchansky et al. (2005).

Clearly, a more in-depth analysis of the different ice age definitions and their impact on the results should be performed. A detailed comparison against satellite retrievals is conditioned to a thorough validation of ice dynamics. In this respect, the addition of oceanic currents (e.g., interpolated from high resolution NEMO-

LIM runs) should increase the match between the observed and the simulated ice drifts, in particular where the ice drift is poorly represented currently: Bering Strait, Nares Strait and Parry Channel (in the Canadian Arctic Archipelago).

The mesh adaptation procedure described in this chapter can serve for Lagrangian models, but also for Eulerian or Arbitrary Lagrangian-Eulerian approaches. This is particularly helpful for numerical simulations based on *a posteriori* error estimators (*h*-adaptation strategy). In these simulations, the mesh is adapted as a function of a particular objective: e.g., minimizing the global error for a given number of elements or minimizing the number of elements while keeping the local error below a given threshold (e.g., Bernard et al., 2007). These error estimators have been implemented in the model. Another ability of the model that has been illustrated is the dynamical mesh refinement along the ice edge.

Several aspects of the model plead for improvements. We have shown that the P_0 interpolant is subject to much more diffusion than higher degree shape functions during the field projection. This statement must be moderated since the benchmark has been designed to involve artificially all the elements of the domain several times in a row. However, P_1^{DG} tracers clearly represent a worth alternative, offering additionally a simple limiting procedure. To solve the sea ice formulation problem (the slope of the ice volume on a given finite element does not affect the ice momentum equations), one can have recourse to penalty terms to enforce the continuity of the solution.

As shown with the sea ice ages comparison, the Lagrangian model constitutes a nice “numerical laboratory”. Any quantity of interest is naturally transported by the model. In particular, the Lagrangian approach delivers a means to follow the evolution of the physical properties of material points in their movement, as opposed to the classical Eulerian point of view. The vertical sea ice age profile has been illustrated for a sea ice particle drifting initially north of Ellesmere Island towards Greenland Sea, showing how local atmospheric and oceanic conditions affect sea ice melt and growth. The same interest has motivated the development of satellite products reconstructing the ice motion and tracking the evolution of ice characteristics (Tschudi et al., 2010). In addition, the inclusion of material properties and the access to the history of deformation are straightforward as we follow material points. Undoubtedly, the Lagrangian model constitutes a natural framework to develop new sea ice rheologies: adding or changing physical parameterizations can be done straightforwardly with a direct insight in the model behaviour. It not only gives a useful point of comparison with the Eulerian model, but it allows to test and understand the Eulerian model. Finally, the comparison and analysis between the trajectory and velocity of ice particles and drifting buoys could serve as the ba-

sis to better understand the modeled ice drift and improve ice dynamics via data assimilation.

CHAPTER 5

CONCLUSION AND PERSPECTIVES

"I know why I am here and not in a city full of people, companies and neon lights. I don't always like myself the way I am when I am with other people. There is something unnatural about the way I have to control, evaluate and observe my reactions. There are certain things that are expected of me - I have to be a son, a friend, a lover, an enemy, a brother, a citizen, a soldier... This means nothing here. I am no God out here. I cannot make the wind blow or the snow fall. Sometimes I cannot even get my sled dogs to obey me. But I am second to God. I am a human being, alive due to my efforts."

—Odd Ivar Ruud, 1978.

During this doctoral research, we have developed a finite element sea ice model based on unstructured grids with Eulerian and Lagrangian descriptions. The model includes a viscous-plastic rheology along with a complete parameterization of the atmospheric fluxes and is driven by daily NCEP/NCAR reanalysis data. It explores the full potential of unstructured grids and features mesh adaptivity. This thesis falls within the framework of the SLIM¹ project that aims at developing a coupled finite element sea ice-ocean model. This research work is thus at the junction between sea ice modeling and applied mechanics, and is therefore original and innovative in many regards:

¹Second-generation Louvain-la-Neuve Ice-ocean Model, <http://www.climate.be/SLIM>.

