

Economic Dynasties with Intermissions.*

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Abstract

We consider a model of successive generations with a fixed proportion of selfish and altruistic members in each individual's offspring. In contrast with the others, selfish members bequeath nothing to their own children. We assume that parents cannot recognize their heirs' types and that negative bequests are forbidden. We study Markov perfect equilibria of this multistage game of incomplete information and their implications for wealth distribution.

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Introduction

Except for a few papers, the economic literature dealing with overlapping generations can be classified in two categories. In the first one, illustrated by Diamond's (1965) classical paper, parents do not show any concern for their grown up children and they bequeath nothing. In the other category, all parents display some kind of dynastic spirit towards their progeny. This is the so called modified Ramsey tradition, where a deciding parent's total utility is a weighted sum of individual descendants' utilities, with exponentially declining weights as the future unfolds.

These two approaches are in vivid contrast when a long run perspective is adopted. Consider an ancestor and his/her descendants after a great many generations have elapsed. Assume that negative bequests are forbidden. Suppose identical labor endowment and preferences both within and across generations. Suppose a competitive economy with stationary prices, including the interest rate, which is assumed larger than the population growth rate.

Then, in the Diamond economy, every agent simply imitates his or her ancestor's behavior if the latter receives no bequest. In the dynastic economy, things may differ sharply, depending on the value of two critical parameters. The first one is the market discount rate and the other is a psychological parameter, viz the rate of discount of individual utility between any generation and the next.

If the stationary market discount rate is lower than the utility discount rate, both wealth and consumption per capita grow permanently, and they grow without bound unless there exists a finite satiation level of consumption. Except for this case, the slightest gap between the two discount rates will result in the accumulation of any desired per capita wealth level, however large, provided enough generations go by.

If, on the other hand, the two discount rates are equal we get an unconstrained stationary outcome. Thus, if the ancestor got no bequest, none of the descendants will leave any. But if the ancestor got a positive bequest, every descendant gets in turn

the same amount from his/her parent. This razor-edge stationarity property proves quite handy in some contexts, but there are other enquiries for which this feature is embarrassing.

The dynastic spirit is too weak if the market discount rate exceeds the psychological discount rate. Then, if the ancestor got no bequest, all the members of each generation would like to borrow for the purpose of increasing their own consumption and to bequeath their debt to the following generation. If this is prohibited by law, we get the same stationary outcome as in the Diamond economy without bequest.

If the nonnegativity constraint on bequests is binding, it can be circumvented by various policies. For instance, one can think of a one shot permanent increase in national debt per capita the proceeds of which are given away to the old generation, whereas all ensuing generations have to bear the burden of debt servicing. One can also think of setting up a pay as you go pension scheme.

As Barro (1974) observed, neither policy is effective if the dynastic spirit is strong enough to warrant positive bequests; indeed, bequests are explained here by altruistic feelings towards descendants: if a third party wants to operate any transfer in the undesired direction, nothing prevents the parent to counteract the undesired transfer by an increase of his or her own gift.

For all their merits, neither the Diamond model nor Barro's can throw light on the evolution of the individual wealth distribution, since all agents are assumed identical.

The range of interest rates consistent with an expected steady state with positive bequests and the evolution of the individual wealth distribution are the main focus of our study, as we attempt to populate the desert stretching between Diamond's viewpoint and Barro's.

We consider a model of successive generations: each individual lives just one period and dies when his or her m children enter economic life. Each mature child gets

from nature a fixed endowment of the single commodity existing in this world and from his or her parent an intergenerational transfer that is nonnegative. These resources are used for the individual's consumption and/or saved. Whatever is saved gets lent on a world market and the full proceeds get bequeathed to the next generation. The consumption/bequest choice is at the discretion of the individual, subject to the resource constraint. Our successive generation model can be interpreted as the reduced form of a model with overlapping generations as long as the interest rate governing the allocation of consumption over the life cycle is stationary. Individual consumption must then be interpreted as the present value of consumption over the life cycle.

There are two types of children within each family: pm children are altruistic, i.e. they value positively not only their own consumption but also the consumption stream of all their descendants, and $(1 - p)m$ children are as selfish as an *homo economicus*. Proportion p is assumed stable over time. Parents cannot recognize their children's types.

Individuals may be fully aware of their family history but they have to make guesses about the future. As altruistic parents cannot check their descendants' behavior, they value their consumption in a purely *ex ante* sense. They have to predict how their descendants will behave and they know that the latter will face an essentially similar problem.

The reader will have recognised in the above sketch a multistage game of incomplete information. An action of a player is his or her consumption and bequest decision. A strategy of a parent playing at the beginning of the game associates with each type an action. At a later stage, people may base their action on the previous history of the game and it is natural in bequest games to summarize the relevant aspects of history as the amount of bequest received by the decision-maker: this is known as the Markov property (see Fudenberg and Tirole (1993)). We shall define

a Markov strategy as a map from the set of types times the set of possible amounts bequeathed to the set of possible actions. In the literature such a strategy is called a bequest function.

We shall be interested in the simplest nontrivial Markov perfect equilibria of our game and in their implications. Our model is inspired by Dutta and Michel (1995), who rely on more purely utilitarian preferences and who get a less tractable model. It is also related to Michel and Pestieau (1994) and to Vidal (1996) where each family is homogeneous in composition, every member being assumed either thoroughly selfish or uniformly altruistic towards their descendants. Despite its stochastic features, our bequest game turns out to be essentially the same as the game studied by Phelps and Pollak (1968) in their pioneering paper and both models share the property of displaying a Pareto-inefficient Markov equilibrium from the viewpoint of actual preferences of altruistic players.

The paper is organized as follows. In section 1, we treat a family of bequest functions as hypothetical social conventions that are randomly violated and we study their implications for wealth distribution. In section 2, we describe the conflict between successive generations and we exhibit circumstances under which social conventions may be considered as selfsustaining because they can be interpreted as Markov perfect equilibrium strategies. We also study the influence of fundamentals on growth and distribution and we unravel the implications of simple exogenous transfers between generations. We point out some open problems in the conclusion. An appendix collects some technical results.

Section 1. Conventional bequest functions.

