



UNIVERSITÉ CATHOLIQUE DE LOUVAIN
École Polytechnique de Louvain
Institute of Information and Communication Technologies,
Electronics and Applied Mathematics (ICTEAM)
1348, Louvain-la-Neuve, Belgium

**Modulated Metasurface Antennas:
Analysis and Direct Numerical Synthesis**

by
Modeste BODEHOU

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requirements for the Ph.D. Degree in Applied Sciences

Examining Committee

Advisors: Prof. Christophe CRAEYE, UCLouvain, Belgium
Prof. Isabelle HUYNEN, UCLouvain, Belgium
Jury: Prof. Stefano MACI, University of Siena, Italy
Prof. Zvonimir SIPUS, University of Zagreb, Croatia
Dr. Thierry GILLES, AGC Glass Europe
Chairperson: Prof. David BOL, UCLouvain, Belgium

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To
my Father Jean BODEHOU, and
my Mother Basilia HOUNNOU
for
their unconditional love, and
sacrifice in my education.

“In many ways, solving a problem somehow quite often has to do with understanding connections between two concepts.”

Caucher Birkar

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Abstract

METASURFACES (MTS) offer an innovative way for the generation and manipulation of electromagnetic (EM) waves. The EM properties of usual surfaces/materials are dictated by their atomic/molecular constituent. Therefore the achievable performance of the resulting EM components (lens, absorbers, waveguides, antennas, etc.) are fundamentally limited by the properties of available materials. MTSs inherit EM characteristics from their internal micro-structure, rather than from the chemical composition of the surface. As a result, the achievable properties can surpass the limitations of those of usual surfaces, thus pushing forward the performance of the feasible EM components.

This thesis is devoted to the analysis and design of aperture antennas making use of artificially engineered boundary conditions provided by modulated MTSs. MTS aperture antennas are getting a considerable interest from the antenna community. Such antennas present the advantage of being extremely flat and thin by nature since they rely on the excitation of SWs with a feed located within the substrate layer (MTS plane). They also exhibit very low ohmic losses and can be easily manufactured with standard printed circuit board (PCB) technology. The radiation characteristics of modulated MTS antennas result from the progressive interaction between the excited SW and the spatially modulated surface impedance.

A class of orthogonal, entire-domain, and divergence-conforming basis

functions (named Fourier-Bessel) is first proposed for the efficient analysis of the surface current distribution, in the Method of Moments (MoM) solution of the electric field integral equation (EFIE). The surface impedance is further expanded into Fourier-Bessel basis functions (FBBF), then allows us to address the inverse problem, which consists of computing the MTS EM properties on the basis of a desired aperture current distribution. The antenna synthesis problem is thereafter formulated, first by computing the objective current distribution based on the desired radiation pattern. Secondly, the relevant integral equation is numerically solved while constraining the tensorial surface impedance to be anti-Hermitian so as to avoid losses on the sheet. This results in a systematic and direct algorithm (optimization is not required) able to provide an arbitrary radiation pattern in amplitude, phase and polarization. The derived impedance profile (described in Fourier-Bessel basis) is no longer necessarily locally sinusoidal, which significantly widens the class of synthesizable radiation patterns as compared to the techniques proposed in the literature. Challenging shaped beams and multiple beams in linear and circular polarizations have been designed and numerically validated using independent electromagnetic (EM) simulation software tools. The developed EFIE based formalism is thereafter extended to deal with multiple beams with multiple feeds synthesis, dual-polarization capability, and multiple frequency-band operation. The thesis also introduced a new MTS beam scanning paradigm in which the antenna is fed at multiple points. Beam scanning is obtained after generating proper embedded element patterns with opposite phase slopes, followed by a phasing of those radiation patterns with respect to each other. Near-field applications are tackled as well through the generation of Orbital Angular Momentum (OAM) beams for near-field multiplexing, and Bessel beams for near-field focusing. Free-space transmittance (power transfer) between Fourier-Bessel fields (a particular class of OAM beams) is analyzed and a closed-form expression of the transmittance (valid for the intermediate-to-far field) is derived. Finally, an antenna prototype for radar applications has been developed, experimentally validated, and the power balance in MTS antennas has been rigorously analyzed.

Résumé

LES métasurfaces (MTSs) offrent une alternative novatrice pour la génération et la manipulation des ondes électromagnétiques. Les propriétés électromagnétiques des surfaces et matériaux usuels sont dictées par leur constitution atomique et moléculaire. Par conséquent, les performances accessibles par les composants électromagnétiques usuels (lentilles, absorbeurs, guides d'onde, antennes, etc.) sont fondamentalement limitées par les propriétés des matériaux disponibles dans la nature. Les MTSs héritent leurs caractéristiques électromagnétiques de leur micro-structure interne, plutôt que de la composition chimique de la surface. De ce fait, les propriétés atteignables peuvent surpasser les limites des surfaces usuelles, repoussant ainsi les performances des composants électromagnétiques réalisables.

Cette thèse est consacrée à la simulation numérique et à la conception d'antennes à ondes de fuite basées sur des conditions aux limites surfaciques artificiellement créées par les MTSs modulées. Les antennes à base de MTS reçoivent actuellement un intérêt considérable de la part de la communauté scientifique. Ces antennes présentent l'avantage d'être plates et très fines par nature puisque leur fonctionnement repose sur l'excitation d'ondes de surfaces avec une alimentation située dans le substrat (le plan de la MTS). Les pertes ohmiques y sont également très faibles, et ces antennes peuvent être aisément fabriquées avec la technologie PCB (Printed Circuit Board). Les

caractéristiques de rayonnement des antennes MTS résultent de l'interaction progressive entre une onde de surface générée par l'alimentation et une impédance de surface spatialement modulée.

Une classe de fonctions de base orthogonales, et couvrant l'entièreté de l'ouverture de l'antenne (nommées Fourier-Bessel) est exploitée pour l'analyse efficace des courants surfaciques, dans une formulation reposant sur l'équation intégrale du champ électrique (EFIE). Le profil de l'impédance de surface a par la suite été également discrétisé dans la base de Fourier-Bessel, ce qui a permis d'adresser le problème inverse consistant à évaluer les propriétés de la MTS à partir des courants surfaciques désirés. La conception de l'antenne est formulée en deux temps. Dans un premier temps, les courants surfaciques sont calculés spectralement (dans l'espace de Fourier) à partir du diagramme de rayonnement désiré. Dans un second temps, l'équation intégrale est numériquement résolue tout en contraignant l'impédance de surface tensorielle à satisfaire à la propriété d'“anti-hermitianité” de façon à éviter les pertes dans la MTS. Le résultat est un algorithme systématique et direct (ne nécessitant pas d'optimisation) capable de générer un diagramme de rayonnement tout à fait arbitraire en amplitude, en phase, et en polarisation. L'impédance de surface ainsi calculée (décrite dans la base de Fourier-Bessel), n'est donc pour la première fois plus nécessairement localement sinusoïdale, ce qui élargit significativement la classe de diagrammes de rayonnement concevables en comparaison avec les solutions proposées dans la littérature. Des faisceaux de forme complexe et des faisceaux multiples en polarisation circulaire et linéaire ont été conçus et numériquement validés avec des logiciels de simulation électromagnétique indépendants. Le formalisme basé sur l'EFIE ainsi développé est par la suite étendu pour adresser la conception d'antennes à alimentations multiples, à double polarisation ou celles opérant simultanément sur plusieurs bandes passantes. La thèse a également introduit un nouveau paradigme pour le balayage continu du faisceau: le balayage est obtenu par génération de diagrammes de rayonnement spécifiques, correspondant à différentes alimentations déphasées. Les applications en champ proche ont été aussi traitées, et illustrées par la conception de faisceaux à moments angulaires orthogonaux (OAM) et la génération de faisceaux de Bessel pour la focalisation

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Notations and Acronyms

Mathematical Notations

$j = \sqrt{-1}$	imaginary unit
x	a scalar
$\mathbf{x}$	a vector
$\hat{\mathbf{x}}$	a unit vector
$ x $	absolute value of x
x^*	complex conjugate of x
$\mathbf{x}^H$	Hermitian transposed of $\mathbf{x}$
$ \mathbf{x} ^2$	the squared Frobenius norm of $\mathbf{x}$: $ \mathbf{x} ^2 = \mathbf{x}^H \mathbf{x}$
$\triangleq$	is defined as
$\approx$	is approximately equal to
$\gg$	is much larger than
$\ll$	is much smaller than
$\tilde{F}$	Fourier transform of F

Acronyms

AIM	Adaptive Integral Method
CFFT	Contour Fast Fourier Transform
CST	Computer Simulation Technology
EFIE	Electric Field Integral Equation
EM	Electromagnetics
FBBF	Fourier-Bessel Basis Functions
FFT	Fast Fourier Transform
FMM	Fast Multipole Method
FO	Flat Optics
FSS	Frequency Selective Surfaces
GMRES	Generalized Minimal RESidual
GO	Geometrical Optics
GRBF	Gaussian Ring Basis Functions
GSTC	Generalized Sheet Transition Conditions
IBC	Impedance Boundary Condition
LHCP	Left Handed Circular Polarization
LW	Leaky Wave
MBF	Macro Basis Functions
MIMO	Multiple-Input Multiple-Output
MoM	Method of Moments
MTM	Metamaterial
MTS	Metasurface
OAM	Orbital Angular Momentum
PCB	Printed Circuit Board
PEC	Perfect Electric Conductor
RHCP	Right Handed Circular Polarization
RWG	Rao-Wilton-Glisson
SFX	Synthetic Functions eXpansion
SW	Surface Wave
TE	Transverse Electric
TM	Transverse Magnetic
ZBF	Zernike Basis Functions

Introduction

CENTURIES ago, communication systems were based on riders (horsemen) carrying documents on horseback. Nowadays, complex signal processing algorithms (modulation, source and channel coding, etc) play the role of the horseman, and the horseback has been replaced by electromagnetic (EM) waves [1], owing to their propagation characteristics (velocity, attenuation, etc) as well as their capability of coding information (channel capacity) [2], [3]. EM based wireless systems play an essential role in our daily life, not only for communication purposes, but also in remote sensing and imaging. In that respect, performance of antennas has a strong impact on the overall system efficiency. On the other hand, the market demands for miniaturization and integration raise serious technological challenges for antenna engineers. Space communications and deep-space exploration are facing strong requirements for low-profile antennas with high gain or presenting beam shaping capability [4]. The 5G communication standard and the upcoming internet-of-things revolution also need new conformal antenna solutions (high gain, beamforming) over a wide bandwidth, to deliver higher data rate, or multiple bands to allow platform sharing among different services [5]. It is clear that such requirements cannot be fulfilled with conventional antennas, such as monopole/dipole or parabolic reflectors.

Antenna arrays are probably the most advanced and mature antenna

technology nowadays. The array solution consists of grouping several scattering elements properly fed with an electronic network [6] - [8]. Phasing and weighting each element enables beam shaping and scanning functionality. Antenna arrays have demonstrated their ability of facing tremendous constraints, as required for example in radar applications [9], radio astronomy [10] - [12] or more recently with MIMO communication systems [13] - [15]. However, this technology (originally developed for defense purposes) appears to be often too expensive to address civilian concerns since it requires a complex and bulky (at millimeter waves) beamforming network implemented with a huge number of active elements (phase-shifters, amplifiers, etc). In light of this, there is a great need for low cost alternatives, still offering very good performance.

Metamaterials (MTM) provide new perspectives for the design of EM devices [16]. The concept of MTM has been introduced a few decades ago as a new way of controlling EM waves. According to the EM Maxwell theory, materials are characterized with their effective permittivity and permeability, which describe respectively their electrical and magnetic response. This is made possible because of the sub-wavelength size of the individual atoms constituting the medium. Following the same reasoning, one can engineer macroscopic properties by arranging periodically a dense array of sub-wavelength elements. A proper synthesis of the elements in terms of composition, shape, orientation then allows one to mimic macroscopic properties that can go beyond those of the materials usually found in the nature (e.g negative permittivity [17]/permeability [18] or both [19]). One of the probably most famous applications of this concept is the super lens that can go beyond the diffraction limit [20].

1.1 Metasurfaces

Despite the fascinating achievement of MTMs, they became less attractive this last decade because of issues relating to fabrication complexity, bulkiness, bandwidth and losses. Therefore a lot of interest has been given to their 2D counterpart, commonly referred to as metasurfaces (MTS) [21]. MTSs are electrically thin materials interacting with waves in a given frequency

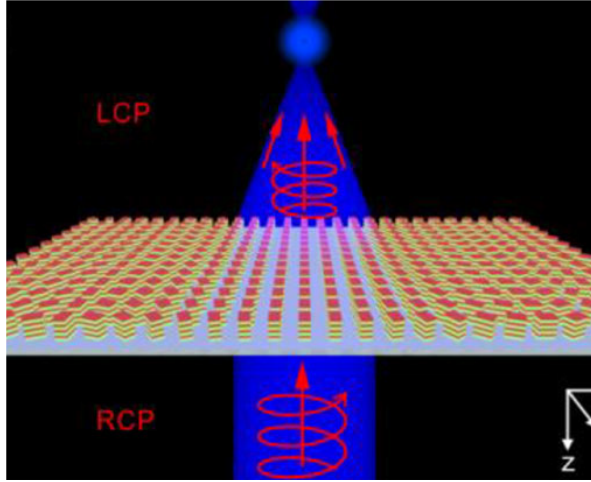


Figure 1.1: From [31], Huygens' metasurface illustration.

band as artificial surfaces [22], (i.e. which properties go beyond those of usual surfaces). They can be used in a wide range of frequencies (from microwaves [23] to optics [24]) to control wave propagation [25]. Space waves manipulation has been recently demonstrated through the so-called Huygens' MTSs [26], [27]. Those MTSs are designed to transform a known incident field into an arbitrary desired transmit one (see Fig. 1.1) through the induction of electrical and magnetic currents at the surfaces. Although this concept is actually similar to transmit-arrays, or frequency selective surfaces (FSS), the sub-wavelength unit cells used in the case of Huygens' MTSs allow the fields to be homogenized, which enables additional functionalities with respect to transmit-arrays. Microwave components such as polarization converters [28], beam splitters, deflectors [29] have been demonstrated in the literature. Inversely, MTSs may also be employed to control reflected fields. This idea has been implemented recently for the design of wideband reflect-array antennas [30].

Another interesting feature of MTSs concerns surface-wave (SW) wavefront manipulation (see Fig. 1.2). This type of application has paved the way to a new class of flat transformation electromagnetics devices [32]. Transformation electromagnetics (initially developed for volumetric materials [36],

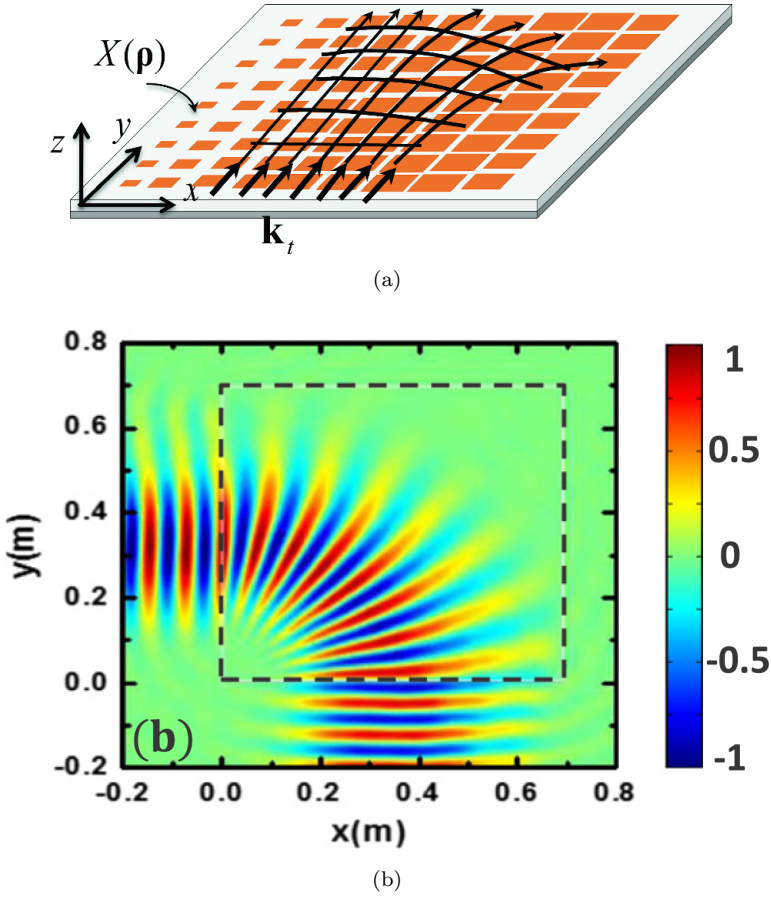


Figure 1.2: Surface wavefront manipulation using metasurface. a) From [33], conceptual illustration. b) From [34], waveguide application.

[37] and thereafter extended to surfaces in [38]) provides a systematic way of manipulating waves through coordinates transformation. This design method relies on the invariance of Maxwell equations with respect to coordinates transformation and on the interpretation of the coordinates transformations as a material transformation in the original space. The material property of MTSs is the boundary condition associated to the surface, the latter can be expressed as an electric or magnetic surface impedance. More elaborated boundary conditions, Generalized Sheet Transition Condition

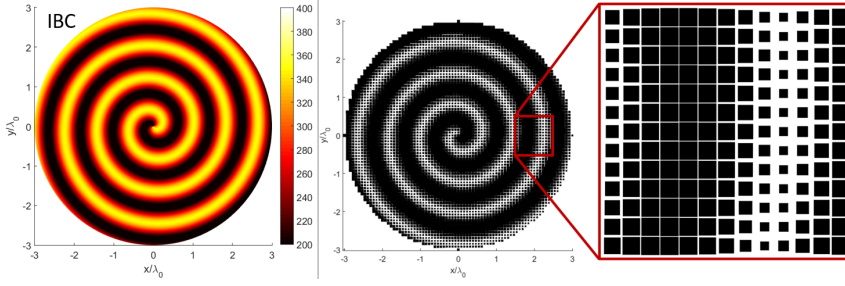


Figure 1.3: Top view of a MTS antenna implemented with printed square patches.

(GSTC) [39] have also been proposed in the literature to achieve exotic properties, such as chirality or non-reciprocity [40]. Famous realizations of transformation EM devices are lenses [41], beam shifters [32], beam splitters [38] etc. Fully metallic MTS lenses implemented with highly symmetric unit cells have been recently demonstrated in [42] for applications requiring larger bandwidth.

1.2 Modulated metasurface antennas

Modulated MTS antennas are a class a leaky-wave (LW) antennas that exploit the modulation of a surface impedance. Such antennas have the advantage of being low profile (flat, thin, low mass) by nature since the antenna is excited with a coplanar feeder. They also exhibit very low power losses and can be easily manufactured with standard PCB techniques [43]. Additionally, the flexibility in achieving shaped beams (active elements are usually used to obtain reconfigurable patterns) makes modulated MTS antennas very attractive in many applications, including space communications and sensing. Those surfaces implement an Impedance Boundary Condition (IBC) on a circular (possibly elliptical [44]) domain, and are usually fed from the center by a SW launcher [45]. The IBC interacts with the excited SW and then generates the desired radiation characteristics. An example of MTS IBC radiating a broadside pencil beam is illustrated in Fig. 1.3. The IBC has been implemented with a dense array of square patches.

The first proof of concept regarding LW antennas based on the surface

impedance modulation has been provided by Oliner and Hessel [46]. It is well known that an inductive constant surface impedance supports the propagation of TM SWs. Such a TM SW does not radiate since its wavenumber falls into the invisible region. By modulating the impedance at the wavelength scale, it becomes possible to create several harmonics of the fundamental SW, some of which can fall into the visible region and then radiate (see Fig. 1.4). This phenomenon constitutes the basic principle behind modulated MTS antennas.

The MTS synthesis problem then consists of finding the appropriate surface impedance modulation for given excitation and pattern requirements. A lot of efforts have been made in this direction. The initial methods were first based on holographic principles [47], [48] and have allowed the control of the phase of the aperture electric field. G. Minatti et al. [49] propose a design procedure inspired from the Oliner paper [46]. An analytical expression of the antenna far field is provided and validated through two practical designs: a grounded dielectric slab with a modulated thickness and a dense array of patches with variable sizes printed on a grounded substrate. The same group further extended the previous approach to anisotropic impedances with the realization of a circularly polarized isoflux radiation pattern for satellite communications [50]. Thereafter, [43] proposes for the first time an amplitude and phase-based synthesis of the aperture field, which significantly widens the class of synthesizable aperture fields. Subsequently, [51] proposes a flat optics (FO) based iterative algorithm for synthesizing a class of radiating aperture electric fields [52]. The used technique consists of locally expanding the fields on the aperture into Floquet harmonics. The surface impedance is then obtained after matching the desired radiating aperture field with the -1 mode of the Floquet expansion. A variant of the FO technique has also been proposed in [53].

1.3 Goals of the thesis

The objective of the present thesis is twofold.

The first goal consists in developing efficient algorithms for the analysis of modulated MTS antennas. Simulation is essential during the design

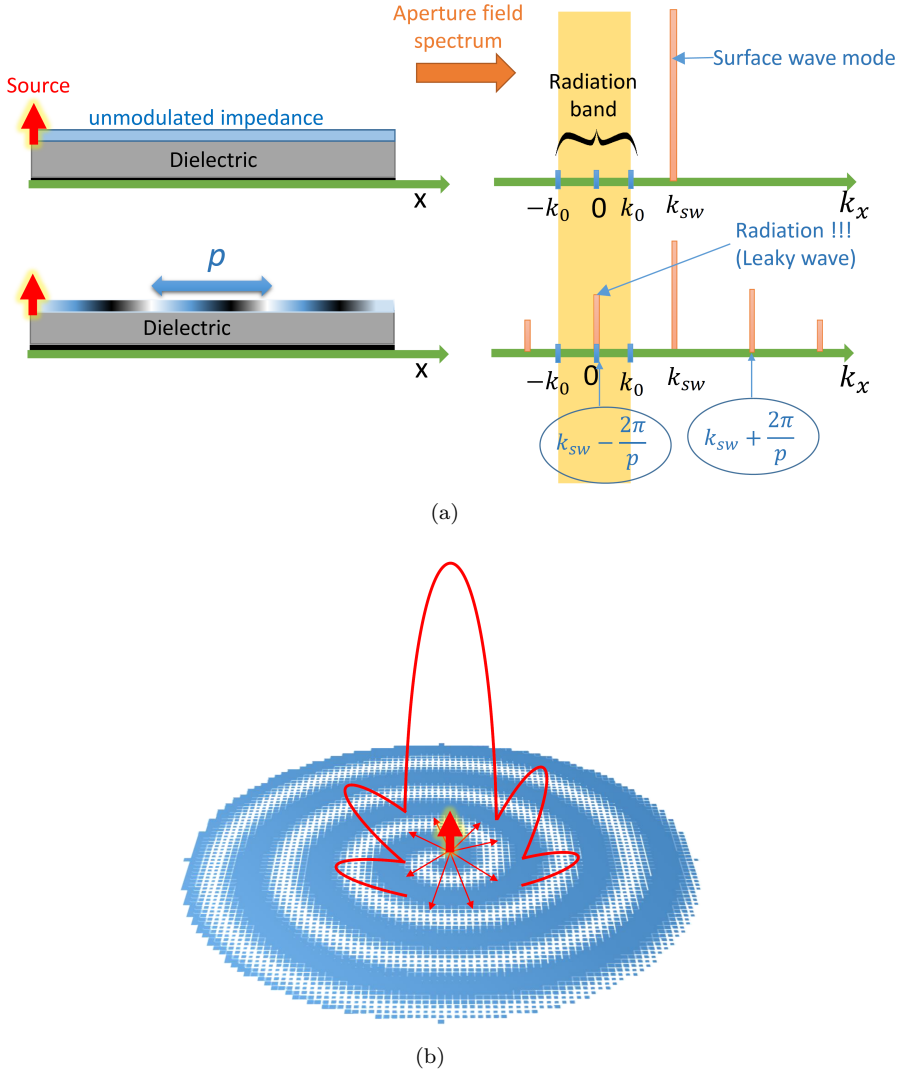


Figure 1.4: a) Basic principle in spectral domain. A periodic modulation (of periodicity p) around an average creates several Floquet modes (spatial harmonics), with the -1 mode falling into the radiation band. b) Corresponding MTS structure and radiation pattern.

procedure and should be carried out relatively fast and accurately to allow possible optimizations. The proposed algorithm should make use of the

physics and symmetry appearing in MTS antennas to efficiently model and discretize the relevant unknowns. This thesis adopted an integral equation formalism because the resulting unknowns (i.e the surface current distribution) can be limited to the MTS plane. This modeling is used in combination with appropriate basis functions to drastically reduce the size of the resulting system of equations matrix.

The second objective is to advance the state of the art on the design of modulated MTS antennas with the aim of opening the way for new functionalities. A systematic algorithm, starting from the desired radiation pattern is proposed for the generation of arbitrary radiation pattern (assumed to be given in amplitude, phase and polarization). Several examples, including multibeam, cosecant squared beam, “fish-like” beam have been designed in circular and linear polarizations. The method is further elaborated to enable reconfigurability functionalities (e.g. multiple beams with multiple feeds, dual polarization, and multi-frequency band operation). Multiple shaped beams at multiple frequencies has been generated for the first time with modulated MTS antennas. More importantly, it has been proven that continuous beam scanning with modulated MTS antennas is possible by feeding the antenna at multiple points, and by phasing the (properly designed) embedded element pattern corresponding to each feed. Finally, near-field synthesis has been demonstrated and discussed through the design of a non-diffractive Bessel beam launcher and multiple Orbital Angular Momentum (OAM) beams.

The power of the proposed algorithms relies on the fact they are numerically inspired. In particular, the developed methods are based on the discretization of the Electric Field Integral Equation (EFIE). Numerical methods are traditionally used for analysis or optimization purposes, but not for synthesis. It has been shown in this thesis that numerical methods can also be employed to provide an efficient and direct solution (without optimization) for the synthesis of modulated MTS antennas, without requiring huge computational resources. All the design results presented in this manuscript have been obtained with a laptop computer with 8 Gb RAM memory and a 2.8 GHz processor and require less than 1 hour computation time.

1.4 Outline and contributions of the thesis

The core of the doctoral dissertation is organized in six chapters, detailed below.

Chapter 2: Numerical simulation of modulated metasurface antennas

This chapter presents a Method of Moments (MoM) algorithm based on the EFIE for the analysis of modulated MTS antennas. Those antennas are usually built on circular (possibly elliptical) domains. A family of orthogonal, and entire-domain basis functions, named Fourier-Bessel [70], is proposed for the efficient discretization of the current distribution on the aperture. It has been proven that this basis allows a more compact representation of the current distribution in comparison with other bases proposed in the literature. The chapter presents an efficient computation of the reaction integrals between Fourier-Bessel basis functions (FBBF) for the EFIE in the presence of layered media. The method is first explained for circular domain antenna and then generalized to elliptical domains, without affecting the computational complexity. Numerical results are compared with other simulation tools and realistic MTS measurements, showing a very good agreement.

Chapter 3: Synthesis of modulated metasurface antennas

The second chapter addresses the synthesis problem of modulated MTS antennas. First, we have demonstrated that the MTS impedance modulation can be computed using the EFIE based on the knowledge of the aperture current distribution. This has been carried out for scalar (isotropic) impedances and then extended to tensorial (anisotropic) surface impedances. However, for antenna applications, only the radiated part of the current distribution is known. The invisible part, corresponding to the antenna near-field is not known a priori. This constitutes the fundamental synthesis problem of modulated MTS antennas. The algorithm proposed here consists of estimating the antenna near-field based on the simulation of the average impedance, and then solve the EFIE, while also constraining the surface impedance to belong to an anti-hermitian space so as to avoid losses on the

sheet. This result in a systematic algorithm able to generate arbitrary radiation patterns in amplitude, phase and polarization. Several challenging examples have been provided and numerically validated.

Chapter 4: On the reconfigurability of metasurface antennas

The previous chapters where based on MTS synthesis with one feeder. Here, we have extended the method developed in Chapter 2 to antennas with multiple feeds and multiple beams. Two approaches are explored. The first one assumes a priori a discrete azimuthal symmetry in the impedance profile, so as to constrain the solution to a subspace which automatically provides multiple beams when illuminated with feeds regularly arranged along azimuth. In the second approach, there are no a priori assumptions on the impedance profile, but the systems of equations corresponding to each beam are stacked and solved simultaneously in the least-squares sense. This second approach can also be used to obtain polarization diversity. It also enables continuous beam scanning through the generation of two embedded patterns in a common azimuthal window with opposite phase slopes, followed by a continuous phasing of the two feed points. To the best of our knowledge, it is the first time that phased MTS antennas are proposed for beam scanning. The EFIE-based method is further extended to deal with multiple frequency band operation through a semi-separable model of the MTS sheet impedance. This capability enables the multiplexing of several applications (with different radiation pattern specifications) sharing the same MTS platform.

Chapter 5: Design and realization

This chapter is devoted to the IBC implementation with printed patches. The obtained physical structure of the MTS is further validated with an in-house full-wave MoM code developed to analyze dense arrays of printed patches. Excellent comparisons have been obtained between the simulation of the IBC and that of the MTS physical structure. A MTS prototype for traffic radar is also realized and experimentally validated. This chapter demonstrates that the class of MTS impedance profiles synthesized in Chapters 3 and 4 can be easily implemented with printed patches and manufactured

with standard PCB technology.

Chapter 6: Near-field shaping using metasurfaces

Although, modulated MTS are leaky-wave antennas, we have shown that the near-field of the MTS can be controlled with a very good accuracy, down to a distance of about one wavelength from the surface. Examples with Bessel beams for near-field focusing and with multiple Orbital Angular Momentum (OAM) beams are given. Fields described with FBBF possess angular momentum because of the harmonic variation along the azimuthal direction. We have provided a detailed study of the transmittance between OAM antennas supporting Fourier-Bessel fields, as well as a closed-form expression of the transmittance in the far-field. Finally, difference between the far-field transmittance of Fourier-Bessel fields and Zernike (another orthogonal basis [68]) aperture fields is discussed.

Chapter 7: Power balance in metasurface antennas

Preceding chapters assume the MTS to be printed on an infinitely extended grounded substrate. This assumption allows us to limit the relevant unknowns to the MTS aperture, through the usage of the substrate Green's function. In practice, the substrate is not infinite, then reflection and diffraction may occur at the substrate edge, thus increasing the sidelobes level. This effect is not taken into account in the simulation model developed in Chapter 2. Therefore, a legitimate question is: To which extent can we believe in the results provided by the infinite substrate model, and how can the error be quantified ? This chapter provides the answer to this question with a rigorous quantification of the substrate truncation effect. An efficient full-wave method is proposed for the calculation of the power delivered by the feed, the radiated power, the ohmic power losses in the MTS, and in turn the antenna efficiency.

CHAPTER 2

Numerical simulation of modulated metasurface antennas

ANTENNAS simulation is a key step during the design process. The present chapter is devoted to the Method of Moments (MoM) analysis of metasurfaces (MTSs) with a special focus in the microwave regime. In the following, it is assumed that the reader is familiar with the MoM for solving Maxwell's equations in surface integral equation form [54].

At microwave frequencies, MTSs are usually implemented as dense textures of sub-wavelength elements (typically $\lambda/10$, with λ being the free-space wavelength) printed on a grounded dielectric slab [50]. The resulting number of printed patches is generally around 10^4 for devices with a 15λ diameter. Indeed, MTS antennas [43] generally consist of circular (possibly elliptical [44]) apertures, where a SW launcher [45] is used to excite the structure. The numerical analysis of MTS devices is very challenging, due to the large number of electrically small printed elements. In fact, this feature leads to a very dense mesh (the example in [43, Sec. III.D] required about 10^6 basis functions) and, hence, a prohibitive memory and simulation time for the resulting system of equations. Besides, the presence of fine mesh details can yield an ill-conditioned system of equations. Therefore,

one has to combine iterative solvers with efficient preconditioners [55], use analytical [56], macro (MBF) [57], or characteristics basis functions, [58] to reduce the number of unknowns, or apply a combination of both [59]. However, analytical and characteristic basis functions may depend on the shape and the orientation of the patches and the aforementioned fast methods also become inefficient for apertures with diameters larger than 20λ .

Given the electrically small size of the patches, the fields in the cell allotted to each patch can be averaged, and the actual MTS structure can thus be represented by an equivalent impedance boundary condition (IBC) [21], [22]. The conditions under which the IBC model is applicable have been discussed in [60]. The synthesis problem then consists of finding the appropriate IBC which will perturb the original SW in order to generate the desired radiation or wave-guiding effect. Therefore, a direct simulation of radiation by the modulated IBC is sufficient to provide an excellent idea about the performance of a given design [43], [61] and by considering the IBC problem, one can optimize a few parameters of the modulation before implementing it. Once the design has been completed, the MTS can be implemented by gradually changing the shape, dimensions and orientation of the sub-wavelength patches, under a local periodicity assumption [43], [50], [53].

MTS structures can be represented as IBCs in two different ways. In both cases, the obtained boundary condition is valid for a given incidence. The first possibility consists of describing the whole metasurface (MTS) as a unique entity, assuming its impenetrability. One can then represent the MTS in the form of an opaque or impenetrable IBC [62], [63] which relates the average tangential electric field to the average tangential magnetic field at the upper interface (see Fig. 2.1(a) from [61]). The second possibility is to treat separately the cladding and the substrate [22], [62], [65]. The resulting sheet transition IBC now relates the average tangential electric field on the MTS sheet to the difference of the tangential magnetic fields on either side of the MTS cladding, which stands for the current flowing on the sheet (see Fig. 2.1(b) from [61]). The sheet transition IBC can be obtained from the opaque one, after removing the contribution of the substrate, as described in [66]. The stability issue of both representations has been discussed in [67].

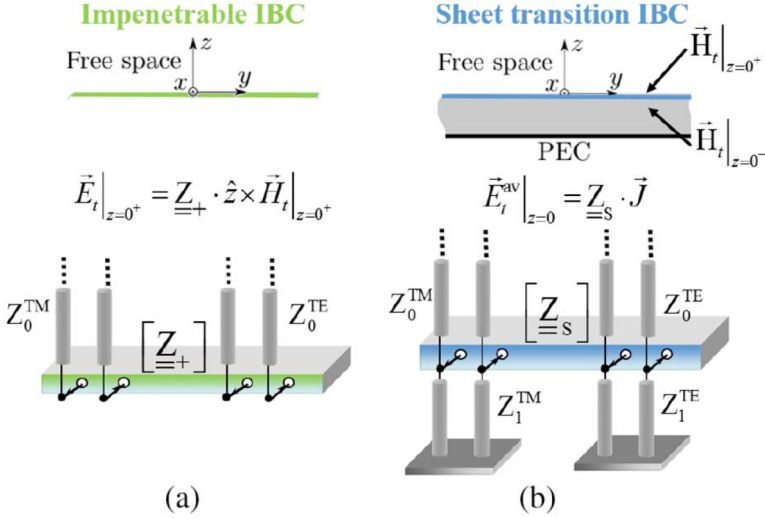


Figure 2.1: From [61], equivalent transmission line circuits for the (a) impenetrable (opaque) IBC and (b) sheet transition IBC for grounded dielectric slab MTS.

Although both approaches present instabilities, the second formulation is more stable and accurate for practical values of the surface impedance. Moreover, in practical optimization problems, the substrate does not change and the optimization parameters are only related to the sheet impedance and to the excitation. For the aforementioned reasons, the sheet transition IBC formulation is selected in this thesis.

The IBCs generally consist of smooth locally periodic modulations, with period of the order of the free-space wavelength. It is thus reasonable to expect that the currents flowing on the MTS sheet be also smooth and with variation at the wavelength scale. It should therefore be possible (by using entire-domain basis functions) to represent the current distribution with a number of basis functions drastically smaller than the one needed to mesh the patches using sub-entire domain basis functions. Such full-domain basis functions provide a good representation of the global field propagation along the surface, while local effects such as interaction with the feeding structure may be less accurate. The authors of [61] propose the use of Gaussian Ring

Basis Functions (GRBF). These basis functions lead to a drastic reduction of the Method of Moments matrix size and a very short computation time. This family of functions tends to provide a good localization of rapid changes in surface currents, owing to the Gaussian decay of the functions along the radial coordinate around a given radius. In opposition, the family of functions proposed here has a relatively homogeneous level over the whole radial domain, which allows a good distinction between different “modes” of propagation over the whole structure. The goal pursued here is to represent more compactly the global evolution of the current distribution by using a set of fully orthogonal and entire-domain basis functions.

Two families of functions are well known, due to their orthogonality on a disk. The first one is the Zernike family of functions [68], [69], which present a polynomial radial dependence. The second one is the Fourier-Bessel family [70]. Both families possess a closed form spectrum, but the Zernike spectrum is much wider. This leads to a non-negligible asymptotic component (in the integrals describing the substrate matrix) that needs to be carefully taken into account by properly choosing the testing operation. Moreover, Zernike basis functions (ZBF) are less suited than the Bessel family to describe typical current distributions on MTS owing to their non-zero admissible edge currents. For the aforementioned reasons, Fourier-Bessel functions are more convenient to describe the current distribution on MTS antennas. The primary goal of this chapter is to demonstrate that an appropriate choice of full-domain basis functions can lead to more compact discretizations than previously used representations [61]. The second objective consists of providing an efficient formulation for the reaction integrals between the proposed basis functions for the electric-field integral equations (EFIE) in the presence of layered media. The method is first explained for circular MTSs, and then generalized to elliptical domain MTSs.

2.1 Circular domain metasurface

This section is devoted to the analysis of circular domain modulated metasurfaces.

2.1.1 Fourier-Bessel Basis Functions (FBBF)

We consider a circular MTS region placed at the air dielectric interface of an infinite grounded dielectric slab, which corresponds to the $z = 0$ plane of a Cartesian coordinate system (x, y, z) with unit vectors $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ (see Fig. 2.1(b)). The observation point on this surface will be denoted by $\boldsymbol{\rho}$. The FBBFs are described in cylindrical coordinates (ρ, ϕ) , with unit vectors $(\hat{\boldsymbol{\rho}}, \hat{\boldsymbol{\phi}})$, and accordingly, the observation vector is $\hat{\boldsymbol{\rho}} = \rho \cos(\phi)\hat{\mathbf{x}} + \rho \sin(\phi)\hat{\mathbf{y}}$.

The Bessel functions are defined on a unit disk ($0 \leq \rho_0 \leq 1$) as follows [70]:

$$F_{m,n}(\rho_0) = J_n(\lambda_n^m \rho_0) \quad (2.1)$$

where λ_n^m is the m -th positive zero of $J_n(x)$ (the Bessel function of the first kind and order n). This class of functions are orthogonal for a fixed value of n (see Appendix A.1). In addition, this family of Bessel functions admits a closed form Hankel transform:

$$\tilde{F}_{m,n}(k_\rho) = -\frac{\lambda_n^m J_{n-1}(\lambda_n^m) J_n(k_\rho)}{(\lambda_n^m)^2 - k_\rho^2} \quad (2.2)$$

Fig. 2.2(a) and Fig. 2.2(b) illustrate the behavior of the Bessel functions in (2.1) for different values of m and n , respectively. The parameter m controls the number of oscillations. The number of maxima and minima with positions different from $\rho_0 = 0$ and $\rho_0 = 1$ is equal to m , for any value of n . On the other hand, the parameter n controls essentially the behavior near to the origin. Fig. 2.3(a) and Fig. 2.3(b), show the spectra which correspond to the Bessel functions plotted in Fig. 2.2(a) and Fig. 2.2(b), respectively. One can see that the spectrum oscillates until the peak at $k_\rho = \lambda_n^m$. From this value, it decays very fast. This results in an easy rule of thumb for the integration domain. It is important to note that the spectrum is finite at λ_n^m since, the numerator is also equal to 0.

The azimuthal dependence is introduced after multiplying $F_{m,n}(\rho_0)$ by $e^{-jn\phi}$, where ϕ is the azimuthal coordinate. The Fourier-Bessel Basis Functions (FBBF) are then defined on a disk of radius a as:

$$R_{m,n}\left(\frac{\rho}{a}, \phi\right) = F_{m,n}\left(\frac{\rho}{a}\right) e^{-jn\phi} \quad (2.3)$$

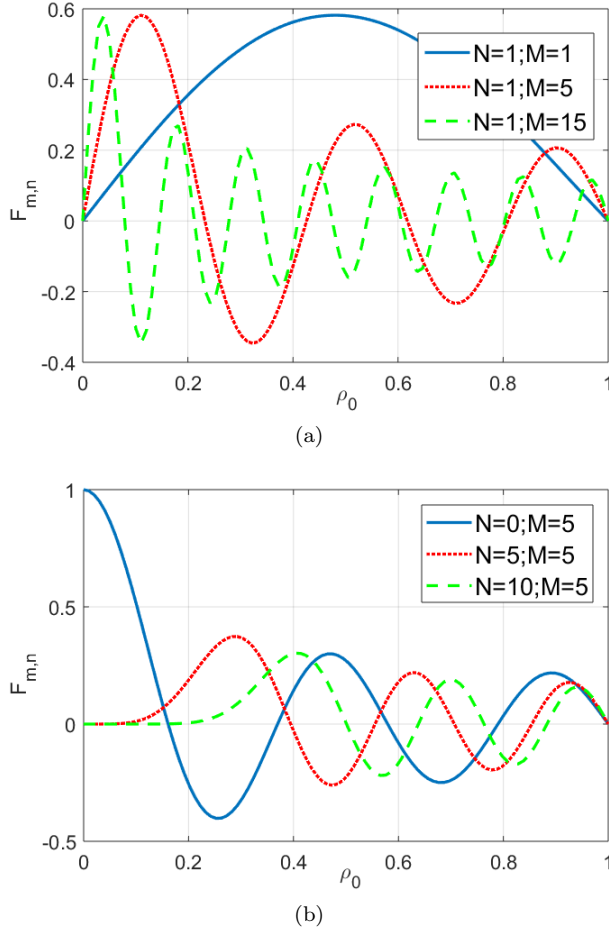


Figure 2.2: Radial part of the FBBFs for (a) a fixed $n = 1$ index and $m = 1, 5$ and 15 , (b) a fixed $m = 5$ index and $n = 0, 5$ and 10 .

This azimuthal behavior generalizes the orthogonality, even for different values of n . The obtained FBBF family is then fully orthogonal (see Appendix A.1) and their closed form Hankel transform is:

$$\tilde{R}_{m,n}(k_\rho, \alpha) = -2\pi j^n e^{-jn\alpha} \left[\frac{a^2 \lambda_n^m J_{n-1}(\lambda_n^m) J_n(k_\rho a)}{(\lambda_n^m)^2 - (k_\rho a)^2} \right] \quad (2.4)$$

where k_ρ and α are the spectral variables in cylindrical coordinates. The

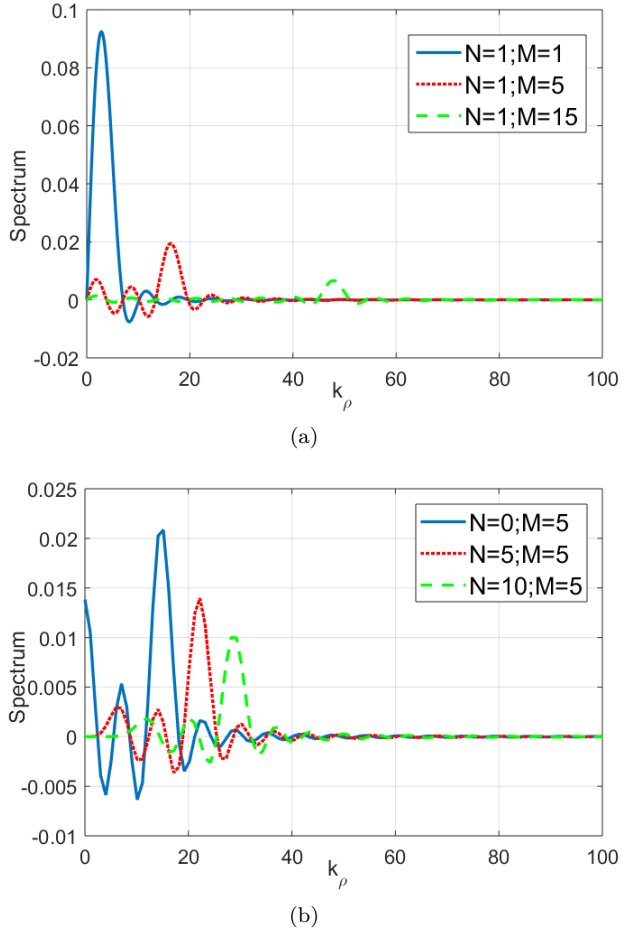


Figure 2.3: Spectral domain counterpart of the radial part of the FBBFs shown in: (a) Fig. 2.2(a) for a fixed n index. (b) Fig. 2.2(b) for a fixed m index.

proof of (2.4) is provided in Appendix. A.2.

As observed, the FBBFs are directly defined in all the circular domain and vanish at the rim of the disc. That is, they do not admit a normal component at the border. This property ensures the absence of line charges at the border in the Integral Equation formulation.

The current distribution can be decomposed in two ways. First, by using

its Cartesian x and y orthogonal components as follows:

$$\mathbf{J}(\boldsymbol{\rho}) = \sum_{n=-N}^N \sum_{m=1}^M i_{mn}^x R_{m,n}^x(\boldsymbol{\rho}) \hat{\mathbf{x}} + i_{mn}^y R_{m,n}^y(\boldsymbol{\rho}) \hat{\mathbf{y}} \quad (2.5)$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors along the x and y directions, respectively.

The second possibility consists in using the $(\hat{\boldsymbol{\rho}}, \hat{\boldsymbol{\phi}})$ decomposition, which is useful to reduce the number of unknowns for isotropic MTS. This decomposition cannot be efficiently and directly used in a MoM formulation, as the resulting dyadic Green's function does not exclusively depend on the vector between the source and observation positions, but on both positions. However, owing to the orthogonality of the basis functions, the second formulation can be extracted from the first one, by decomposing the $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ basis and testing functions into $(\hat{\boldsymbol{\rho}}, \hat{\boldsymbol{\phi}})$ basis and testing functions, as explained in Section 2.1.2.

2.1.2 Method of Moments Formulation

Let us start from the definition of the sheet transition IBC $\underline{\underline{\mathbf{Z}}}_S$

$$\mathbf{E}_t = \underline{\underline{\mathbf{Z}}}_S \cdot \hat{\mathbf{z}} \times (\mathbf{H}_t|_{z=0^+} - \mathbf{H}_t|_{z=0^-}) = \underline{\underline{\mathbf{Z}}}_S \cdot \mathbf{J} \quad (2.6)$$

where $\mathbf{H}_t|_{z=0^+}$ and $\mathbf{H}_t|_{z=0^-}$ are respectively the tangential magnetic fields at the upper and lower sides of the sheet (see Fig. 2.1), $\mathbf{E}_t$ is the tangential electric field on the MTS plane, and it stands for the sum of the E-field radiated by the equivalent currents $\mathbf{J}$ and that from the excitation. Equation (2.6) then leads to the following Electric Field Integral Equation:

$$\hat{\mathbf{n}} \times \left[\iint_{S'} \underline{\underline{\mathbf{G}}}^{EJ}(\boldsymbol{\rho}, \boldsymbol{\rho}') \mathbf{J}(\boldsymbol{\rho}') dS' - \underline{\underline{\mathbf{Z}}}_S(\boldsymbol{\rho}) \mathbf{J}(\boldsymbol{\rho}) \right] = -\hat{\mathbf{n}} \times \mathbf{E}_i \quad (2.7)$$

where $\hat{\mathbf{n}}$ is the normal to the surface, $\boldsymbol{\rho}' = x' \hat{\mathbf{x}} + y' \hat{\mathbf{y}}$ and $\boldsymbol{\rho} = x \hat{\mathbf{x}} + y \hat{\mathbf{y}}$ are the source and observation points, respectively. $\underline{\underline{\mathbf{G}}}^{EJ}$ is the dyadic Green's function of the grounded dielectric slab, $\underline{\underline{\mathbf{Z}}}_S(\boldsymbol{\rho})$ is the modulated sheet transition IBC tensor and $\mathbf{E}_i$ is the excitation electric field. The fields are tested with the complex conjugate of the basis functions. Nevertheless,

testing with the basis functions themselves also provides good results. After introducing the current expansion (2.5) in (2.7) and testing the fields, (2.7) leads to a system of equations that can be written in a compact form as follows:

$$\begin{bmatrix} [Z^{xx}] & [Z^{xy}] \\ [Z^{yx}] & [Z^{yy}] \end{bmatrix} \begin{bmatrix} [i^x] \\ [i^y] \end{bmatrix} = \begin{bmatrix} [v^x] \\ [v^y] \end{bmatrix} \quad (2.8)$$

Identifying each testing function with the couple of parameters (m, n) and each basis function with the couple parameters (m', n') , each element of the submatrix $[Z^{xx}]$ is defined as:

$$Z^{xx}(m, n; m', n') = Z_G^{xx}(m, n; m', n') - Z_{IBC}^{xx}(m, n; m', n') \quad (2.9)$$

with:

$$Z_G^{xx}(m, n; m', n') = \int_0^{2\pi} \int_0^a R_{m,n}^{x,*} \left(\frac{\rho}{a}, \phi \right) \int_0^{2\pi} \int_0^a G_{xx}^{EJ}(|\boldsymbol{\rho} - \boldsymbol{\rho}'|) R_{m',n'}^x \left(\frac{\rho'}{a}, \phi' \right) \rho' d\rho' d\phi' \rho d\rho d\phi \quad (2.10)$$

and the sheet IBC contribution:

$$Z_{IBC}^{xx}(m, n; m', n') = \int_0^{2\pi} \int_0^a R_{m,n}^{x,*} \left(\frac{\rho}{a}, \phi \right) Z_S^{xx}(\rho, \phi) R_{m',n'}^x \left(\frac{\rho}{a}, \phi \right) \rho d\rho d\phi \quad (2.11)$$

The excitation vector $[v_x]$ is given by:

$$v^x(m, n) = - \int_0^{2\pi} \int_0^a R_{m,n}^{x,*} \left(\frac{\rho}{a}, \phi \right) E_i^x(\rho, \phi) \rho d\rho d\phi \quad (2.12)$$

Finally, as the substrate Green's function is known in closed form in spectral domain, and the FBBFs admit a closed form Fourier transform, it is more convenient to evaluate the substrate and the excitation contribution directly in the spectral domain. Expressions (2.10) and (2.12) can be rewritten respectively as:

$$Z_G^{xx}(m, n; m', n') = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^\infty \tilde{R}_{m,n}^{x,*}(k_\rho, \alpha) \tilde{G}_{xx}^{EJ}(k_\rho, \alpha) \tilde{R}_{m',n'}^x(k_\rho, \alpha) k_\rho dk_\rho d\alpha \quad (2.13)$$

$$v_x(m, n) = -\frac{1}{4\pi^2} \int_0^{2\pi} \int_0^\infty \tilde{R}_{m,n}^{x,*}(k_\rho, \alpha) \tilde{E}_i^x(k_\rho, \alpha) k_\rho dk_\rho d\alpha \quad (2.14)$$

where $\tilde{G}_{xx}^{EJ}(k_\rho, \alpha)$ is the xx component of the Green's function spectral dyad and $\tilde{E}_i^x(k_\rho, \alpha)$ is the $\hat{x}$ -oriented component of the excitation spectrum.

The elements in the others submatrices $[Z^{xy}]$, $[Z^{yx}]$ and $[Z^{yy}]$ in (2.8) and that of $[v_y]$ are given by definitions analogous to (2.9), (2.11), (2.13) and (2.14).

Grounded slab contribution

The substrate interaction matrix is directly formulated in the spectral domain. The dyadic spectral Green's function of the grounded slab can be written as follows [72] - [74]:

$$\begin{bmatrix} \tilde{G}_{xx}^{EJ} & \tilde{G}_{xy}^{EJ} \\ \tilde{G}_{yx}^{EJ} & \tilde{G}_{yy}^{EJ} \end{bmatrix} = \tilde{G}_A(k_\rho) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \tilde{G}_V(k_\rho) \begin{bmatrix} k_x^2 & k_x k_y \\ k_x k_y & k_y^2 \end{bmatrix} \quad (2.15)$$

where $\tilde{G}_A$ and $\tilde{G}_V$ are, respectively, the scalar potentials with respect to currents and charges (details in Section A.4). After substituting (A.13) into (2.13) and integrating along α , the substrate matrix elements are given by:

$$Z_G^{xx}(m, n; m', n') = (-j)^n j^{n'} a^4 \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{F}_{m',n'}(k_\rho a) \times \left[2\pi \tilde{G}_A(k_\rho) \delta_{n,n'} + k_\rho^2 \tilde{G}_V(k_\rho) \epsilon_{n,n'}^{cc} \right] k_\rho dk_\rho \quad (2.16)$$

$$Z_G^{xy}(m, n; m', n') = Z_G^{yx}(m, n; m', n') = (-j)^n j^{n'} a^4 \times \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{F}_{m',n'}(k_\rho a) \left[\tilde{G}_V(k_\rho) \epsilon_{n,n'}^{s,c} \right] k_\rho^3 dk_\rho \quad (2.17)$$

$$Z_G^{yy}(m, n; m', n') = (-j)^n j^{n'} a^4 \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{F}_{m',n'}(k_\rho a) \times \left[2\pi \tilde{G}_A(k_\rho) \delta_{n,n'} + k_\rho^2 \tilde{G}_V(k_\rho) \epsilon_{n,n'}^{ss} \right] k_\rho dk_\rho \quad (2.18)$$

with $\epsilon_{n,n'}^{cc} = (\pi/2)(2\delta_{n,n'} + \delta_{n,n'+2} + \delta_{n,n'-2})$, $\epsilon_{n,n'}^{sc} = (\pi/2j)(\delta_{n,n'-2} - \delta_{n,n'+2})$ and $\epsilon_{n,n'}^{ss} = (\pi/2)(2\delta_{n,n'} - \delta_{n,n'+2} - \delta_{n,n'-2})$. The function $\delta_{i,j}$ is the Kronecker delta. These integrals can be efficiently computed using a parabolic contour deformation [75]. In practice, an integration range from 0 to $2\lambda_N^M/a$ is enough to obtain accurate results. For examples discussed in this chapter, $2\lambda_N^M/a \approx 10k_0$, with $k_0 = 2\pi/\lambda$, where λ is the free-space wavelength. This means that each integration can be carried out with about 500 sampling points by using a higher-order quadrature, such as the Gauss-Legendre one. The obtained substrate matrix $\underline{\underline{Z}}_G$ is relatively sparse since it has to be calculated for only few elements. This matrix structure has already been discussed in [61]. The substrate matrix can then be computed extremely fast (less than 1 minute on a laptop computer).

