Regulating Quality by Regulating Quantity:
a Case Against Minimum Quality Standards

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Abstract

We show in a simple model of entry with sunk cost, that a regulator is best advised to limit the output or capacity of the incumbent firm rather than impose a general Minimum Quality Standard in order to maximize industry welfare. The quota amounts to protect the entrant (or low quality firm) from price competition. As a consequence it becomes more profitable to sink money into quality upgrades. As a by-product, our analysis makes a contribution to the study of Bertrand-Edgeworth competition in a market with differentiated products that extends and confirms Krishna (1989) for our particular model of duopolistic competition.

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JEL codes: D43, L13, L51

1   Introduction

The fact that consumers display a high willingness to pay for quality should be a strong driver for competitive firms to constantly improve quality. Yet, Sheshinski (1976) has shown that the optimal quality selection by a monopolist rarely coincides with the efficient one, although the direction in which it departs it is not always clear. In oligopolistic industries, a large literature starting with Gabszewicz and Thisse (1979) has shown the possibility of opting for a low quality when facing a high quality incumbent in order to relax market competition. Adding to this picture the fact that quality is also desirable from a social point of view,[1] we better understand the interventionism of authorities, most often in the form of Minimum Quality Standards (MQS hereafter).

As nicely demonstrated by Ronnen (1991), the adequate selection of a MQS can increase both quality and sales so that the industry welfare unambiguously increases. The intuition for this positive result is quite simple: by constraining the low quality firm to upgrade its quality, the MQS induces the high quality firm to select a higher quality (in order to relax competition). In equilibrium, the price competition is however fiercer so that prices are lower and more consumers end up participating. Crampes and Hollander (1995) establish a qualitatively similar result with a different costs structure.

These two papers obviously make a case for MQS but their conclusions might be challenged on several grounds. Firstly the issue of certification that inevitably goes along with MQS,[2] is neglected. In this respect, Albano and Lizzeri (2001) show that certification does not go without inefficiencies: although certification intermediaries tend to raise firms' incentives to provide quality, they are likely to fail in avoiding quality underprovision. Secondly, the MQS instrument exhibits several drawbacks. Valletti (2000) shows that Ronnen (1991)'s mechanism is not robust to the mode of competition. Scarpa (1998) shows that the welfare enhancing effect might critically depend on the duopolistic structure of the industry. Maxwell (1998) puts MQS in a dynamic perspective and shows that they decrease welfare in the long run because they weaken incentives to innovate. Lutz et al. (2000) provide a model where firms

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[1]It can generate positive externalities as in the case of food or avoid negative externalities as in the case of pollution or the risk linked to foreign sources of energy.

[2]Regarding informational issues raised by quality provision in deregulated markets, we refer the reader to Auriol (1998).

may manipulate the selection of the MQS by the regulator in such a way that industry welfare actually decrease. Glass (2001) reaches similar conclusions in a slightly different setup. Interestingly enough, these cases against MQS are rooted in its most obvious implication: *a MQS undermines industry's profitability.* As a by-product, imposing a MQS might induce the exit of some firms, or reduce entry, a problem also acknowledged in Ronnen (1991) and Crampes and Hollander (1995).[3]

Entry should be a particular concern in those industries which are currently subject to massive deregulation programs. While it is expected that an enhanced competition will ultimately materialize into lower consumers' prices, it is also hoped that this will not be detrimental to quality provision. Needless to say, the recent incidents in the US electricity market or UK railways dramatically suggest that quality is a main concern.[4] Building on the mixed appraisal of MQS reported above and, in particular, on the fact that a MQS might conflict with the possibility of entry in deregulated markets, we explore an alternative way to regulate quality, namely *quantity regulation.* We show in a simple model of entry with sunk cost, that the regulator is best advised to limit the output or capacity of the incumbent rather than impose a MQS in order to maximize industry welfare.

More precisely, we consider a stage game where the regulator (government) can either set a MQS or limit the output of the high quality incumbent firm. Then, the potential entrant selects quality and bears some sunk cost to enter; lastly firms compete in prices. Our (subgame perfect) equilibrium analysis reveals that in order to maximize industry welfare, the government should most often prefer the quantitative regulation over the MQS. The key feature of the quantitative restriction is to relax price competition so strongly in the last stage, that quality differentiation becomes purposeless. Accordingly, the entrant selects a high quality and because the entrant ends up making more profits in equilibrium, entry remains compatible with quality enhancement for a wider range of entry cost.

The particular form of regulation we consider here is clearly not pervasive in actual

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[3]Notice that this mixed theoretical appraisal of MQS is to some extent confirmed by the (limited) empirical evidence. See in particular Chipty and Witte (1997) for a detailed empirical study of the effects of MQS on the quality of child care centers in the US.

[4]Evidences of the negative effect of deregulation in US airline markets on the service quality can also be found in Rhoades and Waguespack (2000).

markets. However, capacity limitation seems quite natural as a tool to invite entry since it ensures the entrant of a protected (though limited) market share. Actually, the current regulation framework in various European industries allows for such a regulation. An example is the italian electricity market where a new law prohibits any generation company from supplying more than one half of the national demand. This measure was successfully taken to induce entry of competitors to challenge the historical incumbent (former monopoly). A comparable provision can be found in the European Regulation on Deregulation of Public Transport whereby the regulator may choose to limit market coverage of an already dominant firm in order to allow for enough competition. More precisely, Article 9 states that *"A competent authority may decide not to award public services contracts to any operator that already has or would, as a consequence, have more than a quarter of the value of the relevant market..."*[5]

Market coverage regulations also exist in traditionally private markets. For instance, specific regulations regarding the maximal size and opening hours for supermarkets or the ceiling on advertising revenues made by publicly owned television operators partly obey to the same logic of guaranteeing sufficient room for competitiors. Needless to say, quantitative regulation is a very old tradition in the area of commercial policy where trade quotas are still common.[6] There is thus a case where capacity limitation may induce entry in deregulated market. In the present paper, we show that such policies also have very nice complementary properties regarding the regulation of quality provision.