We consider an economy consisting of agents who live only one period. When an agent's life comes to an end, he or she is replaced at the beginning of the next period by $m \geq 1$ children. The bequest received by an agent living at t is denoted x_t . The agent further receives a lump sum payment $w > 0$ and spends c_t for his or her consumption; whatever is saved is used to buy one period riskless bonds. The bond market is assumed perfect with a stationary interest factor R . By assumption, the bequeathing parent cannot discriminate among his or her children, so that the total proceeds are distributed evenly among the heirs. Thus the total amount saved at t is $\frac{mx_{t+1}}{R}$. For simplicity, we define $q = \frac{m}{R}$ and write down the agent's budget as

$$w + x_t = c_t + qx_{t+1}. \quad (1)$$

An agent living at period zero who is altruistic is concerned with the consumption stream of his/her descendants. How the latter is affected by the choice of x_1 matters a great deal from his/her point of view. This relation cannot be directly observed but social conventions can be relied on. By assumption, an agent believes that anyone among his or her descendants will bequeath nothing with probability $1 - p$ but that, with probability p , he or she will bequeath x_{t+1} according to the social convention

$$x_{t+1} = \beta x_t + (\beta - 1)H, \quad t \geq 1. \quad (2)$$

Parameters H and β are independent of x_t and deemed stable by agents living at time zero. The shape of the social convention is postulated on account of its simplicity. It does not look unreasonable in an economy with stationary prices, where the proportion of altruistic descendants is stable at each generation in each family and all altruistic agents have the same preferences toward their progeny. Thus, the only source of uncertainty lies in the fact that parents cannot recognize their children's

types.

As outside analysts, we may anticipate that H and β are somehow related to the fundamentals of the model, viz. w, R, m, p and the parameters used to specify altruistic preferences. This relation is explored in the next section in the context of a bequest game, where the decision-makers have to figure out for themselves the true values of both β and H . In particular, it will be possible to show that the only value of H consistent with the fundamentals is given by the following expression

$$H = \frac{w - \mu}{1 - q} \quad (3)$$

where $1 > q, w > \mu$ and μ is another fundamental parameter interpreted as subsistence consumption, so that H stands for the present value of the fraction of the entire wage flow accruing to an individual and all his/her descendants that is not absorbed for subsistence purposes. The discussion of the parameter range is deferred to section 2.

We subtract μ from each side of (1), combine the result with (2) and (3), and after rearranging we get

$$\begin{aligned} c_t - \mu &= w - \mu + x_t - qx_{t+1} \\ &= w - \mu + x_t - q\beta x_t - q(\beta - 1)H \\ &= (1 - q\beta)x_t + (1 - q)H + qH - q\beta H \\ &= (1 - q\beta)(x_t + H), t \geq 1. \end{aligned} \quad (4)$$

Thus the condition $1 > q\beta$ is necessary to have $c_t > \mu$. Consider an altruistic decision-maker at time zero, who is aware of (1), (2) and (3). Straightforward substitutions explain the evolution of both consumption and wealth per capita among persistently altruistic descendants.

Indeed, by (2) and (4), we get, for all $t \geq 1$,

$$\beta = \frac{x_{t+1} + H}{x_t + H} = \frac{c_{t+1} - \mu}{c_t - \mu}$$

$$\begin{aligned} c_t - \mu &= \beta^{t-1}(1 - q\beta)(x_1 + H) \\ &= \beta^{t-1}(1 - q\beta)\alpha(x_0 + H). \end{aligned} \tag{5}$$

In the last expression, we introduced a simplifying notation

$$\alpha \stackrel{\text{def}}{=} \frac{x_1 + H}{x_0 + H}. \tag{6}$$

The consumption at t of selfish children connected to our altruistic decision-maker at time zero by an uninterrupted chain of bequests is also affected by x_1 (or equivalently by α ; it is made up of two elements: w and the bequest received, viz

$$x_t = \beta^{t-1}\alpha(x_0 + H) - H.$$

When selecting α , the altruistic decision-maker can safely forget about his or other descendants, i.e. those who are out of reach because an intermediate parent is selfish, so that the bequest chain (called hereafter dynastic chain) gets interrupted.

Nevertheless, in so far as he or she is concerned with all his or her descendants' consumption levels, the decision-maker is also interested in the evolution of wealth per capita overtime. We shall proceed with a description of the wealth distribution among all descendants living at some period $t > 1$.

In our model, the bequest received by an agent at t depends almost exclusively on the number of links of the dynastic chain ending with him or her. As p is stationary, a fraction $1 - p$ of agents get no bequest at t . So do of course the members of generation $t - 1$, a subfraction p of which have the dynastic spirit and transmit to each child they have a positive bequest so that $x_t + H = \beta H$. This amount corresponds to (2) applied to dynastic chains of length one, an event occurring at t with probability $(1 - p)p$.

Similarly, to get at t a dynastic chain of length 2, we need a selfish parent at $t-3$, followed by two altruistic parents, respectively at $t-2$ and $t-1$. The probability of this compound event is thus $(1-p)p^2$ and the associated wealth level is $x_t + H = \beta^2 H$.

More generally, the probability to get a dynastic chain starting at $t-\tau$ where $t-\tau > 1$ and still going at t is $(1-p)p^\tau$. The associated wealth level is $x_t + H = \beta^\tau H$.

Finally, the probability to get a dynastic chain starting at 1 and still going at t is p^{t-1} . In this ultimate case, the bequest sequence is not broken between x_1 and x_t , and the associated wealth level is $x_t + H = \beta^{t-1}(x_1 + H)$. This is the only case where x_1 has an influence on x_t .

The number of links in a dynastic chain is thus a well defined random variable, whose distribution is parameterized by the number of periods elapsed since the family started. Its development up to $t = 6$ is pictured in the following table. As $t \rightarrow \infty$, we get the geometric distribution, the discrete analogue of the exponential distribution.