Sheet impedance contribution

The analysis is limited here to the Z_{IBC}^{xx} component since the obtained expression is valid for the other components. The sheet contribution can be directly evaluated in space domain as follows:

$$Z_{IBC}^{xx}(m, n; m', n') = \int_0^a F_{m,n}\left(\frac{\rho}{a}\right) F_{m',n'}\left(\frac{\rho}{a}\right) \int_0^{2\pi} e^{j(n-n')\phi} Z_S^{xx}(\boldsymbol{\rho}) d\phi \rho d\rho \quad (2.19)$$

Thanks to the azimuthal periodicity, it is better to expand Z_S^{xx} into a Fourier series as follows:

$$Z_S^{xx}(\boldsymbol{\rho}) = \sum_{r=-\infty}^{\infty} a_r(\rho) e^{jr\phi} \quad (2.20)$$

Then (2.19) reduces to:

$$Z_{IBC}^{xx}(m, n; m', n') = 2\pi \int_0^a F_{m,n}\left(\frac{\rho}{a}\right) F_{m',n'}\left(\frac{\rho}{a}\right) a_{n'-n}(\rho) \rho d\rho \quad (2.21)$$

In practice, the first 10 harmonics of the Fourier series are sufficient to describe accurately the impedance. It means that this integral can be assumed equal to zero for $|n' - n|$ large enough.

Those calculations can be organized differently, in a faster way, but requiring more memory. It consists of directly projecting the sheet impedance in the FBBF basis as follows:

$$Z_S^{xx}(\rho) = \sum_{n''=-N}^{N''} \sum_{m''=1}^{M''} \gamma_{n''m''} R_{m''}^{n''} \left(\frac{\rho}{a}, \phi \right) \quad (2.22)$$

The expansion coefficients are calculated by simple projection:

$$\gamma_{n''m''} = \frac{1}{K(m'', n'')} \int_0^{2\pi} \int_0^a Z_S^{xx}(\rho) R_{m''}^{*n''} \left(\frac{\rho}{a}, \phi \right) \rho \, d\rho \, d\phi \quad (2.23)$$

where K is a normalization factor given by:

$$K(m'', n'') = -\pi J_{n''-1}(\lambda_{n''}^{m''}) J_{n''+1}(\lambda_{n''}^{m''}) a^2 \quad (2.24)$$

After inserting (2.22) into (2.19) and integrating along ϕ , we get:

$$\begin{aligned} Z_{IBC}^{xx}(m, n; m', n') &= 2\pi a^2 \sum_{m''=1}^{M''} \gamma_{(n'-n)m''} \\ &\times \left[\int_0^1 F_{m,n}(\rho) F_{m',n'}(\rho) F_{m'',(n'-n)}(\rho) \rho \, d\rho \right] \end{aligned} \quad (2.25)$$

The expression between brackets is a unique function of $(m, n; m', n'; m'')$ and can therefore be tabulated.

Excitation contribution

The MTS presented in this chapter are antennas excited at the origin by an elementary vertical dipole. The Green's function associated to this excitation can be written in the following form (see Appendix A.5):

$$\tilde{G}_{xz}^{EJ}(\mathbf{k}) = \cos(\alpha) \tilde{f}(k_\rho); \quad \tilde{G}_{yz}^{EJ}(\mathbf{k}) = \sin(\alpha) \tilde{f}(k_\rho) \quad (2.26)$$

Inserting equation (2.26) into (2.14), and integrating along α leads to the following expressions:

$$v_x(m, n) = \frac{(-j)^n}{2} a^2 \lambda_n^m J_{n-1}(\lambda_n^m) (\delta_{n,1} + \delta_{n,-1}) \left[\int_0^\infty \frac{J_n(k_\rho a)}{((\lambda_n^m)^2 - (k_\rho a)^2)} \tilde{f}(k_\rho) k_\rho dk_\rho \right] \quad (2.27)$$

$$v_y(m, n) = \frac{(-j)^n}{2j} a^2 \lambda_n^m J_{n-1}(\lambda_n^m) (\delta_{n,-1} - \delta_{n,1}) \left[\int_0^\infty \frac{J_n(k_\rho a)}{((\lambda_n^m)^2 - (k_\rho a)^2)} \tilde{f}(k_\rho) k_\rho dk_\rho \right] \quad (2.28)$$

When the distance between the source and the observation layer (in this case $d/2$, with d being the substrate thickness) is large enough, integrand between brackets in (2.27) and (2.28) converges very fast due to the exponential decay of the Green's function $\tilde{f}(k_\rho)$. However, when the feed is close to the observation layer (the MTS plane) or for thin substrates ($d \ll \lambda/20$), the integrand does not converge well asymptotically. Therefore, one should extract an asymptotic term which can be integrated analytically before proceeding to the numerical integration. This process is explained below.

Let us assume that one needs to solve integral I between brackets in (2.27) and (2.28). The function $\tilde{f}(k_\rho)$ can be written in the following form

$$\tilde{f}(k_\rho) = \tilde{f}_i(k_\rho) + \tilde{f}_1(k_\rho) + \tilde{f}_{-1}(k_\rho) \quad (2.29)$$

where each term corresponds to the i , -1 , and 1 term in the Green's function (see equation (A.27)). Integral I can therefore be split into 3 integrals, namely I_i , I_1 and I_{-1} .

I_i can be explicitly written as

$$I_i = \frac{\eta}{2k} \int_0^\infty k_\rho^2 e^{-jk_z|z|} \frac{J_n(k_\rho a)}{((\lambda_n^m)^2 - (k_\rho a)^2)} dk_\rho \quad (2.30)$$

where $|z|$ is the distance between the source layer and the observation layer. When $|z| \approx 0$, integrand in I_i decays very slowly. Asymptotically (for

$k_\rho \gg 20k_0$), I_i behaves like:

$$I_i^{asympt} = -\frac{\eta}{2ka^2} \int_0^\infty e^{-k_\rho|z|} J_n(k_\rho a) dk_\rho \quad (2.31)$$

Using identity 6.611-1 of [120], one finds:

$$I_i^{asympt} = -\frac{\eta}{2ka^2} \frac{a^{-n}(\sqrt{|z|+a^2}-|z|)^n}{\sqrt{|z|+a^2}} \quad (2.32)$$

I_1 and I_2 can be computed following a similar procedure.

Polar formulation of the current distribution

The precedent formulation based on the Cartesian $(\hat{x}, \hat{y})$ current expansion leads to a system of equations that can be written in matrix form:

$$(\underline{\underline{\mathbf{Z}}}_G - \underline{\underline{\mathbf{Z}}}_{IBC})\underline{\mathbf{I}} = \underline{\mathbf{V}} \quad (2.33)$$

where $\underline{\underline{\mathbf{Z}}}_G$ is the substrate impedance contribution, $\underline{\underline{\mathbf{Z}}}_{IBC}$ is the sheet impedance tensor written in Cartesian coordinates. The $(\hat{\rho}, \hat{\phi})$ formulation can be written in a similar form:

$$(\underline{\underline{\mathbf{Z}}}'_G - \underline{\underline{\mathbf{Z}}}'_{IBC})\underline{\mathbf{I}}' = \underline{\mathbf{V}}' \quad (2.34)$$

where $\underline{\underline{\mathbf{Z}}}'_G$ is the equivalent substrate contribution and $\underline{\underline{\mathbf{Z}}}'_{IBC}$ is the sheet contribution directly written in polar $(\hat{\rho}, \hat{\phi})$ coordinates, $\underline{\mathbf{I}}'$ is the current expansion in the $(\hat{\rho}, \hat{\phi})$ unit vectors and $\underline{\mathbf{V}}'$ is the excitation.

Starting from equation (2.8) and expanding the basis and testing functions in the $(\hat{\rho}, \hat{\phi})$ basis and test functions, it can be proven that:

$$\underline{\underline{\mathbf{Z}}}'_G = Q^H \underline{\underline{\mathbf{Z}}}_G Q \quad (2.35)$$

and

$$\underline{\mathbf{V}}' = Q^H \underline{\mathbf{V}} \quad (2.36)$$

where superscript (H) in (2.35) and (2.36) represents the Hermitian transposed.

The matrix Q can be written in the following form:

$$\begin{bmatrix} [Q^\alpha] & [Q^\gamma] \\ [Q^\beta] & [Q^\alpha] \end{bmatrix} \quad (2.37)$$

Each block of the Q matrix has a size $(J_{MAX} * J'_{MAX})$, where $J_{MAX} = (2N + 1) * M$ is the number of basis / testing functions used in the $(\hat{x}, \hat{y})$ formulation and $J'_{MAX} = (2N' + 1) * M'$ is the number of basis / testing functions used in the $(\hat{\rho}, \hat{\phi})$ formulation. The block Q^α is defined as:

$$Q^\alpha(m, n; m', n') = \frac{1}{K(m, n)} \int_0^{2\pi} \int_0^a R_{m,n}^{x,*} \left(\frac{\rho}{a}, \phi \right) \cos(\phi) R_{m',n'}^x \left(\frac{\rho}{a}, \phi \right) \rho \, d\rho \, d\phi \quad (2.38)$$

where K is a normalization factor given by:

$$K(m, n) = -\pi J_{n-1}(\lambda_n^m) J_{n+1}(\lambda_n^m) a^2 \quad (2.39)$$

The expressions of Q^β and Q^γ , are obtained by replacing the $\cos(\phi)$ in equation (2.38) by respectively $\sin(\phi)$ and $-\sin(\phi)$. After integrating (2.38) along ϕ , we obtain:

$$Q^\alpha(m, n; m', n') = \frac{\pi a^2}{K(m, n)} (\delta_{n,n'-1} + \delta_{n,n'+1}) \int_0^1 F_{m,n}(\rho) F_{m',n'}(\rho) \rho \, d\rho \quad (2.40)$$

Similar expressions are obtained for the others blocks (see Appendix A.3). The obtained matrix projection Q is then very sparse and can be computed extremely fast. In practice, choosing $J_{MAX} = J'_{MAX}$ is sufficient to describe accurately the current projection. In passing, if one is interested only by the $\hat{\rho}$ oriented currents (as is the case for isotropic MTS antennas), the

projection matrix Q is given by:

$$\begin{bmatrix} [Q^\alpha] \\ [Q^\beta] \end{bmatrix} \quad (2.41)$$

2.1.3 Numerical results

To test the accuracy and the efficiency of the method, we analyze three MTS antennas. The first one is synthesized using an isotropic MTS, i.e. a scalar IBC distribution, whereas the other ones use anisotropic MTSs, i.e. a tensorial IBC distribution. We analyze the FBBFs in terms of convergence (number of basis functions needed to get a stable solution) and computation time, and compare the obtained results with the simulation of the IBC using GRBFs [61], and with the full-wave analysis of MTS antennas implemented with small printed elements.

Broadside pencil beam antenna with isotropic MTS

The analyzed structure is a scalar modulated MTS antenna, designed to radiate a broadside right-handed circularly polarized (RHCP) pencil beam. The design procedure has been discussed in [43] and leads to the following opaque IBC distribution:

$$\begin{aligned} Z_+^{\rho\rho}(\boldsymbol{\rho}) &= jX_0 [1 + M_0 \sin(2\pi\rho/d - \phi)] \\ Z_+^{\rho\phi}(\boldsymbol{\rho}) &= Z_+^{\phi\rho}(\boldsymbol{\rho}) = 0 \\ Z_+^{\phi\phi}(\boldsymbol{\rho}) &= jX_0 [1 + M_0 \sin(2\pi\rho/d - \phi)] \end{aligned} \quad (2.42)$$

where: $X_0 = 0.71\eta_0$, with η_0 being the free-space impedance, $M_0 = 0.27$ and $d = \lambda / \sqrt{1 + (X_0/\eta_0)^2}$. The frequency of operation is $f = 17$ GHz and the antenna radius is $a = 5.65\lambda$ at that frequency. Finally, the dielectric substrate has relative permittivity $\epsilon_r = 3.66$ and thickness $h = 1.524$ mm. The sheet impedance is obtained after removing from the opaque one the substrate contribution, as explained in [67], and the MTS is excited with a vertical electric dipole placed in the middle of the substrate and at the center of the disk ($\rho = 0$).

Fig. 2.4(a) and Fig. 2.4(b) show the absolute value of the current distribution obtained with FBBF and that obtained with GRBF, respectively. In both simulations, we have used the same number of basis functions ($N = 8$; $M = 46$), which leads to a total of 1632 basis functions.

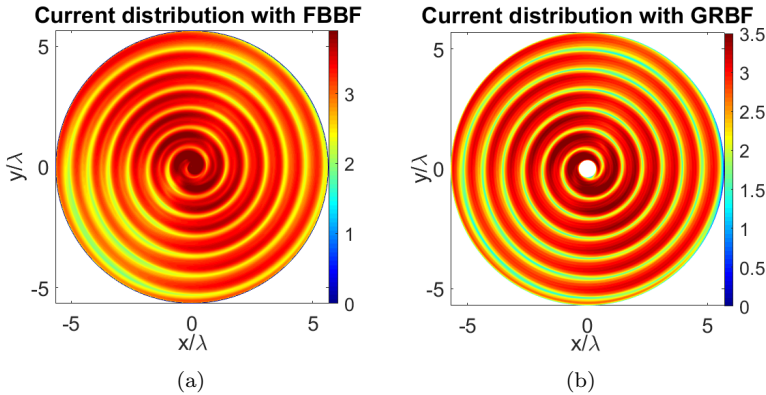


Figure 2.4: Absolute value of the surface current distribution (in log10 scale) on a scalar IBC aperture used to generate a RHCP broadside pencil-beam. The IBC disk has been analyzed with (a) FBBFs and (b) GRBFs, in both cases $N = 8$ and $M = 46$.

We can observe the same appearance in both cases, except close to the center (for $\rho < 0.5\lambda$) given that we have not defined GRBFs in the central region. In this way, one may account for the absence of MTS elements often encountered in the region occupied by the feeder. Indeed, in practice, the feed is not a simple vertical dipole and realistic feeds cover a disk of radius about 0.5λ . On that disk, in the implementation of [43], there are no equivalent electric currents since there are no patches implementing the IBC.

Fig. 2.5(a) shows the normalized co-polar pattern obtained with GRBF and FBBF. One can observe a difference, especially on the secondary lobes. Indeed, close to the center, the current increases very rapidly, leading to a non-negligible contribution to the far field. This may be viewed as a weakness of the FBBF representation which produce nonzero fields everywhere

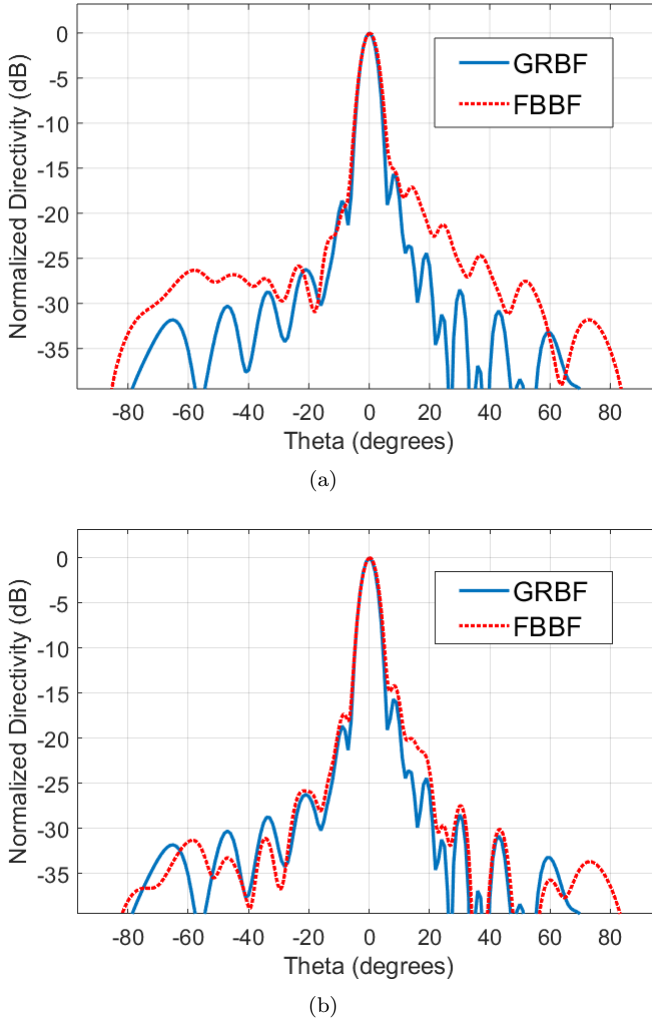


Figure 2.5: RHCP component of the normalized directivity pattern for the structure described by (2.42). The result obtained with GRBFs (solid line) is compared with the FBBFs one (dashed line) in two cases: (a) without center current subtraction. (b) with center current subtraction.

including close to the feed point. This problem can be partly corrected by removing a posteriori the center currents contribution (in the region where the GRBFs are not defined) from the pattern. The obtained pattern is now

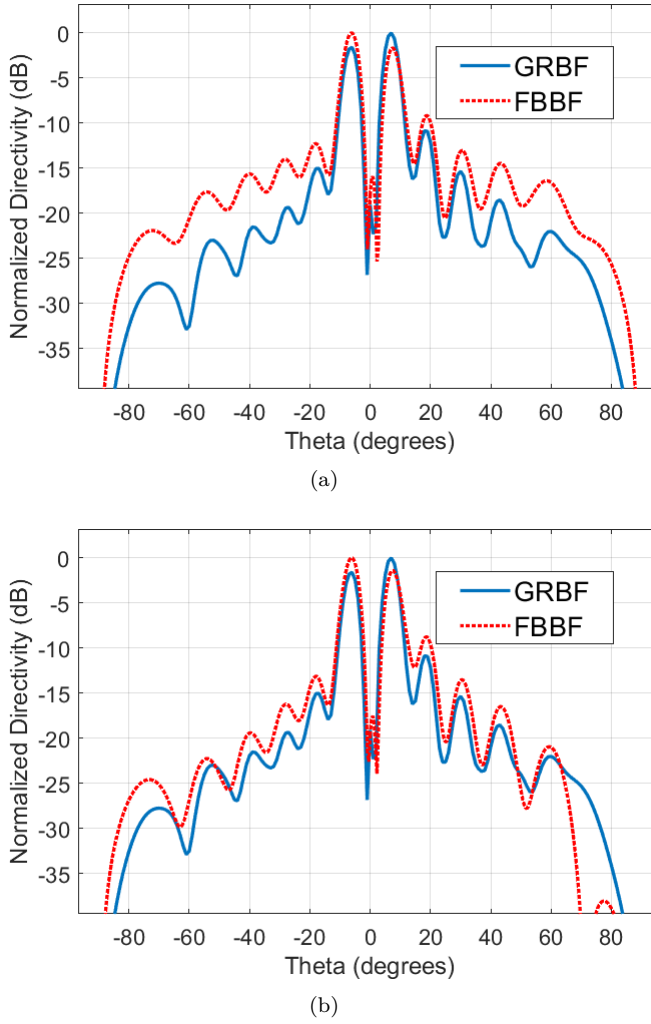


Figure 2.6: LHCP component of the normalized directivity pattern for the structure described by (2.42). The result obtained with GRBFs (solid line) is compared with the FBBFs one (dashed line) in two cases: (a) without center current subtraction. (b) with center current subtraction.

represented in Fig. 2.5(b). A better agreement is observed although it is not perfect since truncating is not strictly equivalent to the use of basis functions which do not admit center currents. The same observation is valid for

the cross polar patterns (see Fig. 2.6). In terms of antenna performances, the results are summarized in Table 2.1. As expected, the performances are

Table 2.1: Computed Co-polar and Cross-polar maximum directivity for the MTS described by (2.42).

Solver	RHCP	LHCP
GRBF	26.5 dBi	18.2 dBi
FBBF without center current subtraction	25 dBi	16.6 dBi
FBBF with center current subtraction	25.9 dBi	18 dBi
Difference w.r.t GRBF	0.6 dB	0.2 dB

much closer to the GRBFs simulation after the center current subtraction. The difference w.r.t the GRBFs Co-polar pattern is 1.5 dB without center current subtraction and 0.6 dB after center current subtraction. The remaining error may be attributed to the fact the two simulated structures are not exactly the same, one of them having a zero sheet admittance over a disk of radius 0.5λ at the center. Nevertheless, the results obtained after center current subtraction provide a good estimation of the radiation performance, both for the Co-polar and the Cross-polar patterns. Next, we have implemented with square patches a smaller version of the MTS (3λ radius). The patches cover all the disk and the structure is excited with the same vertical dipole. The implemented MTS made of square patches is meshed with 154868 RWG basis functions and simulated with the MoM technique in [77]. Fig. 2.7 shows the obtained co-polar pattern. One can observe a good agreement, despite the fact that we did not remove the center current contribution since the implemented MTS is defined on all the disk. It is important to note that for MTSs excited outside the surface or near its border, as is the case for the Luneburg Lens in [76], the IBC is defined on all the circular domain, thus the FBBFs directly provide good results. However, a full modeling of the MTS in presence of realistic feed will require a full-wave analysis of the feeding structure together with its

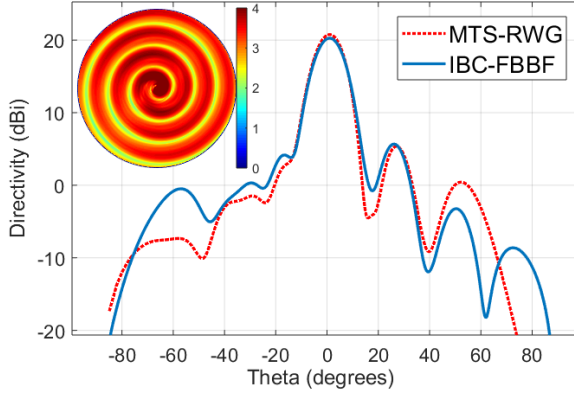


Figure 2.7: Directivity pattern without direct excitation contribution of a smaller version of the structure described by (2.42) and used to obtain a broadside beam. The RHCP pattern obtained with the RWG-MoM simulation of the MTS antenna implemented with square patches (red dotted line) is compared with the pattern obtained using FBBFs and the IBC in the circular domain (blue solid line). The inset disk corresponds to the absolute value of the current distribution (in log10 scale) obtained with FBBF.

immediate vicinity, taking into account the local coupling between the feed and the IBC.

Below, we study the convergence of the solution (2.42) obtained with FBBFs, and compare it with that obtained with GRBFs. Both types of function present identical azimuthal dependence. Therefore, we will just compare their convergence rate with respect to the index M , which controls the radial discretization. To this end, we analyze the evolution of the pattern shape. We define the relative error for a given number M of radial basis functions as follows:

$$err_M = 100 \frac{\sum_{\theta=0^\circ}^{90^\circ} \sum_{\phi=0^\circ}^{360^\circ} \|\mathbf{E}_M(\theta, \phi) - \mathbf{E}_{M+2}(\theta, \phi)\|_2}{\sum_{\theta=0^\circ}^{90^\circ} \sum_{\phi=0^\circ}^{360^\circ} \|\mathbf{E}_{M+2}(\theta, \phi)\|_2} \quad (2.43)$$

The evolution of the relative error obtained with both FBBFs and GRBFs is depicted in Fig. 2.8. Although the convergence rate is approximately the same, the number of basis functions needed to obtain a given relative error is smaller when using FBBFs. In particular, for a relative error of approx-

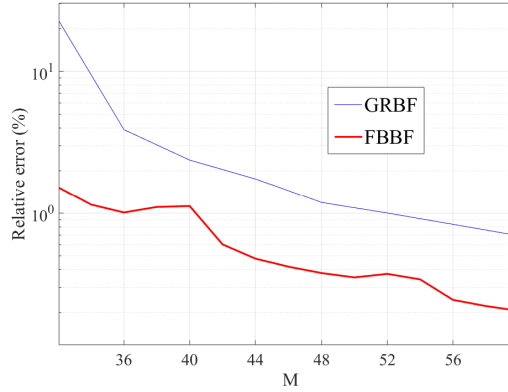


Figure 2.8: Relative error versus index M obtained with (2.43) for the FBBF and the GRBF simulation of the structure in Fig. 2.4 and Fig. 2.5.

imately 1%, we need for FBBFs, $M = 36$ and for GRBF, $M = 48$ where M has been defined in (2.5). As it will be seen later, this difference is more significant for larger MTS. Moreover, for the isotropic MTS case, the orthogonality of the basis allows one to apply the MoM formulation which uses only $\hat{a}_\rho$ directed currents, leading to significantly lower number of basis functions. In this particular example, we only need 612 FBBFs and 1632 GRBFs, provided that we use only $\hat{a}_\rho$ directed currents. In the following sections, we will assume that a stable convergence rate is obtained when the relative error $\approx 1\%$. The computation time needed for the different steps of the simulation is shown in Table 2.2. In order to be consistent in the comparisons, we use now the $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ formulation for the current distribution. The simulations have been done with an Intel Core i7 @ 2.8 GHz computer. One can observe that the simulation based on GRBFs is faster than that

Table 2.2: Simulation times for the MTS described by (2.42).

Solver	Substrate matrix	IBC matrix	Inversion time	Total time
FBBF	6.54 s	1.51 s	0.14 s	9.34 s
GRBF	2.59 s	0.32 s	0.14 s	3.58 s

using FBBFs. This difference is mainly due to the closed-form evaluation of the integrals arising in the GRBF formulation [61]. From the times in Table 2.2, one can conclude that the largest difference lies in the calculation of the substrate matrix $\underline{\underline{Z}}_G$, which cannot be computed in closed form with FBBFs. However, in an optimization process, the substrate matrix needs to be calculated only once and for all, while the matrix related to the sheet transition IBC $\underline{\underline{Z}}_{IBC}$ has to be computed at each optimization step. We can also observe that the computation times of the sheet matrix, remains lower for GRBF than for FBBF. This is due to the fact that the evolution of the sheet is smooth in comparison with the Gaussian width, allowing an asymptotic calculation of the reaction integrals. Finally, it is important to mention that the same simulation takes 2.4 h with FEKO [78], using RWG basis functions and needs 54907 basis functions.

Broadside pencil beam MTS antennas based on anisotropic IBCs

In this subsection, we analyze two anisotropic MTS antennas of different sizes with a RHCP broadside pencil beam. The elements of the impenetrable IBC tensor of the first one are defined as:

$$\begin{aligned} Z_+^{\rho\rho}(\boldsymbol{\rho}) &= jX_0 [1 + M_{\rho\rho} \cos(2\pi\rho/d - \phi)] \\ Z_+^{\rho\phi}(\boldsymbol{\rho}) &= Z_+^{\phi\rho}(\boldsymbol{\rho}) = jX_0 M_{\rho\phi} \sin(2\pi\rho/d - \phi) \\ Z_+^{\phi\phi}(\boldsymbol{\rho}) &= jX_0 [1 - M_{\phi\phi} \cos(2\pi\rho/d - \phi)] \end{aligned} \quad (2.44)$$

where the modulation parameters are $X_0 = 279 \Omega$, $M_{\rho\rho} = M_{\rho\phi} = M_{\phi\phi} = 0.4$ and $d = \lambda / \sqrt{1 + (X_0/\eta_0)^2}$. The frequency of operation is $f = 8.425$ GHz and the antenna radius is $a = 7.6\lambda$ at this frequency. The cladding lies on a dielectric substrate which has a relative permittivity $\epsilon_r = 9.8$ and thickness $h = 1.57$ mm. The obtained current distributions using FBBFs and GRBFs are depicted in Fig. 2.9(a) and Fig. 2.9(b), respectively. In both simulations we have used the same discretization parameters with $N = 16$ and $M = 96$, which leads to a total of 6336 unknowns. Fig. 2.9(c) presents the evolution of the currents obtained with both methods on the $\phi = 0$ axis. We can observe a very good agreement up to $\rho = 0.5\lambda$ since the GRBFs have not been defined in the center region. Fig. 2.10 shows the current distribution

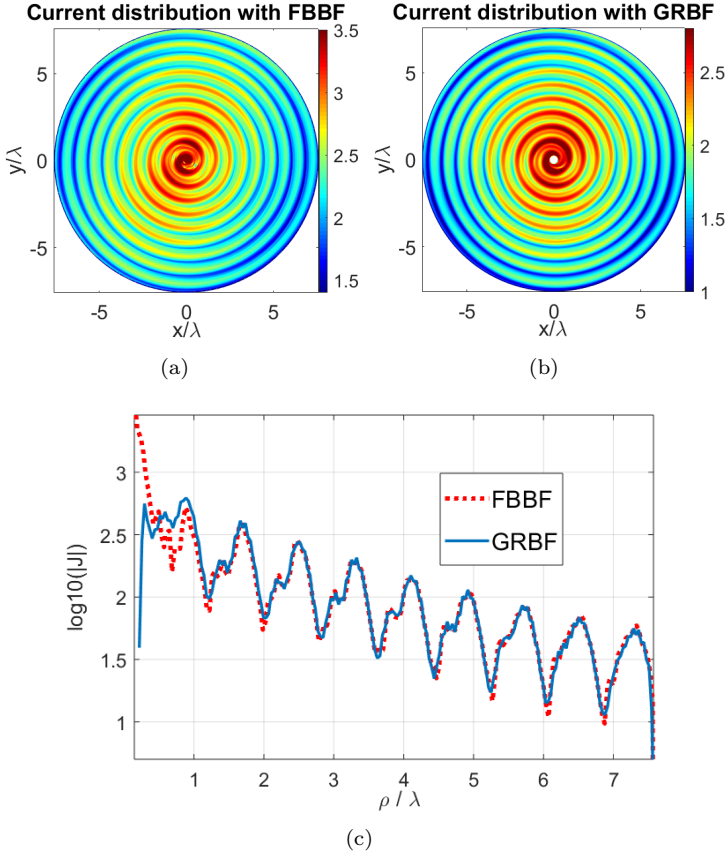


Figure 2.9: Results for the RHCP broadside pencil-beam antenna described by the anisotropic IBC in (2.44). The IBC disk has been analyzed using $N = 16$ and $M = 96$. Absolute value of the surface current distribution obtained: a) on the aperture with FBBFs, b) on the aperture with GRBFs, and c) on the $\phi = 0$ axis both with FBBFs and GRBFs.

with FBBFs obtained with 60% fewer basis functions ($N = 16$; $M = 60$, i.e. 3960 basis functions) in comparison with ($N = 16$; $M = 96$, i.e. 6336 basis functions). We can still observe a very good correspondence despite, using 60% fewer FBBFs (3960 vs. 6336). For this example, choosing ($N = 16$; $M = 65$ i.e. 4290 basis functions) is enough to get a stable convergence. The calculation times, are summarized in Table 2.3. The same simulation, using

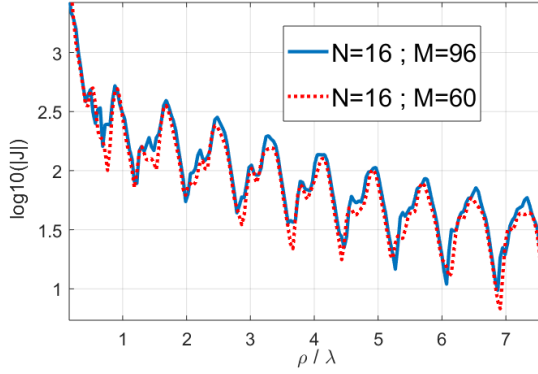


Figure 2.10: Convergence analysis for the RHCP broadside pencil-beam antenna described by the anisotropic IBC in (2.44). Comparison between the absolute value of the currents obtained on the $\phi = 0$ axis with FBBFs and ($N = 16$; $M = 60$); and with FBBFs and ($N = 16$; $M = 96$).

Table 2.3: Simulation times for the MTS described by (2.44).

Solver	Substrate matrix	IBC matrix	Inversion time	Total time
FBBF	21.76 s	12.32 s	3.67 s	39.65 s
GRBF	7.48 s	4.44 s	3.67 s	16.59 s

RWG basis functions to mesh the patches implementing the corresponding IBC requires 1083240 basis functions and 1h35 min of computation time even with the fast full-wave solver adopted in [43].

Next, we analyze a larger MTS aperture. The elements of the opaque IBC tensor are now given by (2.44), with $X_0 = 0.55\eta_0$, $M_{\rho\rho} = 0.2$, $M_{\rho\phi} = 0.2$, $M_{\phi\phi} = 0.15$, and $d = \lambda / \sqrt{1 + (X_0/\eta_0)^2}$. The frequency of operation is $f = 30$ GHz and the antenna radius is $a = 30\lambda$ at this frequency. The cladding lies on a dielectric substrate with relative permittivity $\epsilon_r = 9.8$ and thickness $h = 0.425$ mm. The obtained patterns, without and with center current subtraction are represented in Fig. 2.11(a) and Fig. 2.11(b). One can observe a good agreement between both approaches. However, a stable convergence is obtained with FBBF for ($N = 16$; $M = 90$), i.e. 5940 basis

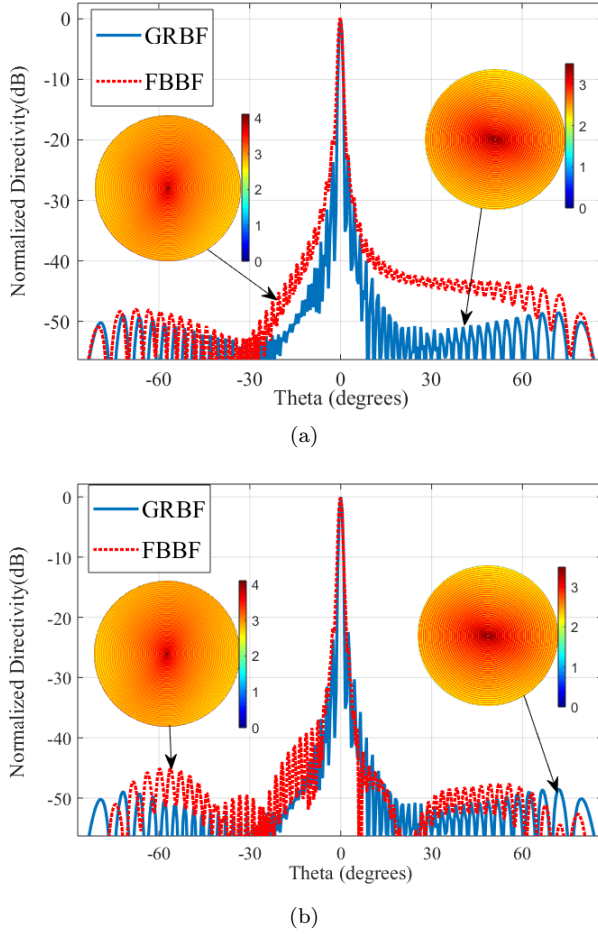


Figure 2.11: RHCP component of the normalized directivity pattern for the structure described by (2.44). The results obtained with GRBFs (solid line) are compared with the FBBFs one (dashed line) in two cases. (a) without center current subtraction. (b) with center current subtraction. The inset disk corresponds to the absolute value of the current distribution (in log10 scale).

functions, whereas in the GRBFs case one needs to use ($N = 16$; $M = 150$) which leads to 9900 basis functions.

Squinted pencil beam MTS antenna based on an anisotropic IBC

The last example is an anisotropic MTS antenna radiating a RHCP squinted pencil beam at $\theta_p = 30^\circ$ and $\phi_p = 0^\circ$. The applied anisotropic IBC is defined as (see Sec. IV-B of [43]):

$$\begin{aligned} Z_+^{\rho\rho}(\boldsymbol{\rho}) &= jX_0 [1 + M_0 \cos \theta_p \cos(2\pi\rho/d - \gamma - \phi)] \\ Z_+^{\rho\phi}(\boldsymbol{\rho}) &= Z_+^{\phi\rho}(\boldsymbol{\rho}) = jX_0 M_0 \sin(2\pi\rho/d - \gamma - \phi) \\ Z_+^{\phi\phi}(\boldsymbol{\rho}) &= jX_0 [1 - M_0 \cos(2\pi\rho/d - \gamma - \phi)] / \cos^2 \theta_p \end{aligned} \quad (2.45)$$

where: $\gamma = k_0 \rho \cos \phi \sin \theta_p$. The other parameters are the same as those considered in Section V-B2 of [61]. Here, a stable convergence (relative error $\approx 1\%$ on the pattern) is obtained with FBBF for $(N = 26; M = 50)$, i.e. 5300 basis functions and with GRBF for $(N = 26; M = 96)$, i.e. 10176 basis functions. The current distributions obtained with FBBFs and GRBFs are represented in Figs. 2.12(a) and 2.12(b), respectively.

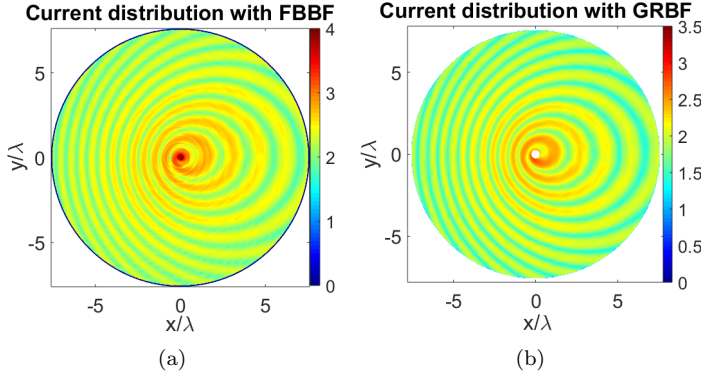
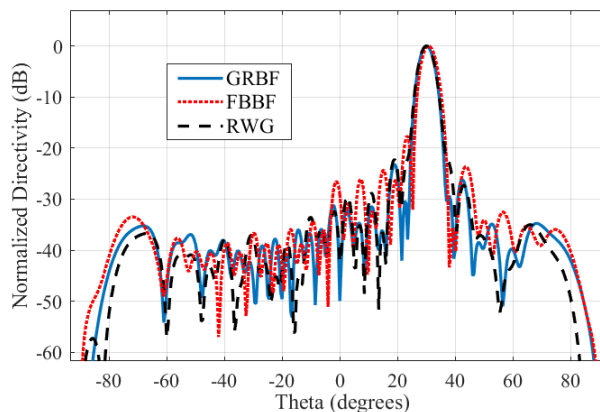


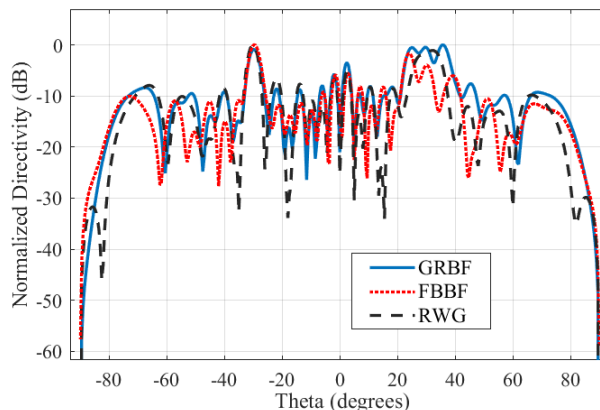
Figure 2.12: Absolute value of the surface current distribution (in log10 scale) on the anisotropic IBC aperture described by (2.45) and used to generate a RHCP squinted pencil-beam. The IBC disk has been analyzed with (a) FBBFs using $(N = 26; M = 50)$ and with (b) GRBFs using $(N = 26; M = 96)$.

The corresponding co-polar and cross-polar patterns are depicted in Figs. 2.13(a) and 2.13(b), respectively. In the same plots, one can find

the radiation pattern from the RWG-MoM simulation of the MTS implemented with screw-head patches in [61]. It is important to note that for the RWG simulation, there are no patches close to the center of the MTS. A good agreement is observed, despite using almost two times fewer basis



(a)



(b)

Figure 2.13: Normalized directivity pattern for the structure described by (2.45) and used to obtain a squinted pencil beam. The results obtained with $N = 26$, $M = 96$ GRBFs (blue solid line) and those obtained after meshing the patches of the MTS with RWG basis functions are compared with the results obtained using $N = 26$, $M = 50$ FBBFs (red dotted line) both for the (a) RHCP component and the (b) LHCP component.

functions (5300 vs 10176), except for the region close to the MTS center, due to reasons we have already discussed in Section 2.1.3. However, one can see that the radiation pattern obtained after center current subtraction provides a good estimate of the pattern obtained with the RWG-MoM solution of the MTS simulated without center patches. This has been observed for all the simulated examples. The authors believe that such modeling is sufficient to achieve a first MTS antenna design. A fine control of the antenna input impedance and far-out sidelobes may require a full-wave analysis of the feeding structure, taking into account the coupling with the immediate IBC vicinity.

2.2 Analysis of elliptical domain metasurface antenna

This section extends the previous formalism to the analysis of elliptical domain metasurface antennas. The used system of coordinates is depicted in Fig. 2.14.

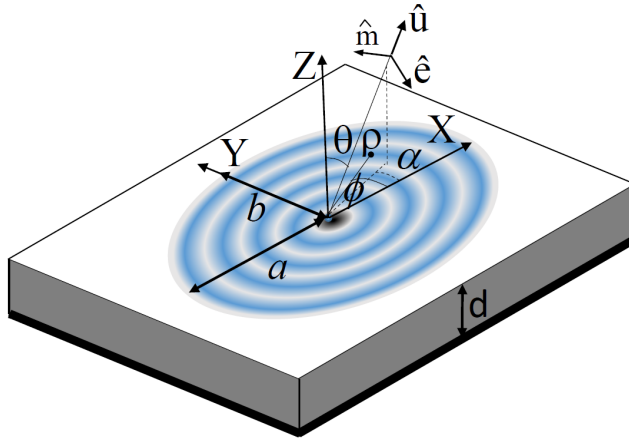


Figure 2.14: Elliptical domain Modulated MTS antenna printed on a grounded dielectric slab substrate.

2.2.1 Fourier Bessel on elliptical domain

Let us consider an elliptical MTS placed at the top ($z=0$) of a grounded dielectric slab as represented in Fig. 2.14. An observation point on the MTS is identified with the vector $\boldsymbol{\rho} = (\rho, \phi)$, where ρ and ϕ are respectively the radial and azimuthal coordinates. It is assumed without loss of generality that the semi-major axis a and the semi-minor axis b of the ellipse are respectively along the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ unit vectors. The FBBFs can be defined in such a structure as follows:

$$R_{m,n}(x, y) = F_{m,n}(\rho') e^{-jn\phi'} \quad (2.46)$$

where:

$$F_{m,n}(\rho') = J_n(\lambda_n^m \rho') \quad (2.47)$$

and $\rho' = \sqrt{(x/a)^2 + (yp/a)^2}$, $\tan(\phi') = yp/x$, $p = a/b > 1$ and λ_n^m is the m -th positive zero of the Bessel function of order n . Primed coordinates refer to a circular domain. The basis (2.46) admits a closed form Fourier Transform:

$$\tilde{R}_{m,n}(k_x, k_y) = -\frac{2\pi j^n e^{-jn\alpha'}}{p} \left[\frac{a^2 \lambda_n^m J_{n-1}(\lambda_n^m) J_n(k'_\rho a)}{(\lambda_n^m)^2 - (k'_\rho a)^2} \right] \quad (2.48)$$

where $k'_\rho = \sqrt{(k_x)^2 + (k_y/p)^2}$, $\tan(\alpha') = k_y/(pk_x)$ and k_x, k_y are the spectral variables written in a cartesian system of coordinates. In this scalar basis, the current distribution on the MTS can be expanded according to its $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ components as follows:

$$\mathbf{J}(\boldsymbol{\rho}) = \sum_{n=-N}^N \sum_{m=1}^M i_{mn}^x R_{m,n}^x(x, y) \hat{\mathbf{x}} + i_{mn}^y R_{m,n}^y(x, y) \hat{\mathbf{y}} \quad (2.49)$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors along the x and y directions, respectively. $R_{m,n}^x(x, y)$ is given by expression (2.46) and $R_{m,n}^y(x, y) = (1/p) R_{m,n}^x(x, y)$.

2.2.2 Method of Moments for elliptical domain MTS

The MoM matrix has the same structure as in the circular domain (see equations (2.8), (2.9)).

Substrate matrix contribution

Each element of the substrate interaction matrix can be written as:

$$Z_G^{xx}(m, n; m', n') = \frac{1}{4\pi^2} \iint \tilde{R}_{m,n}^{x,*}(k_x, k_y) \tilde{G}_{xx}^{EJ}(k_x, k_y) \tilde{R}_{m',n'}^x(k_x, k_y) dk_x dk_y \quad (2.50)$$

where $\tilde{R}_{m,n}^x(k_x, k_y)$ is given by (2.48), $\tilde{G}_{xx}^{EJ}(k_x, k_y)$ is the substrate spectral Green's function associated to the x-directed current and x-directed E-Field. Similar expressions are obtained for the other blocks after replacing the proper Green's function, basis and testing functions. The spectral Green's function $\tilde{G}_{xx}^{EJ}(k_x, k_y)$ can be written as follows [72]- [74]

$$\tilde{G}_{xx}^{EJ}(k_x, k_y) = \tilde{G}_A(k_\rho) + k_x^2 \tilde{G}_V(k_\rho) \quad (2.51)$$

where $k_\rho = \sqrt{k_x^2 + k_y^2}$. The 2D integral (2.50) cannot directly be transformed into a 1D integral as is the case of the circular domain analysis because of the stretching of the basis function in the y-direction only. To overcome this difficulty, the stretching of the basis and testing functions is first translated into a changes in the Green's function expression, according to $k'_y = k_y/p$ and $k'_x = k_x$. The Green's function is no longer symmetrical in (k'_x, k'_y) coordinates.

$$Z_G^{xx}(m, n; m', n') = \frac{p}{4\pi^2} \iint \tilde{R}_{m,n}^{x,*}(k'_x, pk'_y) \tilde{G}_{xx}^{EJ}(k'_x, pk'_y) \tilde{R}_{m',n'}^x(k'_x, pk'_y) dk'_x dk'_y \quad (2.52)$$

Integral (2.52) can be split into two terms $Z_{G_A}^{xx}$ and $Z_{G_V}^{xx}$ according to expression (2.51) of the Green's function.

Next, $\tilde{G}_A(k'_x, pk'_y) = \tilde{G}_A(\sqrt{(k'_x)^2 + (pk'_y)^2})$ is expanded into a har-

monic series:

$$\tilde{G}_A \left(\sqrt{(k'_x)^2 + (pk'_y)^2} \right) \approx \sum_{q=-Q}^Q \tilde{G}_{A_q}(k'_\rho) e^{jq\alpha'} \quad (2.53)$$

where $k'_\rho = \sqrt{(k'_x)^2 + (k'_y)^2}$ and $\tan \alpha' = k'_y/k'_x$. The same technique is used for $(k'_x)^2 \tilde{G}_V \left(\sqrt{(k'_x)^2 + (pk'_y)^2} \right)$, which leads to:

$$\begin{aligned} k_x'^2 \tilde{G}_V \left(\sqrt{(k'_x)^2 + (pk'_y)^2} \right) &\approx \frac{1}{4} (e^{2j\alpha'} + e^{-2j\alpha'} + 2) \\ &\quad k_\rho'^2 \sum_{q=-Q}^Q \tilde{G}_{V_q}(k'_\rho) e^{jq\alpha'} \end{aligned} \quad (2.54)$$

After introducing these harmonic expressions in (2.52), each term of the expansion can be evaluated in closed form along α' , similar to what is done in [79]. In order to smooth the integrand and avoid branch cuts, an integration contour is used. Here, the contour deformation used in [75] is adopted. Each integration is split into two parts. One for $0 \leq k'_\rho \leq 2k_0$, where the contour deformation is performed and the second for $2k_0 < k'_\rho < k_{max} \approx 10k_0 = 20\pi/\lambda$. In practice, $Q = 10$ is sufficient for the $\tilde{G}_A$ expansion on the contour path, and $Q = 5$, is sufficient for the $\tilde{G}_A$ and $\tilde{G}_V$ expansion on the non-contour path. For the $\tilde{G}_V$ expansion on the contour path, we used $Q = 15$, due to the presence of the poles, which renders the integral more oscillatory. All these integrals are computed with a Gauss-Legendre quadrature. Finally, since the stretching (2.53) and (2.54) preserves the parity of the Green's function w.r.t the azimuthal coordinate α' , the odd coefficients of the expansion are equal to 0. We just need to take into account the even coefficients. The same technique is used for Z_G^{xy} , Z_G^{yx} and Z_G^{yy} , after replacing in (2.52), the proper basis function (x or y), testing function (x or y) and the appropriate element of the dyadic Green's function.

Sheet impedance contribution

Similarly to the substrate matrix, the sheet transition matrix has the same block structure as that of the circular domain analysis. Each element of $[Z_{IBC}^{xx}]$ is given by:

$$Z_{IBC}^{xx}(m, n; m', n') = \iint R_{m,n}^{x,*}(x, y) Z_S^{xx}(x, y) R_{m',n'}^x(x, y) dx dy \quad (2.55)$$

where the integration is carried out over the elliptic domain of the MTS, Z_S^{xx} is the xx component of the sheet transition tensorial IBC (see Fig. 2.1), which is obtained from the design. The stretching of the basis functions is then reported on the sheet impedance, which allows rewriting expression (2.55) as

$$Z_{IBC}^{xx}(m, n; m', n') = \frac{1}{p} \iint R_{m,n}^{x,*}(x', y'/p) Z_S^{xx}(x', y'/p) R_{m',n'}^x(x', y'/p) dx' dy' \quad (2.56)$$

where $x' = x$ and $y' = yp$ and the integration is carried out over a unit disk. The integral can then be solved, either after expanding $Z_S^{xx}(x', y'/p)$ into Fourier series or after expanding it into FBBF basis as explained in Section 2.1.2. It is important to mention that, as in practice ($1 < p < 1.5$), the rescaling of the sheet impedance does almost not affect the number of required coefficients of expansion. That is, the computation time of the sheet matrix is not affected by the rescaling. The same procedure is used for the other blocks $[Z_{IBC}^{xy}]$, $[Z_{IBC}^{yx}]$ and $[Z_{IBC}^{yy}]$.

Excitation contribution

When dealing with MTS antennas, excited by a SW launcher, the simple vertical dipole approximation of the feeding is usually sufficient to get an excellent idea about the radiation pattern as shown in [43], [61]. The excitation contribution is structured in two blocks [61], [79]. The first block is related to the x-directed testing functions $[v^x]$ and the second, to the y-directed testing functions $[v^y]$. Each component of $[v^x]$ can be written in

spectral domain as follows:

$$v_x(m, n) = -\frac{1}{4\pi^2} \iint \tilde{R}_{m,n}^{x,*}(k_x, k_y) \tilde{G}_{xz}^{EJ}(k_x, k_y) dk_x dk_y \quad (2.57)$$

where $G_{xz}^{EJ}(k_x, k_y)$ is the spectral x-directed Green's function associated to a vertical dipole placed at the center ($\rho = 0$) and at the middle of the substrate thickness. The technique used for the substrate matrix calculation can then also be used to calculate this integral. The same procedure is used for $[v^y]$.

2.2.3 Numerical results for elliptical domain MTS

We analyze two MTS antennas. The first one is based on a scalar (isotropic) reactance and radiates a pencil beam at 45° , and the second one is based on a tensorial (anisotropic) modulated impedance.

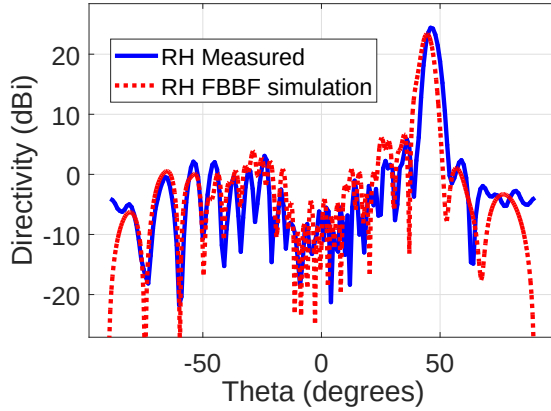
Isotropic MTS antenna

This MTS has been designed in [44]. The obtained opaque (impenetrable) impedance modulation is:

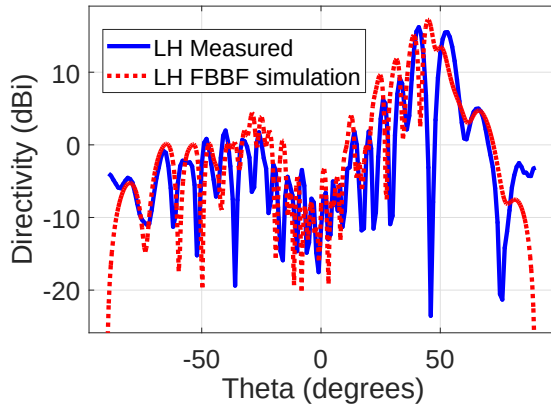
$$Z_s(\boldsymbol{\rho}) = jX_s(1 + M_s \cos(k_t \rho + \phi + k_0 \rho \sin \theta_0 \cos(\phi - \phi_0))) \quad (2.58)$$

where the local periodicity of the modulation is given by $T(\boldsymbol{\rho}) = 2\pi\rho/(k_t\rho + \phi + k_0\rho \sin \theta_0 \cos(\phi - \phi_0))$, $X_s = 290 \, \Omega$, $M_s = 0.18$, $\theta_0 = 45^\circ$, $\phi_0 = 0^\circ$. The operating frequency is 20 GHz and the antenna is built on a substrate with relative permittivity $\epsilon_r = 3.66$ and thickness $d = 1.52$ mm. The semi-major axis of the ellipse is $a = 10\lambda$, and its semi-minor axis is 7.5λ , which leads to $p = 4/3$. The local wave number k_t is obtained after solving the Oliner problem corresponding to the dispersion equation associated to the local periodic structure [46], with the local periodicity $T(\boldsymbol{\rho})$ (periodic quasi-Geometrical Optics (GO) [44] or Flat Optics (FO) [51] approximation). Fig. 2.15(a) and Fig. 2.15(b) show respectively the co-polar (corresponding to the Right Handed Circular Polarization (RHCP)) and the cross-polar pattern (Left Handed Circular Polarization (LHCP)) in the plane $\phi = 0^\circ$, com-

paring our simulation results with the realized MTS measurements in [44]. These solutions have been obtained with $(N = 30, M = 55)$, which corre-



(a)



(b)

Figure 2.15: Directivity pattern for the structure described by (2.58). The result obtained with FBBF (dashed line) is compared with the measurements (solid line) in two cases. (a) RHCP pattern. (b) LHCP pattern.

sponds to 3355/6710 basis functions, depending on the usage of cartesian

or polar formulation for the current expansion. A good agreement is observed, despite using a simplified version of the feeding and the homogenized MTS. The small shift of the beam has already been observed in [44]. The substrate matrix calculation takes 11 min (due to the different harmonics appearing after rescaling the Green's function), and the IBC matrix calculation takes 55s, (same as what we should have with a circular MTS of radius equal to the semi-major axis of the ellipse). In an optimization process, the substrate matrix does not need to be recomputed when only the IBC is modified. For comparison, the direct simulation of the final structure (i.e. implemented with patches) requires more than 10^6 basis functions and takes several hours, even with accelerated techniques [43], [80], [61].