- There are only a few research groups worldwide developing Second-Generation ocean models. If we further focus on those also developing a sea ice component, the list gets even shorter. We can cite the Alfred Wegener Institute (AWI) with the Finite Element Ocean Model (FEOM, Timmermann et al. (2009)) that is also used at the University of Alberta (Terwisscha van Scheltinga et al., 2010), and the model developed by Yakovlev (2003, 2009). Recently, the attractiveness of unstructured grids for sea ice modeling has led to the appearance of a couple of new models: the ICOSahedral Non-hydrostatic general circulation model (ICON) at the Max Planck Institute of Meteorology Hamburg (MPIfM) and the implementation of an unstructured grid version of the Community Ice Code (CICE) model in the unstructured grid Finite Volume Coastal Ocean Model (FVCOM), leading to the so-called UG-CICE model (Gao et al., 2011).
- This thesis presents the first unstructured grid large-scale sea ice model that resolves the narrow straits of the Canadian Arctic Archipelago. A couple of recent studies aim at reproducing this challenging application (Terwisscha van Scheltinga et al., 2010; Gao et al., 2011).
- Though several papers explore the use of Lagrangian or Arbitrary Lagrangian-Eulerian approaches for sea ice modeling, the present study is the first attempt at developing a purely Lagrangian model operational for climate studies.
- To the best of our knowledge, this work is also the first to have recourse to mesh adaptivity for realistic sea ice numerical simulations. In this regard, the ability of the model to track the ice edge has been illustrated.
- Finally, we are not aware of any study investigating systematically the impact of the sea ice age definition on the age estimates.

The key achievements related to this doctoral research can be summarized as follows. In chapter 2, we focus on the thermodynamic evolution of sea ice and propose a vertically-variable sea ice age definition that is interpreted as the time elapsed since the creation of an ice particle at the ice bottom. An analytical solution of an idealized sea ice age problem has been derived and has allowed for assessing two numerical methods. In the first one, the domain is discretized adaptively in space thanks to Lagrangian particles in order to capture the age profile and its discontinuities. The second one focuses on the mean age of the ice using as few degrees of freedom as possible and is based on an Arbitrary Lagrangian-Eulerian (ALE) spatial discretization and the finite element method. We demonstrated the differences in age values obtained with different sea ice age definitions. We also showed that the correlation between ice thickness and ice age is high when averaged over large spatial and time scales, but the scatter is significant at smaller scales.

The last part of this research work has been dedicated to the coupling of the finite element sea ice model with the mesh adaptivity library MAdLib (<http://www.madlib.be>). To this end, several interface routines have been developed to handle mesh conversion, data transfer and additional exchange of information between the two codes. The coupling is robust and even efficient as classical advection test problems take the same order of cpu time to run with the Lagrangian and Eulerian approaches. The mesh adaptation procedure is based on local operations acting on a small number of elements named cavities. Particular emphasis is devoted to the removal of sliver elements. As we consider discontinuous interpolation functions, the field update procedure is limited to the element cavities. The locality of the method makes its efficiency and its originality. As sea ice applications typically require a long time integration, the conservativity of the projection is mandatory. We therefore opted for a local Galerkin projection, and since the shape functions of the source mesh are piecewise polynomials over the elements of the target mesh, we integrate the mixed mass matrix on the intersection between source and target elements. The projection has been validated on several benchmarks. It yields the theoretical orders of convergence, is conservative up to machine precision and is particularly efficient for transporting a large number of tracers.