Table 1. Probability distributions for dynastic chains
initiated at $t = 1$

No. of uninterrupted links	Total wealth per cap. ($x_1 = 0$)	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
0	H	$1 - p$	$1 - p$	$1 - p$	$1 - p$	$1 - p$
1	βH	p	$p - p^2$	$p - p^2$	$p - p^2$	$p - p^2$
2	$\beta^2 H$	0	p^2	$p^2 - p^3$	$p^2 - p^3$	$p^2 - p^3$
3	$\beta^3 H$		0	p^3	$p^3 - p^4$	$p^3 - p^4$
4	$\beta^4 H$			0	p^4	$p^4 - p^5$
5	$\beta^5 H$				0	p^5

Of course, macroeconomists will be rightly interested in the distribution of total wealth per capita. If $x_1 = 0$, it is described in column 2 of Table 1, and an appropriate name for this derived family would be loggeometric. It is the discrete analogue of the Pareto distribution. If $x_1 > 0$, only one element need be changed, ie the one based on an uninterrupted dynastic chain from 1 to t . This event has probability p^{t-1} and is associated with a total per capita wealth level $\beta^{t-1}(H + x_1)$.

Thus the support of the distribution of total per capita wealth creeps out at each generation and only the last element of the support gets influenced by x_1 . We compute next the expected growth factor in total wealth per capita from period 1 to period $t + 1$ assuming that $x_1 = 0$ and $H > 0$.

$$\begin{aligned}
E\left\{\frac{x_{t+1}}{H} + 1\right\} &= \beta^t p^t + (1-p) \sum_{\tau=0}^{t-1} \beta^\tau p^\tau \\
&= \beta^t p^t + (1-p) \frac{1 - \beta^t p^t}{1 - \beta p} \\
&= \frac{1 - \beta p - 1 + p}{1 - \beta p} \beta^t p^t + \frac{1 - p}{1 - \beta p} \\
&= \frac{p(1 - \beta)}{1 - \beta p} \beta^t p^t + \frac{1 - p}{1 - \beta p}.
\end{aligned} \tag{7}$$

If $\beta p > 1$, this expression grows without bound.

If $\beta p < 1$, it converges from below to $\frac{1-p}{1-\beta p}$, so that the expected increase in total wealth per capita remains finite as time goes by. As it turns out, this expression is increasing and convex in β . When the limit exists, we can easily compute the expected value of bequest per capita with help of (7).

$$\text{Lim}_{t \rightarrow \infty} E\{x_{t+1}\} = \left(\frac{1-p}{1-\beta p} - 1\right)H = \frac{\beta - 1}{(1/p) - \beta} H. \tag{8}$$

This expression is obviously increasing in each one of H, β, p .

Towards computing the variance of the growth factor of total wealth per capita from period one to period $t + 1$, we compute first the mean of its square for $x_1 = 0$.

$$\begin{aligned}
E\left\{\left(\frac{x_{t+1}}{H} + 1\right)^2\right\} &= \beta^{2t} p^t + (1-p) \sum_{\tau=0}^{t-1} \beta^{2\tau} p^\tau \\
&= \frac{p(1-\beta^2)}{1-\beta^2 p} \beta^{2t} p^t + \frac{1-p}{1-\beta^2 p}.
\end{aligned}$$

If $\beta^2 p > 1$, this expression grows without bound and so does the variance.

If $\beta^2 p < 1$, it converges to $\frac{1-p}{1-\beta^2 p}$.

In this case, the variance converges to $\frac{1-p}{1-\beta^2 p} - \left(\frac{1-p}{1-\beta p}\right)^2$. As it turns out, this expression is increasing in β if β is large enough or if $p > 1/2$ and $\beta > 1$.

As we pointed out in the introduction, we shall be especially interested in parameter configurations conducive to a steady state in expected wealth per capita with positive bequests, i.e. for which $\frac{1}{p} > \beta > 1$.

Section 2. Validating social conventions.

We shall proceed by introducing formally an array of preferences and an equilibrium concept that can account for the conventional bequest behaviour we have discussed so far. More specifically, we consider the problem of a parent at time zero who assumes that all altruistic descendants uphold the convention, whereas selfish descendants leave no bequest to their own children. We shall exhibit preferences under which the best response bequest of the original parent coincides with the conventional bequest function, provided parameter β is suitably chosen, in relation with the fundamentals of the economy. We shall assume that children cannot commit themselves credibly towards their parent concerning their consumption and bequest behaviour. Moreover, parents cannot tell apart their children's types. Once a member of generation t gets x_t , he or she is to be concerned only with prospective developments, as x_t can no longer get changed. As we have emphasized in the introduction, we are dealing with a multistage game of incomplete information that is played noncooperatively, where the state variable x_t is taken as a sufficient statistic of the past history of the game and the kind of equilibrium we have in mind is known as Markov perfect.

Let us assume that the decision-maker ascribes to every agent he or she is concerned with the same individual utility function

$$u_t = u(c_t - \mu), \quad t \geq 0$$

where u is increasing and concave.

If the decision-maker is concerned only with himself or herself, his/her decision criterion is just $u(c_0 - \mu)$ and there is a single dominating strategy, to wit $x_1 = 0$ and $c_0 = w + x_0$, because negative bequests are prohibited.

Turning next to a decision-maker with dynastic preferences, we assume that he or she is concerned also with every descendant's welfare level and that the aggregation

process can be split in three levels: within the nuclear family of all m children of any parent, between all nuclear families living at the same generation and across all generations.

We shall assume an extreme form of inequality aversion among all children of the same altruistic parent. For all practical purposes, the maximin rule is applied at this aggregation level. The family utility, i.e. the individual utility level eventually ascribed to all children of the same altruistic parent, be they selfish or altruistic, is the one enjoyed by the child with the lowest consumption level, i.e. by an altruistic child.

We move next to aggregating over all nuclear families living at t . At this level, we assume a pure expected family utility representation. In other words, the welfare level ascribed to a particular generation of descendants is a weighted sum of family utilities, with weights corresponding to the full length of each family parent's dynastic chain. Thus, at generation 1, there is only one nuclear family and its weight is one. At generation 2, there are pm families with dynastic chain of length one and $(1-p)m$ families with dynastic chain of length zero, etc.

When aggregation is to be done across generations, our decision maker at period zero is assumed to discount generation t 's expected utility with help of weight γ^t where γ is a psychological parameter, and to add up the weighted expected utilities of all generations from 0 to ∞ . If we were to set $\gamma = m$, there would be a completely symmetrical treatment of all family expected utilities in the objective function. Assuming instead that $\gamma < m$ would mean that egalitarianism gets weaker as we proceed from one level of aggregation to another.