2.2.4 Anisotropic MTS antenna

Here, we show results for the MTS described in [61, Sec. V.B.2], whose analysis with circular FBBFs has been validated in [79, Sec. IV.C] with GRBFs. This circular MTS has been designed to radiate a squinted pencil beam at $\theta_0 = 30^\circ$, $\phi_0 = 0^\circ$. The radius is 7.6λ . Here, we compute the radiation pattern of this structure, restricted to an elliptical MTS, with semi-major axis equal to the original radius and $p = 1.4$. The RHCP and LHCP radiation patterns in the plane $\phi = 0^\circ$ are reported in Fig. 2.16(a) and Fig. 2.16(b).

2.3 Conclusion

A MoM analysis using FBBFs has been proposed to obtain the global evolution of the current distribution on circular modulated MTSs. These basis functions are directly defined over the whole computational domain, without any emphasis on specific sub-domains. The main advantage of the Fourier-Bessel basis with respect to the GRBFs resides in its orthogonality and its completeness, allowing an analysis of a general class of disk current distribution in an effective manner, i.e., more compact. In comparison with the well known family of orthogonal Zernike functions, FBBFs possess a very limited bandwidth, leading to an easy and fast calculation of the reaction integrals in the spectral domain. Moreover, we have observed that

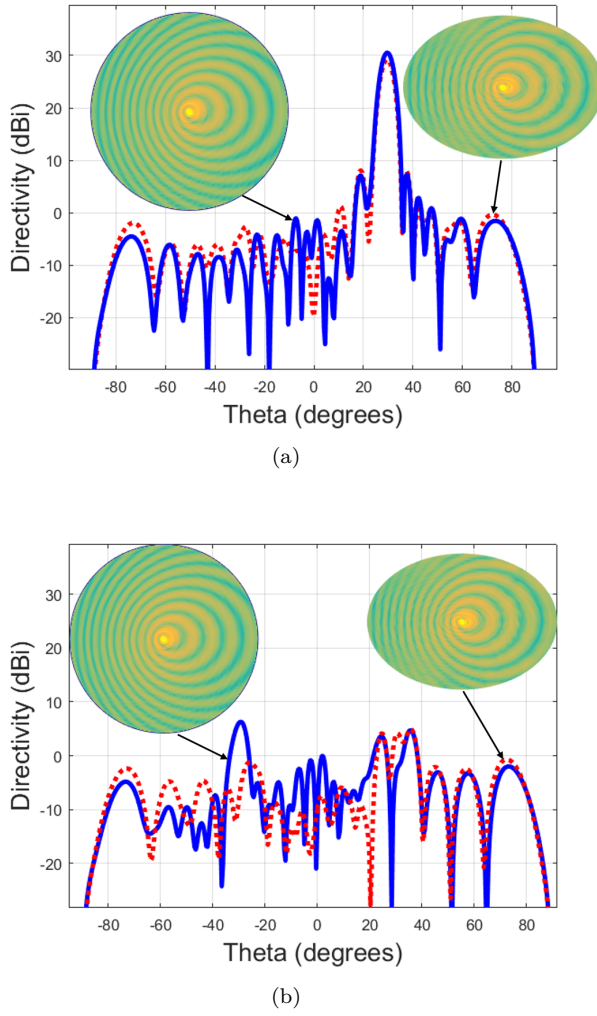


Figure 2.16: Directivity pattern of the structure presented in [61, Sec. V.B.2] is compared with an elliptical version of the same MTS. in two cases. (a) RHCP pattern. (b) LHCP pattern. The disk and the ellipse represent respectively the absolute value of the current distribution on the circular and the elliptical MTS.

the Fourier-Bessel basis is better suited than the Zernike basis in describing typical current distributions on circular MTSs. The proposed basis has been

extended to analyze the more general class of elliptical apertures (with a good comparison with measured data of an MTS antenna made with realistic feed), while maintaining the orthogonality properties. The basis chosen here is not limited to a particular kind of excitation even if the examples proposed in this chapter used a dipole excitation. When the MTS is excited from the center, GRBFs allow a better representation of the currents in the immediate vicinity of the feed.

CHAPTER 3

Synthesis of modulated metasurface antennas

THIS chapter presents a new design algorithm, directly based on the solution of the electric field integral equation (EFIE). Contrary to previous design methods [52], [53] the technique proposed here is global, i.e. it is not based on local periodicity assumptions; but makes use of the global bases mentioned above. The sheet transition impedance boundary condition (IBC) is directly computed based on the knowledge of the current distribution. For antenna applications, only part of the current spectrum is known, i.e. the visible part, which translates into the far-field pattern. This means that the problem is ill-posed by essence. However, as the far-field radiation is due to the perturbation of an initial surface-wave (SW), this a priori knowledge can be used to naturally extend the visible currents toward the invisible spectrum, corresponding to fields constrained to a region very close to the surface. For didactic reasons, the method is first explained for the scalar metasurfaces (MTSs) case and then extended to the more general case of tensorial MTSs.

3.1 Inverse problem based on the EFIE

The main purpose of the present chapter consists of deriving quasi-directly the surface impedance able to provide a given radiation pattern $\mathbf{F}$, assuming that the excitation of the MTS is known a priori. The radiation pattern $\mathbf{F}$ is the desired spatial electric field distribution in the far field. This is known in amplitude, phase and polarization, within a constant multiplicative factor, which depends on the radiated power of the antenna. Through a Fourier transformation, the desired radiation pattern $\mathbf{F}$ can be translated into the visible spectrum of the tangential aperture electric field as follows

$$\begin{bmatrix} \tilde{f}_x \\ \tilde{f}_y \end{bmatrix} = \frac{-2\pi}{jk_0 u_z \sqrt{1 - u_z^2}} \begin{bmatrix} u_y & -u_x u_z \\ -u_x & -u_y u_z \end{bmatrix} \begin{bmatrix} F_{TE} \\ F_{TM} \end{bmatrix} \quad (3.1)$$

where $\eta_0 \approx 377 \, \Omega$ is the free-space impedance. F_{TE} and F_{TM} are the components of the radiation pattern $\mathbf{F}(\theta, \phi)$ respectively parallel to $\hat{\mathbf{m}}$ and $\hat{\mathbf{e}}$ vectors in Fig. 2.14 and $\hat{\mathbf{u}} = (u_x, u_y, u_z) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ is the observation direction.

The visible spectrum is the part of the spectrum for which $\sqrt{k_x^2 + k_y^2} < k_0 = 2\pi/\lambda$, where λ is the free-space wavelength and (k_x, k_y) are the spectral variables in Cartesian coordinates. Then, through the substrate dyadic Green's function, the visible spectrum of the current distribution on the MTS is obtained as $\tilde{\mathbf{J}} = (\underline{\underline{\mathbf{G}}}^{EJ})^{-1} \tilde{\mathbf{f}}$, where $\tilde{\mathbf{f}}$ is the Fourier transform of the aperture tangential electric field and $\underline{\underline{\mathbf{G}}}^{EJ}$ is the dyadic spectral Green's function associated with the substrate. Those currents are described in the form (2.5) through projection on the chosen set of basis functions (parametric closed-form expansion similar to what has been done in [70]). The invisible part of the currents is obtained from the unmodulated impedance $\underline{\underline{\mathbf{Z}}}_0$. The choice of $\underline{\underline{\mathbf{Z}}}_0$ and the scaling between visible and invisible parts will be detailed in Section 3.2.3. In the following two sub-sections, the sheet impedance is also decomposed into entire-domain basis functions and, based on the objective currents referred to above, the sheet impedance coefficients will be obtained by inverting the EFIE (2.7). This will be done first for a scalar impedance, then for a tensorial impedance.

3.1.1 Inverse problem for scalar MTS

For a scalar MTS excited at the center, the current distribution is only along the $\hat{\rho}$ direction. The substrate matrix contribution can then be reduced to the only block referring to the $\hat{\rho}$ oriented basis and $\hat{\rho}$ oriented testing functions. Expression (2.34) then takes the following form:

$$(\mathbf{Z}_G - \mathbf{Z}_{IBC}) \mathbf{I} = \mathbf{V} \quad (3.2)$$

where $\mathbf{I}$ only includes the i_{mn}^ρ coefficients. In this sub-section, i_{mn}^ρ will be denoted as $i_{m_b n_b}$ in order to make the difference between the indices corresponding to the current expansion, field testing, and IBC expansion. Each element of the IBC matrix contribution can be written as follows:

$$Z_{IBC}(m, n; m', n') = \iint_S R_{m,n}^{t,*}(\boldsymbol{\rho}) Z_S(\boldsymbol{\rho}) R_{m',n'}^b(\boldsymbol{\rho}) \rho \, d\rho \, d\phi \quad (3.3)$$

where superscripts t , b and $*$ stand respectively for the testing functions, basis functions, and complex conjugate. Z_S is the scalar sheet impedance, and the integration is carried out over the circular MTS domain. Now, let us write the sheet impedance Z_S in the following general form:

$$Z_S(\boldsymbol{\rho}) = Z_0 + P(\boldsymbol{\rho}) \quad (3.4)$$

where $Z_0 = jX_0$ is the unmodulated reactance value. Expression (3.4) can be expanded into the FBBFs as follows:

$$Z_S(\boldsymbol{\rho}) \approx \sum_{n_S, m_S} (K_S(m_S, n_S) + X_S(m_S, n_S)) R_{m_S, n_S}(\boldsymbol{\rho}) \quad (3.5)$$

The column vector $\mathbf{K}_S$ will contain the $K_S(m_S, n_S)$ values and the column vector $\mathbf{X}_S$ will contain the $X_S(m_S, n_S)$ values. These are the coefficients describing the impedance modulation, i.e. what we are looking for in order to perform the synthesis. Each element of $\mathbf{K}_S$ can be calculated by simple projection of Z_0 as follows:

$$K_S(m_S, n_S) = \frac{1}{K_0(m_S, n_S)} \iint_S Z_0 R_{m_S, n_S}^*(\boldsymbol{\rho}) \rho \, d\rho \, d\phi \quad (3.6)$$

where $K_0(m_S, n_S)$ is a normalization factor given by:

$$K_0(m_S, n_S) = -\pi J_{n_S-1}(\lambda_{n_S m_S}) J_{n_S+1}(\lambda_{n_S m_S}) a^2 \quad (3.7)$$

As Z_0 is a constant, this integral is nonzero only for $n = 0$. In the same way, each element of $\mathbf{X}_S$ corresponds to the projection of $P(\boldsymbol{\rho})$ on a given basis function.

Based on (3.2) and (3.3), $\mathbf{X}_t = \mathbf{Z}_{IBC} \mathbf{I}$ corresponds to the tested surface fields consistent with the sheet IBC. After inserting (3.5) into (3.3), the sheet IBC fields, tested by functions R_{m_t, n_t} , can be rewritten as:

$$\mathbf{X}_t = \sum_{n_b, m_b} i_{m_b n_b} \sum_{n_S, m_S} (K_S(m_S, n_S) + X_S(m_S, n_S)) a^2 \mathbf{Z}_U(m_b, n_b; m_S, n_S; :, :) \quad (3.8)$$

where $\mathbf{Z}_U$ is a 3-D matrix of size $((2N_b + 1) M_b) \times ((2N_S + 1) M_S) \times ((2N_t + 1) M_t)$, whose elements are defined as

$$Z_U(m_b, n_b; m_S, n_S; m_t, n_t) = \iint_{S_U} R_{m_t, n_t}^*(\boldsymbol{\rho}) R_{m_S, n_S}(\boldsymbol{\rho}) R_{m_b, n_b}(\boldsymbol{\rho}) \rho d\rho d\phi \quad (3.9)$$

where the integration domain S_U corresponds to the unit disk. The computation of this matrix has already been discussed in Chapter 2 for simulation acceleration purposes. The nonzero elements of $\mathbf{Z}_U$ can be tabulated in a matrix of size $((2N_b + 1) M_b) \times M_S \times ((2N_t + 1) M_t)$ since integral (3.9) is different from zero only for $n_t = n_S + n_b$. It is interesting to notice that this matrix only depends on the chosen decomposition function on the disk. It is independent from the substrate, frequency or desired radiation pattern. Therefore this function, which is key to the proposed technique, can be computed once and for all, up to any desired upper value for the indices.

After swapping in (3.8), the double summations in (n_b, m_b) with those in (n_S, m_S) and then writing the first double summation in matrix form,

expression (3.8) can be written as:

$$\mathbf{X}_t = \mathbf{Z}^S \mathbf{X}_S + \mathbf{X}_K \quad (3.10)$$

where $\mathbf{Z}^S$ is a $((2N_S + 1) M_S) \times ((2N_t + 1) M_t)$ matrix, where each element is given by

$$Z^S(m_S, n_S; m_t, n_t) = \sum_{n_b, m_b} i_{m_b n_b} Z(m_b, n_b; m_S, n_S; m_t, n_t) \quad (3.11)$$

$\mathbf{Z} = a^2 \mathbf{Z}_U$, and $\mathbf{X}_K$ is a column vector of length $(2N_t + 1) M_t$, given by:

$$\mathbf{X}_K = \mathbf{Z}^S \mathbf{K}_S \quad (3.12)$$

Inserting expression (3.10) into the discretized EFIE (3.2) leads to the derivation of $\mathbf{X}_S$

$$\mathbf{X}_S = (\mathbf{Z}^S)^{-1} (\mathbf{Z}_G \mathbf{I} - \mathbf{V} - \mathbf{X}_K) \quad (3.13)$$

This expression allows the determination of the modulation of the sheet impedance from the knowledge of the currents. Note that there is actually no obligation to have surface impedance described in the same basis as the currents. This means that one can choose another set of basis functions to discretize the surface impedance.

3.1.2 Inverse problem for tensorial MTS

This subsection extends the inversion procedure described above to the case where the sheet impedance $\underline{\underline{Z}}_S$ found in (2.7) becomes fully tensorial. A tensorial MTS is considered, with a sheet IBC $\underline{\underline{Z}}_S$ described by

$$\begin{aligned} Z_S^{\rho\rho}(\boldsymbol{\rho}) &= Z_0^{\rho\rho} + P^{\rho\rho}(\boldsymbol{\rho}) \\ Z_S^{\rho\phi}(\boldsymbol{\rho}) &= -(Z_S^{\phi\rho}(\boldsymbol{\rho}))^* = P^{\rho\phi}(\boldsymbol{\rho}) \\ Z_S^{\phi\phi}(\boldsymbol{\rho}) &= Z_0^{\phi\phi} - P^{\rho\rho}(\boldsymbol{\rho}) \end{aligned} \quad (3.14)$$

where $Z_0^{\rho\rho}$, and $Z_0^{\phi\phi}$ are purely imaginary constants, with negative imag-

inary parts, and $P^{\rho\rho}$ and $P^{\rho\phi}$ are purely imaginary functions of $\boldsymbol{\rho}$. This class of MTS reactances is anti-Hermitian as generally required to be synthesizable with printed patches in the absence of losses [43]. The IBC tensor (3.14) can be expanded as:

$$Z_S^{\rho\rho}(\boldsymbol{\rho}) \approx \sum_{n_S, m_S} [K_S^{\rho\rho}(m_S, n_S) + X_S^{\rho\rho}(m_S, n_S)] R_{m_S, n_S}(\boldsymbol{\rho}) \quad (3.15)$$

$$Z_S^{\rho\phi}(\boldsymbol{\rho}) \approx \sum_{n_S, m_S} X_S^{\rho\phi}(m_S, n_S) R_{m_S, n_S}(\boldsymbol{\rho}) \quad (3.16)$$

$$Z_S^{\phi\phi}(\boldsymbol{\rho}) \approx \sum_{n_S, m_S} [K_S^{\phi\phi}(m_S, n_S) - X_S^{\rho\rho}(m_S, n_S)] R_{m_S, n_S}(\boldsymbol{\rho}) \quad (3.17)$$

Now, similar to expression (3.10), we obtain:

$$\underline{\mathbf{X}}_t = \underline{\mathbf{Z}}_{IBC} \underline{\mathbf{I}} = \underline{\mathbf{Z}}^S \underline{\mathbf{X}}_S + \underline{\mathbf{X}}_K \quad (3.18)$$

or in more structural form:

$$\begin{bmatrix} \mathbf{X}_t^\rho \\ \mathbf{X}_t^\phi \end{bmatrix} = \begin{bmatrix} \mathbf{Z}^{S,\rho} & \mathbf{Z}^{S,\phi} \\ -\mathbf{Z}^{S,\phi} & \mathbf{Z}^{S,\rho} \end{bmatrix} \begin{bmatrix} \mathbf{X}_S^{\rho\rho} \\ \mathbf{X}_S^{\rho\phi} \end{bmatrix} + \begin{bmatrix} \mathbf{X}_K^\rho \\ \mathbf{X}_K^\phi \end{bmatrix} \quad (3.19)$$

where $\mathbf{Z}^{S,\rho}$ is a $((2N_S + 1) M_S) \times ((2N_t + 1) M_t)$ matrix, with elements given by

$$Z^{S,\rho}(m_S, n_S; m_t, n_t) = \sum_{n_b, m_b} i_{m_b n_b}^\rho Z(m_b, n_b; m_S, n_S; m_t, n_t) \quad (3.20)$$

and $\underline{\mathbf{Z}}^{S,\phi}$ is a $((2N_S + 1) M_S) \times ((2N_t + 1) M_t)$ matrix, where each element is given by:

$$Z^{S,\phi}(m_S, n_S; m_t, n_t) = \sum_{n_b, m_b} i_{m_b n_b}^\phi Z(m_b, n_b; m_S, n_S; m_t, n_t) \quad (3.21)$$

$\mathbf{X}_K^\rho$ is a vector of length $(2N_t + 1) M_t$ given by

$$\mathbf{X}_K^\rho = \mathbf{Z}^{S,\rho} \mathbf{K}_S^{\rho\rho} \quad (3.22)$$

and $\mathbf{X}_K^\phi$ is a vector of length $(2N_t + 1) M_t$ given by

$$\mathbf{X}_K^\phi = \mathbf{Z}^{S,\phi} \mathbf{K}_S^{\phi\phi} \quad (3.23)$$

Finally, the perturbation of the anisotropic MTS is obtained as

$$\begin{bmatrix} \mathbf{X}_S^{\rho\rho} \\ \mathbf{X}_S^{\rho\phi} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}^{S,\rho} & \mathbf{Z}^{S,\phi} \\ -\mathbf{Z}^{S,\phi} & \mathbf{Z}^{S,\rho} \end{bmatrix}^{-1} \cdot \underline{\mathbf{U}} \quad (3.24)$$

where $\underline{\mathbf{U}}$ is given by:

$$\underline{\mathbf{U}} = \begin{bmatrix} \mathbf{U}^\rho \\ \mathbf{U}^\phi \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_G^{\rho\rho} & \mathbf{Z}_G^{\rho\phi} \\ \mathbf{Z}_G^{\phi\rho} & \mathbf{Z}_G^{\phi\phi} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I}^\rho \\ \mathbf{I}^\phi \end{bmatrix} - \begin{bmatrix} \mathbf{V}^\rho \\ \mathbf{V}^\phi \end{bmatrix} - \begin{bmatrix} \mathbf{X}_K^\rho \\ \mathbf{X}_K^\phi \end{bmatrix} \quad (3.25)$$

Expression (3.24) allows the determination of the anisotropic IBC modulation introduced in (3.14), from the knowledge of the objective currents.

3.2 MTS synthesis

3.2.1 Development of the solution methodology

Let us assume that one is interested in the synthesis of an arbitrary radiation pattern $\mathbf{F}(\theta, \phi)$, where θ and ϕ are the elevation and azimuthal angles (see Fig. 2.14). This pattern is linked to the visible spectrum of the aperture field $\tilde{\mathbf{f}}$ (see equation (3.1)) and that of the current distribution $\tilde{\mathbf{J}}$ through the spectral Green's function. The power radiated by the MTS is the real part of the integral of the Poynting vector along the surface, which can be calculated in spectral domain as:

$$P_{vis} = \frac{1}{8\pi^2} \int_0^{2\pi} \int_0^{k_0} |\tilde{\mathbf{f}}|^2 \frac{k_z^*}{k_0 \eta_0} k_\rho dk_\rho d\phi \quad (3.26)$$

where, $k_z = \sqrt{k_0^2 - k_\rho^2}$ and η_0 is the free-space impedance (see proof of (3.26) in Section 7.2). Trying to directly find the IBC which produces only the visible spectrum leads to a non-synthesizable IBC, in the sense that the IBC has a non-negligible real part, even for the diagonal elements of the matrix, thus violating the anti-Hermitian property, as required for the

patches implementation process [81], [66], [81]. In a first instance, one may try to solve the problem by finding the invisible part of the current spectrum $\tilde{I}_{invis}$ which will product the desired pattern, and at a same time ensures the anti-Hermitian property of the IBC. Fig. 3.1 displays the x-directed and y-directed current distribution spectrum, obtained with a MoM analysis for a MTS radiating a broadside circularly polarized pencil beam, similar to what has been designed in [43]. It can be seen that the invisible spectrum

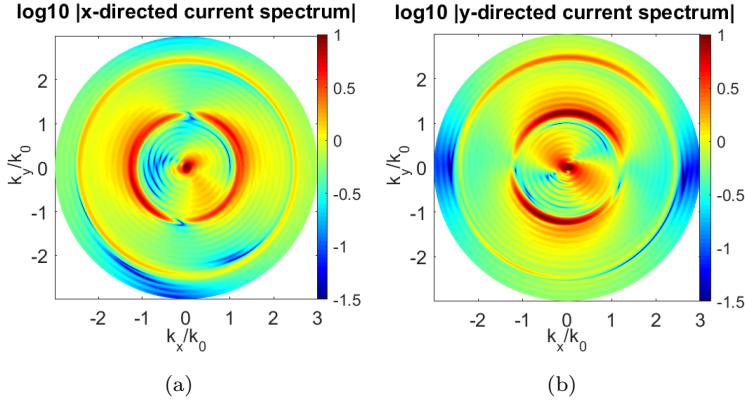


Figure 3.1: Current distribution spectrum on a MTS radiating a broadside circularly polarized pencil beam. (a) x-directed component. (b) y-directed component.

$\tilde{I}_{invis}$ (i.e. outside the unit circle in $(k_x/k_0, k_y/k_0)$ coordinates), corresponding to the evanescent part of the spectrum, cannot be neglected. This is due to the fact that the radiation of the MTS comes from the perturbation of a SW, whose spectrum lies mainly in the invisible part. Fig. 3.2 shows the current spectrum for an unmodulated MTS. As observed, the dominant part of the near-field (or evanescent) spectrum is similar before and after modulation. In fact, the modulation has the effect of creating several Floquet modes. However, outside the visible region, the fundamental mode is the dominant one. This fundamental mode no longer has exactly the same wavenumber i.e. $k_{sw} = \beta_{sw} + \Delta\beta - j\alpha_{sw}$ [46], [51], where k_{sw} and β_{sw} are the supported wavenumber on the modulated and unmodulated reactance,

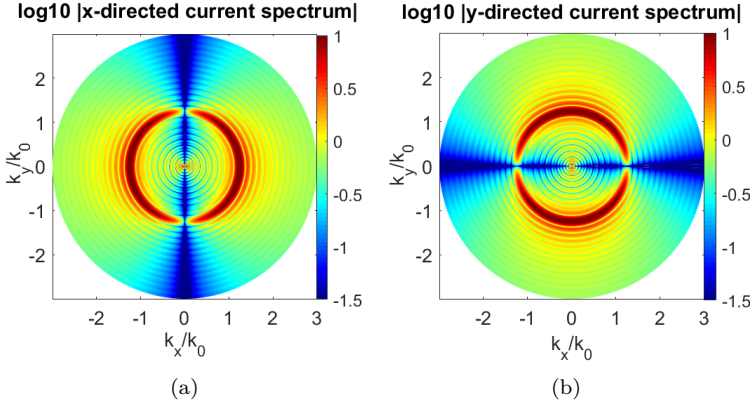


Figure 3.2: Current distribution spectrum on an unmodulated reactance. (a) x-directed component. (b) y-directed component.

respectively. For small modulation depth, as is the case in practice, $\Delta\beta$ can be neglected in comparison with β_{sw} [46]. The attenuation constant α_{sw} is linked with the radiation since the SW must lose energy as radiation leakage goes on. Therefore, the attenuation constant α_{sw} depends on the reactance tensor, which we are actually looking for. This constitutes the fundamental design problem and it can be very difficult to face when dealing with complex patterns. However, since we are looking for a purely imaginary reactance tensor, this constraint can be enforced while solving the EFIE. This is done by imposing a certain symmetry in the expansion of the coefficients of the sheet transition IBC. Indeed, if the IBC is expanded into FBBFs as in (3.15), (3.16), (3.17), the purely imaginary property of the IBC leads to a conjugate symmetry between the coefficients of $X_S^{\rho\rho}$ and $X_S^{\rho\phi}$, along the azimuthal spectral index n_S :

$$X_S^{\rho\rho}(m_S, n_S) F_{m_S, n_S} = - (X_S^{\rho\rho}(m_S, -n_S) F_{m_S, -n_S})^* \quad (3.27)$$

where $F_{m,n}$ corresponds to the radial part of the FBBFs (2.1). After a few algebraic manipulations, (3.27) can be rewritten as:

$$(X_S^{\rho\rho}(m_S, -n_S))^* = - \frac{X_S^{\rho\rho}(m_S, n_S)}{(-1)^{n_S}} \quad (3.28)$$

The same expression is valid for $X_S^{\rho\phi}$. The initial problem, which consists of solving a linear system of equations of the form (3.24):

$$\underline{\underline{\mathbf{Z}}}^S \underline{\underline{\mathbf{X}}}_S = \underline{\underline{\mathbf{U}}} \quad (3.29)$$

can then be reorganized in order to take into account the dependance between each pair of coefficients. In practice, the real and the imaginary parts of each coefficient of expansion $X_S^{\rho\rho}(m_S, n_S)$ and $X_S^{\rho\phi}(m_S, n_S)$ are treated separately as well as the real and the imaginary parts of each element of $\underline{\underline{\mathbf{Z}}}^S$ and $\underline{\underline{\mathbf{U}}}$. This leads to a real system of $4M_t (2N_t + 1)$ equations with $2M_S (2N_S + 1)$ unknowns, where each unknown corresponds to the real or imaginary part of $X_S^{\rho\rho}(m_S, n_S)$ or $X_S^{\rho\phi}(m_S, n_S)$ for $n_S > 0$. For the particular index $n_S = 0$, the coefficients (for every index m_S) need to be purely imaginary. The obtained linear system of equations can then be solved in a least-squares sense. This least-squares EFIE operator will be denoted in the rest of the chapter as *l.s.* $\{EFIE^{-1}\}$:

$$\underline{\underline{\mathbf{X}}}_S = \textit{l.s.} \{EFIE^{-1}(\mathbf{J})\} \quad (3.30)$$

It is important to note that for the proposed formulation, the number of expansion coefficients for the sheet impedance can be chosen independently from that of the currents. Moreover, it is also possible to use more testing functions than basis functions. In practice, it has been observed that the best choice consists of using the same number of current basis and fields testing functions and a bit fewer reactance basis functions. Some realistic choices will be presented in Section 3.3.

The obtained algorithm can then be summarized as follows: we impose in the invisible part of the spectrum the same current as what would be obtained in the absence of modulation and enforce a purely imaginary tensor IBC. This provides the best compromise between the realization of the pattern and near-field constraints. The advantage of proceeding like that is double:

- First, one directly starts with the pattern and not with the aperture tangential electric-field.

- Second, the extension into the near-field is naturally carried out with the least-squares procedure, and can be applied to arbitrary radiation patterns.

3.2.2 MTS feeding

MTSs are in practice fed with a SW launcher. But the feeding is not ideal, in the sense that part of the feeding power is directly converted into space waves and radiates. This part no longer goes through the MTS and can affect the ability of shaping the beam or the radiation performance. This power balance is defined as the power efficiency in [83]. In the same paper, realistic MTS feeds have been studied along with their efficiency. As proven in [43], [84], a simple elementary vertical dipole placed at the center of the MTS ($\rho = 0$) and at the middle of the substrate is sufficient to obtain an excellent estimate of the radiation performance obtained with a realistic feed. We will therefore limit ourselves to the simple vertical dipole approximation of the feeding. However, the proposed method is not limited to this feeding, i.e. it can be used for general MTS excitation. For the vertical elementary dipole feeding, the Green's function providing the electric field can be expressed in closed form:

$$\tilde{G}_{xz}^{EJ}(\mathbf{k}) = \cos \alpha \tilde{g}(k_\rho); \quad \tilde{G}_{yz}^{EJ}(\mathbf{k}) = \sin \alpha \tilde{g}(k_\rho) \quad (3.31)$$

where k_ρ and α are the spectral variables in cylindrical coordinates. The function $\tilde{g}(k_\rho)$ can be found after solving the appropriate transmission line problem [73]. Expression (3.31) is exploited to obtain, in a classical MoM approach, the excitation projection on the FBBFs, yielding the excitation vector $\underline{\mathbf{V}}$ in Section (2.1.2).

3.2.3 Preliminary parameters selection

Impedance modulation

The IBC is modulated around an average reactance $\underline{\mathbf{Z}}_0$. This average reactance is chosen in order to be synthesizable with the adopted patches [53], [81], [66], [82]. In practice, typical values of $X_0^{\rho\rho}$, and $X_0^{\phi\phi}$ vary between

-1000Ω and -200Ω . Note that it is also possible to choose $Z_0^{\rho\rho} \neq Z_0^{\phi\phi}$. Also to facilitate implementation, it is important to limit the modulation depth. The latter is defined for each reactance tensor element as:

$$m = \frac{\max|P|}{Z_0} \quad (3.32)$$

where $\max|P|$ is the maximum of the absolute value of the impedance perturbation P . In this chapter, a maximum modulation depth of 0.6 is assumed. This constraint, in combination with the anti-Hermitian property of the IBC, ensures an easy implementation of the designed IBC using for example the patch shapes adopted in [43].

Invisible current spectrum

As explained previously, the invisible part of the currents spectrum $\tilde{I}_{invis}$ is estimated through the constant sheet impedance $\underline{\underline{Z}}_0$. Those currents must be described in the chosen basis (FBBF), while filtering the visible part. This is carried out by solving the MoM system of equations using that basis, computing the Fourier transform of those currents (the closed-form Fourier transform (2.4) of the basis is exploited here), set to zero the results for $k_x^2 + k_y^2 < k_0^2$, re-project on Fourier transforms of the FBBF basis (2.4), and the resulting coefficients describe the current in the space-domain basis. Note that the orthogonality of the basis is actually not necessary. However, it is very useful in practice since it enables the currents expansion through a simple projection operation (expansion in non-orthogonal bases requires solving a system of equations). As a rule of thumb for the number of needed coefficients of expansion, the following parameters can be considered for a relatively safe upper bound: $M = 10 a/\lambda$ and $N = M/\pi$. They correspond to a maximum significant radial wavenumber equal to 5 times the free-space wavenumber.

Power-based scaling

Another quantity to be determined concerns the total radiated power. As the visible currents are calculated directly from the pattern, the total power

radiated by these currents has to be consistent with the feeding, the substrate and the IBC. Therefore, this power needs to be estimated. We propose to use as a first estimation the total power radiated by a previously designed broadside radiating scalar MTS antenna [43], assuming an average scalar sheet impedance $\underline{\underline{Z}}_0 = \text{diag}(Z_0^{\rho\rho}, Z_0^{\rho\rho})$. The choice of this reference pattern for the radiated power is justified by the simplicity of the design, since an analytical expression of the reactance is available. The derived modulation period is [43] $T = \lambda / \sqrt{1 + (X_{op}/\eta_0)^2}$, where jX_{op} is the corresponding scalar impenetrable (opaque) impedance obtained after adding to $\underline{\underline{Z}}_0$, the substrate contribution [67]. In practice, it can be assumed that the factor $\alpha_{sw}a$ is in the range [0.8 1.3], depending on the desired tapering efficiency. The Oliner problem [46] can then be solved for the chosen values of α_{sw} , X_{op} , and T in order to derive the modulation index m_0 . The obtained scalar modulated IBC $\underline{\underline{Z}}_B = \text{diag}(Z_B^{\rho\rho}, Z_B^{\rho\rho})$ is used to estimate the radiated power P_{vis} . The latter is calculated from expression (3.26), using the MoM solution for that broadside radiating MTS antenna. Once, the radiated power P_{vis} has been estimated, the visible currents I_{vis0} (calculated from the desired pattern) can be rescaled accordingly:

$$I_{vis} = I_{vis0} \sqrt{\frac{P_{vis}}{P_{vis0}}} \quad (3.33)$$

where I_{vis0} , is the visible current obtained directly from the pattern (3.1) and P_{vis0} is the power calculated from the currents I_{vis0} without rescaling. It is interesting to note that an over-estimation of this power will lead to a high modulation depth since it means a strong perturbation of the SW. In the same way, an underestimation of the power will lead to weak pattern fidelity. In our experience, a value of P_{vis} obtained from the broadside pattern forms a good compromise between the above two situations, even for patterns very far from broadside beams. However, an a posteriori tuning of P_{vis} (by about 30%) is an option to slightly improve the solution after going through an iterated process. In the examples treated by the authors so far, a second iteration has never been found necessary. It is also important to note that the impedance derivation process is very stable w.r.t. the radiated power, since a variation of the radiated power by about 10% has

a weak impact on the pattern quality and on the modulation index. The minor correction of P_{vis} and the need to sometimes adapt the number of basis functions (see paragraph 3.2.3) are the only reasons why the method proposed here should be named “quasi-direct” rather than simply “direct”.

3.2.4 Design algorithm

The design algorithm discussed in Section 3.2.3 is summarized in Figs. 3.3 and 3.4.

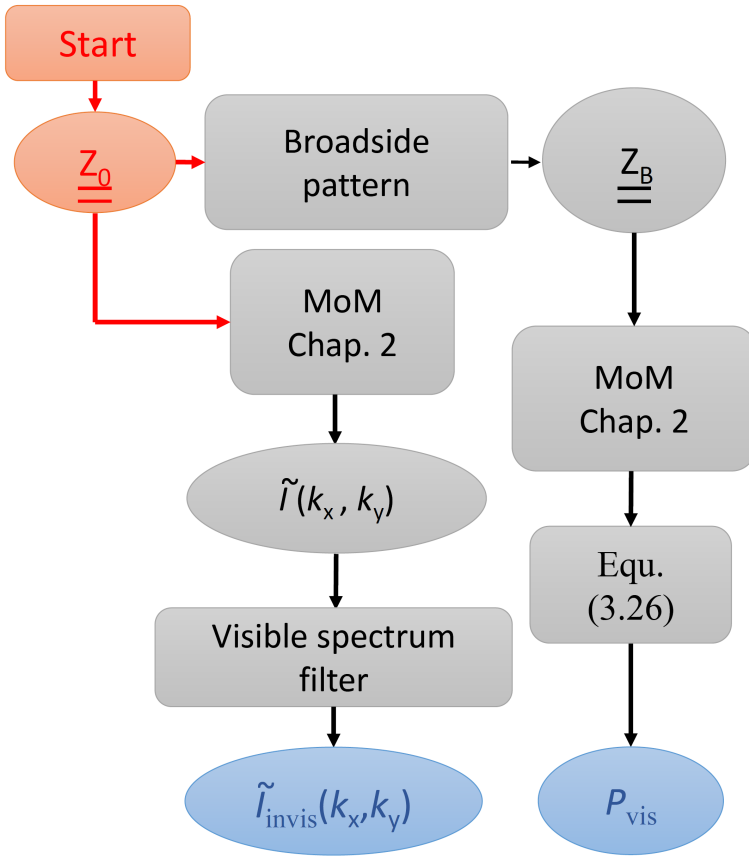


Figure 3.3: Preliminary steps of the algorithm, leading to visible (radiated) power and invisible current spectrum.

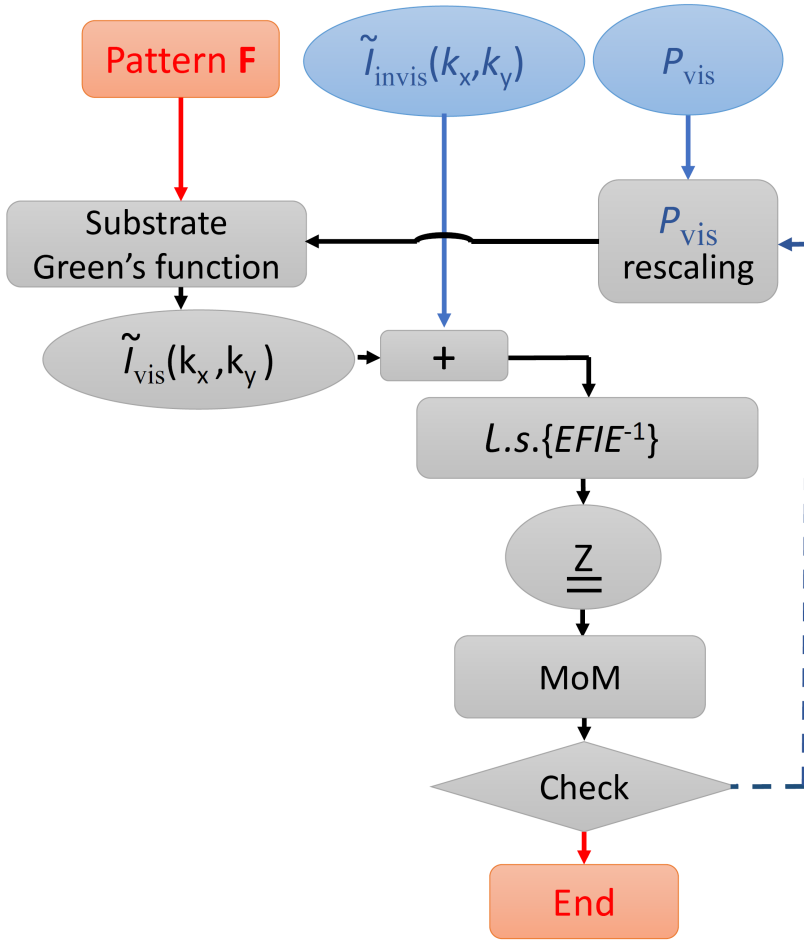


Figure 3.4: Core of the IBC computation algorithm. Lower rhombus: in case the pattern or modulation depth are not satisfactory, P_{vis} is modified by a few percent.

The first part of the algorithm is presented in Fig. 3.3 and describes the determination of the visible (radiated) power P_{vis} and the invisible current spectrum $\tilde{I}_{invis}$. These two quantities are then used in combination with the pattern, in the second part of the algorithm (Fig. 3.4) to compute the IBC. In case the pattern requirements are not satisfied or the modulation depth is too high, the radiated power P_{vis} is rescaled accordingly as dis-

cussed in 3.2.3. It is also important to note that we do not use a mask as done in some classical synthesis processes. The effect of the mask is replaced by the direct determination of desired visible aperture currents -expressed in FBBFs- from desired patterns. Moreover, the proposed algorithm is able to synthesize the radiation pattern $\mathbf{F}$ both in amplitude and phase.

3.3 Numerical results

In this section, five MTS antenna designs are presented. The first one is designed to radiate a circularly polarized pencil beam. The second radiates a rectangular circularly polarized beam in the spectral (k_x, k_y) plane. The third one radiates a cosecant squared pattern. The fourth antenna generates four pencil beams, and the last one is designed to provide a “fish-like” radiation pattern in the (k_x, k_y) plane. For each pattern, the obtained IBC is analyzed and validated numerically with the MoM code developed and validated in Chapter 2.

The four examples have been implemented at an operating frequency of 18 GHz. The substrate thickness is 1.52 mm and the substrate permittivity is chosen to be $\epsilon_r = 3.66$. The unmodulated capacitive sheet reactance is $X_0^{\rho\rho} = X_0^{\rho\phi} = X_0 = -461.5 \, \Omega$. The antenna radius is $10\lambda \approx 0.17$ m. Finally, the maximum indices used for the current basis functions and the fields testing functions expansion corresponds to $N = 15$ and $M = 60$, which yields to 3720 basis functions. For the IBC expansion, we used $N = 15$ and $M = 50$ (3100 basis functions). In practice, we have observed that it is better to use a bit fewer decomposition functions for the IBC than needed for the current and the fields testing. However, the obtained solution remains stable over a wide range of values of the above parameters.

3.3.1 Broadside pencil beam MTS antenna

As a first example, a MTS radiating a broadside circularly polarized pencil beam, similar to what has been designed in [43] is presented. The radiation pattern $\mathbf{F}(\theta, \phi)$ is provided through its TE (F_{TE}) and TM (F_{TM}) components, respectively parallel to $\hat{m}$ and $\hat{e}$ vectors in Fig. 2.14. A broadside circularly polarized pencil beam can be defined as $F_{TE} = \cos \phi - j \sin \phi$

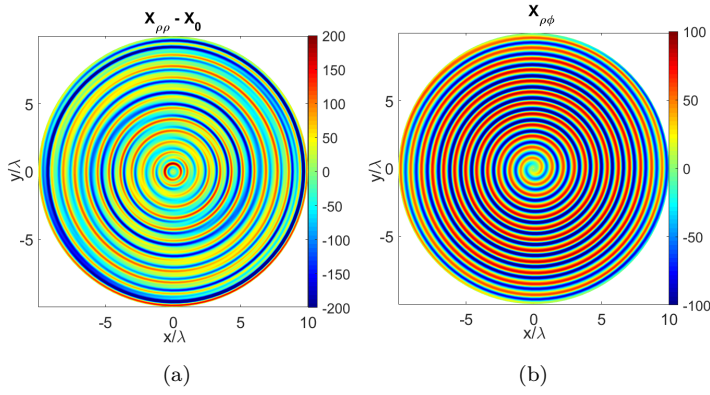


Figure 3.5: Modulated sheet transition IBC for a MTS radiating a broadside circularly polarized pencil beam. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

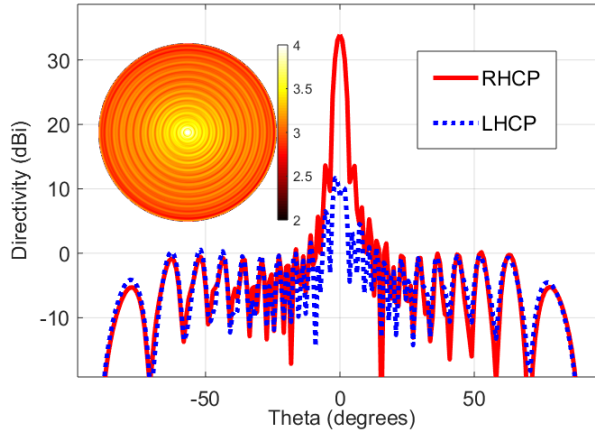


Figure 3.6: Radiation pattern of the designed broadside radiating MTS in the plane $\phi = 0$. The Right Handed Circular Polarization (RHCP) and the Left Handed Circular Polarization (LHCP) corresponds respectively to co-polar and cross polar directivity. The inset disk corresponds to the absolute value of the current distribution on the designed MTS (on $\log_{10}$ scale) for a unit current excitation.

and $F_{TM} = \sin \phi + j \cos \phi$ for $-\Delta k_x < k_x < \Delta k_x$ and $-\Delta k_y < k_y < \Delta k_y$ and $F_{TE} = F_{TM} = 0$ elsewhere. The parameters Δk_x and Δk_y control the beamwidth and have been chosen equal to $0.035 k_0$. The link between the

spectral coordinates and the observation directions is given by $(k_x, k_y, k_z) = k_0 \hat{u} = (k_0 \sin \theta \cos \phi, k_0 \sin \theta \sin \phi, k_0 \cos \theta)$.

The obtained normalized reactance is represented in Fig. 3.5. It is striking to observe that, despite the fact that from the solution of [43], we used only one value (the radiated power), the reactance exhibits a spiral shape, quite characteristic for this kind of radiation pattern. Nevertheless, the obtained reactance shape is not exactly the same as the one designed in [43] since a different spectrum has been used for the invisible current. In fact, depending on the invisible current spectrum, different admissible reactances are obtained. The obtained IBC does not have a constant modulation depth over the aperture. Fig. 3.6 shows the radiation pattern. The procedure proposed here, applied to the example of [43, Sec. IV-A2] (Fig. 8), provides a directivity of 31.5 dBi, against 31.9 dBi in [43]. More degrees of freedom in the invisible part of the spectrum (see Section 3.2.3) may slightly improve the performance of the antennas, at the expense of an iterative optimization procedure. Such improvement would not limit the capability to address arbitrary radiation patterns. Moreover, it is important to stress that the present procedure is more general than the approach presented in [43], not being restricted to a locally sinusoidal modulation. This additional degree of freedom can be useful, for instance, to minimize unwanted lobes due to higher order Floquet modes falling in the visible region for forward LW radiation.

3.3.2 Rectangular beam MTS antenna

In this case, the MTS is designed to produce a circularly polarized rectangular beam in the (k_x, k_y) spectral plane. The desired rectangular shape is defined as in the case of the broadside beam ($|k_x| < \Delta k_x$ and $|k_y| < \Delta k_y$), but with $\Delta k_x = 0.29 k_0$ and $\Delta k_y = 0.09 k_0$. This kind of pattern can be exploited in space applications. The ideal antenna co-polar and cross-polar patterns are represented in Figs. 3.8(a) and 3.8(c); they are obtained after filling the visible spectrum with the desired one and the invisible spectrum with the unmodulated solution, and expanding the current spectrum in the FBBF basis. The cut of this ideal antenna pattern in the $\phi = 0$ plane is represented

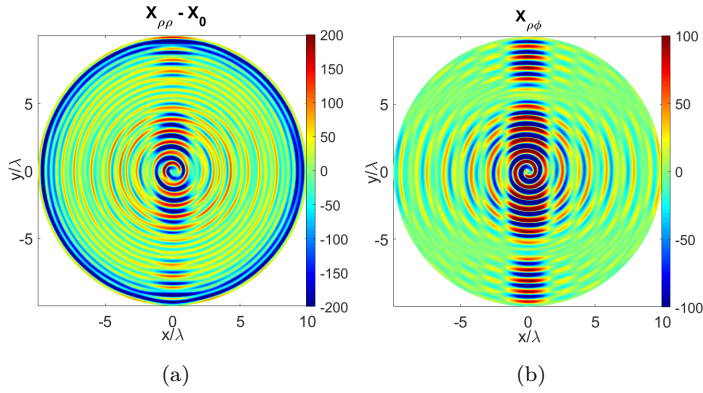


Figure 3.7: Modulated sheet transition IBC for a MTS radiating a react-angular spectral beam. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

in Fig. 3.9(a), and compared with the desired rectangular beam. The obtained impedance modulation is presented in Fig. 3.7. This shape is consistent with the radiated power distribution on the MTS. Roughly speaking, the impedance modulation resembles the inverse Fourier transform of the pattern (it is wide where the pattern is narrow).

Figs. 3.8(b) and 3.8(d) represent the co-polar and cross-polar directivity of the designed MTS, respectively. A cut of this pattern, corresponding to the plane $\phi = 0$ is represented in Fig. 3.9(b), and compared with the desired rectangular beam. One can see a good representation of the desired pattern. The main difference between the desired one, and the designed MTS pattern, relates to the cross-polar level. Other numerical experiments showed that the performance can be improved by increasing the MTS radius.

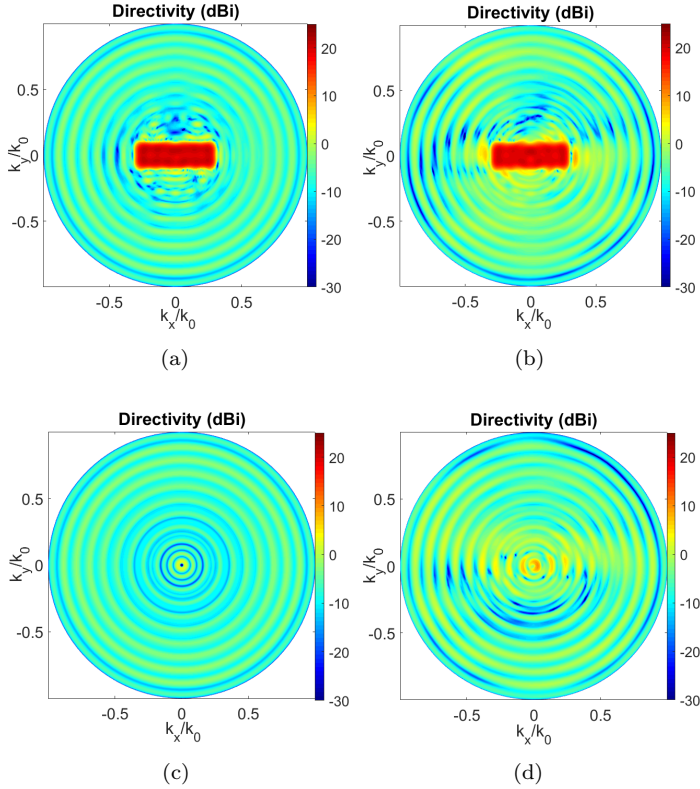


Figure 3.8: Circularly polarized radiation pattern in the (k_x, k_y) plane. (a) Co-polar component for the ideal antenna. (b) Co-polar component for designed MTS. (c) Cross-polar component for the ideal antenna. (d) Cross-polar component for designed MTS.

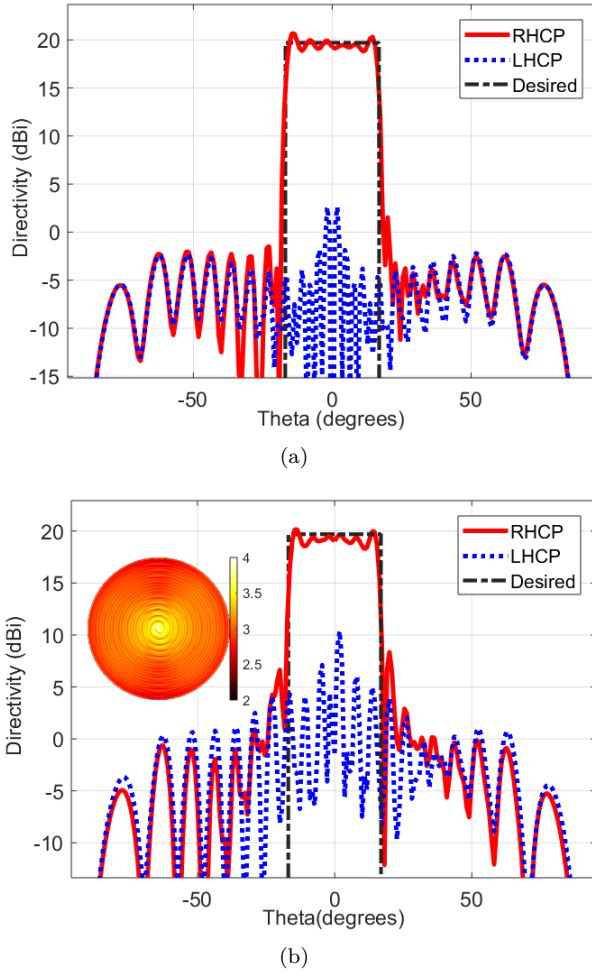


Figure 3.9: Radiation pattern in the plane $\phi = 0$. (a) ideal antenna. (b) designed MTS. The Right Handed Circular Polarization (RHCP) and the Left Handed Circular Polarization (LHCP) respectively corresponds to co-polar and cross-polar directivity. The desired pattern corresponds to the desired rectangular beam. The inset disk corresponds to the absolute value of the current distribution on the designed MTS (on $\log_{10}$ scale) for a unit current excitation.

3.3.3 Cosecant squared pattern MTS antenna

A cosecant squared (CS) radiation is a pattern producing a uniform electric field within a horizontal area, (w.r.t which the MTS aperture plane is perpendicular) in a given range of elevation angles θ [85], [86]. Outside this range, nothing should be radiated. The gain variations in the pattern region compensates the free-space loss and maintain the received power unchanged. This kind of pattern can be used for base station GSM antennas in order to control the field variations in a given coverage area. Another possible application concerns approach radar, where the antenna should illuminate an aircraft flying at a given height above the ground level with a constant power density over the whole approach duration. The CS pattern is defined as:

$$|D(\theta)| = \frac{\text{cosec}^2(\theta)}{\text{cosec}^2(\theta_0)} = \frac{\sin^2(\theta_0)}{\sin^2(\theta)} \quad (3.34)$$

where θ_0 is the angle fixing the maximum distance range. Here, the CS pattern has been designed for $\theta_0 < \theta < \theta_1$, with $\theta_0 = 2.5^\circ$, $\theta_1 = 50^\circ$, and $\phi = 0$. The synthesis is carried out for both circular and linear polarizations.