The literature on quality choice and regulation can be divided into two branches: models that directly change the quality range available to firms and those which act indirectly by altering firms' payoffs at the market competition stage. The first stream has been discussed above. Our effort belongs to the second stream; some previous works are Cremer and Thisse (1994), (1999) and Kemnitz and Hemmasi (2003) who show how taxation or price ceilings can improve average quality. In a trade context, Herguera

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[5]Amended proposal for a regulation of the European Parliament and of the council on action by member states concerning public service requirements and the award of public sector contracts in passenger transport by rail, road and inland waterway, Official Journal of the European Commission, C 151 E/146-183, Article 9-2.

[6]As will become clear in a few pages, our analysis owes much to the strategic trade analysis of quotas, and in particular to Krishna (1989).

et al. (2000) study the impact a quota on quality selection. However their analysis is confined to a Cournot competition framework. To the best of our knowleddge, our paper is the first to deal with the effect of quantitative restrictions on quality selection under price competition.[7]

Our second contribution is related to this last observation. Indeed, our analysis offers some original results regarding the outcomes of Bertrand-Edgeworth competition in markets with differentiated product. As is well-known, capacity constrained price competition has been widely studied in markets with homogeneous goods after the seminal paper of Kreps and Scheinkman (1983). By contrast, very few positive results exist for the case of differentiated goods.[8] In this paper we offer additonals characterization of firms' payoffs for such games which complements the earlier results of Krishna (1989) and Furth and Kovenock (1993). These results should prove useful for further investigations on the nature of price competition in markets with capacity constrained firms and differentiated products.

The paper is organized as follows. The next section presents the model and characterizes the unique subgame perfect equilibrium of the unregulated entry game. Section 3 characterizes the subgame perfect equilibrium of the entry game under a MQS regulation and derives the welfare maximizing level for the MQS. Section 4 performs a similar analysis for the case of a quantity regulation. We then establish the superioty of quantity regulation over MQS. Section 5 concludes.

2 Entry under "Laissez-Faire"

We consider a two-stage game between an entrant e and an incumbent i. The incumbent is already committed to his quality s_i. In the first stage, the challenger decides whether to enter or not, and in case of entry chooses her quality s_e paying a sunk cost $F \geq 0$. In the second stage, the two firms sell indivisible goods differentiated by their quality and compete in prices. We study Subgame Perfect Equilibria of this game. The following assumptions apply in the forthcoming analysis:

H 1 $s_i = 1, F_i = 0, s_e \in [0, 1], F_e = F.$

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[7] Boccard and Wauthy (2003) offers additional results in a model of horizontal differentiation.

[8] Noticeable exceptions are Krishna (1989), Furth and Kovenock (1993) and Cabral et al. (1998)

H 2 *Firms produce at a constant marginal cost, normalized to 0.*

H 3 *There exists a continuum of consumers indexed by x. They are uniformly distributed in the $[0,1]$ interval with a unit density. The indirect utility derived by type x when buying product j is given by $u(x,j) = xs_j - p_j$ with $j = i, e$. Consumers have unit demand and refraining from consuming yields a net utility normalized to 0.*

The following remarks are in order. In H1, we assume that quality is not costly for firms. Moreover, we assume that the incumbent is committed to the best available quality. Accordingly, we focus on the cases where entry does not entail leapfrogging of the incumbent. The implications of this assumptions will be discussed later. Notice that we need to place an upper bound on the admissible qualities in order to ensure that firms' payoffs are bounded. It is possible to perform a similar analysis with a more general convex quality cost without notably affecting the qualitative conclusions of our analysis.[9] H2 is standard in the relevant literature while H3 describes the usual structure of consumers' preferences in the spirit of Mussa and Rosen (1978). This structure is retained in most papers dealing with MQS.

Using H1 and H3 we may characterize demands addressed to the firms as follows:

$$D_e(p_e, p_i) = \begin{cases} 1 - \frac{p_e}{s_e} & if \quad p_e \leq p_i - 1 + s_e \\ \frac{p_i s_e - p_e}{s_e(1-s_e)} & if \quad p_i - 1 + s_e \leq p_e \leq p_i s_e \\ 0 & if \quad p_e \geq p_i s_e \end{cases} \tag{1}$$

$$D_i(p_e, p_i) = \begin{cases} 1 - \frac{p_i}{1} & if \quad p_i \leq \frac{p_e}{s_e} \\ 1 - \frac{p_i - p_e}{1 - s_e} & if \quad \frac{p_e}{s_e} \leq p_i \leq p_e + 1 - s_e \\ 0 & if \quad p_i \geq p_e + 1 - s_e. \end{cases} \tag{2}$$

Firms' profits at the last stage of the game are

$$\Pi_e(p_i, p_e) = p_e D_e(p_i, p_e) \quad \text{and} \quad \Pi_i(p_i, p_e) = p_i D_i(p_i, p_e)$$

The characterization of Nash equilibria in the pricing game is fairly straightforward. Consequently, we limit ourselves to a informal (and mainly graphical argument). The

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[9]We have performed the analysis with the function $k(s) = \frac{s^2}{K}$, with $s \in [0,1]$. The computations are available upon request from the authors. Notice that H1 can be viewed as the extreme form of $k(s)$ with K arbitrarily large.

payoffs are continuous and give rise to continuous best response functions illustrated on Figure 1. Notice in particular that the dotted area characterizes the prices constellation for which both firms enjoy a positive demand. Accordingly, the firms' best response, denoted $\psi_i(p_e)$ and $\psi_e(p_i)$ exhibit kinks when they hit their relevant non-negativity constraint. The unique price equilibrium is $(p_e^*, p_i^*) = \left(\frac{s_e(1-s_e)}{4-s_e}, \frac{2(1-s_e)}{4-s_e} \right)$; it enables to compute the first stage payoffs as function of the entrant's quality s_e and derive the optimal choice.
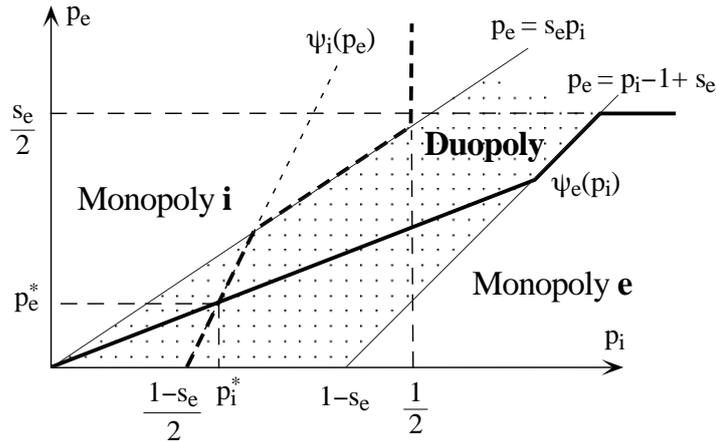