The Lagrangian model has been successfully run on the Arctic Basin. Following Comblen et al. (2010b), the sea ice momentum equations are solved on spherical geometry. This configuration has been used to compare different sea ice age definitions based on the satellite approach, on the vertical sea ice age profile developed in chapter 2 and on modeled sea ice age. Though the study needs further analyses, some key conclusions can be drawn. First, the model is able to reproduce the satellite algorithm and provides an ideal framework to test the sensitivity of ice age estimates to some parameters. We have concluded that considering the maximum area sea ice age gives us the results that are in closest agreement with Lagrangian particles. Second, the satellite approach gives us satisfactory results that are consistent with satellite observations: the presence of very old ice entrapped in the Beaufort Gyre that is eventually exported through Fram Strait and the high variability of first-year ice survival. Several aspects of the study must be further investigated however. In our results, some areas of the Arctic are characterized by very old ice. It should be determined whether this corresponds to a sea ice dynamics discrepancy in the model (e.g., a too strong Beaufort Gyre) or by the influence of old ice export from the Canadian Arctic Archipelago. Some comparison between the model results and the observations must be undertaken and the interannual variability of the age classes must be investigated. Finally, the vertical sea ice age profile integrates the ice growth history and, as the ice column is transported with the ice drift, it illustrates how local atmospheric and oceanic conditions affect the ice.

The Lagrangian model undoubtedly constitutes a valuable development tool. The model naturally transports any quantity of interest and gives a direct access to

material properties and features that makes easier the development of new rheologies. The comparison of different ice age definitions has been considerably simplified in the Lagrangian model. This simplified framework enables the investigation and the understanding of the mechanical behaviour of sea ice as well as the Eulerian model. More than a complementary model, it reveals an efficient numerical laboratory. Such models are likely to promote interest in operational oceanography that is bound to become an increasingly asked discipline.

The model currently lacks an interactive coupling with an ocean model. Clearly, the presence of the oceanic heat advection would considerably reduce the ice extent in winter. The addition of oceanic currents (e.g., interpolated from high resolution NEMO-LIM runs) should correct the simulated ice circulation in areas where these currents play an important role: Bering Strait, Nares Strait and Parry Channel (in the Canadian Arctic Archipelago). The Lagrangian model is not yet parallelized. The major difficulty that one would face is the partition of the mesh between the different processors as the Lagrangian model modifies continuously the element distribution in the domain. Another worthwhile improvement would be to solve the sea ice equations in a regular two-dimensional formulation. In particular, all mesh adaptation operations and the mesh displacement would be simplified in two dimensions. The issue of solving the shallow water equations by means of stereographic coordinates is currently being investigated in our group and could serve as a basis for the sea ice model. Finally, linear discontinuous shape functions should be devised for the ice concentration and volume.

Unstructured meshes have the natural ability to locally increase the mesh resolution and capture the complex oceanic areas formed by coasts and islands. In chapter 3, this ability has been applied to the resolution of the narrow straits of the Canadian Arctic Archipelago (CAA). This feasibility study shows how important the CAA is on the mass balance of the Arctic sea ice and pleads for further investigations with a coupled sea ice-ocean model for a better understanding of the complex processes and interactions taking place in this region. Of particular concern is the impact of the ice export through Nares Strait and more generally from the CAA on the convective overturning downstream in the Labrador Sea. The development of a coupled sea ice-ocean model is part of the future project SLIM.

5.1 Future developments

The future developments of the finite element sea ice model have been settled in the framework of the new Actions de Recherche Concertées (ARC) project "Taking up the challenges of multi-scale marine modelling" that started in October 2010. This research project focuses on multi-scale modeling with particular emphasis on the Canadian Arctic Archipelago. Of particular concern are (1) the interannual vari-

ability of the sea ice cover and freshwater transports in the CAA, and (2) the impact of recent and future climate changes on the sea ice features in this area, including the possible opening of the Northwest Passage.

During this thesis, I have changed twice of model code. It is now highly time to proceed to a third change. Model codes are constantly in evolution, especially for new area of research as the Second-Generation Louvain-la-Neuve ice-ocean model is. It is thus natural to restart from scratch when the code has evolved. The last version of the code was intended to be very generic and flexible to test different types of interpolating functions, various time stepping procedures or several physical parameterizations. After this exploratory work, a new version has been launched two years ago with the aim to focus on the Discontinuous Galerkin (DG) method. The developments that we have undertaken during this thesis are compatible with a Discontinuous Galerkin approach and need to be integrated in the new code.