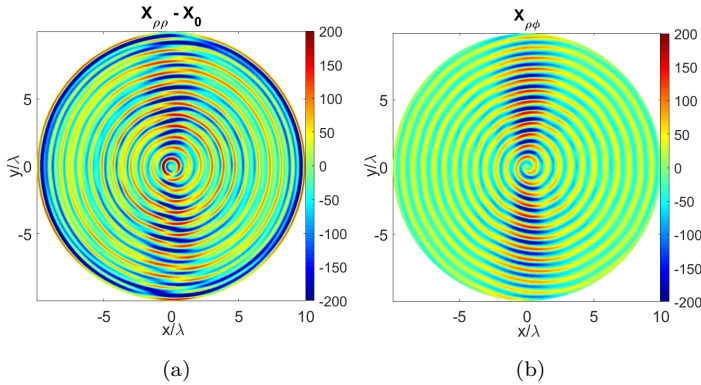


Figure 3.10: Modulated sheet transition IBC for a MTS radiating a circularly polarized cosecant squared pattern. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

For the circular polarization case, the ideal radiation pattern after dis-

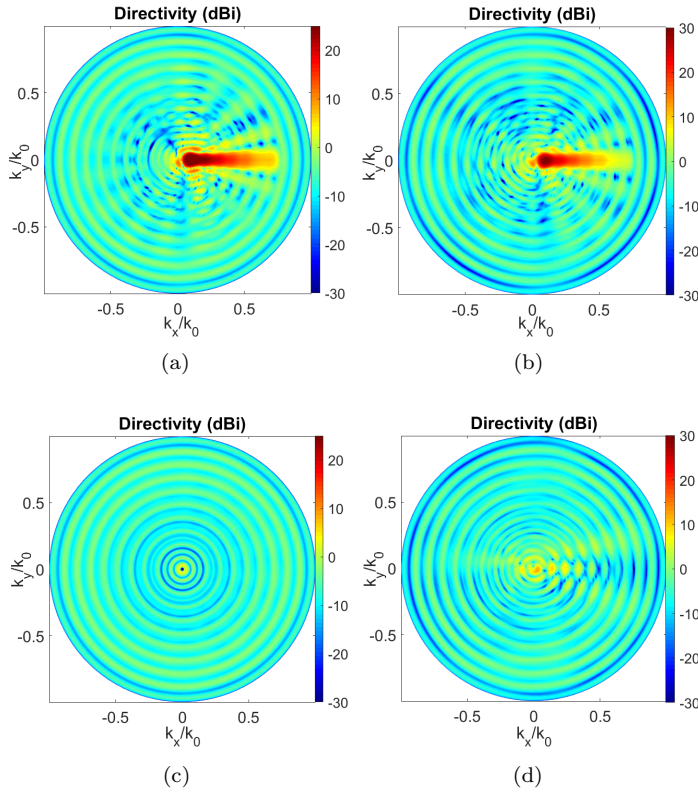


Figure 3.11: Circularly polarized radiation pattern in the (k_x, k_y) plane. (a) Co-polar component for the ideal antenna. (b) Co-polar component for designed MTS. (c) Cross-polar component for the ideal antenna. (d) Cross-polar component for designed MTS.

cretization with FBBFs is represented in Figs. 3.11(a) and 3.11(c). Its cut corresponding to the $\phi = 0$ plane is shown in Fig. 3.12(a). The obtained reactance tensor is illustrated in Fig. 3.10. Finally, the co-polar and cross-polar circularly polarized radiation patterns of the designed MTS are respectively presented in Figs. 3.11(b) and 3.11(d). The cut in the $\phi = 0$ plane is reported in Fig. 3.12(b), and compared with the desired one (given by the cosecant squared law (3.34)). It can be seen that the co-polar pattern is well represented; only the cross polar pattern differs significantly from that of

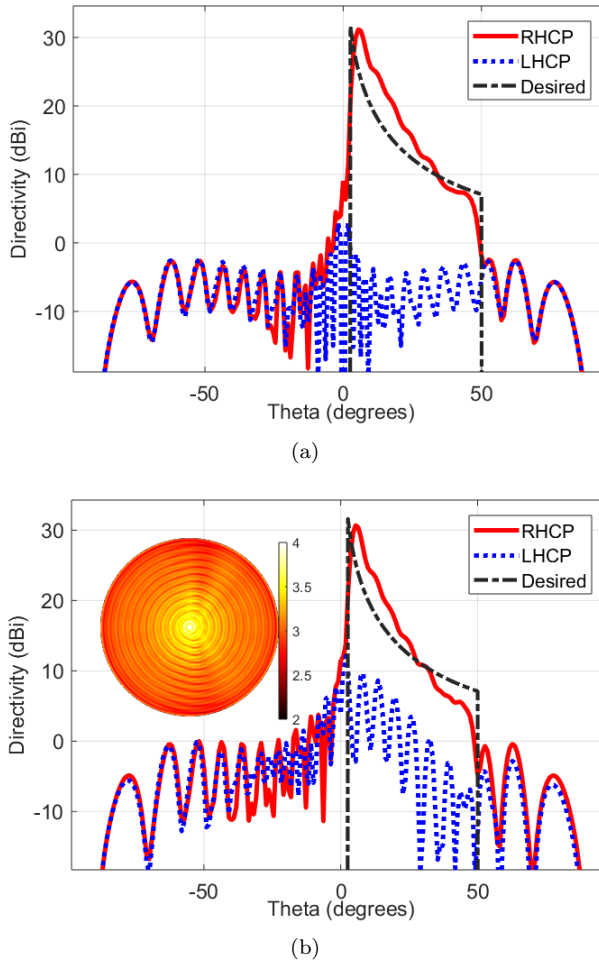


Figure 3.12: Radiation pattern in the plane $\phi = 0$. (a) ideal antenna. (b) designed MTS. The Right Handed Circular Polarization (RHCP) and the Left Handed Circular Polarization (LHCP) respectively correspond to the co-polar and cross-polar directivity. The desired pattern corresponds to the cosecant squared law (3.34). The inset disk corresponds to the absolute value of the current distribution on the designed MTS (on $\log_{10}$ scale) for a unit current excitation.

the ideal antenna. As in the rectangular beam case, the cross-polar pattern can be improved after increasing the antenna radius. Similar performance

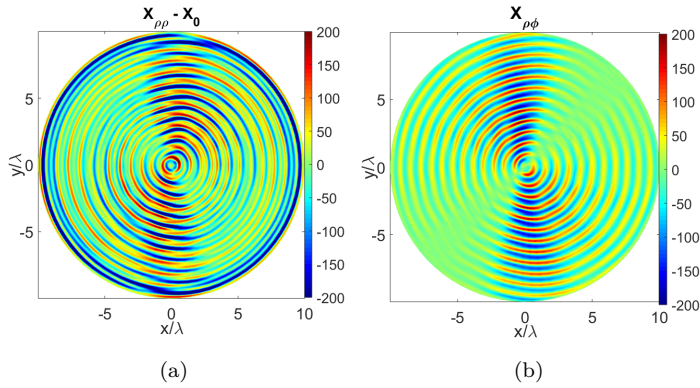


Figure 3.13: Modulated sheet transition IBC for a MTS radiating a linearly polarized cosecant squared pattern. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

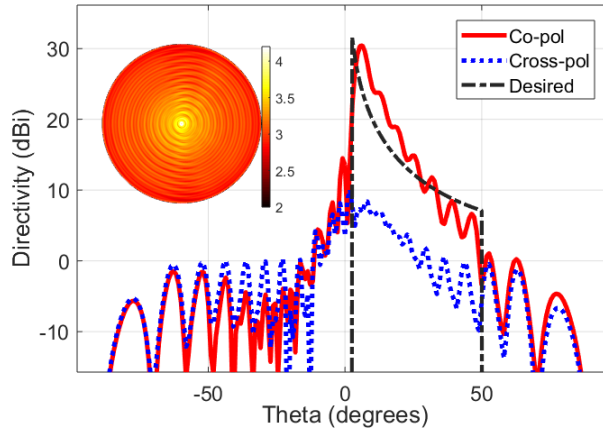


Figure 3.14: Radiation pattern of the designed linearly polarized cosecant squared radiating MTS. The desired pattern corresponds to the cosecant squared law (3.34). The inset disk corresponds to the absolute value of the current distribution on the designed MTS (on $\log_{10}$ scale) for a unit current excitation.

is obtained for the MTS antenna with linear polarization. Fig. 3.13 shows the reactance tensor radiating a linearly polarized CS pattern, and Fig. 3.14 shows the cut (on the axis $\phi = 0$) of the CS linearly polarized designed MTS beam.

3.3.4 Multi-beam MTS antenna

The final example is a tensorial MTS radiating a multi-beam pattern. A similar pattern shape has recently been designed in [53] using holographic techniques and in [87], where different possible configurations have been studied. The radiation pattern designed here consists of four circularly polarized beams, arranged in the spectral plane as $(|k_x|, |k_y|) = (0.5 k_0, 0.5 k_0)$.

The ideal co-polar radiation pattern after discretization with FBBFs is rep-

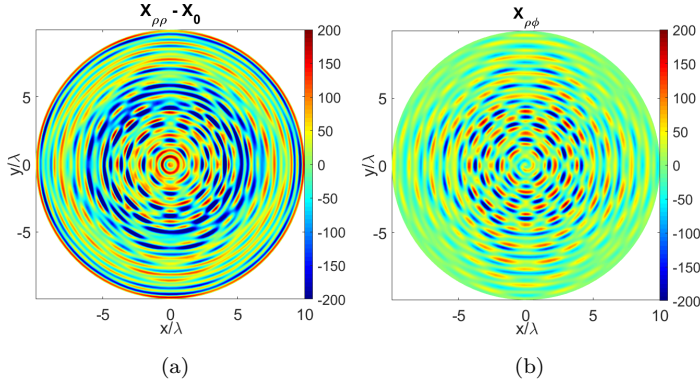


Figure 3.15: Modulated sheet transition IBC for a MTS radiating a multi-beam pattern. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

resented in Fig. 3.16(a). The corresponding ideal mean gain is 25 dBi. This gain could be improved by using a finer azimuthal discretization, i.e. with a larger value of the parameter N . It has not been done here since it requires more memory for loading the 3D matrix reaction integrals (3.9) in the laptop than has been used. The obtained reactance tensor is depicted in Fig. 3.15. Finally, the radiation pattern and the cut in the $\phi = 0$ plane are shown in Figs. 3.16(b), 3.16(d) and 3.17 respectively. The obtained mean gain of the designed MTS is about 23.7 dBi, which corresponds to a loss of about 1 dB after IBC synthesis.

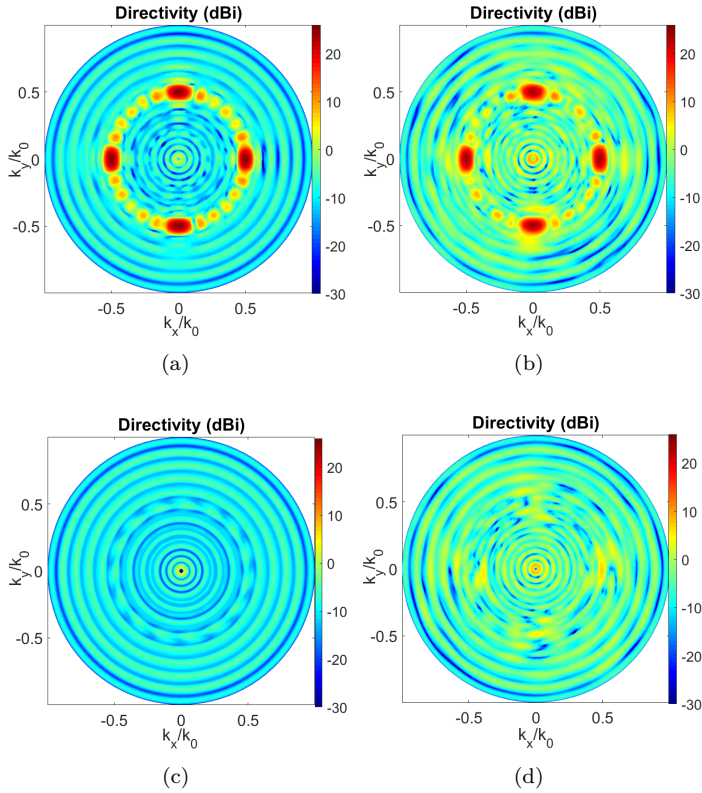


Figure 3.16: Circularly polarized radiation pattern in the (k_x, k_y) plane. (a) Co-polar component for the ideal antenna. (b) Co-polar component for designed MTS. (c) Cross-polar component for the ideal antenna. (d) Cross-polar component for designed MTS.

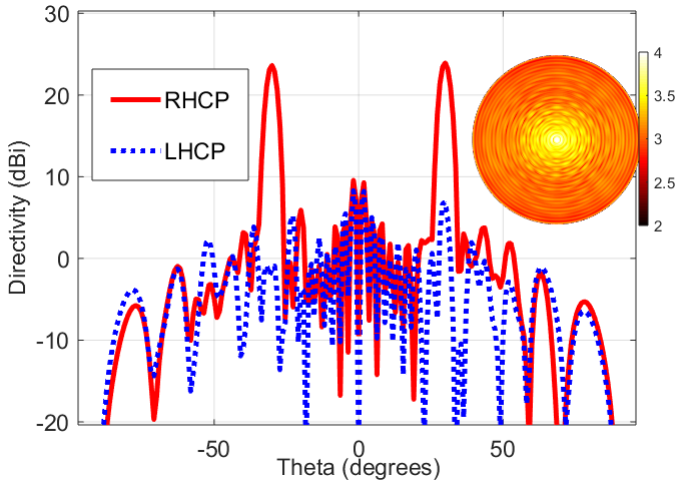


Figure 3.17: Radiation pattern of the designed multi-beam radiating MTS in the plane $\phi = 0$. The Right Handed Circular Polarization (RHCP) and the Left Handed Circular Polarization (LHCP) respectively corresponds to the co-polar and cross-polar directivity. The inset disk corresponds to the absolute value of the current distribution on the designed MTS (on $\log_{10}$ scale) for a unit current excitation.

3.3.5 “fish-like” pattern MTS antenna

Our intention in this section is to demonstrate the robustness of the proposed algorithm. To this end, we have designed a MTS radiating a multi-shaped polarized beam. The objective pattern is a right-handed circularly polarized (RHCP) “fish like” pattern in the spectral $(k_x/k_0, k_y/k_0)$ plane. $k_x = k_0 \sin \theta \cos \phi$ and $k_y = k_0 \sin \theta \sin \phi$ are the spectral variables in Cartesian coordinates. The desired “fish like” radiation pattern $\mathbf{F}(k_x, k_y)$ is

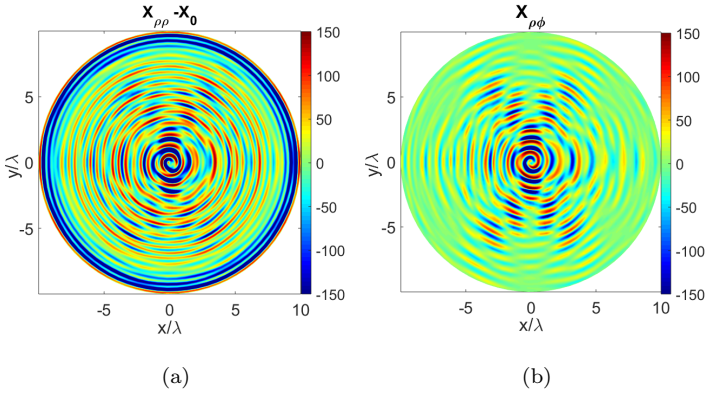


Figure 3.18: Modulated sheet transition IBC for a MTS radiating a “fish like” beam. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

illustrated in Fig. 3.19(a). Basically, the main body of the fish is drawn with 2 triangles. The first triangle is perforated with a small ring to represent the eye. The tail is also drawn with a triangle. The radiation pattern in Fig. 3.19(a) is computed after filling the visible part of the current spectrum with the desired radiation pattern and the invisible part with the invisible spectrum corresponding to the average impedance. The obtained current is discretized in the spectral domain in the Fourier-Bessel basis. The synthesis IBC tensor is illustrated in Fig. 3.18. Figs. 3.19(c), 3.19(d) and Fig. 3.20 show respectively the designed radiation pattern and its cut in the $\phi = 0$ plane. One can observe a very good agreement of the Co-polar (RHCP) pattern w.r.t. the desired one. The main difference are in the cross-pol.

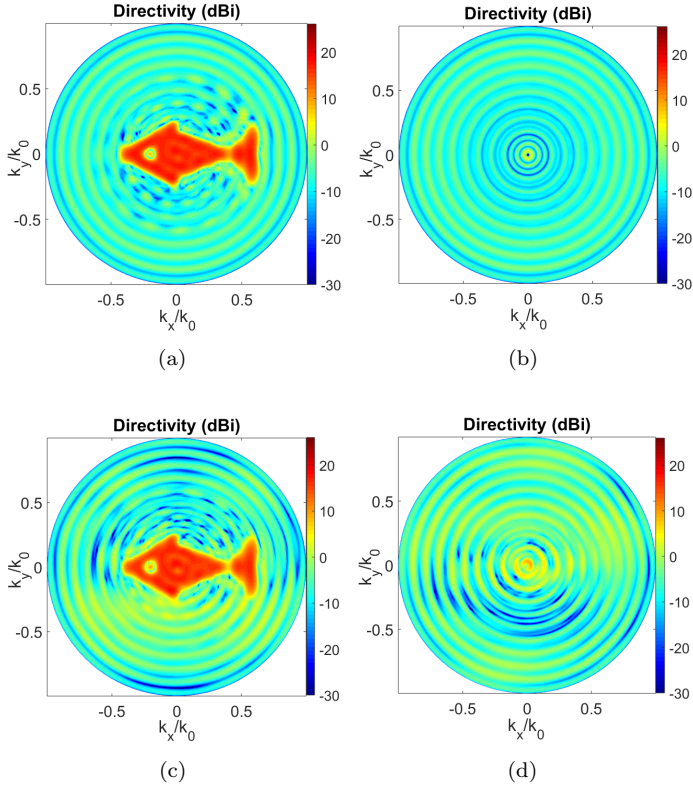


Figure 3.19: Circularly polarized “fish like” radiation pattern in the (k_x, k_y) plane. (a) Co-polar component for the ideal antenna (desired pattern). (b) Cross-polar component for the ideal antenna. (c) Co-polar component for designed MTS. (d) Cross-polar component for designed MTS.

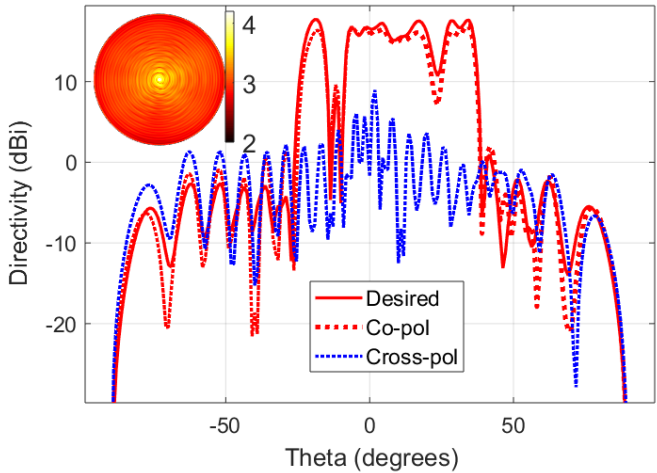


Figure 3.20: Radiation pattern of the designed “fish-like” radiating MTS in the plane $\phi = 0$. The co-polar and the cross-polar pattern corresponds respectively to the Right Handed Circular Polarization (RHCP) and the Left Handed Circular Polarization (LHCP) directivity. The inset disk corresponds to the absolute value of the current distribution on the designed MTS (on $\log_{10}$ scale) for a unit current excitation.

3.4 Elliptical domain metasurface synthesis

Depending on the type of radiation pattern and/or the azimuthal asymmetry of the feed, it may be more convenient to design the MTS on an elliptical aperture [44]. Indeed, in some practical cases, the radiated power density on the MTS is not azimuthally symmetrical. Using an elliptical aperture, one can reduce the antenna footprint without significantly affecting the radiation pattern, and thus improve the antenna aperture efficiency [44]. This section generalizes the procedure previously described to the synthesis of the more general class of elliptical-domain MTSs. Further analysis regarding the excitation treatment and the computation time is provided.

3.4.1 Currents and surface impedance discretization

The FBBFs is defined as in Section 2.2.1 and the currents distribution expansion is given by equation 2.5. Contrary to the circular domain synthesis where $(\hat{\rho}, \hat{\phi})$ polar representation was used for the vector and tensor expansions, we use here a $(\hat{x}, \hat{y})$ formulation. Indeed, in a polar system of coordinates, the unit vectors are no longer orthogonal after stretching the FBBFs. In this context, it has been found necessary to use the Cartesian reference system when dealing with an elliptical domain. For a circular domain, the $(\hat{\rho}, \hat{\phi})$ formulation is preferred, especially for a scalar IBC where the current is only directed along $\hat{\rho}$.

Let us now assume that one is interested in synthesizing a capacitive tensorial sheet impedance of the form 3.14. The reactance tensor 3.14 written in a cartesian system of coordinates then takes the following form:

$$\begin{aligned} Z_S^{xx}(x, y) &= Z_0^{xx}(x, y) + P^{xx}(x, y) \\ Z_S^{xy}(x, y) &= Z_S^{yx}(x, y) = P^{xy}(x, y) \\ Z_S^{yy}(x, y) &= Z_0^{yy}(x, y) - P^{xx}(x, y) \end{aligned} \quad (3.35)$$

where $Z_0^{xx}(x, y) = Z_0^{\rho\rho} - (Z_0^{\rho\rho} - Z_0^{\phi\phi}) \sin^2 \phi$, and $Z_0^{yy}(x, y) = Z_0^{\rho\rho} - (Z_0^{\rho\rho} - Z_0^{\phi\phi}) \cos^2 \phi$ are the average reactances in the Cartesian system of coordi-

nates. The IBC (3.35) is then discretized into FBBFs as follows:

$$Z_S^{xx} \approx \sum_{n_S, m_S} [K_S^{xx}(m_S, n_S) + X_S^{xx}(m_S, n_S)] R_{m_S, n_S} \quad (3.36)$$

$$Z_S^{xy} \approx \sum_{n_S, m_S} X_S^{xy}(m_S, n_S) R_{m_S, n_S} \quad (3.37)$$

$$Z_S^{yy} \approx \sum_{n_S, m_S} [K_S^{yy}(m_S, n_S) - X_S^{xx}(m_S, n_S)] R_{m_S, n_S} \quad (3.38)$$

where the (x, y) dependence has been implicitly omitted and $K_S^{xx}(m_S, n_S)$ and $K_S^{yy}(m_S, n_S)$ correspond respectively to the expansion coefficients of the average reactances Z_0^{xx} and Z_0^{yy} into FBBFs. Since FBBFs are orthogonal, this expansion is carried out through a simple projection.

A spatial FBBF decomposition of a function $f(x, y)$ can be obtained as:

$$a_{mn} = \frac{p}{K_0(m, n)} \iint f(x, y) R_{m, n}^*(x, y) dx dy \quad (3.39)$$

where $K_0(m, n) = -\pi a^2 J_{n-1}(\lambda_{nm}) J_{n+1}(\lambda_{nm})$ and the integration is carried out over the elliptical domain. The superscript $(*)$ corresponds to the complex conjugate. To evaluate this integral, it is more convenient to stretch the ellipse into the circle through the change of variables $x' = x$, and $y' = yp$. One obtains:

$$a_{mn} = \frac{1}{K_0(m, n)} \iint f\left(x', \frac{y'}{p}\right) R_{m, n}^*\left(x', \frac{y'}{p}\right) dx' dy' \quad (3.40)$$

The function $R_{m, n}(x', y'/p)$ corresponds to a circular-domain FBBF of radius a as defined in (2.46), (2.1). It means that, projecting a function $f(x, y)$ over an elliptical domain FBBF is equivalent to projecting the function $f(x, y/p)$ over the corresponding circular-domain FBBF. This operation is much less time consuming since the azimuthal integration can be done with the FFT and the radial part with the Gauss-Legendre quadrature.

The parameters $X_S^{xx}(m_S, n_S)$, $X_S^{xy}(m_S, n_S)$ in (3.36), (3.37), (3.38) are the unknowns expansion coefficients of the surface impedance modulation. Since $P^{xx}(x, y)$ and $P^{xy}(x, y)$ are assumed to be purely imaginary,

the coefficients $X_S^{xx}(m_S, n_S)$ and $X_S^{xy}(m_S, n_S)$ need to satisfy symmetry (3.28). The synthesis problem then consists of finding the reactance modulation, which will provide the desired radiation pattern.

3.4.2 Synthesis of the MTS

Let us assume that one is interested in generating a given known radiation pattern $\mathbf{F}(\theta, \phi)$ in amplitude, phase and polarization, where $\mathbf{F}$ is the spatial electric field distribution in the far field, and θ and ϕ are respectively the elevation and azimuthal angles (see Fig. 2.14). The corresponding visible current spectrum is then obtained as:

$$\tilde{\mathbf{J}} = \left(\tilde{\underline{\underline{\mathbf{G}}}}^{EJ} \right)^{-1} (\tilde{\mathbf{f}} - \tilde{\mathbf{E}}_i) \quad (3.41)$$

where $\tilde{\underline{\underline{\mathbf{G}}}}^{EJ}$ is the dyadic spectral Green's function of the grounded substrate [74], $\tilde{\mathbf{E}}_i$ is the visible spectrum of the MTS excitation, and $\tilde{\mathbf{f}}$ is rescaled w.r.t. the radiated power. The latter has been computed using the technique described in Section 3.2.3. For the synthesized MTSs, we have observed that $\tilde{\mathbf{E}}_i$ can be neglected in comparison with $\tilde{\mathbf{f}}$. This assumption has been implicitly made in the previous sections devoted to circular domain. As an example, Fig. 3.21 compares the radiation pattern obtained in Section 3.3.2, where $\tilde{\mathbf{E}}_i$ has been neglected in the synthesis, with that obtained without neglecting $\tilde{\mathbf{E}}_i$, both for a circular domain MTS. One can observe that a very similar performance has been achieved. When the visible part of $\tilde{\mathbf{E}}_i$ is much smaller than $\tilde{\mathbf{f}}$, it is difficult to say whether the subtraction in (3.41) will be favorable to the final design. This should be decided based on correspondence between final full-wave simulation and initially desired radiation pattern.

At this moment, only the visible part of the current spectrum is known. To synthesize the MTS, one also needs the invisible (near-field) part. This near field part cannot be neglected since the radiation phenomenon comes from the leakage of a SW. For some canonical patterns, analytical expressions of the near-field have been proposed in [43]. However, when one is dealing with an arbitrary radiation pattern, finding the appropriate near-field part is

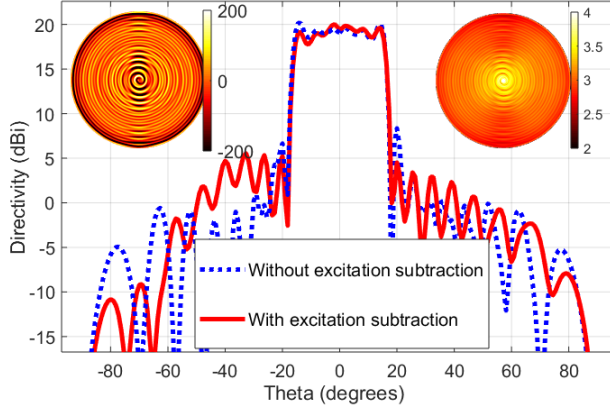


Figure 3.21: Cut in the $\phi = 0$ plane of the co-polar radiation pattern. The result obtained after subtracting the aperture electric field generated by the feed is compared with that obtained without subtracting the feed contribution. The left and the right inset disks correspond respectively to the $\rho\rho$ component of the synthesized reactance (obtained without feed field subtraction), and the corresponding absolute value of the current distribution (in $\log_{10}$ scale).

not an easy task. Therefore, as for the circular domain synthesis, we make in a first instance, the simplest assumption i.e the near-field spectrum is obtained in the absence of modulation. This is the simplest possible estimation since it does not even take into account the leakage rate $\alpha(x, y)$ nor the phase constant variation $\Delta\beta(x, y)$ on the MTS aperture. In passing, finding a priori these parameters for an arbitrary shaped pattern is not a well posed problem. This “simplest” invisible current spectrum, in combination with the visible spectrum part (obtained from the pattern), form a first estimate of the current distribution spectrum $\tilde{\mathbf{J}}$ on the MTS. The corresponding surface impedance can then be obtained after solving the EFIE. Following, the same reasoning as described for circular domain, one obtains that the IBC modulation satisfies the following equation:

$$\begin{bmatrix} \mathbf{X}_S^{xx} \\ \mathbf{X}_S^{xy} \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{S,x} & \mathbf{Z}^{S,y} \\ -\mathbf{Z}^{S,y} & \mathbf{Z}^{S,x} \end{bmatrix} = \underline{\mathbf{U}} \quad (3.42)$$

where $\mathbf{X}_S^{xx}$ and $\mathbf{X}_S^{xy}$ are the expansion coefficients of the IBC modulation

given in (3.36), (3.37), (3.38). $\mathbf{Z}^{S,x}$ and $\mathbf{Z}^{S,y}$ are defined as:

$$Z^{S,x}(m_S, n_S; m_t, n_t) = \sum_{n_b, m_b} i_{m_b n_b}^x Z(m_b, n_b; m_S, n_S; m_t, n_t) \quad (3.43)$$

$$Z^{S,y}(m_S, n_S; m_t, n_t) = \sum_{n_b, m_b} i_{m_b n_b}^y Z(m_b, n_b; m_S, n_S; m_t, n_t) \quad (3.44)$$

with $\mathbf{Z} = (a^2/p) \mathbf{Z}_U$, where $\mathbf{Z}_U$ is tabulated and has been defined in [79], [88].

The vector $\underline{\mathbf{U}}$ is computed as:

$$\underline{\mathbf{U}} = \begin{bmatrix} \mathbf{U}^x \\ \mathbf{U}^y \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_G^{xx} & \mathbf{Z}_G^{xy} \\ \mathbf{Z}_G^{yx} & \mathbf{Z}_G^{yy} \end{bmatrix} \begin{bmatrix} \mathbf{I}^x \\ \mathbf{I}^y \end{bmatrix} - \begin{bmatrix} \mathbf{V}^x \\ \mathbf{V}^y \end{bmatrix} - \begin{bmatrix} \mathbf{X}_K^x \\ \mathbf{X}_K^y \end{bmatrix} \quad (3.45)$$

where $\mathbf{Z}_G^{xx}$, $\mathbf{Z}_G^{xy}$, $\mathbf{Z}_G^{yx}$, and $\mathbf{Z}_G^{yy}$ are the different components of the substrate interaction matrix. They can be computed using the procedure described in Section 2.2.2. $\mathbf{I}^x$ and $\mathbf{I}^y$ are column vectors with the expansion coefficients (into FBBFs) of the previously estimated surface current distribution $\tilde{\mathbf{J}}$, along $\hat{a}_x$ and $\hat{a}_y$, respectively. Those expansion coefficients are computed through a spectral-domain projection of $\tilde{\mathbf{J}}$ (3.48). $\mathbf{V}^x$ and $\mathbf{V}^y$ correspond to the excitation vector. They are computed in the same way as in 2.2.2. Finally, $\mathbf{X}_K^x$ and $\mathbf{X}_K^y$ are defined as:

$$\mathbf{X}_K^x = \mathbf{Z}^{S,x} \mathbf{K}_S^{xx} \quad (3.46)$$

$$\mathbf{X}_K^y = \mathbf{Z}^{S,y} \mathbf{K}_S^{yy} \quad (3.47)$$

where $\mathbf{K}_S^{xx}$, and $\mathbf{K}_S^{yy}$ are vectors respectively containing the expansion coefficients of the known average reactance Z_0^{xx} and Z_0^{yy} . The excitation vectors $\mathbf{V}^x$ and $\mathbf{V}^y$ correspond to the projections of the excitation fields on the entire-domain testing functions. As such, they can be computed for elementary dipole as well as for more realistic extended feeding structures. Equation (3.42) is solved while imposing the symmetry (3.28) in the coefficients of $\mathbf{X}_S^{xx}$ and those of $\mathbf{X}_S^{xy}$ in order to constrain the impedance to be anti-Hermitian. This condition is imposed in the least-squares sense as for

the circular domain synthesis, which in turn leads to significant changes in the current distribution, including in its invisible (i.e. non-radiating) part. The surface currents and far fields are then simulated with the MoM using the derived anti-Hermitian IBC. Depending on the obtained modulation depth, or the generated pattern performance, the radiated power is tuned accordingly.

The resulting algorithm is then the same as for the circular domain synthesis, except that the EFIE operator is given by (3.42), the spatial projection operation is given by (3.40) and the spectral projection of a function $\tilde{f}(k_x, k_y)$ over an elliptical domain FBBF spectrum is equivalent to projecting the function $p\tilde{f}(k_x, k_y p)$ over the following circular-domain FBBF spectrum:

$$\frac{1}{\tilde{K}_0(m, n)} \iint p \tilde{f}(k'_x, k'_y p) \tilde{R}_{m,n}^*(k'_x, k'_y p) dk'_x dk'_y \quad (3.48)$$

where $\tilde{K}_0(m, n) = -4\pi^3 a^2 J_{n-1}(\lambda_{nm}) J_{n+1}(\lambda_{nm})$. This leads to a MTS synthesis process over an elliptical domain that is almost as fast as that for a circular domain, when using the same number of basis functions for sheet impedance and currents. The only difference resides in the computation of the substrate matrix interaction $\underline{\underline{Z}}_G$, which requires for elliptical domain, a harmonic development of the Green's function 2.2.2. However, this matrix needs to be computed only once for any pattern once the aperture size and the substrate are known since $\underline{\underline{Z}}_G$ does not depend on the IBC.

3.4.3 Numerical validation

This section presents a numerical validation (based on the MoM algorithm validated in 2.2) of a synthesized elliptical domain MTS IBC. The MTS has been designed to radiate a spectral circularly polarized rectangular beam and is fed with a vertical elementary dipole placed at the center ($\rho = 0$), and in the middle of the grounded substrate ($z = -d/2$), where d is the substrate thickness (see Fig. 2.14). This beam is defined as $F_{TE} = \cos \phi - j \sin \phi$ and $F_{TM} = \sin \phi + j \cos \phi$ for $-\Delta k_x < k_x < \Delta k_x$, $-\Delta k_y < k_y < \Delta k_y$ and $F_{TE} = F_{TM} = 0$ elsewhere. TE and TM stand respectively for the trans-

verse electric and transverse magnetic components of the electric far field $\mathbf{F}$. We have chosen $\Delta k_x = 0.29 k_0$ and $\Delta k_y = 0.09 k_0$. This beam has already been designed in 3.3.2 considering a circular domain. As can be seen from the results presented in 3.3.2, the surface current distribution is azimuthally no symmetrical since the modulation depth is larger in the y -direction than in the x -direction. This is consistent with the pattern shape. For such a pattern, one can reduce the size of the disk in the y -direction without significantly affecting the radiation performance. However, simply cutting the MTS will not lead to optimal performance since the radiation originates from a global effect. In this section, we designed the MTS on an ellipse with semi-major axis equal to the radius used in 3.3.2, i.e 10λ and a semi-minor axis equal to 7λ , thus leading to $p \approx 1.429$. The operating frequency is 18 GHz and the substrate parameters are the same as in 3.3.2. The design has been carried out using the following values defining the number of basis functions $N_S = 20$, $M_S = 50$, $N_b = N_t = 20$, $M_b = M_t = 55$ and the designed MTS IBC has been analyzed with the method presented in Section 2.2, using $N_b = N_t = 30$ and $M_b = M_t = 60$, which leads to 7320 basis functions. Fig. 3.22 shows the obtained radiation pattern in the spectral (k_x, k_y) plane and Fig. 3.23 represents a cut of this pattern in the $\alpha = 0$ plane. Fig. 3.24 compares the computed co-polar radiation pattern with that obtained in Fig. 3.9, corresponding to a circular domain MTS of radius equal to the semi-major axis of the ellipse. One can see that almost the same gain has been achieved compared to 2.2, but using only 70% of the initial surface.

Finally, the synthesis computation time is summarized in TABLE 3.1, where a comparison between the circular domain synthesis and the elliptical domain synthesis is also made. The code has been developed using the MATLAB language [89] on a conventional laptop computer. One can observe that the main difference when going from circular domain to elliptical domain is related to the substrate matrix computation, which requires a harmonic development of the Green's function. It is worth mentioning that this matrix has to be computed only once in an optimization procedure since it does not depend on the surface impedance nor on the excitation. However, the overall computation time is dominated by the solution of the EFIE

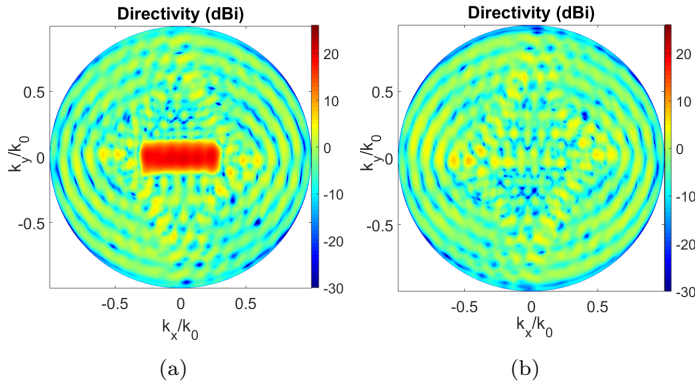


Figure 3.22: Circularly polarized radiation pattern in the spectral (k_x, k_y) plane. (a) Co-polar component for designed MTS. (b) Cross-polar component for designed MTS.

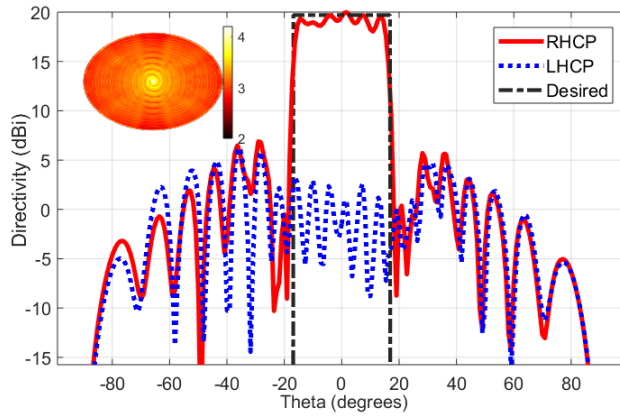


Figure 3.23: Cut in the $\phi = 0$ plane of the radiation pattern for the designed rectangular beam radiating MTS. The Right Handed Circular Polarization (RHCP) and the Left Handed Circular Polarization (LHCP) respectively corresponds to the co-polar and cross-polar directivity. The inset disk corresponds to the absolute value of the current distribution on the designed MTS (in $\log_{10}$ scale) for a unit current excitation.

(including matrix filling) in the LS sense, which allows the derivation of the reactance from the objective currents. In this example, the latter operation is more time consuming for the elliptical domain due to the finer azimuthal

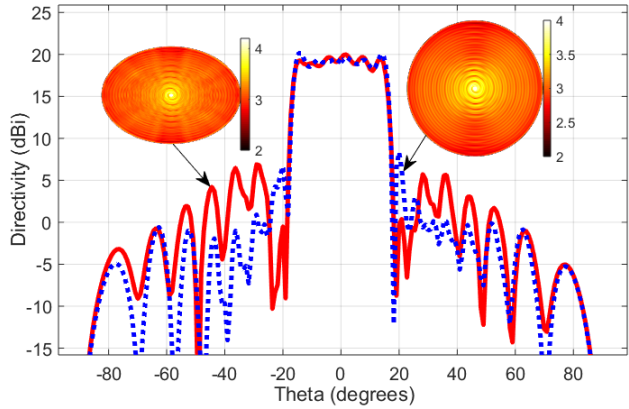


Figure 3.24: Cut in the $\alpha = 0$ plane of the co-polar radiation pattern. The result obtained with the designed elliptical domain MTS is compared with that of the circular domain MTS. The inset disks correspond to the absolute value of the current distribution (in $\log_{10}$ scale) on the elliptical MTS and on the circular MTS for a unit current excitation.

Table 3.1: Computation times for the designed IBC.

Solver	Circular domain	Elliptical domain
Substrate matrix	15 s	5.3 min
Invisible spectrum	46 s	52 s
Radiated power	1.1 min	1.1 min
Visible spectrum	2.3 min	2.4 min
EFIE inversion	3.7 min	8.2 min
Total computation time	8.1 min	17.86 min

discretization required ($N=20$ vs $N=15$).

3.5 Conclusion

A systematic design technique of modulated sheet impedance (IBC) for metasurface (MTS) antennas has been presented. The algorithm is based on the EFIE, after expanding the currents and the IBC into Fourier-Bessel basis. The latter is orthogonal and defined over the entire circular domain. The method remains valid for arbitrary radiation patterns (with arbitrary polarizations), assuming they are compliant with the aperture size. Several radiation patterns, with different polarizations, have been successfully implemented successfully. We demonstrated that the invisible current spectrum of the unmodulated reactance is sufficient to design the IBC, provided that the tensorial reactance is forced (in a least-squares sense) to be anti-Hermitian. Further improvements may be obtained by adding more degrees of freedom in the invisible part of the current spectrum. The fine parametrization of that spectrum is left for later research. The proposed formulation has three main advantages:

- It starts directly from the pattern and not from the aperture fields, i.e. this link is directly incorporated into the algorithm.
- The method can be applied to arbitrary radiation patterns, with any given amplitude and phase distribution and can be naturally extended to near-field applications. In the latter case, the invisible current spectrum just needs to be filled with the desired near-field distribution, up to the wavenumber for which that knowledge is available as long as that spectrum does not start interfering with the surface-wave spectrum.
- Regarding the excitation, although a vertical elementary dipole has been considered in this chapter, the same method can be exploited for more complex excitations, provided that the aperture electric field generated by the feeding structure is known a priori.

The main limitation of the algorithm resides in the 3D matrix tabulation of the reaction integrals, which are computed once and for all but can require a large memory when dealing with complex patterns and very large aper-

tures. The matrix used in this chapter is of size $(1860 \times 50 \times 1860)$. The proposed technique has been generalized to the wider class of elliptical MTSs.

Although, orthogonal entire-domain basis functions have been used here, that property are not mandatory, i.e. the method can be adapted for local, non-orthogonal basis functions. However in those cases, the projection operations need to be carried out by solving a linear system of equations, which is much more time and memory consuming. Local basis functions present the advantage of coping with arbitrary shapes. But discretizing typical current distributions on MTSs with such a basis requires typically more than 50000 unknowns [61], for which direct inversion is unfeasible on conventional personal computers.

CHAPTER 4

On the reconfigurability of metasurface antennas

THE first part of this chapter consists of extending the electric field integral equation (EFIE) formalism (described in Chapter 3) to antennas with multiple beams and multiple feeds. The method also allows us as a particular case, the generation of two radiation patterns with the same amplitudes and polarizations, but with opposite phase slopes. A proper phasing of the feeds then allows continuous beam-steering over an angular range equal to the 3 dB beamwidth.

The second part of the chapter is devoted to multiple frequency-band operation.

4.1 Multiple beams-Multiple feeds metasurface antennas

Generation of multiple beams using a metasurface (MTS) antenna fed at multiple points has already been demonstrated in [87], where it has been shown that assuming a certain orthogonality between the beams, the required modulated impedance corresponding to each excitation can be superimposed. Each impedance modulation is computed analytically based on

the flat optics theory proposed in [51], [52]. The method proposed here aims at addressing shaped beams which are not necessarily orthogonal.

In the previous chapter, it has been shown that the EFIE based MTS synthesis leads to a system of equations, which can be written in compact form as follows:

$$\underline{\underline{\mathbf{Z}}}^S \underline{\mathbf{X}}_S = \underline{\mathbf{U}} \quad (4.1)$$

where $\underline{\mathbf{X}}_S$ is the vector representing the unknown coefficients of the impedance modulation, $\underline{\mathbf{U}}$ contains information regarding the substrate, the excitation as well as the desired radiation pattern and $\underline{\underline{\mathbf{Z}}}^S$ also contains information about the radiation pattern. It is worth noting that, although the solution of the problem leads to the resolution of a linear system of equations, it does not mean that the radiation pattern or the currents depend linearly on the impedance. Indeed, the currents appear in $\underline{\mathbf{U}}$ as well as in $\underline{\underline{\mathbf{Z}}}^S$ in such a way that there is no linear link between them and $\underline{\mathbf{X}}_S$. The reader is referred to Chapter 3 for the mathematical details behind the formulation. In the remainder of the chapter, we identify basis functions for surface impedance, currents and testing functions with subscripts S , b , and t , respectively.

4.1.1 Synthesis for multi-fed MTS antenna

We want now to generalize the formulation presented in Section 3.2 to the case of multiple beams and multiple feeds. Two different approaches are presented in the following. In all the cases, the desired radiation patterns are assumed to be of the general form $\mathbf{F}(\theta, \phi)$, where θ , and ϕ are the elevation and azimuthal angles. It is provided through its transverse magnetic (TM) and transverse electric (TE) field components w.r.t. z , with electric field respectively parallel to the vectors $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\phi}}$ (see an illustration in Fig. 4.1).

Synthesis based on azimuthal symmetry assumption

Let us assume that we want to generate multi-shaped beams $\mathbf{F}_1(\theta, \phi)$, $\mathbf{F}_2(\theta, \phi)$, ... $\mathbf{F}_Q(\theta, \phi)$ satisfying the following symmetry $\mathbf{F}_i(\theta, \phi) = \mathbf{F}_1(\theta, \phi - 2\pi(i-1)/Q)$, $i = 1 \dots Q$. Let us now suppose that the MTS is designed

to radiate the beam $\mathbf{F}_1(\theta, \phi)$ when excited at the point $\boldsymbol{\rho}_1 = (\rho_1, \phi_1)$, with $\rho_1 > 0$. One can easily show that, if the modulation satisfies the same symmetry as the beams, namely:

$$\underline{\underline{\mathbf{Z}}}_S(\rho, \phi) = \underline{\underline{\mathbf{Z}}}_S(\rho, \phi - 2\pi(i-1)/Q), \quad i = 1 \dots Q \quad (4.2)$$

then the MTS automatically radiates the beam $\mathbf{F}_i(\theta, \phi)$ when excited at the point $\boldsymbol{\rho}_i = (\rho_1, \phi_1 + 2\pi(i-1)/Q)$. Since the MTS IBC is discretized into FBBFs, the azimuthal symmetry (4.2) can be imposed by simply restricting the set of FBBFs to those with azimuthal index n_S multiple of Q . We want to stress that for higher values of Q , the subspace in which the modulated impedance is designed is smaller (see expressions (3.15)-(3.17)). The algebraic system to be solved has $4M_t (2N_t + 1)$ equations with $2M_S (2(N_S/Q) + 1)$ unknowns, where it has been implicitly assumed that N_S is multiple of Q . This means that the capabilities of shaping the beams, the aperture efficiency and the computation time decrease for increasing Q . It is important to mention that for stability reasons of the method, one cannot use more basis functions for the impedance than for the currents (i.e $N_S \leq N_b = N_t$ and $M_S \leq M_b = M_t$).

Equations stacking

The second approach consists of over-constraining the system of equations. Since each system of equations corresponds to a given excitation and a given radiation pattern, one can simply simultaneously impose more equations to achieve the multibeam effect. The system of equations to be solved takes the same form as in (3.29), with $\underline{\underline{\mathbf{Z}}}^S$ and $\underline{\underline{\mathbf{U}}}$ respectively written as:

$$\underline{\underline{\mathbf{Z}}}^S = \left[\underline{\underline{\mathbf{Z}}}_1^S; \underline{\underline{\mathbf{Z}}}_2^S; \dots; \underline{\underline{\mathbf{Z}}}_Q^S \right] \quad (4.3)$$

$$\underline{\underline{\mathbf{U}}} = \left[\underline{\underline{\mathbf{U}}}_1; \underline{\underline{\mathbf{U}}}_2; \dots; \underline{\underline{\mathbf{U}}}_Q \right] \quad (4.4)$$

Each index corresponds to a given radiation pattern and a given excitation. Matrix $\underline{\underline{\mathbf{Z}}}^S$ in (4.15) has the same number of columns as the one in

(3.29) for the single feed synthesis, which means that we are using the same number of FBBFs for the IBC expansion in both cases (single feed and multiple feeds). This method can deal with more general multibeam configuration with respect to the azimuthal symmetry based approach. However, one needs to solve a system of $4QM_t (2N_t + 1)$ equations with $2M_S (2N_S + 1)$ unknowns, which means that the size increases with the number of beams Q . Notice that it is essential to add to the visible part of the current spectrum an invisible part corresponding to the exciting surface-wave. Omitting that part may significantly worsen the performance and generally yields non-realizable impedance values. The invisible current spectrum, which corresponds to the antenna near-field, is estimated from a simulation of the currents obtained with the average reactance with each feed. However, one can iterate starting from this first estimation. This means that a first impedance modulation can be computed and then used to update the invisible current for each feed. Results obtained with such a refinement are shown in Section 4.1.2.

For both methods (based on symmetry and equation stacking), the physical degrees of freedom offered by a MTS of given size may become insufficient when the number of desired simultaneous beams is increased. This is expressed in terms of an over-constrained system of equations and explains how the obtained patterns performance may stay behind the requirements when more beams are demanded, as will be shown in Section 4.1.2.

Excitation vector for non-centered fed MTS

In Chapter 2, we have presented the MoM excitation vector computation for centered fed MTS (i.e. with a feed at $\rho = 0$). This section extends the formulation (2.1.2) to non-centered fed MTSs.

Let us assume for example that the feed is placed at $(x = x_0, y = 0)$. The excitation Green's function can be written as $\underline{\underline{\mathbf{G}}}^{EJ}(x - x_0, 0)$, and the corresponding Fourier transform is $\underline{\underline{\tilde{\mathbf{G}}}}^{EJ} e^{-jk_x x_0}$. Inserting the excitation spectrum in (2.14), with $k_x = k_\rho \cos \alpha$, each element of the x-directed exci-

tation for the z-directed infinitesimal dipole can be written as:

$$v_x(m, n) = -\frac{1}{4\pi^2} \int_0^{2\pi} \int_0^\infty \tilde{R}_{m,n}^{x,*}(k_\rho, \alpha) \tilde{G}_{xz}^{EJ}(k_\rho, \alpha) e^{-jk_\rho x_0 \cos \alpha} k_\rho dk_\rho d\alpha \quad (4.5)$$

Using the formal structure of the Green's function in (2.26) and that of the FBBFs spectrum (see (2.2) and (2.4)), (4.5) can be developed as:

$$v_x(m, n) = \frac{a^2(-j)^n}{2\pi} \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{f}(k_\rho) k_\rho \left[\int_0^{2\pi} \cos \alpha e^{jn\alpha} e^{-jk_\rho x_0 \cos \alpha} d\alpha \right] dk_\rho \quad (4.6)$$

Now, based on identity

$$\int_0^{2\pi} e^{jn\alpha} e^{jK r \cos(\alpha - \phi)} d\alpha = 2\pi e^{jn\frac{\pi}{2}} e^{jn\phi} J_n(Kr) \quad (4.7)$$

the integral between brackets in (4.6) can be solved analytically along α , which leads to:

$$v_x(m, n) = \frac{a^2(-j)^n}{2\pi} \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{f}(k_\rho) k_\rho \left[\pi e^{j(n+1)\frac{\pi}{2}} J_{1+n}(-k_\rho x_0) + \pi e^{-j(1-n)\frac{\pi}{2}} J_{n-1}(-k_\rho x_0) \right] dk_\rho \quad (4.8)$$

The y-directed excitation contribution is obtained following a similar reasoning:

$$v_y(m, n) = \frac{a^2(-j)^n}{2\pi} \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{f}(k_\rho) k_\rho \left[\frac{\pi}{j} e^{j(n+1)\frac{\pi}{2}} J_{1+n}(-k_\rho x_0) - \frac{\pi}{j} e^{-j(1-n)\frac{\pi}{2}} J_{n-1}(-k_\rho x_0) \right] dk_\rho \quad (4.9)$$

Similarly, for a feed shifted along the y-direction, each element of the x and

y directed excitation vectors is given by

$$v_x(m, n) = \frac{a^2(-j)^n}{2\pi} \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{f}(k_\rho) k_\rho \cdot \left[\pi e^{j(1+n)\pi} J_{n+1}(-k_\rho y_0) + \pi e^{-j(1-n)\pi} J_{n-1}(-k_\rho y_0) \right] dk_\rho \quad (4.10)$$

$$v_y(m, n) = \frac{a^2(-j)^n}{2\pi} \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{f}(k_\rho) k_\rho \cdot \left[\frac{\pi}{j} e^{j(1+n)\frac{\pi}{2}} J_{1+n}(-k_\rho y_0) - \frac{\pi}{j} e^{-j(1-n)\frac{\pi}{2}} J_{n-1}(-k_\rho y_0) \right] dk_\rho \quad (4.11)$$

4.1.2 Numerical results

This section presents some examples of application of the multibeam synthesis techniques discussed in Section 4.1.1. The IBC design is validated with the MoM code presented in Chapter 2. The chosen frequency of operation is 24 GHz. The substrate relative permittivity is $\epsilon_r = 3.66$ with a thickness $d = 1.52$ mm. The antenna radius is fixed to $a = 10 \lambda$, with λ being the free-space wavelength. Finally, the average reactance is $X_0^{\rho\rho} = X_0^{\phi\phi} = -1260 \Omega$.

Multiple flat beam design

In the first example, the MTS is designed to radiate a flat circularly polarized azimuthal beam in the direction $\theta = 30^\circ$ and ϕ in the range between $\phi_1 = 45^\circ$ and $\phi_2 = 135^\circ$ when it is fed with a vertical elementary dipole placed at $(x = 1.5 \lambda, y = 0, z = -d/2)$. The radiation pattern can then be written as $F_{TE} = \cos \phi - j \sin \phi$ and $F_{TM} = \sin \phi + j \cos \phi$ in the targeted direction. The impedance is designed to exhibit a bi-symmetric pattern ($Q = 2$), which means that the antenna should also automatically radiate a flat beam at $\theta = 30^\circ$ and $\phi \in [225^\circ, 315^\circ]$ when fed by a source placed at $(x = -1.5 \lambda, y = 0, z = -d/2)$. We used for the impedance discretization $M_S = 50$, $N_S = 20$. The desired radiation pattern is represented in Fig. 4.1.

This desired radiation pattern is obtained by adding to the ideal visible current distribution (linked to the radiation pattern), the invisible current distribution obtained with the average impedance, and by discretizing the

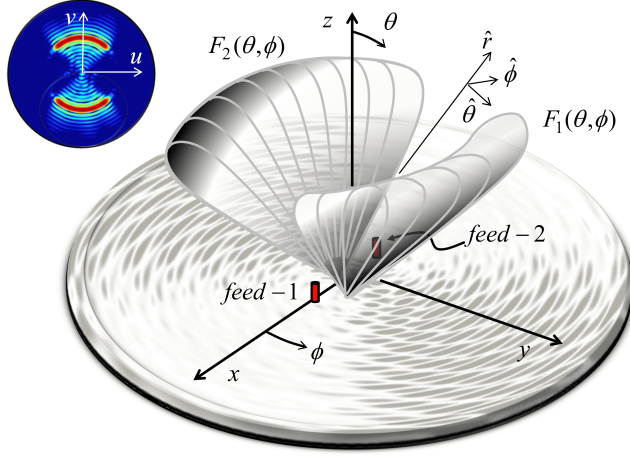


Figure 4.1: Circularly polarized azimuthal bi-symmetric radiation pattern

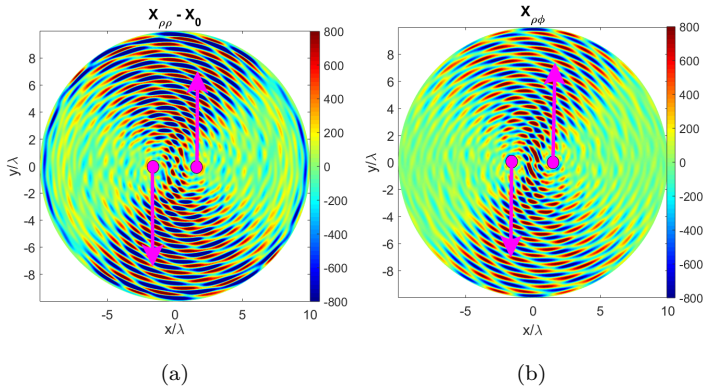


Figure 4.2: Modulated sheet impedance for a MTS antenna radiating two symmetric azimuthal beams at $\theta = 30^\circ$ and $\phi \in [45^\circ, 135^\circ]$ and $[225^\circ, 315^\circ]$. Dots represent the position of the point sources and arrows the direction of the central part of the beam in the azimuthal plane (see also Fig. 4.1). The synthesis is based on the symmetry based method described in Section 4.1.1. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

overall current with FBBFs.

The obtained impedance modulation and the corresponding simulated radiation pattern are depicted in Fig. 4.2 and Figs. 4.3(a)-(c), respectively.