Instead of invoking egalitarian feelings to justify inequality aversion among children of the same altruistic parent, we could also assume that the latter get irritated when some descendants squander what they get from them and violate social bequest

conventions, and these feelings may have the same implications as egalitarianism.¹

To simplify our formal description of dynastic preferences at time zero, we shall be explicit only about variables which depend directly or indirectly on the decision chosen at time zero and we can treat as constant the individual utility level of all descendants whose dynastic chain fails to reach back to period zero. Within reach of our altruistic parent at time zero, there are m children, pm^2 grandchildren, p^2m^3 great grandchildren and so on.

Moreover, in view of (1), (3) and (6), we can write successively

$$c_0 - \mu = x_0 + w - \mu - qx_1 = (x_0 + H) - q(x_1 + H) = (x_0 + H)(1 - q\alpha). \quad (9)$$

We are now in position to put together equations (5) and (9) and our assumptions concerning the dynastic preferences of an agent living at period zero and to provide a simple expression representing the latter as a function of a single control variable, viz. x_1 , or equivalently α .

$$\begin{aligned} W_0^* &= u[(x_0 + H) - q(x_1 + H)] \\ &\quad + \gamma u[(1 - q\beta)(x_1 + H)] \\ &\quad + p\gamma^2 u[\beta(1 - q\beta)(x_1 + H)] \\ &\quad + p^2\gamma^3 u[\beta^2(1 - q\beta)(x_1 + H)] \\ &\quad \vdots \\ &\quad + p^t\gamma^{t+1} u[\beta^t(1 - q\beta)(x_1 + H)] \cdots + \text{constant terms} \\ &= u[(x_0 + H)(1 - q\alpha)] + \frac{1}{p} \sum_{t=1}^{\infty} (p\gamma)^t u[\beta^{t-1}(1 - q\beta)\alpha(x_0 + H)] \\ &\quad \cdots + \text{constant terms.} \end{aligned} \quad (10)$$

¹Alternatively, one can assume that all descendants are altruistic but an uninsurable random event may wholly destroy the bequests they want to transmit with probability $1 - p$.

Let us further specify u as follows: either

$$u(c_t - \mu) = \frac{1}{\sigma - 1}(\sigma c_t - \sigma \mu)^{\frac{\sigma - 1}{\sigma}}, \quad \sigma > 0, \sigma \neq 1 \quad (11)$$

$$\text{or } u(c_t - \mu) = \log(c_t - \mu) \quad (12)$$

As is well known, σ is a key parameter linked with risk aversion and inequality aversion: in our setup, we do not disentangle them at the higher aggregation level. Aversion decreases as σ increases. Indifference curves flatten as $\sigma \rightarrow \infty$ and they get L -shaped as $\sigma \rightarrow 0$. On the other hand, u is bounded below and unbounded above if $\sigma > 1$, whereas it is unbounded below, and bounded above if $\sigma < 1$.

Although our interpretation is quite different, the multistage game played by an altruistic agent and his or her altruistic descendants whose dynastic chain reaches back to him or her turns out to be formally the same as the one studied almost thirty years ago by Phelps and Pollak (1968). To see this, we further adapt (10) with help of (11) and we introduce two temporary pieces of notation, viz. $\tau =_{\text{def}} \frac{1}{p}$ and $\xi =_{\text{def}} p\gamma$. Let

$$\begin{aligned} W_0 &=_{\text{def}} \frac{W_0^* - \text{constant terms}}{(x_0 + H)^{\frac{\sigma - 1}{\sigma}}} \\ &= u(1 - q\alpha) + \tau \sum_{t=1}^{\infty} \xi^t u[\alpha \beta^{t-1} (1 - q\beta)]. \end{aligned} \quad (13)$$

In their pioneering article, Phelps and Pollak (1968) rely on a much similar representation of preferences of typical players who are not perfectly altruistic. In their deterministic context, they interpret ξ as a pure impatience parameter having nothing to do with altruism, a feature they attempt to capture by setting τ close to 1, whereas selfishness implies a τ value close to zero. Understandably, Phelps and Pollak restricted their study to the interval $\tau \in [0, 1]$. They further assumed a stationary linear technology and a family of successive generations with stationary preferences

as in (13). They introduced what amounts to the concept of Markov perfect equilibrium and showed that it solves their conflict of generations in a Pareto inefficient way: every agent would gain by saving more than what he or she would in equilibrium.

We introduced random intermissions in what would otherwise be considered a consistent Ramsey modified dynastic spirit. Hence, we let $\tau = \frac{1}{p} \gg 1$ in (13), and it will not be a surprise to discover that we get too much saving in Markov perfect equilibrium, at least from the viewpoint of the successive altruistic agents.

Let us proceed by defining the best response of an altruistic decision-maker at period zero:

$$\forall x_0 \gg 0, x_1(x_0; \beta) =_{\text{def}} \operatorname{argmax}_{x_1} W_0^*.$$

At a Markov perfect equilibrium, the altruistic decision-maker cannot influence β : equilibrium occurs if

$$\forall x_0 \gg 0, x_1(x_0; \beta) = \beta x_0 + (\beta - 1)H$$

where $H = \frac{w-\mu}{1-q}$.

Alternatively, we can define

$$\begin{aligned} \alpha(\beta) &=_{\text{def}} \operatorname{argmax}_{\alpha} W^0 \text{ and observe that} \\ \alpha(\beta) &= \frac{x_1(x_0; \beta) + H}{x_0 + H}, \text{ so that equilibrium occurs} \end{aligned} \tag{14}$$

if $\alpha(\beta) = \beta$. Thus, our focus is a fixed point of $\alpha(\cdot)$, which we call the decision-maker's reaction function; we interpret it as a natural noncooperative solution of the conflict between an altruistic parent and his or her altruistic descendants.

To illuminate this conflict, we shall use as counterfactuals two alternative hypothetical situations where the decision-maker can influence his or her followers's β .

Let us first imagine that the decision-maker can choose any specific β value and make sure that his or her altruistic descendants will not select another one. Then the best injunction is $\beta^* =_{\text{def}} \arg\max_{\beta} W^0$.

As we rely on CES preferences (11) or (12), β^* does not depend on α . On the other hand, the best pair (α, β) our decision-maker can think of is obviously $(\alpha(\beta^*), \beta^*)$. The conflict between dynastically minded children and their altruistic parent is illustrated in the following.