Several major improvements need to be accomplished in the sea ice model. First, the representation of the physics must be improved. Including subgrid-scale ice thickness distribution (ITD, see Thorndike et al. (1975)) will allow to describe the relative surface coverage of sea ice of different thicknesses. Changes in the ITD are due to thermodynamic (growth and melt) and dynamic (opening and deformation) effects. Models including an interactive ITD improve the representation of ice growth and of the summer ice-albedo feedback, which in turn significantly improves the simulation of the large-scale sea ice characteristics (Holland et al., 2006), speaking for the use of such distribution functions in sea ice models. Furthermore, Vancoppenolle et al. (2009) have shown that, given the importance of the ice salinity on the simulated sea ice mass balance, the model clearly gains in accuracy from a better representation of salinity. The essential of the halo-thermodynamic component included in the LIM3 model has hence to be incorporated in the finite-element model. In this regard, I think that the approach of Kärnä et al. (2011 (subm.)) who coupled SLIM with the turbulence closure model package GOTM, should be investigated to incorporate the one-dimensional ice thermodynamics (LIM1D) to the finite element model.

A new modeling framework for sea ice mechanics has been described by Girard et al. (2011). The study introduces an elasto-brittle constitutive law that has been shown to satisfy the scaling properties characterizing sea ice deformation (Marsan et al., 2004; Rampal et al., 2008; Weiss, 2008). The ice mechanical behaviour is based on continuum mechanics and characterized by progressive damage and long-range elastic interactions. As a result, the model is able to represent very heterogeneous deformation fields with linear kinematic features induced by avalanche of damage. Since the damage and the deformation are material properties, the Lagrangian model constitutes a natural framework to develop this new

approach as we follow material points. A preliminary illustration of the shear rate and the ice velocity obtained from the coupling of the elasto-brittle rheology with the Lagrangian model on the Arctic is shown in Fig. 5.1. The velocity vectors are superposed on the shear rate. Linear kinematic features crossing the Arctic are clearly reproduced and illustrate the localization of the deformation.

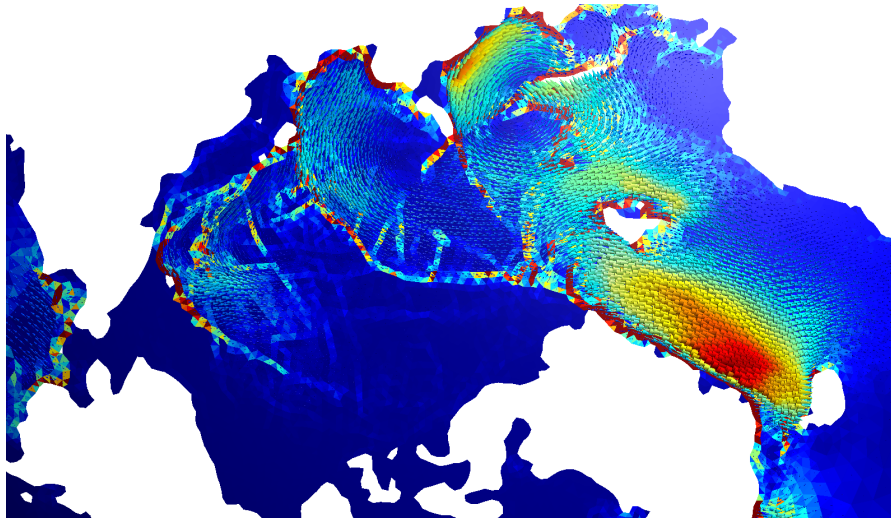


Figure 5.1: Arctic numerical simulation with the Lagrangian sea ice model and the elasto-brittle rheology. The shear rate is shown with color scale varying from 0 (blue) to 0.05 day^{-1} (red). The velocity vectors are superposed on the figure and range from 0 to 30 cm s^{-1} . Figure courtesy of Sylvain Bouillon.