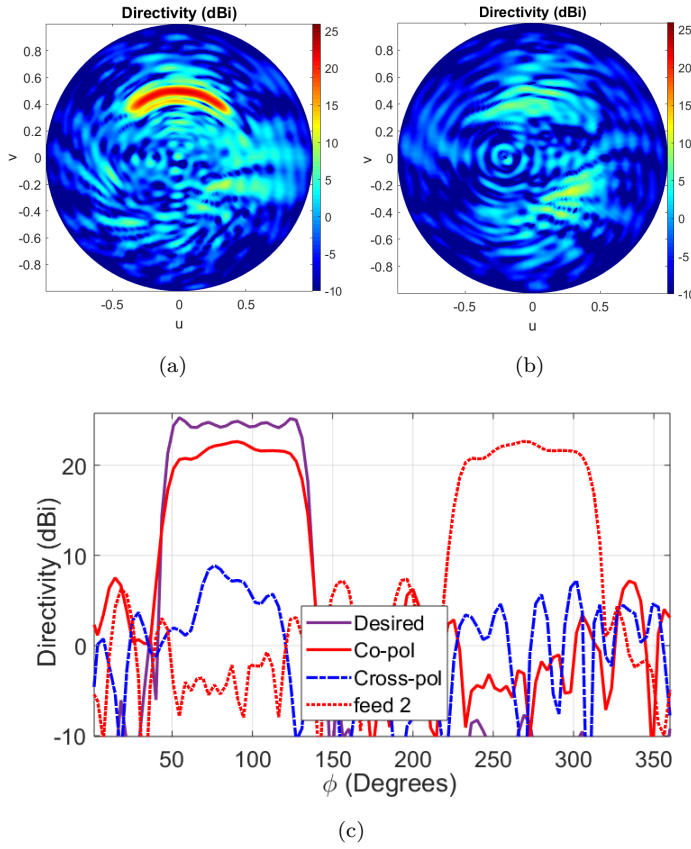


Figure 4.3: Circularly polarized azimuthal radiation pattern in the (u, v) plane. (a) Copolar component for the designed MTS. (b) Cross-polar component for the designed MTS. (c) Cut of the radiation pattern along the cone $\theta = 30^\circ$.

In Fig. 4.2, as well as in the remainder of the chapter, dots represent the position of the feeds, while arrows point toward the central part of the beam in the azimuthal plane. We obtained an average directivity of 21.5 dBi, which corresponds to 49% of the power density of the desired radiation pattern. The same design has now been carried out with the second approach (described in Section 4.1.1). The obtained reactance and the radiation pattern are represented in Fig. 4.4 and Fig. 4.5, respectively. A cut

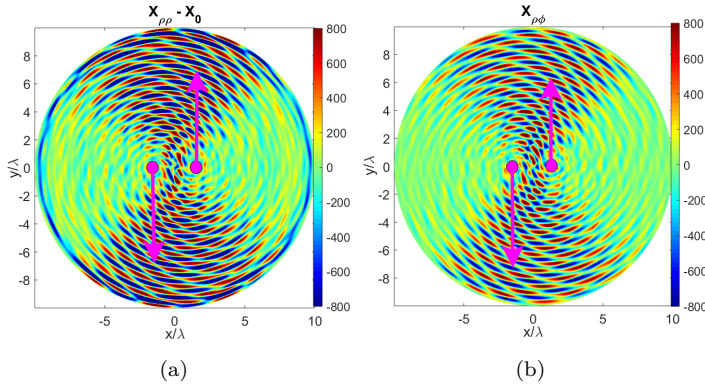


Figure 4.4: Modulated sheet impedance for a MTS antenna radiating two symmetric azimuthal beams at $\theta = 30^\circ$ and $\phi \in [45^\circ, 135^\circ]$ and $[225^\circ, 315^\circ]$. Dots represent the position of the point sources and arrows the direction of the central part of the beam in the azimuthal plane. The synthesis is based on the second method described in Section 4.1.1. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

of this radiation pattern, as well as a comparison with the results obtained using the symmetry technique 4.1.1, are illustrated in Fig. 4.6.

One can see that both methods essentially yield the same performance. However, the second approach is much more time and memory consuming than the first one, since the system of equations matrix to solve is 4 times larger. One can also observe from Fig. 4.4 and Fig. 4.2 that the modulations are quite similar. However, the MTS aperture is not entirely used, which means that one could add more functionalities by exploiting the less modulated part of the aperture. In the next example, we compare the results obtained with $Q = 1$, $Q = 2$, and $Q = 3$. For the case $Q = 1$, the source is located at $(x = 1.5\lambda, y = 0, z = -d/2)$. For the case $Q = 3$, the sources are located at $(x = 1.5\lambda, y = 0, z = -d/2)$, $(x = -0.75\lambda, y = (3\sqrt{3}/4)\lambda, z = -d/2)$ and $(x = -0.75\lambda, y = -3\sqrt{3}/4\lambda, z = -d/2)$. The desired radiation pattern for the case $Q = 3$ is illustrated in Fig. 4.7.

The modulations computed for the cases $Q = 1$ and $Q = 3$ are shown in Fig. 4.8. As can be seen, the case $Q = 3$ uses more efficiently the aperture. The corresponding radiation patterns are compared in Fig. 4.9. As expected,

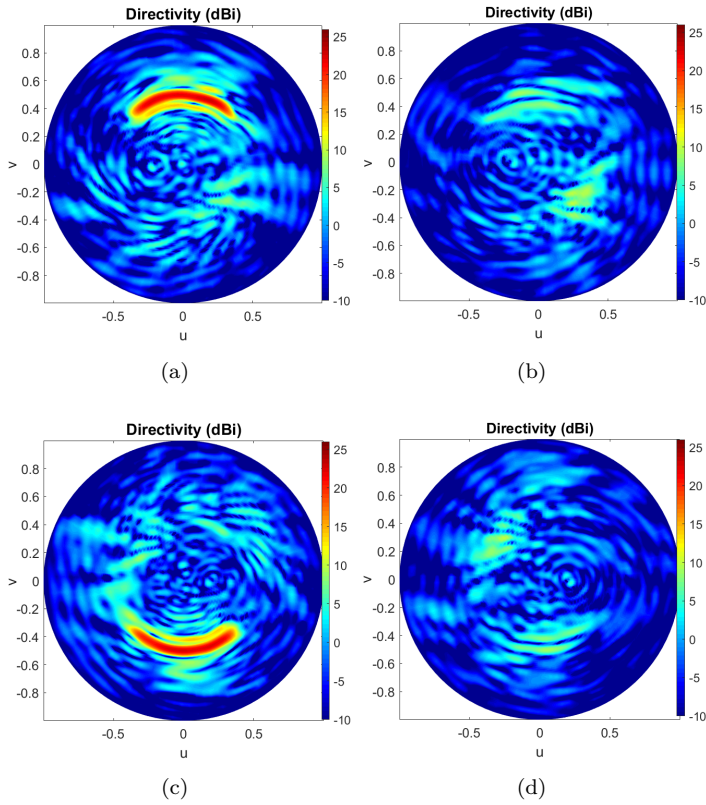


Figure 4.5: Circularly polarized azimuthal radiation pattern in the (u, v) plane based on the stacking method of Section 4.1.1. (a) Copolar component when the MTS is fed at $(x = 1.5 \lambda, y = 0)$. (b) Cross-polar component when the MTS is fed at $(x = 1.5 \lambda, y = 0)$. (c) Copolar component when the MTS is fed at $(x = -1.5 \lambda, y = 0)$. (d) Cross-polar component when the MTS is fed at $(x = -1.5 \lambda, y = 0)$.

the performance decreases when the number of beams increases.

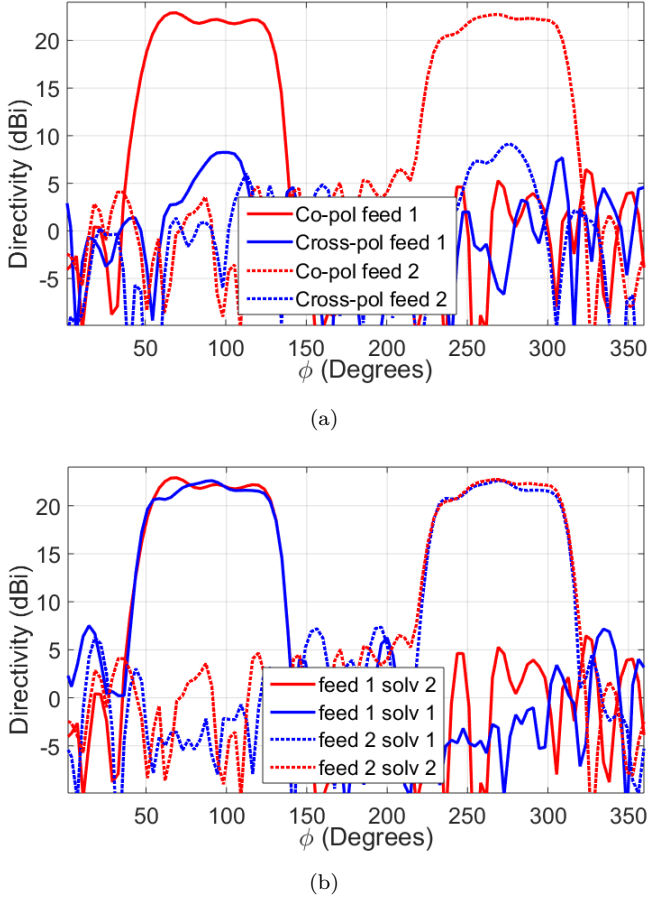


Figure 4.6: Cut of the radiation pattern along the cone $\theta = 30^\circ$ for a MTS radiating an azimuthal bi-symmetric radiation pattern. (a) results from the stacking method described in Section 4.1.1. (b) Comparison between results with the first method (symmetry based) in Section 4.1.1 (solv 1) and the second one (equations stacking) in Section 4.1.1 (solv 2).

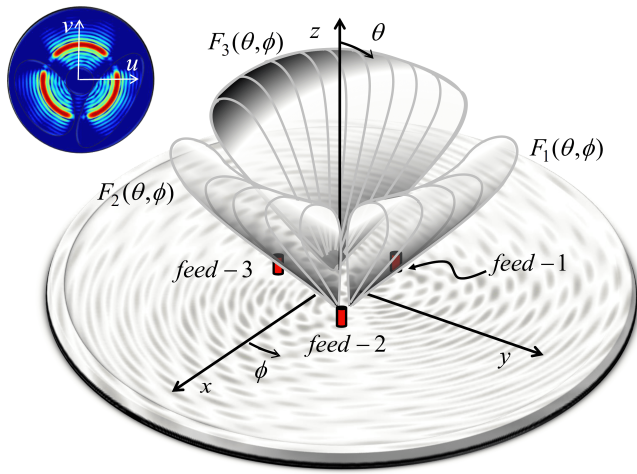


Figure 4.7: Circularly polarized azimuthal tri-symmetric radiation pattern.

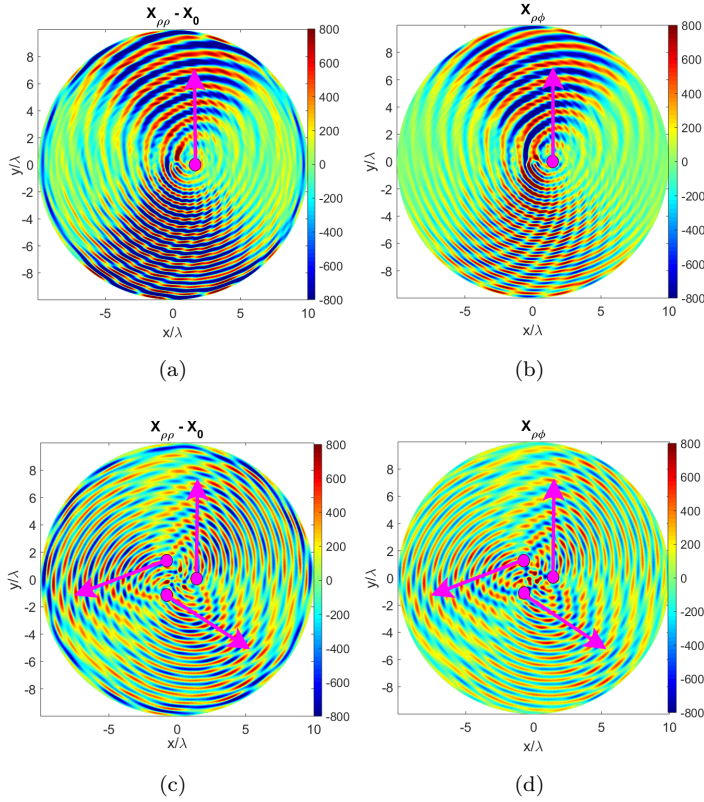


Figure 4.8: Modulated sheet impedance for a MTS antenna radiating azimuthal beams at $\theta = 30^\circ$ based on the method described in Section 4.1.1. (a) $X_{\rho\rho} - X_0$ for a non symmetric beam ($Q = 1$). (b) $X_{\rho\phi}$ for a non symmetric beam ($Q = 1$). (c) $X_{\rho\rho} - X_0$ for a tri-symmetric beam ($Q = 3$) (d) $X_{\rho\phi}$ for a tri-symmetric beam ($Q = 3$). Dots represent the position of the point sources and arrows the direction of the central part of the beam in the azimuthal plane (see also Fig. 4.7).

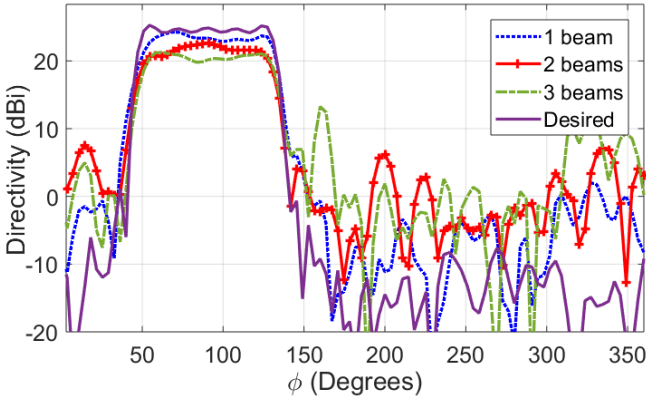


Figure 4.9: Comparison between the radiation patterns for MTS radiating an azimuthally symmetric beam ($Q = 1$, $Q = 2$ and $Q = 3$).

Dual polarization design

Next, we design a MTS antenna capable of radiating two beams with the same amplitude, but with opposite polarizations. Depending on the excited feed, one can select a given polarization. A similar functionality has recently been proposed in [21] using holographic techniques. Here, we feed the MTS as in Section 4.1.2. The IBC is designed in such a way to provide for the first excitation, located at $(x = 1.5 \lambda, y = 0, z = -d/2)$, a flat right-handed circularly polarized (RHCP) radiation pattern in the sector $(\theta = 30^\circ \text{ and } \phi \in [67.5^\circ, 112.5^\circ])$. When the MTS is excited with the second feed, located at $(x = -1.5 \lambda, y = 0, z = -d/2)$, the antenna should radiate a flat left-handed circularly polarized (LHCP) radiation pattern in the same sector as for the first feed. The desired radiation pattern can then be written for the first excitation as $F_{TE} = \cos 4\phi - j \sin 4\phi$ and $F_{TM} = -\sin 4\phi - j \cos 4\phi$. For the second excitation, one needs $F_{TE} = -\cos 4\phi - j \sin 4\phi$ and $F_{TM} = \sin 4\phi - j \cos 4\phi$ in the targeted direction. This beam does not satisfy the symmetry required to apply the method presented in Section 4.1.1. The synthesis has therefore been carried out with the equations stacking method described in Section 4.1.1. We now analyze the impact of iterating over the invisible current spectrum, as explained in Section 4.1.1. Fig. 4.10 represents the obtained impedance modulations. Fig. 4.11 shows the simulated embedded element patterns, corresponding to each excitation taken separately. The results obtained without iteration is compared to those obtained after one iteration. One can see a slight improvement on the radiation pattern. As observed, a good cross-polar level has been obtained after one iteration. The achieved aperture efficiency (computed with the desired radiation pattern) is near 44%. It is important to stress that the iteration process is not necessarily stable, i.e. it may sometimes lead to undesired amplification of the modulation depth after several iterations. The authors recommend two iteration steps and selection of the best performance among the three obtained results.

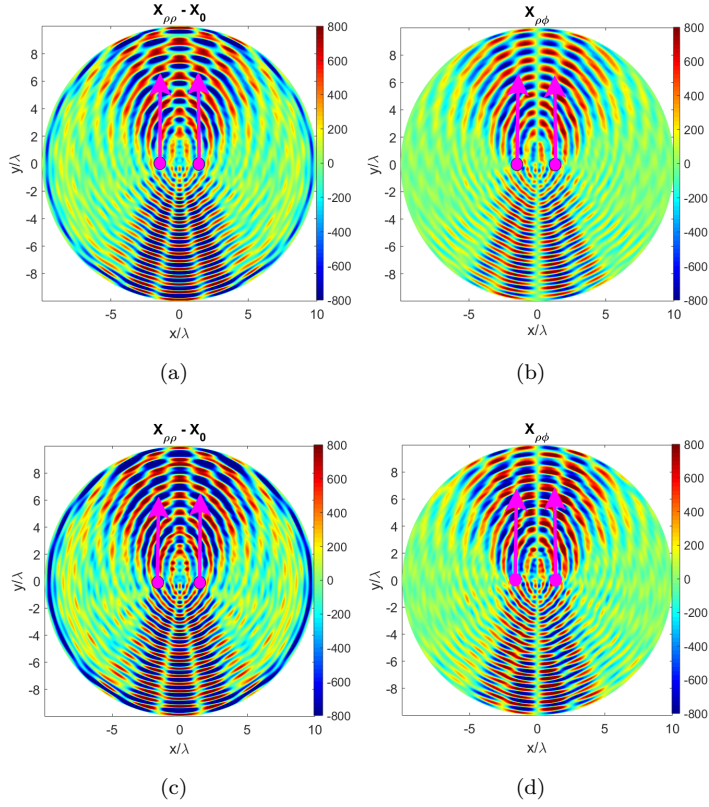
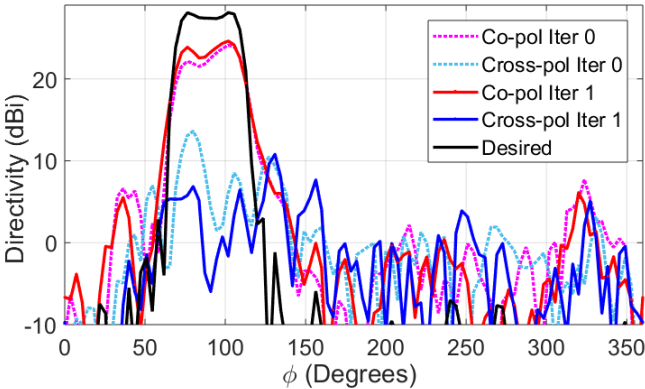
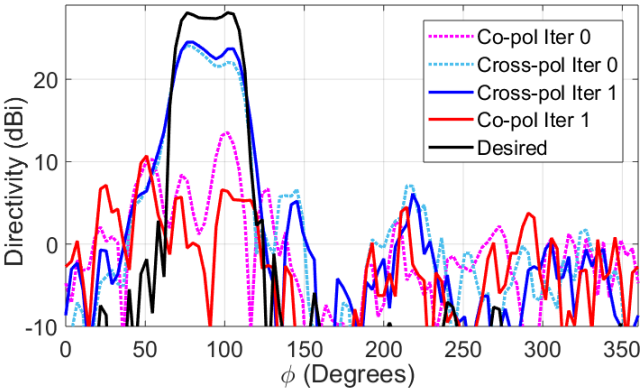


Figure 4.10: Modulated sheet impedance for a MTS antenna radiating two azimuthal beams at $\theta = 30^\circ$ and $\phi \in [67.5^\circ, 112.5^\circ]$ with opposite polarizations. The dots represent the sources location. Synthesis based on the second method described in Section 4.1.1. (a) $X_{\rho\rho} - X_0$ at iteration 0. (b) $X_{\rho\phi}$ at iteration 0. (c) $X_{\rho\rho} - X_0$ at iteration 1 (d) $X_{\rho\phi}$ at iteration 1.



(a)



(b)

Figure 4.11: Comparison between the embedded element patterns obtained at iteration 0 with those obtained after a first iteration along the cone $\theta = 30^\circ$. (a) feed 1. (b) feed 2.

MTS Design for Beam Scanning

The goal of this section consists of numerically demonstrating the possibility of beam scanning using multi-fed phased MTSs. The used parameters (frequency, substrate, radius, feeding points, etc ...) are the same as in the previous section. As opposed to the dual-pol configuration, here we want to generate two embedded element patterns with the same amplitude and polarization (TM), but with different phase progressions. For the first excitation, the phase linearly increases w.r.t. ϕ and covers a phase range of π radian. For the second excitation, the phase linearly decreases, with the same phase range. Exciting the first feed, the desired radiation pattern is then given as $F_{TE} = 0$ and $F_{TM} = \sin 4\phi - j \cos 4\phi$ in the targeted direction. For the second excitation, we want $F_{TE} = 0$ and $F_{TM} = -\sin 4\phi - j \cos 4\phi$. We now excite the two feed points with the same magnitude and a relative phase ψ . The global radiation pattern can then be written as:

$$\mathbf{F}(\theta, \phi) = U_\phi \hat{p} (e^{jk\phi} + e^{j\psi} e^{-jk\phi}) \quad (4.12)$$

where the window U_ϕ is equal to one for $\theta = 30^\circ$ and $\phi \in [67.5^\circ, 112.5^\circ]$ and 0 elsewhere. $\hat{p}$ is the TM unit vector polarization. $k = (\pi/\Delta\phi)$, with $\Delta\phi = (112.5^\circ - 67.5^\circ) \pi/180$. The expression between brackets in (4.12) can be interpreted as an array factor and can be rewritten as $e^{j\psi/2} \cos(k\phi - \psi/2)$, which means that the antenna beam is linearly polarized with a maximum directivity at $\phi = -\psi/(2k)$. Therefore, one can achieve continuous azimuthal scanning by varying the phase ψ of excitation at one of the two feed points. Fig. 4.12 shows the obtained impedance modulation. Fig. 4.13 illustrates the beam scanning capability. One can see that a beam scanning over about 20° has been achieved. The sidelobes level is quite high. Indeed, when the array is scanned, neighboring lobes of $\cos(k\phi - \psi/2)$ in the array factor start to enter into the window U_ϕ . For larger scan angles, the next lobe of $\cos(k\phi - \psi/2)$ appears as important sidelobes. Since there is only one degree of freedom, the method allows a beam scanning over a 3 dB beamwidth only. One can choose independently the beamwidth based on the width of the radiation pattern associated with each excitation.

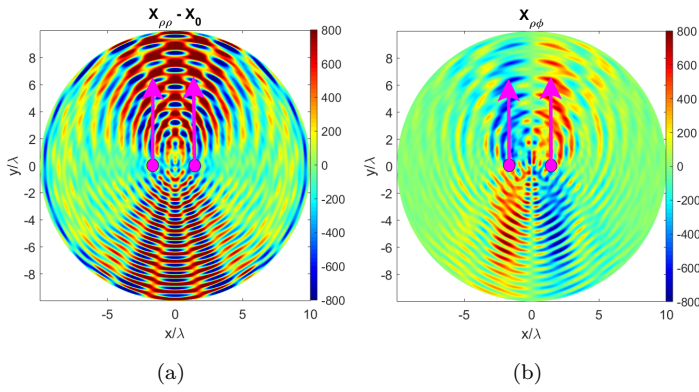


Figure 4.12: Modulated sheet impedance for a MTS antenna radiating two circularly polarized azimuthal beams at $\theta = 30^\circ$ and $\phi \in [67.5^\circ, 112.5^\circ]$ with different phase progression. The dots represent the source locations. The synthesis is based on the second method described in Section 4.1.1. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

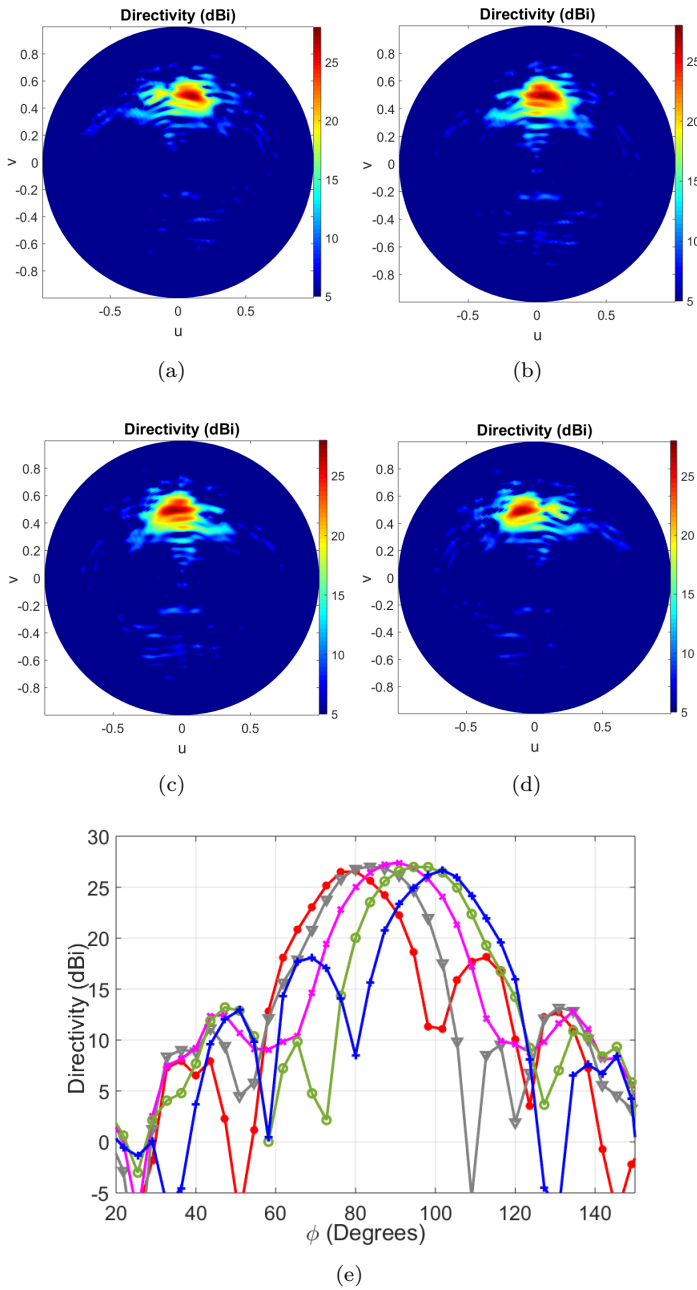


Figure 4.13: Beam scanning capabilities of a two-feeds MTS antenna (a)-(d) illustration in the u - v plane for different values of ψ . (e) Illustration along the cone $\theta = 30^\circ$ for different values of ψ equally spaced in the range $[0, 2\pi]$.

4.2 Multi-frequency bands synthesis of MTS antennas

MTS antennas are relatively narrowband given the strong frequency dispersion of the SW. This means that a change in the frequency of operation strongly impacts the SW mode supported by the MTS. The interaction between the modulation parameters, and the new SW mode no longer produces the desired radiation pattern. In many contexts, multiple-band operation is required since it allows one to multiplex several channels at different frequencies, while sharing the same MTS aperture. [90] proposes a method for the generation of multiple beams at various frequencies with a modulated MTS. That reference deals with isotropic (scalar impedance modulation, which leads to limited performance, in comparison with anisotropic MTS, in terms of polarization control [43]. The used technique is based on the superposition of several holograms. However, the interference between those holograms may significantly affect the antenna performance, then the method requires an optimization procedure. The authors of [91] address the same issue with tensorial impedance modulation. The proposed solution consists of superimposing the modulations required at each frequency. Each modulation is computed using the flat optics synthesis technique [51].

The method presented here has several advantages in comparison with other methods [90], [91]. First it is not based on a superposition technique, and therefore does not suffer from interference between beams. Second, the method is systematic and can be used to generate elaborate, i.e. shaped, multibeam patterns without resorting to an optimization scheme.

4.2.1 EFIE based multi-frequency bands synthesis of MTS antennas

Let us assume now that we want to design a MTS capable of radiating several beams at different frequencies. This requires the characterization of the frequency-dependence of the surface impedance of the patches implementing the impedance boundary condition (IBC). As already mentioned, the opaque surface impedance highly depends on the frequency. However, assuming that the unit cell size is relatively small w.r.t. the wavelength of operation (i.e staying in the homogenizable region), the frequency dependence of the sheet impedance tensor components can be accurately characterized

with a simple semi-separable model:

$$Z_S(\mathbf{p}, \omega) = a(\omega)Z_S(\mathbf{p}) + b(\omega) \quad (4.13)$$

where $\omega = 2\pi\nu$ is the radian frequency, and $\mathbf{p}$ is a vector of parameters characterizing the patch. For a square patch, for example, $\mathbf{p}$ is the size of the patch and its orientation. $a(\omega)$ and $b(\omega)$ are arbitrary (non linear) functions. If the model is provided around a reference frequency ω_1 , $Z_S(\mathbf{p})$ is the impedance at that frequency, while $a(\omega_1) = 1$, and $b(\omega_1) = 0$. As an example, Fig. 4.14 compares the scalar sheet impedance variation (w.r.t. patch size) at 19 GHz with the one obtained after fitting the impedance at 24 GHz to the impedance at 19 GHz, assuming the linear model (4.13). One

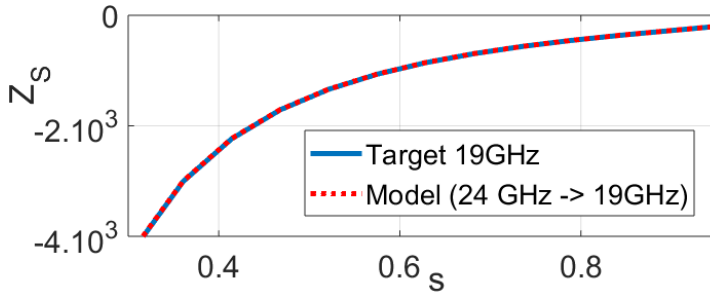


Figure 4.14: Illustration of the sheet impedance of a sub-wavelength square patch in function of the patch size normalized w.r.t unit cell size. The sheet impedance is obtained after removing from the opaque impedance, the contribution of the substrate [67].

can observe an excellent agreement, although the dependence of the sheet impedance on $\mathbf{p}$ is not a priori similar at both frequencies. In our experience, the reduced-order modeling (4.13) is always acceptable (at least in a given range of patch parameters), even for more complex patch shapes. In practice, only a portion of the parameter space characterizing the patches is used since the IBC is slightly modulated around an average value. Assuming a priori the portion of that space used to implement the IBC, one can accurately characterize the frequency behavior at the sampling frequencies of interest using the model (4.13).

The tensorial IBC can then be written at a frequency ω_2 based on its value at the reference frequency ω_1 as:

$$\begin{aligned} Z_S^{\rho\rho}(\omega_2) &= [a^{\rho\rho}(\omega_2)Z_0^{\rho\rho}(\omega_1) + b^{\rho\rho}(\omega_2)] + a^{\rho\rho}(\omega_2)P^{\rho\rho}(\omega_1) \\ Z_S^{\rho\phi}(\omega_2) &= Z_S^{\phi\rho}(\omega_2) = b^{\rho\phi}(\omega_2) + a^{\rho\phi}(\omega_2)P^{\rho\phi}(\omega_1) \\ Z_S^{\phi\phi}(\omega_2) &= [a^{\phi\phi}(\omega_2)Z_0^{\phi\phi}(\omega_1) + b^{\phi\phi}(\omega_2)] - a^{\phi\phi}(\omega_2)P^{\rho\rho}(\omega_1) \end{aligned} \quad (4.14)$$

where $a^{\rho\rho}(\omega_2)Z_0^{\rho\rho}(\omega_1) + b^{\rho\rho}(\omega_2)$, $b^{\rho\phi}(\omega_2)$ and $a^{\phi\phi}(\omega_2)Z_0^{\phi\phi}(\omega_1) + b^{\phi\phi}(\omega_2)$ correspond respectively to the $\rho\rho$, $\rho\phi$, and $\phi\phi$ components of the average impedance at frequency ω_2 . The unknowns are the impedance modulations $P^{\rho\rho}(\omega_1)$, $P^{\rho\phi}(\omega_1)$, which are expressed only at the reference frequency ω_1 , equation (4.13) describing the extension to other frequencies. One can then proceed to the synthesis by solving jointly, in the least-squares sense the systems of equations corresponding to each frequency. The system of equations obtained at a given frequency ω_i is established for a given feed V_i , the average impedance at that frequency, and for the required beam $\mathbf{F}_i$, as explained in Chapter 3. The overall system of equations can then be written in matrix form as in (3.29) with:

$$\underline{\underline{\mathbf{Z}}}^S = \left[\underline{\underline{\mathbf{Z}}}^S(\omega_1, V_1, \mathbf{F}_1); \underline{\underline{\mathbf{Z}}}^S(\omega_2, V_2, \mathbf{F}_2); \dots; \underline{\underline{\mathbf{Z}}}^S(\omega_Q, V_Q, \mathbf{F}_Q) \right] \quad (4.15)$$

$$\underline{\underline{\mathbf{U}}} = \left[\underline{\underline{\mathbf{U}}}(\omega_1, V_1, \mathbf{F}_1); \underline{\underline{\mathbf{U}}}(\omega_2, V_2, \mathbf{F}_2); \dots; \underline{\underline{\mathbf{U}}}(\omega_Q, V_Q, \mathbf{F}_Q) \right] \quad (4.16)$$

where the unknown IBC has been expressed at the reference frequency ω_1 , assuming the model (4.14) for extension at other frequencies. In (4.15) and (4.16), the semi-column is used to stack matrices vertically. The obtained system of equations is then solved in a least-squares sense. Note that, if one is interested in stronger requirements for a given radiation pattern in comparison with the others, one can perform a weighted least-squares. In this process, one should be sure that the exploited value of $\mathbf{p}$ lies in the domain for which the parameters of the model (4.14) is valid. This may require a few iterations. A starting point of these iterations may consist of

considering a frequency-independent IBC (i.e. $a(\omega) = 1, b(\omega) = 0$). At each iteration, one computes the IBC at the reference frequency, implements the IBC with the used patch, and updates the parameters $a(\omega)$ and $b(\omega)$ based on the computed valid range of parameters. We want to stress that the proposed method fully takes into account the frequency variation of the substrate impedance, since the substrate impedance matrix is computed exactly at any given frequency and used to compute the matrix $\underline{\underline{\mathbf{Z}}}^S$ and the vector $\underline{\mathbf{U}}$ at that same frequency.

4.2.2 Numerical results

In this section, we present numerical validation of the proposed synthesis method. The MTS is printed on a substrate with relative permittivity $\epsilon_r = 3.66$ and thickness $d = 1.524$ mm. The average sheet reactance is $X_0 = -1260 \Omega$. The antenna radius is 10λ , where λ is the free-space wavelength at 24 GHz. The antenna is designed to operate at two frequencies ($\nu_1 = 24$ GHz and $\nu_2 = 20$ GHz) and is fed by a vertical elementary dipole placed at the middle of the substrate thickness ($z = -d/2$). This simple modeling of the feed is sufficient to obtain an excellent idea of the radiation pattern of modulated MTS antennas made with a realistic feed, as already proven in several publications [43], [84]. We used $a^{\rho\rho}(\omega_2) = a^{\phi\phi}(\omega_2) = 1.2$, $a^{\rho\phi}(\omega_2) = 1.1$, and $b^{\rho\rho}(\omega_2) = b^{\rho\phi}(\omega_2) = b^{\phi\phi}(\omega_2) = 0$, with a reference frequency at 24 GHz.

In the first example, the antenna is designed to provide a broadside circularly polarized pencil beam with a feed placed at the center of the structure ($\rho = 0$). We assume that the feed is designed to operate at the desired frequencies (wideband or dual-band matching). The feeding design is outside the scope of this chapter. The obtained IBC is depicted in Fig. 4.15.

The IBC is then simulated with the MoM code validated in Chapter 2. Cuts of the obtained radiation pattern in the plane ($\phi = 0$) at 20 GHz and 24 GHz are shown in Fig. 4.16(a) and Fig. 4.16(b). We obtained an aperture efficiency of about 20% at both frequencies. This performance is quite low in comparison with the single-frequency synthesis (about 70%). A

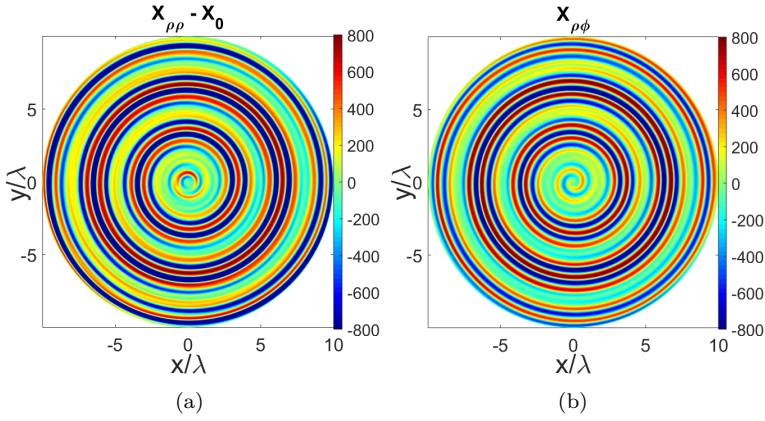


Figure 4.15: Modulated sheet impedance for a MTS antenna radiating a broadside pencil beam at 20 GHz and 24 GHz. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

higher efficiency is obtained when the two frequencies of operation are closer. It is worth mentioning that, since the two frequencies of operation (20 GHz and 24 GHz) are relatively far from each other (in comparison with the bandwidth of such antennas), the radiation performance between the two frequencies of interests is not necessarily good as can be seen from Fig. 4.16(c). Therefore, radiation only occurs in two passbands surrounding 20 GHz and 24 GHz, with approximately 8% and 5% bandwidth for a 3 dB drop of the directivity. This observation also holds for the next example.

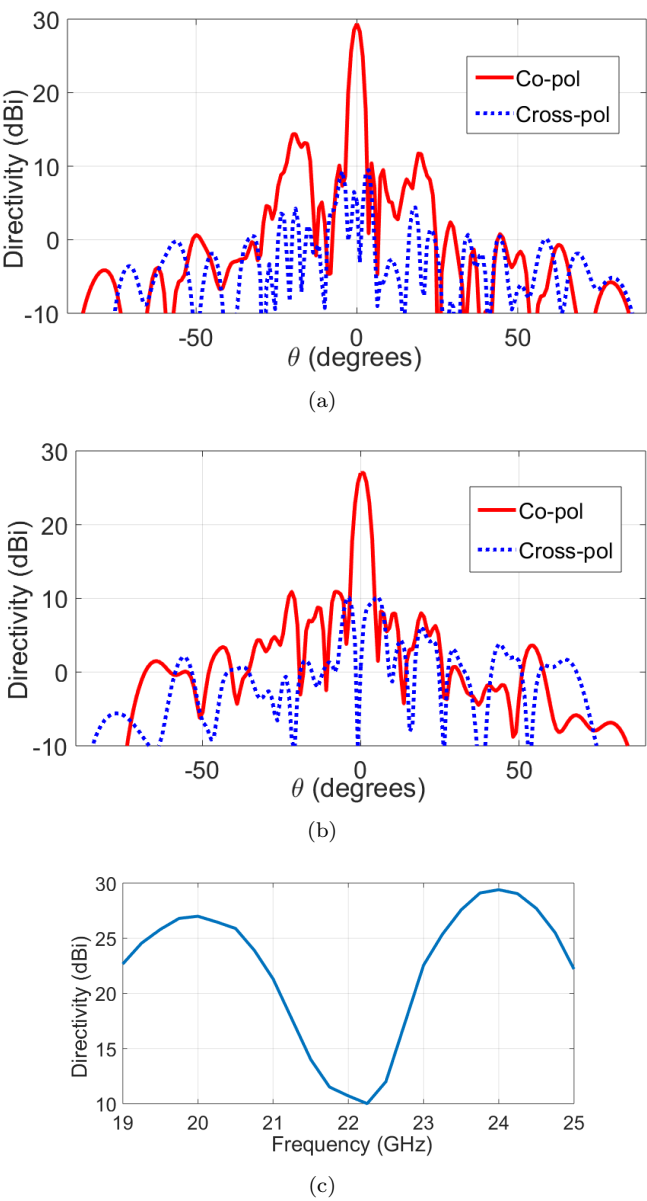


Figure 4.16: Radiation pattern in the plane $\phi = 0$ for a MTS radiating a broadside pencil beam. (a) 24 GHz. (b) 20 GHz. (c) Directivity versus frequency.

In the second example, we present a MTS design capable of radiating two different beams at two different frequencies. The first one is a right-handed circularly polarized (RHCP) conical beam (pointing at $\theta = 20^\circ$ from broad-side), and should be generated at $f_1 = 24$ GHz. The second beam is a RHCP pencil beam generated at $f_2 = 20$ GHz. Fig. 4.17 shows the designed reactance modulation at 24 GHz. Fig. 4.18 illustrates the simulated

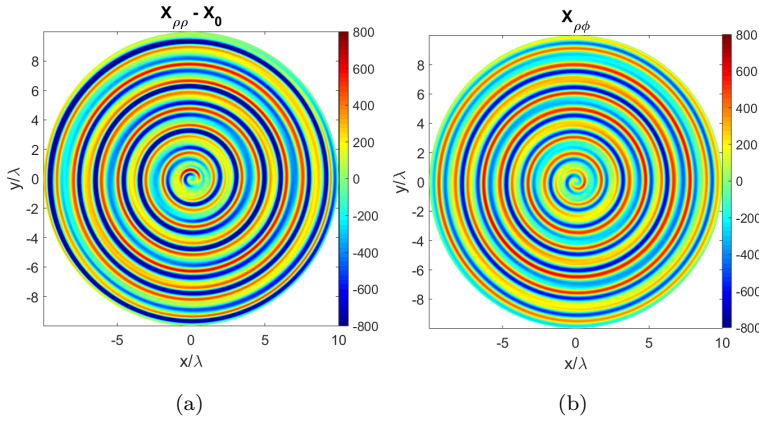


Figure 4.17: Modulated sheet impedance for a MTS antenna radiating a broadside pencil beam at 20 GHz and a conical beam at 24 GHz. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

radiation pattern obtained with two independent MoM codes at $f_1 = 24$ GHz. The first code is based on Gaussian ring basis functions (GRBF) [19] and the second one uses Fourier-Bessel basis functions (FBBF) [15]. GRBF consist in Gaussian-type rings and linear azimuthal phase. One can observe a good agreement between the two simulation results except the side lobes level. The difference is due to the evolution of the currents at a distance very close to the feed. Indeed GRBFs allow a more accurate current evaluation close to the feed because of the possible zero admissible current (see Chapter 2 for a detailed discussion regarding this aspect). The obtained aperture efficiency is about 79%. This efficiency is computed in comparison with the power density radiated by an ideal conical beam antenna of the same radius. Fig. 4.19 shows the radiation pattern at 20 GHz. The direc-

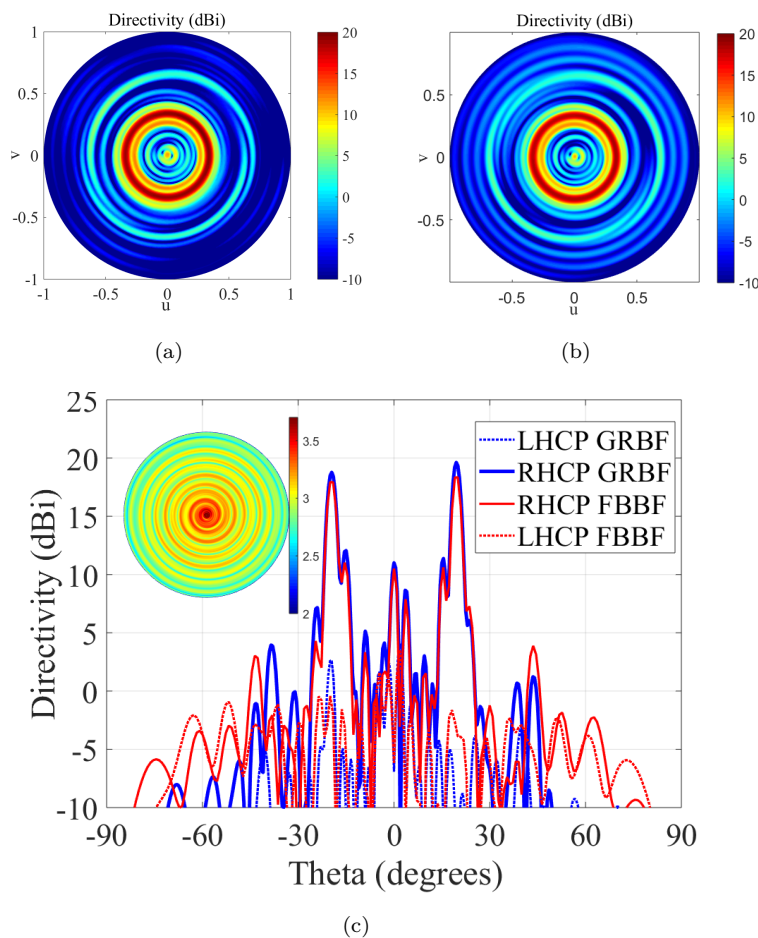


Figure 4.18: Conical beam radiation pattern at 24 GHz.

tivity obtained with GRBF code is 2 dB higher than that of FBBF code due to the better treatment of GRBFs for the currents near the center, as previously mentioned.

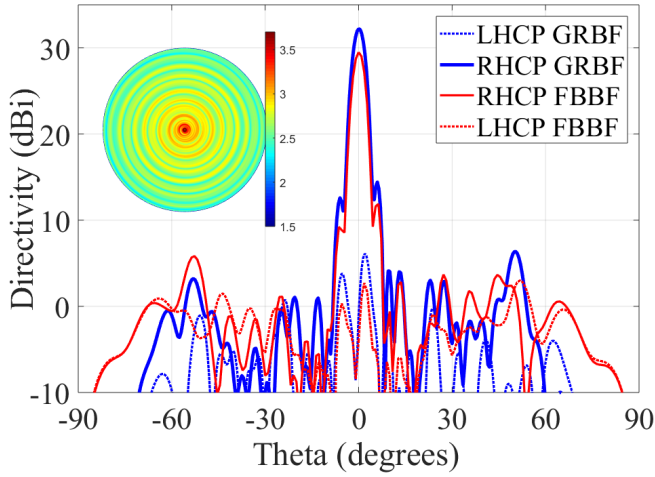


Figure 4.19: Broadside pencil beam radiation pattern at 20 GHz. The inset disk represents the absolute value of the current distribution on the MTS in logarithm scale.

In the third example, the antenna is fed at two points and is designed to generate two shaped beams. The first feed, located at $(x = 1.5\lambda, y = 0)$, should be designed to operate at 24 GHz. The second feed, located at $(x = -1.5\lambda, y = 0)$, should be designed to operate at 20 GHz. Exciting the first feed, the antenna should radiate a flat beam at $\theta = 30^\circ$ $\phi = [45^\circ, 135^\circ]$. When the second feed is excited, the MTS should radiate a flat beam at $\theta = 30^\circ$ $\phi = [225^\circ, 315^\circ]$, i.e in an opposite direction. Fig. 4.20 shows the synthesized IBC. Fig. 4.21 illustrates the two embedded element patterns corresponding to each feed. One can observe from the cut of the radiation pattern in the cone $\theta = 30^\circ$ that the performance is better for the first feed. This can be explained by the larger electrical size of the antenna at the frequency of operation of the first feed. The aperture efficiency is near 49%, in comparison with the power density of the ideal radiation pattern.

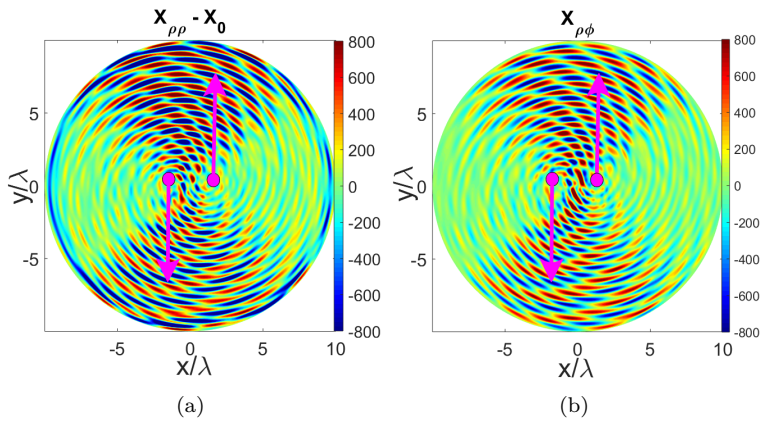


Figure 4.20: Modulated sheet impedance for a MTS antenna radiating two symmetric azimuthal beams at $\theta = 30^\circ$ and $\phi \in [45^\circ, 135^\circ]$ and $[225^\circ, 315^\circ]$. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$. Dots represent the position of the feeds and arrows illustrate the direction of the central part of the beam in the azimuthal plane.

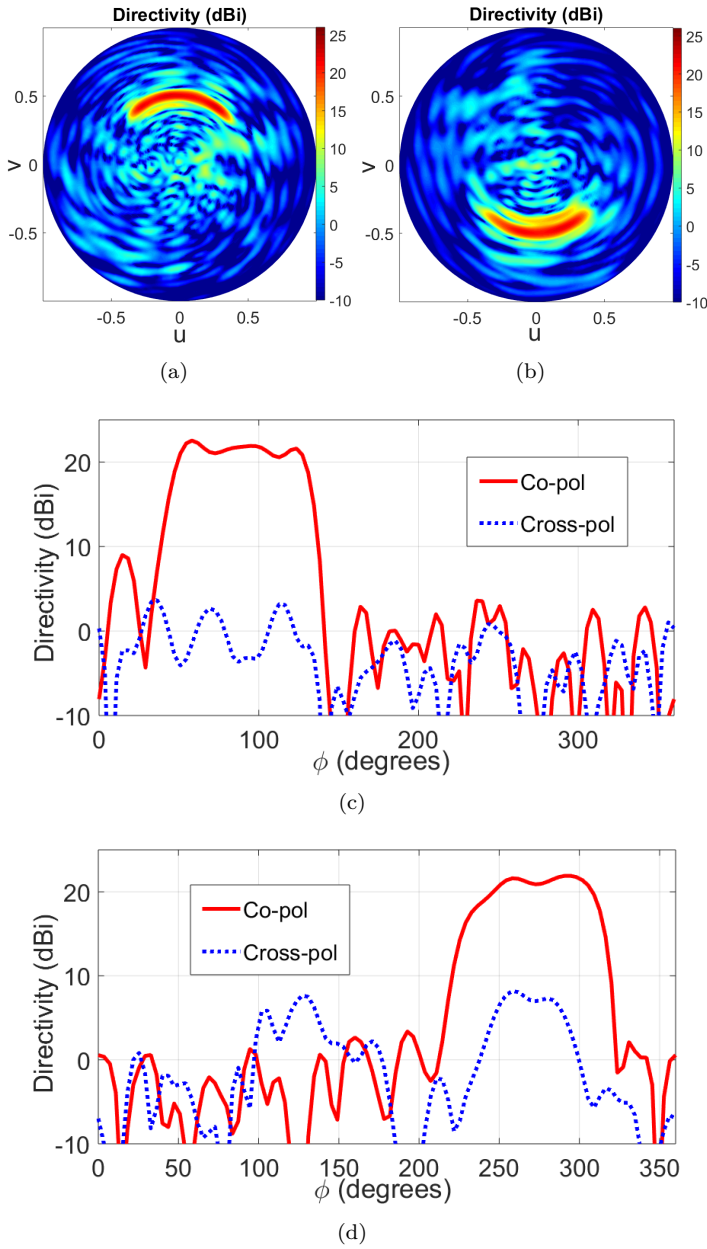


Figure 4.21: Radiation pattern for a MTS radiating two symmetric azimuthal beams at $\theta = 30^\circ$ and $\phi \in [45^\circ, 135^\circ]$ and $[225^\circ, 315^\circ]$. (a) Embedded element pattern (in the u - v plane) corresponding to the first feed (24 GHz). (b) Embedded element pattern (in the u - v plane) corresponding to the second feed (20 GHz). (c) Cut of the radiation pattern of the first feed in the cone $\theta = 30^\circ$. (d) Cut of the radiation pattern of the second feed in the cone $\theta = 30^\circ$.

4.2.3 Improvement of the method

One of the main advantages of the proposed EFIE approach is the fact that the method is systematic and starts directly from the radiation pattern, not from the aperture electric field. That means, we specified at input the desired radiation pattern in amplitude, phase and polarization at the frequencies of interest. Despite this flexibility is useful for near-field applications (see Chapter 6), the phase distribution is usually less or even not important for antenna (far-field) applications. For example, we specified for the conical beam described, a linear phase evolution along the azimuthal direction. The simulated pattern phase evolution of the designed MTS is represented in Fig. 4.22. One can see an excellent agreement with respect to the prescribed phase. It is worth mentioning that the designed pattern amplitude and polarization depends on the prescribed phase. When one is interested in realizing complex radiation patterns at multiple frequencies, with strong requirements, the prescribed phase evolution should be optimized to get better results. The phase optimization problem is a complex issue that deserves further research.

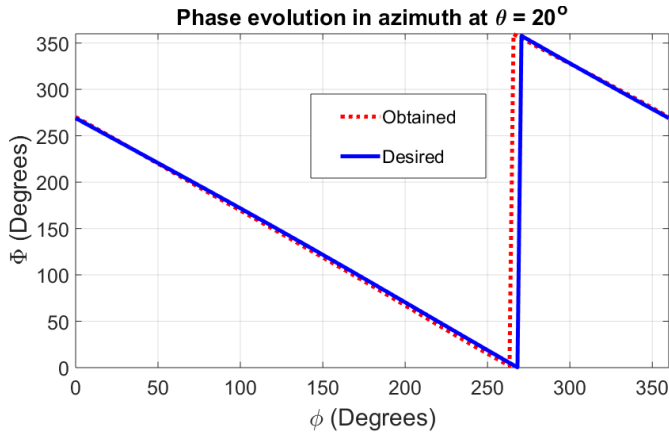


Figure 4.22: Conical beam radiation pattern phase evolution at 24 GHz and $\theta = 20^\circ$

4.3 Conclusion

In a first instance, we have presented the generation of multi-shaped beams with modulated MTS antennas. The proposed methods are based on the EFIE discretization with FBBFs. In particular, we discussed two techniques. The first one relies on the azimuthal symmetry of the impedance to naturally impose an azimuthal symmetry on the radiation pattern. The second method consists of over-constraining the system of equations and can be used for more general multibeam. However, this approach is more time and memory consuming than the first one. Finally, we have proven that a proper control of the amplitude, phase and polarization of the radiation pattern can allow continuous beam steering by phasing the MTS at multiple feed points: two points and azimuthal beam scanning so far. To our best knowledge, it is the first time that phased MTS antennas has been proposed for beam scanning. Further research should allow one to increase the number of feeds in order to improve the scanning capability, for instance obtaining the same scan range with a narrower beam.