Figure 1: The price space

The subgame perfect equilibrium (in pure strategies) of the "Laissez-Faire" game described above is characterized by Choi and Shin (1992) and reproduced in Lemma 1. Straightforward computations show that the operating profit of the entrant is $\frac{1}{48}$ in equilibrium; accordingly, entry will take place if only $F \leq \frac{1}{48}$. The "Laissez-Faire" analysis is summarized in the next Lemma.

Lemma 1 *Suppose quality is not costly and the incumbent sells quality $s_h = 1$, then whenever $F \leq \frac{1}{48}$, the entrant enters and optimally differentiates by selecting quality $\frac{4}{7}$. The price equilibrium of the continuation game is unique and in pure strategies.*

3   Entry and Minimum Quality Standards

In this section, we add a preliminary stage to the game analyzed in the above section. At this zero stage, the government can commit costlessly to a MQS, denoted z before

the challenger decides to enter or not.[10]

Obviously, the MQS has to be lower than the best available quality i.e., we assume $z \leq 1$. Yet, for the MQS to be effective and alter equilibrium choices of the firms, it must be sufficiently high. Using Lemma 1, we impose the further restriction $z \geq \frac{4}{7}$. Relying on the analysis of the "Laissez-Faire" game, we deduce that for an MQS $z \in [\frac{4}{7}, 1]$, the best response for the entrant at the quality stage is to select the lowest admissible quality level z. The resulting price equilibrium is then $p_e^z = \frac{z(1-z)}{4-z}$, $p_i^z = \frac{2(1-z)}{4-z}$ leading to demands $D_e^z = \frac{1}{4-z}$, $D_i^z = \frac{2}{4-z}$ and equilibrium profits $\Pi_e^z = \frac{z(1-z)}{(4-z)^2}$, $\Pi_i^z = \frac{4(1-z)}{(4-z)^2}$.

We may then turn to the study of the optimal MQS to implement in order to maximize industry welfare. Neglecting the sunk entry cost, the industry welfare function is defined as:

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W_{mqs}(z) = \int_{1-D_i^z}^{1} (x - p_i^z)dx \; + \int_{1-D_e^z-D_i^z}^{1-D_i^z} (zx - p_e^z)\,dx \; + \Pi_i^z + \Pi_e^z = \frac{12 - z - 2z^2}{2\,(4-z)^2} \quad (3)
$$

where the first two terms denote the surplus of consumers buying the high and low quality product respectively. Straightforward computations indicate that this function is increasing and concave in z. Notice that $W_{mqs}(z)$ ranges from $\frac{3}{8}$ to $\frac{1}{2}$ over the range $[0; 1]$.

Incidentally, $W_{mqs}(1)$ also defines the first best for this industry, when there are no cost to entry. This result is quite intuitive. The first best corresponds to the case where all consumers buy the best available quality at marginal cost (which is zero in the present case). This is achieved if there are two firms in the market, competing in price with an homogeneous product of top quality $s = 1$. Therefore, in order to implement the first best with a MQS, the regulator must set $z = 1$. However, we obtain the first best market outcome if only entry does take place. This obviously requires $F = 0$ for otherwise the entrant's profit is strictly negative. Should entry not take place, we would be left with a monopoly incumbent and a lower welfare level which is formally equivalent to $W_{mqs}(0) = \frac{3}{8}$. This result illustrates the key weakness of a MQS. While being quite effective in ensuring the selection of a high quality level by the entrant, a MQS dramatically depresses profits. As a consequence, whenever there is an entry cost that must be recouped by the entrant, this cost places an upper bound on the

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[10]We refer the reader to Ronnen (1991) for a detailed analysis of this problem.

available MQS. In other words, in the present model, the level of entry costs defines the actual boundary faced by the government when regulating quality. Formally, we may summarize the previous argument in the following Lemma.

Lemma 2 *For all $F \in [0, \frac{1}{48}]$, there exists an optimal MQS $z^*(F)$ with $\frac{\partial z^*(F)}{\partial F} < 0$ and $\frac{\partial W_{mqs}(z^*(F))}{\partial F} < 0$.*

Proof: The upper bound for the MQS is given by the level $z^*(F)$ for which an entrant's profit, net of the entry cost is zero. By solving $\Pi_e^z = F$, we obtain as the unique relevant root $z^*(F) = \frac{1+8F+\sqrt{1-48F}}{2(1+F)}$ which is a decreasing function of F in the domain $F \in [0, \frac{1}{48}]$. Accordingly, total welfare is a decreasing function of the sunk cost over $[0, \frac{1}{48}]$. ∎

Notice that whenever $F \geq \frac{1}{48}$, welfare drops from 0.46 down to $\frac{3}{8} \simeq 0.38$ since entry is precluded.

4   Sales Restriction and Optimal Quality Provision

We now alter the above game by assuming that, instead of committing to a MQS, the regulator is allowed to impose a quantitative constraint q on the incumbent firm before entry takes place.[11]

4.1   Price Competition with a Quantitative Restraint

By definition, the sales quota q defines the largest demand level the incumbent is allowed to serve. This restriction deeply alters the nature of competition in the pricing game. Indeed, whenever prices are such that the demand $D_i(p_i, p_e)$ is greater than q, the incumbent must turn $D_i(p_i, p_e) - q$ consumers away in order to comply with the sales restriction. In other words, the incumbent rations consumers when demand addressed to him exceeds the quota. The key implication of the sales restriction is thus to induce Bertrand-Edgeworth competition at the pricing stage of the game. As is well-known,

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[11]Such a sales' restriction may actually be implemented by controling the incumbent's production capacity through the limited emission of construction permits or by editing new regulations.

the organization of rationing in the market is a critical issue for such games.[12] We shall assume

H 4 *Whenever $D_i(p_i, p_e) > q$, rationed consumers are those who exhibit the lowest willingness to pay for the good.*

Notice that in our particular case H4 is formally equivalent to the standard efficient rationing rule. We now turn to the analysis of the pricing subgames. Two classes of Bertrand-Edgeworth pricing games have to be distinguished according to the quality selected in the first stage:

- If $s_e = 1$, firms sell homogeneous products in the price game and one of them faces a quantitative constraint. We shall refer to Levitan and Shubik (1972) for a detailed analysis of the price equilibrium in these subgames.

