A substructural analysis of embedded conditionals

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Abstract

The aim of this paper is to give a general solution to the paradoxes of the material conditional, including the paradoxes generated by embedded conditionals. The solution consists in a pragmatic reinterpretation of the formal languages of classical logic *LK* and relevant logic *LR* (which rejects the validity of weakening and splits the connectives into two) as presented in Paoli [2002],[2007]. In particular I argue that the material conditional in the classical logic *LK* captures the truth conditions of ″if...then″, but ignores certain pragmatic enrichments that are associated to it, while relevant logic *LR* can give a systematic diagnostic of the cases in which a conditional is pragmatically enriched and those cases in which it is not. This diagnostic shows the reason why the paradoxes seem unacceptable and why they are, nevertheless, truth-preserving. This reinterpretation will also cover the solution that Paoli [2005] gives to McGee's paradoxes of the Modus Ponens.

1 Introduction

How to formalize ″if...then″ has been a prominent discussion in philosophy of logic. In particular, it has been an important discussion between the classical and the relevant tradition: the so-called paradoxes of the material conditional convinced some philosophers that there is a semantic mismatch between the word ″if″ of natural language and the corresponding classical operator. The main problem of the classical account of conditionals is that it does not semantically capture the connection between antecedent and consequent that is generally associated to indicative conditionals: this generates a wide range of counterintuitive inferences which seem to show that there is a genuine mismatch between the classical $\supset$ and ″if...then″.

A defender of the classical analysis of the conditional has to explain the connection pragmatically, using, for instance, the Gricean theory ([Grice, 1989b]). While this seems to solve simple paradoxes, one can easily construct complex sentences (for instance, when the conditional is not asserted but it is em-

*Acknowledgements: I would like to thank Jos Martnez, for carefully reading different versions of this paper, making fruitful comments and corrections. Thanks also to Elia Zardini for making useful observations on a previous version of the paper, and to two anonymous reviewers of this journal for their very helpful remarks and suggestions. This work was supported by the project FFI2015-70707P of the Spanish Ministry de Economy and Competitiveness and the Eurepean Union on Localism and Globalism in Logic and Semantics, and the project MIS/PGY F.4540.20 funded by the FRS-FNRS, Towards a hyperintensional logical account of explanatory inference.

bedded under the scope of another operator) which escape the Gricean solution. The aim of this paper is to show that a pragmatic solution is still possible for the paradoxes of the material conditional by considering another pragmatic phenomenon, different from the Gricean conversational implicatures, which allows pragmatic contributions to what is said, following Recanati [2004], [Carston, 2002], and [Bach, 1994].

In order to do so, I will adopt a particular view on the meaning of the logical constants in relevant logic. I will reinterpret the formalization of relevant logic in Paoli [2002], [2007], and following Terrés Villalonga [2017] (and also [2018], [2020]) I will argue that relevant logic distinguishes those instances of the logical constants (and in particular the conditional) which capture different pragmatic enrichments of 'if...then'. Hence, the formal language of relevant logic can be used to diagnose the paradoxes, as it shows how the paradoxes are generated, and why they are truth-preserving.

The paper is divided into four parts. Section 2 will be devoted to the presentation of the classical logic *LK*, the paradoxes that emerge in it, the Gricean solution, and the problems that this pragmatic solution faces. In section 3 I will present the relevant logic *LR* and the ambiguity between extensional and intensional connectives, as it is presented in [Paoli, 2002],[2007]. In section 4 I will present a pragmatic interpretation of such language, following Terrés Villalonga [2017], which will be used to solve the paradoxes in section 5. In this last section, I will use the relevant ambiguity of the conditional and the other connectives to diagnose the paradoxes of the material conditional (including McGee's counterexample to Modus Ponens McGee [1985]) as truth preserving but unassertable, reinterpreting pragmatically Paoli's [2005] diagnostic.

2 The paradoxes in the classical framework

The calculus *LK* Given that the paradoxes are generated in relation to other connectives and the relation of consequence itself, it is required to give a general framework in which they emerge. I will present the classical treatment of the conditional in a Gentzen-style calculus *LK*, to highlight the role of weakening as a structural rule in the behavior of the classical conditional.

The proofs in a Gentzen calculus are formed by sequents, with the following form:

$$A, \Gamma \vdash \Delta, B$$

We can read the sequent as "Δ or B follows from A and Γ" or "A and Γ logically imply Δ or B", where Γ and Δ are sequences of formulas, separated by commas, read conjunctively on the left and disjunctively on the right. The rules for *LK* are the following:

Axiom

$$A \vdash A$$

Structural rules

$$\frac{\Gamma \vdash \Delta, A \qquad A, \Pi \vdash \Sigma}{\Gamma, \Pi \vdash \Delta, \Sigma}\ Cut$$

$$\frac{\Gamma \vdash \Delta}{A, \Gamma \vdash \Delta} WL \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, A} WR$$

$$\frac{A, A, \Gamma \vdash \Delta}{A, \Gamma \vdash \Delta} CL \qquad \frac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A} CR$$

$$\frac{\Gamma, A, B, \Delta \vdash \Pi}{\Gamma, B, A, \Delta \vdash \Pi} EL \qquad \frac{\Gamma \vdash \Delta, A, B, \Pi}{\Gamma \vdash \Delta, B, A, \Pi} ER$$

Operational rules

$$\frac{\Gamma \vdash \Delta, A}{\neg A, \Gamma \vdash \Delta} \neg L \qquad \frac{A, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg A} \neg R$$

$$\frac{A, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta} \wedge L_1 \qquad \frac{\Gamma \vdash \Delta, A \qquad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \wedge B} \wedge R$$

$$\frac{B, \Gamma \vdash \Delta}{A \wedge B, \Gamma \vdash \Delta} \wedge L_2$$

$$\frac{A, \Gamma \vdash \Delta \qquad B, \Gamma \vdash \Delta}{A \vee B, \Gamma \vdash \Delta} \vee L \qquad \frac{\Gamma \vdash \Delta, A}{\Gamma \vdash \Delta, A \vee B} \vee R_1$$

$$\frac{\Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \vee B} \vee R_2$$

$$\frac{\Gamma \vdash \Delta, A \qquad B, \Pi \vdash \Sigma}{A \supset B, \Gamma, \Pi \vdash \Delta, \Sigma} \supset L \qquad \frac{A, \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \supset B} \supset R$$

Notice that the calculus is not only composed of the right and left rules for the operators, the *operational rules*, but also of structural rules for the comma, called *structural rules*. These structural rules affect the behavior of the comma and $\vdash$ but also the behavior of the logical constants. In particular, weakening has a crucial role in the derivation of the different paradoxes, as we will see.

2.1 Simple Paradoxes and the Gricean analysis

The acceptance that $\supset$ is the correct analysis of the indicative conditional entails that the correct formalization of conditionals like 1 is 1c.:

1. If the Sun stops shining, darkness will cover the Earth in eight minutes' time.

1c. $A \supset B$

Moreover, it entails that the truth conditions of "if...then" are preserved in *LK*. However, there are certain derivations which are valid for $\supset$ which do not seem to correspond to our use or even to norms for the conditional. Consider the following derivations in natural language:

2. The theory of evolution is correct. Hence, if God created all the animal species, then the theory of evolution is correct.