In a second step, an extension of the EFIE formalism is provided to deal with multiple frequencies operation of modulated MTS antennas. It relies on a semi-separable model of surface impedance versus frequency and patch parameters. A very good agreement has been obtained between the MoM code presented in Chapter 2 and the one developed in [61].

A point deserving further attention concerns the a priori estimation of the antenna near-field. In this chapter, the near-field corresponding to the average impedance has first been used and we have iterated over this first estimation to slightly improve the results. However, a better a priori estimation of the near-field, taking into account the desired multibeam effect may allow one to achieve better performance.

CHAPTER 5

Design and realization

As described in the previous chapters, modulated metasurface (MTS) antennas can be modeled as a continuously spatially varying sheet impedance laying on the top of a grounded dielectric slab (see Fig. 1.3). We have presented in Chapters 3 and 4 algorithms to compute the sheet impedance profile from the antenna radiation pattern specifications. Once the modulated sheet impedance has been computed, one needs to implement the homogenized MTS with sub-wavelength printed patches to get the physical structure of the antenna, as represented for example in Fig. 1.3. The radiation pattern of the resulting dense arrays of printed patches can be simulated with an EM software and compare with the results of the impedance boundary condition (IBC) simulation code (developed in Chapter 2) to ensure that the surface impedance has been accurately implemented.

The purpose of this chapter is not to provide a new method for the IBC implementation, since the latter has already been well treated in several publications [81], [66], [82], [92]. Instead, we will explain step by step the IBC implementation process, based on the literature review and according to our in-house EM codes, with the aim of providing further numerical and experimental validations of the synthesis methods developed in Chapters 3 and 4. As will be shown in the following, the class of synthesized IBC (given by equation (3.14)) can be easily and accurately implemented with

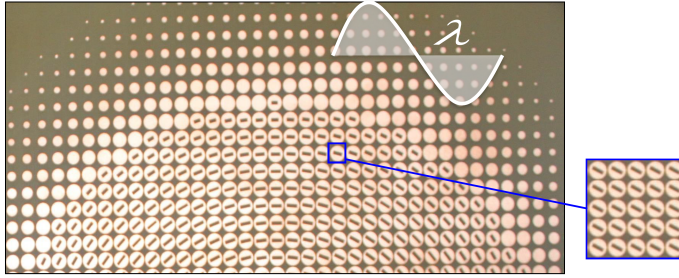


Figure 5.1: Illustration of a surface impedance implemented with sub-wavelength patches, assuming a local-periodicity [94].

printed patches and the corresponding MTS antenna can be manufactured with standard PCB techniques (milling at low frequency (< 20 GHz) and lithography at higher frequencies (> 20 GHz)). All the designs presented in this chapter operate at 24 GHz. We used the substrate Rogers4350B of relative permittivity $\epsilon_r \approx 3.66$, and thickness $d = 1.524$ mm.

The chapter is organized as follows. Section 5.1 discusses the IBC implementation process. Section 5.2 briefly presents an (accelerated) in-house Method of Moments (MoM) code developed to analyze dense arrays made with arbitrary metallizations, and used here for MTS simulation. Finally, Section 5.3 provides some numerical and experimental validations.

5.1 IBC implementation

The MTS IBC we are dealing with are modulated at the wavelength scale, and implemented with subwavelength printed patches (typically $\lambda/10$, with λ being the free-space wavelength). As a consequence, one can reasonably assume each patch to be immersed in an infinitely periodic lattice (see Fig. 5.1). The locally periodic assumption may not be valid very close to the antenna center ($\rho = 0$) or at the boundary of the antenna ($\rho \approx a$). However, it turns out to provide globally excellent performance on the radiation pattern.

If the fields are observed at a vertical distance larger than the unit cell size ($z > \lambda/10$), the higher order harmonics (Floquet modes) required to

satisfy the boundary condition at the patches layer ($z = 0$) can be neglected in comparison with the fundamental mode, with a phasing corresponding to that of the impressed plane wave excitation. Therefore, the periodic unit cell can be modeled as a sheet impedance tensor placed in parallel with a two modal transmission line network (see Fig. 5.2). The MTS impedance

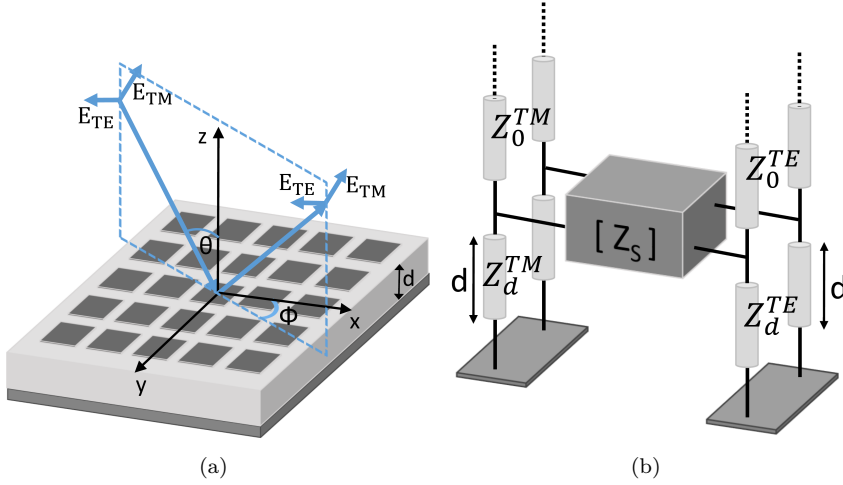


Figure 5.2: Two ports transmission line network modeling of periodic printed patches.

tensor $\underline{\underline{Z}}_S$ relates the fundamental (averaged) electric field to the jump discontinuity of the averaged magnetic field. The latter corresponds to the equivalent current flowing on the MTS sheet (see equation (2.6)). The MTS circuit elements in Fig. 5.2 are thus valid at a given radian frequency ω and for a fixed incident plane wave excitation $\mathbf{k}_i$. Furthermore, the MTS impedance interacts with the ground plane uniquely through the fundamental Floquet mode. Therefore, the distance between the MTS and the ground plane should be at least of the order of the unit cell size p so as to reasonably neglect the interaction between higher order Floquet modes with the ground plane. Reference [56] derives some simple rules of thumb for a proper choice of the unit cell size p and substrate thickness d to ensure that the homogenization assumption is valid.

Several methods have been developed in the literature to extract the impedance tensor from the simulation of the corresponding infinitely periodic structure [81], [66], [82], [92]. In this thesis, we used the method proposed in [92] because of its particularly rigorous formulation. Assuming that one has access to the MoM impedance matrix $\underline{\underline{Z}}_{MoM}$ obtained after meshing the unit cell size and applying a periodic boundary condition, the MTS tensor admittance $\underline{\underline{Y}}_S = \underline{\underline{Z}}_S^{-1}$ can be retrieved as

$$\underline{\underline{Y}}_S = \underline{\underline{Y}}_{FW} (\underline{\underline{Y}}_{GF} - \underline{\underline{Y}}_{FW})^{-1} \underline{\underline{Y}}_{GF} \quad (5.1)$$

where $\underline{\underline{Y}}_{GF}$ is the Green's function impedance and $\underline{\underline{Y}}_{FW}$ is obtained from the MoM impedance matrix as

$$\underline{\underline{Y}}_{FW} = \underline{\underline{Q}} \underline{\underline{Z}}_{MoM}^{-1} \underline{\underline{Q}}^H \quad (5.2)$$

H stands for the Hermitian transposed, and $\underline{\underline{Q}}$ is a projection matrix of the basis functions spectrum used in the MoM on the TE and TM fundamental Floquet modes fields. The reader is referred to [92] for more details. $\underline{\underline{Z}}_S$ has the following structure:

$$\begin{bmatrix} Z_{TM}(\omega, \mathbf{k}) & Z_{TMTE}(\omega, \mathbf{k}) \\ Z_{TETM}(\omega, \mathbf{k}) & Z_{TE}(\omega, \mathbf{k}) \end{bmatrix} \quad (5.3)$$

where $Z_{TM}(\omega, \mathbf{k})$ and $Z_{TE}(\omega, \mathbf{k})$ are respectively the TM and TE scalar impedance components. The non-diagonal elements describe the coupling between the TM and TE modes. In absence of losses, $\underline{\underline{Z}}_S$ must be anti-Hermitian, i.e $\underline{\underline{Z}}_S = -\underline{\underline{Z}}_S^H$, which in turn implies that the off-diagonal elements are related $Z_{TMTE} = -(Z_{TETM})^*$, with $*$ being the complex conjugate operator. Since the TM component is aligned with the wave direction, TM and TE stand respectively for the ρ and ϕ directions used in Chapter 3.

In modulated MTS antennas, the feeder excites a SW in the substrate, which means that the impedance tensor $\underline{\underline{Z}}_S(\omega, \mathbf{k})$ has to be computed at the supported SW mode k_{sw} . The latter can be obtained by enforcing the transverse resonance condition, or equivalently by nulling the determinant

of the MoM matrix $\underline{\underline{\mathbf{Z}}}_{MoM}$.

$$\det(\underline{\underline{\mathbf{Z}}}_{MoM}(\omega, \mathbf{k}_{sw})) = 0 \quad (5.4)$$

The relation $(\omega, \mathbf{k}_{sw})$ is the SW dispersion law. The isofrequency dispersion curve is defined as the locus described by $k_{sw}(\alpha)$ at a given frequency, where α is the azimuthal angle. More precisely, it is a 1D domain in the 2D (k_x, k_y) domain, along which a SW can exist. Hence, it provides a link between $k_{sw,x}$ and $k_{sw,y}$; or equivalently between k_{sw} and α . Since the determinant of the MoM matrix is exactly zero at resonance, the MTS impedance tensor cannot be computed exactly at k_{sw} because it requires the inversion of the MoM matrix (see equations (5.1) and (5.2)). However, in practice, the sheet impedance varies very slowly w.r.t k_{sw} . One can therefore compute the surface impedance for other unit cell phase shifts and interpolate the result to get the impedance at k_{sw} . The research of k_{sw} can be accelerated by interpolating the MoM admittance matrix. That is, the MoM admittance matrix is computed at some sampling points in the Brillouin zone (for k_x and k_y between $-2\pi/p$ and $2\pi/p$, where p is the period of the lattice), and the values at other incident angles are provided by simple interpolation. One can also use analytical model (pole-zero matching for example [93]) of the sheet impedance (calculated from a few unit cell phase shift samplings) to find k_{sw} by enforcing the transverse resonance condition [92].

The process can be summarized as follows:

- Assuming a given incident angle α , compute the supported SW mode by nulling the determinant of the MoM matrix.
- Based on the obtained k_{sw} , compute the sheet impedance in the vicinity using (5.1) and interpolate to get the SW impedance at resonance.

This process can be carried out for different patches geometries, which results in a database that gives the elements of the impedance tensor as functions of the patch geometrical parameters. Several patches geometries have been proposed in the literature to implement anisotropic impedances [95]. In this thesis, we used the coffee bean patch shape (represented in Fig. 5.3) proposed in [50]. This patch is parametrized through its radius b

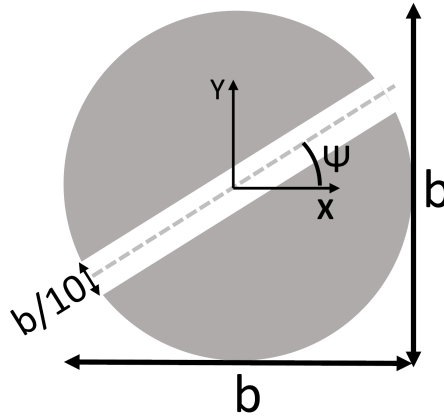


Figure 5.3: Coffee bean patch [50].

and rotation angle ψ .

Fig. 5.4 illustrates the supported SW mode k_{sw}/k_0 versus patch parameters (rotation angle ψ and patches diameter, normalized w.r.t the unit cell size p) for a x-directed excitation. Fig. 5.5 shows a map of the extracted

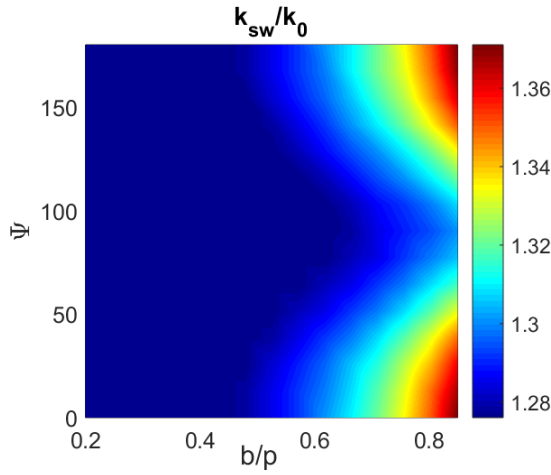


Figure 5.4: Surface-wave mode supported by coffee bean patches.

sheet reactance ($Z = jX$) tensor components (at 24 GHz) as a function

of the patches parameters. Note that the derived impedance components

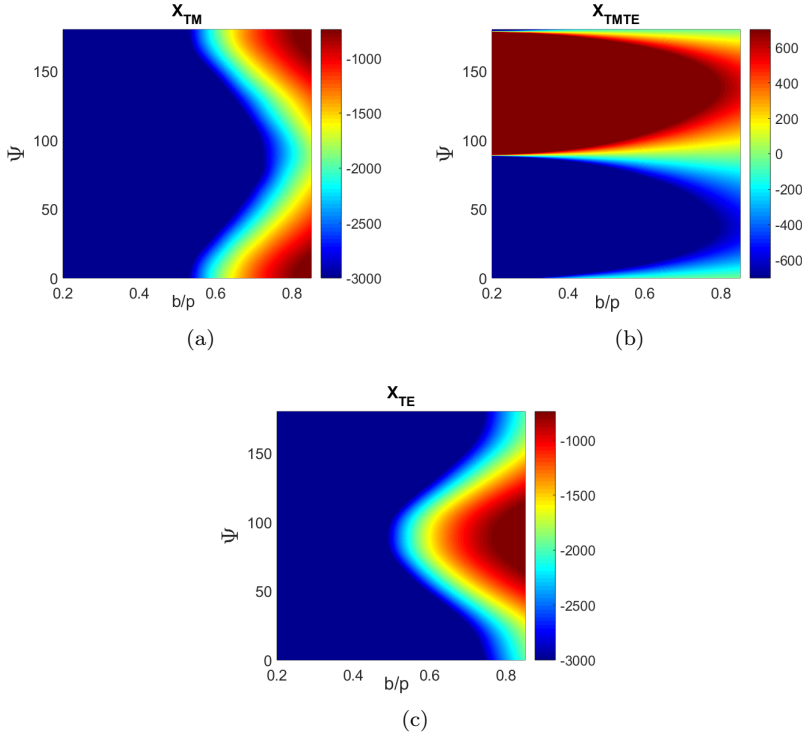


Figure 5.5: Reactance tensor database map. (a) X_{TM} . (b) X_{TMTE} . (c) X_{TE} .

are purely imaginary. Based on the impedance database, we have fixed the average reactance to $X_0 = -1300 \, \Omega$ and the IBC synthesis methods (described in Chapters 3 and 4) are carried out accordingly.

One can therefore proceed to the IBC implementation. First, the MTS aperture is discretized in a Cartesian lattice with unit cell size $p = \lambda/10$. In each cell, the required sheet impedance at the cell center is implemented by means of the pre-computed database shown Fig. 5.5. The required patch in each unit cell is obtained after minimizing the following cost function at the

cell center (x,y):

$$\min \sqrt{(X_S^{\rho\rho}(x,y) - X_{TM})^2 + (X_S^{\rho\phi}(x,y) - X_{TMTE})^2 + \beta (X_S^{\phi\phi}(x,y) - X_{TE})^2} \quad (5.5)$$

where X_{TM} , X_{TE} , and X_{TMTE} are the realizable reactances space in the pre-computed database. Since the coffee bean patches possess two axes of symmetry (then two degrees of freedom), and the class of desirable impedance profile exhibits three degrees of freedom, one cannot fully implement the IBC with the coffee bean patches. Therefore, the coefficient β in (5.5) is used to weight the importance of the TE component w.r.t the other components. Indeed, since the supported SW mode is quasi-TM, one can introduce more tolerance in the TE reactance implementation. We have used $\beta = 0.01$. At the end of the process, one gets a map with the position of each patch, as well as the corresponding geometrical parameters. To quantify the accuracy of the IBC implementation, we define the $\rho\rho$ relative error impedance as:

$$\epsilon^{\rho\rho}(x,y) = 100 \left| \frac{X_S^{TM}(x,y) - X_{realized}^{\rho\rho}(x,y)}{X_S^{TM}(x,y)} \right| \quad (5.6)$$

and the $\rho\phi$ impedance error as

$$\epsilon^{\rho\phi}(x,y) = |X_S^{TMTE}(x,y) - X_{realized}^{\rho\phi}(x,y)| \quad (5.7)$$

where $X_{realized}^{\rho\rho}(x,y)$, and $X_{realized}^{\rho\phi}(x,y)$ are the realized reactance components in the unit cell centered at (x,y). Note that the $\rho\phi$ impedance error is not normalized since the $\rho\phi$ reactance is modulated around a zero value.

5.2 Full-wave simulation of the physical MTS structure

Once, the actual MTS structure described with printed metallizations is obtained, one can simulate the corresponding radiation pattern to ensure that the implemented MTS radiates the required beam. The simulation of such devices is very challenging due to the resulting fine and dense mesh. Even

if commercial software (e.g CST [104]) has already been used in the literature [96], they require huge computational resources. Therefore, efficient numerical methods are required to address this analysis problem on a conventional computer. In [43] for example, the authors adopted an ADF-EMS MoM software combining two accelerations: the sparse matrix-adaptive integral method (SM-AIM) [97] and the synthetic functions expansion method (SFX) [58]. The fast multipole method (FMM) is used in [87] for the simulation of multibeam MTS antennas. In this thesis, we used a Krylov based iterative Contour-FFT (CFFT) acceleration recently developed in the lab by Dr. Simon Hubert [98]. Below, the fundamentals of the iterative CFFT are provided.

5.2.1 The Contour FFT

The CFFT [75] is a recent method that allows the fast and accurate computation of Fourier transforms of singular functions. Starting from the singular function $\tilde{H}$ in the spectral domain, its Fourier transform is expressed as

$$H(x, y) = \frac{1}{4\pi^2} \iint \tilde{H}(k_x, k_y) e^{-j(k_x x + k_y y)} dk_x dk_y \quad (5.8)$$

Due to rapid variation near the spectral poles of $\tilde{H}$, using a simple FFT to evaluate (5.8) will lead to large numerical errors. To smoothen the integrand, one can modify the integration contour to push the integration path further from the poles (see the used contour path in Fig. 5.6). As proven in [75], the Fourier transform expressed in terms of the real spectral variables is written

$$H(x, y) = \frac{1}{4\pi^2} \iint \tilde{H}(k_x^c, k_y^c) e^{-j(k_x^c x + k_y^c y)} e^{(k_x^i x + k_y^i y)} C(k_x, k_y) dk_x dk_y \quad (5.9)$$

with k_x, k_y and k_x^i, k_y^i the real and imaginary parts of the complex wavenumbers k_x^c, k_y^c , respectively, and C a function that depends on the chosen contour. Note that the real exponential term in (5.9) contains both spectral and spatial variables; this expression is thus not strictly speaking a Fourier transform, such that it cannot be evaluated with an FFT. To preserve the

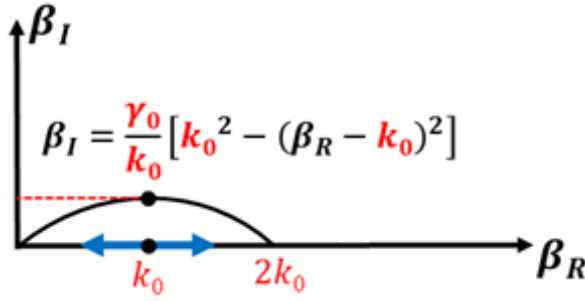


Figure 5.6: Quadratic integration contour [75]. Note that the imaginary part is expressed as a function of the real part.

advantageous complexity of the FFT, that exponential term is expanded into a two-dimensional Taylor series of (x, y) around $(0, 0)$. By rearranging the terms the following expression is obtained

$$\begin{aligned}
 H(x, y) &\approx \left[\text{FFT} \left\{ \tilde{H}' \right\} \right. \\
 &\quad + x \text{FFT} \left\{ \gamma k_x \tilde{H}' \right\} + y \text{FFT} \left\{ \gamma k_y \tilde{H}' \right\} + xy \text{FFT} \left\{ \gamma^2 k_x k_y \tilde{H}' \right\} \\
 &\quad + \frac{x^2}{2} \text{FFT} \left\{ \gamma^2 k_x^2 \tilde{H}' \right\} + \frac{y^2}{2} \text{FFT} \left\{ \gamma^2 k_y^2 \tilde{H}' \right\} \left. \right] \\
 &\triangleq \text{CFFT} \left\{ \tilde{H}' \right\}
 \end{aligned} \tag{5.10}$$

which is the definition of the CFFT operator. γ is a parameter defining the contour height (see Fig. 5.6). The formulation (5.10) is a rapidly converging series of FFTs. Thus it combines the attractive $N \log N$ complexity inherent to the FFT, together with a robust treatment of singularities, thanks to the integration contour deformation. An inverse CFFT operator has also been defined in a similar way, which evaluates Fourier transforms in complex spectral coordinates [77].

5.2.2 The iterative Contour FFT method

The solution of the MoM system of equations

$$\mathbf{Z}\mathbf{x} = -\mathbf{b} \tag{5.11}$$

can be approximated as a linear combination of the columns of

$$\mathbf{Q}_n = \begin{bmatrix} \mathbf{b} & \mathbf{Z}\mathbf{b} & \mathbf{Z}^2\mathbf{b} & \dots & \mathbf{Z}^{n-1}\mathbf{b} \end{bmatrix} \quad (5.12)$$

spanning the Krylov subspace, and which is iteratively constructible by successive products with the interaction matrix $\mathbf{Z}$. Candidate solutions for x are then computed iteratively via the reduced matrix $\mathbf{Q}_n$ using the GMRES procedure described in [99]. The matrix-vector products appearing in (5.12) constitute the complexity bottleneck of such iterative methods. Below, we summarize the building blocks of the iterative Contour-FFT [98].

First the rooftop or Rao-Wilton-Glisson (RWG) [100] basis functions are distributed using a Lagrange interpolation scheme, on a Cartesian grid of points that covers the whole domain of the problem. At each iteration, the current distribution is thus distributed on the grid and its convolution with the Green's function is computed in the spectral domain using the CFFT and its inverse. The key point of the Contour-FFT method is that the spectral Green's function $\tilde{\mathbf{G}}$ used in the convolution is analytical for any layered media. The fields obtained in that way are tested on each testing function in the spatial domain. These operations can be summarized as [77]

$$[\mathbf{Z}\mathbf{b}]_i \approx \iint \hat{\mathbf{f}}_i \cdot \text{CFFT} \left\{ \tilde{\mathbf{G}} \text{CFFT}^{-1} \left\{ \sum_{j=0}^N b_j \hat{\mathbf{f}}_j \right\} \right\} d\mathbf{r} \quad (5.13)$$

with $\hat{\mathbf{f}}_i$ the i -th basis function distributed on the grid and b_j the j -th entry of the excitation vector $\mathbf{b}$. Finally, a correction operation is carried out to compensate for inaccurate short-range interactions, using a pre-computed sparse operator [101].

The resulting method is quite accurate and scales well with the number of unknowns [98]. Also, error models are readily available to determine the size of the grid, the shape of the integration contour and the order of the Taylor expansion, which provides a convenient way to control the error and the complexity of the overall method [102], [103].

5.3 Numerical and experimental validations

This section presents three designs. The first one is a MTS radiating a broadside pencil beam. The second design is a dual-polarized MTS antenna radiating a squinted pencil beam. Those designs have been implemented with coffee-bean patches and numerically validated with the MoM code described in Section 5.2. Finally, the third design is a realized MTS radiating a cosecant squared beam for traffic radar.

5.3.1 Broadside pencil beam MTS antenna

The first example is a MTS radiating a circularly polarized broadside pencil beam. A MTS antenna radiating such beam has already been designed in [43]. Here, we used the method developed in Chapter 3 for the synthesis. The antenna radius is 3λ , with λ being the free-space wavelength. The designed IBC is depicted in Fig. 5.7. Note that we actually got this well

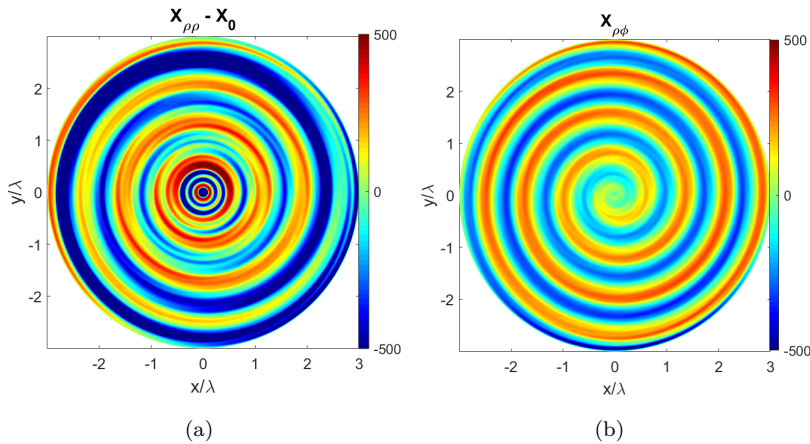


Figure 5.7: Impedance profile for a MTS radiating a circularly polarized broadside pencil beam. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

known spiral shape consistent with the circular polarization. The IBC is implemented with printed patches and the physical structure of the MTS has been meshed with 154868 RWG basis functions and analyzed with the

MoM code [98] briefly described in Section 5.2. Fig. 5.8 shows the error

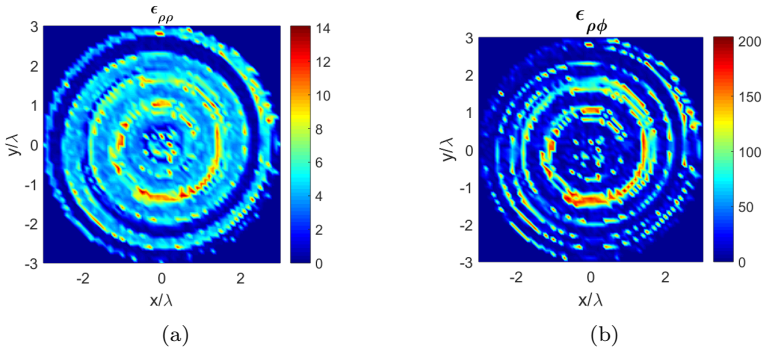


Figure 5.8: Distribution of the error made during the IBC implementation of a MTS radiating a circularly polarized broadside pencil beam. (a) $\epsilon_{\rho\rho}$. (b) $\epsilon_{\rho\phi}$.

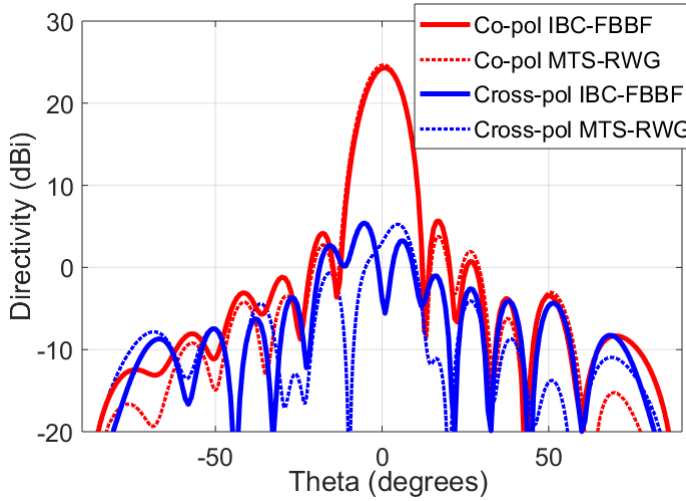


Figure 5.9: Radiation pattern in the plane $\phi = 0$ for a MTS radiating a broadside pencil beam. The IBC simulation is compared to the physical MTS simulation.

distribution made during the IBC implementation process. Fig. 5.9 repre-

sents the simulated radiation pattern in the plane $\phi = 0$. One can observe a very good correspondence between the IBC simulation and the physical MTS simulation.

5.3.2 Dual-polarized squinted pencil beam MTS antenna

Dual polarization is an appealing functionality in many applications since it can be used to increase the channel capacity through multiplexing or to improve the channel reliability through diversity. Papers [121], [122] propose for the first time a dual-polarized broadside beam MTS antenna. The method developed consists of guiding simultaneously two SW modes (TM and TE) in the MTS. The IBC is selected to provide a phase matching between the two modes while the feeding design guarantees the power balance between the modes. A similar idea, but based on a two-layer scalar MTS is also reported in [123]. We demonstrate below that dual-polarization can be achieved with a very good aperture efficiency using two TM feeds. The usage of TM feeders provides two advantages in comparison with a (TM-TE) feeder. First, it makes the antenna feeder simpler since a simple coax terminated by a circular patch can be used (see Fig. 5.10). Second, the IBC implementation is much easier, and the resulting IBC implementation error is lower. Finally, the proposed method (Chapter 4) can be used for a general class of radiation patterns.

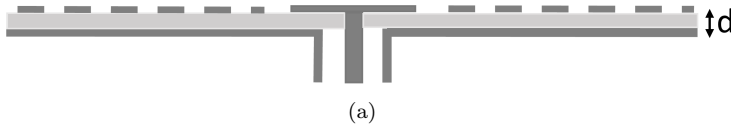


Figure 5.10: Feeder illustration.

We designed a MTS fed in two points ($x = 0$, $y = 1.5\lambda$) and ($x = 0$, $y = -1.5\lambda$). Exciting the first feed, one should get a RHCP pencil beam at ($\theta = 10^\circ$, $\phi = 0$). When the second TM feed is excited, the desired radiation pattern is the same than the first one, but with opposite polarization (LHCP). The MTS radius is fixed at 3.5λ . Fig. 5.11 and Fig. 5.12 show

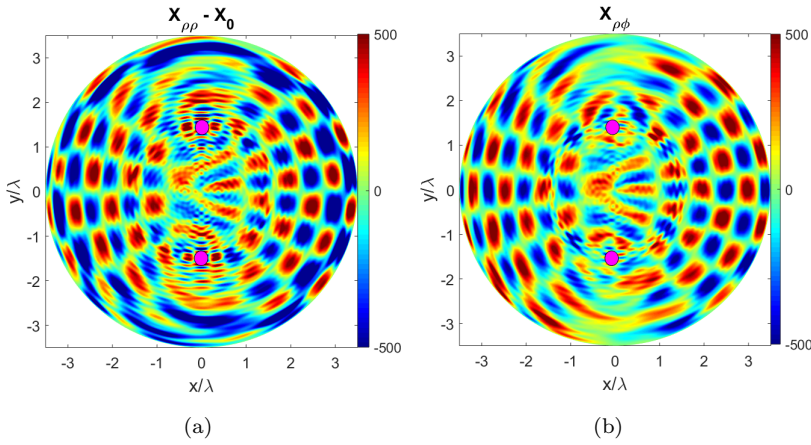


Figure 5.11: Impedance profile for a MTS radiating a dual-polarized squinted pencil beam. Dots represent the position of the point sources. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

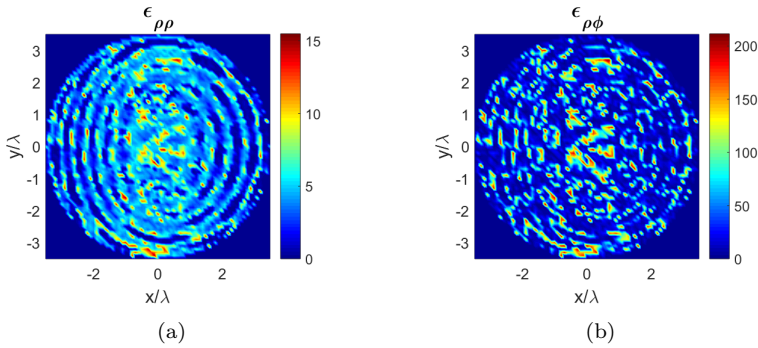
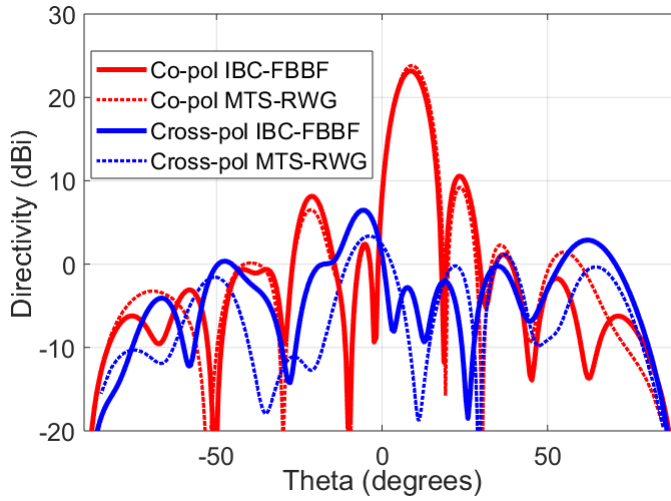
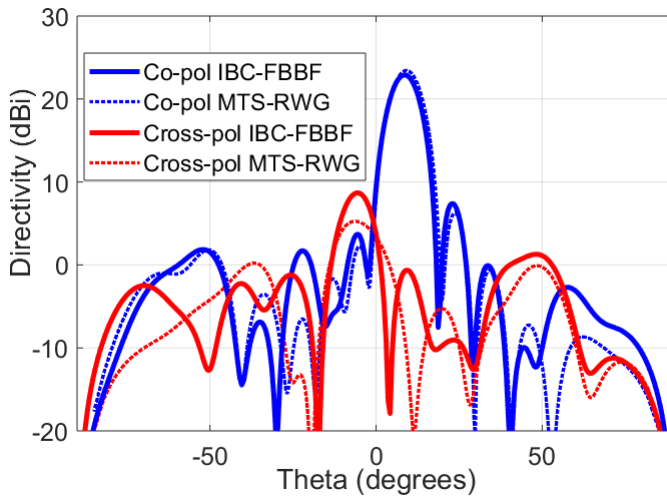


Figure 5.12: Distribution of the error made during the IBC implementation for a MTS radiating a dual-polarized squinted pencil beam. (a) $\epsilon_{\rho\rho}$. (b) $\epsilon_{\rho\phi}$.

the obtained impedance profile as well as the IBC implementation error. As can be seen, the implementation error is relatively low (in average less than 10%). Fig. 5.13 illustrates the radiation pattern in the plane $\phi = 0$. A good comparison is observed between the FBBF and the RWG simulations.



(a)



(b)

Figure 5.13: Radiation pattern in the plane $\phi = 0$ for a MTS radiating a dual-polarized squinted pencil beam. The IBC simulation is compared with the physical MTS simulation. (a) First feed. (b) Second feed.

5.3.3 Cosecant squared beam MTS antenna

This section presents validations of two MTS antennas radiating a circularly polarized cosecant squared beam (see (3.34)) at 24 GHz. The first one has a 3λ radius and has been numerical validated with the MoM RWG code [77]. The second has a 10λ radius and has been realized and experimentally validated.

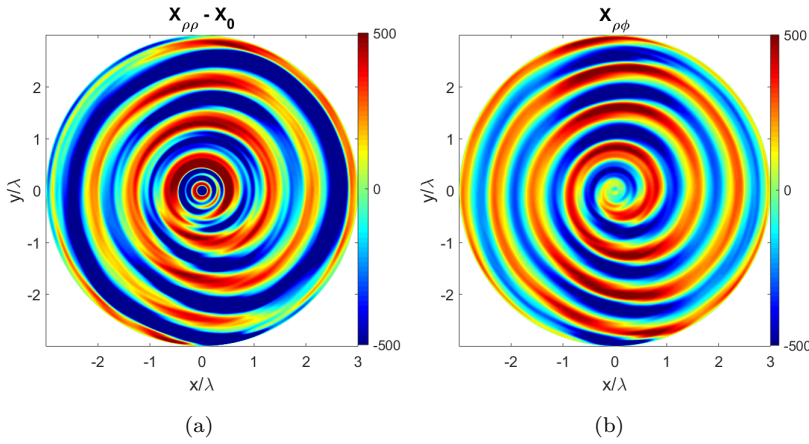


Figure 5.14: Impedance profile for a MTS of radius 3λ radiating a cosecant squared beam. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

Fig. 5.14 shows the reactance profile for the smaller cosecant squared beam MTS antennas. Fig. 5.15 provides a comparison between the IBC-FBBF simulation and that of the implemented MTS structure, meshed with 154868 RWG basis functions. An excellent agreement is obtained.

Now a larger example (10λ radius) has been physically realized (see Fig. 5.18). The designed impedance profile is represented in Fig. 5.16 and the IBC implementation error is shown in Fig. 5.17. Due to the size of the antenna, the measurements cannot be carried out in the far-field in our anechoic chamber. Therefore, the TM and TE components of the fields are measured in the near field (at 1.5 meter distance from the antenna) and compared with the numerically computed (expected) near-field, based on

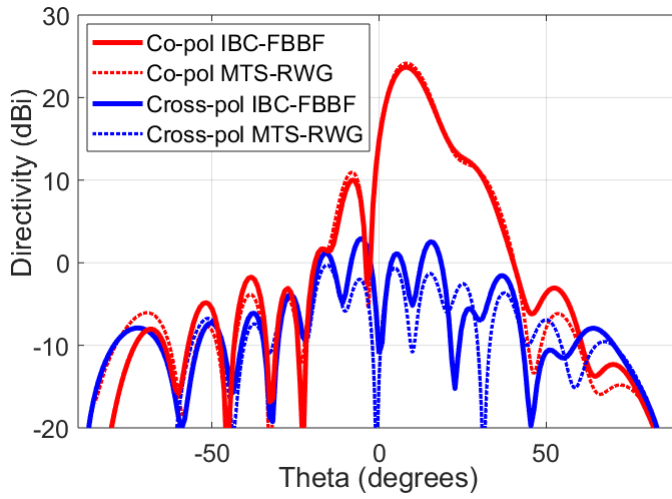


Figure 5.15: Radiation pattern in the plane $\phi = 0$ for a MTS radiating a cosecant squared beam. The IBC simulation is compared with the physical MTS simulation.

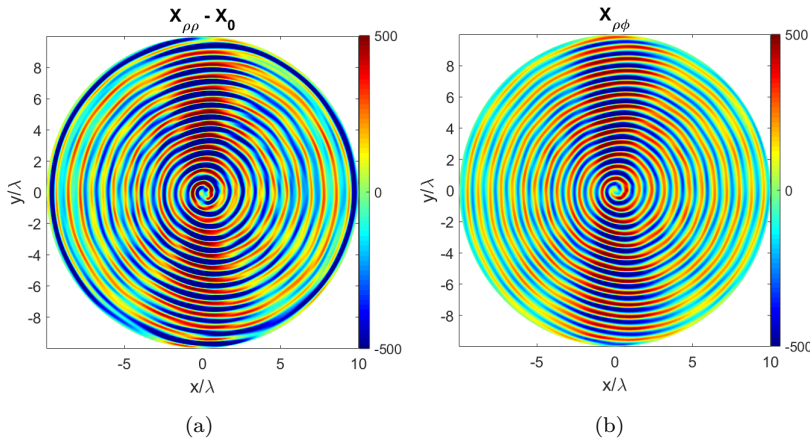


Figure 5.16: Impedance profile for a MTS of radius 10λ radiating a cosecant squared beam. (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

the IBC simulation. Fig. 5.19 shows the comparison between measurements and simulation for the TE and TM components.

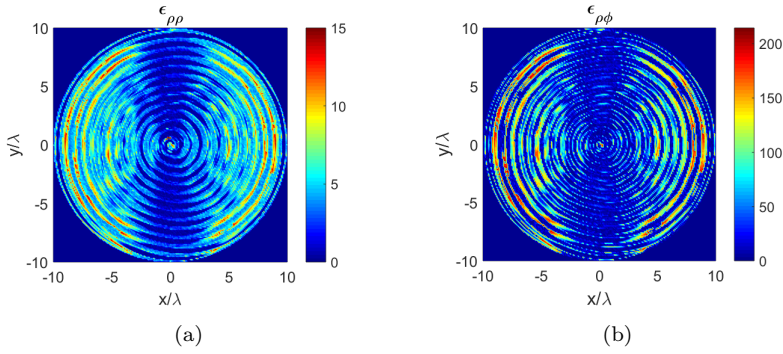


Figure 5.17: Distribution of the error made during the IBC implementation for a MTS radiating a cosecant squared beam. (a) $\epsilon_{\rho\rho}$. (b) $\epsilon_{\rho\phi}$.

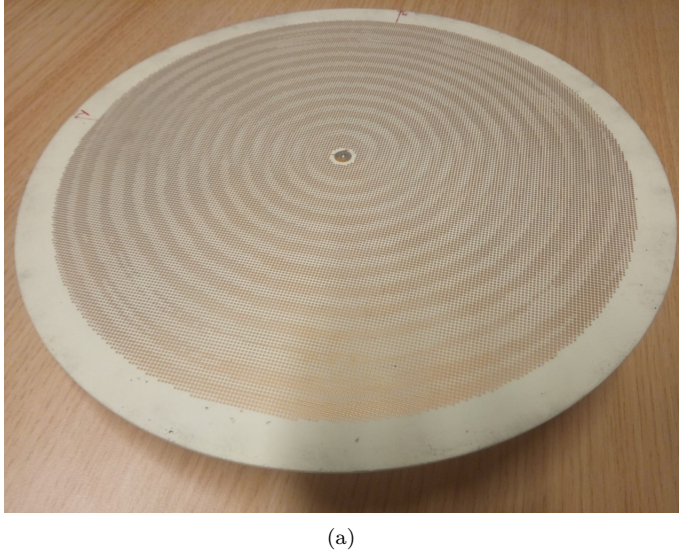
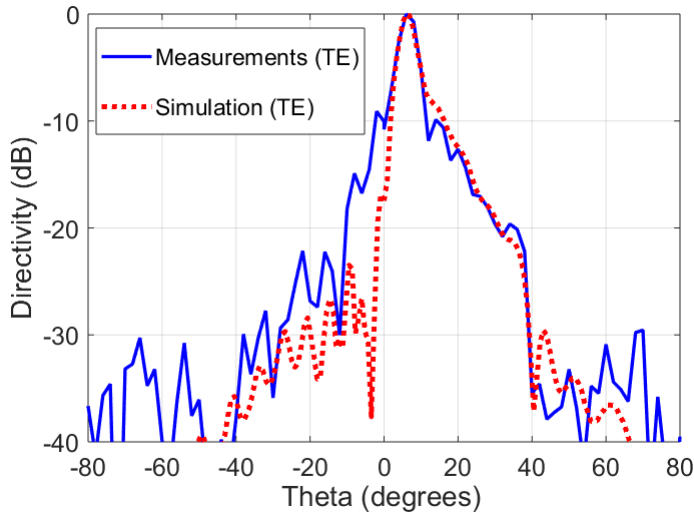


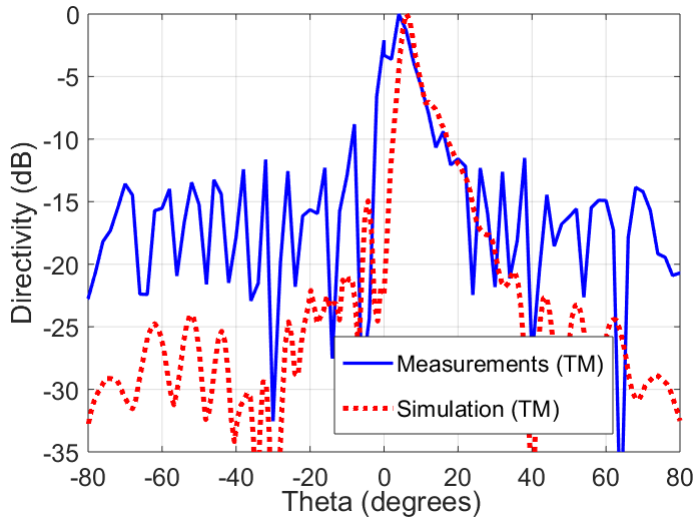
Figure 5.18: Cosecant squared beam MTS antenna realization.

Discussion on the experimental results

One can observe a very good agreement for the TE component. However, the sidelobe level of the TM fields is much larger than expected. This difference can arise from two sources: diffraction at the MTS rim or the limited feeder



(a)



(b)

Figure 5.19: Radiation pattern in the plane $\phi = 0$ for a MTS radiating a cosecant squared beam. The IBC simulation is compared with measurements. (a) TE component. (b) TM component.

efficiency (ratio between the power delivered to the SW and the total input power).

We have noticed that varying the height of the feeder may have a significant impact on the TM radiation pattern. The measured sidelobes of the TM component are similar for different azimuthal angles, which indicates that those sidelobes are impacted by the symmetrical feeder. In modulated MTS antenna design, it is assumed that the feeder maximizes its power into the SW. In practice, part of this power is directly radiated into space without going through the MTS, then affecting the radiation pattern. The feeder should be optimized to minimize the fraction of the power directly radiated into space.

However it has also been observed that the conversion efficiency (see Chapter 7) of this antenna is relatively low (about 20%), which means that the sidelobes are probably dominated by the diffraction effects at the MTS rim. Those issues (feeder efficiency and diffraction effects) can be solved by increasing the average opaque impedance, the required radiated power or by designing directive SW launchers.

5.4 Conclusion

A detailed study of the IBC implementation with printed patches has been provided. It has been shown that the synthesized class of impedance profile can be accurately implemented with printed patches with an average error less below 10%. Excellent comparisons have been obtained between the IBC simulation and the MoM simulation of the implemented MTS for shaped beams and dual-polarized antennas, thus providing further validations of the analysis and synthesis algorithms developed in the previous chapters. However, a MTS realization raised some interrogations about the antenna efficiency. The latter will be treated in Chapter 7. Finally, the feeding optimization issue should be carefully addressed to minimize the undesired space wave directly radiated by the feed.

CHAPTER 6

Near-field shaping using metasurfaces

PREVIOUS chapters focused on metasurface (MTS) antennas for far-field applications. Here, the objective specifically concerns the near-field.

The chapter is organized in two parts. The first part discusses the synthesis of Leaky-wave (LW) radiating MTSs for near-field applications. It has been shown that the near-field of the MTS can be controlled with a very good accuracy, down to a distance of about one wavelength from the surface. Several examples have been successfully designed, which cover Bessel beam radiators for focusing applications and multiple Orbital Angular Momentum (OAM) beams. The second part is devoted to the analysis of the transmittance between OAM beams supporting Fourier-Bessel aperture fields.

6.1 Near-field synthesis

Modern imaging, sensing and communication systems require an accurate shaping of the near-field radiation. Possible applications concern the gener-

ation of OAM beams [105] - [109], non-diffractive Bessel beams for near-field focusing [110] - [112] or Magnetic Resonance Imaging (MRI) systems [113]. Accurately shaping the near-field is still a big challenge nowadays and sometimes requires complex/expensive or specific systems, depending on the required near-field effect. On the other hand, MTSs emerge as a very promising technology given their unique property of controlling waves at sub-wavelength level. The previous chapters demonstrated their effectiveness for aperture antennas through proper surface impedance modulation. As proven in [46] and [51], a sinusoidal or locally sinusoidal modulation of a capacitive-sheet IBC leads to the appearance of several harmonics of the fundamental SW. In antenna applications, one may design the IBC in order to radiate only one of those harmonics. However, in the case of near-field shaping, other harmonics can play an important role if they are very close to the visible region. In the previous chapters, a formalism based on the EFIE has been proposed to design circular domain (or possibly elliptical domain) MTS antennas. In this framework, the IBC modulation is no longer necessarily sinusoidal, but it is simply described in the Fourier-Bessel basis.

Our interest in the present chapter consists of proving that the EFIE formalism can be used to accurately control the near-field of the MTS down to distances very close to the surface (about one wavelength). In particular, we focus our attention on the generation of non-diffractive Bessel beams, and OAM beams. The latter has allowed multiplying the communication channels in near-field conditions.

6.1.1 Synthesis

We assume that the maximum spectral component k_{max} of the aperture electric field $\mathbf{E}(x, y)$ is lower than k_0 , i.e $\tilde{\mathbf{E}}(k_x, k_y) = 0$ for $k_\rho > k_0$ with k_0 being the free-space wavenumber. The difference with the antenna synthesis problem, lies in the way the current $\mathbf{J}$ is found. Indeed, since the desired aperture field is known in spatial domain, one can directly project it into FBBFs and add the contribution of the SW, rather than starting from the radiation pattern and going through a spectral domain as done in Chapters 3 and 4.

6.1.2 Results

The antenna is excited at the center ($\rho = 0$) by a vertical point source, as in Chapter 2. The operating frequency is 20 GHz. The substrate has a relative permittivity of $\epsilon_r = 3.66$ and a thickness $d = 1.52$ mm. The antenna radius is $a = 10\lambda$ and the average reactance is $X_0 = -383 \Omega$.

Non diffractive bessel beams

This section discusses the synthesis of a propagating Bessel beam launcher. Such beams have already been designed in [44] with MTS using a Geometrical Optics (GO) approximation. The desired radiating aperture field is [111]:

$$\begin{aligned} E_\phi &= 0 \\ E_\rho &= j \frac{k_z}{k_\rho} J_1(k_\rho \rho) \end{aligned}$$

where $k_\rho^2 + k_z^2 = k_0^2$, and k_0 is the free-space wavenumber. $J_n(x)$ is the first kind Bessel function of order n . As shown in [111], such a tangential aperture electric field is consistent with a z -directed electric field

$$E_z = J_0(k_\rho \rho) e^{-jk_z z} \quad (6.1)$$

This means that the electric field profile does not change/diffract with respect to z . However, one needs an infinite aperture size to generate (6.1). The truncation of the radiating aperture leads to the definition of a non-diffractive range along z [44]:

$$z_{max} = a \sqrt{\left(\frac{k_0}{k_\rho}\right)^2 - 1} \quad (6.2)$$

where a is the antenna radius. We choose $k_\rho = 0.51k_0$, which leads to $z_{max} \approx 16.8 \lambda$. Once the MTS IBC computed, the z -directed electric field is calculated as follows. First, the MoM algorithm presented in Chapter 2 is used to compute the surface current $\mathbf{J}$. Then the tangential aperture electric field is computed as $\tilde{\mathbf{E}} = \tilde{\mathbf{G}}^{EJ} \tilde{\mathbf{J}} + \tilde{\mathbf{E}}_i$, where $\tilde{\mathbf{G}}^{EJ}$ and $\tilde{\mathbf{E}}_i$ correspond respectively to the spectral Green's function of the substrate and

the spectral electric field of the excitation on the MTS. In spectral domain, the z-component of the electric field is related to the x and y components through $\tilde{E}_x k_x + \tilde{E}_y k_y = -\tilde{E}_z k_z$. From there, one can compute the spatial field E_z through a simple Fourier transform:

$$E_z = -\frac{1}{4\pi^2} \iint \frac{e^{-j(k_x x + k_y y + k_z z)}}{k_z} (\tilde{E}_x k_x + \tilde{E}_y k_y) dk_x dk_y \quad (6.3)$$

Since the fields are TM polarized, ($\tilde{E}_x(k_x, k_y) = \tilde{E}_0(k_\rho) \cos \alpha$, $\tilde{E}_y(k_x, k_y) = \tilde{E}_0(k_\rho) \sin \alpha$), where k_ρ and α are the spectral variables in cylindrical coordinates. After inserting the latter expression in (6.3) and analytically integrating along α , the 2D integral (6.3) reduces to a 1D integral

$$E_z = -\frac{1}{2\pi} \int_0^\infty \frac{J_0(k_\rho \rho)}{k_z} e^{-jk_z z} \tilde{E}_0(k_\rho) k_\rho^2 dk_\rho \quad (6.4)$$

Integral (6.4) can be evaluated over a contour in the complex plane to avoid the poles appearing at $k_z = 0$ and those from the substrate Green's function. Note that the integrand rapidly converges to zero for non-negligible heights z due to the presence of the exponential factor. Fig. 6.1 illustrates a map of E_z in the plane $\phi = 0^\circ$ corresponding to the desired Bessel beam. Fig. 6.2 shows the designed IBC modulation. It can be seen that the $\rho\phi$ component of the IBC is almost zero, which is consistent with the purely TM nature of the Bessel Beam. Fig. 6.3(a) shows a map of E_z in the plane $\phi = 0^\circ$, generated by the designed MTS. One can observe the focusing effect over the whole non-diffractive region ($z \leq 16.8\lambda$). Fig. 6.3(b) compares cuts (corresponding to the axis ($z = 1\lambda$, $\phi = 0^\circ$) and ($z = 16\lambda$, $\phi = 0^\circ$)) with the one obtained from equation (6.1), which assumes an infinite aperture size. One can observe a good agreement on the main lobe and first side lobes at $z = 1\lambda$. Differences are mainly due to the finite size of the structure and the direct space wave radiation of the feed. The latter issue could be solved to a large extent by designing a specific SW launcher. Closely to the non-diffractive range, namely at $z = 16\lambda$, the differences are more important, specially on the side lobes, due to the diffraction effects.

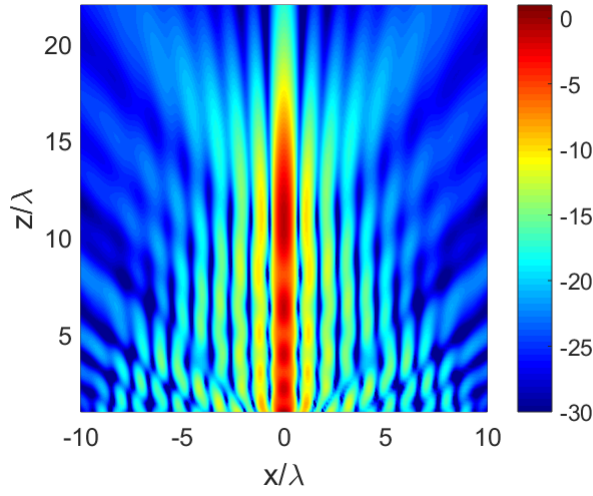


Figure 6.1: Normalized electric field z-component (in dB) of the desired Bessel beam.