Proposition 1: *Let us consider the problem of maximizing W_0 as defined by (13). For parameter configurations satisfying $p, q \in (0, 1), w > \mu > 0, \sigma > 0$ and $\gamma > 0$ with $q \leq p\gamma < q^{1-1/\sigma}$, the maximizers β^* and $\alpha(\beta^*)$ are well-defined and such that $\alpha(\beta^*) > \beta^* = (\frac{p\gamma}{q})^\sigma \geq 1$.*

The proofs of Propositions 1, 2 and 3 can be found in the technical appendix.

It should be remarked that Proposition 1 implicitly relies on (3), our definition of H , and it would be easy to show explicitly that any other definition would be inconsistent with the decision-maker budget constraint.

By virtue of Proposition 1, a dictatorial parent's saving ratio $q\alpha$ would be higher than the one he or she would impose to his or her altruistic descendants. The latter would of course find themselves in the same position as our decision-maker at time zero and they would be tempted not to follow their parent's injunction.

Faced with this problem, one could also imagine a cooperative solution that would be symmetric from each altruistic player's viewpoint. A natural candidate would be $\hat{\beta} =_{\text{def}} \arg\max_{\alpha, \beta} W_0$ subject to $\alpha = \beta$.

But this compromise would not be ideal for the first decision-maker as our next proposition suggests.

Proposition 2: *Let us consider the maximization problem of the last proposition and the same parameter configuration. Then, $\hat{\beta}$ and $\alpha(\hat{\beta})$ are well defined and such that*

$$\alpha(\hat{\beta}) > \hat{\beta} > \beta^* \geq 1.$$

Although $\hat{\beta}$ is the outcome that dominates all symmetric ones from the viewpoint of every coalition of altruistic players, it seems to have little chance to become a social convention because every player in turn would have an incentive to reject it if the others stick to it. It seems to us that the Markov perfect equilibrium is more likely to be stable because no altruistic player is tempted to make a unilateral move away from it.

Proposition 3: *The multistage game defined by means of six parameters $p, q \in (0, 1), w > \mu \geq 0, \sigma$ and $\gamma > 0$ has at least a Markov perfect equilibrium if $q[1 + (1 - q)(\frac{1}{p} - 1)]^{-1} \leq p\gamma < q^{1-1/\sigma}$. Then, there exists a unique MPE based on an affine bequest function of the form (2) with $\beta > 1$. The latter must moreover satisfy $H = \frac{w-\mu}{1-q}$ and*

$$\frac{R}{m} = \frac{1}{q} = (1 - p)\beta + \frac{\beta^{1/\sigma}}{\gamma} \quad (15)$$

and this value of β is larger than both $\hat{\beta}$ and β^ . Finally, in every MPE, selfish agents bequeath nothing.*

Equation (15) is obtained by looking at the first order condition yielding $\alpha(\beta)$ and by setting $\alpha(\beta) = \beta$. As we already pointed out, the MPE value of β is Pareto-inefficient from the viewpoint of successive altruistic family members. They would strictly prefer $\hat{\beta}$ as a stationary value and this would be detrimental to selfish agents, as it would imply a slower capital accumulation. Yet, one can hardly deny that selfish agents are taking undue advantage of the fact that they disregard social conventions. Some inheritance taxation would benefit altruistic agents and what accounts for paradoxical result is their failure to discriminate among altruistic and selfish descendants.

Notice that our uniqueness statement is conditional on the bequest function's belonging to a specific family and as such it may sound little surprising. Let us introduce at this stage a side remark: there exist parameter values for which our model remains interpretable and such that equation (15) yields two meaningful roots. This can be the case if both $\sigma < 0$ in (13) and $\mu > w$ in (10) so that $H < 0$. We have to interpret μ no longer as minimal consumption but as saturation consumption, and equation (2) describes how the bliss wealth level gets approached. For this to make sense over time with positive bequest, we now have to require $\beta \leq 1$, so that a fixed fraction of the gap between actual wealth and the bliss wealth level gets closed at each dynastic link.

In similar fashion, with $\sigma < 0$, the objective function is the opposite of a convex cost function, and $\sigma = -1$ corresponds to the quadratic case. Interestingly, this is symmetrical to the logarithmic utility function (where $\sigma \rightarrow 1, w > \mu$). When roots are multiple in the present context, it looks legitimate to retain only the root yielding the highest total expected utility level, because it Pareto dominates the other one and the actual decision-maker is assuming a leader's role vis-à-vis his or her descendants. But we have not been able to prove that this argument allows to retain only one root in every parameter configuration. Be this as it may, we decided to consider the cases where $\sigma < 0$ as of meager economic interest because they imply increasing absolute risk aversion. In other words, if $\sigma < 0$, the decision-maker becomes more and more reluctant to accept bets of a given size as he or she grows richer and richer.

In the rest of the paper, we shall therefore retain the $\sigma > 0$, and $w > \mu$ assumptions.

We turn next to some exercises in comparative analysis based on (15). We shall focus our attention on changes in the market interest factor R and their implications.

Let us first study the influence of R on β , the growth factor of both consumption and total wealth per capita after netting out minimal consumption μ , between

members of successive generations of a continued dynastic chain.

Inspection of (15) reveals that R is increasing in β . Moreover R is concave (resp. convex) in β if $\sigma > 1$ (resp. $\sigma < 1$), whereas the relation is linear in the logarithmic case. Hence, β is increasing in R ; the function is convex (resp. concave) if $\sigma > 1$ (resp. $\sigma < 1$).

We study next the ratio of free consumption ($c_t - \mu$) to wealth per capita among altruistic agents, viz. $(1 - q\beta)$ as can be inferred from equation (4). For this purpose we divide each side of (15) by β , and observe that $\frac{1}{q\beta}$ is decreasing (resp. increasing) in β if $\frac{1}{\sigma} < 1$, i.e. $\sigma > 1$ (resp. $\frac{1}{\sigma} > 1$, i.e. $\sigma < 1$). Hence $(1 - q\beta)$ is decreasing in R if $\sigma > 1$ and increasing in R if $\sigma < 1$, whereas it is constant in the log case.