3. Santa Claus does not exist. Hence, if Santa Claus exists he does not have a beard.

The conditionals in 2 and 3 seem to express something false, as they seem to express a connection between the antecedent and the consequent which we reject. However, the inferences are classically valid, as q can be arbitrary in the following derivations:

2c. $q \vdash p \supset q$

3c. $\neg p \vdash p \supset q$

If we want the connection between antecedent and consequent to be part of the meaning of "if...then", then $\supset$ cannot capture the meaning of the material conditional. In other words, if we agree that $\supset$ captures the meaning of "if... then", the connection between the antecedent and the consequent of a conditional is not part of its meaning, contrary to our intuitions.

One important and influential solution for this problem is the Gricean theory of implicatures [Grice, 1989b]: according to the Gricean theory, the connection between antecedent and consequent in a conditional is not part of the semantics or "what is said" but is pragmatically derived. The mechanism (for the analysis of the conditional) is sustained in the observation that our discourse is governed by certain rules and maxims, among which we find the maxim of Quantity, which requires the speaker to make any contribution as informative as it is required by the conversation, and the maxim of Relation, which requires the speaker to be relevant to the standars of the conversation.

The Gricean maxims allow us to codify pragmatic information carried by a given utterance. In the case of sentence 1 and given that $A \supset B$ is classically equivalent to $\neg A \vee B$ there are three reasons to assert it [Pérez-Otero, 2001, p.251]:

a The speaker knows that $\neg A$ - That is, the speaker knows that the Sun has not exploded,

b The speaker knows that B - that is, the speaker knows darkness will cover the Earth in exactly eight minutes from now,

c There is a connection between A and B such that A cannot be the case without B being the case - there is a connection between the Sun exploding and the fact that darkness will cover the Earth in eight minutes time.

There are, then, three possible grounds for asserting an utterance of the form "if A then B". However, the first two cases violate the Gricean maxim of quantity, while the third does not. Hence, an utterance of the form "if A then B" is usually understood as carrying an implicature similar to (c), which is the associated connection between antecedent and consequent that is ignored in *LK* formalization.

2.2 Embedded conditionals: difficulties for Grice

The Gricean theory can explain the two previous paradoxes, 2 and 3, but the mentioned schema is only applicable to asserted instances of the conditional, as the analysis is based on the grounds one has to assert a given utterance. In the majority of the paradoxes presented in the literature, however, the conditional is not asserted but embedded in larger sentences. In these cases, the Gricean analysis is not directly applicable. Consider, for instance, the negation of a conditional:

4. It is false that if the Sun explodes darkness will cover the Earth in 8 minutes from now: I am not sure whether the Sun exploded, but darkness would cover the Earth in 28 minutes if that was the case.

Given the classical analysis ($\neg(A \supset B) \equiv A \wedge \neg B$) the negated conditional is equivalent to:

5. The Sun exploded and darkness will not cover the Earth in 8 minutes' time from now.

However, this does not seem to be the intended meaning of the negation; that would make the first sentence in 4 contradictory with the second one. One seems to be able to negate a conditional without knowing the truth value of the antecedent (or the consequent). Or consider a conditional embedded in a disjunction:

6. Either if the Sun explodes life on Earth will last one thousand years more or if life on Earth lasts one thousand years more the Sun will explode. [1]

In this case, the disjunction seems to be a disjunction of the connection between certain facts related to the Sun and life on Earth. Although the connections seem false in both cases, the classical formalization is a tautology, as $\vdash (A \supset B) \vee (B \supset A)$ is valid in *LK*. Moreover, given that the conditionals are not asserted, it is not clear how the previous Gricean schema can solve this problem and restore the classical analysis of "if...then". In this case, the Gricean analysis is unavailable.

These kind of paradoxes are the reason why some philosophers abandon the material analysis of the indicative conditionals and endorse deviant meanings for "if...then".

3 The relevantist interpretation

I argue that the solution to the paradoxes is not limited to a change in the formalization of the conditional. The solution comes from a divergent notion of consequence that does not capture truth preservation but captures the reasons one has to assert a given utterance. In this section, I will present the calculus of *LR* as presented in Paoli [2005] and adopt the interpretation in [Terrés Villalonga, 2017] in order to distinguish different reasons to assert a conditional.

Relevant logic avoids the previous paradoxes of the classical conditional, as the relevant conditional $\rightarrow$ codifies the connection between antecedent and

[1] Modification from [MacColl, 1908].

consequent. The strategy is to reject weakening, and one can easily check how this rejection avoids the introduction of arbitrary information in a derivation, which in turn avoids the lack of connection between antecedent and consequent on a conditional, given the role of Weakening in the derivation of the paradoxes:

$$\dfrac{\dfrac{A \vdash A}{B, A \vdash A}\ WL}{A \vdash B \supset A}\ \supset R \qquad\qquad \dfrac{\dfrac{\dfrac{\dfrac{B \vdash B}{B \vdash B, A}\ \mathrm{WR}}{B, \neg B \vdash A}\ \neg\mathrm{L}}{\neg B, B \vdash A}\ \mathrm{EL}}{\neg B \vdash B \supset A}\ \supset\mathrm{R}$$

Moreover, rejecting weakening avoids related paradoxes that not only affect the conditional but disjunction, conjunction, and the notion of consequence itself. This shows that a solution such as introducing a stronger conditional in the classical framework (consider Lewis's [1960] strict conditional $A \strictif B =_{def} \Box(A \supset B)$) is not sufficient to avoid the lack of relevance in LK:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{B \vdash B}{B \vdash B \vee \neg B}\ \vee\mathrm{R}_1}{\vdash B \vee \neg B, \neg B}\ \neg\mathrm{R}}{\vdash B \vee \neg B, B \vee \neg B}\ \vee\mathrm{R}_2}{\vdash B \vee \neg B}\ \mathrm{CR}}{A \vdash B \vee \neg B}\ \mathrm{WR} \qquad\qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A}{A \wedge \neg A \vdash A}\ \wedge\mathrm{L}_1}{\neg A, A \wedge \neg A \vdash}\ \neg\mathrm{L}}{A \wedge \neg A, A \wedge \neg A \vdash}\ \wedge\mathrm{L}_2}{A \wedge \neg A \vdash}\ \mathrm{CL}}{A \wedge \neg A \vdash B}\ \mathrm{WL}$$

Among the different presentations of relevant logic (see [Anderson et al., 1992],[1977], [Read, 1988], [Mares, 2004]) we are interested in the effect that the lack of weakening has on logical connectives. Hence, we will consider what Paoli calls LR^{ND} [2], which is the result of rejecting weakening in LK[3]. This rejection has an effect on the conditional, and also on the rest of the connectives: each classical connective is split into two relevant connectives, an extensional and an intensional one, which can be proven to be equivalent in LK due to the effect of weakening. In the rest of this section, I will focus on the Gentzen-style presentation of relevant logic LR, which rejects weakening as a structural rule and duplicates the conditional[4], disjunction, and conjunction into two versions, an intensional and an extensional connective, following Paoli [2002, 2007].

[2] The result of rejecting weakening from LK is a non-distributivity logic in which $A \wedge (B \vee C) \nvdash (A \wedge B) \vee (A \wedge C)$. Most relevant logics include this derivation as valid (see [Anderson and Belnap, 1977], [Read, 1988], [Mares, 2004], however this invalidity is not problematic in our case. I am not defending, as relevant logicians do, that LR is the calculus for codifying a unique notion of logical consequence, but it is the calculus for codifying certain pragmatic phenomena which affect logical vocabulary, and the origin of such pragmatic phenomena is explained by the lack of weakening and all the effects that it has on the logical vocabulary. This view on LR is not in tension with the validity of distribution for preserving truth, as I endorse that this is codified in LK.