I hope that this thesis has proven that the Lagrangian approach, combined to mesh adaptivity, constitutes a valuable tool for model development and sea ice applications. As these features are not part of the new research project, my hope is that DG will be soon or later coupled with MAdLib. In this case, the field projection -a missing feature of the mesh adaptation library- developed in this work should be preferentially implemented in MAdLib so that the latter library handles most of the work, with only some interface routines left that have to be written in the DG code. The implementation of a new Lagrangian sea ice model would then pose no difficulty. The major drawback facing the maintenance and the use of mesh adaptivity and Lagrangian model is that it will probably never be developed in a coupled ice-ocean configuration. But “it is amazing the number of investigations that one can perform with uncoupled sea ice models” (C. Bitz, personal communication).

Appendices

APPENDIX A

SUPPLEMENTARY MATERIAL FOR CHAPTER 2

A.1 Coefficients of the system resulting from the spatial ALE discretization

The system resulting from the spatial ALE discretization is the following:

$$\frac{da_i}{dt}(t) = \sum_{j=0}^p A_{ij}(t) a_j(t) + \delta_{0i}, \quad i = 0, 1, \dots, p \quad (\text{A.1})$$

where $a_i(t)$ are the unknown degrees of freedom and p is the maximum degree of the shape functions. The coefficients of the matrix A can be summarized as follows:

$$A_{ij} = \begin{cases} \sqrt{(2i+1)(2j+1)}((-1)^{i+j} q_b^- + q_s^-) & i < j \\ \sqrt{(2i+1)(2j+1)}((-1)^{i+j+1} q_b^+ - q_s^+) & i > j \\ -(i+1)(q_b^+ + q_s^+) + i(q_b^- + q_s^-) & i = j \end{cases} \quad (\text{A.2})$$

for $0 \leq i, j \leq p$. For this study, we need the matrices for Legendre polynomials of degree up to three. For a constant polynomial, we have:

$$A = \begin{pmatrix} -q_b^+ - q_s^+ \end{pmatrix}.$$

For constant and linear polynomials, the matrix reads:

$$A = \begin{pmatrix} -q_b^+ - q_s^+ & \sqrt{3}(-q_b^- + q_s^-) \\ \sqrt{3}(q_b^+ - q_s^+) & -2(q_b^+ + q_s^+) + (q_b^- + q_s^-) \end{pmatrix}.$$

With quadratic elements, we have:

$$A = \begin{pmatrix} -q_b^+ - q_s^+ & \sqrt{3}(-q_b^- + q_s^-) & \sqrt{5}(q_b^- + q_s^-) \\ \sqrt{3}(q_b^+ - q_s^+) & -2(q_b^+ + q_s^+) + (q_b^- + q_s^-) & \sqrt{15}(-q_b^- + q_s^-) \\ \sqrt{5}(-q_b^+ - q_s^+) & \sqrt{15}(q_b^+ - q_s^+) & -3(q_b^+ + q_s^+) + 2(q_b^- + q_s^-) \end{pmatrix},$$

while for polynomials of degree up to three:

$$A = \begin{pmatrix} -q_b^+ - q_s^+ & \sqrt{3}(-q_b^- + q_s^-) & \sqrt{5}(q_b^- + q_s^-) & \sqrt{7}(-q_b^- + q_s^-) \\ \sqrt{3}(q_b^+ - q_s^+) & -2(q_b^+ + q_s^+) + (q_b^- + q_s^-) & \sqrt{15}(-q_b^- + q_s^-) & \sqrt{21}(q_b^- + q_s^-) \\ \sqrt{5}(-q_b^+ - q_s^+) & \sqrt{15}(q_b^+ - q_s^+) & -3(q_b^+ + q_s^+) + 2(q_b^- + q_s^-) & \sqrt{35}(-q_b^- + q_s^-) \\ \sqrt{7}(q_b^+ - q_s^+) & \sqrt{21}(-q_b^+ - q_s^+) & \sqrt{35}(q_b^+ - q_s^+) & -4(q_b^+ + q_s^+) + 3(q_b^- + q_s^-) \end{pmatrix}.$$