Before we turn to stationary paths in expected wealth per capita, we shall study the range of R values for which an equilibrium with positive bequests exists. For H to be positive in (3) we require $R > m$. Positive bequests obtain if $\beta > 1$. Positive consumption is implied by $q\beta < 1$, ie $\beta < \frac{1}{q} = R/m$. The latter inequality turns out to be necessary for convergence of the objective function. Thus, a lower bound for R is obtained after substituting 1 for β in (15):

$$\frac{R}{m} > 1 - p + \frac{1}{\gamma}. \quad (16)$$

It is a restatement, in strict form, of the first inequality in Proposition 3. Another bound can be obtained by substituting similarly $\frac{R}{m}$ for β :

$$\frac{R}{m} < (1 - p)\frac{R}{m} + \frac{(R/m)^{1-\sigma}}{\gamma}.$$

Dividing through by R/m , we get successively

$$\begin{aligned} 1 &< 1 - p + \frac{(R/m)^{\frac{1}{\sigma}-1}}{\gamma} \\ p\gamma &< (R/m)^{\frac{1-\sigma}{\sigma}} = q^{1-1/\sigma} \end{aligned} \quad (17)$$

an inequality already mentioned in Proposition 3. If $\sigma < 1$, the RHS is increasing in its argument and the inverse function is also increasing, hence

$$\frac{R}{m} > (p\gamma)^{\frac{\sigma}{1-\sigma}}.$$

If, on the other hand $\sigma > 1$, the inverse function is decreasing, so that

$$\frac{R}{m} < (p\gamma)^{\frac{\sigma}{1-\sigma}}.$$

For readers who believe that growth of wealth per capita must be bounded in the very long run, we impose stationarity from this viewpoint by adding the requirement $\beta < \frac{1}{p}$. If $\frac{R}{m} = \frac{1}{q} < \frac{1}{p}$, the convergence condition $\beta < \frac{R}{m}$ implies stationarity. Otherwise, we have to impose an upper bound on R/m by substituting $1/p$ for β in (15):

$$\frac{R}{m} < \frac{1}{p} - 1 + \frac{1}{\gamma p^{1/\sigma}}.$$

We are now ready to summarize our findings.

Proposition 4: *The unique MPE based on an affine bequest function described in the last proposition generates a stationary path for expected wealth per capita if $\frac{R}{m}$ lies in an interval defined as follows:*

$$\text{if } \sigma < 1, \text{ } \text{Max}\{1, 1 - p + \frac{1}{\gamma}, (p\gamma)^{\frac{\sigma}{1-\sigma}}\} < \frac{R}{m} < \frac{1}{p} - 1 + \frac{1}{\gamma p^{1/\sigma}};$$

$$\begin{aligned} \text{if } \sigma > 1, \text{ either } \text{Max}\{1, 1 - p + \frac{1}{\gamma}\} < \frac{R}{m} < \text{Min}\{(p\gamma)^{\frac{\sigma}{1-\sigma}}, \frac{1}{p}\} \\ \text{or } \text{Max}\{1 - p + \frac{1}{\gamma}, \frac{1}{p}\} < \frac{R}{m} < \text{Min}\{(p\gamma)^{\frac{\sigma}{1-\sigma}}, \frac{1}{p} + (\frac{1}{\gamma p^{1/\sigma}} - 1)\}. \end{aligned}$$

As $p \rightarrow 0$, the last interval becomes irrelevant and the remaining ones have the same limiting lower bound viz. $1 + \frac{1}{\gamma}$, whereas their upperbound increases boundlessly. In contrast, as $p \rightarrow 1$, we must have by (15)

$$\beta = 1 = \gamma \frac{R}{m}.$$

The observant reader will have noticed that the last clause of Proposition 4 is just the razor edge property that makes the modified Ramsey growth model often unsuitable for studying the incidence of taxation. By setting p close enough to 0, we obtain a range of R values consistent with stationarity as large as we please.

We continue our study of changes induced by a rise in R by looking at the ratio of expected wealth per head along a stationary path to “excess” wage $w - \mu$.

From (7) and the definition of H (3), we obtain

$$\text{Lim}_{t \rightarrow \infty} E\{x_t\}/(w - \mu) = \frac{\beta - 1}{(1/p - \beta)(1 - \frac{m}{R})}.$$

Obviously, the numerator vanishes if $\beta \rightarrow 1$. So does the denominator if either $\beta \rightarrow \frac{1}{n}$ or $R \rightarrow m$. Since β is increasing in R , the whole expression must be increasing in R under the first two assumptions whereas it must be decreasing in R in case $R \rightarrow m$ (from above), a thought experiment that is consistent with the inequalities listed in Proposition 4 only if $\sigma < 1$ and $p\gamma = 1$. Although we believe that a higher R generates a higher value of $\text{Lim}_{t \rightarrow \infty} E\{x_t\}$ under most circumstances, we can show that for any $p, \frac{R}{m}$ and β consistent with the first inequalities in Proposition 4, we can reverse the sign of the derivative by letting σ become small enough and γ correspondingly large enough. The latter assumption may of course be unwarranted, if one considers that altruism is limited.

Let us point out at this stage that $(\frac{\text{Lim}_{t \rightarrow \infty} E\{x_t\}}{w - \mu} + 1)$ tells also something about the spread of the income distribution as it can be interpreted as the ratio of the average

consumption of a selfish agent to the minimal consumption of another selfish agent, after netting out μ . But this is only one part of the story as this ratio remains finite even though the variance of x_t increases boundlessly, i.e. if $\beta \gg \frac{1}{\sqrt{p}}$.

We are now in position to study the implications of a Barro-type experiment. For this purpose, we reinterpret our model in the context of a small economy with overlapping generations of the standard type. This is permitted if the experiment has no influence on R . Consider a one shot increase in public debt, the proceeds of which are given away to members of the old generation in uniform lump-sum fashion, whereas the burden of servicing is spread equally on all agents of all ensuing generations by means of a uniform wage tax.

Suppose first that we start from a stationary path with positive bequests. Every old selfish family member consumes the bonus and gets a utility increase, in contrast with all old altruistic members of the family who increase their bequest to compensate their dynastically linked descendants for the policy change, so that the numerator of W_0 in (13) remains unaffected. Yet they oppose this policy because they foresee that all their descendants who are not dynastically linked to them will suffer a loss. Thus W_0^* as defined by (10) is decreased by the policy change. What about young family members ? Selfish children of an altruistic parent are compensated and they remain indifferent. Altruistic children of an altruistic parent are in the same position as their parent: the numerator in (13) is unchanged but W_0^* gets decreased by the policy change. Finally all the children of a selfish parent suffer from the policy change.