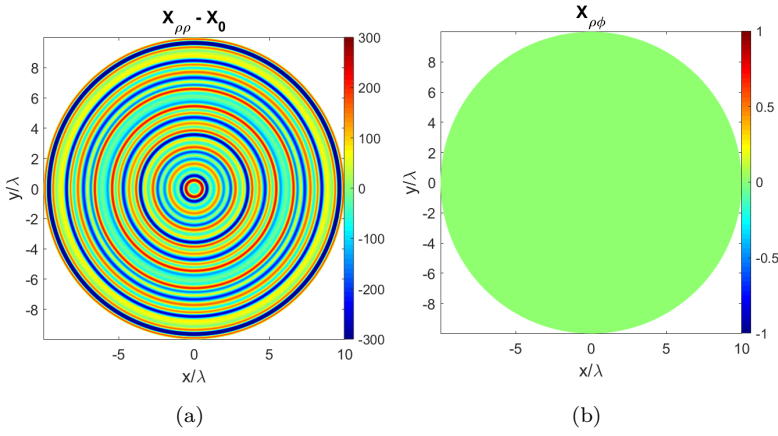


Figure 6.2: IBC Modulation for the synthesized Bessel beam radiating MTS (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

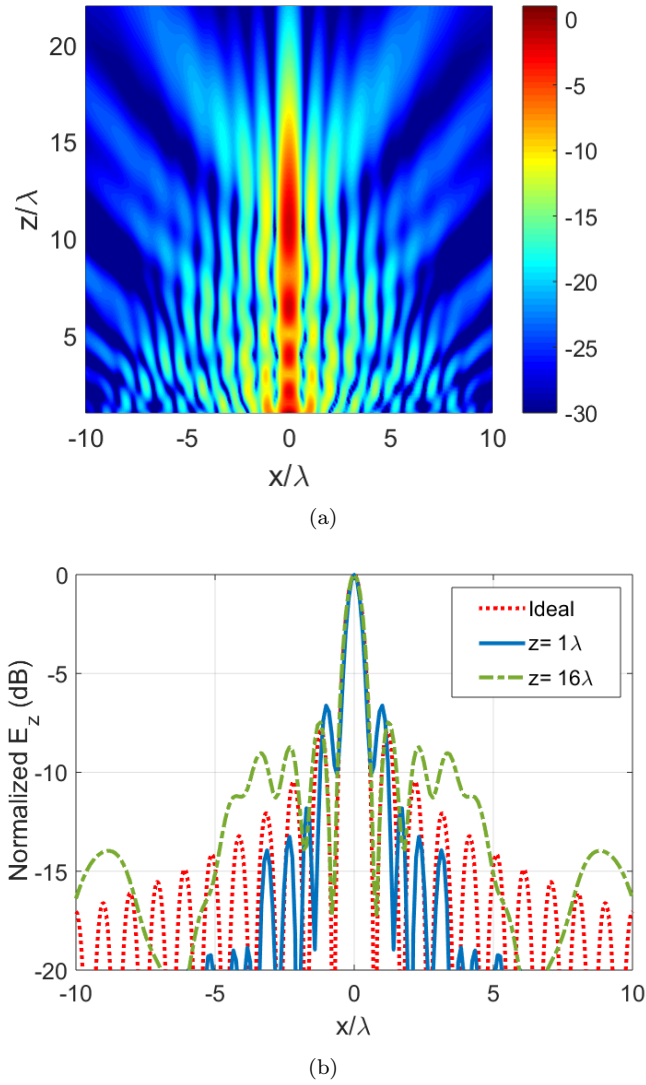


Figure 6.3: Normalized electric field z-component (in dB) of the designed MTS Bessel beam radiator (a) Map in the plane $\phi = 0^\circ$. (b) Comparison of a cut in the axis ($z = 1\lambda$, $\phi = 0^\circ$) and ($z = 16\lambda$, $\phi = 0^\circ$), with the fields (6.1).

Multi-OAM beams

The second example concerns a MTS antenna radiating two circularly polarized OAM beams. Multiplexing with OAM beams is essentially limited to near-field links [107]. The desired radiated aperture field is given as

$$\begin{aligned} E_x &= J_1 \left(\lambda_1^1 \frac{\rho}{a} \right) e^{-j\phi} + 10 J_1 \left(\lambda_1^{10} \frac{\rho}{a} \right) e^{-j\phi} \\ E_y &= jE_x \end{aligned}$$

where λ_n^m is the m-th positive zero of the the first kind Bessel function of order n. Those beams correspond to the Fourier-Bessel Basis functions as defined in (2.3) and are therefore orthogonal on the disk. Fig. 6.4 shows

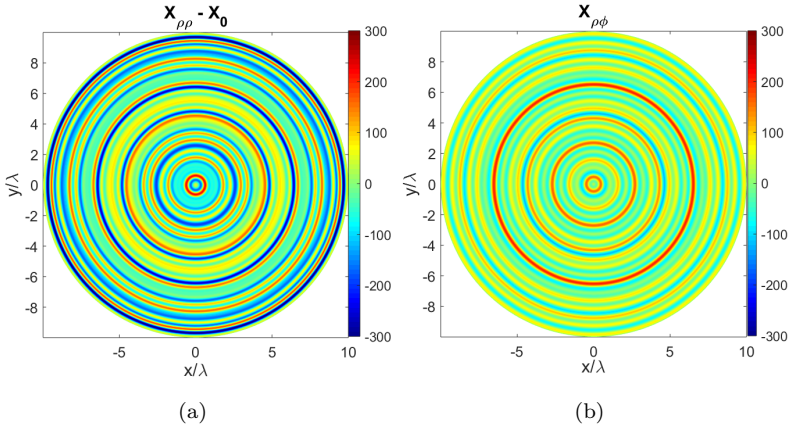


Figure 6.4: IBC Modulation for the synthesized multi-OAM beams radiating MTS (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

the derived modulation required to obtain the multi-OAM beams. Fig. 6.5 (a), (b) illustrates the obtained radiation pattern in the (u, v) plane and Fig. 6.5(c) compares a cut in the plane $\phi = 0^\circ$ with that of an ideal antenna of the same radius, radiating the same beam. This ideal antenna radiation pattern has been computed after discretizing the desired field with Fourier-Bessel basis functions defined over the whole circular domain. One can observe an excellent agreement between the designed MTS radiation pattern

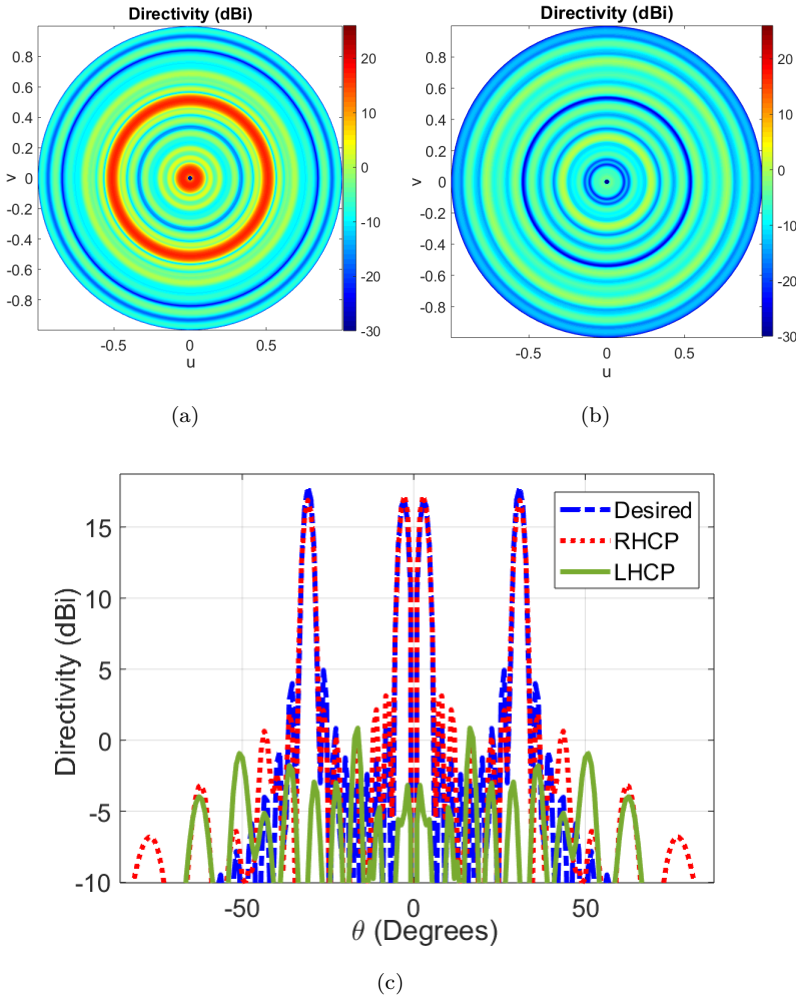


Figure 6.5: Radiation pattern of the designed circularly polarized multi-OAM beams MTS (a) RHCP component in the (u, v) plane. (b) LHCP component in the (u, v) plane. (c) Directivity cut in the plane $\phi = 0^\circ$. $(u, v) = (\sin \theta \cos \phi, \sin \theta \sin \phi)$, where θ and ϕ respectively correspond to the elevation and azimuthal angles.

and the desired one. In the next part of the chapter, the transmittance between antennas radiating such OAM beams will be rigorously analyzed

and a closed form expression of the transmittance will be provided.

6.2 Transmittance between Fourier-Bessel aperture fields

This section first provides an intuitive description of the spectral representations for the transmittance between two apertures. Then, two spectral representations of the transmittance between Orthogonal Angular Momentum antennas are compared. The first one makes use of the Modified Zernike functions and has already been described in a previous publication [108]. The second one is based on the Fourier-Bessel decomposition of the aperture fields. Both families of expansion functions are orthogonal on a disk and admit a closed form spectrum. In both cases, the transmittance in the far-field can be approximated in closed form if the fields are expressed in the chosen basis.

Orbital Angular Momentum (OAM) communication [114] has been presented a few years ago as a possible new way to multiplex signals within a given frequency band. In a nutshell, each channel corresponds to a specific phase roll by a multiple ℓ of 2π versus azimuth. If transmitting and receiving antennas can support several modes, independent signals can be transmitted within the same frequency band, as is already exploited for several decades in optical fiber communication [115]. A first demonstration in free-space RF communications has been described in [109]. It is interesting to notice that the generation and, above all, reception of such modes requires an aperture with substantial size. Moreover, higher order modes decay faster with distance, which limits the process to the near-to-intermediate field [109], [107].

The space-spectrum domain provides a relatively synthetic way to look at this problem. In [108], a general formula has been provided for the transmittance H between two apertures in the form of a mutual impedance. It is generalized below to directly include the vectorial aspect of fields, through the use of the dyadic Green's function $\underline{\underline{\mathbf{G}}}(\mathbf{r}, \mathbf{r}')$ for free-space [116, eq. (7.2)]:

$$H = \frac{-j4k_0\eta}{I_r I_t} \iint_{S_r} \mathbf{J}_r(\mathbf{r}) \iint_{S_t} \mathbf{J}_t(\mathbf{r}') \underline{\underline{\mathbf{G}}}(\mathbf{r}, \mathbf{r}') dS_t dS_r \quad (6.5)$$

where $\mathbf{J}_t$ and $\mathbf{J}_r$ are respectively the equivalent vector current distributions

on the transmitting and receiving antennas, assuming that both antennas are active. I_t and I_r are respectively the current flowing at the transmitting antenna port, η is the free-space impedance and k_0 is the free-space wavenumber. The link between the transmittance, and the power transfer is proven in Appendix A.8.

Equation (6.5) can be interpreted by discretizing the integrals over transmitting and receiving apertures, such that the current distributions can be replaced with virtual antenna arrays, characterized by amplitude distributions $\mathbf{J}_t$ and $\mathbf{J}_r$. If the arrays are placed very far from each other, at the level of the receiving array, the fields from the transmitting array strongly resemble a plane wave, which is the only communication channel between the two apertures. When the curvature of the phase front at the level of the receive-

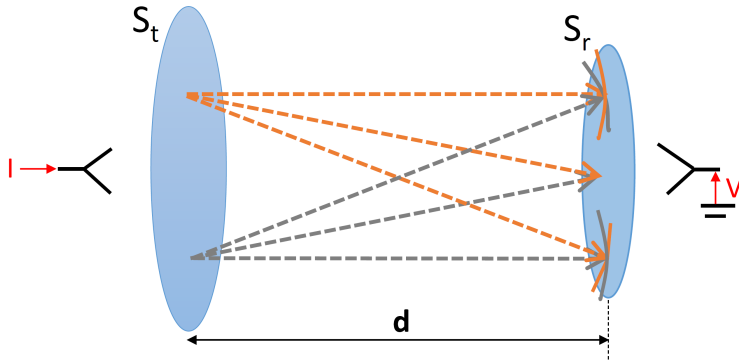


Figure 6.6: Illustration of the different channels created between two apertures

ing antenna becomes non-negligible, this single-channel model is no longer valid. Indeed, the curved phase fronts generated by different transmitting antennas appear as significantly different at the receiving array level (dashed in Fig. 6.6). Hence, after proper beamforming, represented by distribution $\mathbf{J}_r$, signals from different transmitting antennas can be distinguished. This is also true when individual transmitting antennas are replaced by the whole array, with different distributions $\mathbf{J}_t$. Proper sets of distributions $\mathbf{J}_t$ and $\mathbf{J}_r$ create several independent channels (see Fig. 6.6) and this function is also

exploited in free-space MIMO communications [117], [118], [107]. The number of independent channels increases as the two arrays (or apertures) are approached.

It is convenient to provide a spectral-domain description of the field distributions, i.e. along spatial spectral coordinates (k_x, k_y) . Coming back to continuous field distributions, as the apertures are approached, a wider set of samples in (k_x, k_y) plane can be exploited to create independent channels. Each sample corresponds to a different harmonic in space domain. A change of basis can now be operated: instead of decomposing the fields in harmonics versus x and y coordinates, one can decompose them into a family of functions forming a complete set over the disk. If, besides, those functions are orthogonal to each other, the coefficients multiplying each element of the basis can be simply obtained by projection. Two such families of functions exist. The first one corresponds to the Zernike functions [68] and has already been exploited in [108] for the study of the transmittance between OAM antennas. The second family of such functions is the Fourier-Bessel family used in the previous chapters. As will be seen later, although the derivation is similar, the convergence of the transmittance for both families of functions – versus number of modes and versus distance – have fundamentally different behaviors. Depending on the application one may thus prefer one or the other.

6.2.1 Mathematical formulation

Let us consider two circular apertures (transmitting and receiving antennas) as represented in Fig. 6.7. The current distribution on each aperture is assumed to be expanded in Cartesian x and y orthogonal components as in equation 2.5. The expansion coefficients can be obtained by simple projection of the aperture current distribution. Let us now denote by $\gamma_{t_{nm}}$, and $\gamma_{r_{nm}}$ the expansion coefficient of a (x or y directed) equivalent aperture current respectively on the transmitting and receiving antennas, corresponding to the Fourier-Basis basis function $R_{m,n}$. The co-polar transmittance between the two OAM antennas, assuming the same azimuthal behavior (n index) of the current distribution can be decomposed in all pairs of trans-

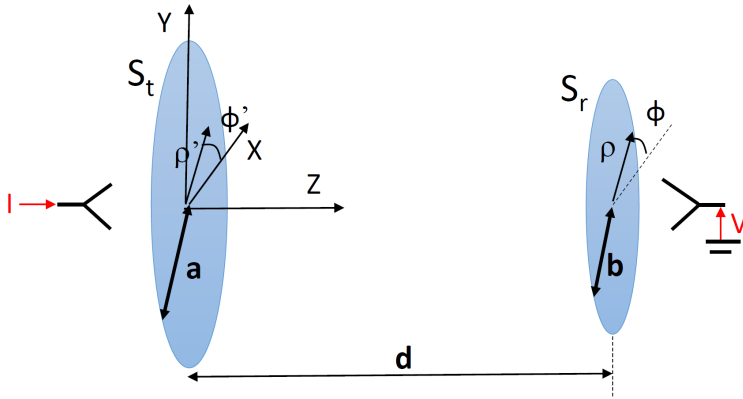


Figure 6.7: Geometrical configuration: two apertures along with the related geometrical parameters.

mittances as follows:

$$H = \sum_m \sum_{m'} \gamma_{t_{nm'}} \gamma_{r_{nm}} H_{nmm'}. \quad (6.6)$$

where each contribution of the co-polar transmittance is given by:

$$\mathcal{H} = H_{nmm'} = C \iint_{S_r} \iint_{S_t} R_{m,n} \left(\frac{\rho}{b}, \phi \right) G(\mathbf{r}, \mathbf{r}') P_S R_{m',n} \left(\frac{\rho'}{a}, \phi' \right) dS_t dS_r \quad (6.7)$$

In this equation, $C = -j4k\eta/(I_r I_t)$ is the constant outside the integral in (6.5) and $G(\mathbf{r}, \mathbf{r}')$ is the scalar free-space Green's function. P_S is a correction factor that stems from Green's dyadic and takes into account the polarization. It is interesting to mention that expression (6.7) resembles the well known reaction integral appearing in the classical Method of Moments formulation.

6.2.2 Plane waves expansion of the Green's Function

The scalar free-space Green's function can be written as a spectrum of plane waves:

$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk_0 \|\mathbf{r} - \mathbf{r}'\|}}{4\pi \|\mathbf{r} - \mathbf{r}'\|} \quad (6.8)$$

$$= \frac{1}{(2\pi)^2} \iint \frac{e^{jk_\rho \rho' \cos(\phi' - \alpha)} e^{-jk_\rho \rho \cos(\phi - \alpha)} e^{-jk_z d}}{2jk_z} d\phi k_\rho dk_\rho$$

where $k_0 = 2\pi/\lambda$ is the free-space wave number, $k_\rho = \sqrt{k_x^2 + k_y^2}$ and $k_z = \sqrt{k^2 - k_\rho^2}$. To take into account the polarization effect (treated through P_S in (6.7)), we need to eliminate the contribution of the source parallel to $\mathbf{k}$, before projecting the field on the polarized current at the receiving antenna. Indeed, (6.8) corresponds to a plane wave decomposition and for each plane wave, the electric field must be orthogonal to the wave vector. It has been proven in [108] that this is equivalent to multiplying each plane wave of the expansion of the scalar free-space Green's function by the factor:

$$P(k_x, k_y) = \left(\mathbf{e}_t - \frac{(\mathbf{e}_t \cdot \mathbf{k}) \mathbf{k}}{k_0^2} \right) \cdot \mathbf{e}_r \quad (6.9)$$

where $\mathbf{e}_t$ and $\mathbf{e}_r$ are respectively the transmitted and received current polarizations. After inserting (6.9) into (6.8) and (6.8) into (6.7), and using the following identity:

$$\int_0^{2\pi} e^{jn\phi'} e^{jk_\rho \rho' \cos(\phi' - \alpha)} d\phi = 2\pi J_n(k_\rho \rho') e^{jn\alpha} j^n \quad (6.10)$$

we obtain (after integrating along α) for the co-polar transmittance:

$$\mathcal{H} = C \frac{\pi}{j} \int_{k_\rho} \int_{\rho'} J_n(k_\rho \rho') F_{m',n} \left(\frac{\rho'}{a} \right) \rho' d\rho' \frac{e^{-jk_z d}}{k_z} \int_{\rho} J_n(k_\rho \rho) \times F_{m,n} \left(\frac{\rho}{b} \right) \rho d\rho k_\rho \left(1 - \frac{k_\rho^2}{2k_0^2} \right) dk_\rho \quad (6.11)$$

Using the closed-form Hankel transform of the Bessel functions (see equation (2.4)), the transmittance can be rewritten as:

$$\mathcal{H} = \frac{C\pi a^2 b^2}{j} \int_{k_{\rho r}=0}^{\infty} \tilde{F}_{m'n}(k_{\rho}a) \tilde{F}_{mn}(k_{\rho}b) \frac{e^{-jk_z d}}{k_z} k_{\rho} \left(1 - \frac{k_{\rho}^2}{2k_z^2}\right) dk_{\rho} \quad (6.12)$$

To avoid the pole at $k_{\rho} = k_0$ and to improve the integration convergence, we choose as in [108] a very simple linear contour given by $k_{\rho} = k_{\rho r} (1 + j)$, where $k_{\rho r}$ is the real part of k_{ρ} . The final expression of the transmittance is the following:

$$\mathcal{H} = \frac{2CS_t S_r}{\pi} \int_{k_{\rho r}=0}^{\infty} \tilde{F}_{m'n}(k_{\rho}a) \tilde{F}_{mn}(k_{\rho}b) \left(1 - j \frac{k_{\rho r}^2}{k_0^2}\right) \frac{e^{-jk_z d}}{k_z} k_{\rho r} dk_{\rho r} \quad (6.13)$$

6.2.3 Numerical Results

The chosen parameters for the examples below are: $D = 2a = 2b = 40$ cm, the wavelength $\lambda = 1$ cm and the distance d is normalized w.r.t. the far-field distance $d_{far} = 2D^2/\lambda$. Fig. 6.8(a) shows the evolution of the transmittance for different values of the n index in a “log-log” scale. One can observe the same trend as with the Zernike basis (see Fig. 6.8(b)). The n index controls the slope of the transmittance in the far-field. However, as can be seen in Fig. 6.9(a), changing the value of m has no effect on the slope with the Bessel decomposition. It can be proven that the dependence of $\mathcal{H}$ versus m in the far-field is proportional to $1/m^3$ for high values of m (see Appendix A.7). This is a fundamental difference with the Zernike family, where n and m both affect the slope of $\mathcal{H}$ versus distance in a “log-log” scale (see Fig. 6.9(b)).

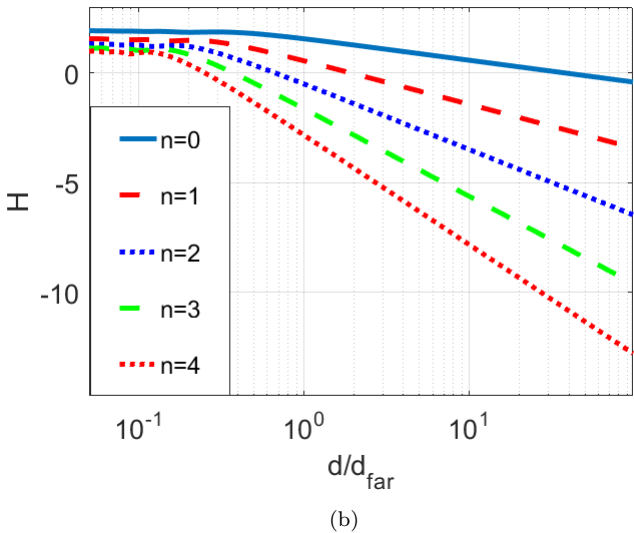
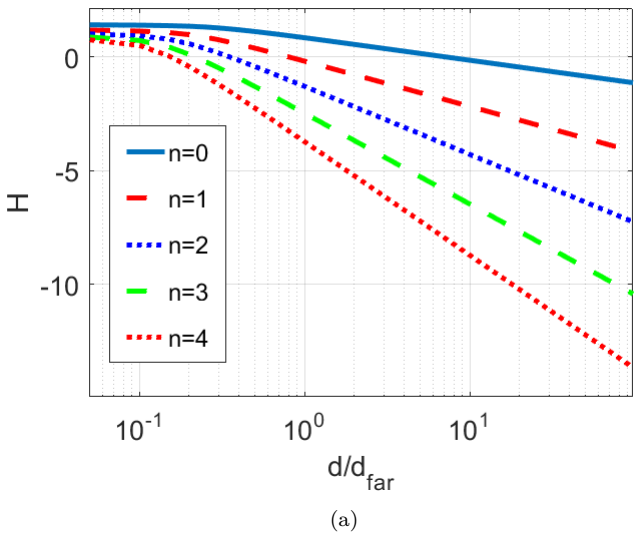


Figure 6.8: Transmittance versus distance w.r.t. far-field distance. $n = 0 \dots 4$ and $m' = m = 0$. (a) Fourier-Bessel basis. (b) Zernike basis.

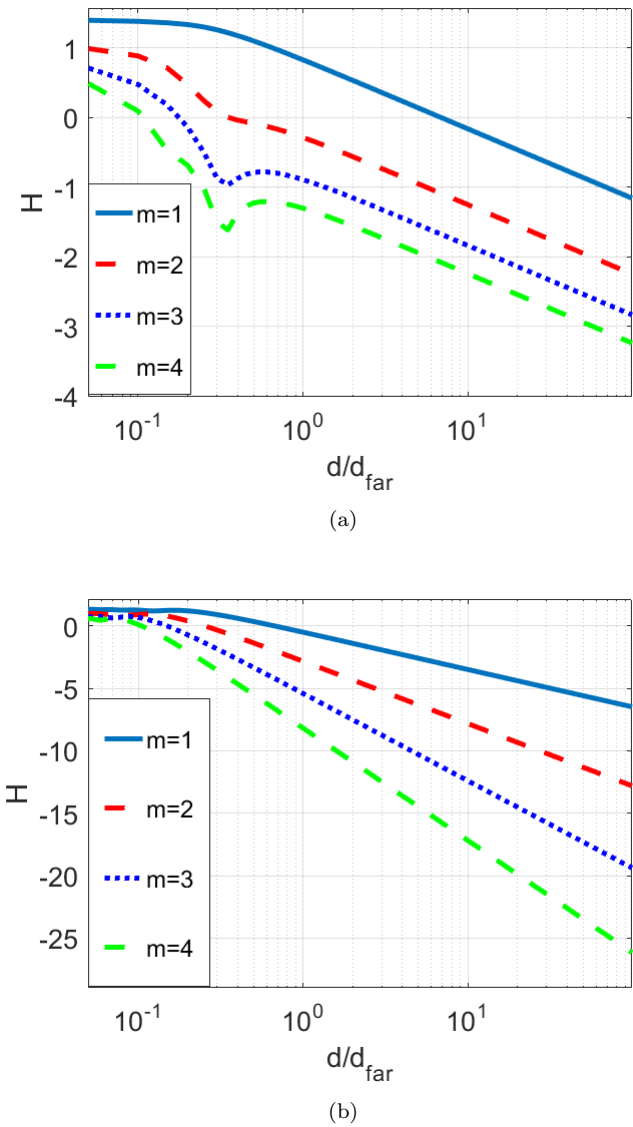


Figure 6.9: Transmittance versus distance w.r.t. far-field distance. $n = 0$ and $m' = m = 1 \dots 4$. (a) Fourier-Bessel basis. (b) Zernike basis.

6.3 Closed-form expression of the transmittance

Under the far-field approximation, the transmittance can be well approximated with a closed-form expression. First, let us replace in (6.13) the spectrum of the Bessel functions (2.2).

$$\mathcal{H} = \frac{2CS_tS_r}{\pi} \int_{k_{\rho r}=0}^{\infty} \frac{\lambda_{nm'} J_{n-1}(\lambda_{nm'}) J_n(k_{\rho} a)}{(\lambda_{nm'})^2 - (k_{\rho} a)^2} \frac{\lambda_{nm} J_{n-1}(\lambda_{nm}) J_n(k_{\rho} b)}{(\lambda_{nm})^2 - (k_{\rho} b)^2} \frac{e^{-jk_z d}}{k_z} k_{\rho r} \left(1 - j \frac{k_{\rho r}^2}{k_0^2}\right) dk_{\rho r} \quad (6.14)$$

For a large distance d , the $e^{-jk_z d}$ factor can be approximated as $e^{-jk_0 d} e^{-k_{\rho r}^2 d/k_0}$, which explains why the integrand converges very fast. As a consequence, the terms βa and βb appearing in the denominator can be neglected, in comparison with $\lambda_{nm'}$ and λ_{nm} respectively and one can assume that the k_z factor appearing in the denominator is almost equal to k . Expression (6.14), can then be approximated as

$$\mathcal{H} \approx \frac{2CS_tS_r J_{n-1}(\lambda_{nm'}) J_{n-1}(\lambda_{nm})}{\pi k \lambda_{nm'} \lambda_{nm}} \int_{k_{\rho r}=0}^{\infty} J_n(k_{\rho} a) J_n(k_{\rho} b) e^{-jk d} \times e^{-k_{\rho r}^2 d/k} k_{\rho r} \left(1 - j \frac{k_{\rho r}^2}{k_0^2}\right) dk_{\rho r} \quad (6.15)$$

Next, for the same reason as before, the Bessel function can be decomposed using its first order approximation ($J_n(x) \approx (x/2)^n/n!$; $n \geq 0$). Finally, using the identity:

$$\int_{k_{\rho r}=0}^{\infty} e^{-k_{\rho r}^2 d/k_0} k_{\rho r}^{1+2n} dk_{\rho r} = \frac{n!}{2} \left(\frac{k_0}{d}\right)^{n+1} \quad (6.16)$$

the transmittance takes the following asymptotic form:

$$\mathcal{H} \approx 4CS_tS_r \left[\frac{J_{n-1}(\lambda_{nm'}) J_{n-1}(\lambda_{nm})}{\lambda_{nm'} \lambda_{nm} n!} \right] \frac{e^{-jk_0 d}}{4\pi d} \left(\frac{abjk_0}{2d}\right)^n \times \left(1 - \frac{j}{(n+1)d}\right) \quad (6.17)$$

Expression $\left(1 - \frac{j}{(n+1)d}\right)$ is the correction factor taking into account the polarization effect stemming from (6.9). One can observe that this factor tends to 1 when the two antennas are far from each other, meaning that in this case, it can be assumed the polarization of the transmitted field being homogeneous over the reception aperture. The expression between brackets shows the m -dependency of the transmittance in the far-field. It is proven in Appendix A.7 that this dependency is proportional to $1/m^3$ for $n \ll m$ and $m = m'$ ¹. Fig. 6.10(a) shows the trend of the transmittance, assuming the obtained far-field closed-form approximation. One can observe a very good agreement down to about half the far-field limit. A similar approximation has been obtained in [108] for the Zernike family (see Fig. 6.10(b)). In the latter case, the propagation exponent was $-(1 + n + 2m)$ instead of $-(1 + n)$.

¹For $m \neq m'$, we have $m^{-3/2}$ and $(m')^{-3/2}$ dependences.

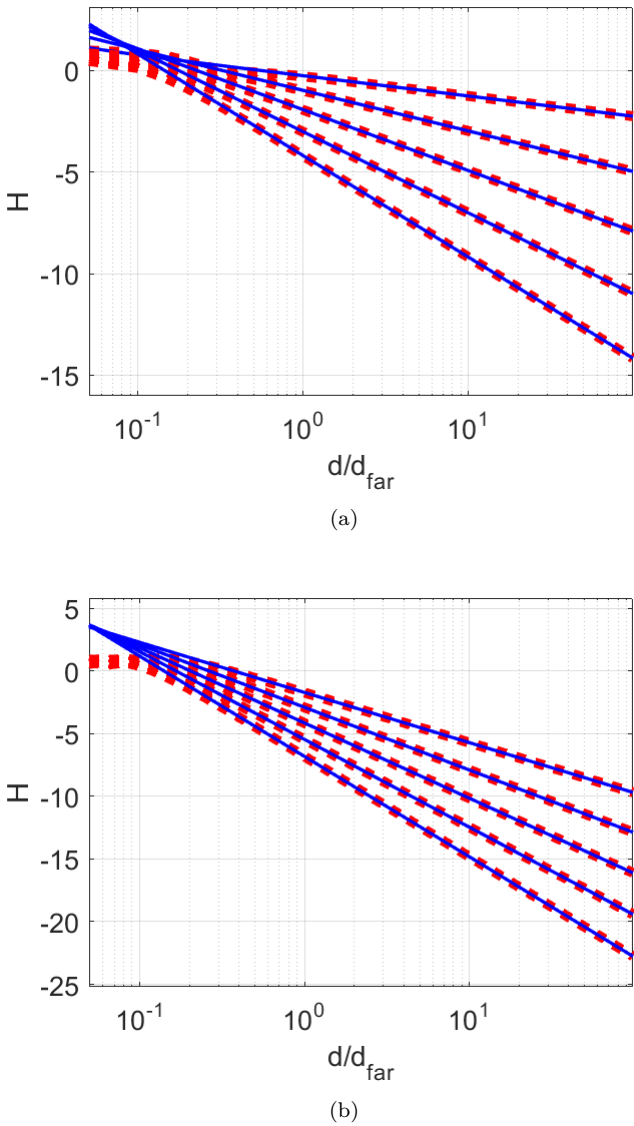


Figure 6.10: Transmittance versus distance normalized w.r.t. far-field limit. $m' = 2$, $m = 1$ and $n = 0 \dots 4$ (top to bottom). Blue lines: far field approximation. Red line: exact expression. (a) Fourier Bessel basis. (b) Zernike basis.

6.4 Conclusion

Near-field shaping with LW modulated MTSs has been discussed through the synthesis of Bessel beams. The obtained results demonstrate the capabilities of modulated MTSs for near-field applications down to distances of about one wavelength from the surface. The effectiveness of the method appears despite the fact the radiation originates from a SW. We have also shown that multiple OAM beams can be efficiently generated with modulated MTS antennas. Finally, transmittance between Fourier-Bessel fields has been analyzed and a closed-form expression of the transmittance (valid for large distances between apertures) has been derived.

CHAPTER 7

Power balance in metasurface antennas

IN Chapter 5, measurements from a realized metasurface (MTS) antenna have shown significant sidelobes for the TM component. In order to localize the problem, one should evaluate the antenna efficiency so as to have a clear understanding of the power balance in the MTS. In [83], the efficiency of MTS antennas has been treated with the flat optics (FO) formalism [51]. However, this approach is limited to locally sinusoidal modulation. This chapter provides a rigorous and efficient full-wave method for the rapid evaluation of the efficiency of MTS antennas, not necessarily based on sinusoidal modulation (i.e. the method can handle arbitrary surface impedance profile).

7.1 Schematic view of the power balance

Let us consider a circular domain MTS printed on a grounded dielectric slab, as represented in Fig. 2.14. The MTS is assumed to be fed at its center ($\rho = 0$) with a vertical elementary dipole. Nevertheless, the method can also be used for any excitation, assuming the excitation fields are a priori known.

The total power delivered by the feed is denoted as P_{tot} . This power is considered to be partially radiated (P_{vis}), partially reflected/diffracted at the MTS substrate boundary (P_{diff}), and partially dissipated as ohmic losses in the substrate (P_{Ω}). A schematic representation of the power balance is shown in Fig. 7.1. The goal of the present chapter consists of

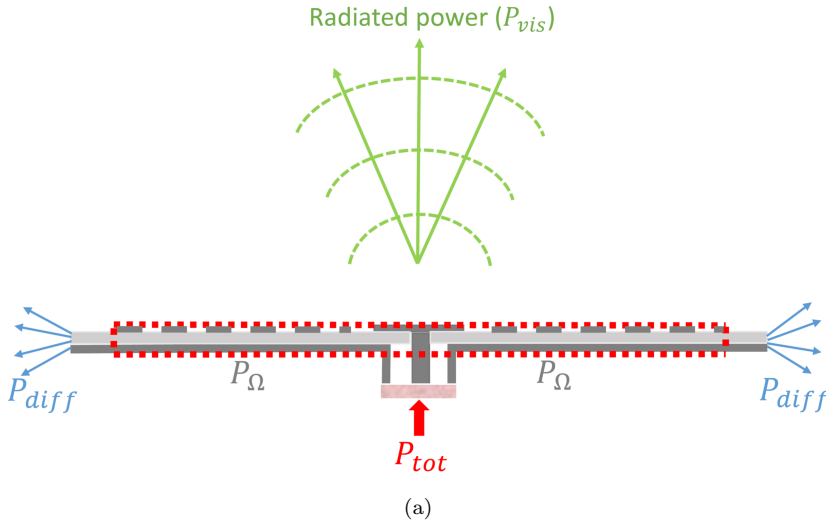


Figure 7.1: Illustration of the power balance.

evaluating the power balance, namely P_{tot} , P_{vis} , P_{diff} , and P_{Ω} for a given circular MTS antenna and excitation amplitude. The surface current distribution on the MTS plane $\mathbf{J}$ is assumed to be known in a Fourier-Bessel basis (FBBF). This surface current can be obtained in the FBBF basis using the MoM tool developed in Chapter 2, or computed with other numerical tools [61], [67], and thereafter projected into FBBFs.

7.2 Radiated (visible) power

The power radiated by the MTS can be derived by integrating the Poynting vector over an infinitely extended disk just above the MTS layer (x-y plane):

$$P_{vis} = \frac{1}{2} Re \left\{ \iint_{-\infty}^{\infty} \mathbf{E}_t \times \mathbf{H}_t^* \cdot \hat{\mathbf{n}} dS \right\} \quad (7.1)$$

where $\mathbf{E}_t$ and $\mathbf{H}_t$ are respectively the total tangential electric and magnetic fields on an infinitely extended disk in the MTS plane, $\hat{\mathbf{n}}$ is the unit vector normal to the MTS, and Re is the real part operator. Using the Parseval theorem, the radiated power can be reformulated in spectral domain as:

$$P_{vis} = \frac{1}{8\pi^2} Re \left\{ \iint_{-\infty}^{\infty} \tilde{\mathbf{E}}_t \times \tilde{\mathbf{H}}_t^* \cdot \hat{\mathbf{n}} dk_x dk_y \right\} \quad (7.2)$$

The magnetic field spectrum is related to electric field spectrum by

$$\tilde{\mathbf{H}} = \frac{\mathbf{k} \times \tilde{\mathbf{E}}}{\omega \mu} \quad (7.3)$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is the spectral vector component. Inserting (7.3) into (7.2), the radiated power can be rewritten as:

$$P_{vis} = \frac{1}{8\pi^2} \int_0^{2\pi} \int_0^{k_0} \|\tilde{\mathbf{E}}\|^2 \frac{k_z^*}{k_0 \eta_0} k_\rho dk_\rho d\alpha \quad (7.4)$$

where $\|\tilde{\mathbf{E}}\|^2$ is the squared Frobenius norm of the spectral electric field $\tilde{\mathbf{E}}$. The latter is the sum of the electric field radiated by the current and that of the excitation, namely:

$$\tilde{\mathbf{E}} = \underline{\underline{\tilde{\mathbf{G}}}}^{EJ} \cdot \tilde{\mathbf{J}} + \tilde{\mathbf{E}}_i \quad (7.5)$$

The calculation of the Green's function $\underline{\underline{\tilde{\mathbf{G}}}}^{EJ}$ and $\tilde{\mathbf{E}}_i$ is explained in Appendixes A.4, and A.5 respectively.

7.3 Power delivered by the feeder

Referring to the Poynting theorem, the total power delivered by a matched feeding source in absence of losses can be computed after integrating the Poynting vector along a closed surface comprising the source. We proposed to use the closed surface represented in dotted red lines in Fig. 7.1. This surface includes the MTS aperture, the rim of the antenna and the ground plane. Since the ground plane is assumed to be a perfect electric conductor, its contribution is zero. Therefore, the power delivered by the feeder can be computed as:

$$P_{tot} = \frac{1}{2} Re \left\{ \iint_{S_1} \mathbf{E}_t \times \mathbf{H}_t^* \cdot \hat{\mathbf{n}} dS + \iint_{S_2} \mathbf{E} \times \mathbf{H}^* \cdot \hat{\boldsymbol{\rho}} dS + \iiint_V \sigma ||\mathbf{E}||^2 dV \right\} \quad (7.6)$$

The first term is referred to as the MTS contribution, the second one as the rim contribution, and the last term is referred to as the ohmic power losses.

7.3.1 MTS contribution

The MTS contribution is explicitly rewritten as:

$$P_{tot} = \frac{1}{2} Re \left\{ \iint_{S_1} E_x \cdot H_y^* - E_y \cdot H_x^* dS \right\} \quad (7.7)$$

The aperture tangential electric field tested with the complex conjugate of the FBBFs can be computed as:

$$\underline{\mathbf{E}}_t = \underline{\mathbf{Z}}_{IBC} \underline{\mathbf{I}} \quad (7.8)$$

where $\underline{\mathbf{Z}}_{IBC}$ and $\underline{\mathbf{I}}$ are respectively the IBC matrix and the surface current in the FBBF basis. The IBC matrix is calculated with the MoM procedure described in Chapter 2. Since FBBFs are orthogonal, the coefficients e_{mn} of the aperture fields projected into FBBFs can be obtained from the tested fields e_{mn}^{tested} as:

$$e_{mn} = \frac{e_{mn}^{tested}}{K(m, n)} \quad (7.9)$$

where $K(m, n)$ is a normalization factor defined in (2.24).

The aperture tangential magnetic field tested with FBBFs is computed as

$$\underline{\mathbf{H}}_t = \underline{\underline{\mathbf{Z}}}_G^H \underline{\mathbf{I}} - \underline{\mathbf{V}}^H \quad (7.10)$$

where $\underline{\underline{\mathbf{Z}}}_G^H$ is the substrate matrix relating the surface current to the aperture magnetic field. $\underline{\mathbf{V}}^H$ is the excitation magnetic field at the MTS plane, tested with the complex conjugate of the FBBFs. $\underline{\underline{\mathbf{Z}}}_G^H$ and $\underline{\mathbf{V}}^H$ are computed using a procedure similar to what has been done for the computation of $\underline{\underline{\mathbf{Z}}}_G$ and $\underline{\mathbf{V}}$ in Chapter 2 (see Appendix A.9 for more details). Note that, in the case the opaque impedance is known, the magnetic field can also be retrieved by simply multiplying the opaque impedance with the electric field.

Once the coefficients (h_{mn}^{tested}) of the magnetic field tested with the complex conjugate of the FBBFs are known, the coefficients h_{mn} resulting from the expansion of the magnetic field into FBBFs are given by

$$h_{mn} = \frac{h_{mn}^{tested}}{K(m, n)} \quad (7.11)$$

The aperture field is therefore written in the FBBF basis as

$$\underline{\mathbf{E}}(\boldsymbol{\rho}) = \sum_{n=-N}^N \sum_{m=1}^M e_{mn}^x R_{m,n}^x(\boldsymbol{\rho}) \hat{\mathbf{x}} + e_{mn}^y R_{m,n}^y(\boldsymbol{\rho}) \hat{\mathbf{y}} \quad (7.12)$$

$$\underline{\mathbf{H}}(\boldsymbol{\rho}) = \sum_{n=-N}^N \sum_{m=1}^M h_{mn}^x R_{m,n}^x(\boldsymbol{\rho}) \hat{\mathbf{x}} + h_{mn}^y R_{m,n}^y(\boldsymbol{\rho}) \hat{\mathbf{y}} \quad (7.13)$$

Inserting expressions (7.12) and (7.13) into (7.7), and using the orthogonality relation of FBBFs (see equation (A.2)), the MTS contribution to the delivered power is given by

$$P_{MTS} = -\frac{\pi a^2}{2} \sum_{n=-N}^N \sum_{m=1}^M J_{n-1}(\lambda_n^m) J_{n+1}(\lambda_n^m) \text{Re} \{ e_{mn}^x h_{mn}^{y*} - e_{mn}^y h_{mn}^{x*} \} \quad (7.14)$$

where λ_n^m is the m -th positive zero of the first kind Bessel function of order n . It means that the calculation of the MTS contribution to the power

delivered by the feed can be carried out extremely fast (in practice, in less than 1 mn).

7.3.2 Rim contribution

The rim contribution in (7.6) can be developed as

$$P_{rim} = \frac{1}{2} Re \left\{ \int_{z=-d}^0 \int_{\phi=0}^{2\pi} E_{\phi}(\rho = a, \phi, z) H_z^*(\rho = a, \phi, z) - E_z(\rho = a, \phi, z) H_{\phi}^*(\rho = a, \phi, z) d\phi dz \right\} \quad (7.15)$$

Since the mode is quasi TM over a grounded plane, it is reasonable to neglect the first term. The calculation of the rim contribution is therefore limited to the second term, namely

$$P_{rim} = -\frac{1}{2} Re \left\{ \int_{z=-d}^0 \int_{\phi=0}^{2\pi} E_z(\rho = a, \phi, z) H_{\phi}^*(\rho = a, \phi, z) d\phi dz \right\} \quad (7.16)$$

which can be rewritten as

$$P_{rim} = \frac{1}{2} Re \left\{ \iint E_z(\rho = a, \phi, z) H_x^*(\rho = a, \phi, z) \sin \phi d\phi dz \right\} - \frac{1}{2} Re \left\{ \iint E_z(\rho = a, \phi, z) H_y^*(\rho = a, \phi, z) \cos \phi d\phi dz \right\} \quad (7.17)$$

The first and second terms in equation (7.17) will be denoted respectively as P_{rim1} , and P_{rim2} . Now, the E_z field is split into three contributions E_{zx} , E_{zy} , and E_{zz} , corresponding respectively to the x -directed surface current, y -directed surface current and the z -directed excitation current. In the same way, we define H_{xx} , H_{xy} , H_{xz} , H_{yx} , H_{yy} , and H_{yz} .

Evaluation of E_{zx}

The electric field E_{zx} can be computed in spectral domain based on the x -directed current distribution (assumed to be known in Fourier-Bessel basis) and the pertaining spectral Green's function $\tilde{G}_{zx}^{EJ}$. One then needs to

calculate the inverse Fourier transform of $\tilde{E}_{zx}$, which can be written as:

$$E_{zx} = \frac{1}{4\pi^2} \sum_{m,n} i_{mn}^x \iint \tilde{G}_{zx}^{EJ} \tilde{R}_{mn} e^{-j(k_x x + k_y y)} dk_x dk_y \quad (7.18)$$

The spectral Green's function $\tilde{G}_{zx}^{EJ}$ and the FBBF spectrum $\tilde{R}_{mn}$ can be respectively written in the form $\tilde{g}_{zx}^E(k_\rho) \cos \alpha$, and $\tilde{F}_{mn}(k_\rho) e^{-jn\alpha}$. After inserting the latter expressions in (7.18), and going through cylindrical coordinates ($k_x = k_\rho \cos \alpha$, $k_y = k_\rho \sin \alpha$), expression (7.18) can be rewritten as

$$E_{zx} = \frac{1}{4\pi^2} \sum_{m,n} i_{mn}^x \int_0^\infty \tilde{g}_{zx}^E \tilde{F}_{mn} \int_0^{2\pi} \cos \alpha e^{-jn\alpha} e^{-jk_\rho \rho \cos(\phi-\alpha)} d\alpha k_\rho dk_\rho \quad (7.19)$$

Integration along α can be carried out in closed form, which leads to:

$$E_{zx} = \frac{1}{4\pi^2} \sum_{m,n} i_{mn}^x \int_0^\infty \tilde{g}_{zx}^E \tilde{F}_{mn} j [f_n^0(k_\rho \rho, \phi, 1) - f_n^0(k_\rho \rho, \phi, -1)] k_\rho dk_\rho \quad (7.20)$$

where the function f_n^0 is defined as

$$f_n^0(k_\rho \rho, \phi, i) = \pi j^n e^{-j(n+i)\phi} J_{n+i}(-k_\rho \rho) \quad (7.21)$$

To simplify the notation, we define the functions:

$$f_{mn}^a(k_\rho \rho, \phi, -1) = -j f_n^0(k_\rho \rho, \phi, -1) \tilde{g}_{zx}^E \tilde{F}_{mn} \quad (7.22)$$

$$f_{mn}^b(k_\rho \rho, \phi, 1) = j f_n^0(k_\rho \rho, \phi, 1) \tilde{g}_{zx}^E \tilde{F}_{mn} \quad (7.23)$$

Then, according to (7.22), and (7.23), E_{zx} is rewritten as

$$E_{zx} = \frac{1}{4\pi^2} \sum_{m,n} i_{mn}^x \int_0^\infty [f_{mn}^a(k_\rho \rho, \phi, -1) + f_{mn}^b(k_\rho \rho, \phi, 1)] k_\rho dk_\rho \quad (7.24)$$

Evaluation of E_{zy}

Following a similar reasoning, E_{zy} is given by:

$$E_{zy} = \frac{1}{4\pi^2} \sum_{m,n} i_{mn}^y \int_0^\infty \tilde{g}_{zy}^E \tilde{F}_{mn} [-f_n^0(k_\rho \rho, \phi, -1) - f_n^0(k_\rho \rho, \phi, 1)] k_\rho dk_\rho \quad (7.25)$$

Note that $\tilde{g}_{zy}^E = \tilde{g}_{zx}^E$. After defining the following functions:

$$f_{mn}^c(k_\rho \rho, \phi, -1) = -f_n^0(k_\rho \rho, \phi, -1) \tilde{g}_{zy}^E \tilde{F}_{mn} \quad (7.26)$$

$$f_{mn}^d(k_\rho \rho, \phi, 1) = -f_n^0(k_\rho \rho, \phi, 1) \tilde{g}_{zy}^E \tilde{F}_{mn} \quad (7.27)$$

E_{zy} , is written in compact form as:

$$E_{zy} = \frac{1}{4\pi^2} \sum_{m,n} i_{mn}^y \int_0^\infty [f_{mn}^c(k_\rho \rho, \phi, -1) + f_{mn}^d(k_\rho \rho, \phi, 1)] k_\rho dk_\rho \quad (7.28)$$

Evaluation of E_{zz}

Since the MTS is assumed to be fed with a vertical elementary dipole placed at the center ($\rho = 0$), the z-component of the excitation electric field does not depend on the azimuthal direction (see Appendix A.5). Therefore, $\tilde{G}_{zz}^{EJ}$ can be written as $\tilde{g}_{zz}^E(k_\rho)$. Then, E_{zz} is computed as:

$$E_{zz} = \frac{i_{00}^z}{4\pi^2} \iint \tilde{g}_{zz}^E e^{-jk_\rho \rho \cos(\phi - \alpha)} d\alpha k_\rho dk_\rho \quad (7.29)$$

where i_{00}^z is the complex excitation amplitude. After integrating along α , we get

$$E_{zz} = \frac{i_{00}^z}{4\pi^2} \int_0^\infty f_{00}^z(k_\rho \rho, \phi, 0) k_\rho dk_\rho \quad (7.30)$$

with

$$f_{00}^z(k_\rho \rho, \phi, 0) = 2f_0^0(k_\rho \rho, \phi, 0) \tilde{g}_{zz}^E \quad (7.31)$$

Evaluation of H_{xx}

The x -directed magnetic field Green's function associated to a x -directed current distribution can be written spectrally as $\tilde{G}_{xx}^{HJ} = \cos \alpha \sin \alpha \tilde{g}_{xx}^H(k_\rho)$. The magnetic field in spatial domain is then derived as

$$H_{xx} = \frac{1}{4\pi^2} \sum_{m,n} i_{mn}^x \int_0^\infty \tilde{g}_{xx}^H \tilde{F}_{mn} \frac{j}{2} [f_n^0(k_\rho \rho, \phi, -2) - f_n^0(k_\rho \rho, \phi, 2)] k_\rho dk_\rho \quad (7.32)$$

Similarly to the previous cases, we defined:

$$f_{mn}^e(k_\rho \rho, \phi, -2) = \frac{j}{2} f_n^0(k_\rho \rho, \phi, -2) \tilde{g}_{xx}^H \tilde{F}_{mn} \quad (7.33)$$

and

$$f_{mn}^f(k_\rho \rho, \phi, 2) = -\frac{j}{2} f_n^0(k_\rho \rho, \phi, 2) \tilde{g}_{xx}^H \tilde{F}_{mn} \quad (7.34)$$

Expression (7.32) is then written compactly as follows

$$H_{xx} = \frac{1}{4\pi^2} \sum_{m,n} i_{mn}^x \int_0^\infty [f_{mn}^e(k_\rho \rho, \phi, -2) + f_{mn}^f(k_\rho \rho, \phi, 2)] k_\rho dk_\rho \quad (7.35)$$

Evaluation of H_{xy} , H_{yx} , H_{yy} , H_{xz} , and H_{yz}

The x -directed magnetic field spectral Green's function corresponding to a y -directed current takes the form $\tilde{G}_{xy}^{HJ} = \tilde{g}_{xy1}^H(k_\rho) \cos^2 \alpha + \tilde{g}_{xy2}^H(k_\rho) \sin^2 \alpha$. Therefore, the magnetic field can be written as the sum of two contributions, namely H_{xy1} and H_{xy2} corresponding to each term of the Green's function. Those contributions as well as the other magnetic fields components are computed in a manner similar to that of the previous fields (see Appendix A.10).

Computation of the rim contribution

Let us consider the first term in the rim power contribution, namely P_{rim1} , given by

$$P_{rim} = \frac{1}{2} \text{Re} \left\{ \iint E_z(\rho = a, \phi, z) H_x^*(\rho = a, \phi, z) \sin \phi d\phi dz \right\} \quad (7.36)$$

The method is explained for the first term in the rim contribution. The second term is computed similarly.

The computation of E_z and H_x has been previously explained and written compactly as:

$$E_z = \frac{1}{4\pi^2} \int_0^\infty \left\{ \sum_{m,n} \left[i_{mn}^x \left(f_{mn}^a(k_\rho \rho, \phi, -1) + f_{mn}^b(k_\rho \rho, \phi, 1) \right) + i_{mn}^y \left(f_{mn}^c(k_\rho \rho, \phi, -1) + f_{mn}^d(k_\rho \rho, \phi, 1) \right) \right] + i_{00}^z f_{00}^z(k_\rho \rho, \phi, 0) \right\} k_\rho dk_\rho \quad (7.37)$$

$$H_x = \frac{1}{4\pi^2} \int_0^\infty \left\{ \sum_{m,n} \left[i_{mn}^x \left(f_{mn}^e(k_\rho \rho, \phi, -2) + f_{mn}^f(k_\rho \rho, \phi, 2) \right) + i_{mn}^y \left(f_{mn}^g(k_\rho \rho, \phi, 0) + f_{mn}^h(k_\rho \rho, \phi, -2) + f_{mn}^i(k_\rho \rho, \phi, 2) + f_{mn}^j(k_\rho \rho, \phi, 0) + f_{mn}^k(k_\rho \rho, \phi, -2) + f_{mn}^l(k_\rho \rho, \phi, 2) \right) \right] + i_{00}^z f_{00}^{x1}(k_\rho \rho, \phi, -1) + i_{00}^z f_{00}^{x2}(k_\rho \rho, \phi, 1) \right\} k_\rho dk_\rho \quad (7.38)$$

Inserting (7.37) and (7.38) into (7.36), the first term in the rim power can be expressed as the sum of several contributions of the form:

$$x_{uvmm'n'} = C \iiint f_{mn}^u(k_\rho a, \phi, w1) f_{m'n'}^{v*}(k'_\rho a, \phi, w2) \sin \phi d\phi dk_\rho dk'_\rho dz \quad (7.39)$$

where ρ has been replaced by the MTS radius a , and:

$$C = \frac{i_{mn}^u i_{m'n'}^{v*}}{32\pi^4} \quad (7.40)$$

Integral (7.39) can be evaluated in closed form along ϕ due to the harmonic azimuthal variation of f_{mn} . Moreover, (7.39) is non-zero only if $|n - n' + w1 - w2| = 1$, which significantly speeds up the numerical computation.

After azimuthal integration, expression (7.39) can be written in the form:

$$x_{uvmm'n'} = \frac{i_{mn}^u i_{m'n'}^{v*}}{32\pi^4} \int \left[\int g_{mn}^u(k_\rho a) dk_\rho \cdot \int g_{m'n'}^{v*}(k'_\rho a) dk'_\rho \right] dz \quad (7.41)$$

Therefore, one can pre-compute integrals along k_ρ , which significantly accelerate the computation. Integration along k_ρ is performed using a complex contour deformation to avoid the poles of the Green's function. Integration along z can be carried out accurately with very few sampling points (about 4) since the integrand along z is relatively smooth.