- If $s_e < 1$, we have a Bertrand-Edgeworth pricing game with product differentiation. To the best of our knowledge, Krishna (1989) is the first paper which provides a characterization for a price equilibrium in such games.[13] We shall rely on and extend her analysis in the following.

We start by analyzing subgames where products are differentiated ($s_e < 1$) and then pass to the case of homogeneous products before concluding with the optimal quality choice by the entrant.

4.2 Differentiated Products

Let us start with an informal discussion of the nature of equilibria in pricing games where products are vertically differentiated. We may partition the price space according to whether the constraint is binding or not. Solving $D_i(p_i, p_e) = q$ for p_e yields a function $\beta(p_i)$ which defines two competition regimes. If $p_e < \beta(p_i)$, the traditional Bertrand analysis, as developed in section 2 applies while if $p_e \geq \beta(p_i)$, the constraint is binding.

Notice then that a consumer wishing to buy the high quality product but being rationed by the incumbent always prefers to buy the low quality product of the entrant

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[12]See Davidson and Deneckere (1986) for a classical analysis of this last issue.

[13] Furth and Kovenock (1993) also provide some characterization of equilibrium payoffs in Bertrand-Edgeworth games of product differentiation.

instead of refraining from consuming. Thus, when at the prevailing prices the demand addressed to the incumbent exceeds the quota q, all rationed consumers are recovered by the entrant which faces a residual market $1 - q$. Therefore, within this domain, the entrant's payoff does not depend on p_i. The optimal price, which we denote p_e^s is then independent of the incumbent's one and is referred to as the "security" strategy.[14] As for the incumbent, sales are constant in this domain and equal to the quota q. Thus, the optimal price is $p_i = \beta^{-1}(p_e)$, the highest price for which the quota is binding.

The keypoint then is to note that the possibility of rationing breaks the concavity of the entrant's profit function whereas that of the incumbent's remains concave in the domain where its demand is positive. This will preclude the existence of pure strategy equilibria in many pricing subgames. While the existence of mixed strategy equilibria is not an issue here because of the continuity in payoffs, the characterization of mixed strategy equilibria in Bertrand-Edgeworth games with product differentiation is to a large extent an open problem. To the best of our knowledge, Krishna (1989) was the first paper providing a characterization of equilibrium in such a case. The mixed strategy equilibrium she identifies can be used within our setup. It takes the following form: the entrant will mix over two atoms (the security price and some lower price) while the incumbent will play a pure strategy. However, in many subgames, this equilibrium does not exist because a crucial non-negativity constraint is not satisfied for the incumbent. While we do not characterize equilibria for such cases, we are able to characterize the entrant's equilibrium payoff.

The following lemma constitutes the technical contribution of this article to the literature on Bertrand-Edgeworth competition with product differentiation.

Lemma 3 *Assume $s_e < s_i = 1$, there exists a critical values for the quota, denoted $\bar{q}(s_e)$ such that*

▶ *if $q > \bar{q}(s_e)$, the "Laissez-Faire" equilibrium prevails.*
▶ *if $q \leq \bar{q}(s_e)$, there exists a mixed strategy equilibirum where the entrant obtains the security payoff $\frac{1}{4}s_e(1 - q)^2$ in equilibrium.*

Proof We proceed through four steps. Firstly, we derive firms' best reponse. Secondly, we identify the range in which the "Laissez-Faire" equilibrium applies. Thirdly,

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[14]The term "security" refers to the fact that this strategy yields the minimax payoff

we characterize a mixed strategy equilibrium where only the entrant firm mixes over two atoms and characterize the associated payoffs. Finally, we show that the former mixed strategies equilibrium does not exists for very tight quotas; in that case, we prove that any equilibrium (involving non-degenerated mixed strategies for the two firms) yields the security payoff to the entrant.

Step 1 The frontier between price constellations where the constraint is binding or not is found by equating the incumbent's demand $D_i(p_e, p_i)$ (the second line of (1)) with the sales quota q. We obtain $p_e = \beta(p_i) \equiv p_i - (1-q)(1-s_e)$. Firms sales' functions are therefore

$$S_i(p_e, p_i) = \begin{cases} D_i(p_e, p_i) & mboxif p_e \leq \beta(p_i) \\ q & \text{if } p_e > \beta(p_i) \end{cases} \tag{4}$$

and

$$S_e(p_e, p_i) = \begin{cases} D_e(p_e, p_i) & \text{if } p_e \leq \beta(p_i) \\ 1 - q - \frac{p_e}{s_e} & \text{if } p_e > \beta(p_i) \end{cases} \tag{5}$$

In the binding regime (second branch of the above equations), the incumbent faces a constant demand. Accordingly, its profit is increasing in p_i. Therefore, the best response consists of choosing the maximal price which is by definition the frontier price $\beta^{-1}(p_e)$. Using the continuity of payoffs, we note that this price is itself weakly dominated by the best response of the non binding regime. The latter is $\psi_i(p_e) = \frac{p_e + 1 - s_e}{2}$, whenever it is attainable. Solving $\psi_i(p_e) = \beta^{-1}(p_e)$ yields $\bar{p}_e \equiv (2q-1)(1-s_e)$. The best response of the incumbent is therefore continuous with a kink at $\bar{p}_e$. Formally, we obtain the best response[15]

$$\phi_i(p_e) = \begin{cases} \psi_i(p_e) & \text{if } p_e \leq \bar{p}_e \\ \beta^{-1}(p_e) & \text{if } p_e \geq \bar{p}_e \end{cases} \tag{6}$$