[3] It is important to note that, although I am using Paoli's presentation of LR, this is not the logic that Paoli considers as the correct logic. Paoli defends linear logic, LL, a logic without weakening and contraction, which differs from LR in that the extensional and intensional connectives are completely independent whereas in LR the intensional imply the extensional ones. This is natural given our interpretation of LR as the enrichment of a connective entails its non-enriched content, but not the other way around.

[4] Paoli introduced in [2007] an extensional version of the conditional, $\rightsquigarrow$, which will be a key connective in the diagnostic of the paradoxes.

The calculus *LR*

Axiom

$$A \vdash A$$

Structural rules

$$\frac{\Gamma \vdash \Delta, A \qquad A, \Pi \vdash \Sigma}{\Gamma, \Pi \vdash \Delta, \Sigma}\ Cut$$

$$\frac{\Gamma, A, B, \Delta \vdash \Pi}{\Gamma, B, A, \Delta \vdash \Pi}\ EL \qquad \frac{\Gamma \vdash \Delta, A, B, \Pi}{\Gamma \vdash \Delta, B, A, \Pi}\ ER$$

$$\frac{A, A, \Gamma \vdash \Delta}{A, \Gamma \vdash \Delta}\ CL \qquad \frac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A}\ CR$$

Operational rules

$$\frac{\Gamma \vdash \Delta, A}{\neg A, \Gamma \vdash \Delta}\ \neg L \qquad \frac{A, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg A}\ \neg R$$

$$\frac{\Gamma \vdash \Delta, A \qquad B, \Pi \vdash \Sigma}{A \to B, \Gamma, \Pi \vdash \Delta, \Sigma} \to L \qquad \frac{A, \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \to B} \to R$$

$$\frac{\Gamma \vdash \Delta, A \qquad B, \Gamma \vdash \Delta}{A \rightsquigarrow B, \Gamma \vdash \Delta} \rightsquigarrow L \qquad \frac{A, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, A \rightsquigarrow B} \rightsquigarrow R_1$$

$$\frac{\Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \rightsquigarrow B} \rightsquigarrow R_2$$

$$\frac{A, \Gamma \vdash \Delta \qquad B, \Gamma \vdash \Delta}{A \sqcup B, \Gamma \vdash \Delta}\ \sqcup L \qquad \frac{\Gamma \vdash \Delta, A}{\Gamma \vdash \Delta, A \sqcup B}\ \sqcup R_1$$

$$\frac{\Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \sqcup B}\ \sqcup R_2$$

$$\frac{A, \Gamma \vdash \Delta \qquad B, \Pi \vdash \Sigma}{A + B, \Gamma, \Pi \vdash \Delta, \Sigma}\ +L \qquad \frac{\Gamma \vdash \Delta, A, B}{\Gamma \vdash \Delta, A + B}\ +R$$

$$\frac{A, \Gamma \vdash \Delta}{A \sqcap B, \Gamma \vdash \Delta}\ \sqcap L_1 \qquad \frac{\Gamma \vdash \Delta, A \qquad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \sqcap B}\ \sqcap R$$

$$\frac{B, \Gamma \vdash \Delta}{A \sqcap B, \Gamma \vdash \Delta}\ \sqcap L_2$$

$$\frac{A, B, \Gamma \vdash \Delta}{A \times B, \Gamma \vdash \Delta}\ \times L \qquad \frac{\Gamma \vdash \Delta, A \qquad \Pi \vdash \Sigma, B}{\Gamma, \Pi \vdash \Delta, \Sigma, A \times B}\ \times R$$

The Relevant Conditional and the Indirectness condition The relevant conditional $\rightarrow$ captures the connection between antecedent and consequent on the different examples of natural language that we have seen above. Hence, among the three different reasons that one had to assert a conditional 'if A then B' in the Gricean schema, the relevant conditional $\rightarrow$ codifies the third one:

c There is a connection between A and B such that A cannot be the case without B being the case.

In [1989a] Grice discusses what he calls the *the Indirectness Condition*, a condition for asserting 'if A then B' which would negate his analysis of the material conditional. It expresses a stronger connection between antecedent and consequent of a conditional that is part of the meaning of 'if...then' and defended by those that reject the classical material analysis of the indicative conditionals. It can be expressed as follows:

> [T]here are no truth-conditional reasons for saying 'if A then B'.

Grice argues, in order to reject that the indirectness condition is part of the meaning of 'if...then', that it does not seem to be the case that a conditional is always asserted on grounds of some non truth-conditional reason that connects the antecedent and the consequent. There are some cases in which one seems to be able to assert 'if A then B' without the indirectness condition being true. Consider the following example by Grice:

7. I know just where Smith is and what he is doing, but all I will tell you is that if he is in the library he is working [Grice, 1989a]

The same observations can be made with respect to the clause [c] as the only reason for asserting a conditional: the relevantist rejection of [a] or [b] as reasons for asserting a conditional finds counterexamples in natural language.

4 Pragmatic Logical Pluralism

4.1 Implicatures and Enrichments

So far we have seen, on the one hand, that the counterexamples to classical logic seen above are counterintuitive because the pragmatic content that is linked to the conditional is embeddable, and neither classical logic nor the Gricean theory is capable of explaining this fact. On the other hand, it seems that the relevant conditional can solve this problem, but the price is to reject some other natural uses of the conditional such as cases similar to 7.

Our solution to the problem is motivated by the observation that the problem of the embedding of pragmatic content in larger sentences is not exclusive of conditionals. Other connectives can carry pragmatic content whenever they are embedded. Consider the following example in [Carston, 2002]:

8. Driving home and drinking several beers is better than drinking several beers and driving home. [Carston, 2002]

The natural way to understand 8 is the following, in which the pragmatic content associated to "and" contribute to the meaning of the whole expression:

8′ Driving home *and then* drinking several beers is better than drinking several beers *and then* driving home.

The phenomenon is widely recognized (although the explanations it has received diverge), and even Grice was aware that pragmatic enrichments were a problem for his theory:

> It certainly does not seem reasonable to subscribe to an absolute ban on the possibility that an embedding locution may govern the standard non-conventional implicatum rather than the conventional import of the embedded sentence [Grice, 1989c, p.375]

The solution comes from philosophers of language such as Recanati [2004], Carston [2002], or Bach [1994], who have argued that there can be pragmatic contributions different from the Gricean conversational implicatures, and which contribute to "what is said" by an utterance. I will argue that the connection between antecedent and consequent is one of such pragmatic contributions and that it is codified by $\rightarrow$.

From embedded implicatures to enrichments Given that embedded implicatures are a different phenomenon from Gricean implicatures, we should abandon the term 'implicature' for them. Hence, following Recanati [2004][5] let us call these embedded contents *pragmatic enrichments*, or simply *enrichments*. One simple mechanism to distinguish implicatures from enrichments is to acknowledge that a pragmatic content is an enrichment rather than an implicature *if* it is embeddable. This is what Recanati calls the *Scope Principle*:

> Scope Principle: A pragmatically determined aspect of meaning is part of what is said (and, therefore, not a conversational implicature) if - and, perhaps, only if - it falls within the scope of logical operators such as negation and conditionals. [Recanati, 2004]

Carston has a similar diagnostic,

> The interesting fact is that some pragmatically derived meaning does fall in the scope of logical operators and some do not so that we have a test for distinguishing pragmatic contributions to the proposition expressed from conversational implicatures. [Carston, 2002]

Two senses of what is said According to Recanati and Carston, there is no sense of *what is said* before the pragmatic enrichment. However, in the case of logical connectives there seems to be a literal sense that can be communicated as a complete utterance: we can grasp a sense of "and" in 8 which we could equate to $\wedge$, and another sense of "and" which is pragmatically enriched and expresses an order of the conjuncts. And the same can be argued for the conditional, as we will see in detail in what follows.