A.2 Approximation of the analytical solution with piecewise polynomials

In this appendix, we treat the question as to whether it is better to use several piecewise polynomials of low order or one polynomial of higher order. Therefore, we approximate the analytical solution in a leastsquare sense with piecewise polynomials of degree $p = 0$ to $p = 3$ and on an increasing number of elements (see Fig. A.1). When no discontinuity is present in the solution (case $N = 1$, where N corresponds to the number of annual growth layers), the expected spatial rate of convergence $p + 1$ is reached for constant and linear approximations (Fig. A.2a), while the least-square fit is exact to machine precision for higher degree since the solution is of degree 2. For larger values of N (panels b and c), due to the discontinuities in the solution, the order of the method is only $O(h)$ where h is the vertical grid size (i.e., the inverse of the number of elements). In conclusion, increasing the number of elements does not enhance significantly the approximation and a linear polynomial on one element seems to be a good candidate.

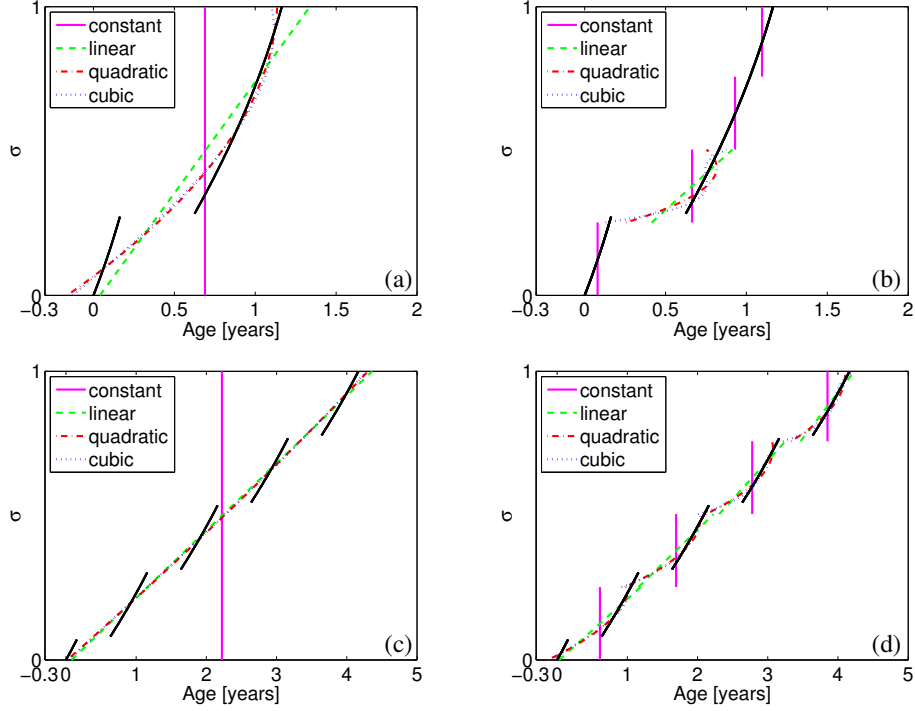


Figure A.1: Leastsquare approximation of the analytical solution (solid black line, cf. profiles in Fig. 2.6) with piecewise polynomials (the degree is mentioned in the legend). Panels (a) and (b): case $N = 2$ with 1 and 4 elements, respectively. (c-d) Same for the case $N = 5$.