The analysis is more involved for the entrant. The optimal behavior in the two regimes are quite different. In the binding regime, the entrant acts as a monopoly over a market of maximal size $1 - q$. We therefore have $\pi_e = (1 - q - \frac{p_e}{s_e})p_e$. This payoff

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[15]As can be seen on Figure 2, the β line crosses the frontier between duopoly and monopoly for the incumbent at $p_i = 1 - q$ and $p_e = s_e(1 - q) = 2p_e^s$. As shown in the study of the entrant's best reply, p_e^s is the highest price he might use in equilibrium, thus we need not worry for the exact shape of the β curve above $p_i = 1 - q$.

reaches a maximum of $\frac{s_e(1-q)^2}{4}$ at the security price $p_e^s \equiv \frac{(1-q)s_e}{2}$. In the non binding regime, the best response candidate is $\psi_e(p_i) = \frac{p_i s_e}{2}$. After simplifications, we obtain $\Pi_e(\psi_e(p_i), p_i) = \frac{s_e p_i^2}{4(1-s_e)}$ which is increasing in p_i. It then remains to choose between those two candidate best responses by solving $\frac{s_e(1-q)^2}{4} = \frac{s_e p_i^2}{4(1-s_e)} \Leftrightarrow p_i = \mu(q, s_e) \equiv (1-q)\sqrt{1-s_e}$. Finally, we obtain the following best response correspondence:

$$\phi_e(p_i) = \begin{cases} p_e^s & \text{if } p_i \leq \mu(q, s_e) \\ \psi_e(p_i) & \text{if } p_i \geq \mu(q, s_e) \end{cases} \tag{7}$$

Step 2 Notice that $\phi_e(.)$ is discontinuous, so that we cannot ensure the existence of a pure strategy equilibrium. It can be checked, using equation (4) that the payoff of firm i remains concave in the domain where sales are non-negative. Therefore, its best response to any pure strategy by firm e is a pure strategy. Thus, the only candidate for a pure strategy equilibrium is the "Laissez-Faire" equilibrium (p_i^*, p_e^*). For this equilibrium to exist, it must be true that $p_i^* > \mu(q, s_e) \Leftrightarrow q > \bar{q}(s_e) \equiv 1 - \frac{2\sqrt{1-s_e}}{4-s_e}$.
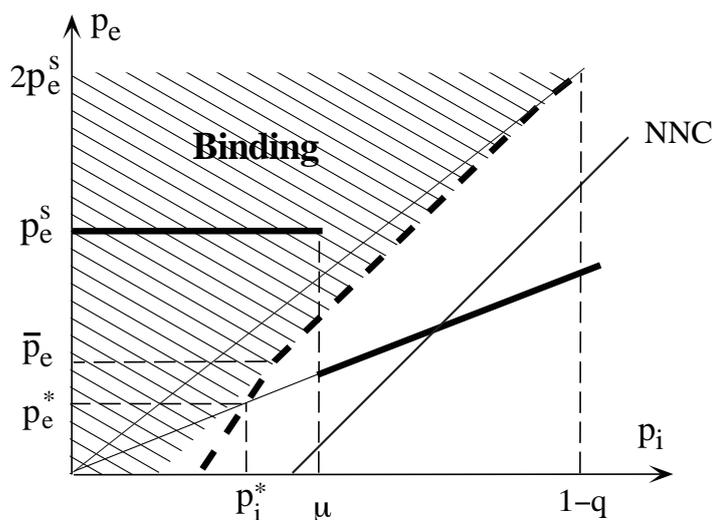

Figure 2: The price space with quota

Step 3 Suppose $q < \bar{q}(s_e)$. As illustrated on Figure 2, there exists no equilibrium in pure strategies. Since payoffs are continuous and the strategy space is compact, we know that a mixed strategy equilibrium must exist. A natural candidate is the one originally proposed by Krishna (1989). The incumbent plays the pure strategy $\mu(q, s_e)$ and the entrant plays the pair p_e^s and $\psi_e(\mu(q, s_e))$. The entrant chooses the weights over those two atoms so as to ensure that $\mu(q, s_e)$ is indeed a best response for firm i

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against the mixture. Notice that the entrant's equilibrium profit can be computed with any of the prices in the support of its strategy. Against the pure strategy $\mu(q, s_e)$, when quoting the security price $p_e^s = s_e \frac{(1-q)}{2}$, the entrant sells $\frac{1-q}{2}$ thus its equilibrium profit is $\Pi_e(q, s_e) \equiv \frac{s_e(1-q)^2}{4}$. This equilibrium exists provided that $D_i\left(\mu, \psi_e(\mu)\right) > 0$, i.e. the non-negativity constraint (NNC) is not binding on $D_i(.)$. Solving this inequality for q we obtain the restriction $q \geq \underline{q}(s_e) \equiv 1 - \frac{2\sqrt{1-s}}{2-s}$.

Step 4 Next, we show that for $q < \underline{q}(s_e)$, in every mixed strategy equilibrium, the entrant earns $\frac{s_e}{4}(1 - q)^2$.

In order to see this, consider Figure 3. It depicts a configuration where the non-negativity constraint (NNC) is binding for the mixed strategy equilibrium candidate identified in Step 3. Recall that the frontier between the binding and non-binding quota regimes is identified with $\beta(.)$. Best responses are drawn in bold face. Denote F_i and F_e the firm's mixed strategies in a Nash equilibrium.
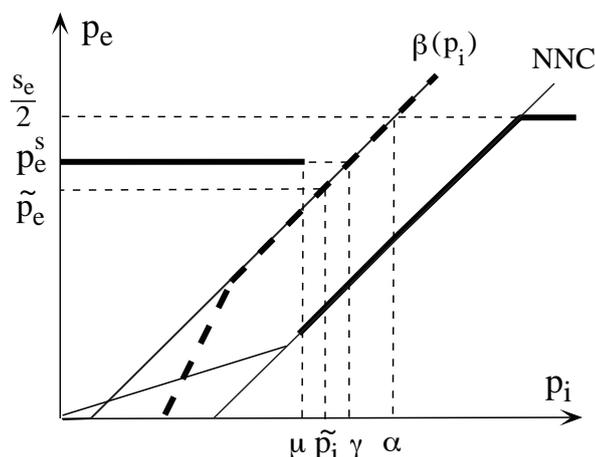