The approach to pragmatic enrichment that we will endorse, hence, follows the distinction made by Bach [1994] between two different senses of what

[5] Recanati in [2003] refers to them as 'embedded implicatures'; however, he abandons this term in [2004].

is said. According to Bach, there is a minimal sense of what is said (what the utterance says$_{min}$), and an expanded sense of what is said, in which the logical connectives (and other components of the utterance) can differ from their literal meaning (what the utterance says$_{prag}$). Moreover this last pragmatic content needs to be distinguished from the Gricean *implicatures* (which are derived *after* the expansion of the utterance). This is why Bach refers to the pragmatic content which contributes to what is said as an *impliciture*.

In what follows we will show that the contribution to the truth conditions of "if...then" can be formalized with $\supset$ (we will refer to this as the literal sense of the conditional) and also with $\rightarrow$ (we will refer to it as the enriched sense of the conditional), expressing a connection between antecedent and consequent, and which can be embedded in larger sentences.

4.2 Mechanisms of Enrichment

Although the present proposal rejects the Gricean view according to which the mismatch between logical connectives in natural language and their classical counterparts is explained with conversational implicatures, the Gricean maxims will have an important role in deriving their pragmatic enrichment. The role of maxims is not limited to generating implicatures but may also help to enrich the meaning of the utterance.

The importance of reasons We can see how the enrichment (at least for the logical vocabulary) is produced by assuming certain *reasons* to assert the literal meaning of an utterance: the process of enrichment of an utterance is to choose one among many reasons to assert it, and implicitly assume that those reasons are also communicated. Consider the following two utterances:

9. Charles Darwin joined the HMS Beagle for a trip around the world and wrote *The Voyage of the Beagle*.

10. Charles Darwin wrote *The Voyage of the Beagle* and joined the HMS Beagle for a trip around the world.

Both 9 and 10 say$_{min}$ exactly the same: Darwin joined the HMS Beagle for a trip around the world and ($\wedge$) Darwin wrote *The Voyage of the Beagle*. However, they do not say$_{prag}$ the same:

9e. Charles Darwin joined the HMS Beagle for a trip around the world and (then) wrote *The Voyage of the Beagle*.

10e. Charles Darwin wrote *The Voyage of the Beagle* and (then) joined the HMS Beagle for a trip around the world.

One mechanism to derive what is said$_{prag}$ from what is said$_{min}$ is by a simple assumption along the following lines:

9r. There are reasons for asserting 9 which observe the submaxim of orderliness.

10r. There are reasons for asserting 10 which observe the submaxim of orderliness.

Moreover, these assumptions can be embedded under the scope of another operator. Whenever one wants to negate what is said$_{prag}$ by 10, that is, one wants to negate 10e, one is also negating 10r.:

11. It is not the case that there are reasons for asserting 10 which observe the submaxim of orderliness.

Consider now the embedding of 9 under the scope of a conditional:

12. If Charles Darwin joined the HMS Beagle for a trip around the world and wrote *The Voyage of the Beagle*, it must be the case that the trip was inspiring.

If the speaker intends to convey that the trip inspired the book, the utterance is, strictly speaking, false. The utterance only makes sense if the antecedent is embedded with its order implicature:

13. If there are reasons to assert that Charles Darwin joined the HMS Beagle for a trip around the world and wrote *The Voyage of the Beagle*, observing the submaxim of orderliness, it must be the case that the trip was inspiring.

What generates, then, what is said$_{prag}$ from what is said$_{min}$ are assumptions about the reasons for asserting the utterance. We can generalize the previous examples and say, for any maxim M, and any connective $\bullet$, that we can distinguish the following reasons to assert $p \bullet q$:

- There are reasons for asserting $p \bullet q$ which observe the maxim M.
- There are reasons for asserting $p \bullet q$ which violate the maxim M.

In both cases what is said$_{min}$ is equivalent, but what is said$_{prag}$ varies. And these reasons both enrich the meaning of $\bullet$ and can be embedded under the scope of other operators, helping to capture the enriched sense of utterances. We will see below how relevant logic distinguishes different reasons to assert a conditional and hence, captures enriched senses of 'if...then'.

4.3 A pragmatic interpretation of LR

One possible interpretation of the divergence of the behavior of the logical constants between LK and LR is a pragmatic approach towards LR. As it is defended in [Terrés Villalonga, 2017], LR's notion of "follows from" avoids the violation of Relation and Quantity maxims (given the rejection of weakening as a structural rule), and the language (conditional, conjunction and disjunction) can be pragmatically enriched.

The presence of weakening makes it possible to add irrelevant information to a derivation, either in the premises or in the conclusion. And this irrelevant information invalidates those inferences in which 'follows from' observes the maxims of Relation and Quantity. First, if the irrelevant information is on the premises, and given their conjunctive reading on a sequent, the conclusion does not require all of them to be derived. Hence, whenever the speaker knows that Δ follows only from Γ, the affirmation that $A, \Gamma \vdash \Delta$ violates the Gricean maxim of Relation.

$$\frac{\Gamma \vdash \Delta}{A, \Gamma \vdash \Delta}\ WL$$

Second, if the irrelevant information is in the conclusion, and given its disjunctive reading in a sequent, it also follows something stronger from the same premises, which implies that whenever the speaker knows that Δ follows from Γ, the affirmation that $\Gamma \vdash \Delta, A$ violates the Gricean maxim of Quantity.

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, A}\ WR$$

Given this, *LK* and *LR* encode two senses of logical consequence. On the one hand, *LK*, which ignores the connection between premises and conclusion, codifies truth preservation. On the other hand, *LR* is pragmatically enriched and codifies a connection between premises and conclusion.

These two different senses of "follows from" are connected to different behaviors of the logical connectives. On the one hand, the consequence relation in *LK* is not affected by the reasons why a logical connective is used, given that it captures truth preservation. On the other hand, we have seen that in *LR* connectives are split into two, and this corresponds to two different reasons for using the connective. Given the particular reading of "follows from" in *LR* it is required that some connectives are pragmatically enriched (or not) in order to establish a relevant connection between them.

In more detail, the distinction between intensional and extensional connectives in *LR* corresponds to a distinction between those instances of the classical connectives that are introduced violating a Gricean maxim ($\rightsquigarrow$, $\sqcup$, $\sqcap$) and those that are not ($\rightarrow$, $+$, $\times$)[6]:

Natural Language	Reasons to assert	*LK*	*LR*
"if A then B"	not A	$\supset$	$\rightsquigarrow$
	B		
	connection: B is inferrable from A		$\rightarrow$
"A or B"	A	$\vee$	$\sqcup$
	B		
	connection: the rejection of one implies the other		$+$
"not (A and B)"	not A	$\wedge$	$\sqcap$
	not B		
	connection: incompatibility between A and B		$\times$
"if (A and B) then C"	if A then C	$\wedge$	$\sqcap$
	if B then C		
	connection: both A and B are required for C		$\times$

Following [Terrés Villalonga, 2017], we can distinguish two reasons to assert each logical constant, in particular for the conditional, which correspond to each conditional in *LR*:

[6]The violation of the Gricean maxims of Relation and Quantity are related to the *assertion* of a disjunction or a conditional. The case of the conjunction is a bit more complex as *LR* does not capture a violation of relevance or quantity whenever the conjunction is asserted, but whenever it is embedded under the scope of another operator such as the negation or the conditional.