Asymptotic component extraction for the excitation fields evaluation

When evaluating integrals along k_ρ in (7.41) for the excitation fields at (or very close to) the excitation dipole layer ($z = -d/2$, with d being the substrate thickness), the integrand does not converge rapidly. In this case, accurate results can be obtained after removing from the integrand, an asymptotic term corresponding to the homogeneous medium Green's function. As an example, one needs to calculate:

$$E_{zz} = \frac{i_{00}^z}{2\pi} \int_0^\infty J_0(k_\rho a) \tilde{g}_{zz}^E k_\rho dk_\rho \quad (7.42)$$

Integral (7.42) is reformulated as

$$E_{zz} = \frac{i_{00}^z}{2\pi} \int_0^\infty J_0(k_\rho a) \cdot \left[\tilde{g}_{zz}^E - \tilde{g}_{zz}^{Ehom} \right] k_\rho dk_\rho + i_{00}^z E_{zz}^{hom} \quad (7.43)$$

where $\tilde{g}_{zz}^{Ehom}$ is zz the spectral Green's function in the homogeneous medium of relative permittivity corresponding to that of the MTS substrate, and E_{zz}^{hom} is the spatial Green's function of the homogeneous medium given by:

$$E_{zz}^{hom} = i_{00}^z \frac{e^{-jkr}}{4\pi r} \left[-j \eta k \sin^2 \theta \left(1 - \frac{j}{kr} - \frac{1}{(kr)^2} \right) + \frac{2\eta}{r} \cos^2 \theta \left(1 - \frac{j}{kr} \right) \right] \quad (7.44)$$

where r is the distance between the source and the observation, θ is the elevation angle (see Fig. 2.14). $\eta = 376.7/\sqrt{\epsilon_r}$ and $k = 2\pi\sqrt{\epsilon_r}/\lambda$ are the homogeneous medium impedance and the wavenumber, respectively.

The same technique is used for the evaluation of the excitation magnetic field. This allows limiting the integration versus k_ρ to about $20k_0$, where $k_0 = 2\pi/\lambda$ is the free-space wavelength.

7.3.3 Ohmic Power losses

This section is devoted to the ohmic power losses evaluation in the MTS substrate. It is assumed that the losses in the metallization are negligible compared to the substrate losses, which is a good assumption at frequencies lower than 30 GHz [83]. Considering a substrate with electric conductivity σ , the ohmic power losses is given by

$$P_\Omega = \frac{1}{2} \iiint_V \sigma ||\mathbf{E}||^2 dV \quad (7.45)$$

where the integration is carried out over the cylindrical substrate of radius equal to the MTS radius a , and height corresponding to the substrate thickness d . $||\mathbf{E}||$ is the Frobenius norm of the electric field. In order to efficiently compute the electric field in the substrate, we have defined a substrate interaction matrix $\underline{\underline{\mathbf{Z}}}_G^S(z)$ between the surface current $\mathbf{J}$ and the scattered electric field in a given planar layer of the substrate (fixed height z) of radius a . The interaction matrix $\underline{\underline{\mathbf{Z}}}_G^S(z)$, as well as the excitation fields vector $\underline{\mathbf{V}}^E(z)$, can be computed using a procedure similar to what has been described in Chapter 2. Therefore, the total electric field at a given height z in the substrate, tested with the complex conjugate of the FBBFs $\underline{\mathbf{E}}(z)$ can be calculated as

$$\underline{\mathbf{E}}(z) = \underline{\underline{\mathbf{Z}}}_G^S(z) \underline{\mathbf{I}} - \underline{\mathbf{V}}^E(z) \quad (7.46)$$

Based on (7.46), the coefficients e_{mn} of the total electric field in FBBFs are derived from the tested fields using expression (7.9). After expanding each component of the electric field into FBBFs and using the orthogonality relation of the FBBFs, the ohmic power losses in (7.45) can be rewritten as:

$$P_\Omega = \frac{\sigma}{2} \sum_{m,n} \int_{z=-d}^0 (e_{mn}^{x*} e_{mn}^x + e_{mn}^{y*} e_{mn}^y + e_{mn}^{z*} e_{mn}^z) K(m,n) dz \quad (7.47)$$

which means that the ohmic power losses can be computed extremely fast assuming that the fields expansion coefficients are known. The computation time for the fields coefficients is dominated by the computation time of the underlying substrate matrix $\underline{\underline{Z}}_G^S$, which requires in practice less than 1 mn (per sampled layer) on a traditional laptop computer.

7.4 Efficiencies under consideration

From the previous sections, the total power delivered by the feed can be calculated as

$$P_{tot} = P_{rim} + P_{MTS} + P_{\Omega} \quad (7.48)$$

Now, we define the following efficiencies.

7.4.1 Conversion efficiency

The conversion efficiency ϵ_{conv} is defined as the ratio between the radiated power P_{vis} and the total power delivered by the feed

$$\epsilon_{conv} = \frac{P_{vis}}{P_{tot}} \quad (7.49)$$

Note that the radiated power is computed taking also into account the space waves directly radiated by the feed. Therefore, the conversion efficiency is only affected by the losses and the reflection/diffraction effects at the MTS rim.

7.4.2 Aperture efficiency

The aperture efficiency ϵ_{ap} is the ratio between the antenna directivity and the maximum directivity that can be achieved considering the antenna area. The maximum directivity is obtained with an uniform illumination of the aperture. For a broadside beam, the aperture efficiency is

$$\epsilon_{ap} = \frac{\lambda^2 D}{4\pi S} \quad (7.50)$$

where S is the physical antenna area and D is the antenna directivity. Note that the directivity is computed assuming an infinitely extended substrate. Hence, contrary to the conversion efficiency, diffraction effects originating from the substrate truncation and the losses are not really taken into account in the aperture efficiency.

7.4.3 Loss factor

The loss factor L_{loss} quantifies the impact of the losses on the conversion efficiency. It is defined as:

$$L_{loss} = \frac{P_{\Omega}}{P_{tot}} \quad (7.51)$$

A small loss factor means that the conversion efficiency is mainly limited by the diffraction effects at the substrate rim.

7.4.4 Diffraction factor

The diffraction factor L_{diff} evaluates the impact of the substrate truncation effects on the conversion efficiency. More precisely, it quantifies the power lost in SW propagating beyond the MTS.

$$L_{diff} = \frac{P_{diff}}{P_{tot}} = \frac{P_{rim} + P_{MTS} - P_{vis}}{P_{tot}} \quad (7.52)$$

A small diffraction factor means that the conversion efficiency is mainly due to the ohmic losses.

7.4.5 Compound efficiency

There is no universal figure of merit to define the efficiency of an antenna, since the compromise between the previous metrics will depend on the application of interest. In this thesis, we define a “compound efficiency” as the product of the conversion efficiency with the aperture efficiency.

$$\epsilon = \epsilon_{ap} \epsilon_{conv} \quad (7.53)$$

This choice is roughly justified by the fact a zero (or very small) value of the aperture/conversion efficiency automatically leads to a zero (or very small) compound efficiency.

7.5 Numerical results

This section provides numerical results and validation of the efficiency computation method developed previously. We consider a MTS antenna modeled as an impenetrable scalar impedance modulation given by [61]

$$\begin{aligned} Z_+^{\rho\rho}(\boldsymbol{\rho}) &= jX_0 [1 + M \sin(2\pi\rho/d - \phi)] \\ Z_+^{\rho\phi}(\boldsymbol{\rho}) &= Z_+^{\phi\rho}(\boldsymbol{\rho}) = 0 \\ Z_+^{\phi\phi}(\boldsymbol{\rho}) &= Z_+^{\rho\rho}(\boldsymbol{\rho}) \end{aligned} \quad (7.54)$$

where: $X_0 = 0.71 \eta_0$, with η_0 being the free-space impedance, and $d = \lambda / \sqrt{1 + (X_0/\eta_0)^2}$. The frequency of operation is 17 GHz. The MTS radius is fixed to $a = 5.65\lambda$ and the sheet impedance lays on the substrate ROGERS 4350B of relative permittivity $\epsilon_r = 3.66$ and $\tan \delta = 0.0037$.

7.5.1 In absence of losses

First, we neglect the substrate losses and we compute the conversion and aperture efficiencies versus the modulation index M (see Fig. 7.2). As expected, for relatively low modulation depths, the conversion efficiency is small. However, the conversion efficiency is not zero at $M = 0$ due to the space-waves directly radiated by the feed. Moreover, the aperture efficiency is also relatively small for small M because the aperture efficiency takes also into account the space-waves of the feeder. For small modulation depths, the radiation contribution is dominated by those space waves, which are not present at broadside because the feeder is a TM vertical dipole. Increasing the modulation depth, the conversion efficiency improves, but the aperture efficiency gets to a maximum at approximately $M = 0.2$. From there, the aperture efficiency starts to decrease because the uniformity of the aperture illumination rapidly degrades. The compound efficiency (defined in Section 7.4.5) is maximum at $M \approx 0.3$.

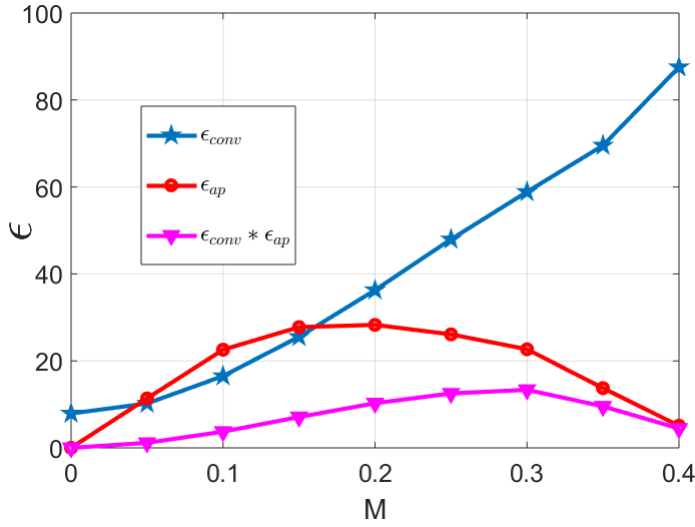


Figure 7.2: Efficiencies w.r.t. modulation index in absence of losses.

For validation purposes, the conversion efficiency has been computed at $M = 0.27$ using the FEKO commercial software [78], and the obtained result is $\epsilon_{conv} \approx 59\%$. With our method, we got $\epsilon_{conv} \approx 53\%$. Differences may be explained by the fact that, in our case, we did not exclude the portion of the MTS occupied by the feeder and by possible inaccuracies in the rim power evaluation due to the zero value of the FBBF basis/testing functions at the MTS rim.

To further validate the results, we have computed the rim power (7.15), which is the most complex part in the efficiency calculation, as a function of the radius, in two different ways. The first method is the one explained in Section 7.3.2. The second method consists of defining at each layer z of interest, an interaction matrix between the surface current of the MoM and the electric or the magnetic field. From there, the tested fields are computed and the projected fields are obtained through renormalization, similarly to the technique presented in Section 7.3.3 concerning the Ohmic power losses. Note that this approach cannot be used to evaluate the rim power exactly at $\rho = a$ because the FBBFs (also used as testing functions) are exactly equal to zero at $\rho = a$. Fig. 7.3 shows a comparison between both methods

for $M = 0.27$. One can observe that in the average, the two methods give

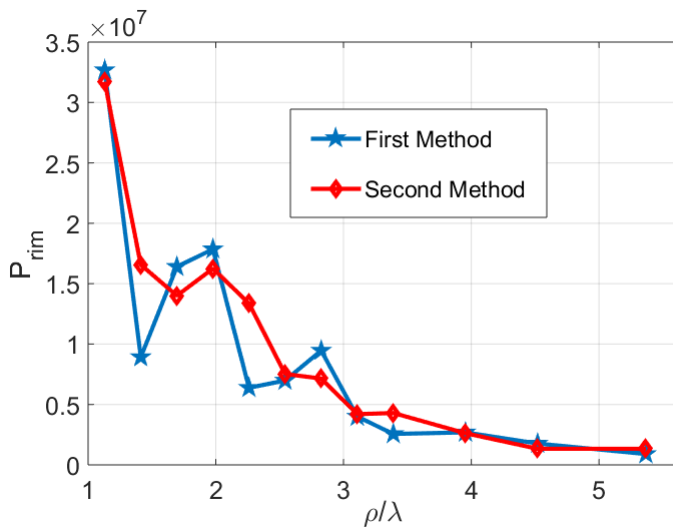


Figure 7.3: Comparison of the rim power traveling the MTS with two methods.

similar results. Differences may be explained by numerical errors originating from the projection from Cartesian unit vectors to polar unit vectors in the alternative method presented here. The r.m.s relative error is about 29%, which translates into an error of about 10% on the efficiency evaluation, in this case $(53 \pm 5)\%$. A finer estimation will be looked for using more basis functions.

7.5.2 In presence of losses

Now, we consider a lossy substrate with $\tan \delta = 0.0037$. Fig. 7.4 depicts the efficiency w.r.t. M . The obtained trend is similar to the lossless case. However, it is observed that the losses are very low (about 5% of the total power delivered by the feed) and relatively constant w.r.t M . This constitutes one of the main advantages of modulated MTS antennas. However, at high frequencies (≥ 30 GHz), the losses in the metallizations become non negligible and should also be taken into account. This issue is not treated in the

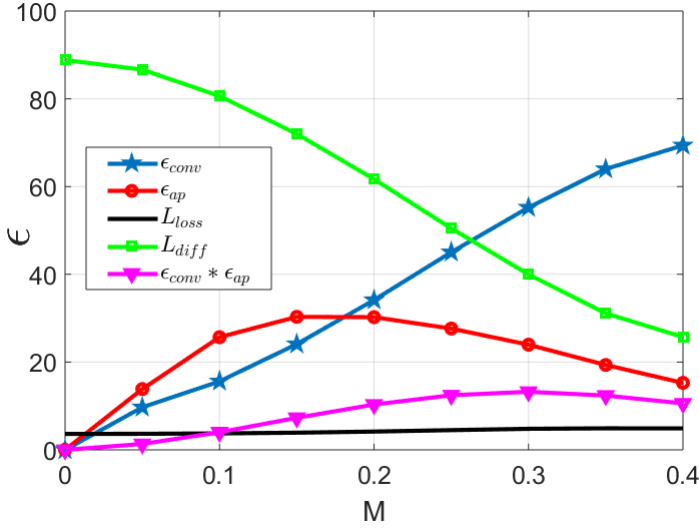


Figure 7.4: Efficiencies w.r.t. modulation index using a lossy substrate.

present thesis. The metal losses can be computed for example by defining an equivalent lossy (i.e non anti-Hermitian) sheet impedance tensor and a lossy ground plane.

7.6 Conclusion

An efficient full-wave method has been developed to compute the power balance in MTS antennas implementing arbitrary surface impedance modulations. The results yield more physical insights on the physics involved in those antennas. The method proposed here is not limited to azimuthally symmetrical feeders. Indeed, any feed excitation can be decomposed in Fourier harmonics along the azimuthal direction, thus allowing analytical azimuthal integration of the power as for the symmetrical feed used in this chapter. The chapter also highlights the compromise that should be made between the aperture efficiency and the conversion efficiency in order to get the best solution for a given application. In this sense, this work provides a useful tool for the design of MTS antennas.

Conclusion and Future Works

8.1 Conclusion

This thesis aimed at providing efficient algorithms for the analysis and design of modulated metasurface (MTS) antennas.

The numerical simulation of MTS devices is very challenging, due to the large number of electrically small printed elements. Given the electrically small size of the scatterers immersed in a locally periodic lattice, the fields in the cell allotted to each element can be averaged (homogenized), and the actual MTS structure can, thus, be represented by an equivalent anisotropic impedance boundary condition (IBC). The IBC scattering has been analyzed in a Method of Moments (MoM) formalism through the discretization of the electric field integral equation (EFIE) with appropriate basis functions. In the methods already proposed in the literature, local basis functions (RWG) [67], or Gaussian type basis functions are used to provide a good localization of rapid changes in the aperture surface currents [61]. Instead, the basis functions suggested in this thesis (Fourier-Bessel) have a relatively homogeneous level over the whole MTS domain, which allows a good distinction between different “modes” of propagation over the whole structure. Consequently, Fourier-Bessel basis functions (FBBF) provide a very compact representation of the surface current distribution. Orthogo-

nality between the FBBF modes further enables to speed up the reaction integrals computation in the MoM. Numerical results from the proposed method are validated in Chapter 2 with other simulation software and experimental data associated with specific MTS antenna devices, showing a very good accuracy.

In most of the cases presented in the literature, modulated MTS antennas implement a capacitive sheet impedance on top of an inductive substrate. The overall structure exhibits at resonance an inductive behavior, sinusoidally modulated around an average. The fundamental radiation characteristics of such apertures have been addressed by Oliner and Hessel several decades ago [46]. The synthesis methods proposed so far are therefore inspired from the Oliner formulation and exploit a locally sinusoidal tensorial surface impedance modulation [43], [48], [52], [53]. Those works have demonstrated impressive performances, in particular for the generation of high-gain polarized beams. However, all those references approximate locally the waves-MTS interaction with the Oliner formulation.

This thesis has introduced for the first time a class of tensorial impedance, not necessarily locally sinusoidal. The developed synthesis technique is also no longer inspired from the Oliner problem, but from the integral equation formalism. In that approach, the impedance profile is expanded into Fourier-Bessel functions, which form a complete base. The inverse problem based on the EFIE is addressed and numerically solved to derive the MTS properties on the basis of the aperture current distribution. This allows us to formulate the synthesis problem as follows. First, the desired radiating current distribution is derived based on the radiation pattern specifications in amplitude, phase and polarization. This desired current is then combined with an estimate of the non-radiating (invisible) current on the antenna to form the objective surface current. The inverse problem based on the EFIE is then solved to derive the corresponding impedance profile. However, the resulting impedance is not necessarily anti-Hermitian and may require losses or even active elements on the MTS. This problem is tackled during the solution of the EFIE by constraining, in a least-squares sense, the surface impedance to belong to an anti-Hermitian space. The resulting algorithm is, first, systematic and starts directly from the desired radiation pattern. Sec-

ond, optimization procedures are not required and arbitrarily shaped beams can be designed on a 10λ MTS radius device in less than 20 minutes. Since, the class of synthesized impedance distribution is wider than that proposed in the literature, the class of achievable radiation pattern is also wider, as demonstrated in Chapter 3.

Next, reconfigurable MTS designs are addressed in the integral equation formalism, either by imposing some symmetries on the MTS or by over-constraining the resulting system of equations. In this framework, we have demonstrated multiple-fed MTS antennas with multiple beams, dual-polarized MTS antennas, and multi-frequency band operation. The latter functionality can be used to integrate on the same MTS platform, several applications (with different radiation pattern specifications), as required for the next generation of communication and sensing systems. The thesis has also introduced a new continuous beam scanning paradigm: beam scanning is obtained through a proper design (in amplitude, phase, and polarization) of the embedded element patterns associated with several feeds sharing the same MTS platform. Phasing each embedded element pattern with respect to the others then allows one to achieve continuous beam scanning over a limited sector. This idea has been demonstrated in Section 4.1.2 with two feeds and has allowed beam scanning over a 3 dB beam width.

In Chapter 5, it has been proven that the synthesized class of impedance profile can be easily implemented with anisotropic printed patches (such as the coffee bean patches used in [50]). The excellent agreement between the IBC-based simulation and the full-wave simulation of the physically implemented MTSs shows the practical applicability of the developed synthesis methods. A shaped beam radar antenna has also been developed and experimentally validated.

Chapter 6 has been devoted to near-field manipulation by MTS. Two examples have been presented and numerically validated. The first example concerns the generation of Bessel beams, which find applications in near-field focusing. In the second example, multiple OAM beams are generated for near-field multiplexing. Free-space transmittance (power transfer) between apertures supporting Fourier-Bessel fields is also analyzed.

Finally, Chapter 7 provides an efficient algorithm for the evaluation of MTS antennas efficiency and highlights the compromise that should be made between the conversion efficiency and the aperture efficiency, when designing MTS antennas.

8.2 Future Works

Although significant progress has been made through the present thesis, important challenges remain to be addressed to face the specifications of the next-generation of communication systems.

8.2.1 Efficient analysis and design of the feeder

Throughout this thesis, the MTS feed has been modeled as an infinitesimal vertical dipole. This assumption is not always accurate in practice and does not provide the antenna impedance. This calls for an accurate modeling of the feeder, taking into account the coupling with the MTS. Such analysis tool should allow one to optimize the feed so as to minimize the power directly radiated as space waves. The accurate feeding analysis will also provide a better aperture field excitation for the proposed MTS synthesis method. It is worth mentioning that this synthesis method can cope with a complex feeder, provided that the aperture field generated by the feed is known a priori.

8.2.2 Dual polarized/multiband/wideband shaped beams

Dual-polarization, multiband, and wideband operation limit the capability of MTS antennas in terms of beam shaping and aperture efficiency. In space applications for example, dual-band isoflux patterns are required. This problem may be tackled in a multi-layer MTS configuration.

8.2.3 Large angle beam scanning

Continuous azimuthal beam scanning has been demonstrated in Section 4.1.2 with two feeds. However, extending this idea to more than two feeds may enable a wider beam scanning range.

8.2.4 Conformal metasurface antennas

Conformability is an appealing functionality since it makes the antenna profile lower. However, conformal MTSs design is still a huge challenge and may require a deep theoretical effort to get a better understanding of SW excitation along curved substrates.

APPENDIX A

Additional proofs and relations

A.1 Fourier-Bessel orthogonality

The Bessel functions satisfy the orthogonality relation (for the same n index) on the unit disk:

$$\int_0^1 F_{mn}(\rho_0) F_{m'n}(\rho_0) \rho_0 d\rho_0 = -0.5 \delta_{mm'} J_{n-1}(\lambda_n^m) J_{n+1}(\lambda_n^m). \quad (\text{A.1})$$

After adding the azimuthal term, the Fourier-Bessel functions become fully orthogonal and the orthogonality relation is expressed as

$$\begin{aligned} \int_0^{2\pi} \int_0^1 R_{mn}(\rho_0, \phi) R_{m'n'}^*(\rho_0, \phi) \rho_0 d\rho_0 d\phi = -\pi \delta_{mm'} \delta_{nn'} \\ \times J_{n-1}(\lambda_n^m) J_{n+1}(\lambda_n^m). \end{aligned} \quad (\text{A.2})$$

A.2 Fourier-Bessel Hankel transform

The Fourier transform of (2.3) in cylindrical coordinates is

$$\tilde{R}_{m,n}(k_\rho, \alpha) = \int_0^{2\pi} \int_0^a J_n\left(\lambda_n^m \frac{\rho}{a}\right) e^{-jn\phi} e^{jk_\rho \rho \cos(\alpha-\phi)} \rho d\rho d\phi \quad (\text{A.3})$$

Based on identity (4.7), integral (A.3) can be analytically solved along the azimuthal coordinate ϕ , which leads to

$$\tilde{R}_{m,n}(k_\rho, \alpha) = 2\pi j^n e^{-jn\alpha} \int_0^a J_n\left(\lambda_n^m \frac{\rho}{a}\right) J_n(k_\rho \rho) \rho d\rho \quad (\text{A.4})$$

Now, using the identity

$$\int J_n(ax) J_n(bx) x dx = \frac{x(bJ_n(ax)J_{n-1}(bx) - aJ_{n-1}(ax)J_n(bx))}{a^2 - b^2} \quad (\text{A.5})$$

The hankel transform of Fourier-Bessel basis functions (A.4) is given by

$$\tilde{R}_{m,n}(k_\rho, \alpha) = -2\pi j^n e^{-jn\alpha} \left[\frac{a^2 \lambda_n^m J_{n-1}(\lambda_n^m) J_n(k_\rho a)}{(\lambda_n^m)^2 - (k_\rho a)^2} \right] \quad (\text{A.6})$$

A.3 Projection matrix

The Q^β matrix is given as:

$$Q^\beta(m, n; m', n') = \frac{j\pi a^2}{K(m, n)} (\delta_{n, n'+1} - \delta_{n, n'-1}) \int_0^1 F_{m,n}(\rho) F_{m',n'}(\rho) \rho d\rho, \quad (\text{A.7})$$

where $K(m, n)$ is obtained using expression (2.39) and

$$Q^\gamma(m, n; m', n') = -Q^\beta(m, n; m', n'). \quad (\text{A.8})$$

A.4 Substrate Green's functions at the current layer

Derivation procedure of the spectral Green's functions in layered media can be found in [119, Appendix A], [72] - [74]. This section simply provides the relevant Green's functions used for the substrate matrix computation in section 2.1.2. The substrate scalar potential Green's functions used in (A.13) is given by:

$$\tilde{G}_A = -jk_0\eta_0 \frac{\sin(k_{z,1}d)}{T_e} \quad (\text{A.9})$$

$$\tilde{G}_V = j \frac{\eta_0}{k_0} \frac{\sin(k_{z,1}d)}{k_\rho^2} \left(\frac{k_0^2}{T_e} - \frac{k_{z,1}k_{z,2}}{T_m} \right) \quad (\text{A.10})$$

with

$$T_e = jk_{z,2} \sin(k_{z,1}d) + k_{z,1} \cos(k_{z,1}d) \quad (\text{A.11})$$

$$T_m = \epsilon_{r,1}k_{z,2} \cos(k_{z,1}d) + j\epsilon_{r,2}k_{z,1} \sin(k_{z,1}d) \quad (\text{A.12})$$

Here, index 1 and 2 correspond respectively to the substrate and the free-space (see Fig. A.1). The dyadic spectral electric Green's function of the

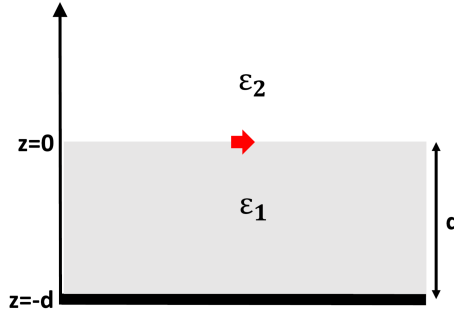


Figure A.1: Layered media configuration for the MTS substrate.

grounded slab can then be written as follows:

$$\begin{bmatrix} \tilde{G}_{xx}^{EJ} & \tilde{G}_{xy}^{EJ} \\ \tilde{G}_{yx}^{EJ} & \tilde{G}_{yy}^{EJ} \\ \tilde{G}_{zx}^{EJ} & \tilde{G}_{zy}^{EJ} \end{bmatrix} = \tilde{G}_A(k_\rho) \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{k_x}{k_z} & -\frac{k_y}{k_z} \end{bmatrix} + \tilde{G}_V(k_\rho) \begin{bmatrix} k_x^2 & k_x k_y \\ k_x k_y & k_y^2 \\ -\frac{k_x k_\rho^2}{k_z} & -\frac{k_y k_\rho^2}{k_z} \end{bmatrix} \quad (\text{A.13})$$

The spectral magnetic field Green's function of the grounded slab looking from the top of the surface can be easily derived from that of the electric field, using the plane wave fields representation ([119, Sec. A.3.1]) and is given below

$$\tilde{G}_{xx}^{HJ} = -\frac{k_x k_y}{\eta} \left[\frac{\tilde{G}_A}{k k_z} + \frac{k \tilde{G}_V}{k_z} \right] \quad (\text{A.14})$$

$$\tilde{G}_{yy}^{HJ} = -\tilde{G}_{xx}^{HJ} \quad (\text{A.15})$$

$$\tilde{G}_{xy}^{HJ} = -\frac{1}{\eta} \left[k_x^2 \frac{k_z \tilde{G}_A}{k_\rho^2 k} + k_y^2 \left(\frac{k \tilde{G}_A}{k_\rho^2 k_z} + \frac{k \tilde{G}_V}{k_z} \right) \right] \quad (\text{A.16})$$

$$\tilde{G}_{yx}^{HJ} = \frac{1}{\eta} \left[k_y^2 \frac{k_z \tilde{G}_A}{k_\rho^2 k} + k_x^2 \left(\frac{k \tilde{G}_A}{k_\rho^2 k_z} + \frac{k \tilde{G}_V}{k_z} \right) \right] \quad (\text{A.17})$$

One should note that the z-component of the electric field as well as the magnetic field as derived are valid only looking the surface from the top and are not continuous through the surface.

A.5 Excitation spectral Green's functions

Let us consider a vertical source placed in a layered media as represented in Fig. A.2. The source layer corresponds to the plane $z = 0$. The total electric field in the source region (between $z = d_1$ and $z = -d_{-1}$) can be written in spectral domain as:

$$\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_i + \tilde{\mathbf{E}}_1 + \tilde{\mathbf{E}}_{-1} \quad (\text{A.18})$$

with:

$$\tilde{\mathbf{E}}_i = \tilde{E}_i e^{-jk_z|z|} \hat{\mathbf{e}}^\pm \quad (\text{A.19})$$

$$\tilde{\mathbf{E}}_1 = \tilde{E}_1 e^{+jk_z z} \hat{\mathbf{e}}^- \quad (\text{A.20})$$

$$\tilde{\mathbf{E}}_{-1} = \tilde{E}_{-1} e^{-jk_z z} \hat{\mathbf{e}}^+ \quad (\text{A.21})$$

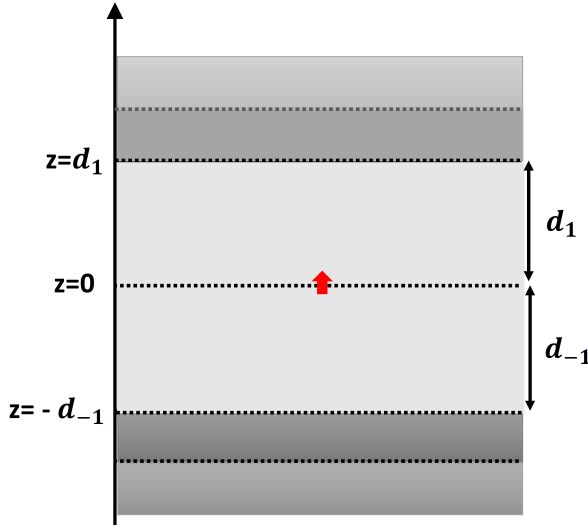


Figure A.2: Layered media configuration for the excitation fields.

where $\hat{e}^\pm$ is the TM unit vector defined as

$$e^\pm = \frac{1}{k_\rho k} \begin{bmatrix} \pm k_x k_z \\ \pm k_y k_z \\ -k_\rho^2 \end{bmatrix} \quad (\text{A.22})$$

The subscripts + and - denote respectively waves going upward and downward. $\tilde{E}_i$ is the homogeneous media scalar Green's function given by

$$\tilde{E}_i = \frac{k_\rho \eta}{2k_z} \quad (\text{A.23})$$

where $\eta = \sqrt{\mu/\epsilon}$ is the impedance in the source layer.

Imposing the boundary condition at interfaces 1 and -1 (corresponding respectively to $z = d_1$ and $z = -d_{-1}$) allows to express $\tilde{E}_1$ and $\tilde{E}_{-1}$ as function of $\tilde{E}_i$

$$\tilde{E}_1 = \frac{1 + \Gamma_{-1}^u e^{2jk_z d_{-1}}}{-1 + \Gamma_{-1}^u \Gamma_1^d e^{2jk_z (d_1 + d_{-1})}} \tilde{E}_i = \frac{\Gamma_0^u (1 + \Gamma_0^d)}{(1 - \Gamma_0^d \Gamma_0^u)} \tilde{E}_i \quad (\text{A.24})$$

$$\tilde{E}_{-1} = \frac{\Gamma_0^d(1 + \Gamma_0^u)}{(1 - \Gamma_0^d\Gamma_0^u)} \tilde{E}_i \quad (\text{A.25})$$

where the coefficients Γ_i^j are the reflection coefficients at interface i as defined in Appendix A of [119]. Since $\tilde{E}_i$, $\tilde{E}_1$, and $\tilde{E}_{-1}$ depend only on k_ρ , and not explicitly on k_x and k_y , the x and y directed Green's function can be written in the form:

$$\tilde{G}_{xz}^{EJ}(\mathbf{k}) = \cos(\alpha) \tilde{f}(k_\rho); \quad \tilde{G}_{yz}^{EJ}(\mathbf{k}) = \sin(\alpha) \tilde{f}(k_\rho) \quad (\text{A.26})$$

Similarly to the electric field, the magnetic field is given by

$$\widetilde{\mathbf{H}} = \widetilde{\mathbf{H}}_i + \widetilde{\mathbf{H}}_1 + \widetilde{\mathbf{H}}_{-1} \quad (\text{A.27})$$

with

$$\widetilde{\mathbf{H}}_i = \frac{\tilde{E}_i}{\eta} e^{-jk_z|z|} \hat{\mathbf{m}} \quad (\text{A.28})$$

$$\widetilde{\mathbf{H}}_1 = \frac{\tilde{E}_1}{\eta} e^{+jk_z z} \hat{\mathbf{m}} \quad (\text{A.29})$$

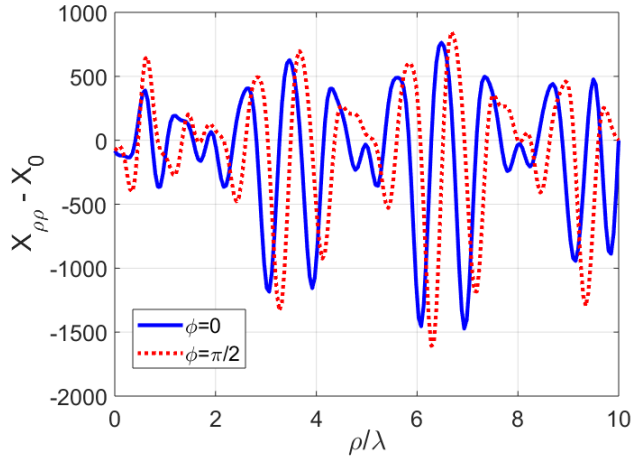
$$\widetilde{\mathbf{H}}_{-1} = \frac{\tilde{E}_{-1}}{\eta} e^{-jk_z z} \hat{\mathbf{m}} \quad (\text{A.30})$$

where $\hat{\mathbf{m}}$ is the TE unit vector defined as

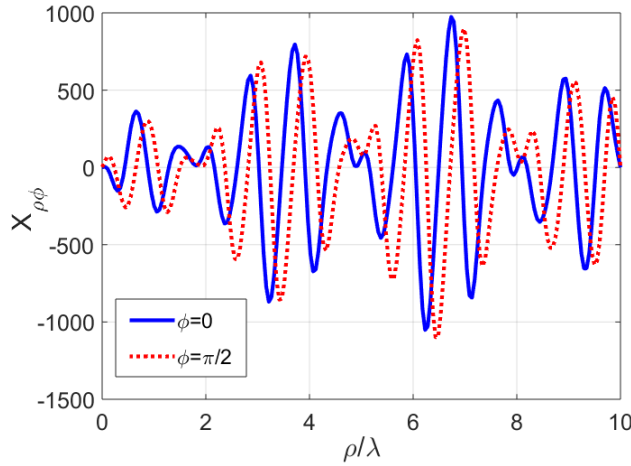
$$\hat{\mathbf{m}} = \frac{1}{k_\rho} \begin{bmatrix} -k_y \\ k_x \\ 0 \end{bmatrix} \quad (\text{A.31})$$

A.6 Additional 2D figures

To help for reproducibility purposes, additional data are provided below representing cuts of some impedances computed in Chapter 4.

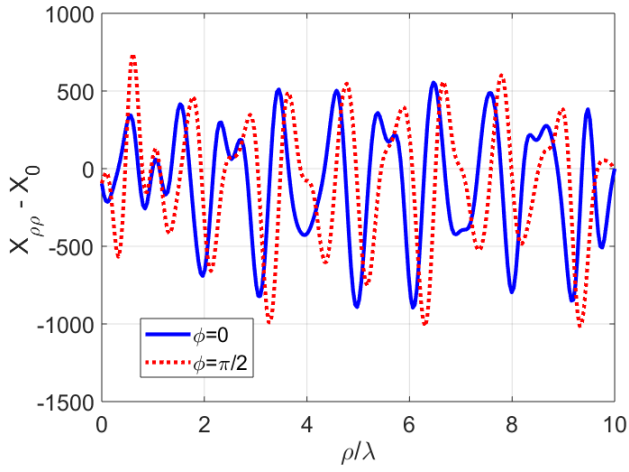


(a)

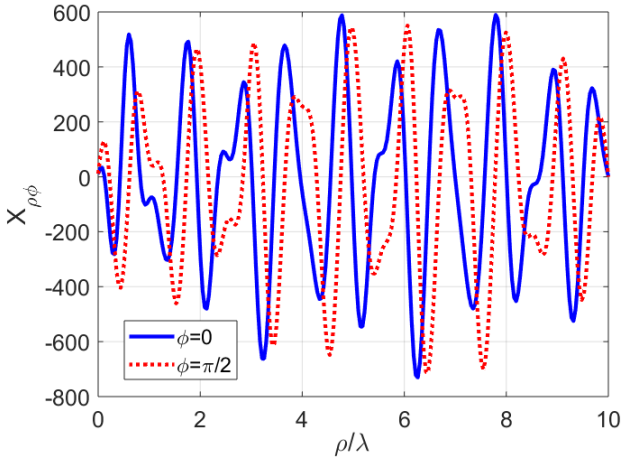


(b)

Figure A.3: Cuts of a modulated sheet impedance for a MTS antenna radiating a broadside pencil beam at 20 GHz and 24 GHz (Computed in section 4.2.2). (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.



(a)



(b)

Figure A.4: Cuts of a modulated sheet impedance for a MTS antenna radiating a broadside pencil beam at 20 GHz and a conical beam at 24 GHz (computed in Section 4.2.2). (a) $X_{\rho\rho} - X_0$. (b) $X_{\rho\phi}$.

A.7 Fourier-Bessel fields transmittance

Starting from the asymptotic approximation of the Bessel functions (valid for high value of the argument):

$$J_n(x) \approx \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right), \quad (\text{A.32})$$

the m -th positive zero of $J_n(x)$ can be approximated (for high values of m) as:

$$\lambda_{nm} \approx \frac{\pi}{2}(n+1) + \frac{\pi}{4} + (m-1)\pi. \quad (\text{A.33})$$

After inserting (A.33) in the asymptotic form of the Bessel function (A.32), we get:

$$\begin{aligned} J_{n-1}(\lambda_{nm}) &\approx \sqrt{\frac{2}{\pi \lambda_{nm}}} \cos\left(\frac{\pi}{2}(n+1) + \frac{\pi}{4} + (m-1)\pi - \frac{(n-1)\pi}{2} - \frac{\pi}{4}\right) \\ &= \sqrt{\frac{2}{\pi \lambda_{nm}}} \cos(\pi + (m-1)\pi) \\ &= \sqrt{\frac{2}{\pi \lambda_{nm}}} (-1)^m. \end{aligned} \quad (\text{A.34})$$

Now, the dependence of the transmittance on m (expression between brackets in (6.17)) can be written as:

$$\mathcal{H}_m = \frac{2}{\pi} \frac{1}{(\lambda_{nm} \lambda_{nm'})^{\frac{3}{2}} n!} (-1)^{m+m'}. \quad (\text{A.35})$$

Finally, using the approximation (A.33) of λ_{nm} , for high values of m in comparison with n , the dependence on those parameters are $m^{-3/2}$ and $(m')^{-3/2}$.

A.8 Link between transmittance and power transfer

Let us consider two antennas. The first one as the transmitter and the second one, the receiver (see Fig. A.5). Referring to the Poynting theorem,

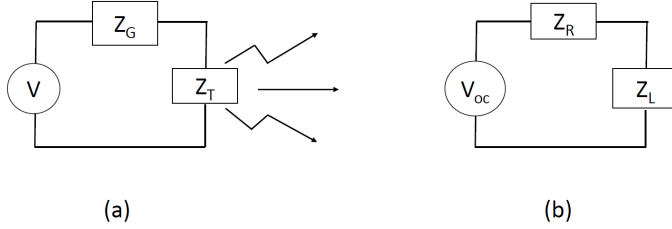


Figure A.5: Monoport modeling of two antennas. a): the transmitting antenna with impedance Z_T connected to a voltage source V , with Z_G being the source impedance. b): the receiving antenna with impedance Z_R on which an impinging electric-field modeled with a voltage source V_{oc} , and Z_L being the load impedance.

the transmitted power is defined as:

$$P_T = \frac{1}{2} R_e \{ V_T I_T^* \}, \quad (\text{A.36})$$

where R_e stands for the real part, $I_T = V / (Z_G + Z_T)$ and V_T are respectively the current and the voltage at the transmitted antenna port. Expression (A.36) can then be rewritten as:

$$P_T = \frac{1}{2} R_e \{ Z_T \} \frac{|V|^2}{|Z_T + Z_G|^2}. \quad (\text{A.37})$$

The available power P_{av} , i.e. the power delivered by the source can be obtained from (A.37), after inserting the matching condition $Z_T = Z_G^*$, which leads to:

$$P_{av} = \frac{1}{8} \frac{|V|^2}{R_e \{ Z_G \}} = \frac{1}{8} \frac{|I_T|^2 |Z_T + Z_G|^2}{R_e \{ Z_G \}}. \quad (\text{A.38})$$

The current $|I_T|$ at the transmitted antenna port can then be written as

$$|I_T|^2 = 8 P_{av} \frac{R_e \{ Z_G \}}{|Z_T + Z_G|^2}. \quad (\text{A.39})$$

Similarly, the received power on the load is:

$$P_R = \frac{1}{2} R_e \{Z_L\} \frac{|V_{oc}|^2}{|Z_R + Z_L|^2} = \frac{1}{2} \frac{R_e \{Z_L\}}{|Z_R + Z_L|^2} \left[\frac{|V_{oc}|^2}{|I_T|^2} \right] |I_T|^2. \quad (\text{A.40})$$

Expression $|V_{oc}|^2/|I_T|^2$ between the brackets is the square of the absolute value of the transmittance $|H|^2$. After replacing expression of $|I_T|^2$ given in (A.39), we can then rewrite (A.40) as

$$P_R = 4|H|^2 P_{av} \frac{R_e \{Z_L\}}{|Z_R + Z_L|^2} \frac{R_e \{Z_G\}}{|Z_T + Z_G|^2}. \quad (\text{A.41})$$

Thereafter, the following identities can be easily proven:

$$\frac{R_e \{Z_L\}}{|Z_R + Z_L|^2} = \frac{1 - |\Gamma_R|^2}{4R_e \{Z_R\}}, \quad (\text{A.42})$$

$$\frac{R_e \{Z_G\}}{|Z_T + Z_G|^2} = \frac{1 - |\Gamma_T|^2}{4R_e \{Z_T\}}, \quad (\text{A.43})$$

where $\Gamma_R = (Z_L - Z_R^*)/(Z_R + Z_L)$ and $\Gamma_T = (Z_T - Z_G^*)/(Z_T + Z_G)$ are the reflection coefficients.

Inserting (A.42) and (A.43) into (A.41), the received power P_R can be related to the available power P_{av} as follows:

$$P_R = \frac{1}{4} |H|^2 P_{av} (1 - |\Gamma_T|^2)(1 - |\Gamma_R|^2) \frac{1}{R_e \{Z_T\} R_e \{Z_R\}}. \quad (\text{A.44})$$

This expression allows us to link the received and the available power with the transmittance given in (6.5).

A.9 Computation of $\underline{\underline{Z}}_G^H$

The dyadic spectral Green's function relating the surface current to the magnetic field can be written in the form

$$\begin{bmatrix} \tilde{G}_{xx}^{HJ} & \tilde{G}_{xy}^{HJ} \\ \tilde{G}_{yx}^{HJ} & \tilde{G}_{yy}^{HJ} \end{bmatrix} = \begin{bmatrix} \cos \alpha \sin \alpha \tilde{g}_{xx}^H(k_\rho) & \cos^2 \alpha \tilde{g}_{xy1}^H(k_\rho) + \sin^2 \alpha \tilde{g}_{xy2}^H(k_\rho) \\ -\cos^2 \alpha \tilde{g}_{xy2}^H(k_\rho) - \sin^2 \alpha \tilde{g}_{xy1}^H(k_\rho) & -\cos \alpha \sin \alpha \tilde{g}_{xx}^H(k_\rho) \end{bmatrix} \quad (\text{A.45})$$

Following the same reasoning and using the same notations as in Section 2.1.2, each element of $\underline{\underline{Z}}_G^H$ is obtained as

$$Z_G^{Hxx}(m, n; m', n') = (-j)^n j^{n'} a^4 \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{F}_{m',n'}(k_\rho a) \tilde{g}_{xx}^H \times \epsilon_{n,n'}^{sc} k_\rho dk_\rho \quad (\text{A.46})$$

$$Z_G^{Hyy}(m, n; m', n') = -Z_G^{Hxx}(m, n; m', n') \quad (\text{A.47})$$

$$Z_G^{Hxy}(m, n; m', n') = (-j)^n j^{n'} a^4 \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{F}_{m',n'}(k_\rho a) \times \left[\tilde{g}_{xy1}^H \epsilon_{n,n'}^{cc} + \tilde{g}_{xy2}^H \epsilon_{n,n'}^{ss} \right] k_\rho dk_\rho \quad (\text{A.48})$$

$$Z_G^{Hyx}(m, n; m', n') = (-j)^n j^{n'} a^4 \int_0^\infty \tilde{F}_{m,n}(k_\rho a) \tilde{F}_{m',n'}(k_\rho a) \times \left[-\tilde{g}_{xy1}^H \epsilon_{n,n'}^{ss} - \tilde{g}_{xy2}^H \epsilon_{n,n'}^{cc} \right] k_\rho dk_\rho \quad (\text{A.49})$$

where $\epsilon_{n,n'}^{sc}$, $\epsilon_{n,n'}^{cc}$, and $\epsilon_{n,n'}^{ss}$ have been defined in section 2.1.2.

A.10 Expressions of H_{xy} , H_{yx} , H_{yy} , H_{xz} , and H_{yz}

$$H_{xy1} = \frac{1}{4\pi^2} \sum_{m,n} i_{mn}^y \int_0^\infty \left[f_{mn}^g(k_\rho \rho, \phi, 0) + f_{mn}^h(k_\rho \rho, \phi, -2) + f_{mn}^i(k_\rho \rho, \phi, 2) \right] k_\rho dk_\rho \quad (\text{A.50})$$

with:

$$f_{mn}^g(k_\rho \rho, \phi, 0) = f_n^0(k_\rho \rho, \phi, 0) \tilde{g}_{xy1}^H \tilde{F}_{mn} \quad (\text{A.51})$$

$$f_{mn}^h(k_\rho \rho, \phi, -2) = -\frac{1}{2} f_n^0(k_\rho \rho, \phi, -2) \tilde{g}_{xy1}^H \tilde{F}_{mn} \quad (\text{A.52})$$

$$f_{mn}^i(k_\rho \rho, \phi, 2) = -\frac{1}{2} f_n^0(k_\rho \rho, \phi, 2) \tilde{g}_{xy1}^H \tilde{F}_{mn} \quad (\text{A.53})$$

Similarly:

$$H_{xy2} = \frac{1}{4\pi^2} \sum_{m,n} i_{mn}^y \int_0^\infty \left[f_{mn}^j(k_\rho \rho, \phi, 0) + f_{mn}^k(k_\rho \rho, \phi, -2) + f_{mn}^l(k_\rho \rho, \phi, 2) \right] k_\rho dk_\rho \quad (\text{A.54})$$

with:

$$f_{mn}^j(k_\rho \rho, \phi, 0) = f_n^0(k_\rho \rho, \phi, 0) \tilde{g}_{xy2}^H \tilde{F}_{mn} \quad (\text{A.55})$$

$$f_{mn}^k(k_\rho \rho, \phi, -2) = \frac{1}{2} f_n^0(k_\rho \rho, \phi, -2) \tilde{g}_{xy2}^H \tilde{F}_{mn} \quad (\text{A.56})$$

$$f_{mn}^l(k_\rho \rho, \phi, 2) = \frac{1}{2} f_n^0(k_\rho \rho, \phi, 2) \tilde{g}_{xy2}^H \tilde{F}_{mn} \quad (\text{A.57})$$

For the x -directed magnetic field contribution of the feed:

$$H_{xz} = \frac{i_{00}^z}{4\pi^2} \int_0^\infty \left[f_{00}^{x1}(k_\rho \rho, \phi, -1) + f_{00}^{x2}(k_\rho \rho, \phi, 1) \right] k_\rho dk_\rho \quad (\text{A.58})$$

with:

$$f_{00}^{x1}(k_\rho \rho, \phi, -1) = -f_0^0(k_\rho \rho, \phi, -1) \tilde{g}_{xz}^H \tilde{F}_{mn} \quad (\text{A.59})$$

$$f_{00}^{x2}(k_\rho \rho, \phi, 1) = -f_0^0(k_\rho \rho, \phi, 1) \tilde{g}_{xz}^H \tilde{F}_{mn} \quad (\text{A.60})$$

For the y -directed magnetic field contribution of the feed:

$$H_{yz} = \frac{i\tilde{z}_{00}}{4\pi^2} \int_0^\infty \left[f_{00}^{y1}(k_\rho\rho, \phi, -1) + f_{00}^{y2}(k_\rho\rho, \phi, 1) \right] k_\rho dk_\rho \quad (\text{A.61})$$

with:

$$f_{00}^{y1}(k_\rho\rho, \phi, -1) = -j f_0^0(k_\rho\rho, \phi, -1) \tilde{g}_{yz}^H \tilde{F}_{mn} \quad (\text{A.62})$$

$$f_{00}^{y2}(k_\rho\rho, \phi, 1) = j f_0^0(k_\rho\rho, \phi, 1) \tilde{g}_{yz}^H \tilde{F}_{mn} \quad (\text{A.63})$$

H_{yy} and H_{yx} have the same structure than H_{xx} and H_{xy} .

APPENDIX B

Awards and Publications

Awards

A1. Best Student Paper Award

13th European Conference on Antennas and Propagation (EUCAP), Krakow, Poland, April 2019.

A2. First Prize Student Paper Competition Award

IEEE MTT-S International Conference on Numerical Electromagnetic, Multiphysics Modeling and Optimization for RF, Microwave, and Terahertz Applications (NEMO), Seville, Spain, May 2017.

Journal Papers

- J1.** M. Bodehou, C. Craeye, and I. Huynen, “Multifrequency band synthesis of modulated metasurface antennas,” *IEEE Antennas and Wireless Propagation Letters*, vol. 19, no. 1, pp. 134-138, Jan. 2020.
- J2.** M. Bodehou, E. Martini, S. Maci, I. Huynen, and C. Craeye, “Multi-beam and beam scanning with modulated metasurfaces,” *IEEE Trans. Antennas Propag.*, vol. 68, no. 3, pp. 1273-1281, March 2020.
- J3.** M. Bodehou, C. Craeye, and I. Huynen, “Electric field integral equa-

- tion based synthesis of elliptical domain metasurface antennas,” *IEEE Trans. Antennas Propag.*, vol. 67, no. 2, pp. 1270-1274, Feb. 2019.
- J4. M. Bodehou**, C. Craeye, E. Martini, and I. Huynen, “A Quasi-direct method for the surface impedance design of modulated metasurface antennas,” *IEEE Trans. Antennas Propag.*, vol. 67, no. 1, pp. 24-36, Jan. 2019.
- J5. M. Bodehou**, C. Craeye, H. Bui-Van, and I. Huynen, “Fourier-Bessel basis functions for the analysis of elliptical domain metasurface antennas,” *IEEE Antennas and Wireless Propagation Letters*, vol. 17, no. 4, pp. 675-678, April. 2018.
- J6. M. Bodehou**, D. González-Ovejero, C. Craeye, and I. Huynen, “Method of Moments simulation of modulated metasurface antennas with a set of orthogonal entire-domain basis functions,” *IEEE Trans. Antennas Propag.*, vol. 67, no. 2, pp. 1119-1130, Feb. 2019.
- J7. M. Bodehou**, and C. Craeye, “Another field decomposition for the transmittance between OAM antennas,” *FERMAT Journal*, vol. 24, no. 1, Nov-Dec. 2017.
- J8. M. Bodehou**, et al., “Power balance in metasurface antennas,” in preparation for submission on *IEEE Trans. Antennas Propag.*
- J9. M. Bodehou**, et al., “Method of Moments as an efficient framework for the synthesis of modulated metasurface antennas: numerical and experimental validations,” under preparation.

Conference Papers in IEEE Proceedings

- C1. M. Bodehou**, D. González-Ovejero, C. Craeye, and I. Huynen, “Dual-band beams generation with metasurface based on the EFIE,” *European Conf. Antennas Propag., EUCAP*, Copenhagen, Denmark, March 2020. (**Convened paper**)
- C2. M. Bodehou**, C. Craeye, and I. Huynen, “Near-field shaping by leaky-waves metasurface,” *Int. Conf. Electromagn. Advanced App., ICEAA*, Granada, Spain, Sept 2019.

- C3.** M. Bodehou, C. Craeye, E. Martini, and S. Maci, “Full-wave synthesis procedure for the design of innovative metasurface devices,” *IEEE Int. Symp. Antennas and Propag. USNC-URSI Radio Science Meeting*, Atlanta, Georgia, July 2019.
- C4.** M. Bodehou, C. Craeye, E. Martini, and I. Huynen, “Full-Wave synthesis of modulated metasurface antennas,” *European Conf. Antennas Propag., EUCAP*, Krakow, Poland, April 2019.
- C5.** M. Bodehou, C. Craeye, S. Hubert, H. Ali Kayani, and I. Huynen, “Analysis of elliptical aperture metasurface antennas,” *Advanced Electromagn. Materials in Microwaves and Optics (Metamaterials)*, Espoo, Finland, Sept. 2018.
- C6.** S. Hubert, M. Bodehou, and C. Craeye, “Fast spectral iterative technique for the analysis of planar metasurface-type structures,” *European Conf. Antennas Propag., EUCAP*, London, United Kingdom, April 2018.
- C7.** M. Bodehou, D. González-Ovejero, C. Craeye, and I. Huynen, “Numerical analysis of modulated metasurface antennas using Fourier-Bessel basis functions,” *IEEE MTT-S Int. Conf. Numer. Electromagn. Multiphys. Modeling Optim., for RF, Microwave, and THz App. (NEMO)*, Seville, Spain, May 2017.
- C8.** M. Bodehou, and T. Gilles, “Cosecant squared pattern synthesis of linear antenna arrays based on an adaptive least square algorithm,” *IEEE MTT-S Int. Conf. Numer. Electromagn. Multiphys. Modeling Optim., for RF, Microwave, and THz App. (NEMO)*, Sevilla, Spain, May 2017.
- C9.** T. Gilles, and M. Bodehou, “Beamforming for base station GSM antennas,” *IEEE RADIO and Antennas Days of the Indian Ocean*, Oct 2016.

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