Figure 3: Best responses in prices

Observe that by construction of the best response, the entrant's profit is decreasing in own price over $[\frac{s_e}{2}, 1]$, hence the average over F_i is likewise decreasing over the same range so that the support of F_e is included in $[0, \frac{s_e}{2}]$. For $p_e \in [0, \frac{s_e}{2}]$, the incumbent's profit is decreasing in own price over $[\alpha, 1]$, hence the average over F_e is likewise decreasing over the same range so that the support of F_i is included in $[0, \alpha]$. For $p_i \in [0, \alpha]$, the entrant's profit is decreasing in own price over $[p_e^s, \frac{s_e}{2}]$ (because he needs not consider the area on the right of the NNC), hence the average over F_i is likewise decreasing over the same range so that the support of F_e is included in $[0, p_e^s]$. By the

same token the support of F_i is included in $[0; \gamma]$.

Let $\tilde{p}_e \leq p_e^s$ be the supremum of F_e's support. If $\tilde{p}_e < p_e^s$, the previous reasoning applies again telling us that Π_i is decreasing over $[\tilde{p}_i, \gamma]$ for every $p_e \in [0, \tilde{p}_e]$, hence the incumbent does not play prices above $\tilde{p}_i$ in equilibrium. Now recall that in a mixed strategy equilibrium the payoff of a player can be computed at any of the prices belonging to the support of his optimal strategy; let us then consider $\tilde{p}_e$ for the entrant. For any $p_i \in [0, \tilde{p}_i]$, the incumbent is constrained by the quota so that the entrant is a monopoly over a market of size $1 - q$, hence her optimal behavior is to try to reach the price p_e^s. This stands in contradiction to the fact that $\tilde{p}_e$ is the highest optimal price. We have thus proven that $\tilde{p}_e = p_e^s$ and as a consequence that the equilibrium payoff is $\Pi_e(\tilde{p}_e, F_i) = \frac{s_e}{4}(1 - q)^2$ since $p_i \in [0, \gamma]$. ∎

4.3   Homogeneous Products

We may now analyze the equilibrium in the second class of pricing subgames where firms sell identical qualities ($s = 1$) at the pricing stage. In this case, the vertical differentiation model degenerates into a standard Bertrand-Edgeworth duopoly over the demand $D(p) = 1 - p$, but with a quantity constraint q for one firm. Recalling that H4 amounts to assume efficient rationing in the market for an homogeneous good, we notice that the corresponding game has been studied by Levitan and Shubik (1972). Using their results and defining $\lambda(q) \equiv \frac{1 - \sqrt{q(2-q)}}{2}$, we may directly state:

Lemma 4 *In a Nash equilibrium of the pricing game where $s_i = s_e = 1$, firms play a mixed strategy with common support $[\lambda(q), \frac{1-q}{2}]$ and cumulative distributions $F_e(p) = 1 - \frac{\lambda(q)}{p}$ and $F_i(p) = \frac{p(1-p) - \lambda(q)(\lambda(q)-1)}{pq}$.*

The interested reader is referred to Levitan and Shubik (1972) and in particular their appendix for a formal proof of this result. Observe that $F_i(\lambda(q)) = 0$, $F_i\left(\frac{1-q}{2}\right) = 1$, $F_e(\lambda(q)) = 0$ and $F_e\left(\frac{1-q}{2}\right) < 1$, thus only the entrant has an atom at the upper price $\frac{1-q}{2}$. In this equilibrium, the incumbent's profit is $\Pi_i(q) = q\lambda(q)$ (at the lowest price he gets the whole demand $1 - \lambda(q)$ thus sells q because $\lambda(q) < \frac{1-q}{2} < 1 - q$ implies that his capacity constraint is binding) while the entrant earns $\Pi_e(q) = \frac{(1-q)^2}{4}$ (at the highest price she receives the residual demand $1 - q$). Notice last that this latter payoff

is $\Pi_e(q, 1)$, the limit of the equilibrium payoff obtained in Lemma 3 when product differentiation tends to 0.

4.4   Optimal Quality Choice for the Entrant

With the help of Lemma 3 and 4 we may now turn to the selection of the quality by the entrant given the sales restriction q.

Lemma 5 *The entrant selects $s_e = 1$ in a SPE whenever $q < q^* \equiv 1 - \frac{1}{2\sqrt{3}}$ and $s_e = \frac{4}{7}$ otherwise.*

Proof In the domain, $q > \bar{q}(s_e)$, where the "Laissez-Faire" equilibrium exists (see Figure 4), the best response in quality is given by the "Laissez-Faire" candidate $s_e = \frac{4}{7}$ (or $s_e = \bar{q}^{-1}(q)$ whenever $\frac{4}{7}$ lies outside the relevant domain). Whenever, $q \leq \bar{q}(s_e)$, the price equilibrium is in mixed strategies and the entrant's payoff is $\Pi_e(q, s_e) = \frac{s_e(1-q)^2}{4}$, so that the best response is obviously the top quality; we refer to this as the "imitation" strategy.
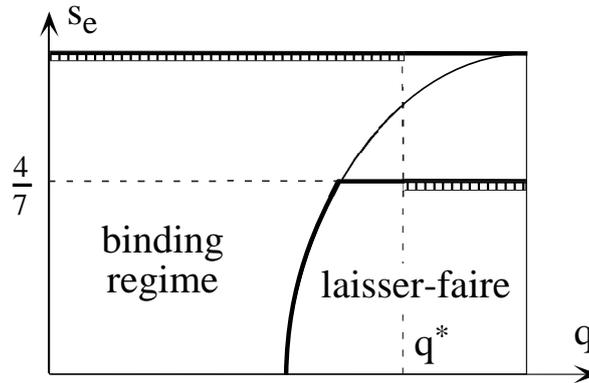