- I-reasons: they are the intensional reasons to assert a connective, that is, the reasons for using a connective without violating a Gricean maxim (i.e. the connection between the antecedent and consequent): $\rightarrow$.
- E-reasons: they are those reasons (I will call them extensional reasons) for using a connective but violating a Gricean maxim (i.e. the assumption that the antecedent is false or that the consequent is true): $\rightsquigarrow$.

In sum, the classical consequence ($\vdash_k$) is a relation between truth conditions and truth conditions. But if we reject weakening we have a relevant relation ($\vdash_r$) that connects reasons to assert the premises to reasons to assert the conclusion. And given that *LR*'s language distinguishes different reasons to assert the connectives, we have a general diagnostic of the reasons to assert any classical formalization, including the paradoxes. *LR* offers what was required in the previous section of this paper: a formal language that distinguishes the reasons to assert a conditional, which is embeddable in larger sentences and can give a systematic diagnostic for the paradoxes, as will be seen in the following section.

5 The paradoxes solved

5.1 A general diagnostic

In this section, we will see how *LR* provides the necessary tools to diagnose the paradoxes in *LK*. For each paradox, we will see four things. First, we will introduce the paradox in natural language, as presented in the literature, which motivates the intuition that we should endorse the premises but reject the conclusion. Second, we will see its classical formalization, which makes the paradox valid, given that *LK* does not distinguish the reasons to assert the connectives. Third, we will present at least one relevant formalization with the intensional conditional (which expresses the intended meaning of the paradox) and show that it is invalid in *LR*. This explains why the paradoxes are counterintuitive: the intended meaning of the premises does not give reasons to assert the intended meaning of the consequent. Finally, we will see that there is at least one relevantly valid version of the paradox with an extensional conditional: if some conditional in the paradox is translated as 'there are E-reasons for asserting an utterance of the form 'if A then B' the derivation is valid.

Although the paradoxes can be of any syntactic complexity, we can divide them into the following main groups:

- Unambiguous paradoxes: paradoxes with just one relevantly valid formalization. These, in turn, can be divided into three groups:
  - Paradoxes with a conditional in the premises,
  - Paradoxes with a conditional in the conclusion.[7]
- Ambiguous paradoxes: Paradoxes with more than one relevantly correct formalization.

Let us see them in detail.

[7] This group includes those tautologies from classical logic that are paradoxical, considering them as inferences without premises.

5.2 Unambiguous paradoxes

5.2.1 Paradoxes with a conditional in the premises

In the first group of paradoxes, we find those in which a conditional is embedded in the premises. When a conditional is embedded what we usually mean is the embedding of its pragmatic enrichment. In these cases, the paradox emerges because we seem to endorse the premise, with an embedded conditional, but not the conclusion. However, what we actually endorse is just the embedding of the implicature. Hence the classical formalization is usually a bad formalization of the intended meaning of the natural language presentation of the paradox.

Negation Consider the following argument in natural language:

1. It is not the case that if there is a good God the prayers of evil people will be answered. Hence, there is a good God. [Cooper, 1968]

In classical logic the negation of the consequent (and also the antecedent) follows from the negation of a conditional:

1c. $\neg(p \supset q) \vdash_k p$

The derivation is straightforward:

$$\dfrac{\dfrac{\dfrac{p \vdash p}{p \vdash p, q}\,\text{WR}}{\vdash p \supset q, p}\,\supset\text{R}}{\neg(p \supset q) \vdash p}\,\neg\text{L}$$

This validity shows that the argument is classically valid. However, our use of the conditional does not usually derive either the antecedent or the negation of the consequent of a conditional just from the negation of a conditional. In fact, it even seems misleading to do so, given that what we tend to express with "it is not the case that "if p then q"" is something along the following lines: "there is no I-reason to assert "if p then q"" (i.e., there is no connection between being a good God and the prayers of evil people being answered). That is, what we tend to negate is that there is a connection, and from this, the negation of the conclusion does not follow:

1r. $\neg(p \to q) \nvdash_r p$

Notice that adding $\neg(p \to q) \vdash p$ to the calculus would reestablish weakening:

$$\dfrac{\dfrac{\dfrac{p \vdash p \qquad q \vdash q}{p \to q, p \vdash q}\,\supset\text{L}}{p \vdash q, \neg(p \to q)}\,\neg\text{R} \qquad \neg(p \to q) \vdash p}{p \vdash p, q}\,\text{Cut}$$

However, the argument is classically valid, that is, it preserves truth. The reason is that $\supset$ does not distinguish between E/I-reasons to assert a conditional, and the premise can express something else, from which the conclusion does follow. That is, if we just endorse the fact that there is no connection between being a good God and the prayers of evil people being answered, and we remain skeptical about the existence of God, then we do not really endorse the fact "it is not the case that if there is a good God the prayers of evil people will be answered": there can be an E-reason to assert the conditional, and we do not endorse its negation. In *LR* it is valid to infer the antecedent of a negated conditional if the conditional is asserted with an E-reason.

1r′. $\neg(p \rightsquigarrow q) \vdash_r p$

The derivation is straightforward:

$$\dfrac{\dfrac{p \vdash p}{\vdash p \rightsquigarrow q, p}\ {\rightsquigarrow}\mathrm{R}}{\neg(p \rightsquigarrow q) \vdash p}\ \neg\mathrm{L}$$

This shows that there are cases in which one can derive p from the negation of a conditional: when the negated conditional is asserted for E-reasons, for instance: "it's not the case that if Gödel was a logician then pigs can fly", or in the context of a cards game in which someone negates the utterance 'if I have 5 As I am going to Paris', just to assert that one *does* have 5 As.

Conditional nested in the antecedent Another case of a paradox in which there is a conditional embedded but which is not a good formalization of what we want to express is that of a conditional nested as an antecedent of another conditional:

2. If the cup breaks if dropped, then it is fragile; and the cup was not dropped. Hence, the cup is fragile. [8]

The classical formalization validates the argument:

2c. $(p \supset q) \supset r, \neg p \vdash_k r$

However this derivation seems valid only if the embedded conditional is true due to the falsity of the antecedent, in no case does it seem valid given a connection between antecedent and consequent. What we usually try to express when we embed a conditional as an antecedent is "if there are I-reasons to assert $A \supset B$, then...", but from this and the falsity of the antecedent, the conclusion does not follow. The formal counterpart in LR confirms this:

2r. $(p \rightarrow q) \rightarrow r, \neg p \nvdash_r r$

Note that the validity of this sequent would reestablish weakening, as we could derive $\neg p \vdash r, q$ from $\neg p \vdash r$:

[8] This is a modification from the discussion in [Gibbard, 1980, p.237] about the compound of conditionals, in which he discusses the sentence "If the cup broke on being dropped, then it was fragile.", whose diagnostic we will see in what follows.

$$
\begin{prooftree}
\AxiomC{$p \vdash p$}
\RightLabel{$\neg$R}
\UnaryInfC{$\vdash p, \neg p$}
\AxiomC{$\neg p \vdash r$}
\RightLabel{Cut}
\BinaryInfC{$\vdash p, r$}
\RightLabel{+R}
\UnaryInfC{$\vdash p + r$}
\AxiomC{$p \vdash p$}
\AxiomC{$q \vdash q$}
\RightLabel{$\to$L}
\BinaryInfC{$p, p \to q \vdash q$}
\AxiomC{$r \vdash r$}
\RightLabel{+L}
\BinaryInfC{$p + r, p \to q \vdash r, q$}
\RightLabel{$\to$R}
\UnaryInfC{$p + r \vdash (p \to q) \to r, q$}
\AxiomC{$(p \to q) \to r, \neg p \vdash r$}
\RightLabel{Cut}
\BinaryInfC{$p + q, \neg p \vdash r, q$}
\RightLabel{Cut}
\BinaryInfC{$\neg p \vdash r, q$}
\end{prooftree}
$$

Nevertheless, the extensional reading of the conditional makes the derivation valid (but again, this is not what we tend to express with a conditional nested in the antecedent of another conditional).