In conclusion, it is easy to reckon that the losing family members outnumber the gainers if $m \geq 1$ and $p \geq 0$ if two generations overlap.

However, if we consider a family whose altruistic members do not transmit bequests because R is too low, the situation is slightly different. If no old member gets any positive bequest, each one would like to obtain a transfer from the ensuing generation, because the dynastic spirit is too weak even among altruists. Yet the latter

would oppose what they would consider excessively large transfers from the young. On the other hand, every young agent would suffer from the change. Thus among agents living at the same period, the losers outnumber the gainers even though the transfer is moderate whenever $m > 1$.

Interested readers may easily guess the implications of a pay as you go pension scheme along similar lines. Our conclusions about gainers and losers would remain unchanged.

Conclusion

We believe our model offers an attractive middle ground between Diamond economies and purely dynastic setups. In particular, it generates an interesting income distribution, even though labor income is the same for all individuals. If generations overlap in the standard way, policies designed to circumvent the prohibition of negative bequests can easily be studied if R is unaffected but it would seem that the losers outnumber the gainers if all members of any generation are treated uniformly and population is increasing. We conjecture that this conclusion can be reversed if transfer policies also involve an element of redistribution among members of the same generation.

On the other hand, we have observed that all altruistic agents would gain by instituting an inheritance tax, the proceeds of which would be uniformly distributed. This is explained by their inability to discriminate between selfish and altruistic children in their bequest policy, which generates excess saving in Markov perfect equilibrium, at least from their viewpoint. We hope to come back to this subject in future work.

Technical Appendix

Proof of Proposition 1.

In view of (11) and (13), we can write

$$W_0 = \frac{1}{\sigma - 1}(\sigma - \sigma q\alpha)^{1-\frac{1}{\sigma}} + \frac{1}{p} \sum_{t=1}^{\infty} (p\gamma)^t \frac{(\sigma\alpha\beta^{t-1}(1-q\beta))^{1-\frac{1}{\sigma}}}{\sigma - 1}$$

where $q\beta > 1$ if consumption exceeds μ .

For convergence of the infinite sum, we require $p\gamma\beta^{1-\frac{1}{\sigma}} < 1$.

Under these conditions, we seek to maximize

$$W_0 = \frac{1}{\sigma - 1}(\sigma - \sigma q\alpha)^{1-\frac{1}{\sigma}} + \frac{\gamma(\sigma\alpha)^{1-\frac{1}{\sigma}}}{(\sigma - 1)} \frac{(1 - q\beta)^{1-\frac{1}{\sigma}}}{1 - p\gamma\beta^{1-\frac{1}{\sigma}}}. \quad (A.1)$$

Let us define

$$\Psi(\beta) \stackrel{\text{def}}{=} \frac{(1 - q\beta)^{1-\frac{1}{\sigma}}}{1 - p\gamma\beta^{1-\frac{1}{\sigma}}} \quad (A.2)$$

and differentiate W_0 .

$$\frac{\partial W_0}{\partial \alpha} = -q(\sigma - \sigma q\alpha)^{-\frac{1}{\sigma}} + \gamma\Psi(\beta)(\sigma\alpha)^{-\frac{1}{\sigma}} \quad (A.3)$$

$$\frac{\partial W_0}{\partial \beta} = \gamma \frac{(\sigma\alpha)^{1-\frac{1}{\sigma}}}{\sigma - 1} \Psi'(\beta) \quad (A.4)$$

where

$$\Psi'(\beta) = \frac{-q}{\sigma} \frac{(1 - q\beta)^{-\frac{1}{\sigma}}(\sigma - 1)}{(1 - p\gamma\beta^{1-\frac{1}{\sigma}})} + \frac{p\gamma}{\sigma} \frac{(1 - q\beta)^{1-\frac{1}{\sigma}}(\sigma - 1)\beta^{-\frac{1}{\sigma}}}{(1 - p\gamma\beta^{1-\frac{1}{\sigma}})^2} \quad (A.5)$$

Rearranging (A.5), we get

$$\Psi'(\beta)\left(\frac{\sigma}{\sigma-1}\right)(1-q\beta)^{\frac{1}{\sigma}}(1-p\gamma\beta^{1-\frac{1}{\sigma}})^2\beta^{\frac{1}{\sigma}} = -q\beta^{\frac{1}{\sigma}}(1-p\gamma\beta^{1-\frac{1}{\sigma}}) + p\gamma(1-q\beta). \quad (A.6)$$

$$\text{As } \beta^* =_{\text{def}} \arg\max_{\beta} W_0, \quad \Psi'(\beta^*) = 0$$

$$p\gamma(1-q\beta^*)\beta^{*- \frac{1}{\sigma}} = q(1-p\gamma\beta^{*1-\frac{1}{\sigma}})$$

$$p\gamma\beta^{*- \frac{1}{\sigma}} - p\gamma q\beta^{*1-\frac{1}{\sigma}} = q - p\gamma q\beta^{*1-\frac{1}{\sigma}}$$

$$p\gamma\beta^{*- \frac{1}{\sigma}} = q \quad (A.7)$$

$$\beta^* = \left(\frac{p\gamma}{q}\right)^{\sigma}. \quad (A.8)$$

We want consumption above μ to be positive by setting $\beta < \frac{1}{q}$. By (A.7), $1-p\gamma\beta^{*1-\frac{1}{\sigma}} = 1-q\beta^*$ so that free consumption is positive iff the convergence condition holds. For desired bequest to be nonnegative we require $1 \leq \beta$.

By (A.8), $1 \leq \beta^* < 1/q$ if $q \leq p\gamma < q^{1-\frac{1}{\sigma}}$.

We turn next to $\alpha(\beta)$, which is obtained by setting $\frac{\partial W_0}{\partial \alpha} = 0$ in (A.3). Rearranging and simplifying, we get successively

$$\begin{aligned} q(\sigma - \sigma q\alpha)^{-\frac{1}{\sigma}} &= \gamma\Psi(\beta)(\sigma\alpha)^{-\frac{1}{\sigma}} \\ \frac{q}{\gamma\Psi(\beta)} &= \left(\frac{\sigma - \sigma q\alpha}{\sigma\alpha}\right)^{\frac{1}{\sigma}} = \left(\frac{1}{\alpha} - q\right)^{\frac{1}{\sigma}} \\ \frac{1}{\alpha} - q &= \left(\frac{q}{\gamma\Psi(\beta)}\right)^{\sigma} \end{aligned} \quad (A.9)$$

To complete the proof of Proposition 1, we establish that $\alpha(\beta^*) > \beta^*$.