Figure 4: The quota-quality space

In order to characterize the SPE as a function of the quota, we now have to compare the profits under "Laissez-Faire" and "imitation". Solving for $\Pi_e^*(\frac{4}{7}) = \frac{1}{48} = \frac{(1-q)^2}{4}$, the imitation payoff, we obtain the cut-off quota $q^* \equiv 1 - \frac{1}{2\sqrt{3}} \simeq 71\%$. ∎

4.5   Optimal Sales Restriction for the Regulator

The previous analysis shows that if a government's objective was simply to ensure the provision of the best available quality by both firms, it would be sufficient to impose

a sales restriction at a level $q < q^*$. Notice that this level does not look unreasonably restrictive. It is actually larger than the equilibrium sales' level of the incumbent in the "Laissez-Faire" case ($\simeq 58\%$). It remains then to perform a welfare analysis in order to identify the optimal quota. The intuition underlying the welfare comparison is easy to grab. In the domain where the quota induces an optimal quality selection of $s_e = 1$, increasing the level of the quota reduces industry profits. Indeed, the looser the binding quota, the closer we are to the standard Bertrand equilibrium with zero profits. In other words, a looser quota generates a fiercer competition at the price stage and a greater consumer surplus. Computations show that the consumer surplus gain dominates the industry profit loss. Accordingly, the optimal quota consists of imposing the maximal sales restriction compatible with the selection of $s_e = 1$.

Proposition 1 *The optimal Sales Restriction for the regulator is $q^* = 1 - \frac{1}{2\sqrt{3}}$.*

Proof: For $q \leq q^*$, we know from Lemma 5 that the entrant chooses the highest quality and competition takes places in a market for an homogeneous good. In this equilibrium the incumbent profit is $\Pi_i(q) = q\lambda(q)$ while the entrant obtains $\Pi_e(q) = \frac{(1-q)^2}{4}$.

The surplus of the consumer with type $x \in [0, 1]$ is best understood by separating 2 cases:

- if $x > 1 - q$, then $x > p_e$ because $p_e \leq \frac{1-q}{2}$. The incumbent price p_i is the lowest with probability $F_i(p_e)$ in which case the consumer buys at the price p_i (because $x > p_e > p_i$ and the incumbent is not constrained) so that we need to compute an expectation. With complementary probability, the consumer buys at the entrant, thus the surplus of consumer x is

$$H(x, p_e) \equiv (x - p_e)\left(1 - F_i(p_e)\right) + \int_{\lambda(q)}^{p_e} (x - p_i)dF_i(p_i)$$

- if $x < 1 - q$, the consumer is rationed by the incumbent; then either $x < p_e$ so that he does not buy at all, or $x > p_e$ and he buys from the entrant deriving a surplus of $x - p_e$.

Integrating with respect to the distribution of the entrant's prices, we have three cases according to the respective positions of x and the upper price limit:

- if $x < \frac{1-q}{2}$, $\underline{\underline{W}}(q,x) \equiv \int\limits_{\lambda(q)}^{x} (x - p_e) dF_e(p_e)$

- if $\frac{1-q}{2} < x < 1 - q$, $\underline{W}(q,x) \equiv \int\limits_{\lambda(q)}^{\frac{1-q}{2}} (x - p_e) dF_e(p_e) + \left(x - \frac{1-q}{2}\right) \left(1 - F_e\left(\frac{1-q}{2}\right)\right)$

- if $1 - q < x$, $\overline{W}(q,x) \equiv \int\limits_{\lambda(q)}^{\frac{1-q}{2}} H(x, p_e) dF_e(p_e) + H\left(x, \frac{1-q}{2}\right) \left(1 - F_e\left(\frac{1-q}{2}\right)\right)$

Integrating with respect to the uniform distribution of consumers over the range of potential buyers i.e., $x \geq \lambda(q)$, we obtain the consumer surplus expression:

$$W_C(q) \equiv \int\limits_{\lambda(q)}^{\frac{1-q}{2}} \underline{\underline{W}}(q,x)dx + \int\limits_{\frac{1-q}{2}}^{1-q} \underline{W}(q,x)dx + \int\limits_{1-q}^{1} \overline{W}(q,x)dx = \frac{1}{8} + q\frac{4 - 3q + 2\sqrt{q(2-q)}}{8} \quad (8)$$

which is an increasing and concave function. Observe that $W_C(1) = \frac{1}{2}$, is the total surplus at the outcome of Bertrand competition between two identical products where no consumer refrains from buying, all consumers buy the best available quality and firms capture no rent. The total surplus is

$$W(q) = W_C(q) + \Pi_i(q) + \Pi_e(q) = \frac{3}{8} + q\frac{4 - q - 2\sqrt{q(2-q)}}{8} > \frac{3+q}{8}$$

This function is increasing and concave in q and defines a welfare level which is above above the "Laissez-Faire" one, $\frac{3}{8}$, for any q. Accordingly, the Welfare is maximized at q^*, the highest quota which is compatible with $s_e = 1$ in a SPE. ∎

4.6 Comparing Sales Restriction and MQS

We now come to our last result, the comparison of the respective merits of Sales Restrictions and Minimum Quality Standards in our model of entry with sunk cost. Notice from lemma 5 and proposition 1 that the entrant's operating profits are exactly equal to $\frac{1}{48}$ at the optimal quota. Therefore, the presence of the entry cost F does not constrain the government's possibilities, as compared to Laissez-Faire and more importantly, as compared to the case of a MQS policy. However, the quota may not always dominate the MQS in terms of welfare because if F is very low, a very high MQS is feasible and therefore dominates the Sales Restriction instrument. Formally, we may state:

Proposition 2 *There exists a lower bound $\underline{F}$ such that when $F > \underline{F}$, a sales restriction induces a higher industry welfare than a minimum quality standard.*

Proof The proof is only a matter of computations. By Lemma 2, we know that in order to induce entry, the MQS is bounded above by $z^*(F)$, a decreasing function of F. Now, since $W_{mqs}(z)$ is increasing in z, the welfare at the limit, $W_{mqs}(z^*(F))$, is a decreasing function of the sunk cost F; solving for equality with our Sales Restriction candidate $W(q^*)$, we obtain $\underline{F} \simeq 4.765 \times 10^{-3}$. ∎

The economic intuition underlying this proposition is straightforward. A sales restriction relaxes price competition by inducing a less aggressive behaviour of the constrained firm. Recall then that in a vertically differentiated duopoly, one firm selects a low quality in order to relax competition. However, in the presence of the sales restriction this is no more necessary. The sales restriction is actually quite effective at reducing competition. Accordingly, the entrant has no more incentive to downgrade quality and both firms end up selecting a high quality. Moreover, because price competition is less fierce, equilibrium profits for any quality pair tend to be larger. There exists however a limit to the effective level of the quota. Should it be to loose, the entrant would prefer to differentiate optimally rather than playing on the other's capacity limit. The mechanism at work may therefore be summarized as follows: the quota alters the payoffs in the second stage in such a way that the entrant's incentive at the first stage are put in the "right" direction, i.e. quality upgrades.