2r′. $(p \rightsquigarrow q) \to r, \neg p \vdash_r r$

The conclusion only follows if the reasons to assert the first conditional are E-reasons. This analysis is a generalization of the one given by Gibbard:

> [C]onsider first the antecedent "if the cup was dropped, it broke". Such an indicative conditional may have an obvious *basis*,
>
> > [*basis*] The cup was disposed to break on being dropped. [Gibbard, 1980, p.237]

The present solution subsumes Gibbard's view: "basis" is just the expression of an I-reason to assert the conditional.

5.2.2 Paradoxes due to a conditional in the conclusion

Let us turn to the second kind of paradoxes: those generated by an embedded conditional in the conclusion. In this case, the general diagnostic is that we endorse the premises, but they only provide an E-reason for a conditional. That is, the premises only give reasons to assert in the conclusion some conditional for E-reasons, which means that the conditional in the conclusion could be substituted by the negation of the antecedent or the affirmation of the consequent. However, in no case, do they imply a connection between them. This explains why we seem to reject the conclusion.

Affirmed conditional Among this subgroup of paradoxes we find those in which a conditional is asserted, either as a simple conditional or embedded in a conjunction:

3. There are planets. Hence, if there are no planets anywhere, the solar system has at least eight planets. [Bennett, 2003]

The classical formalization of the argument is valid:

3c. $A \vdash_k \neg A \supset B$

But our intended meaning of the conclusion is that there is a connection between the fact that there are no planets and the fact that the solar system has at least eight planets. This is why it seems invalid: the connection seems plainly false. However, the premise does not give any reason to affirm a connection between these two facts:

3r. $A \nvdash_r \neg A \rightarrow B$

The only reason we can assert $\neg A \supset B$ in the conclusion is that given the truth of A, that is, the truth of "there are planets", we have an E-reason to affirm that "if there are no planets then the solar system has at least eight planets": the known falsity of the antecedent assures that it is a true conditional, independently of what the consequent says:

3r'. $A \vdash_r \neg A \rightsquigarrow B$

Disjunction of conditionals (I) When a conditional is embedded under the scope of a disjunction, it can easily generate a paradox. Consider the following disjunction in natural language:

4. Either if it is raining the sun is shining or if the sun is shining it is snowing.

In *LK* the following formula is valid:

4c. $\vdash_k (A \supset B) \vee (B \supset C)$

This expression is usually understood as affirming that either one or the other disjunct is the case. That is, there are I-reasons to assert the disjunction (recall that the I/E reasons can be distinguished for all the connectives). Moreover, we also understand that the conditionals embedded are intensional, that is, that either there is an I-reason to assert "if A then B" or there is an I-reason to assert "if B then C". However, this is invalid in *LR*, which shows that the disjunction of I-reasons is not a relevant tautology.

4r'. $\nvdash_r (A \rightarrow B) + (B \rightarrow C)$

Why, then, is this argument a tautology in classical logic? Given that $B \vee \neg B$ is a logical truth, either there are E-reasons to assert $A \supset B$ or there are E-reasons to assert $B \supset C$. From the relevant perspective, this fact is confirmed:

4r. $\vdash_r (A \rightsquigarrow B) + (B \rightsquigarrow C)$

Disjunction of conditionals (II) Consider the following inference:

5. If you close switch x and switch y the light will go on. Hence, it is the case either that if you close switch x the light will go on, or that if you close switch y the light will go on. [Priest, 2008]

The classical formalization validates the argument:

5c. $(A \wedge B) \supset C \vdash_k (A \supset C) \vee (B \supset C)$

However, the premise does not give reasons for the disjunction of intensional conditionals. That is, the premise does not imply that either there is a connection between A and C or a connection between B and C:

5r. $(A \times B) \rightarrow C \nvdash_r (A \rightarrow C) + (B \rightarrow C)$

Let us see a relevantly valid formalization of the argument:

5r'. $(A \times B) \rightarrow C \vdash_r (A \rightsquigarrow C) + (B \rightarrow C)$

Note that either A is true or false, that is, $A \vee \neg A$. If it is false, that gives an E-reasons to assert $A \supset C$, that is, $A \rightsquigarrow C$. However, if A is true, then there is a connection between B and C. Given that if both A and B are true, then C is true, the fact that A is true creates a connection between B and C, that is, $B \rightarrow C$ is true. The connection is dependent on the truth of A, as the formalization suggests.

5.3 Ambiguous paradoxes

All the paradoxes seen so far have just one valid relevant formalization. This means that there is just one reason why the argument is truth-preserving. However, there are other paradoxes which have at least two relevantly valid formalizations. This means that there is not just one diagnostic, but more. This is the case because the reasons to assert some other connective in the derivation affect the reasons to assert the rest of the connectives. Let us see these dependencies in detail.

Interderivability In order to solve combined paradoxes, one needs to bear in mind the relation between the different connectives. Consider the following equivalences between the intensional/extensional disjunction and conjunction and the intensional/extensional conditional in LR:

	E-connectives	conditional equiv.	I-connectives	conditional equiv.
Disjunction/conditional	$\neg A \sqcup B$	$A \rightsquigarrow B$	$\neg A + B$	$A \rightarrow B$
Conjunction/conditional	$\neg(A \sqcap B)$	$A \rightsquigarrow \neg B$	$\neg(A \times B)$	$A \rightarrow \neg B$
Conjunction/conditional	$(A \sqcap B) \rightarrow C$	$A \rightsquigarrow (B \rightarrow C)$ $A \rightarrow (B \rightsquigarrow C)$	$(A \times B) \rightarrow C$	$A \rightarrow (B \rightarrow C)$

Given the equivalence relations between the different connectives in LR, it is clear that E or I reasons for asserting an utterance with a particular connective imply E or I reasons for asserting their equivalents. For instance:

6. It is false that ($2 + 2 = 5$ and dinosaurs existed). Hence, if $2 + 2 = 5$ then dinosaurs existed - (both the negated conjunction and the conditional are asserted for E-reasons).

7. Either Paris is in France or Gödel was a physicist. Hence, if Paris is not in France then Gödel was a physicist - (both the disjunction and the conditional are asserted for E-reasons).

This fact will be crucial for the solution to McGee's paradox.

5.3.1 Conditional in the consequent: McGee

McGee's counterexamples to Modus Ponens are based on a conditional embedded as the consequent of another conditional. McGee presents three counterexamples, and I will develop the claim that only the first one ("The Election") allows for a diagnostic in LR, although the diagnostic made is common to the three examples. Let us see them in detail:

The Election

Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the polls believed, with good reason:

a If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

b A Republican will win the election.

Yet they did not have reason to believe:

c If it's not Reagan who wins, it will be Anderson.