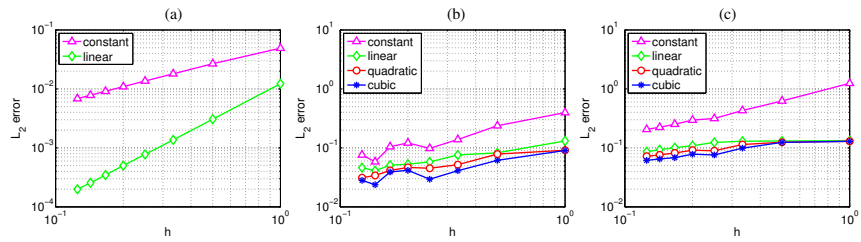


Figure A.2: Convergence analysis of the leastsquare approximations (L_2 norm in function of the vertical grid size h). Case $N = 1$ (a), $N = 2$ (b) and $N = 5$ (c). Note the different axes scales.

APPENDIX B

SUPPLEMENTARY MATERIAL FOR CHAPTER 3

B.1 A simple testcase for the viscous-plastic rheology

In this appendix, we illustrate the viscous-plastic rheology from Hibler (1979) in a one-dimensional testcase. Assume that a uniform sea ice layer of $L = 2500$ km lying on a ocean at rest is forced against the right domain boundary. To this end, we apply a constant and uniform wind stress on the ice and a linear oceanic drag is applied. No thermodynamical interaction is included in this problem and the sea ice concentration is kept fixed to a value of 1. The equations that we solve are the following:

$$\frac{\partial h}{\partial t} = -\frac{\partial(hu)}{\partial x} \quad (\text{B.1})$$

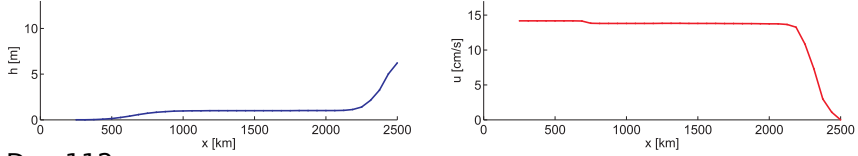
$$\rho h \frac{\partial u}{\partial t} = \tau_a - C_w u + \frac{\partial}{\partial x} \left(2\eta \frac{\partial u}{\partial x} + (\zeta - \eta) \frac{\partial u}{\partial x} - \frac{p^* h}{2} \right) \quad (\text{B.2})$$

where h is the ice thickness, u is the ice velocity, x the spatial coordinate and t the time. η and ζ are the shear and bulk viscosities, respectively. The following parameters are used in this numerical simulation:

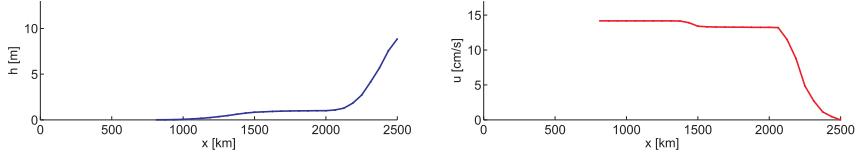
Ice strength parameter	p^*	20000 N m^{-2}
Wind stress	τ_a	0.1560 N m^{-2}
Water drag coefficient	C_w	1.1 kg s^{-1}

Some results are illustrated in Fig. B.1. We get a progressive accumulation of the

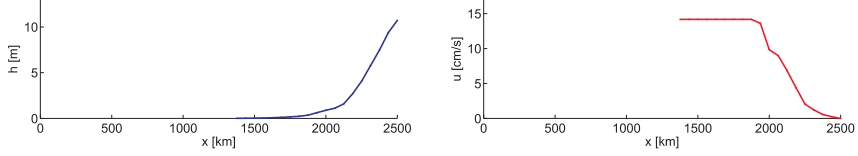
Day 56



Day 112



Day 168



Day 450

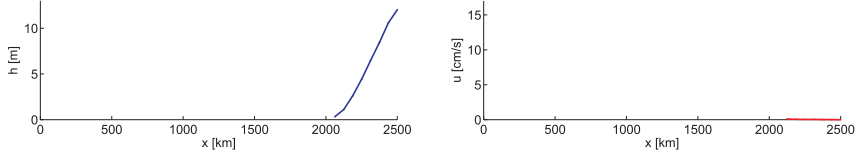


Figure B.1: Results of the one-dimensional testcase. Left: ice thicknesses (m); right: ice velocities. The ice is progressively accumulated against the right boundary.

ice along the right boundary that is coupled to a decrease in the ice velocity. Indeed, the slope of the ice accumulation induces a gradient in ice strength that counter-balance the wind stress. Asymptotically, one can show that the solution is a linear thickness profile.