This mechanism should be constrasted with that underlying the MQS. The MQS directly constrains the firms' strategy space at the quality stage. By definition, in order to be effective, the MQS must run *against* firms' incentives. By leaving less room for differentiation, the MQS undermines firms' profits in equilibrium and therefore impedes entry. As shown in Proposition 2, it is only when the entry costs are negligible that a government should prefer the MQS to sales restrictions. In this case indeed, the fact that operating profits sharply decrease because of a very high MQS is not a concern anymore. By contrast, the residual market power that must be left to firms in order to induce quality upgrades does not depend on F.

5 Final Remarks

In vertically differentiated industries, MQS are often used to control for quality provision. Within a very simple model, we have shown that sales restrictions might be more efficient than MQS. Our example is quite specific, although it should be stressed that it is quite in line with the received literature on MQS. Several generalizations can be contemplated. Notice that the introduction of positive quality costs that would be sunk before price competition does not alter our conclusion. Obviously, we do not expect minimal differentiation anymore. However, average quality bought by consumers increases and industry welfare increases as well. Similarly, our result are likely to remain valid if we do not impose any exogenous quality hierarchy between the entrant and the incumbent.[16] All in all, the qualitative mechanism that drives our result is robust. It is the intrinsic nature of quantitative restraints to relax price competition. In vertically differentiated industries, this almost immediately implies that firms do not need to relax competition by differentiating products. Accordingly, average quality may rise. Regarding quality selection, the chief merit of the sales restriction is thus quite clear: it gives to all firms an incentive to select a high quality for its product.

Appendix

Proof of Lemma 1

If quality is not costly and the incumbent sells quality $s_i = 1$, then whenever $F \leq \frac{1}{48}$, the entrant enters and optimally differentiates by selecting quality $\frac{4}{7}$. The price equilibrium of the continuation game is unique and in pure strategies.

Proof Recall that

$$D_e(p_e, p_i) = \begin{cases} 1 - \frac{p_e}{s_e} & if \quad p_e \leq p_i - 1 + s_e \\ \frac{p_i s_e - p_e}{s_e(1 - s_e)} & if \quad p_i - 1 + s_e \leq p_e \leq p_i s_e \\ 0 & if \quad p_e \geq p_i s_e \end{cases} \tag{9}$$

---

[16]See Boccard and Wauthy (1997) for a more detailed analysis.

$$
D_i(p_e, p_i) \;=\; \begin{cases} 1 - p_i & if \quad p_i \leq \frac{p_e}{s_e} \\ 1 - \frac{p_i - p_e}{1 - s_e} & if \quad \frac{p_e}{s_e} \leq p_i \leq p_e + 1 - s_e \\ 0 & if \quad p_i \geq p_e + 1 - s_e \end{cases} \tag{10}
$$

and that profits are $\Pi_e(p_i, p_e) = p_e D_e(p_i, p_e)$ and $\Pi_i(p_i, p_e) = p_i D_i(p_i, p_e)$.

The solution to $\frac{\partial \Pi_e}{\partial p_e} = 0$ over the range where both demands are non-negative is $\psi_e(p_i) \equiv \frac{p_i s_e}{2} \leq p_i s_e$; thus, the low quality best response function is $\phi_e(p_i) = \psi_e(p_i)$. In the incumbent monopoly region $(p_e > p_i s_e)$, the incumbent's best response is the monopoly price $\frac{1}{2}$ which is feasible if and only if $p_e > \frac{s_e}{2}$. Otherwise, Π_i is strictly increasing in the monopoly region and we always reach the duopoly region where the profit is $p_i \left[1 - \frac{p_i - p_e}{1 - s_e} \right]$ leading to a candidate best response $\psi_i(p_e) \equiv \frac{p_e + 1 - s_e}{2}$. Whenever $p_e \leq \frac{s_e(1 - s_e)}{2 - s_e}$ then $\psi_i(p_e) \leq \frac{p_e}{s_e}$ meaning that ψ_i is the best response, otherwise it is the frontier price $\frac{p_e}{s_e}$ which is optimal. As we have $\frac{s_e(1 - s_e)}{2 - s_e} < \frac{s_e}{2}$, the (kinked) best response of firm h is

$$
\phi_i(p_e) = \begin{cases} \psi_i(p_e) & if \quad p_e \leq \frac{s_e(1 - s_e)}{2 - s_e} \\ \frac{p_e}{s_e} & if \quad \frac{s_e(1 - s_e)}{2 - s_e} \leq p_e \leq \frac{s_e}{2} \\ \frac{1}{2} & if \quad \frac{s_e}{2} \leq p_e \end{cases} \tag{11}
$$

As one can see on Figure 1 in the text p.7, the Laissez-Faire equilibrium $(p_e^*, p_i^*) = \left(\frac{s_e(1 - s_e)}{4 - s_e}, \frac{2(1 - s_e)}{4 - s_e} \right)$ is given by the intersection of ψ_e and ψ_i.

In the quality stage we have $\Pi_i(s_e) \equiv p_i^* D_i^* = \frac{4(1 - s_e)}{(4 - s_e)^2}$ and $\Pi_e(s_e) \equiv p_e^* D_e^* = \frac{s_e(1 - s_e)}{(4 - s_e)^2}$. It is a matter of calculations to check that Π_e reaches its maximum for $s_e = \frac{4}{7}$. ∎

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