Lungfish

I see what looks like a large fish writhing in a fisherman's net a ways off. I believe

a If that creature is a fish, then if it has lungs, it's a lungfish

That, after all, is what one means by "lungfish". Yet, even though I believe the antecedent of this conditional [(b) that creature is a fish], I do not conclude

c If that creature has lung, it's a lungfish.

Lungfish are rare, oddly shaped, and, to my knowledge, appear only in freshwater. It is more likely that, even though it does not look like one, the animal in the net is a porpoise.

Uncle Otto

Having learned that gold and silver were both once mined in his region, Uncle Otto has dug a mine in his backyard. Unfortunately, it is virtually certain that he will find neither gold nor silver, and it is entirely certain that he will find nothing else of value. There are simple reasons to believe

a If Uncle Otto doesn't find gold, then if he strikes it rich, it will be by finding silver.

b Uncle Otto won't find gold.

Since, however, his chances of finding gold, though slim, are no slimmer than his chances of finding silver, there is no reason to suppose that

c If Uncle Otto strikes it rich, it will be by finding silver.

General structure The general classical formalization of McGee's counterexamples is:

8c. $A \supset (B \supset C), A \vdash_k B \supset C$

The analysis in *LR* shows that from an I-reason to assert a conditional in the premises follows an I-reason to assert the conditional in the conclusion:

8r. $A \to (B \to C), A \vdash_r B \to C$

Hence, the paradoxes seem to escape the above analysis, and it is necessary to give another explanation for why the conclusions of the arguments seem unacceptable while the premises do not.

We need to realize that the three examples have a common characteristic: in the three examples, there is a tension between the truth of the antecedent of the conclusion (B) and the minor premise (A). To be more specific, in the three vignettes one seems to assert A given that one ignores whether B is the case or even assuming that B is *not* the case. Let us see this tension for each case: first, in the Election, the grounds for saying A in the minor premise is that Reagan is winning, that is, $\neg B$. A is then in tension with B because knowing B would rule out the assertion of A in that particular scenario. Second, in the lungfish example, the grounds for saying A are the shape of the fish. However, given that lungfish are rare and it is much more probable that if the creature has lungs it is something different from a fish, the knowledge of B (the creature has lungs) would prevent one of asserting A (it is a fish) as a minor premise, again, in this particular scenario. Third, knowing that uncle Otto is rich (B) would make one refrain from asserting A, that is, that he has not found gold.

With this tension in mind, we can now explain why the premises are highly assertable but the conclusion is not. In each case, C is a very special situation that only happens when A together with B happen, which is also a very special situation because B is in tension with A. That is, 'if B then C' is hardly assertible because A is also required but B rules it out. In other words, the conditional in the conclusion seems to be false given that the antecedent usually rules out one of the premises (A), which is necessary (together with B) for the truth of the consequent of the conclusion.

Hence, in the three examples, and assuming that A is the case, one would suspect that the conditional ($B \supset C$) in the conclusion is true because it has a false antecedent. However, this is not captured in *LR* (that is, the solution to the paradox is not explained by the presence of an extensional conditional, $\rightsquigarrow$, as we have seen above). This is so because the premise that creates this tension (that is, A) cannot be analyzed by the formal language *LR*. However, the Election will be an exception to this fact, as we will see in what follows.

Before this, consider an alternative argument with a similar structure in which this tension between the minor premise and the antecedent of the conclusion is not present:

a If a car runs for 1 hour, then, if it runs at 80 kph, it will cover a distance of 80 km,

b I have been driving my car for 1 hour,

c Hence, if my velocity was 80 kph then I have covered 80 km.

The consequent of the conditional in the conclusion does not seem to follow from the antecedent alone, but it requires that the minor premise be also true, that is, there is no reason to assert the last conditional unless the second premise is true. However, in this case, the antecedent of the conditional is not in tension with the minor premise, and the argument seems completely correct.

It is not surprising that one can transform McGee's vignettes in illustrations without the tension mentioned above: adding to the illustration a DNA analysis of the creature which confirms (before knowing whether it has lungs or not) that the creature is a fish; an analysis of uncle Otto's terrain which confirms that there is no gold; or the knowledge that the Elections are manipulated by the Republican party in order to make one of the candidates the winner of the election. In the three modifications, the arguments seem as regular as the car example.

In sum, the premises of the arguments support a conditional in the conclusion that seems to be true because of the falsity of the antecedent rather than the connection between antecedent and consequent. However, we have seen that the inference is valid for the intensional conditional in *LR*. This is because the responsible for the tension are the grounds for saying A, which are not analyzed in *LR* given that they are not related to the use of a logical connective.

The Election: an ambiguous paradox While in the Uncle Otto and the Lungfish vignettes there is just one way of formalizing A, (although there might be different grounds for it), in the case of the Election we can disambiguate these different grounds for saying "a' Republican will win the election", which will allow us to give a similar diagnostic to the previous cases of the paradox.

This diagnostic rests on Paoli's solution to the counterexample in [2005], which is based on the distinction between an intensional and an extensional existential quantifier, which is also based on the distinction between an intensional and an extensional disjunction. While Paoli argues for the distinction in *LL*, we use *LR* for the same task[9]. In particular, Paoli's diagnostic is that the second premise of McGee's counterexample is using a different quantifier ($\bigsqcup$) from the first (Σ). This paper reinterprets this ambiguity pragmatically, maintaining that the argument is classically valid but violates a Gricean maxim.

Moreover, in this paper it is defended that in the Election, *LR* codifies the different senses for saying the minor premise, and this allows us to better justify the general diagnostic that we have seen for the three paradoxes.[10]

In particular, I will argue that the Election paradox is generated by the connection between the second premise and the conclusion in the following sense: the tension between the minor premise and the antecedent of the conditional in the conclusion is generated because the minor premise violates the Gricean maxim of quantity and this can only entail a conclusion which also violates the

[9]Paoli's diagnostic based on *LL*, but we can consider the same interpretation of the quantifiers in *LR*, with the difference that the intensional implies the extensional, contrary to Paoli's view.

[10]Zardini [2015] argues against Paoli's diagnostic of the paradox, in which the present solution is sustained, considering an equally problematic counterexample to the Election in which the minor premise is substituted by 'Carston is not going to win the Election'. The diagnostic in this paper cannot formalize and analyze this version of the counterexample, but this is not surprising: in this case the example is similar to the uncle Otto or the Lungfish: the tension between the minor premise and the antecedent of the conditional in the conclusion is present but our formal language does not support a correct way of formalizing it.

Gricean maxim of quantity, while our reading of the conclusion is pragmatically enriched.

Paoli [2005] argues that the indefinite article "a" (as used in "a Republican will win") is ambiguous, which transforms McGee's paradox into a fallacy of equivocation. Let D be a finite domain of discourse $\{a_1...a_n\}$. There are two reasons to assert "there is an $x \in D$ such that Px":

a. One specific object $a \in D$ such that Pa,

b. if not Pa_1 then, if not Pa_2 ... then Pa_n.

The first reason corresponds to the extensional existential quantifier $\sqcup xPx$ while the second to the intensional existential quantifier Σ. Unlike Paoli, who defends a semantic interpretation of this difference, I argue for a pragmatic perspective, arguing that the difference between these reasons motivates the pragmatic distinction between $\sqcup$ and Σ. Given that [a] violates the maxim of Quantity, "there is an x that P" is usually enriched with [b].