In view of (A.7), we get $q\beta^* = p\gamma\beta^{*1-\frac{1}{\sigma}}$.

Therefore, by virtue of (A.2) and (A.7), we obtain

$$\begin{aligned}\Psi(\beta^*) &= (1 - q\beta^*)^{-\frac{1}{\sigma}} \\ \frac{\gamma\Psi(\beta^*)}{q} &= \frac{1}{p}\left(\frac{1}{\beta^*} - q\right)^{-\frac{1}{\sigma}} \\ \left(\frac{q}{\gamma\Psi(\beta^*)}\right)^\sigma &= p^\sigma\left(\frac{1}{\beta^*} - q\right).\end{aligned}$$

Thus, unless $p = 1$ or $\sigma = 0$,

$$\frac{1}{\alpha(\beta^*)} - q < \frac{1}{\beta^*} - q \text{ and } \frac{1}{\alpha(\beta^*)} < \frac{1}{\beta^*}.$$

■

Proof of Proposition 3.

We first establish (15). For this purpose, we combine (A.2) with (A.9) where α is set equal to β so that $\alpha(\beta) = \beta$.

$$\frac{1}{\beta} - q = \left(\frac{q}{\gamma}\right)^\sigma \frac{(1 - p\gamma\beta^{1-\frac{1}{\sigma}})^\sigma}{(1 - q\beta)^{\sigma-1}} = \frac{1}{\beta}(1 - q\beta).$$

Dividing through by $(1 - q\beta)$ and rearranging, we get successively

$$\beta = \left(\frac{\gamma}{q}\right)^\sigma \left(\frac{1 - q\beta}{1 - p\gamma\beta^{1-\frac{1}{\sigma}}}\right)^\sigma \quad (\text{A.10})$$

$$\beta^{1/\sigma} = \frac{\gamma}{q} \left(\frac{1 - q\beta}{1 - p\gamma\beta^{1-\frac{1}{\sigma}}}\right) \quad (\text{A.11})$$

$$\beta^{\frac{1}{\sigma}} - p\gamma\beta = \frac{\gamma}{q} - \gamma\beta$$

$$\frac{1}{q} = (1 - p)\beta + \frac{\beta^{\frac{1}{\sigma}}}{\gamma} \quad (15).$$

Next, we show that if (15) holds, $1 - q\beta$ and $1 - p\gamma\beta^{1-\frac{1}{\sigma}}$ must be of the same sign. Indeed, we require β to be positive and (15) proceeds from (A.10), where the r.h.s. must be positive. Inequality (17) is sufficient to make sure that $1 > q\beta$.

Similarly, in weak form, inequality (16) is sufficient to make sure that $\beta \geq 1$. Multiplying both sides by $\frac{1}{p}$, we get

$$\begin{aligned}\frac{1}{qp} &\geq \frac{1-p}{p} + \frac{1}{\gamma p} \\ \frac{1}{\gamma p} &\leq \frac{1-q(1-p)}{qp} = \frac{1}{q} \left[\frac{1}{p} - q \left(\frac{1}{p} - 1 \right) \right] = \frac{1}{q} \left[1 + \left(\frac{1}{p} - 1 \right) (1-q) \right].\end{aligned}$$

Rearranging this expression, we obtain the first inequality in proposition 3.

By definition, β^* is such that the partial derivative $\frac{\partial W_0}{\partial \beta}$ vanishes. As (A.8) indicates, β^* does not depend on α . By definition, $\hat{\beta}$ is such that the directional derivative along the bissectrix in the (α, β) plane vanishes:

$$\frac{\partial W_0}{\partial \alpha} \Big|_{\alpha=\beta} + \frac{\partial W_0}{\partial \beta} \Big|_{\beta=\alpha} = 0. \quad (\text{A.12})$$

We want to evaluate the LHS of (A.13) at the MPE value of β , at which $\alpha(\beta) = \beta$ as expressed by (15). Now, the best response $\alpha(\beta)$ is characterized by $\frac{\partial W_0}{\partial \alpha} = 0$ for every β in the relevant domain. Thus, there remains to evaluate $\frac{\partial W_0}{\partial \beta}$ at the fixed point (15). By (A.4), we know that $\frac{\partial W_0}{\partial \beta}$ has the sign of $\frac{\Psi'(\beta)}{\sigma-1}$, which is also the sign of the LHS of (A.6). We combine this with (A.11) and we get

$$p\gamma(1-q\beta) - q\beta^{\frac{1}{\sigma}}(1-p\gamma\beta^{1-\frac{1}{\sigma}}) = (p-1)\gamma(1-q\beta)$$

where the RHS is negative if $p < 1$.

We conclude that both the partial derivative $\frac{\partial W_0}{\partial \beta}$ and the LHS of (A.13) are negative, so that the MPE value of β is larger than both β^* and $\hat{\beta}$.

The proof of the uniqueness of the definition (3) if (2) is a maintained assumption, is obtained by showing that for every other definition, the FOC needed for a maximum of W_0^* as defined by (10) are inconsistent with the requirement that H and β in (2) be independent of x_t in MPE. ■

Proof of Proposition 2.

Again we analyse the sign of the directional derivative along the bisectrix, as expressed by the LHS of (A.13). The sign of the sum must be the same as the signs of its components when the latter are the same. By proposition 1, they are both negative if $\alpha = \beta < \beta^*$ and both positive if $\alpha = \beta$ to the right of the fixed point $\alpha(\beta) = \beta$. Hence the LHS of (A.13) must vanish in between, to the right of β^* so that $\frac{\partial W_0}{\partial \beta}$ must be negative, whereas $\frac{\partial W_0}{\partial \alpha} < 0$. In other words, $\hat{\beta}$ is located strictly between β^* and the MPE value of β if $p < 1$, and $\alpha(\hat{\beta}) > \hat{\beta}$. ■

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