Let us now focus on a material point situated at the middle of the basin ($x = L/2$) at $t = 0$. γ (see Eq. 3.6) can be interpreted as a measure of the strain rate determining whether the material undergoes plastic deformation or not. In Fig. B.2, we show the evolution of the stress state and the corresponding γ value of the material point. At the begin of the simulation (day 6), the stress state does not undergo a strong deformation and the value γ is weak. While we approach the right boundary (days 34 and 61), the deformation increases progressively (the stress state reaches the ellipse) but is still under the γ_0 value. The material point is thus still in a viscous

behaviour. From day 75 onwards, γ is greater than the threshold value γ_0 and the stress state enters plasticity (it belongs to the elliptical curve).

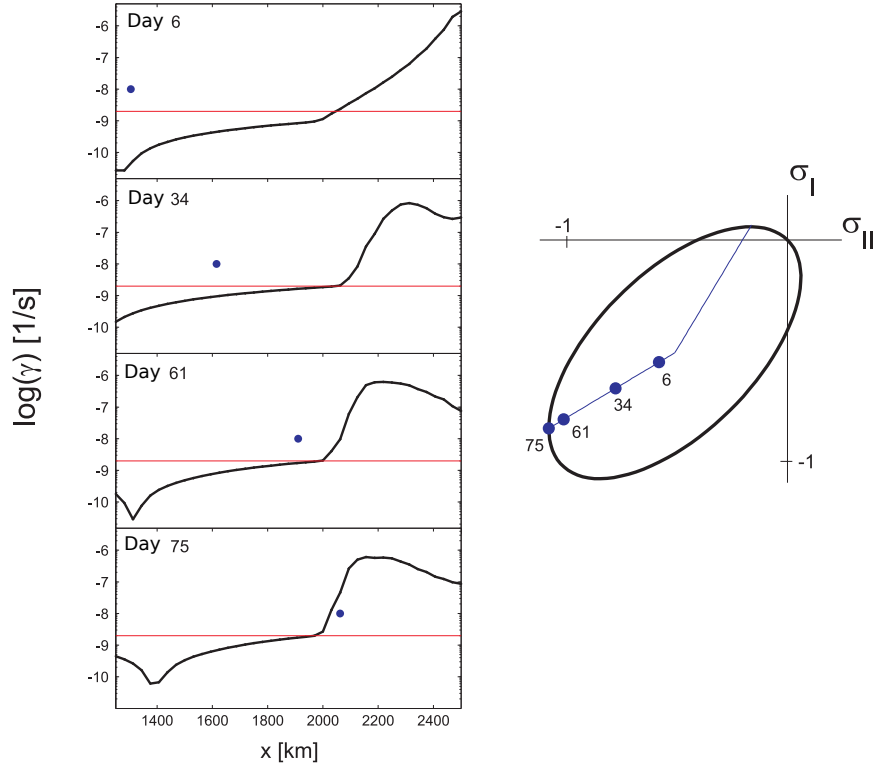


Figure B.2: Right: evolution of the stress state (in the principal axes) of a material point initially situated in $x = L/2$. In one dimension, the stress states evolve on the two blue lines. Left: evolution of the measure of the strain rate γ (black line). The red line corresponds to the value γ_0 above which the material undergoes plasticity. The display is reduced to the interval $[L/2, L]$. The location of the material point in the domain is illustrated (blue dot, its ordinate has no meaning).

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