That is, the intensional quantifier is pragmatically enriched by means of Grice's maxim of Quantity. This enrichment is similar to the enrichment that a disjunction may have, according to which we assume that the speaker establishes a connection between disjuncts. Hence, equivalently to the rest of the connectives, we can distinguish two reasons to assert the existential quantifier $\exists$, which corresponds to two formalizations of the logical connective[11] :

Natural Language	Reasons to assert	*LK*	*LR*
"There is an x in D such that Px"	Pa	$\exists$	$\sqcup$
	if not Pa_1 then, if not Pa_2 ... then Pa_n		Σ

A formalization for the two existential quantifiers can be that presented by Zardini [2011], accepting the restriction to finite domains for Σ:

$$\frac{\Gamma, \phi(a) \vdash \Delta}{\Gamma, \sqcup x\phi \vdash \Delta}\sqcup\mathrm{L} \qquad\qquad \frac{\Gamma \vdash \phi(t), \Delta}{\Gamma \vdash \sqcup x\phi, \Delta}\sqcup\mathrm{R}$$

[11] This distinction resembles the distinction de re/de dicto (I want to thank to a referee for pointing this out). A sentence is de re just in case it permits substitution of co-designating terms without changing its truth-value, and de dicto whenever this substitution might change its truth-value. Hence, "a republican" said de dicto would correspond to ΣxPx, and said de re would correspond to $\sqcup xPx$.

However, the ambiguity de dicto/de re is generated within the scope of an attitude report or a modal operator and the opaque contexts which they create, while the extensional/intensional ambiguity is present also outside such contexts: "A republican will win the election" can be said observing or flouting a Gricean maxim, outside any attitude report or modal operator. The ambiguity is originated by the particle "a" and not by the opaque context which these operators generate. But there surely is a connection between the two distinctions: Once one reports one's believe that a republican will win the election we can distinguish the de re/de dicto interpretation of such belief. And this will depend on the quantifier in which one should formalise one's thought in the first place. In particular, whenever an extensional existential quantifier is embedded under the scope of a modal operator or attitude report, the natural reading of the belief is de re. And whenever an intensional existential quantifier is embedded under the scope of a modal operator or an attitude report, the natural reading of the report is de dicto.

$$\frac{\Gamma, \phi(t_1) \vdash \Delta \qquad \Gamma', \phi(t_2) \vdash \Delta' \qquad ...}{\Gamma, \Gamma', ..., \Sigma x\phi \vdash \Delta, \Delta', ...}\Sigma L \qquad \frac{\Gamma \vdash \phi(t_1), \phi(t_2), ..., \Delta}{\Gamma \vdash \Sigma x\phi, \Delta}\Sigma R$$

Let us see how this pragmatic distinction applies to the Election case. First, note the different reasons to assert the premises in the argument: the first premise does not violate any Gricean maxim, while the second violates the maxim of Quantity: one should say "Reagan will win the Election" instead of "a Republican will win the Election". Now recall that E-reasons to assert some connective entail E-reasons to assert their equivalences (an E-reason to assert a negated conjunction entails an E-reason to assert a conditional, and the same for disjunction and the other equivalences saw above). The same applies to the existential quantifier: an E-reason to assert an existential quantifier only entails an E-reason to assert a disjunction, which only entails an E-reason to assert the equivalent conditional. We can easily check that the two premises in the example do not entail an I-reason to assert the conditional in the conclusion:

8r′. $\Sigma xPx \to (\neg Pa \to Pb), \sqcup xPx \nvdash_r \neg Pa \to Pb$

We can see the invalidity of the sequent observing that adding $\Sigma xPx \to (\neg Pa \to Pb), \sqcup xPx \vdash_r \neg Pa \to Pb$ as an axiom, weakening would be reestablished (see figure 1).

However, as in the above cases, the argument is truth preserving. In this case, we can give two relevantly valid arguments that explain why:

8r″. $\Sigma xPx \to (\neg Pa \to Pb), \Sigma xPx \vdash_r \neg Pa \to Pb$

8r‴. $\sqcup xPx \to (\neg Pa \rightsquigarrow Pb), \sqcup xPx \vdash_r \neg Pa \rightsquigarrow Pb$

8r″ is an instance of Modus Ponens which relies on the validity of $\Sigma xPx \vdash \neg Pa \to Pb$:

$$\cfrac{\cfrac{\cfrac{Pa \vdash Pa \qquad Pb \vdash Pb}{\Sigma xPx \vdash Pa, Pb}\Sigma L}{\Sigma xPx, \neg Pa \vdash Pb}\neg L}{\Sigma xPx \vdash \neg Pa \to Pb}\to R$$

8‴ is an instance of Modus Ponens which relies on the validity of $\sqcup xPx, \vdash \neg Pa \rightsquigarrow Pb$:

$$\cfrac{\cfrac{\cfrac{Pa \vdash Pa}{\sqcup xPx \vdash Pa}\sqcup L}{\sqcup xPx, \neg Pa \vdash}\neg L}{\sqcup xPx, \vdash \neg Pa \rightsquigarrow Pb}\rightsquigarrow R$$

If the second premise in McGee's counterexample is asserted without violating the Gricean maxim of quantity the conditional in the conclusion can be asserted for I-reasons, which 8″ captures. Consider the alternative scenario mentioned above in which the speaker is sure that a Republican is going to win the Election, but not on grounds of the knowledge that a particular candidate is the favorite one, but because the Election is manipulated by the Republican party. In that case, the existential quantifier in the second premise (asserted for I-reasons) gives I-reasons to assert the conditional in the conclusion.

$$
\dfrac{
\dfrac{Pa \vdash Pa}{Pa \vdash \sqcup x Px}\,\sqcup\mathrm{L}
\qquad
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\dfrac{Pa \vdash Pa \qquad Pb \vdash Pb}{\Sigma x Px \vdash Pa, Pb}\,\Sigma\mathrm{L}
}{\Sigma x Px, \neg Pa \vdash Pb}\,\neg\mathrm{L}
}{\Sigma x Px \vdash \neg Pa \to Pb}\,{\to}\mathrm{R}
}{\vdash \Sigma x Px \to (\neg Pa \to Pb)}\,{\to}\mathrm{R}
\qquad
\Sigma x Px \to (\neg Pa \to Pb), \sqcup x Px \vdash \neg Pa \to Pb
}{\sqcup x Px \vdash \neg Pa \to Pb}\,\mathrm{Cut}
\qquad
\dfrac{
\dfrac{Pa \vdash Pa}{\vdash \neg Pa, Pa}\,\neg\mathrm{R}
\qquad
Pb \vdash Pb
}{\neg Pa \to Pb \vdash Pa, Pb}\,{\to}\mathrm{L}
}{\sqcup x Px \vdash Pa, Pb}\,\mathrm{Cut}
}{Pa \vdash Pa, Pb}\,\mathrm{Cut}
$$

Figure 1

However, in the scenario presented by McGee, the second premise is asserted for E-reasons. Hence, no connection between antecedent and consequent can be derived from it, as captured in 8′′′. In conclusion, the E-reasons to assert the existential quantifier only gives E-reasons to assert the conditional in the conclusion, as with the other connectives.

6 Conclusion

This paper has defended that it is possible to give a pragmatic solution to the paradoxes of the material conditional by using the substructural logic *LR*. We have seen that *LR* can formally distinguish two reasons to assert a conditional: those that violate a Gricean maxim and those that do not. These reasons codify different pragmatic enrichments for the logical connectives and the conditional in particular. Hence, *LR* gives a systematic and formal analysis of the reasons why the conditionals are embedded in the different paradoxes, showing why they are both unassertable and truth-preserving.

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